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Equilibrium Income and Interest Elasticities of the Demand for M1 in Japan

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This study investigates the equilibrium relationship between real M1 balances, real GNP and short-term interest rates in Japan since 1955. Although each of these variables appears to be nonstationary, the evidence suggests that there exists a stable, stationary linear combination of the three variables over the entire sample period. The estimated coefficients of this vector suggest that the long-run income elasticity of real M1 is not significantly different from one, and the long-run interest elasticity is around 0.5.

I. Introduction

Instability of empirical money demand equations appears to be as perplexing a problem in Japan in the 1980s as it has been in the United States since the mid 1970s. In Japan in recent years a broad concept of money (M2+CD) has received more attention than the comparable monetary aggregates in the United States, but the observed behavior of "transactions money" has proven as troublesome to understand as the behavior of the broader concept.

In particular, estimates of M1 demand functions have been constructed by Hamada and Hayashi (1983), Ueda (1988), and Ueda (1989) (these studies are available in English; a number of additional studies are published only in Japanese, in particular Tsutsui and Hatanaka (1983)). These studies have estimated lagged dependent variable specifications of the "real" and "nominal" adjustment form investigated by Goldfeld (1973) for the U.S. The failure of these specifications in the Japanese context is similar to their failure in the U.S. experience. The estimated "speeds of adjustment" are very slow, the equations produce systematic out-of-sample forecast errors ("missing money"), and the parameter estimates are highly sensitive to the choice of sample period.

The primary focus of this study is on the equilibrium or long-run demand function for real M1 balances in Japan. There is general agreement among economists on the

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specification of such a demand function, and our available macroeconomic theories appear to provide a sufficient number of restrictions to identify the parameters of such a function. In contrast, there is neither a well specified theory nor a consensus of opinion on the proper specification of a dynamic or short-run demand function for real balances. There are serious questions on the feasibility of identifying such a short-run demand function given the current state of theory and knowledge of macroeconomic structures (e.g. Laidler, 1982 and Gordon, 1984).

This study is organized as follows. In Section II the concepts, sources, and characteristics of the relevant time series data for Japan are discussed. In Section III a model of the demand for and supply of transactions balances in Japan is outlined. In Section IV it is shown that the parameters of an equilibrium money demand equation are identified by this model in terms of the parameters of a reduced form error correction model (Engle and Granger, 1987), but that the model does not contain sufficient restrictions to identify a short-run money demand function. In Section V the statistical analysis of this model is presented, and the impact on the estimated parameters of adding data from the period of rapid change in Japanese financial markets is evaluated. In Section VI a summary of the major conclusions of this analysis is presented.

II. Data

A. Measurement and sources

Official Bank of Japan data on M1 are available monthly from the beginning of 1955. This M1 measure is the most comprehensive measure of transactions money available for Japan. However, it excludes some of transactions deposits, since not all financial institutions are included in the coverage. In particular, deposits in the postal savings system; agricultural, fishery and credit cooperatives; and labor credit associations are not included in official Bank of Japan M1 (Suzuki, 1987, Table 3.2). These depository institutions are excluded because of a significant lag in the reporting of their statistics. At the end of March, 1988 (end of fiscal year 1987) transactions deposits at these financial institutions amounted to 206,088 (100 million yen) or about 19 percent of measured M1 at that time (see Table 1). The largest portion of the transactions deposits omitted from M1 appear to be ordinary deposits at agricultural cooperatives and current deposits in the postal savings system. In March, 1955 transactions deposits at the same set of financial intermediaries amounted to approximately 23 percent of measured M1. Thus the fraction of covered transactions deposits in measured M1 has increased slightly over the sample period, but this increase appears to be sufficiently small that it is not a serious measurement error for purposes of the analysis in this study.

Official measures of M1 are currently available monthly on two bases: end-of-month figures and daily averages figures. The latter series is available only from the beginning of 1963, so to produce quarterly data from the mid 50s on an average basis it is necessary to

Table 1. Transactions Deposits at Various Financial Intermediaries
End of March 1988 (100 million ¥)

Intermediary	Current Deposits	Ordinary Deposits	Deposits at Notice	Special Deposits
Credit Cooperatives[1]	3,913	17,004	3,997	1,742
Labor Credit Associations[2]	180	6,505	386	282
Agriculture Cooperatives[3]	918	78,424	1,299	2,121
Fishery Cooperatives[4]	69	1,876	342	1,042
Postal Savings[5]	85,988			
Total	91,068	103,809	6,024	5,187
M1[6]	1,064,102			

Source: [1] Bank of Japan, *Economic Statistics Monthly,* Sept. 1988 p. 51
[2] Bank of Japan, *Economic Statistics Monthly,* Sept. 1988 p. 53
[3] Bank of Japan, *Economic Statistics Monthly,* Sept. 1988 p. 55
[4] Bank of Japan, *Economic Statistics Monthly,* Sept. 1988 p. 57
[5] Bank of Japan, *Economic Statistics Annual,* 1988 p. 99
[6] Bank of Japan, *Economic Statistics Monthly,* Jan. 1989 p. 17

link the two series. In this analysis the quarterly average series for M1 is measured as the *geometric* average of the three end-of-month observations during the years 1955-62 and the *geometric* average of the monthly daily average data for 1963 to the present. The series are chained at the first quarter of 1963 by multiplying 1955-62 data by the ratio of the daily average measure for 1963:1 to the end-of-month average measure for the same quarter. The money stock data used are seasonally adjusted.

Many interest rates in Japan have been regulated or administered rates during much or all of the sample period considered in this study (Suzuki, 1987, pp. 142-159). The most significant market determined short-term interest rate during the sample period is the call rate. The interest rate used in this analysis is the monthly average unconditional call rate at Tokyo banks (also known as the one-day notice call rate in the early years of the sample).[1] During 1957-59, the published data in the *Economic Statistics Monthly* of the Bank of Japan indicate a range of values for this rate. During these months the midpoint of the published range has been used as the short-term rate observation for that month. Quarterly average rates are constructed as *geometric* averages in the specifications which use logs of interest rates. *Arithmetic* averages of the monthly data are used in the specifications with

[1]The sources of the monthly call rate data are as follows:
1955: Bank of Japan, *Economic Statistics Monthly*, 1955, p. 165
1956: Bank of Japan, *Economic Statistics Monthly*, 1956, p. 161
1957: Bank of Japan, *Economic Statistics Monthly*, 1957, p. 168
1958: Bank of Japan, *Economic Statistics Monthly*, 1958, p. 161
1959: Bank of Japan, *Economic Statistics Monthly*, 1959, p. 165
1960 and subsequently: Bank of Japan data base.

levels of interest rates.

The real income data are the official quarterly seasonally adjusted GNP estimates for Japan published by the Economic Planning Agency. The GNP deflator is used to construct real M1.

B. Data characteristics

It is interesting to compare the behavior of Japanese GNP relative to transactions deposits with the comparable data from the U.S. economy during 1955-88. The estimates of the velocity of M1 in both countries are shown in Figure 1. It is evident from that graph that the experience in the two countries differs considerably over much of the sample period. In the U.S., M1 velocity drifted up during the 1955-80 period and during the most recent decade generally drifted downward. In Japan, M1 velocity generally drifted down

Figure 1. Japanese and U.S. GNP-M1 Velocity

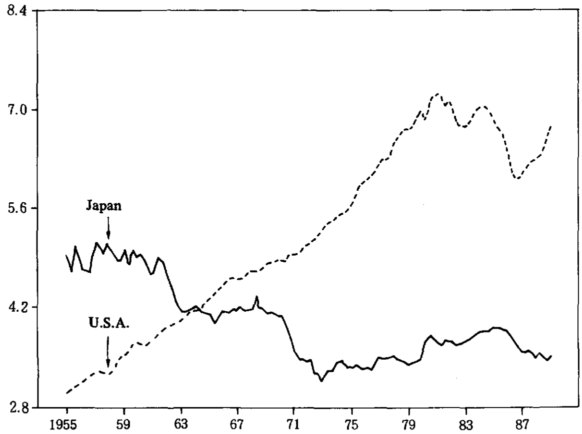
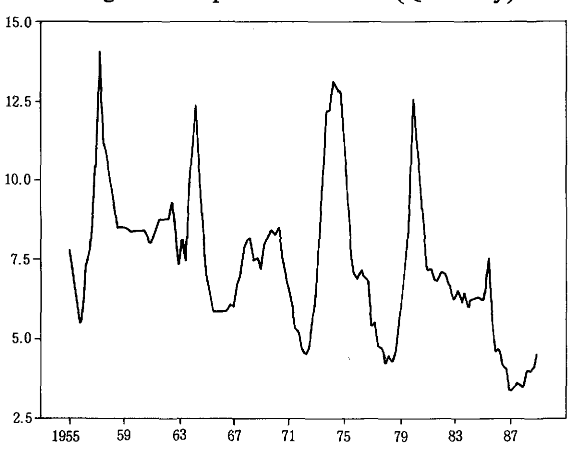


Figure 2. Japanese Call Rate (Quarterly)



during the 1956-80 period and has been relatively trendless during the past decade. In addition the behavior of short-term interest rates is remarkably different between the two countries, at least through the 1970s. The Japanese call rate exhibits significant cyclical variation during this period, but gradually drifts downward (Figure 2). U.S. Treasury bill and commercial paper rates show cyclical variation around an upward drift prior to 1980. In the 1980s the difference between short-term interest rate behavior between the two countries is less pronounced. The differences prior to 1980 are not surprising given the significant capital controls that were in place in Japan during that period (Suzuki, 1987, pp. 42-43). With the passage of the Foreign Exchange and Trade control Act of 1980, the process of liberalizing controls on all capital transactions began (Suzuki, 1987, pp. 49-50).

A significant element in recent discussion (an disputes) in the analysis of economic time series data is the question of whether or not such data possess "unit roots." The economic implication of a unit root is that shocks to the economy are permanent rather than transitory: in the presence of a unit root at each point in time an economic measurement is just the sum of all past shocks to that variable.

In an extensive study of macroeconomic time series from the U.S. economy, Nelson and Plosser (1982) conclude that the unit root hypothesis cannot be rejected for most of the data series that they consider. Subsequently an extensive econometric literature has advanced numerous tests of the hypothesis of a unit root (Dickey and Fuller, 1979; Phillips and Perron, 1986; Phillips, 1987; Schwert, 1987). It is recognized that all the available tests of the unit root hypothesis have low power against the alternative hypothesis of a near unit (but stationary) root. Consequently separate analyses of identical economic data come to opposite conclusions on the existance of a unit root (Cochrane, 1988; DeLong and Summers, 1988; Campbell and Mankiw, 1988) and other analysts conclude that it is impossible to discriminate a unit root from a near unit root (Christiano and Eichenbaum, 1989).

The data used in this analysis have similar characteristics to the data from the U.S. economy that have been studied so extensively. In Table 2 the results of a series of unit root tests are presented for two sample periods. The shorter sample period starts in the mid 50s and ends in 1979, before the period of financial liberalization and innovation in Japan. The longer sample includes the data from the 1980s. The results are the same for both samples. All of the tests fail to reject a unit root in real income and real M1. For these series the only cases where the computed values of the test statistics come close to the estimated critical values are for the augmented Dickey-Fuller tests (τ_μ) when no provision is made for a deterministic trend in the data. The unit root tests on the call rate data are ambiguous. The unit root hypothesis is decisively rejected based on some test statistics, while it is not rejected on the basis of competing test statistics.

The question of a second unit root in these data series is examined by performing the battery of unit root tests (assuming no deterministic trend) on the first differences of each data series. The computed test statistics are reported in Table 3. In contrast to the tests on

Table 2. Unit Root Test Statistics (Logarithms)

Test Statistic	AR	.05 Critical Values $\theta = -.5$	$\theta = 0$	$\theta = .5$	GNP (SA) 1955:2 -89:3	Real M1 (SA) 1955:2 -89:3	Rcall 1955:2 -89:3	GNP (SA) 1955:2 -79:4	Real M1 (SA) 1955:2 -79:4	Rcall 1955:2 -79:2
τ_μ	4	-3.02	-2.87	-2.93	-2.81	-2.67	-3.83*	-1.96	-1.63	-3.95*
τ_T	4	-3.61	-3.41	-3.49	- .90	- .48	-5.32*	- .25	- .22	-4.69*
$T(\rho_\mu-1)$	4	-29.2	-14.4	-9.8	- .76	- .71	-16.69	- .53	- .39	-16.34
$T(\rho_\mu-1)^c$	4	-19.9	-16.6	-17.4	-1.10	-1.36	-49.74*	- .77	- .72	-74.73*
$T(\rho_T-1)$	4	-44.2	-22.4	-15.6	- .98	- .45	-27.90	- .46	- .37	-21.16
$T(\rho_T-1)^c$	4	-33.8	-28.3	-30.4	-1.44	- .84	-138.45*	- .66	- .69	-142.13*
$t_\alpha*$ $[Z_{\tau\mu}]$	4	-5.30	-2.93	-2.73	-3.68	-2.97	-2.93	-2.40	- .66	-2.92
$t_{\hat\alpha}$ $[Z_{\tau\tau}]$	4	-6.59	-3.53	-3.15	- .66	- .48	-3.52	.14	- .21	-3.12
$Z_\alpha*$	4	-44.9	-14.3	-11.9	-1.09	-1.27	-1.24	- .73	5.68	-16.75*
Z_α	4	-65.8	-21.8	-17.6	- .81	- .73	-29.01	.27	-- .52	-19.12

Table 3. Unit Root Test Statistics (First Difference of Logarithms)

Test Statistic	AR	.05 Critical Values $\theta = -.5$	$\theta = 0$	$\theta = .5$	GNP (SA) 1955:3 -89:3	Real M1 (SA) 1955:3 -89.3	Rcall 1955:3 -89.3	GNP (SA) 1955:3 -79.4	Real M1 (SA) 1955:3 -79.4	Rcall 1955:3 -79.2
τ_μ	4	-3.02	-2.87	-2.93	-3.02	-3.35*	-6.21*	-2.91	-3.11*	-5.11*
$T(\rho_\mu-1)$	4	-29.2	-14.4	-9.8	-53.79*	-46.62	-101.14*	-48.31*	-37.58*	-66.92*
$T(\rho_\mu-1)^c$	4	-19.9	-16.6	-17.4	-24.42*	-35.61*	-850.13*	-25.32*	-30.47*	-2487.6*
$t_\alpha*$ $[Z_{\tau\mu}]$	4	-5.30	-2.93	-2.73	-10.23*	-4.94	-6.02*	-3.36	-4.89	-4.62
$Z_\alpha*$	4	-44.9	-14.3	-11.9	-25.79	-24.01	-34.32	-19.51	-18.66	-23.75

Note: τ_μ and τ_T are the augmented Dickey-Fuller test statistics of the hypothesis $\rho_\mu = 1$ or $\rho_T = 1$ in the regressions.

$$y_t = \alpha + \rho_\mu y_{t-1} + \sum_{i=1}^{q} \gamma_i \Delta y_{t-i} + \epsilon_t \quad \text{or} \quad y_t = \alpha + \beta t + \rho_T y_{t-1} + \sum_{i=1}^{q} \gamma_i \Delta y_{t-i} + \epsilon_t$$

$T(\rho_\mu-1)$ and $T(\rho_T-1)$ are normalized bias tests of the same hypotheses. $t_{\alpha*}$ and $t_{\hat\alpha}$ are adjusted Dickey-Fuller tests suggested by Phillips. $Z_{\alpha*}$ and Z_α are Phillips corrected normalized bias tests.

the levels data, the maintained hypothesis of a unit root is rejected strongly here for all the time series. Therefore we conclude that if the time series used in this analysis are integrated (possess unit roots), they are integrated of order less than 2.[2] For the econometric analysis that is utilized in this study to be valid, it is critical that the time series be integrated of order <2. The statistical model is valid for trend stationary data (I(0)) or difference stationary data (I(1) or a single unit root), and the analysis provides an additional test of the unit root hypothesis. The econometric technique is discussed in Section IV below.

III. An Economic Model of Long-run Demand for Money in Japan

A. Theoretical specification of the long-run demand for transactions money

There exists a significant consensus on the theoretical sepecification of the long-run (or equilibrium) demand for transactions money. There are several different approaches to the microfoundations of the long-run demand for transactions balances. One approach emphasizes the costs of exchanging nonmonetary for monetary assets (Baumol, 1954; Tobin, 1956) and derives equilibrium microeconomic demand functions for transactions money from cost minimization behavior.

More recent theoretical analyses start from traditional utility maximization models of consumer behavior and emphasize that the transactions services available from holding money balances provide savings of real resources (additional consumption and/or leisure opportunities) for the consumer (McCallum and Goodfriend, 1987; Lucas, 1988).

The conclusion from all these analyses is that in equilibrium the demand for real transactions balances is a function of real income and an opportunity cost variable.[3] Opportunity costs are variously measured as nominal interest rates on alternative assets less the nominal interest rate on transactions money (own rates) and/or brokerage charges. Since explicit deposit rates in Japan are highly regulated and have been maintained at very low levels for those deposits where the permitted maximum is greater than zero, the opportunity cost in this study is represented by a short-term market-determined rate of interest.

A general specification of the demand for real balances that is consistent with these theoretical conclusions is:

$$1\text{n}(M/P)_t = -c_1(B)R_t + c_2(B)y_t + c_3(B)1\text{n}(M/P)_t + \mu_{3t} \tag{1}$$

[2]See Engle and Granger (1987) for a discussion of orders of integration.

[3]Lucas derives the demand for real balances in terms of real permanent income, but in full equilibrium transitory income is zero. McCallum and Goodfriend derive a relationship between the demand for real balances and real consumption. If consumption is proportional to permanent income (or real wealth) then in equilibrium the demand for real balances can again be expressed in terms of real income.

where

$$c_1(B) = c_{10} + c_{11}B + c_{12}B^2 + \ldots + c_{1n}B^n$$
$$c_2(B) = c_{20} + c_{21}B + c_{22}B^2 + \ldots + c_{2n}B^n$$

and

$$c_3(B) = c_{31}B + c_{32}B^2 + \ldots + c_{3n}B^n$$

are finite polynomials in the lag operator B and μ_{3t} is a stationary (I(0)) error term.[4]

This equation can be rewritten in an algebraically equivalent expression in first differences as:

$$\Delta\ln(M/P)_t = -c_{10}\Delta R_t - a_{31}(B)\Delta R_t + c_{20}\Delta y_t + a_{32}(B)\Delta y_t + a_{33}(B)\Delta\ln(M/P)_t$$
$$-c_1(1)R_{t-n-1} + c_2(1)y_{t-n-1} + [c_3(1)-1]\ln(M/P)_{t-n-1} + \mu_{3t} \tag{2}$$

where $c_i(1)$ is the sum of the coefficients in the $c_i(B)$ polynomials (i=1,..,.3) and the coefficient of the i^{th} order term in the polynomials $a_{3j}(B)$ is:

$$a_{3ji} = \sum_{k=0}^{i} c_{jk}, \quad i = 1, \ldots, n.$$

The theoretical restrictions on the long-run demand for real balances are: $c_3(1)<1.0$; $c_1(1)>0.0$; and $c_2(1)>0.0$. Under these restrictions the long-run interest and real income elasticities are given by:

$$\frac{-c_1(1)}{[1-c_3(1)]} \quad \text{and} \quad \frac{c_2(1)}{[1-c_3(1)]} \quad \text{respectively.}$$

B. Bank of Japan operating procedures and the supply of transactions money

A critical question for any attempt to estimate a demand function for money (transactions balances or more broadly defined money) is the issue of whether the demand function is identified. This question cannot be answered independently of the operating procedure used by the monetary authorities, since a specification of this procedure is required to complete the requisite economic model.

Historically the operating procedure of the Bank of Japan has focused on achieving a desired call or bill rate. For example a report on monetary control technique by the Bank of Japan (Fukui, 1986) states:[5]

> The Bank of Japan, while carefully observing the general 'surplus or shortage of funds' trend in the money market, influenced supply and

[4]To simplify notation y_t represents real income and R_t represents either the interest rate or the log of the interest rate.

[5]See also Suzuki (1987): "The Bank of Japan conducts its financial adjustments paying attention to the interest rate movements in the interbank markets as its operating variables" (p. 327).

> demand by adjusting its credit to financial institutions on a day-to-day basis, which allowed it to maintain control of the call and bill rates. (p. 4)

and

> ...the Bank of Japan controlled the call and bill rates by means of influencing, ex-ante, the supply of and demand for, high powered money...(p. 4)

Changes in the Bank of Japan official discount rate play a major role in these operating procedures:

> The main thrust of the Bank of Japan's operations in adjusting money market conditions has been, first the fine-tuning of the call and bill rates by means of creating either 'stringent' or 'relaxed' conditions in the markets on a day to day basis; and secondly, changes in the official discount or reserve rates, should a change in the underlying call and bill rates be deemed necessary. (Fukui, 1986, p. 8)

Finally, this operating procedure has remained stable over the sample periods studied here even after the Bank of Japan began the policy of announcing estimated values of the future growth rate of the money stock (initially M2, subsequently M2+CD):

> The Bank of Japan's framework for controlling money market conditions since the mid-1970s has basically remained the same as the traditional framework in which the call and bill rates were controlled by means of adjusting the 'reserve progress ratio of reserve deposits' (Fukui, 1986, p. 15)

Quantitatively it is appropriate to represent the short-run behavior of interbank interest rates in Japan as:

$$R_t - R_{t-1} = (R_t^* - R_{t-1}^*) + \epsilon_{1t} \tag{3}$$

where R_t is the interbank interest rate, R_t^* is the rate objective of the Bank of Japan and ϵ_{1t} is an error term. The role of the official discount rate in the implementation of the interbank rate objective is modeled as:

$$R_t^* = \alpha_t + \beta R^{\mathrm{dis}}{}_t \tag{4}$$

so that the short run behavior of interbank rates is:

$$(R_t - R_{t-1}) = \Delta\alpha_t + \beta\,(R^{\mathrm{dis}}{}_t - R^{\mathrm{dis}}{}_{t-1}) + \epsilon_{1t}. \tag{5}$$

In the longer run there are potential feedbacks from the behavior of real output growth and the growth rate of real balances onto the interbank rate objective of the Bank of Japan. We allow for this by specifying:

$$\Delta\alpha_t = a_{11}(B)\Delta R_t + [a_{120}+a_{12}(B)]\Delta y_t + [a_{130}+a_{13}(B)]\Delta 1n(M/P)_t+\epsilon_{2t} \tag{6}$$

where

$$a_{11}(B) = a_{111}B + a_{112}B^2 + \ldots$$
$$a_{12}(B) = a_{121}B + a_{122}B^2 + \ldots$$

and

$$a_{13}(B) = a_{131}B + a_{132}B^2 + \ldots$$

are polynomials in the lag operator B. The important identifying restriction for the long-run demand function for real balances is that changes in nominal interest rates are independent of *levels* of real output and real balances in the long run (equilibrium). This restriction appears in the constraint of (6) to a first difference specification. In the long run the level of nominal interest rates is assumed to reflect both the real rate of interest in the economy and inflationary expectations. The latter may be closely related to the growth rate of the nominal money stock, but is consistent with any level of real balances or real income.

The particular specification of (6) models changes in the official discount rate as an intervention term in the interbank rate equation. The specification also allows for contemporaneous and lagged feedback from changes in real income and real balances onto changes in interbank interest rates reflecting the Bank of Japan's operating procedures through adjustment of "stringent" or "relaxed" conditions in the markets on a day-to-day basis, independent of changes in the official discount rate. Thus the specification of (6) is not sufficiently restrictive to identify the short-run demand function for real balances. (This is discussed in more detail below). Furthermore, this specification does not impose unidirectional "Granger Causality" on interest rates, real income or real money balances.

Finally, the error term in the interest rate specification $\mu_{1t}=(\epsilon_{1t}+\epsilon_{2t})$, is assumed to be stationary (I(0)).

IV. Model Specification, Identification, Estimation and Testing

A. Complete model specification

Equations (2) and (6) specify the demand for and supply of transactions balances in Japan. A complete model requires the specification of the determination of real output. Economic theory suggests that in the long run (equilibrium) real output reflects the productive capacity of the economy which is determined by the real resources that are available to the economy and the available technology to utilize these resources, and should be independent of the price level, the inflation rate, nominal interest rates and real transactions balances. We wish to allow for transactions, adjustment or information costs that produce less than complete nominal wage and price adjustment in the short run. Thus interest rates and/or real balances can affect the short run path of real output. Finally, we incorporate the failure to reject the unit root hypothesis for real GNP (Table 2) by specifying the real output equation in first difference (growth rate) terms:

$$\Delta y_t = -a_{210}\Delta R_t - a_{21}(B)\Delta R_t + a_{22}(B)\Delta y_t + a_{230}\Delta \ln(M/P)_t + a_{23}(B)\Delta \ln(M/P)_t + \mu_{2t}. \quad (7)$$

The error term, μ_{2t} is assumed to be stationary (I(0)). Equations (2), (7) and (6) can be written as a complete model of interest rates, real output and real balances as:

$$\begin{bmatrix} 1.0 & -a_{120} & -a_{130} \\ a_{210} & 1.0 & -a_{230} \\ c_{10} & -c_{20} & 1.0 \end{bmatrix} \begin{bmatrix} \Delta R_t \\ \Delta y_t \\ \Delta \ln(M/P)_t \end{bmatrix}$$

$$= \begin{bmatrix} \beta \\ 0 \\ 0 \end{bmatrix} \Delta R_t^{\text{dis}} + \begin{bmatrix} a_{11}(B) & a_{12}(B) & a_{13}(B) \\ -a_{21}(B) & a_{22}(B) & a_{23}(B) \\ -a_{31}(B) & a_{32}(B) & a_{33}(B) \end{bmatrix} \begin{bmatrix} \Delta R_t \\ \Delta y_t \\ \Delta \ln(M/P)_t \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -c_1(1) & c_2(1) & c_3(1) \end{bmatrix} \begin{bmatrix} R_{t-n-1} \\ y_{t-n-1} \\ \ln(M/P)_{t-n-1} \end{bmatrix} + \begin{bmatrix} \mu_{2t} \\ \mu_{2t} \\ \mu_{3t} \end{bmatrix} \quad (8)$$

Let $A^* = [a^*_{ij}] = \begin{bmatrix} 1.0 & -a_{120} & -a_{130} \\ a_{210} & 1.0 & -a_{230} \\ c_{10} & -c_{20} & 1.0 \end{bmatrix}^{-1}$

Then the reduced form model of (8) is

$$\begin{bmatrix} \Delta R_t \\ \Delta y_t \\ \Delta \ln(M/P)_t \end{bmatrix} = A^* \begin{bmatrix} \beta \\ 0 \\ 0 \end{bmatrix} \Delta R_t^{\text{dis}} + A^* \begin{bmatrix} a_{11}(B) & a_{12}(B) & a_{13}(B) \\ -a_{21}(B) & a_{22}(B) & a_{23}(B) \\ -a_{31}(B) & a_{32}(B) & a_{33}(B) \end{bmatrix} \begin{bmatrix} \Delta R_t \\ \Delta y_t \\ \Delta \ln(M/P)_t \end{bmatrix}$$

$$+ \begin{bmatrix} -a^*_{13}c_1(1) & a^*_{13}c_2(1) & a^*_{13}[c_3(1)-1] \\ -a^*_{23}c_1(1) & a^*_{23}c_2(1) & a^*_{23}[c_3(1)-1] \\ -a^*_{33}c_1(1) & a^*_{33}c_2(1) & a^*_{33}[c_3(1)-1] \end{bmatrix} \begin{bmatrix} R_{t-n-1} \\ y_{t-n-1} \\ \ln(M/P)_{t-n-1} \end{bmatrix} + A^* \begin{bmatrix} \mu_{1t} \\ \mu_{2t} \\ \mu_{3t} \end{bmatrix} \quad (9)$$

In particular note that under the restrictions that are imposed on the structure of the model the reduced form coefficient matrix of the lagged levels terms has rank=1 and can be written as:

$$\begin{bmatrix} a^*_{13} \\ a^*_{23} \\ a^*_{33} \end{bmatrix} \left[-c_1(1) \;\; c_2(1) \;\; [c_3(1)-1] \right] \quad (10)$$

Alternatively, the second vector in this product can be scaled so that the element corresponding to lagged real balances is -1.0 and the reduced form coefficient matrix can be expressed equivalently as:

$$\begin{bmatrix} a^*_{13}[1-c_3(1)] \\ a^*_{23}[1-c_3(1)] \\ a^*_{33}[1-c_3(1)] \end{bmatrix} \begin{bmatrix} \frac{-c_1(1)}{1-c_3(1)} & \frac{c_2(1)}{1-c_3(1)} & -1.0 \end{bmatrix} \tag{11}$$

When this normalization is applied, the first two elements of the row vector are the long-run interest rate and real income elasticities of the demand for real balances respectively. Hence the restrictions that are outlined above on the long-run structure of the model are sufficient to identify the long-run demand function for real balances.

It is important to note that the restricted model does not identify the short-run demand for money function. This requires identification of all the a^*_{ij}, $i,j=1,\ldots,3$ elements of the inverse matrix A^*. From the reduced form (9) it is possible to determine the relative magnitudes of the elements in the first and third columns of A^*, but full identification of this matrix requires five additional restrictions.[6]

The additional assumption of no contemporaneous feedback from $\ln(M/P)_t$ to R_t and y_t is *not* sufficient to identify the short-run money demand function. Under these restrictions the contemporaneous interactions are represented by the matrix:

$$\begin{bmatrix} 1.0 & -a_{120} & 0 \\ a_{210} & 1.0 & 0 \\ c_{10} & -c_{20} & 1.0 \end{bmatrix} \tag{12}$$

The inverse of this matrix is of the form:

$$\begin{bmatrix} a^*_{11} & a^*_{12} & 0 \\ a^*_{21} & a^*_{22} & 0 \\ a^*_{31} & a^*_{32} & a^*_{33} \end{bmatrix} \tag{13}$$

and the reduced form coefficient matrix of the lagged levels variables is still of rank=1 and can be written as:

$$\begin{bmatrix} 0 \\ 0 \\ a^*_{33}[1-c_3(1)] \end{bmatrix} \begin{bmatrix} \frac{-c_1(1)}{1-c_3(1)} & \frac{c_2(1)}{1-c_3(1)} & -1.0 \end{bmatrix} \tag{14}$$

Thus the restrictions on contemporaneous feedbacks provide no additional identifying information with respect to the short-run money demand function.[7]

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[6]The reduced form coefficients of the ΔR^{dis}_t are βa^*_{i1} while the reduced form coefficients of the lagged levels terms (under the restriction that this matrix is of rank=1) give $[1-c_3(1)]a^*_{i3}$ as noted above. From these six reduced form coefficients four ratios of the elements a^*_{ij} are determined, leaving five underidentified a^*_{ij} coefficients.

[7]Restrictions on the covariance structure of the model are a potential source of information with which to achieve identification of the short-term structure (Bernanke, 1986).

The reduced form equations of this model specify a multivariate error correction model (Granger, 1986; Engle and Granger, 1987). The model implies several hypotheses that can be tested within the error correction framework. First, and most important, is a rank restriction on the coefficient matrix of the error correction terms (lagged levels variables). The rank of this matrix must be greater than zero if a long-run relationship exists between the levels of real balances, real income and interest rates given the nonstationarity of the data series. Second, the rank of this matrix must be one to identify the long-run income and interest elasticities of the model. Third, if there is no contemporaneous feedback from changes in real balances to changes in real income and interest rates, then all elements in the matrix of coefficients of the lagged levels variables corresponding to the real income and interest rate variables must not be significantly different from zero (equation (14)).

B. Estimation and testing

Recently Johansen (1988, 1989a, 1989b) and Johansen and Juselius (1988, 1989) have examined the problem of maximum likelihood estimation of error correction models such as (9). Johansen has developed likelihood ratio tests for the rank of the coefficient matrix on the lagged levels variables (the π matrix in his notation). In addition he has developed likelihood ratio tests for linear restrictions on the elements of the matricies α and β defined as $\pi=\alpha\beta'$, when $0<\text{rank}(\pi)<\text{full rank}$. Additionally he has developed Wald tests for linear restrictions on the elements of β subject to a normalization such as that in (11). These procedures have been applied to the analysis of money demand in Denmark and Finland by Johansen and Juselius (1988, 1989) and in the United States by Hoffman and Rasche (1989a, 1989b). The techniques, which are described in detail in those studies, are used for the analysis of post-war Japanese data in this study.

V. Empirical Analysis

A. Estimates of the basic model

The initial estimation was performed over a sample period from 1955:2 through 1979:4. This sample was chosen to provide the maximum number of observations but to avoid using data from the period in which Japanese financial markets were undergoing substantial and rapid changes. The results of this estimation are presented in Table 4. Here there is strong evidence in support of the hypothesis that the data series for real M1, real GNP, and the call rate are cointegrated, since the trace test strongly rejects the hypothesis that the rank of π is zero. The trace test also fails to reject the hypothesis that the rank of π is $<=2$, and therefore is consistent with the assumption that the data series are nonstationary. The results of both the trace and maximum eigenvalue tests are consistent with the hypothesis of a single cointegration vector among the three variables that is developed above. Finally, the signs of the individual elements of the cointegration vector

Table 4. Tests for Cointegration Real M1, Real GNP, Call Rates
Constant Elasticity Specification

Sample Period 1955:2 – 79:4
(k = 2, T = 97)

	Unconstrained		
	Johansen Test Statistics		
H_0	$r = 0$	$r \leq 1$	$r \leq 2$
Trace Test:	35.97	16.90	3.10
Maximum Eigenvalue Test:	19.07	13.80	3.10
Estimated Eigenvalues	.1785	.1325	.0315
	M/P	y	R
Estimated Cointegration Vector	14.86	−16.89	2.87
Estimated Standard Errors (LR)	(8.53)	(10.18)	(1.79)
	Income Elasticity Constrained to 1.0		
Estimated Eigenvalues	.1674	.0848	
$\chi^2_{(1)}$ Test Statistic For Constraint = 1.30			
	M/P	y	R
Estimated Cointegration Vector	6.65	−6.65	3.76
Estimated Standard Errors (LR)	(2.73)	(2.73)	(1.38)
Estimated Interest Elasticity of Velocity		.57	
Estimated Standard Error (Wald)		(.16)	

are consistent with a positive equilibrium real income elasticity and negative equilibrium interest elasticity of the demand for real balances.

It should be noted that the precision with which the elements of the cointegration vector are estimated is low. This can be seen from the individual estimated asymptotic standard errors of these elements.[8] This lack of precision appears to be attributable to multicolinearity between real balances and real income. Since the unrestricted implied income elasticity is 1.14, the restriction of a unitary equilibrium income elasticity (a long-run velocity function) is also tested by the likelihood ratio test. The computed χ^2 value of 1.3 (with one degree of freedom) fails to reject this restriction. Under the unitary equilibrium income elasticity restriction, the precision of the individual elements of the

[8]These asymptotic standard errors are constructed from likelihood ratio tests of the restriction of each of the individual elements of the cointegration vector to zero. Johansen (1988, 1989) shows that such likelihood ratio statistics are asymptotically distributed as χ^2 with one degree of freedom. Hence the square root of the computed likelihood ratio is asymptoticly normal and provides an estimate of the desired asymptotic standard error.

cointegration vector improves considerably. The resulting estimate of the equilibrium interest elasticity of velocity of .57 is significantly greater than zero and significantly less than 1.0.

The autoregressive part of the error correction model is truncated at $t-1$ ($k=2$ in Johansen's notation) because there is no evidence of autocorrelation in the residuals of either the real balances equation or the real income equation, and only small autocorrelations in the residuals of the interest rate equation.

The results reported in Table 4 are unaffected by extending the sample period through 1981.[9] The detailed results are reported in Table 5 for this extended sample period. The trace test rejects the hypothesis of no cointegration at the 0.025 level, but fails to reject the hypothesis of one or fewer cointegration vectors at the 0.10 level. The maximum eigenvalue test rejects the hypothesis of no cointegration at the 0.05 level. However the estimates of the individual elements of the unrestricted cointegration vector again are very imprecise. The implied (unrestricted) point estimate of the real income elasticity of 0.99 which is not significantly different from 1.0. As with the shorter sample period, the restriction of the equilibrium real income elasticity to unity improves the precision of the estimated elements of the cointegration vector substantially, and the resulting implied equilibrium interest elasticity of velocity of 0.60 is significantly greater than zero and significantly less than 1.0.

The full set of estimates of the parameters of the restricted error correction model are also shown in Table 5. The estimated parameters are for equations (9) and (11), so $\hat{\alpha}$ is shown corresponding to the normalization of $\hat{\beta}_3$ to -1.0. The $\hat{\Lambda}$ matrix measures the estimated variance-covariance matrix of the residuals of the error correction model (μ_1 μ_2 μ_3).

The stability of these estimates is *not* maintained as the sample period is extended beyond the end of 1981. This can be seen in Figure 3 where the estimates of the unrestricted real income and interest elasticities are plotted for sample periods ending in 1982:1 through 1985:4. As additional observations are added after 1981, the estimated equilibrium income elasticity becomes progressively smaller and the estimated equilibrium interest elasticity becomes progressively larger in absolute value. When the sample is extended to 1986:1, the estimated equilibrium income elasticity actually becomes negative. When the sample is extended beyond 1986:1, the estimates of the income and interest elasticities fluctuate widely because the estimated coefficient of real balances in the cointegration vector is very close to zero. This is not the result of colinearity between real balances and real income. Constraining the income elasticity to unity does not produce stable estimates of the equilibrium interest elasticity as the sample period is extended through the 1980s. Some

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[9]The extension of the sample period was actually performed by recursive regression, adding one observation at a time from 1980:1 through 1981:4. To save space only the results from the 1955:2-81:4 sample are reported here.

Table 5. Tests for Cointegration Real M1, Real GNP, Call Rates
Constant Elasticity Specification

Sample Period 1955:2 – 81:4
(k = 2, T = 105)

	Unconstrained		
	Johansen Test Statistics		
H_0	$r=0$	$r<1$	$r<2$
Trace Test:	35.59	15.03	2.46
Maximum Eigenvalue Test:	20.87	12.57	2.46
Estimated Eigenvalues	.1778	.1128	.0232
	M/P	y	R
Estimated Cointegration Vector	−6.23	6.17	−3.89
Estimated Standard Errors (LR)	(5.93)	(6.95)	(1.46)

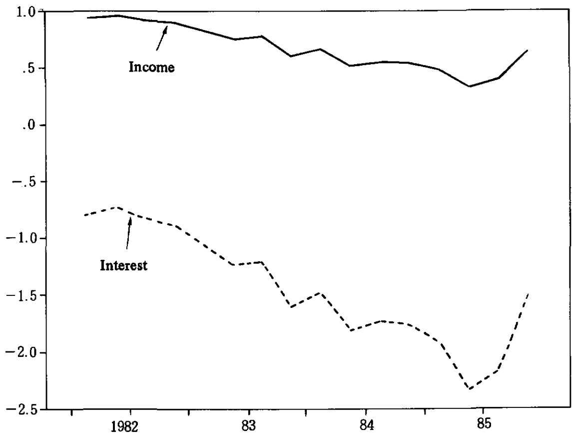
	Income Elasticity Constrained to 1.0		
Estimated Eigenvalues	.1778	.0928	
$\chi^2_{(1)}$ Test Statistic For Constraint = .001			
	M/P	y	R
Estimated Cointegration Vector	6.50	−6.50	3.88
Estimated Standard Errors (LR)	(2.62)	(2.62)	(1.32)
Estimated Interest Elasticity of Velocity			.60
Estimated Standard Error (Wald)			(.16)

$$\hat{\alpha} = \begin{bmatrix} -.2035 \\ -.0206 \\ -.0221 \end{bmatrix} \qquad 10^3 * \hat{\Lambda} = \begin{bmatrix} 9.310 & -.008 & .035 \\ -.008 & .181 & -.065 \\ .035 & -.065 & 1.248 \end{bmatrix}$$

$$\begin{array}{cc} & \begin{array}{ccc} R_{t-2} & y_{t-2} & (M/P)_{t-2} \end{array} \\ \hat{\pi} = & \begin{bmatrix} .1214 & .2034 & .2034 \\ .0123 & .0206 & .0206 \\ .0132 & .0221 & .0221 \end{bmatrix} \end{array}$$

$$\begin{array}{cc} & \begin{array}{ccccc} \text{const} & \Delta R_t^{dis} & \Delta R_{t-1} & \Delta y_{t-1} & \Delta (M/P)_{t-1} \end{array} \\ \hat{\Gamma} = & \begin{bmatrix} .4431 & .5089 & .2015 & .1309 & -.6482 \\ .0602 & -.0100 & .0015 & -.0948 & .1588 \\ .0556 & -.0676 & -.0312 & .2258 & .3244 \end{bmatrix} \end{array}$$

Figure 3. Income and Interest Elasticities



kind of change has occurred in the relationship between real M1, real GNP, and short-term interest rates in Japan since 1981. The question is what kind of change has occurred and if the answer to this question can be found, what has caused the change?

Hoffman and Rasche (1989a) found that the addition of a dummy variable that allows a change in the constant term of a reduced form error correction model for real M1, real personal income, and short-term interest rates is sufficient to produce a stable equilibrium demand function for real balances in the United States during the post-war period. Subsequently they argue (Hoffman and Rasche, 1989b) that this change in the constant term is consistent with the hypothesis of a break in inflation expectations from a regime in which inflation was expected to accelerate to one in which inflation was expected to remain constant.

The same dummy variable to shift the constant term was added to the Japanese error correction model.[10] The impact of the addition of this variable in stabilizing the estimated parameters of the error correction model for Japanese real M1 is remarkable. This can be seen in Figure 4 which plots the unrestricted estimates of the equilibrium real income and interest elasticities as the terminal data of the sample period is extended from 1982:1 through 1988:4. In sharp contrast to Figure 3, the estimates of the real income elasticity vary only from 0.98 to 1.01. the estimates of the equilibrium interest elasticity fluctuate over a wider range, from 0.49 to 0.71, but this is not particularly large relative to the estimated asymptotic standard error for this coefficient of 0.16 reported in Table 5 for the sample period ending in 1981:4.

The stability of the estimated values characterizes all of the parameters of the error correction model, once the D82 dummy variable is added to the specification. This can be

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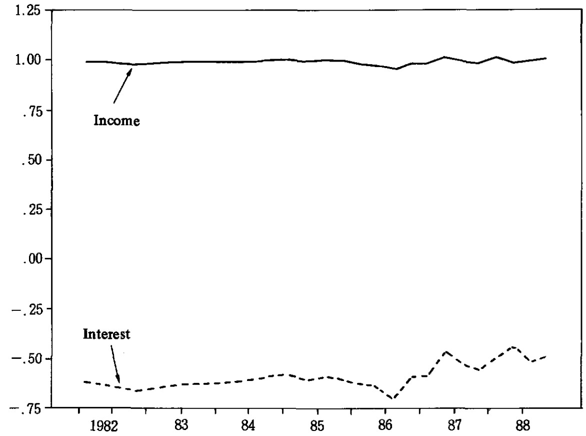
[10]In particular the dummy variable, D82, is 0.0 for all quarters through 1981:4 and 1.0 for all subsequent quarters.

Figure 4. Income and Interest Elasticities



seen by comparing the results reported in Table 6 for the sample period ending in 1988:4 with those reported in Table 5. With the augmented specification, the data again support the hypothesis of a single cointegration vector among the three variables, and the hypothesis of unitary real income elasticity is not rejected (the unconstrained income elasticity is 1.003). Under the constraint on the real income elasticity, the point estimate of the equilibrium interest elasticity is 0.50 with an estimated standard error of 0.12. The estimated values of $\hat{\alpha}$, $\hat{\Gamma}$, and $\hat{\pi}$, shown in Table 6 are virtually identical to the estimates from the shorter sample period in Table 5. In addition, it can be seen from a comparison of the estimates of $\hat{\Lambda}$ in Tables 5 and 6 that the residual variances of interest rates, real GNP, and real M1 are of the same order of magnitude regardless of whether the data from the 80s are included in the sample.[11]

The implications of this model are more readily understood if it is rewritten in alternative, but equivalent forms. Engle and Yoo (1987) show that an error correction model such as (9) has an equivalent representation as a standard VAR in the levels of the individual variables or a moving average (MA) representation in terms of the differences of the variables. The VAR representation is shown at the end of Table 6 based on the parameter estimates in that table. The model in this representation is probably the most useful for forecasting since standard computer programs for the analysis of VAR systems can be applied directly.

[11]It does not appear necessary to increase the length of the lags in the system beyond k=2. For k=2 there is no evidence of autocorrelation in the residuals of real M1 or real GNP, though there is some evidence of autocorrelation remaining in the residuals of the call rate equation. As a check on the robustness of the specification, k was set at 4 and the estimation was repeated for the samples ending in 1981:4 and 1988:4. The resulting estimates of the interest elasticity of M1 velocity are 0.81 (0.25) and 0.67 (0.19) respectively. Increasing the lag length produced only minor reductions in the standard errors of the residuals.

Table 6. Tests for Cointegration Real M1, Real GNP, Call Rates
Constant Elasticity Specification

Sample Period 1955:2 – 88:4
(k = 2, T = 133)
With D82 Dummy Variable

	Unconstrained		
	Johansen Test Statistics		
H_0	$r = 0$	$r \leq 1$	$r \leq 2$
Trace Test:	41.81	16.87	3.65
Maximum Eigenvalue Test:	24.94	13.22	3.65
Estimated Eigenvalues	.1710	.0946	.0271
	M/P	y	R
Estimated Cointegration Vector	−7.82	7.84	−3.87
Estimated Standard Errors (LR)	(5.50)	(6.47)	(1.23)

	Income Elasticity Constrained to 1.0		
Estimated Eigenvalues	.1710	.0766	
$\chi^2_{(1)}$ Test Statistic For Constraint = 1.10			
	M/P	y	R
Estimated Cointegration Vector	7.69	−7.69	3.87
Estimated Standard Errors (LR)	(2.49)	(2.49)	(1.15)
Estimated Interest Elasticity of Velocity			.50
Estimated Standard Error (Wald)			(.12)

$$\hat{\alpha} = \begin{bmatrix} -.2037 \\ -.0245 \\ -.0265 \end{bmatrix} \qquad 10^3 * \hat{\Lambda} = \begin{bmatrix} 9.651 & -.010 & -.046 \\ -.010 & .151 & .013 \\ -.046 & .013 & .212 \end{bmatrix}$$

$$\hat{\pi} = \begin{array}{c} \begin{array}{ccc} R_{t-2} & y_{t-2} & (M/P)_{t-2} \end{array} \\ \begin{bmatrix} .1025 & -.2037 & .2037 \\ .0123 & -.0244 & .0244 \\ .0133 & -.0265 & .0265 \end{bmatrix} \end{array}$$

$$\hat{\Gamma} = \begin{array}{c} \begin{array}{cccccc} \text{const} & \text{D82} & \Delta R_t^{dis} & \Delta R_{t-1} & \Delta y_{t-1} & \Delta (M/P)_{t-1} \end{array} \\ \begin{bmatrix} .4057 & -.0238 & .5126 & .1339 & .0946 & -.6585 \\ .0648 & -.0098 & -.0146 & -.0012 & -.1029 & .1298 \\ .0697 & -.0086 & -.0685 & -.0240 & .2468 & .2994 \end{bmatrix} \end{array}$$

Table 6. (Continued)

Autoregressive VAR in Levels

$$\begin{bmatrix} R_t \\ y_t \\ (M/P)_t \end{bmatrix} = \begin{bmatrix} 1.1339 & .0946 & -.6585 \\ -.0012 & .8971 & .1298 \\ -.0240 & .2468 & 1.2994 \end{bmatrix} \begin{bmatrix} R_{t-1} \\ y_{t-1} \\ (M/P)_{t-1} \end{bmatrix}$$

$$+ \begin{bmatrix} -.2364 & .1091 & .4548 \\ -.0111 & .1273 & -.1542 \\ .0107 & -.2203 & -.3259 \end{bmatrix} \begin{bmatrix} R_{t-2} \\ y_{t-2} \\ (M/P)_{t-2} \end{bmatrix}$$

$$+ \begin{bmatrix} .4057 & -.0238 & .5126 \\ .0648 & -.0098 & -.0146 \\ .0697 & -.0086 & -.0685 \end{bmatrix} \begin{bmatrix} \text{const} \\ \text{D82} \\ \Delta R_t^{\text{dis}} \end{bmatrix} + \begin{bmatrix} \mu_{1t} \\ \mu_{2t} \\ \mu_{3t} \end{bmatrix}$$

B. Semi-log specification

An alternative specification to the model discussed above is to use semi-logarithmic equations involving levels and first differences of short-term interest rates and first differences of the official Bank of Japan discount rate. This alternative is illustrated in Table 7 for the specification with the equilibrium real income elasticity constrained to one and for the sample periods ending in 1981:4 and 1988:4. The data do not distinguish between the logarithmic and semi-logarithmic alternative specifications. With the levels rather than the logs of interest rates, the D82 dummy variable again stabilizes all the parameter estimates as the sample is extended into the 1980s. Furthermore, the estimated values of the various parameters (except the coefficients of the interest rate levels) are almost identical to those in Tables 5 and 6. In particular, the standard errors of the residuals are no different between the two specifications in either sample period. With levels of interest rates and k=2, there is no significant autocorrelation in the estimated residuals of any of the three equations.

C. "Real adjustment" vs. "nominal adjustment" models

The literature on single equation short-run money demand functions contains many examples of comparisons of "nominal adjustment" verses "real adjustment" models. Both models make identical assumptions about the specification of the equilibrium money demand function as a real balance equation:

$$\ln(M/P)_t^* = \alpha + \beta_1 \ln y_t - \beta_2 R_t \tag{15}$$

but assume different short-run adjustment mechanisms (Goldfeld, 1973). The difference in these models is the inclusion or exclusion of $\Delta \ln P_t$ from the regression equation:

$$\ln(M/P)_t = a + b_1 y_t - b_2 R_t + b_3 \ln(M/P)_{t-1} + b_4 \Delta \ln P_t. \tag{16}$$

Table 7. Estimated Cointegration Models

Semi Logarithmic Specification
Long-run Real Income Elasticity Restricted to 1.0

Sample Period 1955:2 – 81:4
(k = 2, T = 105)

	M/P	Y/P	R
Estimated Cointegration Vector	−6.18	−6.18	.48
Estimated Standard Errors (LR)	(2.07)	(2.07)	(.15)
Estimated Interest Semielasticity of Velocity			.078
Estimated Standard Error (Wald)			(.022)

$$\hat{\alpha} = \begin{bmatrix} -1.6221 \\ -.0214 \\ -.0220 \end{bmatrix} \qquad 10^3 * \hat{\Lambda} = \begin{bmatrix} 750.1 & -.694 & -1.083 \\ -.694 & .178 & -.007 \\ -1.083 & -.007 & .253 \end{bmatrix}$$

$$\begin{array}{cc} & \begin{array}{ccc} R_{t-2} & y_{t-2} & (M/P)_{t-2} \end{array} \\ \hat{\pi} = & \begin{bmatrix} .1271 & -1.6224 & 1.6224 \\ .0017 & -.0214 & .0214 \\ .0017 & -.0220 & .0220 \end{bmatrix} \end{array}$$

$$\begin{array}{cc} & \begin{array}{ccccc} \text{const} & \Delta R_t^{\text{dis}} & \Delta R_{t-1} & \Delta y_{t-1} & \Delta\ M/P)_{t-1} \end{array} \\ \hat{\Gamma} = & \begin{bmatrix} 2.5642 & .6857 & .1504 & 3.1938 & -6.5657 \\ .0498 & -.0037 & .0005 & -.1138 & .1522 \\ .0439 & -.0106 & .0036 & .2088 & .3087 \end{bmatrix} \end{array}$$

Sample Period 1955:2 – 88:4
(k = 2, T = 133)

	M/P	Y/P	R
Estimated Cointegration Vector	6.86	6.86	.53
Estimated Standard Errors (LR)	(2.17)	(2.17)	(.12)
Estimated Interest Semielasticity of Velocity			.077
Estimated Standard Error (Wald)			(.019)

$$\hat{\alpha} = \begin{bmatrix} -1.6488 \\ -.0224 \\ -.0237 \end{bmatrix} \qquad 10^3 * \hat{\Lambda} = \begin{bmatrix} 665.8 & -.171 & -1.219 \\ -.171 & .150 & -.009 \\ -1.219 & -.009 & .216 \end{bmatrix}$$

$$\begin{array}{cc} & \begin{array}{ccc} R_{t-2} & (Y/P)_{t-2} & (M/P)_{t-2} \end{array} \\ \hat{\pi} = & \begin{bmatrix} .1275 & -1.6489 & 1.6489 \\ .0017 & -.0224 & .0224 \\ .0018 & -.0237 & .0237 \end{bmatrix} \end{array}$$

$$\begin{array}{cc} & \begin{array}{cccccc} \text{const} & \text{D82} & \Delta R_t^{\text{dis}} & \Delta R_{t-1} & \Delta(Y/P)_{t-1} & \Delta(M/P)_{t-1} \end{array} \\ \hat{\Gamma} = & \begin{bmatrix} 2.6099 & -.2081 & .6663 & .1310 & 2.3168 & -6.6110 \\ .0516 & -.0098 & -.0034 & .0002 & -.1171 & .1310 \\ .0463 & -.0078 & -.0109 & -.0033 & .2332 & .2934 \end{bmatrix} \end{array}$$

The "real adjustment" model restricts $b_4=0.0$; the "nominal adjustment" model restricts $b_4=-b_3$. The analogy in terms of a system of error correction equations such as (9) is a four variable system:

$$\begin{bmatrix} \Delta R_t \\ \Delta y_t \\ \Delta \ln(M/P)_t \\ \Delta \ln P_t \end{bmatrix} = A_4^* \begin{bmatrix} \beta \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta R_t^{\mathrm{dis}} + A_4^* \begin{bmatrix} a_{11}(B) & a_{12}(B) & a_{13}(B) & a_{14}(B) \\ a_{21}(B) & a_{22}(B) & a_{23}(B) & a_{24}(B) \\ a_{31}(B) & a_{32}(B) & a_{33}(B) & a_{34}(B) \\ a_{41}(B) & a_{42}(B) & a_{43}(B) & a_{44}(B) \end{bmatrix} \begin{bmatrix} \Delta R_t \\ \Delta y_t \\ \Delta \ln(M/P)_t \\ \Delta \ln P_t \end{bmatrix}$$

$$+ \begin{bmatrix} -a_{13}^* c_1(1) & a_{13}^* c_2(1) & a_{13}^*[c_3(1)-1] & 0 \\ -a_{23}^* c_1(1) & a_{23}^* c_2(1) & a_{23}^*[c_3(1)-1] & 0 \\ -a_{33}^* c_1(1) & a_{33}^* c_2(1) & a_{33}^*[c_3(1)-1] & 0 \\ -a_{43}^* c_1(1) & a_{43}^* c_2(1) & a_{43}^*[c_3(1)-1] & 0 \end{bmatrix} \begin{bmatrix} R_{t-n-1} \\ y_{t-n-1} \\ \ln(M/P)_{t-n-1} \\ \ln P_{t-n-1} \end{bmatrix} + A_4^* \begin{bmatrix} \mu_{1t} \\ \mu_{2t} \\ \mu_{3t} \\ \mu_{4t} \end{bmatrix} \quad (17)$$

Equation (17) assumes that $\Delta \ln P_t$ is a stationary process, and that in equilibrium there is no trade-off between real output and the price level or nominal interest rates and the price level. These assumptions are necessary to identify the long-run equilibrium demand function for real balances (Hoffman and Rasche, 1990).

Equation (17) is the reduced form analogy to the single equation "nominal adjustment" model. In the short run in this system, real balances respond to inflation, interest rates and real income, but equilibrium real balances are determined only by real income and nominal interest rates. If the polynomials $a_{14}(B)$, $a_{24}(B)$ and $a_{34}(B)$ are restricted to zero, then the equation system decomposes into two separate subsystems, one for real balances, real income and interest rates that is independent of the inflation rate, and an equation for inflation that is affected by changes in all three of the other variables. In fact these restrictions impose unidirectional "Granger causality" from real balances, real income and interest rates to inflation. Under these restrictions, the first three equations in (17) are just the equation system of (9). In this system real balances respond only to real income and interest rates, both in the short run and the long run, so (9) is the error correction analogy to the single equation "real adjustment" model. It is easily seen that equation (9) is nested within the "nominal adjustment" system (17), though the restrictions to get (9) are complicated nonlinear functions of the parameters obtained from the Johansen estimation procedure.

Equation (17) was estimated for sample periods 1955:2-81:4 and 1955:2-88:4 without restrictions on the equilibrium real income elasticity.[12] In this specification there is no evidence of autocorrelation in the estimated residuals of any of the four equations in either sample period with $k=2$.

[12]The point estimates of the equilibrium real income elasticities are 1.10 and 1.08 respectively. In neither case are these estimates significantly different from one. The estimate long-run interest elasticities under the long-run velocity restriction are 0.52 and 0.46 respectively.

Table 8. Goodness of Fit Statistics (R)
Double Log Specifications

Variable	"Real Adjustment" Model (9)	"Nominal Adjustment" Model (17)
Sample Period 1955:2 – 81:4		
$\Delta ln\ (M/P)_t$	.43	.47
Δy_t	.17	.18
ΔR_t	.33	.32
Sample Period 1955:2 – 88:4		
$\Delta ln\ (M/P)_t$	.45	.47
Δy_t	.21	.21
ΔR_t	.26	.26

Goodness-of-fit statistics for the estimated equations of (17) compared to the estimated equations of (9) are given in Table 8. The "nominal adjustment" model is only marginally better for changes in real balances in both sample periods, and both models are equally good (or bad) in terms of real GNP changes and interest rates. Thus it appears that the data do not reject the "real adjustment" model in favor of the "nominal adjustment" model.

D. Interpretation of the change in the VAR structure implied by the D82 dummy variable

The critical question that remains is what economic factors in the 1980s are responsible for the change in the error correction model between real M1, real GNP and the call rate that is described so satisfactorily by the D82 variable? An understanding of the role of the constant terms in the reduced-form error correction model is critical to this question. If r *stable* cointegration vectors exist among p nonstationary variables, *and* if the variables have nonzero deterministic trends, then only $(p-r)$ or these trends can be independent. Let μ_0 be the vector $(p\times 1)$ of deterministic trends and β'_0 be the $(r\times 1)$ vector of means of the cointegration vectors.

When β is stable, estimates of these two vectors can be determined from the estimated parameters of the error correction model (Yoshida and Rasche, 1990). The presence of *a* dummy variable such as D82 in the error correction model, with coefficient vector δ, indicates a shift in the constant vector to $\mu+\delta$. This reflects either a change in the deterministic trends of the variables, a change in the level (constants) of the cointegration vector, or both.

Hoffman and Rasche (1989b) conjecture that the presence of dummy variables in an error correction model for U.S. M1 represent changes in the interdependent deterministic trends of the nonstationary variables with no shift in the mean of the cointegration vector.

This also appears to be the situation for Japan. For the double log specification in Table 6, the estimated shift in the mean of the cointegration vector after 1981 is −0.112. For the semilog specification in Table 7, the estimated shift in the mean of the cointegration vector after 1981 is −0.0105.

The estimates of the reduced-form error correction model under the nonlinear restriction that there is no shift in the mean of the cointegration vector are reported in Table 9 for the constant elasticity specification and Table 10 for the semilog specification. In the former case the likelihood ratio test statistic for the restriction is 0.01 which is distributed as χ^2 with 1 degree of freedom. The corresponding test statistic for the latter case is also 0.01. Thus, in both cases the hypothesis that the D82 dummy variable reflects a "shift in the drift" of real M1, real GNP and the call rate, *without a shift in equilibrium real balances* is not rejected.

In the U.S. experience, a likely source of a shift in the drift of nominal interest rates is a revision of inflation expectation from a regime of accelerating expected inflation to one of stable (though nonzero) expected inflation. Hoffman and Rasche (1989b) show that survey data on inflation expectations in the U.S. are consistent with this hypothesis. Does this conclusion extend to Japan? The history of inflation in Japan is considerably different from that of the U.S. Unfortunately survey data that directly measure inflation expectations are not available for Japan. The best available data are the proportion of respondents who expect increases or decreases in purchase or selling prices found in the Bank of Japan *Short-Term Survey of Economic Enterprises in Japan.*[13] These data suggest that after 1980 a substantial proportion of respondents expected price stability or declining prices (but in the latter case the magnitude of the expected decline is not measured). After 1982 there are no observations where more than 11 percent of the respondents reported that they expected price increases in either purchase or selling prices. In contrast, in 1974-76 and 1979-80 substantial proportions of respondents reported expectations of price increases. However, during 1978 most respondents reported expectations of stable or falling prices. Therefore it does not appear possible to confirm or falsify the inflation expectations hypothesis from the available Japanese survey data.

A possible alternative explanation is the internationalization of Japanese financial markets after the change in the Foreign Exchange and Foreign Trade Control Act in 1980 (Suzuki, 1987, pp. 29-30). During earlier years, Japanese short-term interest rates had drifted lower as illustrated in Figure 2, while at the same time in the U.S. (and Western Europe) short rates generally drifted upward. With the internationalization of capital markets, a plausible hypothesis is that Japanese short-term rates are expected to be less independent of short-rates internationally. This is consistent with a change in the drift of Japanese short rates in the early 80s and a corresponding change in the drift of Japanese M1

[13]Bank of Japan, Research and Statistics Department. The survey began in 1974.

Table 9. Estimates for Constant Equilibrium Real Balance
Relation in the Presence of Changes in Drift
Constant Elasticity Specification

Sample Period 1955:2 – 88:4

	M/P	Y	R
Estimated Cointegration Vector	7.718	−7.718	3.856
Estimated Interest Elasticity			.50

$$\hat{\alpha} = \begin{bmatrix} -.2036 \\ -.0246 \\ -.0267 \end{bmatrix} \qquad 10^3 * \hat{\Lambda} = \begin{bmatrix} 9.762 & .013 & -.046 \\ .013 & .151 & -.010 \\ -.046 & -.010 & .212 \end{bmatrix}$$

$$\hat{\pi} = \begin{array}{c} \begin{array}{ccc} R_{t-2} & Y_{t-2} & (M/P)_{t-2} \end{array} \\ \begin{bmatrix} .1017 & -.2036 & .2036 \\ .0123 & -.0246 & .0246 \\ .0133 & -.0267 & .0267 \end{bmatrix} \end{array}$$

$$\hat{\Gamma} = \begin{array}{c} \begin{array}{cccccc} \text{const} & \text{D82} & \Delta R_t^{\text{dis}} & \Delta R_{t-1} & \Delta y_{t-1} & \Delta(M/P)_{t-1} \end{array} \\ \begin{bmatrix} .3895 & .0033 & .5478 & .1282 & .2822 & -.5795 \\ .0648 & -.0098 & -.0145 & -.0012 & -.1033 & .1294 \\ .0616 & -.0085 & -.0684 & -.0241 & .2465 & .2992 \end{bmatrix} \end{array}$$

Table 10. Estimates for Constant Equilibrium Real Balance
Relation in the Presence of Changes in Drift
Semi-Log Specification

Sample Period 1955:2 – 88:4

	M/P	y	R
Estimated Cointegration Vector	6.892	−6.892	.5297
Estimated Interest Elasticity			.0769

$$\hat{\alpha} = \begin{bmatrix} -1.6500 \\ -.0225 \\ -.0239 \end{bmatrix} \qquad 10^3 * \hat{\Lambda} \quad \begin{bmatrix} 672.5 & -.171 & -1.218 \\ -.171 & .150 & -.009 \\ -1.218 & -.009 & .216 \end{bmatrix}$$

$$\hat{\pi} = \begin{array}{c} \begin{array}{ccc} R_{t-2} & y_{t-2} & (M/P)_{t-2} \end{array} \\ \begin{bmatrix} .1268 & -1.6500 & 1.6500 \\ .0017 & -.0225 & .0225 \\ .0018 & -.0239 & .0239 \end{bmatrix} \end{array}$$

$$\hat{\Gamma} = \begin{array}{c} \begin{array}{cccccc} \text{const} & \text{D82} & \Delta R_t^{\text{dis}} & \Delta R_{t-1} & \Delta y_{t-1} & \Delta(M/P)_{t-1} \end{array} \\ \begin{bmatrix} 2.5023 & -.0010 & .6988 & .1285 & 3.7480 & -5.9948 \\ .0518 & -.0098 & -.0034 & .0002 & -.1175 & .1307 \\ .0468 & -.0078 & -.0108 & -.0033 & .2329 & .2931 \end{bmatrix} \end{array}$$

velocity through its common deterministic trend with short-term nominal rates. This conjecture warrants further investigation, but is beyond the scope of the present analysis.

VI. Conclusions

This study has examined the stability of the equilibrium demand for real transactions balances in Japan since 1955. Once allowance is made for a change in the deterministic trends in the nonstationary variables: real M1, real GNP and the nominal call rate, it is found that estimates of equilibrium real income elasticities and interest elasticities are robust during the period of rapid financial market change in Japan. The change in the deterministic trends in these variables is not unique to Japan, but is very similar to the experience in the U.S. even though the simple trends in the data are quite dissimilar between the two countries prior to 1980. For Japan the equilibrium real income elasticity of real M1 is not significantly different from one, and the equilibrium interest elasticity is on the order of -0.5 to -0.6.

Alternative models that allow for variable interest elasticities (semilog specifications) and short-run "nominal adjustment" rather than "real adjustment" mechanisms are also examined. Neither of the alternative models is more consistent with the data than a basic constant interest elasticity, "real adjustment" model.

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References

Baumol, W.J., "The Transactions Demand for Money: An Inventory Theoretic Approach," *Quarterly Journal of Economics* 66, 1952, pp. 545-556.

Bernanke, B.S., "Alternative Explanations of the Money-Income Correlation," *Carnegie Rochester Conference Series on Public Policy* 25, Autumn 1986, pp. 49-100.

Campbell, J.Y., and G.W. Mankiw, "Are Output Fluctuations Transitory?," *Quarterly Journal of Economics* 102, 1987, pp. 856-880.

Christiano, L.J., and M. Eichenbaum, "Unit Roots in Real GNP: Do We Know and Do We Care?," NBER Working Paper 3130, October 1989.

Cochrane, J.H., "How Big is the Random Walk in GNP?," *Journal of Political Economy* 96, 1988, pp. 797-814.

DeLong, J.B., and L.H. Summers, "On the Existence and Interpretation of a 'Unit Root' in U.S. GNP," NBER Working Paper 2716, 1988.

Dickey, D.A., and W.A. Fuller, "Distribution of the Estimates for Autoregressive Time Series with Unit Roots," *Journal of the American Statistical Association* 74, 1979, pp. 427-431.

Engle, R.F., and C.W. Granger, "Cointegration and Error Correction: Representation, Estimation and Testing," *Econometrica* 55, 1987, pp. 257-276.

————, and B.S. Yoo, "Forecasting and Testing in Cointegrated Systems," *Journal of Econometrics* 35, 1987, pp. 143-159.

Fukui, T., "Recent Developments of the Short-term Money Market in Japan and Changes in Monetary Control Techniques and Procedures by the Bank of Japan," Bank of Japan, Research and Statistics Department, Special Paper No. 130, January 1986.

Goldfeld, S., "The Demand for Money Revisited," *Brookings Papers on Economic Activity* 4, 1973, pp. 577-638.

————, "The Case of Missing Money," *Brookings Papers on Economic Activity* 7-3, 1976, pp. 683-730.

Gordon, R.W., "The Short-Run Demand for Money: A Reconsideration," *Journal of Money, Credit and Banking* 16, 1984, pp. 403-434.

Granger, C.W., "Developments in the Study of Cointegrated Economic Variables," *Oxford Bulletin of Economics and Statistics* 18-3, 1986, pp. 213-228.

Hamada, K., and F. Hayashi, "Monetary Policy in Postwar Japan," in A. Ando, *et al.*, eds., *Monetary Policy in Our Times*, Cambridge, MA: MIT Press, 1985.

Hoffman, D., and R.H. Rasche, "Long-Run Income and Interest Elasticities of Money Demand in the United States," NBER Working Paper 2949, April 1989a.

————, and ————, "The Demand for Money in the United States During the Great Depression: Estimates and Comparison with the Post-war Experience," NBER Working Paper 3217, December 1989b.

Johansen, S., "Statistical Analysis of Cointegration Vectors," *Journal of Economic Dynamics and Control* 12-2/3, 1988.

————, "Estimation and Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models," Institute for Mathematical Statistics, University of Copenhagen, 3, January 1989a.

————, "Cointegration in Partial Systems and the Efficiency of Single Equation Analysis," Institute for Mathematical Statistics, University of Copenhagen, mimeo, December 1989b.

————, and K. Juselius, "Hypothesis Testing for Cointegration Vectors with an Application to the Demand for Money in Denmark and Finland," Institute for Mathematical Statistics, University of Copenhagen, 2, March 1988.

————, and ————, "The Full Information Maximum Likelihood Procedure for Inference on Cointegration—with Applications," Institute for Mathematical Statistics, University of Copenhagen, 4, January 1989.

Laidler, D., *Monetarist Perspectives*, Cambridge, MA: Harvard University Press, 1982.

Lucas, R.E., "Money Demand in the United States: A Quantative Review," *Carnegie-Rochester Conference Series on Public Policy* 29, 1988, pp. 137-168.

McCallum, B.T., and M.S. Goodfriend, "Demand for Money: Theoretical Studies," *The New Palgrave Dictionary of Economics*, London: MacMillan Press, I, 1987, pp. 715-781.

Phillips, P.C.B., "Time Series Regression with a Unit Root," *Econometrica* 55, 1987, pp. 227-301.

———, and P. Perron, "Testing for Unit Root in Time Series Regression," Cowles Foundation Discussion Paper No. 795, Yale University, New Haven, CT, 1986.

Schwert, G.W., "Effects of Model Specification on Tests for Unit Roots in Macroeconomic Data," *Journal of Monetary Economics* 20, 1987, pp. 73-103.

Suzuki, Y., *The Japanese Financial System*, Oxford: Clarendon Press, 1987.

Tobin, J., "The Interest Elasticity of Transactions Demand for Cash," *Review of Economics and Statistics* 38, 1956, pp. 241-247.

Tsutsui, Y., and M. Hatanaka, "On the Stability of Money Demand in Japan and the United States (in Japanese)," *Contemporary Economics* 50, 1982, pp. 124-135.

Ueda, K., "Financial Deregulation and the Demand for Money in Japan," Conference on Monetary Aggregates and Financial System Behavior in Interdependent Economics, Washington, DC: Board of Governors of the Federal Reserve System (forthcoming), 1988.

———, "Financial Deregulation and the Demand for Money in Japan," Faculty of Economics, Osaka University, mimeo, 1989.

Yoshida, T., and R.H. Rasche, "The M2 Demand in Japan: Shifted and Unstable?," *Monetary and Economic Studies* 8-2, Institute for Monerary and Economic Studies, Bank of Japan, September 1990.