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Bank Failures and Optimal Bank Audit Policy

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This paper deals with the issue of efficient and effective banking policy, and focuses upon the determination of optimal audit policies under two different assumptions about the nature of information obtained through bank audits. In Part I of the paper, it is assumed that once an audit is taken, the quality of each bank as of the audit date is perfectly known to the authority. In Part II, an alternative assumption is made that information obtained through audits contains noise so that the authority never knows the true quality of each bank for sure no matter how many audits are taken. Decision rules for the authority's choice of optimal audit policies are derived, in analytical and numerical ways respectively.

Introduction

This paper is motivated by the recent increase in bank failures in the United States. It is instructive, therefore, to take a brief look at the reality of bank failures, as well as institutional elements behind them, to obtain some background knowledge.

Since the serious financial disorder associated with the Great Depression subsided in the mid 1930s, the banking industry in the United States enjoyed a fairly stable period of half a century. Especially from 1943 through 1974, bank failures per year were at a negligible, single digit level. Although the number slightly increased in the seven years following 1974, they did not stand out as a significant deviation from the preceding thirty years. The picture was roughly the same during this period in terms of the size of bank failures (Table 1).[1,2]

However, this stability was apparently lost in the 1980s when the number of bank

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[1]One notable exception is 1974, when only four banks failed, but the total asset size of failed banks was much larger than ordinary years due to the failure of Franklin National Bank of New York (asset size: $1.4 billion).

[2]For the histrorical description of bank failures in the U.S. from the early nineteenth century up to the 1970s, see Sinkey (1979).

Table 1. Bank Failures and the Deposits of Failed Banks, 1934–1987

	Banks	Deposits		Banks	Deposits
1934	61	37	1961	9	11
1935	32	14	1962	3	4
1936	72	28	1963	2	23
1937	84	34	1964	8	24
1938	81	61	1965	9	45
1939	72	160	1966	8	106
1940	48	143	1967	4	11
1941	17	30	1968	3	23
1942	23	20	1969	9	40
1943	5	13	1970	8	55
1944	2	2	1971	6	132
1945	1	7	1972	3	100
1946	2	0	1973	6	971
1947	6	7	1974	4	1,576
1948	3	11	1975	14	341
1949	9	9	1976	17	866
1950	5	6	1977	6	205
1951	5	6	1978	7	854
1952	4	3	1979	10	111
1953	5	45	1980	10	216
1954	4	3	1981	10	3,826
1955	5	12	1982	42	9,908
1956	3	12	1983	48	5,442
1957	3	13	1984	79	2,883
1958	9	10	1985	120	8,059
1959	3	3	1986	138	6,471
1960	2	8	1987	183	n.a.

Note: The deposits are in $ million.
Source: FDIC (1987, p.53).

failures started to rise, broke the post-Great-Depression record in 1985, and kept increasing to 183 failures in 1987. Not only the number, but also the size of troubled banks, sometimes became staggeringly large. Bailout operations carried out by the monetary authority included the nation's biggest banks such as Continental Illinois National Bank in 1984 (asset size: approximately $40 billion) and First Republic Bank in 1988 (asset size: approximately $20 billion). A similar phenomenon has been observed in the thrift industry, where the pace at which savings and loan associations with serious financial problems disappear through mergers accelerated in the 1980s.

These developments attracted attention of many observers of the U.S. economy. Because the majority of the failed banks were located in the states whose economy is dependent on agriculture or oil production, it is customary to attribute the surge in failures in the 1980s primarily to the poor performance of these two sectors spilled over to financial intermediaries in the form of nonperforming loans. There are other views, however, which emphasize structural rather than cyclical factors. For example, many economists have criticized the current deposit insurance system (to be outlined below) as encouraging excessive risk taking by banks.[3] The process of financial deregulation in the 1980s, such as the abolition of Regulation Q (ceilings upon deposit rates), is likely to aggravate the distortive impact of deposit insurance, which may have resulted in increased bank failures.

Whether or not it is a cause of failures, the deposit insurance system is an important factor in the analysis of post-war financial intermediation. First established in 1934, it has been quite successful in eliminating contagious runs and panics characteristic of prewar financial crises. The system was once praised by Friedman and Schwartz (1963) as "the structural change most conducive to monetary stability since state banknote issues were taxed out of existence immediately after the Civil War" (p.434). It is rather ironical that the system is now criticized by some economists as a major source of instability in the banking industry in the 1980s.

Deposit insurance is provided by different government agencies for different categories of intermediaries. For example, Federal Deposit Insurance Corporation (FDIC) insures bank deposits, while Federal Savings and Loan Insurance Corporation (FSLIC) covers thrift institutions. The current insurance system charges the insurance premium of 1/12% of total deposits every year, and insures deposits up to $100,000 per person per bank. However, the effective coverage of the insurance is larger than this statutory limit if a method of handling failed banks called "purchase and assumption" is used. In this method, all the claims against the failed bank, regardless of their size, are transferred to a healthy bank together with the performing part of the failed bank's assets. This method in fact has been applied to the majority of actual failure cases.

The current insurance system may be called "flat-rate" insurance, because the premium does not reflect the level of risk each bank is taking. The opposite concept of the flat-rate insurance is the "risk-related" insurance, a system preferred by most economists as being less distortive. In spite of suggestions repeatedly made by economists to make the current system more risk-related, insurance agencies have been reluctant to take such a step citing as the reason the difficulty of assessing the riskiness of individual banks accurately.[4]

In exchange for the insurance obligations, the monetary authority (which includes insurance agencies as a part) is endowed with power which enables it to keep track of,

[3]See Kane (1985) and citations therein. Kane's book is a recent, and perhaps the most extensive, example of criticisms along this line.

[4]See FDIC (1983) and Federal Home Loan Bank Board (1983) for the agencies' view about risk-related insurance and other possible measures of reform.

and influence if necessary, day-to-day business operations of insured intermediaries. The authority receives from insured intermediaries detailed financial reports called Call Report quarterly, conducts audits to obtain first-hand information, forces insured intermediaries to take actions which the authority thinks are appropriate (such as an increase in the level of equity capital or replacement of incompetent management), and in extreme cases revokes the charter of insured intermediaries, which results in their failure. With these policy tools, the authority can control the level of its insurance loss, at least to a certain extent, without explicitly introducing risk-related insurance premium.

Due to the recent bank and thrift problems, however, insurance funds are being drained away rather quickly. FSLIC became *de facto* bankrupt, and a rescue package was approved by Congress in the fall of 1987 which involved massive (but yet insufficient) infusion of fresh funds.[5] FDIC is in better financial condition, but its premium income has been below the insurance losses and expenses three years in a row starting from 1984.[6] Therefore, the necessity to use the policy tools described above efficiently and effectively to control insurance loss is more important today than ever before.

This paper deals precisely with the issue of efficient and effective banking policy implemented by the monetary authority. We focus upon the determination of optimal audit frequency. The paper is divided into two parts according to the difference in assumptions made about the nature of information obtained through audits.

In Part I, it is assumed that once an audit is taken, the quality of each bank as of the audit date is perfectly known to the authority. The authority chooses either Poisson or deterministic audit policy, and determines under each type of policy the frequency of audits which maximizes the deposit rate each depositor can earn net of fair insurance premium. The main issue here is how to balance the costs of auditing too frequently (increased audit costs) against the costs of auditing too infrequently (increased deposit insurance payments). A simple decision rule specifying the condition under which the authority should choose one or the other type of audit policy is derived analytically.

In Part II, an alternative assumption is made that information obtained through audits contains noise so that the authority never knows the true quality of each bank for sure no matter how many audits are taken. In this case, the authority not only has to determine audit frequency, but also faces a non-trivial decision problem of whether or not to close a bank after each audit. Bayes rule is used to update the authority's belief about unobservable quality of the bank, and its policy choice is given as a function of the

[5]Congress authorized FSLIC to borrow $10.8 billion in new bonds, which are to be repaid by increased insurance premium income. However, (i) it is estimated that the amount is insufficient by $10 billion to $25 billion to handle more than 500 S&Ls that are, or likely to be, in trouble, and (ii) FSLIC is said to be overestimating its insurance premium income by assuming a very high rate of increase in S&L deposits. Therefore, it is not clear whether the rescue package can achieve its objective without taxpayers bearing a subsitantial burden in the end.

[6]See FDIC (1987). Since the agency has other sources of income such as interest income earned on assets accumulated in the past, net income is still positive as of 1986.

updated subjective probability. The optimal policy is derived numerically by solving a stochastic dynamic programming problem.

Part I

The subject of Part I of this paper, most generally stated, is the analysis of the length of contract written by a principal and an agent when there is asymmetric information between them. We explicitly designate the principal as the monetary authority and the agent as a banker, with the authority representing the true principals (depositors) behind the scene. This specific setting is chosen because our main interest is in the optimal audit policy of the monetary authority. However, the framework presented in this part is applicable to a more general principal-agent relationship involving fund management by the agent. The outline of the agency relationship implicit in this part is the following.

An agent manages the fund of his principal, and receives a fixed wage income. The rate of return the agent earns on the trusted fund can deteriorate permanently at any moment, and the deterioration is not costlessly observable by the principal. Therefore, the principal has to pay audit costs and check the agent's investment portfolio once in a while to see if the deterioration has already occurred. If it has, he terminates the agency contract immediately, and if it has not, the contract is renewed. Hence, the audit interval determines the contract length.

When the principal chooses the audit interval, he has to balance the cost of auditing too frequently (increased audit costs) and the cost of auditing too infrequently (increased loss of yields earned on his fund). Also, because the agent observes the rate of return deterioration costlessly, the principal may be able to design a mechanism by which the agent is induced to disclose his private information voluntarily. Such a disclosure mechanism is compared with auditing mechanisms without disclosure to determine the optimal choice of the principal.

Having stated the general form of our problem, we now discuss its more specific characteristics by relating them to the existing literature. There are three lines of literature which motivated our paper. They are (i) the pricing of the monetary authority's deposit insurance liability (Merton 1977, 1978), (ii) asymmetric information and costly state verification (Townsend 1979; Gale and Hellwig 1985), and (iii) the determination of the optimal contract length (Dye 1985; Harris and Holmstrom 1987).

Merton has presented a model of the monetary authority's contingent liability pricing, in which he assumes the authority audits a bank either at a certain deterministic date (his 1977 paper) or at a stochastic date governed by a Poisson arrival parameter (his 1978 paper). In spite of the fact that the audit date and the Poisson parameter are of crucial importance for the price of the authority's liability, no explanation is given about how they are determined. In particular, it is not clear how these two types of audit policy are related with each other.

The model in this part gives an answer to this question. The authority in our model chooses audit policy which maximizes the rate of return depositors can earn net of fair insurance premium. In doing so, the authority has two choices; either to implement Poisson audit policy or deterministic audit policy. The authority compares the net rate of return on deposits under these two choices, and picks the one which gives a higher net deposit rate. A simple decision rule is derived for the authority's choice, as well as the optimal Poisson parameter, the optimal deterministic audit interval, and the level of fair insurance premium to be charged under each type of policy.[7]

As a representative of the literature on asymmetric information and costly state verification, consider the work by Gale and Hellwig (1985). They prove that if an outsider who finances an insider's investment project cannot costlessly observe the outcome of the project, the debt contract is optimal in the sense that it minimizes the deadweight audit costs. In other words, they show that an outsider-insider financial relationship necessarily becomes a lender-borrower relationship. Although the monetary authority does not make loans to banks except for a small amount of central bank loans which are out of considerations of our paper, actual lenders (i.e., depositors) are protected by deposit insurance provided by the authority. Because of this, the authority bears some of the important attributes of actual lenders. Therefore, the relationship between the authority and a banker can be thought of as a lender-borrower relationship without stretching our imagination too much.

Unfortunately, Gale and Hellwig limit their arguments to a one-period model, because the mechanism which they claim will save the deadweight audit costs does not survive an extension of the model to multi-period setting. This fact significantly reduces the model's relevance for understanding more realistic, long-lasting lender-borrower relationship. Nonetheless, we find their basic idea that an optimal contract minimizes the state verification cost quite persuasive and attractive. One way which we propose to save the deadweight audit costs in a multi-period setting (and thus not discussed by Gale and Hellwig) is the adjustment of the date of maturity. Our model can be interpreted as assuming that audits by the lender take place at every maturity date whether or not the borrower defaults on his debt (i.e., the Gale-Hellwig cost saving mechanism does not exist). However, audit costs can still be saved over time by lengthening the loan maturity and reducing the frequency of audits. At the same time, longer-term loans may create a larger loss for the lender once their default occurs. Therefore, the optimal maturity is determined balancing the two effects against each other.

Finally, the comparison of our paper with the literature on the optimal contract length (in particular, Harris and Holmstrom (1987) which is formulated in terms of the

[7]Note, however, that because of several assumptions we make for tractability, such as non-existence of equity capital and the deterministic bank asset return, our model is in fact quite different from Merton's. Therefore, our results are self-contained, and cannot be plugged into Merton's pricing formula.

lender-borrower relationship) is the following. There are two major differences between the Harris-Holmstrom approach and ours; (i) they incorporate Bayesian inference into their model, whereas we assume there is no interim information between audits which can be utilized to update priors; (ii) our framework includes not only deterministic contract length (which is the only type of contract length Harris and Holmstrom consider), but also stochastic contract length.

First, although Bayesian inference is probably an important ingredient of reality, it is not necessary for an optimal contract length problem to be well-defined. Because of this additional complication introduced to their framework, Harris and Holmstrom cannot go much further than the numerical approximation of the optimal recontract intervals. Thus, the informativeness of their result is rather limited. On the other hand, by removing all kinds of inference problems from our model for simplicity, we succeed in obtaining closed-form solutions for the optimal audit interval as well as the optimal Poisson arrival parameter. The nature of these solutions can be studied easily by means of straightforward comparative statics.

Second, stochastic contract length is a promising extension of the optimal contract length literature, because it presupposes the existence of a principal-agent problem which is an important ingredient of contract relationship. If the principal can punish the agent's action cheaply, the former may be able to use this mechanism to reduce the frequency of recontracting which typically involves costly state verification. To be more specific, if the principal uses stochastic audit policy and punishes the agent if the latter is caught hiding some important information, this strategy may induce the agent to disclose the private information voluntarily before a surprise audit takes place.

Summarizing the discussions above, we state the following two possible contributions of Part I of this paper. First, our result provides a certain normative guideline for the monetary authority to follow, though its usefulness is limited by rather strong assumptions we need to derive our solutions. Second, on the positive side, it presents a simple example which illustrates how a term of contract between a principal and an agent (maturity, or more generally, the length of contract) is determined when there is asymmetric information between them.

In what follows, we specify the assumptions of our model in section I–I. Section I–II solves a representative banker's optimization problem under the condition that the authority's audit occurs as Poisson events. From that solution, we derive the range of the Poisson audit parameter consistent with the banker's disclosure of insider information. In section I–III, we calculate the level of fair insurance premium for a given Poisson audit parameter assuming the disclosure. Section I–IV derives the level of fair insurance premium when the authority chooses deterministic audit policy instead of Poisson policy. We then compare the two levels of fair insurance premium, and show the decision rule of the authority about which of the two policies to choose. Section I–V concludes Part I of this paper.

I–I. Assumptions of the Model

There are two types of people, "bankers" and "depositors," and the monetary authority in our economy. All bankers are homogeneous and risk-neutral, each endowed with a project which requires initial investment normalized to one. They have no wealth of their own, so they have to rely completely on outside finance (deposits) to fund their projects. Each project yields a rate of return i at least at first. The yield is reinvested continuously on the same project; namely, we let the project size grow over time just enough to allow the reinvestment of the yield, and no more, for simplicity.

In addition to the rate of return i, the project yields a constant wage income to bankers. Bankers have a special skill in fund management, which allows them to earn a wage rate higher than the one prevailing in the competitive labor market. We express the difference in the wage rates by w. Once a bank fails and a banker loses his job, he loses the opportunity to earn the wage differential w forever.

The market for bank deposits is competitive, which makes bankers to offer the deposit rate equal to their investment return i to depositors. Depositors can come in and out of a bank anytime changing the identity of the depositor group of the bank over time. However, we impose a condition that the net deposit flow is zero to avoid unnecessary complications; namely, if some amounts of deposits are withdrawn, they are offset by a deposit inflow of the same amounts at the same moment.

As time goes by, the rate of return i on the investment project of a banker permanently deteriorates to i^* ($<i$) at any moment with positive probability density. In other words, a "good" bank can suddenly turn into a "bad" bank. We specify the probability as exponential with its density $\lambda \exp(-\lambda t)$, where λ is a positive constant and t denotes time. The risk of the rate deterioration is assumed to be uncorrelated across bankers. The occurrence of the deterioration is immediately known to the banker himself at no cost, but cannot be observed costlessly by depositors and the monetary authority.

We also allow the possibility that the value of bank assets drops suddenly and discretely. It is unnecessary to specify the stochastic process for the value drops as long as it satisfies the following three conditions; (i) the probability density for the event that the rate of return deterioration and a discrete drop in the asset value occur simultaneously is strictly positive at any moment in time; (ii) once a drop occurs, the asset value (say, A_t) declines to ηA_t in an instant, where η is a random variable distributed over [0, 1] according to an arbitrary continuous distribution; (iii) the probability that a discrete drop occurs in any finite interval is so small that the effect of the drop can be completely ignored in the calculation of fair insurance premium.[8] An example of stochastic processes

[8]The third condition is unnecessary for our discussion, but is included because it helps us focus upon the rate of return deterioration in which we are most interested. Although discrete jumps have only a negligible impact upon the level of fair insurance premium, they make a significant difference in the way the monetary authority's audit policy is carried out. We will see this later in section I–II.

satisfying the three conditions given above is a Poisson process with parameter ε, where ε is arbitrarily small, and η distributed uniformly over [0, 1].

Because of the small size of wealth individual depositors hold, it is prohibitively costly for them to conduct audits independently and separately to detect the rate of return deterioration of their bank. They thus delegate the task to some outside organization (the monetary authority in our case). Further, they are assumed to be risk averse, so they are strictly better off if actuarially fair deposit insurance is available. Providing such insurance is feasible for the monetary authority, or for any private insurance company for that matter, since the risk of the rate of return deterioration is perfectly diversifiable by assumption.

The monetary authority, acting on behalf of depositors, attempts to maximize the deposit rate which the depositors of each bank can earn net of fair insurance premium. This can be done in two ways; to implement either stochastic (Poisson) audit policy, or deterministic audit policy.[9] The nature of each policy is further specified in the later sections. The authority continuously collects insurance premium from depositors at a constant rate p per unit of deposits.

If the authority conducts an audit, it obtains perfect information about the value of bank assets as well as the rate of return on the assets.[10] If the rate is found to have deteriorated, the authority immediately closes the bank and pays off the difference between the promised amount of deposits and the actual value of assets. If it is not, the bank remains in business and the insurance commitment of the authority is renewed. While it is never optimal for the authority to keep a "bad" bank open, we assume that closing a "good" bank is sufficiently costly (perhaps because of ensuing litigation costs) so that the authority never closes any bank without auditing.

Having set out the assumptions of our model, we now turn to the optimization problem of our representative banker under the Poisson audit policy, and the determination of the Poisson parameter range which induces the disclosure of the insider information held by the banker.

I–II. The Banker's Problem

We analyze the representative banker's choice when the monetary authority imple-

[9]The choice of stochastic audit policy for the authority is not limited to Poisson policy, and the form of the solution to our problem is greatly affected by the specific probability distribution to be used. It is beyond the scope of our paper to determine which distribution of the audit arrival is optimal among all distributions.

[10]If the rate of return is not directly observable, the authority, having observed negative net worth of a bank, cannot determine whether it is the result of the rate of return deterioration or that of a discrete jump in the asset value. It faces an inference problem, although the problem is trivial in our particular case because the probability of a discrete jump is assumed to be arbitrarily small. However, as we mentioned in the introduction for Part I, we avoid any type of inference problems in this part, and for that purpose assume explicitly the observability of the rate of return. This assumption is removed in Part II.

ments Poisson policy which has the following characteristics. Audits take place as Poisson events with parameter μ. The rate of return deterioration and audits are distributed independently. μ is chosen by the authority, announced publicly, and strictly observed. At the same time, the authority introduces an amnesty rule to induce the disclosure of the insider information held by bankers. Namely, if a banker voluntarily and immediately notifies the authority the deterioration of his rate of return, there is no penalty imposed on him. On the other hand, if an audit is taken strictly before the confession, and the deterioration of the rate of return is found, the authority knows for sure that the banker failed to notify the fact immediately. Then, a penalty whose pecuniary value is denoted by X is imposed on the banker costlessly. The size of X is a finite, exogenous constant.[11]

Although the policy requires immediate confession, the authority cannot tell whether particular confession is immediate or not unless an audit follows the rate deterioration and precedes the confession. If a banker hides his rate of return deterioration for a while and then confesses before the next audit, the authority is unable to determine from the *ex post* observation of the asset value whether or not and how long the crucial information has been kept secret by the banker, because there is small but non-zero probability over any time interval that the asset value jumps discretely, and the downward jump can be of any size.

This fact implies that the disclosure mechanism never works under the deterministic audit policy. Bankers have an incentive to keep their banks open to maximize their wage income, so long as they can escape the penalty. Therefore, a banker who experiences the rate of return deterioration always discloses the fact a moment before the known audit date, and claims that the discrete jump in the asset value and the rate of return deterioration just occurred simultaneously. Although such a claim is most likely a lie, the authority does not have evidence to falsify it. If the criminal law requires that for a suspect to be punished, the punisher be absolutely sure that the suspect committed a crime, the banker in our example cannot be punished because the probability density of the simultaneous occurrence of the two events is strictly positive (though arbitrarily small).[12]

A question we now ask is whether the Poisson audit policy combined with the

[11]If punishment can be controlled by the authority over some finite range, it will be set at its upper bound (this will be shown in the following section). Therefore, we can set X equal to the upper bound of the range without loss of generality. If X can be made infinitely large, the deadweight cost of auditing can be reduced arbitrarily close to zero. We exclude this possibility by assumption.

[12]Another possible criterion for punishability is the negative net worth. In this case, all the unlucky innocent who truly experience the simultaneous occurrence of the rate of return deterioration and a discrete asset value jump will be punished, while some (or all) of those who might attempt to take advantage of the amnesty rule under the deterministic audit policy as described in the text will be discouraged to do so.

If the determination of the criminal law is a part of our problem, the negative net worth criterion is clearly better than the punishment criterion given in the text, as long as it is assumed that discrete jumps in the asset value very rarely occur. However, without this assumption which is made solely to simplify our framework, it is not obvious which of these criteria is better.

amnesty rule induces voluntary disclosure. The answer is given as follows. Normalize the moment the deterioration of a banker's rate of return occurs to zero. The banker, realizing the deterioration, chooses the moment to disclose this fact to the authority. Denoting by A(T) the expected discounted present value (hereafter referred to as "discounted value") of his wealth conditional on the occurrence of the rate deterioration at time zero and the disclosure of the fact at time T, we have the following expression.

$$A(T) = -\int_0^T Xe^{-\tau t}\,\mu e^{-\mu t}dt + \int_0^T\left[\int_0^t we^{-\tau t'}dt'\right]\mu e^{-\mu t}dt + \int_0^T we^{-\tau t}dt \cdot \int_T^\infty \mu e^{-\mu t}dt \tag{1}$$

The first term in equation (1) is the expected penalty in the event that an audit takes place by time T, i.e., before the confession. τ is the discount rate of the banker. The second term is the expected wage income earned in the same event as above up to the moment of the audit. Finally, the third term is the expected wage income earned in the event that the audit does not take place by time T. Taking a derivative of equation (1) with respect to T, we obtain;

$$A'(T) = e^{-(\tau+\mu)T}(w - \mu X). \tag{2}$$

Equation (2) shows that the optimal disclosure time for the banker is given as two corner solutions.[13] If $w \leq \mu X$, the disclosure takes place on the moment the deterioration occurs. We assume that when the banker is indifferent between notifying and not notifying the truth, he chooses the former. If, on the other hand, $w > \mu X$, the banker never discloses the fact until it is found out in an audit.

For given values of w and X, the authority has to choose μ larger than or equal to w/X to secure the disclosure by the banker. If the authority is not willing to set μ this high, the result is the Poisson policy without disclosure. It can be shown that this policy is dominated by the deterministic audit policy.[14] Therefore, we only need to compare the Poisson policy with disclosure and the deterministic policy (which is necessarily without disclosure) to determine the authority's optimal choice. In the following two sections, we further analyze each of these two types of audit policy.

I–III. The Poisson Audit Policy

The monetary authority's sole objective is to maximize the net deposit rate the

[13]The fact that two solutions are given at both corners is specific to the exponentially distributed next audit arrival. We will have a very different characterization of the banker's optimal behavior if, for example, the next audit is uniformly distributed.

[14]It can be shown that the minimized level of fair insurance premium under the Poisson policy without disclosure is approximately $\sqrt{2}$ times as large as that under the deterministic audit policy.

depositors can earn. For this purpose, the authority goes through the following procedure. First, it calculates the expected present discount value of the insurance premium it can collect throughout the stochastic lifetime of a bank. Second, it calculates the discounted value of audit costs for given $\mu(\geq w/X)$. Third, the two discounted values are equated and solved for p to obtain the level of fair insurance premium as a function of the Poisson parameter μ. The premium is then minimized with respect to μ, which is equivalent to maximizing the net deposit rate i–p.

Note that because of the voluntary disclosure by the banker, his bank is closed at the moment the rate of return deterioration occurs by his own confession. Thus, the deposit insurance payment is identically equal to zero, and the insurance premium is collected solely to cover audit expenses. Also, notice that audits are just a ceremony. The authority knows that with probability one it will find any bank being audited solvent. Still, it continues to audit to make the whole policy credible.

We first calculate the discounted value of the insurance premium revenue over the lifetime of a bank, denoted by Φ. If the moment when the rate of return deterioration takes place is given by t, Φ is expressed as;

$$\Phi = \int_0^\infty \left[\int_0^t pe^{(i-r)t'}dt'\right]\lambda e^{-\lambda t}dt = \frac{p}{\lambda-(i-r).} \tag{3}$$

Recall that the premium is charged proportionally to the value of deposits at each moment. The latter value is given by $e^{it'}$ for any time t′ in [0, t], normalizing the size of deposits at time zero to unity. The insurance income is discounted back to time zero by r, the rate of return on safe investment to which the authority has an access. r is strictly smaller than i, for otherwise it is optimal for the authority to ban all private banking business and take over the whole deposits in the economy. We also assume $r>i^*$, i.e., the safe rate is larger than the deteriorated level of investment return, to avoid similar perverse implications. Finally, expected value is taken with respect to the moment of the rate deterioration, t, over $[0, \infty]$. For the discounted value of the premium income to be positive and finite, it is necessary that $i-r-\lambda<0$, which we assume throughout Part I.

Next, the discounted value of audit costs per bank at time zero, denoted by $\Omega_0(\mu)$, can be expressed as follows.

$$\Omega_0(\mu) = \int_0^\infty e^{-rt}\left[ce^{it} + \Omega_t(\mu)\right]e^{-\lambda t}\,\mu e^{-\mu t}\,dt \tag{4}$$

Equation (4) is interpreted as follows. Given the moment t of the next audit, the audit cost is zero if the rate of return deterioration takes place before t, and the term drops out of equation (4). If, on the other hand, the deterioration does not take place by t, which is an event that occurs with probability $e^{-\lambda t}$, the authority expends the cost for the first audit and acquires with probability one the discounted value of audit costs evaluated at time t, or $\Omega_t(\mu)$. We assume that the audit costs are a constant fraction c of the value of deposits at the moment each audit is taken.[15] The whole expression is discounted back to time zero at rate r. Finally, expected value is taken with respect to the stochastic next audit date t over [0, ∞].

Note that because of the memoryless property of the exponential and Poisson distributions, and the assumption that the audit costs are proportional to the value of deposits, $\Omega_t(\mu)$ is equal to $\Omega_0(\mu)e^{it}$ as long as exogenous parameters are expected to be the same at time t, where the multiplication by e^{it} reflects the growth of deposits over [0, t].[16] Using this fact, we can solve equation (4) for $\Omega_0(\mu)$ to get;

$$\Omega_0(\mu) = \frac{i-r-\lambda-\mu}{i-r-\lambda} c\mu \int_0^H e^{(i-r-\lambda-\mu)t}\, dt = \frac{c\mu}{\lambda-(i-r).} \tag{5}$$

Equation (5) shows that the expected audit costs are increasing in c and μ, which is quite natural; decreasing in λ, because larger λ implies the bank is likely to fail earlier (i.e., before it is audited many times); and increasing in $i-r$, because a larger value of $i-r$ means the present value of future audit costs is larger. Equating (3) and (5) and solving for p, we obtain the level of fair insurance premium.

$$p = c\mu \tag{6}$$

Because of the assumed independence of the rate of return deterioration process and the Poisson audit process, the fair insurance premium is not affected by λ. From equation (6), it is clear that minimizing p is equivalent to minimizing μ. Thus, μ=w/X is the authority's best choice of μ consistent with the banker's voluntary disclosure. The validity of our former claim that if X is controllable over a range it will be set at its upper bound is obvious from this result.

I–IV. Deterministic Audit Policy

In this section, we analyze the deterministic audit policy. As we have seen in section I–II, bankers have no incentive to disclose their insider information more than a moment before the next audit date, which essentially means the voluntary disclosure mechanism does not work at all under the deterministic audit policy. As the result, the expected insurance loss of the authority is no longer zero.

As in the previous section, for a given audit interval T, the authority calculates the expected present discount value of insurance premium collected over the stochastic lifetime of a bank, equates it with the discounted value of its deposit insurance costs and audit costs arising from the bank, and solves it for the rate of fair insurance premium, p.

[15]In reality, it is likely that there are increasing returns to scale in auditing technology. This fact can be roughly captured by taking c to be smaller for banks with large assets.

[16]We can think of this formulation as degenerate dynamic programming in which the value function takes only one value.

The solution is then minimized with respect to the audit interval T, which is equivalent to maximizing the net deposit rate i–p depositors can earn.

First, we calculate the discounted value of the authority's insurance premium income at time zero, denoted by $\Phi_0(T)$. We limit our attention to the class of policy in which audits are equally spaced in time, which turns out to be the class containing the optimal policy in our particular setting. Namely, there is only one value for T as long as exogenous parameters such as i, i*, and λ are expected to stay constant, and audits take place at time T, 2T, 3T, and so on with time zero being the day when a bank begins its business. $\Phi_0(T)$ can then be expressed as follows.

$$\Phi_0(T) = e^{-rT}\,\Phi_T(T)\,e^{-\lambda T} + \int_0^T p\,e^{(i-r)T}\,dt \tag{7}$$

Equation (7) is interpreted as follows. First, with probability $e^{-\lambda T}$ the failure of the bank does not occur by the first audit date, and the authority obtains the discounted value of premium income evaluated at time T, or $\Phi_T(T)$ (the first term in equation (7)). Second, the authority obtains the premium income up to time T with probability one (the second term in equation (7)). As before, both terms are discounted back to time zero at rate r. By the same argument as the one used in the previous section, we can express $\Phi_T(T)$ as $\Phi_0(T)e^{iT}$, which allows us to solve equation (7) for $\Phi_0(T)$ as;

$$\Phi_0(T) = \frac{p(e^{(i-r)T}-1)}{(i-r)(1-e^{(i-r-\lambda)T})} \tag{8}$$

as long as $i-r-\lambda<0$.

Next, we analyze the costs born by the monetary authority as the auditor and the deposit insurer. Denote by $\Omega_0(T)$ the discounted value of the authority's costs arising from a bank with deposits equal to one at time zero given that the next audit takes place at time T. $\Omega_0(T)$ is calculated as;

$$\Omega_0(T) = e^{-rT}\left[ce^{iT} + \Omega_T(T)e^{-\lambda T} + \int_0^T (e^{iT}-e^{it+i^*(T-t)})\,\lambda e^{-\lambda t}dt\right]. \tag{9}$$

Equation (9) is interpreted as follows. The first term in the square bracket is the cost of next audit. As we have noted in the previous section, the audit cost is assumed proportional to the value of deposits when the audit is taken. If the audit shows that the bank's rate of return has not declined, the probability of which event is given by $e^{-\lambda T}$, the authority renews its insurance commitment and obtains $\Omega_T(T)$. Hence, the second term in the square bracket is the value of the commitment renewal. The third term is the expected value of deposit insurance payments when the bank is found to be "bad." While e^{iT} in the integrand represents the promised value of deposits at time T, $e^{it+i^*(T-t)}$ is the realized value of assets in the event that the rate deterioration takes place at time t. Expected value is taken with respect to t, which is distributed over the interval [0, T].

Finally, the whole expression is discounted back to time zero with the safe rate r.

Again, we replace $\Omega_T(T)$ by $\Omega_0(T)e^{iT}$ and solve equation (9) for $\Omega_0(T)$, which yields the following expression.[17]

$$\Omega_0(T) = \frac{1}{(1-e^{(i-r-\lambda)T})}\Big[ce^{(i-r)T} + e^{(i-r)T}(1 - e^{-\lambda T}) - \frac{\lambda}{i-i^*-\lambda}(e^{(i-r-\lambda)T} - e^{(i^*-r)T})\Big] \tag{10}$$

Equating (8) and (10), we obtain the expression for the fair insurance premium p as a function of audit interval T.

$$p(T) = \frac{i-r}{e^{(i-r)T}-1}\Big[ce^{(i-r)T}+e^{(i-r)T}(1 - e^{-\lambda T}) - \frac{\lambda}{i-i^*-\lambda}(e^{(i-r-\lambda)T} - e^{(i^*-r)T})\Big] \tag{11}$$

Inspection of equation (11) reveals the following two facts. First, if the audit interval T approaches infinity, i.e., if the authority does not audit at all, p(T) approaches $(i-r)(1+c)$. In this case, the net deposit rate depositors can earn, or i–p, is $r-c(i-r)$ which is lower than r. This is clearly suboptimal, because depositors would be able to earn a higher rate of return on their deposits if the authority itself operated a bank. Second, if T approaches zero, the first term in the square bracket blows up, pushing p(T) to infinity. In other words, conducting audits too often is again suboptimal for any $c>0$. We thus seek for some intermediate T which minimizes the value of p(T).

Taking a derivative of equation (11) with respect to T, we obtain the following first order condition;

$$\frac{\partial p(T)}{\partial T} = \frac{i-r}{(e^{(i-r)T}-1)^2}\Big[\frac{\lambda(i-i^*)}{i-i^*-\lambda}(e^{(2i-2r-\lambda)T} - e^{(i+i^*-2r)T}) - (1+c)(i-r)e^{(i-r)T} + \frac{(i-r-\lambda)(i-i^*)}{(i-i^*-\lambda)}e^{(i-r-\lambda)T} + \frac{\lambda(r-i^*)}{i-i^*-\lambda}e^{(i^*-r)T}\Big] = 0. \tag{12}$$

The formula is not solvable analytically for T, and is not very informative as it is. To get a better idea about the behavior of the optimal audit interval when exogenous parameters change, we take a Taylor series expansion around unity of the exponentials in the first order condition up to the second order. This approximation is justified by the fact that exponents appearing in equation (12) are small fractions for practically relevant values of T, λ, and three interest rates. The procedure yields a quadratic equation in T,

[17] To be precise, the following formula is valid only when $i-i^*-\lambda \neq 0$. We omit the analysis of the special case where $i-i^*-\lambda=0$ since it adds nothing to our basic arguments.

the only positive solution of which is the following.[18]

$$T = \frac{Rz + \sqrt{2z - R^2z^2}}{1 - R^2z} \tag{13}$$

where $R = i - r$, and $z = \dfrac{c}{\lambda(r - i^*)}$.

z can be interpreted as the ratio of the cost of auditing and the cost of not auditing. The formula tells us that the authority should look only at this ratio as a measure of cost. As expected, T is increasing in our cost measure z. Further, as z approaches zero, T also approaches zero; in the limit, the authority will be auditing the bank continuously.

The fact that the terms in equation (13) involving R have very little influence upon the optimal value of T for realistic exogenous parameter values motivates further approximation of the solution in which we ignore all such terms. This gives us the following simple expression;

$$T = \sqrt{2Z} = \sqrt{\frac{2c}{\lambda(r - i^*)}}. \tag{14}$$

It is interesting to note that replacing λ with y (the rate of consumption expenditure) and setting $i^*=0$, we obtain the familiar Baumol-Tobin inventory demand for cash formula. This result is reasonable in view of the similarity of the structure of our model with theirs.

Applying the second order approximation to the definition of the fair insurance premium (equation (11)), and substituting the approximate solution for T derived in equation (13), we can express the minimized value of the insurance premium as a function of exogenous parameters as follows;

$$p = \frac{2\lambda c(i - i^*)}{\sqrt{2\lambda c(i - i^*) - R^2c^2}}. \tag{15}$$

By the same reasoning as given above, we may further approximate equation (15) as $p=\sqrt{2\lambda c(i-i^*)}$. As expected, p is increasing in λ, c, and $i-i^*$.

The comparison between the numerical solution for the original first order condition and the approximate solutions given by equations (13) and (14) is made in Table 2 for some hypothetical parameter values. The approximate solutions are always reasonably close to the numerical solution. Also, the minimized fair insurance premium (both numerical and approximate analytical values) is given in Table 3.[19]

[18] The discriminant is positive for all realistically conceivable parameter values.

[19] As a matter of comparison, the average audit interval in reality is about once a year, with some variations across banks reflecting such factors as the size and the asset quality of individual banks. The level of actual deposit insurance premium is 1/12% (i.e., 0.0833%) per annum.

Table 2. The Solution for the Optimal Audit Interval (deterministic audits, in years)

λ	(a)			(b)		
	(i)	(ii)	(iii)	(i)	(ii)	(iii)
0.015	2.62	2.62	2.58	0.82	0.82	0.82
0.020	2.28	2.26	2.24	0.71	0.71	0.71
0.025	2.05	2.02	2.00	0.64	0.63	0.63
0.030	1.87	1.84	1.83	0.58	0.58	0.58
0.035	1.73	1.70	1.69	0.54	0.54	0.53
0.040	1.63	1.59	1.58	0.50	0.50	0.50
0.045	1.53	1.50	1.49	0.48	0.47	0.47
0.050	1.45	1.42	1.41	0.45	0.45	0.45
0.055	1.39	1.36	1.35	0.43	0.43	0.43
0.060	1.33	1.30	1.29	0.41	0.41	0.41
0.065	1.27	1.25	1.24	0.40	0.39	0.39
0.070	1.23	1.20	1.20	0.38	0.38	0.38
0.075	1.19	1.16	1.15	0.37	0.37	0.37
0.080	1.15	1.12	1.12	0.36	0.35	0.35
0.085	1.13	1.09	1.08	0.35	0.34	0.34
0.090	1.09	1.06	1.05	0.34	0.33	0.33
0.095	1.07	1.03	1.03	0.33	0.33	0.32
0.100	1.03	1.01	1.00	0.32	0.32	0.32

Notes: (a) : $c = .001, i - r = .01, r - i^* = .01$
(b) : $c = .0001, i - r = .01, r - i^* = .01$
(i) : numerical solution derived from equation (12)
(ii) : approximate solution derived from equation (13)
(iii): approximate solution derived from equation (14)

Table 3. The Minimized Value of Fair Insurance Premium (deterministic audits, % per annum)

λ	(a)			(b)		
	(i)	(ii)	(iii)	(i)	(ii)	(iii)
0.015	0.0773	0.0775	0.0775	0.0245	0.0245	0.0245
0.020	0.0891	0.0895	0.0894	0.0283	0.0283	0.0283
0.025	0.0995	0.1000	0.1000	0.0316	0.0316	0.0316
0.030	0.1089	0.1096	0.1095	0.0346	0.0346	0.0346
0.035	0.1175	0.1183	0.1183	0.0373	0.0374	0.0374
0.040	0.1255	0.1265	0.1265	0.0399	0.0400	0.0400
0.045	0.1330	0.1342	0.1342	0.0423	0.0424	0.0424
0.050	0.1400	0.1414	0.1414	0.0446	0.0447	0.0447
0.055	0.1468	0.1483	0.1483	0.0468	0.0469	0.0469
0.060	0.1532	0.1549	0.1549	0.0488	0.0490	0.0490
0.065	0.1594	0.1613	0.1613	0.0508	0.0510	0.0510
0.070	0.1653	0.1673	0.1673	0.0527	0.0529	0.0529
0.075	0.1710	0.1732	0.1732	0.0546	0.0548	0.0548
0.080	0.1765	0.1789	0.1789	0.0563	0.0566	0.0566
0.085	0.1819	0.1844	0.1844	0.0581	0.0583	0.0583
0.090	0.1870	0.1897	0.1897	0.0597	0.0600	0.0600
0.095	0.1921	0.1949	0.1949	0.0614	0.0616	0.0616
0.100	0.1970	0.2000	0.2000	0.0629	0.0633	0.0633

Notes: (a) : $c = .001, i - r = .01, r - i^* = .01$
(b) : $c = .0001, i - r = .01, r - i^* = .01$
(i) : numerical solution derived from equation (11)
(ii) : approximate solution derived from equation (15)
(iii): approximate solution derived from $\sqrt{2\lambda c(i - i^*)}$

Having determined the optimal choice of the monetary authority under both Poisson and deterministic audits, we can now compare the relative performance of the two types of policy. This is quite straightforward to do because our criterion of comparison is simply the minimized value of the fair insurance premium. Namely, the Poisson audit policy is strictly preferable to the deterministic policy if and only if the following inequality holds.

$$\sqrt{2\lambda c(i-i^*)} > c\mu \tag{16}$$

The right-hand side of equation (16) is taken from equation (6) of the previous section, while the left-hand side is the approximate solution for p derived above. From equation (16), we obtain the following decision rule of the authority.

Choose the Poisson auditing if and only if

$$\sqrt{\frac{2\lambda(i-i^*)}{c}} > \mu = w/X. \tag{17}$$

This rule is quite reasonable, and can be interpreted as follows. Suppose that equation (17) is holding with equality so that the authority is indifferent between Poisson and deterministic audits. Comparing these two policies, we note that Poisson policy has less insurance costs (in fact, it is zero), while it involves more frequent auditing, and hence higher audit costs. Since the parameters λ and $i-i^*$ are the determinants of insurance costs, their increase does not affect the costliness of the Poisson policy at all, while it raises the fair insurance premium under the deterministic policy. Thus, it breaks the assumed equality of equation (17) in favor of the Poisson policy. A similar argument can be made for changes in the audit cost c, wage differential w, and the punishment X.

I–V. Concluding Remarks

We succeeded in this part in deriving either exact or approximate closed-form solutions for the optimal policy of the monetary authority under the Poisson and deterministic auditing. From these, we obtained a decision rule for the authority to follow in choosing from the two types of auditing methods. The important variables we should look at turned out to be the risk of failure λ, the size of the adverse impact of bank failure $i-i^*$, the cost of auditing c, and the ratio between the wage differential w a banker can earn and the punishment X the authority can inflict on him when he is found to be hiding the deterioration of his rate of return.

Our success in deriving tangible and easily interpretable results depends critically upon our decision to eliminate all kinds of inference problems from our framework. For example, adding a stochastic component (e.g., a Wiener process) to the rate of return on investment, which is a natural and realistic extension of our framework to be discussed in Part II, introduces a formidable inference problem. Under this condition, having observed a low asset value of a bank in an audit, the authority cannot tell whether the mean rate of return on the bank assets permanently deteriorated (i.e., a failure occurred by our definition) or it is just the result of an unlucky draw of the Wiener process. The difficulty may be reduced if we assume, as we did in Part I, that the rate of return is observable through an audit. However, the plausibility of this assumption is much more questionable when the rate of return has a stochastic component such as a Wiener process.

The optimal audit interval in these circumstances is contingent upon the observed asset value at each audit date. We have no obvious bank-closing rule, and determining it is a part of our problem. The voluntary disclosure mechanism will be difficult to implement even under the Poisson audit policy, because detecting a banker's lie will be harder for the authority. Let us further discuss some of these issues in Part II.

Part II

In Part I of this paper, we solved the monetary authority's problem of determining the optimal bank audit policy. An important simplifying assumption made there was the absence of Bayesian inference. This was achieved by assuming that the rate of return each bank earned on its assets, either i or i^* ($<i$), was perfectly observable at the moment each audit was taken, and that there was no additional information available between audits. With this assumption, the authority was able to eliminate those banks, and only those banks, whose rate of return deteriorated over the past audit interval without mistakes. We argued that this assumption, though unrealistic, had a compensating benefit that it allowed us to obtain a closed-form solution from which we were able to derive several interesting comparative static insights.

The purpose of Part II is to introduce the Bayesian inference process into our previous framework. We assume first that not the rate of return on bank assets, but the value of bank assets, is observable through audits.[20] Second, to give substance to the first assumption, we assume that the rate of return has an additional stochastic component expressed by a Wiener process. The second assumption implies that having conducted audits at two or more points in time and obtained information about the asset value at each point, the authority still cannot tell for sure whether the (unobservable) mean rate of return on bank assets is i or i^*.

The cost of complicating our model by Bayesian inference is the loss of clear-cut results; we have very little to say analytically about our model, much less its exact solution. The best we can hope for is to numerically solve a stochastic dynamic programming problem to be specified in the following section. On the other hand, its benefit is that we can better approximate the actual problem faced by the monetary authority. In particular, unlike in the previous part, the monetary authority not only determines audit frequency, but also faces a non-trivial decision problem of whether or not to close a bank after each audit, due to the fact that the authority never knows for sure the true quality of

[20] The "value" here refers to scrap value. See section II–I below for its more precise meaning.

the bank. This is a potentially useful extension of our previous model, because the bank closure decision is probably a matter of more practical importance for the authority in reality than the determination of audit intervals.

Unfortunately, the introduction of Bayesian inference eliminates many of the basic distinctions between our model and that of Harris and Holmstrom (1987). One of the major remaining differences is the way the Bayesian updating takes place. In our model, information used for the updating comes exclusively through audits. By contrast, they assume that the lender uses successful repayment by the borrower of his debt each period as a source of interim information between audits, while audits, once they are taken, completely reveal the true quality of the borrower. Our model is more robust than theirs when the borrower can conceal his poor project outcome by borrowing from the third party, which nullifies the information content of repayment in each period.

In what follows, we formulate the stochastic dynamic programming problem faced by the monetary authority in section II–I. Section II–II presents the numerical solution of the problem. Section II–III concludes Part II of this paper.

II–I. Formulation of the Model

The model we deal with in Part II is similar to the machine replacement problem often discussed in textbooks of dynamic programming. In its typical form, the problem is stated as follows.

A machine is used to produce goods every period. It can be in either of two states, good or bad. If it is in good (bad) state, output produced by it is of good (bad) quality. A good machine can become bad in any period with some probability. Once it becomes bad, it stays bad permanently until it is replaced. There are three types of costs associated with this machine. They are output costs (i.e., costs arising from poor quality output), inspection costs, and machine replacement costs. Output costs provide the controller of the system with an incentive to check the state of the machine and replace it if it is found to be bad. Inspection costs, however, prevent him from checking the machine too frequently. Machine replacement costs also affect his decision, especially when inspection does not reveal the state of the machine completely and thus the controller can make mistakes. The higher the replacement costs are, the more expensive are the mistakes which lead to replacement of the machine when it is in fact good.

We now translate this framework into a bank audit model. There are bankers, depositors, and the monetary authority in our economy. A representative banker is endowed with bank charter which can be revoked any time at the monetary authority's will, some exogenous amounts of initial deposits, and a skill to find an opportunity to make a loan for an investment project. We assume that he relies completely on deposits to fund his loan, and that he holds only one loan in his bank's asset portfolio at any moment in time. The latter assumption is made because diversification at the level of individual banks does not add anything to our analysis.

Each project requires input of investment goods, and yields a stochastic real rate of return expressed as the sum of the mean rate and fluctuations around it. The yield on the project is reinvested continuously on the same project. We assume for simplicity that there is no net inflow or outflow of deposits once a bank begins its operation. Thus, the reinvestment of the yield, whether it is positive or negative, is the only source of changes in the value of the project and bank assets.

If the amount of goods invested in the project at some point in time is given as A(0), the (scrap) value of the project after time τ has passed, $A(\tau)$, is assumed to be given by the following stochastic process;

$$A(\tau) = A(0) \exp\left[\left(\iota - \frac{\sigma^2}{2} \right) \tau + \sigma Z(\tau) \right] \tag{18}$$

where ι is the mean rate of return, $Z(\tau)$ a standard Wiener process, and σ an exogenous constant representing *instantaneous standard deviation of stochastic fluctuations around the mean*. If a bank is closed at time τ, its loan is recalled and the project financed by it is terminated immediately. Then, the amount $A(\tau)$ of investment goods are left and are divided among the claim holders against the bank.

The mean rate of return, ι, is itself a random variable which takes the value of either i or i* ($<$i). If ι is i, it can deteriorate to i* at any moment according to exponential distribution with parameter λ. Once ι is i*, it remains i* forever.[21] Besides the investment projects described above, there are safe assets in this economy, whose rate of return r satisfies $i>r>i^*$. We take the safe rate to be the rate the monetary authority uses to discount future costs and earnings.

The bank deposits are insured by the monetary authority and are perfectly safe. The existence of deposit insurance transfers the burden of bank auditing from individual depositors to the authority, eliminating the possibility of redundancy in information production. We assume that the authority can pool many small banks in the economy and diversify away the insurance risk.

If the authority takes an audit, it obtains perfect information about the value of bank assets at the moment the audit is taken. We denote by t_j the length of time between the j th and the j+1 st audits, and by τ_j the moment at which the j th audit is taken. Then, we have;

$$\tau_j = \sum_{k=0}^{j-1} t_k \tag{19}$$

[21]If the banker finds out the deterioration of the mean rate of return on his current investment, he may try to find some other investment project which yields the mean rate i and shift his loan to it. As long as he is successful in his search for a good new project, the mean rate does not deteriorate. Therefore, we can think of the exponential distribution mentioned in the text as describing the banker's loss of skill to find good investment projects one after another.

and $\tau_0=0$. Using this notation, our assumption is that the authority learns $A(\tau_j)$ through the j th audit.[22] For expositional convenience, the expression A_j is used for $A(\tau_j)$ in what follows. The same rule applies to variables other than A.

Because there are two stochastic elements (i.e., ι and Z), knowledge of the bank asset value at two or more points in time is not enough for the authority to determine whether the deterioration of the mean rate of return from i to i* has taken place. The authority has prior subjective probability p_0 for each bank that its mean rate of return is i at time 0, and updates it to p_j, $j=1,2,\ldots$, by the Bayes rule utilizing asset value information obtained through successive audits.

Given the latest information about the asset value of a bank and the updated probability, the authority determines whether or not to close the bank, and if it chooses not to close the bank, when to take the next audit. We express the authority's policy choice by μ (or μ_j if it is time-dependent), which is a mapping from the sequence of state variables observed up to the time the choice is made, say τ_j, to the two-dimensional space of control variables, (u_j, t_j). u_j takes the value of either one (no closure) or zero (closure),[23] whereas t_j, the audit interval, is a positive real number.[24] We also denote by x_j the pair of state variables, (p_j, A_j).

Suppose that an audit has just been taken, or alternatively, that a new bank begins its operation. We set our time subscript equal to zero, and consider the authority's policy sequence $\pi=(\mu_0, \mu_1, \mu_2, \ldots)$ to be implemented now and into infinite future at each successive audit date.

The authority's objective in our model is, loosely speaking, to get as much gain to the economy as a whole as possible from banking operations (and underlying investment projects), taking into account the cost of auditing and the deadweight loss arising from the eventual termination of the projects.[25] We are not concerned in this paper how this gain is distributed among people in the economy. Our problem is formulated as that of

[22]Note that unlike bankers, the authority in our model does *not* have the skill to judge quality of investment projects. The only skill it has is to measure the amount of goods currently invested in a project, which in itself has nothing to do with the future profitability of the project. This is why the authority has to go through the Bayesian inference process to be discussed below. It is obvious from this observation that the authority cannot take over the role played by bankers in our model economy.

[23]The authority's decision to close a bank in our framework is based on economic efficiency, and has nothing to do with ordinary bank failure criteria such as negative net worth. It is quite possible in our model economy that a bank with large positive (negative) net worth is closed (kept open). Therefore, we use the word "closure" rather than "failure" to mean termination of a bank's operation. One interpretation of our bank closure is the replacement of incompetent bank management, which can be initiated by the authority solely upon the basis of economic efficiency in reality.

[24]The choice of $t_j=0$ would be suboptimal, if it was technically feasible; the authority gains nothing in terms of information gathering because the state does not change at all in an instant, while it incurs non-zero audit costs.

[25]In the spirit of the interpretation given in footnote 21 above, this sentence can be restated as: The authority's objective is to get as much gain to the economy as a whole as possible from the skill of bankers to find good investment projects, taking into account the fact that their skill can deteriorate over time.

cost minimization. Suppose that the authority implements some policy sequence π. For a given pair of current state variables, x_0, the loss function of the authority under this policy sequence, denoted by J_π is expressed in the familiar recursive form;

$$J_\pi(x_0) = E\left[g(x_0, x_1, \mu(x_0)) + \beta_1 J_\pi(x_1) \mid x_0, \pi \right] \tag{20}$$

where β_1 is the discount factor over the period between now and the next audit date, and g( ) is the one-stage cost function. The expectation operator E[ ] is conditional upon the value of the current state variables and the policy sequence which affects the distribution of future state variables through state transition equations to be discussed below. Recursive substitution yields an alternative expression for J_π.

$$J_\pi(x_0) = \sum_{j=0}^{H} E\left[\prod_{k=0}^{j} (\beta_k)\, g(x_j, x_{j+1}, \mu(x_j)) \mid x_0, \pi \right] \tag{21}$$

where $\beta_0=1$. We specialize the discount factor and the one-stage cost function as follows.

$$\beta_j = u_{j-1}\, e^{-rt_{j-1}} \qquad \text{for } j \geq 1 \tag{22}$$

$$g(x_j, x_{j+1}, \mu(x_j)) = (1-u_j)\delta A_j + u_j c A_{j+1} e^{-rt_j} + u_j(A_j - A_{j+1}e^{-rt_j}) \tag{23}$$

Equation (22) implies that the size of the discount factor per audit interval is affected by the choice of control variables. In particular, once the bank closure decision is made ($u_j=0$ for some j), all the one-stage costs afterwards are discounted to zero and become irrelevant, which is obvious because the bank ceases to exist.[26]

Equation (23) is interpreted as follows. The first term on the right-hand side is the cost of closing a bank, assumed to be a fraction δ (≥ 0) of the bank asset value. The second term is the cost of the j+1 st audit. It is assumed to be a fraction c of the value of bank assets at time τ_{j+1} (i.e., when the audit is actually taken), and is discounted back to time τ_j at rate r. The third term represents the value to be lost (or gained if it is negative) by letting the bank operate for one more audit interval, again expressed in terms of value at time τ_j. These three terms correspond to the replacement costs, inspection costs, and output costs mentioned earlier, respectively.

Define the optimal loss function $J^*(x_0)$ by;

$$J^*(x_0) = \inf_{\pi \in \Pi} J_\pi(x_0) \tag{24}$$

[26]From equation (22), we can derive;

$$\prod_{k=0}^{j} \beta_k = \prod_{k=1}^{j} u_{k-1} e^{-rt_{k-1}} = \left(\prod_{k=0}^{j-1} u_k\right) \exp\left[-r \sum_{k=0}^{j-1} t_k\right] = \left(\prod_{k=0}^{j-1} u_k\right) \exp[-r\tau_j]$$

$$= \begin{cases} \exp[-r\tau_j] & \text{if } u_k = 1 \text{ for all } k = 0, 1, \ldots, j-1 \\ 0 & \text{if } u_k = 0 \text{ for some } k \end{cases}$$

where Π is the set of all policy sequences. The authority's objective is to find a policy sequence π which achieves $J^*(x_0)$ under the constraints imposed by state transition equations. The latter equations are given by equation (18) for A_j (where $A(0)$ and $A(\tau)$ are replaced with A_{j-1} and A_j) and the following Bayesian updating equation for p_j.

$$p_j(x_{j-1}, \mu(x_{j-1}), A_j) = \frac{p_{j-1}Y_1(A_j)}{p_{j-1}(Y_1(A_j)+Y_2(A_j))+(1-p_{j-1})Y_3(A_j)} \tag{25}$$

where the definition of $Y_k(A_j)$ (k=1,2,3) is given by,[27]

$Y_1(A_j)$; probability density of the event that A_j is observed and the deterioration of the mean rate of return does not occur in the interval $[\tau_{j-1}, \tau_j]$, conditional upon x_{j-1}, $\mu(x_{j-1})$ and $\iota=i$ at time τ_{j-1}.

$Y_2(A_j)$; probability density of the event that A_j is observed and the mean deterioration occurs at some time τ in the interval $[\tau_{j-1}, \tau_j]$, conditional upon x_{j-1}, $\mu(x_{j-1})$ and $\iota=i$ at time τ_{j-1}.

$Y_3(A_j)$; probability density of the event that A_j is observed, conditional upon x_{j-1}, $\mu(x_{j-1})$ and $\iota=i^*$ at time τ_{j-1}.

The actual formula for Y_k's are given as follows.

$$Y_1(A_j) = e^{-\lambda t_{j-1}}\phi\left[\frac{1}{\sigma\sqrt{t_{j-1}}}\left\{\ln\left(\frac{A_j}{A_{j-1}}\right) - \left(i-\frac{\sigma^2}{2}\right)t_{j-1}\right\}\right] \tag{26a}$$

$$Y_2(A_j) = \int_0^{t_{j-1}} \lambda e^{-\lambda t}\phi\left[\frac{1}{\sigma\sqrt{t_{j-1}}}\left\{\ln\left(\frac{A_j}{A_{j-1}}\right) - \left(i^*-\frac{\sigma^2}{2}\right)t_{j-1} - (i-i^*)t\right\}\right]dt \tag{26b}$$

$$Y_3(A_j) = \phi\left[\frac{1}{\sigma\sqrt{t_{j-1}}}\left\{\ln\left(\frac{A_j}{A_{j-1}}\right) - \left(i^*-\frac{\sigma^2}{2}\right)t_{j-1}\right\}\right] \tag{26c}$$

where ϕ is the standard normal density function, and the expression inside the curly bracket is the value of the Wiener process Z_j consistent with the observed value of A_j under the event specified for each Y_k, k=1, 2, 3. It is assumed that the Wiener process and the deterioration of the mean rate ι are independently distributed.

$J^*(x_0)$ is clearly bounded from above by δA_0, because closing the bank immediately regardless of the state is an admissible policy and guarantees the cost of δA_0. If we have $J^*(x_0)$ strictly less than δA_0, the bank remains open if it already exists. However, if a decision is to be made about whether a permission to open a new bank should be issued to a banker, the critical level of loss function is zero rather than δA_0. It is not desirable to create a bank for which $J^*(x_0)$ is calculated as positive, because the net rate of return such a bank can earn on its assets is expected to be lower than the safe rate. Obviously, if $\delta=0$,

[27] A_j which appears as the argument of Y_k should be interpreted as a particular realization of the asset value process whose value is identified by the authority through the j th audit.

the two types of decision coincide.

II–II. Numerical Solution

To obtain the numerical solution of the dynamic programming problem discussed in the preceding section, we need to discretize the state and control spaces. In general, the finer the discretized spaces (or the "gridlines"), the better approximated solution one can obtain, but at the same time, the longer time is required for each iteration. The following grid, which we believe is reasonably fine, is chosen as a compromise between these two considerations.

We use a three-dimensional grid, (p_0, t_0, A_1). The number of gridlines for p_0 (the authority's prior probability) is 51 and the interval between two adjacent gridlines is .02. Naturally, the grid covers the probability space from zero to one. Another state variable A_0 (initial bank asset value) is normalized to unity. This normalization is motivated by the linearity of the loss function in A_0. What we derive numerically, therefore, is the loss function per unit of investment goods at time 0. The control variable t_0 (the first audit interval) has 20 gridlines, and the gridline interval is a quarter of a year. This means that the shortest (longest) audit interval is restricted to a quarter of a year (five years). A_1 (bank asset value at time τ_1) is given 50 gridlines. The gridline interval and the position of the grid on the positive real line should depend on exogenous parameters (i, i^*, r, λ and σ) as well as the authority's choice of t_0, because these variables determine the probability distribution of A_1. Our Fortran program chooses the position of the grid so that the grid covers the segment on the positive real line corresponding to three standard deviations above and below the mean of A_1 (more precisely, $\ln(A_1/A_0)$) for each set of exogenous parameters and each possible value of t_0, and determines the gridline interval by dividing the segment by 50. Finally, note that we do not need a grid for p_1, because p_1 is uniquely determined from p_0, A_0 $(=1)$, t_0 and A_1 by the state transition equation (25).

Iterative calculations are carried out thirty to fifty times until the maximum amount of changes in the loss function per iteration (where maximum is taken over all grid values of p_0) is less than .001, or .1% of the initial investment goods for which the loss function is defined. Our results are given in Table 4 and Figures 1 through 7. Table 4 compares various optimal loss functions obtained in our numerical exercise.[28] Each figure corresponds to one of the seven parameters in our model (i, i^*, r, λ, c, σ, and δ), and shows how the optimal policy choice is affected when the value of the selected parameter changes keeping the other six parameters constant. The results are summarized as follows.

(i) The shape of the loss function is flat at first for small value of p_0 and monotonically decreasing in p_0 thereafter, where the flat part corresponds to bank closure. The bank closure threshold level of subjective probability is quite low, around .1 or less, for most of

[28]For clarity and simplicity, we present the optimal loss function evaluated at only eleven grid points (p_0=0, 0.1, 0.2, etc.) out of the total of 51 grid points.

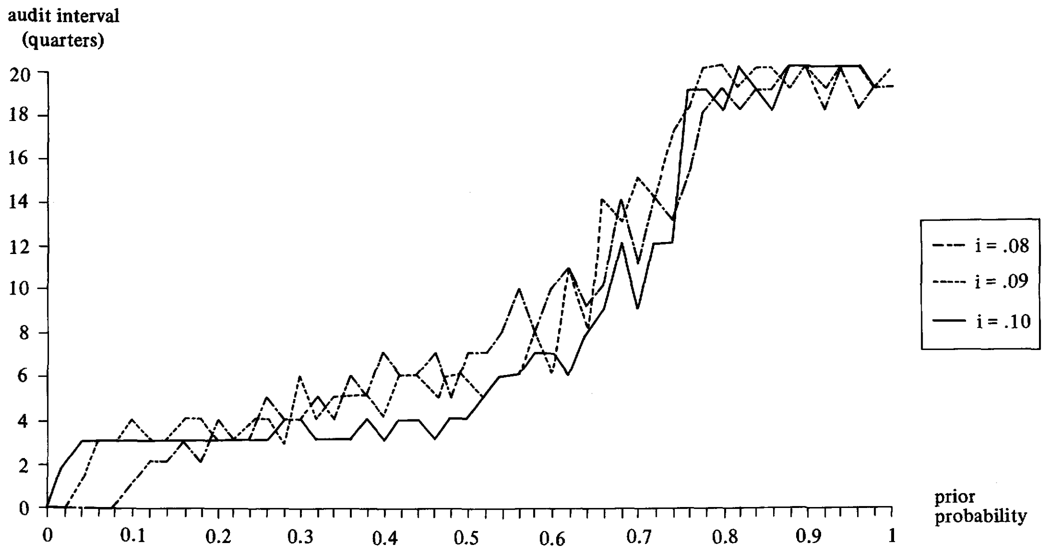
Table 4. The Optimal Loss Functions

p_0	i = .08	i = .09	i = .10	i* = 0	i* = .02	i* = .04	r = .05	r = .06	r = .07	λ = .03	λ = .05	λ = .07
0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
0.1	0.097	0.055	-0.039	0.097	0.097	0.097	0.047	0.097	0.1	0.061	0.097	0.1
0.2	0.068	-0.033	-0.241	0.067	0.069	0.068	-0.046	0.068	0.1	-0.030	0.068	0.089
0.3	0.030	-0.130	-0.461	0.031	0.034	0.030	-0.148	0.030	0.1	-0.132	0.030	0.069
0.4	-0.011	-0.233	-0.693	-0.010	-0.007	-0.011	-0.251	-0.011	0.094	-0.241	-0.011	0.045
0.5	-0.055	-0.338	-0.929	-0.052	-0.049	-0.055	-0.360	-0.055	0.086	-0.355	-0.055	0.019
0.6	-0.101	-0.446	-1.174	-0.096	-0.094	-0.101	-0.472	-0.101	0.075	-0.473	-0.101	-0.009
0.7	-0.149	-0.559	-1.421	-0.142	-0.140	-0.149	-0.587	-0.149	0.062	-0.594	-0.149	-0.038
0.8	-0.199	-0.673	-1.680	-0.189	-0.188	-0.199	-0.699	-0.199	0.048	-0.716	-0.199	-0.068
0.9	-0.248	-0.784	-1.927	-0.236	-0.236	-0.248	-0.810	-0.248	-0.032	-0.836	-0.248	-0.099
1.0	-0.297	-0.891	-2.163	-0.283	-0.284	-0.297	-0.919	-0.283	-0.014	-0.953	-0.297	-0.129

p_0	c = .005	c = .001	c = .0001	σ = .01	σ = .03	σ = .05	δ = .08	δ = .10	δ = .12
0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.08	0.1	0.12
0.1	0.1	0.099	0.097	0.064	0.097	0.1	0.077	0.097	0.116
0.2	0.083	0.071	0.068	0.012	0.068	0.098	0.049	0.068	0.087
0.3	0.050	0.035	0.030	-0.045	0.030	0.078	0.011	0.030	0.049
0.4	0.012	-0.006	-0.011	-0.103	-0.011	0.051	-0.031	-0.011	0.007
0.5	-0.028	-0.049	-0.055	-0.164	-0.055	0.019	-0.075	-0.055	-0.037
0.6	-0.070	-0.095	-0.101	-0.225	-0.101	-0.018	-0.121	-0.101	-0.083
0.7	-0.115	-0.142	-0.149	-0.292	-0.149	-0.057	-0.170	-0.149	-0.131
0.8	-0.162	-0.191	-0.199	-0.357	-0.199	-0.098	-0.219	-0.199	-0.181
0.9	-0.207	-0.239	-0.248	-0.415	-0.248	-0.141	-0.269	-0.248	-0.230
1.0	-0.252	-0.287	-0.297	-0.472	-0.297	-0.185	-0.319	-0.297	-0.278

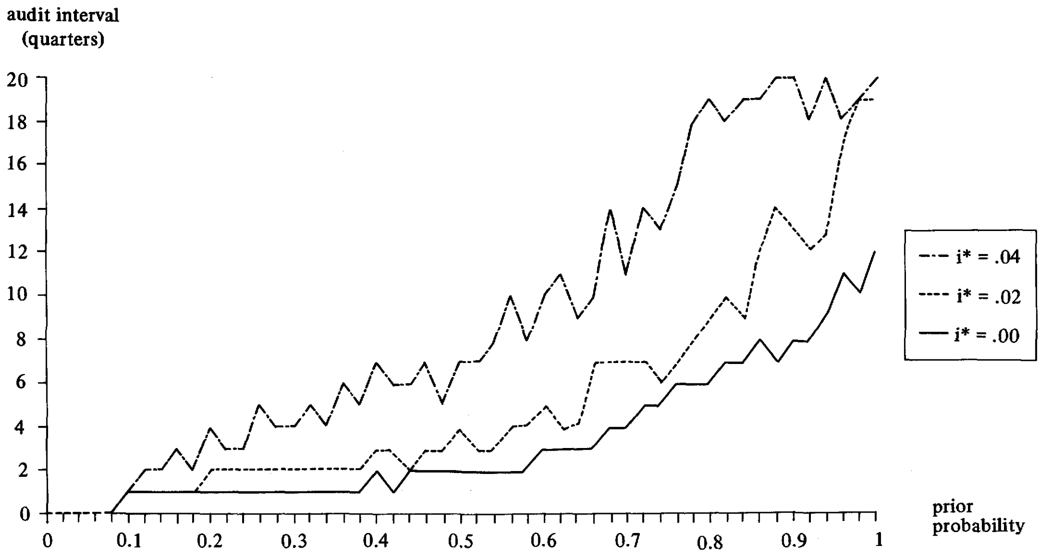
Note: The baseline parameter set is given as, i = .08, i* = .04, r = .06, λ = .05, c = .0001, σ = .03, δ = .1, from which one parameter is varied keeping the rest constant.

Figure 1. Good Rate of Return on Bank Assets (i)



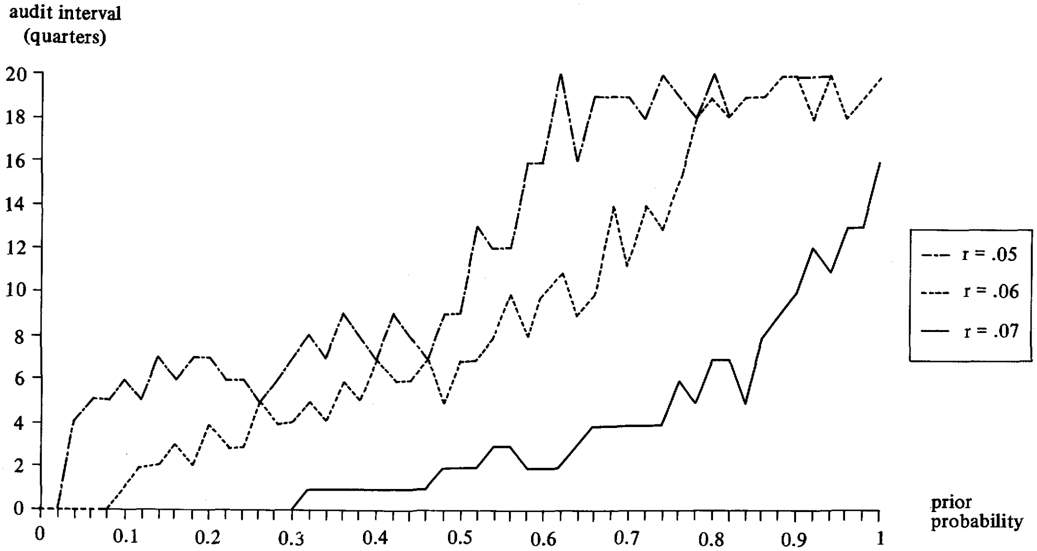
the rest of the parameters
i* = .04, r = .06, λ = .05, c = .0001, σ = .03, δ = .1

Figure 2. Bad Rate of Return on Bank Asset (i*)



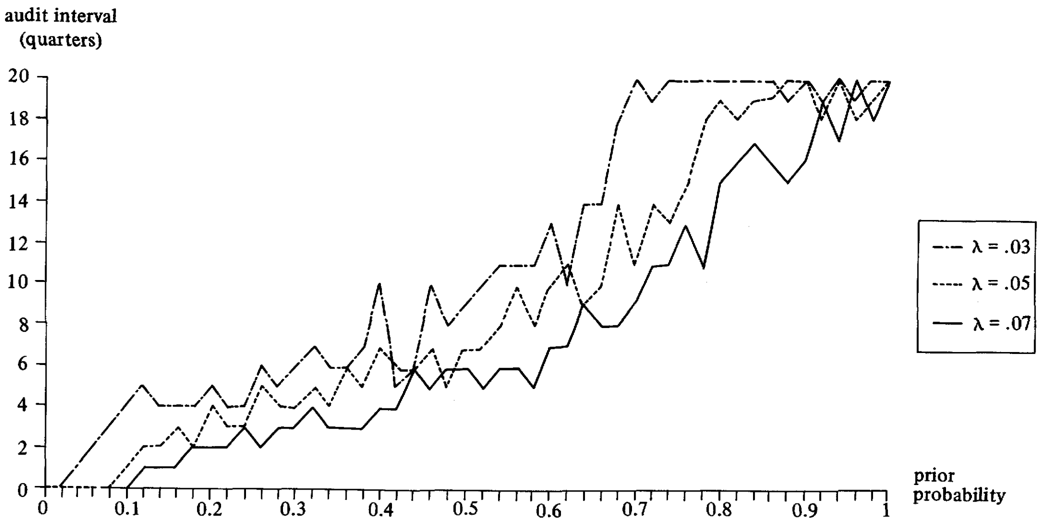
the rest of the parameters
i = .08, r = .06, λ = .05, c = .0001, σ = .03, δ = .1

Figure 3. Safe Rate of Return (r)



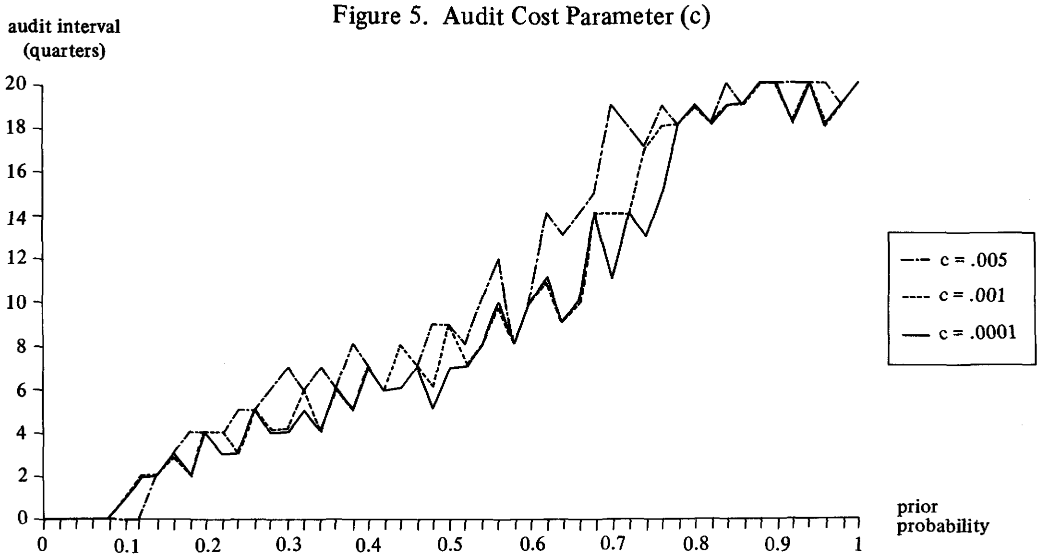
the rest of the parameters
i = .08, i* = .04, λ = .05, c = .0001, σ = .03, δ = .1

Figure 4. Parameter for the Rate of Return Deterioration (λ)



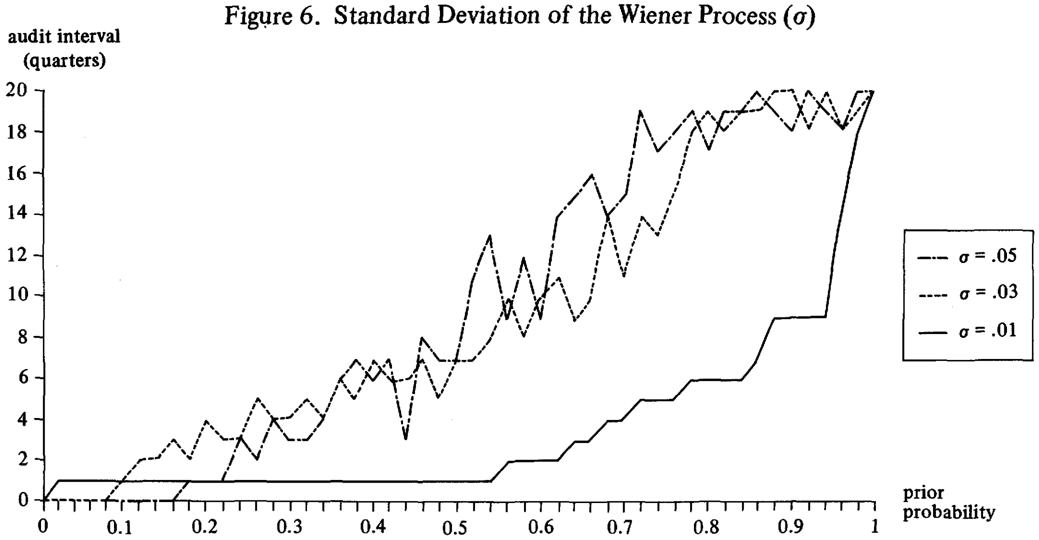
the rest of the parameters
i = .08, i* = .04, r = .06, c = .0001, σ = .03, δ = .1

Figure 5. Audit Cost Parameter (c)



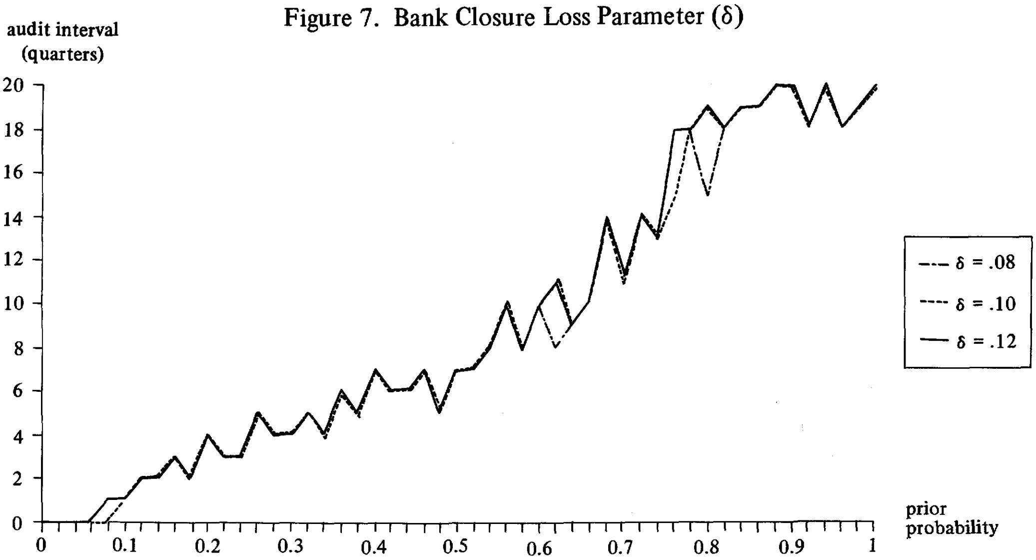
the rest of the parameters
i = .08, i* = .04, r = .06, λ = .05, σ = .03, δ = .1

Figure 6. Standard Deviation of the Wiener Process (σ)



the rest of the parameters
i = .08, i* = .04, r = .06, λ = .05, c = .0001, δ = .1

Figure 7. Bank Closure Loss Parameter (δ)



the rest of the parameters
i = .08, i* = .04, r = .06, λ = .05, c = .0001, σ = .03

the parameter sets examined.

(ii) The higher the prior probability of the authority that a bank's rate of return is high, the longer the audit interval, at least globally. The fact that this relationship does not always hold locally is likely to be the result of insufficient grid points.[29]

(iii) Changes in i (good rate of return on investment, Figure 1) do not seem to have significant impact upon the optimal policy choice, except perhaps upon the bank closure threshold level of the prior probability. However, they do affect the optimal loss function significantly, and the direction of their influence is consistent with our intuition.

(iv) The impact of changes in i* (bad rate of return on investment, Figure 2) on the optimal policy choice is quite intuitive; the smaller the value of i*, the shorter audit intervals are chosen. What is not so intuitive is their effect on the optimal loss function. The loss function does not change very much in response to changes in i*, and the direction of its (small) change is not even monotonic. This result, counterintuitive as it may appear at first sight, can be interpreted as follows. Suppose the value of i* declines. While this clearly increases the costs arising from the difference between r and i* (what we called "output" costs), it has a compensating benefit that it makes it easier for the authority to tell the occurrence of the rate of return deterioration, because lower i* means that the distribution of A_1 is more polarized (i.e., more distinctively bi-modal). This enables the authority to cut the period of time shorter during which it mistakenly

[29]By increasing the gridline number for p_0 from 51 to 401, we were able to reduce the number and the size of local irregularities for the baseline parameter set considerably.

keeps a bad bank open, which in turn reduces the value of the optimal loss function. The net impact of these two effects is unclear, and our result in Table 4 reflects this point. Note that the argument is not symmetrically true for the case of i, which is also reflected in our result.

(v) The safe rate r affects the loss function and the policy choice by changing the opportunity cost of banking operation. Given i and i*, the higher the value of r, the less profitable the banking operation becomes relative to safe investment, and hence the larger loss function and the more frequent audits (Figure 3). Table 4 shows that if r is as high as 7% (while i=8% and i*=4%), issuing new bank charter is uneconomical even when the authority believes that the banker applying for the charter has a good loan opportunity with probability one.

(vi) Changes in λ (the parameter of exponential distribution which the rate of return deterioration follows, Figure 4) have an impact on the loss function and the policy choice in the direction which conforms with our intuition. Namely, the lower the level of λ (i.e., the less risky is the bank), the smaller the optimal loss function, and the longer the audit interval.

(vii) The larger the audit cost parameter c, the less frequently audits are taken (Figure 5), and the larger the optimal loss function, although the impact of changes in c upon the policy choice and the loss function is rather small. This is in sharp contrast with the results obtained in the previous chapter where the optimal audit interval was proportional to the square root of c. This difference is explained by the presence of incomplete information; if information is incomplete, the optimal audit frequency problem is well-defined *without* the existence of audit costs.

(viii) Figure 6 shows that the lower the value of σ (the standard deviation of the Wiener process term), the lower the threshold level of prior probability below which the bank is closed, and at the same time, the shorter audit intervals are chosen as long as the bank closure decision is not taken. This result is interpreted as follows. If the value of σ is small, less noise is created by the Wiener process so that the authority can make a better judgement about the quality of the bank by taking more audits. Thus, the authority chooses to take another audit at a short interval rather than closing the bank, even when its subjective belief is that the bank is very likely to be bad.

(vii) Finally, changes in δ (the parameter for the deadweight loss caused by bank closure, Figure 7) have almost no influence upon the optimal policy choice. They affect the optimal loss function, however, and the direction of their impact is consistent with our intuition.

II–III. Concluding Remarks

The purpose of Part II of this paper has been to make a step closer toward reality by adding Bayesian inference to our previous framework. This yielded several interesting results. For example, the fact that some exogenous parameters such as i* and σ affect the loss function and the policy choice by changing the noisiness of audit information is a feature totally absent in our model formulated in Part I.

As usual, a step closer to reality is still a long way to reality. However, we believe there is a lesson to be learned even from our somewhat limited numerical exercises. The importance of the lesson is increasing in the 1980s when the government agencies in charge of bank examination are experiencing erosion of experienced bank examiners and are being forced to look for more efficient ways of auditing banks.[30]

[30]The erosion is primarily due to the agencies' inability to provide compensations to the examiners comparable to the one offered by the private sector. See GAO (1981) for General Accounting Office's proposals to cope with this adverse development.

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