Nonstationary Covariance Structure of Detrended Economic Time Series: a Time Varying Model Approach*

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I. Introduction

One of the basic approach to estimate the principal features of time series data with relatively complex structure is to apply a statistical model based on the stochastic process. A premise of stationary process permits the robust modeling of time series as the established stochastic process theory, which emphasizes the effectiveness of economic time series analysis. Detrending or differencing of original economic series are so far widely used for this purpose, because almost all economic series are nonstationary. In addition to this, if we can apply a sufficiently practical time evolving model expressing the nonstationary state of time series, then it would be natural to expect that such a model would produce more implications.

Typical examples of nonstationary economic time series are nonstationary in the mean, that is the series of which trends are changing, and nonstationary in the covariance, that is, in which the statistical characteristics of the fluctuation around the trend are changing over time. The changing pattern of detrended series may occur slowly or abruptly. Abrupt change may suggest structural changes. Accordingly, to attempt to distinguish whether the series is stationary or nonstationary will be important in economic analysis when applying a model based on the stochastic process. The purpose of this paper is to examine the nonstationarity of detrended economic time series from the statistical viewpoint.

The approaches used in this paper are based on Kitagawa-Gersch (1984, 1985a)

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as can be seen in II., which aim to estimate stochastic trend and to find changing statistical evidence detrended time series using time evolving autoregressive coefficient model. Also we attempt to examine structural changes observing the statistical abrupt changes in the series. It is often suggestive of a change in the economic structure.

From III. to V., the application results fitted to the above-mentioned methods are shown. In III., Japan's and the U.S. money supply and GNP are examined. In IV., the movement of Marshallian k of Japan and in V., real money stock for Japan and the U.S. are treated, respectively. Results are quite suggestive in the following sense, that first in Japan, since around the year 1975, the range of the fluctuation of the M_2+CD has been decreasing while that of nominal and real GNP have similarly continued to decrease from nearly the conterminous period, which suggests that there have been structural changes around that period. This period corresponds almost to the same period when the Bank of Japan made much account of money supply. The results obtained here are similar views by Oritani (1981), Okubo (1983), and Suzuki (1983), though they analysed from another point of view, that in Japan, the stabilization of money supply brought about the stableness of the growth rate of real GNP. In contrast to this, the U.S. money supply (M_1, M_2) and nominal and real GNP have been continuously showing big fluctuations. These differences between Japan and the U.S. confirm the assertions of Friedman (1985).

Second, the Japanese real money stock, M_1 and M_2+CD change in the trend during the 1973–1974 period. This period was so far generally stated that a structural change seemed to occur. Besides, the time evolving model reveals that changes can be seen in the detrended series. Noteworthy is that after 1980 there are suggestive movements of structural changes, especially in M_1. In this regard, further studies and observation will be required in relation to the assessment of the effect of financial innovation and liberalization.

In a strict sense, it is not likely to state clearly whether a detrended series is stationary or nonstationary on actual data. Naturally, the statistical features of detrended series as well as the trend crucially depend upon the sample period. Although the advantages to analyse economic time series on a stationary assumption have to be recognized, the results in this paper reveal that it is also necessary to analyse the series as a nonstationary process in order to obtain further implications. To capture the relationship among economic variables, a multivariate model will be necessary. Even in such cases, first of all, comparative statistical analyses of economic time series of every kind are made. In this sense, the methods presented in this paper is of great significance to observe the statistical features of economic time series as a first step in a proper economic analysis.

II. A Statistical Approach to the Nonstationary Economic Time Series[1]

1. The Kitagawa-Gersch Approach

As mentioned previously, we examine the statistical properties of the economic time series that are nonstationary in the mean and the detrended time series that are nonstationary in the covariance.[2] The analysis of mean nonstationary series is to estimate the trend and examine its change. For the nonstationary in the covariance time series, the series which exclude estimated trend are examined. For this purpose, we use the Kitagawa-Gersch method in this paper. Critical ideas in their methods are as follows:

First, trends are estimated stochastically using a smoothness priors-state space modeling. Trend may vary depending on the object, purpose, and period of analysis. However, intuitively fitted deterministic trend is not always appropriate when trend is likely to change over time, particularly when we pay much attention to the fluctuation around the trend.[3] If sufficiently practical stochastic trend can be estimated, we can expect up-to-date implications such as changes in tendencies which were not realized enough in deterministic trends.[4]

Second, the covariance structure of detrended time series is examined by means of time varying autoregressive coefficient model.[5] As the general assumption, the statistical characteristics of detrended series may change smoothly over time,

1. Takeuchi (1974) emphasized regarding the stationary and nonstationary state of economic time series that, though assumption on stationary state often seems unrealistic, we may not be able to apply the statistical method if we do not assume some 'repeated observed value' in a time change. He also anticipated to proceed concerning this problem with mathematical study and empirical analysis based on the actual data.

2. The outliers or the missing values are also other examples of nonstationary time series. Discussions based on the unit root are also well known. But, in this paper, we focus mainly on the changing features of the detrended series, which are used more often in recent empirical analyses depend on weak stationarity.

3. See, for example, Nelson-Plosser (1982) concerning the problems when deterministic trends are excluded.

4. There are several attempts concerning the estimation of stochastic trends (for example, Stultz-Wasserfallen (1984), Watson (1985)). Therefore, an estimation of a stochastic trend is not enough features of the Kitagawa-Gersch method. Their idea is connected to the use of the likelihood of a Bayesian model for model selection. See note 6.

5. In this paper we use the term not "varying parameter model", but "time varying coefficient model" because, generally, a system in which input-output state does not change with time is called "time invariant system" and a system which changes with time is called "time varying system".

moreover, step changes or shifts are also considered. The smoothness priors constraints are imposed on the evolution of the time varying autoregressive coefficients.

Third, a smoothness priors methodology and the use of the likelihood of a Bayesian model is used as a measure of the goodness of fit of the model. The ideas aim at constructing a highly practicable statistical model. As the smoothness priors, the information with regard to the time series components and their stochastic changes are used. The best model is selected by using the minimum AIC procedure.[6] In addition, the model assuming slow changes over time and the model considered step shifts at some period are compared.

There are a number of studies regarding the varying coefficient models, but the examples of the application of the time varying autoregressive coefficient model to economic analysis are rare. Doan, Litterman and Sims (1984) attempts to apply the time varying autoregressive model, but their approach is different from the methodology adopted in this paper.[7] Kitagawa and Gersch aim for a relatively straightforward method for the modeling of nonstationary covariance data. In their method, Kalman filter is used to facilitate the likelihood computation. However, it is well known that reliable estimation cannot be obtained from Kalman filter unless the model is well defined and also identification of the model is essential. A consequence of Kitagawa-Gersch's idea of the use of a smoothness priors and the adoption of model selection criterion encourages more extensive applications.

The followings are the outline of the methods used in this paper.

2. Nonstationary Time Series in the Mean—Estimating Stochastic Trend[8]

Observed time series y(n) can be decomposed into trend, stationary factor, seasonal factor, trading-day factor, and observation noise. Each component is considered to follow the stochastic process.[9] Then the trend component satisfies a k-th

6. In this regard, Kitagawa-Gersch (1985-a) state in their survey that "In none of the above mentioned papers does the concept of the likelihood of a Bayesian model or of a hyperparameter nor anything related to smoothness priors appear."

7. The so-called time varying models so far tried to apply in economic analysis may be broadly classified into two types: the one is multivariate regression type and the other is an application of Kalman filter. Both methods deal with nonstationary time series. Already in the 1940s and 50's, studies of time varying coefficient models can be seen. For historical brief survey, see, for example, Pagan (1980), Nicholls-Pagan (1985) and Los-Kell (1985). Concerning the examples of time varying autoregressive coefficient models, see the references in Kitagawa-Gersch (1985-a).

8. In detail, see Kitagawa-Gersch (1984), and also on the application to Japan's economic series, see Naniwa (1985).

9. For simplification, trading-day factor can be considered deterministically.

order stochastically perturbed difference equation

$$\nabla^k t(n) = w_1(n) : w_1(n) \sim N(0, \tau_i^2) \tag{1}$$

where ∇ denotes a difference operator such as $\nabla t(n)=t(n)-t(n-1)$, and $w_1(n)$ is an i.i.d. sequence. With other components, stochastic trend is represented by a state space model

$$x(n) = \begin{bmatrix} F_1 & & & \\ & F_2 & & \\ & & F_3 & \\ & & & F_4 \end{bmatrix} x(n-1) + \begin{bmatrix} G_1 & & & \\ & G_2 & & \\ & & G_3 & \\ & & & G_4 \end{bmatrix} w(n) \tag{2}$$

$$y(n) = [H_1 H_2 H_3 H_4(n)]\, x(n) + \varepsilon(n) \tag{3}$$

where $(F_1\ G_1\ H_1)$, $(F_2\ G_2\ H_2)$, $(F_3\ G_3\ H_3)$, $(F_4\ G_4\ H_4(n))$ represent coefficient matrix of trend factors, stationary factors, seasonal factors, and trading-day factors, respectively. $x(n)$ is a state vector which includes each factor and $w(n)=(w_i(n)\ (i=1,\ldots,4))'$ are assumed to be normal i.i.d. stochastic terms with zero means and unknown variances τ^2_i $(i=1,\ldots,4)$, which correspond to each factor. $(')$ denotes transposition. Observation noise $\varepsilon(n)$ is assumed to be normal i.i.d. with zero mean and unknown variance σ^2.

In Equations (2) and (3), trend factors can be given using the notion of Equation (1) as

$$x_1(n) = [t(n), t(n-1), \cdots\cdots, t(n-k+1)]'$$

$$F_1 = \begin{bmatrix} C_1 \cdots\cdots C_{k-1} & C_k \\ 1 \cdots\cdots 0 & 0 \\ \vdots \ \ddots \ \vdots & \vdots \\ 0 \cdots\cdots 1 & 0 \end{bmatrix}, G_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, H_1 = [1\ 0 \cdots 0]$$

where $x_1(n)$ denotes the trend part of state vector components, C_i $(i=1,\ldots,k)$ reflects smoothness constraint determined in difference Equation (1).

The unknown parameters in the state space representation (2) and (3) are order k in stochastic difference equation, order p in the autoregressive process

$$\upsilon(n) = \sum_{i=1}^{p} \alpha_i \upsilon(n-i) + w_2(n) \tag{4}$$

which is represented in F_2 of Equation (2), variance τ^2_i(i= 1, 2, 3), σ^2 of stochastic term and coefficient α_i(i=1,...,p) of Equation (4). Recursive Kalman filtering and smoothing yield the estimates of the state vector x(n) and the likelihood for the unknown variances. Then

$$\text{AIC} = -2 \times \begin{pmatrix}\text{maximum log}\\ \text{likelihood}\end{pmatrix} + 2 \times \begin{pmatrix}\text{number of}\\ \text{parameters}\end{pmatrix} \tag{5}$$

are obtained. According to Akaike (1974), AIC is used as a criterion for model selection.

3. Nonstationary Time Series in the Covariance — Time Varying Features of Detrended Series[10]

Let the estimated trend at time n from N observations y(1),...,y(N) be t(n|N), then the detrended series

$$z(n) = y(n) - t(n|N) \tag{6}$$

is assumed to be nonstationary time series in the covariance structure. The detrended observations z(1),...,z(N) can be expressed as a time varying autoregressive (AR) coefficient model

$$z(n) = \sum_{i=1}^{m} a(i, n) z(n-i) + \varepsilon(n) \tag{7}$$

where m is the model order, and ε (n) is white noise with mean zero and variance σ^2. Coefficients a(i,n) are assumed to change gradually over time. A useful constraint model for time varying coefficients is obtained by the stochastically perturbed difference equation. If the coefficients behave like a random walk and the successive values of the coefficients a(i,n) and a(i,n−1) are approximately equal, then the time varying coefficient can be expressed by the first order stochastic difference equation with the stochastic term δ (i,n) as

$$a(i,n) = a(i,n-1) + \delta(i,n) \tag{8}$$

where δ (i,n) is a Gaussian white noise with mean zero and variance τ^2. Likewise, if

10. See Appendix I.

the change of the coefficients is gradual and is approximated by a local linear function, a second order stochastic difference equation

$$a(i,n) = 2a(i,n-1) - a(i,n-2) + \delta(i,n) \tag{9}$$

can be considered.

In general, time varying AR coefficient model is obtained by k-th order stochastic difference equation

$$\nabla^k a(i,n) = \delta(i,n) \tag{10}$$

where ∇ denotes difference operator defined by $\nabla a(i,n)=a(i,n)-a(i,n-1)$ and $\delta(i,n)$ is assumed to be Gaussian white noise sequence with mean zero and variance τ^2.

Defining the state vector x(n) by

$$x(n) = [a(1,n),\cdots,a(m,n),\cdots,a(1,n-k+1),\cdots,a(m,n-k+1)]'$$

($'$ denotes transposition), then the time varying AR coefficients can be expressed in the form of a state space model.

$$\begin{aligned} x(n) &= Fx(n-1) + Gw(n) \\ z(n) &= H(n)x(n) + \varepsilon(n) \end{aligned} \tag{11}$$

In (11), w(n) is the vector $w(n)=(\delta(i,n),\ldots, \delta(m,n))'$, F,G,H(n) are coefficient matrices and z(n) is the detrended observation.

Using Equations (7) and (10), state space model (11) is defined in general form as

$$x(n) = \begin{bmatrix} a(1,n) \\ \vdots \\ a(m,n) \\ \hline a(1,n-1) \\ \vdots \\ a(m,n-1) \\ \hline \vdots \\ \hline a(1,n-k+1) \\ \vdots \\ a(m,n-k+1) \end{bmatrix} = \left[\begin{array}{c|c|c|c} C_1 I_m & \cdots & C_{k-1} I_m & C_k I_m \\ \hline I_m & \cdots & 0 & 0 \\ \hline \vdots & \ddots & \vdots & \vdots \\ \hline 0 & \cdots & I_m & 0 \end{array}\right] \begin{bmatrix} a(1,n-1) \\ \vdots \\ a(m,n-1) \\ \hline a(1,n-2) \\ \vdots \\ a(m,n-2) \\ \hline \vdots \\ \hline a(1,n-k) \\ \vdots \\ a(m,n-k) \end{bmatrix} + \begin{bmatrix} I_m \\ \hline 0 \\ \hline \vdots \\ \hline 0 \end{bmatrix} \begin{bmatrix} \delta(1,n) \\ \vdots \\ \delta(m,n) \end{bmatrix}$$

$$z(n) = [z(n-1),\cdots\cdots,z(n-m),0\cdots\cdots 0]x(n) + \varepsilon(n) \tag{12}$$

with m × m identity matrix I_m and coefficients $C_i(i=1,\ldots,k)$ which constraints the smoothness of the time evolving AR coefficients determined by order k in Equation (10).

The unknown parameters in the state space model (12) are the orders m, k and the ratio of the variances $\mu^2=\tau^2/\sigma^2$. The last ratio can be interpreted as the trade-off parameter between the (in)fidelity to the difference equation and the (in) fidelity to the data.

Given the state space representation in Equation (12) for the smoothness priors constraints on the time varying coefficient AR model, the fitting of the model to the data z(1),..., z(N) is achieved by maximizing the likelihood

$$\log L(\mu^2|m,k) = -\frac{N}{2}\log 2\pi - \frac{1}{2}\sum_{n=1}^{N}\log r(n) - \sum_{n=1}^{N}\frac{v(n)^2}{2r(n)} \tag{13}$$

by the Kalman filter, specified by the orders m, k and the trade-off parameter μ^2. In Equation (13), v(n) and r(n) denote innovation and the covariance of the state vector.

For the fitted model, the AIC (5) is used, using the maximum likelihood estimate of the parameters of the model which is obtained by maximizing (13). Based on the fitted model, changing cyclical components are estimated from the instantaneous spectral density of a time varying AR coefficient process by

$$p(f,n) = \frac{\sigma^2(n)}{\left|1-\sum_{j=1}^{m} a(j,n)\exp(-2\pi ijf)\right|^2}, \quad -\frac{1}{2}\leqq f\leqq\frac{1}{2} \tag{14}$$

where f denotes frequency, i is imaginary unit and $\sigma^2(n)$ is time varying variance.

In time varying AR coefficient model (10), the changes of the coefficients are considered to evolve gradually and smoothly over time. To represent abrupt or step changes, smoothness constraint of the coefficients which is defined in stochastic vector w(n) in Equations (11) or (12) are removed. That is, when abrupt change occurred at time j, the model assumes that the variance τ^2_{j1} and τ^2_{j2}, which are estimated from time 1 to time j−1 and the rest of time, respectively, are not equal. If AIC obtained from the abrupt change model is sufficiently smaller than that of the gradual changing model, then we consider that the model suggests the structural shift at time j, which reflects the abrupt change of the actual data more naturally.[11]

11. See Appendix II. Also, in Equation (11) we assume the normality of w(n) and ε (n), but there is a possibility that structural changes can be expressed more clearly by assuming non-normality. See Kitagawa (1985) concerning this.

To select the model using the AIC when the abrupt change occurred, we consider three cases. First, the change in the covariance structure is always gradual and there is no steep shift. Second, there are structural shifts and the time of the shifts are known in advance. Third, on the assumption that there is shift in the series, the time, at which the shift occurs, is searched by the model which described abrupt change. In the last case, we proceed on the assumption that there is the possibility of shift at each period in a statistical sense. In this paper, we consider the first and the third case because the time of the structural shift is unknown. The AICs obtained from the fitted model on the third case are compared with the AIC on the first case. One criterion we adopt here to assess these models is the AIC. If the AIC on the third case model is as smaller than the AIC on the first case as the extent of the log value of the number of samples,[12] then we select the third case model.

Traditional approaches via fitting of locally stationary models will also be helpful to evaluate the nonstationary of the covariance structure. Even in this application, the use of the AIC is helpful as a criterion for model selection.[13]

In the following chapters the analysis of the trend and the detrended series of the GNP, money supply and money stock of Japan and the U.S. are shown.

III. Application to Money Supply and the GNP

Friedman (1985) asserted that stability in the economy and a steady reduction of inflation in Japan post-1973 was due to the stable monetary growth, while the unstable monetary policy of the U.S. caused instability in the economy. Such assertion has always been advanced by monetarists and it is also a fact that the observed series in both countries reflected their insistence.[14]

Following is a statistical examination of Friedman's assertion concerning GNP

12. If the number of samples is n, then the probability of choosing the one time at which the shift occurs is 1/n. If this point in time is j, the probability that likelihood L(j) occurs is $p(j)L(j)=L(j)/n$. When considering log likelihood, then the right hand side is $\log L(j)-\log n$. On the other hand, if we assume the probability without any shift as $p(0)=1$, then log likelihood becomes $\log L(0)$. So, we consider that, when $j=0$, difference is about $-\log n$. In this regard strict statistical procedures will be required which take into consideration concerning the search for outliers. In this paper we use the values introduced above as a yardstick.

13. See Appendix 3.

14. In Japanese case, studies regarding stable money supply and the stability of GNP growth are shown in Oritani (1981), from the point of "The negative effect of inflation on the economic growth rate", and Okubo's (1983) causality analysis. Based on their works, Suzuki (1983) furthermore discusses the effects of money supply on prices and the business activities. In the U.S., on the other hand, the disputes of real business cycles are flourished.

and money supply of the two countries using the stochastic trends and time varying AR coefficient models.

1. Japan's Money Supply and GNP

(1) Changes in trends and in detrended series

Trends of money supply (M_2+CD average balance), nominal and real GNP are shown in Figure 1.[15] In order to estimate the trend, the rate of change over the previous year is used.[16] This figure illustrates that, after the rate of change in M_2+CD showed great increase around 1972–74, it declined gradually along with the declining of the growth rate of the nominal GNP. However, the growth rate of real GNP has not declined since 1975.

The deviation around the estimated trend, that is, the detrended series of the rate of change[17] is shown in Figure 2. Before 1975, M_2+CD showed a wide range of fluctuation and also the fluctuations in the nominal and real GNP are great and unstable. However, following around 1975, fluctuations in the rate of change of M_2+CD have become small and, at the same time, the similar pattern can be seen in the growth of nominal and real GNP. These tendencies suggest a structural change.

The period of the structural change is one of the major concerns to review or to foresee economic activities and economic policies. The structural change may arise from institutional, organizational or some economic situation. We try to analyse it here from the change of the covariance structure using time varying AR coefficient model. Estimated AICs obtained from the time varying AR coefficient models applying to the detrended series of M_2+CD, nominal and real GNP are shown in Figure 3. A straight line parallel to the horizontal axis in Figure 3 is AIC of Case 1 and broken line is AIC of Case 2 in Table 1, respectively. Model of Case 1 assumes a

15. Order k in stochastic difference Equation (1) and p in autoregressive process (4) are given respectively from 1 to 3 when estimating the trends. The results we adopted here are based on the minimum AIC. This is the same in the following unless we stipulate otherwise. Orders (k, p) of M_2+CD, nominal GNP, and real GNP on Figure 1 are respectively (2, 2), (2, 3), (3, 2).

16. The method adopted in this paper should be applied to original series which include seasonal factors. It may not be appropriate to apply it to the rate of change over the previous year in the strict sense. But here, we use the rates of change according to established usage and they are observed attentively in actual survey. The periods of the data are 1968 I-1984 IV for M_2+CD, and 1966 I-1984 IV for nominal and real GNP. Also, Friedman (1985) uses the average of end of quarter. In this paper we use average balance to smooth irregular movements. The latter trend does not greatly diverge from the former.

17. The autoregressive component plus irregular component estimated from Equations (2) and (3). The same is in the following.

Figure 1 Rates of change of Japan's M_2 +CD and GNP over previous year and their trends

------ rate of change over previous year
——— trend

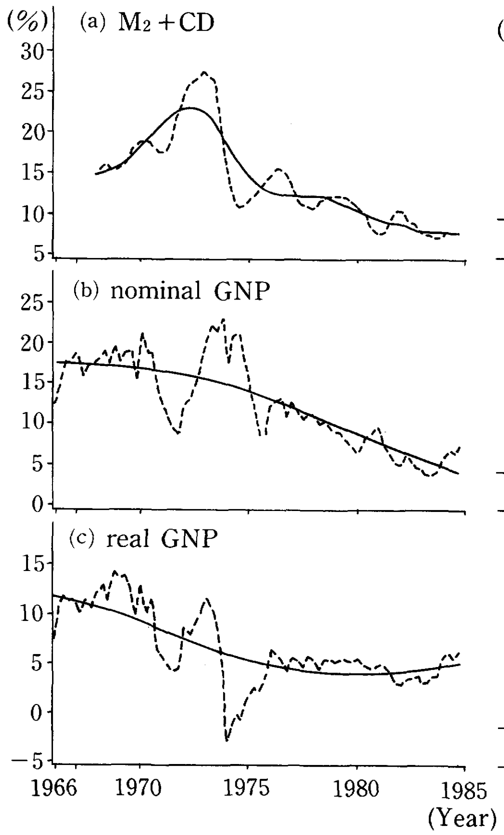
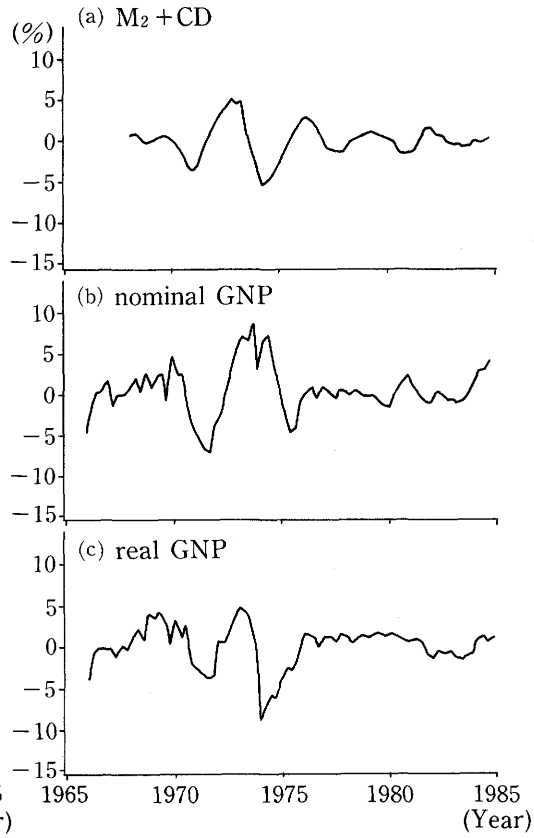


Figure 2 Detrended series of Japan's M_2 +CD and GNP

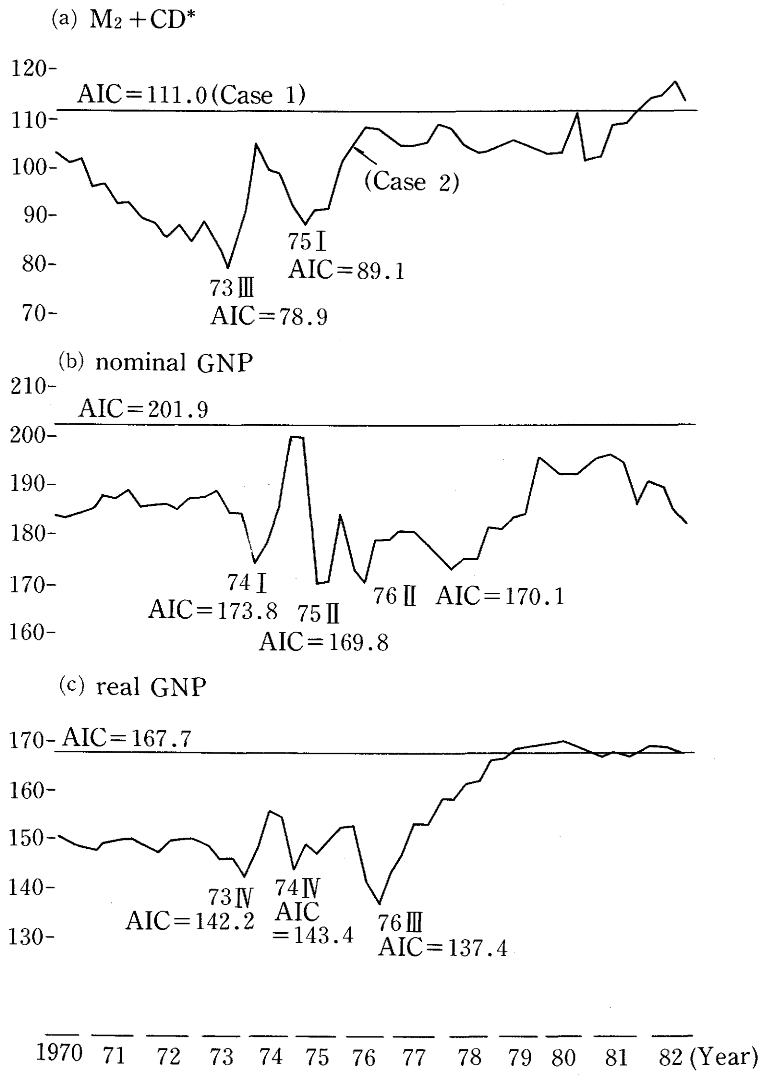


gradual change in the covariance structure and models of Case 2 assume a shift at each period.[18] Figure 3 implies that if AIC in Case 2 is sufficiently smaller than that of Case 1, the models in Case 2 reflect the actual movements more clearly than the model of Case 1 and minimum AIC model in Case 2 suggests[19] the period of step change in the covariance structure.

18. AICs of Case 2 in Table are shown in the 1970–82 period for considering to observe the post-70's movement.

19. To compare AIC in Case 1 and Case 2, we consider the log of the number of samples (See note 12).

Figure 3 AICs estimated from time varying AR coefficient models (see Table 1)



* In charts (b) and (c), and hereafter, the indication of cases 1 and 2 are left out.

Observing each series in Figure 3 from above point of view, the AICs of Case 2 in each period in 70's are lower than the AIC of Case 1, especially in mid-70's. These facts imply that substantial change has occurred in the covariance structure. For further details, the AICs of money supply in Case 2 are smaller in the period 1973–75, and those of nominal and real GNP are smaller in 1973–76 relative to other periods and than the AICs in Case 1. When we notice the minimum AIC, nominal and real GNP are seen in 1975–76, lagged after M_2+CD of which minimum AIC can be seen in 1973–75. These results suggest the periods of big change in the features of the fluctuations.

(2) The statistical features of detrended series

On the basis of the previous results, time evolving statistical features can be more fully expressed by the instantaneous spectral density using the fitted models. As illustrative examples, the figures of changing spectral analyses can be helpful to discern the evolving cyclical components.

Figure 4 shows the changing cyclical component based on the fitted model selected by the AIC.[20] The horizontal axis represents the cycle (in the figures scale is shown as frequency: the inverse of the cycle) and the vertical axis shows the relative strength of the cyclical components (energy of power spectrum). The individual curves which are laid on a time axis are estimates of the log power spectral density versus frequency using the time varying AR coefficients and variances. The results of the estimated model are more fully expressed to reveal changing phase of cyclical components. Figure 4 portrays that since 1973 power of the cyclical components of money supply have moved with stability in contrast to changes in pre-1973 and the peak frequency decreases smoothly. More close inspection reveals that recently not only the longer term cycles (around 0.1 frequency (about 10 periods)) but also medium and shorter term cycles (around 0.3–0.4 frequencies (about 2.5–3 periods)) have been stronger in some degree though the changes are gradual. On the other hand, the powers of nominal and real GNP decrease greatly after the time of shift, and thereafter, the fluctuations become smaller.

Figure 5 shows changing variance. The models used are the same as those used in Figure 4. The variance of M_2+CD and of real GNP tend to decrease after the shift period though the variance of nominal GNP does not decrease greatly after shift period.[21]

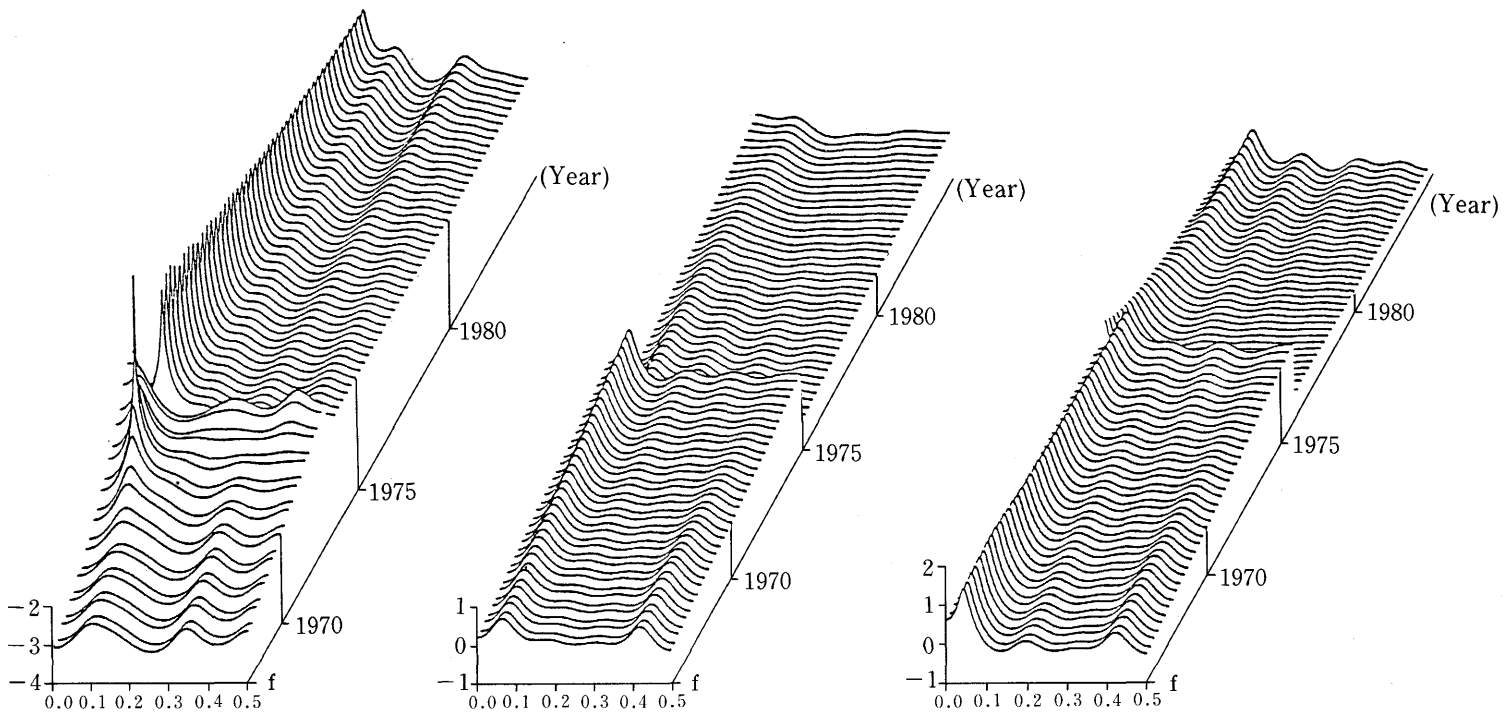
Above results demonstrate that after the Bank of Japan began to watch the money supply as an intermediate target in the mid-1970s, there occurred a structural shift in the direction of the stable monetary growth and, after that, nominal and real GNP shifted structurally in the direction of stabilization.

20. Figure 4 was based on the model that a shift occurred only once in a period. However, as can be seen in Figure 3, there are other periods with relatively smaller AIC. When we assume there were shifts in two periods, AIC of M_2+CD does not become so small, and with regard to nominal and real GNP, the periods which indicate smaller AIC are so close to that we showed the figures as examples that big shift occurs only once.

21. To modulate the relatively fast wiggles of a nonstationary covariance time series, data are transformed by the envelope function. See Appendix II.

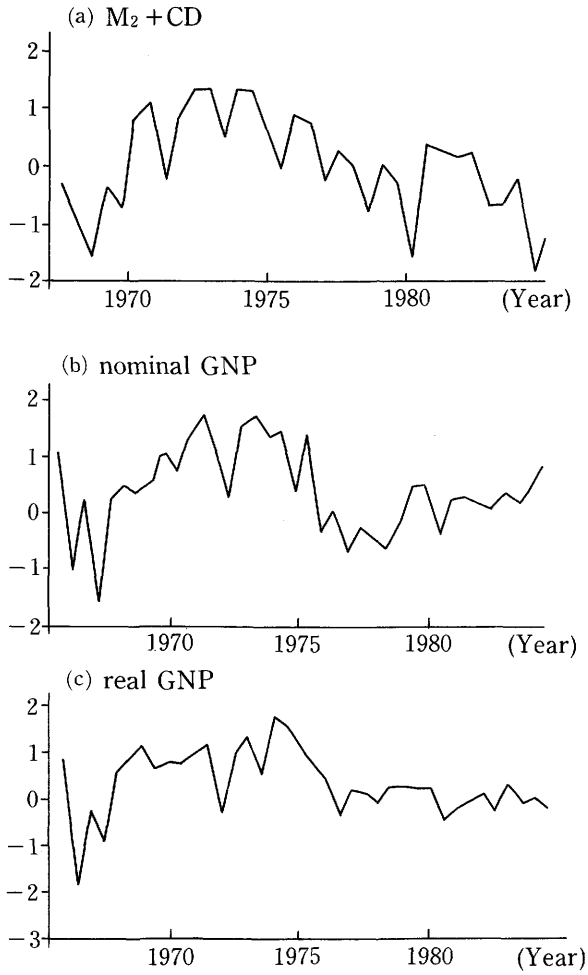
Figure 4 Changing cyclical component

(a) Japan's M_2 +CD (1973Ⅲ) (b) Japan's nominal GNP (1975Ⅱ) (c) Japan's real GNP (1976Ⅲ)



Note: 1. Period in parenthesis shows assuming shift period estimated from the model.
2. Fluctuation of M_2+CD seems to be larger than GNP because the scale of power is smaller.
3. Horizontal axis shows frequency.

Figure 5 Changing variance



Note: Vertical scale shows the power of ten.

2. The U.S. Money Supply and GNP

(1) Changes in trends and in detrended series

Bearing in mind above results of money supply and GNP in Japan, let us look at the U.S. series. To measure of money, M_1 is assigned greater importance in the U.S. However, noting the correspondence with Japan, M_2 is examined at first, then M_1 is treated. The sample period is the same as in Japan's case.

Figure 6 shows the rate of change over the previous year of M_2, nominal and real

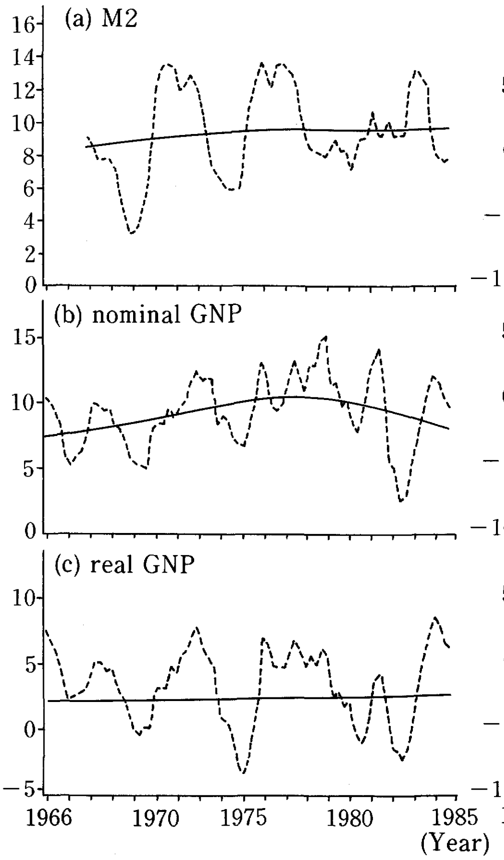
GNP and their trends. The detrended series are shown in Figure 7.

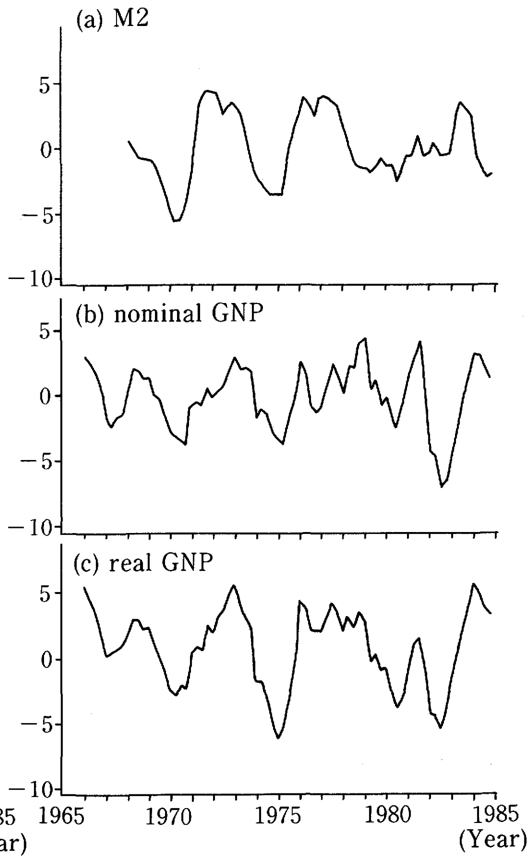
Comparing with Japanese series, it can be said that the changes of trends and of detrended series are different from those of Japan. The trends of the U.S. M_2 and real GNP continue to move broadly stationary, while the trends of Japan's monetary and real GNP growth rate changed from high growth to low growth as a whole. As to the movement around the trends, each series continues to exhibit wide range fluctuations. It is also contrast to Japanese detrended series which have decreased the range of fluctuation since mid-1970s as seen in Figure 2.

Figure 6 Rates of change of the U.S. M_2 and GNP over previous years and their trends

Figure 7 Detrended series of the U.S. M_2 and GNP

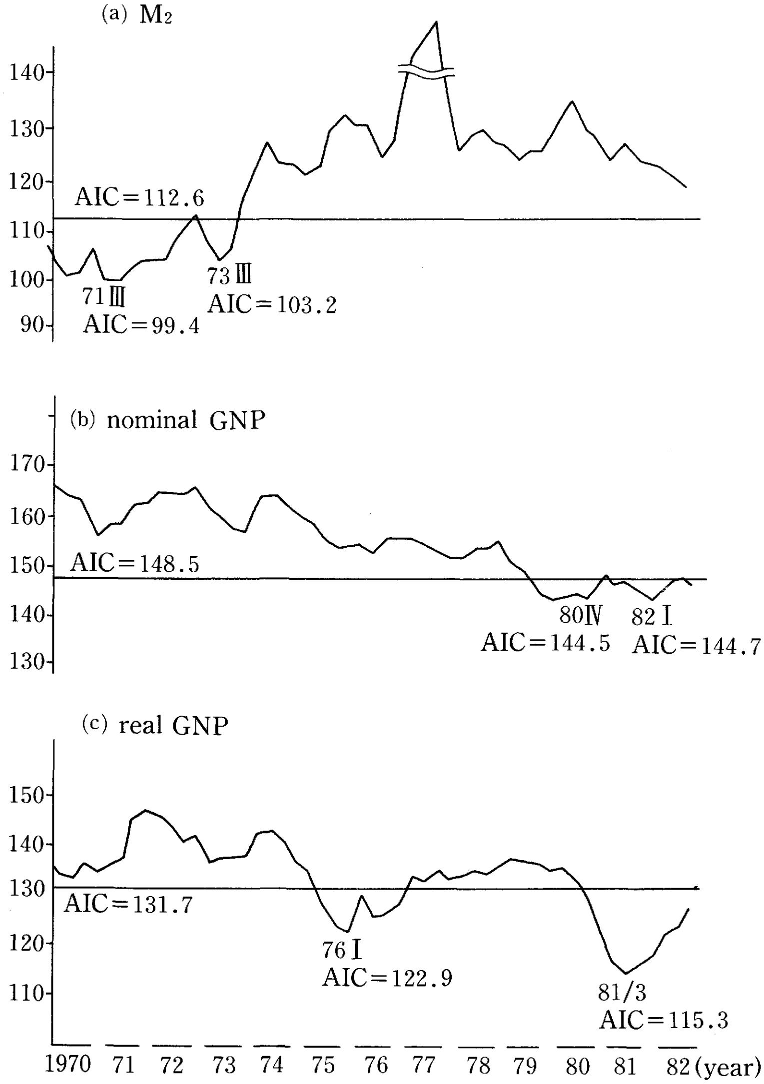
------ rate of change over previous years
——— trend





The AICs of the fitted time varying AR model are shown in Figure 8. The AICs indicate that after big change occurred in M_2 in the first half of the 1970's, the changes of fluctuations seem to be gradual after the mid-1970s. Also, the AICs of nominal GNP imply in general, that the change does not occur so abruptly though relatively big changes seems to rise after 1980. On the other hand, big changes can be seen in real GNP in 1975–76 and after 1980.

Figure 8 AIC estimated from time varying autorègressive coefficient models (see Table 2)



(2) The statistical features of detrended series

Changing cyclical components and changing variance are shown in Figures 9 and 10, respectively. As the AIC in Figure 8 implies, M_2 shows the big change in the covariance structure in the first half of the 1970s. However, the variance of M_2 in Figure 10(a) depicts similar pattern even in the first half of the 1970s. This indicates that larger fluctuations around the trend have been continuing. In the late 1970s, the variance of M_2 has been likely to begin to reduce, but in the 1980s it increases again. Thus, it may be said that M_2 in the U.S. did not shift in the direction of stability as seen in Japan's M_2+CD.[22] Meanwhile, as seen in Figure 9(a) the cyclical components of M_2 shows the strong power in the longer cycle term (around 0.08 frequency (about 12 periods)) at the early 1970s. However, after 1980 the medium term cycles around 0.12 frequency (about 8 periods) become gradually stronger. Moreover, after the late 1970s shorter term-cycles (around 0.4–0.5 frequencies (about 2 periods)) also become stronger.

In contrast, nominal GNP does not reveal great changes in the pattern of fluctuations as seen in Figure 9(b) but Figure 10(b) imply a gradual increase of variance. It is also opposite tendency to the decrease of fluctuation as seen in Japan's nominal GNP. While, the cyclical components of real GNP shows great changes before and after the shifts in the two periods as seen in Figure 9(c), and Figure 10(c) suggests the variance of real GNP also showing increasing tendency.

Above results show more clearly that there seemed to be no structural changes in the direction of stability of money supply in the U.S., and the range of fluctuations with regard to actual economic activity also remains bigger. In this respect the results also accord well with Friedman's argument.[23]

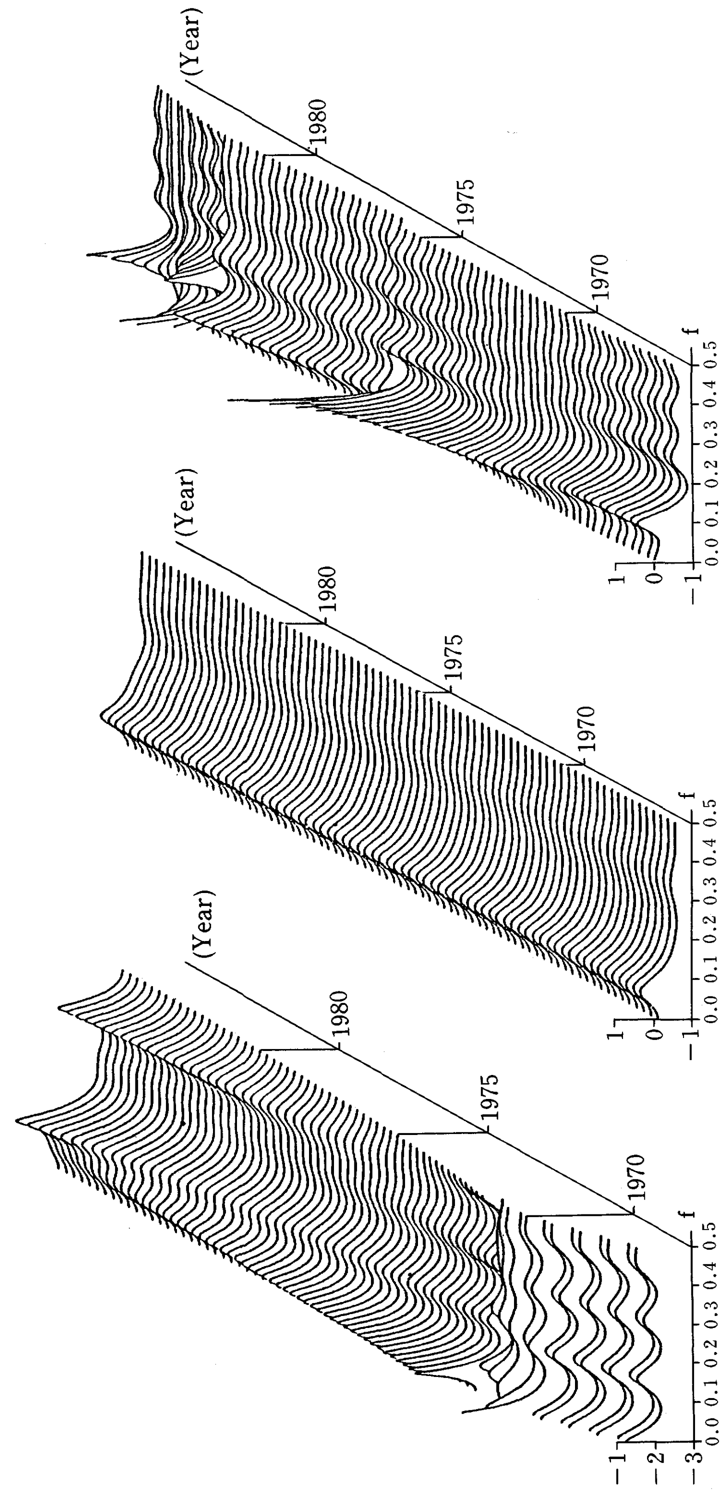
22. As seen in Figure 5, variance of Japan's M_2+CD has declined to below 10^{-1}. But the variance of the U.S. M_2 remains greater than 10^{0} though once the variance decreased at the beginning of 1980s. It indicates that the fluctuation of money supply is larger than Japan.

23. "In so far as U.S. policy can be said ever have to been monetarist, it was so solely in rhetoric, never in performance. Since the Fed adopted temporarily the rhetoric of monetarism in 1979, monetary growth has been more unstable than in any other postwar period of comparable length." (Friedman (1985)).

Also Bomhoff (1983) states concerning the U.S. money supply between 1979–82 as "... in retrospect it is clear that the Federal Reserve never adopted the monetarist proposal for a gradual and planned elimination of inflation, but opted instead for policies that were much more variable and unpredictable than before. Policymakers found the "monetarist" label convenient, both because it absolved them from direct responsibility for the high interest rates that could be caused by a more restrictive policy, and because it would help to deffuse any criticisms from Karl Brunner, Milton Friedman and Allan Meltzer, the three best-known independent experts on U.S. monetary policy and three advocates of planned money growth." (p.219) See, also Meltzer (1986).

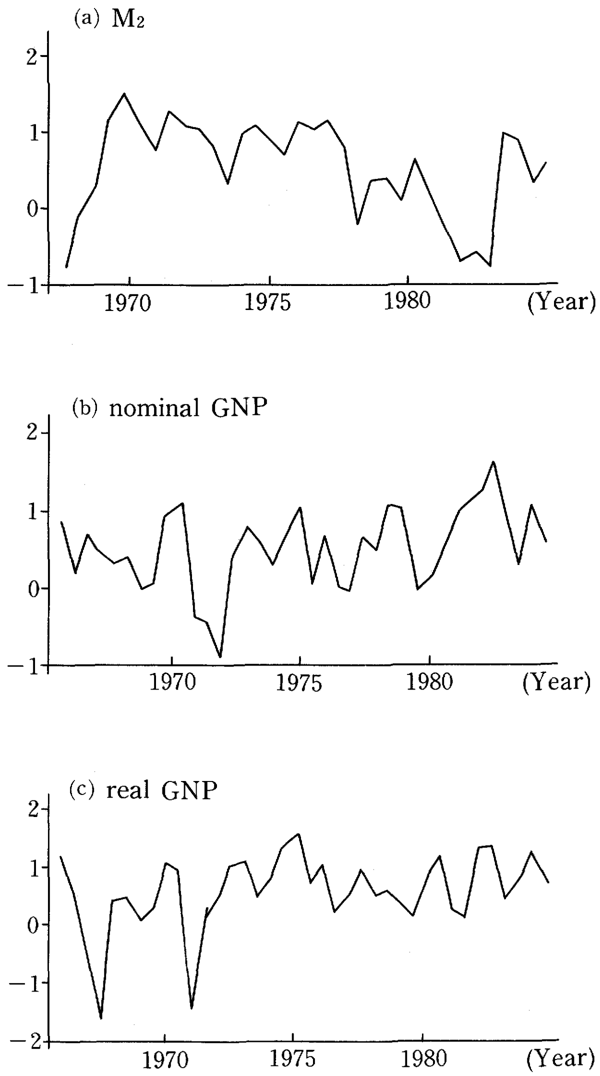
Figure 9 Changing cyclical component

(a) The U.S. M_2 (1971 III) (b) The U.S. nominal GNP (c) The U.S. real GNP (1976 I , 1981 III)



Note: See Notes of Figure 4.

Figure 10 Changing variance

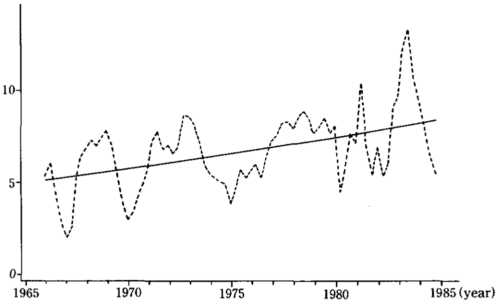


(3) Changes in the U.S. M_1

As a measure of the U.S. money, M_2 was examined in the previous section to compare with Japan. Here, we try to observe the U.S. M_1 movement which is assigned greater importance in the U.S. and used widely in empirical analyses.

The estimated trend and the fluctuations around trend are shown in Figure 11. The trend of M_1 continues to increase monotonically and the change in the trend itself is not likely to appear. Also, the movements around the trend continues to

Figure 11 Rates of change over the previous year of M_1 and its trend (a) and detrended series (b)



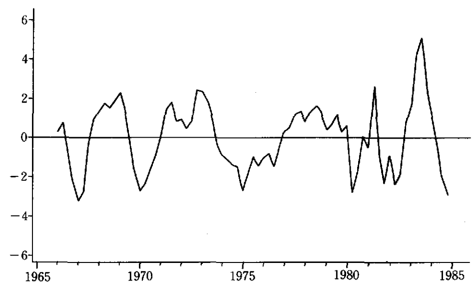


Figure 12 AIC of the U.S. M_1 estimated from time varying AR coefficient model

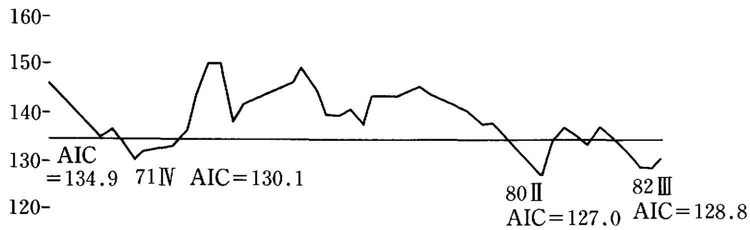


Figure 13 Changing variance

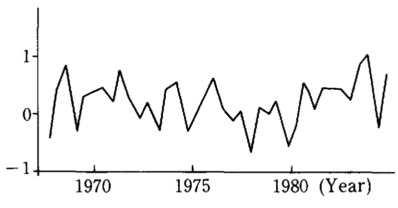


Figure 14 Changing cyclical components

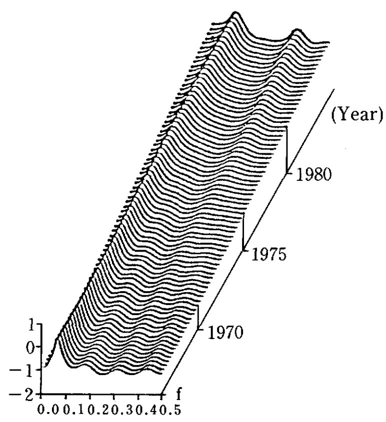


exhibit wide range fluctuations and its range seems to increase gradually in recent years. The AIC in Figure 12 implies that the change of the covariance structure is not so remarkable in 1970s, but some bigger changes are suggested after late 1970s. The change in the variance in Figure 13 indicates that the variance continues similar phase which implies that fluctuations of M_1 continues throughout. Accordingly, the movement of cyclical components shows similar pattern, but it may be noticeable that the power of the longer term cycles (around 0.1 frequency (about 10 periods)) and the shorter term cycles (around 0.3–0.4 frequencies (about 2–3 periods)) are gradually increasing especially after the beginning of 1980s which the AICs in Figure 13 suggested. The changes of the U.S. M_1 seems to be larger than the U.S. M_2.

IV. Application to Marshallian k

Taking the results in III. into consideration, let us observe the fluctuation of Marshallian k in Japan as one indicator to observe a relation between Japanese monetary and real economy. The movement around the trend especially noted to evaluate the level of liquidity in the national economy. In general, the ratio of money to nominal income is used. We attempt to use here the sum of nominal aggregate demand and the intermediate transactions. Thus, Marshallian k adopted here can be expressed as the ratio of M_2+CD to nominal aggregate demand plus intermediate transactions.[24]

The sample period is from the first quarter of 1967 to the second quarter of 1985. The series are seasonally adjusted. The estimated results are shown in Figures 15 and 16. The trend has been rising right along and in recent years it shows more upward tendency. When the trend is estimated by a linear deterministic function, time evolving change of the trend cannot be examined, but only fitting the local linear trends by separating the sample periods are examined. The estimated stochastic trend reveals that the trend analysis is also important in addition to the analysis of the deviation from the trend. As seen in Figure 15(b), the trend itself shows distinct upswing post-1980, which suggests a need for the trend analysis as to whether this reflects a change in the economic structure. Around the trend, a cyclical fluctuation throughout the sample period can be seen. To interpret this behavior, further examination will be interesting following the analysis of the movements of both money and actual economy, but here we shall look into it mainly from the statistical point of view.

Figure 16 shows that the fluctuation seems to be smaller after 1975. In fact, the fitting result of time varying AR coefficient model in Figure 17 implies that at the

24. The reason for the use of the nominal aggregate demand and intermediate transactions is described in the special paper of the Bank of Japan (1983).

Figure 15 Marshallian k and its trend

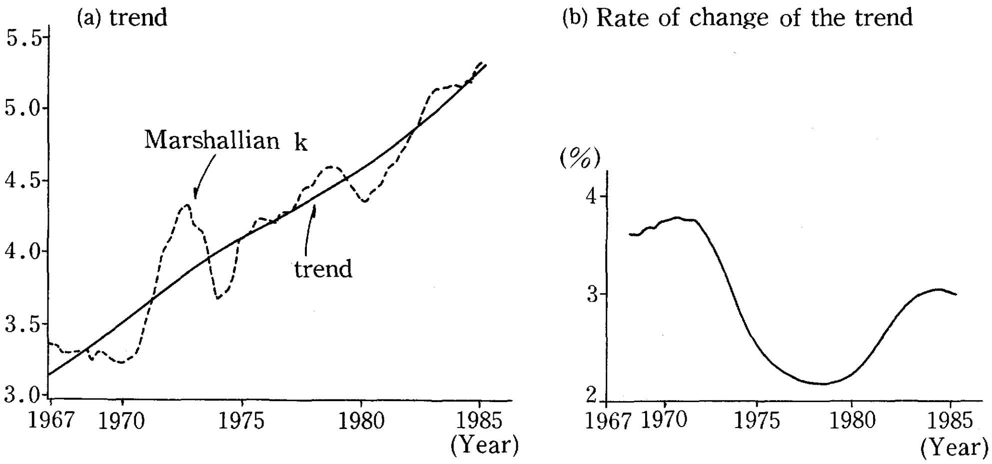


Figure 16 Deviation from the trend of Marshallian k

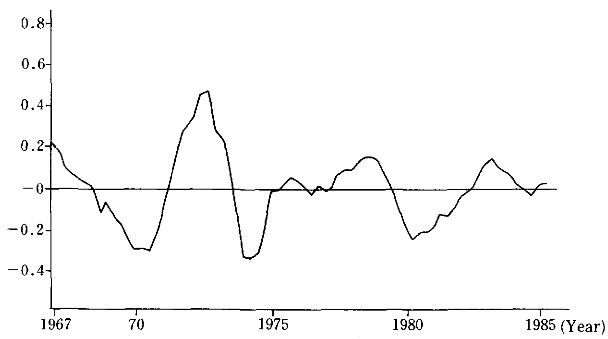
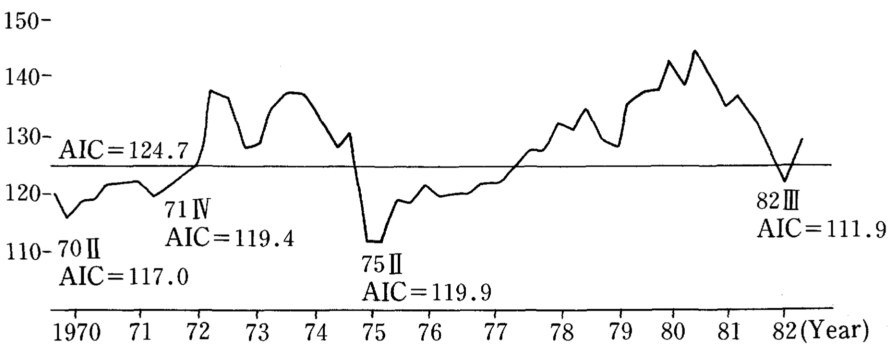


Figure 17 AIC estimated from the time varying AR coefficient model of Marshallian k (see Table 3)



beginning of the 1970s there seemed to occur relatively big change, but the biggest change arose around 1975,and except for those two periods, AIC has been relatively stable.

Changing cyclical components are illustrated in Figure 18. The estimated model is considered a shift occurred in the second quarter of 1975, when the AIC became minimum since 1970. Power of the deviation becomes evidently smaller after the shift, and subsequently, stable movement can be observed. The changing variance shown in Figure 19 tend to behave almost in similar pattern since the time of the shift, but the AICs in Figure 17 suggest some bigger changes after 1980. One example is seen in 1982. The AIC in Case 2 becomes smaller than that of Case 1 in the third quarter of 1982. This tendency appears in longer term cycle in Figure 18. In this regard, also, further examination will be interesting taking into consideration such as the effect of financial innovation in progress or liquidity in the economy. Assuming there have been two step changes, inferring from the AIC of Case 2, two shift change models are estimated. The AICs of the fitted models are shown in Case 3 in Table 3. The smaller AIC implies that the periods are the fourth quarter of 1971 and the second quarter of 1975. But, the AIC of this Case (113.6) is greater than the minimal AIC in Case 2, which suggests that there were no two big changes in the 1970s.

The results of the Japan's Marshallian k also indicate that in the mid-1970s there

Figure 18 Changing cyclical components of Marshallian k (a shift assumed in 1975 II)

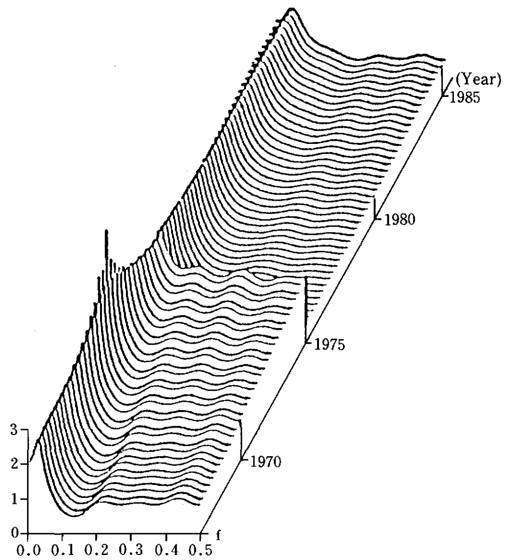
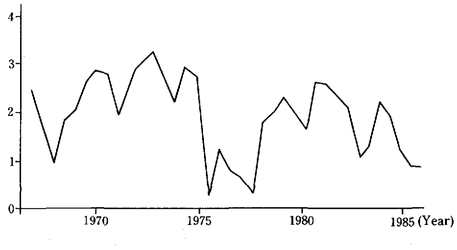


Figure 19 Changing variance



is likely to be a structural shift in the direction of stability as observed in money supply and real GNP.

V. Application to the Real Money Stock

There have been a number of issues regarding the money stock stability through the arguments of the demand for money function of which various approaches are considered.[25] To argue the factors for the stable real money stock, empirical and theoretical approach about specifications have to be taken, and to corroborate by actual data, multivariate model will be helpful. In this section, however, we shall not treat with these problems further but pay attention solely to the statistical features of the real money stock behavior.

1. M_1 and M_2+CD in Japan

In Japanese case, Tsutsui-Hatanaka (1982) showed that, first, the real money stock M_1/P (P denotes GNP deflator, hereafter the same) seems to exhibit remarkable similar behavior to the missing money in the U.S., though as a whole it is more stable compared with M_1/P in the U.S., and the behavior suggests a structural change. Second, $(M_2+CD)/P$ is more stable than M_1/P. Hamada-Hayashi (1983) also state that they noticed a change in the trend of M_1/P and M_2+CD/P around 1973 and 1974, which is suggestive of a structural change and after those periods movement become stable. On the other hand, Ishida (1984) examined both the simple sum and Divisia index of money supply and indicated that the instability of money could be seen after 1977 in the simple sum by contrast with the Divisia index.[26] Kama (1986) also shows the instability of M_2+CD than M_1 after 1977. Thus, there is divided assertion on whether there was a shift after 1974.

In the meantime, there are arguments concerning the method used in the examination of a structural shift. Tsutsui-Hatanaka used the confidence belt ,[27] and Kama attempts to use the method proposed by Brown,Durbin and Evans (1975). However, the Chow-Fisher test is used in many cases. Regarding this, for example,

25. See for example, Tsutsui and Hatanaka's survey (1982) concerning Japan's demand for money function and also see Komura (1985).

26. According to Ishida (1984), the year 1977 was the period when the prediction error of demand for money function using the Divisia Index became larger.

27. Hatanaka (1974)

Cargil (1985) points out that this test is not always appropriate for the examination of cumulative movements over a long period of time, particularly in the case of Japan, where gradualism is generally accepted. Cargil proposes the application of a time varying parameter model to examine the structural change.[28]

Bearing the above points in mind, we shall now examine the statistical features of the real money stock using a time varying AR coefficient model.

(1) Trend and the fluctuations around the trend

The estimated trends of M_1/P and M_2+CD/P and their rates of changes over the previous years are shown in Figures 20 and 21, respectively. Sample periods are from the first quarter of 1965 to the fourth quarter of 1984 for M_1/P, and from the first quarter of 1976 to fourth quarter of 1984 for M_2+CD/P. The figures show that the upward tendency of M_1/P began to slow down after 1973, and in 1974 the rate of change became negative, which indicates that there was a prominent change in the trend around 1973–74. Further in 1980 the rate of change became negative again which also shows a big change in the trend.[29] On the other hand, the rate of change of the trend of M_2+CD/P, began to decrease since 1975 and became negative in 1976. But, thereafter, the rate of change increased and continued a stable movement that is different from the M_1/P.

The trend of both M_1/P and M_2+CD/P change rather bigger around 1973–74 that corresponds to the period of structural change according to previous studies. However, as for the change in the trend of M_1/P seen in 1980, further examination will be necessary whether it was a structural change or not .[30]

The detrended series of M_1/P and M_2+CD/P are shown in Figure 22.[31] In the figure, M_1/P shows that there is a shorter term fluctuation, while M_2+CD/P has moved stably with a smaller fluctuation after the bigger cycle fluctuation in the 1970s.

The AIC estimated from the time varying AR coefficient model is shown in Table 4. At first glance, the AIC of M_1/P suggests that there have been big changes in 1974 and after 1980 as seen in Case 2 of Table 4(1). These years nearly correspond to

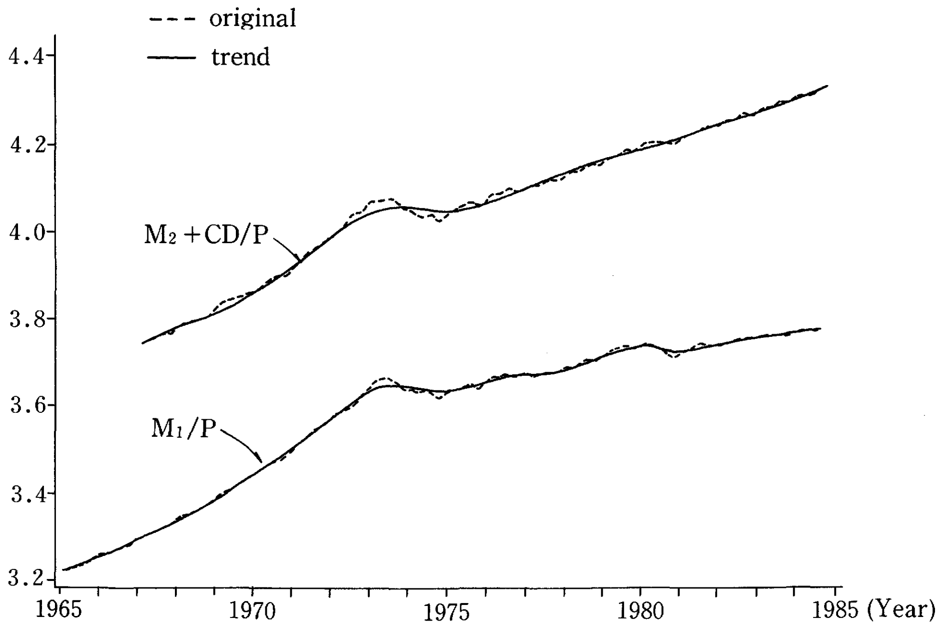
28. Cargil's indication is concerning the demand for money function. In this respect, the analysis shown in this paper can be regarded as a preliminary study of the issues.

29. The year 1980 is said to be a period when the velocity of circulation of M_1 changed. In 1980 trend began to rise.

30. There is a view that M_2+CD shows similar behavior when applying the Divisia index (see, for example, Ishida (1984)).

31. Also seasonally factors are removed to stress fluctuating movements. Seasonal factors are estimated from the method mentioned in II (2).

Figure 20 Trends of M_1 /P and M_2 +CD/P in Japan



* estimated by log value.

Figure 21 Rate of change of the trend over the previous years

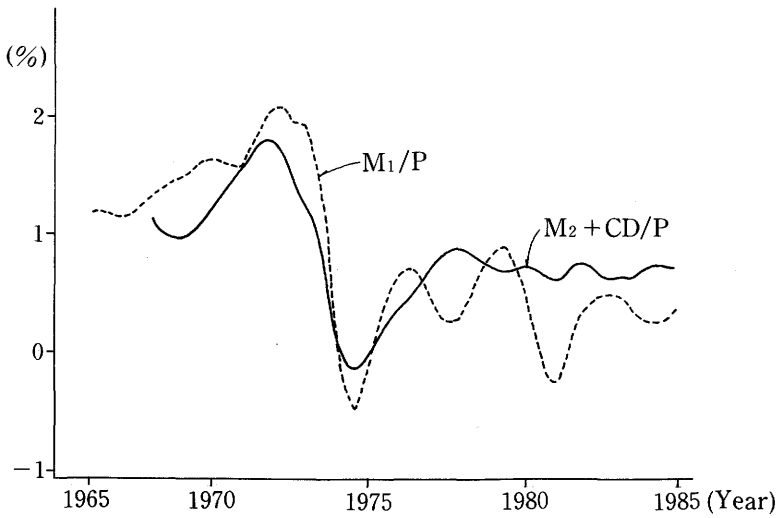
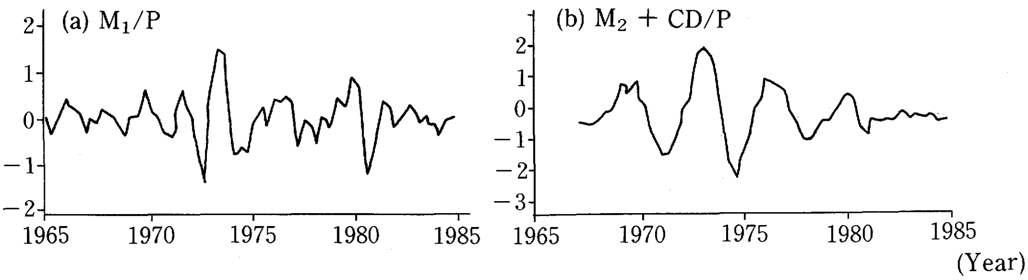


Figure 22 Detrended series of M_1 /P and M_2 +CD/P



* Both figures are 100 times as large.

the period of big changes in trend as seen in Figure 21 when the rate of change of the M_1/P was negative. Likewise, the AICs in Case 3, estimated on the assumption that there were two step changes and the periods of changes were selected from the smaller AIC in Case 2, imply changes after 1980 are also rather bigger. As to M_2+CD/P, the AIC of Case 3 are greater than the minimal AIC in Case 2, which suggests that there was no big shift after 1974.

(2) Statistical features of detrended series

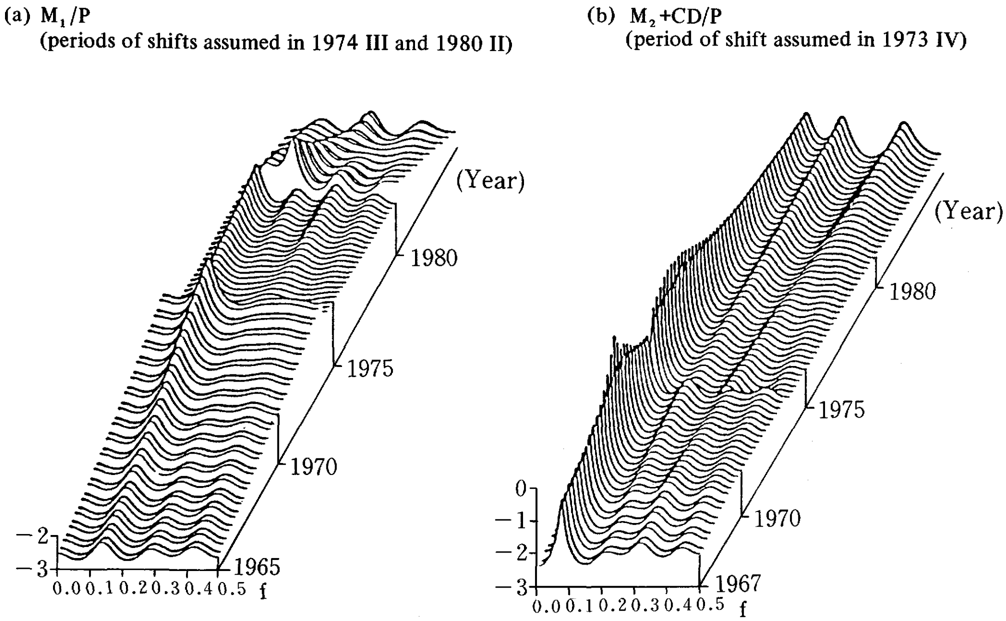
Looking at the changing cyclical components in Figure 23, a distinctive feature for M_1/P is that a big change occurs even after 1980 by comparison with the periods before that year. As the sample periods after 1980 are so short that we cannot conclude definitely. Thus, let us examine it by another way.

When looking at detrended series of M_1/P in Figure 22(a), the fluctuations seem to be exceptionally larger around 1973 and 1980 than other periods. It may not be so strange that we assume the behavior in those two periods are unusual caused by some temporary shocks and except for these periods there were no structural changes as a whole. One way is to apply a locally stationary model, at first, to the whole period, after excluding outliers which are the periods of unusual behavior. Next, after excluding outliers, the series are divided into three periods where dividing points are the periods of excluding outliers. Then, to each period the locally stationary models are fitted. If the covariance structures are not changed before and after the shifts, the stationarity of the fitted model will be similar. For the divided periods we select; (1) from first quarter of 1965 to first quarter of 1972, (2) from first quarter of 1974 to fourth quarter of 1979 and (3) from third quarter of 1980 to fourth quarter of 1984.[32]

The results show that when the model is fitted to the whole period, from the first

32. As to locally stationary AR model used here, see Appendix 3.

Figure 23 Changing cyclical components



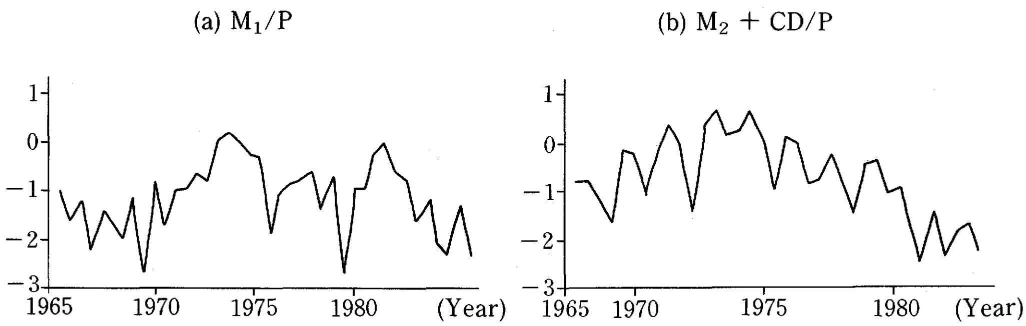
quarter of 1965 to the fourth quarter of 1984 excluding outliers, the AIC became −114.4. While when the model is fitted separately to each three periods, the AIC became (1) −37.8, (2) −38.0, (3) −42.2, respectively. On the latter case, the sum of the AIC of the three periods is −118.0: This result suggests that the fitted locally stationary model explains better when the series were divided, which implies that it is better to think that there were different structural characteristics in the divided periods rather than that there was not structural change as a whole.[33]

As to the changing variance of M_1/P and M_2+CD/P, decreasing tendency can be seen as a whole since around 1973–74 (Figure 24). A closer look, however, reveals that M_2+CD/P in Figure 23(b) does not show big change as M_1 after 1974.[34] However, it is noticeable that there are signs of big change such as the gradual expansion of

33. As outliers are chosen by visual inspection, there remains some lack of strictness on statistical criteria.

34. If there are structural changes in M_1, then M_2+CD which include M_1 will be affected. In this regard, there are views that such changes are hard to come to the fore in M_2+CD because "the share of high-liquid money, such as cash and demand deposits, is exhibiting a declining tendency" (Bank of Japan, special paper (1985)). Okubo (1983) also traces the differences in the movements of M_1 and M_2+CD after 1980 from the standpoint of nominal transaction demand in the actual economic activity.

Figure 24 Changing variance



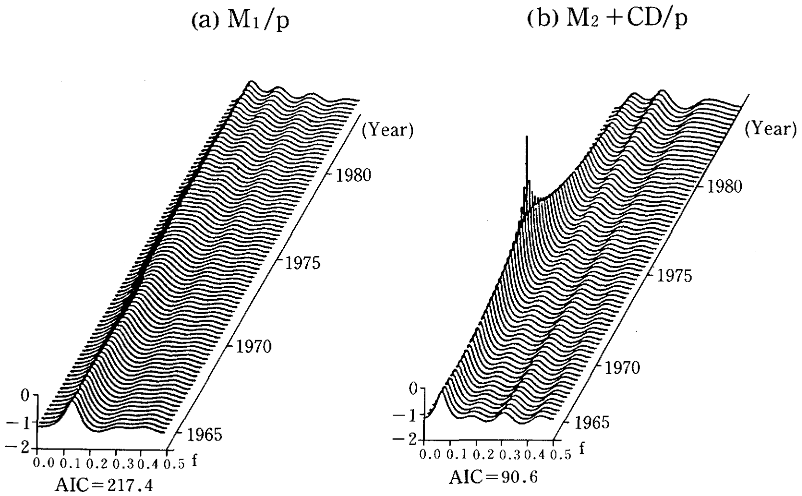
the power in the shorter or medium terms cycles in post-1980.[35]

2. Changes by Type of Money

To see more detail of the behavior of Japan's real money stock, the behavior of money stock by type are examined.

The rates of changes of the trends over the previous year of cash, deposit currency, and quasi-money+CD are shown in Figure 25. Series are real base respec-

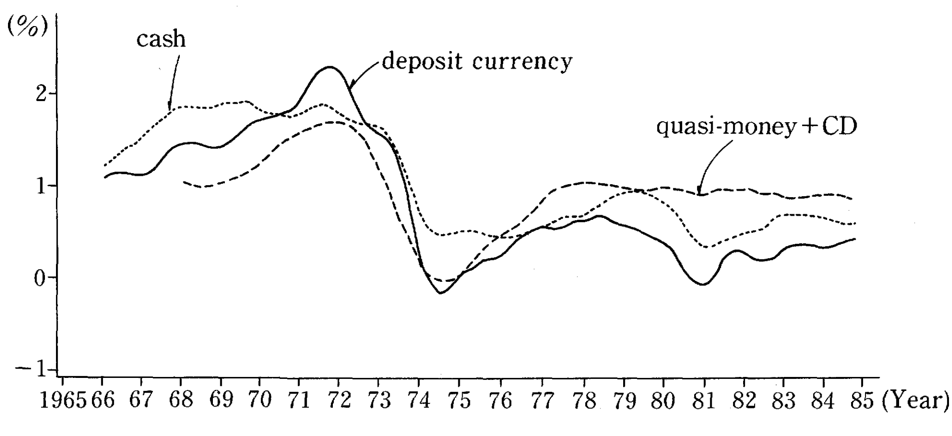
35. As a reference, the changing cyclical components of M_1/P and M_2+CD/P when the model assumed no shift, are shown below. The AICs of the models shown in Figure 23(a) and (b) are larger by 42.4 to M_1/P and by 19.6 to M_2+CD/P. Thus, the fitted models do seem to be much inferior to those which assumed there were shifts (see Table 4).



tively. In each series, great decrease are seen between 1972 and 74, but after 1975, each of them shows different movement. The rates of changes of cash and deposit currency continued to increase after 1975 but declined in 1979–80. Especially, the deposit currency growth becomes negative in early 1980s. In contrast, the rate of change of quasi-money+CD increase after 1975, but since then it becomes stable. The decline of quasi-money+CD in 1980 is smaller than that of cash or deposit currency. As to the detrended series shown in Figure 26, the fluctuations in the 1980s have become smaller on the whole than the big fluctuations in the 1970s. However, the movements of quasi-money+CD in the 1980s do not seem to be similar to those of cash and deposit currency.

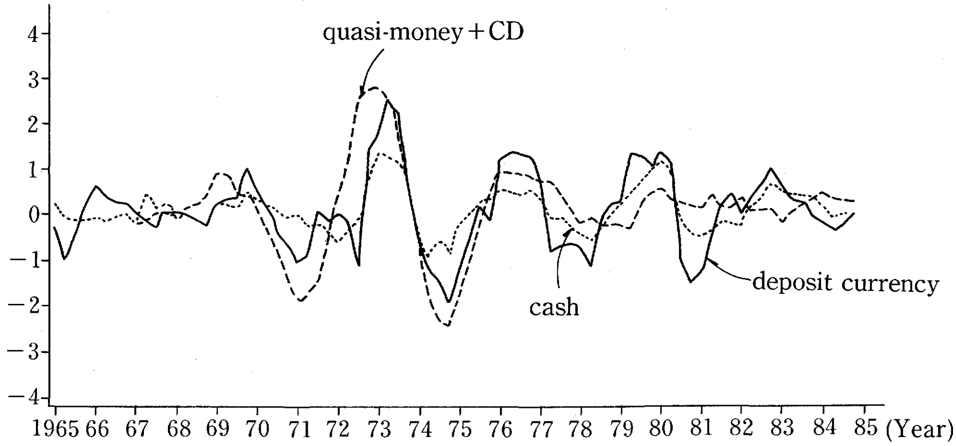
In fact, the results of the fitting of time varying AR model reveal, as shown in Table 5, that cash has shown a big change around 1975, and also a sign of change in 1980 and deposit currency has shown shifts at two periods in 1973 and 80. On the other hand, quasi-money+CD has changed around 1975, but the changes are not always clear after 1980. When observing more closely, the trend of quasi-money+CD shows a slight declining tendency after 1980 as seen in Figure 25, and also the fluctuation of the detrended series is somewhat different from the one before 1980 as seen in Figure 26. In this point, further analysis will be necessary about the behavior in the 1980s.

Figure 25 Rate of change over previous years of the trends of M_2 +CD by type of money



* Estimated by log value.

Figure 26 Detrended series in M_2 +CD by type of money



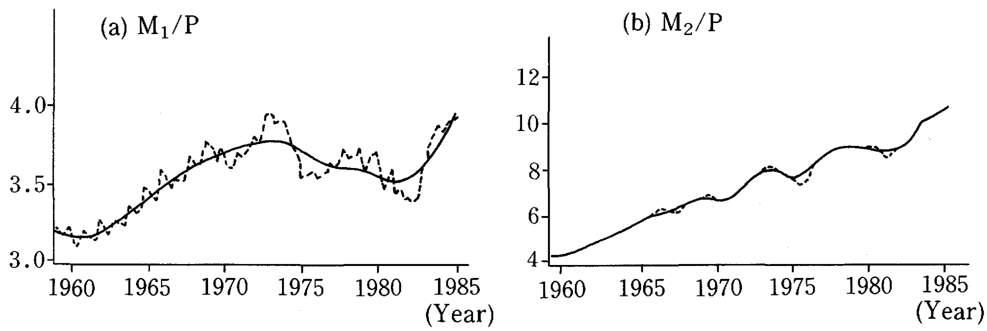
* Figures are 100 times as large.

3. M_1 and M_2 in the U.S.

There have been many studies by means of the demand for money function on the movements of real stock of the U.S. M_1, on which the U.S. monetary authority has put more emphasis nowadays. In recent debates, the attention is paid especially on the instability after 1980.[36] Here we shall examine the statistical features of the U.S. M_1/P and M_2/P. Sample period is from the first quarter of 1959 to the first quarter of 1985.

The estimated trends are shown in Figure 27. The trend of M_1/P had been on the

Figure 27 US M_1 and M_2 /P and their trends



* M_1/P is estimated with a log value.

36. On the recent survey, see, for example, Roley (1985), Rose (1985).

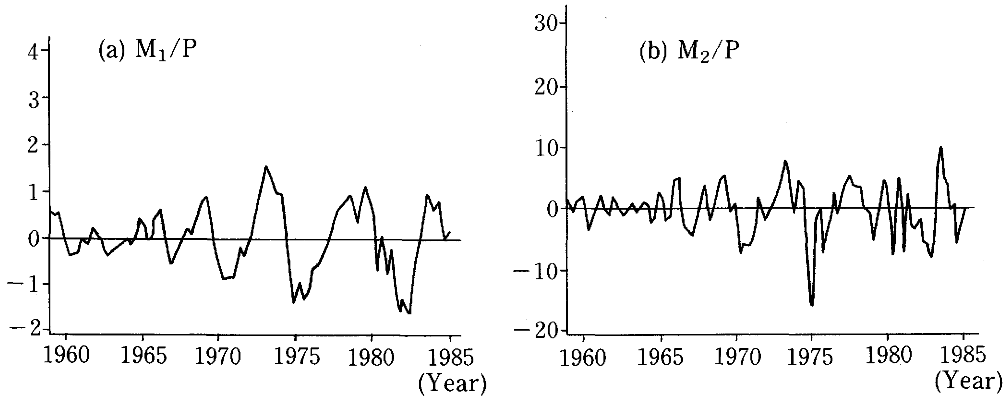
rise until 1974. Subsequently trend began to decline but in 1982 began to rise again. These times have been so far indicated as the unstable periods for money stock movements. On the other hand, the changes of trend of M_2/P are not so distinctive as seen in M_1/P.

The movements around the trend are shown in Figure 28. Both M_1/P and M_2/P continue to show the bigger fluctuations since 1970s, which are similar behavior to nominal M_1, M_2 as noted in III.(2).

Fitted time varying AR model to this series reveals that almost no big change like a shift can be seen in both M_1/P and M_2/P in the covariance structure as shown in Table 6 and Figure 29. In particular, M_1/P continue almost the same pattern of fluctuation. The AIC of Case 1 of M_1/P, assuming no structural shift, was smaller throughout the 1970s. On the other hand, M_2/P shows rather big changes in 1976. However, the differences of AIC between Cases 1 and 2 are not so large. Noteworthy is that, in both series, the AIC in Case 2 tends to grow smaller after 1981.

The changing cyclical components and changing variance of M_1/P reflect above results, which are shown in Figures 30 and 31. Changing cyclical components show that relatively longer cycles are continuing to be dominant and its range of fluctuations of longer term cycles are gradually expanding with medium or shorter terms cycles.

Figure 28 The movements around the trends



* The scale is 100 times.

Figure 29 AIC estimated from time varying AR coefficient model (see Table 6)

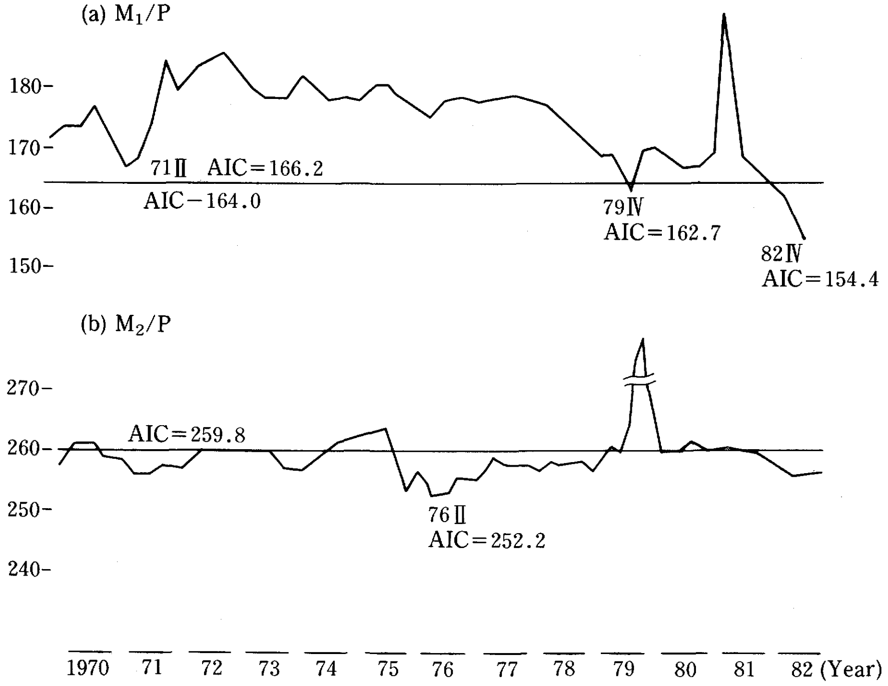


Figure 30 Changing cyclical components of the US M_1/P

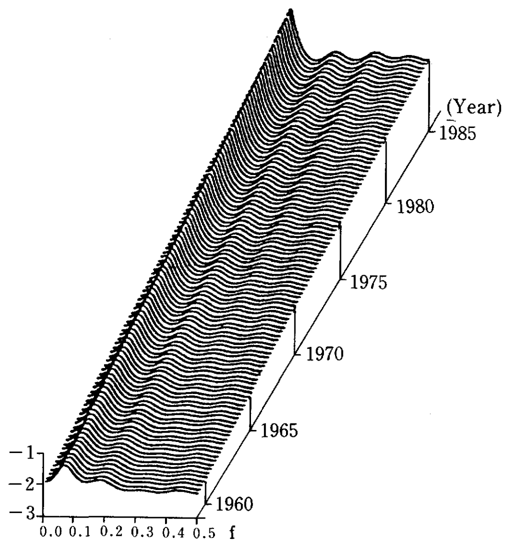
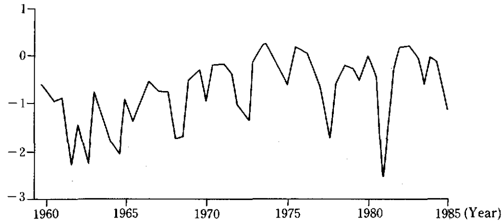


Figure 31 Changing variance of the US M_1/P

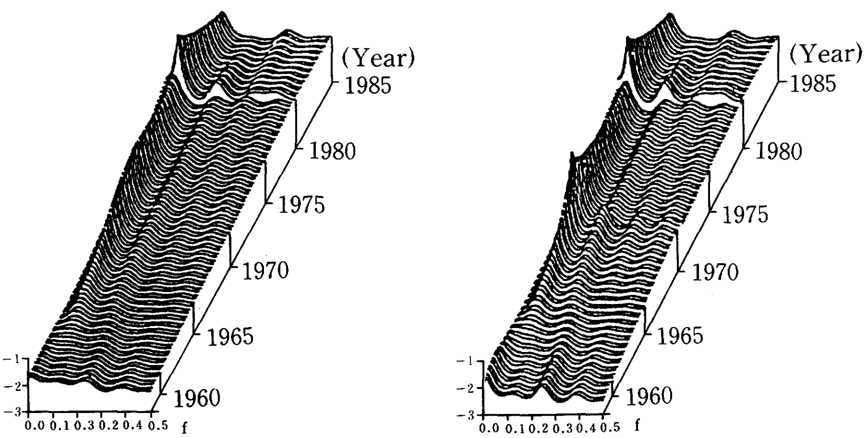


The subtle shifting in the 1980s may reflect the era of the financial reform, when considering about financial innovation and the progress of liberalization, such as introduction of the new scheme for financial adjustment in October 1979, liberalization of deposit interest rates, introduction of new financial commodities, and a change in the definition of money supply.[37]

37. In the 1970s, relatively smaller AIC can be seen in Case 2 of Table 6, for example, the second quarter of 1971 and the fourth quarter of 1979. Those periods correspond to the times when the U.S. economy entered a new phase. Therefore, for reference, we estimate the models assuming there have been shifts in those periods. Changing cyclical components are as following figures. However, in both cases, the AIC is not so smaller (see note 12 and Table 6).

(a) Shift in 1979 IV, AIC=162.7

(b) Shifts in 1971 II and 1979 IV, AIC=166.8



AIC Estimated from Time Varying Autoregressive Coefficient Model

Table 1 Money supply and GNP in Japan

(1) M_2+CD (rate of change over previous years) (period 1968 I -1984 IV)

(Case 1 assuming no structural shift) 111.0

(Case 2 assuming a structural shift (in one period))

Year/Quarter	AIC	Year/Quarter	AIC	Year/Quarter	AIC
1970 I	102.4	1975 I	89.1	1980 I	103.5
II	100.4	II	91.3	II	103.9
III	101.6	III	91.6	III	111.4
IV	95.3	IV	100.3	IV	102.1
1971 I	96.3	1976 I	104.1	1981 I	102.7
II	92.5	II	108.6	II	109.2
III	91.9	III	108.0	III	109.8
IV	89.0	IV	106.3	IV	112.7
1972 I	88.2	1977 I	105.0	1982 I	114.9
II	86.0	II	104.9	II	115.8
III	88.3	III	105.9	III	118.4
IV	84.6	IV	109.5	IV	114.3
1973 I	88.9	1978 I	108.0		
II	84.6	II	105.4		
III	78.9	III	103.9		
IV	86.5	IV	103.9		
1974 I	105.0	1979 I	104.7		
II	99.3	II	106.0		
III	98.9	III	104.8		
IV	93.0	IV	104.0		

(Case 3 assuming structural shifts (specified 2 periods))

Year/Quarter	AIC
1973 III, 1975 I	79.2
II	82.1
III	81.3
IV	82.5

(2) Nominal GNP (rate of change over previous years) (period 1966 I -1984 IV)

201.9

Year/Quarter	AIC	Year/Quarter	AIC	Year/Quarter	AIC
1970 I	183.3	1975 I	199.8	1980 I	193.4
II	183.0	II	169.8	II	191.9
III	183.6	III	170.4	III	192.2
IV	184.1	IV	184.3	IV	194.0
1971 I	187.8	1976 I	173.9	1981 I	195.3
II	186.9	II	170.1	II	196.4
III	189.0	III	179.0	III	194.6
IV	185.4	IV	179.1	IV	186.8
1972 I	185.5	1977 I	180.6	1982 I	191.1
II	185.9	II	180.6	II	189.0
III	185.3	III	177.8	III	185.4
IV	187.2	IV	174.9	IV	182.5
1973 I	187.5	1978 I	173.3		
II	188.7	II	177.4		
III	184.4	III	177.4		
IV	184.0	IV	181.5		
1974 I	173.8	1979 I	180.9		
II	178.4	II	183.0		
III	184.6	III	183.8		
IV	199.8	IV	195.9		

Year/Quarter	AIC
1974 I, 1975 II	168.6
" ,1976 II	165.9
1975 II, "	163.7
" , 1976 III	160.8

(3) Real GNP (rate of change over previous years) (period 1966 I -1984 IV)

167.7

Year/Quarter	AIC	Year/Quarter	AIC	Year/Quarter	AIC
1970 I	150.6	1975 I	148.7	1980 I	169.7
II	148.9	II	146.8	II	169.7
III	147.7	III	149.4	III	168.8
IV	147.6	IV	152.3	IV	167.6
1971 I	149.4	1976 I	152.8	1981 I	166.5
II	150.0	II	141.5	II	167.4
III	150.2	III	137.4	III	167.1
IV	148.8	IV	143.3	IV	167.5
1972 I	147.1	1977 I	146.9	1982 I	168.9
II	149.6	II	153.3	II	169.1
III	150.0	III	153.2	III	168.4
IV	150.1	IV	157.5	IV	167.6
1973 I	149.1	1978 I	158.0		
II	146.4	II	160.7		
III	146.1	III	161.7		
IV	142.2	IV	166.1		
1974 I	146.8	1979 I	166.6		
II	155.3	II	168.2		
III	154.5	III	168.9		
IV	143.4	IV	169.0		

Year/Quarter	AIC
1973 IV, 1974 IV	144.5
" , 1976 III	133.5
1974 IV, 1976 III	131.3

Table 2 Money supply and GNP in the US

(1) M_2 (rate of change over previous years) (period 1968I-1984 IV)

(Case 1 assuming no structural shift) 111.0

(Case 2 assuming a structural shift (one period))

Year/Quarter	AIC	Year/Quarter	AIC	Year/Quarter	AIC
1970 I	108.0	1975 I	121.9	1980 I	124.2
II	102.4	II	120.0	II	127.7
III	100.7	III	121.7	III	133.2
IV	101.6	IV	128.9	IV	127.9
1971 I	106.1	1976 I	131.8	1981 I	125.9
II	99.5	II	129.5	II	121.5
III	99.4	III	129.6	III	125.0
IV	102.0	IV	123.5	IV	122.4
1972 I	103.9	1977 I	126.4	1982 I	120.7
II	104.2	II	140.3	II	120.2
III	104.0	III	163.2	III	118.3
IV	109.8	IV	240.5	IV	116.5
1973 I	113.5	1978 I	138.0		
II	106.9	II	124.4		
III	103.2	III	127.5		
IV	105.7	IV	128.5		
1974 I	115.4	1979 I	126.2		
II	122.1	II	124.8		
III	127.0	III	122.3		
IV	122.3	IV	124.0		

(Case 3 assuming structural shifts (specified 2 periods))

Year/Quarter	AIC
1971 II, 1973 III	94.9
III, "	95.4

(2) Nominal GNP (rate of change over previous years) (period 1966I-1984 IV)

148.5

Year/Quarter	AIC	Year/Quarter	AIC	Year/Quarter	AIC
1970 I	167.1	1975 I	161.4	1980 I	144.5
II	165.9	II	160.0	II	145.0
III	163.7	III	156.3	III	146.0
IV	163.3	IV	155.0	IV	144.5
1971 I	156.5	1976 I	155.0	1981 I	149.1
II	158.8	II	155.5	II	147.8
III	159.1	III	153.3	III	148.2
IV	162.9	IV	156.9	IV	146.5
1972 I	162.8	1977 I	156.7	1982 I	144.7
II	165.0	II	156.6	II	147.9
III	165.1	III	155.9	III	149.2
IV	165.0	IV	154.9	IV	147.8
1973 I	166.2	1978 I	152.9		
II	161.9	II	152.6		
III	160.0	III	154.7		
IV	157.9	IV	155.0		
1974 I	154.7	1979 I	156.3		
II	164.9	II	152.1		
III	165.1	III	150.5		
IV	163.9	IV	146.5		

(3) Real GNP (rate of change over previous years) (period 1966I-1984 IV)

131.7

Year/Quarter	AIC	Year/Quarter	AIC	Year/Quarter	AIC
1970 I	136.5	1975 I	136.4	1980 I	135.5
II	133.9	II	134.7	II	136.3
III	133.5	III	128.3	III	133.9
IV	136.2	IV	124.1	IV	129.5
1971 I	134.7	1976 I	122.9	1981 I	122.6
II	135.5	II	130.2	II	117.4
III	137.3	III	126.1	III	115.3
IV	145.4	IV	126.4	IV	116.8
1972 I	146.9	1977 I	128.7	1982 I	118.0
II	146.3	II	134.1	II	123.1
III	144.6	III	133.4	III	123.9
IV	140.7	IV	135.2	IV	127.5
1973 I	142.0	1978 I	133.3		
II	136.5	II	134.4		
III	137.2	III	135.9		
IV	137.2	IV	135.0		
1974 I	137.8	1979 I	136.7		
II	143.0	II	137.7		
III	143.0	III	137.2		
IV	140.7	IV	136.9		

Year/Quarter	AIC
1976 I, 1981 I	108.5
II	107.4
III	106.0
IV	105.5

Table 3 Marshallian k (M_2 +CD/Nominal transactions) in Japan (period 1967 I-1985 II)

(Case 1 Assuming no structural shift) 124.7

(Case 2 Assuming a structural shift) (one period)

Year/Quarter	AIC	Year/Quarter	AIC	Year/Quarter	AIC
1970 I	120.7	1975 I	130.2	1980 I	136.5
II	117.0	II	111.9	II	136.5
III	119.9	III	112.2	III	141.8
IV	119.9	IV	119.6	IV	137.3
1971 I	122.8	1976 I	118.4	1981 I	143.8
II	122.2	II	122.0	II	138.4
III	122.5	III	119.7	III	134.5
IV	119.4	IV	120.5	IV	136.2
1972 I	121.6	1977 I	120.4	1982 I	132.1
II	122.9	II	122.3	II	127.2
III	125.1	III	122.0	III	121.9
IV	137.7	IV	124.8	IV	128.1
1973 I	136.3	1978 I	127.1		
II	127.8	II	127.6		
III	128.5	III	132.2		
IV	134.8	IV	130.4		
1974 I	137.0	1979 I	134.5		
II	137.2	II	129.1		
III	133.1	III	127.4		
IV	127.6	IV	134.4		

(Case 3 assuming structural shifts (specified two periods)

Year/Quarter	AIC
1970 II, 1975 II	112.3
1971 IV	113.6

Table 4 Real money stock in Japan

(1) M_1 / GNP deflator (period 1965 I -1984 IV)

(Case 1 assuming no structural shift) 217.4

(Case 2 assuming a structural shift (one period))

Year/Quarter	AIC	Year/Quarter	AIC	Year/Quarter	AIC
1970 I	201.2	1975 I	194.2	1980 I	200.3
II	205.4	II	193.4	II	199.2
III	205.5	III	191.7	III	199.9
IV	201.8	IV	198.1	IV	202.0
1971 I	202.4	1976 I	200.4	1981 I	202.0
II	195.8	II	202.4	II	199.1
III	196.0	III	201.4	III	199.4
IV	193.0	IV	202.7	IV	201.2
1972 I	193.2	1977 I	201.9	1982 I	200.7
II	192.0	II	203.5	II	200.7
III	193.3	III	202.8	III	199.4
IV	190.7	IV	205.0	IV	202.2
1973 I	193.2	1978 I	203.1		
II	193.8	II	206.3		
III	193.0	III	205.5		
IV	194.1	IV	204.4		
1974 I	194.5	1979 I	203.4		
II	191.3	II	204.3		
III	188.9	III	203.5		
IV	193.1	IV	201.3		

(Case 3 assuming structural shifts (specified 2 periods))

Year/Quarter	AIC	Year/Quarter	AIC
1974 III, 1979 IV	177.9	1974 III, 1981 I	184.5
1980 I	176.9	II	181.1
II	175.0	III	181.2
III	176.7	IV	180.7
IV	183.8		

(2) M_2+CD / GNP deflator (period 1967 I -1984 IV)

(Case 1 assuming no structural shift) 90.6

(Case 2 assuming a structural shift (one period))

Year/Quarter	AIC	Year/Quarter	AIC	Year/Quarter	AIC
1970 I	84.0	1975 I	79.3	1980 I	91.1
II	85.0	II	77.1	II	94.6
III	88.0	III	74.8	III	93.8
IV	89.9	IV	85.1	IV	91.5
1971 I	92.8	1976 I	87.4	1981 I	93.9
II	90.1	II	91.6	II	96.5
III	85.9	III	90.8	III	96.4
IV	81.1	IV	89.8	IV	91.2
1972 I	79.0	1977 I	89.2	1982 I	89.3
II	78.4	II	88.3	II	90.5
III	82.2	III	89.6	III	90.4
IV	80.8	IV	90.1	IV	91.6
1973 I	82.5	1978 I	92.7		
II	79.3	II	93.4		
III	79.1	III	92.6		
IV	71.0	IV	90.1		
1974 I	78.6	1979 I	88.4		
II	85.3	II	88.3		
III	89.0	III	89.5		
IV	84.5	IV	89.4		

(Case 3 assuming structural shifts (specified 2 periods))

Year/Quarter	AIC	Year/Quarter	AIC
1973 IV, 1979 IV	75.4	1973 IV, 1981 I	84.2
1981 I	74.3	II	77.6
II	77.7	III	86.2
III	74.4	IV	77.8
IV	83.8		

Table 5 Real money stock in Japan (by type of currency)

	(1) cash currency (period 1965 I -1984 IV)	(2) deposit currency (period 1965 I -1984 IV)	(3) quasi-money + CD (period 1967 I -1984 IV)
(Case 1 assuming no structural shift)	134.3	186.4	105.5

(Case 2 assuming a structural shift (one period))

(1) cash currency

Year/Quarter	AIC	Year/Quarter	AIC	Year/Quarter	AIC
1970 I	131.4	1975 I	121.9	1980 I	138.6
II	131.7	II	120.6	II	133.0
III	132.2	III	119.8	III	131.0
IV	131.2	IV	126.9	IV	134.3
1971 I	132.3	1976 I	128.9	1981 I	133.4
II	131.8	II	129.6	II	131.1
III	132.9	III	126.8	III	129.8
IV	132.2	IV	130.2	IV	130.6
1972 I	133.9	1977 I	129.5	1982 I	130.2
II	129.0	II	132.1	II	133.4
III	129.0	III	133.4	III	134.1
IV	133.8	IV	136.7	IV	135.9
1973 I	138.5	1978 I	137.7		
II	132.0	II	137.4		
III	128.8	III	135.4		
IV	126.5	IV	134.1		
1974 I	127.8	1979 I	135.5		
II	130.8	II	137.0		
III	126.4	III	139.0		
IV	128.3	IV	137.9		

(2) deposit currency

Year/Quarter	AIC	Year/Quarter	AIC	Year/Quarter	AIC
1970 I	165.1	1975 I	166.0	1980 I	186.3
II	176.8	II	168.6	II	183.2
III	215.2	III	168.4	III	183.5
IV	170.6	IV	172.8	IV	187.7
1971 I	171.5	1976 I	177.0	1981 I	185.2
II	171.2	II	181.1	II	179.2
III	169.1	III	179.5	III	198.4
IV	164.8	IV	180.5	IV	177.3
1972 I	164.3	1977 I	179.0	1982 I	181.0
II	165.3	II	182.3	II	184.4
III	166.8	III	183.0	III	183.5
IV	172.1	IV	184.0	IV	186.2
1973 I	174.4	1978 I	185.1		
II	171.5	II	187.8		
III	169.5	III	186.1		
IV	165.8	IV	186.4		
1974 I	172.3	1979 I	186.9		
II	167.4	II	189.7		
III	166.6	III	188.9		
IV	169.7	IV	187.1		

(3) quasi-money + CD

Year/Quarter	AIC	Year/Quarter	AIC	Year/Quarter	AIC
1970 I	100.4	1975 I	106.6	1980 I	99.9
II	102.3	II	96.3	II	105.0
III	102.9	III	95.4	III	103.6
IV	102.1	IV	75.6	IV	98.1
1971 I	98.1	1976 I	79.4	1981 I	95.7
II	94.9	II	75.4	II	100.1
III	94.4	III	75.7	III	101.9
IV	89.6	IV	77.0	IV	102.0
1972 I	90.3	1977 I	80.1	1982 I	101.3
II	95.5	II	82.6	II	104.5
III	93.9	III	89.3	III	103.7
IV	98.5	IV	87.3	IV	103.6
1973 I	97.5	1978 I	87.5		
II	95.4	II	89.2		
III	95.7	III	90.6		
IV	96.0	IV	94.4		
1974 I	96.1	1979 I	96.1		
II	92.3	II	98.6		
III	100.3	III	101.8		
IV	103.4	IV	97.2		

(Case 3 assuming structural shifts (specified 2 periods))

Year/Quarter	AIC	Year/Quarter	AIC	Year/Quarter	AIC	Year/Quarter	AIC	Year/Quarter	AIC
1973 III, 1980 I	134.5	1975 III, 1980 I	123.4	1973 IV, 1980 I	168.9	1974 II, 1980 I	111.4	1976 II, 1980 I	87.6
II	129.5	II	115.2	II	163.7	II	103.6	II	84.0
III	122.6	III	108.3	III	163.7	III	102.0	III	82.4
IV	136.4	IV	120.7	IV	170.5	IV	96.3	IV	76.5
						1981 I	95.0	1981 I	73.6

Table 6 Real money stock in the US

(1) M_1 / GNP deflator (period 1959 I-1985 I)

(Case 1 assuming no structural shift) 164.0

(Case 2 assuming a structural shift (one period))

Year/Quarter	AIC	Year/Quarter	AIC	Year/Quarter	AIC
1970 I	171.5	1975 I	178.1	1980 I	168.4
II	174.0	II	177.5	II	169.3
III	173.6	III	179.8	III	172.7
IV	177.0	IV	180.3	IV	165.8
1971 I	172.3	1976 I	177.3	1981 I	166.2
II	166.2	II	176.2	II	168.3
III	167.8	III	174.7	III	192.1
IV	174.5	IV	177.1	IV	167.6
1972 I	184.1	1977 I	178.2	1982 I	164.8
II	179.4	II	177.7	II	162.7
III	182.7	III	178.0	III	160.9
IV	184.5	IV	178.8	IV	154.4
1973 I	186.3	1978 I	178.3		
II	183.2	II	177.8		
III	179.0	III	174.5		
IV	177.7	IV	173.0		
1974 I	177.7	1979 I	170.1		
II	181.9	II	168.4		
III	180.2	III	168.1		
IV	177.8	IV	162.7		

(Case 3 assuming structural shifts (specified 2 periods))

Year/Quarter	AIC
1971 II, 1979 IV	166.8
III, 1980 I	170.0

Year/Quarter	AIC
1979 III, 1982 IV	165.2
IV ,	161.4

(Case 4 assuming structural shifts (specified 3 periods))

Year/Quarter	AIC
1971 II, 1979 IV, 1982 IV	166.4

(2) M_2+CD / GNP deflator (period 1959 I-1985 I)

(Case 1) 259.8

(Case 2)

Year/Quarter	AIC	Year/Quarter	AIC	Year/Quarter	AIC
1970 I	257.2	1975 I	261.6	1980 I	260.1
II	260.5	II	262.7	II	259.3
III	261.1	III	263.0	III	260.2
IV	259.5	IV	252.5	IV	260.5
1971 I	258.0	1976 I	255.9	1981 I	259.7
II	255.5	II	252.2	II	260.3
III	256.2	III	252.6	III	259.8
IV	257.1	IV	255.7	IV	258.9
1972 I	257.0	1977 I	255.4	1982 I	257.6
II	259.7	II	258.8	II	256.5
III	259.8	III	257.8	III	255.8
IV	260.5	IV	257.5	IV	255.9
1973 I	260.3	1978 I	256.4		
II	260.0	II	257.8		
III	259.4	III	257.1		
IV	256.2	IV	257.9		
1974 I	256.4	1979 I	256.3		
II	257.8	II	259.8		
III	259.2	III	260.0		
IV	260.8	IV	329.3		

(Case 3)

Year/Quarter	AIC
1973 IV, 1979 IV	75.4
1980 I	74.3

Year/Quarter	AIC
1973 IV, 1981 I	84.2
II	77.6

Appendix I. Time Varying Autoregressive Coefficient Model

Let z(1),..., z(N) be detrended time series defined by

$$z(n) = y(n) - t(n|N) \qquad (n = 1, \cdots, N), \tag{A-1}$$

where y(n) is original series and t(n|N) is the trend estimated from N samples.

The detrended series can be assumed nonstationary in the covariance. Then, the series can be expressed by a time varying autoregressive (AR) coefficient model,

$$z(n) = \sum_{i=1}^{m} a(i,n)z(n-i) + \varepsilon(n) \tag{A-2}$$

with smoothness constraint on the time evolving AR coefficients a(i,n) which are assumed to change gradually over time. In (A–2), m is the model order, ε (n) is a Gaussian white noise with mean 0 and variance σ^2. A model for time varying AR coefficient is obtained by a k-th order stochastically perturbed difference equation constraint model,

$$\nabla^k a(i,n) = \delta(i,n) \tag{A-3}$$

where ∇ is the difference operator defined by ∇ a(i, n)=a(i, n)−a(i, n-1), δ (i, n) is a Gaussian white noise such that,

$$\begin{aligned} &E\delta(i,n) = 0 \\ &E\delta(i,n)\cdot\delta(j,m) = \begin{cases} \tau^2, & n = m,\ i = j \\ 0, & \text{otherwise} \end{cases} \end{aligned} \tag{A-4}$$

where E denotes expectation. Equations (A-3) and (A-4) show that the coefficient changes over time stochastically and that its movement depends on parameter τ^2. By defining the state vector as,

$$x(n) = [a(1,n), \cdots, a(m,n), \cdots, a(1,n-k+1), \cdots, a(m,n-k+1)]' \tag{A-5}$$

where (′) denotes the transpose of the vector.
The linear difference model for the coefficients (A-3) can be expressed in the form of a state equation.

$$x(n) = Fx(n-1) + Gu(n) \tag{A-6}$$

For the difference equation orders k=1, 2, 3, the matrices F and G in equation (A–6) are,

$$
\begin{aligned}
k = 1 &: F = (I_m), & G &= (I_m) \\
k = 2 &: F = \begin{bmatrix} 2I_m & -I_m \\ I_m & 0 \end{bmatrix}, & G &= \begin{bmatrix} I_m \\ 0 \end{bmatrix} \\
k = 3 &: F = \begin{bmatrix} 3I_m & -3I_m & I_m \\ I_m & 0 & 0 \\ 0 & I_m & 0 \end{bmatrix}, & G &= \begin{bmatrix} I_m \\ 0 \\ 0 \end{bmatrix}
\end{aligned}
\qquad \text{(A−7)}
$$

with m × m identity I_m and zero matrices 0.

This time varying AR coefficient model has a state space representation,

$$
\begin{aligned}
x(n) &= Fx(n-1) + Gu(n) \\
z(n) &= H(n)x(n) + \varepsilon(n)
\end{aligned}
\qquad \text{(A−8)}
$$

where

$$u(n) = (\delta(1,n), \cdots, \delta(m,n))' \qquad \text{(A−9)}$$

and H (n) is the km dimensional vector.

$$H(n) = (z(n-1), \cdots, z(n-m), 0, \cdots, 0), \qquad \text{(A−10)}$$

The vector u(n) and ε (n) are assumed to be independent with time,

$$
\begin{aligned}
& u(n) \sim N(0, \Sigma), \quad \varepsilon(n) \sim N(0, \sigma^2) \\
& \Sigma = \begin{bmatrix} \tau^2 & & 0 \\ & \ddots & \\ 0 & & \tau^2 \end{bmatrix}
\end{aligned}
\qquad \text{(A−11)}
$$

In general, state space representation of the time varying AR coefficient model can be expressed by (12) in this paper. For example, if k=2, Equation (A–8) is as follows,

$$\begin{bmatrix} a(1,n) \\ \vdots \\ a(m,n) \\ \hdashline a(1,n-1) \\ \vdots \\ a(m,n-1) \end{bmatrix} = \left[\begin{array}{ccc:ccc} 2 & & & -1 & & \\ & \ddots & & & \ddots & \\ & & 2 & & & -1 \\ \hdashline 1 & & & & & \\ & \ddots & & & 0 & \\ & & 1 & & & \end{array}\right] \begin{bmatrix} a(1,n-1) \\ \vdots \\ a(m,n-1) \\ \hdashline a(1,n-2) \\ \vdots \\ a(m,n-2) \end{bmatrix} + \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \\ \hdashline & & \\ & 0 & \\ & & \end{bmatrix} \begin{bmatrix} \delta(1,n) \\ \vdots \\ \delta(m,n) \end{bmatrix}$$

$$Z(n) = [Z(n-1),\cdots,Z(n-m),0,\cdots,0] \begin{bmatrix} a(1,n) \\ \vdots \\ a(m,n) \\ \hdashline a(1,n-1) \\ \vdots \\ a(m,n-1) \end{bmatrix} + \varepsilon(n) \qquad \text{(A-12)}$$

where

$$\begin{bmatrix} \delta(1,n) \\ \vdots \\ \delta(m,n) \\ \hdashline \varepsilon(n) \end{bmatrix} \sim N\left[\begin{pmatrix} 0 \\ \vdots \\ 0 \\ \hdashline 0 \end{pmatrix}, \left(\begin{array}{ccc:c} \tau^2 & & & 0 \\ & \ddots & & \vdots \\ & & \tau^2 & 0 \\ \hdashline 0 & \cdots & 0 & \sigma^2 \end{array}\right)\right] \qquad \text{(A-13)}$$

Given the state space model (A–8), the fitting of the model to the data is achieved by the Kalman Filter, given observations z(1),..., z(N) and initial conditions x(0|0) and V(0|0). Then, the one step ahead predictor x(n|n-1) and its error covariance matrix V(n|n-1) and filter x(n|n) of the state vector x(n) is computed. For time update,

$$x(n|n-1) = Fx(n-1|n-1)$$
$$V(n|n-1) = FV(n-1|n-1)F' + GQ(n)G' \qquad \text{(A-14)}$$

where F, G are coefficient matrices in Equation (A–8) and Q(n) is derived from Equations (A–11) and (A–13),

$$\begin{bmatrix} u(n) \\ \varepsilon(n) \end{bmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & \\ & \sigma^2 \end{pmatrix}\right], \quad Q = \begin{bmatrix} \tau^2 & & \\ & \ddots & \\ & & \tau^2 \end{bmatrix} \qquad \text{(A-15)}$$

For the initial values of x(0|0) and V(0|0), zero and a diagonal matrix with large diagonal values are used. For observation update,

$$K(n) = V(n|n-1)H(n)'[H(n)V(n|n-1)H(n)'+\sigma^2]^{-1}$$
$$x(n|n) = x(n|n-1)+K(n)[z(n)-H(n)x(n|n-1)] \qquad (A-16)$$
$$V(n|n) = [I-K(n)H(n)]V(n|n-1).$$

In the above, H(n) is the coefficient matrix of Equation (A–8) and K(n) is the Kalman filter. Using these estimates, the smoothed value of state vector is obtained by the smoothing algorithm.

$$A(n) = V(n|n)F'V(n+1|n)^{-1}$$
$$x(n|N) = x(n|n)+A(n)[x(n+1|N)-x(n+1|n)] \qquad (A-17)$$
$$V(n|N) = V(n|n)+A(n)[V(n+1|N)-V(n+1|n)]A(n)'$$

The joint density of the observation is,

$$f(z(1),\cdots,z(n)) = \prod_{n=1}^{N} f(z(n)|z(1),\cdots,z(n-1)) \qquad (A-18)$$

with

$$f(z(n)|z(1),\cdots,z(n-1)) = (2\pi r(n))^{-\frac{1}{2}}\cdot\exp\left[-\frac{v(n)^2}{2r(n)}\right] \qquad (A-19)$$

That yields the log likelihood

$$\log L(\mu^2|m,k) = -\frac{N}{2}\log 2\pi - \frac{1}{2}\sum_{n=1}^{N}\log r(n) - \sum_{n=1}^{N}\frac{v(n)^2}{2r(n)} \qquad (A-20)$$

v(n) and r(n) in Equations (A–19) and (A–20) are,

$$v(n) = z(n)-H(n)x(n|n-1)$$
$$r(n) = H(n)V(n|n-1)H(n)'+\sigma^2 \qquad (A-21)$$

where v(n) is the innovations and r(n) is its variance conditioned on the values up to time n-1.

The ratio of the variances $\mu^2=\tau^2/\sigma^2$ can be interpreted as the trade-off parameter between the (in) fidelity to the linear difference constraint and the (in) fidelity to the data. The minimum AIC model is then defined by the values of m,k and trade-off parameter μ^2, for which the AIC is the smallest.

In respect of the periods of shifts in the covariance structure, smoothness constraints of the stochastic term in Equation (A–3) which assumes the coefficient changes smoothly over time, are removed. For example, when k=1, the difference equation

$$a(i,n) = a(i,n-1)+\delta(i,n)$$

depends on the stochastic term one period before because a(i, n-1)=a(i, n-2) + δ (i, n-1). In estimation, constraint is removed.

Appendix II. Changing Variance and Cyclical Components

1. Changing Variance

To capture the local and global statistical relationships in the detrended time series, the time varying AR coefficient model described in Appendix I. is used by imposing priors constraint. In practical data situations, the relatively fast irregular movements of a nonstationary covariance time series seem to be modulated by a slowly changing envelope function. That envelope function can be interpreted as a change of a scale associated with the changing innovations variance of the time varying AR coefficient model.

The changing variance is estimated in the following way. Consider a realization of white noises S(n) (n=1,...N) (S(n)~N(0, σ^2(n)) with unknown time varying variance σ^2(n).

The stochastic process χ^2(m) defined by

$$\chi^2(m) = \frac{S^2(2m-1)+S^2(2m)}{2}, \left(m = 1,\cdots,\frac{N}{2}\right) \qquad \text{(A-22)}$$

constitutes an independent sequence of chi-square random variables with two degrees of freedom. Then, the transformation,

$$t(m) = \ln\chi^2(m)+\gamma, \left(m = 1,\cdots,\frac{N}{2}\right) \qquad \text{(A-23)}$$

leaves the independent random variable t(m) with distribution that is almost normal and with the moments E[t(m)]= ln σ^2(m), var[t(m)]= $\pi^2/6$ and γ =0.57721 is Euler constant.[38] To obtain a smooth estimate of the variance $\sigma^2(n)$, k-th order difference equation constraint on the log variance defined by

$$\nabla^k t(m) = w(m) \tag{A-24}$$

are considered. In (A-24) ∇ is a shift operator defined by $\nabla t(m)=t(m)-t(m-1)$, and $w(m) \sim N(0, \tau^2)$ i.i.d. Then Equation (A-24) can be expressed with a state space model,

$$x(m) = Fx(m-1)+Gw(m) \qquad \text{(A-25)}$$
$$t(m) = Hx(m)+\varepsilon(m)$$

where,

$$\begin{bmatrix} w(m) \\ \varepsilon(m) \end{bmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \tau^2 & \\ & \sigma^2 \end{pmatrix}\right]. \tag{A-26}$$

For example, when k=2, then the state vector and the matrices in (A–25) are

$$x(m) = \begin{bmatrix} t(m) \\ t(m-1) \end{bmatrix}, \tag{A-27}$$

$$F = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}, G = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, H = (1 \quad 0) \tag{A-28}$$

Applying the Kalman filter and smoothing algorithms yields the smoothed value of $t(m|N)$, the logarithm of smoothed estimate of the changing variance,

$$\sigma^2(2m|N) = \sigma^2(2m-1|N) = \exp[t(m|N)] \tag{A-29}$$

is then smoothed estimate of the changing variance. According to the symbols used in the paper the smoothed estimate is calculated as,

$$y^2(m) = \ln[z^2(2m-1)+z^2(2m)]/2, \left(m = 1, \cdots, \frac{N}{2}\right) \tag{A-30}$$

38. Concerning the approximation to χ^2 distribution, the Wilson-Filferty approximation is known. In the Kitagawa-Gersch method, on account of the stochastic term in the Kalman method assumes the normal distribution, transformation is done through (2-1) and (2-2) in consideration of practical use.

and time varying covariance is estimated by

$$\hat{\sigma}^2(2m-1|N) = \sigma^2(2m|N) = \exp[e^2(m|N)+\gamma], \left(m = 1, \cdots, \frac{N}{2}\right) \quad \text{(A-31)}$$

where

$$e^2(m) = \ln\left[\frac{v^2(2m-1)}{r(2m-1)}+\frac{v^2(2m)}{r(2m)}\right], \left(m = 1, \cdots, \frac{N}{2}\right) \quad \text{(A-32)}$$

v(m), r(m) in (A–32) are obtained from Equation (A–21).

2. Changing Cyclical Components

In order to examine the time series with irregular movements, analysis of regular cycles is first necessary.

The power spectrum is a convenient way to see in the frequency domain what cyclical components are dominant. The power spectrum of the stationary time series defined by the AR model,

$$z(n) = \sum_{i=1}^{m} a(i)z(n-i)+\varepsilon(n) \quad \text{(A-33)}$$

is

$$p(f) = \frac{\hat{\sigma}^2}{\left|1-\sum_{j=1}^{m} \hat{a}(j)\cdot\exp(-2\pi ijf)\right|^2}, \left(-\frac{1}{2} \leqq f \leqq \frac{1}{2}\right) \quad \text{(A-34)}$$

Kitagawa-Gersch defines the instantaneous spectrum of a time varying coefficient AR process by

$$p(f,n) = \frac{\hat{\sigma}^2(n)}{\left|1-\sum_{j=1}^{m} \hat{a}(j,n)\cdot\exp(-2\pi ijf)\right|^2}, \left(-\frac{1}{2} \leqq f \leqq \frac{1}{2}, n = 1, \cdots, N\right) \quad \text{(A-35)}$$

This definition is motivated by the relationship between a stationary AR process (A–33) and its theoretical spectrum (A–34). The autoregressive coefficient $\hat{a}(j)(j=1, \ldots, m)$ and variance σ^2 are replaced by coefficient $\hat{a}(j,n)(j=1,\ldots, m, n=1,\ldots, N)$ and the time varying variance $\sigma^2(n)$ $(n=1, \ldots, N)$ which are estimated by the maximum likelihood method.

In (A–35), the time varying AR coefficient a(j, n) is obtained from

$$\begin{bmatrix} \hat{a}(1,n) \\ \vdots \\ \hat{a}(m,n) \end{bmatrix} = \begin{bmatrix} 1 & & \vdots \\ & \ddots & \vdots\, 0 \\ & 1 & \vdots \end{bmatrix} \cdot \hat{x}(n|N) \tag{A-36}$$

where $\hat{x}(n|N)$ is the smoothed estimate of the state vector when z(1),..., z(N) are given. Their method is particularly relevant for the estimation of the spectral density with a relatively small amount of data. The use of smoothness priors alleviates this problem.[39]

Appendix III. Locally Stationary Autoregressive Model

The modeling and application of nonstationary covariance time series has been approached via fitting of locally stationary models. This model assumes that even though time series are as a whole nonstationary, if the stationary process can be observed locally, the locally stationary model can be fitted.

Consider a time series being divided into two parts, which we call part 1 and part 2, and the number of samples are N and M, respectively. Each part is assumed stationary. Then let the autoregressive models applied to each part to be AR_1 and AR_2, respectively, such as,

$$AR_1 : x(n) = \sum_{i=1}^{P_1} a_1(i)x(n-i) + \varepsilon_1(n), (n = 1, \cdots, N) \tag{A-37}$$

$$AR_2 : x(n) = \sum_{i=1}^{P_2} a_2(i)x(n-i) + \varepsilon_2(n), (n = 1, \cdots, M) \tag{A-38}$$

where, $\varepsilon_1(n)$ and $\varepsilon_2(n)$ are Gaussian white noises with covariance S_1, and S_2, and P_1 and P_2 denote autoregressive order, respectively.

At the same time, we assume stastionary process to whole series with N+M sample and fit the autoregressive model,

$$AR_0 : x(n) = \sum_{i=1}^{P_0} a_0(i)x(n-i) + \varepsilon_0(n), (n = 1, \cdots, N+M) \tag{A-39}$$

where, $\varepsilon_0(n)$ is a Gaussian white noise with covariance S_0, and P_0 is an autoregressive order.

The estimated models are evaluated by,

$$AIC = N \log|S_1| + M \log|S_2| + 2(P_1 + P_2 + 2) \tag{A-40}$$

with regard to Equations (A–37) and (A–38) and by,

39. Kitagawa-Gersch (1985-b).

$$AIC_0 = (N+M)\log|S_0| + 2(P_0+1) \qquad (A-41)$$

with regard to Equation (A–39).

From the viewpoint that the best model is selected using the minimum AIC, AR_1 and AR_2 models are adopted when $AIC < AIC_0$, and we choose AR_0 model when $AIC > AIC_0$. The former means that the time series is nonstationary as a whole, but locally stationary, the latter means that time series is stationary on the whole.

In practice it is not easy to find the period of division because there are the possibilities of divisions for all period. A Bayesian locally stationary autoregressive model is suggested in this case using prior information to select the best model.[40]

40. Kitagawa-Akaike (1978).

REFERENCES

Akaike, Hirotsugu, "A New Look at the Statistical Model Identification," *IEEE Trans. Automat. Contr.*, Vol. AC-10, 1974.

———, "A Bayesian Extension of the Minimum AIC Procedure of Autoregressive Model Fitting," *Biometrika*, 1979.

Bar-Shalom, Y., "Optimal Simultaneous State Estimation and Parameter Identification in Linear Discrete-Time Systems," *IEEE Trans. Automat, Contr.*Vol. AC-17, 1972.

Bendat, J. S. and Piersol, A. G., *Measurement and Analysis of Random Data,* John Wiley, 1966.

Bomhoff, Eduard J., *Monetary Uncertainty*, North-Holland, 1983.

Brown, R. L., Durbin, J., and Evans, J. M., "Techniques for Testing the Constancy of Regression Relationships Over Time," *Journal of Royal Statistical Society*, Series B, 1975.

Cargil, Thomas F., "A U.S. Perspective on Japanese Financial Liberalization," *Monetary and Economic Studies*, Bank of Japan, Vol. 3, No.1, 1985.

Cooley, T. F. and Prescott E. C., "Estimation in the Presence of Stochastic Parameter Variation," *Econometrica* 44, 1976.

Chow, G. C., "Test of Equality Between Sets of Coefficients in Two Linear Regression," *Econometrica*, Vol. 28, July 1960.

Doan, T., Litterman, R., and Sims, C., "Forecasting and Conditional Projection using Realistic Prior Distribution," *Paper presented Econometrics and Statistics Colloquim*, University of Minnesota, 1984.

Duncan, D. B. and Horn, S. D., "Linear Dynamic Regression Estimation from the Viewpoint of Regression Analysis," *JASA,* Vol. 67, 1972.

Friedman, Milton, "Monetarism in Rhetoric and in Practice," *Monetary and Economic Studies*, Bank of Japan, Vol. 1, No.2, 1983.

———, "The Feds' Monetarism was Never Anything but Rhetoric," *Wall Street Journal*, December 18, 1985.

Furukawa, Akira, *The Financial Analysis of Modern Japan* (in Japanese), Toyo Keizai Shinposha, 1985.

Goldfeld, S. M., "The Case of the Missing Money," *Brookings papers on Economic Activity*, 1976.

Hamada, Koichi and Hayashi, Fumio, "Monetary Policy in Postwar Japan," Bank of Japan, *1st International Conference Paper*, 1983.

Hannan, E. J., *Time Series Analysis*, Methuen, 1960.

Harrison, P. T. and Stevens, C. F., "Bayesian Forecasting," *Journal of the Royal Statistical Society*, Series B, 38, 1976.

Hatanaka, M., "An Efficient Two-Step Estimator for the Dynamic Adjustment Model with Autoregressive Errors," *Journal of Econometrics*, Vol.2, 1974.

Havenner, A. and Swamy, P.A.V.B., "A Random Coefficient Approach to Seasonal Adjustment of Economic Time Series," *FRB Special Studies Paper* 125, 1978.

Hino, Mikio, *Spectrum Analysis* (in Japanese), Asakura Shobo, 1977.

Ishida, Kazuhiko, "Divisia Monetary Aggregates and Demand for Money," *Monetary and Economic Studies*, Bank of Japan Vol.2, No.1, 1984.

Judge, G. G., Griffiths, W. E., Hill, R. C., and Lee T. C., *Theory and Practice of Econometrics*, John Wiley, 1980.

Kalman, R. E., "A New Approach to Linear Filtering and Prediction Problems," *ASME J. Basic Eng.* 82, 1960.

Kama, Kunio, "Stability of the Demand for Money Function in Japan," *The Soka Economic Studies* Vol.16, No.1.2, 1986.

Kitagawa, Genshiro, "Non-Gaussian Smoothness Prior Modeling of Nonstationary Time Series," *Research Memo* No.290, The Institute of Statistical Mathematics, 1985.

————, "Changing Spectrum Estimation," *Journal of Sound and Vibration*, 89, 1983.

Kitagawa, Genshiro and Akaike, Horitsugu, "A Procedure for the Modeling of Nonstationary Time Series," *Ann, Inst. Statist. Math*, 1978.

————, "On TIMSAC-78, Applied Time Series Analysis II," in D. Findley edited, Academic Press, 1981.

Kitagawa, Genshiro and W. Gersch, "A Smoothness Priors-State Space Modeling with Trend and Seasonalities," JASA, Vol.79, 1984.

————, "A Smoothness Priors Time Varying AR Coefficient Modeling of Nonstationary Covariance Time Series," *IEEE Trans. Automat., Contr.*, Vol. AC-30, January 1985-a.

————, "A Smoothness Priors-Long AR Model Method for Spectral Estimation," *IEEE Trans. Automat., Contr.*, 1985-b.

Komura, Chikara, "The Demand for Money Reconsidered, The Case of Japan," *Keizaigakubu Ronshu*, Seikei University, 1985.

Los, C. A., "Regression Analysis and Diagnosis by an Evolutionary Parameter Estimator," FRB of New York, 1985.

Los, C. A. and Kell, C. M., "Monte Carlo Studies of Econometric Estimators of Evolutionary Parameter Structures," *FRB of New York research paper* No.8502, March 1985.

Mark, W. D., "Spectral Analysis of Convolution and Filtering of Non-Stationary Stochastic Process," *Journal of Sound and Vibration*, 1970.

Meltzer, Allan H., "Variability of Prices, Output and Money Under Fixed and Fluctuating Exchange Rates," *Monetary and Economic Studies*, Bank of Japan, Vol.3, No.3 1985.

———, "Limits of Short-Run Stabilization Policy," *Presidential Address to the Western Economic Association*, 1986.

Morimune, Kimio, "The Estimation and Statistical Test of Economic Models," (in Japanese), Kyoritsu Shuppan, 1985.

Naniwa Sadao, "Trends Estimation via Smoothness Priors-state Space Modeling," *Monetary and Economic Studies*, Bank of Japan, No.1, Vol.4, 1986.

Nelson, C. R. and Plosser, C. L., "Trends and Random Walk in Macroeconomic Time Series," *Journal of Monetary Economics*, October, 1982.

Nicholls, D. F. and Pagan, A. R., "Varying Coefficients Regression," in E. J. Hannan, P. R. Krishnaiah and M. M. Rao edited, *Handbook of Statistics,* Vol.5, 1985.

Okubo, Takashi, "Money Supply and Financial Policy," (in Japanese), Toyo Keizai Shinposha, 1983.

Oritani, Yoshiharu, "The Negative Effects of Inflation on the Economic Growth in Japan," Bank of Japan Discussion Paper Series, No.5, 1981.

Pagan, A. R., "Some Identification and Estimation Results for Regression Models with Stochastically Varying Coefficients," *Journal of Econometrics*, 1980.

Page, C. H., "Instantaneous Power Spectra," *Journal of Applied Physics*, 1952.

Priestly, M. B., "Evolutionary Spectra and Nonstationary Process," J. of Royal Statistical Society, Series B, 1965.

Research Department, the Bank of Japan, "On the Importance of Money Supply in Japan," (in Japanese), *Monthly Economic Review,* July 1975.

———, "On Marshallian k's Trend to Rise," (in Japanese), *Monthly Economic Review*, November 1977.

Research and Statistics Department, Bank of Japan, "The Liberalization of Deposit Interest Rates in the U.S. and Financial Policy," (in Japanese), *Monthly Economic Review*, November 1977.

———, "Recent Developments in the Money Supply," Special Paper No.105, May 1983 and No.120, June 1985.

Roley, V. V., "Money Demand Predictability," *NBER Working Paper* No.1580, March 1985.

Rose, A., "An Alternative Approach to the American Demand for Money," *Journal of Money Credit and Banking*, 1985.

Rosenberg, B., "The Estimation of Stationary Stochastic Regression Parameters Re-examined," JASA 67, September, 1972.

————, "The Analysis of a Cross Section of Time Series by Stochastically Convergent Parameter Regression," *Annals of Economic and Social Measurement 2*, October, 1973.

Sarris, A. H., "A Bayesian Approach to Estimation of Time Varying Regression Coefficients," *Annals of Economic and Social Measurement 2*, October, 1973.

Seisaku Koso Forum (ed.), "Debate: Is Economic Policy Effective?" (in Japanese), Toyo Keizai Shinposha, 1984.

Stulz, R. M. and Wasserfallen, W., "Macroeconomic Time-Series, Business Cycles and Macroeconomic Policies," *Carnegie-Rochester Conference Series on Public Policy*, 22, Spring 1984.

Suzuki, Yoshio, "Essay on Japan's Financial Economy," (in Japanese), Toyo Keizai Shinposha, 1983.

————, "Financial Innovation and Monetary Policy in Japan," *Monetary and Economic Studies*, Bank of Japan, Vol.2, No.1, 1984.

————, "Japan's Monetary Policy over the Past 10 Years," *Monetary and Economic Studies*, Bank of Japan, Vol.3, No.2, 1985.

Swamy, P.A.V.B. and Tinsley, P. A., "Linear Prediction and Estimation Methods for Regression Models with Stationary Stochastic Coefficients," *Journal of Econometrics*, 12, 1980.

Takeuchi, Kei, "Preface to the Japanese edition of Hannan's Time Series Analysis," (in Japanese), Baifukan, 1974.

————, "The Approximation of Probability Distribution," (in Japanese), Kyoiku Shuppan, 1975.

Tamura, Yoshiyasu, "On Spectral Analysis of Nonstationary Process," The Institute of Statistical Mathematics, No.1, Vol.32, 1984.

Tsutsui, Yoshio and Hatanaka, Michio, "On the Stability of the Money Demand Function in Japan and the U.S.," (in Japanese), Gendai Keizai, 1982.

Watson, M. W., "Univariate Detrending Method with Stochastic Trends," *Discussion Paper* No.1158, Harvard University, 1985.

West, M., Harrison, P. J., and Migon, M. S., "Dynamic Generalized Linear Models and Bayesian Forecasting," JASA, 80, 1985.

Yomo, Hiroshi, "Japan's Successful Monetarism," (in Japanese), Toyokeizai, *Modern Economics Series*, 72, May 1985.