Economies of Scope: Theory and Application to Banking*

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I. Introduction

Traditionally, production theories have been concerned primarily with single-product output. For this reason, arguments in industrial organization theory have generally been based on the production function for single-product output. Real business enterprises, however, generally produce multiple products. The emergence of large-scale multiproduct firms called conglomerates is one manifestation of this fact. Accordingly, growing attention is being focused on the phenomena that distinguish the multiproduct case from that of the single product. Multiproduct output cannot be explained solely by the single-product output theory or by the industrial-structure theory based upon it. Economies of scope, the economic efficiency in multiproduct output technology, refer to situations in which one firm can produce multiple products at lower cost than can several single-product firms.[1] This paper is an attempt to empirically analyze multiproduct output theory and apply it to the banking industry, which is now undergoing rapid diversification in an environment of deregulating financial markets.

During the period characterized by rapid economic growth and considerable regulation of the capital market, banks sought to reduce fixed unit costs by expanding

* The author is grateful to the valuable comments received from Kazumi Asako of Yokohama National University, Ken Ariga of Kyoto University, Masahiro Kuroda of Keio University, Yoshiro Tsutsui of Nagoya City University, Masao Baba of Kyoto University, Akiyoshi Horiuchi of Tokyo University, Yozo Honda of Kobe University of Commerce, Akira Yamazaki of Hitotsubashi University and Kanji Yoshioka of Keio University.

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1. Panzar and Willig first introduced the concept of economies of scope in 1975. The present paper analyzes the concept and its relationships with other concepts mainly through a graphic exposition to deepen geometrical understanding. For more detailed discussions, see Baumol, Panzar, and Willig (1982) and Fuss – McFadden (1978).

their landing and deposit bases. In such an environment, this quantitative expansion—supported by the then prevailing competitive parity among banks—could increase profits. It is assumed that at this time banks were seeking to maximize a kind of economy of scale by increasing output of single products.

As deregulation of financial markets has progressed, however, it has become clear that while the economies-of-scale theory retains validity and explanatory significance, one-sided dependence on quantitative expansion cannot guarantee profit increase. The argument is advanced in many quarters that banks will diversify as a way to manage the more intensely competitive environment brought about by deregulation. They will need to enter new areas of business that will enable them to make the most of their information and accumulated know-how. The argument is that diversification and entry into new business areas will yield economies of scope and improve profitability.

The assumption is that economies of scale—the theoretical base of banking in the high-growth era—will give way to the concept of economies of scope. Economies of scope will provide the theoretical foundation for the diversification of the banking industry in an era of slower economic growth and advances in information technologies.

1. Multiproduct output yields two types of economic efficiency: the effect of production technology on expanding the scale of production (economies of scale) and the effect of diversification of production (economies of scope as a static concept, or cost complementarity as a marginal concept). In this paper, these two types of efficiency are together referred to as economies of multiproduct output.

2. Economies of scope come into being when a production factor (public inputs) associated with the production of a given product is used in the production of another product without additional cost. Two examples of such factors are information and know-how.

Assuming that one significant role of banking is to yield information related to specific borrowers and industries (Leland et al. (1977)), banks can achieve economic efficiency by applying this information and know-how to other business fields for instance, securities).

3. Multiproduct production opens up the possibility for natural monopolies (conditions determined solely by production technology), in that one company can produce multiple products at a lower total cost than can several single-product companies. Moreover, natural monopolies can be based on both types of efficiencies: scale expansion or diversification of business lines. To determine whether banks are candidates for natural monopolies, one must consider both types of economies.

4. Measurement of the cost structure of city banks reveals economies of scope (cost complementarities). And there are indications that cost complementarities are proliferating as diversifying banks accumulate public inputs and incorporate technical advances that enable them to enhance their use of information and know-how.

In contrast, local banks have not achieved economies of scope (cost complementarities) until very recently. The probable reason is that these banks have accumulated fewer public inputs and their customer information has less horizontal application. But it is noteworthy that, lately, local banks have begun to achieve economies of scope.

5. It is likely that financial institutions will handle a wider range of products in the future. It is also likely that they will be entering new businesses, in which they can apply the information and know-how gained in existing or newly-developed operations. In doing so, banks will achieve economies of scope. This is one of the benefits of deregulation of the financial market.

6. City banks, and more recently local banks, are exhibiting economies of scale and economies of scope. There are signs, then that banks are becoming natural monopolies, but this is a measured result that is only one of many factors determining market performance. It would therefore be inappropriate to construe these results as signs of a noncompetitive market situation.[2] Still, during the course of deregulation, it would seem appropriate to guard against the formation of actual monopolies by conducting prior consultations and subsequent monitoring of the effects of diversification.

II. The Concept of Economies of Scope

The concept of economies of scope is appearing increasingly in the literature. This introductory section defines the concept and then discusses related concepts.

1. Definition of Economies of Scope

When the total production costs for one company to produce multiple products is lower than those that would be incurred by several single-product companies, the economic effect is called "economies of scope." In a two-product case,[3] economies of

2. Even a market that is evidently monopolized can generate the same effects as those obtained by a competitive market so long as newcomers have the opportunity to enter and contest that market. This theory of the contestable market (Baumol et al. (1982)) has recently received considerable attention.

3. This paper deals with a two-product case as a simplified model, but it is possible to expand the argument into n numbers of products without changing the essential points of arguments.

scale exist at point $Y(=[y_1, y_2])$ if and only if:

$$C\ (y_1, y_2) < C\ (y_1, 0) + (0, y_2), \tag{1}$$

where y_1 and y_2 are the outputs of product y_1 and product y_2, respectively,[4,5] and C is the minimum cost for a given amount of production.

Equation (1) is illustrated in Figure 1.[6] In Figure 1, the y_1 coordinate of any given point indicates the output of product y_1, and the y_2 coordinate indicates the output of product y_2. The C coordinate indicates the costs to produce the products given by the y_1 or y_2 coordinates.

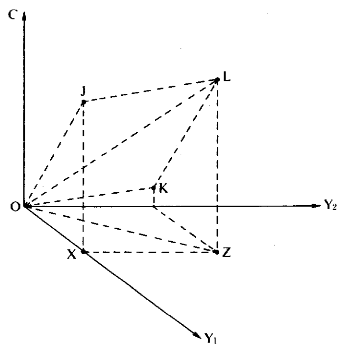
First of all, let points J and K indicate the respective minimum costs $C(y_1, 0)$ and $C(0, y_2)$ to separately produce products y_1 and y_2. In this case, point L has the sum of the coordinates vector of point $J(y_1, 0, c[y_1, 0])$ and the coordinates vector of point $K(0, y_2, c[0, y_2])$ as its coordinate vector, and is expressed as $(y_1, y_2, C[y_1, 0]) + C[0, y_2])$. C coordinate of point $L(C[y_1, 0] + C[0, y_2])$ indicates the sum of the costs to produce products y_1 and y_2 separately. Next, suppose that product y_1 and y_2 are produced by a single firm. Point M, which has the output of product y_1 and y_2 as its y_1 and y_2 coordinates (y_1, y_2), is given where C coordinate indicates the minimum cost $C(y_1, y_2)$. According to the above definition of economies of scope, if point M falls below point L, there are economies of scope.[7]

4. Under the prescribed conditions, information about production technology that the production function possesses can be described by the cost function. For simple verification, the analysis of this paper will use the cost function (see Appendix I).

5. $C=C(y_1, y_2)$ is the abbreviated form of the cost function $C=C(y_1, y_2, w_1, w_2 \ldots)$, omitting production factor price w_j, which is defined as a constant and positive value.

6. It is provided that production factor prices take constant positive values. The same is applied to all other three=dimensional figures in this paper.

7. Economies of scope are different from the concept that the total average cost will decrease by increasing certain products. For instance, in the following figure, if point X moves to point Z by simply increasing the production of y_2, which has lower average cost (this results in multiproduct production), the average cost will decrease from $\angle$ JOX to $\angle$ LOZ without economies of scope (see the figure below).

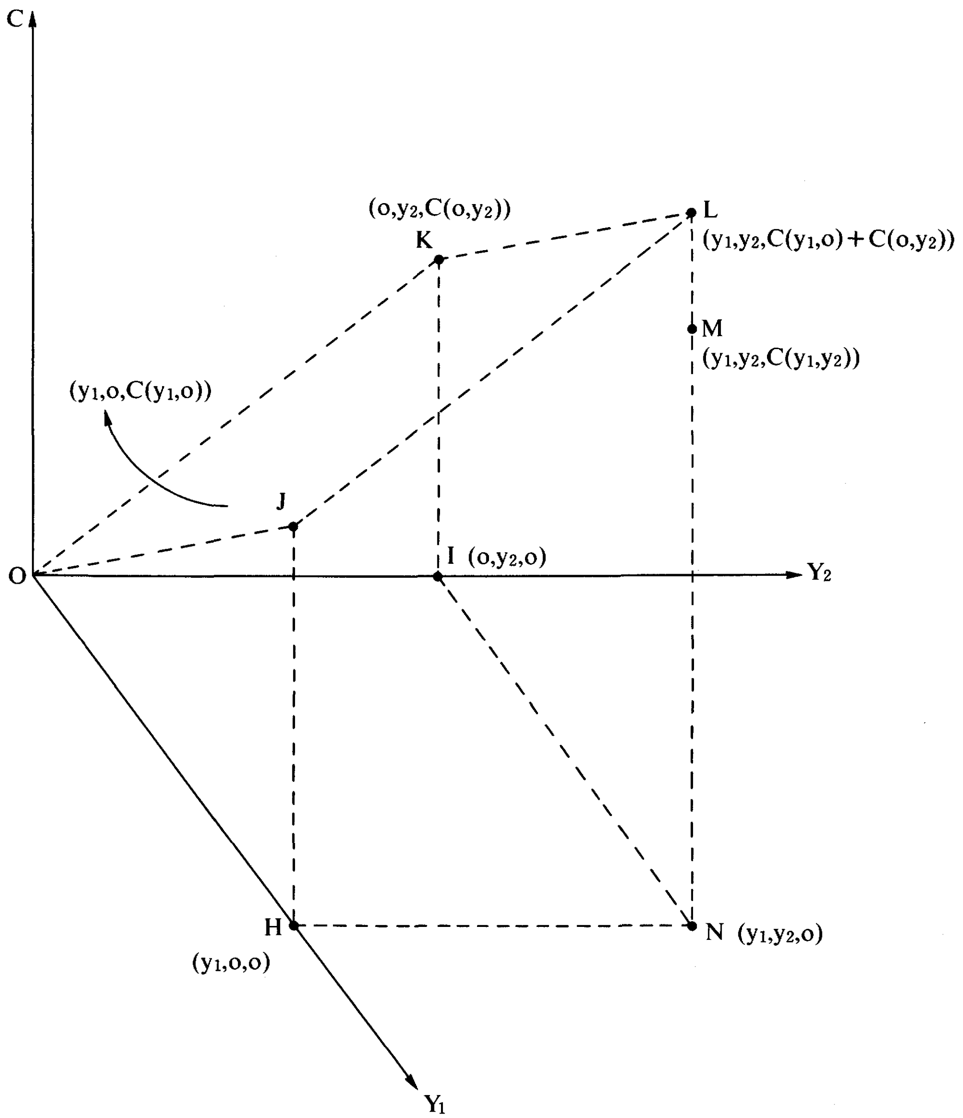


Economies of scope exist, for example, in hand calculator and digital watch manufacture, the natural and synthetic textile industry, production of two- and four-wheel small vehicles, and retail and credit businesses.

2. Cost Complementarities

Economies of scope are intuitively understandable as an indicator of multipro-

Figure 1 Economies of Scope



duct economies. Since, however, this concept is a static concept given as points on the curved surface of the cost function, it is not easy to use it to communicate information about the cost function, and it causes difficulties in collecting data for empirical analysis.[8] It is therefore appropriate to introduce a new concept to indicate economies of multiproduct production—that is, cost complementarities.

Cost complementarities are defined by the following equation, where the twice-differentiable cost function

$$C = C(y_1,\ y_2), \quad \frac{\partial^2 C}{\partial y_1 \partial y_2} < 0 \ . \tag{2}$$

That is, cost complementarities are said to exist when the marginal cost of providing one product declines because of increases in the production of a different product.

This concept is illustrated in Figure 2. Figure 2b represents Figure 2a viewed from the direction of the y_1 coordinate axis. The largeness of the marginal cost of y_2 at point $A(y_1{}^o,\ y_2{}^o,\ C^o)$ is given as θ_A in Figure 2b. If y_2 is then fixed, and y_1 is increased by Δy_1, the marginal cost of y_2 at point $B(y_1{}^o + \Delta y_1,\ y_2{}^o,\ C^o + \Delta C)$ is shown as θ_B. If $\theta_B < \theta_A$, cost complementarities are said to exist (The same is applied when y_1 is fixed and y_2 is increased by Δy_2.).

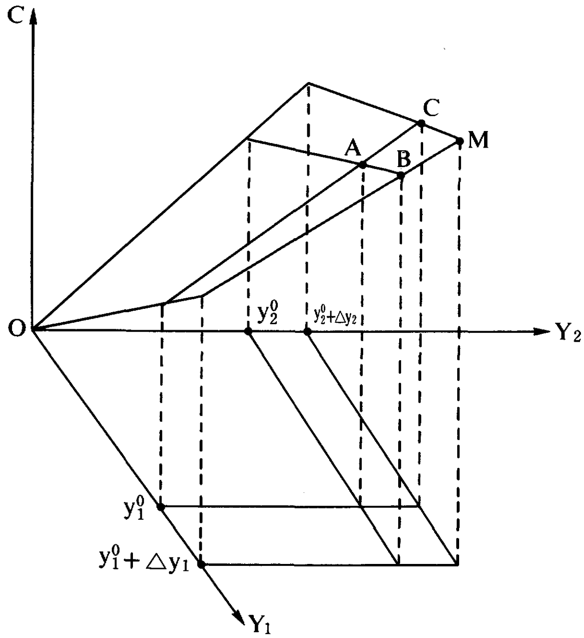
Next, consider the relationship between economies of scope and cost complementarities defined above. In Figure 3, J and K indicate the costs of separate production of each of the two products, and L gives the sum of those costs. Also, point M gives the total production cost if both products are produced by a single firm. For a simple explanation, suppose that the outputs of the products are increased in the order O → H → N. In this case, MN can be divided into JH(= PN) and MP. JH(= PN) is the production cost of product y_1 incurred on the incremental production process from O to H, and MP is the production cost of product y_2 incurred in the incremental production process from H to N. That is,

$$\begin{aligned} MN &= PN + MP = JH + MP \\ &= \int_0^{y_1} C_1(x,\ 0)dx + \int_0^{y_2} C_2(y_1,\ z)dz \\ &= \int_0^{y_1} C_1(x,\ 0)dx + \int_0^{y_2} \left[\int_0^{y_1} C_{12}(x,\ z)dx + C_2(0,\ z) \right] dz \\ &= \int_0^{y_1} C_1(x,\ 0)dx + \int_0^{y_2} \int_0^{y_1} C_{12}(x,\ z)dxdz + \int_0^{y_2} C_2(0,\ z)dz, \end{aligned} \tag{3}$$

8. In order to directly exhibit economies of scope by using actual data, data showing the output of a certain product at zero is always required.

Figure 2 Cost Complementarities

a.



b.

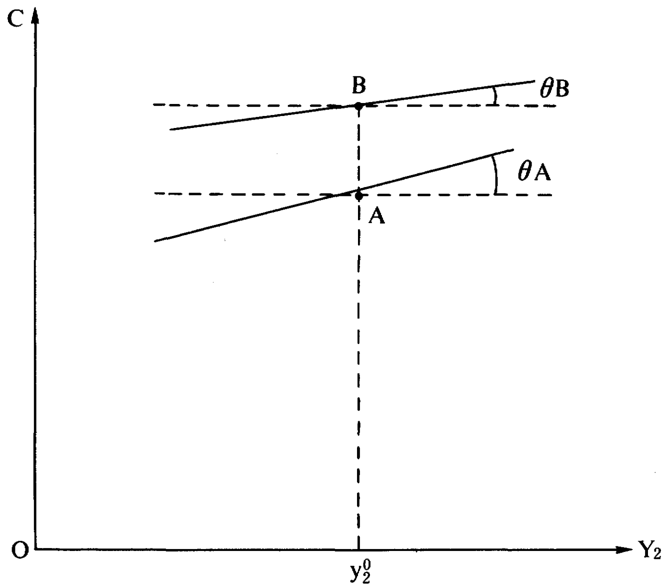
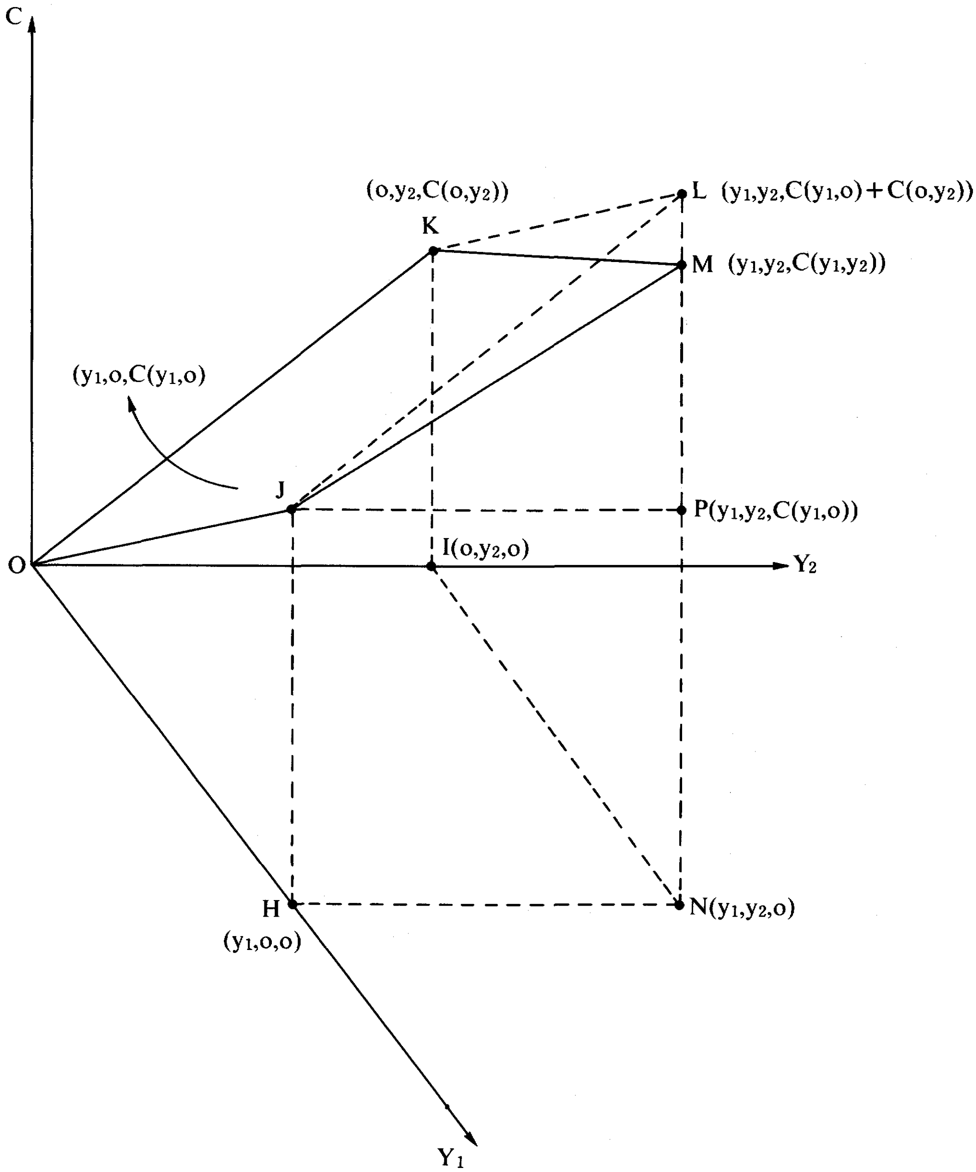


Figure 3 Relation between Cost Complementarities and Economies of Scope



provided that $C_1=\partial C/\partial y_1$, $C_2=\partial C/\partial y_2$, $C_{12}=\partial^2C/\partial y_1\partial y_2$, and x and z are integral variables.

On the other hand, the total cost of separate production is given by the following equation:

$$LN=PN+LP=JH+KI$$

$$=\int_0^{y_1}C_1(x,\ 0)dx+\int_0^{y_2}C_2(0,\ z)dz\ . \tag{4}$$

Subtracting equation (4) from equation (3), we obtain

$$MN-LN=MP-KI, \tag{5}$$

or

$$MN-LN=\int_0^{y_2}\int_0^{y_1}C_{12}(x,\ z)dxdz\ . \tag{6}$$

Based on equation (5), point M will be below point L (MN − LN < 0) if MP is smaller than KI. Here, KI represents the value of $\partial C/\partial y_2$ integrated from 0 to y_2, when $y_1=0$, and MP represents the value of $\partial C/\partial y_2$ integrated from 0 to y_2, when $y_1=y_1$. As shown in equation (6), this condition will be met if $C_{12}(=\frac{\partial^2C}{\partial y_1\partial y_2})<0$ throughout the given path. In other words, cost complementarities are the sufficient condition for economies of scope.[9,10]

9. As a simplified model, the lines from O to J and from J to M are used here. Generally speaking, however, if a smooth and monotonous integral line from O to M meets the condition of $\partial^2C/\partial y_1\partial y_2<0$, economies of scope can be said to exist.

10. "Joint production" has been used as a concept to indicate economies of multiproduct production (Henderson, Quandt et al. (1971)). This concept means that the isocost section on the cost curve surface is concave towards the origin.
Let $\bar{C}=C(y_1,\ y_2)$ denote the equal cost section in a two-product case of y_1 and y_2. If the isocost section is concave to the origin, it means that the marginal variation rate, that is $-dy_2/dy_1 = (\partial C/\partial y_1)/(\partial C/\partial y_2)$ is positive. That is:

$$-\frac{d^2y_2}{dy_1{}^2}=\frac{1}{(\partial C/\partial y_2)^3}\left\{\frac{\partial^2C}{\partial y_1{}^2}\cdot\left[\frac{\partial C}{\partial y_2}\right]^2-2\frac{\partial^2C}{\partial y_1\partial y_2}\cdot\ \frac{\partial C}{\partial y_1}\cdot\frac{\partial C}{\partial y_2}+\frac{\partial^2C}{\partial y_2{}^2}\cdot\left[\frac{\partial C}{\partial y_1}\right]^2\right\}>0\ .$$

According to this condition, the fact that the isocost section is concave to the origin does not necessarily mean the existence of cost complementarities ($\partial C/\partial y_1\partial y_2<0$). This is because the isocost section can be concave for the origin (that is, $-d^2y_2/dy_1{}^2>0$) even without $\partial C/\partial y_1\partial y_2<0$ when the values of $\partial^2C/\partial y_1{}^2$ and $\partial C/\partial y_2{}^2$ are large enough.

3. Sources of Economies of Scope

Why do economies of multiproduct output such as economies of scope occur? Cost function (C), which is a function of production outputs (yi) and production factor prices (wi), can also be expressed as a sum total of the products obtained by multiplying the quantity of each production factor (xj) by the corresponding production factor price (The example shows the case in which i=1, 2, j=1, 2, 3.). Here, $xj(y_1, y_2)$ is the function of optimal production factor quantity for (w_1, w_2, w_3)

$$C = C(y_1, y_2, w_1, w_2, w_3)$$

$$= \sum_{j=1}^{3} wj \cdot xj(y_1, y_2) \quad . \tag{7}$$

Supposing that economies of scope exist between product y_1 and y_2, $C(y_1, y_2)<C(y_1, 0)+C(0, y_2)$,

$$\sum_{j=1}^{3} wj \cdot xj(y_1, y_2) < \sum_{j=1}^{3} wj \cdot xj(y_1, 0) + \sum_{j=1}^{3} wj \cdot xj(0, y_2).$$

That is,

$$\sum_{j=1}^{3} wj \cdot [xj(y_1, y_2) - xj(y_1, 0) - xj(0, y_2)] < 0. \tag{8}$$

The production factor that is commonly used in the production process of both product y_1 and product y_2 and, as a result, creates economies of production is defined here as a public input between the y_1 and y_2 production process of y_1 and y_2. If a public input exists in the y_1 and y_2 production process, $xj(y_1, y_2)-xj(y_1, 0) -xj(0, y_2)< 0$ in one or more xj for all sets of production factor prices.

Employing this concept, since $wj<0$, it can be inferred from equation (8) that at least one public input must be included in a production process for an economy of scope to occur.

Information and know-how are examples of public input. Specific examples include customer information in the retail and credit industries and electronics technology in handy calculators and digital watch manufacture. Once a firm has obtained information and know-how, it can employ these factors over and over without additional cost.

Leland et al. (1977) and others hold the increasingly accepted view that banking is characterized by the production of information (searching and monitoring). Assuming that is the case, it becomes predictable that, as banks enter other businesses, they will employ their accumulated information and know-how and will thus achieve economies of scope:

The intangible assets of financial institutions, including collected information, credit assessment capability, and long-term customer relationships, have a variety of possible uses. They can be used for short-term and long-term lending, for providing management consulting and investment advice to customers, for investment in securities, and other functions. The nature of these intangible assets is not such that, after being employed in short-term lending, they can not still be used for long-term lending. They can be used for both. Their character is that of a public road, which can simultaneously accommodate transportation of goods and pedestrians (Tachi (1985)).

Although public inputs yield economies of scope, if those public inputs were marketable (at least with a price in line with the marginal revenue product), a firm would not decide to begin multiproduct output. This is because, if the sales margin of an acquired public input is equal to or greater than the profit that could be gained by employing that production factor in another production process, entry into multiproduct production would have no merit.

But information and know-how can be reused without additional cost, and their values are uncertain before marketing. In addition, these factors are generally associated with employees, and thus are often difficult to transfer. For these reasons, it is generally difficult to create a market for them. It is reasonable then for a firm to begin multiproduct output when it can achieve economies of scope based on public input such as information and know-how.[11,12]

4. Relationship Between Economies of Scope and Economies of Scale

It is also important to clarify several concepts in relation to economies of scope. First, in multiproduct production, economies of scale can be defined as how many times the total production cost will be multiplied when the output of all products becomes t times larger. Let S_N represent such scale elasticity with respect to all products. S_N can be expressed as follows:

$$S_N = \frac{\partial \ln C(ty^0{}_1,\ ty^0{}_2)}{\partial \ln t}$$

$$= \sum_{j=i}^{2} \partial \ln C / \partial \ln yj \tag{9}$$

(where $y_1{}^o$ and $y_2{}^o$ are unit vector, $y_1 = ty_1{}^o$, $y_2 = ty_2{}^o$).

11. See Williamson (1975) for discussion of the idea that a firm absorbs within itself products that are hard to market owing to high sales cost.

12. It has been concluded that common production factors are often actually intermediate products—produced within the company and not placed on the external market. This view does not affect the essentials of the foregoing argument.

Here, economies of scale can be said to exist if $S_N<1$, and also, in multiproduct production, product-specific economies of scale, that is, economies of scale achieved by increasing the output of a certain product, y_i, and keeping the output of other products unchanged. For instance, in a two-product case, let S_1 denote the scale elasticity with respect to a specific product, y_1, S_1 can be written as follows:

$$S_1=\frac{\partial C/(C(y_1,\ y_2)-C(0,\ y_2))}{\partial y_1/y_1} \tag{10}$$

or

$$S_1=\frac{\partial \ln C}{\partial \ln y_1}\cdot\frac{C}{C(y_1,\ y_2)-C(0,\ y_2)}\ . \tag{11}$$

If $S_1<1$, it can be said that economies of scale exist with respect to a specific product, y_i.

Figure 4 describes relationships among economies of scope, economies of scale with respect to ① all products, ② product-specific economies of scale of y_1 and ③ y_2. In Figure 4a, economies of scale with respect to all products appear in the O-N direction. In Figure 4c, which shows the section including O-N, the scale elasticity with respect to all products are expressed as follows:

$$\frac{\triangle C/C}{\triangle Y/Y}=\frac{\triangle C}{\triangle Y}\cdot\frac{Y}{C}\frac{MN}{QN}\cdot\frac{ON}{MN}=\frac{ON}{QN}\ .$$

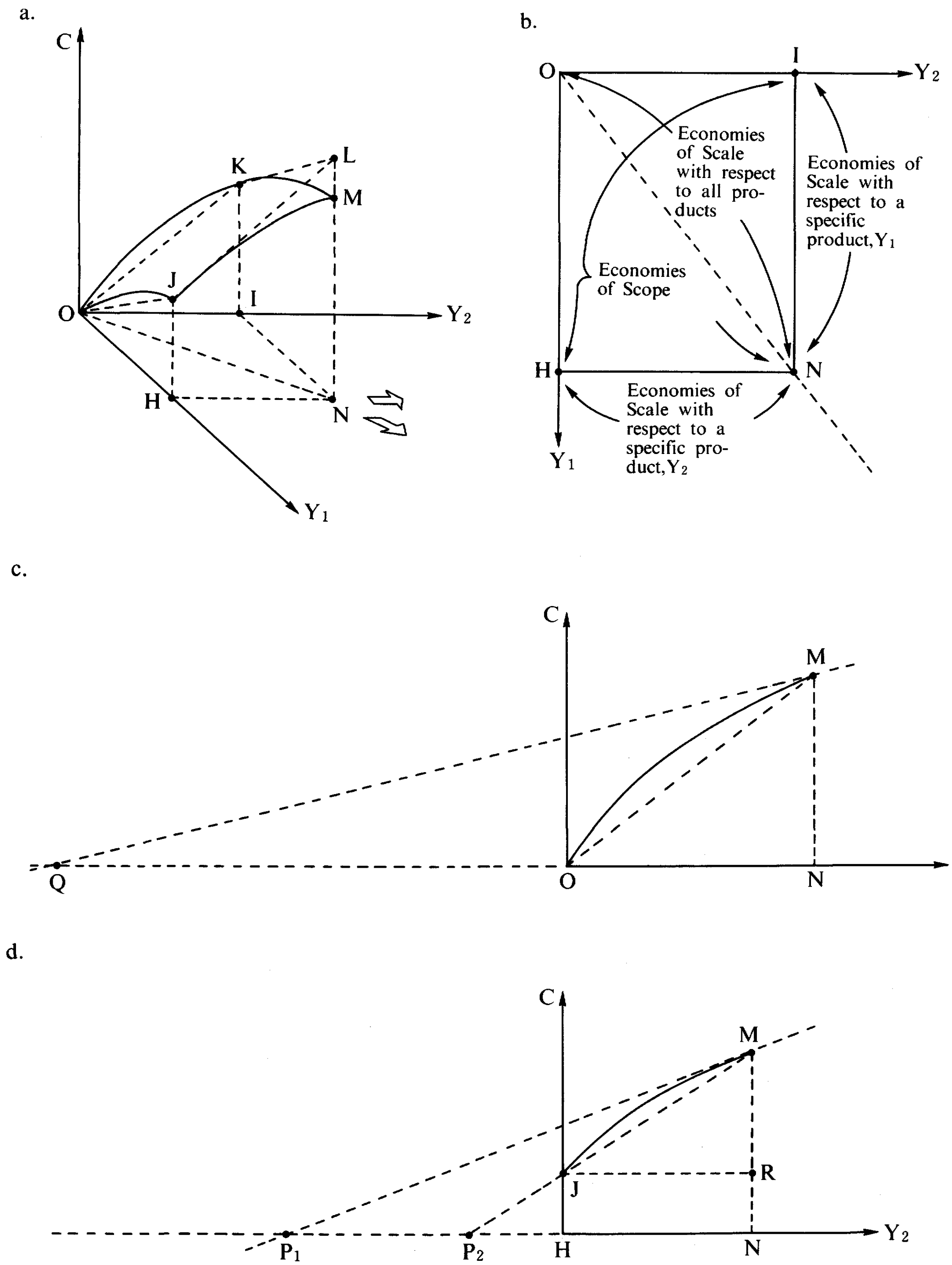
Product-specific economies of scale appear in the H-N direction in Figure 4a. In Figure 4b which shows the section including H-N, scale elasticity with respect to a specific product, y_2, is given by the following equation:

$$\frac{\triangle C/(C(y_1,\ y_2)-C(y_1,\ 0))}{\triangle y_2/y_2}=\frac{\triangle C}{\triangle Y_2}\cdot\frac{Y_2}{C(y_1,\ y_2)-C(y_1,\ 0)}$$

$$=\frac{MN}{P_1N}\cdot\frac{HN}{MN-RN}=\frac{MN}{P_1N}\cdot\frac{P_2N}{MN}=\frac{P_2N}{P_1N}\ .$$

The same is true for product y_1. All ①-③ are illustrated together in Figure 4b. As shown in this figure, economies of scope, economies of scale with respect to all products, and product-specific economies of scope are closely related. In a two-product case, S_N can be written as follows, using S_1, S_2, and economies of scope Sc ($\equiv C(y_1,\ y_2)-C(y_1,\ 0)-C(0,\ y_2)$), based on equations (9)(10).

Figure 4 Economies of Scope and Economies of Scale



$$S_N = S_1 \times \left\{\frac{C(y_1, y_2) - C(0, y_2)}{C(y_1, y_2)}\right\} + S_2 \times \left\{\frac{C(y_1, y_2) - C(y_1, 0)}{C(y_1, y_2)}\right\}$$

$$= \left(1 + \frac{S_C}{C(y_1, y_2)}\right) \times [S_1 \times \alpha + S_2 \times (1-\alpha)],$$

provided that

$$\alpha = \frac{C(y_1, y_2) - C(0, y_2)}{2C(y_1, y_2) - C(y_1, 0) - C(0, y_2)} \cdot$$

Regarding economies of scale with respect to all products (①), product-specific economies of scale for y_1 (②) and y_2 (③) , and economies of scope, if these are determined, the unknown can be automatically determined through α .

5. Economies of Scope and Natural Monopoly

When a single firm can produce multiple products at lower cost than several single-product firms separately, a natural monopoly exists. From this definition, one can see that natural monopolies, economies of scope, and economies of scale are all related.

In this paper, a natural monopoly is defined as subadditivity of the cost function following Faulhaber et al. (1975)[13] (see Figure 5). In Figure 5, point P indicates the total production costs when each of two firms produces products y_1 and y_2 in any possible output mix, (y_1^H, y_2^H) and (y_1^I, y_2^I), respectively. Point M indicates a total cost when a single firm produces $(y_1^H + y_1^I, y_2^H + y_2^I)$. If M is below P, it can be said that the cost function is subadditive, or that a natural monopoly exists.

Figure 6 illustrates the relationship between natural monopolies and economies of scope and economies of scale. In Figure 6a at point M neither economies of scope nor economies of scale (with respect to all products as well as a specific product) exist. In this case, at point M the production cost, MN, by a single firm is larger than the total production cost, LN, when two firms produce the products separately at

13. Using a two-product case as a simplified model, let $y=(y_1, y_2)$ denote the output vector of product y_1 and y_2, $y^*=(y_1^*, y_2^*)<y$, $y^* \neq 0$, if:
$$C(y^*) + C(y - y^*) > C(y)$$
that is,
$$C(y_1^*, y_2^*) + C(y_1 - y_1^*, y_2 - y_2^*) > C(y_1, y_2),$$
$C(y_1, y_2)$ can be said to be subadditive at $y = (y_1, y_2)$. At this point, the production of the given products by a single firm is cheaper than the production of the same products by two or more firms, and as a result, natural monopoly in production technology occurs.

points J and K. MN is also larger than the total cost of CN when two firms produce the products separately at points A and B or any point within the cost-curve surface. Therefore, natural monopoly does not exist.

Figure 6b illustrates the other extreme. At point M, so the economies of scope

Figure 5 Natural Monopoly (Subadditivity of the Cost Function)

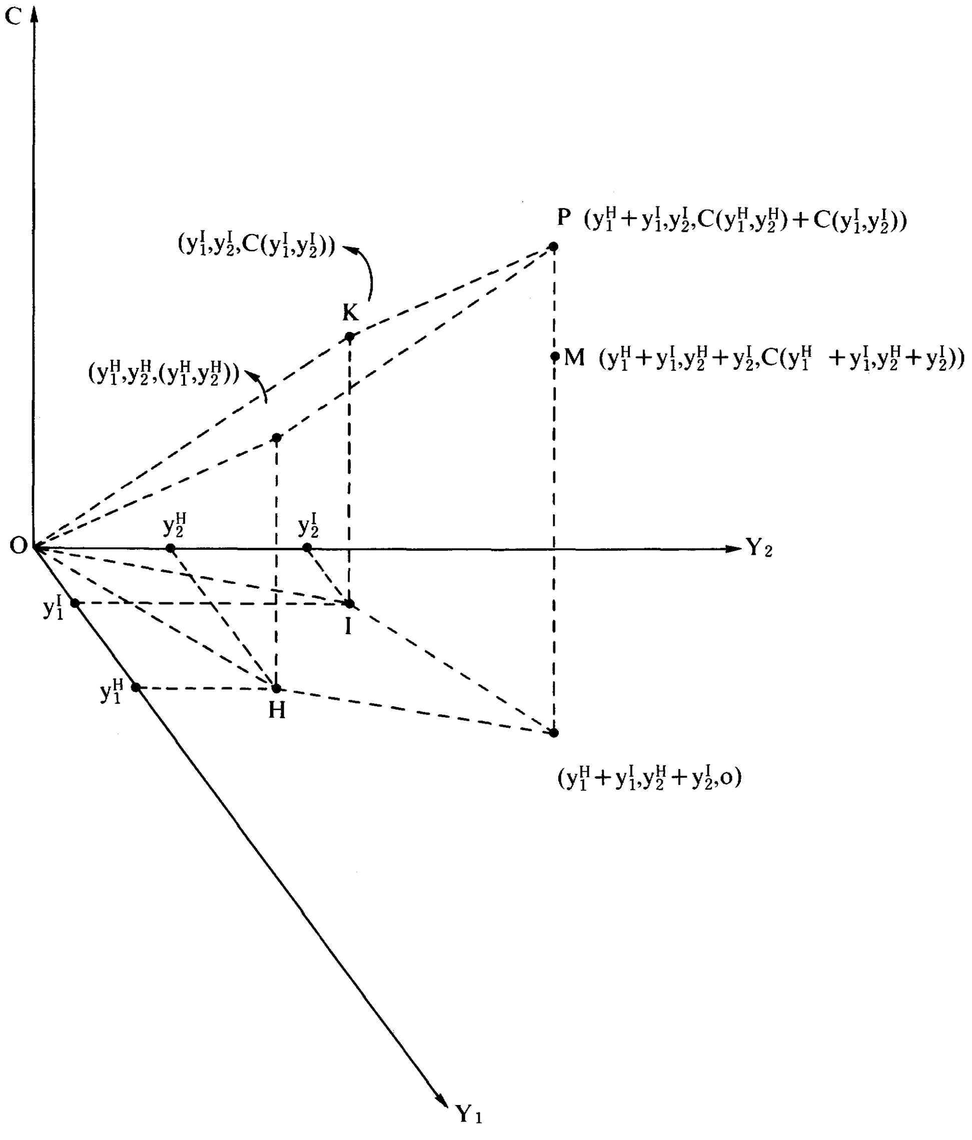
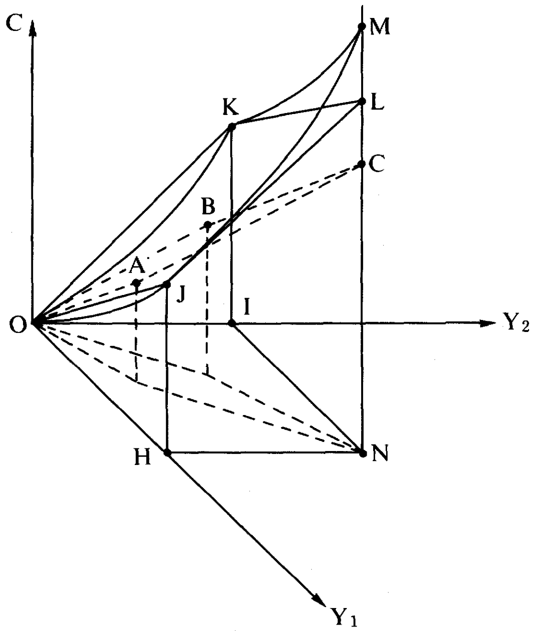
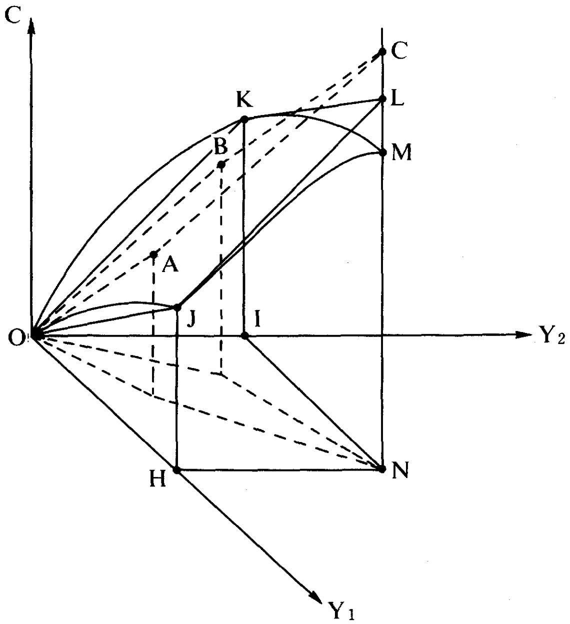


Figure 6 Natural Monopoly in Relation to Economies of Scope and Economies of Scale

a. When neither economies of scope nor economies of scale exist



b. When both economies of scope and economies of scale exist



and economies of scale (with respect to all products as well as a specific product) Simultaneously exist. The production cost, MN, when a single firm produces products at point M is smaller than the total cost, LN, when two firms produce them separately at points J and K. Moreover, the production cost, MN, is smaller than the total cost, CN, when products are produced separately at any given point between A and B on the cost-curve surface. In this case, natural monopoly can there done be said to exist. It is exist but easy to illustrate cases in which economies of scope exist but economies of scale do not (no natural monopoly) and in which economies of scale exist but economies of scope do not (no natural monopoly).

As clearly indicated by these examples, natural monopoly exists when both economies of scope and economies of scale exist. More detailed explanation is given in Appendix II, but, in general, natural monopoly (subadditivity of the cost function) exists under the following circumstances:[14]

(i) Economies of scope exist at given point M, and, at the same time, economies of scale with respect to a specific product exist up to point M.
(ii) On a certain cross-section of the cost-curve surface, which includes the given point M, cost complementarities are large enough, and, at the same time, economies of scale exist with respect to all products up to the section.

In this paper, at the time of empirical verification of economies of scope, natural monopoly is also measured as a derivative. For convenience, the measurement employs (ii). as a condition that uses cost complementarities and economies of scale with respect to all products.

III. Empirical Analysis

1. Summary of the Empirical Literature on Banking

Studies of the cost structure of the banking industry have been conducted mainly in the United States—partly because empirical proofs have been required in arguments on regulatory easing and tightening (see Table 1). A survey of such studies is useful for the purposes of this paper. Prior to measuring economies of scope (cost complementarities) and economies of scale, it is necessary to determine the concept

14. Nevertheless, "natural monopoly" in production technology (subadditivity of the cost function) is not equal to the noncompetitive market monopoly situation in the actual market. For instance, under circumstances in which a firm is free to enter into and retreat from the market, and unretrievable (sunk) cost at time of retreat is zero, effective competition is possible even if there is a natural monopoly of production technology (Baumol et al. (1982)). This idea has recently attracted attention as the theory of the contestable market.

of products in banking. In the empirical literature, (banking products have been classified in to three categories:

(i) Outstanding working assets (outstanding deposits)
(ii) Loan cases (number of deposit accounts)
(iii) Revenue.

Alhadeff (1954), Horvits (1963), Schweiger – McGee (1961) and Gramley (1962) employed working assets (or outstanding deposits) as products. They did so mainly because working assets are an important index of bank size. There are, however, some problems with this method, which employs a concept of stocks. First, various types of working assets, each of which yields different production costs, are given equal weight in the calculation. Second, the method does not accurately reflect sales performance during the period under review.

Benston (1965), Bell – Murphy (1968), and Longbrake (1974) focused on the production process in each service operation within banks. They attempted to measure economies of scale obtaining between operating costs tied to each service operation and the output of each of those operations. They considered personnel expenses (including document processing costs—the greatest single operating cost) as a function of loan cases (or number of deposit accounts) instead of a function of the dollar amounts of loans (or dollar amounts of deposits). They thus used the number of loan cases (or accounts) as the bank's output.

Use of the number of loan cases (or deposit accounts) as output, however, does not allow the influence of indirect cost levels to be captured precisely. In another approach, Greenbaum (1967), Powers (1969), Schweitzer (1972), and Clark (1984) attempted to determine economies of scale for an entire bank; they included indirect costs in their calculations and employed net earnings as output. The advantage of this method is that different types of output can be given appropriate weights in the measurement.[15]

The above studies were all concerned with economies of scale. Beginning in the 1980s, studies have also been made of economies of scope. These studies include Benston, Beager, Hanweck, and Humphrey (1983) (hereafter referred to as BBHH), Murray – White (1983) (MW), Gilligan – Smiflock (1984) (GS), and Gilligan, Smirlock, and Marshall (1984) (GSM). These studies share a common methodology for estimating economies of multiproduct output by defining cross terms among multiple

15. Since revenues are the product of price and quantity, if earnings are employed as output, the monopolizing power in setting production price may possibly influence measurement. Therefore, at the time of measurement, it is necessary to exclude such influence by introducing average cost in one way or another (Greenbaum (1967) and others) or to assume that the monopolizing power is negligible based on the observation of the real market (Royama (1982) and others). The measurement of this paper assumes the latter.

Table 1 Earlier Measurements of the Cost Structure of Financial Institutions (1/3)

Major Theses	Outputs			Costs		Samples	Method of Measurement	Results	
	A	B	C	D	E			Economies of Scale	Economies of Scope
Alhadef 1954	○			○		Member banks of the Federal Reserve Bank in the San Francisco area	Comparison between production scale and average cost	Exist	–
Schweiger & McGee 1961	○			○		Member banks of the Federal Reserve Bank in the Chicago area	Regression of average cost by production scale	Exist	–
Gramley 1962	○			○		Member banks of the Federal Reserve Bank in the Kansas City area	Regression of average cost by production scale	Exist	–
Horvitz 1963	○			○		All member firms of the Federal Deposit Insurance Corporation and all member banks of the Federal Reserve Bank	Comparison between production scale and average cost	Exist	–
Benston 1965		○			○	Banks in the Boston area participating in the FCA program*	Estimation of Cobb-Douglas in each operation	Exist	–
Greenbaum 1967			○	○		Member banks of the Federal Reserve Bank in District 10 and District 5	Regression of cost by earnings (cubic expression)	Small scale – exist Large scale – do not exist	–
Bell & Murphy 1968		○			○	Banks in Boston, New York City, and Philadelphia participating in the FCA program	Estimation of Cobb-Douglas in each operation	Exist	–
Powers 1969			○	○		Member banks of the Federal Reserve Bank	Regression of cost by earnings (cubic expression)	Small scale – exist Large scale – do not exist	–

Notes: A : The balance of working assets (or deposits)
B : The number of loans (the number of accounts)
C : Revenue
D : Current expenses
E : Direct expenses attributed to each operation

* The FCA (Functional Cost Analysis) program is a research operation conducted by the Federal Reserve Bank for its member banks. Member banks independently decide whether or not to participate in the program; 10 to 20 percent of member banks ordinarily participate.

Table 1 Earlier Measurements of the Cost Structure of Financial Institutions (2/3)

Major Theses	Outputs			Costs		Samples	Method of Measurement	Results	
	A	B	C	D	E			Economies of Scale	Economies of Scope
Schweitzer 1972			○	○		Member banks of the Federal Reserve Bank	Regression of cost by earnings (cubic expression)	Small scale – exist Large scale – do not exist	–
Nishikawa 1972	○			○		Japanese city and local banks	Estimation of Cobb-Douglas	Exist	–
Longbrake 1974		○			○	Member banks of the Federal Reserve Bank participating in the FCA program	Estimation of Cobb-Douglas	Exist	–
Longbrake & Haslem 1975		○			○	Member banks of the Federal Reserve Bank participating in the FCA program	Estimation of Cobb-Doublas	Exist	–
Mullineaux 1975		○			○	Member banks of the Federal Reserve Bank participating in the FCA program	Estimation of Cobb-Douglas	Exist	–
Benston, Hanweck & Humphrey 1982		○			○	Member banks of the Federal Reserve Bank participating in the FCA program	Estimation of translog cost function	Small scale – exist Large scale – do not exist	–
Flannery 1983		○			○	Member banks of the Federal Reserve Bank participating in the FCA program	Estimation of translog cost function	Small scale – exist Large scale – do not exist	–
Royama 1983			Δ	Δ		Japanese city and local banks	Estimation of Cobb-Douglas	Exist	–

Notes: A : The balance of working assets
B : The number of loans (the number of accounts)
C : Revenue
D : Current expenses
E : Direct expenses attributed to each operation

* The FCA (Functional Cost Analysis) program is a research operation conducted by the Federal Reserve Bank for its member banks. Member banks independently decide whether or not to participate in the program; 10 to 20 percent of member banks ordinarily participate.

1. Revenue – expenses of procurement
2. Personnel expenses + non-personnel expenses

Table 1 Earlier Measurements of the Cost Structure of Financial Institutions (3/3)

Major Theses	Outputs			Costs		Samples	Method of Measurement	Results	
	A	B	C	D	E			Economies of Scale	Economies of Scope
Murray & White 1983	○			○		Credit unions in British Columbia, Canada	Estimation of multiproduct translog cost function	Exist	Exist in a part of the operation
Benston, Berger, Hanweck, & Humphrey 1983		○			○	Member banks of the Federal Reserve Bank participating in the FCA program	Estimation of multiproduct translog cost function	Small scale – exist Large scale – Δ	Exist in a part of the operation
Gilligan, Smirlock, & Marshall 1984		○			○	Member banks of the Federal Reserve Bank participating in the FCA program	Estimation of multiproduct translog cost function	Small scale – exist Large scale – do not exist	Exist
Gilligan & Smirlock 1984	○			○		Unit banks for which Federal Reserve Bank in Kansas City data is available	Estimation of multiproduct translog function, constant production factor prices	Small scale – exist Large scale – do not exist	Exist
Clark 1984			○	○		Unit banks in 57 major U.S. cities	Estimation of Box-Cox cost function	Exist	–
Kuroda & Kaneko 1985	○			○		Japanese city and local banks	Exponential approach	Exist	–

Notes: A : The balance of working assets
B : The number of loans (the number of accounts)
C : Revenue
D : Current expenses
E : Direct expenses attributed to each operation

* The FCA (Functional Cost Analysis) program is a research operation conducted by the Federal Reserve Bank for its member banks. Member banks independently decide whether or not to participate in the program; 10 to 20 percent of member banks ordinarily participate.

products as a variable of the cost function. Each of these studies verified the existence of economies of scope.

In defining outputs, BBHH and GSM employed number of lending cases (or accounts); MW and GS employed operating assets (or deposit balance). The obtained results (cost function of MW and GS can be considered essentially well-defined.[16] In contrast, BBHH and GSM have problems related to the conditions of monotonic increase of production factor prices, and their results are not necessarily well-defined as a cost function.

2. Measurement Based on Actual Data

Based on the above literature, this paper will use data from Japanese city banks and local banks to measure the cost function in order to verify economies of scope. The bank is defined here as a firm which does such production activities as lending and securities investment by joining collected fund (more strictly speaking, fiduciary right over funds), capital, and labor force together.

Using cross section data (annual data) for thirteen city banks[17] and sixty-four local banks (sixty-three banks for the terms before March 1981), the measurement was conducted for eleven terms from the year ending in March 1975 to the year ending in March 1985. First, it is necessary to explain the selection of outputs, production costs, and production factor prices.

A. Selection of Variables

a. Output (Y_i)

For the reasons mentioned in Section 1, outputs are measured by revenue. For actual calculation, if outputs are fractioned, an adequate theoretical model that corresponds to reality can be made, but the difficulty of calculating the cost function will increase. Therefore, it is necessary to divide outputs so that the model data is close to actual banking data but is still numerically verifiable.

In fact, the measurement is quite difficult if the data are classified into three

16. The well-defined cost function means that the function is economically meaningful. For details, refer to Chapter III, 2B. Functional Form, in this paper.

17. In this paper, we include the Bank of Tokyo, which specializes in foreign exchange business, in our analysis due to the following reasons;
i) we have assume the production function that allows diversified business activities, ii) the patterns of behavior across the city banks, including the Bank of Tokyo, have turned to be similar in recent years. According to the estimation excluding the Bank of Tokyo, we get the similar results, although the stability of parameters get a little bit worse.

products. The author attempted to calculate the data of a three-product case, but he failed to obtain satisfactory results that can be considered well-defined a cost function. Therefore, this paper takes the position that the total outputs are represented by the following two sets of variables:[18]

Y_1 : Revenue from loans (interest on loans and discount on bills)

Y_2 : Ordinary revenue (after deducting selling and redemption margin of securities held for investment and profit on foreign exchange transactions, etc.) $-Y_1$.

Y_1 is the revenue from lending activities, traditionally viewed as the primary business of banks. Y_2 is the revenue from other business activities, including securities investment. Selling and redemption margin of securities held for investment and revenue from foreign exchange transactions are deducted because they are not generally considered as revenue naturally obtained from ordinary operating activities (production factor inputs).

b. Production costs (C) and production factor prices (P_j)

As previously explained, if the bank is considered as a firm that carries out production activities by joining collected fund (fiduciary right of fund), real capital, and labor force together, production costs will be represented by the total sum of expenses for raising fund, nonpersonnel expenses, and personnel expenses. Three production factor prices are identified: yield on fund-raising (P_1), cost of nonpersonnel expenses (P_2), and cost of labor (P_3). Yield on fund-raising will be calculated by the following equation:

$$\text{yield on fund-raising } (P_1) = \frac{\text{fund-raising expenses}}{\text{the average outstanding balance of raised fund}} .$$

For the price of nonpersonnel expenses (P_2), it is necessary to consider the problem of the vintage of real capital. For instance, when there is a positive correlation between the production scale and the years of vintage (or briskness of real capital investment), larger productions will demonstrate a higher rate of economies. Such differences in economies, however, reflect the difference in the economies of production technology and cannot be considered as the difference in economies caused by production scale.

18. Other than this classification, for instance, it is possible to separate into "revenue from working assets and all other outputs." Using this classification, however, measurement results which can be considered as a cost function with adequate characteristics could not be obtained. This is partly because the sum of the parameters of each product becomes the parameter of the outputs after classification. Therefore, the division of a small number of outputs and all other outputs (earnings excluding profit from trust fund constitute only 2 to 6 % of total revenue) makes interaction terms and linear terms of the output of small quantity too small, and measurement becomes difficult.

In order to verify this, the investment ratio of real capital (I/K = the amount of investments in movables and immovables / the average outstanding balance of movables and immovables) is given regression by outputs. The cross section data for each term between the years ending in March 1976 and March 1960 are used, and the results are shown in Table 2. As shown in this table, no significant relationship between the production scale and the investment ratio of real capital was identified. On the contrary, the constant term shows a significant correlation for most of the periods.[19] This suggests that there is no bias with respect to the vintage of real capital of the given data and therefore, that the problem of vintage is rather negligible. The price of labor will therefore be calculated by the following equation:

$$\text{the price of non personnel expenses } (P_2) = \frac{\text{nonpersonnel expenses}}{\text{the average outstanding balance of movables and immovables}}.$$

Table 2 Investment Ratio to Real Capital (I/K) and Production Scale (Y)

Equation for Measurement I/K = a + bY

year	City Banks				Local Banks			
	a	t value	b	t value	a	t value	b	t value
1976/3	0.134	3.09	-2.36×10^{-8}	−0.26	0.139	8.86	-1.24×10^{-7}	−0.42
1977/3	0.108	3.53	-4.77×10^{-8}	−0.76	0.126	7.92	-3.37×10^{-7}	−1.19
1978/3	0.119	3.17	-9.53×10^{-8}	−1.19	0.088	6.45	-6.06×10^{-8}	−0.26
1979/3	0.069	2.42	-3.25×10^{-8}	−0.32	0.054	4.12	1.70×10^{-7}	0.75
1980/3	0.045	1.47	8.58×10^{-9}	0.19	0.045	4.34	6.58×10^{-8}	0.41
1981/3	0.009	0.234	4.82×10^{-8}	1.15	0.063	6.05	1.09×10^{-7}	0.97
1982/3	0.067	3.02	-2.27×10^{-8}	−1.23	0.064	5.34	1.40×10^{-7}	1.16
1983/3	0.067	2.23	-9.00×10^{-8}	−1.99	0.016	0.90	-5.27×10^{-8}	−0.30
1984/3	−0.086	−1.53	1.46×10^{-8}	0.30	0.057	4.28	-1.73×10^{-7}	−1.40
1985/3	0.039	2.59	-3.63×10^{-8}	−0.21	0.053	5.60	-1.68×10^{-8}	−0.21

19. In addition to the correlation between investment-income ratio of real capital (I/K) and production scale (Y), the correlation between investment-income ratio of real capital (I/K) and multiple products (linear, quadratic, and interaction terms in y_1 and y_2) was measured. But, no significant correlation was observed, and thus, the calculation method employed in this paper was not invalidated.

Next, the price of personnel expenses (P_3) will be measured by calculating the amount of work done per employee. P_3 will be obtained by the following equation:

$$\text{the price of labor } (P_3) = \frac{\text{personnel expenses}}{\text{the average number of employees}} \quad .$$

B. Functional Form

The translog cost function will be employed for estimation because it imposes no a priori restrictions on the elasticities of substitution and thus is more flexible than the Cabb-Douglas production function.[20,21]

$$\ln C = \alpha_0 + \sum_{i=1}^{n} \alpha i \cdot \ln yi + \sum_{j=i}^{m} \beta j \cdot \ln Pj + \frac{1}{2} \sum_{i=1}^{n} \sum_{k=1}^{n} \sigma ik \cdot \ln yi \cdot \ln yk$$

$$+ \frac{1}{2} \sum_{j=1}^{m} \sum_{h=1}^{m} \gamma jh \cdot \ln Pj \cdot \ln Ph \tag{12}$$

However, in order to accept the estimation result of equation (12) as a well-defined cost function, equation (12) should be (i) symmetric in cross terms, (ii) monotonic (positive marginal cost), (iii) linearly homogeneous in all input prices, and (iv) should satisfy conditions for second order (conditions for stability).[22]

Given equation (12) cost function, cost complementarities are given by the following equation, according to partial differential with respect to yi and yk:

$$\frac{\partial^2 C}{\partial yi \partial yk} = \frac{C}{yi \cdot yk} \cdot \left[\frac{\partial^2 \ln C}{\partial \ln yi \partial \ln yk} + \frac{\partial \ln C}{\partial \ln yi} \cdot \frac{\partial \ln C}{\partial \ln yk} \right]$$

$$= \frac{C}{yi \cdot yk} \cdot \left[\sigma ik + \left\{ \alpha i + \sum_{k=1}^{n} \sigma ik \cdot \ln yk \right\} \times \left\{ \alpha k + \sum_{i=1}^{n} \sigma ik \cdot \ln yi \right\} \right] \quad . \tag{13}$$

In addition, scale elasticity in all outputs, S_N, is given by the following equation.

$$S_N = \sum_{i=1}^{n} \frac{\partial \ln C}{\partial \ln yi}$$

$$= \sum_{i=1}^{n} \left\{ \alpha i + \sum_{k=1}^{n} \sigma ik \cdot \ln yk \right\} \tag{14}$$

20. Measurement is possible by simultaneously using the cost equation (12) and the following cost-share equation derived from Shephard's lemma, which states that "the marginal cost in Pj equals Xj (optimal production factor quantity of the given quantity)":

$$S_j = \frac{\partial \ln C}{\partial \ln Pj} = \beta j + \sum_{h=1}^{m} \gamma jh \cdot \ln Pj$$

(Provided that one of the above equations is redundant, since Σ Si=1.).

21. For actual estimates, yi and pj will be normalized about their sample means.

3. Estimation Results

At the time of actual calculation of the data of local banks, in addition to such restrictions as symmetry and linear homogeneity in production factor prices,[23] various restrictions associated with conditions for second order were imposed,[24] and

22. a. Symmetry with respect to interaction terms
In order for equation (12) to be a second-derivative cost function, the symmetry of the interaction terms as shown below is required:

$$\sigma ik = \sigma ki$$
$$\gamma jh = \gamma hj \quad .$$

b. Monotonicity
In order to satisfy the conditions for the marginal cost of outputs, $(\partial C/\partial yi) > 0$, and the marginal cost of production factor prices, $(\partial C/\partial pj) > 0$, they should be satisfied by the marginal cost at least at the approximate point of yi=1 and pj=1 as shown below:

$$\alpha i > 0$$
$$\beta j > 0 \quad .$$

c. Linear homogeneity of production factor prices with respect to production costs
The linear homogeneity of production factor prices to production costs, which contends that the change in production factor price unit will not affect production technology, is given by the following equations:

$$\sum_{j=1}^{m} \beta j = 1$$

$$\sum_{j=1}^{m} \gamma jh = 0 \quad .$$

Furthermore, for the homogeneity of output to production cost, the following restriction is imposed:

$$\sum_{i=1}^{n} \sigma ik = 0 \quad .$$

d. Conditions for second order
In order for equation (12) to represent the cost function indicating the minimized costs associated with a given production technology, the condition for second order needs to be satisfied. That is, Hessian matrix, which is expressed by the following equation, has to be a negative semidefinite matrix.

$$Hp = \left(\frac{\partial^2 C}{\partial P_j \partial Ph}\right)$$

$$= \begin{bmatrix} \gamma_{11}+\beta_1(\beta_1-1) & \gamma_{12}+\beta_1\cdot\beta_2 & \gamma_{13}+\beta_1\cdot\beta_3 \\ \gamma_{21}+\beta_2\beta_1 & \gamma_{22}+\beta_2(\beta_2-1) & \gamma_{23}+\beta_2\cdot\beta_3 \\ \gamma_{31}+\beta_3\beta_1 & \gamma_{32}+\beta_3\cdot\beta_2 & \gamma_{33}+\beta_3(\beta_3-1) \end{bmatrix}$$

then, conditions for monotonous increase and other conditions were verified after estimation. According to the estimation results, a cost function with adequate characteristics was found only where $\gamma jh(j,h=1, 2, 3)=0$. In contrast, for city banks, the restriction associated with the second-order condition, that is, $\gamma jh(j, h=1, 2, 3)=0$ was imposed from the beginning because of the limited degree of freedom. All other parts of the estimation procedure for city banks were the same as for local banks.

The estimation results are shown in Tables 3 and 4. Previously stated conditions, including symmetry, monotonicity, linear homogeneity in production factor prices, and the second-order condition, are all satisfied (or at least there is no case in which it is shown that these conditions do not have statistical significance). Thus, the employed estimation equation can be generally considered as a well-defined cost function.

In the column of economies of scope (cost complementarities), the values of $(\partial \ln C/\partial \ln y_1)\cdot(\partial \ln C/\partial \ln y_2)-(\partial^2 \ln C/\partial \ln y_1\ \partial \ln y_2)$ are shown (the value at the approximate point). For city banks, economies of scope (cost complementarities) are found to be statistically significant, excluding a certain period.[25] Also, economies of scope at city banks seem to be increasing. If so, it is probably related to the fact that

23. In addition to these restrictions, the estimation assumed the homogeneity of outputs to increase the degree of freedom. The estimation without this assumption was also made and it revealed economies of scope and scale although its parameters were not stable.

24. Let Bjh represent the matrix that should be negative semidefinite.
If Bjh is given Cholesky factorization,
Bjh=TDT'

$$
=\begin{bmatrix} 1 & 0 & \cdots\cdots & 0 \\ \lambda_{21} & 1 & \cdots\cdots & 0 \\ \vdots & \vdots & & \\ \lambda_{m1} & \lambda_{m2} & \cdots\cdots & 1 \end{bmatrix}
\begin{bmatrix} D_1 & 0 & \cdots\cdots & 0 \\ 0 & D_2 & \cdots\cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots\cdots & D_m \end{bmatrix}
\begin{bmatrix} 1 & \lambda_{21} & \cdots\cdots & \lambda_{1m} \\ 0 & 1 & \cdots\cdots & \lambda_{2m} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots\cdots & 1 \end{bmatrix}
$$

$$
=\begin{bmatrix} D_1 & \lambda_{21} & \cdots\cdots & \lambda_{m1}D_1 \\ \lambda_{21}D_1 & \lambda_{21}\lambda_{21}D_1+D_2 & \cdots\cdots & \lambda_{m1}\lambda_{21}D_1+\lambda_{m2}D_2 \\ \vdots & \vdots & & \vdots \\ \lambda_{m1}D_1 & \lambda_{m1}\lambda_{21}D_1+\lambda_{m2}D_2 & \cdots\cdots & \lambda_{m1}\lambda_{m1}D_1+\lambda_{m2}\lambda_{m2}D_2+\cdots+D_m \end{bmatrix}
$$

The estimation was conducted with various restriction for Dj=0. When all Dj's are zero, the part of production factor prices will become a Cobb-Douglas function.

25. A certain period refers to the several years after the First and the Second Oil Crises. During these periods, an especially tight money policy was enforced, so banks' behaviors for these periods may have been different from those more during ordinary periods.

Table 3 City Banks

	α_0	α_1	α_2	σ_{11}	σ_{12}	σ_{22}	β_1	β_2	β_3	R^2	S.E.	Scale	Scope
1975/3	0.287×10^{-5} (0.45×10^{-3})	0.642 (11.52)	0.313 (5.85)	0.201 (2.66)	−0.201 (2.66)	0.201 (2.66)	0.918 (6.08)	0.045 (0.52)	0.038 (0.19)	0.998	0.023	0.955*	0.000
1976/3	0.383×10^{-5} (0.65×10^{-3})	0.536 (7.48)	0.411 (6.22)	0.206 (2.41)	−0.206 (2.41)	0.206 (2.41)	0.901 (3.97)	0.069 (0.57)	0.032 (0.10)	0.998	0.021	0.947*	0.014
1977/3	0.990×10^{-3} (0.14×10^{-3})	0.413 (4.69)	0.518 (6.22)	0.256 (2.38)	−0.256 (2.38)	0.256 (2.38)	0.389 (1.70)	−0.038 (0.47)	0.650 (2.20)	0.997	0.022	0.931*	−0.043*
1978/3	0.416×10^{-5} (0.52×10^{-3})	0.412 (3.60)	0.513 (4.79)	0.305 (1.87)	−0.305 (1.87)	0.305 (1.87)	0.513 (3.37)	−0.057 (0.57)	0.543 (2.44)	0.995	0.029	0.926*	−0.093*
1979/3	0.183×10^{-5} (0.25×10^{-3})	0.529 (4.45)	0.393 (3.20)	0.386 (1.35)	−0.386 (1.35)	0.386 (1.35)	0.343 (1.99)	−0.083 (1.70)	0.740 (4.41)	0.996	0.026	0.922*	−0.178*
1980/3	0.586×10^{-7} (0.681×10^{-5})	0.635 (3.50)	0.286 (1.54)	0.175 (0.58)	−0.175 (0.58)	0.175 (0.58)	0.495 (2.49)	0.013 (0.11)	0.492 (2.14)	0.996	0.031	0.921*	0.007
1981/3	-0.786×10^{-5} (0.12×10^{-2})	0.591 (2.93)	0.351 (1.96)	0.269 (1.22)	−0.269 (1.22)	0.269 (1.22)	0.524 (2.54)	0.031 (0.18)	0.445 (1.41)	0.997	0.024	0.941	−0.062
1982/3	0.103×10^{-5} (0.16×10^{-3})	0.575 (4.65)	0.358 (3.10)	0.214 (1.05)	−0.214 (1.05)	0.214 (1.05)	0.524 (3.91)	−0.030 (0.34)	0.505 (3.32)	0.998	0.023	0.933*	−0.008*
1983/3	0.579×10^{-7} (0.14×10^{-4})	0.618 (5.26)	0.337 (2.97)	0.346 (1.93)	−0.346 (1.93)	0.346 (1.93)	0.456 (3.18)	0.127 (2.29)	0.417 (2.40)	0.999	0.015	0.955*	−0.138*
1984/3	0.397×10^{-5} (0.84×10^{-3})	0.502 (4.03)	0.460 (3.62)	0.461 (2.27)	−0.461 (2.27)	0.461 (2.27)	0.343 (2.61)	0.159 (2.51)	0.498 (4.45)	0.999	0.017	0.965*	−0.229*
1985/3	0.136×10^{-6} (0.29×10^{-4})	0.471 (3.48)	0.458 (1.43)	0.578 (1.43)	−0.578 (1.43)	0.578 (1.43)	0.448 (4.76)	0.049 (0.867)	0.504 (6.24)	0.999	0.017	0.928*	−0.362*

Notes: 1. The figures in parentheses are the absolute values for t value.

2. Scale = $\sum_{i=1}^{2} \partial \ln C/\partial \ln Y_i$ (Economies of scale with respect to all products)

3. Scope = $\dfrac{\partial^2 \ln C}{\partial \ln Y_1 \partial \ln Y_2} + \dfrac{\partial \ln C}{\partial \ln Y_1} \cdot \dfrac{\partial \ln C}{\partial \ln Y_2}$ (Cost complementarities x $(Y_1 \cdot Y_2/C)$).

4. * Indicates that economies of scale with respect to all products and cost complementarities are significant under likelihood ratio test of 5% level.

Table 4 Local Banks

	α_0	α_1	α_2	σ_{11}	σ_{12}	σ_{22}	β_1	β_2	β_3	R^2	S.E.	Scale	Scope
1975/3	0.419×10^{-5} (0.92×10^{-3})	0.530 (3.43)	0.410 (2.64)	0.140 (1.43)	−0.140 (1.43)	0.140 (1.43)	0.667 (11.54)	0.063 (3.11)	0.270 (4.70)	0.997	0.036	0.941*	0.077
1976/3	0.861×10^{-5} (0.21×10^{-2})	0.532 (3.42)	0.419 (2.67)	0.168 (1.54)	−0.168 (1.54)	0.168 (1.54)	0.614 (11.34)	0.052 (3.06)	0.334 (5.89)	0.998	0.033	0.951*	0.055
1977/3	0.199×10^{-5} (0.47×10^{-3})	0.513 (2.65)	0.438 (2.25)	0.206 (1.37)	−0.206 (1.37)	0.206 (1.37)	0.591 (11.20)	0.052 (2.74)	0.357 (6.33)	0.997	0.034	0.951*	0.019
1978/3	-0.603×10^{-7} (0.13×10^{-4})	0.711 (3.94)	0.251 (1.38)	0.062 (0.40)	−0.062 (0.40)	0.062 (0.40)	0.625 (11.21)	0.054 (2.48)	0.320 (5.33)	0.997	0.037	0.962*	0.116
1979/3	0.378×10^{-6} (0.83×10^{-4})	0.721 (5.47)	0.242 (1.81)	0.043 (0.31)	−0.043 (0.31)	0.043 (0.31)	0.654 (12.94)	0.020 (0.95)	0.326 (5.80)	0.997	0.036	0.963*	0.132
1980/3	0.858×10^{-5} (0.21×10^{-2})	0.643 (6.35)	0.302 (2.96)	0.105 (1.00)	−0.105 (1.00)	0.105 (1.00)	0.689 (15.00)	−0.001 (0.05)	0.312 (6.40)	0.998	0.032	0.945*	0.090
1981/3	-0.313×10^{-5} (0.87×10^{-3})	0.586 (5.47)	0.359 (3.33)	0.130 (1.35)	−0.130 (1.35)	0.130 (1.35)	0.674 (14.48)	0.027 (1.43)	0.299 (6.09)	0.998	0.029	0.945*	0.080
1982/3	0.327×10^{-5} (0.77×10^{-3})	0.608 (6.01)	0.342 (3.44)	0.121 (1.28)	−0.121 (1.28)	0.121 (1.28)	0.657 (13.6)	0.012 (0.57)	0.331 (6.64)	0.998	0.034	0.950*	0.088
1983/3	-2.60×10^{-5} (0.54×10^{-3})	0.587 (6.61)	0.365 (4.17)	0.141 (1.65)	−0.141 (1.65)	0.141 (1.65)	0.669 (14.34)	0.019 (0.93)	0.312 (6.22)	0.998	0.032	0.952*	0.073
1984/3	-0.280×10^{-5} (0.89×10^{-3})	0.506 (6.22)	0.451 (5.60)	0.231 (2.37)	−0.231 (2.37)	0.231 (2.37)	0.753 (18.46)	0.032 (1.90)	0.215 (4.98)	0.999	0.025	0.957*	−0.003*
1985/3	0.322×10^{-5} (0.88×10^{-3})	0.490 (5.53)	0.455 (5.19)	0.250 (2.64)	−0.250 (2.64)	0.250 (2.64)	0.683 (15.74)	0.025 (1.27)	0.292 (6.40)	0.998	0.029	0.945*	−0.027*

Notes: 1. The figures in parentheses are the absolute values for t value.

2. Scale = $\sum_{i=1}^{2} \partial \ln C/\partial \ln Y_i$ (Economies of scale with respect to all products)

3. Scope = $\dfrac{\partial^2 \ln C}{\partial \ln Y_1 \partial \ln Y_2} + \dfrac{\partial \ln C}{\partial \ln Y_1} \cdot \dfrac{\partial \ln C}{\partial \ln Y_2}$ (Cost complementarities x $(Y_1 \cdot Y_2/C)$).

4. * Indicates that economies of scale with respect to all products and cost complementarities are significant under likelihood ratio test of 5% level.

diversification is occurring at the same time that public inputs are accumulating and the technology for applying them is advancing. For instance, Iwata – Horiuchi (1985) points out that "due to the remarkable development of computer and telecommunication technologies, it has become possible to concentrate, store, analyze, and process customer information more quickly and at lower cost."

Taking a look at economies of scope (cost complementarities) of local banks, no significant economies are found except in recent data. This means that at local banks economies of scope were previously minimal or not present. One possible explanation for this is that local banks have accumulated fewer public inputs like know-how and information about various business fields than have city banks. Local banks have been able to remain profitable even though they have pursued a conservative lending policy. Another probable reason is that local banks generally have fewer customers than city banks, so their customer information services have more narrow applicability. Cost complementarities are found in recent data from local banks, however, because increasing deregulation has forced even local banks to diversify their business under the pressure of intensified competition. These deregulation pressures, and the adoption by local banks of the same technologies as used by city banks, has positively influenced the business of local banks.

Both city banks and local banks show statistically significant economies of scale with respect to all outputs. The degree of economies fluctuates somewhat annually, but for city banks, the value remains within 0.92 – 0.97, and for local banks, within 0.94 –0.96 (The lower the values, the larger the economies of scale. See pp. 69 of this paper.).

Considering the fact that economies of scale previously measured in other Japanese industries showed values of 0.9 – 1.0,[26] economies of scale in banks is noticeable but not significantly large. (The above figure may include the effect of financial regulations based primarily on the size of banks, so it is inferred that the actual degree of economies of scale is still smaller than the results.) Therefore, with respect to such problem as what the bank's management should be and what measures related authorities should take after monetary deregulation, it is more appropriate to strengthen banks' functions by expanding economies of scope than to take business expansion measures such as mergers, which tend to cause friction.

Evidence of economies of scale and cost complementarities in city banks and, more recently, in local banks, suggests the existence of natural monopoly in production technology. However, it would be inappropriate to seek from this finding signs of a noncompetitive situation in the real market, for the following reasons:

1) These measurement results are concerned with an industrial segment, such as city banks and local banks, in a certain period, and thus provide information with

26. For instance, refer to Yoshioka (1982).

limited scope, although "natural monopoly" is a concept applicable to the whole industry.

2) In actual competition in the market, in addition to production technology, the unique characteristics of each bank (locality, customer relations) play significant roles.
3) Even when there is a natural monopoly in production technology, effective competition is possible if entrance into and exit from the market is completely unrestricted and sunk cost is zero (theory of the contestable market).[27,28]

Indeed, the Japanese financial market place is open not only to domestic but also to foreign competition, as can be seen in the active participation by foreign institutions (e.g., entrance by foreign banks into the trust banking market). And even with increasing deregulation, local shinkin banks (credit associations) continue to enjoy the competitive advantage afforded by their locations. Lowering of the barriers between banks and securities companies will further contribute to increasing market contestability. Nevertheless, it will be necessary to make appropriate prior consultations and subsequent monitoring of the effects of diversification prevent the development of noncompetitive market conditions (This concern will be more critical for regional markets than for the national financial market as a whole.).[29]

IV. Conclusion

This paper introduced the concept of economies of scope and presented the results of an empirical study applying this concept to the Japanese banking business. As observed in this study, it is certain that economies of multiproduct production exist in city banks and the most modern regional banks. This observation is backed up by the fact that banks have been actively participating in international financial operations and promoting bank dealings. Furthermore, these diversifications have been made under various regulations, including the so-called "agreement by three bureaus," other administrative guidance provided by Article 65 of the Securities and Exchange Law, and other regulations stipulated by law. It is expected that the

27. Refer to Baumol et al. (1982).

28. In the real banking industry (broadly defined market), new entry to the market is extremely difficult because of regulations. At the level of each operation and each product of banks (narrowly defined market), however, various competitive operations and products may be provided by nonbank participants.

29. Although Sudo (1985) also tries to verify the existence of economies of scope for Japanese banks, it fails to obtain the stable estimates because she used inadequate output data, ignored input prices and assumed strong separability in making output index.

business deversification of banks will accelerate when the business field restriction that has restrained competition is abolished in the future under the policy of monetary deregulation. For banks, this means that they will positively make use of information and know-how accumulated through lending business for the development of new business fields. By doing so, they will enjoy economies of scope.

There are many who fear that bank profits will decrease with the advancement of monetary deregulation. This fear, however, is based on the premise that all conditions work adversely, and it should not be overlooked that there are some indications that bank revenue will not necessarily be negatively affected. The possible development of the widely spread banking system and the reduction of implicit interest rate may have positive effects. This paper, for example, points out that economies of scope are becoming more widespread with the development of monetary deregulation.

Needless to say, no type of bank will always experience economies of scope to a large extent. First of all, there are problems related to management resources. For instance, the managers of a small bank have an advantage in that they can easily grasp its whole business. Therefore, it may be more appropriate to continue to develop local business than to expand the bank's business into other fields. Large-scale banks are more likely to enjoy the merits of business diversification, because the managers of larger banks have more diversified functions. Questions about whether a bank should diversify its business in situations where economies of scope exist, or should concentrate its established business and become a financial institution specialized in a particular field should be answered by the management of each bank. There are also reasons why it is inappropriate to unconditionally promote economies of scope. It has already been pointed out that it is necessary to check strictly whether economies of scope are exhibited in ways that lead to the establishment of local monetary monopoly. The problem of conflicting interests should also be mentioned here. There are many reasons why the restriction of business fields was introduced in the 1930s. For instance, it was feared that bank profits would become unstable to because of the bank's dealings. Today, however, an increasing number of people have become skeptical of this view. It is said that the grounds for the business field restriction, which is still considered valid, is the prevention of conflicting interests from the standpoint of customer (investor) protection. It is widely recognized that it is necessary to control information flow in an organization in order to prevent the problem of conflicting interests, even though the restriction of business fields tends to be eased. As a matter of fact, in the United States, where there are no regulations about the separation of commercial and trust banks, a so-called "Chinese Wall" has been established to strictly ban information and personnel transfer (self-imposed control). This type of restriction will negatively affect economies of scope. It is, however, unquestionable that restrictions to avoid conflicting interests are necessary for the maintenance of a sound monetary system in the same sense that the restriction of the ratio of net worth to total capital will be tightened while various regulations that restrict competition are being relaxed and abolished.

Based on the above argument, it can be concluded that in the era of monetary deregulation, it is appropriate to expand the business fields of financial institutions and allow them to enjoy economies of scope while devising necessary minimum restrictions.

Appendix I: Production Function and Cost Function

In the analysis of this paper, the cost function is used to analyze production technology. It is verified here that the cost function provides the same information as the production function under the prescribed conditions.

Let $\mathbf{y}=(y_1, y_2, \ldots, y_n)$ represent the vector of outputs, $\mathbf{x}=(x_1, x_2, \ldots, x_m)$ represent the vector of production factors, and $\mathbf{w}=(w_1, w_2, \ldots, w_m)$ represent the vector of production factor prices. An effective combination of technology for producing $\mathbf{y}$ from $\mathbf{x}$ is given by the following production function:

$$f(\mathbf{x}, \mathbf{y}) = 0, \tag{A-1}$$

subject to the following assumptions:

(1) If $f(0, \mathbf{y}) = 0$, $\mathbf{y} = 0$.

(2) f is monotonous nondecreasing in all xj and monotonic nonincreasing in all y_i.

(3) f is a semiconcave function in $\mathbf{x}$.

Here, the cost function which gives the minimum cost is defined as follows:

$$C(\mathbf{y},\mathbf{w}) = \underset{\mathbf{x}}{\mathrm{Min}} \{\mathbf{w}\cdot\mathbf{x} \mid f(x, y) = 0\}. \tag{A-2}$$

1. Economies of Scale

In addition to the above assumptions (1) - (3), it is assumed that f is differentiable. Let Sp denote the scale elasticity that indicates how many times larger the production vector $\mathbf{y}$ becomes when the production factor vector $\mathbf{x}$ is multiplied by t. Since

$f(t\mathbf{x}^\circ, k\mathbf{y}^\circ) = 0$ ($\mathbf{x}^\circ$ and $\mathbf{y}^\circ$ are unit vectors),

$$\left(\Sigma_j \frac{\partial f}{\partial xj}\cdot\frac{\partial txj^0}{\partial t}\right)\cdot dt+\left(\Sigma_i \frac{\partial f}{\partial yi}\cdot\frac{\partial kyi^0}{\partial k}\right)\cdot dk = 0$$

$$\frac{dk}{dt} = -\frac{\Sigma_j \dfrac{\partial f}{\partial xj}\cdot xj^0}{\Sigma_i \dfrac{\partial f}{\partial yi}\cdot yi^0} .$$

Therefore, Sp can be expressed as follows.

$$Sp = -\frac{\Sigma_j xj \cdot \frac{\partial f}{\partial xj}}{\Sigma_i yi \cdot \frac{\partial f}{\partial yi}} \tag{A-3}$$

Considering the problem of minimizing cost C, that is,

$$\underset{\mathbf{x}}{\text{Min}}\ \mathbf{w} \cdot \mathbf{x}$$
$$\text{s. t. } f(\mathbf{x}, \mathbf{y}) = 0\ ,$$

the second order condition is satisfied here because f is a quasi=concave function. According to the first=order condition,

$$wj + \mu \frac{\partial f}{\partial xj} = 0$$

(μ is Lagrange's multiplier).

In addition to the above equation, $C = \Sigma_j wj \cdot xj$.
Therefore,

$$C = -\mu \Sigma_j xj \cdot \frac{\partial f}{\partial xj}\quad . \tag{A-4}$$

Here,

$$dc = \Sigma_j wjdxj$$
$$= -\mu \Sigma_j \frac{\partial f}{\partial xj} \cdot dxj \equiv (*)\quad .$$

Also, since $f(\mathbf{x}, \mathbf{y}) = 0$, $$\Sigma_j \frac{\partial f}{\partial xj} \cdot dxj + \Sigma_i \frac{\partial f}{\partial yi} \cdot dyi = 0\quad .$$

Therefore,

$$(*) = \mu \Sigma_i \frac{\partial f}{\partial yi} \cdot dyi\quad .$$
$$\therefore\quad \frac{\partial C}{\partial yi} = \mu \frac{\partial f}{\partial yi} \tag{A-5}$$

Taken from equations (A-4) and (A-5),

$$Sp = -\frac{\Sigma_j xj \cdot \dfrac{\partial f}{\partial xj}}{\Sigma_i yi \cdot \dfrac{\partial f}{\partial yi}} = \frac{C}{\Sigma_i yi \cdot \dfrac{\partial C}{\partial yi}} \tag{A-6}$$

or

$$\frac{1}{Sp} = \Sigma_i \frac{\partial \ln C}{\partial \ln yi} \quad . \tag{A-7}$$

Based on the above, the scale elasticity, Sp, in production function f can be expressed by the cost function.

2. Superadditivity of the Production Function and Subadditivity of the Cost Function

In addition to the assumptions (2) to (3) with regard to the production function, assume the following:

(4) $(\partial yi/\partial xj) > 0$, and at the same time,
$(\partial yi/\partial xj) \neq \infty$.

The set of necessary production factors A (y) and the set of inefficient production factors A°(y) are defined as follows:

$$A(\mathbf{y}) \equiv \{\mathbf{x} | (\mathbf{x}, \mathbf{y}) \in T\}$$

$$A^{\circ}(\mathbf{y}) \equiv \{\mathbf{x} | (\mathbf{x}, \mathbf{y}) \in T, \exists \mathbf{x}', \mathbf{x}' \leq \mathbf{x}, \mathbf{x}' \neq \mathbf{x}, (\mathbf{x}', \mathbf{y}) \in T\}$$

where T is the production possibility set.
Then, $\bar{A}(\mathbf{y})$, which is the set of efficient production factors, is defined as follows:

$$\bar{A}(\mathbf{y}) = A(\mathbf{y}) - A^{\circ}(\mathbf{y}) \quad .$$

According to assumption (3), A(y) is a convex set, and also, based on (2) and (4), it strictly satisfies monotoneity separation theorem of separate hyperplane, in all $x \in \bar{A}(\mathbf{y})$, the positive combination of production factor prices W>0 exists, which satisfies

$$\mathbf{w} \cdot \mathbf{z} \geq k = \mathbf{w} \cdot \mathbf{x}, \ \forall \mathbf{z} \in A(\mathbf{y})$$

(where $\mathbf{w}$ is the vector of efficiency prices, and $k = \mathbf{w}\cdot\mathbf{x}$ is the hyperplane of equal cost).

a. Suppose that superadditivity exists in the production factor as follows:
If $\mathbf{x}_1 \in A(\mathbf{y}_1), \mathbf{x}_2 \in A(\mathbf{y}_2)$, then $\mathbf{x}_1+\mathbf{x}_2 \in A^\circ(\mathbf{y}_1+\mathbf{y}_2)$.
Here, if $C(\mathbf{y}_1, \mathbf{w})=\mathbf{w}\cdot\mathbf{x}_1^*$, $C(\mathbf{y}_2, \mathbf{w})=\mathbf{w}\cdot\mathbf{x}_2^*$ then $\mathbf{x}_1^* \in A(\mathbf{y}_1), \mathbf{x}_2^* \in A(\mathbf{y}_2)$.
Since for $\mathbf{x}_1^*+\mathbf{x}_2^*$, $\mathbf{x}^1$ exists which satisfies $\mathbf{x}^1 \leqq \mathbf{x}^*+\mathbf{x}_2^*$, $\mathbf{x}^1 \neq \mathbf{x}_1^*+\mathbf{x}_2^*$, $\mathbf{x}^1 \in A(\mathbf{y}_1+\mathbf{y}_2)$ for all positive combinations of production factor prices $\mathbf{w}>0$.

$$C(\mathbf{y}_1+\mathbf{y}_2, \mathbf{w}) \leq \mathbf{w}\cdot\mathbf{x}' < \mathbf{w}\cdot(\mathbf{x}_1^*+\mathbf{x}_2^*) = \mathbf{w}\cdot\mathbf{x}_1^* + \mathbf{w}\cdot\mathbf{x}_2^*$$
$$= C(\mathbf{y}_1, \mathbf{w})+C(\mathbf{y}_2, \mathbf{w}) .$$

b. Next, suppose that subadditivity of the cost function exists in all positive combinations of production factor prices $\mathbf{w}>0$. That is,

$$C(\mathbf{y}_1+\mathbf{y}_2, \mathbf{w}) < C(\mathbf{y}_1, \mathbf{w})+C(\mathbf{y}_2, \mathbf{w}) .$$

Here, if $\mathbf{x}_1 \in A(\mathbf{y}_1)$ and $\mathbf{x}_2 \in A(\mathbf{y}_2)$, then $\mathbf{w}\cdot\mathbf{x}_1 \geqq C(\mathbf{y}_1, \mathbf{w})$, $\mathbf{w}\cdot\mathbf{x}_2 \geqq C(\mathbf{y}_2, \mathbf{w})$.

Therefore,

$$C(\mathbf{y}_1+\mathbf{y}_2, \mathbf{w}) < \mathbf{w}\cdot(\mathbf{x}_1+\mathbf{x}_2), \ \forall \mathbf{w}>0 .$$

Here, if $\mathbf{x}_1+\mathbf{x}_2 \notin A(\mathbf{y}_1+\mathbf{y}_2)$, then

$$C(\cdot) = \min_{\mathbf{x}} \{\mathbf{w}\cdot\mathbf{x} \mid \mathbf{x} \in A(\mathbf{y}_1+\mathbf{y}_2)\} .$$

Nevertheless, $\exists\, \mathbf{w}$, $C(\mathbf{y}_1+\mathbf{y}_{21}\ \mathbf{w}) > \mathbf{w}\cdot(\mathbf{x}_1+\mathbf{x}_2)$.
This is contradictory.

On the other hand, if $\mathbf{x}_1+\mathbf{x}_2 \in \bar{A}(\mathbf{y}_1+\mathbf{y}_2)$, then $\mathbf{w}>0$ exists, which satisfies $\mathbf{w}\cdot\mathbf{z} \geqq k = \mathbf{w}\cdot(\mathbf{x}_1+\mathbf{x}_2)$, $\forall \mathbf{z} \in A(\mathbf{y}_1+\mathbf{y}_2)$.
For all $W>0$,

$$C(\mathbf{y}_1+\mathbf{y}_2, \mathbf{w}) = \mathbf{w}\cdot(\mathbf{x}_1+\mathbf{x}_2) = C(\mathbf{y}_1, \mathbf{w})+C(\mathbf{y}_2, \mathbf{w}) .$$

But this is also contradictory. Therefore,

$$\mathbf{x}_1+\mathbf{x}_2 \in A^\circ(\mathbf{y}_1+\mathbf{y}_2) .$$

This implies that, under the given assumption, the cost function is subadditive for all the positive vectors of production factor prices if, and only if, the production function is superadditive.

Appendix II: Natural Monopoly, Economies of Scope, and Economies of Scale

1. The sufficient condition for subadditivity of the cost function, which employs trans-ray convexity and economies of scale in all products.

For ease of exposition, Figure 7 illustrates a two-product case. Figure 7b is the figure when looking at Figure 7b from the directrion of C axis.

For a production vector **y**, let $\mathbf{y}^1$ represent any production vector $\mathbf{y}^*$ which satisfies $\mathbf{y}^*<\mathbf{y}$, $\mathbf{y}^*\neq 0$, and let $\mathbf{y}^2$ represent $\mathbf{y}-\mathbf{y}^*$. The production costs of $\mathbf{y}^1$ and $\mathbf{y}^2$ are given by the C coordinate value of points K and J, respec=fively. Subadditivity of the cost function is indicated if point M, which gives $C(\mathbf{y}^1+\mathbf{y}^2)=C(Y)$ is below point L, which gives $C(\mathbf{y}^1)+C(\mathbf{y}^2)$.

If $\mathbf{y}^1$ and $\mathbf{y}^2$ are multiplied by V_1 and V_2, respectively, and the results are on the line AB which passes Y, in order for point M to be below point L, economies of scale with respect to all products should strictly exist in the process from $\mathbf{y}^1$ to $V_1\mathbf{y}^1$, and from $\mathbf{y}^2$ to $V_2\mathbf{y}^2$, and at the same time, the trans-ray which stands on the line AB:w·y=wo (elements of W, W_1 and W_2, are positive) should exhibit convexity in a weak sense. That is,

$$C(\mathbf{y}^1)+C(\mathbf{y}^2)>C(V_1\mathbf{y}^1)/V_1+C(V_2\mathbf{y}^2)/V_2 \tag{A-8}$$

and at the same time,

$$C(\mathbf{y})=C\left(\frac{1}{V_1}\cdot V_1\mathbf{y}^1+\left(1-\frac{1}{V_1}\right)\cdot V_2\mathbf{y}^2\right)\leq\frac{1}{V_1}\cdot C(V_1\mathbf{y}^1)+\left(1-\frac{1}{V_1}\right)\cdot C(V_2\mathbf{y}^2)\,.$$

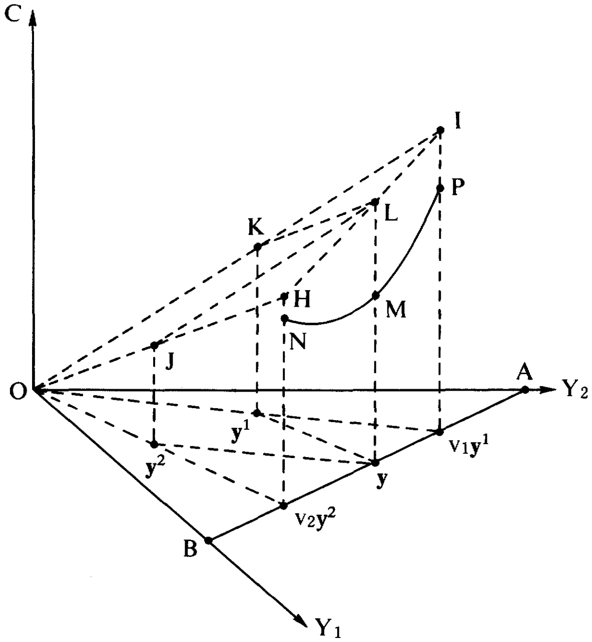
$$\because\quad \mathbf{w}\cdot(V_1\mathbf{y}^1)=\mathbf{w}\cdot(V_2\mathbf{y}^2)=\mathbf{w}\cdot\mathbf{y}=\omega_0,\ \ \mathbf{y}^1+\mathbf{y}^2=\mathbf{y}$$

$$\frac{1}{V_1}+\frac{1}{V_2}=1 \tag{A-9}$$

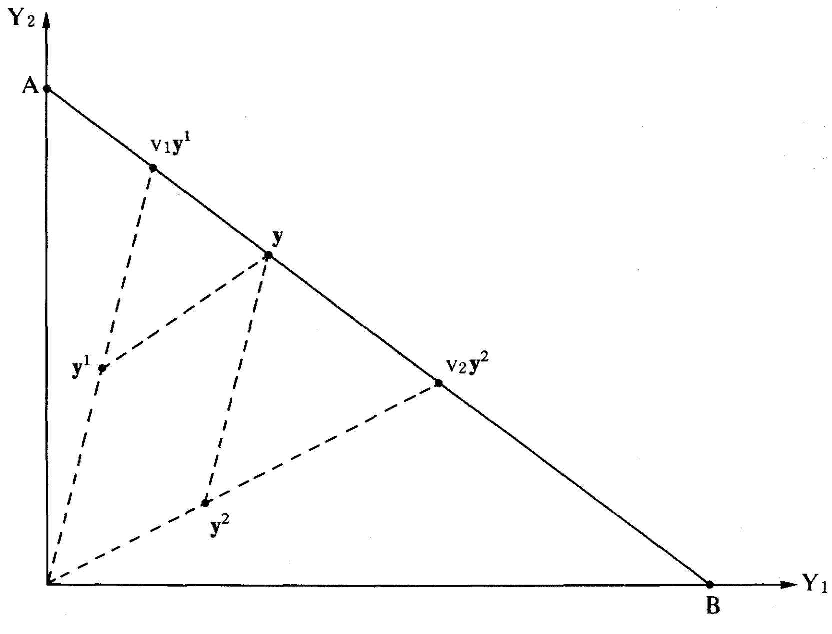
Here, let $T(t)=C(t\cdot\frac{\omega_0}{\omega_1}, (1\text{-}t)\cdot\frac{\omega_0}{\omega_2})$ represent the trans-ray on the line connecting point $A(0, \frac{\omega^0}{\omega_2})$, and point $B(\frac{\omega_0}{\omega_1}, 0)$. In order to satisfy equation (A-9), that is, for the trans-ray convexity to hold:

Figure 7 Natual Monopoly in Relation to Cost Complementarities and Economies of Scale with Respect to All Products

a.



b.



$$\frac{d^2T}{dt^2}=\left(\frac{\omega_0}{\omega_1\omega_2}\right)^2\cdot\left\{\omega^2{}_2\frac{\partial^2C}{\partial y_1{}^2}+\omega_1{}^2\frac{\partial^2C}{\partial y_2{}^2}-2\omega_1\omega_2\frac{\partial^2C}{\partial y_1\partial y_2}\right\}\geqq 0\ .$$

That is,

$$\omega_2{}^2\frac{\partial^2C}{\partial y_1{}^2}+\omega_1{}^2\frac{\partial^2C}{\partial y_2{}^2}-2\omega_1\omega_2\frac{\partial^2C}{\partial y_1\partial y_2}\geqq 0 \qquad \text{(A-10)}$$

$$(\because \quad \omega_0,\ \omega_1,\ \omega_2 \neq 0)\ .$$

Therefore, trans-ray convexity will appear either when $\partial^2 C/\partial y_1{}^2$, and $\partial^2 C/\partial y_2{}^2$ take positive values that are large enough or when cost complementarities ($\partial C/\partial y_1\partial y_2<0$) are large enough.

2. The sufficient condition for subadditivity of the cost function, which employs economies of scope and economies of scale with respect to a particular product.

As in Section 1, Figure 8 shows a two-product case. Figure 8b is the figure when Figure 8b is seen from the direction of C coordinate axis.

When $\mathbf{y}^1=(y_1{}^1,\ y_2{}^2)$, and $\mathbf{y}^2=(y_1{}^2,\ y_2{}^2)$

consider the situation in which both products are produced separately:

$$\mathbf{y}^A=(y_1{}^1+y_1{}^2,\ 0),\ \mathbf{y}^B=(0,\ y_2{}^1+y_2{}^2).$$

Then, the sufficient condition for subadditivity of the cost function is that economies of scale with respect to both specific products hold for the entire process from $C(\mathbf{y}^1)$ + $C(\mathbf{y}^2)$ to $C(\mathbf{y}^A)$ + $C(\mathbf{y}^B)$, and at the same time, economies of scope exist at $\mathbf{y}(=\mathbf{y}^A+\mathbf{y}^B)$.
Thus, the sufficient condition can be written as follows:

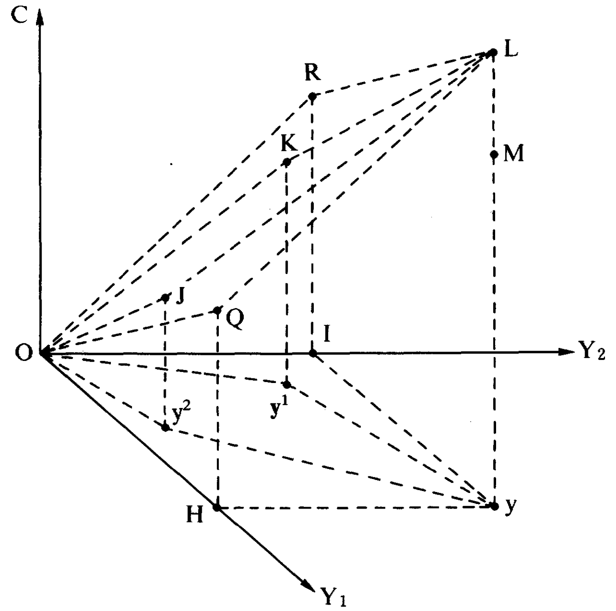
$$\begin{aligned}C(y_1{}^1,\ y_2{}^1)+C(y_1{}^2,\ y_2{}^2)&>C(y_1{}^1+y_1{}^2,\ y_2{}^1)+C(0,\ y_2{}^2)\\&>C(y_1{}^1+y_1{}^2,\ 0)+C(0,\ y_2{}^1+y_2{}^2)\end{aligned}$$

and at the same time,

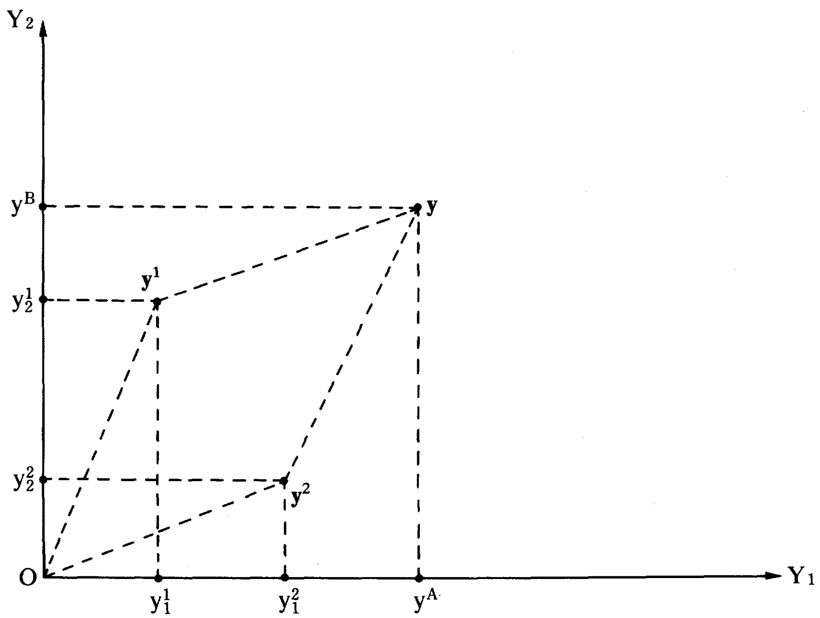
$$C(y_1{}^1+y_1{}^2,\ 0)+C(0,\ y_2{}^1+y_2{}^2)>C(y_1{}^1+y_1{}^2,\ y_2{}^1+y_2{}^2)=C(\mathbf{y})\ .$$

Figure 8 Natural Monopoly in Relation to Economies of Scope and Economies of Scale with Respect to a Specific Product

a.



b.



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