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On the Statistical Properties of Floating Exchange Rates: A Reassessment of Recent Experience and Literature

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The paper reviews the statistical behavior of major currency exchange rates during 1975–86. A close inspection indicates small deviations of recent exchange rate behavior from random walks and some systematic movements in monthly data, possibly corresponding to the relatively infrequent arrivals of information concerning major macroeconomic variables. The distributional characteristics of exchange rate changes differ between daily and monthly data and thus imply the possible presence of heterogeneity in underlying factors. These and other observations suggest care in the use of daily data in empirical work and the usefulness of explicit modeling of heterogeneity among market participants and in information structure.

I. Introduction

The paper will review the statistical properties of nominal exchange rates between major currencies in the light of some 15 years of experience with floating exchange rates and the relevant literature that has emerged in recent years. The paper is intended both as a summary description of empirical nominal exchange rate behavior and as a review of the existing literature on this subject. As a summary description, it will present basic statistical measures of the spot and forward exchange rates between four major currencies during 1975–86; as a review, it will trace recent theoretical and empirical developments that are important to our understanding of the nature of exchange rate behavior.

Basic understanding of the statistical behavior of exchange rates is important because it forms the basis for discussion and analysis of other pertinent issues involving exchange rates. In particular, the empirical behavior of exchange rates is important for at least three principal reasons. First, the time-series behavior of exchange rates has implications for the question of market efficiency that has received far more attention in the

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literature. Second, the distributional property of exchange rates in part determines the riskiness of the foreign exchange market and the validity of statistical inference in empirical work. Third, the statistical behavior of exchange rates provides insight into the nature of the process governing exchange rate determination. A systematic review of empirical exchange rate behavior in these three principal areas might also provide useful implications for the direction of future research.

The paper will first clarify the often confused relationship between market efficiency and exchange rate behavior; it will show that market efficiency imposes certain constraints on the relationship among endogenous variables but not necessarily on the stochastic process governing the time-series behavior of exchange rates. It will then discuss the statistical properties of nominal exchange rates, which are divided for analytical convenience into time-series and distributional properties. This division corresponds to the two important elements of the traditional random walk theory of asset prices (Fama 1965): (1) the serial correlation of successive price changes and (2) the type of probability distribution to which those price changes conform. For each type of statistical behavior, the paper will review the literature and draw implications for the nature of forces that underlie the exchange rate generating process.

The paper will show that, although the level of the empirical exchange rates of major currencies followed a process that is closely approximated by a random walk, some serial dependence in successive exchange rate changes was almost always present on a closer examination. Moreover, monthly data showed generally greater serial dependence than daily data, possibly suggesting the presence of systematic information in low frequency data corresponding to macroeconomic variables. Regarding the distributional properties, the paper confirmed that the distribution of daily exchange rate changes was in general too "peaked" and "fat-tailed" to be normal (i.e., leptokurtic); in contrast, the distribution of monthly changes could be characterized as approximately normal. The normal with an autoregressive conditional heteroskedasticity (ARCH) process for innovations has shown some promise as a model of the empirical distribution of exchange rate changes. However, the most one can say with confidence is that the distributional characteristics of daily and monthly exchange rate changes point to the presence of heterogeneity among market participants as well as changing parameters over time.

The paper is organized as follows. Section II clarifies the concept of efficiency in foreign exchange markets and presents an example of the way market efficiency is related to the time-series behavior of nominal exchange rates using a simple monetary-type model. Section III discusses the time-series properties of exchange rates, including random walk tests of the levels of exchange rates and various serial correlation tests of the first differences of their logarithms. Section IV then discusses the distributional properties of the first logarithmic differences of exchange rates and critically analyzes some of the possible hypotheses that have been proposed to explain the observed behavior. Finally, section V presents concluding remarks.

II. Market Efficiency and Exchange Rate Behavior

1. Efficiency in Foreign Exchange Markets

It is sometimes thought that market efficiency necessarily specifies the type of time-series behavior of exchange rates, such as a random walk. In reality, however, market efficiency can be consistent with many types of statistical behavior. What market efficiency does specify is the kind of relationship that must exist between certain endogenous variables. The time-series behavior of exchange rates in an efficient market would be in turn determined by the nature of the time-series behavior of the exogenous variables underlying that relationship.

In general, an efficient capital market is defined as a market in which prices fully reflect all available information and, consequently, investors cannot systematically earn an "unusual" profit on the basis of information available in the market (Fama 1976). This definition of market efficiency, however, lacks testable content in and of itself unless an operational definition is given for what constitutes an "unusual" profit and "available" information. The first operational definition requires a formal economic model of asset price determination; otherwise, one cannot determine what is "normal," hence what is "unusual." The second operational definition, on the other hand, simply requires a characterization of the information set against which the question of market efficiency is tested.

In the empirical literature on stock markets, broadly two types of characterization have been made, corresponding to what is called weak form efficiency and semi-strong form efficiency.[1] Weak form efficiency requires that an asset's price should not be predicted on the basis of the past history of its own prices. Popular forms of weak form efficiency tests have involved testing either for the existence of a trading rule based on observed prices that would yield a higher return than a simple buy-and-hold strategy, or for the existence of serial correlation in securities returns on the assumption that the expected returns are constant.

Semi-strong form efficiency requires that the asset price should be the market's best prediction based on all publicly available information. Popular empirical tests have involved analysis of the residuals calculated as the difference between actual prices and the prices predicted by a model of market equilibrium (e.g., the Market Model or the Capital Asset Pricing Model). Others have tested the profitability of different strategies to trade on published information.

In the context of foreign exchange markets, the term "efficiency" has come to acquire the two additional meanings of covered interest parity and forward market effi-

1. The paper does not discuss strong form efficiency. At least in the context of the foreign exchange market, very little work has been done in this area.

ciency. First, covered interest parity is a condition that riskless arbitrage yields no profit, i.e.:

$$ {}_{t}f_{t+1}-e_{t}=R_{t}+u_{t}, \tag{1}$$

where ${}_{t}f_{t+1}$ is the one-period ahead forward exchange rate and e_t is the spot exchange rate, both expressed in logarithm as the domestic currency price of the foreign currency at time t (i.e., an increase in f or e means a depreciation of the domestic currency); R_t is a differential between the domestic and foreign one-period interest rates formed at t, both expressed as the log of one plus the nominal interest rate; and u_t is a random deviation at t.[2] In general, no restrictions need to be placed on the distribution of u_t; it may or may not be white noise. For example, in the presence of capital controls, deviations from covered interest parity can be in one direction, corresponding to one-sided restrictions on either outflows or inflows; thus, u_t can be serially correlated.

Equation (1) states that, in a state where no exploitable profit opportunity exists, a forward premium must be offset by the corresponding nominal interest rate differential, except for a deviation (u) whose magnitude must be within the cost of arbitrage. In this case, an "unusual" profit means a profit in excess of transactions costs provided that domestic and foreign assets are perfect substitutes. There is, however, a conceptual difficulty regarding the interpretation of the transactions cost arising from capital controls. For example, the deviation from covered interest parity can be large in the presence of capital controls; however, the market can be considered efficient in the sense of equation (1) as long as there is no exploitable profit opportunity within that given regulatory framework. Available evidence suggests that, at least for the 1980s, deviations from covered interest parity between similar short-term instruments in major industrial countries have been extremely small and serially uncorrelated, suggesting that the market has been unambiguously efficient with the recent elimination of capital control measures in these countries.[3]

Second, forward market efficiency expresses the idea that the forward exchange rate incorporates all available information about the expected future spot rate, i.e.:

$$ {}_{t}f_{t+1}=E_{t}e_{t+1}+r_{t} \tag{2}$$

where E_t is a mathematical expectations operator based on the set of information available at t, and r is a deviation term that reflects, at most, the possible presence of a risk premium plus random mean-zero error. In the empirical literature, most studies have

2. An analogous expression holds for the relationship between the n-period forward rate and the n-period interest rate differential for any arbitrary n.

3. Both Japan and the United Kingdom lifted capital controls in the late 1970s. However, covered interest parity has always held as a matter of course in the Eurocurrency market; it is said that Euro interest rates and forward rates are set according to the covered interest parity formula.

made the operational assumption of risk neutrality and tested the hypothesis that the forward exchange rate is an unbiased predictor of the future spot rate. Although not conclusive, empirical evidence generally points to the rejection of the joint hypothesis of risk neutrality and forward market efficiency, suggesting either market inefficiency or the existence of a risk premium (for a survey of the empirical literature, see Levich 1985; Boothe and Longworth 1986; and Isard 1987).[4]

Covered interest parity and forward market efficiency jointly imply a more familiar condition of market efficiency,

$$E_t e_{t+1} - e_t = R_t + u_t - r_t. \tag{3}$$

This condition amounts to uncovered interest parity if u_t is white noise and r_t (the risk premium) is zero. Equation (3) corresponds to the usual formulation of market efficiency in stock markets, because the left-hand-side variable can be interpreted as the expected rate of return from speculation in the spot exchange market. That is to say, covered interest parity and forward market efficiency are components of the more traditional concept of market efficiency. One can interpret condition (3) as weak form efficiency if one assumes risk neutrality and a constant nominal interest differential; in this case, the presence of serial dependence in expected exchange rate changes will reject the efficiency hypothesis.[5] More generally, one can also interpret condition (3) as semi-strong form efficiency; in this case, empirical testing of market efficiency requires a knowledge of how the risk premium and the nominal interest differential are determined.

Compared with tests of market efficiency in stock markets, empirical testing of equation (3) in foreign exchange markets involves two types of limitations (Levich 1985). First, in testing weak form efficiency, one has less justification for assuming that the nominal interest differential plus the risk premium are constant particularly for monthly or quarterly data. Second, as a test of semi-strong form efficiency, one has no satisfactory model of market equilibrium in the foreign exchange market comparable to the Capital Asset Pricing Model. In the absence of a satisfactory model of market equilibrium, foreign exchange market efficiency possesses little testable content.[6]

4. One conceptual difficulty encountered in testing the hypothesis of forward market efficiency is the so-called "peso problem." It is the small sample problem of a large change with a small probability not occurring during the sample period, causing the forward rate to show an apparent bias.

5. In addition to market inefficiency, capital controls can produce serial correlation in the left-hand-side variable by introducing serial correlation in the residuals.

6. One direct way of testing for market efficiency is to use a filter rule suggested by Alexander (1961). Dooley and Shafer (1983), for example, found that 1, 3, and 5 percent filter rules would have yielded large profits for several major currencies during 1973–81. This type of efficiency tests have at least two conceptual problems: (1) one must know the extent of transactions costs in currency trading; and (2) one must know the riskiness of speculation in the foreign exchange market in order to judge what constitutes excessive profits.

2. Examples of Time-series Behavior

The preceding discussion suggests that the behavior of expected changes in exchange rates has little to do with market efficiency; rather, it depends on the structure of underlying economic variables that influence the risk premium (r) and the joint determination of e and R. This idea can be made explicit by specifying general forms of market equilibrium in the goods and money markets and imposing them on equation (3). As a simple illustration, let us assume the following form of international price linkage as the goods market equilibrium condition:

$$e_t=\beta p_t+\eta_t, \tag{4}$$

where the exchange rate is influenced by both a systematic relative price factor (p) and a non-systematic factor (η), both expressed in logarithm; if $\beta=1$ and $\eta=0$, equation (4) corresponds to purchasing power parity.

For the money market, let us assume that the logarithmic difference between domestic and foreign money supplies (m) is related to the relative price in the consolidated world market in the following manner:

$$m_t=p_t-\gamma R_t, \tag{5}$$

where γ, interpreted as the global interest elasticity of money demand, assumes that the country elasticities are identical for simplicity. Alternatively, equation (5) can be interpreted as a characterization of a less specific equilibrium condition in which the relative price is related to other real and nominal variables; in this case, m can be more broadly interpreted as a fundamental economic variable.

Substituting (4) and (5) into (3), we obtain:

$$e_t=[\frac{\beta}{(1+\phi)}]m_t+[\frac{\phi}{(1+\phi)}]E_te_{t+1}+[\frac{\phi}{(1+\phi)}](r_t-u_t)+\eta_t, \tag{6}$$

where $\phi\equiv\beta\gamma$. This intermediate solution, which relates the current exchange rate to the current value of the underlying fundamental variable, the expected value of the future exchange rate and random errors, is a general solution form obtained from most models of exchange rate determination in which an intertemporal variable (such as R) plays a role (Genberg 1984; and Frenkel and Mussa 1985).[7] A solution of equation (6) can be obtained by forward iteration:

$$e_t=[\frac{\beta}{(1+\phi)}]\sum_{j=0}^{\infty}[\frac{\phi}{(1+\phi)}]^jE_tm_{t+j}+\sum_{j=0}^{\infty}[\frac{\phi}{(1+\phi)}]^{j+1}E_tr_{t+j}+z_t, \tag{7}$$

where $z\equiv\eta-(\frac{\phi}{1+\phi})u$. This solution assumes that all expected future values of z are zero and rules out the existence of a rational bubble.

Equation (7) makes clear the forward-looking nature of the solution for the ex-

7. Mussa (1984) shows that the same solution form can be obtained by concentrating on balance of payments flows rather than money market conditions.

change rate. However, it does not suggest that e must follow any particular stochastic process to satisfy market efficiency. For example, e can be functionally related to its past values if they help predict its current and future values. For example, suppose that there is no risk premium and m follows a first-order autoregressive process, i.e., AR(1), with a coefficient k. Then, equation (7) can be simplified to,

$$e_t=\{\frac{\beta}{1+\phi(1-k)}\}m_t+z_t. \tag{8}$$

Clearly, if m is AR(1) and z_t is white noise, e is ARMA (1, 1). As to the behavior of the forward rate, we see from equation (3) that the forward rate is simply the appropriately updated spot exchange rate plus a risk premium. Consequently, if we assume that the risk premium is zero and m follows AR(1), we have:

$$_tf_{t+n}=\{\frac{\beta k^n}{1+\phi(1-k)}\}m_t. \tag{9}$$

Thus, if m is AR(1), the forward rate is also AR(1). Yet, in either case, the stationarity of the exchange rate process in no way implies market inefficiency because the behavior of the exchange rate satisfies the condition of market efficiency given by equation (3).

III. Time-Series Properties of Exchange Rates

1. Random Walk Tests

Despite the lack of any theoretical necessity for e or f to follow any particular stochastic process, recent empirical investigations have almost unanimously found that the exchange rate follows a process that is closely approximated by a random walk. One straightforward way to see this "random walk" nature of empirical exchange rate behavior is to calculate the "F" and "t" statistics proposed by Dickey and Fuller (1979, 1984) that are extensions of the conventional F and t tests to nonstationary time-series. Like the conventional tests, Dickey and Fuller's F test involves a test of the joint restriction that the intercept is zero and the coefficient is unity when e or f is regressed on its lagged value; the statistic in excess of a critical value would result in a rejection of the random walk hypothesis. Dickey and Fuller's t test is also analogous to the conventional one, involving a test of the hypothesis that the slope coefficient is unity in the above regression.

Table 1 and Table 2 report Dickey-Fuller test statistics based on daily and monthly exchange rates over four subperiods during 1975–86 for six bilateral exchange rates.[8] Tests were performed for spot exchange rates as well as 3–month and 12–month forward exchange rates. Two observations emerge from the tables. First, daily and monthly data, as well as the spot and forward rates, show broadly similar patterns. Second, except in a few cases, one cannot reject the hypothesis that the spot and forward exchange rates

8. Of the six bilateral exchange rates reported in the paper, only three are independent.

Table 1. Dickey-Fuller Statistics on Daily Exchange Rates

	U.S. Dollar			Sterling		Deutsche mark
	Sterling	Deutsche mark	Yen	Deutche mark	Yen	Yen
			January 1, 1975 to December 31, 1977			
Spot	3.0 (−1.9)	1.0 (0.7)	7.5**(2.6*)	2.8 (−1.0)	5.0**(−1.2)	1.9 (−1.7)
3-month	2.2 (−1.7)	1.1 (0.8)	7.9**(2.6*)	2.2 (−1.0)	4.2* (−1.2)	2.3 (−1.9)
12-month	1.9 (−1.7)	2.0 (1.4)	9.0**(2.9**)	2.1 (−1.2)	3.5 (−1.2)	2.3 (−1.9)
			January 1, 1978 to December 31, 1980			
Spot	1.2 (−0.8)	2.1 (−2.0)	1.2 (−1.3)	0.7 (0.5)	0.3 (−0.7)	0.3 (−0.5)
3-month	1.2 (−0.5)	2.4 (−2.0)	1.2 (−1.3)	0.7 (0.5)	0.3 (−0.7)	0.3 (−0.5)
12-month	1.0 (−0.5)	2.4 (−2.2)	1.2 (−1.3)	0.4 (0.2)	0.3 (−0.7)	0.3 (−0.6)
			January 1, 1981 to December 31, 1983			
Spot	5.3**(−2.0)	4.2* (−2.4)	2.8 (−2.3)	1.4 (−1.3)	2.2 (−0.5)	1.0 (−0.2)
3-month	11.3**(−2.0)	4.7* (−2.5)	2.8 (−2.3)	1.3 (−1.2)	1.9 (−0.6)	1.0 (−0.6)
12-month	5.6**(−1.8)	4.8**(−2.4)	2.3 (−2.0)	1.0 (−1.1)	1.8 (−0.8)	1.1 (−0.9)
			January 1, 1984 to June 30, 1986			
Spot	0.5 (−0.9)	0.8 (1.5)	4.2* (1.8)	1.2 (−0.9)	1.1 (−0.3)	1.0 (−1.2)
3-month	0.5 (−1.0)	0.6 (0.1)	4.6**(1.9)	1.3 (−1.3)	1.2 (−0.2)	1.9 (−1.8)
12-month	0.6 (−1.1)	0.6 (−0.0)	3.3 (1.5)	1.4 (−1.3)	1.2 (−0.4)	2.2 (−2.0)

Note: a) **(*) indicates that the statistic is significant at 5(10) percent.
b) For each exchange rate, the first entry is the "F" statistic and the second entry in the parenthesis is the "t" statistic (with an intercept).

Table 2. Dickey-Fuller Statistics on Monthly Exchange Rates

	U.S. Dollar			Sterling		Deutsche mark
	Sterling	Deutsche mark	Yen	Deutche mark	Yen	Yen
			January 1975 to December 1977			
Spot	2.1 (−1.7)	0.0 (−0.1)	4.6* (2.1)	2.0 (−1.0)	5.3**(−1.2)	2.1 (−2.2)
3-month	1.6 (−1.6)	0.0 (0.1)	4.8* (2.2)	1.8 (−1.0)	4.8* (−1.2)	2.4 (−2.1)
12-month	1.0 (−1.4)	0.5 (0.6)	5.2**(2.4)	1.5 (−1.1)	3.7 (−1.2)	2.4 (−2.1)
			January 1978 to December 1980			
Spot	0.9 (−1.0)	2.9 (−2.4)	1.4 (−1.6)	0.1 (−0.3)	0.1 (−0.8)	0.5 (−0.9)
3-month	0.8 (−0.8)	2.9 (−2.4)	1.3 (−1.5)	0.0 (−0.2)	0.1 (−0.8)	0.5 (−0.9)
12-month	0.7 (−0.6)	3.0 (−2.5)	1.3 (−1.5)	0.0 (−0.1)	0.1 (−0.8)	0.5 (−0.9)
			January 1981 to December 1983			
Spot	5.9**(−1.9)	0.7 (−2.4)	2.8 (−2.4)	0.0 (−1.4)	1.4 (−0.6)	0.0 (−0.7)
3-month	6.4**(−2.0)	0.9 (−2.6)	2.7 (−2.3)	0.0 (−1.4)	1.4 (−0.7)	0.0 (−0.8)
12-month	6.1**(−1.8)	1.4 (−2.6)	2.2 (−2.1)	0.0 (−1.3)	1.3 (−0.8)	0.0 (−1.1)
			January 1984 to June 1986			
Spot	0.2 (−1.0)	0.1 (−0.2)	2.7 (1.5)	1.1 (−1.0)	0.9 (−0.4)	1.1 (−1.5)
3-month	0.3 (−1.1)	0.1 (−0.2)	2.6 (1.5)	1.1 (−1.0)	0.9 (−0.5)	0.9 (−1.4)
12-month	0.5 (−1.3)	0.1 (−0.3)	2.3 (1.3)	1.2 (−1.1)	1.0 (−0.6)	0.9 (−1.4)

Note: a) **(*) indicates that the statistic is significant at 5(10) percent.
b) For each exchange rate, the first entry is the "F" statistic and the second entry in the parenthesis is the "t" statistic (with an intercept).

followed random walks during different subperiods of 1975–86.[9] It is interesting to note that two of the cases where the null hypothesis of a random walk was rejected involved the Japanese yen during the first subperiod, although it is not clear to what extent the presence of capital controls might have been a contributing factor.[10]

Similar unit root tests on other frequency data or on currency futures prices reached essentially the same conclusion. On the basis of weekly data for the Swiss franc, deutsche mark, and Canadian dollar during 1976–81, Meese and Singleton (1982) found similar random walk patterns in the spot and forward exchange rates of these currencies against the U.S. dollar. More recently, Diebold and Nerlove (1986) reached the same conclusion for the weekly spot rates of the deutsche mark, Japanese yen and Canadian dollar against the U.S. dollar during 1973–85. As to the behavior of foreign currency futures prices, Doukas and Rahman (1987) found similar random walk behavior on the basis of the daily U.S. dollar exchange rates of the deutsche mark, Canadian dollar, Swiss franc and Japanese yen during 1977–83.

Caution should be exercised, however, before concluding that the exchange rate should of necessity follow or did in fact follow a random walk. For one thing, we have already established that a random walk is not a necessary implication of market efficiency. In fact, a random walk requires quite stringent conditions about the time-series process of underlying economic variables that are unlikely to be met in practice.[11] For another, it has been argued that the Dickey-Fuller types of unit root tests have low power against borderline stationary alternatives. Hakkio (1986) has shown, on the basis of a Monte Carlo study, that four popular types of random walk tests, including the Dickey-Fuller test, have an extremely low rejection rate when the true model follows a stationary process that is close to a random walk. Frankel and Meese (1987) have suggested a rough rule of thumb that a sample size of over 700 is required for Dickey-Fuller's t test to be able to distinguish between a random walk and an autoregressive process with a coefficient of .99.

2. Autocorrelations of Exchange Rate Changes

To examine the possibility that the exchange rate displays a small deviation from a random walk, it may be informative to obtain basic descriptive statistics of innovations in exchange rate series. Following standard practice, we define innovations to be the first differences of the logarithms of nominal exchange rates, which can be roughly interpreted as percentage changes. For the spot rate, the innovation can be expressed as:

9. We also note that the frequency of rejection is greater for the "F" test, suggesting that the "F" test has more power than the "t" test.

10. Japan almost completely liberalized foreign exchange transactions in December 1980.

11. Takagi (1986) has shown in a formal model that a random walk implies the absence of correlation between the exchange rate and the interest rate differential.

$$\Delta e_t=(\frac{\beta}{1+\phi})\sum_{j=0}^{\infty}(\frac{\phi}{1+\phi})^j(E_t m_{t+j}-E_{t-1}m_{t+j-1})+$$
$$\sum_{j=0}^{\infty}(\frac{\phi}{1+\phi})^{j+1}(E_t r_{t+j}-E_{t-1}r_{t+j-1})+\Delta z_t, \qquad (10)$$

where Δ is a first difference operator. For the forward rate, we have:

$$\Delta f_t=(\frac{\beta}{1+\phi})\sum_{j=0}^{\infty}(\frac{\phi}{1+\phi})^j(E_t m_{t+j+1}-E_{t-1}m_{t+j})+$$
$$\sum_{j=0}^{\infty}(\frac{\phi}{1+\phi})^j(E_t r_{t+j}-E_{t-1}r_{t+j-1}). \qquad (11)$$

Comparison of equation (11) with equation (10) reveals that Δf is simply an updated version of Δe except that Δf has the additional term of (r_t-r_{t-1}) and does not have Δz_t. Both expressions become white noise if r and m follow random walks; otherwise, there is no reason for them to be white noise. Note that the celebrated martingale property of speculative prices refer to the property of $(E_t m_{t+j}-E_{t-1}m_{t+j})$ and not of $(E_t m_{t+j}-E_{t-1}m_{t+j-1})$. In general, therefore, exchange rate changes reflect changes in expectations about the future path of relevant economic variables as well as changes in risk premia, whatever their stochastic processes might be.

Autocorrelations of Δe and Δf based on daily and end-of-month data were calculated up to five lags each (Table 3 and Table 4). We find that the magnitude of autocorrelations is generally small and statistically insignificant. There are, however, exceptions to this observation, although some of the statistically significant coefficients may admittedly reflect type II errors or heteroskedasticity.[12] For daily time-series, we find a few sample autocorrelations at different lags that exceed 0.2, implying that at least 4 percent of the variance of daily changes can be explained by the linear relationship between present and lagged changes. For monthly time-series, some sample autocorrelations are as large as 0.3–0.5, implying that between 10 and 25 percent of the variance of a current exchange rate change is linearly related to one of its lagged values.

Three conclusions can be drawn from the tables. First, the sample autocorrelations of both daily and monthly changes in exchange rates are virtually identical for spot, 3–month forward, and 12–month forward rates of the same period. Second, the values of sample autocorrelations are generally small and imply that much less than 1 percent of the variance can be explained by the linear relationship between present and lagged

12. Heteroskedasticity can understate the true variance (Hsieh 1987). Tschoegl (1987) has also shown that, when the observed price consists of the equilibrium price and noise, observed prices can show first-order negative serial correlation. Thus, some of the statistically significant negative first-order daily serial correlations may simply reflect the presence of noise in the observed time-series. If this is the case, the absence of statistically significant autocorrelations may in some cases mean that negative correlations due to noise are largely offset by positive correlations. Another possible source of bias in daily data is leptokurtosis, which may create more "significant" t-statistics when autocorrelation coefficients are estimated (see Friedman and Vandersteel 1982; and the next section).

Table 3. Autocorrelations of Daily Exchange Rate Changes

Lag	U.S. Dollar/Sterling			U.S. Dollar/Deutsche mark			U.S. Dollar/Yen		
	Spot	3-month	12-month	Spot	3-month	12-month	Spot	3-month	12-month
				January 1, 1975 to December 31, 1977					
1	–0.01	–0.03	–0.01	–0.02	–0.00	0.01	0.11**	0.08**	0.07*
2	–0.04	–0.05	–0.05	–0.09**	–0.08**	–0.07**	–0.08**	–0.07*	–0.06*
3	0.04	0.07**	0.08**	0.03	0.01	0.02	0.06	0.05	0.03
4	–0.03	–0.04	–0.04	0.14**	0.13**	0.10**	0.02	–0.01	0.02
5	0.03	0.02	0.04	0.02	0.02	0.01	0.03	0.06	0.07*
				January 1, 1978 to December 31, 1980					
1	–0.01	–0.01	–0.01	0.01	–0.01	–0.00	0.07**	0.03	0.03
2	–0.04	–0.04	–0.04	–0.01	–0.00	0.02	–0.02	–0.01	–0.00
3	–0.01	–0.01	0.00	0.03	0.03	–0.06*	0.02	0.02	0.02
4	–0.03	–0.04	–0.02	–0.06	–0.07*	–0.14**	0.04	0.03	0.01
5	0.03	0.02	–0.02	–0.02	–0.02	–0.02	–0.02	–0.03	–0.03
				January 1, 1981 to December 31, 1983					
1	0.04	0.04	0.05	–0.04	–0.04	–0.03	–0.04	–0.12**	–0.13**
2	–0.02	–0.03	–0.02	0.07*	0.07*	0.08**	0.03	0.03	0.02
3	–0.08**	–0.08**	–0.06	0.00	0.02	0.02	0.02	0.03	0.01
4	0.07*	0.08**	0.06	–0.00	0.01	0.02	0.03	0.02	0.05
5	0.02	0.01	–0.01	0.02	0.01	0.00	0.09**	0.08**	0.06*
				January 1, 1984 to June 30, 1986					
1	0.03	0.03	0.03	0.01	0.01	–0.00	0.04	0.03	–0.10**
2	0.00	0.00	–0.00	0.00	0.00	0.01	0.03	0.02	0.03
3	0.06	0.05	0.05	0.12**	0.12**	0.12**	0.17**	0.18**	0.15**
4	–0.06	–0.06	–0.06	–0.01	–0.00	–0.00	0.05	0.05	0.05
5	–0.02	–0.02	–0.03	–0.02	–0.01	–0.01	0.02	0.02	0.02

Lag	Sterling/Deutsche mark			Sterling/Yen			Deutsche mark/Yen		
	Spot	3-month	12-month	Spot	3-month	12-month	Spot	3-month	12-month
				January 1, 1975 to December 31, 1977					
1	–0.07*	–0.07**	–0.05	–0.06	–0.08**	–0.06*	–0.00	–0.05	–0.06
2	–0.05	–0.05	–0.05	–0.06*	–0.06	–0.06	–0.09**	–0.08**	–0.05
3	0.08**	0.10**	0.12**	0.04	0.06*	0.06	0.02	–0.01	0.01
4	0.00	–0.02	–0.02	–0.02	–0.03	–0.04	0.14**	0.12**	0.09**
5	–0.01	–0.01	0.01	–0.04	–0.03	0.01	0.00	0.00	–0.03
				January 1, 1978 to December 31, 1980					
1	0.05	0.04	0.03	0.09**	0.08**	0.08**	0.18**	0.15**	0.10**
2	–0.01	–0.00	0.00	–0.05	–0.05	–0.05	–0.00	–0.01	–0.00
3	–0.00	–0.02	–0.10**	0.05	0.06	–0.05	0.02	0.02	–0.07*
4	0.05	0.06*	–0.00	0.10**	0.09**	0.07*	0.08**	0.09**	0.06
5	0.07*	0.07*	0.07*	–0.03	–0.05	–0.05	–0.03	–0.04	–0.05
				January 1, 1981 to December 31, 1983					
1	0.02	0.01	0.04	0.07*	–0.01	–0.01	0.13**	0.00	0.00
2	–0.02	–0.03	–0.01	–0.03	–0.05	–0.03	0.02	0.02	–0.00
3	0.09**	0.09**	0.08**	0.02	0.03	0.01	0.04	0.02	0.03
4	0.05	0.05	0.06	0.05	0.07*	0.08**	0.02	0.01	0.01
5	0.06*	0.06*	0.09**	0.04	0.01	0.00	–0.00	0.01	0.03
				January 1, 1984 to June 30, 1986					
1	0.04	–0.22**	–0.18**	0.12**	0.11**	0.03	0.03	–0.21**	–0.26**
2	0.06	0.03	0.01	0.06	0.05	0.05	0.04	0.05	0.08**
3	–0.01	0.04	0.04	0.05	0.05	0.06	0.08*	0.07*	0.04
4	–0.02	–0.05	–0.04	–0.11**	–0.11**	–0.10**	–0.08**	–0.10**	–0.10**
5	0.03	0.01	–0.00	–0.05	–0.05	–0.05	–0.01	–0.01	0.02

Note: **(*) indicates that the statistic is significant at 5(10) percent.

Table 4. Autocorrelations of Monthly Exchange Rate Changes

Lag	U.S. Dollar/Sterling			U.S. Dollar/Deutsche mark			U.S. Dollar/Yen		
	Spot	3-month	12-month	Spot	3-month	12-month	Spot	3-month	12-month
				January 1975 to December 1977					
1	0.21	0.22	0.22	–0.02	–0.03	–0.03	0.23	0.25	0.27*
2	0.15	0.12	0.14	0.11	0.14	0.18	–0.02	0.05	0.12
3	–0.09	–0.08	–0.09	–0.03	–0.03	–0.02	0.09	0.13	0.19
4	0.01	0.07	0.12	0.15	0.13	0.11	0.15	0.12	0.09
5	0.20	0.19	0.22	–0.01	–0.00	0.01	0.10	0.10	0.08
				January 1978 to December 1980					
1	–0.23	–0.26*	–0.28*	–0.38**	–0.41**	–0.43**	0.03	0.01	–0.01
2	–0.05	0.02	0.11	0.19	0.24	0.31**	–0.07	–0.02	0.08
3	–0.21	–0.20	–0.20	–0.11	–0.13	–0.15	0.21	0.22	0.20
4	–0.18	–0.18	–0.19	–0.08	–0.05	0.00	0.24	0.26*	0.28*
5	0.24	0.19	0.15	0.07	0.03	–0.07	–0.01	–0.02	–0.04
				January 1981 to December 1983					
1	–0.04	–0.03	0.01	0.23	0.24	0.28*	0.03	0.04	0.07
2	0.09	0.10	0.11	–0.21	–0.20	–0.17	–0.14	–0.13	–0.11
3	–0.14	–0.11	–0.04	0.01	0.04	0.08	0.06	0.06	–0.02
4	–0.21	–0.21	–0.22	–0.16	–0.14	–0.13	–0.18	–0.16	–0.10
5	0.11	0.14	0.04	–0.24	–0.23	–0.27*	0.11	0.10	0.05
				January 1984 to June 1986					
1	–0.11	–0.10	–0.09	–0.34**	–0.29**	–0.26	–0.03	–0.03	–0.03
2	0.18	0.16	0.09	0.41**	0.38**	0.34**	0.51**	0.52**	0.52**
3	0.08	0.08	0.08	0.07	0.10	0.11	0.14	0.15	0.15
4	0.34**	0.35**	0.38**	0.21	0.22	0.21	0.31*	0.31*	0.28*
5	–0.07	–0.10	–0.15	0.12	0.12	0.11	0.37*	0.38**	0.38**

Lag	Sterling/Deutsche mark			Sterling/Yen			Deutsche mark/Yen		
	Spot	3-month	12-month	Spot	3-month	12-month	Spot	3-month	12-month
				January 1975 to December 1977					
1	0.06	0.08	0.12	0.07	0.11	0.17	–0.09	–0.09	–0.11
2	–0.08	–0.06	–0.04	–0.07	–0.10	–0.09	–0.04	–0.09	–0.14
3	–0.32**	–0.32**	–0.33*	–0.35**	–0.36**	–0.33**	–0.26*	–0.20	–0.18
4	0.04	0.03	0.00	0.08	0.14	0.13	0.05	0.01	–0.03
5	0.19	0.14	0.14	0.22	0.30*	0.31**	0.04	0.03	0.02
				January 1978 to December 1980					
1	–0.02	0.03	0.10	0.14	0.18	0.27*	0.04	0.05	0.03
2	0.05	0.06	0.12	–0.02	–0.02	0.08	0.31**	0.30**	0.31**
3	–0.17	–0.15	–0.13	0.47**	0.48**	0.44**	0.24	0.24	0.21
4	–0.06	–0.04	–0.00	0.18	0.23	0.31**	–0.11	–0.07	–0.04
5	–0.11	–0.13	–0.17	0.00	0.00	0.00	0.14	0.16	0.17
				January 1981 to December 1983					
1	–0.07	–0.01	0.09	–0.00	0.03	0.11	–0.14	–0.11	–0.04
2	–0.10	–0.09	–0.08	0.07	0.09	0.13	–0.14	–0.12	–0.08
3	0.12	0.12	0.15	–0.05	–0.07	–0.11	0.15	0.14	0.10
4	–0.10	–0.14	–0.19	–0.08	–0.11	–0.12	–0.04	–0.02	0.04
5	–0.44**	–0.44**	–0.48**	–0.36**	–0.33**	–0.37**	0.18	0.17	0.10
				January 1984 to June 1986					
1	0.05	0.15	0.14	0.00	–0.00	–0.03	–0.36**	–0.32**	–0.30*
2	0.06	0.00	–0.06	0.06	0.03	0.01	–0.01	–0.03	–0.01
3	–0.12	–0.14	–0.20	0.05	0.05	0.01	–0.03	0.00	0.01
4	0.04	0.01	0.04	0.15	0.18	0.22	0.08	0.07	0.08
5	–0.05	–0.07	–0.07	–0.00	–0.03	–0.06	0.05	0.06	0.05

Note: **(*) indicates that the statistic is significant at 5(10) percent.

changes, although each exchange rate series has at least one statistically significant sample autocorrelation. Third, when sample autocorrelations are statistically significant, their values tend to be much larger for monthly data (amounting to as much as 0.5) than for daily data (amounting to no more than 0.26). This probably indicates the presence of low frequency movements which are not detected by daily data.[13]

The presence of some time dependency in high frequency data is also apparent from the comparison of the standard deviations of daily and monthly changes (see Tables 9 and 10 in the following section). If we think of a month as consisting of about 22 trading days, strict independence implies that the monthly variance would be 22 times the daily variance; that is, the monthly standard deviation should be roughly 5 times the daily standard deviation. However, some of the monthly standard deviations were in practice somewhat smaller than what independence would suggest, implying that the sample autocorrelations were on average negative. This conclusion would be strengthened if we think of a month as consisting of 30 calendar days.[14]

3. Time-series Representations of Exchange Rate Changes

Deviations from random walks may be better detected if one simply fits an n^{th} order AR process to the exchange rate series as,

$$d_t = \sum_{j=1}^{n} \lambda_j d_{t-j} + \varepsilon_t, \tag{12}$$

where d_t can be either Δe_t or Δf_t, λ_j's are coefficients, and ε_t is a white-noise shock. Table 5 and Table 6 report the t-values of autoregressive coefficients with order 5. We find not surprisingly that the estimated AR coefficients provide similar information as that provided by the estimated autocorrelations. As we found from autocorrelation analysis, the presence of statistically significant coefficients in monthly data indicates the presence of systematic low frequency movements in the exchange rates which could not be detected by daily data. For both daily and monthly exchange rate changes, one can almost always find an AR representation that fits the data better than pure white noise. However, the deviations of the first-differenced time-series from white noise are generally very small.

Any AR process can alternatively be expressed as a Moving Average (MA) process of infinite order or, as a practical matter, approximated as an MA process of high order (ℓ) as,

$$d_t = \varepsilon_t + \sum_{j=1}^{\ell} \theta_j \varepsilon_{t-j} \tag{13}$$

13. Some of the larger absolute values of monthly coefficients relative to those of daily coefficients when the coefficients are negative can be a statistical artifact. It can be shown that, if the true process is AR(1) in daily data with the AR coefficient close to unity, first-order autocorrelation coefficients are negative and the monthly coefficients are larger in absolute value than the daily coefficients.

14. It should also be noted that a few of the monthly standard deviations were larger than what independence would suggest, implying that the sample autocorrelations were on average positive in these cases.

Table 5. Autoregressive Representations of Daily Exchange Rate Changes (t-values)

Lag	U.S. Dollar/Sterling			U.S. Dollar/Deutsche mark			U.S. Dollar/Yen		
	Spot	3-month	12-month	Spot	3-month	12-month	Spot	3-month	12-month
				January 1, 1975 to December 31, 1977					
1	–0.29	–0.61	0.05	0.41	–0.15	0.36	3.45**	2.89**	2.10**
2	–1.26	–1.38	–1.54	–2.14**	–2.03**	–1.83*	–2.95**	–1.91*	–1.52
3	1.17	1.93*	2.28**	0.84	0.22	0.54	2.32**	2.02**	1.38
4	–0.71	–1.05	–1.19	3.52**	3.47**	2.68**	–0.31	–0.83	0.16
5	0.95	0.82	1.36	0.74	0.57	0.33	1.49	2.11**	2.14**
				January 1, 1978 to December 31, 1980					
1	–0.22	–0.28	–0.27	0.40	–0.15	–0.29	1.85*	0.97	0.70
2	–1.05	–1.05	–1.25	–0.21	–0.09	0.76	–0.57	–0.25	–0.02
3	–0.16	–0.34	–0.04	0.99	0.77	–1.71*	0.65	0.43	0.59
4	–0.84	–1.30	–0.64	–1.85	–2.11**	–4.40**	0.92	0.92	0.15
5	1.10	0.69	–0.42	–0.39	–0.70	–0.64	–0.75	–0.81	–0.84
				January 1, 1981 to December 31, 1983					
1	1.29	1.45	1.67*	–0.80	–0.77	–0.69	–1.14	–3.10**	–3.79**
2	–0.58	–0.83	–0.42	2.01**	1.95*	2.34**	0.64	0.50	0.03
3	–1.86*	–1.71*	–1.34	0.15	0.63	0.70	0.49	0.87	0.58
4	2.14**	2.58**	1.90*	–0.15	0.23	0.47	0.83	1.03	1.63
5	0.37	–0.13	–0.38	0.47	0.30	–0.01	2.52**	2.33**	2.01**
				January 1, 1984 to June 30, 1986					
1	0.86	0.80	0.89	0.24	0.19	–0.08	0.57	0.40	–2.94**
2	0.02	0.03	–0.07	0.04	0.12	0.42	0.47	0.35	0.99
3	1.44	1.33	1.22	3.08**	3.08**	2.99**	4.23**	4.51**	4.18**
4	–1.57	–1.56	–1.48	–0.29	–0.14	0.00	1.12	1.22	1.99**
5	–0.44	–0.48	–0.62	–0.39	–0.38	–0.41	0.15	0.24	0.49

Lag	Sterling/Deutsche mark			Sterling/Yen			Deutsche mark/Yen		
	Spot	3-month	12-month	Spot	3-month	12-month	Spot	3-month	12-month
				January 1, 1975 to December 31, 1977					
1	–1.90*	–2.02**	–1.29	–1.68*	–2.18**	–1.70*	–0.13	–1.35	–1.53
2	–1.35	–1.42	–1.38	–1.75*	–1.71*	–1.66	–2.07**	–1.91*	–1.24
3	2.13**	2.50**	3.05**	1.03	1.51	1.54	0.53	–0.23	0.30
4	0.30	–0.21	–0.31	–0.44	–0.86	–0.91	3.76**	3.21**	2.45**
5	–0.15	0.02	0.48	–0.88	–0.87	0.28	0.24	0.37	–0.62
				January 1, 1978 to December 31, 1980					
1	1.23	1.00	0.87	2.81**	2.38**	2.40**	5.25**	4.39**	3.11**
2	–0.17	–0.05	0.33	–1.49	–1.56	–1.57	–0.97	–1.00	–0.28
3	–0.14	–0.69	–2.75**	1.35	1.57	1.30	0.23	0.34	–1.99*
4	–1.35	1.64	0.04	2.39**	2.23**	1.70*	2.41**	2.55**	2.19**
5	1.81*	1.83*	1.89*	–1.31	–1.52	–1.58	–1.77*	–1.79*	–1.74*
				January 1, 1981 to December 31, 1983					
1	0.44	0.17	0.86	2.02**	–0.33	–0.26	3.57**	0.01	0.02
2	–0.66	–0.93	–0.44	–0.80	–1.04*	–0.69	0.09	0.71	–0.07
3	2.43**	2.63*	2.22**	0.79	0.85	0.21	0.81	0.51	1.03
4	1.16	1.31	1.26	1.15	2.00**	2.26**	0.45	0.36	0.39
5	1.70*	1.87*	2.27**	1.18	0.49	0.08	–0.18	0.16	0.74
				January 1, 1984 to June 30, 1986					
1	0.91	–5.23**	–4.47**	2.82**	2.58**	0.74	0.84	–5.10**	–6.07**
2	1.58	–0.10	–0.50	1.08	1.07	1.31	1.00	0.71	0.71
3	–0.31	0.83	0.70	1.29	1.25	1.53	1.89*	1.73*	1.20
4	–0.65	–0.77	–0.81	–2.94**	–2.92**	–2.70**	–2.23**	–1.99**	–1.90*
5	0.89	–0.27	–0.41	–0.68	–0.87	–1.27	–0.21	–1.21	–0.62

Note: **(*) indicates that the t-value is significant at 5(10) percent.

Table 6. Autoregressive Representations of Monthly Exchange Rate Changes (t-values)

Lag	U.S. Dollar/Sterling			U.S. Dollar/Deutsche mark			U.S. Dollar/Yen		
	Spot	3-month	12-month	Spot	3-month	12-month	Spot	3-month	12-month
				January 1975 to December 1977					
1	1.18	1.20	1.06	−0.06	−0.08	−0.13	1.39	1.57	1.62
2	0.88	0.59	0.63	0.48	0.70	0.97	−0.33	−0.24	0.02
3	−0.48	−0.41	−0.82	−0.15	−0.09	−0.05	0.50	0.50	0.73
4	−0.01	0.32	0.46	0.80	0.57	0.34	0.42	0.17	0.03
5	1.17	0.88	1.06	0.01	0.05	0.10	0.22	0.22	0.17
				January 1978 to December 1980					
1	−1.81*	−1.85*	−1.33	−1.96*	−2.08**	−2.04**	0.03	−0.10	−0.19
2	−1.06	−0.73	−0.04	0.22	0.54	0.96	−0.37	−0.11	0.42
3	−1.80*	−1.68	−1.51	−0.49	−0.24	0.08	1.21	1.35	1.31
4	−1.65	−1.66	−1.78*	−0.72	−0.68	−0.75	1.28	1.52	1.65
5	0.22	0.12	0.14	0.00	−0.13	−0.56	0.02	−0.03	−0.26
				January 1981 to December 1983					
1	−0.18	−0.04	0.10	1.39**	2.26**	2.87**	0.47	0.45	0.44
2	0.73	0.80	0.82	−2.02**	−2.14**	−2.44**	−1.01	−0.91	−0.68
3	−0.88	−0.67	−0.27	1.08	1.74*	2.49**	0.56	0.51	0.07
4	−0.72	−0.66	−0.71	−0.41	−1.50	−1.81*	−1.11	−0.96	−0.57
5	0.72	0.83	0.30	−0.39	−0.14	0.03	0.89	0.81	0.38
				January 1984 to June 1986					
1	−0.65	−0.55	−0.49	−2.09**	−1.83*	−1.65	−0.69	−0.75	−0.43
2	0.68	0.62	0.40	1.60	1.54	1.42	2.34**	2.48**	2.38**
3	0.75	0.72	0.38	1.83*	1.67	1.54	0.33	0.43	0.52
4	1.91*	2.02**	1.90*	1.37	1.37	1.30	0.47	0.49	0.42
5	−0.11	−0.25	−0.42	0.58	0.57	0.50	1.13	1.25	1.39

Lag	Sterling/Deutsche mark			Sterling/Yen			Deutsche mark/Yen		
	Spot	3-month	12-month	Spot	3-month	12-month	Spot	3-month	12-month
				January 1975 to December 1977					
1	0.34	0.52	0.71	0.40	0.68	1.01	−0.34	−0.54	−0.76
2	−0.13	−0.05	0.10	0.06	0.08	0.06	−0.41	−0.73	−1.15
3	−0.95	−0.96	−1.36	−1.00	−1.08	−1.02	−1.54	−1.21	−1.23
4	0.35	0.42	0.35	0.61	1.04	1.12	−0.04	−0.24	−0.60
5	0.81	0.57	0.63	0.91	1.18	1.12	0.09	−0.12	−0.35
				January 1978 to December 1980					
1	−0.13	0.18	0.58	0.79	1.06	1.31	0.03	0.06	−0.03
2	0.19	0.34	0.57	−0.60	−0.78	−0.30	2.08**	1.79*	1.88
3	−0.99	−0.83	−0.71	4.06**	4.23**	3.10**	1.14	1.16	0.92
4	−0.32	−0.09	0.12	0.21	0.22	0.64	−1.12	−1.14	−0.72
5	−0.48	−0.55	−0.86	0.16	0.14	−0.58	−0.04	0.07	0.21
				January 1981 to December 1983					
1	−0.70	−0.28	0.37	−0.14	0.00	0.39	−0.80	−0.62	−0.27
2	−0.53	−0..41	−0.33	0.43	0.58	0.72	−0.95	−0.79	−0.43
3	0.40	0.60	1.07	−0.12	−0.21	−0.47	0.87	0.89	0.62
4	−0.44	−0.53	−0.96	−0.45	−0.57	−0.44	0.01	0.06	0.22
5	−3.08**	−3.03**	−2.28**	−1.80*	−1.62	−1.65	1.14	0.97	0.54
				January 1984 to June 1986					
1	0.31	0.80	0.80	−0.03	−0.03	−0.13	−2.03**	−1.81*	−1.66
2	0.26	−0.04	−0.31	0.24	0.14	0.05	−0.79	−0.70	−0.52
3	−0.56	−0.66	−0.90	0.27	0.24	0.09	−0.30	−0.11	0.02
4	0.22	0.29	0.49	0.86	1.03	1.34	0.30	0.39	0.50
5	−0.14	−0.27	−0.40	−0.04	−0.13	−0.23	0.55	0.62	0.63

Note: **(*) indicates that the t-value is significant at 5(10) percent.

where θ_j's are coefficients. The advantage of an MA representation for our purpose is that it has a convenient interpretation as a series of news items reaching the market (Granger and Newbold 1986). Table 7 and Table 8 report the estimated MA coefficients for daily and monthly first-differenced exchange rate series, where the moving average process was arbitrarily truncated at lag 5; the arbitrary choice of 5 lags was made on the *a priori* ground that, for daily series, information should not take more than a week to fully affect the market; if there is serial correlation in the news, however, information of higher lags could affect the market. Although there are statistically significant coefficients in almost all time-series, comparison with Table 5 and Table 6 show that the MA coefficients are in many cases mirror images of the AR coefficients, confirming that the changes in exchange rates were close to white noise, i.e., $d_{t-j} \approx \varepsilon_{t-j}$.

4. Comparison with Previous Studies

These results are broadly consistent with the results of various types of serial correlation tests. Starting with the pioneering work of Poole (1967) on European exchange rates in the 1920s and the Canadian dollar rate in the 1950s, most empirical studies that directly tested for serial correlation have shown that nominal exchange rate changes have weak but statistically significant serial dependency. For example, Giddy and Dufey (1975) found such serial dependency in daily data from the early 1970s as well as the 1920s for the U.S. dollar exchange rates of the Canadian dollar, pound sterling and French franc; MA models performed marginally better than a random walk model, although the degree of serial dependency was too small to generate profitable opportunities. Burt, Kaen and Booth (1977) also found some significant serial correlation in daily spot rate changes of the Canadian dollar, deutsche mark, and pound sterling against the U.S. dollar during 1973–75, although they attributed it to type II errors. Finally, Dooley and Shafer (1983) also found significant departures from random walks based on the Q statistics (i.e., weighted sums of squared serial correlations) on daily U.S. dollar exchange rates of nine major currencies during 1973–81.

It remains true, however, that the empirical deviations of exchange rate changes from random walks were small and that the first-differenced exchange rate series were close to stationary. This broadly random walk property of nominal exchange rates has at least two implications. First, it implies that the underlying fundamental economic variables followed a process that is almost indistinguishable from a random walk (Nelson and Plosser 1982; and Campbell and Mankiw 1987). However, at the same time, evidence suggests the presence of serial correlation in low frequency data which was not detected in high frequency data. The systematic movements in low frequency data may correspond to the news concerning GNP, money supplies, and other macroeconomic variables whose arrival in the market are at most monthly. Second, the approximately random walk property of exchange rates casts some doubt on the presence of bubbles in the foreign exchange market, because the presence of a rational bubble in a time-series implies in

Table 7. Moving Average Representations of Daily Exchange Rate Changes (t-values)

Lag	U.S. Dollar/Sterling			U.S. Dollar/Deutsche mark			U.S. Dollar/Yen		
	Spot	3-month	12-month	Spot	3-month	12-month	Spot	3-month	12-month
				January 1, 1975 to December 31, 1977					
1	0.31	0.60	−0.07	−0.48	0.17	0.36	−3.47**	−2.72**	−1.96**
2	1.01	1.38	1.58	2.19**	2.08**	1.86*	2.54**	2.17*	1.78*
3	−1.31	−2.01*	−2.34**	−0.79	−0.24	0.50	−1.67*	−1.49	−1.25
4	0.75	1.10	1.15	−3.83**	−3.73**	2.87**	−0.49	0.34	−0.41
5	−0.76	−0.66	−1.14	−0.62	−0.51	0.28	−0.94	−1.67*	−1.91*
				January 1, 1978 to December 31, 1980					
1	0.22	0.28	0.27	−0.42	0.16	0.33	−1.85*	−0.97	−0.70
2	1.05	1.05	1.26	0.21	0.10	−0.83	0.45	0.22	0.00
3	0.14	0.32	0.01	−0.78	−0.86	1.06	−0.58	−0.42	−0.59
4	0.80	1.26	0.62	1.80*	2.17**	4.54**	−1.22	−1.15	−0.19
5	−1.15	−0.76	0.43	0.49	0.42	0.77	0.66	0.76	0.84
				January 1, 1981 to December 31, 1983					
1	−1.29	−1.45	−1.73*	1.00	0.77	0.86	1.16	3.14**	3.83**
2	0.51	0.75	0.33	−1.90*	−1.99**	−2.19**	−0.84	−1.03	−0.63
3	1.93*	1.79*	1.04	−0.01	−0.49	−0.52	−0.45	−0.73	−0.52
4	−1.97*	−2.42**	−1.78*	0.02	−0.32	−0.69	−0.83	−0.88	−1.52
5	−0.66	−0.24	0.15	−0.55	−0.36	−0.05	−2.52**	−2.24**	−1.69*
				January 1, 1984 to June 30, 1986					
1	−0.86	−0.80	−0.89	−0.20	−0.15	0.08	−0.56	−0.39	2.95**
2	−0.04	−0.06	0.04	−0.04	−0.12	−0.35	−0.62	−0.48	−1.22
3	−1.45	−1.35	−1.23	−3.14**	−3.13**	−3.04**	−4.42**	−4.71**	−4.08**
4	1.49	1.49	1.40	0.25	0.10	0.01	−1.22	−1.27	−1.21
5	0.65	0.69	0.88	0.48	0.43	0.41	−0.42	−0.47	−0.44

Lag	Sterling/Deutsche mark			Sterling/Yen			Deutsche mark/Yen		
	Spot	3-month	12-month	Spot	3-month	12-month	Spot	3-month	12-month
				January 1, 1975 to December 31, 1977					
1	1.90*	2.02**	1.28	1.69*	2.20**	1.69*	0.14	1.44	1.52
2	1.22	1.27	1.33	1.66*	1.55	1.57	2.14**	2.01**	1.17
3	−2.32**	−2.73**	−3.22**	−1.29	−1.84*	−1.49	−0.56	0.05	−0.39
4	0.00	0.54	0.56	0.47	1.01	1.00	−4.06**	−3.10**	−2.51**
5	0.39	0.21	−0.15	0.97	0.92	−0.28	−0.10	−0.04	0.94
				January 1, 1978 to December 31, 1980					
1	−1.24	−1.01	−0.90	−2.83**	−2.41**	−2.43**	−5.28**	−4.43**	−3.38**
2	−0.11	0.01	−0.37	1.25	1.41	1.40	0.04	0.43	−0.07
3	0.18	0.70	2.80**	−1.29	−1.58	−1.25	0.00	−0.14	1.63
4	−1.36	−1.62	0.14	−2.77**	−2.62**	−2.03*	−2.49**	−2.68**	−1.93*
5	−1.96**	−1.99**	−2.26**	0.98	1.34	1.45	1.15	1.08	1.44
				January 1, 1981 to December 31, 1983					
1	−0.42	−0.17	−0.95	−2.10**	0.31	0.26	−3.57**	−0.01	−0.03
2	0.72	1.02	0.42	0.53	1.30	0.70	−0.65	−0.73	0.07
3	−2.77**	−3.00**	−2.45**	−0.69	0.99	−0.19	−0.89	−0.56	−0.95
4	−1.43	−1.59	−1.58	−1.34	1.92*	−2.31**	−0.60	−0.41	−0.38
5	−1.00	−1.06	−1.89*	−1.37	0.35	−0.03	0.04	−0.11	−0.73
				January 1, 1984 to June 30, 1986					
1	−0.91	5.23**	4.47**	−3.06**	−2.80**	−0.67	−0.84	5.11**	6.07**
2	−1.62	−1.00	−0.30	−1.58	−1.52	−1.49	−1.04	−1.76*	−2.21**
3	0.20	−0.67	−0.74	−1.72*	−1.64	−1.75	−1.99**	−1.25	−0.48
4	0.57	1.34	1.30	2.10**	2.12**	2.09**	2.11**	2.61**	2.28**
5	−0.99	−0.17	0.09	1.35	1.51	1.40	0.24	0.13	0.74

Note: **(*) indicates that the t-value is significant at 5(10) percent.

Table 8. Moving Average Representations of Monthly Exchange Rate Changes (t-values)

Lag	U.S. Dollar/Sterling			U.S. Dollar/Deutsche mark			U.S. Dollar/Yen		
	Spot	3-month	12-month	Spot	3-month	12-month	Spot	3-month	12-month
				January 1975 to December 1977					
1	-1.29	-1.30	-1.36	0.06	0.13	0.12	-1.39	-1.47	-1.52
2	-1.28	-0.96	-1.18	-0.58	-0.77	-0.98	0.26	-0.10	-0.42
3	0.71	0.64	0.82	0.16	0.12	0.09	-0.38	-0.63	-0.94
4	0.17	-0.16	-0.41	-0.72	-0.64	-0.51	-0.70	-0.50	-0.40
5	-0.59	-0.53	-0.65	0.04	0.02	-0.07	-0.29	-0.29	-0.23
				January 1978 to December 1980					
1	1.39	0.90	0.96	2.20**	2.24**	2.02**	0.00	0.22	0.44
2	0.41	0.08	-0.63	-0.54	-0.94	-1.65	0.31	0.04	-0.52
3	1.11	1.34	1.96*	0.90	0.90	0.85	-1.13	-1.34	-1.43
4	0.72	0.86	0.95	0.32	0.21	0.15	-1.55	-1.75*	-1.95*
5	-1.93*	-1.63	-0.91	-0.42	-0.17	0.53	0.05	0.10	0.19
				January 1981 to December 1983					
1	0.17	0.04	-0.09	-2.30**	-2.62**	-3.79**	-0.77	-0.61	-0.54
2	-0.89	-1.05	-0.96	1.86*	1.75*	1.47	1.61	1.31	0.68
3	1.09	0.94	0.29	0.21	-0.04	-0.05	-0.66	-0.53	0.02
4	0.72	0.78	0.74	0.64	0.40	-0.15	1.19	0.97	0.53
5	-0.68	-0.88	-0.22	0.98	0.93	1.25	-0.61	-0.56	-0.26
				January 1984 to June 1986					
1	0.85	0.74	0.59	1.90*	1.66	2.46**	2.55**	1.07	2.95**
2	-1.16	-1.07	-0.65	-4.21**	-3.59**	-2.73**	-2.73**	-3.08**	-2.73**
3	-0.49	-0.51	-0.93	-1.11	-1.32	-1.40	-0.97	-2.14**	-0.59
4	-2.17**	-2.31**	-2.49**	-2.49**	-2.12**	-1.20	-0.29	-0.91	-0.57
5	0.41	0.59	0.47	-1.05	-0.99	-1.04	-4.10**	-4.94**	-2.45**

Lag	Sterling/Deutsche mark			Sterling/Yen			Deutsche mark/Yen		
	Spot	3-month	12-month	Spot	3-month	12-month	Spot	3-month	12-month
				January 1975 to December 1977					
1	-0.47	-0.63	-1.09	-0.74	-2.23**	-2.14**	0.57	0.41	0.60
2	-0.01	-0.10	-0.34	-0.41	-1.28	-1.18	0.30	0.67	1.07
3	1.23	1.17	1.07	1.68	0.41	1.45	1.65	1.14	1.01
4	-0.15	-0.13	0.08	-0.43	-0.75	-0.49	-0.28	-0.10	0.15
5	-1.18	-0.84	-0.77	-1.49	-1.55	-2.18**	-0.23	-0.16	-0.08
				January 1978 to December 1980					
1	0.17	-0.18	-0.36	-0.82	-1.07	-1.09	-0.04	-0.06	0.03
2	-0.19	-0.35	-0.73	0.46	0.42	-0.09	-1.98	-1.78*	-1.77**
3	0.95	0.82	0.67	-6.95**	-7.84**	-6.26**	-1.29	-1.29	-0.99
4	0.18	0.11	-0.08	-0.28	-1.25	-2.04**	0.65	0.44	0.25
5	0.59	0.65	1.00	-0.00	-0.01	-0.02	-0.81	-0.87	-0.95
				January 1981 to December 1983					
1	1.10	0.60	-0.21	0.19	-0.00	-0.47	0.78	0.61	0.30
2	0.88	0.49	-0.13	-0.47	-0.73	-1.00	0.88	0.76	0.50
3	-0.40	-0.75	-1.46	0.15	0.22	0.38	-1.20	-1.11	-0.67
4	0.94	1.53	1.53	0.56	0.66	0.56	0.11	0.02	-0.26
5	0.44	1.60	2.32**	1.55	1.85*	2.12**	-1.05	-0.95	-0.58
				January 1984 to June 1986					
1	-0.34	-0.80	-0.80	0.03	0.03	0.12	1.63	1.45	1.33
2	-0.32	-0.09	0.18	-0.24	-0.14	-0.05	0.01	0.09	0.00
3	0.61	0.83	1.24	-0.27	-0.23	-0.08	-0.06	-0.21	-0.23
4	-0.20	-0.09	-0.24	-0.87	-1.02	-1.32	-0.56	-0.51	-0.54
5	0.12	0.28	0.29	0.01	0.12	0.29	-0.28	-0.33	-0.28

Note: **(*) indicates that the t-value is significant at 5(10) percent.

certain cases that the time-series obtained by differencing should not have stationary means (Diba and Grossman 1985; Obstfeld 1987).

IV. Distributional Properties of Exchange Rate Changes

1. Leptokurtosis in Exchange Rate Changes

The distributional characteristics of empirical exchange rates have important implications for statistical inference in empirical work. It is well known that the empirical distribution of short-run changes in asset and commodity prices are often too "peaked" and too "fat-tailed" to fit the normal distribution (Mandelbrot 1963), and the empirical distribution of exchange rate changes has not been an exception. One quantitative measure of such a distributional characteristic can be given by kurtosis,

$$a_4 \equiv \frac{m_4}{\sigma^4},$$

where m_4 is the fourth moment about the mean and σ is the standard deviation. A distribution that is too "peaked" and too "fat-tailed" relative to the normal has a value of kurtosis (a_4) greater than 3; such a distribution is called leptokurtic.

Leptokurtosis was first observed in the context of the foreign exchange market by Giddy and Dufey (1975) on the basis of the daily exchange rate changes of the U.S. dollar against the Canadian dollar, pound sterling, and French franc during the first few years of the recent floating period as well as during the 1920s. More recently, Islam (1982) found significant leptokurtosis in the daily exchange rates of the deutsche mark against the U.S. dollar during 1973–81, and Friedman and Vandersteel (1982) for the daily exchange rates of nine major currencies against the U.S. dollar during 1973–79. As to weekly exchange rate changes, Westerfield (1977) calculated the values of kurtosis in the spot, 1–month, 2–month, and 3–month forward exchange rates of the Canadian dollar, pound sterling, deutsche mark, Swiss franc, and Dutch guilder relative to the U.S. dollar during the 1973–75 period, and concluded that the weekly distributions were indeed leptokurtic.[15]

Table 9 presents basic statistical measures of daily changes in spot, 3–month and 12–month forward exchange rates during 1975–86. We note that, although the distributions were almost always symmetrical, all values of kurtosis were significantly greater than 3, indicating that the distributions were more leptokurtic than the normal distribution would imply.[16] Although leptokurtosis was a broad characteristic of the empirical

15. Westerfield (1977) also analyzed the distribution of weekly exchange rate changes for the fixed exchange rate period. Her analysis shows that the degree of leptokurtosis was greater for fixed exchange rates; the estimated characteristic exponents were lower than the estimates for the flexible period, ranging between 1.00 and 1.44, when a stable Paretian distribution was fitted (see the next subsection).

16. So (1987) found similar leptokurtic distributions for the daily futures prices of the U.S. dollar against the pound sterling, deutsche mark and Japanese yen during 1974–82.

Table 9. Basic Statistics of Daily Exchange Rate Changes

	U.S. Dollar/Sterling			U.S. Dollar/Deutsche mark			U.S. Dollar/Yen		
	Spot	3-month	12-month	Spot	3-month	12-month	Spot	3-month	12-month
				January 1, 1975 to December 31, 1977					
Size	774	774	762	774	774	762	773	773	762
Mean	−0.00	−0.00	−0.00	0.00	0.00	0.00	0.00	0.00	0.00
Standard deviation	0.005	0.005	0.006	0.004	0.004	0.004	0.003	0.003	0.003
Skewness	−0.59	−0.69	−0.69	−0.39	−0.30	−0.36	0.92	0.71	1.00
Kurtosis	17.72**	16.24**	13.74**	5.24**	5.57**	5.81**	6.94**	7.52**	9.49**
				January 1, 1978 to December 31, 1980					
Size	767	767	767	767	767	767	767	767	767
Mean	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Standard deviation	0.006	0.006	0.006	0.006	0.006	0.007	0.007	0.007	0.008
Skewness	−1.51	−1.58	−1.31	−1.83*	−1.98**	−1.06	−0.17	−0.16	−0.32
Kurtosis	14.52**	15.40**	14.79**	24.61**	26.66**	26.81**	7.92**	8.09**	8.03**
				January 1, 1981 to December 31, 1983					
Size	758	758	758	758	758	758	758	758	758
Mean	−0.00	−0.00	−0.00	−0.00	−0.00	−0.00	−0.00	−0.00	−0.00
Standard deviation	0.007	0.007	0.006	0.007	0.007	0.007	0.007	0.007	0.007
Skewness	−0.37	−0.46	−0.63	0.14	0.14	0.12	0.39	0.15	0.42
Kurtosis	5.07**	5.16**	5.80**	3.39**	3.39**	3.52**	3.95**	9.20**	5.84**
				January 1, 1984 to June 30, 1986					
Size	631	638	638	631	639	639	632	638	638
Mean	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Standard deviation	0.009	0.009	0.009	0.008	0.008	0.008	0.006	0.006	0.007
Skewness	0.47	0.46	0.44	0.69	0.68	0.65	1.17	1.15	0.89
Kurtosis	6.53**	6.48**	6.42**	5.96**	5.97**	5.93**	10.13**	9.86**	10.87**

	Sterling/Deutsche mark			Sterling/Yen			Deutsche mark/Yen		
	Spot	3-month	12-month	Spot	3-month	12-month	Spot	3-month	12-month
				January 1, 1975 to December 31, 1977					
Size	774	774	762	773	773	762	773	773	762
Mean	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Standard deviation	0.006	0.006	0.007	0.005	0.006	0.006	0.004	0.004	0.004
Skewness	0.82	0.77	0.79	0.60	0.70	0.60	0.39	0.35	0.32
Kurtosis	13.73**	13.22**	12.62**	12.38**	11.73**	10.17**	4.81**	5.05**	4.94**
				January 1, 1978 to December 31, 1980					
Size	767	767	767	767	767	767	767	767	767
Mean	−0.00	−0.00	−0.00	−0.00	−0.00	−0.00	0.00	0.00	0.00
Standard deviation	0.005	0.005	0.006	0.007	0.007	0.008	0.006	0.006	0.007
Skewness	0.53	0.51	0.48	0.24	0.24	0.24	0.46	0.33	−0.16
Kurtosis	6.08**	5.70**	13.51**	4.60**	4.72**	4.48**	4.74**	4.94**	12.14**
				January 1, 1981 to December 31, 1983					
Size	758	758	758	758	758	758	758	758	758
Mean	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Standard deviation	0.006	0.006	0.006	0.006	0.007	0.007	0.005	0.006	0.006
Skewness	0.28	0.25	0.40	0.52	0.37	0.79	0.12	−0.02	0.11
Kurtosis	5.41**	5.16**	5.38**	4.64**	9.55**	7.32**	4.92**	14.88**	9.01**
				January 1, 1984 to June 30, 1986					
Size	631	596	596	631	638	638	631	596	596
Mean	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Standard deviation	0.005	0.007	0.008	0.007	0.008	0.009	0.006	0.008	0.008
Skewness	0.54	0.12	−0.03	0.26	0.22	0.05	−0.34	−0.03	−0.06
Kurtosis	5.26**	7.47**	6.80**	5.47**	5.36**	6.50**	5.22**	6.01	6.94**

Note: For skewness and kurtosis, **(*) indicates a significant departure from normality at 5(10) percent.

distributions of daily and weekly exchange rate changes, however, an important qualification should be made: the values of kurtosis were close to 3 in a few cases. When we recognize that the large degrees of freedom tend to increase the power of the test and make it more likely to reject the null hypothesis of normality, we may characterize these distributions as approximately normal. Moreover, it is evident that the values of kurtosis varied over time even for the same exchange rate.

2. Three Popular Distributional Hypotheses[17]

Broadly speaking, three types of popular theories have been advanced to account for the leptokurtosis of the empirical distributions of daily and weekly exchange rate changes. First, Westerfield (1977) followed Mandelbrot (1963) in suggesting that, as in the stock market, empirical distributions could be best described as a family of symmetric stable Paretian distributions, of which the normal distribution is a special case. The log characteristic function of symmetric stable Paretian distributions can be given by,

$$F(\tau)=ic\tau-s|\tau|^{\alpha}, \tag{14}$$

where τ is any real number, c is the location parameter, s is the scale parameter and α is the characteristic exponent. Of these parameters, the most important is the characteristic exponent, which ranges between 0 and 2 in value and determines the kurtosis of the distribution; the characteristic exponent of the normal distribution is 2. The estimates of the characteristic exponent for weekly exchange rate changes have ranged between 1.25 and 1.85, while the estimates for daily changes have been somewhat smaller, ranging between 1.11 and 1.75 (Westerfield 1977; Islam 1982; McFarland, Pettit and Sung 1982; Friedman and Vandersteel 1982; and Boothe and Glassman 1987). An important attribute of stable Paretian distributions is that they are invariant under addition. That is to say, if the distribution of daily returns is stable Paretian, the distribution of weekly and monthly returns must also be stable Paretian with the same characteristic exponent.

While the stable Paretian distribution is consistent with the notion that the economic forces underlying exchange rate determination are subject to large and abrupt changes, the Paretian distribution has one undesirable implication: finite variance exists only if the characteristic exponent is 2.[18] Operationally, what this means is that a sample variance is likely to be unstable and does not converge as the sample size is increased. The non-normal stable Paretian distribution means that the sample variance is no longer an adequate measure of foreign exchange risk; the greater probability of large losses implicit in a market characterized by such a probability distribution suggests that the market is inherently riskier than one characterized by the normality of returns. Moreover, the

17. In a general distribution of Bookstaber and McDonald (1987), all the three hypotheses can be considered as special cases of the mixed distribution hypothesis.

18. Pareto's law is a generalization of the central limit theorem to the case where there is no finite variance.

absence of finite variance makes many standard statistical concepts less than appropriate, although, as long as the characteristic exponent is greater than unity, the sample serial correlation coefficient remains a consistent and unbiased estimate of the true population serial correlation.

Second, some have followed Blattberg and Gonedes (1974) in proposing the Student distribution as an alternative probability distribution for exchange rate changes. The Student distribution is described by three parameters: location, scale, and degrees of freedom. It approaches the normal distribution as the degrees of freedom approaches infinity. The Student distribution has an important property that it is "fat-tailed" relative to the normal and has well-defined moments whose order is less than the degrees of freedom. Moreover, with the Student distribution, most statistical inference is not substantially altered because t and F tests are valid and ordinary least squares (OLS) estimators remain maximum likelihood estimators.

Some studies have indicated that the Student distribution fits the empirical data better than the stable Paretian distribution in terms of the log-likelihood test or chi-square test: based on the Westerfield data, Rogalski and Vinso (1978) argued that the best fitting distribution was a Student distribution with about 4 degrees of freedom; Boothe and Glassman (1987) suggested degrees of freedom that ranged between 2 and 4 based on daily, weekly, monthly, and quarterly data for the pound sterling, Canadian dollar, deutsche mark and Japanese yen against the U.S. dollar during 1973–84. Although both distributions have a similar shape, the Student distribution with finite variance differs from the non-normal Paretian distribution in one important respect: sums of independently distributed Student variables converge toward normality under temporal aggregation, i.e., the differencing interval is increased as from weekly to monthly.

Third, another popular attempt has been to explain the leptokurtosis in terms of mixed normal distributions.[19] For example, the empirical distribution may consist of several normal distributions of the same mean and different variances. One possible explanation for changing variance discussed in the context of the stock market is the weekend effect: if the amount of information is proportional to the lapse of chronological time, there is more information between Friday and Monday than between other successive trading days; that is to say, weekend distributions have greater variance. Although Islam (1982) presents evidence that this type of weekend effect may be insignificant in the deutsche mark/U.S. dollar rate, it is nonetheless plausible that the foreign exchange

19. Both the stable Paretian and Student distributions can be thought of as special cases of the class of mixed normal distributions: if the variance follows an inverted gamma-2 distribution, the resulting distribution becomes a Student distribution; if the variance follows a strictly positive stable distribution, the resulting distribution becomes a symmetric stable Paretian distribution (Islam 1982). For a more comprehensive discussion on mixed distributions, see Bookstaber and McDonald (1987) and McDonald and Butler (1987).

market is, by its nature, subject to different types of distributional shocks. For example, the variance may be higher when there is significant news or central bank intervention. A mixture of normal distributions with different variances is known to have "fat-tailed" unconditional density.

Another kind of mixed distribution is a normal with changing parameters. One possible case for this hypothesis is found in the day of the week effect discussed by McFarland, Pettit and Sung (1982). Based on the spot rates (and 3–month forward rates for the major currencies only) of seven currencies during 1975–79, they found apparently different distributional characteristics for different days of the week. For example, they found a tendency for Tuesday to Wednesday changes to be positive, and for Wednesday to Thursday changes to be negative, and attributed this effect to settlement practice in the foreign exchange market. This type of day of the week effect would result in differences in the location of the distribution and not in its scale (or dispersion).[20] Based on the daily spot exchange rates of nine currencies against the U.S. dollar during 1973–79, Friedman and Vandersteel (1982) found evidence in support of such a mixed distribution. In a more comprehensive study based on ten years of daily data for the U.S. dollar rates of the pound sterling, Canadian dollar, deutsche mark, Japanese yen and Swiss franc, Hsieh (1987) found some "day of the week" effect as well as changing means and variances over a longer time horizon. The possibility of changing parameters is also evident from inspecting Table 9: the values of kurtosis for the same exchange rate vary over time.[21] According to Hsieh (1987), however, these time varying parameters did not entirely account for the leptokurtosis in daily data.

3. Convergence to Normality Under Temporal Aggregation

There is an important observed difference between the behavior of longer-run exchange rate changes and that of short-run changes. Islam (1982) and Boothe and Glassman (1987) noted that, while daily and weekly changes were leptokurtic, monthly and quarterly changes were close to normal. Comparison of Table 9 and Table 10 confirms this tendency for the value of kurtosis to fall as the differencing interval is increased. Although part of the difference between daily and monthly changes may reflect the difference in sample size, we find that the distributions of monthly changes were either slightly platykurtic (i.e., the value of kurtosis is less than 3) or normal during 1975–86. This convergence to normality under temporal aggregation is certainly inconsistent with

20. However, McFarland, Petit and Sung (1982) consistently found the estimated characteristic exponent to be considerably less than 2 for different daily distributions and concluded that the mixed distribution was not a normal but a stable Paretian with changing parameters.

21. It is noteworthy that some of the values of kurtosis in daily data were close to 3 when the sample period was three years, whereas the values of kurtosis in longer sample periods in other studies were hardly ever smaller than 10. This suggests the possibility that even the daily distribution can be normal during a short enough period and that the leptokurtosis is generated by mixing several normals with different parameters.

Table 10. Basic Statistics of Monthly Exchange Rate Changes

	U.S. Dollar/Sterling			U.S. Dollar/Deutsche mark			U.S. Dollar/Yen		
	Spot	3-month	12-month	Spot	3-month	12-month	Spot	3-month	12-month
January 1975 to December 1977									
Size	36	36	36	36	36	36	36	36	36
Mean	–0.01	–0.01	–0.00	0.00	0.00	0.00	0.01	0.01	0.01
Standard deviation	0.027	0.030	0.034	0.025	0.024	0.024	0.016	0.017	0.019
Skewness	–0.13	–0.18	–0.17	–1.26	–1.11	–0.79	0.80	0.51	0.56
Kurtosis	2.81	2.74	2.73	6.86**	6.73**	6.93**	4.26*	3.86	4.43*
January 1978 to December 1980									
Size	36	36	36	36	36	36	36	36	36
Mean	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.01
Standard deviation	0.031	0.030	0.031	0.040	0.040	0.041	0.044	0.045	0.047
Skewness	–0.55	–0.57	–0.44	–0.05	0.00	0.15	–0.26	–0.19	–0.12
Kurtosis	2.96	3.31	3.70	4.89**	5.22**	5.65**	3.16	2.96	2.86
January 1981 to December 1983									
Size	36	36	36	36	36	36	36	36	36
Mean	–0.01	–0.01	–0.01	–0.01	–0.01	–0.01	–0.00	–0.00	–0.00
Standard deviation	0.029	0.028	0.027	0.029	0.028	0.028	0.036	0.035	0.033
Skewness	0.33	0.13	0.04	0.12	0.14	0.24	1.21	1.22	1.11
Kurtosis	3.65	3.55	3.30	2.79	2.99	3.03	4.01	4.02	3.81
January 1984 to June 1986									
Size	30	30	30	30	30	30	30	30	30
Mean	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01
Standard deviation	0.042	0.043	0.046	0.045	0.044	0.045	0.033	0.033	0.034
Skewness	1.10	1.08	1.06	0.18	0.22	0.31	0.76	0.78	0.76
Kurtosis	4.17	4.05	3.94	2.09	2.11	2.19	2.67	2.67	2.56

	Sterling/Deutsche mark			Sterling/Yen			Deutsche mark/Yen		
	Spot	3-month	12-month	Spot	3-month	12-month	Spot	3-month	12-month
January 1975 to December 1977									
Size	36	36	36	36	36	36	36	36	36
Mean	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00
Standard deviation	0.030	0.032	0.037	0.023	0.024	0.028	0.023	0.022	0.022
Skewness	0.21	0.42	0.39	0.24	0.24	0.10	0.88	0.99	0.85
Kurtosis	4.72**	4.40*	4.19	3.05	3.44	3.65	5.45**	5.74**	5.36**
January 1978 to December 1980									
Size	36	36	36	36	36	36	36	36	36
Mean	–0.00	–0.00	–0.00	–0.00	–0.00	–0.00	0.00	0.00	0.00
Standard deviation	0.030	0.029	0.031	0.040	0.040	0.041	0.044	0.044	0.045
Skewness	0.58	0.57	0.53	0.17	0.19	0.08	0.37	0.32	0.29
Kurtosis	3.28	3.32	3.11	3.63	3.78	3.57	2.52	2.42	2.33
January 1981 to December 1983									
Size	36	36	36	36	36	36	36	36	36
Mean	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01
Standard deviation	0.033	0.033	0.036	0.036	0.036	0.037	0.028	0.028	0.028
Skewness	–0.10	–0.07	0.05	1.18	1.21	1.13	0.16	0.07	–0.25
Kurtosis	3.43	3.54	3.96	5.05**	5.11**	4.79*	3.06	3.09	3.26
January 1984 to June 1986									
Size	30	30	30	30	30	30	30	30	30
Mean	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00
Standard deviation	0.027	0.026	0.028	0.036	0.037	0.039	0.028	0.028	0.029
Skewness	–0.49	–0.29	–0.44	–0.58	–0.60	–0.72	–0.46	0.66	–0.72
Kurtosis	3.62	3.32	3.83	5.00**	4.90**	4.83**	3.96	4.13	4.25*

Note: For skewness and kurtosis, **(*) indicates a significant departure from normality at 5(10) percent.

the stable Paretian distribution, which implies stability under addition.

Although sums of most variables that are independently distributed with finite variance are known to converge to normality,[22] recent research has suggested a promising distributional hypothesis that is not based on statistical independence: a normal distribution with an autoregressive conditional heteroskedasticity (ARCH) process for innovations, originally proposed by Engle (1982) for inflation data. Diebold and Nerlove (1986) have shown that such a distribution is consistent with both leptokurtosis in original frequency data and convergence to normality under temporal aggregation. One evidence supportive of such conditional heteroskedasticity in exchange rate data was earlier reported by Cumby and Obstfeld (1982) in the context of weekly forecasting errors of major currency exchange rates during 1976–81. The normal with ARCH can be considered to be a special case of subordinated normal distributions first considered by Clark (1973); one such subordinated distribution is a normal distribution x(T(t)) in which the conditional distribution of T(t) is a function of volume and follows a log-normal distribution.

The ARCH(q) process is a process whose conditional variance depends on the history of past innovations,

$$(w_t|w_{t-1},\ldots\ldots, w_{t-q})\sim N(0,\sigma_t^2) \tag{15}$$

$$\sigma_t^2=f(w_{t-1},\ldots\ldots, w_{t-q}), \tag{16}$$

where $\sigma^2{}_t$ is the variance of w_t. A popular parameterization of (16) is simply to assume an AR process of the form,

$$\sigma_t^2=b_0+\sum_{j=1}^{q} b_j w^2{}_{t-j}, \tag{17}$$

where b_0 and b_j's are all positive parameters.[23] Such a distribution has normal conditional density but its unconditional density is fat-tailed.

The ARCH process implies that exchange rate innovations are uncorrelated but not independent, because the current variability depends linearly on the past variabilities, leading to the situation where large changes tend to be followed by large changes and, small changes by small changes, of random sign, the type of dependence suggested by Mandelbrot (1963) as a possible explanation for the leptokurtosis of empirical distributions. In fact, leptokurtosis is a natural outcome of the conditional variance persistence

22. White (1984, Chapter 5) reviews several versions of the central limit theorem that specify the conditions needed for finite variance distributions to converge to normality.

23. Diebold and Nerlove (1986) suggested the estimated conditional variance as a measure of volatility in the foreign exchange market. Domowitz and Hakkio (1985) applied the ARCH process to explain the risk premium. They hypothesized that the risk premium is a positive function of the conditional variance of the forecasting error. Based on monthly data for 1973–82, however, they concluded that the conditional variance did not entirely explain the risk premium.

the ARCH process implies, as periods of high volatility leads to increased "tail action" and periods of low volatility to increased "center action" (Diebold 1986). As long as successive changes are of random sign, however, this type of dependence is nonlinear and does not show up in serial correlation coefficients.[24]

This type of nonlinear dependence is consistent with the empirical observation that the foreign exchange market is frequently characterized by successive periods of volatility and stability. Fama (1965) presented a plausible explanation for this type of nonlinear dependence. When a new piece of important information comes, the market cannot always evaluate it correctly. If the initial reaction is too large, the price has to be corrected by a large subsequent change of opposite sign. If the initial reaction is too small, the price will continue to move little by little in the same direction. However, this explanation suggests negative correlations for large changes and positive correlations for small changes.

Diebold and Nerlove (1986) give an alternative explanation based on the assumption of heterogeneous agents. Greater volatility results when there is more disagreement about new information, whereas smaller volatility results when there is greater agreement. The observed exchange rate behavior would result, if information is serially correlated. Such a behavior is also consistent with the environment where agents believe with time-varying degreee of confidence that a major discrete change may occur in the stochastic process generating the fundamental economic variables (Singleton 1987). Recent evidence on the behavior of foreign exchange options prices gives support to the view that market participants have held expectations of a large discrete jump in the spot exchange rate (Borensztein and Dooley 1987; Bodurtha and Courtadon 1987).

The preceding discussion is not meant in any way to imply that the ARCH distribution is necessarily the true underlying distributional process for exchange rate changes. It is true that the ARCH model has some attractive features and shows promise as a useful analytical model. At the same time, some have questioned the ability of ARCH models to explain changes in exchange rates in some applications, having found little evidence of ARCH particularly in monthly data (Hodrick 1987). In fact, the distribution of exchange rate changes must be endogenous to the underlying economic environment, and there is no law of nature that prescribes *a priori* the type of distribution to which exchange rate changes must conform. What one can infer with some confidence from the distributional characteristics of exchange rate changes, however, is the complex nature of the exchange rate determination process. The superior ability of different types of mixed distributions, including the ARCH model, to explain the observed distribution must be a reflection of the heterogeneity of information and agents as well as of the nonstationarity of the process itself. This in turn points to the direction of future research in exchange rate economics.

24. Moreover, the asymptotic distribution of the Dickey-Fuller statistics is invariant to ARCH.

V. Conclusion

The paper has reviewed the statistical behavior of the nominal exchange rates between major currencies in the recent period of floating exchange rates. While market efficiency does not place restrictions on the time-series behavior of exchange rates, a random walk was consistently shown to characterize the recent behavior of most empirical exchange rates fairly well. This probably means that the underlying economic variables and the expectations concerning the future path of those variables followed a process that is indistinguishable from a random walk. A closer inspection, however, indicated that an alternative stationary stochastic process (such as an autoregressive process) could almost always explain both daily and monthly data better than a simple random walk, although the deviations from random walks were generally small. Moreover, we often found the presence of systematic movements in monthly data which were not detected in daily data, possibly corresponding to the relatively infrequent arrivals of information concerning major macroeconomic variables.

Although the distribution of almost all daily exchange rate changes was found to be too leptokurtic to be normal, that of most monthly changes could be well described as normal. Thus, it is inappropriate to explain the leptokurtosis of empirical distributions by means of stable Paretian distributions, which imply stability under temporal aggregation. Most finite variance distributions are consistent with convergence to normality under temporal aggregation, and there may be little merit in deciding which distribution "fits" the observed distribution better. However, one distributional hypothesis that is promising in terms of analytical tractability is to assume that exchange rate changes are normal with ARCH innovations. Such a normal distribution is not only consistent with leptokurtosis in original frequency data and convergence to normality under temporal aggregation but also with the observation that the foreign exchange market is often characterized by successive periods of volatility and stability.

From these observations, we can draw at least two useful implications for the direction of future research in exchange rate economics. First, our discussion has suggested that the use of daily or weekly data in empirical work should be made with care because the daily or weekly distributions are not "well-behaved" for statistical analysis. Moreover, our discussion on the time-series properties has suggested that perhaps the use of daily or weekly data should be altogether avoided in empirical work involving structural modeling because high frequency data do not seem to detect the presence of important information about macroeconomic variables; it may be that there is too much noise in daily or weekly data.

Second, our discussion has suggested the complex nature of the exchange rate determination process. It is possible, for example, that the exchange rate market is subject to different types of news with potentially different distributional properties and that the market is inhabited by different types of economic agents with potentially different behavioral characteristics; the presence of heterogenous information and heterogeneous agents can account for the ability of mixed distributions—including the ARCH model—to explain the behavior of short-run exchange rate changes. The foreign exchange market has also exhibited the type of non-linear dependence that can be well accounted for by the ARCH process for innovations, whereby large changes tend to be followed by large changes and small by small. This may reflect the process in which expectations cautiously adjust to new and uncertain information. Thus, future research should involve investigations of how exchange rates respond to different types of news in high frequency data (Ito and Roley 1987) as well as modeling of foreign exchange markets with heterogeneous agents (Singleton 1987). Some work has already begun; and more work is needed.

Appendix: Sources of Data

The daily time-series of the U.S. dollar exchange rates of the pound sterling, deutsche mark, and Japanese yen between January 1, 1975 and June 30, 1986 were all obtained from the Financial and Credit Statistics Information Service of Data Resources, Inc. The rates are 10 a.m. opening rates in New York as quoted by the Bank of America.

The rates actually used in the paper are averages of the bid and ask rates, and the cross rates were calculated from these average U.S. dollar rates. The monthly rates are end-of-month rates, taken from the daily rates reported on the last trading day of each month.

REFERENCES

Adams, Charles and Boyer, Russell S., "Efficiency and a Simple Model of Exchange-Rate Determination", *Journal of International Money and Finance* 5, June 1986, pp. 285–302.

Alexander, Sidney S., "Price Movements in Speculative Markets: Trends or Random Walks", *Industrial Management Review* 2, May 1961, pp. 7–26.

Blattberg, Robert C. and Gonedes, Nicholas J., "A Comparison of the Stable and Student Distributions as Statistical Models for Stock Prices", *Journal of Business* 47, April 1974, pp. 244–280.

Bodurtha, James N. and Courtadon, Georges R., "Tests of an American Option Pricing Model on the Foreign Currency Options Market", *Journal of Financial and Quantitative Analysis* 22, June 1987, pp. 153–167.

Bookstaber, Richard M. and McDonald, James M., "A General Distribution for Describing Security Price Returns", *Journal of Business* 60, July 1987, pp. 401–424.

Boothe, Paul and Glassman, Debra, "The Statistical Distribution of Exchange Rates: Empirical Evidence and Economic Implications", *Journal of International Economics* 22, May 1987, 297–319.

——— and Longworth, David, "Foreign Exchange Market Efficiency Tests: Implications of Recent Empirical Findings", *Journal of International Money and Finance* 5, March 1986, pp. 135–152.

Borensztein, Eduardo R. and Dooley, Michael P., "Options on Foreign Exchange and Exchange Rate Expectations", Working Paper 87/13, International Monetary Fund, 1987.

Burt, John, Kaen, Fred R. and Booth, G. Geoffrey, "Foreign Exchange Market Efficiency under Flexible Exchange Rates", *Journal of Finance* 32, September 1977, pp. 1325–1330.

Campbell, John Y. and Mankiw, N. Gregory, "Permanent and Transitory Component in Macroeconomic Fluctuations", NBER Working Paper No. 2169, February 1987.

Clark Peter K., "A Subordinated Stochastic Process Model with Finite Variance for Speculative Prices", *Econometrica* 41, January 1973, pp. 135–155.

Cumby, Robert E. and Obstfeld, Maurice, "International Interest Rate and Price Level Linkages under Flexible Exchange Rates: A Review of Recent Evidence", in John F.O. Bilson and Richard C. Marston, eds., *Exchange Rate Theory and Practice*, Chicago: University of Chicago Press, 1984, pp. 121–151.

Diba, Behzad T. and Grossman, Herschel I., "Rational Bubbles in Stock Prices?", NBER Working Paper No. 1779, October 1985.

Dickey, David A. and Fuller, Wayne A., "Distribution of the Estimators for Autoregressive Time Series with a Unit Root", *Journal of the American Statistical Association* 74, June 1979, pp. 427–431.

——— and ———, "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root", *Econometrica* 49, July 1981, pp. 1057–1072.

Diebold, Francis X., "Temporal Aggregation of ARCH Processes and the Distribution of Asset Returns", Special Studies Paper No. 200, Federal Reserve Board, August 1986.

——— and Nerlove, Marc, "The Dynamics of Exchange Rate Volatility: A Multivariate Latent Factor ARCH Model", Special Studies Paper No. 205, Federal Reserve Board, November 1986.

Domowitz, Ian and Hakkio, Craig S., "Conditional Variance and the Risk Premium in the Foreign Exchange Market", *Journal of International Economics* 19, August 1985, pp. 47–66.

Dooley, Michael P. and Shafer, Jeffrey R., "Analysis of Short-Run Exchange Rate Behavior: March 1973 to November 1981", in David Bigman and Teizo Taya, eds., *Exchange Rate and Trade Instability*, Cambridge, Massachusetts: Ballinger, 1983, pp. 43–69.

Doukas, John and Rahman, Abdul, "Unit Roots Test: Evidence from the Foreign Exchange Futures Market", *Journal of Financial and Quantitative Analysis* 22, March 1987, pp. 101–108.

Engle, Robert F., "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation", *Econometrica* 50, July 1982, pp. 987–1008.

Fama, Eugene F., "Mandelbrot and the Stable Paretian Hypothesis", *Journal of Business* 36, October 1963, pp. 420–429.

———, "The Behavior of Stock Market Prices", *Journal of Business* 38, January 1965, pp. 34–105.

———, *Foundations of Finance*, New York: Basic Books, 1976.

Frankel, Jeffrey A. and Meese, Richard, "Are Exchange Rates Excessively Variable?", NBER Working Paper No. 2249, May 1987.

Frankel, Jacob A., "Flexible Exchange Rates, Prices, and the Role of 'News:' Lessons from the 1970s", *Journal of Political Economy* 89, August 1981, pp. 665–705.

Frenkel, Jacob A. and Mussa, Michael, "Asset Markets, Exchange Rates and the Balance of Payments", in Ronald W. Jones and Peter B. Kenen, eds., *Handbook of International Economics* Vol. II, Amsterdam: North-Holland, 1985.

Friedman, Daniel and Vandersteel, Stoddard, "Short-Run Fluctuations in Foreign Exchange Rates: Evidence from the Data 1973–79", *Journal of International Economics* 13, August 1982, pp. 171–186.

Fuller, Wayne A., *Introduction to Statistical Time Series*, New York: John Wiley and Sons, 1976.

Genberg, Hans, "Properties of Innovations in Spot and Forward Exchange Rates and the Role of Money Supply Processes", in John F.O. Bilson and Richard C. Marston, eds., *Exchange Rate Theory and Practice*, Chicago: University of Chicago Press, 1984, pp. 153–173.

Giddy, Ian H. and Dufey, Gunter, "The Random Behavior of Flexible Exchange Rates: Implications for Forecasting", *Journal of International Business Studies* 6, Spring 1975, pp. 1–30.

Granger, C.W.J. and Newbold, Paul, *Forecasting Economic Time Series*, second edition, Orlando: Academic Press, 1986.

Hakkio, Craig S., "Does the Exchange Rate Follow a Random Walk?: A Monte Carlo Study of Four Tests for a Random Walk", *Journal of International Money and Finance* 5, June 1986, pp. 221–229.

Hodrick, Robert J., "Risk, Uncertainty and Exchange Rates", J.K. Kellogg Graduate School of Management, Northwestern University, 1987.

Hsieh, David A., "The Statistical Properties of Daily Foreign Exchange Rates: 1974–1983", Graduate School of Business, University of Chicago, 1987.

Isard, Peter, "Lessons from Empirical Models of Exchange Rates", *IMF Staff Papers*, Vol. 34, March 1987, pp. 1–28.

Islam, Shafiqul, "Statistical Distribution of Short-Term Exchange-Rate Variations", Research Paper No. 2815, Federal Reserve Bank of New York, August 1982.

Ito, Takatoshi and Roley, V. Vance, "News from the U.S. and Japan: Which Moves the Yen/Dollar Exchange Rate?", *Journal of Monetary Economics* 19, March 1987, pp. 255–277.

Levich, Richard M., "Empirical Studies of Exchange Rates: Price Behavior, Rate Determination and Market Efficiency", in Ronald W. Jones and Peter B. Kenen, eds., *Handbook of International Economics* Vol. II, Amsterdam: North-Holland, 1985, pp. 979–1040.

Logue, Dennis E. and Sweeney, Richard James, "White-noise in Imperfect Markets: The Case of the Franc/Dollar Exchange Rate", *Journal of Finance* 32, June 1977, pp. 761–768.

Mandelbrot, Benoit, "The Variation of Certain Speculative Prices", *Journal of Business* 36, 1963, pp. 394–419.

McDonald, James B. and Butler, Richard J., "Some Generalized Mixture Distributions with an Application to Unemployment Duration", *Review of Economics and Statistics* 69, May 1987, pp. 232–240.

McFarland, James W., Pettit, R. Richardson and Sung, Sam K., "The Distribution of Foreign Exchange Price Changes: Trading Day Effects and Risk Measurement", *Journal of Finance* 37, June 1982, pp. 693–715.

Meese, Richard A. and Singleton, Kenneth J., "On Unit Roots and the Empirical Modeling of Exchange Rates", *Journal of Finance* 37, September 1982, pp. 1029–1035.

Mussa, Michael, "Empirical Regularities in the Behavior of Exchange Rates and Theories of the Foreign Exchange Market", *Carnegie-Rochester Conference Series on Public Policy* 11, 1979, pp. 9–57.

————, "The Theory of Exchange Rate Determination", in John F.O. Bilson and Richard C. Marston, eds., *Exchange Rate Theory and Practice*, Chicago: University of Chicago Press, 1984, pp. 13–78.

Nelson, Charles R. and Plosser, Charles I., "Trends and Random Walks in Macroeconomic Time Series", *Journal of Monetary Economics* 10, September 1982, pp. 139–162.

Obstfeld, Maurice, "Peso Problems, Bubbles, and Risk in the Empirical Assessment of Exchange-Rate Behavior", NBER Working Paper No. 2203, April 1987.

Poole, William, "Speculative Prices as Random Walks: An Analysis of Ten Time Series of Flexible Exchange Rates", *Southern Economic Journal* 33, April 1967, pp. 468–478.

Rogalski, Richard J. and Vinso, Joseph D., "Empirical Properties of Foreign Exchange Rates", *Journal of International Business Studies* 9, April 1978, pp. 69–79.

Singleton, Kenneth, "Speculation and the Volatility of Foreign Currency Exchange Rates", *Carnegie-Rochester Conference Series on Public Policy* 26, Spring 1987, pp. 9–56.

So, Jacky C., "The Sub-Gaussian Distribution of Currency Futures: Stable Paretian or Nonstationary?", *Review of Economics and Statistics* 69, February 1987, pp. 100–107.

Takagi, Shinji, "Real and Monetary Factors in the Joint Determination of the Exchange Rate and the Interest Rate", Working Paper 86/17, International Monetary Fund, December 1986.

Tschoegl, Adrian E., "Noise in Observed Returns: The Case of the London Gold Market", University of Michigan, March 1987.

Westerfield, Janice M., "An Examination of Foreign Exchange Risk under Fixed and Floating Rate Regimes", *Journal of International Economics* 7, May 1977, pp. 181–200.

White, Halbert, *Asymptotic Theory for Econometricians*, New York: Academic Press, 1984.