Trend Estimation via Smoothness Priors-State Space Modeling*

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I. Introduction

A trend is often considered to be a phenomenon that continues monotonically or one whose direction is irreversible. Nevertheless, it is not easy to formulate or estimate a trend in a strictly statistical manner.[1] Although business cycles or fluctuations around trends have been subjected to rigorous analyses, attempts to estimate a particular economic trend using traditional statistical methods have often engendered significant problems. A lower-order polynominal regression is conventionally used to investigate trends.[2] Havenner and Swamy (1978), however, have suggested that although a lower-order polynominal trend may describe a totally deterministic movement, longer-term trends must be estimated stochastically. Because trends are usually irregular over time, it may not be appropriate to formulate them using deterministic polynominals.

In recent empirical economic studies based on the stationary stochastic process,

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1. "In 'eliminating a trend', we must take care not to throw out the baby along with the bath water" (Samuelson (1980); p. 237).

2. Although the moving average is often used to estimate trends, this is not considered here because of the problem of missing values at the beginning and end of the series and other problems connected with this method.

the problems related to trends have been described as the "arbitrariness of stationarity" on detrending (Yamamoto (1983)). Stationary stochastic model-based analyses have great significance for the study of a dynamic system, compared with traditional econometric analyses which have not worked well. The nonstationarity of economic time series, however, has had to be approximated to the stationary process. In most cases residuals obtained by detrending series from the nonstationary series are used. But the methods used to remove the trend are not always convincing. Chan, Hayya, and Ord (1977), Nelson and Kang (1981), and Nelson and Plosser (1982) have studied on the problems caused by trend removal, especially the removal of polynominal trends by regression. Nelson and Plosser (ibid.) state that the problems stemmed from the deterministic trends when the trends should actually be regarded as stochastic in nature. That is, if the secular movement is that of a stochastic, models based on deterministic time-trend residuals are misspecified. Also, a series obtained by removing deterministic trends may produce pseudo-periodic behavior with long lags.

To avoid the arbitrariness of trend estimation, Box and Jenkins (1976) suggested the adoption of first- or higher- order differencing for a series with trends. Since then, the variate differencing method (including the issue of seasonal adjustment) has been widely employed in analyses based on the stationary stochastic model, above all in the empirical analyses in the United States (see, for example, Pierce (1975); Box, Hillmer, and Tiao (1978); Hillmer and Tiao (1982); and Box and Pierce (1981)). Havenner and Swamy (1978), however, have pointed out that although it is essential to analyze a time series as a stochastic process, the problems caused by over- or under-differencing, will make the series' statistical attributes more complicated[3] and there is still no sufficient method for determining the order of differencing.[4]

Analysis around trends bears significantly on the method chosen for empirical research. It is therefore imperative to consider it from different perspectives and avoid the tendency to limit one's inquiry to specific methodologies. This paper describes a statistical method for estimating stochastic trends that takes into consideration the problems discussed above. To find the best may be hard, but it should be defined as clearly as possible from the statistical point of view. The method presented in this paper was developed by Kitagawa and Gersch (1984). As a basic premise, their method follows a stochastic process. These are its main features: (1) trends are characterized by a perturbed stochastic difference equation, which differs from the conventional method of estimating trends which often observe an original series and

3. Mizoguchi and Kariya (1983) discuss the problems associated with the detrending method in the ARIMA model.

4. Ozaki (1977) gives a criterion for the order determination of ARIMA models.

applies the deterministic curves that are assumed to exist. (2) a smoothness prior based on the Bayesian probability distribution is used in the formulation and is represented by a state space model.[5] The model's validity is evaluated by a statistical criterion using actual data. By using a smoothness prior, the Bayesian approach attempts to reach a more appropriate solution that covers the traditional sampling theory.[6] A statistical criterion is used to objectively evaluate the subjective uncertainty inherent in prior information and to construct a more persuasive statistical model. Combining the Bayesian approach with the model criterion gives their method great practical applicability. (3) Stochastic trends and other components of original series are estimated simultaneously. The components have stationary factors that can be applied to the analysis of the stationary process. This is distinguished from the conventional method, which intuitively fits the deterministic trend and eliminates it or makes differencing the series to obtain stationary process.

Kitagawa and Gersch tested their method by applying it to time series in the U.S. economy. In this paper the method is applied to Japanese macroeconomic series: real GNP, the money supply, and the money-income ratio. The paper will also consider the relationships between real GNP and the money supply and between real GNP and the wholesale price index (WPI).

Although further examination is necessary, results suggest that the stochastic trend developed by Kitagawa and Gersch is quite practical for analyses of real economic time series. Compared to former analyses, the features of the statistical relationship between variables are more clear. The implication is that continued examination of the problems related to economic trends is necessary.

Section 2 contains a general review of time series with trends. Section 3 is a summary of the method developed by Kitagawa and Gersch. In Section 4, we examine the results of the application of the method to Japanese macroeconomic time series.

5. We used the expression "Bayesian approach" instead of "Bayesian method" in this paper because there are various perspectives in the Bayesian formula. The Kitagawa-Gersch model actively uses all possible prior information on observed time series by combining it with sample information.

6. "Non-Bayesians, as well as Bayesians, use a good deal of prior information in building and using models" (Zellner (1985)).

II. A Bayesian Solution to the Smoothing Problem of Economic Time Series

Let the time series with trend be

$$y(n) = f(n) + \varepsilon(n) \quad (n = 1, \ldots, N), \tag{1}$$

where f (n) is an unknown smooth function and ε (n) is i.i.d. from N (0, σ^2), σ^2 is unknown. The problem is then to estimate f (n) in some satisfactory statistical manner. Whittaker (1923) suggested that the solution will satisfy

$$\min_f \left[\sum_{n=1}^{N} (y(n)-f(n))^2 + \lambda^2 \sum_{n=1}^{N} (\nabla^k f(n)^2\right] \tag{2}$$

for fixed values of k and λ.[7] The first term in Equation (2) is the infidelity to the original series y (n), which is the cost of f (n) being separate from the actual data. The second term is the infidelity to the smoothness constraint, which is the cost incurred by rapid change over time. Whittaker left the choice of the smoothness parameter λ to the investigator. This formulation is based on the notion that f (n) will be estimated from the balance between the trade-off terms in Equation (2).

In the traditional formulation, Equation (2) was solved by treating f (n) as a deterministic function and y (n) and ε (n) as stochastic functions. A recent tendency is to treat f (n) as a stochastic process (See, Box, Hillmer and Tiao (1978)). A noteworthy example of this tendency is the Bayesian approach. Akaike (1980), fully develops Shiller's (1973) smoothness prior idea and gives a solution by maximizing the likelihood function

$$l(f) = \exp\left\{\frac{-1}{2\sigma^2}\sum_{n=1}^{N}(y(n)-f(n))^2\right\}\cdot\exp\left\{\frac{-\lambda^2}{2\sigma^2}\sum_{n=1}^{N}(\nabla^k f(n))^2\right\} \tag{3}$$

and An Bayesian Information Criterion (ABIC) procedure is used to select the best model. The λ in Equation (3) is referred to as a hyperparameter in the Bayesian literature.[8] Kitagawa and Gersch extended Akaike's concept; the smoothness prior

7. The ∇ in the second term in Equation (2) is the difference operator; that is, $\nabla f(n) = f(n) - f(n-1)$.

8. When f (n) is divided into trend factor t (n) and seasonal factor s (n), Equation (2) is expressed as:

$$\sum_{n=1}^{N}\left[\{y(n)-t(n)-s(n)\}^2 + \lambda^2\{(\nabla^k t(n))^2 + \gamma^2(s(n)-s(n-L))^2 + z^2(\sum_{i=0}^{L-1} s(n-i)^2)\}\right]$$

(L is for the seasonal cycle). Weight γ^2 and z^2 in this equation are further strengthened by hyperparameter λ^2 (Akaike (1980); Akaike and Ishiguro (1983)).

problems correspond to the maximization of

$$
\begin{aligned}
l(f) = & \exp\left\{\frac{-1}{2\sigma^2}\sum_{n=1}^{N}(y(n)-t(n)-v(n)-s(n))^2\right\} \\
& \times \exp\left\{\frac{-\tau_1{}^2}{2\sigma^2}\sum_{n=1}^{N}(\nabla^k t(n))^2\right\} \\
& \times \exp\left\{\frac{-\tau_2{}^2}{2\sigma^2}\sum_{n=1}^{N}(\sum_{i=0}^{P}\alpha_i v(n-i))^2\right\} \\
& \times \exp\left\{\frac{-\tau_3{}^2}{2\sigma^2}\sum_{n=1}^{N}(\sum_{i=0}^{L-1}s(n-i))^2\right\}
\end{aligned}
\tag{4}
$$

The first term is the fidelity to the actual data; the second to fourth terms express the smoothness constraint of trend t (n), stationary v (n) and seasonal s (n) components, respectively. P in the third term is the order of the autoregressive process, α_i, (i = 1, ..., P) are coefficients and L in the fourth term is the number of period of seasonal cycle. The role of hyperparameters τ_i^2 (i = 1, 2, 3) expresses uncertainty in the priors. Equation (4) explicitly involves stationary factors as an autoregressive (AR) process. This concept is outlined in the next section.

III. Smoothness Priors of Economic Time Series and Their State Space Representation

1. Smoothness Priors of Economic Time Series Components

Observed time series y (n) can be decomposed into trend t (n), globally stationary stochastic factor v (n), seasonal factor s (n), trading-day effect factor d (n) and observation noise ε (n) as

$$y(n) = t(n) + v(n) + s(n) + d(n) + \varepsilon(n). \tag{5}$$

A smoothness prior of each component on the right side of Equation (5) is as follows:[9] Trend component t (n) satisfies a k-th order stochastically perturbed difference equation

$$\nabla^k t(n) = w_1(n);\ w_1(n) \sim N(0,\ \tau_1^2), \tag{6}$$

where $w_1(n)$ is an i.i.d. sequence and ∇ denotes a difference operator $\nabla t(n) = t(n) - t(n-1)$. For k = 1, Equation (6) is a random walk model and a stochastic term is effective up to two previous periods for k = 2. τ_1^2 implies the relative degree

9. When an original series is used logarithmically, Equation (5) is treated as an additive-type by log model.

of smoothness, which is estimated from the actual series.

A smoothness prior of the stationary stochastic component v (n) is assumed to satisfy an AR model of order p. That is,

$$v(n)= \alpha_1 v(n-1) + \ldots + \alpha_p v(n-p) + w_2(n);\ w_2(n) \sim N(0,\ \tau_2^2). \tag{7}$$

In Equation (7), $w_2(n)$ is an i.i.d. sequence. The AR model is constrained to be stationary.

The seasonal factor s (n) may be nearly the same in every year. The stochastic term is introduced to accommodate a changing seasonal pattern. Then the seasonal model's stochastic process is

$$s(n) = -\{s(n-1) + s(n-2) + \ldots + s(n-L+1)\} + w_3(n); \tag{8}$$
$$w_3(n) \sim N(0,\ \tau_3^2),$$

where $w_3(n)$ is an i.i.d. sequence.

Trading-day effect model is

$$d(n) = \beta_1(n)d_1(n) + \beta_2(n)d_2(n) + \ldots + \beta_6(n)d_6(n), \tag{9}$$

where $\beta_i(n)$ denotes trading-day factor and $d_i(n)$ is the number of i-th day of the week at time n. Furthermore, the constraint $\sum_{i=1}^{7} \beta_i(n) = 0$ so that $\beta_7(n) = -\sum_{i=1}^{6} \beta_i(n)$ is applied. The non-perturbed difference equation constraint on the trading-day

$$\beta_i(n) = \beta_i(n-1), \qquad (i=1, \ldots, 6), \tag{10}$$

is also considered.[10] Smoothness priors of each component is represented by a state space model, as shown in the next section.

2. State Space Representation of Smoothness Priors[11]

The state space model for the observations y(n) is

$$x(n) = Fx(n-1) + Gw(n)$$
$$y(n) = H(n)\,x(n) + \varepsilon(n), \tag{11}$$

where F, G, and H(n) are matrices, respectively. w (n) and ε (n) are assumed to be zero mean i.i.d. normal random variables. x (n) is a state vector that includes trend, stationary, seasonal, and trading-day components.

10. The trading-day effect may change stochastically in every month. But, in this model, we deal with it deterministically.

11. See Appendix 1.

The general state space model for the time series y (n) that includes components and observation errors is written in the following form:

$$x(n)=\begin{bmatrix}F_1 & & & O\\ & F_2 & & \\ & & F_3 & \\ O & & & F_4\end{bmatrix}x(n-1)+\begin{bmatrix}G_1 & & & O\\ & G_2 & & \\ & & G_3 & \\ O & & & G_4\end{bmatrix}w(n)$$

$$y(n)=[H_1H_2H_3H_4(n)]x(n)+\varepsilon(n) \tag{12}$$

In Equation (12), matrices F, G, and H(n) in Equation (11) are constructed by the component models (F_j, G_j, H_j) (j=1, ...,4). In order (j=1, ...,4), these models represent the trend, stationary AR, and seasonal and trading-day effects component models, respectively.

An example of a state space model that incorporates each of the above constraints is given as follows:

$$\begin{bmatrix} t(n)\\ t(n-1)\\ \vdots \\ t(n-k+1)\\ v(n)\\ v(n-1)\\ \vdots\\ v(n-p+1)\\ s(n)\\ s(n-1)\\ \vdots\\ s(n-L+2)\\ \beta_1(n)\\ \vdots\\ \beta_6(n)\end{bmatrix} = \begin{bmatrix} \begin{matrix}C_1 \cdots C_{k-1} & C_k\\ 1 \ \cdots 0 & 0\\ \ddots & \vdots\\ 0 \ \cdots 1 & 0\end{matrix} & & & \\ & \begin{matrix}\alpha_1 \cdots \alpha_{p-1} & \alpha_p\\ 1 \ \cdots 0 & 0\\ \ddots & \vdots\\ 0 \ \cdots 1 & 0\end{matrix} & & \\ & & \begin{matrix}-1 \cdots -1 & -1\\ 1 \cdots 0 & 0\\ \ddots & \vdots\\ 0 \cdots 1 & 0\end{matrix} & \\ & & & \begin{matrix}1 \cdots 0\\ \ddots\\ 0 \cdots 1\end{matrix}\end{bmatrix} \cdot \begin{bmatrix} t(n-1)\\ t(n-2)\\ \vdots \\ t(n-k)\\ v(n-1)\\ v(n-2)\\ \vdots\\ v(n-p)\\ s(n-1)\\ s(n-2)\\ \vdots\\ s(n-L+1)\\ \beta_1(n-1)\\ \vdots\\ \beta_6(n-1)\end{bmatrix} + \begin{bmatrix}100\\000\\ \vdots\\000\\010\\000\\ \vdots\\000\\001\\000\\ \vdots\\000\\000\\ \vdots\\000\end{bmatrix}\cdot\begin{bmatrix}w_1(n)\\w_2(n)\\w_3(n)\end{bmatrix}$$

$$y(n)=[1\cdots01\cdots01\cdots0\ d_1(n)\cdots d_6(n)]\ x(n)+\varepsilon(n) \tag{13}$$

where

$$x(n)=[t(n)\cdots t(n-k+1)v(n)\cdots v(n-p+1)s(n)\cdots s(n-L+2)\beta_1(n)\cdots\beta_6(n)]',$$

and C_i ($i = 1, \ldots, k$) reflects the trend constraint in Equation (6). System noise vector w (n) and observation noise ε (n) are assumed to be normal i.i.d. with zero mean and diagonal covariance matrices

$$\begin{bmatrix} w(n) \\ \varepsilon(n) \end{bmatrix} \sim N\left[\begin{pmatrix} O \\ O \end{pmatrix}, \begin{pmatrix} Q & \\ & \sigma^2 \end{pmatrix}\right] \tag{14}$$

where

$$w(n)=\begin{bmatrix} w_1(n) \\ w_2(n) \\ w_3(n) \end{bmatrix}, \quad Q=\begin{bmatrix} \tau_1{}^2 & & \\ & \tau_2{}^2 & \\ & & \tau_3{}^2 \end{bmatrix} \tag{15}$$

Recursive Kalman filtering and smoothing yields estimates of the state vector x (n) and the likelihood for the unknown variances. Likelihoods are computed for different constraint order models. The unknown variances τ_1^2, τ_2^2, τ_3^2, σ^2 and unknown AR coefficients α_i (i=1, ..., p) in the state space model are estimated by the maximum likelihood method. τ_i^2 (i = 1,2,3) is a hyperparameter, which is a measure of the uncertainty of belief in the priors. In order to select the best model fitted to the data in the alternative model classes, the criterion

$$\text{AIC}=-2(\text{maximum log likelihood})+2(\text{the number of parameters}) \tag{16}$$

is used from the log likelihood function

$$l(\theta,\ x(o))=\prod_{n=1}^{N}(2\pi\ v(n|n-1))^{-\frac{1}{2}}\cdot\exp\left\{\frac{-v(n)^2}{2v(n|n-1)}\right\} \tag{17}$$

The v (n) and v (n|n−1) in Equation (17) are the innovation and conditional innovation variance at time n, respectively. The parameters for Equation (16) are determined by order k of the trend, order p of the AR process, seasonal cycle L, and trading-day effects d in the state space model (Equation (13)). For the best model, the filtered data are smoothed.

3. Numerical Examples by Kitagawa and Gersch

Gersch and Kitagawa (1983) and Kitagawa and Gersch (1984) have analyzed U.S. data by their method, and compared the results with those obtained by the Census X-11 procedure and a different method in Hillmer and Tiao (1982). The series chosen was one of the set provided for analysis by Sandra McKenzie of the

Census Bureau to participants in the 1981 ASA-CENSUS-NBER Conference on Applied Time Series Analysis of Economic Data.

When comparing the results obtained by the Kitagawa and Gersch method with those obtained by the Census X-11 procedure, using the Bureau of Labor Statistics data for all employees in the U.S. food industries, trend plus AR process is very similar to the trend obtained by the Census X-11 procedure. The trend removed AR component is much smoother, and the AIC selects the AR order-2 model for the trend estimation.[12] In the increasing horizon prediction, the prediction performance of the model including AR component is superior to that of the model ignoring AR component. There is an increasing divergence between true and predicted data when the model ignoring AR component is used.[13] Similar results were observed when the Bureau of Labor Statistics data for male agricultural workers twenty years and older were applied.

The trends estimated by Hillmer and Tiao (1982), using the ARIMA model on the Bureau of Labor Statistics data for unemployed 16-19 year old males in the United States were more wiggly than those obtained by the Census X-11 procedure. Kitagawa and Gersch also pointed out that the model analysis by Hillmer and Tiao paid insufficient attention to the best model. The ARIMA model approach often results in a relatively poor increasing horizon prediction performance.

Based on their results, Kitagawa and Gersch concluded that their approach transcends the Census X-11 and ARIMA model approach in the treatment of the estimation of trends and seasonal components from the vantage point of the model analysis.

IV. Application to Japanese Macroeconomic Time Series

In this section the Kitagawa and Gersch method is applied to Japanese economic series. First, trends in the real GNP, money supply and the ratio of money to nominal GNP are estimated as the representative macroeconomic series. Second, the relationship among economic variables is analyzed by estimated AR component.[14]

12. X-11 is based on the default option.

13. Let one-step-ahead prediction be $y(n+1|n)$ ($n = N, N+1, \ldots, N+M-1, 1 \leqq n \leqq N$, M denotes prediction period), then increasing horizon prediction is estimated by $y(N+i|N)$ ($i=1, \ldots, M$).

14. In this paper the stationary process is estimated by p-th order AR model. Estimated AR process, however, may evolve over time. In this regard, the non-stationality of the detrended series has to be discussed (See, Naniwa (1986)).

1. Trend Estimation

A. Real GNP

The rate of change over the previous year and its trend are widely used because they offer a simple method for observing the current state of national growth and eliminating the seasonal effects. From the analytical point of view, however, year-to-year comparisons of trends may not ensure optimum results because such comparisons do not explicitly account for the intrinsic trends or the seasonal and other components of the original series. The analysis presented below suffers from these comparative problems.[15] Figure 1 illustrates the trends estimated when the order of the perturbed difference equation and the order of the AR model are given from 1 to 3.[16] The parentheses at the upper right-hand corner of each figure indicate the order of the trends and the AR model. The figures are arranged from the upper-left corner downward based on the AIC. The seasonal factors are not estimated because we are using the year-to-year rate of change.[17] The sample period is from the first quarter of 1966 to the first quarter of 1984. Sample size is 73.

The trend estimated from the minimum AIC model is Figure 1-i, but the difference of the AIC between Figure 1-h and Figure 1-i is 0.8. The AIC suggests that the two models are approximately the same, although trend in each is different.[18] Trend in Figure 1-i exhibits a downward movement on the whole, but the trend in Figure 1-h reveals no marked downward tendency in the latter half of the sample period. Subsequently, it becomes difficult to accurately predict which trend will be adopted.

Figure 2 shows the trend that is directly estimated from the original series with seasonal factors and other components. The sample period is from the first quarter of

15. Ito and Sasaki (1984) noted that the rate of change over the previous year is often misleading in evaluating business activities, because in transitional periods of cyclical changes in the original series, the rate of change is sometimes reversed.

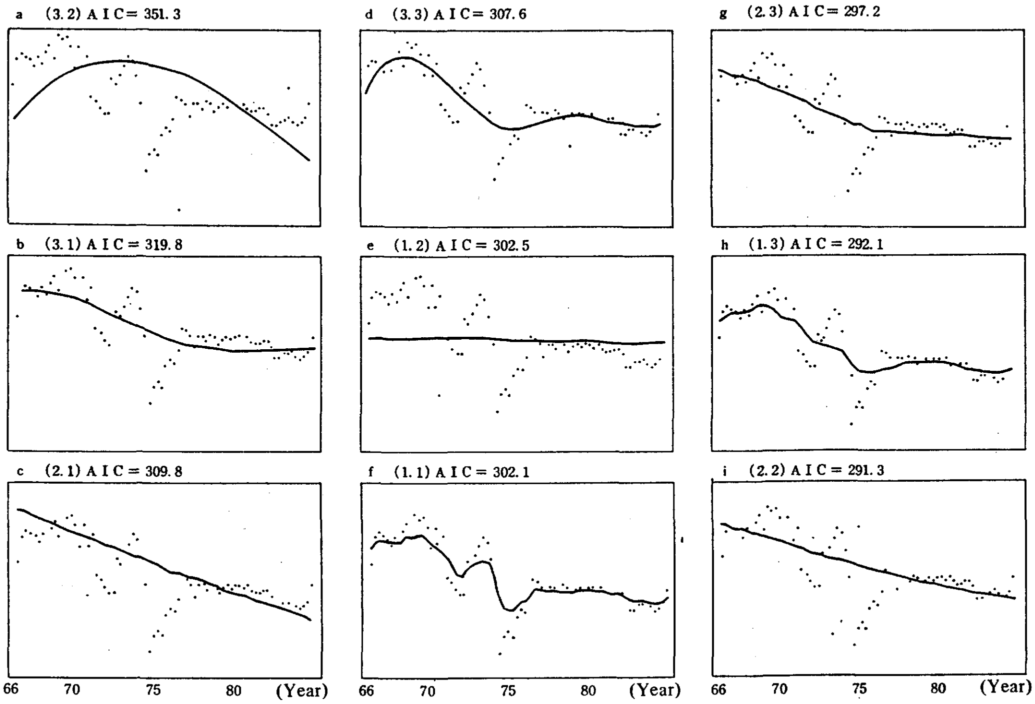
16. The orders of the trend and AR models are 1 through 3. In our experience, these orders are appropriate for the economic data discussed in this paper.

17. Figure 1 presents the results of the model, which are composed of trend, the stationary AR process and irregular component. The results of the models composed of trends and irregularities were excluded from the figures because of the large AIC and the wiggly trends (See, Table 1-(1) for the estimated parameters and AIC).

18. AIC is a criterion for determining the best model among the alternative model classes. When the absolute value of difference of AIC between the two models is less than 1, the fitted models are considered to be similar (Sakamoto, Ishiguro, and Kitagawa (1986)).

Figure 1 Real GNP, Rate of Change over the Previous Year and Estimated Trends

(. . . Rate of Change over the Previous Year — Estimated Trend)



1965 to the first quarter of 1984. Sample size is 77.[19] Figure 2-a illustrates the original series for real GNP and the estimated trend, and Figure 2-b is the series of the AR component.[20] The trend estimated from the original series is smoother, and the continuously downward trend observed in Figure 1-i is not seen. The selected model's AIC is significantly smaller than those of other models.[21] In this case there was no

19. Unless specifically noted, the results of the minimum AIC models are taken up in the following discussion.

20. Log values are used. The autoregressive series shown in Figure 2-b is set at 100 times. Outlier analysis may produce better information for time series analysis, especially around 1973 and 1974 as seen in Figure 2-b. But, we do not use any adjustment here to see the whole span of the process.

21. The parameters and AIC are shown in. In the best model, (AIC = −1239.1), τ_1^2 is 0.19, and τ_2^2 is 0.99. The trend's stochastic fluctuations are rather minor (See, Table 1-(2)).

Figure 2 Real GNP, Original Series, Trend and AR Process

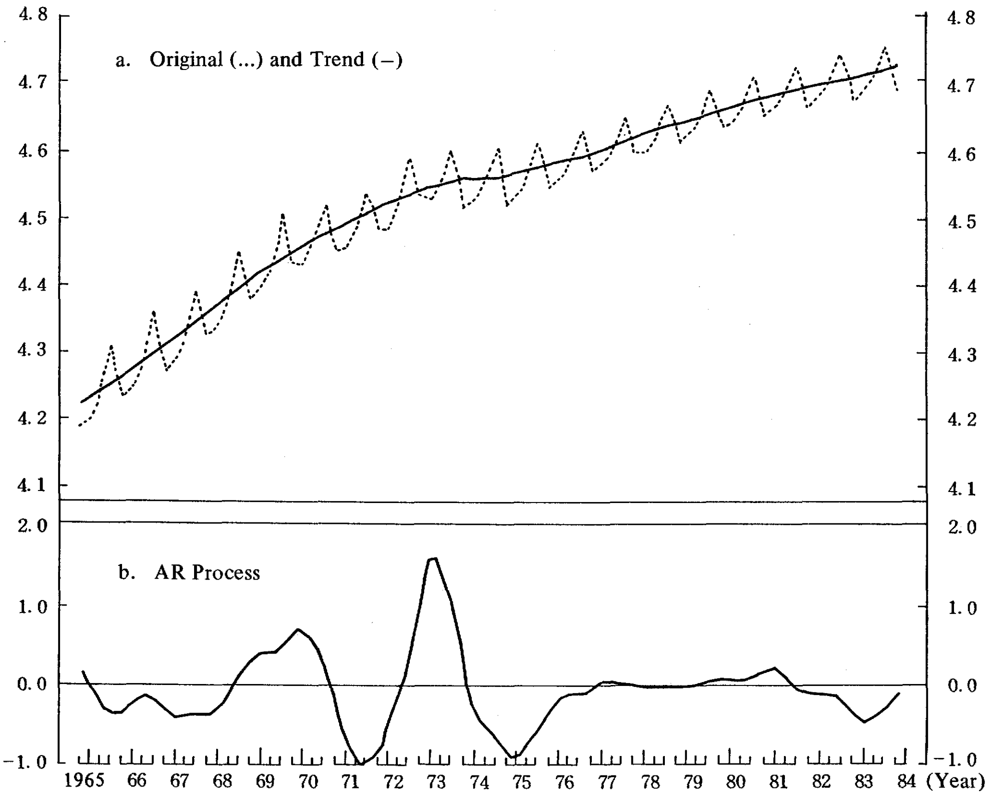
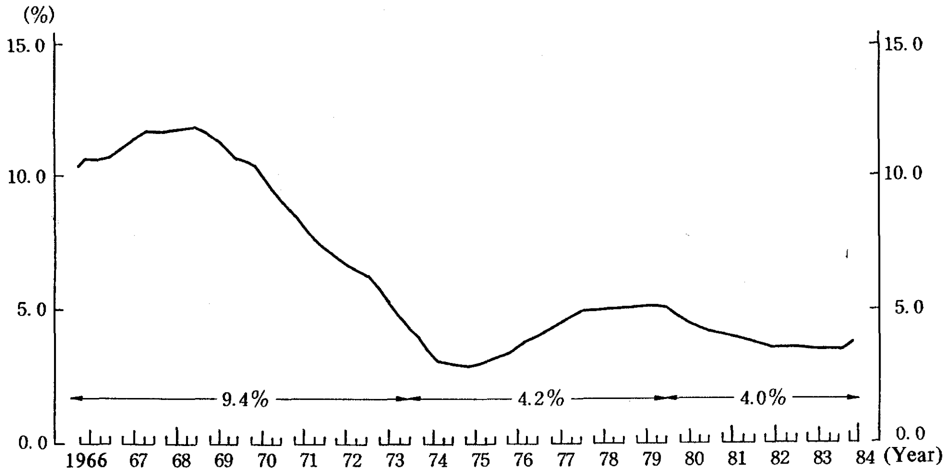


Figure 3 Rate of Change of Estimated Trend over Previous Year and Average Growth Rates



difficulty in selecting a model. To more clearly determine the economic implications of this trend, Figure 3 illustrates the rate of change in comparison to that of the previous year, time is divided into three periods: (1) 1965 to 1973 (just prior to the First Oil Crisis); (2) 1974 to 1979 (just before the Second Oil Crisis); and (3) 1980 and thereafter following the Second Oil Crisis.[22] The average growth rate during period (1) was 9.4 percent; the average growth rates during periods (2) and (3) were 4.2 and 4.0 percent, respectively. The result shows that growth rate following the Second Oil Crisis did not remarkably decline and depression in business activities in 1981 to 1982 also appeared in the AR component in Figure 2-b.

B. Money Supply

In addition to real GNP, the rate of change of M_2 +CD (average balance) quarterly series over the previous year and its trend are also observed attentively. As mentioned in the previous section on real GNP, estimating stochastic trends from the rate of change over the previous year by the method discussed in this paper would result in problems. Following established procedure, however, we have estimated the trend of the rate-of-change over the previous year. The sample period is from the first quarter of 1968 to the first quarter of 1984. Sample size is 66.

Figure 4 illustrates the trends estimated when both the orders of the trend and the autoregressive series are upgraded from 1 to 3 and arranged from the upper-left corner downward in accordance with AIC. Figure 4-i indicates the minimum AIC. In this instance, there is no problem in selecting the best model, and the trend of M_2 + CD shows the downward direction from around 1972 to 73.

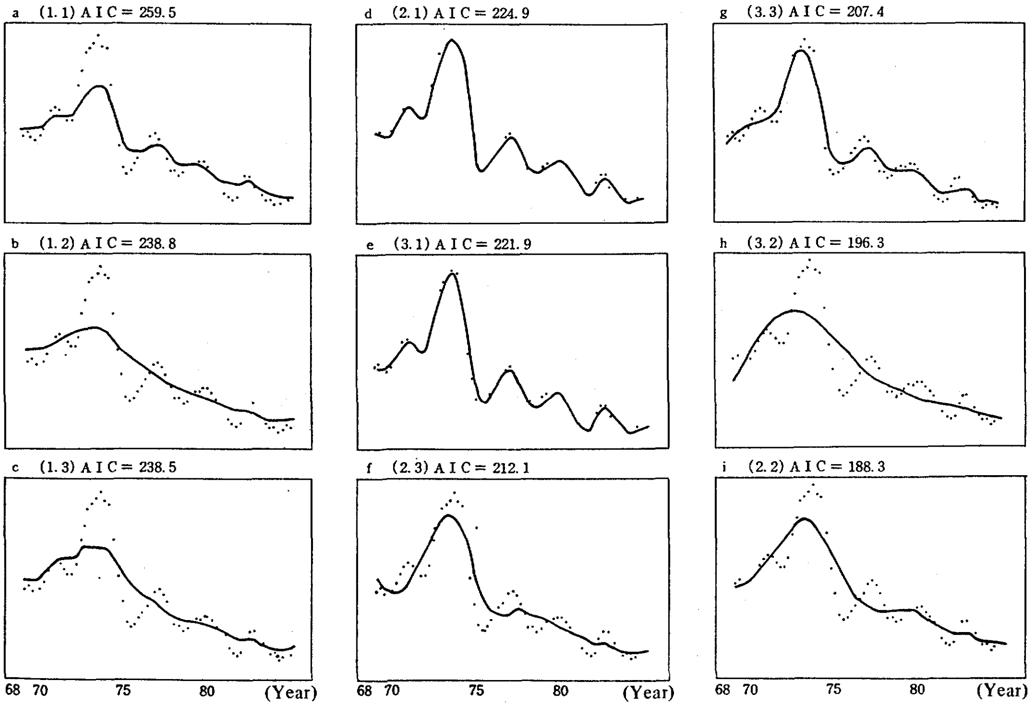
The trends estimated directly from the original series are shown in Figure 5.[23] Because the seasonal factors have very little influence, the trends show virtually identical movements for the original series. The rate of change of the trend over the previous year is shown in Figure 5-2. The gradually declining trend may be interpreted as a reflection of the Bank of Japan's policy stance. The Bank takes into

22. The rate of change is computed by converting the estimated trend into antilog.

23. Each factor is estimated by log value. The rates of change over the previous year are calculated after conversion to antilog. The parameters and AIC of estimated models are shown in Tables 1-(3) and (4).

Figure 4 M_2 +CD, Rate of Change over the Previous Year and Estimated Trends

(. . . . Rate of Change over the Previous Year, — Estimated Trend)



account the money supply after the mid-1970s.[24]

24. For purposes of comparison to Figure 4, we estimate the trend in the year-to-year change as shown in Figure below which shows the gradually declining tendency.

Rate of Change (dotted line) over the Previous Year and its Trend (solid line) in the M_2+CD Quarterly Average (original series – seasonal factor)

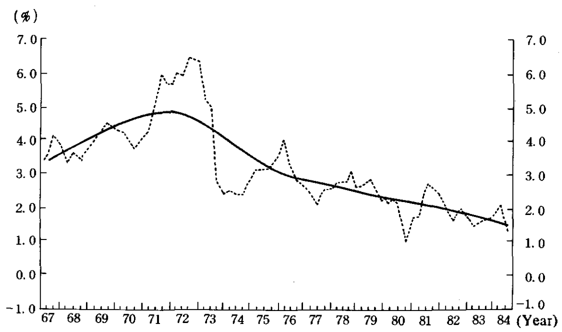


Figure 5-1 M_2 +CD, Original Series (Quarterly Average) and Trend

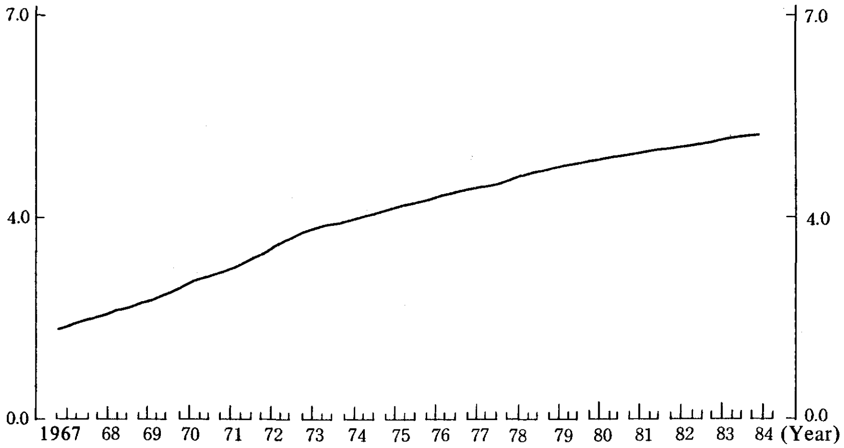
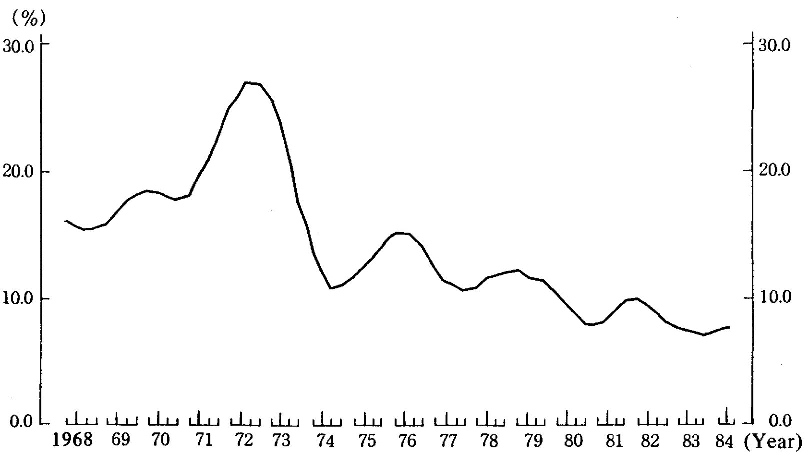


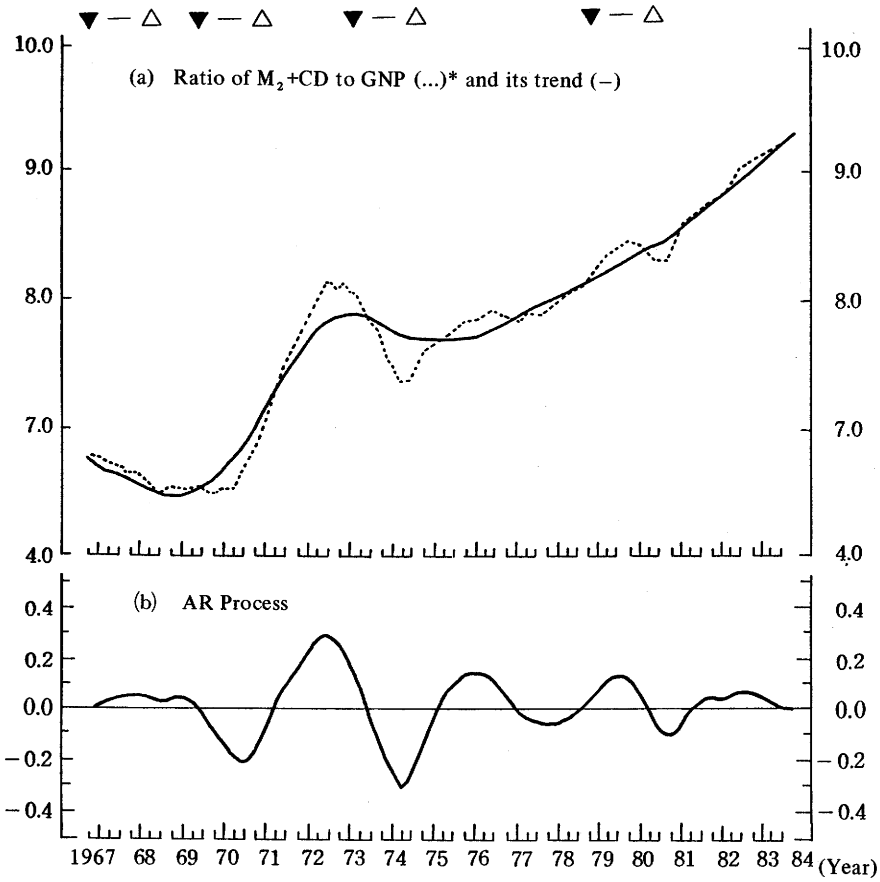
Figure 5-2 Rate of Change of Trend over Previous Year



C. Ratio of Money to Nominal GNP

The ratio of the money to nominal income is used as the Marshallian k to evaluate the level of liquidity in the national economy; the movement around the trend is especially noted. We use the ratio of M_2 + CD to nominal GNP as the Marshallian k.[25] The series are seasonally-adjusted.

Figure 6 Ratio of M_2 +CD to Nominal GNP, Trend and AR Process

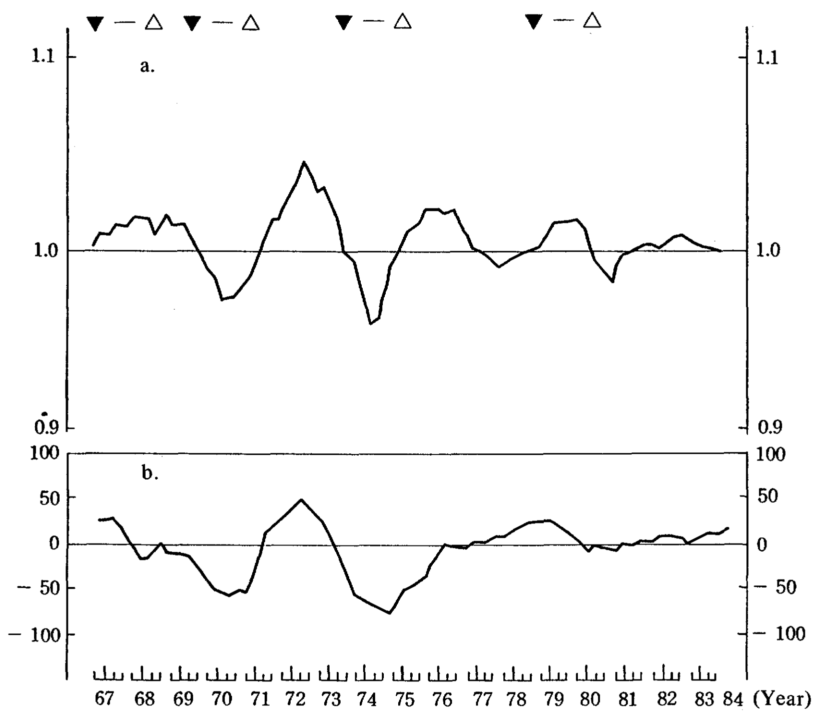


* Seasonally-adjusted
▼ — △ indicates monetary restraint period

25. As mentioned before, there are problems associated with estimating a stochastic trend by using seasonally adjusted ratios. We assumed here that the series are subjected stochastic processes.

Figure 6 shows the trend and AR series of the Marshallian k for the period from the first quarter of 1967 to the first quarter of 1984. Figure 7-a illustrates the detrending series, which reveals an almost identical movement to that of the AR series of Figure 6-b. This similarity results because the seasonally-adjusted detrending series is a composite of AR and irregular factors. For economic implications, it should be noted that AR plus irregular series tendency resembles the Financial Position Diffusion Index of the Short-term Economic Survey of Enterprises in Japan (all industries)[26] conducted by the Bank of Japan, which is an index of the tightness and easiness of money (See Figure 7-b).

Figure 7 a. Deviation Rate of Ratio of M_2 +CD to Nominal GNP
b. D.I. of Judgment of Fund Management*

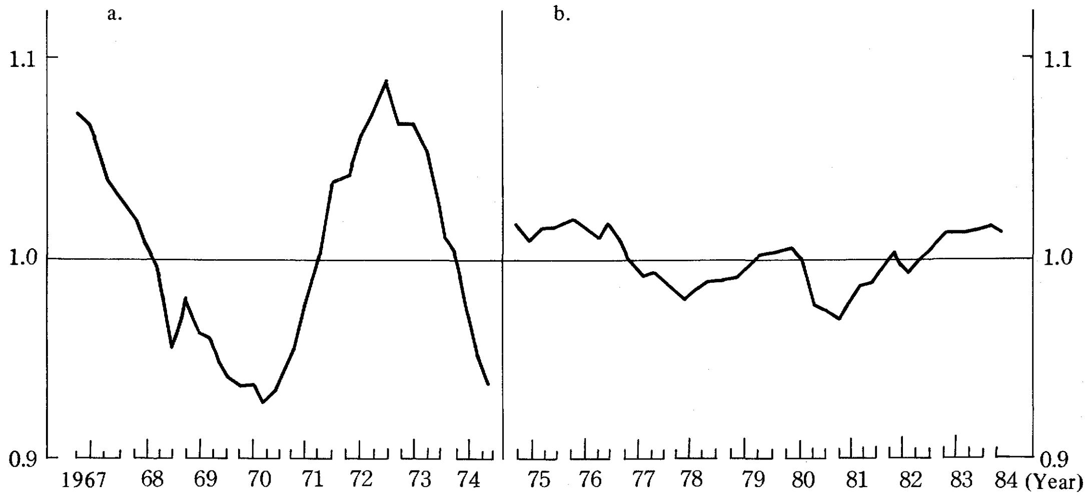


* Short-term Economic Survey of principal enterprises, all industries, "Easy"–"Tight"

26. The number of enterprises that responded as financially "tight" was subtracted from the number that responded as "easy".

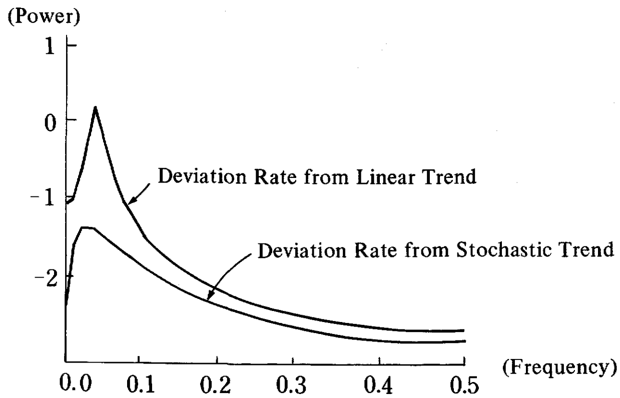
The detrending series from the deterministic linear trends determined by the least squares method are shown in Figure 8. For convenience, the sample period is divided into the periods 1967–74 and 1975–84. The results indicate that there is a certain degree of difference in the levels of the most recent trends.[27] Further analyses must be conducted to interpret these variations, but it is imperative that the analysis around the trend be entirely dependent upon the estimated trend.

Figure 8 Deviation Rates of Ratio of M_2 +CD to Nominal GNP from Linear Trends*



* estimated period: (a) 1967:I–1974:IV, (b) 1975:I–1984:I.

27. The power spectrum of deviation from stochastic trend shown in Figure 7-a decreases its power in lower frequency (longer cyclical) area, compared with that of a linear trend, as shown in the Figure below.

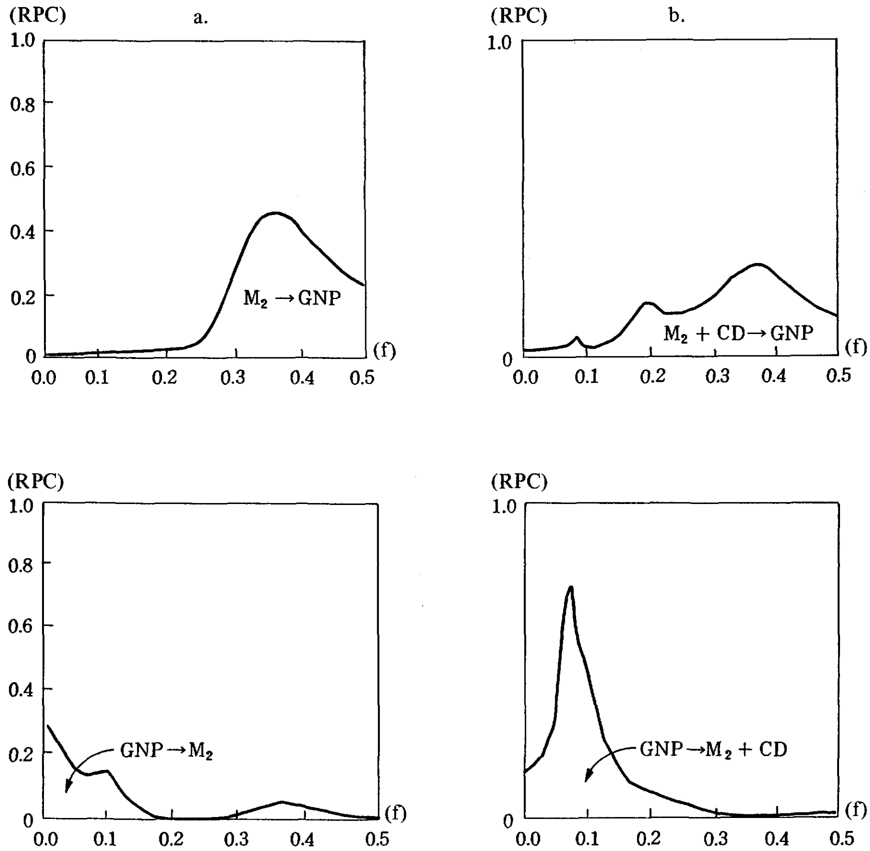


2. Application of Stationary Factors: Relationship Among Macroeconomic Variables

Relative Power Contribution (RPC) is one statistical way to search for relationships among variables in the stationary feedback system.[28] The economic system, however, is far from stationary. Therefore, we had to approximate to stationary series. Using a detrending series is one way to do this. In the following example we use AR plus irregular components for the stationary process.

Figure 9 presents RPC using the deviates from deterministic linear trends of the

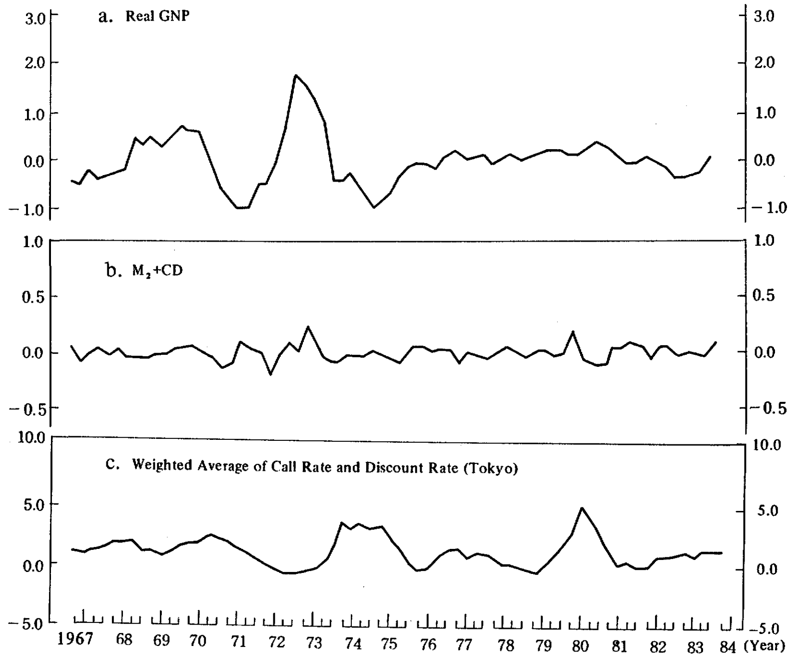
Figure 9 RPC between Real GNP and Money



a. Real GNP and M_2 (Oritani (1979), 1956: I–1978: I)
b. Real GNP and M_2+CD, 1967: I–1984: I
f denotes frequency.

28. See Akaike (1967) for details of RPC. For RPC applied to economic variables, see Oritani (1979), Okubo (1982), and also see Appendix 2.

Figure 10 AR plus Irregular Components



year-to-year change estimated by Oritani (1979). This analysis attempts to analyze the causal relationship between real GNP and M2. (Sample period is from the first quarter of 1956 to the first quarter of 1978.) Figure 9-b illustrates the results of RPC using the AR plus irregular components estimated by the Kitagawa and Gersch method. (Sample period is from the first quarter of 1967 to the first quarter of 1984.)[29] The series of this period is shown in Figure 10-a and b. RPC assumes that the residuals obtained from the models are independent. In our estimation, the correlation coefficient of these two variables is −0.024. The residuals of both variables can be said to be almost independent.

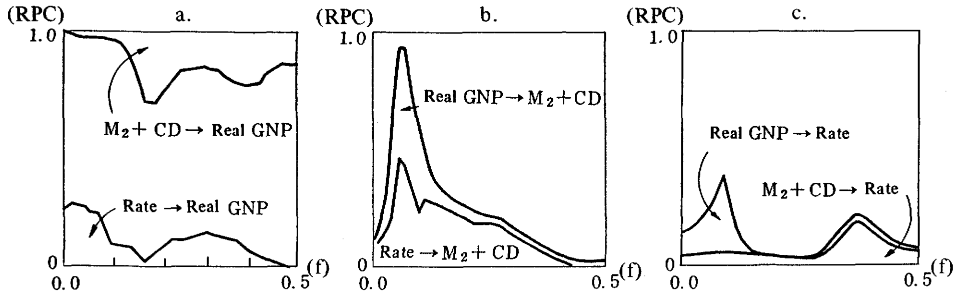
Figure 9-a and b show that the money supply is influencing real GNP mainly in the shorter- and medium-term cycles. Figure 9-b shows a much more significant effect of GNP on the money supply in the longer-term cycles than Figure 9-a.

The result not only reveals that real GNP is affected by the money supply but also shows that fluctuations in the money supply are not completely independent of real GNP movements in longer cyclical terms. The longer-term influence observed in Figure 9-a appears to be more significant. In a strict sense, of course, these two

29. AR plus irregular components are the results of trend estimations that are discussed in the previous section.

Figure 11 RPC among Real GNP, M_2+CD and Interest Rate

Contribution to a. Real GNP, b. M_2+CD, and c. Rate



figures cannot be compared, because the sample period and the money supply used for measurement are not identical.[30]

RPC, in which interest rates are included in the model with real GNP and M_2 + CD, is shown in Figure 11. As for interest rates, the weighted average quarterly series of call rate and discount rate bills (both in Tokyo) is employed. The composite of AR and irregular series of the interest rates is shown in Figure 10-c.[31] The correlation coefficients of the residuals in the models of the three variables are generally small: 0.25 for interest rates and M_2 + CD, −0.21 for interest rates and real GNP, and −0.06 for M_2 + CD and real GNP. The results of RPC are shown in Figure 11. Figure 11-a indicates that the effect of M_2 + CD on real GNP is still observed as shown in the two-variable model. The influence of interest rates on real GNP is also indicated. The influence on M_2 + CD is shown in Figure 11-b. The effect of real GNP on M_2 + CD can be seen as in the results of the two-variable model. In addition to this, the impact of interest rates in the middle and longer-term cycles can also be observed. The influence of M_2 + CD and real GNP on interest rates is shown in Figure 11-c and indicates interdependence among variables.

In his analysis of the causality between money and nominal income in the United States, Sims (1972; 1980) initially concluded that on the basis of the two-variable model there was a causal relationship from money to nominal income. Since then,

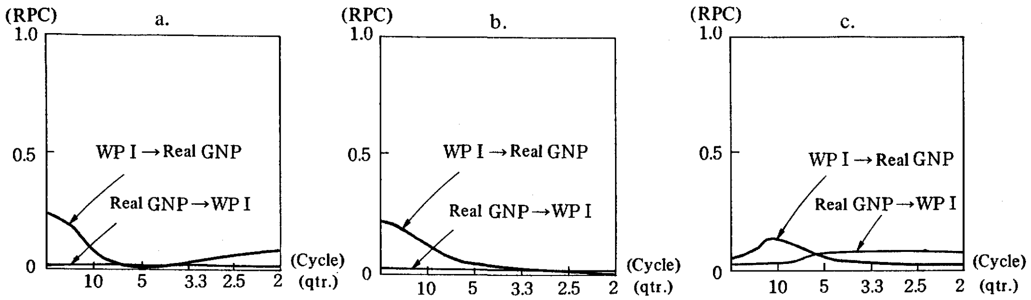
30. The seasonal factors are eliminated. If the original series are used, the relationship based on quarterly seasonal cycles becomes dominant, and information on nonseasonal factors is less revealed. Although there are problems with the seasonal adjustment method, for purposes of the analysis, it may be necessary to take into consideration situations in which there are seasonal effects as well as ones in which there are no such effects.

31. When the seasonal factors are not estimated, the AIC indicates smaller values.

however, Sims has conducted an analysis using both a three-variable model consisting of production, money, and WPI for the U.S. economy before and after World War II and a four-variable model consisting of these three variables plus short-term interest rates. From, these analyses, Sims argues that whereas in the three-variable model the money is significant in explaining the production before and after the war, in the four-variable model money has little impact on production. The findings show that money's influence on production declined, but both these variables were strongly influenced by interest rates. Nevertheless, it is highly probable that the relative degree of influence of each variable will be changed by the addition of a new variable. As for the existence or nonexistence of causality, the results of analyses of these statistical data should be used as one clue in further examination. It should also be kept in mind that the method used in Sims 1972 has a methodological problem in that different results are obtained when the order of the variables contained in the model is changed.

Another example of RPC is to compare the results gained when differencing series are used with those gained when AR plus irregular components are used. Okubo (1982) analyzed the relationship between prices and real GNP by RPC (See Figure 12). The analysis was conducted for the period from the second quarter of 1956 to the fourth quarter of 1981. Two approaches were taken: the first considered the overall period; the second conpensated for the sharp rise in oil prices and other factors in 1973–74 by dividing the period into two parts. The period from the second quarter of 1956 to the first quarter of 1973 (Figure 12-b) was designated Period 1; the period from the first quarter of 1974 to the fourth quarter of 1981 (Figure 12-c) was disignated Period 2. The outstanding characteristic of the various components of Figure 12 is that the relative contributions of WPI to real GNP were generally larger for longer cycles; the greatest influence in Figure 12-a and b was exerted on the longest term period (infinite cycle). In contrast, the relative contributions of real

Figure 12 RPC between WPI and Real GNP

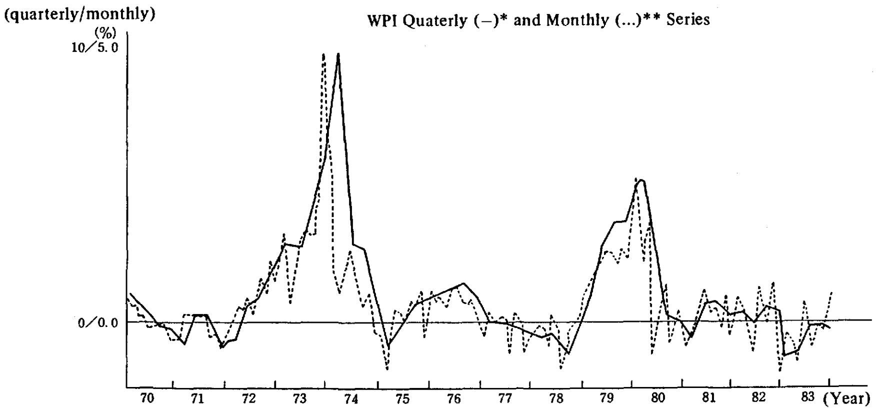


Source: Okubo (1982)

GNP to WPI are minimal. A glance at the stationary series used in this analysis reveals that real GNP and WPI are both based on seasonally adjusting and differencing their log values. In terms of the components of the time series, this would mean that the composite series of AR and irregular factors are converted into log values and then differenced. As a result, the relative deviation in the series may become smaller, and the shorter-term movements observed in the monthly WPI seem to disappear as a result of the conversion monthly series into quarterly series.[32]

RPC by estimated AR plus irregular components is summarized in Figure 13. The composite values of the AR plus irregular series are shown in Figure 14. The sample period is from the first quarter of 1965 to the first quarter of 1984. The effect of WPI on real GNP is seen in the medium-term cycle, but the effect on longer terms, (demonstrated in Figure 12-a) is not observed. The effect of real GNP on WPI is more significant than that shown in Figure 12-c. In this analysis reexamination of the models will be imperative to resolve this problem, because the correlation coefficient of the residuals in the models between the two variables is very high (−0.44).

32. In discussing the statistical distortion of time series type conversion (for example, monthly to quarterly), Nelson and Plosser (1982) state that "it is well known that time aggregation amplifies low frequency movements relative to high frequency movements." Although they admit that, in general, the influence is not very significant, it is possible that such distortions may affect the stationary series. The figure below indicates that minor fluctuations in the monthly series disappear in the quarterly series.



* First difference of log seasonally adjusted series.
** Month to month change of seasonally adjusted series.

Figure 13 RPC between real GNP and WPI

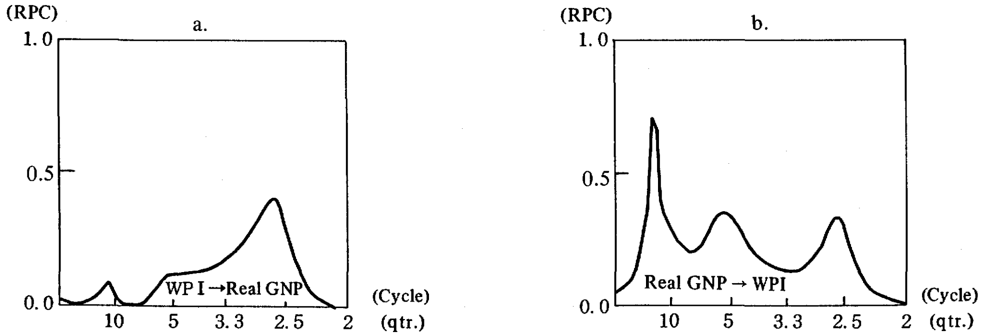


Figure 14 AR plus Irregular Components

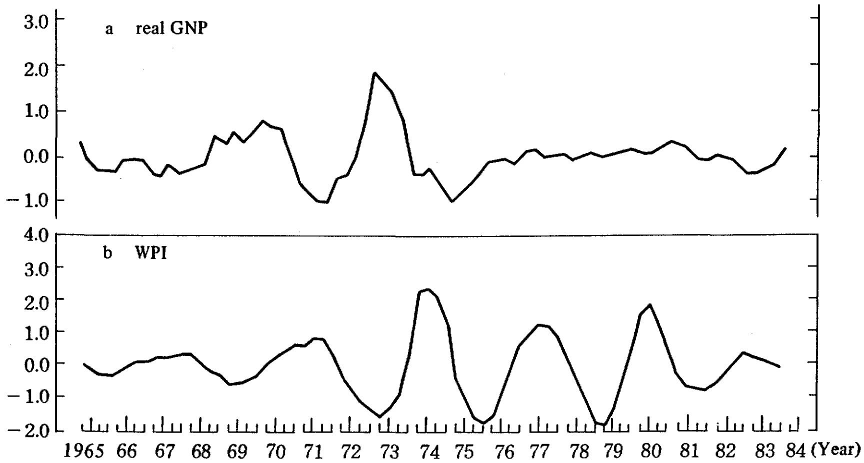


Table 1. Parameters and AIC of Fitted Model to Japanese Real GNP and M_2 +CD

Trend order	AR order	AR coefficients AR(1)	AR(2)	AR(3)	τ_1^2	τ_2^2	τ_3^2	σ^2	AIC
(1) Real GNP, year-to-year percentage change (period 1966/1–1984/1)									
1	0				1.00			1.39	305.2
	1	0.857			0.99	0.99		0.88	302.1
	2	1.365	−0.540		0.01	0.92		1.10	302.5
	3	1.549	−0.688	−0.071	0.82	0.44		0.75	292.1
2	0				0.93			0.91	309.2
	1	0.899			0.45×10^{-4}	0.94		1.45	309.8
	2	1.416	−0.599		0.87×10^{-4}	0.92		0.88	291.3 *
	3	0.896	0.312	−0.555	0.91×10^{-3}	0.99		0.95	297.2
3	0				0.19			1.13	331.7
	1	0.738			0.11×10^{-3}	0.85		1.61	319.8
	2	1.391	−0.569		0.51×10^{-6}	0.92		1.94	351.3
	3	1.282	−0.170	−0.435	0.10×10^{-2}	0.99		0.72	307.6
(2) Real GNP, original series (period 1965/1–1984/1)									
1	0				0.99		0.03	0.70×10^{-4}	−1122.2
	1	0.898			0.99	0.68	0.08	0.54×10^{-4}	−1112.0
	2	−0.156	−0.470		0.79	0.38	0.08	0.83×10^{-4}	−1085.7
	3	−0.537	0.840	0.443	0.87	0.01	0.12	0.16×10^{-3}	−1038.1
2	0				0.99		0.53	0.53×10^{-5}	−1237.3
	1	0.857			0.84	0.99	0.61	0.43×10^{-5}	−1235.0
	2	1.504	−0.673		0.19	0.99	0.73	0.40×10^{-5}	−1239.1 *
	3	1.169	−0.890	0.311	0.99	0.37	0.61	0.41×10^{-5}	−1232.0
3	0				0.12		0.30	0.83×10^{-5}	−1216.4
	1	0.885			0.11×10^{-2}	0.95	0.17	0.11×10^{-4}	−1221.4
	2	1.580	−0.767		0.01	0.79	0.97	0.37×10^{-5}	−1233.7
	3	1.577	−1.571	−0.764	0.24	0.02	0.47	0.50×10^{-5}	−1215.9

Trend order	AR order	AR coefficients A R(1)	A R(2)	A R(3)	τ_1^2	τ_2^2	τ_3^2	σ^2	A I C
(3) M_2+CD year-to-year percentage change (Average, quarterly) (period 1968/1–1984/2)									
1	0				1.00			1.21	267.4
	1	0.888			0.86	0.97		0.74	259.5
	2	1.576	-0.751		0.34	0.37		0.65	238.8
	3	0.783	0.850	-0.900	0.99	0.99		0.38	238.5
2	0				1.00			0.36	220.9
	1	-0.309			1.00	0.22×10^{-7}		0.36	224.9
	2	1.706	-0.899		0.54	0.99		0.14	188.3 *
	3	0.692	0.772	-0.900	0.50	0.99		0.32	212.1
3	0				1.00			0.19	218.4
	1	0.863			0.99	0.52		0.17	221.9
	2	1.708	-0.898		0.15×10^{-2}	0.99		0.19	196.3
	3	1.902	-1.450	0.381	0.99	0.99		0.09	207.4
(4) M_2+CD original series (Average, quarterly) (period 1967/1–1984/2)									
1	0				0.93	-	0.03	0.24×10^{-3}	-1141.8
	1	-0.871			0.78	0.01	0.12	0.28×10^{-3}	-1119.8
	2	-0.254	-0.364		0.99	0.04	0.10	0.22×10^{-3}	-1128.9
	3	-0.529	0.566	0.895	0.74	0.85	0.07	0.13×10^{-3}	-1139.2
2	0				0.97	-	0.97	0.22×10^{-4}	-1424.5
	1	0.835			0.96	0.01	0.12	0.22×10^{-5}	-1418.0
	2	0.114	0.752		0.99	0.01	0.12	0.22×10^{-5}	-1416.0
	3	1.666	-0.835	-0.024	0.99	0.85	0.20	0.13×10^{-2}	- 929.6
3	0				0.98	-	0.13	0.11×10^{-5}	-1428.0
	1	0.752			0.99	0.12	0.14	0.10×10^{-5}	-1423.8
	2	1.686	-0.898		0.50	0.83	0.18	0.73×10^{-6}	-1434.7 *
	3	0.202	0.444	-0.110	0.97	0.22×10^{-3}	0.19	0.19×10^{-5}	-1373.7

Notes: 1. τ_i^2 (i=1,2,3) denote hyperparmeters in Equations(13) and (15).
2. σ^2 is the variance of $\Sigma(n)$ in Equation(13).
3. * in AIC denotes minimum value.

Appendix 1:
State Space Modeling

The details of Kitagawa and Gersch's state space modeling in Equation 12 are as follows:

1. (F_1, G_1, H_1): Stochastic Trend Component

Trend component satisfies the k-th order stochastically perturbed difference Equation given in Equation 6. For k=1, 2, 3, those constraints are

k = 1: $t(n) = t(n-1) + w_1(n)$,

k = 2: $t(n) = 2t(n-1) - t(n-2) + w_1(n)$,

k = 3: $t(n) = 3t(n-1) - 3t(n-2) + t(n-3) + w_1(n)$.

The corresponding F_1, G_1 and H_1 matrices and state vector components are

$$k=1:\ F_1=[1],\ G_1=[1],\ H_1=[1],\ x_1(n)=t(n),$$

$$k=2:\ F_1=\begin{bmatrix}2 & -1\\ 1 & 0\end{bmatrix},\ G_1=\begin{bmatrix}1\\0\end{bmatrix},\ H_1=[1\ 0],\ x_1(n)=\begin{bmatrix}t(n)\\ t(n-1\end{bmatrix},$$

$$k=3:\ F_1=\begin{bmatrix}3 & -3 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix},\ G_1=\begin{bmatrix}1\\0\\0\end{bmatrix},\ H_1=[1\ 0\ 0],$$

$$x_1(n)=\begin{bmatrix}t(n)\\ t(n-1)\\ t(n-2)\end{bmatrix}.$$

In general, matrices and state vector components are given as follows:

$$F_1=\begin{bmatrix}C_1\cdot & \cdot\ \cdot & C_{k-1} & C_k\\ 1 & & 0 & 0\\ \cdot & & \cdot & \cdot\\ \cdot & & \cdot & \cdot\\ \cdot & & \cdot & \cdot\\ 0 & \cdots\cdots & 1 & 0\end{bmatrix},\ G_1=\begin{bmatrix}1\\0\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ 0\end{bmatrix},\ H_1=[1\ 0\ \cdots\ 0],\ x_1(n)=\begin{bmatrix}t(n)\\ t(n-1)\\ \cdot\\ \cdot\\ \cdot\\ t(n-k+1)\end{bmatrix}$$

2. (F_2, G_2, H_2): Stationary Stochastic Component

The stationary stochastic component is assumed to satisfy AR process of order p as in Equation 7. Then matrices F_2, G_2 and H_2 are

$$F_2=\begin{bmatrix} \alpha_1 & \cdot & \cdot & \alpha_{p-1} & \alpha_p \\ 1 & \cdot & \cdot & 0 & 0 \\ \cdot & & & \cdot & \cdot \\ \cdot & & & \cdot & \cdot \\ \cdot & & & \cdot & \cdot \\ 0 & \cdot & \cdot & 1 & 0 \end{bmatrix},\quad G_2=\begin{bmatrix} 1 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix},\quad H_2=[1\ 0\ \cdot\ \cdot\ \cdot\ 0],\quad x_2(n)=\begin{bmatrix} v(n) \\ v(n-1) \\ \cdot \\ \cdot \\ \cdot \\ v(n-p+1) \end{bmatrix}$$

3. (F_3, G_3, H_3): Seasonal Component

The seasonal component of period n becomes almost identical to the previous year. That is:

$$(1-B^L)\, s(n) \approx 0,$$

where B^L is the backward shift operator defined by $B^L s(n) = s(n-L)$ and L is the seasonal cycle. Considering the change of seasonal effect, stochastic term $w_3(n) \sim N(O, \tau_3^2)$ i.i.d. is included. From $(1-B^L) = (1-B)\,(1+B+\ldots+B^{L-1})$, the seasonal stochastic model is given[33] as

$$\sum_{i=0}^{L-1} B^i s(n-i) = w_3(n).$$

The matrices and state vector components of seasonal model are

$$F_3=\begin{bmatrix} -1 & \cdot & \cdot & \cdot & -1 & -1 \\ 1 & \cdot & \cdot & \cdot & 0 & 0 \\ \cdot & \cdot & & & \cdot & \cdot \\ \cdot & & & & \cdot & \cdot \\ \cdot & & & & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & 1 & 0 \end{bmatrix}, G_3=\begin{bmatrix} 1 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}, H_3=[1\ 0\ \cdot\ \cdot\ \cdot\ 0],\quad x_3(n)=\begin{bmatrix} s(n) \\ s(n-1) \\ \cdot \\ \cdot \\ \cdot \\ s(n-L+2) \end{bmatrix}$$

33. If the changing effects are large, another seasonal model can be:

$$\left(\sum_{i=0}^{L-1} B^i\right)^2 s(n) = w_3(n).$$

4. (F_4, G_4, H_4 (n)): Trading-day Effect Component

The trading-day effect component $\beta_i(n)$ (i = 1,...,7) of i-th day of the week at month n has the constraint

$$\sum_{i=1}^{7} \beta_i(n) = 0, \text{ so that } \qquad \beta_7(n) = -\sum_{i=1}^{6} \beta_i(n).$$

Then trading-day effect is expressed by

$$\sum_{i=1}^{7} \beta_i(n) d_i^*(n) = \sum_{i=1}^{6} \beta_i(n)(d_i^*(n) - d_7^*(n)),$$

where $d_i^*(n)$ denotes the number of i-th days of the week in the nth month. The nonperturbed difference equation constraint on the trading days is

$$\beta_i(n) = \beta_i(n-1) \quad (i = 1,\ldots,6).$$

Matrices F_4, G_4 and H_4 (n) and state vector components are

$$F_4 = \begin{bmatrix} 1 & & & \\ & \cdot & & \\ & & \cdot & \\ & & & \cdot \\ & & & & 1 \end{bmatrix}, \quad G_4 = \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}, \quad H_4(n) = [d_1(n) \cdots d_6(n)], \quad x_4(n) = \begin{bmatrix} \beta_1(n) \\ \\ \\ \beta_6(n) \end{bmatrix}$$

where d_i (n) = d_i^* (n) − d_7^* (n) (i = 1,...,6).

Equation 13 summarizes the above relationships. For a specific example, k=3, p=2, L=12, then

$$x(n) = \begin{bmatrix} t(n) \\ t(n-1) \\ t(n-2) \\ v(n) \\ v(n-1) \\ s(n) \\ s(n-1) \\ \cdot \\ \cdot \\ \cdot \\ s(n-10) \end{bmatrix} = \begin{bmatrix} 3 & -3 & 1 & & & & & & \\ 1 & 0 & 0 & & & & & & \\ 0 & 1 & 0 & & & & & & \\ & & & \alpha_1 & \alpha_2 & & & & \\ & & & 1 & 0 & & & & \\ & & & & & -1 & \cdots & -1 & -1 \\ & & & & & 1 & \cdots & 0 & 0 \\ & & & & & \cdot\;\cdot & & \cdot & \cdot \\ & & & & & \cdot & \cdot & \cdot & \cdot \\ & & & & & \cdot & & \cdot & \cdot \\ & & & & & 0 & \cdots & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} t(n-1) \\ t(n-2) \\ t(n-3) \\ v(n-1) \\ v(n-2) \\ s(n-1) \\ s(n-2) \\ \cdot \\ \cdot \\ \cdot \\ s(n-11) \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} w_1(n) \\ w_2(n) \\ w_3(n) \end{bmatrix}$$

$$y(n) = [1\ 0\ 0\ 1\ 0\ 1\ \cdot\ \cdot\ \cdot\ 0]x(n) + \varepsilon(n).$$

Recursive Kalman filter and smoothing procedure are used to estimate state vector x(n) in Equation 13, which yields the likelihoods of the unknown hyperparameters. Likelihoods are computed for different constraint order models.

Appendix 2: Statistical Relations among Economic Variables

In a stationary dynamic feedback system, the irregular fluctuation of system components often causes a great move of the system that influences other components. RPC attempts to observe these relations in the frequency domain in the system. In this context, RPC is also used to analyze causal relationships among variables by means of the mutual influence of the components. Naturally, actual causality cannot be determined only by statistical investigation because of the difficulties of modeling an economic system. A cognitive and theoretical discussion on the existence or nonexistence of empirically observed causality may also be urgently required.[34] Akaike (1967), who proposed RPC, stressed the importance of analysis from a comprehensive perspective. He also repeatedly carried out simulations based on the results obtained by RPC to observe relations at the operational level.

As a tool to explore causality, the RPC concept attempts to give more attention to feedback relations than traditional statistical methods do. For example, even when the causal relation $x(t) \rightarrow y(t+1)$ is obtained statistically over time t, x may not be a sufficient condition to cause y. This is because relationship $y(t) \rightarrow x(t+1)$ may also exist simultaneously. Both x(t) and y(t) may be subject to changes caused by the effects of other common variables, which may result in the relation $x(t) \rightarrow y(t+1)$. Furthermore, when there is an autocorrelation for each x and y—that is, when there is the correlation $x(t) \rightarrow x(t+1)$ and $y(t) \rightarrow y(t+1)$—the cause of y's movement is not always clear. The same problem will occur even when both variables have the same trends. In a multivariate dynamic feedback system, the correlations among variables would obviously be more complicated. For instance, when considering causal order $x \rightarrow y \rightarrow z$, changes $y \rightarrow y'$ and $z \rightarrow z'$ occur against change $x \rightarrow x'$. Such feedback relations among variables have to be analyzed differently depending on whether x' or y' is the major cause for z'.

One should anticipate that various problems will accompany the process of statistically verifying the existence of causality. When true causal relationships do exist, however, they should be reflected in the data, and the usefulness of exploring the implications of causality by analysis of these data cannot be denied. But since many of the series used in economic analyses are not only modified but are also nonstationary, differences can easily occur among the results of analyses conducted using approximation methods to stationarity. Also, the results are dependent on the particular model and data used in the analytical issues. Therefore, it is not unusual for conflicting results to emerge in inquiries into similar subjects. In statistical analyses, it is necessary both to be familiar with the features of the statistical methods applied and to be aware of the traits of the models used. It is also necessary to analyze the reasons why differences appear in the results and use this knowledge to interpret their statistical significance. In that sense there is a need for further examination of the findings presented in this text. These problems should always be kept in mind when analyzing the results of any statistical method, they are not solely associated with RPC.

34. In natural sciences, where it is possible to do repeated experiments, causal relationships are considered to be relatively easy to identify. This is because the subjects treated in the natural sciences are governed by the unchanging principles of nature. In opposition to this view, Suzuki (1975) argues that even in physics, the most precise of the natural sciences, it is well known that in the world of quantum mechanics, the pillar of modern physics, principles cannot exist without the concept of fortuity on probability. Such being the case, the basic question then becomes what is "inevitability" for "causality". Inevitability exists from the viewpoint of probability. In discussing causality in economic phenomena, McClelland (1975) also points out these problems. He also cites Nagel and the strange phenomenon that the word "cause" is seldom used in the developed sciences. He adds that although the word "cause" is hardly ever used in research papers and scholarly reports in the field of natural science, the concept of "cause" is widely recognized. These can be considered as examples of epistemological level thinking.

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