Economies of Scale in Japan's Banking Industry*

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I. Introduction

During the decade between the mid-1960s and the mid-70s, the problem of restructuring the financial sector was subject to heated debate. One of the disputed points was to what extent the economies of scale existed in the banking industry. Some concluded that if the economies of scale significantly functioned, administrative guidance favoring large scale mergers would lead to favorable results for both borrowers and lenders of funds through improved productivity of financial services. Conversely, it was concluded that if the economies of scale were not very significant, more attention should be paid to the incomplete competition factor enhanced by bank mergers.[1]

The decade from the mid-80s is, and will continue to be, a period of financial internationalization and liberalization. With the deregulation of the financial industry, some fear that competition will become extremely intense between large, medium and small banks, between banks and securities companies and between

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1. As for this point, the detail is mentioned in Nishikawa (1973).

domestic and foreign financial institutions. This may, in some cases, lead the financial institutions to bankruptcy or liquidity problems. Indeed, in a market economy system, firms that sell unproductive services at high prices are replaced by firms that sell cheaper services. Bankruptcies are usually and should be, as a general rule, considered a form of friction in the process of resource distribution. However, financial services are different from other service industries, because they are essentially transactions between different times and different places. Therefore, information may become incomplete or asymmetrical, and if one financial institution loses its credibility a chain reaction may cause more credit erosion, and one could endanger the entire financial system.

Traditional regulations, unadapted to technological innovation and social changes, should be changed. Internationalization and liberalization of the financial sector are now irreversible trends.[2] However, one cannot overlook the fact that the number of bankruptcies of banks are increasing and the deposit insurance system is being reviewed in the United States where the liberalization process is more advanced.[3] Taking into account these present circumstances, the discussion on the economies of scale in the financial sector is an inevitable angle when considering a new financial system. The objective of this report is to present "a verification analysis of the economies of scale in Japan's banking industry" as a basic document.

The following can be said after observing domestic and foreign research activities in this field. First there is the problem of specifying the production function and cost function in order to estimate the economies of scale. If these specification are biased, errors might arise in the economies of scale indicator. As a matter of fact, the specification methods have changed, from the simple Cobb-Douglas type and others of the mid-60s to a more universal analysis based on definition methods such as CES, Translog, Box-Cox types that are more recent.[4] Considering this point, special atten-

2. As for this point, see Iwata-Horiuchi (1985) and Kinyu Mondai Kenkyu Kai (1986).

3. According to a report of FDIC (1985), the number of bankruptcies of banks, which was 10 in 1980 and 1981 respectively, increased to 42 in 1982, 48 in 1983 and 79 in 1984. In parallel with this phenomenon, due to financial difficulties the deposit insurance has been faced with and other reasons, discussions to review the deposit insurance system have been held. For detail, see Kinoshita (1985) and Ohta (1984).

4. Calculation of the elasticity of scale using the Cobb-Douglas production function or cost function was also made in this analysis. However, since strong multi-collinearity occurred and reliability of the parameter was remarkably decreased the calculation was omitted from this report. Recently, calculation of elasticity of scale based upon the Translog type cost function has been frequently conducted, including one conducted by Benston et al (1982,1983). As for the Box-Cox type, no analysis has yet been found in relation to the field, but a study was conducted by Bendt et al (1979) in relation to the manufacturing industry. The Box-Cox type production function is a more flexible production function than the Translog type in terms of how well it fits.

tion was paid to the problems of function specification. In this analysis, we used a calculation method of the economies of scale indicator in a general homogeneous production function, in order to avoid specifications. The second problem is that measurement criteria of production volume of financial services, the volume of production factor outlay, differ depending on the analyst. And the results of the analysis show that the degrees of economies of scale are scattered depending on the measurement criteria, making their interpretation very difficult.

This is considered to be reflecting the difficulty to define the production process, the products and the production factors of financial services compared to other industries. In this analysis, revenues from banking activities are considered to be the product, and capital, labour and funding are used as the production factors. Moreover, a comparative examination was made with analyses using other measurement criteria.

When observing the actual banking business, automation and mechanization, including the second and third phases of on-line system implementation, advanced at a very rapid pace along with the progress in information-related high technology of the past 10 years. This trend suggests a strengthening of the effects of the economies of scale. This point was taken into our consideration, thus we did not only measure the past single year but we repeatedly measured over a relatively long period of time. We also observed how the economies of scale changed through a time series. In addition, we tried a cross-section analysis of city banks and regional banks for each year between 1974 and 1984.

Conclusions that were drawn through this analysis can be summarized as the following.

(1) It is possible to clearly discern a rise in productivity along with magnification of the size, when using the aggregate factor productivity, which is a comprehensive productivity indicator that considers all the production factors.

(2) The elasticity of output with respect to the scale of inputs is called elasticity of scale.[5] This value was determined as significantly larger than 1 in most cases. Consequently, one can conclude that there are economies of scale in banking business with almost no exceptions. Moreover, the estimates of profit elasticity, which are the elasticity of profits to the scale of production factor outlays,[6] have the larger values. For example in the case when labour, capital and funding are taken as the production factors and ordinary revenues as the product, the elasticity of scale was between

5. Regarding elasticity of scale in the production, there are many other expressions. In this analysis, however, only the term of "elasticity of scale" used by Griliches-Ringstad (1971) is used.

6. Elasticity of profit is used here particularly to emphasize elasticity of profit with respect to the input scale of the production factor.

1.06–1.15 for city banks and between 1.02–1.06 for regional banks, and the elasticity of profits was between 1.36–2.42 for city banks and between 1.12–1.40 for regional banks.

(3) In terms of the time series, the trend of elasticity of scale is difficult to observe. Conversely, there is a net upward trend in elasticity of profits from the 70s through the 80s, showing an enlarged difference in profit levels between financial institutions of different sizes.

(4) The difference in productivity among different scales is often debated separately between the economies of scope and the economies of scale to accomodate diversification. However, as long as it is based on the past observations made against a background where financial activities are restricted to a certain extent, it is difficult to observe the effects of diversification to productivity differentials.[7]

(5) The problems of the economies of scale of the banking industry are often debated together with the problem of mergers. On the basis of documents of large-scale merger experiences in the past, it appears that the economies of scale cannot derive immediately from mergers. In that sense, it was verified that the so-called aftermaths of mergers were felt to a certain extent.

II. Chart Analysis Concerning the Bank Scale and Productivity

1. Bank Size and Single-factor Productivity

What kind of indicators should be used when one wants to make a diagnosis of the productivity among different production scales? The easiest answer would be the share of profits in total revenues, or the profitability. However, usually, since price effects of production factors directly influence the differential in profits, diagnosing productivity only on the basis of profitability is not recommended. Other indicators that may be used are productivity concerning one single production factor. This is expressed by $Y/X_1, \ldots, Y/X_n$, when production factors are $X_1, X_2, \ldots, X_n$ and production is Y. We took the 1984 data as an example and calculated the labour productivity (productivity of staff), capital productivity (productivity of liquid and fixed assets) and procured-fund productivity, when production is the operating income. Figures 1 to 3 are graphs in which single-factor productivity is expressed by a vertical axis and production scale by a horizontal axis.

7. This result was obtained in accordance with an approach based upon the index theory mainly to measure the economy of scale. In the report made by Kasuya (1986) using the multiproduct Translog production function, the economy of diversification has been studied by testing the complementary effect of the marginal cost among plural products. In his study, the complementary effect of the marginal cost was found.

Figure 1 Gaps Among Scales of Labor Productivity (1984)

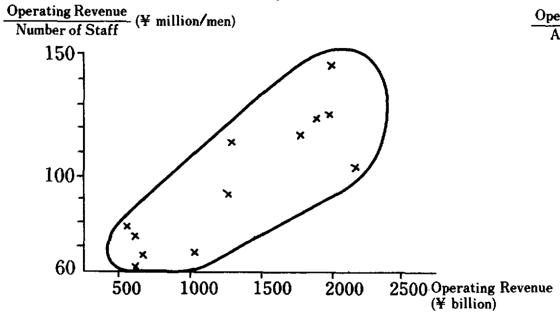


Figure 2 Gaps Among Scales of Capital Productivity (1984)

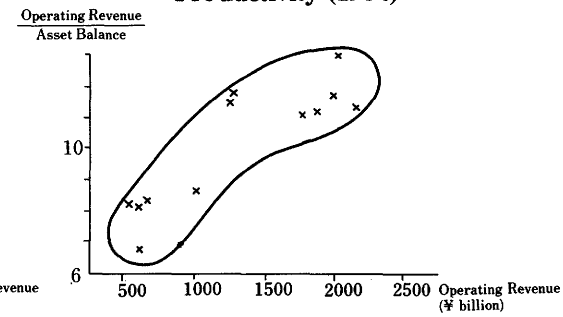


Figure 3 Gaps Among Scales of Funding Productivity (1984)

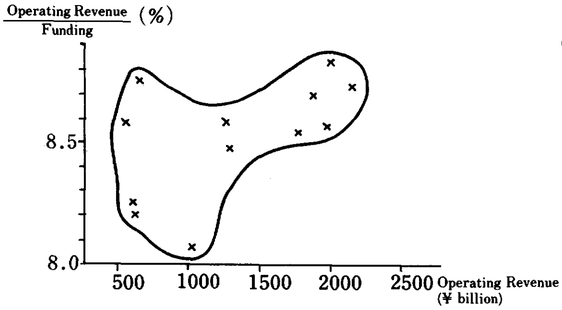
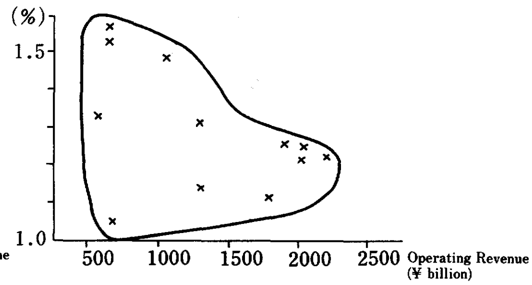


Figure 4 Gaps Among Scales of Spread Yield – Funding Cost (1984)



By comparing these figures, one can see that labour productivity and capital productivity increase according to the scale, showing higher productivity for a larger scale. Meanwhile, the productivity of procured funds does not necessarily show a clear tendency to ascend on the right-hand side; rather, in many cases higher productivity can be observed in smaller scale entities. As a result, one can see that it is difficult to make a diagnosis on profitability differentials among different production scales on the basis of single-factor profitabilities. This is because it is not clear which factor to use as a criterion for productivity. And in some ways of choosing criteria, it is not excluded that different conclusions are drawn from the same data.

Figure 4 is also based on 1984 figures and took the spread (return on placements —funding cost) as the vertical axis and revenues as the horizontal axis. It shows that there are many banks with high spreads in spite of their small scale. According to the data, this is due to low funding cost instead of a high return on placements[8] and this can be considered as reflecting the banks' efforts to procure cheap funds (capital) through relatively numerous branches and staff (labour). These efforts are con-

8. For example, the top city bank in terms of the spread among city banks in 1984 ranked 11th in terms of the average yield on assets. Thus, in comparison with other banks, its yield was not high, but its funding cost was the lowest among city banks.

sidered to indicate substitution between labour, capital, and funding capital in the production function. This also shows that judging differentials in productivity among different scales with a single-factor productivity is problematic. From all of the above points, it is deduced that relying on single-factor productivity in productivity analysis is inappropriate and the necessity to invent new indicators emerges.

2. Bank Scale and Total Factor Productivity

One can name total factor productivity as a new indicator which complements the insufficiency of single-factor productivity. The total factor productivity is a comprehensive productivity indicator that covers all factors by indexing. It enables a comprehensive measurement of outlay effects of each factor. On the basis of the data from the years between 1974 and 1984, we calculated the total factor productivity against product (ordinary revenue) by totalling and indexing the three factors of staff (labour), current and fixed assets (capital), and funding through Translog index.[9] Figures 5 a–k show the aggregate factor productivity as the vertical axis and ordinary revenue as the horizontal axis.[10]

Upon examination of Figures 5 a–k, one can see a noticeable upward trend on the right-hand side for each year. It can be conjectured that every year, the aggregate factor productivity of large banks was higher by 15–28% compared to that of smaller banks.

3. Economies of Scope and Economies of Scale on Productivity

By calculating the total factor productivity, we found a clue to visualize the existence of effects of scales in terms of productivity. However, it is impossible to simply discuss the effects of the economies of scale on the basis of a graph showing the scale and productivity. If it is certain that only the scale contributes to the productivity differentials, there is no problem in admitting the existence of the economies of scale. However, as far as the banking industry is concerned, it is more or less necessary to discuss the effects of the economies of scope, which has been the subject of many research studies in recent years. The economies of scope refer to an effect of diversification of products leading to improved productivity and a successive

9. The definition of the input factor and the product in this calculation of total factor productivity are based upon Model 3. As for Model 3, see III.3. The value of total factor productivity was calculated by dividing the income (the product) by the Translog. For details see III.1.

10. We also made the calculation of total factor productivity on the basis of the definition of input-output of Model 4. However, it was deleted since the result was not different from the nature of this debate concerning the productivity.

Figure 5 Gaps Among Scales of Aggregate Factor Productivity and Degree of Diversification

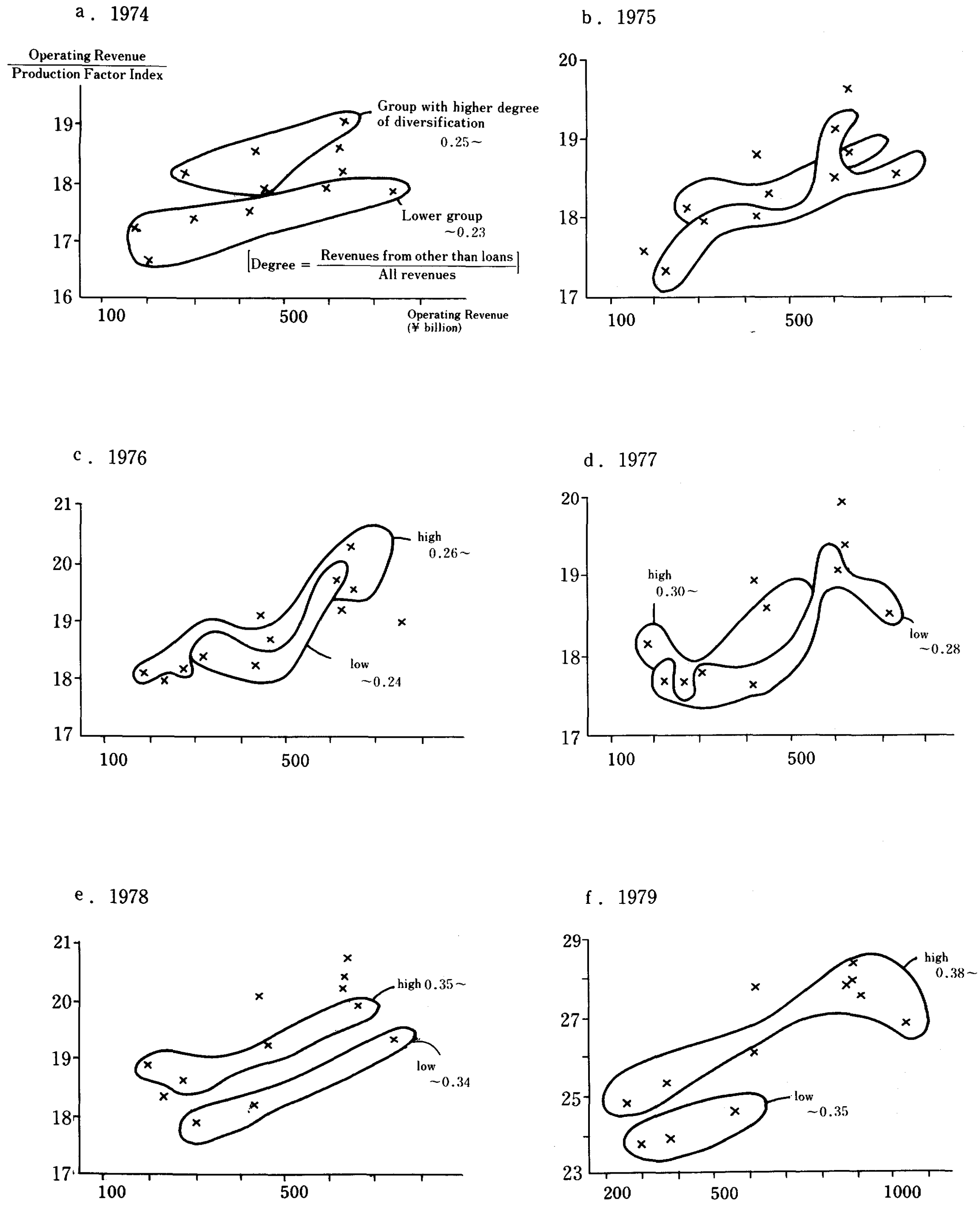


Figure 5 continued

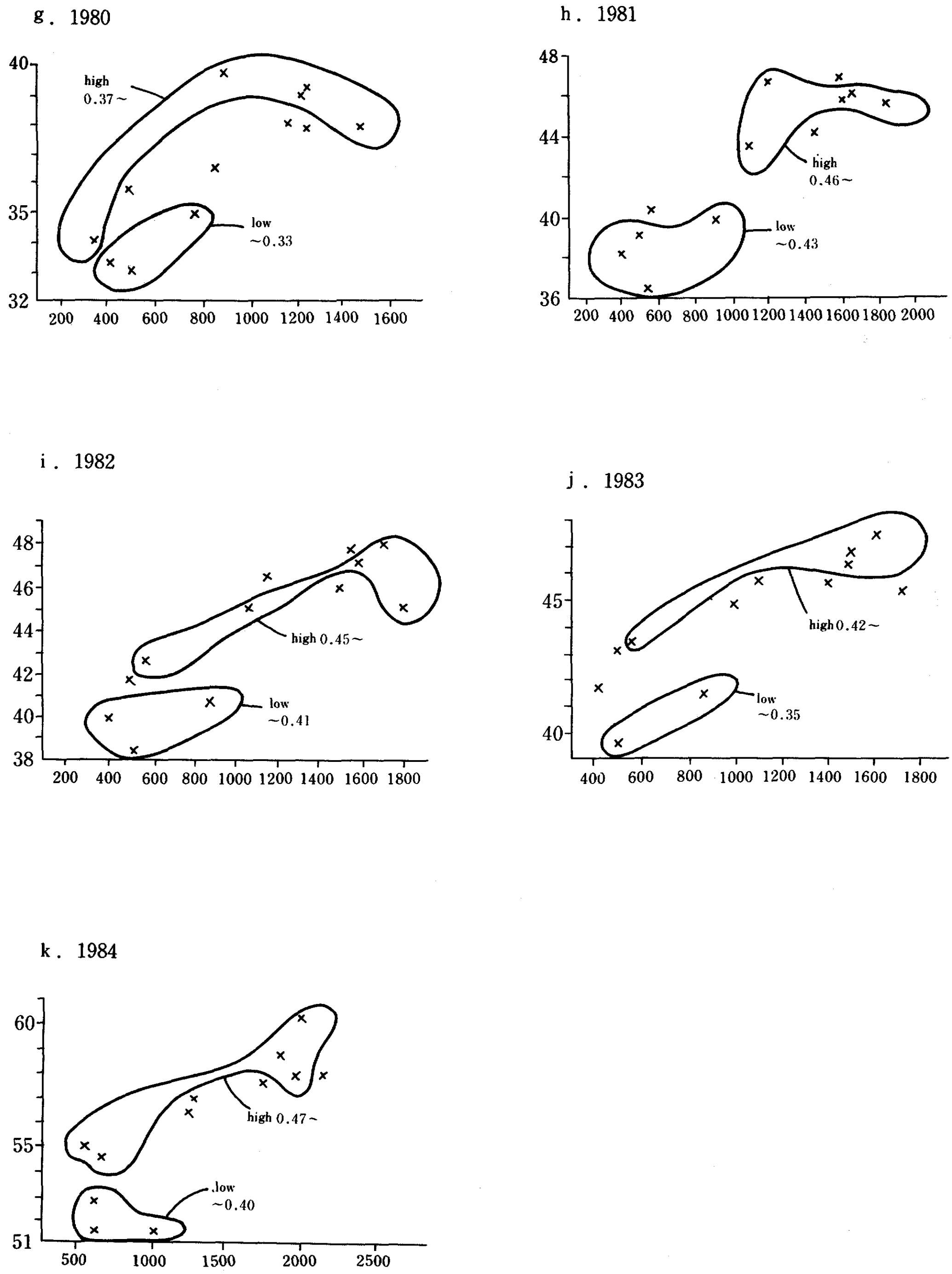
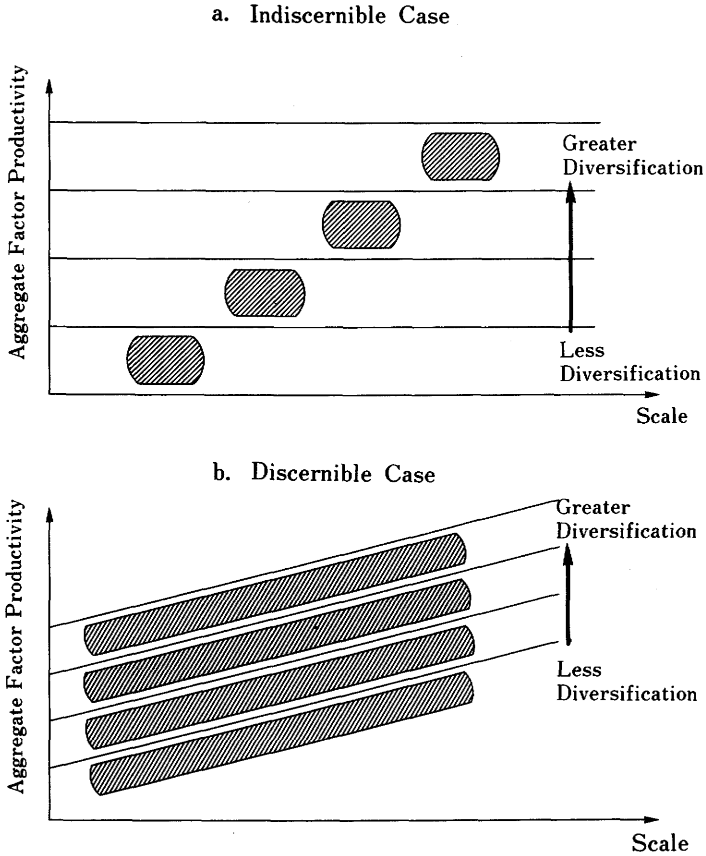


Figure 6 Image of Effects of Scale and Diversification



diminution of marginal costs, which induces a rise in productivity.[11] If this effect exists, the upward trend of the right hand side of Figures 5 a–k should be disucussed separately, in view of the effects of scale and diversification upon productivity. We tried to approach this problem of discerning by utilizing the graph for the aggregate factor productivity in 2. Following are the approach method and analysis results in a chronological order.

(a) First, a distribution pattern of the aggregate factor productivity as in Figure 6a

11. From the economics point of view, economies of scope are defined as,

$$C(Y_1, Y_2) < C(Y_1, 0) + C(0, Y_2)$$

when C(.)=cost function, Y_1, Y_2=products.

In real analysis, a Translog type multi-product cost function is settled, and an inequality from the viewpoint of marginal cost, $\delta^2 C / \delta Y_1 \delta Y_2 < 0$, is set as a standard for existence of scope. For details explanations on this point, see Panzar—Willig (1981), Kasuya (1986). Also, among the bibliography listed below, there are many studies where economies of scope are treated simultaneously.

is assumed. This figure shows that the enlargement of production scale and the degree of diversification [12] are correlated and it shows a case in which the effects of the scale and diversification on productivity improvement is not identifiable.

(b) Figure 6b shows a case where the identification is possible. Productivity improvement along with the enlarged scale can be observed, even when the degree of diversification is controlled. Conversely, when the scale is fixed, productivity improvement along with the development of diversification can also be observed alongside the vertical axis.

(c) When the above two conceptual figures are applied to Figures 5 a–k, Figure 5a corresponds to the case where the identification is possible, as in Figure 6b, and Figure 5h is in conformity with the case where the identification is difficult (as Figure 6a shows). There are years, such as shown in Figure 5b, when the effects are difficult to tell because of interrelations of diversification levels.

(d) From the above analysis, although it is difficult to identify the effects of diversification in some years, the effects of scale could be clearly observed when the level of diversification was under control; an almost stable existence of the economies of scale could be seen.

III. Theoretical Model

1. Estimation Methodology of Scale Elasticity[13]

In this section we introduce how to measure the degree of scale economies. Here we adopt "passus coefficient" of production function as the measure of scale economies. Under the assumptions of producers' rationality and of homogeneous production function, the upper and lower bounds of the passus coefficient are derived. Secondly these two bounds are translated in terms of the index number theory. As a result the upper and lower bounds are equivalently calculable by using Paasche and Laspeyres indices of factor inputs respectively. For investigating the nature of these bounds in empirical analysis, we also prepare the passus coefficient of the specific production function, which contain the desirable property of flexibility; Translog form and Diewert's quadratic form of homogeneous function. Their passus coefficients are calculable by using the Theil-Törnqvist index of input quantity and

12. The degree of diversification used here indicates the share of revenue from activities other than fund lending in total revenue. In other words, diversification means that banks tend to rise the share of profits deriving from activities other than those from fund lending, and are not dependent on fund lending only, as it used to be the case.

13. See Yoshioka (1984, 1985) for details concerning the approximation method according to index theory.

Fischer's ideal index if inputs respectively.

We suppose the situation where a single output is made from n sorts of factor inputs. And assume the production function

$$Y = f(V) \tag{1}$$

where Y and V are the quantity of output and the inputs vector. The elasticity of the output with respect to the scale of the inputs has been applied as the measure of the scale economies in such a single-ware production function. Frisch(1965) named this elasticity "passus coefficient" and defined it as the elasticity of the output quantity with respect to the scale of input quantity, when all the inputs vary proportionally. Denoting V^o as a given relative inputs combination and the scalar μ as the scale factor so as to $V=\mu V^o$, we may express the passus coefficient as follows:

$$k = \frac{dY/Y}{d\mu/\mu}, \tag{2}$$

where

$$Y = f(V) = f(\mu V^o).$$

This coefficient is also rewritten as the sum of the marginal elasticities of inputs evaluated at V, if the function is differentiable at V. That is to say,

$$dY = \nabla f(V)'dV = \nabla f(V)'V^0 d\mu = \nabla f(V)'V \cdot d\mu/\mu, \tag{3}$$

then

$$k = \frac{dY/Y}{d\mu/\mu} = \nabla f(V)'V/Y.$$

In estimating this coefficient based on observed economic data, we have occasionally assumed some specification about the production function and have estimated several parameters which in turn are integrated to get the estimates of the passus coefficient. With regard to Douglas production function, for instance, these estimates are derived from the sum of the estimated marginal elasticities of inputs that appear in the above equation. Approaches like this have, needless to say, a great advantage of acquiring integrated information with the production structure under the considerations, such as the marginal elasticity of substitutions, the marginal elasticity of each input and so on. However, when focusing the analysis of scale economies of production, these approaches are not always the best. Whenever analysing a specific industry empirically, we are often faced with the situation that the observed inputs data are mutually highly correlated. In such cases, it is so difficult to identify the isoquant curve of production function. Our focus is in this "multi-collinearity" problem of mutual inputs. Therefore, we concentrate our attention mainly on two points. The first is to estimate the only one parameter of the passus coefficient directly, not through the estimated production function. And the second is to specify the functional form of production as general as possible, especially regarding the curvature of the isoquant.

Although it is not quite feasible to estimate jointly all the relevant parameters of a production function in the situation that the data of inputs are mutually closely correlated, such a situation may be in effect suitable for evaluating the degree of scale economies. As the extreme case, for instance, if all the inputs were kept strictly proportional and if the average productivity of any input in larger scale of production is sufficiently greater than that in smaller scale of production, we usually insist that this is the evidence of scale economies between these two productions. Frisch also gives the following relation as the approximate estimates of the passus coefficient.

$$k=\frac{dY/Y}{d\mu/\mu}=\frac{\ln x^2-\ln x^1}{\ln \mu^2-\ln \mu^1}=\frac{\ln x^2-\ln x^1}{\ln v_i^2-\ln v_i^1} \qquad (\text{for } \forall v_i^r \in v^r) \tag{4}$$

where the superscript "1" denotes the observation of the smaller scale and "2" does that of larger scale, and $V^1=\mu^1V^o$, $V^2=\mu^2V^o$ hold(Figure 7). Looking at this equation precisely, we can find the good natures for empirical study in it. The first is that this approximation is quite adaptable as the measure for the observations mentioned above. Because it indicates greater than 1, if and only if $x^2/V_i^2>x^1/V_i^1$ hold, that is the indicator of scale economies in the above-mentioned observations. And when it indicates less than 1, we can use it as the indicator for scale diseconomies, vice versa. Secondly, this approximation is the mean of passus coefficient in the relevant interval.[14]

However the data, generated within economic mechanism, seldom show all inputs vary proportionally. This is the matter of our primary concern, and thus the theoretical framework in this paper is constructed under the next presuppositions:

[A] Producers' Rationality: Under the given output level producers minimize their cost of production.

[B] Homogeneity of Production Function.

Then the production function is given to

$$Y=\lambda^{-k}f(\lambda V),\ \lambda>0,\ V>0,\ Y>0 \tag{5}$$

where the constant k is the degree of homogeneity of function, and λ is any positive constant scalar.[15]

14. See Yoshioka (1985).

15. A theoretical model based on homothetic production function is shown by Yoshioka (1985). In this analysis, homothetic cases were also examined, but the measured values vary widely and were not suitable for analysis, therefore, this analysis has been conducted with a presumption of a homogeneous function.

Figure 7 Factor Space Based on Two Samples whose Production Factors are in Equal Proportion

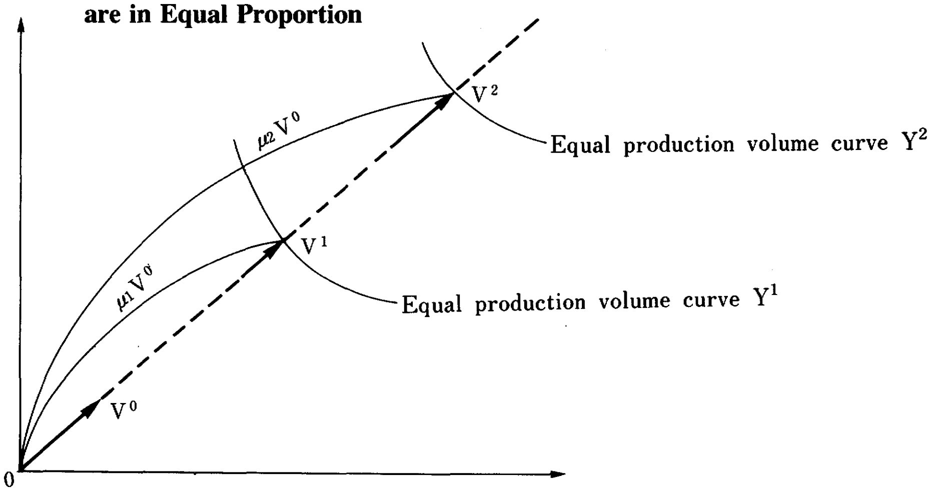
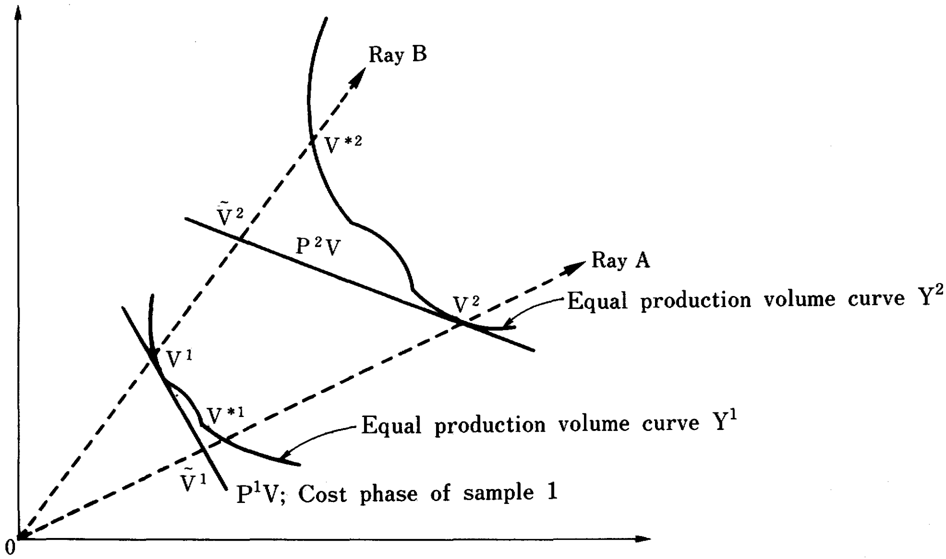


Figure 8



Suppose there are two samples containing different quantities of inputs and output, and corresponding prices of inputs. Let's denote P as these price vector and write the samples as (x^1, V^1, P^1), (x^2, V^2, P^2). Figure 8 is an illustration of the isoquants and the cost hyperplane, depending on these samples, where the dotted line A is the ray on which all the inputs are kept proportional as to V^2 and similarly B is defined as the ray on which all the inputs are kept proportional as to V^1. In addition $\tilde{V}^1$ is defined as the intersecting point of ray A on the cost hyperplane of sample 1 (P^1V) and V^{*1} is that on the isoquant surface of x^1. $\tilde{V}^2$ and V^{*2} are also defined for the sample 2 in the same manner. Here it should be noted that V^{*1} and V^{*2} are not observable, while $\tilde{V}^1$ and $\tilde{V}^2$ are calculable, then observable, such that:

$$\tilde{V}^1 = \frac{P^1V^1}{P^1V^2}V^2,\ \tilde{V}^2 = \frac{P^2V^2}{P^2V^1}V^1. \tag{6}$$

Let's denote the positive constant λ so that $V^2 = \lambda V^{*1}$ on ray A, Equation (5) shows that $f(V^{*1}) = \lambda^{-k} f(\lambda V^{*1}) = \lambda^{-k} f(\lambda V^2)$ and then,

$$k = \frac{\ln Y^2 - \ln Y^1}{\ln v_i^2 - \ln v_i^{*1}}.$$

And similarly on ray B, Equation (5) shows

$$k = \frac{\ln Y^2 - \ln Y^1}{\ln v_i^{*2} - \ln v_i^1}.$$

Since we have no information about the value of $V^{*r}(r=1,2)$, we change those to the observable values of $\tilde{V}^r$. Under the condition of Producers' Rationality [B], $\tilde{V}^r \leq V^{*r}$ is derived as follows:

> Let's denote λ^r so that $\tilde{V}^r = \lambda^r V^{*r}$. And from the definition of $\tilde{V}^r$ and V^{*r}, $P^rV^r = P^r\tilde{V}^r$ and $Y^r = f(V^{*r})$ are filled. On the other hand the producers' rationality is shown as $P^rV^r = \min_V [P^rV;\ Y^r \leq f(V)]$. Then
>
> $$\lambda^r P^r V^{*r} = P^r \tilde{V}^r = P^r V^r \leq P^r V^{*r}.$$
>
> Turning to the definition of λ^r, we get $\tilde{V}^r \leq V^{*r}(r=1,2)$.

Therefore, if we define the values k_1, k_u such that

$$k_1 = \frac{\ln Y^2 - \ln Y^1}{\ln v_i^2 - \ln v_i^1},\ k_u = \frac{\ln Y^2 - \ln Y^1}{\ln v_i^2 - \ln v_i^1}, \tag{7}$$

these values show the boundary of the passus coefficient of homogeneous production function as $k_1 \leq k \leq k_u$.

If we regard the sample "1" as the base point and "2" as the compared point in this cross-section study, the passus coefficient of proper production function will be calculable by proper index number. For instance, the lower and upper bounds are derived by Laspeyres and Paasche indices of inputs respectively such that:

Laspeyres index: $Q_L = \frac{P^1V^2}{P^1V^1}$, Paasche index: $Q_P = \frac{P^2V^2}{P^2V^1}$.

Putting them in Equation (7), then,

$$k_l = \frac{\ln Y^2 - \ln Y^1}{\ln Q_L}, \quad k_u = \frac{\ln Y^2 - \ln Y^1}{\ln Q_P} \tag{8}$$

Here, we call k_l as the lower bound of passus coefficient k and k_u as the upper bound of k.

As a theoretical conclusion of the above, if the estimates of lower bound k_l indicate a large value enough to accept the hypothesis $k_l > 1$, the considered production process will express the scale economies. And the estimates of upper bound k_u will be measure of decreasing return to scale, vice versa. When these two estimates are so near as to accept the hypothesis $k_u = k_l$, this method is efficient for estimating the passus coefficient of production.

As the other examples, we can derive the passus coefficients of Translog production function and Diewert's quadratic production with k-th degree homogeneity. The passus coefficient of the former production function corresponds to the k in the Equation (8) when we use Translog index (Theil-Törnqvist index) instead of Laspeyres or Paasche index. And the passus coefficient of latter one equals to the k using the Fischer index in Equation (8).[16] In this analysis we took into consideration these two cases. Then we added to the two following equations.

$$k_l = \frac{\ln Y^2 - \ln Y^1}{\ln Q_i}, \quad k_u = \frac{\ln Y^2 - \ln Y^1}{\ln Q_d} \tag{9}$$

where

Fischer index: $Q_i = \sqrt{Q_l \cdot Q_u}$,

Translog index: $Q_d = \prod_i \left(\frac{V_i^2}{V_i^1}\right)^{\frac{1}{2}\left(\frac{P_i^1 V_i^1}{P^1 V^1} + \frac{P_i^2 V_i^2}{P^2 V^2}\right)}$.

When we estimate the elasticity of scale according to this methodology, we have to consider the random disturbance. Let's denote u_j, the random disturbance term, then we may write it as follows:

$$Y_j = f(V_j) \cdot e^{u_j} \tag{10}$$

where the vector of the term u_j has the next stochastic property;

$$E(u) = 0, \quad E(u \cdot u') \frac{1}{2} \sigma^2 I$$

In this stochastic process, the Equations (8) and (9) are rewritten as follows:

16. See Diewert (1976).

$$\begin{aligned} \ln Y_{j+1} - \ln Y_j &= k_l \cdot \ln Q_l^{\,jj+1} + u_{j+1} - u_j \\ \ln Y_{j+1} - \ln Y_j &= k_u \cdot \ln Q_u^{\,jj+1} + u_{j+1} - u_j \\ \ln Y_{j+1} - \ln Y_j &= k_i \cdot \ln Q_i^{\,jj+1} + u_{j+1} - u_j \\ \ln Y_{j+1} - \ln Y_j &= k_d \cdot \ln Q_d^{\,jj+1} + u_{j+1} - u_j \end{aligned} \tag{11}$$

Defining the m dimensional vectors e, y and q, whose j-th elements are such that;

$$e_j = u_{j+1} - u_j, \ \ Y_j = \ln Y_{j+1} - \ln Y_j, \ \ q_{\cdot j} = \ln Q_{\cdot}^{\,jj+1} \tag{12}$$

we reach the next regression models for estimating the elasticity of scale.

$$Y = k_{\cdot} \cdot q_{\cdot} + e, \tag{13}$$

where the changed disturbance e has the next property.

$$E(e) = 0, \ \ E(e \cdot e') = \sigma^2 \Omega,$$

$$\Omega = \begin{bmatrix} 1 & -1/2 & \cdots & 0 \\ -1/2 & \ddots & \ddots & -1/2 \\ 0 & \cdots & -1/2 & 1 \end{bmatrix},$$

and Ω is a positive definite matrix whose trace equals to m. Therefore, the Aitken estimators are the best linear unbiased estimators of the elasticity of scale, which are shown as follows:

$$\hat{k}_{\cdot} = \frac{q_{\cdot}' \Omega^{-1} y}{q_{\cdot}' \Omega^{-1} q_{\cdot}} \tag{14}$$

As an alternative way, the elasticity of scale can be calculated by OLS, using a chain index of inputs. When defining the chain index of inputs as CQ_j, we can derive the next equation from Equation (11).

$$\ln Y_j = a + k_{\cdot} \cdot \ln (CQ)_j + u_j \tag{15}$$

where

$$CQ_j = CQ_{j-1} \cdot Q^{j-1j}, \ \ CQ_1 = 1.$$

In Equation (15) u_j is an original random disturbance, so the estimation value of the elasticity of scale k. can be estimated by OLS.

2. Definition of the Elasticity of Profit

The elasticity of profit is defined as the elasticity of profit with respect to inputs scale. It is quite difficult to compare our measurements of the scale elasticities from four alternative models each other, because of different definitions of inputs and output. We prepare the measurement of the elasticity of profit with respect to inputs scale.

Here we use the production function;

$$Y = f(V),$$

and the cost definition equation;

$$C = \sum_i P_i V_i = C(V),$$

where P_i is the price of input V_i. Then profit is defined as

$$\pi = P_Y \cdot f(V) - C(V) = G(V) = G(\lambda V^0). \tag{16}$$

According to this definition of profit we can derive the elasticity of profit with respect to inputs scale λ as follows:

$$k_{pr} = \frac{d\pi/\pi}{d\lambda/\lambda} = \frac{\nabla G(V)'V}{\pi}$$

Differenciating Equation (16) we get

$$d\pi = \nabla G(V)'V^0 d\lambda = \nabla G(V)'V d\lambda/\lambda = [P_Y \cdot \nabla f(V) - \nabla C(V)]'V d\lambda/\lambda.$$

Using this equation and Equation (3) the definition of the elasticity of profit can be easily rewritten as

$$k_{pr} = \frac{k \cdot P_Y \cdot f(V) - C(V)}{P_Y \cdot f(V) - C(V)} = \frac{k \cdot P_Y \cdot f(V) - P'V}{P_Y \cdot f(V) - P'V} \tag{17}$$

When we calculate k_{pr} with data in Equation (17), we use the theoretical value of output calculated using Equation (15), constant price of inputs, and inputs index each for f(V), P, and V in order to eliminate price change of costs and fluctuations caused by random disturbance. Considering price of output is unity for all banks, then, Equation (17) will be

$$\hat{k}_{prj} = \frac{\hat{k} \cdot a \cdot (CQ)_j^{\hat{k}} - P_j \cdot (CQ)_j}{a \cdot (CQ)_j^{\hat{k}} - P_j \cdot (CQ)_j} \tag{18}$$

Empirically, we use Fischer inputs index of $(CQ)_j$ and estimates of the elasticity of scale $\hat{k}_i$, which are consistent with Fischer index Q_i^{jj+1}.

3. Application to the Banking Industry: 4 Models

In this analysis, the methodology indicated in 1. and 2. is applied to the banking industry. But as expressed in the introduction, in the banking industry, there are many different ways of considering production factors and products. So, by taking into consideration the discussions in the introduction, it was decided to use the 4 models indicated in the following table as far as possible with the data available.

	Production Factors	Products
Model 1	staff, current and fixed assets	procured funds
Model 2	staff, current and fixed assets	operating revenues – funding costs
Model 3	staff, current and fixed assets, procured funds	operating revenues
Model 4	staff, current and fixed assets, funding costs	operating revenues

Model 1 is a model where, of the past studies about the economies of scale, we took into consideration the deposit amount and loan amount as products in terms of stock. As a background of the studies that take stocks as products, it is considered that there was an intention to consider as banks products the benefits given to depositors or those needing funds. In fact, in similar verification studies in the United States, such as Bell – Murphy (1968), Benston, Hanweck and Humphrey (1982), Benston, Berger, Hanweck and Humphrey (1983), Gilligan, Smirlock and Marshall (1984), a number of transactions in deposit and loan businesses or a number of deposit accounts are often used as an indicator of customer services.

Model 2 is the model to measure the economies of scale in Royama (1982). This Royama model is based on the idea that a bank is a financial intermediary between final lenders and final borrowers, and that it renders service production by servicing both. Therefore, the product is remuneration as a financial intermediary, which is the operating revenue minus funding cost (value-added).

Models 3 and 4 are generalized production functions based on the added value of Model 2. In the case of the production function based on added value, the value is produced out of labour and capital. And crude product (operating revenue) is a special item obtained by added value and raw materials (funding). But in Models 3 and 4, this kind of separability hypopthesis is avoided. Here, the revenue produced by banks[17] is considered as their product from the standpoint that if competition among the banks becomes severe due to the present financial liberalization, this competition would be on revenue.

From the above 4 models, the elasticity of scale can be estimated according to the index-theory method.

4. Observation Data and Period

The basic data for this analysis is the analysis of financial statements of all banks in Japan. Additional explanation concerning products and production factors is made here. Funding is a part of liabilities in the balance sheet (Deposits + CD + call money + bills sold + borrowed money + foreign exchange + others). The operating revenue is, in a strict sense, the amount of operating revenue as shown in the profit and loss account minus the profit and loss related to marketable securities transactions. The funding cost signifies interest payments, including interest on deposits. The breakdown of cost items include personnel remunerations of the staff, costs incurring to current and fixed assets, and funding costs. The price of products in each model is common to all banks (=1).

Here, the operating revenue of the banking industry is the total figure of all revenues as shown in the profit and loss statement. It is checked whether conformity with the measurement methodology is not lost when this total amount assumed the product index. For instance, when considering a production function as follows,

$$Y(Y_1, Y_2, Y_3,\dots,Y_m)=F(X_1, X_2,\dots, X_n) \tag{19}$$

our product index corresponds to $\Sigma\ P_iY_i$. Here if the analysis is limited to cross section, we can consider that the price of each bank's product Y_i is common to all banks. In this case if P_iY_i has a strong mutual proportionate relation, (if a high degree of positive interrelation is observed), this $\Sigma\ P_iY_i$ corresponds as a proxy variable of the product. As a matter of fact, during the 11 years between 1974 and 1984, each revenue interrelation was very strong and constant, as shown in Tables 1 to 4. Therefore, if we use here $\Sigma\ P_iY_i$ as a proxy variable of the product index, no bias would occur in the measured value of the economies of scale in this cross section

17. The concept of income is used in the same sense of profit in the field of economics. In this analysis, however, the income is used similarly as is used in banks' financial statements. Accordingly, the current income means the revenue used in economics.

Table 1 Coefficient of Y2=a Y1 and t-value of $\hat{a}$

FY	City Banks		Regional Banks	
	Coefficient	t-value	Coefficient	t-value
1974	0.9806	25.610	0.9651	41.424
1975	0.9834	27.779	0.9673	42.833
1976	0.9870	31.363	0.9723	46.636
1977	0.9916	39.168	0.9756	49.777
1978	0.9946	48.972	0.9797	54.761
1979	0.9956	54.262	0.9835	60.864
1980	0.9958	55.722	0.9759	50.127
1981	0.9975	71.790	0.9519	35.325
1982	0.9967	63.018	0.9640	41.077
1983	0.9978	76.756	0.9622	40.073
1984	0.9988	105.003	0.9594	38.591

Y1 : Revenues from loans

Y2 : Revenues from other than loans

Table 2 Coefficient of Y2′=a Y1′ and t-value of $\hat{a}$

FY	City Banks		Regional Banks	
	Coefficient	t-value	Coefficient	t-value
1974	0.9808	25.767	0.9586	37.889
1975	0.9825	26.984	0.9625	39.901
1976	0.9880	32.777	0.9668	42.498
1977	0.9925	41.524	0.9708	45.421
1978	0.9953	52.478	0.9751	49.314
1979	0.9957	54.870	0.9787	53.418
1980	0.9961	57.685	0.9695	44.358
1981	0.9976	73.173	0.9400	31.410
1982	0.9967	62.646	0.9534	35.883
1983	0.9978	76.721	0.9521	35.400
1984	0.9988	105.382	0.9473	33.636

Y1′ : Revenues from loans

Y2′ : Interest and dividend payments from securities + other interest payments

Table 3 Coefficient of Y3′=a Y2′ and t-value of $\hat{a}$

FY	City Banks		Regional Banks	
	Coefficient	t-value	Coefficient	t-value
1974	0.9979	78.048	0.8764	20.966
1975	0.9968	63.954	0.9247	27.602
1976	0.9961	57.290	0.9081	24.747
1977	0.9937	45.312	0.9244	27.529
1978	0.9913	38.476	0.9397	31.093
1979	0.9931	43.140	0.9374	30.468
1980	0.9925	41.508	0.9253	27.718
1981	0.9895	35.078	0.9131	25.730
1982	0.9927	41.929	0.9146	25.981
1983	0.9906	36.965	0.9227	27.422
1984	0.9920	40.092	0.8977	23.510

Y2′ : Interest and dividend payments from securities + other interest payments

Y3′ : Other operating revenues

Table 4 Coefficient of Y3′=a Y1′ and t-value of $\hat{a}$

FY	City Banks		Regional Banks	
	Coefficient	t-value	Coefficient	t-value
1974	0.9979	23.993	0.8984	23.413
1975	0.9847	28.884	0.9349	29.848
1976	0.9798	25.086	0.9312	28.971
1977	0.9832	27.546	0.9459	32.946
1978	0.9848	29.038	0.9584	37.816
1979	0.9892	34.551	0.9630	40.178
1980	0.9875	32.040	0.9631	40.212
1981	0.9873	31.770	0.9731	47.779
1982	0.9905	36.724	0.9717	46.473
1983	0.9893	34.671	0.9737	48.283
1984	0.9914	38.612	0.9658	42.158

Y1′ : Revenues from loans

Y3′ : Other operating revenues

analysis.[18]

The cross-section analysis was made for city banks and regional banks for the observation period of 11 years between 1974 and 1984.

IV. Measurement Result

1. Measurement Results of Scale Elasticity

Values of elasticity of scale estimated by an approach based on the index theory are shown in Tables 5 to 8. The results are compiled as follows.

	Production Factors*	Products*	City Banks	Regional Banks
Model 1	L, K	X	1.19 ~ 1.51	1.18 ~ 1.23
Model 2	L, K	Y-Px•X	1.05 ~ 1.34	1.11 ~ 1.15
Model 3	L, K, X	Y	1.05 ~ 1.16	1.02 ~ 1.06
Model 4	L, K, Px•X	Y	1.00 ~ 1.03	1.02 ~ 1.05

*L : employees K: current and fixed assets X: total liabilities
Px: funding cost Y: operating revenues

The above shows:

(a) In Model 4 where funding cost is used as the production factor instead of total liabilities, estimates of the elasticity of scale were less than 1 in the data on city banks for business year 1974. Except for these cases, however, estimates were larger than 1 in all other cases. This seems to attest the existence of the economies of scale in both city and regional banks.

(b) Comparing the estimated values of the elasticity of scale among the models, that of Model 1, where the stock amount of total liabilities is used as the product, is the largest. The second largest value of the elasticity of scale is shown in Model 2, the

18. In dealing with the capital stock, the difference in vintage by samples is often taken up as a problem. As this is a cross-sectional analysis, however, if the increase rate of capital($\Delta K/K$) of each bank is very close among banks irrespective of the production scale, there is no strong need to explicitly take the difference in vintage of capital stock of each bank when the economies of scale are measured. In other words, it is sufficient that the significance of α and the insignificance of β can be shown by estimating the parameter cross-sectionally with the formula $\Delta K/K = \alpha + \beta \cdot Y$. By actual measurement, it was made clear that the hypothesis $\alpha = 0$ was rejected at the significance level of 5% in both cases of city and regional banks and that the hypothesis $\beta = 0$ was accepted.

Table 5 Estimated Elasticity of Scale (Model 1)

FY	City Banks				Regional Banks			
	Laspeyres	Fischer	Translog	Paasche	Laspeyres	Fischer	Translog	Paasche
1974	1.17193	1.18904	1.18946	1.20645	1.19451	1.18556	1.18362	1.17634
	(4.55605)	(5.13711)	(5.15743)	(5.70622)	(91.7314)	(85.0656)	(83.7910)	(78.1867)
1975	1.23272	1.23550	1.23539	1.23825	1.12539	1.16728	1.16713	1.21118
	(13.8742)	(13.7956)	(13.7963)	(13.7110)	(56.6897)	(86.0860)	(85.8366)	(116.370)
1976	1.22915	1.23896	1.23900	1.24888	1.12365	1.17987	1.18088	1.23982
	(10.4711)	(10.8694)	(10.8708)	(11.2529)	(54.1598)	(92.7585)	(93.3634)	(132.230)
1977	1.26564	1.27283	1.27315	1.28005	1.17156	1.21138	1.21523	1.25283
	(8.52063)	(8.70271)	(8.71440)	(8.87753)	(86.4051)	(114.709)	(116.954)	(142.807)
1978	1.29632	1.30412	1.30416	1.31196	1.13661	1.18702	1.18485	1.24092
	(7.96467)	(8.12850)	(8.13502)	(8.28491)	(74.3891)	(114.593)	(112.656)	(154.874)
1979	1.34214	1.34424	1.34333	1.34626	1.13681	1.18237	1.18404	1.23086
	(10.2793)	(10.2980)	(10.2885)	(10.3109)	(67.1572)	(101.131)	(102.368)	(136.217)
1980	1.33351	1.33078	1.33003	1.32804	1.15987	1.19176	1.19220	1.22514
	(10.5737)	(10.5203)	(10.5148)	(10.4654)	(69.0573)	(89.0377)	(89.2335)	(109.531)
1981	1.38082	1.39069	1.39042	1.40059	1.17352	1.19788	1.19833	1.22308
	(8.15679)	(8.29739)	(8.29275)	(8.42739)	(66.1485)	(79.2093)	(79.3172)	(92.4457)
1982	1.44352	1.45606	1.45630	1.46873	1.17489	1.19742	1.19843	1.22068
	(14.9806)	(15.2150)	(15.2233)	(15.4277)	(66.2411)	(78.2254)	(78.6976)	(90.3670)
1983	1.49661	1.50587	1.50585	1.51520	1.20576	1.21021	1.20801	1.21443
	(12.3368)	(12.4456)	(12.4474)	(12.5469)	(101.854)	(104.480)	(103.109)	(106.768)
1984	1.50076	1.51103	1.51099	1.52139	1.23114	1.22889	1.22983	1.22622
	(21.8996)	(22.2972)	(22.2995)	(22.6799)	(104.452)	(103.746)	(104.208)	(102.636)

Figures in parenthesis show F value when the value of elasticity of scale is 1.

Table 6 Estimated Elasticity of Scale (Model 2)

FY	City Banks				Regional Banks			
	Laspeyres	Fischer	Translog	Paasche	Laspeyres	Fischer	Translog	Paasche
1974	1.04178	1.05166	1.05168	1.06162	1.12288	1.14456	1.14308	1.16677
	(0.64521)	(0.94197)	(0.94322)	(1.27739)	(46.3011)	(58.1789)	(57.1039)	(70.0915)
1975	1.14574	1.15764	1.15764	1.16968	1.11598	1.14635	1.14351	1.17792
	(3.97479)	(4.42963)	(4.42554)	(4.88565)	(39.3071)	(56.2839)	(54.4560)	(74.4826)
1976	1.16261	1.17307	1.17322	1.18361	1.09739	1.14519	1.14521	1.19569
	(9.93195)	(10.6408)	(10.6543)	(11.3308)	(33.7629)	(62.5907)	(62.3295)	(94.1700)
1977	1.14082	1.14810	1.14776	1.15542	1.05438	1.11217	1.11246	1.17389
	(3.15110)	(3.41262)	(3.40488)	(3.67899)	(10.6848)	(36.2287)	(36.2910)	(68.6240)
1978	1.12941	1.13006	1.12977	1.13069	1.05358	1.11074	1.10713	1.17273
	(2.70111)	(2.76431)	(1.75786)	(2.82811)	(7.59574)	(26.0728)	(24.7663)	(50.4206)
1979	1.16197	1.16378	1.16461	1.16555	1.08100	1.14523	1.14451	1.21494
	(4.28071)	(4.40165)	(4.43199)	(4.52279)	(20.0529)	(50.5908)	(50.1969)	(85.9615)
1980	1.22136	1.21932	1.21986	1.21728	1.11739	1.16219	1.16183	1.20991
	(6.45693)	(6.41978)	(6.43337)	(6.38158)	(47.1864)	(76.2802)	(75.9766)	(107.449)
1981	1.22896	1.23542	1.23540	1.24185	1.11069	1.12260	1.12228	1.13464
	(7.00429)	(7.24487)	(7.24371)	(7.47676)	(28.1850)	(33.1508)	(33.0115)	(38.2968)
1982	1.28159	1.29464	1.29480	1.30788	1.09575	1.12048	1.12064	1.14615
	(7.45504)	(7.93119)	(7.93981)	(8.40836)	(27.7684)	(40.6457)	(40.7766)	(55.1452)
1983	1.32840	1.33949	1.33955	1.35068	1.13955	1.14289	1.14562	1.14608
	(11.6071)	(12.1814)	(12.1859)	(12.7584)	(61.6120)	(64.4210)	(66.2379)	(67.0373)
1984	1.32175	1.33565	1.33594	1.34978	1.13123	1.12925	1.13247	1.12699
	(9.93808)	(10.3381)	(10.3489)	(10.7258)	(55.4122)	(54.2393)	(56.2284)	(52.7667)

Figures in parenthesis show F value when the value of elasticity of scale is 1.

Table 7 Estimated Elasticity of Scale (Model 3)

FY	City Banks				Regional Banks			
	Laspeyres	Fischer	Translog	Paasche	Laspeyres	Fischer	Translog	Paasche
1974	1.05235	1.05590	1.05602	1.05947	1.04046	1.04202	1.04167	1.04354
	(4.91841)	(5.47258)	(5.48104)	(6.04166)	(57.5392)	(61.8931)	(60.4393)	(66.0883)
1975	1.06217	1.06471	1.06482	1.06725	1.01861	1.02481	1.02553	1.03100
	(16.7507)	(17.6683)	(17.7051)	(18.5697)	(10.1650)	(18.0019)	(18.7630)	(27.9073)
1976	1.06767	1.06939	1.06956	1.07111	1.02936	1.03764	1.03850	1.04591
	(14.5929)	(15.0054)	(15.0422)	(15.4035)	(21.4115)	(34.1849)	(35.4789)	(49.2251)
1977	1.06904	1.06881	1.06906	1.06857	1.03083	1.03822	1.03865	1.04558
	(13.3413)	(12.9316)	(12.9837)	(12.5284)	(18.3109)	(27.4628)	(27.8841)	(38.0018)
1978	1.07815	1.07614	1.07639	1.07413	1.03071	1.03839	1.03819	1.04613
	(10.7325)	(10.2245)	(10.2614)	(9.72330)	(14.6763)	(22.2827)	(22.0538)	(31.1868)
1979	1.11969	1.11865	1.11868	1.11761	1.04368	1.05339	1.05445	1.06320
	(23.3584)	(22.9025)	(22.8864)	(22.4480)	(38.0660)	(55.7419)	(57.7147)	(76.3191)
1980	1.11843	1.11679	1.11696	1.11515	1.04124	1.05157	1.05184	1.06203
	(18.2893)	(17.8732)	(17.8951)	(17.4559)	(47.6057)	(72.1868)	(72.6825)	(100.918)
1981	1.16352	1.16222	1.16278	1.16091	1.04484	1.04466	1.04468	1.04444
	(24.1396)	(23.9003)	(24.0197)	(23.6577)	(40.4856)	(40.4629)	(40.4338)	(40.3513)
1982	1.12866	1.12938	1.12973	1.13009	1.05179	1.04868	1.04827	1.04555
	(15.8372)	(16.0406)	(16.0948)	(16.2413)	(74.4251)	(66.4494)	(65.2503)	(58.7421)
1983	1.09388	1.09537	1.09566	1.09687	1.04971	1.04876	1.04877	1.04779
	(13.5392)	(14.0309)	(14.0886)	(14.5311)	(98.8901)	(96.2467)	(96.1301)	(93.4182)
1984	1.08260	1.08323	1.08345	1.08385	1.06492	1.06126	1.06200	1.05760
	(12.3612)	(12.6926)	(12.7195)	(13.0286)	(160.291)	(145.782)	(148.027)	(131.397)

Figures in parenthesis show F value when the value of elasticity of scale is 1.

Table 8 Estimated Elasticity of Scale (Model 4)

FY	City Banks				Regional Banks			
	Laspeyres	Fischer	Translog	Paasche	Laspeyres	Fischer	Translog	Paasche
1974	0.99345	0.99825	0.99829	1.00309	1.03331	1.03950	1.03934	1.04574
	(0.55420)	(0.03861)	(0.03702)	(0.11657)	(42.8192)	(58.2393)	(57.3886)	(75.2224)
1975	1.02475	1.02762	1.02771	1.03050	1.01523	1.02802	1.02906	1.04107
	(8.65356)	(10.2959)	(10.3437)	(11.9642)	(6.13427)	(19.8722)	(21.1619)	(40.6732)
1976	1.02917	1.03170	1.03185	1.03424	1.02470	1.03935	1.03997	1.05426
	(9.54900)	(10.6431)	(10.7218)	(11.7142)	(15.3652)	(37.1574)	(38.3138)	(67.0431)
1977	1.03230	1.03378	1.03403	1.03526	1.02882	1.04346	1.04411	1.05828
	(6.11686)	(6.31655)	(6.40051)	(6.49941)	(13.7537)	(29.6056)	(30.4745)	(50.2642)
1978	1.02310	1.02520	1.02545	1.02730	1.01984	1.03610	1.03606	1.05276
	(2.87887)	(3.26564)	(3.32491)	(3.65173)	(3.75624)	(11.7223)	(11.7372)	(23.7433)
1979	1.01860	1.01868	1.01870	1.01876	1.02794	1.04546	1.04653	1.06437
	(5.34393)	(5.14599)	(5.14502)	(4.95363)	(11.4376)	(28.2708)	(29.5993)	(51.3036)
1980	1.02450	1.02422	1.02437	1.02394	1.02723	1.04509	1.04528	1.06343
	(8.61772)	(8.29113)	(8.38781)	(7.97184)	(28.3729)	(71.7583)	(72.6623)	(130.168)
1981	1.01648	1.01749	1.01795	1.01849	1.01277	1.02364	1.02391	1.03467
	(4.78843)	(5.35612)	(5.64821)	(5.93672)	(3.34447)	(10.9480)	(11.2240)	(22.4556)
1982	1.02163	1.02371	1.02404	1.02580	1.01663	1.02614	1.02585	1.03577
	(6.96402)	(8.53607)	(8.77223)	(10.2686)	(8.99569)	(21.3938)	(21.0240)	(38.4614)
1983	1.02768	1.03036	1.03065	1.03305	1.02323	1.02978	1.02973	1.03640
	(17.9908)	(23.3323)	(23.8287)	(29.7418)	(21.6163)	(35.0066)	(34.9785)	(51.4119)
1984	1.02174	1.02315	1.02341	1.02457	1.02293	1.03170	1.03259	1.04057
	(17.1565)	(20.3890)	(20.7255)	(24.0127)	(21.0873)	(38.5165)	(40.5952)	(60.1495)

Figures in parenthesis show F value when the value of elasticity of scale is 1.

so-called Royama Model, in which the product is the current income minus the funding cost. The third one is shown in Model 3 using the current income as the product, which is followed by Model 4.

(c) The significance of the economies of scale is shown by a hypothesis testing rejection or acceptance of the null hypothesis k=1. When the significance level is set at 5%, the economies of scale are significant in almost all cases.[19] The F value is highest in Model 3, which is followed by Model 1, Model 4 and Model 2. In Model 2, a comparatively larger estimated elasticity of scale could be obtained, but the standard deviation is also large. Accordingly, Model 2 is inferior to other models in terms of statistical fitness.

(d) Analyzing the estimated values of the elasticity of scale in a time series, the estimated values of city banks are on the upward trend in the case of Models 1 and 2, and differentials among banks expand depending upon the scale. In Models 3 and 4, however, a remarkable time serial trend is not discernible. It cannot be necessarily said, therefore, that the economies of scale have been increasing in recent years. As for regional banks, the estimated values of elasticity of scale are considerably stable in all models throughout the observation period and no time serial trend can be found. In the measurement based upon a concept of the elasticity of profits shown below, however, differentials in profits increase in time series depending upon the scale of banks.

2. Measurement Results of Estimates of Elasticity of Profits

What is of vital concern for banking activities seems to be the elasticity of profits. Measurement results of values of the elasticity of profits are shown in Table 9 and Figures 9a through 9c, the horizontal axis denotes the current income of 12 city banks, while the vertical axis denotes the value of the elasticity of profits which corresponds to the current income of each bank. Figures 9a–c are year-based observation line graphic charts. Their results are:

(a) The estimated values of the elasticity of profits in each model are larger than those of the elasticity of scale mentioned before. In particular, the value of the elasticity of profits is large in Model 3 in which the current income is used as the product. Even in terms of the mean value, a 1% increase in the production factor surely contributed to a 2.4% rise in the profit in 1981.

(b) By replacing the economies of scale with profit, a term common among models,

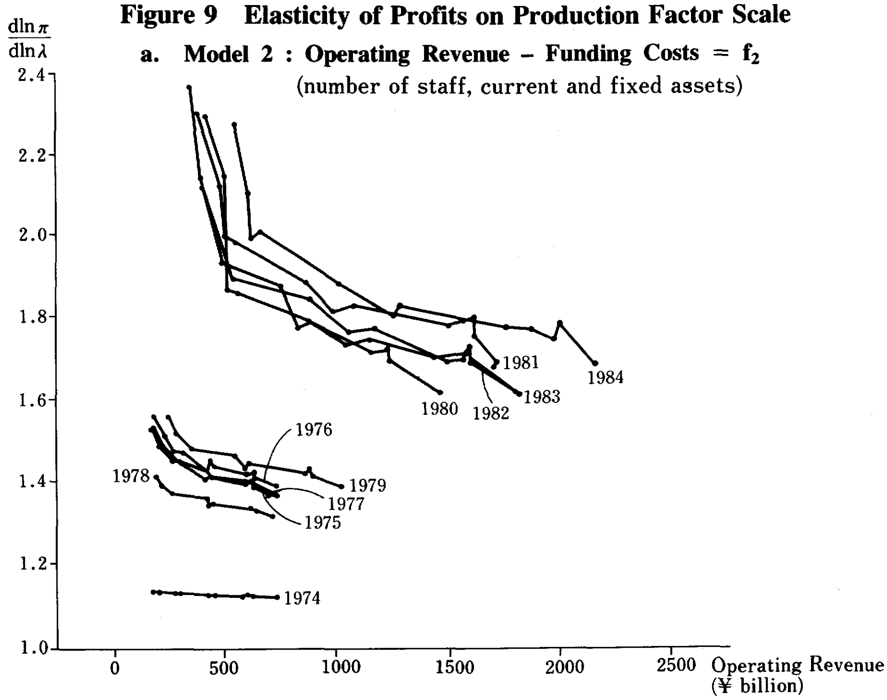
19. The point here is whether the economies of scale exist or not. Testing is conducted by checking the F value when the value of the elasticity of scale k is 1 (likelihood ratio test). In the case of city banks, the economies of scale were significant when the F value was more than 4.84 and the significance level was set at 5%. In the case of regional banks, the economies of scale were significant when the F value was more than 4.00

Table 9 Average Estimated Elasticity of Profits

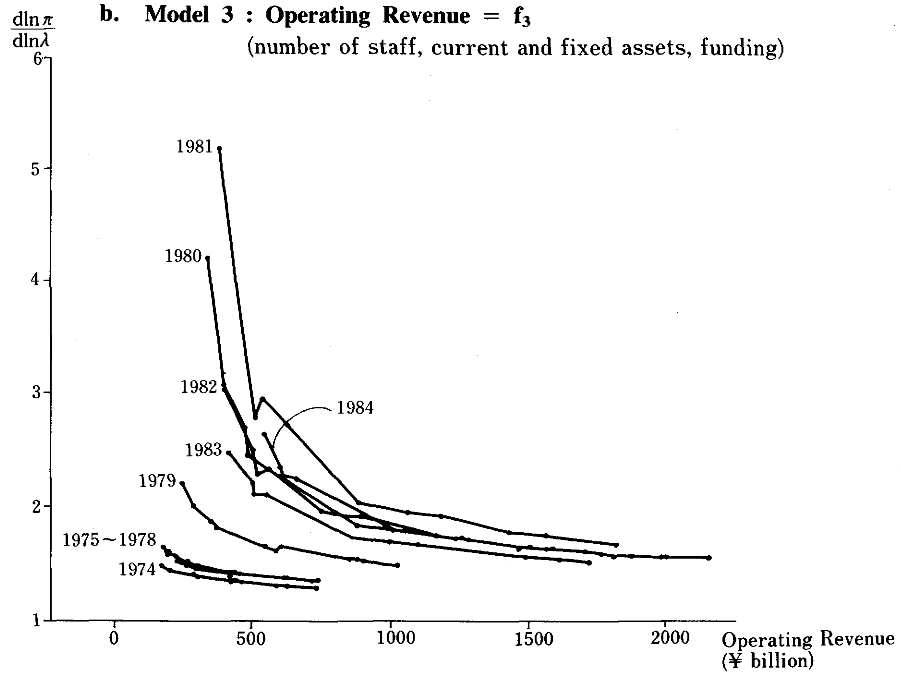
FY	Model 2		Model 3		Model 4	
	City Banks	Regional Banks	City Banks	Regional Banks	City Banks	Regional Banks
1974	1.12790	1.25491	1.36368	1.16859	0.98481	1.15985
1975	1.42501	1.27816	1.46348	1.12074	1.23467	1.13453
1976	1.45035	1.28447	1.44166	1.17392	1.23449	1.18108
1977	1.42890	1.26887	1.46102	1.21112	1.26140	1.23431
1978	1.35160	1.26208	1.43394	1.19904	1.17148	1.18860
1979	1.45454	1.31242	1.71737	1.27270	1.16702	1.23652
1980	1.85868	1.39444	2.24371	1.35358	1.45686	1.32432
1981	1.84003	1.35659	2.41507	1.40284	1.33495	1.26152
1982	1.79405	1.28588	1.97976	1.32848	1.29111	1.20759
1983	1.89581	1.31868	1.82631	1.30909	1.35951	1.21427
1984	1.88788	1.31168	1.90061	1.39221	1.35911	1.25198

Figure 9 Elasticity of Profits on Production Factor Scale

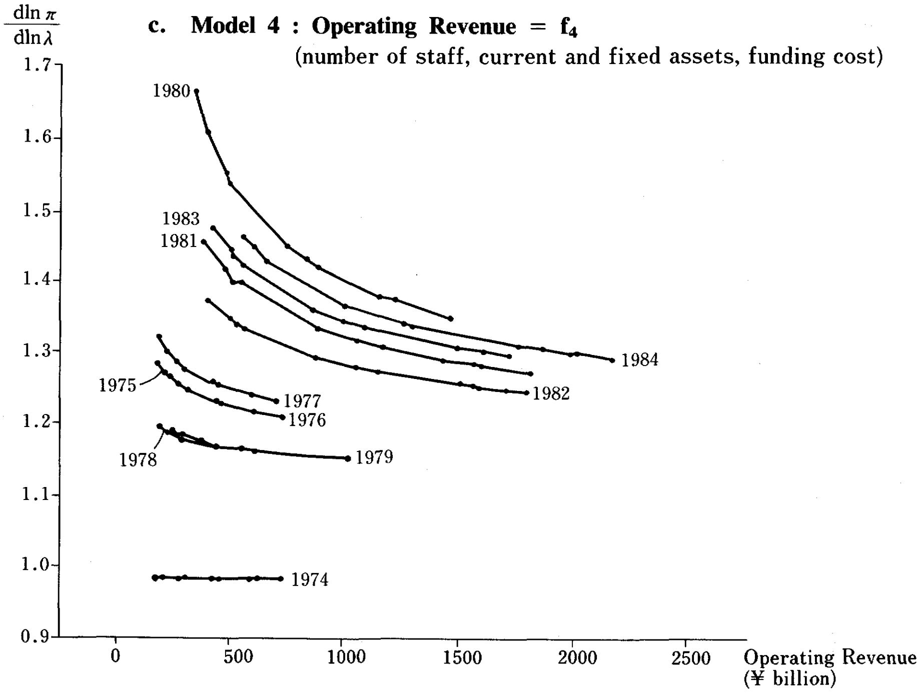
a. Model 2 : Operating Revenue – Funding Costs = f_2
(number of staff, current and fixed assets)



b. Model 3 : Operating Revenue = f_3
(number of staff, current and fixed assets, funding)



c. Model 4 : Operating Revenue = f_4
(number of staff, current and fixed assets, funding cost)



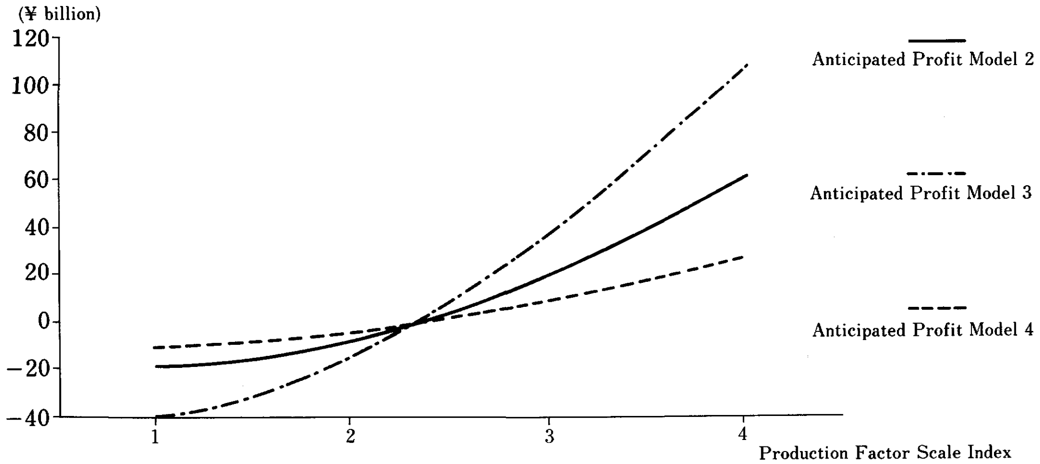
comparison of the value of the elasticity of profits among models is possible. Differing from the case of the value of the elasticity of scale, the values of the elasticity of profits in Models 2 (Royama Model) and 3 are rather close to each other. Comparing the two models in terms of profits, the common yardstick, there is not a large difference between the two models. On the other hand, in Model 4 in which funding cost is used as the production factor and the estimated value of the elasticity of scale was the smallest among the four models, the estimated value of the elasticity of profits is also smaller than those in the other two models. It suggests that differentials in profit among scales of banks are estimated smaller in Model 4.

(c) In terms of time series, there has been a trend that the value of the elasticity of profits of city and regional banks has been expanding from the 1970s through the 1980s. It can be surmised, therefore, that latent differentials in profit among scales of banks have enlarged in the 1980s.

3. The Value of Elasticity of Profit and Differentials in Profits

We shall examine here the real profit differential among the banks' values. This is brought out by the difference in the value of the elasticity of profits, based upon the estimates in this analysis. First of all, the production factor cost is increased up to the point where the profit gained by a bank with the production factor of intermediate size should become zero, without varying any other conditions. Then the profit to be gained by other banks is calculated in accordance with Models 2, 3, and 4. The relations between the scale of banks and their estimated profits are shown in the graph (Figure 10). Since the value of the elasticity of scale is larger than 1 in every

Figure 10 Anticipated Profits According to 1984 Data*



* Anticipating the case where deposit rate rises to make middle size bank profit zero

model, every model depicts that curves ascending to the right show a gradual increase in profits corresponding to the expansion of the scale of banks. Even in Model 4 in which the value of the elasticity of profits is smallest, the standard bank (its production factor scale index is 1) has a loss of about ¥10 billion. In Model 3, the standard bank has a loss of about ¥40 billion, while the four times larger bank has a profit of about ¥120 billion.

4. Economies of Scope and Economies of Scale

Prior to measuring the value of the elasticity of scale, we attempted to identify the effect of the economies of scope and of the economies of scale on productivity by utilising graphs (as mentioned in Figure 6). Here the problem is approached by using a formula based upon the method of estimating the value of the elasticity of scale applied in this analysis.

We are interested in three points: whether or not diversification would contribute to increased productivity when the scale of the bank is controlled; and whether an enlargement in scale would boost production when diversification is controlled; and whether the value of elasticity of scale estimated in this analysis does not include any bias brought out by diversification. The following is a formula to inspect these three points.

$$\ln(CY) = a + a'D + b \cdot \ln(CQ) + b'D \cdot \ln(CQ) \qquad (20)$$

CY: product index (CY=1)
CQ: production factor index (Translog index)
D : scope dummy variable

The prototype of this formula is a formula to estimate the elasticity of scale using chain indices. Accordingly, the indices here mean chain indices. Definition of the product and of the production factor is based upon Model 3. The scope dummy variable D is 1 as for a bank in which diversification is comparatively highly promoted (a comparatively high rate of profits gained by means other than loans against the total profits) and 0 as for a bank in which diversification is comparatively low. When the parameter a′ is estimated to be significant, it can be verified that diversification has a favorable impact on productivity. When the parameter b′ is under the significant level, it can be shown that the scale of the bank has a stable impact on the differential in productivity, irrespective of the degree of diversification. When the value of b is estimated to be larger than 1, the impact is favorable (that is to say, the economy of scale exists).

The estimated results of the calculation according to Equation (20) are shown in

	City Banks	Regional Banks
sign a′	+ in the preceding year	mixture of ±
significance of a′*	no significance each year	significant with + in 1978, 79
sign b′	mixture of ±	mixture of ±
significance of b′*	no significance each year	significant with + in 1978
value of b	larger than 1 in the preceding year	larger than 1 in the preceding year

* 5% significance level

Table 10 by groups of city banks and of regional banks.

As a result, the following were made clear:

(a) The estimates of a′ are positive and not significant in almost all cases. It seems that diversification does not have an effect on productivity.

(b) Judging from the values of b′ and t values of estimates of b′, it can be said that the impact given by the scale of the bank on productivity is not influenced by the degree of diversification.

(c) The estimates of b show the stable existence of the economies of scale here. The value of b is very close to the value of elasticity of scale which was already estimated (see Table 7). It was made clear that the values of elasticity of scale which were estimated by us scarcely include any bias caused by economies of scope.

5. Merger of Banks and the Economies of Scale

As mentioned in the introduction, the economies of scale of banking were often discussed together with the issue of merger of banks. In this section, we will examine what influence bank mergers have on productivity of banking business and what relations it has with the economies of scale. The method used here is that of applying a dummy variable as was used in the examination of the scope in the previous section. Fortunately, in Japan there have been large-scale mergers of banks such as the merger of Daiichi Bank and of Kangyo Bank into Dai-ichi Kangyo Bank and the merger of Taiyo Bank and of Kobe Bank into Taiyo Kobe Bank. Data on such mergers can be used with the following formula.

$$\ln(CY) = a + a' \cdot GD + b \cdot \ln(CQ) \qquad (21)$$

CY: product index (CY=1)
CQ: production factor index (Translog index)
GD: merger dummy variable

Table 10 Model 3 : Translog Index

FY	City Banks					Regional Banks				
	a	a′	b	b′	R-SQ	a	a′	b	b′	R-SQ
1974	-0.0178 (-1.309)	0.0567 (1.788)	1.0433 (66.59)	-0.0132 (-0.408)	0.9983	-0.0571 (-2.994)	-0.0199 (-0.966)	1.0362 (105.7)	0.0105 (0.775)	0.9974
1975	-0.0218 (-1.115)	0.0220 (0.862)	1.0678 (53.18)	0.0030 (0.110)	0.9982	-0.0659 (-3.973)	0.0199 (0.763)	1.0297 (122.4)	-0.0107 (-0.797)	0.9975
1976	-0.0211 (-0.888)	0.0067 (0.217)	1.0636 (43.14)	0.0200 (0.594)	0.9974	-0.0578 (-3.603)	0.0071 (0.251)	1.0358 (117.2)	0.0033 (0.215)	0.9972
1977	-0.0532 (-1.745)	0.0277 (0.702)	1.0728 (33.58)	0.0070 (0.159)	0.9956	-0.0739 (-4.412)	0.0619 (1.678)	1.0398 (107.1)	-0.0227 (-1.252)	0.9967
1978	-0.0721 (-1.977)	0.0551 (1.160)	1.0961 (27.90)	-0.0225 (-0.416)	0.9933	-0.0446 (-2.688)	0.1139 (2.929)	1.0399 (102.1)	-0.0430 (-2.271)	0.9969
1979	-0.1031 (-2.608)	0.1036 (2.060)	1.1966 (23.10)	-0.1124 (-1.858)	0.9940	-0.0281 (-1.970)	0.0658 (2.068)	1.0524 (127.2)	-0.0208 (-1.330)	0.9977
1980	-0.0358 (-1.133)	0.0606 (1.274)	1.1184 (24.26)	-0.0300 (-0.523)	0.9945	-0.0364 (-2.829)	-0.0078 (-0.294)	1.0470 (144.4)	0.0084 (0.644)	0.9982
1981	-0.0138 (-0.566)	0.1352 (1.282)	1.1126 (28.04)	-0.0587 (-0.607)	0.9952	-0.0426 (-2.204)	-0.0029 (-0.099)	1.0391 (97.99)	0.0075 (0.505)	0.9970
1982	-0.0242 (-0.779)	0.0729 (1.154)	1.1474 (25.97)	-0.0648 (-0.968)	0.9930	-0.0444 (-2.739)	-0.0095 (-0.350)	1.0409 (117.9)	0.0121 (0.887)	0.9976
1983	-0.0167 (-0.663)	0.0416 (0.802)	1.0830 (25.01)	-0.0152 (-0.257)	0.9953	-0.0341 (-2.530)	-0.0050 (-0.235)	1.0423 (139.2)	0.0089 (0.829)	0.9984
1984	-0.0795 (-3.321)	0.0719 (2.211)	1.1091 (33.17)	-0.0503 (-1.259)	0.9971	-0.0510 (-3.423)	0.0039 (0.169)	1.0542 (127.2)	0.0083 (0.708)	0.9982

Note: 1. Figures in parenthesis are tvalue.
2. R-SQ is a coefficient of determination of which degrees of freedom were adjusted.

Table 11 City Bank Model 3: Translog Index

FY	a	a′	b	R-SQ
1974	-0.0091 (-0.551)	-0.0396 (-1.839)	1.0670 (57.74)	0.9970
1975	-0.0093 (-0.759)	-0.0317 (-2.003)	1.0739 (76.44)	0.9983
1976	-0.0187 (-1.588)	-0.0460 (-3.069)	1.0833 (79.54)	0.9984
1977	-0.0390 (-2.757)	-0.0644 (-3.543)	1.0890 (65.23)	0.9976
1978	-0.0423 (-2.243)	-0.0705 (-2.849)	1.0981 (48.20)	0.9957
1979	-0.0426 (-2.110)	-0.0733 (-2.716)	1.1392 (47.54)	0.9955
1980	-0.0202 (-0.951)	-0.0509 (-1.739)	1.1302 (44.90)	0.9950
1981	-0.0214 (-0.889)	-0.0565 (-1.709)	1.1754 (43.78)	0.9946
1982	-0.0077 (-0.413)	-0.0817 (-3.129)	1.1476 (54.68)	0.9965
1983	-0.0133 (-0.912)	-0.0633 (-3.019)	1.1089 (65.79)	0.9976
1984	-0.0450 (-3.020)	-0.0562 (-2.544)	1.0948 (62.04)	0.9973

Note: See Note of Table 10

Table 12 City Bank Model 3: Translog Index

FY	a	a′	b	b′	c	R-SQ
1974	-0.0184 (-1.271)	0.0573 (1.706)	1.0483 (49.74)	-0.0182 (-0.498)	-0.0088 (-0.386)	0.9981
1975	-0.0240 (-1.264)	0.0242 (0.979)	1.0788 (50.52)	-0.0080 (-0.283)	-0.0226 (-1.242)	0.9983
1976	-0.0256 (-1.283)	0.0112 (0.434)	1.0837 (47.64)	-0.0001 (-0.004)	-0.0395 (-2.102)	0.9982
1977	-0.0603 (-2.671)	0.0347 (1.194)	1.1036 (42.48)	-0.0238 (-0.700)	-0.0593 (-2.787)	0.9976
1978	-0.0795 (-2.834)	0.0625 (1.715)	1.1327 (34.09)	-0.0592 (-1.352)	-0.0707 (-2.584)	0.9961
1979	-0.1011 (-3.749)	0.0940 (2.729)	1.2099 (34.02)	-0.1037 (-2.507)	-0.0683 (-3.194)	0.9972
1980	-0.0351 (-1.205)	0.0532 (1.205)	1.1296 (26.17)	-0.0263 (-0.496)	-0.0442 (-1.547)	0.9953
1981	-0.0137 (-0.564)	0.0914 (0.807)	1.1250 (27.26)	-0.0279 (-0.276)	-0.0355 (-1.035)	0.9952
1982	-0.0197 (-0.844)	0.0476 (0.988)	1.1620 (34.69)	-0.0468 (-0.928)	-0.0765 (-2.701)	0.9961
1983	-0.0173 (-0.894)	0.0311 (0.778)	1.1053 (32.21)	-0.0177 (-0.390)	-0.0592 (-2.569)	0.9972
1984	-0.0735 (-4.492)	0.0627 (2.815)	1.1131 (48.89)	-0.0407 (-1.487)	-0.0507 (-3.210)	0.9986

Note : See Note of Table 10

As for CY and CQ, the definition is completely the same as in the former section. The merger dummy variable GD is 1 in the cases of Dai-ichi Kangyo Bank and of Taiyo Kobe Bank and 0 in the cases of other city banks. In order to examine the effects of a large-scale merger of banks, only data on city banks are used. If the parameter a′ in the above formula is estimated to be significant, it will be confirmed that the merger of banks has an impact on the productivity of banking business. Whether the impact is favorable or negative may depend upon whether the value of the parameter is plus or minus.

The results of the estimation are shown in Table 11. The estimated value of a′ was minus in all years and was significant when the significance level was set at 5% in seven years excluding 1974, 1975, 1980 and 1981. Accordingly, as far as these data are concerned, merger of banks does not lead to a rise in productivity, when the current income is the product. It can be said that the merger of banks does not contribute to boosting the economies of scale.[20] Table 12 shows the results of a similar calculation using both a scope dummy variable and a merger dummy variable.[21] Even if these two factors are included in the above-mentioned formula simultaneously, the result scarcely differs, leading to the same conclusion.

V. Conclusion

As far as the above mentioned econometric analysis is concerned, it was confirmed that Japanese banks have the economies of scale to a considerable extent in terms of production of service. The value of the elasticity of production scale was 1.05 to 1.06 for city banks and 1.02 to 1.06 for regional banks. The value of the elasticity of profits was 1.44 to 2.42 for city banks and 1.12 to 1.40 for regional banks (the estimated values were based upon Model 3). All these figures were larger than we expected. This fact was clearly shown by our simulation experiment using data on city banks for fiscal 1984.

When we supposed the profit of an intermediate-size city bank as zero as a simulation test, it was shown in the calculation based upon Model 2 (Royama Model), that smaller banks have a loss of about ¥20 billion, while banks which are four times larger than the standard bank still gain a ¥60 billion profit. The estimates of the elasticity of scale in Model 3, in which the current income is the product, showed that smaller banks' loss and bigger banks' profit are about two times larger

20. In accordance with a similar calculation with Model 1 in which total liabilities were used as the product, the coefficient of the merger dummy variable was estimated to be significantly negative.

21. The calculation formula in this case is,

$$\ln(CY)=a+a'\cdot D+b\cdot\ln(CQ)+b'\cdot D\cdot\ln(CQ)+c\cdot GD.$$

than those calculated in Model 2 respectively (¥40 billion loss and ¥120 billion profit). It was also confirmed by our analysis that merger of banks with an aim of reorganization of the financial industry brings out the so-called side effects of merger and that it will take a considerably long time for the merged banks to achieve a favorable impact in terms of the economies of scale. These points suggest that liberalization of the financial industry, in particular the problem of deregulation of interest rates for small deposits, should carefully be dealt with.

The Kinyu Mondai Kenkyu Kai (Study Group on Financial Issues) said in its report that, "In a backdrop of internationalization of the Japanese economy and financial industry and technological innovation progressing, liberalization of the Japanese financial industry seems to be inevitable. If regulation of interest rates is continued under these circumstances, it is likely to cause funds to shift from products of which interests are regulated, to products out of the regulation and to adversely hinder the smooth financial activities. In worse cases, there is a likelihood that it will have a negative impact on the order of credit." (Kinyu Mondai Kenkyu Kai (1986)). As mentioned in the report, deregulation of interest rates for small deposits is unavoidable for the Japanese banking business.

Under these circumstances, there are voices that interest rates for small deposits should also be synchronized with the market rates, but on the other hand, that 100-percent deregulation of interest rates for small deposits under which each bank can decide freely the interest rate for small deposits is preferable. The former argument is based upon a recognition that complete deregulation has a danger of bringing out cooperative oligopoly of banks, as Royama pointed out (Royama (1986), chapter 6) that in the case of small deposits, financial institutions can reduce the threat which they pose each other through coordination as a whole.

That is to say, complete deregulation of interest rates does not necessarily mean free competition. Accordingly, in order to avoid cooperative oligopoly of banks, it is necessary that the interest rates for small deposits should be fixed at a level as though they were determined through a competition on prices (idem). Royama argued that it would be preferable that the level of interest rates for small deposits should be set in correspondence with the market rate. Royama's view seems to agree with that of Edgeworth who advocated that the problem of oligopoly may be solvable by realizing an efficient market with auctioneers' function, even if the competition is evolved among a small number of competitors (Edgeworth (1881)).

Concluding this analysis of the economies of scale, we would like to further support the former argument from another point of view. As discussed already, small depositors' concern about interest rates may be less than that of large depositors, and financial information that small depositors can obtain is apt to be incomplete. Accordingly, there is a large possibility that cooperative oligopoly of financial institutions would be brought up when regulation on financial industry is completely lifted.

Taking into consideration the fact that there are such significant economies of scale in the banking industry, there is also, however, a likelihood that cooperative oligopoly of financial institutions would suddenly change into competitive oligopoly based upon the conjectual variation of price (for example, deposit rates). As well known, it is said that financial oligopoly tends to turn to a Bertrand-Edgeworth oligopoly rather than to a Cournot style. In order to avoid such situations, this analysis suggests that the most vital problems the Japanese financial institutions should tackle, for the time being, are to streamline the interest rate system for small deposits corresponding with the market and at the same time to maintain the order of credit and to carefully discuss the problem of complete deregulation of interest rates for small deposits.

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