The Financing of the Government Budget in Japan and Its Relation to Macroeconomic Variables*

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I. Introduction

This paper deals with the relation of government finance in Japan to the rate of inflation and other macroeconomic variables. For the past decade, Japan's anti-inflation record has been one of the best in the world: its annual inflation rate came down very quickly from 25% in 1974, and subsequently declined gradually, reaching 2% in 1985. This has been brought about by the action of the Bank of Japan which limited the growth rates of high powered money (to be denoted by H), M_1, and M_2 + CD. Velocities of circulation and money multipliers did not change rapidly during the past decade. This suggests that it was a fairly simple task conceptually (though perhaps not simple practically, since so few countries succeeded!) to control inflation by keeping the average growth rate of the money supply close to the average growth

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rate of real output.

Beginning in 1975 the Japanese government budget has gone heavily into deficit, with deficits in the neighborhood of 4 or 5% of GNP. Had these deficits been financed by high powered money creation, the result would have been a poorer anti-inflation record. Instead, they were financed largely by the sale of government bonds to investors outside the Bank of Japan. So far there appear to have been no serious adverse effects of these deficits, financed in this way. However, the experience of many countries shows that there is such a thing as borrowing too much, relative to the capacity of the country's tax system to raise the revenue needed to service the debt. In general, as Table 1 and Figure 1 indicate, countries with severe inflation are countries with government deficits in excess of 3% of GNP, while countries without severe inflation are countries with government deficits below 3% of GNP. This is not an inviolable rule: Japan has been an exception for the past 5 or 6 years. But in many countries, when the deficit becomes large, heavy political pressure falls on the central bank to issue high powered money faster so as to help finance the deficit. And this can only result in inflation.

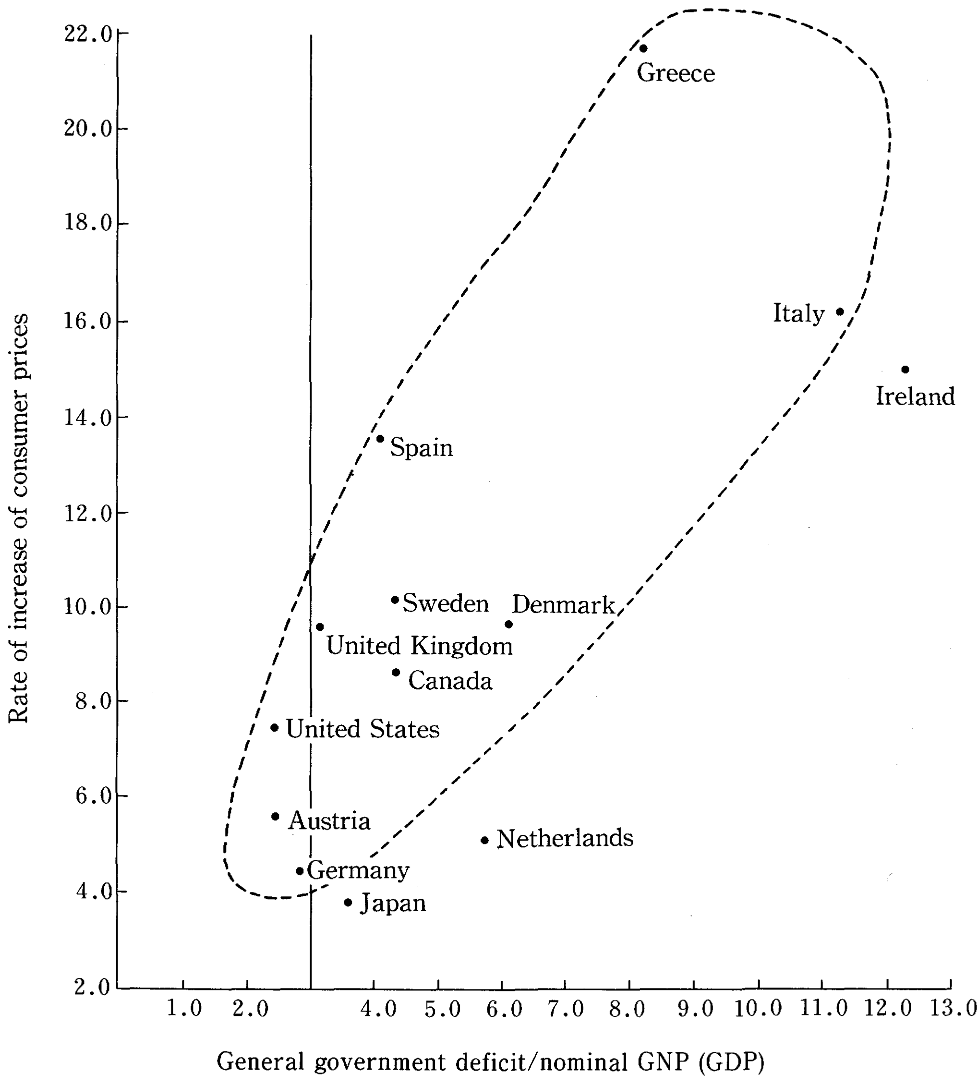
Table 1 Relationship between Government Deficit and Inflation

(Average of 1980–1984, %)

		General government deficit/nominal GNP(GDP) X 100	Rate of increase of consumer prices
Countries with high government deficit and high inflation	Ireland	△12.4	15.0
	Italy	△11.4	16.2
	Greece	△ 8.3	21.8
	Sweden	△ 4.4	10.3
	Spain	△ 4.2	13.6
	Denmark	△ 6.2	9.5
	United Kingdom	△ 3.2	9.6
	Canada	△ 4.4	8.7
Countries with low government deficit and low inflation	Germany	△ 2.9	4.6
	Austria	△ 2.5	5.5
	United States	△ 2.5	7.5
Exceptions	Japan	△ 3.6	3.9
	Netherlands	△ 5.8	5.1

Source: OECD *Economic Outlook* May 1986

Figure 1 Relationship between Government Deficit and Inflation
(Average of 1980-1984,%)



There is considerable discussion in the literature about government debt. At least two strands can be distinguished. One strand deals with how to incorporate government debt into macroeconomic analysis, and centers on the government budget restraint (GBR). The GBR equation says that the total of government expenditures must be equal to the total of financing by all methods, one of those methods being the sale of government debt to private and foreign investors. This is one of the ways government debt enters into the analysis. Other ways include its effect on taxes and transfer payments (since government interest is a transfer and is typically taxed), and its effect on the behavior of its holders.

Another strand deals with the question of whether government debt really has any significance in macroeconomic analysis at all, in spite of the GBR. This strand argues that under appropriate stringent assumptions investors do not regard government debt as net wealth, because the present value of expected future taxation to service the debt is a liability which is exactly equal to the debt's current market value. In such a world tax finance and debt finance of government expenditure are equivalent; when the paths of government expenditure and the high powered money stock have been chosen, the behavior of interest rates, output, and prices is determined, regardless of what mix of taxation and debt sale to investors is used to satisfy the GBR. Then the only role of the GBR is to determine investors' holdings of government debt, but these holdings have no feedback effect on interest rates, prices, or output. This is known as the Ricardian equivalence theorem, recently brought to prominence by Barro (1974, 1976). The assumptions include the following. All taxes are of the lump-type, having no allocative effects. When government debt is issued, it is certain that the interest and principal payments required to service the debt will be covered by future taxes (not by issuing money). And everyone lives forever and maximizes an intertemporal utility function. This last assumption may be relaxed by assuming that everyone has ancestors and descendants, and family members engage in intergenerational wealth transfers so as to maximize the intertemporal utility function of the family.

While the Ricardian equivalence theorem is well established as a theoretical proposition, the issue of its applicability to actual economies is still not decided in the minds of many economists, and is the subject of active research and debate.

This paper is in three parts. The first part II presents data for money multipliers and velocities in Japan since 1965, in support of the claim that they varied little, and that therefore it was relatively easy to control inflation by controlling the growth of monetary aggregates. The second part III-V describes the GBR, assembles data for it, and shows how the government deficits in Japan have been financed. It also shows the theoretical relation of the deficit to the rate of inflation. It then estimates the inflation rate that can be expected in long-run balanced-growth equilibrium, as a function of the ratio of the deficit to GNP. The third part VI-XII introduces a government debt variable and a GBR into a simple macroeconomic model. It then shows how the GBR interacts with the model when the Ricardian equivalence theorem does not hold, and when it does hold. XIII is a brief conclusion.

II. Recent Short-Run Behavior of Velocity and Money Multipliers

Table 2 shows the high powered money stock H, M_1, and M_2 + CD for Japan, together with their velocities, and the money multipliers for M_1 and M_2 + CD, and

Table 2 Japanese Monetary Aggregates at End of Fiscal Year and Their Velocities and Money Multipliers

F.Y.	H	M_1	M_2+CD	m(M_1)	m(M_2+CD)	Y	y	p	V(H)	V(M_1)	V(M_2+CD)
1965	2655.4	10065.1	25687.4	3.79	9.67	33602.3	92027.6	36.5	12.654	3.338	1.308
1966	2979.8	11377.6	29731.0	3.82	9.98	39508.9	102209.9	38.7	13.259	3.473	1.329
1967	3565.7	12945.2	34169.2	3.63	9.58	46239.4	113182.4	40.9	12.968	3.572	1.353
1968	4118.3	14817.0	39435.1	3.60	9.58	54760.4	127709.1	42.9	13.297	3.696	1.389
1969	5039.5	17798.2	46612.7	3.53	9.25	64920.0	142993.8	45.4	12.882	3.648	1.393
1970	5799.2	21167.3	55002.0	3.65	9.48	75152.0	153915.4	48.8	12.959	3.547	1.366
1971	6601.5	27062.5	68224.8	4.10	10.33	82806.3	161688.1	51.2	12.544	3.060	1.214
1972	8840.7	34475.3	85346.2	3.90	9.65	96539.1	176627.9	54.7	10.920	2.800	1.131
1973	11229.6	39778.8	98235.8	3.54	8.75	116679.2	184569.3	63.2	10.390	2.933	1.188
1974	12977.4	43671.2	109374.8	3.37	8.43	138155.8	183797.8	75.2	10.646	3.164	1.263
1975	13503.8	49755.2	126234.7	3.68	9.35	152209.4	190874.7	79.7	11.272	3.059	1.206
1976	15036.1	54854.6	142350.0	3.65	9.47	171152.5	199630.1	85.7	11.383	3.120	1.202
1977	16150.6	58814.5	157331.8	3.64	9.74	190034.8	210234.4	90.4	11.766	3.231	1.208
1978	17853.2	66619.9	177587.5	3.73	9.95	208780.9	221243.0	94.4	11.694	3.134	1.176
1979	20274.7	71005.8	194734.9	3.50	9.60	225452.6	232878.3	96.8	11.120	3.175	1.158
1980	20639.0	68023.3	208097.4	3.30	10.08	245162.7	242130.9	101.3	11.879	3.604	1.178
1981	21272.0	74484.0	230485.7	3.50	10.84	259668.8	250158.8	103.8	12.207	3.486	1.127
1982	23021.0	78675.1	247926.5	3.42	10.77	272382.9	258240.9	105.5	11.832	3.462	1.099
1983	24431.3	80512.1	267172.4	3.30	10.94	284121.0	267782.0	106.1	11.629	3.529	1.063
1984	25543.2	87675.6	291609.6	3.43	11.41	303155.7	281102.2	107.8	11.866	3.458	1.040

Source: See the Appendix

Units: For stocks : billion yen
For Y : billion yen per year
For y : billion 1980 yen per year
For money multipliers : none
For velocities : per year

real and nominal GNP and the GNP deflator, annually for fiscal years 1965-1984. Sources are given in the Appendix. Figure 2 shows the behavior of the velocities and Figure 3 shows the money multipliers through time. Neither the velocities nor the money multipliers have changed rapidly in the past decade, since the Bank of Japan began its successful anti-inflationary policy. $V(M_1)$ (the velocity of M_1) shows the most variation, but its annual growth rate for 1975-84 averaged only +1.4%, and did

Figure 2 Velocities of M_1, M_2 + CD, and H (High Powered Money)

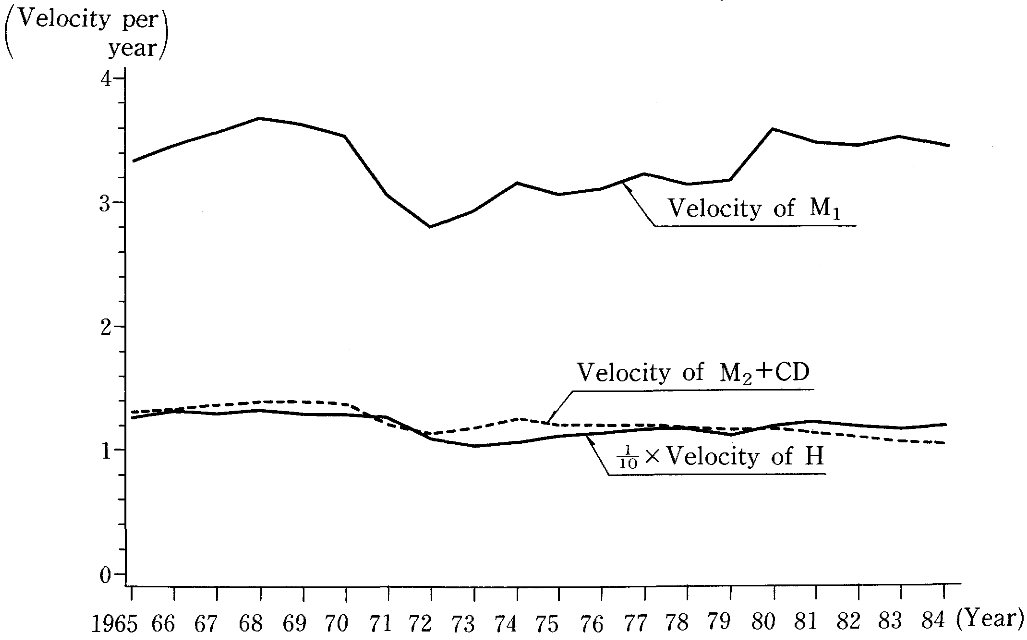
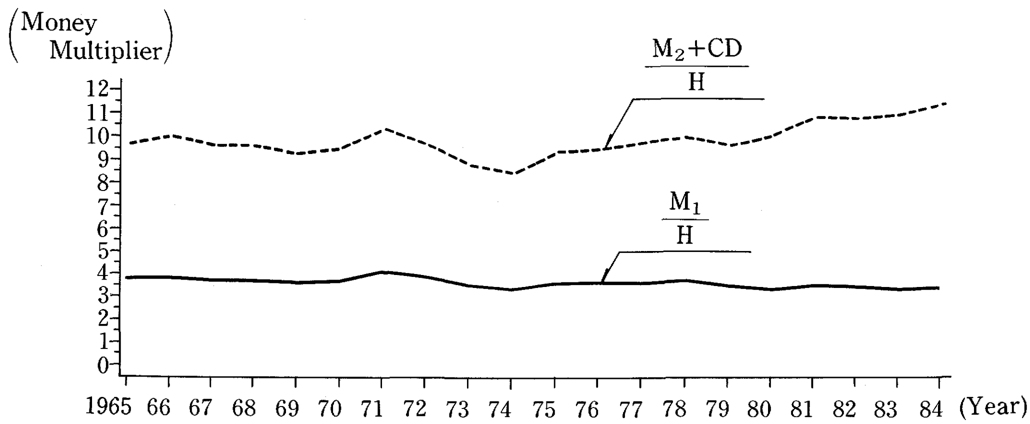


Figure 3 Money Multipliers for M_1 and M_2 + CD



not deviate much from that average. Even for the period 1972-84 its growth rate averaged only 1.8% a year. This evidence about Japan supports the view that rather slow average growth of a monetary aggregate, at approximately the growth rate of the real economy, with occasional adjustments to account for changes in velocity, will control inflation.

III. Bonds and the Government Budget Restraint in a Macaroeconomic Model

Many macroeconomic models contain government purchases of goods and services (henceforth to be called simply government purchases), government transfer payments, tax variables, and the money supply, but not government debt. Sometimes the omission of government debt is defended by stating that the bond equation, being dependent on the other equations of the model by Walras' Law, can be omitted.

In general this is not correct, as the following reasoning shows. Any equation that is dependent upon other equations can be derived from those other equations. Hence, every variable that appears in such a dependent equation must also appear somewhere among the other equations. Therefore, when a dependent equation that explains the bond variable is omitted by Walras' Law, the bond variable does not drop out of the model. If government debt is important in the economy, a variable representing it should appear in the model. That is where the GBR comes in.

The GBR says that total government expenditure for purchases and for transfer payments must be financed by some combination of taxes, borrowing from the central bank (that is, issuing high powered money), borrowing from others, and depleting stocks of foreign assets such as gold and foreign exchange reserves. This means that the authorities cannot independently choose the values of government purchases, transfers, taxes, high powered money, government debt held outside the central bank, and foreign assets. At least one of these policy variables will be chosen endogenously by the economy to satisfy the GBR, when the rest of them have been chosen exogenously.

Analysis of models containing the GBR has shown that the path of the economy, following a policy change, will depend upon which of the policy variables is allowed to adjust endogenously. The stability of the path may also depend on this choice. Several authors have obtained results suggesting that in simple models the adjustment path may be unstable if government debt is the endogenous variable. See, for example, Tobin – Buiter (1976) and Christ (1978, 1979).

In an open-economy model, there is at least one more endogenous policy variable: under fixed exchange rates the balance of payments will be endogenous, whereas under flexible exchange rates the exchange rate will be endogenous. And foreign holdings of government debt should be accounted for.

The GBR can be expressed in an equation. Let g stand for real government purchases, tr for real transfers other than interest, B for government debt (assumed to be perpetuities) held outside the central bank, r_1 for the interest rate on that debt, P for the price level, tx for real tax receipts, F for government and central bank holdings of gold and foreign exchange expressed in domestic currency, r^* for the interest rate on such foreign assets, H for the high powered money stock, and D for the derivative with respect to time. Then the GBR says spending equals sources of finance, thus:

$$g+tr+r_1\frac{B}{P}=tx+r^*\frac{F}{P}+\frac{DH+DB-DF}{P}. \quad (1)$$

Here DH, DB, and DF are to be understood as flows, taking account of asset quantity changes but not changes in asset prices.[1] It will be convenient to write t = taxes less net debt interest less other transfers, thus:

$$t = tx+r^*\frac{F}{P}-tr-r_1\frac{B}{P}. \quad (2)$$

Then the GBR can be written as

$$g = t + \frac{DH+DB-DF}{P}. \quad (3)$$

IV. Some Data Regarding the GBR in Japan

Tables 3, 4, and 5 show data for three definitions of the government budget deficit for Japan, together with the three major components of its financing. In each table, the three components are the increase in the high powered money stock held outside the government and the Bank of Japan, the increase in government debt held outside the Bank of Japan, and the decrease in gold and net foreign assets held by the government and the Bank of Japan.

The three definitions of the government deficit are as follows. In Table 3 the central government sector is as defined by the flow of funds accounts of the Bank of Japan. In Tables 4 and 5 the government sectors are as defined by the Economic Planning Agency in the System of National Accounts, Table 4 being for the central government only, and Table 5 for the general government sector including central

1. Of course the price of H does not change—it is always 1. But the price of a perpetuity that yields 1 yen per year is $1/r_1$, and it changes when r_1 changes. Denote the number of such perpetuities by b and their value by b/r_1. Then the change in value, $D(b/r_1)$, is $(Db)/r_1-(b/r_1^2)Dr_1$. The expression DB in the text denotes $(Db)/r_1$, excluding the capital gain $-(b/r_1^2)Dr_1$. The symbol B in the text stands for b/r_1.

and local government and the social security system. In each table, the definitions of all the variables correspond to the chosen definition of the government sector.

I have been unable to find quarterly data for government deficits defined as I wish to define them (namely, the decrease in financial net worth of a government sector, where government sectors are defined as in the System of National Accounts, and their deficits are measured by the transactions flows in their asset and liability accounts). Also, I have been unable to find annual data for such variables extending back before 1970. (The Bank of Japan prepares quarterly data for the flow of funds starting in 1955, but I prefer the national accounting data because the national accounting definitions of the government sectors correspond better to the distinction between public and private goods.) The national accounting data for fiscal years 1970

Table 3 Financing of Japan's Central Government Deficit (Flow of Funds Data)

F.Y.	Deficit	DH	DB	−DF
1965	−63.4	363.8	−321.2	−106.0
1966	209.8	324.4	−168.6	54.0
1967	176.3	585.8	−480.7	71.2
1968	−277.0	552.6	−219.5	−610.1
1969	−438.8	921.3	−951.9	−408.2
1970	−1156.0	759.6	−914.6	−1001.0
1971	−693.6	802.3	2434.2	−3930.1
1972	−776.4	2239.2	−726.0	−2289.6
1973	−2090.9	2388.9	−6745.9	2266.1
1974	193.9	1747.8	−1709.4	155.5
1975	4375.1	526.4	3971.9	−123.2
1976	5517.1	1532.3	4682.3	−697.5
1977	7999.0	1114.5	10090.8	−3206.3
1978	10244.8	1702.6	9293.2	−751.0
1979	9609.6	2421.5	5453.0	1735.1
1980	8869.6	364.3	12583.7	−4078.4
1981	10197.1	633.0	8255.2	1308.9
1982	10279.6	1749.0	9718.6	−1188.0
1983	11207.8	1410.3	10028.5	−231.0
1984	9231.4	1116.9	8346.6	−232.1

Source: See the Appendix
Units: Billion yen per year

Table 4 Financing of Japan's Central Government Deficit (SNA Data)

F.Y.	Deficit	DH	DB	−DF	RESIDUAL
1970	−244.9	744.2	−29.3	−626.2	−333.6
1971	399.4	901.5	3868.1	−3875.1	−495.1
1972	1054.4	2376.3	798.0	−352.6	−1767.3
1973	−124.5	2406.3	−4852.6	1333.9	987.9
1974	2426.4	1867.5	32.0	−196.3	723.2
1975	6222.1	687.4	5638.8	−21.0	−83.1
1976	7536.6	1574.5	6556.5	−752.9	158.5
1977	10079.4	1286.5	12149.5	−2696.1	−660.5
1978	12566.6	1071.1	11364.9	380.8	−250.2
1979	12186.5	3877.3	6561.3	2376.7	−628.9
1980	13203.2	−692.6	15688.5	−3104.0	1311.3
1981	14242.6	780.7	12266.6	405.9	789.5
1982	14234.8	1667.2	13561.2	545.4	−1539.0
1983	13723.1	838.8	13438.4	−408.2	−145.9
1984	12245.4	1887.4	12155.3	−263.6	−1533.7

Table 5 Financing of Japan's General Government Deficit (SNA Data)

F.Y.	Deficit	DH	DB	−DF	RESIDUAL
1970	−1638.7	739.0	−1391.8	−626.2	−359.7
1971	−1004.7	890.4	2462.0	−3875.1	−482.0
1972	−374.8	2303.0	−471.0	−353.0	−1853.8
1973	−1423.6	2353.1	−6049.0	1333.7	938.6
1974	66.9	1760.6	−2186.8	−196.5	689.6
1975	5246.8	586.0	4782.6	−21.9	−99.9
1976	5968.5	1598.5	4974.1	−754.4	150.3
1977	7757.0	1246.9	9905.7	−2699.5	−696.1
1978	10779.6	1325.5	9313.2	369.0	−228.1
1979	9369.6	2709.8	5091.1	2370.9	−802.2
1980	10295.9	229.2	11899.3	−3110.6	1278.0
1981	10182.9	755.4	8121.3	388.2	918.0
1982	9711.4	1753.1	8970.4	526.0	−1538.1
1983	9545.8	1097.3	9057.7	−424.6	−184.6
1984	8123.5	1372.7	8680.3	−293.8	−1635.7

Source: See the Appendix
Units: Billion yen per year

through 1984 are all from the Economic Planning Agency, either directly from their published reports on National Accounts (1985, 1986), or indirectly by way of the master file of data maintained on the computers of the Bank of Japan. For more detail, see the Appendix.

For the purpose of studying the GBR, it seems preferable to consider the central government rather than general government because only the central government creates high powered money by borrowing from the Bank of Japan. Therefore Table 4 is the preferable one. However, because there may be some interest in the role of general government, Table 5 is presented as well.

Figure 4 shows the deficits according to each definition. Figures 5, 6, and 7 show the deficits and their financing, as percentages of GNP, for each of the three definitions. All show that the budget had small deficits (or even surpluses) through 1974, and that the deficit/GNP ratio rose rapidly beginning in 1975, reaching a peak in 1978, and declining somewhat thereafter. All also show that the ratio of newly issued high powered money to GNP reached a peak of around 2% in 1972, which was about the time of the inflationary surge, and has been kept fairly steady at modest levels (generally under 1%) since 1975. The main source of financing of the large deficits has been the sale of government debt to investors outside the Bank of Japan, that is, increases in B.

V. The Relation of Deficits to the Equilibrium Rate of Inflation

The nominal deficit is financed by issuing high powered money and debt and by spending foreign exchange reserves:

$$\text{nominal deficit} = DH+DB-DF=H\left(\frac{DH}{H}+\frac{DB}{B}\frac{B}{H}-\frac{DF}{F}\frac{F}{H}\right). \qquad (4)$$

The inflation rate is related to the growth rate DH/H of the high powered money stock, through the growth rates of real output and of the velocity of H. That is, the inflation rate $\frac{DP}{P}$ is given by

$$\frac{DH}{H}+\frac{DV_H}{V_H}=\frac{DP}{P}+\frac{Dy}{y} \qquad (5)$$

where V_H is the velocity of the high powered money stock H and y is real output. Hence, Equation (4) indicates that deficits do not necessarily cause inflation if they are financed mainly by depleting foreign exchange reserves and issuing debt. But these two methods, alone or together, cannot finance a large deficit indefinitely. The depletion of foreign exchange reserves can continue only until they are used up. And pure debt finance cannot continue indefinitely, as an appendix shows. Hence a

Figure 4 Three Definitions of the Japanese Government Deficit as Percentages of GNP

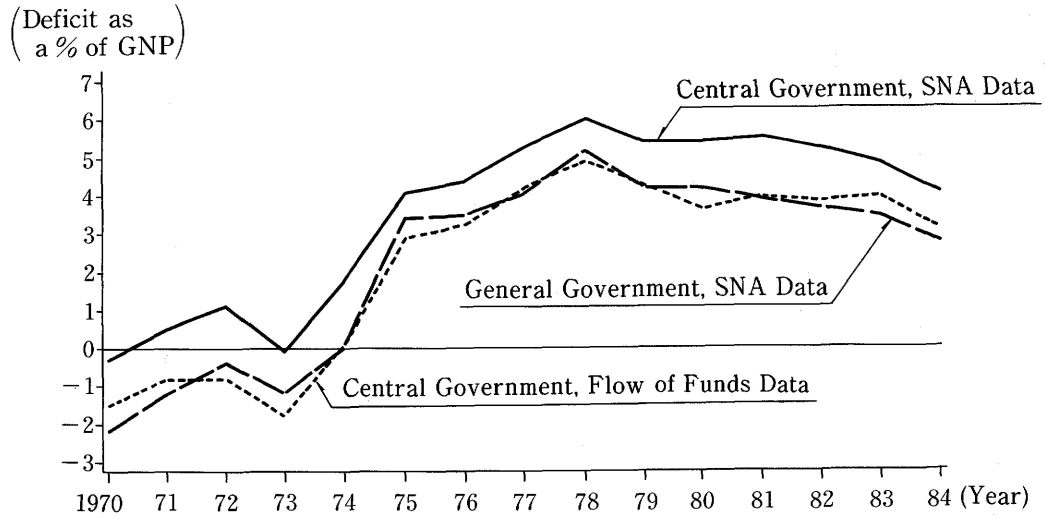


Figure 5 Financing of Japan's Central Government Deficit (Flow of Funds Data)

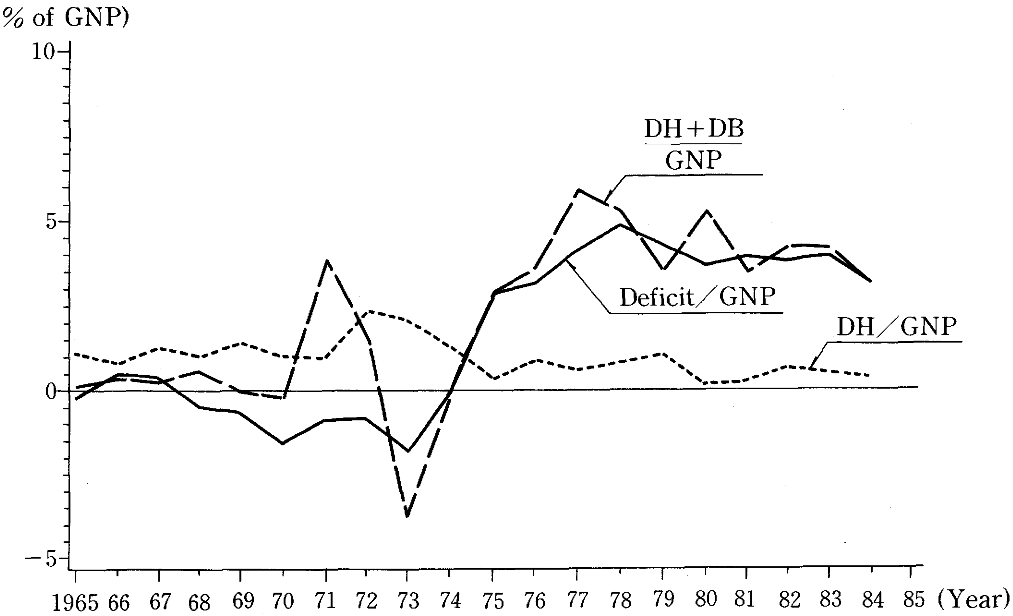


Figure 6 Financing of Japan's Central Government Deficit (SNA Data)

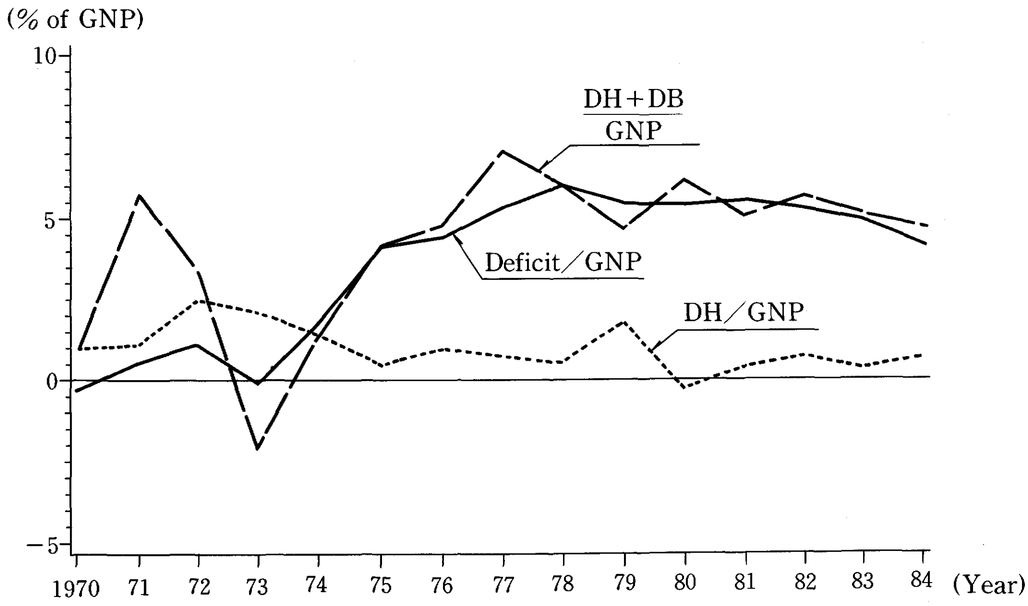
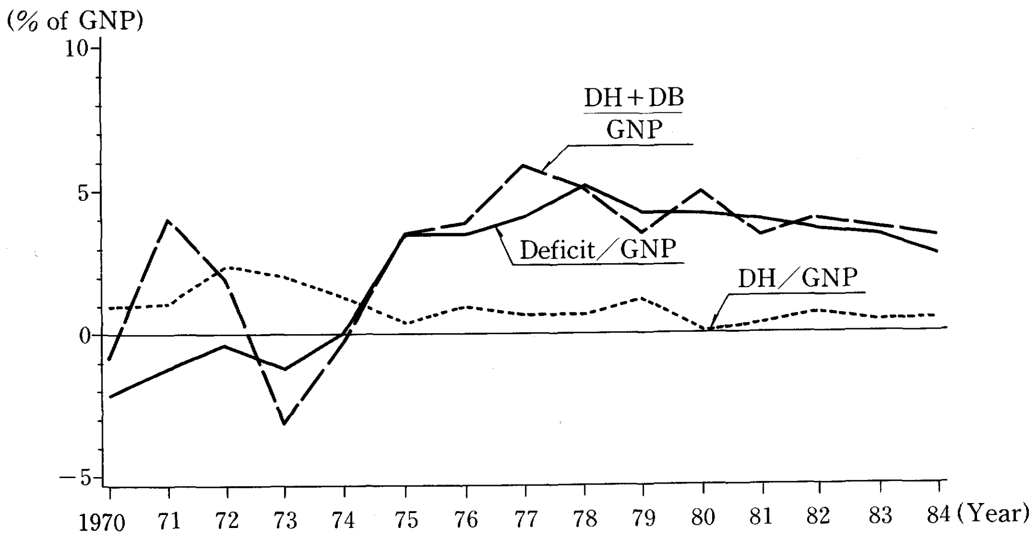


Figure 7 Financing of Japan's General Government Deficit (SNA Data)



sustainable path with perpetual large budget deficits involves some creation of high powered money.

On a sustainable long-run steady-state balanced-growth equilibrium path, the nominal stocks of high powered money, government debt, and foreign exchange reserves all have the same growth rate. Denote this rate by g_H. Then Equation (4) becomes

$$\text{nominal deficit} = g_H(H + B - F). \tag{6}$$

Dividing by nominal GNP yields

$$\text{deficit/GNP} = g_H(H + B - F)/\text{GNP}. \tag{7}$$

On such an equilibrium path, the ratio (H + B −F)/GNP is constant. And because the velocity of the high powered money stock is also constant on such a path, the growth rate of H equals the sum of the growth rates of the price level and real output, that is $g_H = g_P + g_y$. Therefore we find that on such an equilibrium path the inflation rate is given by

$$g_P = \frac{\text{deficit}}{\text{GNP}} \frac{\text{GNP}}{H+B-F} - g_y. \tag{8}$$

A higher deficit/GNP ratio leads to higher inflation, for a given GNP/(H+B−F) ratio and a given real growth rate g_y. The GNP/(H+B−F) ratio can decline for a time, if debt grows faster than GNP, but cannot fall below a certain limit, as we have seen. Furthermore, GNP/(H+B−F) is likely to increase during inflation, because the velocity of H will increase, and this will worsen the inflation. At present, in Japan, the ratio of GNP to (H+B−F) is about 2.6 when central government data are used, and g_y is about .05 per year. Therefore, if the Japanese economy were to follow a steady state equilibrium path starting now, with those magnitudes, the equilibrium inflation rate would be equal to about 2.6 times the central government's deficit/GNP ratio, minus .05. The following table gives some examples.

Central Government Deficit GNP	g_H =2.6 (Deficit/GNP)	inflation rate g_P $=g_H - .05$
.01	.026	−.024
.02	.052	.002
.03	.078	.028
.04	.104	.054
.05	.130	.080

These calculations suggest that if inflation is to be prevented in Japan in the long run, it will be necessary to keep the deficit/GNP ratio at a moderate level on the average.

VI. The Definitions of Wealth and Disposable Income

The introduction of government debt into a macroeconomic model requires a careful definition of wealth. Let us suppose that the private sector holds B_P yen's worth of government bonds, and regards them as λB_P yen's worth of net wealth, where λ is a constant between 0 and 1, inclusive. The fraction $1 - \lambda$ represents the extent to which the private sector regards its wealth as diminished by the present value of the future taxes that are required to service the debt. If the private sector fully accounts for these taxes, $\lambda = 0$ and government debt is not regarded as wealth at all. If the private sector ignores these taxes, $\lambda = 1$ and government debt is regarded as wealth at full value.

Now suppose that foreign investors hold an amount B_f of domestic government debt, and that the taxes required to service this foreign-held debt must be paid by the private domestic sector. Then the private sector's wealth will be diminished by $1-\lambda$ times the foreign-held debt B_f.

Therefore we will define the real wealth of the private sector as the sum of its holdings of real physical capital k, real high powered money H/P, λ times its real holdings of government debt B_p/P, and real net claims against the rest of the world A/P, minus $(1-\lambda)$ times foreign-held domestic government debt B_f/P, thus:

$$w = k + \frac{H + \lambda B_p - (1-\lambda)B_f + A}{P} \tag{9}$$

Therefore the change Dw in wealth is given by

$$Dw = DK + \frac{DH + \lambda DB_p - (1-\lambda)DB_f + DA}{P} + cg \tag{10}$$

where DH, DB_p, DB_f, and DA are nominal flows of assets at the current period's asset prices, and cg, the real capital gain on the holdings of financial assets, is

$$cg = -\frac{H + \lambda B_p - (1-\lambda)B_f + A}{P^2}DP - \frac{\lambda B_p - (1-\lambda)B_f}{Pr_1}Dr_1. \tag{11}$$

The two terms in this expression reflect the real capital gains arising from changes in the price level P and the market value of long term bonds due to changes in interest rates Dr_1. An additional term could be included to reflect changes in the value of foreign assets due to changes in the exchange rate.

We will use the national income identity

$$y = c + DK + g + nx \tag{12}$$

where y = real NNP, c = real consumption, and nx = real exports.

Disposable income is the sum of consumption plus additions to wealth. Therefore, using first the expression for Dw and then the national income identity, it can be expressed as

$$yd = c+Dw=c+Dk+\frac{DH+\lambda DB_p-(1-\lambda)DB_f+DA}{P}+cg \tag{13}$$

$$=y-g-nx+\frac{DH+\lambda DB_p-(1-\lambda)DB_f+DA}{P}+cg.$$

Notice that this expression defines disposable income in terms of the resources remaining after government purchases and net exports are subtracted from income, namely, $y-g-nx$, and after certain adjustments to assets. It does not involve taxes or transfer payments. (We will come back to this expression later, when we discuss a form of the model in which debt and taxes don't matter.) Alternatively, the GBR can be used to express g in terms of taxes and transfers and other variables. (Note that DB in the GBR is equal to $DB_p + DB_f$.) When this is done, the expression $(DA+DF-DB_f)/P-$ nx appears, but it drops out because it is zero by virtue of the balance of payments equation. Then disposable income can be written in terms of taxes less transfers, thus:

$$yd = y-t-\frac{(1-\lambda)(DB_p+DB_f)}{P}+cg. \tag{14}$$

If government bonds are regarded as wealth, so that $\lambda=1$, bonds drop out of this definition of disposable income. And if the price level and the interest rate are constant, cg=0. Then disposable income becomes simply $y-t$, as in simple textbook models.

VII. A Simple Model of an Open Economy

Consider an open economy containing 4 sectors: the government including the central bank, the private domestic sector, the foreign sector, and a fictitious sector to be called the capital sector, whose liabilities are the physical capital held by the other sectors, and whose assets are the net worths of the other sectors. Consider 5 types of assets: physical capital k, domestic high powered money H, domestic government debt of which B_p is net private domestic holdings and B_f is foreign holdings, foreign-issued debt A, and gold and foreign exchange reserves F. Domestic private debt is assumed to be held only within the domestic private sector, and to be a perfect substitute for domestic government debt, so that the private sector's net holdings of both private and government debt are equal to B_p, its net holdings of government debt alone.

Table 6 shows the intersectoral claims among these 4 sectors. The entries in any column are the assets of the sector named at the top of the column, and the entries in any row are the liabilities of the sector named at the left of the row. Each sector's balance sheet equation is given by equating the total assets at the bottom of its column to the total liabilities at the right of its row. Only 3 of the 4 balance sheet equations are independent, because any one of them can be obtained by appropriate addition or subtraction from the other three. Let us omit the capital sector's balance sheet. Then, after the other equations of the model have been specified, it will turn out that the government's net worth variable appears only in the government's balance sheet equation, and similarly, the foreign sector's net worth variable appears only in that sector's balance sheet equation. Therefore those two equations do not affect the behavior of the rest of the model: they can be set aside and ignored, unless and until one wishes to use them to find the net worths of their two sectors.

Table 6 Nominal Intersectoral Claims, in Yen

assets of / liabilities of	private	government and central bank	foreign	capital	total
private	–	$(1-\lambda)(B_p+B_f)$		wP	$(1-\lambda)(B_p+B_f)$
government and central bank	$H+B_p$	–	B_f	w_gP	$H+B_p+B_f$ $+w_gP$
foreign	A	F	–	w_fP	$A+F+w_fP$
capital	kP	$k'P$		–	$kP+k'P$
total	$H+B_p$ $+A+kP$	$(1-\lambda)(B_p+B_f)$ $+F+k'P$	B_f	$wP+w_gP$ $+w_fP$	$\Sigma\,\Sigma$

The model then has 14 equations. It will be assumed that government debt B_p is the endogenous policy variable in the GBR, and that exchange rates are allowed to float freely so that e is endogenous and F is exogenous.

Then the 14 endogenous variables are

A = private net holdings of foreign debt, in yen
B_p = private net holdings of government debt (perpetuities)
c = private real consumption
e = exchange rate (yen per unit of foreign currency)
k = private real physical capital stock
nx = real net exports

p = price level
r_1 = nominal interest rate on domestic debt (government and private)
r_2 = nominal rate of return on equity (titles to physical capital)
r_3 = real marginal product of capital
t = real taxes less net government interest payments less non-interest transfer payments
w = real private net wealth
y = real national product
yf = real productive capacity

The exogenous variables are

B_f = foreign holdings of domestic government debt (perpetuities)
$(De/e)^e$ = expected rate of growth of the exchange rate
F = government and central bank holdings of gold and foreign exchange, in yen
g = real government purchases of goods and services
H = high powered money stock
k' = government's real physical capital stock
P^* = foreign price level
$(dP/P)^e$ = expected inflation rate
r^* = foreign interest rate
rt = rate of taxes less transfer payments

Different policy choices would lead to different selections of endogenous variables. In particular, the endogenous macroeconomic policy variables might be H or g or a tax rate or a transfer rate instead of B_p. And the endogenous foreign exchange policy variable might be the balance of payments instead of the exchange rate. Indeed, more complex policies can be considered, for example, a rule that says what fraction of a deficit is to be financed by money creation and what fraction by borrowing; or a rule for intervention in the foreign exchange market. Such a rule constitutes an additional equation for the model, and causes an additional policy variable to become endogenous. But for simplicity only one policy choice will be considered here, namely the one corresponding to the above list of endogenous variables.

The equations of the model are displayed in Table 7. Equation 1 is an expectations-adjusted Phillips curve. It says that the inflation rate will be the same as, or less than, or greater than expected, according to whether real output is equal to, or less than, or greater than capacity. Equation 2 says that investment depends on the difference between the real marginal productivity of capital and the expected real rate of return on equity. Equation 3 is the balance of payments equation. It says that the net change in domestic holdings of foreign assets is equal to net exports (which

Table 7 Equations of the Model

Associated Endogenous Variables	Equation Number	Equation	Description
P	1	$\frac{DP}{P} = f_1\left[\frac{y}{yf}\right] - 1 + \left[\frac{DP}{P}\right]^e$	inflation
k	2	$DK = f_2\ [r_3 - r_2 + (DP/P)^e]$	investment
A	3	$DA + DF - DB_f = P \cdot nx$	balance of payments
B	4	$g = t + \frac{DH + DB_p + DB_f - DF}{P}$	GBR
yf	5	$yf = f_5\ (k + k')$	production function
r_3	6	$r_3 = f_6\ (k + k')$	marginal production of k
w	7	$w = k + \frac{H + \lambda B_p - (1 - \lambda)B_f + A}{P}$	private wealth
e	8	$\frac{A}{P} = f_8\left[y, w, r_1, r_2, e, \left[\frac{DP}{P}\right]^e, \left[\frac{De}{e}\right]^e\right]$	foreign bond demand
r_2	9	$\frac{\lambda B_p}{P} = f_9\left[y, w, r_1, r_2, e, \left[\frac{DP}{P}\right]^e, \left[\frac{De}{e}\right]^e\right]$	domestic bond demand
r_1	10	$\frac{H}{P} = f_{10}\left[y, w, r_1, r_2, e, \left[\frac{DP}{P}\right]^e, \left[\frac{De}{e}\right]^e\right]$	money demand
c	11	$c = f_{11}\left\{\left[y - g - nx + \frac{DH + \lambda DB_p - (1 - \lambda)DB_f + DA}{P} - \frac{H + \lambda B_p - (1 - \lambda)B_f + A}{P^2} DP - \frac{\lambda B_p - (1 - \lambda)B_f}{Pr_1} Dr_1\right], r_1, r_2, \left[\frac{DP}{P}\right]^e, w\right\}$	consumption
nx	12	$nx = f_{12}\left[y, \frac{P}{P^*e}, \frac{F + A}{P} r^* - \frac{B_f r_1}{P}\right]$	net exports
t	13	$t = f_{13}\left[y, \frac{r_1(B_p + B_f) - r^*F}{P}, rt\right]$	taxes less transfers
y	14	$y = c + Dk + g + nx$	income identity

include net interest from abroad). Equation 4 is the GB$\dot{R}$, which has already been described. Recall that DB in Equation (3) has been replaced by its equivalent, $DB_p + DB_f$, here. Note that these 4 equations are dynamic, that is each of them determines the rate of change of a state variable (P, k, A, or B_p) from one period to the next. We will speak of the past levels of these variables as predetermined.

Equation 5 is the production function. It gives real capacity as a function of the total (private and government) capital stock. It can be thought of as a long-run aggregate supply curve, in the sense that if enough time is allowed or the price level to come to equilibrium, then the quantity of output supplied will be equal to the normal capacity level yf. Equation 6 says that the marginal productivity of capital depends on the total capital stock. Equation 7 is the definition of private wealth, discussed earlier. Equation 8, 9, and 10 are the private portfolio demand equations for real foreign assets, domestic debt, and high powered money. Each depends on income, interest rates, wealth, the exchange rate, and expectations about inflation and changes in the exchange rate.A fourth portfolio demand equation, for physical capital, is dependent upon Equations 7-10, since it can be obtained by subtracting Equations 8, 9, and 10 from Equation 7. Hence this physical capital demand equation has been omitted. (Either Equation 7, 8, 9 or 10 could have been omitted instead, without affecting the model's behavior.) Equation 11 says that consumption depends on disposable income as defined in Equation (13) above, as well as on expected real interest rates and wealth. Equation 12 says that net exports depend on income, the terms of trade, and net interest received from abroad. Equation 13 says that taxes less net government interest and other transfers depend on income, tax-transfer rates, and net government interest payments $(r_1(B_p+B_f)-r^*F)/P$. Equation 14 is the national income identity mentioned above.

It would be possible to endogenize the expected inflation rate and the expected change in the exchange rate, by introducing two more equations. If those two equations were to contain no new variables in adition to those that already appear in Equation 1-14, then most of the analysis that follows would be unaffected.

The static equilibrium form of the model is obtained by setting all time-derivatives equal to zero, that is, the exogenous variables DB_f, DF, and DH, the exogenous expectations $(De/e)^e$, and $(DP/P)^e$, and the endogenous rates of change DP, DK, DA, DB_p, and Dr_1. This means that in static equilibrium output y is equal to capacity yf in Equation 1, the marginal product of capital r_3 is equal to the expected real rate of return on equity $r_2-(DP/P)^e=r_2$ in Equation 2, the current account is balanced in Equation 3, and the budget is balanced in Equation 4.

Of course, it is only for simplicity that equilibrium is defined as having zero growth, zero inflation, and a balanced budget. One could consider a balanced-growth equilibrium with a steady rate of inflation and a continuous budget deficit or surplus, and transform all variables so as to express the model in terms of deviations from such a balanced-growth equilibrium. The resulting transformed model would look very much like Equations 1-14.Christ (1978) gives an example of such a transformation when the equilibrium has zero real growth but steady nominal growth and a constant real budget deficit (that is, steady inflation).

VIII. Recursive Dynamics

A model of this sort generates the time paths of its endogenous variables, given the paths of its exogenous variables, in a two-step recursive manner. The model has two subsets of variables, and two subsets of equations. The dynamic variables are the ones whose rates of change from period t-1 to period t are determined by dynamic equations, given the values of all variables for period t-1. The static variables are the ones whose levels at time t are determined by static equations, given the levels of the dynamic variables for period t. The dynamic equations are a subset of the equations, containing time derivatives of the dynamic variables. There should be as many equations in this subset as there are dynamic variables. The remaining equations will be called the static equations (even though they may contain some time derivatives).

Consider first a simple case in which the static equations don't contain any time derivatives. Then the model may be expressed in terms of vector equations, as follows:

$$F(y_{1,t-1},y_{2,t-1},Dy_{1t},x_t) = 0 \text{ (dynamic equations)} \quad (15)$$
$$y_{1t} = y_{1,t-1} + Dy_{1t} \text{ (updating equations)} \quad (16)$$
$$G(y_{1t},y_{2t},x_t) = 0 \text{ (static equations)} \quad (17)$$

Earlier lagged variables may be included, but are not shown. Here y_1 is a vector of n dynamic endogenous variables, y_2 is a vector of m static endogenous variables, x is a vector exogenous variables, and F and G are vector functions of order n and m respectively. The first vector equation (15) represents the dynamic part of the model, and the third vector equation (17) represents the static part. At the beginning of period t, the levels of all variables for period t-1 have been predetermined. The values for period t are generated as follows. The n dynamic equations (15) involving F are solved or Dy_{1t}, the change in y_1 from period t-1 to period t. Then y_1 for period t is obtained from Equation (16) by adding to the level for period t-1 the change so obtained. Then the m static quations (17) involving G are solved for y_2 to obtain its value for period t. Then the cycle repeats for the next period, t+1, and so on.

In a more general case some time derivatives of dynamic variables appear in the static equations. For example, P, k, A, and B_p are the dynamic variables in the model discussed earlier, Equations 1-4 are the dynamic equations, and Equations 5-14 are the static equations. Observe from Table 7 (to be described in the next section) that DP, Dk, DA, and DB_p appear in the static equations 11 and 14. In such a case, the recursive dynamic process described just above works in almost the same way as before, with one exception: Replace the Dy_1 that appears in the static equations by a function of y_1, y_2, and x, which is obtained by solving the dynamic equations involving F for Dy_1. Then the static part no longer contains any dynamic variables, and the recursive dynamic process can proceed as before. (Note that although Equation 11 contains Dr_1, Equation 11 is not a dynamic equation, and r_1 is not a dynamic variable: its level at time t is determined by Equations 5-14, given the values of the dynamic variables P, k, A, and B_p for period t, and the lagged value of r_1 (and perhaps other variables) from period t-1.)

In the 14-equation model presented above, the process works as follows. At the start of period t, all variables for period t-1 are predetermined. Given these predetermined variables, the dynamic equations 1-4 determine the changes DP, Dk, DA, and DB_p between periods t-1 and t. These are added to the old levels of P, k, A, and B_p to get the new levels for period t. Then the static equations 5-14 determine the values of the remaining 10 endogenous variables for period t, given the now predetermined levels of P, k, A, and B_p for period t. Then the cycle repeats for later periods.

IX. The Causal Structure of the Model

In order to make it easy to visualize the structure of the model, it is useful to construct a table showing which endogenous variables appear in each equation. This is done twice, once for the dynamic version of the model in Table 8 and once for the static equilibrium version in Table 9. It is convenient to associate each endogenous variable with one of the equations in which it appears (there is typically more than one way to do this, unless the model is fully recursive). The variable so associated with each equation is shown by an x in the variable's column and the equation's row in Tables 8 and 9. Other endogenous variables appearing in an equation are shown by circles in the appropriate columns in that equation's row.

Consider first Table 8, for the dynamic version of the model.[2] Each of its first 4 rows has only a single entry in the first 4 columns corresponding to the 4 dynamic variables. This means that each of the first 4 equations can be regarded as a single-equation model that determines one dynamic variable, in terms of values of static variables and levels of the dynamic variables. Further inspection of the next 3 rows shows that each of them has only a single entry in the first 14 columns. This means that each of the 3 equations 5, 6, and 7 can be regarded as a single-equation model determining one variable in terms of exogenous variables and the levels of state variables. The 3 variables so determined are yf, r_3, and w.

Inspection of the last 7 rows of Table 8 does not obviously reveal whether the last 7 equations are fully simultaneous or not. McElroy (1978) gives a simple method for determining the causal structure of a model. It reveals that 6 of these 7 equations

2. Table 8 does not have a column for Dr_1, because Dr_{1t} has been replaced by $r_{1t}-r_{1,t-1}$. Thus r_1 is treated like the other static variables.

Table 8 Endogenous Variables in the Dynamic Version of the Model

	Equation	Endogenous Variables														Predetermined Variables			
		Dynamic				Static													
		DP	Dk	DA	DB_p	yf	r_3	w	e	r_2	r_1	c	nx	t	y	p	k	A	B_p
1	inflation	×				○									○	○			
2	investment		×				○			○									
3	balance of payments			×									○			○			
4	GBR				×									○		○			
5	production function					×											○		
6	marginal production of k						×										○		
7	private wealth							×								○	○	○	○
8	foreign bonds demand							○	×	○	○				○	○		○	
9	domestic bonds demand							○	○	×	○				○	○			○
10	money demand							○	○	○	×				○	○			
11	consumption	○		○	○			○		○	○	×	○		○	○		○	○
12	net exports								○				×		○	○		○	
13	taxes-transfers										○			×	○	○			○
14	income identity		○									○	○		×				

Table 9 Endogenous Variables in the Equilibrium Version of the Model

	Equation	y	r_2	nx	t	yf	r_3	k	A	B_p	r_1	w	e	P	c
1	inflation	X				O									
2	investment		X				O								
3	balance of payments			X											
4	GBR				X										
5	production function					X		O							
6	marginal production of k						X	O							
7	private wealth							X	O	O		O		O	
8	foreign bonds	O	O						X		O	O	O	O	
9	domestic bonds	O	O							X	O	O	O	O	
10	money	O	O								X	O	O	O	
11	consumption	O	O								O	X			O
12	net exports	O		O					O				X	O	
13	taxes-transfers	O			O					O	O			X	
14	income identity	O		O											X

(all except Equation 13) are fully simultaneous, determining the 6 variables e, r_2, r_1, c, nx, and y. Then Equation 13 determines t, given the other variables.

Hence the model is not fully simultaneous, but has 9 causal segments, as follows:

1–4: Equations 1-4 are from separate segments, which determine respectively DP, dk, DB_p, and DA, given the values of variables from the previous period.

5–7: Equations 5, 6, and 7 are three separate segments, which determine respectively, yf, r_3, and w, given the new levels of P,k, A, and B_p that have been determined by adding to their previous levels their first differences obtained from Equations 1-4.

8: Equations 8-12 and 14 are a simultaneous segment that determines the 6 endogenous variables e, r_1, r_2, c, nx, and y, given the values of the 7 variables that have been determined by Equations 1-7.

9: Equation 13 is a one-equation segment that determines t, given the values of the other already-determined variables.

The same method can be applied to Table 9 to find the causal structure of the equilibrium version of the model. Equation 3 (the balance of payments) is a complete one-equation model that implies nx = 0. Equation 4 (the balanced-budget GBR) is a complete one-equation model that determines taxes less transfers t. The remaining 12 equations are a simultaneous system, which does not decompose into causal segments.

X. The Reduction of the Model to Two Static and Four Dynamic Equations

It is useful to reduce the static subset of equations algebraically to two equations, a long-run aggregate supply curve and an aggregate demand curve (AS and AD). The AS curve is Equation 5, giving the level of productive capacity as a function of the real capital stock of the economy. The AD curve gives the level of real GNP demanded in terms of exogenous variables and the endogenous levels of dynamic variables that have been predetermined by the dynamic equations. One of these predetermined variables is the price level P; it is the one that is usually plotted against output y in a graph of the AD curve. The AD curve is obtained in two steps. First, substitute Equations 1-4 for DP,Dk, DA, and DB_p wherever they appear in Equations 6-14 (that is, in Equations 11 and 14). Second, by repeated substitution, eliminate from this new version of Equations 6-14 all static variables except y, obtaining the AD equation which expresses y in terms of predetermined variables. The result, ignoring lags that may appear, and supposing at expectations have been endogenized as described in VII, is

$$y = f(P,k,A/P,B_p/P,B_f/P,H/P,F/P;g,rt,k',r^*,P^*) \tag{18}$$

The semicolon separates exogenous variables (which are listed after it) from predetermined endogenous variables. The reason that B_f/P,l H/P and F/P are endogenous is that, while B_f and H and F are exogenous, P is endogenous. This equation can be called partially reduced-form equation, based on the static equations 6-14. It contains only one static variable, y. It is not a reduced form equation of the whole model, because it contains current values of the dynamic endogenous variables P, k, A, and B_p which have been predetermined by the previous operation of the dynamic part of the model. It is identified, since all its explanatory variables are predetermined.

Note that this equation contains more variables than the celebrated St. Louis equation, which has sometimes been regarded as a reduced form equation. See Anderson – Carlson (1970).

Suppose that now, in Equations 1-4, the level of each static variable (other than real output and capacity) is replaced by its partially reduced-form solution in terms of exogenous and predetermined variables from Equations 5-14. Then the model has been reduced to 6 equations in 6 endogenous variables, P, k, A, B_p, y, and yf. Its operation can be thought of as follows. Given the previous period's levels of all variables, the four dynamic equations determine the current period's changes DP, Dk, DA, and DB_p. The new levels of P, k, A, and B_p are obtained by adding the changes to the old levels. Then the AS and AD curves determine current real output and capacity in terms of the current exogenous variables and the current levels of P, k, A, and B_p. Then the cycle repeats.

XI. The Reduced Form of a Dynamic Model

One can consider two kinds of reduced form for a dynamic model. One expresses the current period's endogenous variables in terms of their previous values and exogenous variables. This is in effect a system of difference or differential equations. To find the explicit path of each variable as a function of time, one needs to find the solution of these difference or differential equations. Such a solution requires that the path of each exogenous variable be specified as a function of time (in a purely static model the function of time that emerges for this purpose is a constant).

The second kind of reduced form of a dynamic model comes from the static equilibrium form of the model. It gives the static equilibrium values of all endogenous variables in the model, as functions of the exogenous variables.

XII. How the Model Changes If Government Bonds Are Not Net Wealth

Suppose that the private sector does not regard government bonds as part of its wealth, that is, $\lambda = 0$. Then B_p disappears from Equations 7,9, and 11, and appears in the model only in the GBR equation 4 which determines DB, and in the tax-transfer equation 13. Now suppose that all taxes and transfer payments are of the lump-sum type, so that they have no allocative effects. Note that in Equation 11 (the consumption equation) disposable income is expressed in terms of government purchases g instead of taxes less transfers t. Therefore t, like B_p, appears only in Equations 4 and 13. It is determined by Equation 13 in lump-sum fashion. However, foreign-held debt B_f does not disappear.

Therefore the 12 equations excluding 4 and 13 are a complete subsystem that determines all endogenous variables except t and B_p. This illustrates the point made by Barro (1974, 1976), namely, that if government bonds are not wealth, and if taxes and transfers are lump-sum, and if it is certain that debt charges will be covered by future taxes, then the only macroeconomic policy variables that matter are government purchases and the money supply; once they are chosen, it makes no difference whether the remaining government expenditures are financed by taxes now, or by borrowing from the market now and taxes later. Interest rates, investment, capacity, real output, and prices are determined with no dependence whatever upon taxes or upon market borrowing. In such a model, the values of t and B_p are of no consequence. Their values can be obtained from Equations 4 and 13 if desired, but nothing in the economy is affected by them. In such a case, the AD equation (18) would not contain variables for B_p or tax-transfer rates.

Therefore a test of the Ricardian equivalence proposition could be based on whether or not the coefficients of B_p and taxes in the aggregate demand equation are zero. If they are, the Ricardian proposition is supported.[3,4]

If debt and taxes do not influence the other variables in any way, it would mean that the government could cut its taxes to zero, and perpetually finance all its expenditures by issuing debt, without any change in the money stock, prices, interest rates, or any other variable besides debt and taxes. As indicated in V, the interest burden of the debt would then grow to exceed the taxing capacity of the economy, and hence such a policy could not be sustained.

XIII. Conclusion

Japan has had rather large budget deficits since 1975, sometimes as high as 5% of GNP. As the government budget restraint shows, deficits need not lead to inflation in the short run. Indeed, Japan has an enviable record of price stability in the last decade, because the deficits were not financed to an excessive degree by issuing money. But large persistent deficits entail inflation on a sustainable balanced-growth equilibrium path. A macroeconomic model containing the government budget restraint can be helpful in understanding the effects of different methods of financing the government budget.

3. Estimates of several versions of the aggregate demand equation (18) were attempted, without success. Data on a consistent basis were available only annually and only for 1970-1984, as noted in the text. Lagged variables were required in order to begin to obtain good fits and some significant coefficients, but then the number of degrees of freedom became very small: only 1 or 2. And when approximate data for 1969 (constructed by assuming zero values for some small unavailable components) were included in the sample, the results were drastically changed. I conclude that the available sample is so small, relative to the length of the lags required to represent the economy's adjustment process, that there are not enough degrees of freedom to yield credible estimates of an aggregate demand equation such as Equation (18).

4. Hirschhorn (1984) shows that a positive correlation between government debt and output does not imply that government debt is wealth, if individuals have imperfect information about future debt issue. This is because the unanticipated part of the debt issue will lead to an increase in output even if government debt is not wealth.

DATA APPENDIX

I. General Comments Regarding the Data

Data are for fiscal years. Fiscal year t ends on March 31 of calendar year t + 1.

Where the same letter is used both in upper case and lower case form, such as Y and y, upper case denotes current dollars and lower case denotes constant dollars in calendar year 1980 prices.

Unless otherwise stated, the units for flows are billions of yen per year; the units of stocks are billions of yen; price deflators have the value 1.00 in calendar year 1980.

Three definitions of the government sector are used in what follows. One is central government as defined in the Flow of Funds accounts; it is used in IV of this Appendix. The other two are central government and general government as defined in the National Accounts; they are used in III.A and III.B of this Appendix.

II. Data from the Master File of the Bank of Japan

Y = nominal GNP.
y = real GNP.
P = GNP deflator.
H = high powered money stock outside the central government (as defined by the Flow of Funds accounts) and the Bank of Japan, year end.
M_1 = narrow money stock, year-end.
M_2+CD = broad money stock, year-end.
CCA = capital consumption allowances.
GDCF = gross domestic capital formation.
gdcf = real gross domestic capital formation.
Dk = real net domestic capital formation, estimated by (GDCF – CCA)/(GDCF/gdcf)
k = real capital stock, computed by cumulating Dk starting from a benchmark value of zero at the end of 1964. (Because of the zero benchmark, the intercept of any linear equation containing k will be in error, but the slopes will not.)
m = money multiplier = (monetary aggregate)/H in Table 2.
V = velocity of circulation = Y/(monetary aggregate) in Table 2.

III. Data from EPA Reports on National Accounts (1985, 1986)

The sources given here are for the 1980-1984 data obtained from EPA (1986), but those for 1970-1979 from EPA (1985) are similar because the formats of the two reports are similar.

Each of the variables defined in this section is computed twice, once for each of two National-Accounts definitions of the consolidated Bank-of-Japan-plus-government sector. One definition is the Bank of Japan plus the central government (including nonprofit institutions controlled by government (jigyodan)), and the other is the Bank of Japan plus general government. The EPA National Accounts tables contain columns for the sectors and subsectors needed for this purpose.

A. Current Transactions Data from EPA (1986), Part 1 Flow, [3] Supporting Tables, Table 6 Current and Capital Transactions by the Subsectors of General Government, pp. 228-231

G = government purchases of goods and services = line 12 (final consumption expenditure) + line 28 (gross capital formation).

INT = government net interest payments = line 13 (interest paid) − line 1 (interest received).

TX = tax receipts = line 11 (current receipts) − line 1 (interest receipts) − lines (8 + 9 + 10) (current transfers received).

TR = net transfers paid (other than interest) = line 22 (current disbursements) − line 12 (final consumption expenditure) − line 13 (interest paid) − (lines 8 + 9 + 10) (current transfers received) − (lines 25 + 26) (capital transfers received).

TXR = taxes less non-interests transfer payments = TX − TR.

T = taxes less all transfer payments = TX − TR − INT = TXR − INT.

DEF(NF) = government deficit based on nonfinancial transactions = G − T− consumption of fixed capital + net land purchases = (−1) times (line 30).

g = real government purchases = G/P.

rtxr = average rate of tax less non-interest transfers = TXR/Y.

rt = average rate of tax less all transfers = T/Y.

B. Financial Transactions Data from Part 1 Flow, [3] Supporting Tables, Table 21 Financial Tfransactions by Subsectors, pp. 262-271 and 294-298

Each quantity defined in this section is obtained from the sum of increases in certain liability items minus the sum of increases in the same asset items or the relevant sectors in Table 21.

DH = increase in high powered money outside the Bank of Japan and the government = line 2(1) (currency) + line 2(3) (deposits at the Bank of Japan) + line 2(4) (government current deposits).

DB = $DB_p + DB_f$ = increase in government debt outside the Bank of Japan =

lines 3(1) + 3(3) + 3(4) (time deposits + trust + negotiable CD's) + lines 4 through 10 (short term securities + long term bonds + corporate shares + loans + life insurance + transfers from general government + trade credits) + lines 11(1) + 11(2) + 11(3) (deposits in Trust Fund Bureau + investments by government + nonlife insurance) + line 2(2) (deposits on demand). (The last item, deposits on demand, does not strictly belong in any of the categories defined here, but it is so small that rather than define a separate category for it I have simply included it in DB.)

DF = increase in foreign exchange and gold held by the government and Bank of Japan = (−1) times (lines 11(5) through 11(9)) (foreign direct investments + foreign trade credits + foreign loans + foreign securities + other foreign claims and debts) + (liabilities in lines 1 + 11(4) for the rest-of-the-world column) (gold and SDR's and other foreign exchange reserves, which are shown only in total in the rest-of-the-world column, not separately in the government and Bank of Japan columns).

RESIDUAL = line 11(10) (others).

Note that lines 1, 3(2), and 11(4) for the Bank of Japan, for central government, and for general government have not been assigned to any of the foregoing variables; however, this does not matter since their published values are always zero for those sectors.

DEF(F) = government deficit based on financial transactions = (−1) times (last line, giving the increase in financial assets minus liabilities). [This item is also equal to (−1) times (line 31 of Table 6).]

Note that DEF(F) is equal to DH + DB − DF + RESIDUAL, in Tables 6 and 7.

DA = increase in private sector holdings of claims against the rest of the world = − DF − increase in foreign sector's financial net worth = − DF − (last entry in rest-of-the-world column of Table 21). This makes use of the balance sheet of the rest-of-the-world sector.

IV. Data from Flow of Funds Accounts

These data are for the consolidated Bank of Japan and central government sector as defined in the Flow of Funds accounts and presented in Bank of Japan (1982). Units are 100 million yen.

Each variable defined below is obtained from the sum of increases in certain liability items minus the sum of increases in the same asset items for the two sectors in question. Because some items are not shown separately, it is necessary to get their values from other columns of line e in the same table, as explained below.

DH = increase in high powered money= lines A + B (deposits with the Bank of

Japan + currency).

DB = DB_p + DB_f = increase in government debt held outside the Bank of Japan = lines I + S + U + V + Z (securities + Bank of Japan loans + bills + loans + deposits with Trust Fund Bureau) − (line e for public financial institutions and for public corporations and local authorities) (equity advanced by central government to those two sectors).

DF = increase in the Bank of Japan and central government holdings of gold and foreign exchange reserves = (−1) times (lines d + e) (other foreign claims and debts + others) + (−1) times (line e for public financial institutions and for public corporations and local authorities) (equity advanced by central government to those two sectors).

DEF(F of F) = central government deficit from Flow of Funds accounts = (−1) times (line f) (financial surplus).

Note that DEF(F of F) is equal to DH + DB − DF, in Table 5.

APPENDIX II

On the Impossibility of Perpetual Pure Debt Finance of Deficits

To see why pure debt finance of continuous government deficits cannot go on indefinitely, consider the following. Let real national income grow at the constant rate q, so that $y_t = y_0(1 + q)^t$. Suppose that there is no government debt and no deficit until the beginning of period 0, at which time the government borrows an amount d_0, and uses it to finance a deficit of d_0 in period 0. Suppose that in each subsequent period the government's deficit other than for interest payments is $d_t = d_0(1 + q)^t$, that is, the non-interest deficit is a constant fraction d_0/y_0 of national income. Suppose that at the beginning of each period the government borrows enough to finance that period's non-interest deficit, plus enough to pay the interest on its outstanding debt at the constant rate r. Then the debt outstanding at the beginning of periods 0, 1, 2, and t is as follows:

$$b_0 = d_0 \qquad b_1 = d_0(1 + q) + b_0(1 + r) = b_0(2 + q + r)$$
$$b_2 = d_0(1 + q)^2 + b_1(1 + r) = b_0(1 + q)^2 + b_0(2 + q + r)(1 + r)$$
$$b_t = d_0(1 + q)^t + b_{t-1}(1 + r)$$

Inspection shows that the debt always grows at a rate that exceeds the economy's growth rate q, and exceeds the interest rate r. Note that if the interest rate exceeds the economy's growth rate, $r > q$, then the term $b_{t-1}(1 + r)$ will eventually dominate, and the debt's growth rate will approach r from above. If $q > r$, the term $d_0(1 + q)^t$ will eventually dominate, and the debt's growth rate will approach q from above. If $r = q$, we easily see that

$$b_0 = d_0 \qquad b_1 = d_0(1 + r) + b_0(1 + r) = 2b_0(1 + r)$$
$$b_2 = d_0(1 + r)^2 + 2b_0(1 + r)(1 + r) = 3b_0(1 + r)^2$$
$$b_t = d_0(1 + r)^t + tb_0(1 + r)^{t-1}(1 + r) = (t + 1)b_0(1 + r)^t$$

In this case the debt's instantaneous growth rate is $r + 1/(1 + t)$, which approaches r from above.

We can now show that if the interest rate equals or exceeds the economy's growth rate, $r \geqq q$, the path just described is not sustainable. We do so by showing that it would require the ratio of government debt interest divided by national income, rb/y, to grow without limit. But this cannot happen, because no government would be able to find lenders willing to lend amounts so large that the entire national income would be insufficient to cover the annual interest due.

Consider first the case where $r > q$. Then, as noted above, the debt always grows at a rate that approaches r from above, and since $r > q$, the ratio of the debt interest to income grows without limit.

Consider now the case where r = q. Then the ratio of debt interest to inicome is

$$\frac{rb_t}{y_t} = \frac{r(t+1)b_0(1+r)^t}{y_0(1+r)^t} = r(t+1)\frac{b_0}{y_0}.$$

This ratio grows without limit, even though the debt's growth rate approaches r.

What of the case where $r < q$, the interest rate is less than the economy's growth rate? This can be ruled out if the intertemporal allocation of resources has been chosen optimally. The argument is as follows. The interest rate is the marginal product of capital. Phelp's Golden Rule of Economic Growth shows that in the neoclassical growth model, if the capital/labor ratio is so large that the marginal product of capital exceeds the growth rate, consumption per person can be increased and maintained at a permanently higher level by reducing the capital stock per person until the marginal product of capital becomes equal to the growth rate.

This completes the argument that pure debt finance of continuous government deficits cannot go on indefinitely. It has assumed that the non-interest deficit is a constant fraction of national income. If the non-interest deficit is a declining fraction of national income, the argument is almost as strong, but not quite. Consider the case where the deficit is constant at d_0 per year, instead of growing in proportion to national income as above. Then the debt at the beginning of each year is

$$b_0 = d_0 \qquad b_1 = d_0 + b_0(1+r) = b_0(2+r)$$
$$b_2 = d_0 + b_1(1+r) = b_0 + b_0(2+r)(1+r)$$
$$b_t = d_0 + b_{t-1}(1+r) = b_0[(1+r)^{t+1} - 1]/r.$$

The general result given by the latter expression can be verified by mathematical induction. The instantaneous growth rate of the debt is $r(1+r)^{t+1}/[(1+r)^{t+1} - 1]$, which always exceeds r, but approaches r from above. The ratio rb/y, debt interest divided by national income, is

$$\frac{rb_t}{y_t} = \frac{b_0[(1+r)^{t+1} - 1]}{y_0(1+q)^t}$$

if $r > q$, this ratio grows without limit and so the path is not sustainable. However, in the borderline case where r = q, this ratio is always less than $(1+r)b_0/y_0$, but approaches $(1+r)b_0/y_0$ from below. Hence if r = q, but not if $r > q$, debt financing can be used indefinitely to cover a deficit that decreases geometrically as a proportion of national income.

Finally, consider the case where there is a non-interest deficit for only a single period, $d_t = 0$ when t = 0; thereafter, $d_t = 0$ and additional debt finance is used only to cover the interest on the outstanding debt. Then the debt at the start of year t is $b_t = d_0(1+r)^t$. Then, as in the preceding case, the path is sustainable if r = q but not if $r > q$.

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