Economies of Scale and Lending Behavior in the Banking Industry*

MASAHIRO KURODA**
TAKASHI KANEKO***

I. Introduction

The purpose of this paper is to present an empirical model of the lending behavior of banks. First, we will theoretically derive the supply curve for loans from the viewpoint of the rational behavior of bank under conditions of default risk. The supply curve will then be estimated in the form of a structural equation. We treat loans as the output of banks and examine particularly the relationship between the scale of production and cost in the banking industry. To avoid problems of multicollinearity, we employ an alternative to the traditional approach of obtaining the scale elasticity by estimating the cost function directly. The existence of economies of scale in bank loans has important implications for the formulation of lending behavior.

Let us proceed by summarizing our basic approach and some features of our model.

First, traditional theories of the lending behavior of banks can be broadly classified into "aggregative models" and "bilateral negotiation models". While "aggregative models" focus on explaining the determination of the total lending of a bank, that is the sum of the individual loans,"bilateral negotiation models" concentrate on

* This paper was written when the authors were Visiting Scholars at the Institute for Monetary and Economic Studies, Bank of Japan. We have benefited from discussions with Professor Ryuichiro Tachi (Chief Councillor of the Institute), Professor Hidekazu Eguchi of the Hitotsubashi University (former Director of the Institute), Dr. Yoshio Suzuki (Director of the Institute) and other Institute staff. It goes without saying that we alone are responsible for any deficiencies that remain.

** Professor, Keio University

*** Associate Professor, Keio University

the bilateral negotiations between individual bank and customer in order to determine their specific lending conditions. This paper aims at an integration of these two approaches, a point which is discussed in detail in Section II.

Second, in treating loans as the single output of banks, we assume that all loans have common structures of production and cost from the viewpoint of bank as a producer. When determining the allocation of output (loans) among borrowers, however, we assume that the terms of individual loan (such as the loan rate) are determined as the result of bilateral negotiation taking into account the heterogeneity of borrowers. Specifically, this heterogeneity refers to the difference in the ability to repay, or to put it the other way round, their default risk.

Third, we assume that loan rates are competitively determined. As we shall see in Section II, traditional "bilateral negotiation models" depict a situation in which a bank acts as a discriminatory monopolist or one in which bilateral monopoly prevails. It is not necessarily reasonable, however, to regard negotiated transactions as monopolistic ones. So long as banks and customers are free to choose their own transacting partners, their search for partners offering the most favorable terms could result in a situation of competitive equilibrium even if all transactions were bilaterally negotiated.

In the following section of the paper, traditional theoretical models of bank loans are broadly classified into "aggregative models" and "bilateral negotiation models" and some problems associated with each of these two approaches are discussed. In Section III we construct our own model. We argue that banks' subjective probability distributions regarding default risk together with loan cost structure are the two major determinants of the supply of loans.

In Section IV, we estimate the cost function of banks. First, in IV.1, we show that loans exhibit significant economies of scale with respect to the cost of production (specifically, operating expenses) when they are treated as the output of banks. To avoid problems of multicollinearity, we estimate the scale elasticity by using the index number approach.

When determining whether there exist economies of scale in the production activities of banks, we face the problem of defining the input and output of banks. Regarding loans as output is only one among many possible alternatives. We also estimated the scale elasticity for more broadly defined inputs and outputs (see Appendix), but we rely on the loan-output results for the estimation of other parameters. In Section IV.2 we use the scale elasticity estimated in IV.1 to specify the cost function of banks. In Section V.1, we estimate the parameters of the subjective probability of default for different types of borrowers (identified by industry and firm size). In Section V.2 the supply curves for loans for different types of borrowers are estimated.

Finally, in Section VI, we discuss the implications of our empirical results and point to the direction for further research.

II. Overview: Approaches to Lending Behavior of Banks

There exist a number of models on bank lending behavior. In terms of the determination of bank loans, these can be broadly classified into two groups. First, there is a class of models which explain the determination of the total lending of a bank, that is, the sum of all individual loans. In what follows, these models will be referred to as "aggregative models". Many models of bank loans in Japan are of this type, including Suzuki (1966), Iwata and Hamada (1980) and Tachi (1982).[1] The second type of models emphasizes the bilateral nature of loans transaction and attempts to explain the determination of loans to individual borrowers. Such models will be referred to as "bilateral negotiation models". Models of this type in Japan include Kaizuka and Onodera (1974) and Teranishi (1982).[2]

In "aggregative models", all loans are taken to be homogeneous and in almost all cases, the market for bank loans is implicitly assumed to be perfectly competitive. The focus is on how much a bank chooses to lend as a whole, given the loan rate. The marginal cost relevant to the determination of the optimal amount of lending includes the opportunity cost as measured by the money market rate and the marginal operating expenses for lending. Given that all loans are assumed homogeneous, all have the same cost structure.[3]

This approach is extremely useful in constructing general equilibrium models of the financial market and in analyzing the transmission mechanism of monetary policy. In addition, when compared with the "bilateral negotiation models" that follow, aggregative models also have the advantage that they can be easily formulated from the viewpoint of the design of experiments.

In reality, however, since borrowers are of heterogeneous quality, the terms of loans (such as the loan rate) are determined by bilateral negotiation. The simplicity of the aggregative approach has been achieved at the cost of ignoring such detail.

1. The original form of these models can be traced back to Tobin's Manuscript (1958). See Tobin (1982).

2. Kaizuka and Onodera (1974) is a direct application of the models in Jaffee and Modigliani (1969) and Jaffee (1971) to the case of Japan.

3. As explicitly shown in Tachi (1982), P. 62, this amounts to assuming a common production structure (or production function) for all loans.

"Bilateral negotiation models" may be superior in this sense. Let us examine two typical "bilateral negotiation models" as presented in Jaffee-Modigliani (1969) (to be referred to as the J-M model) and Teranishi (1982). While both models belong to the "bilateral" type, they differ from each other in many aspects. If we limit ourselves to the common features of these two models, we note that all loan contracts are assumed to be bilaterally negotiated, with either or both of the transacting parties exercising monopoly power. Such models highlight the optimizing behavior of the monopolist(s) in determining the loan rate.

The major difference between these two models lies in the types of heterogeneity among borrowers. While the J-M model explicitly considers the difference in borrowers' ability to repay (or default risk), Teranishi emphasizes the difference in borrowers' bargaining power as reflected by their positions in the market for long-term funds.

When compared to the "aggregative models", "bilateral negotiation models" also have some shortcomings. The J-M model discusses the determination of the size of each individual loan but leaves the determination of the volume of total lending and other balance sheet items unclear. It also ignores operating expenses in determining loan cost. In reality, however, the balance sheet constraint and operating expenses are certainly relevant to a bank's decisioin to lend.

The Teranishi model takes the balance sheet constraint and operating expenses into explicit account. Teranishi, however, posits separate cost functions for individual loans; this is the same as saying that each loan has its own production structure. Furthermore, since Teranishi ignores the opportunity cost of loan,[4] there is no link among bilateral negotiated transations. This means that bilateral transactions are carried out in markets that are perfectly segmented from one another.

We next attempt to bridge the gap between the "aggregative models" and "bilateral negotiation models". We construct a J-M type model that takes the balance sheet constraint and operating expenses into explicit account. However, pointed out earlier, although all transactions will be taken to be bilaterally negotiated, we shall not assume that banks have monopoly power.

III. Theoretical Framework of Lending Behavior of Banks

The basic assumptions of our model are as follows:

i) Although all loan contracts are determined by bilateral negotiation, compe-

4. In Teranishi's model, there exists no money market where banks can invest their funds instead of lending (or raise funds for lending). In its place, it is implicitly assumed that banks can borrow whatever amount they want from the Bank of Japan at the official discount rate.

tition among bilaterally negotiated transactions works so that interest rates are competitively determined.

ii) Loans are the output of banks and are homogeneous from the producers' point of view. The cost of loans, which includes such operating expenses as the costs of labor and capital, depends on total lending rather than the size of each individual (bilaterally negotiated) loan.

iii) The default risk of borrowers is explicitly modelled by introducing the banks' subjective probability distribution of borrowers' returns.

The internal equilibrium of a typical bank can then be derived as follows:

First, the balance sheet of the bank at the beginning can be written as,

$$R+\sum_{i=1}^{n} L_i+M \equiv D_p+\sum_{i=1}^{n} k_i L_i, \tag{1}$$

where R = reserves,
L_i = loan to customer i, where n = the total number of customers,
$\sum_{i=1}^{n} L_i$ = total lending,
M = investment in money market ($\gtrless 0$),
D_p = primary deposit,
k_i = proportion of loan to derivative deposit for customer i, (to be referred to as the deposit-loan ratio).

Then, the size of reserves can be expressed as

$$R = \alpha\ (D_p + \Sigma k_i L_i), \tag{2}$$

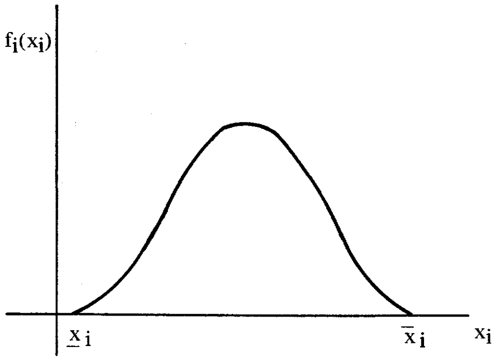
where the reserve ratio α is assumed to be a fixed constant exogenously.

When the bank receives a request for loan from any customer (i), it forms a subjective probability distribution of the return (x_i) on the customer's investment project. This can be expressed in the form of density function $\{f_i(x)\}$ as shown in Figure 1. We assume that the bank's subjective probability distributions of the return to different customers are independent of one another, and that they are independent of the size of the loans concerned.[5]

It is a common practice in Japan for a borrowing firm to provide the bank with collateral. Assume that the collateral offered by customer i is evaluated at a price T_i by the bank (T_i is assumed to be given for the bank). Let the loan rate be r_{Li}. If the return to the borrowing firm x_i exceeds the sum of principal and interest $(1 + r_{Li})\ L_i$, then the bank receives the contract repayment. On the other hand, if x_i turns out to be smaller than $(1 + r_{Li})\ L_i - T_i$, then the bank can get back no more than $(x_i + T_i)$,

5. The latter assumptioin means that the customers' investment oppotunity is independent of the size of the loan. It is possible to formulate our model under the assumption that the probability distribution of return depends on the size of loan. See Jaffee (1971), pp.957-62.

Figure 1 Probability Distribution of Return to Customer i as Evaluated by Bank



Note: $\underline{x}_i$ and $\overline{x}_i$ denote, respectively, the minimum and maximum level of expected return.

which is the case of default. Thus,

$x_i \geqq (1 + r_{Li})L_i$; both principal and interest can be recovered,

$(1+r_{Li})L_i > x_i \geqq (1+r_{Li})L_i - T_i$; principal and interest can only be recovered by disposing the collateral,

$(1 + r_{Li})L_i - T_i > x_i$; no more than $(x_i + T_i)$ can be recovered—the case of default.

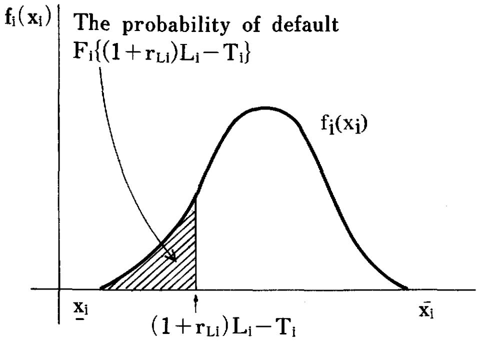
The subjective probability of default when a bank lends L_i to customer i is given by the probability that x_i falls below $(1 + r_{Li}) L_i - T_i$. This is represented by the shaded area in Figure 2. As can be seen from the figure, for any given loan and probability distribution of return, the risk of default becomes smaller as T_i (the value of the collateral) becomes larger. Note also that the expected risk of default depends very much on the shape of the subjective probability distribution of return as summarized by its mean and variance.

We present the operating cost function of the bank in the following general form.

$$C = C(D_p, \Sigma L_i). \quad (3)$$

Operating costs include the costs labor and capital employed in obtaining deposits and making loans. Here, operating cost is formulated as a function of total lending ΣL_i. Alternative formulations which account for the different cost structures of different loans might be also feasible. If the cost of loans is linearly homogeneous with respect to the size of loans, we could employ either L_i or ΣL_i in the cost function. However, these two formulations carry different implications when economies or

Figure 2 The Probability of Default



diseconomies of scale are significant. It is therefore important to know the cost structure of bank loans. We shall come to this point again in Section IV.

We further assume that the money market is perfectly competitive so that banks can lend or borrow at an interest rate r_M, and that the deposit interest rate r_D is exogenously given.

The bank maximizes expected total profit by choosing optimal values of L_i ($i = 1, \ldots\ldots, n$), D_p, and M. Expected total profit can be written as,

$$E[\Pi(L_1, \cdots, L_n, D_p, M)] = \sum_{i=1}^{n}\{(1+r_{L_i})L_i\int_{(1+r_{Li})L_i-L_i}^{\bar{X}_i} f_i(x_i)dx_i + \int_{X_i}^{(1+r_{Li})L_i-L_i}(x_i+T_i)f_i(x_i)dx_i - L_i\} + r_M M - r_D D_p - C(D_p, \sum_{i=1}^{n} L_i). \quad (4)$$

For simplicity, the interest rate on derivative deposits is assumed to be zero. Substituting (1) and (2) into (4), and rearranging, we have[6]

$$E[\Pi(L_1, \cdots, L_n, D_p, M)] = \sum_{i=1}^{n}\{(1+r_{Li})L_i)L_i - F_i(X_i)dx_i - L_i\} + r_M M\{(1-\alpha)(D_p+\sum k_i L_i) - \sum L_i\} - r_D D_p - C(D_p, \sum_{i=1}^{n} L_i), \quad (5)$$

6. Equation (5) is obtained by partial integration of the second term in { } on the right-hand side.

where $$F_i(x_A) = \int_{\underline{X}_i}^{X_i} f_i(x_i)\,dx_i \tag{5}$$
is the cumulative distribution of $f_i(x_i)$.

Maximizing expected profit in (5) yields the following first order conditions.

$$\frac{\partial E(\Pi)}{\partial L_i} = (1+r_{Li}) - (1+r_{Li})F_i\{(1+r_{Li})L_i - T_i\} - 1 + (1-\alpha)k_i r_M - r_M$$
$$-C_L(D_p, \Sigma L_i) = 0 \qquad (\text{for } i=1, \ldots, n), \tag{6}$$

$$\frac{\partial E(\Pi)}{\partial D_p} = (1-\alpha)r_M - r_D - C_D(D_p, \Sigma L_i) = 0, \tag{7}$$

where $C_L(D_p, \Sigma L_i)$, $C_D(D_p, \Sigma L_i)$ denote, respectively, the marginal cost of loans and the marginal cost of obtaining primary deposit.

Equation (6) can be rewritten as

$$\overbrace{(1+r_{Li})\underbrace{[1-F_i\{(1+r_{Li})L_i-T_i\}]}_{①}}^{②} - 1 = \underbrace{\{1-(1-\alpha)k_i\}r_M + C_L(D_p, \Sigma L_i)}_{③}. \tag{8}$$

The term labelled ① on the left-hand side denotes the probability of recovering both the principal and interest, so that ② denotes the expected marginal revenue of loans. In the absence of uncertainty, $F_i[\cdot] = 0$, so that marginal revenue on the left-hand side becomes r_{Li}. The right-hand side shows the marginal cost of loans which is composed of opportunity cost and operating cost. Thus Equation (8) is nothing but the subjective equilibrium of an ordinary firm when deciding its level of output (here interpreted as loans).

Equation (8) can be rewritten as

$$F_i\{(1+r_{Li})L_i - T_i\} = \frac{r_{Li} - \{1-(1-\alpha)k_i\}r_M - C_L}{1 + r_{Li}}. \tag{9}$$

Here, the left-hand side shows the risk of default for a loan L_i with loan rate r_{Li}. The parameters of the probability distribution of return and those of the cost function can be estimated empirically. Given T_i, k_i, α, and r_M, Equation (9) can then be used to solved the combinations of loan rate and loan size (r_{Li}, L_i) that equate customer i's probability of default to the value of the right-hand side. This gives the bank's optimal loan to customer i as a function of the loan rate.

Then we show the relationship between the size of loan L_i and the probability of default F_i, given some value of the loan rate r_{Li}. The curve AB in Figure 3 presents the relationship. Other things being equal, an increase in r raises the risk of default so that the AB curve shifts upward (say to A'B'). On the other hand, if marginal cost C_L

is an increasing function of L_i for given r_{Li}, then the right-hand side of Equation (9) as a whole is a decreasing function of L_i, as shown in Figure 4.[7] Other things being equal, the CD curve shifts upward (say to C'D') as the loan rate increases.

When imposing Figure 3 on Figure 4, a combination of r_{Li} and L_i that maximizes expected profit can be obtained at the intersection of the AB and CD curves. Optimal combinations of (r_{Li}, L_i) can be generated by varying r_{Li}. This is shown in Figure 5. As r_{Li} increases, AB and CD shift to A'B' and C'D' respectively, so that the equilibrium point shifts from E to E'. The larger the shift in the CD curve relative to that of the AB curve, the more likely that the new equilibrium lies to the north-east

Figure 3 Defalut Risk and the Size of Loan

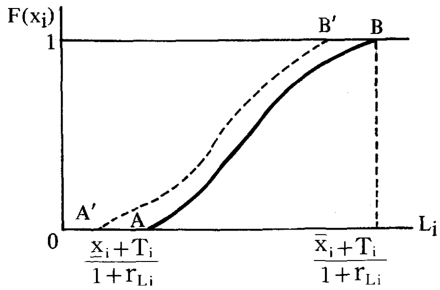
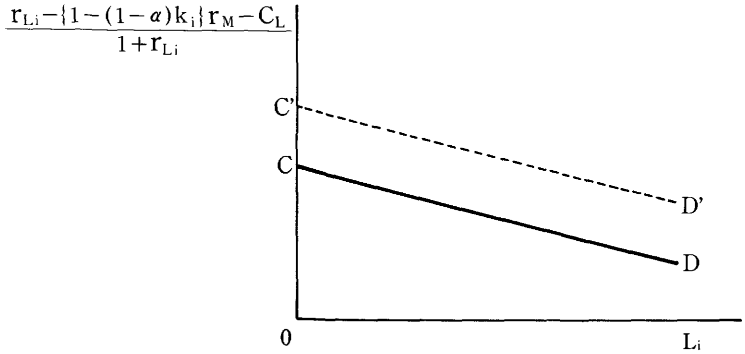


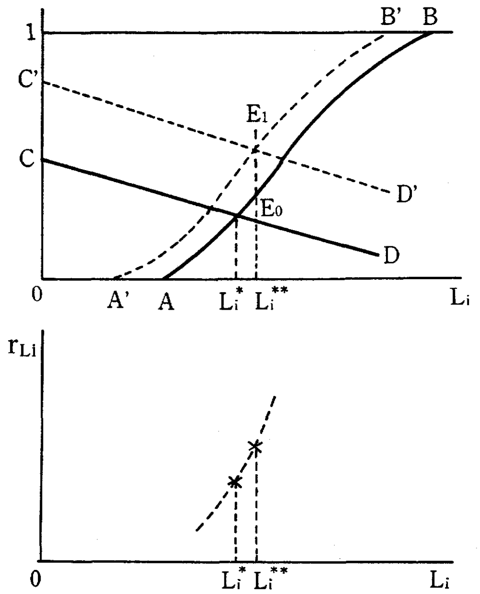
Figure 4 Value of Right-hand Side of Equation (9)



7. As we shall see in Section IV, when the capital stock and the availability of funds for lending are given in the short run, it is highly probable that marginal cost of bank loans exhibits increasing. This may occur even if the long-term cost function is characterized by constant or diminishing marginal cost.

of the initial equilibrium, that is, the more likely that the supply curve relating the optimal size of loan to the loan rate slopes upward to the right. However, above a certain level, further increase in the loan rate may shift the AB curve more than the CD curve so that the optimal loan size decreases. This is the case of the backward bending supply curve for loans that appears in the J-M model discussed above.

Figure 5 Derivation of the Supply Curve for Loans



To summarize, the property of the supply curve for a customer depends on the parameters of the probability distribution of return as evaluated by the bank and the parameters of the bank's cost function. It also depends on the treatment of loans as bilaterally negotiated transactions which determines the properties of these parameters.

IV. Estimation of the Economies of Scale and Cost Function of Banks

As we have seen in Section III, to derive the subjective equilibrium of a bank, we have to estimate the marginal cost involved in obtaining deposits and extending loans. Here, let us examine some alternative ways of formulating the activities and cost of banks in economic models.

When an output x is produced with inputs $\{v_1, ..., v_n\} = V$, the production function can be written as

$$x = f(v_1, \ldots, v_n) = f(V). \tag{10}$$

The scale elasticity k is defined as the elasticity of output x to the scale factor of input μ, when factor proportions are kept constant. That is,

$$k=\frac{d\ln x}{d\ln\mu}=\frac{\frac{dx}{x}}{\frac{d\mu}{\mu}}, \tag{11}$$

where

$$x = f(v_1, \ldots, v_n) = f(\mu v_1^0, \ldots, \mu v_n^0).$$

Here

$k > 1$ denotes economies of scale

$k = 1$ denotes constant return to scale

and $k < 1$ denotes diseconomies of scale.

In models that treat loans as the output of the banking industry, the input and cost structures of production have important implications for the formulation of lending behavior. If the cost function exhibits constant return to scale, then marginal cost C_L in Equation (9) is constant and independent of the size of loans. If we limit ourselves to cost considerations in this case, the optimal size of loan to each individual customer can be determined independently of other (bilaterally negotiated) loan transactions. However, if the cost function exhibits economies or diseconomies of scale, marginal cost changes with the size of lending so that the marginal cost of any transaction depends on the size of other bilaterally negotiated transactions preceding it.

In principle, the scale elasticity k can be obtained by specifying and then estimating the production function in Equation (10) directly. In many cases, however, multicollinearity among inputs makes estimation difficult. To avoid this, Frisch suggests that the scale elasticity can be estimated by using the following approximation:[8]

$$k=\frac{dx/x}{d\mu/\mu}=\frac{\ln x^2-\ln x^1}{\ln\mu^2-\ln\mu^1}=\frac{\ln x^2-\ln x^1}{\ln v_i^2-\ln v_i^1}, \tag{12}$$

where

$$v_i^2 = \mu^2 v_i^0,\ v_i^1 = \mu^1 v_i^0.$$

Our estimation of the scale elasticity follows Yoshioka (1984), which is an extension of the method used by Frisch. Instead of specifying the production function, we estimate the scale elasticity k using observable data of output, input, and relative input price. This assumes that 1) producers minimize cost and 2) the production function is homogeneous of degree k so that:

$$x = \lambda^{-k} f(\lambda V),$$

8. See Frisch (1965).

where λ is a positive scalar. (13)

1. Estimation of Scale Elasticity in the Banking Industry

A. The Method of Estimation

Here, we summarize the method of estimating the scale elasticity used in Yoshioka (1984).[9]

Consider a producer that minimizes cost subject to a homogeneous production function. The relationship between output x, inputs v_i, and relative input price P_i (which are observable) can be depicted on the input plane as shown in Figure 6. For simplicity, we assume that there are only two inputs and that two levels of production x^1 and x^2 ($x^1<x^2$) have been observed.

In Figure 6, the output levels (x^1 and x^2), the input levels, as well as the relative input prices at A and B can be directly observed. Rays A and B show, respectively, the expansion paths corresponding to the constant relative factor proportions (and thus constant relative input prices) at point A and point B. Let V^{*2} denote the vector of inputs at the point where Ray A cuts the isoquant x_2 and V^{*1} denote the vector of inputs at the intersection of x_1 and Ray B. The scale elasticity k can then be defined as follows:

Let λ be a constant that satisfies $V^{*2}= \lambda V^1$ on Ray A. For any i (i = 1, ..., n), the scale elasticity k is defined as

$$k^1= \frac{\ln f(V^{*2}) - \ln f(V^1)}{\ln \lambda} = \frac{\ln x^2 - \ln x^1}{\ln v_i^{*2} - \ln v_i^2}. \tag{14}$$

Likewise, on Ray B, we have

$$k^2= \frac{\ln f(V^2) - \ln f(V^{*1})}{\ln \lambda} = \frac{\ln x^2 - \ln x^1}{\ln v_i^2 - \ln v_i^{*1}}, \tag{15}$$

by the assumption of homogeneity, $k^1 = k^2 = k$.

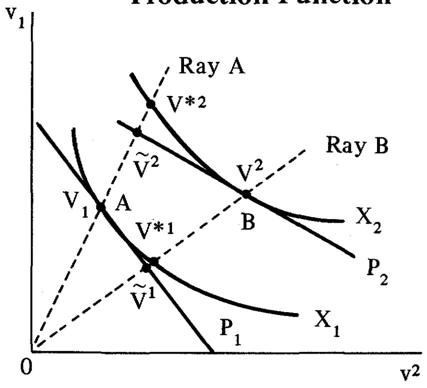
As can be seen from Figure 6, V^{*1} and V^{*2} are not directly measurable since we have not specified the shape of the isoquants. However, since $\tilde{V}^1$ and $\tilde{V}^2$ can be easily derived from other measurable data, they can be substituted for V^{*1} and V^{*2} in Equation (14) and (15) to derive approximations for the scale elasticity.
from

$$\tilde{V}^1 = \lambda_1 V^2,\ P^1\tilde{V}^1 = P^1V^1,$$

we have

9. See Yoshioka (1977, 1979, 1982 1984).

Figure 6 Scale Elasticity and the Production Function



$$P^1\tilde{V}^1 = \lambda_1 P^1V^2 = P^1V^1,$$

so that $\lambda_1 = \dfrac{P^1\,V^1}{P^1\,V^2}$ or $\tilde{V}^1 = \dfrac{P^1\,V^1}{P^1\,V^2} \cdot V^2$. (16)

Likewise, from

$$\tilde{V}^2 = \lambda_2 V^1,\ P^2V^2 = P^2\tilde{V}^2,$$

we have

$$P^2\tilde{V}^2 = \lambda_2 P^2V^1 = P^2V^2,$$

so that $\lambda_2 = \dfrac{P^2V^2}{P^2V^1}$ or $\tilde{V}^2 = \dfrac{P^2V^2}{P^2V^1} \cdot V^1$. (17)

Substituting $\tilde{V}_i^2$ in Equation (17) for $V^*{}_i^2$ in Equation (14), and $\tilde{V}_i^1$ in Equation (16) for $V^*{}_i^1$ in Equation (15), we define

$$k_u = \frac{\ln x^2 - \ln x^1}{\ln \tilde{v}_i^2 - \ln v_i^1} , \tag{18}$$

$$k_\ell = \frac{\ln x^2 - \ln x^1}{\ln v_i^2 - \ln \tilde{v}_i^1} . \tag{19}$$

As can be seen from the figure, since

$$v_i^{*2} > \tilde{v}_i^2, \text{ and } v_i^{*1} > \tilde{v}_i^1,$$

we have

$$k_u > k > k_\ell . \tag{20}$$

Under the assumption of homogeneity, k_u and k_ℓ give respectively the estimated upper and lower limits of k.

From Equations (16) and (17), the denominator on the right-hand side of Equation (18) can be written as

$$\ln \tilde{v}_i^2 - \ln v_i^1 = \ln \lambda_2 = \ln Q_P = \ln \left(\frac{P^2 V^2}{P^2 V^1}\right), \tag{21}$$

where Q_L, Q_P, Q_I, Q_D are, respectively, the Laspeyres, Paasche, Fisher and Divisia quantity indices of input derived from the cross section data.
Likewise, the denominator on the right-hand side of Equation (19) can be written as

$$\ln v_i^2 - \ln \tilde{v}_i^1 = -\ln \lambda_1 = \ln Q_L = \ln \left(\frac{P^1 V^2}{P^1 V^1}\right), \tag{22}$$

They show, respectively, the rates of change of the Paasche and Laspeyres indices of the input. Thus, the estimated upper and lower limits of the scale elasticity k_u and k_ℓ correspond, respectively, to the scale elasticities derived when the Paasche index and the Laspeyres index are used to represent the aggregation of i inputs.

Other approximations of the scale elasticity k corresponding to other aggregator functions can also be defined. Thus, for the Fisher ideal index Q corresponding to the quadratic root function, and for the Divisia index corresponding to the Translog function, we have

$$k_I = \frac{\ln x^2 - \ln x^1}{\ln Q_I} = \frac{\ln x^2 - \ln x^1}{\ln \sqrt{Q_L \cdot Q_P}}, \tag{23}$$

$$k_D = \frac{\ln x^2 - \ln x^1}{\ln Q_D} = \frac{\ln x^2 - \ln x^1}{\Pi \left(\frac{V_i^2}{V_i^1}\right)^{\frac{1}{2}\left\{\left(\frac{P_i^1 V_i^1}{P^1 V^1}\right) + \left(\frac{P_i^2 V_i^2}{P^2 V^2}\right)\right\}}}, \tag{24}$$

respectively.[10]

B. Estimation of Economies of Scale in the Banking Industry

Based on the method discussed above, we estimated the scale elasticity for Japan's city banks (12 banks, excluding the Bank of Tokyo). The scale elasticity depends on the way in which the input and output are defined. We define the output of banks narrowly to include only loans. In line with this, input cost will be interpreted as operating expenses, including the costs of labor and capital. We justify our narrow definition of bank output by noting that the ratio of loans outstanding to total earning assets has stabilized over time at a level above 80% for the twelve city banks, both individually and in the aggregate.

10. See Diewert (1976).

It is also interesting, however, to examine whether there exist economies of scale in the city banks when the definition of output is extended to include other activities. Extensions in this direction are presented in the Appendix.

Our plan for estimating the scale elasticity can be summarized as follows.

Sample: 12 city banks, excluding the Bank of Tokyo.

Sample Period: March 1973(the second half of 1972)- September 1983(the first half of 1983), semi-annual basis.

Source: Analysis of Financial Statement of All Banks

Variables: x_i = loans outstanding (average balance).

$v_1{}^i$ $(=K^i)$ = bank premises and real estate (book value).

$v_2{}^i$ $(=N^i)$ = number of employees.

C_{ik} = non-personnal expenses (capital cost).

C_{iN} = personnal expenses (labor cost).

C_i $(=C_K + C_N)$ = operating expenses.

$r_i = C_{ik}/K^i$.

$w_i = C_{iN}/N^i$.

The scale elasticity for each of the 22 periods from March 1973 to September 1983 was estimated using cross-section data of the 12 city banks according to the following procedure.

a) For each period, loans outstanding L_i of all banks are ranked according to their relative size, with the smallest coming first.

b) For each pair of adjacent banks in the above ranking (there are 11 pairs), we can derive the following scale elasticities.

$$\hat{k_l} = \frac{\ln x_i - \ln x_j}{\ln Q_L},$$

$$\hat{k_u} = \frac{\ln x_i - \ln x_j}{\ln Q_P},$$

$$\hat{k_I} = \frac{\ln x_i - \ln x_j}{\ln Q_I} = \frac{\ln x_i - \ln x_j}{\ln\sqrt{Q_P \cdot Q_L}},$$

$$\hat{k_D} = \frac{\ln x_i - \ln x_j}{\ln Q_D}.$$

c) For each period, the scale elasticities $\hat{k}_j$ $(j = \ell$, u, I, D) so derived may be subject to sampling errors. To reduce these errors, we estimate the average scale elasticities $\tilde{k}_j$ by regressing output directly on each of the quantity indices of input. That is,

$$\ln x_i = \tilde{k}_j \ln Q_j i + k_{0j} + u. \tag{25}$$

The results of estimating Equation (25) are summarized in Table 1. Four scale elasticities $\tilde{k}_j$ (j = L, I, D, P) corresponding to the four indices (Laspeyres, Fisher, Divisia and Paashe) of input are estimated for each of the 22 periods. Here k_L and k_P

correspond, respectively, to the lower and upper limits of the true scale elasticity, as we have seen in the theoretical discussion above.

According to Table 1, Equation (25) fits well in all cases. All scale elasticities (regression coefficients) are statistically significant at the one percent level, and the hypothesis that the production function is homogeneous cannot be rejected. In addition, both k_L and k_P are significantly larger than 1.0 at the one percent level, showing that there exist economies of scale. In all 22 periods, $k_P > k_L$. Among them, k_P is significantly larger than k_L at the 1% level in 3 periods and at the 5% level in 11 periods. This is consistent with the assumptions of a homogeneous production function and cost minimization. It also shows the appropriateness of the estimation method used here.[11]

Our results indicate the economies of scale resulting from loan expansion may be quite large for the city banks of Japan. The results in Table 1 also show that the scale elasticities have been increasing over time, suggesting that productivity has been improving as a result of technical progress.

Since the relation $\hat{k}_\ell < \hat{k}_I < \hat{k}_D < \hat{k}_u$ is empirically stable, in what follows, the scale elasticity for the city banks will be taken as the estimated value of $\tilde{k}_D$. In order to estimate the cost function, we take the scale elasticity as its average value in the period under study. That is,

$$\frac{1}{22}\sum_{t=1}^{n}\tilde{k}_D^t = 1.40075 \ .$$

2. Estimation of the Cost Function

Our results in Section IV. 1 show that there exist substantial economies of scale in the cost structure of city banks which produce loans as their output. At the same time, they also support our characterization of the production structure of banks as homogeneous production functions.

The cost function of city banks can be specified as follows.

First let us consider the production functions

$$D_P = a e^{g_1 t} K_P^b N_P^c , \tag{26}$$

$$L = \alpha e^{g_2 t} K_L^\beta N_L^\gamma D_P^\eta = (\alpha a^\eta) e^{(g_1\eta + g_2)t} K_L^\beta K_P^{b\eta} N_L^\gamma N_P^{c\eta} , \tag{27}$$

11. A necessary condition for homogeneity is that k_L (lower limit) and k_p (upper limit) are sufficiently close to each other. However, even if our sample shows that this condition holds, this does not necessary imply global homogeneity. When multicollinearity between inputs are high in the sample, this caveat becomes particularly important.

Table 1 Estimation of the Scale Elasticity of City Banks

output: average loans outstanding input: operating expense items

(Year)	Laspeyres Index			Fisher Index			Divisia Index			Paasche Index		
	C	LNCQL	R^2	C	LNCQI	R^2	C	LNCQD	R^2	C	LNCQP	R^2
1973 - 3	13.7502	1.24821	0.9869	13.7303	1.29326	0.9861	13.7302	1.29346	0.9860	13.7096	1.34094	0.9845
	(330.4)	(26.04)		(314.4)	(25.24)		(314.2)	(25.23)		(293.2)	(23.99)	
1973 - 9	13.8851	1.21774	0.9855	13.8764	1.23482	0.9859	13.8764	1.23492	0.9859	13.8676	1.25222	0.9862
	(321.5)	(24.78)		(333.4)	(25.14)		(333.3)	(25.13)		(333.7)	(25.38)	
1974 - 3	13.9375	1.23864	0.9849	13.9170	1.27607	0.9861	13.9169	1.27638	0.9861	13.8957	1.31511	0.9868
	(319.3)	(24.23)		(326.9)	(25.30)		(326.9)	(25.30)		(329.4)	(26.00)	
1974 - 9	14.0352	1.22495	0.9814	14.0219	1.24170	0.9828	14.0217	1.24206	0.9828	14.0085	1.25848	0.9839
	(298.6)	(21.80)		(306.9)	(22.71)		(306.8)	(22.71)		(313.1)	(23.49)	
1975 - 3	14.0395	1.30442	0.9660	14.0301	1.32029	0.9709	14.0301	1.32026	0.9672	14.0206	1.33641	0.9720
	(209.4)	(16.04)		(211.4)	(16.34)		(211.4)	(16.35)		(283.3)	(16.64)	
1975 - 9	14.0739	1.32724	0.9794	14.0610	1.34480	0.9799	14.0616	1.34487	0.9800	14.0494	1.36246	0.9803
	(268.6)	(20.71)		(269.5)	(21.01)		(269.7)	(21.03)		(268.8)	(21.21)	
1976 - 3	14.1086	1.35416	0.9717	14.0952	1.38142	0.9752	14.0951	1.38172	0.9722	14.0814	1.40950	0.9724
	(225.1)	(17.59)		(224.2)	(17.75)		(224.3)	(17.76)		(222.5)	(17.84)	
1976 - 9	14.1607	1.35470	0.9793	14.1503	1.36893	0.9799	14.1502	1.36912	0.9799	14.1398	1.38320	0.9804
	(267.6)	(20.60)		(269.1)	(20.98)		(269.2)	(20.98)		(269.8)	(21.24)	
1977 - 3	14.2018	1.34425	0.9886	14.1938	1.37081	0.9885	14.1935	1.37169	0.9884	14.1855	1.39834	0.9882
	(361.3)	(27.90)		(357.1)	(27.79)		(356.9)	(27.78)		(351.4)	(27.56)	
1977 - 9	14.2343	1.35662	0.9918	14.2314	1.37201	0.9917	14.2313	1.37252	0.9917	14.2285	1.38773	0.9915
	(425.8)	(33.00)		(422.2)	(32.84)		(422.0)	(32.84)		(417.9)	(32.60)	
1978 - 3	14.2977	1.34335	0.9930	14.2913	1.36828	0.9927	14.2911	1.36887	0.9927	14.2846	1.39406	0.9923
	(473.4)	(35.80)		(460.1)	(34.98)		(459.8)	(34.97)		(445.2)	(34.06)	
1978 - 9	14.3325	1.36074	0.9915	14.3208	1.40225	0.9907	14.3205	1.40361	0.9907	14.3085	1.44690	0.9899
	(426.9)	(32.35)		(405.5)	(31.05)		(404.9)	(31.08)		(383.1)	(29.67)	
1979 - 3	14.4201	1.34194	0.9901	14.4050	1.39094	0.9891	14.4046	1.39209	0.9891	14.3888	1.44340	0.9878
	(406.8)	(29.90)		(382.5)	(28.59)		(381.9)	(28.60)		(356.8)	(27.10)	
1979 - 9	14.4438	1.40786	0.9798	14.4348	1.44524	0.9783	14.4347	1.44575	0.9783	14.4253	1.48460	0.9768
	(278.8)	(20.95)		(266.8)	(20.20)		(266.6)	(20.21)		(254.7)	(19.47)	
1980 - 3	14.5205	1.40374	0.9780	14.5096	1.44319	0.9772	14.5096	1.44359	0.9772	14.4981	1.48489	0.9763
	(273.5)	(20.04)		(265.6)	(19.66)		(265.5)	(19.64)		(257.6)	(19.26)	
1980 - 9	14.6177	1.34151	0.9768	14.6051	1.37949	0.9777	14.6049	1.38037	0.9777	14.5919	1.41947	0.9786
	(285.0)	(19.48)		(287.6)	(19.91)		(287.5)	(19.91)		(289.7)	(20.32)	
1981 - 3	14.6467	1.36027	0.9750	14.6379	1.39734	0.9739	14.6379	1.39783	0.9739	14.6286	1.43647	0.9727
	(271.6)	(18.76)		(263.4)	(18.35)		(263.4)	(18.35)		(255.2)	(17.94)	
1981 - 9	14.6532	1.43532	0.9710	14.6452	1.46875	0.9699	14.6452	1.46923	0.9699	14.6369	1.50374	0.9687
	(242.2)	(17.40)		(235.7)	(17.06)		(235.6)	(17.05)		(229.1)	(16.71)	
1982 - 3	14.7115	1.48079	0.9688	14.7015	1.50805	0.9680	14.7015	1.50834	0.9680	14.6912	1.53622	0.9672
	(231.1)	(16.75)		(226.1)	(16.55)		(226.1)	(16.55)		(220.9)	(16.32)	
1982 - 9	14.7536	1.53123	0.9724	14.7461	1.55695	0.9716	14.7461	1.55707	0.9716	14.7384	1.58351	0.9706
	(421.9)	(17.84)		(236.5)	(17.56)		(236.6)	(17.57)		(231.2)	(17.29)	
1983 - 3	14.8061	1.58651	0.9732	14.8031	1.63193	0.9719	14.8031	1.63182	0.9719	14.8003	1.67946	0.9700
	(252.7)	(18.13)		(245.7)	(17.69)		(245.7)	(17.67)		(245.6)	(17.68)	
1983 - 9	14.8618	1.59780	0.9652	14.8564	1.61101	0.9650	14.8564	1.61093	0.9650	14.8509	1.62440	0.9649
	(220.3)	(15.83)		(218.6)	(15.79)		(218.6)	(15.79)		(216.9)	(15.75)	

where

D_p = primary deposit = $D - \Sigma k_i L_i$ (k_i = deposit-loan ratio, D = total deposit),
k_i = (cash and deposits)/(long- and short-term borrowings from financial institutions)
calculated by industry and firm size (data from Quarterly Report of Incorporated Enterprise Statistics),
K_p = capital stock devoted to obtaining primary deposits,
K_L = capital stock devoted to loan transactions,
K = $K_p + K_L$ = bank premises and real estate,
N_p = number of employees engaging in deposit transactions,
N_L = number of employees engaging in loan transactions,
N = $N_p + N_L$ = total number of employees,
t = time trend.

In the above formulation, data for K_p, K_L, N_p, and N_L, are not available and therefore Equations (26) and (27) cannot be estimated directly.

Equation (27) assumes that the size of output (that is, loans) is related to inputs through a Douglas-type homogeneous production function. Thus the scale elasticity is equal to the sum of the exponents of the terms from K_L to N_p, that is, $\beta + b\eta + \gamma + c\eta$. Recalling our estimation of the scale elasticity in Section IV. 1, we have $\beta + b\eta + \gamma + c\eta = 1.40075$.

Cost is defined as

$$C = r(K_p + K_L) + w(N_p + N_L), \tag{28}$$

where r and w denote, respectively, per unit capital cost and wage per employee, and C denotes total operating expenses.

The cost equation can be derived by the minimization of Equation (28) subject to the constraints of Equation (26), (27). We have

$$C = [\beta + b\eta + \gamma + c\eta)\{(\alpha a^{\eta})(\beta^{\beta})(b\eta)^{b\eta}(\gamma^{\gamma})(c\eta)^{c\eta}\}^{-\frac{1}{\beta+b\eta+\gamma+c\eta}}],$$
$$\times e^{-\frac{(g_1\eta+g_2)}{\beta+b\eta+\gamma+c\eta}t} L^{\frac{1}{\beta+b\eta+\gamma+c\eta}} r^{\frac{\beta+b\eta}{\beta+b\eta+\gamma+c\eta}} w^{\frac{\beta+b\eta}{\beta+b\eta+\gamma+c\eta}}, \tag{29}$$

In addition, from the first order conditions of cost minimization, we have

$$K_L = \frac{\beta}{\beta + b\eta} K, \tag{30}$$

and

$$N_L = \frac{\gamma}{\gamma + c\eta} N. \tag{31}$$

Substituting Equations (30) and (31) into (27), we have

$$L=\left\{\alpha\left(\frac{\beta}{\beta+b\eta}\right)^{\beta}\left(\frac{\gamma}{\gamma+c\eta}\right)^{\gamma}\right\}e^{g_2 t}K^{\beta}N^{\gamma}D_P^{\eta}. \tag{32}$$

We estimate the parameters of the cost function by constraining $\beta - b\eta + \gamma + c\eta$ to equal 1.40075, the average estimated scale elasticity. Given the scale elasticity, we isolate the effect of technical progress by using time series data for the average of all city banks (excluding the Bank of Tokyo) in estimating Equations (29) and (32). The deflator for investment goods (SNA base) is used to approximate the deflator for the capital stock K.

The results of estimation are summarized as follows.

Corresponding to Equation (29), we have

$$\begin{aligned} \ln C = {} & 0.543164 - \underset{(23.40)}{0.0202728}\, t + \underset{(11.74)}{0.7139032} \ln L + \underset{(2.34)}{0.492982} \ln \gamma \\ & + \underset{(3.20)}{0.507018} \ln w \end{aligned} \tag{33}$$

$$R = 0.9916,$$

where

$$0.7139032 = \frac{1.0}{1.40075} = \frac{1}{\beta + b\eta + \gamma + c\eta} = \frac{1}{\tilde{k}_D},$$

and corresponding to Equation (32), we have

$$\begin{aligned} \ln L = {} & \underset{(141.41)}{1.72105} + \underset{(25.37)}{0.0270816}\, t + 0.55048 \ln(K/P_k) + 0.44952 \ln N \\ & + 0.181749 \ln D_p \end{aligned}$$

$$R = 0.9863. \qquad (34)^{12}$$

From the reduced-form parameters of Equations (33) and (34), the structural parameters of Equations (26) and (27) can be exactly identified. That is,

$\hat{\alpha} = 7.7790723$, $\hat{a} = 0.00352731$,
$\hat{\beta} = 0.55048$, $\hat{b} = 0.7706481$,
$\hat{\gamma} = 0.44952$, $\hat{c} = 1.4343152$,
$\hat{\eta} = 0.1817491$, $\hat{g}_1 = 0.007238$,
$\hat{g}_2 = 0.0270816$.

12. To avoid multicollinearity between the parameters of ln (K/P_K) and ln N in Equation (34), we use cross-section data of the city banks to estimate the equation

$$\frac{L}{N} = \alpha\left(\frac{K}{N}\right)^{\beta} \cdot D_P{}^{\eta}$$

for semi-annual period from September 1973 to September 1983. The β and η are taken to be the average of their estimated values over the sample period.

Thus technical progress takes place at an annual rate of 0.72% for deposit transactions and 2.70% for loan transactions. The combined effect is that, other things being equal, total cost decreases at an annual rate of 2.02%.

V. Estimation of the Supply Curve for Loans

1. Estimation of the subjective probability distribution of return

The estimated parameters of the production function indicate the existence of increasing returns to scale (or diminishing marginal cost) in bank lending. In this case, the marginal cost of loan C_L (the last term in the numerator of Equation (9)) depends on the scale of lending. Even though each loan transaction is bilaterally negotiated between a bank and the borrower concerned, its marginal cost depends on the size of the bank's other loan transaction. By altering the scale of production (and thus marginal cost), bilaterally negotiated transactions are no longer independent of one another.

Equation (33) corresponds to the estimation of the long-run cost curve. Given the relative input price ratio w/r and the state of technology, this cost function exhibits diminishing marginal cost. The long-term marginal cost function is given by

$$MC_L = [\{(\alpha a^{\eta})(\beta^{\beta})(b\eta)^{b\eta}(\gamma(^{\gamma}(c\eta)^{c\eta}\}^{-\frac{1}{\beta+b\eta+\gamma+c\eta}}]e^{-\frac{(g_1\eta+g_2)t}{\beta+b\eta+\gamma+c\eta}}L^{\frac{1}{\beta+b\eta+\gamma+c\eta}-1}$$

$$\times r^{\frac{\beta+b\eta}{\beta+b\eta+\gamma+c\eta}} w^{\frac{\gamma+c\eta}{\beta+b\eta+\gamma+c\eta}}. \tag{35}$$

The long-run cost curve described by Equation (29) forms an envelope around the short-run cost curves which correspond to different levels of K_p, K_L, and D_p. Given K_p, K_L, D_p, w and r, the short-run cost function is given by

$$C_S = r(K_p + K_L) + wN_p + w\left\{\frac{L}{\alpha e^{g_2 t} K_L^{\beta} D_p^{\eta}}\right\}^{\frac{1}{\gamma}} \tag{36}$$

and the short-run marginal cost function is given by

$$MC_S = (\frac{1}{\gamma})(\alpha^{-\frac{1}{\gamma}})e^{-g_2 t}L^{\frac{1}{\gamma}-1}K_L^{-\frac{\beta}{\gamma}}D_p^{-\frac{\eta}{\gamma}}w. \tag{37}$$

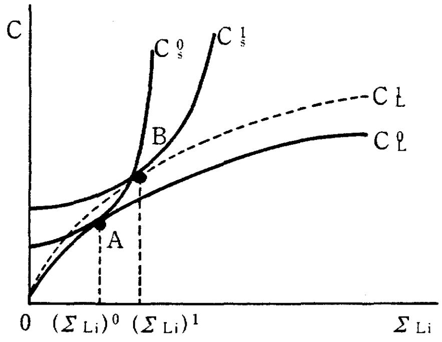
Using the estimated structural parameters estimated in Section IV, the short-run and long-run marginal cost functions can be derived as

$$MC_L = 1.2289343\ e^{-0.202728t}\ L^{-0.286096}\ r^{0.492982}\ w^{0.507018}\ , \tag{38}$$

and

$$MC_s = 0.0231126\ e^{-0.0270816t}\ L^{1.2245951}\ K_L^{-1.2245951}\ D_p^{-0.4043179}\ w\ , \tag{39}$$

Figure 7 Short-run and Long-run Cost Curves



respectively.

As shown in Figure 7, given K_L, D_p, and w, the short-run cost function exhibits increasing marginal cost. Assume that at the end of each period, the bank operates at a point on the envelope surface of all short-run and long-run cost curves so that short-run and long-run total cost are equal; that is, given the relative input price ratio w/r, capital stock K, number of employees L, and loans outstanding ΣL_i have all fully adjusted to long-run equilibrium. The available time series data of banks correspond to the end-of-period equilibrium at A and B in Figure 7. From the parameters of the long-run cost curves at point A and B, the marginal cost of total lending ΣL_i at the end-of-period equilibrium can be derived.

Reproducing Equation (9) here for convenience,

$$F_i\{(1 + r_{Li})L_i - T_i\} = \frac{r_{Li} - \{1 - (1 - \alpha)k_i\}r_M - C_L}{1 + r_{Li}} . \tag{9}$$

we note that it determines the end-of-period equilibrium loan level for each customer. If C_L can be derived from our estimation of the cost function above, and if the loan, rate r_{Li}, the money market rate r_M, the deposit-loan ratio k_i, and the reserve ratio α can be estimated, we can calculate the right-hand side of equation (9) for each bilaterally negotiated transaction. This measures indirectly the bank's subjective evaluation of default risk for each customer.

Unfortunately, it is not possible to obtain information on the terms and size of each individual bilaterally negotiated transaction for each city bank. When estimating the parameters of the cost function above, we have used time series data for the average of 12 city banks. This amounts to assuming a representative city bank.

Time series data for loans to firms by industry and by firm size, classified further into working funds and equipment funds, are available from the Economic Statistics

Monthly published by the Bank of Japan. Here, we divide the average balance of total loan outstanding of the city banks into twelve groups according to the industry (manufacturing and non-manufacturing industries) and size (small, medium, and large) of the borrowing firms, as well as the purpose of loans (working funds and equipment funds). The deposit-loan ratio k_i for borrowers of different industries and sizes can be derived from data available in the Quarterly Report of Incorporated Enterprise Statistics.[13]

Loan rates for individually negotiated loans r_{Li} are difficult to obtain. However, if our assumption that transactions take place along the long-run cost curve holds at end-of-period equilibrium, we may further assume that as a result of arbitrage, the average loan rate applies to all transactions.[14]

$$r_{Li} \doteqdot r_L$$

We can then calculate time series data for the right-hand side of Equation (9), generating the representative bank's subjective evaluation of the probability of default for different groups of borrowers.

Denoting the estimated probability of default by $\tilde{y}_i$, we have

$$F_i\{(1 + r_{Li})L_i - T_i\} = \tilde{y}_i. \tag{40}$$

Using {(cash and deposits) + (securities held) − (corporate bonds)} as a proxy for the collateral T_i, the collateral ratio can be calculated as

$$\theta_i = \frac{\{(\text{cash and deposits}) + (\text{securities held}) - (\text{corporate bonds})\}}{\{\text{borrowings from financial institutions}\}}$$

13. The advantage of using data in the Quarterly Report of Incorporated Enterprise Statistics is that k_i by industry and firm size can be derived. The disadvantage is that we cannot distinguish between different types of financial institutions. We may instead approximate k_i by the deposit ratios of borrowers in the Tabular Report on Compensating balances from Ministry of Finance. This data distinguishes between different types of financial institutions but provides no disaggregative information by industry.

14. It is difficult to test whether annual data correspond to long-run end-of-period equilibrium in reality and whether the mechanism of arbitrage works to equalize all r_{Li}. These assumptions are necessary to make the estimation feasible, given limited data availability.

All data necessary for calculating this ratio can be obtained from the Quarterly Report of Incorporated Enterprise Statistics.[15]

For each borrower, let

$$x_i = (1 + r_L - \theta_i)L_i . \tag{41}$$

Assume that in x_i is normally distributed random variable with mean μ and variance σ^2, we have

$$F(\frac{\ln x_i - \mu}{\sigma}) = \tilde{y}_i, \tag{42}$$

where F : N (0, 1).

Assume that the mean μ shifts over time according to

$$\mu = \mu_0 + \mu_1 \ln L_i^{-1}, \tag{43}$$

or

$$\mu = \mu_0 + \mu_1 L_i^{-1}, \tag{44}$$

where L_i^{-1} denotes the size of the loan to customer i's at the end of the previous period.

We now estimate the parameters of the banks' subjective probability distribution of the return to different categories of borrowers assuming either Equation (43) or (44) holds.

Let the inverse function of the cumulative distribution function F in Equation (42) be Φ^{-1} so that

15. The collateral ratio θ_i can be calculated from several sources. First, it can be derived from the financial statements of individual banks as the ratio of secured and guaranteed loans to total lending. This does not yield collateral ratios by industry and by firm size. Alternatively, we can derive the collateral ratio by industry and firm size from the Quarterly Report of Incorporated Enterprise Statistics by using the following definitions.
    1. { cash and deposit } / { borrowings from financial institutions }
    2. { (cash and deposit) – (corporate bonds) } / { borrowings from financial institutions }
    3. { (cash and deposit) + (securities) } / { borrowings from financial institutions }
    4. { (cash and deposit) + (securities) – (corporate bonds) } / { borrowings from financial institution }
    5. { tangible fixed assets } / { borrowings from financial institutions }
    6. { (tangible fixed assets) – (corporate bonds) } / { borrowings from financial institutions }

    Definition 5 or 6 may better correspond to usual practice. In order to avoid bias due to the book valuation of tangible fixed assets, however, definition 4 is used here.

$$\Phi^{-1}(\tilde{y}_i) = \frac{\ln x_i - \mu}{\sigma}.$$

The equation to be estimated is either

$$\ln x_i = \mu_0 + \mu_1 \ln L_i^{-1} + \sigma \Phi^{-1}(\tilde{y}_i)$$

or

$$\ln x_i = \mu_0 + \mu_1 L_i^{-1} + \sigma \Phi^{-1}(\tilde{y}_i),$$

which correspond to the assumptions in Equations (43) and (44) respectively. Here we constrain that parameter σ is positive in estimation.

The results of estimation are summarized in Tables 2 and 3. Quarterly data have been used in our estimation. The sample period extends from the third quarter of 1973 to the fourth quarter of 1983. The standard deviation of the probability distribution of return σ is statistically significant in all cases except for working funds and total loans for medium-size firms in the manufacturing industry. The results of estimation do not differ significantly when either Equation (43) or (44) is used to represent the trend in μ. In what follows we shall limit ourselves to the specification in Equation (43).

Our results show that the standard deviation of return declines significantly as firm size increases. This tendency is observed in all cases save equipment funds for medium-sized firms. Thus the standard deviation of return is larger for small firms than for medium-size firms, and large firms have the smallest standard deviation of all. Within the same size group, firms in the manufacturing industry tend to have a smaller standard deviation of return. The standard deviation of the return for equipment funds for large firms in the manufacturing industry is only about one tenth of that for small firms in the non-manufacturing industry. The observation that the standard deviation (or variance) of return as evaluated by banks is smaller for large firms (for which reliable information is available) than for small firms is a reflection of an important property of the market for bank loans.

The mean of the probability distribution of return shifts upward over time at a rate that depends on the level of loans at the end of the previous period. This elasticity is represented by μ_1 in Equation (43). Like the standard deviation, it is small for large firms and large for small firms. This probably reflects the fact that loans to large firms do not depend so much on the level of past loans outstanding because reliable information is readily available. For small firms, however, banks tend to emphasize the level of past loans in determining the size of current loans.

On the assumption that total lending at the end of each period is in equilibrium, we have used the time series data to estimate the parameters of the probability distribution of return by industry and firm size. In what follows, we use these parameters to derive the supply curves for different categories of loans and then examine

Table 2 The Mean and Standard Deviation of the Subjective Probability Distribution of Return for Manufacturing Firms as Evaluated by Banks (by firm size)

		$\ln x = (\mu_0 + \mu_1 \ln L^i_{-1}) + \sigma z$				$\ln x = (\mu_0 + \mu_1 L^i_{-1}) + \sigma z$			
Manufacturing industry		σ	μ_0	μ_1	R	σ	μ_0	μ_1	R
large firms	equipment funds	0.041108 (3.77)	8.16839 (6.61)	0.274764 (2.57)	0.8889	0.043461 (0.83)	11.0642 (95.30)	0.2709×10^{-5} (2.47)	0.8774
	working funds	0.045422 (1.74)	5.25528 (11.34)	0.599571 (17.63)	0.9862	0.024557 (0.94)	12.7416 (383.9)	0.7531×10^{-6} (19.48)	0.9958
	total	0.042539 (1.59)	5.68989 (11.39)	0.57095 (15.69)	0.9850	0.024841 (0.93)	12.8913 (356.6)	0.6389×10^{-6} (17.03)	0.9947
medium-sized firms	equipment funds	0.752255 (3.05)	−0.248716 (0.26)	1.09062 (11.86)	0.9448	0.76102 (2.82)	9.65812 (100.33)	0.4526×10^{-4} (11.12)	0.9427
	working funds	0.0606735 (0.26)	2.12906 (1.75)	0.77783 (7.83)	0.9979	0.0293173 (0.12)	10.8022 (106.97)	0.3779×10^{-5} (7.94)	0.9995
	total	0.077504 (0.31)	1.54006 (1.12)	0.829773 (7.46)	0.9964	0.048676 (0.19)	10.8949 (96.50)	0.3606×10^{-5} (7.51)	0.9986
small firms	equipment funds	0.379256 (2.46)	−1.41118 (3.17)	1.12029 (27.85)	0.9894	0.67121 (4.37)	10.4227 (191.27)	0.1625×10^{-4} (20.25)	0.9647
	working funds	0.315422 (1.76)	−1.31711 (2.84)	1.08592 (30.66)	0.9945	0.562077 (3.04)	12.2257 (250.06)	0.2192×10^{-5} (24.06)	0.9814
	total	0.315691 (1.82)	−1.40243 (2.99)	1.09157 (30.83)	0.9943	0.585018 (3.30)	12.3924 (250.27)	0.1935×10^{-5} (23.62)	0.9790

Note: When the cumulative distribution F is written as . . . . $F(\frac{\ln x_i - \mu}{\sigma}) = \tilde{y}_i$ its inverse function ϕ^{-1} can be written as $\phi^{-1}(\hat{y}) = \hat{z}$.

Table 3 The Mean and Standard Deviation of the Subjective Probability Distribution of Return for Firms in the Non-Manufacturing Industry as Evaluated by Banks (by firm size)

		$\ln x = (\mu_0 + \mu_1 \ln L^i_{-1}) + \sigma z$				$\ln x = (\mu_0 + \mu_1 L^i_{-1}) + \sigma z$			
		σ	μ_0	μ_1	R	σ	μ_0	μ_1	R
Non-Manufacturing industry									
large firms	equipment funds	0.04004 (5.83)	0.729674 (1.31)	0.942092 (20.02)	0.9671	0.01947 (2.46)	10.8444 (316.52)	0.6793×10^{-5} (29.42)	0.9917
	working funds	0.025799 (4.40)	0.282686 (0.51)	0.97961 (24.46)	0.9803	0.01321 (2.74)	12.7262 (485.15)	0.1027×10^{-5} (39.22)	0.9946
	total	0.02762 (4.73)	0.355737 (0.62)	0.97494 (23.87)	0.9784	0.01405 (2.79)	12.8687 (478.09)	0.8921×10^{-6} (38.10)	0.9942
medium-sized firms	equipment funds	0.340016 (4.31)	−1.72035 (1.46)	1.17367 (11.17)	0.9016	0.380275 (4.60)	10.2462 (84.11)	0.1700×10^{-4} (10.81)	0.8758
	working funds	0.275727 (3.36)	−0.065306 (0.07)	1.01271 (14.05)	0.9481	0.237412 (3.16)	12.2072 (194.35)	0.1835×10^{-5} (16.05)	0.9620
	total	0.283257 (3.49)	−0.309423 (0.29)	1.03196 (13.25)	0.9400	0.257798 (3.35)	12.3623 (175.71)	0.1639×10^{-5} (14.45)	0.9507
small firms	equipment funds	0.482552 (4.70)	−0.92000 (2.06)	1.09408 (32.55)	0.9863	0.290199 (1.79)	12.0175 (300.21)	0.1882×10^{-5} (32.58)	0.9951
	working funds	0.29421 (2.25)	−1.95615 (4.38)	1.14000 (35.11)	0.9942	0.412821 (2.73)	12.7238 (275.44)	0.1193×10^{-5} (27.94)	0.9882
	total	0.36561 (3.18)	−1.67936 (3.69)	1.12537 (35.19)	0.9913	0.372458 (2.55)	13.1352 (310.54)	0.7286×10^{-6} (30.55)	0.9909

the implications.

2. Derivation of the supply curve for loans

The supply curves for different categories of loans can be derived from the estimated parameters of the cost function of bank loans and the corresponding subjective probability distributions of return. When estimating the parameters of the distribution of return in the previous section, we assumed that banks operate on their long-run cost curve at the end of each period. This assumption, however, need not hold in the short run. Given the capital stock K_L^{-1} and primary deposit D_p^{-1} at the end of the previous period, cost increases along the short-run cost curve as lending increases. The estimated short-run cost curve is given by Equation (39) above, which is reproduced here.

$$MC_s = 0.0231126\, e^{-0.0270816t} L^{1.2245951}\, K_L^{-1.2245951}\, D_P^{-0.4043179} w. \tag{39}$$

Over time, marginal cost curve shifts downward as the capital stock K_L expands. An increase in primary deposit also reduces marginal cost. Technical progress in the banking industry carries the same effect. Wage increases work in the opposite direction to offset these cost-reducing effects.

The variable L in Equation (39) denotes the total of all negotiated loan transactions. The position of the supply curve for loans for a particular firm, then, depends on the allocation of loans among other firms. In this sense, negotiated transactions are no longer independent. Here we concentrate to deduce the supply curve for each individual transaction, assuming all other transactions hold constant. Although this does not show the interdependence of the market among each bilateral transactions, it seems to clearly show the sorts of characteristics of the supply behavior of loan to each categorized borrowers.

We continue to divide loans into 12 categories as before. When considering the supply curve for loans that applies to a particular category, we assume that loans belonging to other categories are fixed at their level at the end of the previous period. From Equation (9), we have

$$F_1\{(1+r_{Li}-\theta_i^t)L_i\} = \frac{r_{Li} - \{1-(1-\alpha^t)\cdot k_i^t\} r_M^t - C_L^S\left(\sum_{j=1}^{n\,(j\neq 1)} L_j + L_i\right)}{1+r_{Li}}, \tag{45}$$

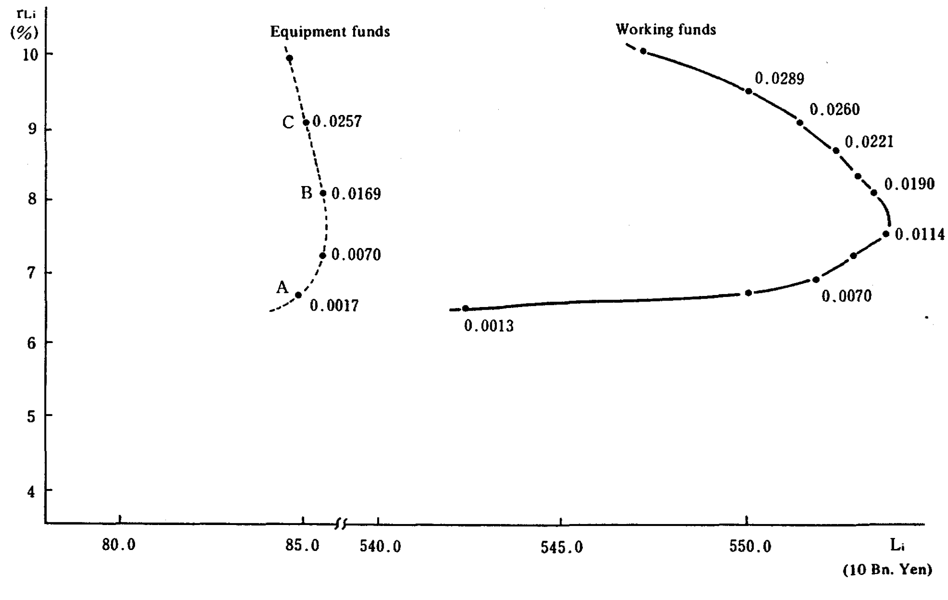
where L_i denotes the loans outstanding of the category under consideration, and $\sum_{j=1}^{n}$ $L_j (j \neq i)$ denotes the sum of all other loans at the end of the previous period level. Given the parameters of the distibution F_i (σ_i, μ_{oi}, and μ_{li}), and C_L^S, and the values of θ_i^t, α_i^t and k_i^t for period t, and with the capital stock K_L^t and primary deposit D_p^t at the beginning of the current period known, the partial equilibrium supply curve for loans corresponding to that in Figure 5 can be derived.

Figure 8 shows the supply curve for loans for equipment funds and working funds for large manufacturing firms in December 1973. The vertical axis shows the loan rate while the horizontal axis shows the size of loans. Since we have ignored the number of firms, the size of loans for working funds exceeds that for equipment funds.

Because the return to equipment funds has relatively small variance, the risk of default rises sharply as the loan rate increases. The supply curve slopes down to the right when the loan rate exceeds 8%. Since the right-hand side of Equation (45) cannot be negative, the lower limit of this supply curve is around 6.5%. The same tendency also characterizes the supply curve for loans for working funds. This curve has positive slope only when the loan rate lies in the 6.5% – 8% range. This range is far narrower than usually expected.

The numbers in the figure show the probability of default for some points along the supply curves. In the case of loans for equipment funds, a loan rate of 6.5% corresponds to a probability of default of 0.17% (point A). The probability of default rises to 1.69% with an 8% loan rate (point B), and further to 2.57% when the loan

Figure 8 Loan Supply Curves for Large Manufacturing Firms (December 1973)

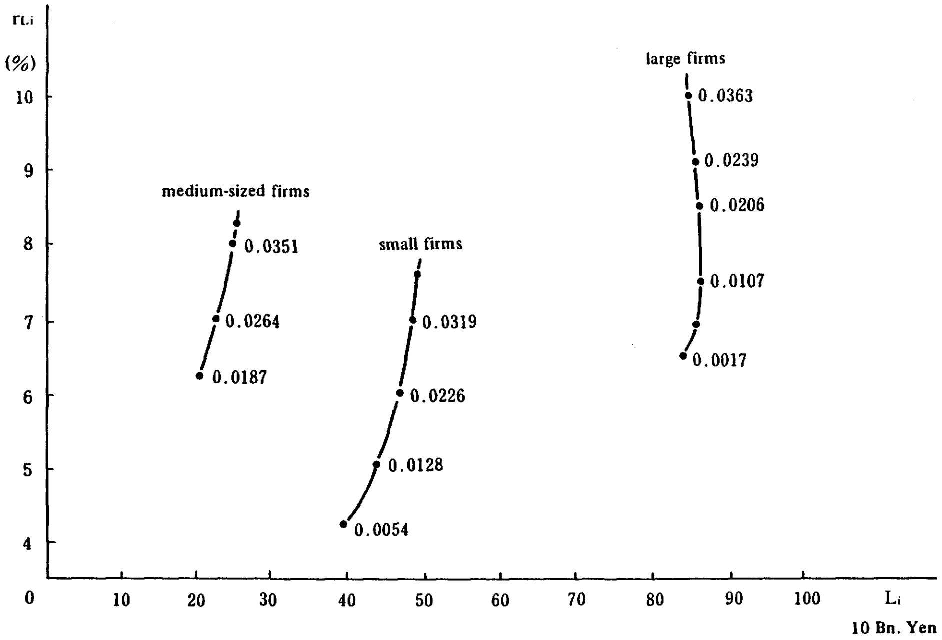


rate reaches 9% (point C). In the case of loans for working funds, the probability of default as evaluated by banks is usually higher, and the figures corresponding to loan rates at A, B, and C are 0.13%, 1.9%, and 2.6%, respectively.

To determine the equilibrium level of loans we also need to know the position of the demand curve for loans. For large firms, given the collateral ratio and the deposit-loan ratio, an increase in the loan rate beyond 8% would reduce the supply of loans due to sharp increase in default risk. Thus, the loan rate facing large firms tends to be rigid and constrained to the 6.5% – 8.0% range. However, this conclusion depends on the partial equilibrium nature of our analysis, and the supply curve itself may shift as other terms of loans such as the collateral ratio and the deposit-loan ratio change.

In Figure 9 we plot the supply curves for equipment funds for manufacturing firms by firm sizes in December 1973. The standard deviation of return differs significantly depending on the size of firms (0.0411108 for large firms, 0.752255 for medium-sized firms, and 0.379256 for small firms). When the standard deviation of return is large, the increase in default risk accompanying an increase in the loan rate is relatively small, so that it is unlikely that the supply curve bends backward. Thus

Figure 9 Supply Curves for Equipment Funds for Manufacturing by Firm Size (December 1973)

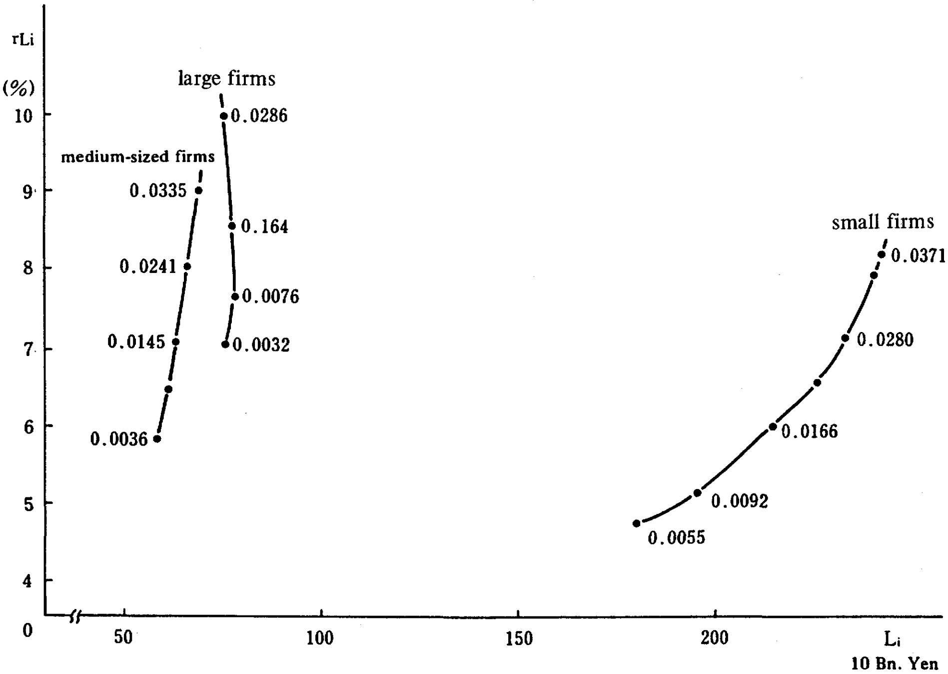


the supply curves of loans for small and medium-sized firms have the normal positive slope over a wide range of loan rates. For any given loan rate, however, the risk of default decreases with firm size. For example, when the loan rate is at the 6.5 percent level, the probability of default is 0.17% for large firms, 1.87% for medium-sized firms and 2.5% for small firms. It shows that banks may prefer to lend to large firms from the viewpoint of reducing default risk.

Supply curves for equipment funds for non-manufacturing firms are shown in Figure 10. The standard deviation of the probability distribution of return is 0.04004 for large firms, 0.340016 for medium firms and 0.482552 for small firms. Reflecting this, the supply curve for large firms is backward bending when the loan rate exceeds 8%. It is upward sloping only when the loan rate lies within the narrow range between 7% and 8%. On the other hand, the supply curves for small and medium-sized firms slope upward over a wide range of loan rates. As in the manufacturing case, the risk of default diminishes with firm size. For example, the probability of default corresponding to a loan rate of 7% is 0.32% for large firms, 1.45% for medium-sized firms, and 2.8% for small firms. Again banks may prefer to lend to large firms out of default risk considerations.

Finally, Figures 11 and 12 show the shift over time of the supply curves for

Figure 10 Supply Curves for Equipment Funds for Non-Manufacturing by Firm Size (December 1973)



equipment funds and working funds for large manufacturing firms. The curves bend back to the left at high loan rates. Since December 1973, growth in capital stock and primary deposits has shifted the supply curves to the right. The lower bound of the supply curve has been changing mainly reflecting the changes in the money market rate. And also the probability of default for any given loan rate has been changing.

Let us interpret the above observations in terms of Equation (45) which implicitly defines the supply curve for loans.

Let $x_i = (1 + r_{Li} - \theta_i)L_i$. Standardizing the probability distribution of $\ln x_i$ to N (0, 1), we have

$$F_i\left\{\frac{\ln x_i-(\mu_{0i}-\mu_{1i}\ln L_i^{-1})}{\sigma_i}\right\}=\frac{r_{Li}-\{1-(1-\alpha)k_i\}r_M-C_L^S(\overset{(j\neq i)}{\Sigma L_j}+L_i)}{1+r_{Li}} \quad . \tag{46}$$

Figure 11 Point-in-time Supply Curves for Equipment Funds for Large Manufacturing Firms

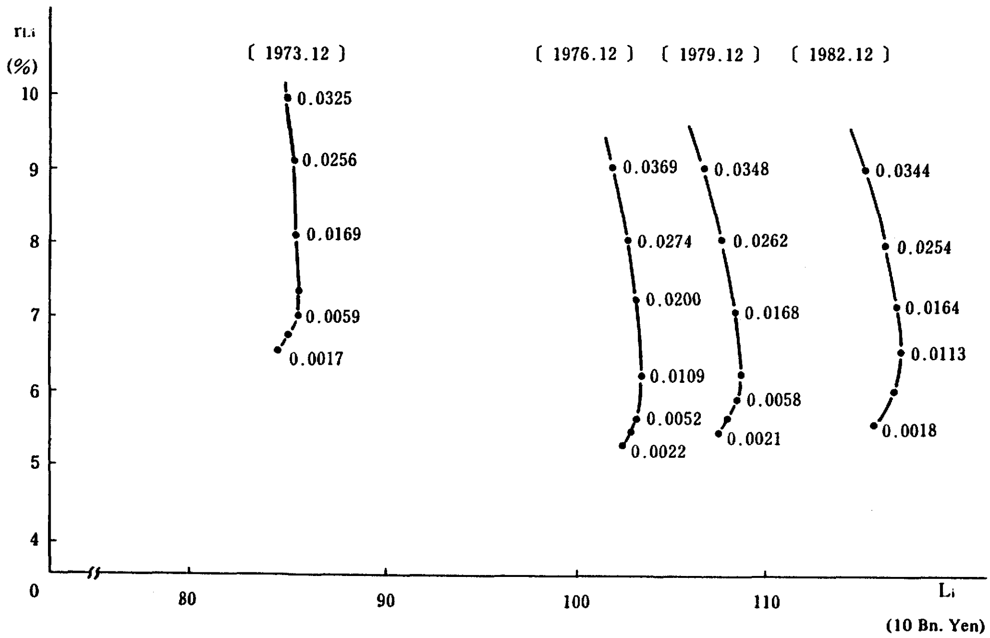
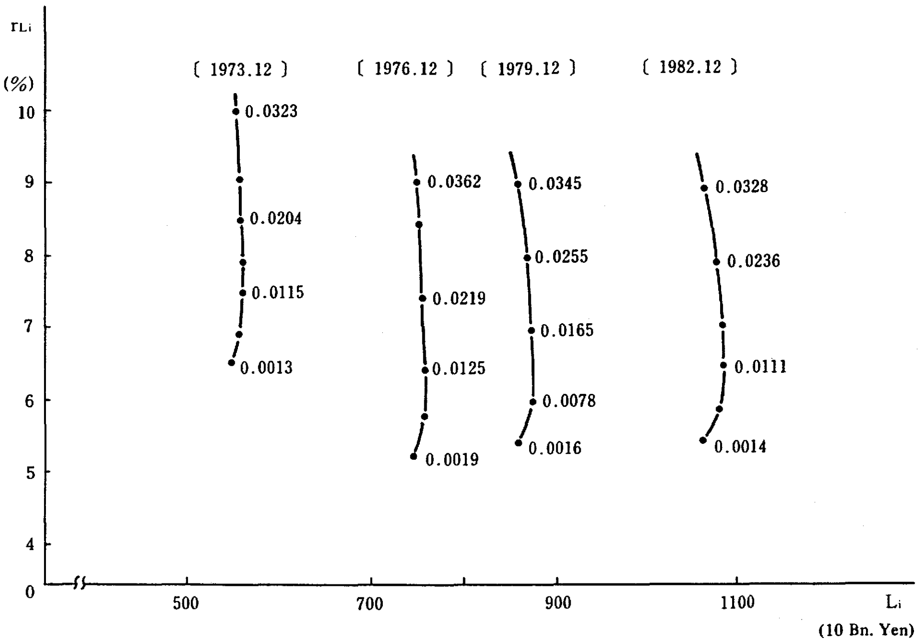


Figure 12 Point-in-time Supply Curves for Working Funds for Large Manufacturing Firms

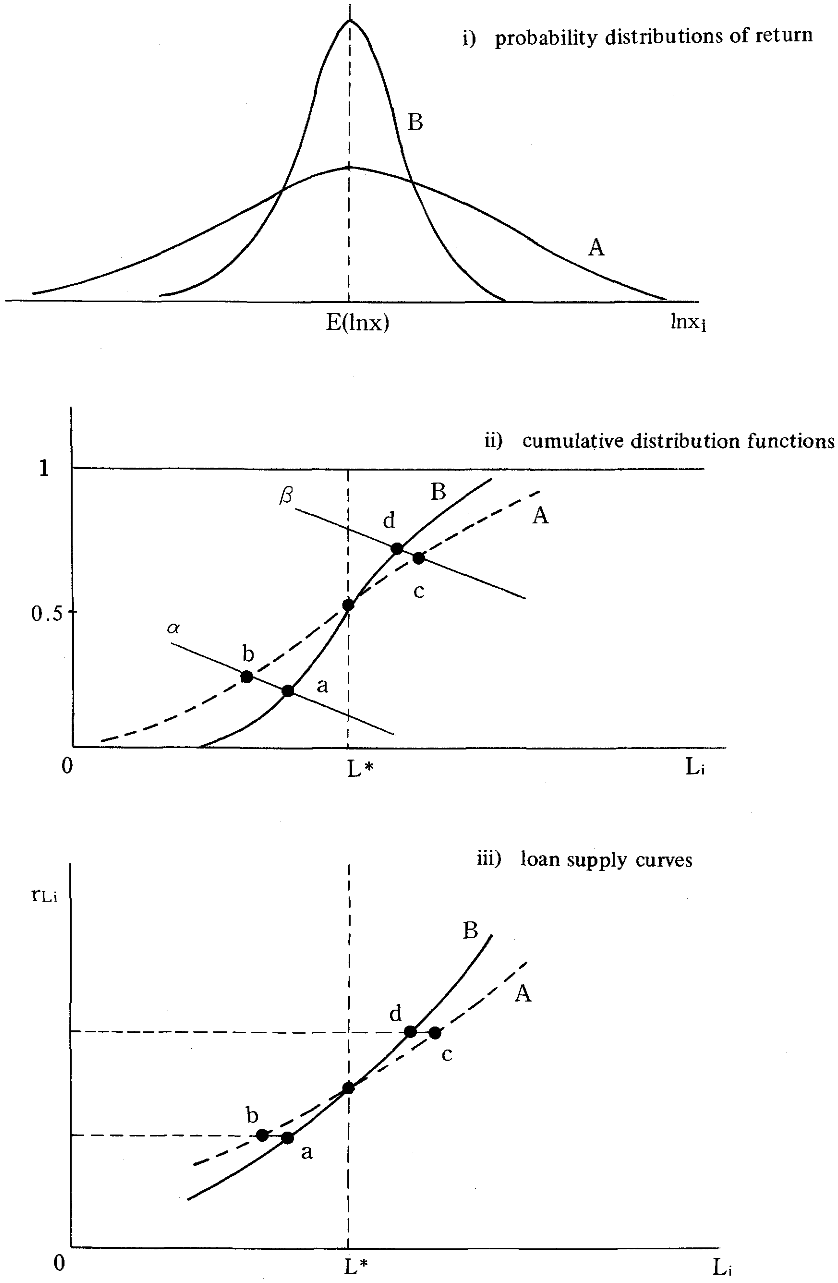


A) The effect of the standard deviation of return

Consider the probability distributions of return of two firms which have the same mean but different standard deviations ($\sigma_A > \sigma_B$), as represented by A and B in i) of Figure 13. The corresponding cumulative distributions are shown as A and B in ii). When the right-hand side of Equation (46) is plotted for given values of the loan rate r_{Li}, we obtain such curves as α and β in ii). To the left of L^*, the supply of loan is smaller for firm A (point b) than firm B (point a), and to the right of L^*, it is larger for firm A (point c) than firm B (point d). This is also clear from iii) which depicts the supply curves for the two firms.

The larger is the standard deviation of return, the larger will be the supply elasticity of loans. Granted that banks operate at low default risk, the area to the left of L^* is relevant. Small and medium-sized firms which have larger standard deviations of return can then borrow less than large firms, even if the means of their return are the same.

Figure 13 Return Distribution and the Shape of the Loan Supply Curve



B) The effect of the collateral ratio

As can be seen from Equation (46), the supply of loans L_i changes with the collateral ratio θ_i. Other things being equal, an increase in the collateral ratio reduces X_i, shifting the supply curve to the right. For a given loan rate, the size of loan expands. If the collateral ratio is endogenously determined in each negotiated loan contract together with the loan rate, this mechanism should also be incorporated into our model. In the partial equilibrium analysis above, the supply curve for equipment funds for large firms is backward bending at high loan rates. When the collateral ratio is endogenous, the possibility that an increase in demand may change the collateral ratio in the short-run and shift the supply curve to the right should also be considered.

C) The effect of the deposit-loan ratio

Given the loan rate, an increase in the deposit-loan ratio k_i increases the right-hand side of Equation (46). As a result, the α line in ii) of Figure 13 shifts upward, and the supply curve shifts to the right. Other things being equal, the volume of loans expands. As in the case of the collateral ratio, it is desirable to treat the deposit-loan ratio as endogenous, and our model should be reformulated accordingly.

D) The effects of the money market rate

An increase in the money market rate r_M reduces the right-hand side of Equation (46). Such change in the opportunity cost of loans has well-known effect in the loan market. It is interesting to note that in our model, since the properties of the probability distributions differ from one borrower to another, the impact on the supply curves also differs for different borrowers, even if the money market is competitive.

VI. Concluding Remarks

Our major findings can be summarized as follows:

1) The cost structures of bank loans is not characterized by linear homogeneity. When total cost is limited to operating expenses, we find significant economies of scale in bank loans.

2) As a result, the marginal cost of each negotiated loan is not constant but depends on the size of total lending. A representative bank's marginal cost of loan to a particular firm depends on the size of its loans to other firms. Contrary to the

assumptions of the Teranishi model or the J-M model, negotiated transactions are not independent of or segmented from one another. Rather, there exists a market for loans where bilaterally negotiated transactions take place interdependently.

3) In the short-run, the supply curve for loans depends on the capital stock and the size of primary deposits. By improving productivity and enlarging the availability of lending funds, growth in capital stock and primary deposits shift the short-run supply curve to the right.

4) The subjective probability distribution of return differ significantly for different types of borrowers. The standard deviation is small for large firms, reflecting the reliability of information. On the other hand, the mean shifts over time positively depending on the level of past loans. This elasticity of this shifts decreases with the level of firms. For small firms, the level of past loans is of particular importance in determining the mean and therefore the supply curve for loans.

5) The partial-equilibrium supply curve for loans to large firms bends backward to the left when the loan rate has risen above a certain level. In the short run, given the collateral ratio and the deposit-loan ratio, the supply curve for large firms slopes upward to the right only when the loan rate lies within the narrow range between 6.5% and 8.0%. For small and medium-sized firms, however, the supply curve slopes upward to the right over a much wider range.

6) When comparing the partial-equilibrium supply curves for different groups of firms, we note that the probability of default corresponding to a given loan rate is smaller for large firms than small firms, and therefore banks may prefer to lend to large firms.

It has become clear from the above observations that, in the short-run, negotiated transactions are not independent. Rather, they are closely related to one another through the level of the marginal cost of loans. Thus, for a bank facing n firms, the order in which loan contracts are negotiated is important for both the bank and the firms in determining the terms. At the present stage, when the demand curve for loans has not been estimated, it is still too early to draw any definite conclusions from our model. At the same time, the relation between primary deposits and the market for loans, as well as the effects of the money market on the loan market, deserve further study in the future.

Appendix. A Re-examination of the Economies of Scale in the Banking Industry

In Section IV we have attempted to measure the economies of scale in the banking industry by limiting the concept of output to loans and that of input to operating expense items. Here we examine the implications for the scale elasticity when input and output are more broadly defined.

On the input side, total cost may be extended to include interest on deposits, CD_s, call money, bills sold, Bank of Japan credit, and borrowings from other financial institutions, in addition to operating expenses considered in the text.

On the other hand, output may be extended to include earning assets such as securities and call loans, in addition to loans narrowly defined.[16]

The composition of the input and output sides may differ from one city bank to another. Assuming separability, the production function can be written as

$$X_i(x_{i1}, x_{i2}, x_{i3}) = Y_i(y_{i1}, y_{i2}, \cdots, y_{i8}) \ ,$$

where $x_{ij}(j = 1, 2, 3)$ denotes the three earning assets of bank i, and $y_{ij}(j = 1, \ldots.., 8)$ denotes the eight items of total cost. There are a variety of methods of aggregating x_{ij} and y_{ij} into x_i and y_i respectively. Here we shall limit ourselves to the Laspeyres index, Passche index, Fisher index, and Divisia index, as in the text of this paper.

As depicted in the text, loans make up 80% of the total of the three earning assets. The 12 city banks are ranked according to the size of loans, and scale elasticities are calculated for each pair of adjacent banks. That is,

$$\tilde{\tilde{k}}_u = \frac{\ln x_i^P - \ln x_j^P}{\ln Q_i^P - \ln Q_j^P} \ ,$$

$$\tilde{\tilde{k}}_\ell = \frac{\ln x_i^L - \ln x_j^L}{\ln Q_i^L - \ln Q_j^L} \ ,$$

$$\tilde{\tilde{k}}_I = \frac{\ln x_i^I - \ln x_j^I}{\ln Q_i^I - \ln Q_j^I} \ ,$$

$$\tilde{\tilde{k}}_D = \frac{\ln x_i^D - \ln x_j^D}{\ln Q_i^D - \ln Q_j^D} \ ,$$

16. For consistency, we have formulated the output of banks as a stock concept. For an alternative formulation in terms of the flow of output, see Royama (1982).

where x_i^P, x_i^L, x_i^I, x_i^D .... correspond to the different quantity indices of output and Q_i^P, Q_i^L, Q_i^I, Q_i^D correspond to the different quantity indices of input. To remove the random disturbances in the scale elasticities, we estimate the following equation.

$$\ln x_i^m = \tilde{\tilde{k}}_m \ln Q_i^m + \tilde{\tilde{k}}_{0m} + u_i \, . \qquad (m = P, L, I, D)$$

The results of estimation are summarized in Table A1. These equations fit well statistically. The coefficients of determination exceed 99% for all periods (September 1973 – September 1983) and all parameters are statistically significant at the 1% level. The estimated scale elasticities are significant and the hypothesis that the production function is homogeneous cannot be rejected.

The estimated scale elasticity corresponding to the Laspeyres quantity indices of input and output is significantly larger than 1 at the 5% or 1% level for all periods from March 1976 to March 1981 (except for September 1979). For all other periods, however, it is not significantly different from one (except for March 1983 when it is significantly smaller than 1.)

On the other hand, the scale elasticity corresponding to the Paasche quantity indices is significantly larger than 1 at the 5% or 1% level for all periods from September 1975 to September 1980 (except for September 1979). For all other periods, (except for September 1973 and September 1982), however, it is not significantly different from 1. In particular, it is smaller than one for September 1982. The scale elasticity K^L is significantly smaller than k^P at the 5% level for 9 out of the 21 periods under study.

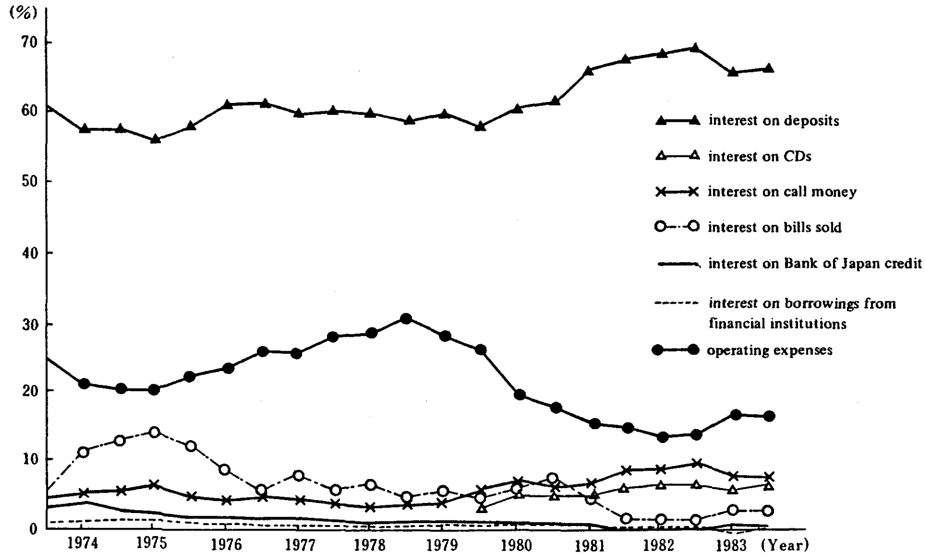
When our sample period is divided into three sub-periods according to September 1973 – September 1974, March 1975 – March 1979, and September 1979 – September 1983, we note that the scale elasticity declines in the first period, rises in the second period and then declines again in the third period. Figure A1 shows the share of various cost items in total cost. During the third period the share of operating expenses had declined significantly, while the shares of interest on deposits, interest on call money, and interest on CD_s had increased significantly.

Reflecting the declining share of operating expenses over time, the estimated scale elasticity exceeds 1.0 significantly when input is limited to operating expenses items, as we have seen in the text. However, when the definition of input is extended to include other items, as we do in this appendix, the scale elasticity for city banks is estimated to be around 1.02 – 1.05. With financial liberalization progressing rapidly, the estimated scale elasticity seldom exceeds 1.0 significantly in recent years.

Table A1 Estimation of the Scale Elasticity of City Banks Using Input and Output Indices

	Laspeyres Index			Paasche Index			Fisher Index			Divisia Index		
	C	LNQL	R^2	C	LNQP	R^2	C	LNQI	R^2	C	LNQD	R^2
1973 - 9	0.009678 (0.5904)	1.01750 (60.72)	0.997025	0.006530 (0.3825)	1.03736 (58.31)	0.996774	0.008069 (0.4867)	1.02738 (60.04)	0.996957	0.007940 (0.4172)	1.02986 (52.34)	0.996000
1974 - 3	0.015324 (0.7609)	0.99737 (49.20)	0.995474	0.011096 (0.5349)	1.02429 (47.92)	0.995230	0.013211 (0.6485)	1.01068 (48.71)	0.995804	0.011063 (0.4297)	1.02834 (38.61)	0.992671
1974 - 9	0.034973 (1.8686)	1.00367 (51.75)	0.995907	0.046936 (2.0859)	0.99196 (42.50)	0.993944	0.040570 (2.0702)	0.99828 (49.13)	0.995461	0.021566 (0.3507)	1.09077 (15.87)	0.957972
1975 - 3	0.025236 (1.3289)	1.00296 (50.89)	0.995769	0.011603 (0.6066)	1.03828 (51.33)	0.995840	0.018472 (0.9786)	1.02042 (51.60)	0.995884	0.016701 (0.6692)	1.04030 (39.09)	0.992848
1975 - 9	0.015305 (0.9938)	1.01505 (62.69)	0.997008	0.001285 (0.0768)	1.05509 (58.56)	0.996801	0.008347 (0.5296)	1.03480 (61.70)	0.997118	0.008246 (0.4627)	1.04137 (54.55)	0.996316
1976 - 3	0.002422 (0.1626)	1.03482 (65.40)	0.997434	−0.005646 (−0.3534)	1.05192 (61.58)	0.997107	−0.001618 (−0.1058)	1.04336 (64.01)	0.997322	−0.034108 (−0.2183)	1.04798 (62.74)	0.997213
1976 - 9	−0.004742 (−0.3265)	1.04093 (66.58)	0.997524	−0.012101 (−0.7641)	1.05599 (61.51)	0.997100	−0.008424 (−0.5586)	1.04844 (64.36)	0.997351	−0.009298 (−0.6078)	1.05028 (63.48)	0.997277
1977 - 3	−0.012481 (−0.8740)	1.03473 (67.50)	0.997591	−0.021016 (−1.3021)	1.05974 (60.12)	0.996965	−0.016739 (−1.1115)	1.04713 (64.22)	0.997339	−0.016916 (−1.1026)	1.04578 (63.03)	0.997238
1977 - 9	−0.026556 (−1.7244)	1.04620 (62.25)	0.997168	−0.028997 (−1.7210)	1.06354 (56.92)	0.996615	−0.027781 (−1.7277)	1.05481 (59.63)	0.996915	−0.026909 (−1.6745)	1.05208 (59.59)	0.996911
1978 - 3	−0.026120 (−1.9557)	1.04386 (70.58)	0.997796	−0.027629 (−1.8374)	1.05239 (62.57)	0.997198	−0.025893 (−1.9023)	1.04813 (66.62)	0.997527	−0.026002 (−1.8498)	1.04442 (66.87)	0.997545
1978 - 9	−0.030827 (−2.0157)	1.04969 (60.70)	0.997023	−0.030431 (−1.8973)	1.05136 (57.88)	0.996727	−0.030663 (−1.9710)	1.05057 (59.68)	0.996920	−0.026950 (−1.7924)	1.05117 (61.52)	0.997107
1979 - 3	−0.021897 (−1.4002)	1.05770 (58.14)	0.996756	−0.015435 (−1.0768)	1.03441 (62.87)	0.997224	−0.018832 (−1.3159)	1.04621 (63.27)	0.997259	−0.011261 (−0.7874)	1.05143 (62.84)	0.997221
1979 - 9	−0.014872 (−1.0193)	1.03109 (63.46)	0.997276	−0.006399 (−0.4567)	1.02424 (64.63)	0.997370	−0.010719 (−0.7666)	1.02780 (65.49)	0.997441	−0.000720 (−0.0491)	1.01801 (61.88)	0.997135
1980 - 3	−0.007230 (−0.4825)	1.03885 (61.31)	0.997082	−0.002242 (−0.1801)	1.02539 (73.17)	0.997952	−0.004821 (−0.3593)	1.03222 (68.20)	0.997640	0.010788 (0.74193)	1.02136 (61.75)	0.997122
1980 - 9	0.019408 (1.2742)	1.02257 (58.06)	0.996747	0.023600 (1.6341)	1.03377 (61.41)	0.997091	0.021497 (1.4513)	1.02815 (59.79)	0.996931	0.027295 (1.4221)	1.03588 (45.74)	0.994767
1981 - 3	0.030094 (2.6741)	1.02975 (77.48)	0.998171	0.024655 (2.4860)	1.01277 (89.21)	0.997662	0.027334 (2.6001)	1.02117 (83.57)	0.998427	0.046767 (3.3355)	1.03463 (62.39)	0.997181
1981 - 9	0.018163 (1.1036)	1.02106 (53.27)	0.996137	0.011069 (0.8125)	0.97334 (65.21)	0.997423	0.014425 (0.9744)	0.99670 (59.63)	0.996915	0.035545 (2.1562)	1.00633 (52.79)	0.996067
1982 - 3	0.017148 (1.1791)	1.01232 (62.35)	0.997178	0.009071 (0.6857)	0.97228 (68.90)	0.997691	0.012968 (0.9465)	0.99199 (66.36)	0.997508	0.039580 (2.1995)	0.99244 (49.45)	0.995521
1982 - 9	−0.004830 (−0.3321)	1.00053 (63.36)	0.997266	−0.009220 (−0.6998)	0.96548 (70.20)	0.997774	−0.007105 (−0.5186)	0.98274 (67.38)	0.997582	0.014879 (0.9456)	1.00473 (57.89)	0.996727
1983 - 3	−0.029691 (−2.3911)	0.95537 (72.76)	0.997926	−0.013025 (−1.0324)	1.02233 (70.51)	0.997290	−0.021753 (−1.8210)	0.98787 (75.08)	0.998052	−0.018223 (−0.9930)	0.99113 (48.74)	0.995389
1983 - 9	0.004510 (0.3767)	0.98555 (73.74)	0.997981	0.003988 (0.3146)	0.98929 (69.74)	0.997741	0.004244 (0.3450)	0.98742 (71.81)	0.997871	0.006751 (0.5658)	0.98139 (73.61)	0.997973

Figure A1 Share of Various Cost Items in Total Cost



REFERENCES

[1] Diewert, W.Erwin, "Exact and Superative Index Numbers," *Journal of Econometrics,* Vol. 4, 1976.

[2] Frisch, R., *Theory of Production,* 1965.

[3] Iwata, Kazumasa and Hamada, Kouichi, *Kinyu-seisaku to Ginko Kodo* (*Monetary Policy and Bank Behavior*), Toyo-Keizai-Shimposha, 1980.

[4] Jaffee, Dwight M., *Credit Rationing and the Commercial Loan Market,* John Wiley & Sons, 1971.

[5] Jaffee, Dwight M. and Modigliani, Franco, "A Theory and Test of Credit Rationing," *American Economic Review,* Vol. 59, No. 5, December 1969.

[6] Kaizuka, Keimei and Onodera, H., "Shinyo-wariate ni tsuite" ("On Credit Rationing,") *Keizai Kenkyu,* Vol. 25, No. 1 Jan. 1974.

[7] Royama, Shouichi, *Nihon no Kinyu Shisutemu* (*The Financial System of Japan*), Chapter 8, Toyo-Keizai Shimposha, 1982.

[8] Suzuki, Yoshio, *Kinyu-seisaku no Koka* (*The Effects of Monetary Policy*), Toyo-Keizai Shimposha, 1966.

[9] Tachi, Ryuichiro, *Kinyu-seisaku no Riron* (*The Theory of Monetary Policy*), Tokyo University Press, 1982.

[10] Teranishi, Juro, *Nihon no Keizai-hatten to Kinyu* (*Economic Development and Finance in Japan*), Chapter 9, Iwanami Shoten, 1982.

[11] Tobin, James, "The Commercial Banking Firm: A Simple Model," *Scandinavian Journal of Economics,* Vol. 84, No. 4, 1982.

[12] Yoshioka, Kanji, "Wagakuni Kogyo-tokei ni motozuku Kibo-dansei no Suitei ("An Estimation of Scale Elasticity Based on the Industrial Statistics of Japan"), *Keizai Tokei Kenkyu,* Vol. 15, 1977.

[13] ———, "A Method for Measuring the Scale Elasticity of Production: An Empirical Study of Japanese Manufacturing Industries from 1964 to 1972," Keio Economic Studies, Vol. 15, No. 1, 1979.

[14] ———, "A Measurement of Return to Scale in Production: A Cross-section Analysis of the Japanese Two-digit Manufacturing Industries from 1964 to 1978," *Keio Economic Observatory Discussion Paper,* December 1982.

[15] ———, "Seisan ni okeru Kibo no Keizaisei no Sokuteiho— Shisuronteki Sekkin ni yoru Hohoron" ("The Methodology of Estimating the Economies of Scale in Production—the Index Number Theory Approach",) *Mita Shogaku Kenkyu,* Vol. 27, No. 1, April 1984.