Fluctuations in Yields on Bonds —A Reassessment of the Expectations Theory Based on Japanese and U.S. Data*

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I. Purpose, Structure and General Outline

In Japan in recent years, the yields on bonds have been subject at times to sharp fluctuations due to vigorous trading resulting from the steady progress in liberalization and internationalization of financial markets . Since particularly in recent days the volume of trading in long-term bonds in a very short period of time has substantially increased, fluctuations in the yields on long-term bonds have exceeded those in short-term interest rates. Many writers have drawn the comparison with the United States, which also has experienced marked fluctuations in yields since the second half of the 1970s.

The most prevalent explanation of fluctuations in yields on bonds (long-term interest rates) has long been the expectations theory of the term structure, which posits that the yields reflect the market expectations for the future short-term interest rates. The "pure expectations theory" argues that the yields are determined as the weighted average of the current and future short-term interest rates, while the "constant risk premium hypothesis" asserts that a constant risk premium should be added to the weighted average. These standard theories had been relatively valid in explaining the variability in yields until the early 1980s in Japan and until the mid-1970s in the United States (Kuroda(1982), Shikano(1984), and Modigliani—Shiller(1973)). However, is it still possible to account for the large fluctuations recorded in recent years satisfactorily by these conventional hypotheses? If these hypotheses are in-

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adequate, then what kinds of alternative hypotheses could prove to be adequate for an explanation of the phenomenon? This paper aims to present a possible solution to these questions through the empirical analyses of yields in both Japan and the United States.

The analysis in this paper contains several innovative approaches. First, we employ the variance bounds test, that is used in studying the wildly fluctuating asset prices, such as exchange rates and stock prices. This test serves to examine whether the sample variance of yields on bonds (holding period yields) exceeds the theoretical upper bound in the context of the rational expectations model. If the theoretical bound is exceeded, the fluctuation in yields is judged to be volatile. By carrying out this test, we are able to verify the rationality of market expectations, which has, heretofore, not been subject to independent discussion. This paper makes a detailed study of this issue.[1]

Second, a time-varying risk premium (hereafter referred to as the "variable risk premium") is introduced explicitly into the empirical analysis.[2] Traditional analyses had treated the risk premium as a constant; but in this paper, we consider it as a time-variable in order to gauge the volatility of yields on bonds.

Third, a series of empirical works is conducted in the same procedure in both Japan and the United States, and the comparison of the results is also made. This is done for the purpose of determining whether the variability in yields on bonds in both countries could be explained by the same hypothesis, or whether the different hypothesis would be needed. The comparative study is helpful to drawing the implications for monetary policy of each country.

The contents of this paper are organized as follows. In II, we summarize the standard expectations theory of the term structure, and make intuitive study on its validity. Then the variance bounds inequalities are examined for the sample data sets of holding period yields. In III, based on the results of the variance bounds tests, we present a theoretical interpretation of volatility of the yields; at the same time, the rational expectations hypothesis and the standard theory of the term structure are

1. The exsisting econometric analysis on the term structure equations must assume the rational expectations hypothesis because it satisfies the orthogonality requirement, which ensures the consistency and unbiasedness of estimated parameters, in the ordinary least squares estimation (cf. III. 1. B.). But when the expectations of market participants are not rationally formed, the reliability of the empirical results may be seriously questioned. From this standpoint, there is significance in testing the rational expectations hypothesis independently.

2. Recently, in the analyses of investor behaviors under uncertainty, such as the capital asset pricing model (CAPM) and so forth, the determination of bond yeild as an asset price, as well as the role of the variability of risk premium, has been frequently discussed. The possibility of the existence of the variable risk premium has been suggested in Japan's bond market since mid-1981 (see Shikano (1984)).

tested independently.[3] Consequently, IV analyzes whether the variable risk premium can explain the failure of the yields to behave according to the standard rational expectations model. Finally, in V, we draw some concluding remarks and discuss future problems. The analysis of this paper covers the period of approximately ten years, April 1977 to June 1986 (hereafter referred to as the "whole period"); however, where necessary, it also covers the last five years (April 1981 to June 1986, hereafter referred to as the "recent period").

The major conclusions resulting from the above analyses can be summarized as follows.

(1) Results of the variance bounds tests applied to the ten years, 1977–86, show that the yields on bonds in Japan and the United States are too volatile to accord with the standard rational expectations model. The volatility of Japanese bonds is noticeable especially in recent years (1981–86). The degrees of excess volatility, moreover, are in general higher in the United States than in Japan. From this study, it is evident that applying the standard expectations theory to the explanation of fluctuations in yields is highly inadequate in the United States, and is becoming increasingly difficult in the case of Japan.

(2) The volatility of the yields in both Japan and the United States may be attributable to the irrational expectations in the bond markets. In other words, it is possible that the investors do not forecast the yields efficiently by using all information available and that the data are inconsistent with the rational expectations hypothesis (the market efficiency hypothesis). But, at the same time, we cannot exclude the scenario that the variable risk premium plays a crucial role in making the yields volatile. This scenario would be especially applicable to the data of recent period in Japan. In this instance, it means that in addition to the current and the future short-term interest rates, the other information (such as yields on U.S. bonds and foreign exchange rates) strongly influences the determination of yields taking the form of variable risk premium.

(3) In Japan, with the data of 1977 to 86, the hypothesis that the variable risk

3. Once the standard theory of the term structure turns out to be invalid, one of the following alternative hypotheses has the possibility to explain the fluctuations in yeilds, and we need to test these alternatives; ① the hypothesis that variable risk premiums may exist; ② the "myopic expectations hypothesis" based on the assumption that investors may over react to the current information (short-term interest rates); ③ the "static expectations hypothesis" which assumes that the expected yield of the next period is equal to the current yield.

premium causes the excess volatility of yields on bonds is generally accepted.[4] This indicates that, if we permit the risk premium to vary over time, the fluctuations in yields on Japanese bonds are consistent with the rational behavior of investors. However, regarding the observation that the yields have seemed much more volatile in recent days, the volatility may have been chiefly ascribed to the extremely short-term trade of bonds promoted by the Japan's distinctive system of settling bond transactions (settled on the tenth, twentieth, and the end of each month). It can be interpreted as a sort of "quasi-bubble" phenomenon, whereby the yields including the risk premiums, deviate greatly from the theoretical value.

(4) On the contrary, in the case of the United States, it is difficult to attribute the excess volatility of yields solely to fluctuations in the variable risk premium. In other words, the yields on U.S. bonds are so volatile as to be inconsistent with the rational behavior of investors, even if the variability of risk premiums is hypothesized. The irrational behavior of market participants might explain the large volatility of yields; although, on the other hand, "information shocks" to the market, which can not be included in the equilibrium model on this paper, may occur to make the yields move violently in a direct manner.

(5) We can point out the implications for monetary policy in each country. For monetary authorities in Japan, to evaluate the transmission mechanism of interest rates, it is inadequate only to see about the short-term interest rates but necessary to consider various informations (such as yields on U.S. bonds, foreign exchange rates, etc.) that influence the formation of the variable risk factors and to estimate their effects on the mechanism. The results of analysis on U.S. bonds yields implies that if monetary authorities are aiming at stabilizing fluctuations in yields, to minimize information shocks would be needed. Suppose information shocks are caused generally by monetary policy uncertainty, it is of great consequence that monetary policy is implemented with stability to remove it.

4. Kuroda (1982) and Shikano (1984) have made the empirical analyses (for the period 1977–81) of Japan, which indicate the validity of the "pure expectation theory" or the "constant risk premium hypothesis." These results of the two papers may seem inconsistent with the "variable risk premium hypothesis" in our study. However, the inconsistency can be reconciled under the following consideration: Their estimations surely assume the rational expectations hypothesis. Supposing the validity of this premise, their studies are not inconsistent with our results. This is because my empirical work shows that the fluctuations in risk premiums are much smaller in the period prior to 1981 than in the recent period, and we can interpret the risk premium in their studies as zero or constant. On the other hand, even if the the rational expectaions hypothesis is invalid and the reliability of their results is weakend, the existence of variable risk premium can not be denied.

II. The Standard Expectations Theory of the Term Structure and Its Validity

1. The Standard Expectations Theory of the Term Structure

The standard expectations theory of the term structure is designed to explain long-term interest rates (yeilds on bonds) in terms of the investor's forecasts or expectations on future short-term interest rates. There are two major theories, the "pure expectations theory" and the "constant risk premium hypothesis" depending on efficacy of expectations factor in fluctuations in long-term interest rates. The term structure theory in this paper is premised on the expectations theory, which assumes the existence of constant risk premiums (hereafter referred to as the "standard theory").[5] The constant risk premium is what the investor receives to compensate for the risk entailed, which, for instance can be interpreted as the price fluctuations risk in an uncertain world. If the risk of price fluctuations becomes larger in proportion to the years to maturity, the constant risk premium will be described as the "liquidity premium" of Hicks'.

From the viewpoint of the standard theory, as a result of the investor's arbitrage behavior, the expected one-period holding yield on an n-period bond equals the short-term interest rate corresponding to the holding period plus the constant risk premium in the equilibrium:

$$E_t[H_{t+1}^n \mid I_t] = r_t + \phi^n \tag{1}$$

where

$$H_{t+1}^n = \frac{P_{t+1}^{n-1} - P_t^n + C}{P_t^n} \tag{2}$$

$E_t\,[\cdot|I_t]$: expectations operator conditional on information available at time t, I_t

H_{t+1}^n: one-period ($t \rightarrow t+1$) holding yield on an n-period bond (the ratio of holding period return, which is coupon plus any capital gains or losses, to its purchase price)

r_t : one-period short-term interest rate (yield to maturity on a one-period bond)

ϕ^n: constant risk premium of bonds on an n-period bond

P_t^n : price of an n-period bond

5. This is due to the reasons as follows: Our analysis aims at clarifying what kind of theoretical framework serves to explain the fluctuations (variance) in yields on bonds. In this context, the zero risk premium hypothesis and the constant risk premium hypothesis should be treated as being of equivalent importance and the former is considered as a special case of the latter.

C : coupon payment (equals coupon rate multiplied by redemption price)

In order to obtain the term structure equation (the equation relating yield to a weighted average of expected future short-term interest rates), one-period holding yield, H^n_{t+1}, must be expressed by the yield to maturity on n-period bonds R^n_t.

The relation between the price of bond and compound yield can be written as follows:

$$P^n_t = \frac{C}{R^n_t} + \frac{R^n_t - C}{R^n_t(1+R^n_t)} \cdot \tag{3}$$

Equation (2) may be rewritten, using(3), in terms of R^n_t and R^{n-1}_{t+1}. As Shiller (1979) shows, the approach will be to linearize around $R^n_t = R^{n-1}_{t+1} = C$ (that is, take a Taylor expansion truncated after the linear term) to give us a linearized one-period holding yeild.

$$H^n_{t+1} = \frac{R^n_t - \bar{\gamma} R^{n-1}_{t+1}}{1-\bar{\gamma}} \tag{4}$$

where

$$\bar{\gamma} = \frac{\gamma - \gamma^n}{1-\gamma^n}, \qquad \gamma = \frac{1}{1+\bar{R}}$$

$\bar{R}$: average yield of bonds at time t and t−1.

Substituting this expression for H^n_{t+1} in place of H^n_{t+1} in Equation (2) and rearranging gives:

$$R^n_t = \bar{\gamma} E_t[R^{n-1}_{t+1} | I_t] + (1-\bar{\gamma})(r_t + \phi^n) \tag{5}$$

which is a first-order linear expectations difference equation in R^n_t with variable coefficients. Such a model can be solved by a method of recurvise substitution that is familiar in the rational expectations literature. The resulting solution is the term structure equation expressed as follows.[6]

$$R^n_t = \frac{1-\gamma}{1-\gamma^n} \sum_{s=0}^{n-1} \gamma^s E_t[r_{t+s} | I_t] + \Phi^n \tag{6}$$

where,

$$\Phi^n = \frac{1-\gamma}{1-\gamma^n} \sum_{s=0}^{n-1} \gamma^s \phi^{n-s}$$

6. For the details of derivation of these equations, see Kuroda (1982) and Shikano (1984).

2. The Validity of the Standard Theory

A. Ex-post Rational Rates and Observed Data — Figure 1 and Figure 2

We first try to understand intuitively whether the actual fluctuations in yields on bonds in Japan and the U.S. can be explained validly by the model employing the standard theory (that is, Equation (6) or Equation (1)).[7] The intuitive understanding can be dealt with thus: "Supposing the model on Equation (6) is able to account for the fluctuations in yields, that is to say, yields are basically expressed as the weighted average of short-term interest rates, the fluctuations in yields should be smoother than the fluctuations in short-term interest rates, unless the market expectations to the future short-term interest rates fluctuate violently."

In order to check the existence of such a relationship, we shall compare the theoretical value of yields under perfect foresight with the actual yields.

From Equation (6), the yield under perfect foresight, R_t^{n*}, can be expressed as follows:

$$R_t^{n*} = \frac{1-\gamma}{1-\gamma^n}\sum_{s=0}^{n-1}\gamma^s r_{t+s} \ . \tag{7}$$

The relationship between R_t^n and R_t^{n*} is therefore given by:

$$R_t^n = E_t[R_t^{n*}] + \Phi^n . \tag{8}$$

It is possible to obtain R_t^{n*} by using Equation (7). However, in this case, we require the short-term interest rate $n-1$ periods later. Therefore, we choose to modify Equation (7) and figure up the perfect-foresight yield recursively from the end of the sample (referred to as the "ex-post rational rate" for the sake of convenience).

By applying Equation (4), Equation (7) is easily rewritten as,

$$R_t^{n*} = (1-\bar{\gamma})\sum_{s=0}^{n-1}\gamma^s r_{t+s} \tag{7)'$$

where we can obtain:

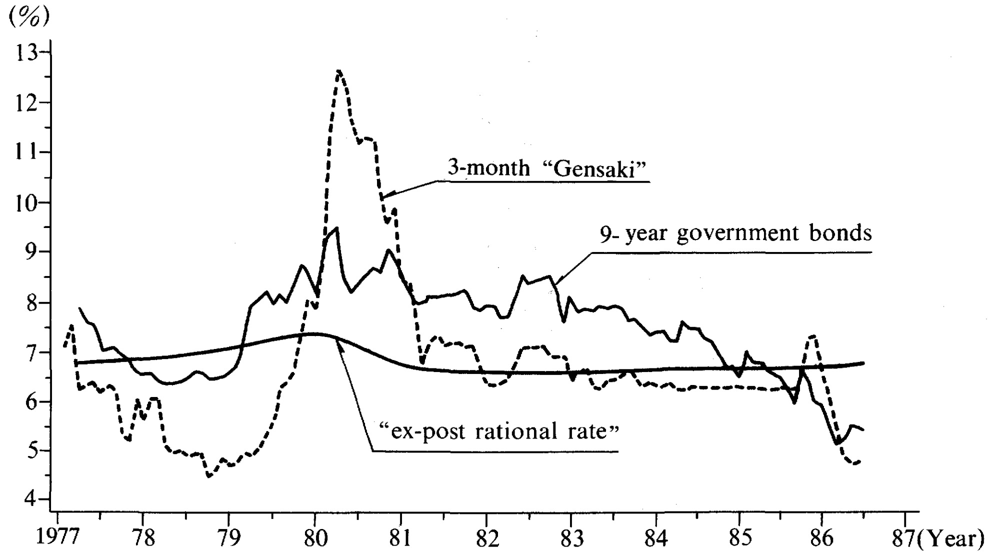
$$R_t^{n*} = \bar{\gamma} R_{t+1}^{n-1*} + (1-\bar{\gamma}) r_t \ . \tag{9}$$

Here, we propose a terminal value conditon that R* of the end of the sample equals,

7. The yields adopted in this paper are the compound yield to maturity on government bonds both in Japan and the United States. In Japan, their years to maturity are constituted of 3 to 9 years (less than 3 years are not available) and in the United States, 2, 5, 10 and 20 years. Regarding the short-term interest rates, in Japan 3-month Gensaki rate and in the United States 3-month Treasury Bill rate are employed respectively.

Figure 1 "Ex-Post Rational Rate" and Actual Yield

(1) Japan



(2) U.S.

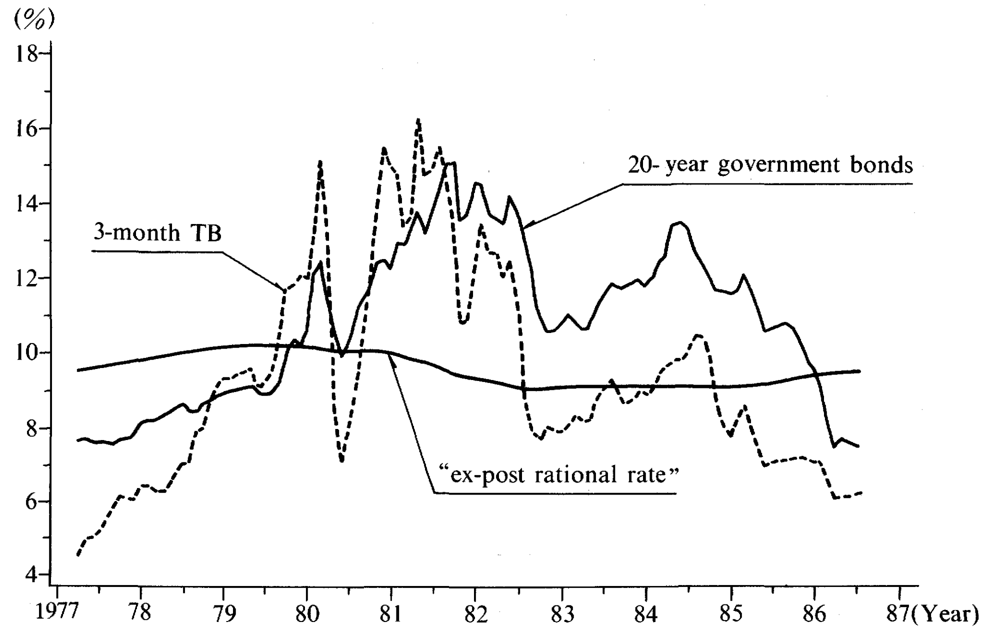
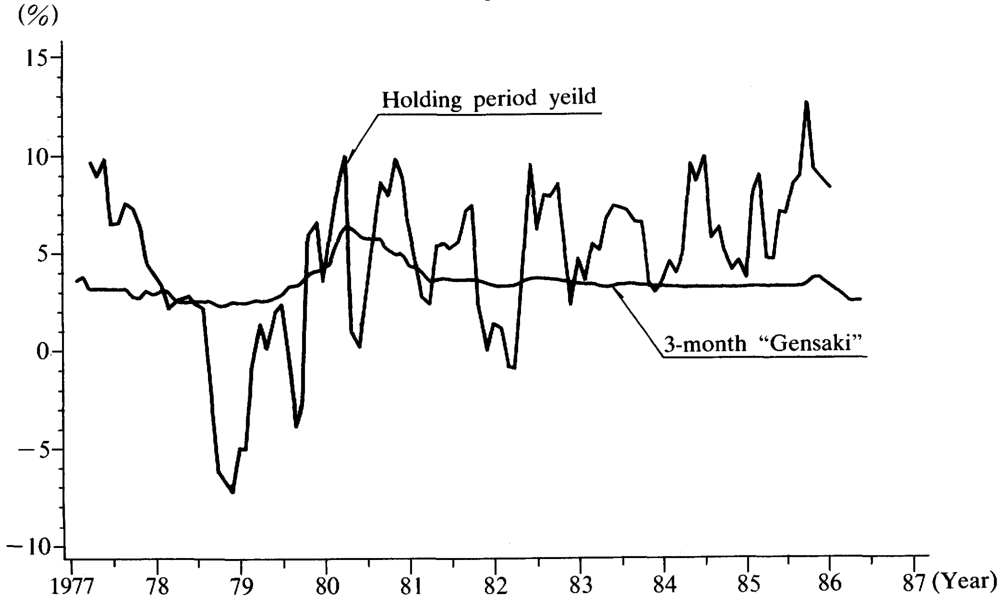


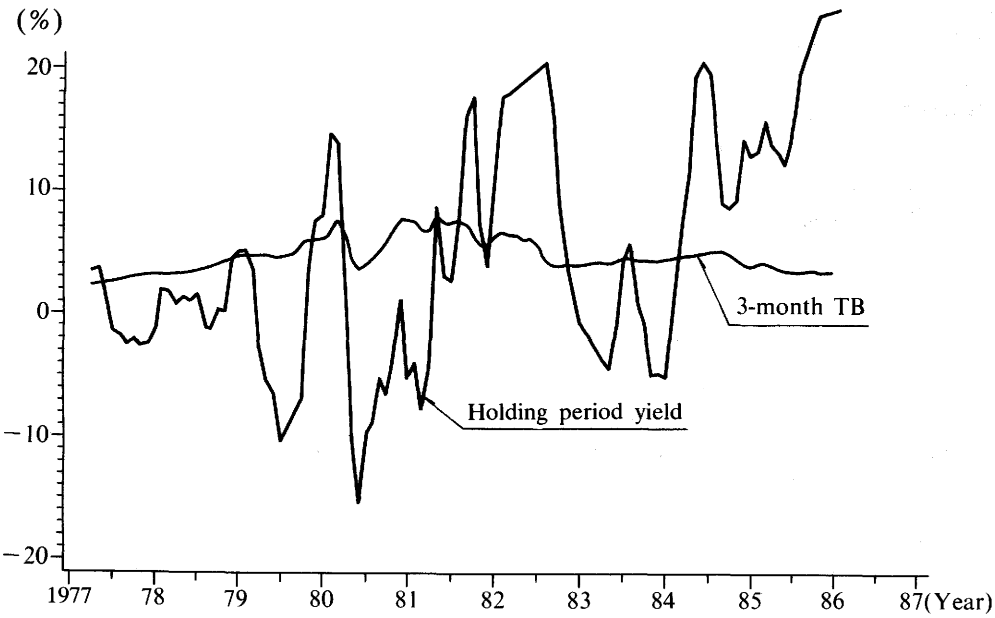
Figure 2 Holding Period Yield (H) with Short-Term Interest Rate (r)

(six months case for the holding period)

(1) Japan



(2) U.S.



for instance, the average short rate over the sample and use Equation (9), working backward from the terminal value, to calculate the ex-post rational rate R_t^{n*}. While we have already observed that the relation between this ex-post rational rate (R_t^{n*}) and the yield (R_t^n) presented by the model of standard theory is expressed by Equation (8), by plotting the ex-post rational rates and actual yields (see Figure 1), we note the dramatically reduced amplitude for the series R_t^{n*} compared with the actually observed series for R_t^n, and that the movements in R_t^n seem totally absent from R_t^{n*} in both Japan and the United States.

On the other hand, since Equation (9) designed to figure the ex-post rational rate can be regarded as having been solved by assuming perfect foresight in Equation (1), $H_{t+1}^n = r_t$,the comparison of the holding period yield on bonds with the short-term interest rate serves as another experiment in the validity of the standard theory. (see Figure 2). The figure shows that the fluctuation in the holding period yield is much more volatile than that in short-term interest rate, which coincide with the deviation of the ex-post rational rate from actual yield observed above.

In other words, Figure 1 and Figure 2 suggest that the models of the standard theory, Equation (6) or Equation (1), do not explain sufficiently the fluctuations in actual yields on bonds.

B. Variance Bounds Tests

a. Derivation of variance bounds inequality for the holding period yield

Regarding the wide fluctuations in yields as shown in the previous paragraph, we now test whether the movements of holding period yields[8] should be considered as volatile under a certain criterion. The criterion adopted here refers to "imposing a theoretical upper bound on the variance of the holding period yield and then comparing it with the sample variance of observed holding period yield ." This is called the "variance bounds tests."[9]

8. The empirical works starting with variance bounds tests are given on the "holding period yields (H_{t+1}^n)" (except the analyses on the standard theory in III) because the variance bounds tests on "yield (R_t^n)" are inevitably arbitrary (see Appendix II). From the relation between Equation (1) and (6), the conclusions on H_{t+1}^n could be treated as those on R_t^n.

9. The variance bounds inequalities have often been applied to the tests on the volatility of asset prices such as stock prices, exchange rates, etc. (see Okina(1985), Ueda et al. (1986), Leroy–Porter (1981), Shiller (1981), Kleidon (1986)). However, the following two technical issues have been pointed out: ① The stationarity of the random variable that provides the upper bound should be examined, and ② the small sample bias must be taken into consideration. In spite of these difficulties, we still consider the variance bounds tests to be effective because, regarding ①, the rational expectations model would be essentially based on stationary stochastic variables and regarding ②, the sample size is large enough to keep the reliability of the test results even when the bias is taken into account (for further information, see Appendix III).

The upper bound is derived under the joint hypothesis that supposes:

① The holding period yields are determined by the Equation (1) model of the standard theory,

② The market expectations are formed rationally.

If it turns out that the sample variance of actual holding period yields exceed the upper bound significantly, the holding period yields can be judged to be volatile, deviating from the equilibrium of rational expectations model of the term structure in which constant risk premium is hypothesized (hereafter referred to as the "standard rational expectations model").

Let us now confirm the concept of rational expectations. The "rational" means that the market participants use all the information available up to time t efficiently, and if yields realized at time t+1 deviate from anticipated values at time t (if any forecast errors occur), we consider that it is caused by the information unavailable at time t. (In this sense, it is possible to interpret the rational expectations hypothesis as the efficient market hypothesis.) This understanding means that the forecast error must be a function of the innovations which take place on and after time t+1 and that the innovations up to time t do not include the information concerning the innovations on and after time t+1. Thus, the rational expectations hypothesis can be regarded statistically, by using Equation (1), as satisfying:

$COV(H^n_{t+1}-r_t-\phi^n, R^n_t)=0$ (cov(·) represents covariance).

The variance bounds inequality derived under the above-mentioned joint hypothesis by Shiller (1979) is

$$VAR(H^n_{t+1}) \leqq VAR(r_t)\cdot\rho^2{}_{rR}\cdot\frac{1}{(1-\bar{\gamma}^2)} \tag{10}$$

where

$VAR(H^n_{t+1})$: variance of the holding period yield on bonds (H^n_{t+1})

$VAR(r_t)$: variance of short-term interest rate (r_t)

ρ_{rR} : correlation coefficient between short-term interest rates (r_t) and yields on bonds (R^n_t).

(For further information on the derivation, see Appendix I. Regarding the concept of variance bounds for the yield R^n_t, see Appendix II.)

b. Results of variance bounds tests—Table 1

The inequality (10) is examined for the "whole period" (April 1977 through June 1986) both in Japan and the United States in Table 1 (as for the holding periods, we assume one year, six months and three months[10]), and our results are as follows:

① For all the data sets in Japan and the United States, the variance of the

Table 1 Results of Variance Bounds Tests

(inequality : $V(H) \leqq V(r) \cdot \rho^2_{rR} \cdot 1/(1-\bar{\gamma}^2)$)

(1) Japan

① t→t+1 : 1 year

m	V(H)	Vm(H)	V(r)	ρ^2_{rR}	$1-\bar{\gamma}^2$	B	α
9	30.01	21.78	3.33	0.527	0.272	6.45	4.65
8	24.62	17.87		0.591	0.294	6.69	3.68
7	20.31	14.74		0.514	0.321	5.33	3.81
6	16.10	11.68		0.458	0.357	4.27	3.77
5	13.04	9.46		0.520	0.406	4.26	3.06
4	11.60	8.42		0.572	0.477	3.99	2.91
3	8.92	6.41	3.56	0.663	0.587	4.03	2.21

② t→t+1 : 6 months

m	V(H)	Vm(H)	V(r)	ρ^2_{rR}	$1-\bar{\gamma}^2$	B	α
9	15.20	11.22	0.73	0.488	0.144	2.46	6.18
8	13.65	10.08		0.552	0.156	2.57	5.31
7	12.80	9.45		0.492	0.172	2.08	6.15
6	8.46	6.24		0.437	0.192	1.65	5.13
5	6.68	4.93		0.492	0.221	1.62	4.12
4	5.85	4.32		0.555	0.262	1.54	3.80
3	4.48	3.28	0.77	0.656	0.331	1.53	2.93

③ t→t+1 : 3 months

m	V(H)	Vm(H)	V(r)	ρ^2_{rR}	$1-\bar{\gamma}^2$	B	α
9	8.86	6.56	0.17	0.433	0.074	0.99	8.95
8	7.63	5.65		0.496	0.081	1.04	7.34
7	7.18	5.32		0.449	0.089	0.86	8.35
6	5.25	3.89		0.398	0.100	0.68	7.72
5	4.45	3.30		0.447	0.110	0.69	6.45
4	3.85	2.85		0.520	0.131	0.67	5.75
3	3.40	2.50	0.18	0.631	0.164	0.69	4.93

(2) U.S.A.

① t→t+1 : 1 year

m	V(H)	Vm(H)	V(r)	ρ^2_{rR}	$1-\bar{\gamma}^2$	B	α
20	268.55	195.72	8.96	0.571	0.216	23.67	11.35
10	172.02	125.37		0.629	0.284	19.83	8.67
5	81.82	59.69		0.710	0.428	14.86	5.51
2	18.29	13.20	7.58	0.825	0.776	8.06	2.27

② t→t+1 : 6 months

m	V(H)	Vm(H)	V(r)	ρ^2_{rR}	$1-\bar{\gamma}^2$	B	α
20	116.12	85.36	1.90	0.558	0.115	9.20	12.62
10	76.51	56.24		0.625	0.153	7.75	9.87
5	39.75	29.22		0.712	0.235	5.74	6.93
2	12.09	8.81	1.64	0.838	0.468	2.93	4.13

③ t→t+1 : 3 months

m	V(H)	Vm(H)	V(r)	ρ^2_{rR}	$1-\bar{\gamma}^2$	B	α
20	58.36	42.67	0.47	0.561	0.060	4.40	13.26
10	37.86	27.69		0.630	0.079	3.76	10.07
5	21.41	15.66		0.718	0.123	2.75	7.79
2	7.46	5.41	0.41	0.844	0.256	1.36	5.49

Note : V(H) : Sample variance of holding period yields on bonds
Vm(H) : Lower bound of 99% confidence interval of the population variance of holding period yields
V(r) : Sample variance of short-term interest rates (per unit term)
B : $V(r) \cdot \rho^2_{rR} \cdot 1/(1-\bar{\gamma}^2)$ (theoretical upper bound)
α : V(H)/B = excess volatility
m : number of years to maturity
Sample period : April 1977 to June 1986

holding period yield (V(H)) exceeds the upper bound (B), and the violation of the variance bounds inequality is significant at 1% level (tested by whether the lower bound of 99% confidence interval of the population variance of the holding period yield (Vm(H)) exceeds significantly the upper bound (B) supposing V(H) distributed as a chi-square random variable).

② The longer the years to maturity and the shorter the holding period, the larger tends to grow the degree of excess volatility (α =V(H)/B).[11]

③ Excess volatility is greater in the United States compared with in Japan. In other words, the holding period yields on Japanese bonds are judged"less volatile" according to the property of the standard rational expectations model.

III. Hypotheses Related to Fluctuations in Yields on Bonds

The analyses in the preceding chapter show that the holding period yields on bonds are too volatile to accord with the standard rational expectations model both in Japan and the United States. As variance bounds inequalities are derived under the joint hypothesis (the rational expectations hypothesis and the standard theory), the violation of the inequalities enables us to make two different interpretations depending on which of the two hypotheses is not satisfied. In this chapter, we would like to test independently each component of the joint hypothesis based on such interpretations. In 1. below, we make some theoretical explanations and in 2. we go into the empirical results.

1. Implication of the Results of Variance Bounds Tests

A. Rational Expectations Hypothesis

The first interpretation of the result whereby the variance bounds inequality is not satisfied is that, although the standard theory serves to explain the movements in yields, the market expectations are not formed rationally. It means that the forecast error of market participants is correlated with the innovations up to time t. Statistically it can be considered that the requirement of $COV(H^n_{t+1}-r_t-\phi^n, R^n_t)=0$ is not

10. It is obvious that the holding period tends to become shorter reflecting the recent behavior of investors who are aiming mainly at capital gains. However, our empirical analyses exclude the transaction made in less than 3 month. This is because in the case of Japan securities transaction taxes and in the case of the United States capital gain taxes on transactions less than 6 months are levied respectively, and it is necessary to make precise adjustments of the taxes levied.

11. α (excess volatility), which is the ratio of the variance (point estimate) of the holding period yield to the upper bound, represents the proportion of the fluctuations that cannot be explained by the standard rational expectations model to the whole fluctuations.

satisfied (see Appendix I). Therefore, this interpretation (hypothesis) is able to be tested by the significance test (t-test) of $\hat{b}$ (estimate of b) subsequent to OLS estimation on

$$H^n_{t+1} - r_t = a + bR^n_t + \delta_t. \tag{11}$$

In estimating Equation (11), however, it is necessary to note a technical point. If we are given the reliable information that the residual of OLS estimation is a white noise, t-test of $\hat{b}$ makes it possible to ascertain whether the volatility of the holding period yields is caused purely by the rational expectations hypothesis or by the standard theory. In case the residual of OLS estimation is not a white noise (that is, when it has a serial correlation), regardless of the result of t-test of $\hat{b}$, the possibility cannot be denied that the OLS estimated Equation (11) lacks another variable; it can be considered as a variable risk premium.

B. Standard Theory

Another possible interpretation of the violation of the variance·bounds inequality is that, although the market expectations are formed rationally, the holding period yields are not determined in accordance with the model of the standard theory, and in other words, there is a failure in the model. Test concerning this interpretation is possible by OLS estimation on Equation (13) (mentioned below), which constitutes the essence of the standard theory obtained from Equation (1), and by hypothesis testing for the estimates of the parameter.

Let us show the process of deriving Equation (13). First of all, since Equation (1) representing the standard theory can be expressed as:

$$E_t[H^n_{t+1}] = E_t[(R^n_t - \gamma R^{n-1}_{t+1})/(1-\bar{\gamma})] = r_t + \phi^n ,$$

(expectations operator $E_t[\cdot|I_t]$ shall be described as $E_t[\cdot]$ for the sake of simplification)

and since R^n_t is already known at time t, we can obtain:

$$R^n_t - \bar{\gamma} E_t[R^{n-1}_{t+1}] = (1-\bar{\gamma})\cdot(r_t + \phi^n) .$$

Here, supposing that $R^{n-1}_{t+1} = E_t[R^{n-1}_{t+1}] + \varepsilon'_{t+1}$, we will have

$$R^n_t = \bar{\gamma}(R^{n-1}_{t+1} - \varepsilon'_{t+1}) + (1-\bar{\gamma})\cdot(r_t + \phi^n),$$

and this can be modified into:

$$R_{t+1}^{n-1}-R_t^n=-\{(1-\bar{\gamma})/\bar{\gamma}\}\phi^n+\{(1-\bar{\gamma})/\bar{\gamma}\}(R_t^n-r_t)+\varepsilon'_{t+1}\,. \tag{12}$$

Equation (12) is easily rewritten as:

$$R_{t+1}^{n-1}-R_t^n=\alpha+\beta(R_t^n-r_t)+\delta_{t+1}. \tag{13}$$

However, it is clear that the theoretical values of α and β (respectively shown as $T\alpha$ and $T\beta$) are respectively:

$$T\alpha=-(1-\bar{\gamma})/\bar{\gamma}\cdot\phi^n$$

$$T\beta=(1-\bar{\gamma})/\bar{\gamma}\left(>0,\ \bar{\gamma}=\frac{\gamma-\gamma^n}{1-\gamma^n},\ \gamma=\frac{1}{1+\bar{R}}\right).$$

The essence of the standard theory expressed by Equation (13) is that the path of yields on bonds (long-term interest rates) can be predicted by changes in the spread and constant terms. On condition that the risk premium is almost zero (the case of the pure expectations theory), the positive spread in a current period will raise the yields in the next period.

The t-test on $\hat{\alpha}$ (estimate of α) represents a hypothesis testing of whether the risk premium is constant or not. However, as mentioned above, the zero risk premium hypothesis is considered to be a special case of the constant risk premium hypothesis. For this reason, we do not deal with the test on $\hat{\alpha}$ and do concentrate our attention on the hypothesis testing on $\hat{\beta}$ (estimate of β).[12]

The hypothesis testing on $\hat{\beta}$ must be carried out based on the hypothesis (Ha) that it is equal to the theoretical value ($T\beta$) as well as the hypothesis (Hb) that it is zero, and the theoretical interpretations of the results of the testing are the following (see Appendix IV):

① $\hat{\beta}\fallingdotseq T\beta$ (the case where the hypothesis Ha that β is equal to $T\hat{\beta}$ is not rejected)

—In this case, it can be judged that the standard theory model of Equation (1) is substantially supported.

② $\hat{\beta}>T\beta$ (the case where the hypothesis Ha is rejected since $\hat{\beta}$ exceeds significantly $T\beta$)

—In this case, market participants place a greater weight on the current short-term interest rate than is imposed by the standard theory model. In other words, the market expectations overreact to the current level of the

12. See Mankiw–Summers (1984), Mankiw–Miron (1985), and Pesando–Plounde (1986) for further details.

short-term interest rates (or news affecting the short-term interest rates), and in that sense the myopic expectations are formed.

③ $\hat{\beta} < T\beta$ and $\hat{\beta} \doteqdot 0$ (while Ha is rejected with $\hat{\beta}$ being significantly below $T\beta$, Hb is not rejected[13])

—This case indicates that the market expectations are static and the rationally predicted future yields may be equal to the current yields. (Supposing the risk premium is almost zero, the series of yields are expressed as a random walk model.) And generally, in addition to the static expectations, there is a possibility of variable risk premium fluctuating slightly. In empirical analyses white noise test of the residual in regression is useful to investigate the presence of variable risk premium.

④ $\hat{\beta} < T\beta$ and $\hat{\beta} < 0$ (Ha is rejected with $\hat{\beta}$ being significantly below $T\beta$ and Hb is also rejected with $\hat{\beta}$ falling significantly below zero)

—This is interpreted due to a large downward bias to $\hat{\beta}$, suggesting a strong possibility of the presence of widely fluctuating variable risk premium.

Ordinary Least Squares (OLS) produces consistent and unbiased estimates only if the error term in regression is uncorrelated with the variables on the right-hand side. The assumption of rational expectations implies that this condition (orthogonality) is satisfied in Equation (13). The error term ε'_{t+1} in Equation (12) measures the "news" that arrives between time t and time t+1. The right-hand side variable, $(R^n_t - r_t)$, is known at time t. If expectations are rationally formed, news should not be predictable from known information. In this case, the rational expectations imply that $(R^n_t - r_t)$ and ε'_{t+1} are uncorrelated, and we can thus estimate Equation (13) using OLS.

The rationality implied by Equation (1) would suggest the following property for ε'_{t+1}:

$$\varepsilon'_{t+1} = -\left(\frac{1-\bar{\gamma}}{\bar{\gamma}}\right)\varepsilon_{t+1}. \tag{14}$$

This can be shown as follows: When the use is made of Equation (1) and $H^n_{t+1} = E_t[H^n_{t+1} | I_t] + \varepsilon_{t+1}$, we get:

$$\varepsilon_{t+1} = (R^n_t - \bar{\gamma} R^{n-1}_{t+1})/(1-\bar{\gamma}) - r_t - \phi^n. \tag{15}$$

13. When $T\beta$ is large (when it is close to 1), $\hat{\beta}$ is sometimes significantly positive and furthermore is significantly below the theoretical value. In this case also, it is identified as the result of downward bias due to the presence of variable risk premium, and the white noise test will be effective.

Then modified and rearranged by using Equation (12), Equation (15) becomes:

$$\varepsilon_{t+1} = \left(\frac{\bar{\gamma}}{1-\bar{\gamma}}\right)\left\{\frac{1-\bar{\gamma}}{\bar{\gamma}}(R_t^n - r_t) - (R_{t+1}^{n-1} - R_t^n) - \frac{1-\bar{\gamma}}{\bar{\gamma}}\phi^n\right\} \tag{16}$$

$$= -\left(\frac{\bar{\gamma}}{1-\bar{\gamma}}\right) \cdot \varepsilon'_{t+1}.$$

Thus in the case that the rational expectations hypothesis is not satisfied in Equation (1) (when ε_{t+1} is correlated with R_t^n), Equation (13) does not satisfy the orthogonality condition (ε'_{t+1} is also correlated with R_t^n). Therefore, OLS estimation does not guarantee consistency and unbiasedness of the parameter estimates in Equation (13).

2. Empirical Analyses of the Two Hypotheses

The results of OLS estimating on Equation (11) (for the rational expectations hypothesis) and Equation (13) (for the standard theory) show the strong positive serial correlation in their respective error terms (Durbin-Watson statistics are 0.093–0.745). Since OLS may provide inefficient parameter estimates, and furthermore, the coefficient of determination is low, we can draw no clear conclusion as to the hypothesis testing (OLS estimations being thus invalid, the results shall not be printed).

However, the serial correlation in error terms is inferred to be a "moving-average (hereafter referred to as "MA") type serial correlation" from the property as follows: The longer the holding period is (three months→six months→one year), the stronger the serial correlation becomes (D.W. statistic declines). Hence, in this section we first theoretically show the possibility that the estimations on Equations (11) and (13) bring about the "MA-type serial correlation." After recognizing the actual data having the MA-type error structure, we correct this serial correlation and return to the hypothesis testing.

As for the sample period, in addition to the "whole period," we test the "recent period" (after April 1981). The purpose is, on one hand, to analyze the effects of recent financial liberalization in Japan (in April 1981, the syndicate was permitted to sell government bonds in 100 days after issuance), and on the other hand, to verify the results of the examination by Shikano (1984) who pointed out that the variable risk premium is possible to exist in the "recent years." From the viewpoint of comparing Japan with the United States, we also conduct the tests for the "whole period" and the "recent period" in the United States.

A. Existence of Moving-Average (MA) Type Serial Correlation and Its Correction

It is known that the "MA type serial correlation" is produced when the forecasting interval is longer than the data sampling interval (see Hansen-Hodick (1980)). The existence of the MA-type serial correlation for the case of the hypothesis testing on the rational expectations in this paper (estimation of Equation (11)) can be demonstrated easily.

To regress $H_{t+i}-r_t$ on R_t means

$$E_t[H_{t+i} - r_t] + \nu_t = \alpha + \beta R_t + \delta_t \tag{17}$$

(ν_t represents white noise).

After $\frac{R_t - \bar{\gamma} R_{t+i}}{1-\bar{\gamma}}$ is substituted for H_{t+i} and parameters with no effects on the conclusion are simplified, Equation (17) yields:

$$\delta_t = \lambda (R_t - E_t[R_{t+i}]) - r_t + \nu_t \tag{18}$$

If we assume that R_t, r_t are approximated by simple AR (1) process ($R_t = \phi R_{t-1} + \varepsilon_t, r_t = kr_{t-1} + \varepsilon_t', \varepsilon_t, \varepsilon_t'$ is white noise), we obtain:

$$\delta_t = \sum_{j=0}^{i-1} \hat{\theta}(j)\varepsilon_{t+j} + \nu_t - \varepsilon_t' + Z \tag{19}$$

where

$$Z = \hat{\omega} R_{t-1} - kr_{t-1},$$

and it would be found that the error term (residual) in the OLS estimation of Equation (11) (the same in the estimation of Equation (13)) constructs MA ($i-1$) type model. In other words, if $i>2$ (in the case of monthly data, the forecasting interval or the holding period of bonds being more than two months), the serial correlation in the error term could exist (for further information, see Appendix V).

Since the possibility has been shown that MA-type serial correlation could be provided for the OLS estimation of Equations (11) and(13), in order to recover efficiency of the parameter estimates, we run generalized least squares estimation assuming the MA-type error term (correction of MA-type serial correlation). The following is the procedure for the analyses.

① We first examine the auto-correlation function (hereafter referred to as "ACF") and the partial auto-correlation function (hereafter referred to as "PACF") of the residual in the OLS estimation to make diagnostic checking of the model and to corroborate if they are of the theoretically proven MA ($i-1$) type.

② If ACF and PACF reflect MA ($i-1$) process, specify the degrees of the MA model (specify variance and covariance matrices of the error term) and proceed to a re-estimation of Equations (11) and (13)[14] using the unconditional least squares estimation method.

First, we undertake the diagnostic checking of the model. When we have removed the residuals of the OLS estimation on Equations (11) and (13) and then examined their respective ACF and PACF, we have acquired the results that all the samples have supported MA (i−1) type ACF (cuts-off) and PACF (tails-off).[15] Therefore, the error terms have proved to be generally expressed by the MA (i−1) type model.

B. Empirical Results after MA-type Serial Correlation Correction—Tables 2 and 3

In accordance with the diagnostic checking we re-estimate Equations (11) and (13) correcting the serial correlation of error terms.[16] The results are summarized as follows:

<Japan>

For the test on the rational expectations hypothesis (estimation of Equation (11)) (see Table 2),

① The hypothesis is rejected for the "whole period", while it is hardly rejected for the "recent period" except some cases.

② The null hypothesis that the residual in the estimation after MA-type serial correlation correction is white noise is rejected at 1% or 5% level of significance. In other words, the existence of the variable risk premiums would not be denied.

As to the standard theory (estimation of Equation (13)), we re-estimate only for the "recent period" when the orthogonality requirement would be generally satisfied (see Table 3). The results are:

① On almost all the samples, the possibility of presence of variable risk premiums ($\hat{\beta} < T\beta$) can be recognized.

② However, when the holding period is assumed to be one year, the static

14. The re-estimation here refers to the estimation of the parameters after specifying the structure of the error term. When the error term has a serial correlation, the question of efficiency of parameter estimates is pivotal, and therefore some analysts propose that only the variances (or standard errors) of parameter estimates should be estimated and that parameter estimates themselves should not be re-estimated. However, in case that the serial correlation of the error term is very strong, parameters themselves are very likely to be affected and therefore we have opted this re-estimation.

15. For ACF and PACF, we used SAS program ARIMA procedure.

16. For MA type serial correlation correction, we ran SAS program, and white noise tests (autocorrelation checks of the error term) were by Ljung=Box Q statistics.

Table 2 Testing of the Rational Expectations Hypothesis after Moving-Average Type Serial Correlation Correction

1. April 1977 to June 1986

(1) Japan

m	① t→t+1 : 1 year $\hat{a}$	$\hat{b}$	Q(6)	② t→t+1 : 6 months $\hat{a}$	$\hat{b}$	Q(6)	③ t→t+1 : 3 months $\hat{a}$	$\hat{b}$	Q(3)
9	−24.125	3.403	12.92*	−11.412	3.712	14.38*	−5.860	3.701	15.96*
	(−3.942)	(4.475)		(−2.434)	(3.105)		(−1.533)	(1.869)	
8	−12.199	2.306	54.79**	−12.478	4.012	33.66**	−4.243	2.777	21.77**
	(−1.996)	(3.052)		(−3.249)	(4.122)		(−1.333)	(1.685)	
7	−12.073	2.302	38.33**	−9.094	3.102	21.51**	−3.241	2.257	20.75**
	(−2.341)	(3.734)		(−2.285)	(3.073)		(−1.161)	(1.567)	
6	−3.052	1.531	294.33**	−13.559	4.191	30.49**	−3.958	2.415	16.97**
	(−0.476)	(2.001)		(−3.785)	(4.554)		(−1.934)	(2.266)	
5	−11.009	2.211	105.58**	−10.395	3.191	15.82*	−3.661	2.254	16.91**
	(−2.898)	(4.727)		(−4.021)	(4.771)		(−2.276)	(2.674)	
4	−9.243	1.735	29.72**	−9.640	3.095	27.83**	−2.993	1.884	12.69**
	(−2.956)	(4.469)		(−4.237)	(5.185)		(−1.708)	(1.990)	
3	−12.969	1.865	13.05*	−4.427	1.401	16.54*	−3.372	2.033	20.12**
	(−6.451)	(7.666)		(−2.742)	(3.319)		(−2.186)	(2.392)	

(2) U.S.A.

m	① t→t+1 : 1 year $\hat{a}$	$\hat{b}$	Q(6)	② t→t+1 : 6 months $\hat{a}$	$\hat{b}$	Q(6)	③ t→t+1 : 3 months $\hat{a}$	$\hat{b}$	Q(3)
20	−29.817	3.543	48.49**	−14.819	4.585	134.19**	−4.452	2.475	24.51**
	(−2.263)	(3.159)		(−1.555)	(2.755)		(−1.513)	(2.075)	
10	−19.767	2.441	41.11**	−11.338	3.438	107.18**	−6.942	2.690	22.76**
	(−2.122)	(3.082)		(−1.621)	(2.820)		(−2.126)	(2.319)	
5	−10.943	1.400	32.81**	−9.221	2.234	42.93**	−7.747	3.063	18.00**
	(−2.085)	(3.151)		(−2.343)	(3.204)		(−3.223)	(3.435)	
2	−6.895	0.771	30.57**	−6.962	1.475	28.47**	−5.311	2.086	17.13**
	(−4.304)	(5.962)		(−3.868)	(4.754)		(−3.611)	(3.916)	

2. April 1981 to June 1986

(1) Japan

m	① t→t+1 : 1 year $\hat{a}$	$\hat{b}$	Q(6)	② t→t+1 : 6 months $\hat{a}$	$\hat{b}$	Q(6)	③ t→t+1 : 3 months $\hat{a}$	$\hat{b}$	Q(3)
9	18.554	−1.744	23.21**	−6.923	1.885	17.26**	1.704	−0.554	8.19*
	(1.492)	(−1.122)		(−0.770)	(0.882)		(0.499)	(−0.309)	
8	2.149	0.286	13.17*	−7.819	1.818	24.99**	−0.032	0.636	8.26*
	(0.305)	(0.319)		(−0.933)	(0.837)		(−0.010)	(0.369)	
7	−7.149	1.328	26.28**	−5.236	1.632	17.39**	1.482	−0.345	8.89*
	(−0.744)	(1.214)		(−0.768)	(0.915)		(0.570)	(−0.256)	
6	0.164	0.458	15.64*	−6.010	1.339	42.76**	−0.405	0.755	8.17*
	(0.048)	(1.026)		(−0.769)	(0.673)		(−0.154)	(0.551)	
5	−6.859	1.385	17.72**	−12.547	3.530	19.52**	−1.705	1.297	9.31*
	(−1.543)	(2.368)		(−2.483)	(2.642)		(−0.842)	(1.219)	
4	−2.256	0.783	148.57**	−7.861	2.411	16.40*	−3.658	2.235	20.91**
	(−0.499)	(1.322)		(−2.331)	(2.659)		(−1.254)	(1.400)	
3	−12.138	1.898	12.68*	−7.746	2.209	17.91**	−3.605	2.140	14.00**
	(−3.677)	(4.257)		(−2.592)	(2.715)		(−1.927)	(2.077)	

(2) U.S.A.

m	① t→t+1 : 1 year $\hat{a}$	$\hat{b}$	Q(6)	② t→t+1 : 6 months $\hat{a}$	$\hat{b}$	Q(6)	③ t→t+1 : 3 months $\hat{a}$	$\hat{b}$	Q(3)
20	−13.445	7.847	32.25**	−30.014	5.227	18.89**	−21.538	7.983	13.63**
	(−6.733)	(8.010)		(−1.532)	(1.681)		(−2.175)	(2.483)	
10	−76.796	5.779	38.64**	−20.855	3.453	14.19*	−7.390	3.224	9.07*
	(−5.853)	(6.952)		(−1.587)	(1.707)		(−0.969)	(1.301)	
5	−36.404	2.786	96.52**	−12.426	2.339	12.59*	−8.721	3.505	11.58**
	(−4.906)	(5.421)		(−1.686)	(1.979)		(−1.860)	(2.263)	
2	−10.177	0.785	29.80**	−5.394	1.130	18.71**	4.840	2.022	9.93*
	(−3.492)	(4.585)		(−2.693)	(3.505)		(2.423)	(2.978)	

Note : Q(P) : Ljung=Box Q−statistic (distributed as a chi-square random variable with p degrees of freedom)

**(*)indicates the hypothesis that the residual of the estimated equation is white noise is rejected at 1% (5%) level of significance.

t-value in parentheses

Table 3 Testing of the Standard Theory after Moving-Average Type Serial Correlation Correction

(Japan, April 1981–June 1986)

① t→t+1 : 1 year

m	$\widehat{\alpha}$ (Standard error) [t-value]	$\widehat{\beta}$ (Standard error) [t-value]	Ha : $\widehat{\beta}=T\beta$
9	− 0.448 0.189 − 2.370	− 0.127 0.183 − 0.694	− 1.628
8	− 0.640 0.183 − 3.497	− 0.010 0.175 0.057	− 1.137
7	− 0.546 0.164 − 3.329	− 0.113 0.146 − 0.774	− 2.226*
6	− 0.615 0.155 − 3.970	− 0.068 0.141 − 0.482	− 2.234*
5	− 0.491 0.154 − 3.189	− 0.241 0.161 − 1.497	− 3.341**
4	− 0.378 0.155 − 2.427	− 0.424 0.189 − 2.232	− 4.259**
3	− 0.309 0.285 − 1.082	− 0.698 0.175 − 3.970	− 7.097**

② t→t+1 : 6 months

m	$\widehat{\alpha}$ (Standard error) [t-value]	$\widehat{\beta}$ (Standard error) [t-value]	Ha : $\widehat{\beta}=T\beta$
9	0.038 0.192 0.198	− 0.458 0.169 − 2.710	− 3.189**
8	− 0.049 0.086 − 0.569	− 0.247 0.153 − 1.614	− 2.190*
7	− 0.020 0.097 − 0.206	− 0.331 0.150 − 2.207	− 2.866**
6	− 0.081 0.088 − 0.920	− 0.155 0.176 − 0.881	− 1.519
5	0.031 0.211 0.147	− 0.538 0.162 − 3.321	− 4.141**
4	− 0.019 0.078 − 0.244	− 0.399 0.174 − 2.293	− 3.236**
3	0.030 0.090 0.332	− 0.615 0.161 − 3.810	− 5.199**

③ t→t+1 : 3 months

m	$\widehat{\alpha}$ (Standard error) [t-value]	$\widehat{\beta}$ (Standard error) [t-value]	Ha : $\widehat{\beta}=T\beta$
9	− 0.007 0.037 − 0.202	− 0.139 0.121 − 1.149	− 1.475
8	0.030 0.058 0.517	− 0.320 0.128 − 2.495	− 2.834**
7	0.013 0.041 0.317	− 0.257 0.108 − 2.380	− 2.816**
6	0.028 0.052 0.538	− 0.332 0.120 − 2.767	− 3.214**
5	0.015 0.046 0.326	− 0.290 0.117 − 2.478	− 2.995**
4	0.015 0.043 0.348	− 0.348 0.125 − 2.784	− 3.368**
3	0.010 0.273 0.370	− 0.351 0.127 − 2.764	− 3.495**

Note : $T\beta$: Theoreticall value of $\beta(=1-\bar{\gamma}/\bar{\gamma})$

**(*)indicates that "Ha" is rejected at 1% (5%) level of significance.

expectations ($\hat{\beta} \fallingdotseq 0$) would also be relevant.

<United States>

The results of the test on the rational expectations hypothesis show (see Table 2),

① The hypothesis is basically rejected both in the "whole period" and the "recent period."

② As a consequence of the white noise test for the residual after the MA-type serial correlation correction, the null hypothesis is rejected at 1% or 5% level of significance for the both periods. Same as in Japan, this could represent the existence of variable risk premiums.

Since the rational expectations hypothesis has been generally rejected for the both periods, the test of the standard theory cannot be made.

IV. Time Varying Risk Premium (Variable Risk Premium) and Its Validity

1. Concept of Variable Risk Premium and Procedure of Test

We have seen that the volatility of holding period yields would rather be attributable to the irrational expectations (the inefficient market) as long as the standard theory is hypothesized while, on the other hand, the fluctuation in risk premiums should not be neglected. Especially the data for the "recent period" in Japan would suggest strongly that risk premiums are not time-invariant.

The possibility of the existence of variable risk premiums is consistent with the intuitive understanding obtained from Figure 1 and 2 in II, that is, unless the market expectations fluctuate violently, the standard theory that posits risk premiums as being time-invariant may not be able to explain the fluctuations in yields.

On the other hand, according to the recent asset pricing theories (CAPM etc.), the risk premium identified as the result of the investors behavior to maximize their expected utility depends upon sample information and generally varies over time.

Therefore, as the next step, we will deal with the joint hypothesis of the expectations theory of term structure with the variable risk premium (hereafter referred to as the "variable risk premium model") and the rational expectations hypothesis. The test on this joint hypothesis shall be done by the following procedure.

① Assume that excess volatility of the holding period yields can be wholly ascribed to the variability of risk premiums, and compute backward "the minimum value of the variance" of variable risk premiums consistent with the rational expectations hypothesis that would account for the fluctuations in the yields from the results of the variance bounds tests.

② Measure the variance of the theoretical variable risk premiums based on intertemporal asset pricing model.

③ If the theoretical value in ② exceeds (or falls below) the minimum value of the variance calculated in ①, the volatility of the holding period yield can (cannot) be explained analytically by the variability of risk premiums. In other words, the joint hypothesis of the "variable risk premium model" and the rational expectations hypothesis would not (would) be rejected.

A. Variability of Risk Premiums Obtained from Variance Bounds Tests

First we assume that excess volatility of the holding period yields obtained from variance bounds tests is due to the variability of risk premiums, and compute the minimum value of the variance of variable risk premiums consistent with the rational expectations hypothesis.

The expectations model of the term structure with time-varying risk premium is given by:

$$E_t[H^n_{t+1}|I_t] = r_t + \phi^n_t. \qquad (20)$$

(ϕ^n_t represents variable risk premium)

When the risk premium is permitted to vary, then $H^n_{t+1} - r_t - \phi^n_t$ must have the property of a forecast error, implying that $COV(H^n_{t+1} - r_t - \phi^n_t) = 0$ (see Appendix I), and the derivation of Equation (10) generalizes in a straight forward manner to yield:

$$VAR(H^n_{t+1}) \leqq \{VAR(r_t) + VAR(\phi^n_t) + 2COV(r_t, \phi^n_t)\} \cdot \rho^2_{rR} \cdot 1/(1-\bar{\gamma}^2). \qquad (21)$$

Using inequatity (21), a point estimate of the minimum value of $VAR(\phi^n_t)$, $Vs(\phi^n_t)$, that is necessary *if their presence is to reverse evidence of excess volatility*, is obtained as:

$$Vs(\phi^n_t) = VAR(H^n_{t+1}) \cdot \frac{1}{\rho^2_{rR}} \cdot (1-\bar{\gamma}^2) - VAR(r_t) - 2COV(r_t, \phi^n_t). \qquad (22)$$

Now, if one hypothesizes $COV(r_t, \phi^n_t) = 0$ (see Appendix VI), Equation (22) is rewritten as[17]:

$$Vs(\phi^n_t) = VAR(H^n_{t+1}) \cdot \frac{1}{\rho^2_{rR}} \cdot (1-\bar{\gamma}^2) - VAR(r_t). \qquad (23)$$

Since $VAR(H^n_{t+1})$, $VAR(r_t)$, ρ^2_{rR} and $(1-\bar{\gamma}^2)$ are already known from

17. With regard to the variance of the holding period yield ($VAR(H^n_{t+1})$), we use point estimate instead of the bound of a confidence interval.

the results of the variance bounds tests in II, it would be possible to calculate the minimum value of the variance of the variable risk premium consistent with the rational expectations hypothesis from Equation (23)[18].

B. Theoretical Motivation of Variable Risk Premium

In order to measure the theoretical value of the variance of the variable risk premium, we formularize the variable risk premium based on intertemporal asset pricing model. The well-known essence of this theoretical model is that investors maximize their expected discounted utility defined on future levels of consumption subject to sequential budget constraints, and in equilibrium, asset prices are set such that the marginal utility of a unit current consumption foregone (by purchasing an asset j) equals the expected discounted utility of the return from investing that unit of the consumption good (for detailed discussions, see Shikano (1984), Brock (1982) and Lucas (1978)). That is:

$$\frac{u'(C_t)}{P_t} = \beta E_t[\frac{u'(C_{t+1})}{P_{t+1}} \cdot V^j_{t+1} | I_t] \tag{24}$$

where $u'(\cdot)$: marginal utility of consumption
C_t : real consumption at time t
P_t : price level at time t
V_{t+1} : one-period nominal return on asset j
β : constant time discount rate
$E_t[\cdot|I_t]$: expectations operator conditioned on information available at time t (agents are assumed to be rational).

Using the equilibrium condition (24) and Equation (20) the variable risk premium is formulated. The risk premium can be defined as the difference between the expected holding period yield on risky assets and the yield on risk-free assets and is given by from Equation (20):

$$\begin{aligned} \hat{\phi}_t &= E_t[H_{t+1}|I_t] - r_t \\ &= E_t[H_{t+1}+1|I_t] - (r_t+1). \end{aligned} \tag{25}$$

18. When Equation (23) is divided by $VAR(r_t)$, it becomes:
$V_s\ (\phi^n_t)/VAR\ (r_t)$
$= VAR\ (H^n_{t+1}) \cdot \frac{1}{\rho^2_{rR}} \cdot (1-\bar{\gamma}^2) \cdot \frac{1}{VAR(r_t)} - 1$
$= (VAR\ (H^n_{t+1})/B) - 1 = \alpha - 1.$
This equation shows that when excess volatility ($= \alpha$) is larger than 2, the greater part of the fluctuation in the holding period yield is explained by the variable risk premium with the assumption of rational expectations.

When $E_t[H_{t+1}+1|I_t]$ and r_t+1 obtained by substituting $H_{t+1}+1$ and r_t+1 for V^j_{t+1} in Equation (24) are used in Equation (25), the risk premium resulting from the maximizing behavior of rational agent can be expressed as follows: (for the derivation, see Appendix VII).

$$\hat{\phi}_t=\frac{E_t[\Omega|I_t]E_t[H_{t+1}+1|I_t]-E_t[\Omega\cdot(H_{t+1}+1)|I_t]}{E_t[\Omega|I_t]} \tag{26}$$

where

$$\Omega=\frac{u'(C_{t+1})}{u'(C_t)}\cdot\frac{P_t}{P_{t+1}}$$

Thus, it has been shown that the risk premium can be described as a function of the expected utility, price level and holding period yield on risky assets. The conditional expectation can depend on sample information and need not be time invariant, hence, the risk premium can fluctuate over time. The revision of expectations would occur more frequently due to active exchanges of information incidental to the liberalization and internationalization of financial transactions. Particularly in the case of Japan, the volatile movements of yields on U.S. bonds and the fluctuations in foreign exchange rates, etc. can be the information which considerably affects the revision of investors' expectations on the holding period yield as well as the price level.

The variable risk premium expressed by Equation (26) is not suitable for empirical analyses because it includes the investor's utility function. Therefore, in order to make the empirical works practicable, we hypothesize the constant relative risk averse for the utility function. The relative risk averse refers to the measure of changes in marginal utility with the level of income or assets (here consumption) weighted by the scale of income or assets. Consequently, the constant relative risk averse means that the proportion of risky assets in total assets is constant irrespective of the scale of assets.

The utility function of investors that satisfies the condition of constant relative risk averse is generally expressed by (relative risk averse is $-u''(C_t)\cdot C_t/u'(C_t)=\mu$=constant, a risk lover ($\mu\geq 0$) is excluded a priori):

$$u(C_t)=\frac{C_t^{1-\mu}}{1-\mu}\ . \tag{27}$$

From Equation (27), we obtain

$$u'(C_t)=C_t^{-\mu}, \tag{28}$$

and therefore by substituting this for Ω, Equation (26) becomes:

$$\hat{\phi}_t = \frac{E_t[\Omega'|I_t]E_t[H_{t+1}+1|I_t]-E_t[\Omega'\cdot(H_{t+1}+1)|I_t]}{E_t[\Omega'|I_t]} \tag{29}$$

where

$$\Omega'=(\frac{C_t}{C_{t+1}})^{\mu}\cdot\frac{P_t}{P_{t+1}}.$$

Thus, supposing that the utility function of investors satisfies the constant relative risk averse, the variable risk premium is determined by the expected real rate of consumption change, expected inflation rate, the expected holding period yield and the relative risk averse.

It should be noted in this connection that the conditions for the pure expectations theory (zero risk premium hypopthesis) are the following to be satisfied simultaneously: ① Investors are risk neutral ($\mu=0$) and ② no uncertainty exists and the price fluctuations can be perfectly forecasted ($E_t[P_{t+1}/P_t=P_{t+1}/P_t$).

C. Estimation of the Variable Risk Premium

In Equation (29) we have formulated the variable risk premium based on intertemporal asset pricing model. In order to estimate this theoretical variable risk premium by use of actual data, it is necessary to find the average relative risk averse in the market. It is known that the relative risk averse can be expressed as Equation (30) under the hypotheses 1. to 5. below (specifically, see Friend – Blume (1975) and Maru (1976)).

(Hypotheses) 1. Investors are risk averse.
2. Yields on assets are normally distributed.
3. Investors have homogeneous expectations.
4. The market is perfectly competitive.
5. Asset returns are exempt from taxation.

In this case, the relative risk averse is given by:

$$\mu=\frac{E(Rm-Rf)}{\sigma m^2}\cdot\frac{1}{\alpha} \tag{30}$$

where

E : expectation
Rm : yields on all risky assets in the market
σm^2 : variance of yields on all risky assets in the market
Rf : yields on risk free assets in the market
α : proportion of risky assets in total assets.

It is difficult to observe the precise values for Rm, σm^2, Rf and α. However, supposing that risky assets consist of public and corporate bonds, stocks and investment trusts, while risk free assets are composed of deposits (including postal savings in the case of Japan) and short-term financial market assets (maturity in less than one year), we have estimated by data after 1977 that the relative risk averse is around 2.0 in both Japan and the U.S. This finding was obtained from a very rough estimation which did not take account of taxation on assets (if it was taken into account, the figure possibly rising to about 3.0 in both countries) and in which risky and risk free assets were chosen arbitrarily. Therefore, in estimating[19] the theoretical variable risk premiums based on Equation (29)[20], we set the value of relative risk averse within the range of 0.5 to 5.0.[21]

2. Empirical Results in Japan and the United States (Variance Comparison of Risk Premiums)—Table 4, Figure 3

A. For the "Whole Period" in Both Countries

For the "whole period" both in Japan and the United States, we calculated the minimum values of the variances of the variable risk premium consistent with the rational expectations hypothesis, $Vs(\phi^n_t)$, from Equation (23) and estimated the variances of the theoretical variable risk premium ($\hat{\phi}_t$), $V(\hat{\phi}_t)$, in accordance with Equation (29) respectively. We then compared these values.[22] The summarized re-

19. Regarding $E_t[\cdot|I_t]$ of Equation (29) (conditional expectations operator), we employed the method that combines a time trend with an autoregressive model and uses a stepwise method to select the lags to use for the autoregressive process (SAS program FORECAST procedure). Concerning the time trend model, we selected the constant trend model and the linear trend model.

20. As for the level of consumption, we made use of the household budget survey in Japan and the survey of current business in the United States. With regard to price level, CPI was used in both countries.

21. The utility function of an investor whose relative risk averse is 5.0 is, according to Equation (27) mentioned above, $u(C_t)=-1/4\cdot C_t^{-4}$, where $u'(C_t)=1/C^5_t$. We can see that this utility function is extremely convex upwards, in other words, when considering the level of a large C_t (or asset) even to a limited extent (if wealthy even to a limited extent), it is obvious that an additional unit C_t (asset) will hardly increase the marginal utility. This means that such investors have very little incentive to invest in risky assets. Using the relationship of Equation (30) and assuming the average yield in risk free assets is 5%, the risky assets have to yield extremely high expected rate of return (around 20%) with stable fluctuations (about 5% for variance) to induce the investors to hold half of their assets in risky assets. Such a restriction may not be plausible when the investors manage their assets in stock or bond market or both. Furthermore, the empirical studies (in stock market in the United States) by Brown–Gibbon (1985), Friend–Blume (1975), Grossman–Shiller (1981), Hansen–Singleton (1982, 1983), etc. have shown that the relative risk averse falls within a range of 2.0 to 4.0.

22. In the strict sense, it is necessary to specify the distribution of variance of the variable risk premium and compare its bound of a confidence interval with the minimum value. In this paper, however, we adopt point estimates.

Table 4 Variance Comparison of Risk Premiums

(1) Japan

① t→t+1 : 1 year

m	$V_s(\phi)$	$\hat{V}_a$	$\hat{V}_b$	$\hat{V}_c$	$\hat{V}_d$	$\hat{V}_e$	$\hat{V}_f$	$\hat{V}_g$
9	12.16	0.41	1.10	1.72	1.77	2.13	11.19	15.97
8	8.92	0.97	1.39	2.12	2.19	2.35	11.30	15.77
7	9.36	0.73	1.37	1.83	1.91	2.26	11.32	10.16
6	9.22	0.65	1.06	1.67	1.64	1.83	9.67	13.43
5	6.85	0.36	0.72	1.22	1.19	1.41	9.68	9.93
4	6.35	0.55	0.93	1.29	1.19	1.55	9.73	9.43
3	4.33	0.45	0.80	1.23	1.39	1.35	9.69	13.02

② t→t+1 : 6 months

m	$V_s(\phi)$	$\hat{V}_a$	$\hat{V}_b$	$\hat{V}_c$	$\hat{V}_d$	$\hat{V}_e$	$\hat{V}_f$	$\hat{V}_g$
9	3.76	0.63	1.38	3.53	4.94	6.48	6.64	9.79
8	3.13	0.49	1.03	3.42	4.66	5.80	7.66	9.57
7	3.75	0.46	1.53	4.24	4.53	7.25	8.11	8.52
6	2.99	0.41	0.82	2.70	3.83	6.03	6.33	6.68
5	2.28	0.42	0.82	2.78	3.85	4.56	6.03	5.91
4	2.04	0.47	0.76	2.71	3.66	3.65	4.39	5.31
3	1.49	0.50	0.73	2.33	3.73	3.83	4.51	5.51

③ t→t+1 : 3 months

m	$V_s(\phi)$	$\hat{V}_a$	$\hat{V}_b$	$\hat{V}_c$	$\hat{V}_d$	$\hat{V}_e$	$\hat{V}_f$	$\hat{V}_g$
9	1.34	0.38	1.56	2.22	3.12	3.91	5.09	5.67
8	1.08	0.78	1.60	2.31	2.95	3.20	5.22	6.17
7	1.25	0.43	1.61	2.05	2.77	3.53	4.62	6.89
6	1.15	0.37	1.37	1.73	1.74	2.67	3.96	4.73
5	0.93	0.34	1.22	1.49	2.15	2.92	2.93	3.01
4	0.80	0.34	1.15	1.56	1.70	2.06	2.62	2.63
3	0.70	0.56	1.36	1.81	1.81	3.09	3.54	3.43

(2) U.S.A.

① t→t+1 : 1 year

m	$V_s(\phi)$	$\hat{V}_a$	$\hat{V}_b$	$\hat{V}_c$	$\hat{V}_d$	$\hat{V}_e$	$\hat{V}_f$	$\hat{V}_g$
20	92.63	0.06	0.09	1.92	2.23	2.59	3.94	5.34
10	68.71	0.08	0.18	1.12	1.17	1.49	3.77	4.92
5	41.06	0.08	1.25	1.61	1.67	2.39	2.40	2.53
2	9.62	0.53	0.53	0.99	1.34	1.65	1.70	2.10

② t→t+1 : 6 months

m	$V_s(\phi)$	$\hat{V}_a$	$\hat{V}_b$	$\hat{V}_c$	$\hat{V}_d$	$\hat{V}_e$	$\hat{V}_f$	$\hat{V}_g$
20	22.04	0.04	1.44	2.30	2.91	3.94	4.58	6.43
10	16.83	0.04	1.00	1.25	1.45	2.19	3.00	3.33
5	11.22	0.07	0.20	0.44	0.80	1.74	2.00	2.49
2	5.11	0.09	0.22	0.69	0.65	0.63	0.63	2.38

③ t→t+1 : 3 months

m	$V_s(\phi)$	$\hat{V}_a$	$\hat{V}_b$	$\hat{V}_c$	$\hat{V}_d$	$\hat{V}_e$	$\hat{V}_f$	$\hat{V}_g$
20	5.77	0.06	0.23	1.20	1.80	3.17	3.51	4.01
10	4.28	0.08	0.26	0.61	1.58	2.36	3.06	2.95
5	3.20	0.15	0.52	1.49	1.96	2.42	2.50	3.54
2	1.85	0.39	0.39	0.47	0.73	0.92	0.96	1.09

Note : $V_s(\phi)$: variance of the variable risk premium calculated retroactively from the results of variance bounds tests

$\hat{V}_a$: variance of the theoretical risk premium, assuming the investor's RRA to be 0.5 (RRA : relative risk averse)

$\hat{V}_b$: assuming RRA = 1.0

$\hat{V}_c$: assuming RRA = 1.5

$\hat{V}_d$: assuming RRA = 2.0

$\hat{V}_e$: assuming RRA = 3.0

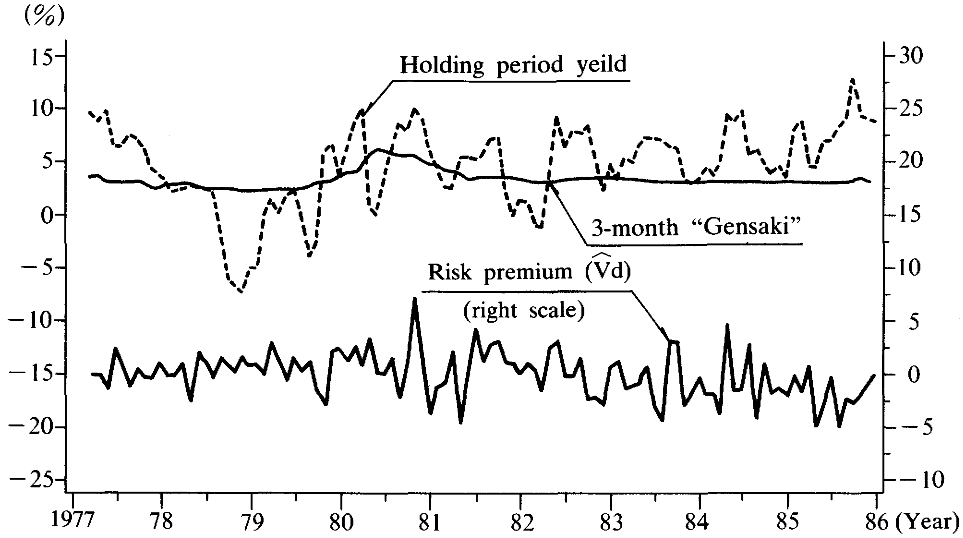
$\hat{V}_f$: assuming RRA = 4.0

$\hat{V}_g$: assuming RRA = 5.0

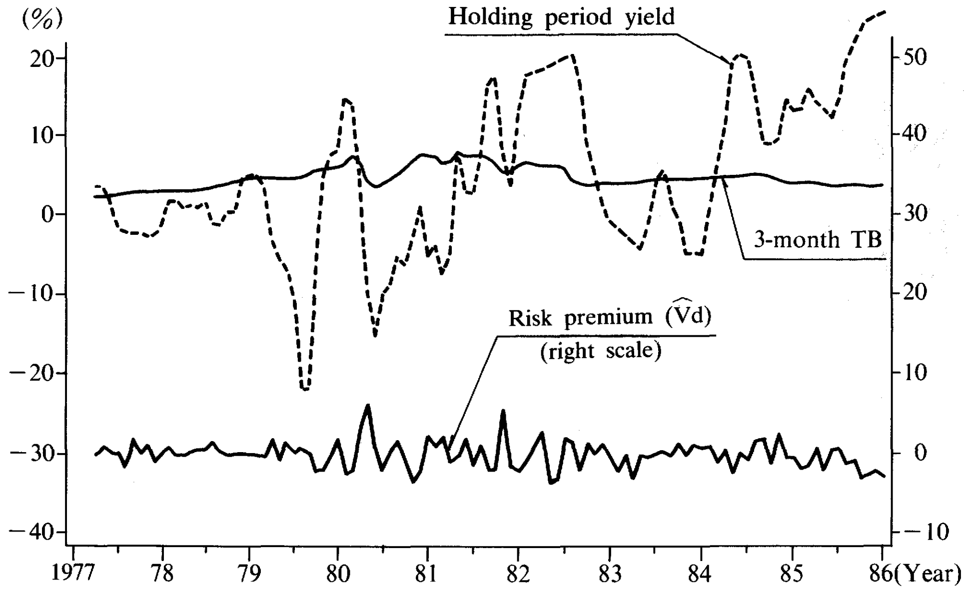
Figure 3 Fluctuations in Variable Risk Premiums

–the case that relative risk averse is 2.0 and the holding period is six months

(1) Japan



(2) U.S.



sults are:

① In most cases of Japan, $V(\hat{\phi}_t)$ can cover $Vs(\hat{\phi}^n_t)$ when the relative risk averse is assumed to be 2.0 (in the cases of one-year holding period, the relative risk averse should be assumed 3.0 to 4.0). In other words, the excess volatility of the holding period yields can be validly explained by the variability of variable risk premiums consistent with the rational expectations hypothesis. Therefore the joint hypothesis consisting of the "variable risk premium model" and the rational expectations hypothesis would not be rejected, and the fluctuations in yields in Japan are consistent with the rational behavior of investors as long as one assumes the presence of variable risk premiums.

② As far as the United States is concerned, even if one assumes that the relative risk averse is 5.0, in almost all cases $Vs(\phi^n_t)$ cannot be exceeded by $V(\hat{\phi}_t)$. In particular, when the holding period is one year or six months, $V(\hat{\phi}_t)$ is less than half of $Vs(\phi^n_t)$. Consequently, it is quite possible that the joint hypothesis will be rejected. In other words, the fluctuations in yields in the United States are not consistent with the rational behavior of agents even if the existence of variable risk premiums is assumed.

The above empirical results are clearly demonstrated in Figure 3, in which the theoretical variable risk premium (where the relative risk averse is assumed to be 2.0 and the holding period to be 6 months) is plotted along with the holding period yield and the short-term interest rate. To explain the fluctuations in the holding period yield by the variability of variable risk premium consistent with the rational expectations hypothesis ($Vs(\phi)$), the variance of the theoretical variable risk premium ($\hat{V}d$) plotted in Figure 3 must be more than approximately 20% (50% in the case of standard deviation) of the variance of the holding period yield (V(H)) (Japan...$V_s(\phi)=3.76$, $V(H)=15.20$, United States... $V_s(\phi)=22.04$, $V(H)=116.12$). Observed from Figure 3, in the case of Japan, $\hat{V}d$ would satisfy the requirement, while in the case of the United States it would not.

B. For the "Recent Period" in Japan[23]—Table 5

Here we carry out the variance bounds tests and the subsequent comparison of the variances of variable risk premiums for the "recent period" in Japan since the

23. Since the sample size of the "recent period" is about 50, it may be necessary to consider the small sample bias. As regards variance bounds tests, however, since excess volatility ($=\alpha$) is sufficiently large, this difficulty can be ignored. Furthermore, for the variance comparison of risk premiums, as the downward bias on the upper bound of variance bounds inequality brings about the upward bias in the minimum value of the variance of risk premiums consistent with the rational expectations model, there will be no problem if the joint hypothesis consisting of the "variable risk premium model" and the rational expectations hypothesis are difficult to reject.

Table 5 Reference: Variance Bounds Tests and Variable Risk Premiums

(Japan, April 1981–June 1986)

(1) Results of variance bounds tests

① t→t+1 : 1 year

m	V(H)	Vm(H)	V(r)	ρ^2_{rR}	$1-\bar{r}^2$	B	α
9	5.55	3.57	0.14	0.563	0.270	0.30	18.8
8	4.41	2.84		0.662	0.292	0.32	13.7
7	4.82	3.10		0.771	0.320	0.34	14.1
6	3.13	2.01		0.714	0.357	0.28	11.0
5	2.50	1.61		0.618	0.406	0.22	11.6
4	2.21	1.42		0.716	0.476	0.21	10.3
3	1.57	1.01	↓	0.746	0.585	0.18	8.7

② t→t+1 : 6 months

m	V(H)	Vm(H)	V(r)	ρ^2_{rR}	$1-\bar{r}^2$	B	α
9	7.38	4.94	0.03	0.526	0.143	0.12	62.5
8	7.51	5.02		0.622	0.156	0.14	55.2
7	9.22	6.17		0.757	0.171	0.14	64.9
6	5.44	3.64		0.704	0.192	0.12	46.5
5	4.23	2.83		0.610	0.220	0.09	47.6
4	3.26	2.18		0.688	0.262	0.08	38.8
3	2.52	1.69	↓	0.753	0.330	0.07	34.6

③ t→t+1 : 3 months

m	V(H)	Vm(H)	V(r)	ρ^2_{rR}	$1-\bar{r}^2$	B	α
9	5.10	3.44	0.008	0.229	0.074	0.03	204.1
8	5.44	3.67		0.297	0.080	0.03	181.3
7	5.79	3.91		0.436	0.089	0.04	148.5
6	4.22	2.85		0.397	0.100	0.03	131.9
5	3.31	2.23		0.311	0.110	0.02	143.8
4	2.88	1.94		0.382	0.131	0.02	125.1
3	2.43	1.64	↓	0.497	0.163	0.02	101.0

Note : See note for Table 1.

(2) Variance comparison of risk premiums

① t→t+1 : 1 year

m	$V_S(\phi)$	$\widehat{Va}$	$\widehat{Vb}$	$\widehat{Vc}$	$\widehat{Vd}$	$\widehat{Ve}$	$\widehat{Vf}$	$\widehat{Vg}$
9	2.52	0.78	1.11	1.68	1.69	1.72	1.79	1.82
8	1.80	0.81	1.86	2.22	2.20	2.21	2.23	2.29
7	1.86	0.76	1.77	1.71	1.77	1.78	1.79	1.80
6	1.42	0.58	1.21	1.26	1.29	1.28	1.26	1.29
5	1.50	0.42	0.43	0.43	0.44	0.44	0.45	0.46
4	1.32	0.69	0.74	0.76	0.77	0.77	0.78	0.79
3	1.09	0.47	0.55	0.57	0.58	0.59	0.59	0.60

② t→t+1 : 6 months

m	$V_S(\phi)$	$\widehat{Va}$	$\widehat{Vb}$	$\widehat{Vc}$	$\widehat{Vd}$	$\widehat{Ve}$	$\widehat{Vf}$	$\widehat{Vg}$
9	1.97	0.70	2.30	3.30	3.96	6.37	6.51	9.91
8	1.73	0.80	1.42	2.49	2.52	2.61	3.00	4.36
7	2.05	0.85	2.12	3.44	3.77	4.62	4.71	4.93
6	1.46	0.68	1.07	1.54	1.56	1.57	1.57	1.57
5	1.49	0.64	1.11	1.44	1.79	2.01	2.49	2.45
4	1.21	0.59	1.07	1.06	1.08	1.11	1.32	1.33
3	1.07	0.65	0.86	0.87	0.89	0.93	0.94	0.95

③ t→t+1 : 3 months

m	$V_S(\phi)$	$\widehat{Va}$	$\widehat{Vb}$	$\widehat{Vc}$	$\widehat{Vd}$	$\widehat{Ve}$	$\widehat{Vf}$	$\widehat{Vg}$
9	1.62	0.72	1.45	2.58	2.57	2.58	3.12	3.81
8	1.44	0.68	1.22	2.57	2.60	2.65	2.67	2.68
7	1.18	0.82	1.73	2.44	3.09	3.69	3.73	3.77
6	1.05	0.47	0.88	1.70	1.73	1.75	1.77	1.79
5	1.14	0.85	1.25	1.51	1.52	1.53	1.54	2.24
4	0.99	0.51	1.13	1.36	1.37	1.39	1.39	1.40
3	0.80	0.67	1.18	1.33	1.34	1.35	1.35	1.34

Note : See note for Table 4.

preceding chapter has shown the presence of the variable risk premium. The results are summarized as follows.

a. Results of variance bounds tests

As for the results of the variance bounds tests, in all the cases, the lower bound of 99% confidence interval of the variance of the holding period yield (V(H)) exceeds the theoretical upper bound, and the joint hypothesis of the standard theory and the rational expectations hypothesis is rejected at 1% level of significance.

b. Variance comparison of the risk premiums

Since the variances of the short-term interest rates are extremely small compared with the case of the "whole period", the ratio of the minimum value of the variance of the variable risk premium consistent with the rational expectations hypothesis, Vs(ϕ_t^n), to the variance of the holding period yield, V(H), would be relatively large. However, except for some cases where the holding periods are one year, the variance of the theoretical variable risk premium, V($\hat{\phi}_t$), can generally explain the minimum value of the variance of the variable risk premium consistent with the rational expectations hypothesis, Vs(ϕ_t^n). Therefore, the joint hypothesis consisting of the "variable risk premium model" and the rational expectations hypothesis would not be rejected.

V. Conclusion and Future Tasks

1. Conclusion

The empirical results of the variance comparison in the preceding chapters are considered to be related to the conclusion obtained from the variance bounds tests assuming the standard rational expectations model in II which states that the fluctuation in yields is less volatile in Japan than in United States. In other words, the volatility of yields on Japanese bonds is able to be explained by the fluctuations in risk factors consistent with the rational behavior of investors, and therefore it is possible to mention that the volatility is theoretically explicable. On the other hand, the yields on U.S. bonds are too volatile to accord with the rational expectations model including such a time variant risk factor.

What causes the volatility of U.S. bonds? One possible explanation is that it is simply due to the irrational behavior of market participants. (It may be possible to interpret the volatility as a phenomenon similar to the rational bubble. This point shall be discussed in detail in the next section.) In other words, the market yields on U.S. bonds are not formed effciently by making use of past information including the short-term interest rates and the variables related to the risk factors. In addition another interpretation that external information shocks affect the holding period yields directly in the form of large forecast errors would be possible. It should be noticed that monetary uncertainty would bring about such external shocks which, however, are unaccountable within the framework of the equilibrium model in this paper.

2. Future Tasks

In this section, we would like to suggest some of the remaining tasks that need to be tackled regarding the future analyses.

The first problem concerns the implications for the monetary policy in Japan. The conclusion in this respect is that it is important for the monetary authorities to improve their understanding of the variable risk factors and fluctuations involved. This means, in regard to theory, that there is a need to clarify these factors by developing further the theory of variable risk premium. In this connection, the formulation of models expressing actual variable risk factors precisely, the analysis of transmission mechanism of interest rate policies based on such models, etc., will hereafter be just some of the important tasks for our study.

Secondly, we have the tasks of elucidating the volatility of yields on U.S. bonds. Three essential points need to be pointed out.

① Since, in the case of United States, we have been unable to explain the fluctuations in yields even by the variability of risk premium, it will be our first work to test alternative hypotheses such as the "myopic expectations hypothesis" (in the United States, this can be interpreted as an overreaction to the money supply announcement) and the "static expectations hypothesis" (see III. 1–B).

The appropriate procedure involves newly derived variance bounds tests under a joint hypothesis of the expectations model of the term structure with the myopic expectations hypothesis or the static expectations hypothesis and the rational expectations hypothesis. In connection with the test of the joint hypothesis consisting of the myopic expectations and the rational expectations, it should be noted that Mankiw – Summers (1984) have already obtained the empirical results that the myopic expectations hypothesis is rejected (that is, there is no evidence that the yields are overreacting to the current short-term interest rate) in the process of testing the standard rational expectations model. If this result is correct, to examine the joint hypothesis of the static expectations and the rational expectations will have greater significance.

② If the two joint hypotheses mentioned above are rejected (this would be possible), the volatility of yields on U.S. bonds is basically regarded as being ascribed to the irrational market expectations (the inefficient market). It means that the fluctuations in yields on U.S. bonds are inconsistent with the rational behavior of agents whatever expectations model of the term structure may be assumed. The reason why we can state like this is that, in the case of bond yield (or price), due to the presence of maturity period, the rational bubble (bubble consistent with the rational expectations) cannot theoretically be generated (see Appendix VIII). However, if one assumes that the holding period of investors is extremely short in comparison with the years to maturity, it is very unlikely that investors will forecast all the future bond prices rationally until the maturity. In such a situation, the generation of a quasi-bubble in yields, which represents a large deviation from the equilibrium of the rational expectations model, may not be always denied. And we should also refer to the empirical result that the longer the years to maturity is, the greater becomes the excess volatility. If one is permitted to interpret in this way, it would be possible to approach the fluctuations in yields from the viewpoint of quasi-rational bubble, and this may be considered to be one of the tasks to be examined in the future.

③ For the possibility that the yields overreact to information shocks resulting from monetary uncertainty, the given framework of equilibrium analysis in this paper is not available. However, the test by VAR model, for example, which investigates how much the variance (fluctuation) of the expected growth rate of money supply can explain the variance (fluctuation) of yields, may be one of the possible approaches to the problem.

Appendix I. The Variance Bounds Inequality for the Holding Period Yield on Bonds

The expectations model of the term structure adopted in this paper is:

$$E_t[H^n_{t+1}|I_t] = r_t + \phi^n, \tag{A–1}$$

and the forecast error (ε_{t+1}) for the holding period yield (H^n_{t+1}) can be expressed as:

$$\begin{aligned}\varepsilon_{t+1} &= H^n_{t+1} - E_t[H^n_{t+1}|I_t] \\ &= H^n_{t+1} - r_t - \phi^n.\end{aligned} \tag{A–2}$$

Since H^n_{t+1} is defined as:

$$H^n_{t+1} = (R^n_t - \bar{\gamma}R^{n-1}_{t+1})/(1-\bar{\gamma}), \tag{A-3}$$

ε_{t+1} can be seized as a function of the forecast error for the yield R^{n-1}_{t+1} that is realized at time t+1. Suppose all the innovations available at time t are reflected in the yield R^n_t, the rational expectations hypothesis which assumes that market participants rationally predict the holding period yields based on the information available up to time t, is required to satisfy that the forecast error ε_{t+1} is uncorrelated with R^n_t.

This means statistically: $COV(\varepsilon_{t+1}, R_t^n)=0$ ($COV(\cdot)$ represents covariance). The use was made of Equation (A–2) and the requirement above becomes:

$$COV(H_{t+1}^n - r_t - \phi^n, R_t^n) = COV(H_{t+1}^n - r_t, R_t^n) = 0 \,. \tag{A–4}$$

Using the definition (A–3) of H_{t+1}^n, we then see that:

$$\begin{aligned} COV(R_{t+1}^{n-1}, R_t^n) &= \frac{1}{\bar{\gamma}} VAR(R_t^n) - \frac{1-\bar{\gamma}}{\bar{\gamma}} COV(R_t^n, r_t) \\ &= \frac{1}{\bar{\gamma}} VAR(R_t^n) - \frac{1-\bar{\gamma}}{\bar{\gamma}} \cdot \rho_{rR} \cdot \sqrt{VAR(R_t^n)} \cdot \sqrt{VAR(r_t)}, \end{aligned} \tag{A–5}$$

where ρ_{rR} is the correlation coefficient between r_t and R_t^n.

Now, from Equation (A–3) the variance of the holding period yield, $VAR(H_{t+1}^n)$, is given by:

$$\begin{aligned} VAR(H_{t+1}^n) &= VAR\{(R_t^n - \bar{\gamma} R_{t+1}^{n-1})/(1-\bar{\gamma})\} \\ &= \left(\frac{1}{1-\bar{\gamma}}\right)^2 \cdot VAR(R_t^n - \bar{\gamma} R_{t+1}^{n-1}) \\ &= \left(\frac{1}{1-\bar{\gamma}}\right)^2 \cdot \{VAR(R_t^n) + \bar{\gamma}^2 VAR(R_{t+1}^{n-1}) - 2COV(R_t^n, R_{t+1}^{n-1})\}. \end{aligned} \tag{A–6}$$

Here, the assumption that $VAR(R_t^n)$ is equal to $VAR(R_{t+1}^{n-1})$ (this implies the stationarity of H_{t+1}^n and is necessarily satisfied) makes Equation (A–6) to be

$$VAR(H_{t+1}^n) = \left(\frac{1}{1-\bar{\gamma}}\right)^2 \{(1+\bar{\gamma}^2) VAR(R_t^n) - 2\bar{\gamma} \cdot COV(R_t^n, R_{t+1}^{n-1})\}. \tag{A–6$'$}$$

By the use of Equation (A–5), Equation (A–6)′ yields:

$$VAR(H_{t+1}^n) = \left(\frac{1}{1-\bar{\gamma}}\right)^2 [(1+\bar{\gamma}^2) VAR(R_t^n) - 2\{VAR(R_t^n) - (1-\bar{\gamma})\rho_{rR} \cdot \sqrt{VAR(R_t^n)} \cdot \sqrt{VAR(r_t)}\}]. \tag{A–7}$$

Thus, the variance of the holding period yield is indicated by the variances of yield and short-term interest rate at time t and sample informations as $\bar{\gamma}$, ρ_{rR}. The maximization of $VAR(H_{t+1}^n)$ would be made with respect to $VAR(R_t^n)$. From Equation (A–7),

$$\frac{\partial VAR(H_{t+1}^n)}{\partial VAR(R_t^n)} = \frac{(\bar{\gamma}^2 - 1) + (1-\bar{\gamma}) \cdot \rho_{rR} \cdot \sqrt{VAR(r_t)}}{(1-\bar{\gamma})^2 \cdot \sqrt{VAR(R_t^n)}} = 0 \,.$$

Accordingly, $VAR(R_t^n)$ that gives the maximum value of $VAR(H_{t+1}^n)$ can be expressed as:

$$\mathrm{VAR}(R_t^n) = \frac{\rho^2_{rR}\cdot \mathrm{VAR}(r_t)}{(1+\bar{\gamma})^2} \tag{A-8}$$

and by substituting Equation (A–8) for Equation (A–7), we find that the maximum value ($V_X(H^n_{t+1})$) of $\mathrm{VAR}(H^n_{t+1})$ is

$$V_X(H^n_{t+1}) = \left\{\frac{(\bar{\gamma}^2-1)+2(1-\bar{\gamma})\cdot(1+\bar{\gamma})}{(1-\bar{\gamma})^2(1+\bar{\gamma})^2}\right\}\cdot \mathrm{VAR}(r_t)\cdot \rho^2_{rR}$$

$$= \mathrm{VAR}(r_t)\cdot \rho^2_{rR}\cdot 1/(1-\bar{\gamma}^2). \tag{A-9}$$

Therefore,

$$\mathrm{VAR}(H^n_{t+1}) \leqq V_X(H^n_{t+1}) = \mathrm{VAR}(r_t)\cdot \rho^2_{rR}\cdot 1/(1-\bar{\gamma}^2)$$ [24]

becomes the variance bounds inequality which imposes an upper bound on the variance of the holding period yield.

Appendix II. Concept of the Variance Bounds for Yield (R_t^n)

Although this paper is working on the variance bounds inequality for the holding period yield on bonds, it is also possible to derive it for the yield R_t^n. The tests of variance bounds for present-value relations are shown by Leroy–Porter (1981), Singleton (1980) and others.

For the sake of simplification, when the term structure equation of Equation (6) can be expressed as:

$$R_t = \alpha \sum_s \beta_s E_t[r_{t+s}|I_t] + L \tag{A-10}$$

and the perfect-foresight yield R_t^* is:

$$R_t^* = \alpha \sum_s \beta_s r_{t+s}\,. \tag{A-11}$$

Then, it is obvious that:

24. The upper bound derived here is affected by the sample correlation coefficient ρ_{rR}. It should be noted that ρ_{rR} would specify the process of short-term interest rates, because the yield R_t^n is expressed in terms of the current and future short-term interest rates. If one supposes only $\rho_{rR} \leqq 1$ and does not make use of any specific ρ_{rR} for the upper bound, any process of the short-term interest rates, including AR(1) ($r_{t+1} = \lambda r_{t+1} + \varepsilon_{t+1}$, $E_t(r_{t+s}) = \lambda^s r_t$, $s>1$), would not be excluded.

$$R_t = E_t[R_t^*] + L. \tag{A–12}$$

Now, let us denote δ_t as the forecast error on the yield R_t. δ_t is defined as:

$$\delta_t = \alpha \sum_s \beta_s (r_{t+s} - E_t[r_{t+s}|I_t]), \tag{A–13}$$

subsequently, we obtain

$$R_t^* = R_t + \delta_t - L. \tag{A–14}$$

Assuming that the market is rationally forecasting the future short-term interest rates, the forecast error δ_t can not be correlated with R_t which represents the innovations up to time t, and statistically, the following requirement should be made

$$COV(\delta_t, R_t) = 0. \tag{A–15}$$

Therefore, the variance of Equation (A–14) is proved to be:

$$VAR(R_t^*) = VAR(R_t) + VAR(\delta_t). \tag{A-16}$$

Here, since we would be allowed to exclude the possibility of perfect foresight for the future short-term interest rates, that is there exists the forecast error (VAR (δ_t)>0), the variance of R_t can be bounded as follows:

$$VAR(R_t) < VAR(R_t^*). \tag{A–17}$$

The inequality (A–17) shows that if the yield R_t is the equilibrium of the rational expectations model of the term structure, the upper bound on the variance of R_t would be the variance of the perfect-foresight yield R_t^*.

However, we have a potential problem to obtain the perfect-foresight yield R_t^* in carrying out the variance bounds tests. A simplified method, as shown in this paper, is to compute R_t^* recursively under a certain terminal value condition that R_t^* at the end of the sample equals, for instance, the sample mean of the short-term interest rate r_t. But, as a matter of course, we should note that this manner of working is grounded on an arbitrary condition.

Appendix III. Problems in the Variance Bounds Tests

Not a few economist have conducted the variance bounds tests to analyse the volatile movements of asset prices such as stock prices, foreign exchange rates, etc.

However, it has been pointed out that the analysts should overcome a few technical problems. So long as the analyses in this paper are based on the variance bounds tests, we would have to give our ideas on these problems.

1. The Stationarity of Short-term Interest Rates

In making the results of variance bounds tests reliable, it is necessary to testify that the stochastic process of random variable which imposes an upper bound, that is short-term interest rates, is stationary, and the variances are constant for all time.

However, regarding the variables of which the number of data is limited such as economic indicators, since it is difficult to validate the stationarity of the process without taking their differences,[25] here we will consider this problem from the generalized idea of consistency between the stochastic process stationarity and the rational expectations.

If a stochastic process is stationary, the auto-covariance of the process depends only on a lag but not on the particular time point, which means that the random variables are generated by a system with a uniform structure. Then, if a stochastic process is not stationary, it can be inferred that there is no system with a uniform structure that generates the stochastic process. Therefore, the rational expectations hypothesis which posits that the stochastic process is forecasted by making an efficient use of all information available is not compatible with the property of unstationarity. Thus, in the rational expectations model, the stationarity of the process should be approved, and in this paper we assume that the short-term interest rate is stationary.

2. Small Sample Bias

Flavin (1983) pointed out that in small samples the variance bounds tests tend to be biased, often severely, toward rejection of the null hypothesis of market efficiency. In other words, there would be a downward bias on $VAR(r_t)$ on the right side of the variance bounds inequality (10) in this paper. Thus the apparent violation of market efficiency may be reflecting the sampling properties of the volatility measures, rather than a failure of the market efficiency hypothesis itself, and he showed the necessity of adjusting the bias.

According to Flavin, although an unbiased estimate of the variance of the holding period yield can be obtained simply by taking the sum of squares of the deviations

25. For example, one possible method to testify the stationarity of a process is under the hypothesis of the short-term interest rate process (Z_t) being AR(1), to OLS estimate $Z_t = \phi Z_{t-1} + \varepsilon_t$ and make a hypothesis testing whether $\hat{\phi}$ (estimate of parameter) satisfies $|\hat{\phi}| < 1$ (if it is satisfied, the variance of the process, $V(Z_t)$, is $\sigma^2/(1-\hat{\phi}^2)$ where σ^2 is the variance of white noise). However, it is rare to obtain the results which state the process being stationary with regard to data that apparently do not show any cyclical movements.

of H^n_{t+1} form its sample mean and dividing by degrees of freedom rather than sample size (T), the short-term interest rate, r_t, is highly serially correlated, so that the same correction for degrees of freedom will not eliminate the downward bias in the sample variance of r_t. Using the notation $var(r_t)$ for the population variance of r_t and $\hat{var}(r_t)$ for the sample variance of r_t (computed by taking deviations from the sample mean), Flavin denotes the relative bias of $\hat{var}(r_t)$ by

$$\frac{E[\hat{var}(r_t)]}{var(r_t)}=1-\frac{var(\bar{r}_t)}{var(r_t)},$$

where $var(\bar{r}_t)$ is the variance of the sample mean of r_t. If the short-term interest rate is generated by an AR(1) process ($r_t=\rho r_{t-1}+e_t$), equation above can be evaluated by straight-forward algebra:

$$\frac{E[\hat{var}(r_t)]}{var(r_t)}=1-\frac{1+\rho}{(1-\rho)T}+\frac{2\rho(1-\rho T)}{(1-\rho)^2T^2}$$

Here, we can calculate and correct for the bias to the upper bound of the holding period yield. I retained the assumption that the short-term interest rates are well approximated by an AR(1) process with an autoregressive parameter of 0.90 and the sample size is 110, and obtained the bias around 0.15. In other words, we need to correct 15% bias to the upper bound. However, we employ the sample variance in the analyses because of the following reasons: 1) the correction for the bias is based on the assumption that the process of short-term interest rates can be expressed by a simple AR(1) model, 2) even if a downward bias of about 15% is taken into account, the variance bounds inequalities are still sufficiently violated.

Appendix IV. Hypothesis Testing on the Standard Theory

We shall now present more detailed examination with respect to the hypothesis testing on $\hat{\beta}$.

Since it is clear from the definition that the theoretical value of $\hat{\beta}(=\frac{1-\bar{\gamma}}{\bar{\gamma}})$ is positive without specifying $\bar{\gamma}$, if $\hat{\beta}$ is proved to be significantly positive, then the standard expectations model of the term structure basically has no problem. Consequently, we must first consider the case where $\hat{\beta}$ is significantly negative. In this case, the expectations model of the term structure with constant risk premium is not valid fundamentally, and it is likely that α is not a constant, in other words, the risk premium is not time-invariant but varies over time. Let us denote the variable risk premium as θ^n_t, one can find that its presence produces a downward bias on $\hat{\beta}$ described as follows (case ④):

$$-\alpha \cdot \mathrm{COV}(R_t^n - r_t, \theta_t^n)/\mathrm{VAR}(R_t^n - r_t).$$

Next, if $\hat{\beta} \fallingdotseq 0$, that is, if we cannot reject $\hat{\beta}$ being zero, how should we think? It is possible that in such a situation the market expectations are static and given by,

$$E_t[R_{t+1}^{n-1} | I_t] = R_t^n.$$

Here, if we use $R_{t+1}^{n-1} = E_t[R_{t+1}^{n-1} | I_t] + \varepsilon'_{t+1}$, the equation becomes:

$$R_{t+1}^{n-1} = R_t^n + \varepsilon'_{t+1},$$

and by substituting this for Equation (12), we obtain

$$\varepsilon'_{t+1} = -\{(1-\bar{\gamma})/\bar{\gamma}\}\phi^n + \{(1-\bar{\gamma})/\bar{\gamma}\}(R_t^n - r_t) + \varepsilon'_{t+1}.$$

Hence, it is obvious that nobody can reject the null hypothesis that $\hat{\beta}$ is zero. Furthermore, if we assume that the risk premium is constant, the standard error of $\hat{\beta}$ will be infinite. Then, in the case that the standard error of $\hat{\beta}$ obtained from estimation takes a fixed value, the risk premium would vary over time even if not widely. This can be testified by the white noise test of the residual of OLS estimation. And if the risk premium is significantly zero, the process of yields is a random walk (case ③).

Finally, what has happened if $\hat{\beta}$ is significantly above the theoretical value? As the theoretical value T_β is $\frac{1-\bar{\gamma}}{\bar{\gamma}}$, the fact that $\hat{\beta}$ exceeds T_β means the estimate of $\bar{\gamma}$ is below the theoretical value, which is equivalent to the situation that the weight of the current short-term interest observed in data is larger than the theoretical value. In this connection, it is useful to return to Equation (6). Namely,

$$R_t^n = \frac{1-\gamma}{1-\gamma^n} \sum_{s=0}^{n-1} \gamma^s E_t[r_{t+s} | I_t] + \Phi^n.$$

From $\bar{\gamma} = \frac{\gamma - \gamma^n}{1-\gamma^n}$, it would be obvious that we acquire

$$R_t^n = (1-\bar{\gamma}) r_t + (1-\bar{\gamma}) \sum_{s=1}^{n-1} \gamma^s E_t[r_{t+s} | I_t] + \Phi^n$$

which shows the theoretical value of the weight of the current short-term interest rate r_t is $1-\bar{\gamma}$. Therefore, if $\hat{\beta}$ exceeds significantly the theoretical value, the weight estimated from the term structure model will exceed its theoretical value. This could be considered as so-called myopic expectations in the sense that the market overreacts to the current short-term rate (case ②).

Appendix V. OLS Estimation and Moving-Average Type Error

Equation (18) shows the OLS estimation error term can be primarily expressed as:

$$\delta_t = \lambda(R_t - E_t[R_{t+i}]) - r_t + \nu_t . \tag{A-18}$$

The assumption that, for the sake of simplification, R_t process would be approximated as AR(1) ($R_t = \psi R_{t-1} + \varepsilon_t$, ε_t: white noise) can be rewritten as:

$$\begin{aligned} R_{t+i} &= \psi^i R_t + \sum_{j=0}^{i-1} \psi^i \varepsilon_{t+i-j} \\ &= \psi^{i+1} R_{t-1} + \sum_{j=0}^{i} \psi^i \varepsilon_{t+i-j} . \end{aligned} \tag{A-19}$$

From Equation (A-19), we then obtain:

$$\begin{aligned} R_t - E_t[R_{t+i}] &= (\psi R_{t-1} + \varepsilon_t) - (\psi^{i+1} R_{t-1} + \sum_{j=0}^{i} \psi^i \varepsilon_{t+i-j}) + \varepsilon_{t+i} \\ &= \omega R_{t-1} + \sum_{j=0}^{i-1} \theta(j) \varepsilon_{t+j} \end{aligned} \tag{A-20}$$

where

$\omega = (\psi - \psi^{i+1})$

$\theta(j)$: weights of ε_{t+j}.

When Equation (A-20) is used, and the condition that r_t process would also be approximated as AR(1) ($r_t = kr_{t-1} + \varepsilon_t'$, ε_t': white noise) is added, Equation (A-18) yields:

$$\begin{aligned} \delta_t &= \lambda(\omega R_{t-1} + \sum_{j=0}^{i-1} \theta(j) \varepsilon_{t+j}) - (kr_{t-1} + \varepsilon_t') + \nu_t \\ &= \sum_{j=0}^{i-1} \hat{\theta}(j) \varepsilon_{t+j} + (\nu_t - \varepsilon_t') + (\hat{\omega} R_{t-1} - kr_{t-1}) \end{aligned} \tag{A-21}$$

where $\hat{\theta}(j) = \lambda \cdot \theta(j)$, $\hat{\omega} = \lambda \cdot \omega$.

Accordingly, the process of the error term δ_t is given by MA(i−1) model.

Appendix VI. On the Hypothesis of $COV(r_t, \phi_t) \fallingdotseq 0$

In calculating $Vs(\phi_t^n)$ in Equation (23), we hypothesized $COV(r_t, \phi_t^n) \fallingdotseq 0$, that is, the short-term interest rate and the variable risk premium (formed as the difference between expected holding period yield and short-term interest rate) are generally uncorrelated. However, if the short-term interest rate and the variable risk premium actually have a positive or negative correlation, there would be a bias on $Vs(\phi_t^n)$. As Equation (22) shows clearly, in the case of $COV(r_t, \phi_t^n)<0$, $Vs(\phi_t^n)$ will be under-estimated, and therefore, the bias works in the direction in which it will be difficult to reject the joint hypothesis. In the case of $COV(r_t, \phi_t^n)>0$, on the other hand, $Vs(\phi_t^n)$ will be over-estimated, and the bias will work in the direction in which it is easier to reject the joint hypothesis. Although it is necessary to be prudent in hypothesizing $COV(r_t, \phi_t^n)\fallingdotseq 0$, we believe that $COV(r_t, \phi_t^n)\fallingdotseq 0$ is generally approved in this paper for the following two reasons.

(1) Both Kessel (1965) and Nelson (1970) regressed the variable forward risk premium on the short-term interest rate and the results showed both positive and negative correlation. It means we have no consensus on this point.

(2) Although a positive correlation was found both in Japan and the United States using the theoretical variable risk premium of the sample, its magnitude was about 1 to 7% of $Vs(\phi_t^n)$ and negligible.

Appendix VII. Formulation of the Variable Risk Premium

The study of risk premiums based on the intertemporal asset pricing theory began with Lucas (1978) in the form of analysis of agents' subjective behavior under uncertainty and developed thereafter particularly in relation to tests on the efficiency hypothesis in the forward foreign exchange market. (see Hansen – Hodrick (1983), Hodrick (1981) and so forth).

The under-mentioned formulation of the variable risk premium followed the procedure in Shikano (1984), Mark (1985) etc. From Equation (24) we see

$$\frac{U'(C_t)}{P_t} = \beta E_t\left[\frac{U'(C_{t+1})}{P_{t+1}}\cdot V^j_{t+1}|I_t\right]. \tag{A–22}$$

Since P_t, $U'(C_t)$ is considered to be known at time t, dividing both sides of Equation (A–22) by $1/P_t$, $U'(C_t)$, we then obtain:

$$1 = \beta E_t\left[\frac{U'(C_{t+1})}{U'(C_t)}\cdot\frac{P_t}{P_{t+1}}\cdot V^j_{t+1}|I_t\right]. \tag{A–23}$$

Now, V^j_{t+1} (one-period nominal return on asset j) is defined as:

$$V^j_{t+1} = (P^j_{t+1}+C^j)/P^j_t \tag{A–24}$$

where,

P_t^j : price of asset j at time t

C^j : coupon payment for asset j .

The equilibrium condition when an investor invests in a risky asset (bonds, holding period yield H_{t+1}) is expressed as:

$$1 = \beta E_t\left[\frac{U'(C_{t+1})}{U'(C_t)} \cdot \frac{P_t}{P_{t+1}} \cdot (H_{t+1}+1)|I_t\right] \tag{A–25}$$

and likewise when he invests in a risk-free asset (short-term interest rates, yield r_t), it becomes:

$$1 = \beta E_t\left[\frac{U'(C_{t+1})}{U'(C_t)} \cdot \frac{P_t}{P_{t+1}} \cdot (r_t+1)|I_t\right]. \tag{A–26}$$

Since the risk premium $\hat{\phi}_t$ is defined as the difference between the expected holding period yield on risky assets and the yield on risk-free assets, that is:

$$\hat{\phi}_t = E_t[H_{t+1}|I_t] - r_t = E_t[H_{t+1}+1|I_t] - (r_t+1), \tag{A–27}$$

we need to be provided with $E_t[H_{t+1}+1|I_t]$ and r_t+1 from Equations (A–25) and (A–26). Let us denote here:

$$\Omega = \frac{U'(C_{t+1})}{U'(C_t)} \cdot \frac{P_t}{P_{t+1}}.$$

From Equation (A–25) it is obvious:

$$\frac{1}{\beta} = E_t[\Omega|I_t]E_t[H_{t+1}+1|I_t] + COV_t[\Omega \cdot (H_{t+1}+1)|I_t]$$

therefore, we get:

$$E_t[H_{t+1}+1|I_t] = \frac{1-\beta\{E_t[\Omega|I_t]E_t[H_{t+1}+1|I_t] - E_t[\Omega \cdot (H_{t+1}+1)|I_t]\}}{\beta E_t[\Omega|I_t]} \tag{A–28}$$

On the other hand, as r_t is known at time t, Equation (A–26) produces,

$$r_t+1 = \frac{1}{\beta E_t[\Omega|I_t]}. \tag{A–29}$$

When Equations (A–28) and (A–29) are substituted, Equation (A–27) becomes:

$$\hat{\phi}_t = \frac{\beta\{E_t[\Omega|I_t]E_t[H_{t+1}+1|I_t] - E_t[\Omega \cdot (H_{t+1}+1)|I_t]\}}{\beta E_t[\Omega|I_t]}. \tag{A–30}$$

Appendix VIII. Bond Price and Rational Bubble

It is generally known that in asset prices such as stock prices, foreign exchange rates, etc. generate rational bubbles (bubbles consistent with the rational expectations). However, we will demonstrate that rational bubbles cannot be generated in the bond pricing model.

Let us consider a model where the price of an asset X at time t, x(t), is determined by the economic fundamentals at time t, z(t), and the expected price of the asset X at time t+1. That is,

$$x(t) = \alpha E_t\, x(t+1) + \beta\ z(t),\ 0 < \alpha < 1 \tag{A–31}$$

where $E_t x(t+1)$ is the expected value of x(t+1) given information at time t. Rational expectations models usually assume that the current price x(t) reflects only current and future expected values of economic fundamentals. Consequently, the relationship in (A–31) is expected to hold at time t+1 and t+2 as well. And repeating the procedure for the asset price until time t+n−1, the following equation is obtained:

$$x(t) = \alpha^n E_t\, x(t+n) + \beta \sum_{i=0}^{n-1} \alpha^i E_t\, z(t+i). \tag{A–32}$$

Here, when we take a limit of Equation (A–32), we are given:

$$x(t) = \lim_{n\to\infty} \alpha^n E_t\, x(t+n) + \beta \sum_{i=0}^{\infty} \alpha^i E_t\, z(t+i)\ . \tag{A–33}$$

Since $0 < \alpha < 1$ and the bonds have some limited years to maturity, that is, $E_t x(t+n)$ is always finite, $\lim_{n\to\infty} \alpha_t^n E_t x(t+n) = 0$. Consequently, Equation (A–31) would be:

$$x(t) = \beta \sum_{i=0}^{\infty} \alpha^i E_t\, z(t+i) \tag{A–34}$$

Thus, in the case of price of bonds, x(t) can be written as a weight average of current and future values of economic fundamentals (specifically short-term interest rates) and the rational bubble would not generate.

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