Expectations Theory and Term Structure of Interest Rates*

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I. Introduction

The purpose of this paper is to provide a certain degree of theoretical insight into the term structure of interest rates, and to examine its validity in Japan. It is common practice to distinguish between theories on the term structure of interest rates by representing them in the form of two alternative hypotheses: (a) expectations theory that emphasizes market expectations on future interest rates; and (b) the market segmentation hypothesis that stresses the supply and demand conditions of each respective market. When tracing the development of theories on the term structure of interest rates, one will discover that the most influential works had been conducted during mid- 1960s and early 1970s. Modigliani and Sutch (1966, 1967), Modigliani and Shiller (1973), and Shiller (1972) have examined the movement of U.S. interest rates in the postwar period and obtained results favorable to expectations theory. These results drew considerable attention not only from scholars in general but also from policy-makers, since the implication of expectations theory is that long-term interest rates are essentially the averages of the expected short-term interest rates in the future and cannot be changed as long as the market expectations of future interest rates remain constant. These arguments have especially challengd the effectiveness of the so-called "operation twist" policy of the Kennedy era and forced the reconsideration of debt management policy. Kuroda and Okubo (1981, 1982) exam-

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ined the term structure of interest rates in Japan by applying the expectations theory in the style of Modigliani and Shiller. They concluded that expectations theory has high-level explanatory power in Japan as well.

Thus, in American academic circles, the dominant view until mid-1970s was that the term structure of interest rates was well-described by expectations theory. However, such a view on the term structure of interest rates underwent dramatic change around the late 1970s, as Shiller (1979) and Singleton (1980) provided new insight regarding this particular theory. They examined expectations theory using the technique of the variance bounds test and rejected it empirically. Their results were accepted, owing to the fact that short-term interest rates have increased in terms of their volatility and that long-term interest rates remained extremely high in spite of the fact that inflation had been curbed. Intensive efforts are now being made to elaborate this expectations theory. Above all, among the recent arguments, the role of risks involved are emphasized in the determination of the term structure of interest rates.

In this paper, we shall summarize recent discussions on the theory of the term structure of interest rates by presenting sufficient conditions for expectations theory to hold, and then re-examine its validity in Japan. In the following, expectations theory is analyzed in terms of its relationship to the capital asset pricing model in Section II, and, in Section III, the optimizing behavior of investors is further investigated to derive sufficient conditions for the theory to hold. Lastly, in Section IV, the validity of expectations theory in Japan is empirically tested.

The main conclusions of the paper are as follows:

Theoretically, expectations theory can be roughly divided into pure expectations theory (unbiased expectations theory) and expectations theory with risk premium (risk premium hypothesis), the former of which can be considered as a special case (with zero risk premium) of the latter. In addition, reflecting different assumptions on the time series behavior of risk premium, the risk premium hypothesis can be further sub-divided into (a) the constant risk premium hypothesis, such as the liquidity premium hypothesis of Hicks; and (b) the time-varying risk premium hypothesis. The former hypothesis again can be regarded as a special case (in which the risk premium does not change over time) of the latter. Thus, in general, we can assume from expectations theory that long-term interest rates are determined by expectations of future interest rates and the risk premium that changes over time.

By analyzing the optimizing behavior of investors in a stochastic environment, we examined the sufficient conditions for expectations theory, especially unbiased expectations theory, to hold. It is shown that two conditions must be satisfied for unbiased expectations theory to hold; investors are risk-neutral, and price level movements can be perfectly predicted. Although these two conditions are too strict to be fulfilled in general, unbiased expectations theory approximately holds when investors are nearly-risk-neutral and roughly able to predict future price level movements.

We examined empirically unbiased expectations theory and the constant risk premium hypothesis by using data on interest rates in Japan. Between yields on government bonds and the gensaki rate, both hypotheses cannot be rejected in the whole period of April 1977 to June 1984, but are rejected during the recent period of October 1981 to June 1984. We observe that interest rate arbitrage transactions were not carried out smoothly until the mid-1980 reflecting the restrictions on transactions of money market instruments. This indicates that the recent period is more suitable as a sample. Taking this fact into consideration, then, we can reject both hypotheses. Thus, we can conclude that the risk premium that changes over time does exist and plays an important role in the determination of term structure between yields on government bonds and the gensaki rate. On the other hand, between gensaki rates of different maturities, while unbiased expectations theory is rejected in both periods, the constant risk premium hypothesis is not rejected for both periods; the term structure between gensaki rates are well described by the constant risk premium hypothesis.

II. Expectations Theory and Capital Asset Pricing Model

A. Expectations Theory

Expectations theory on the term structure of interest rates explains the movements of long-term interest rates by market expectations for future short-term interest rates. It means that, in order to maximize the expected discount value of a certain asset, investors try to choose an optimum portfolio among the alternatives: (a) purchasing and consecutive holding of bonds whose maturity equals their planned investment period; (b) rolling-over of short-term instruments; and (c) purchasing of longer-term bonds and selling of such bonds before their maturity. In expectations theory, expectations regarding future interest rates are supposed to play a significant role in the determination of their portfolio and eventually the term structure of interest rates. Depending on the explanatory power of the expectations factors assumed, the theory can be classified into the so-called unbiased expectations theory and the risk premium hypothesis. Unbiased expectations theory maintains that long-term interest rates are perfectly determined by market expectations for future short-term interest rates, whose first advocator can be traced back to Fisher (1930). On the other hand, the risk premium hypothesis assumes that, in addition to expected future short-term interest rates, the risk premium, as well, plays an important role in the determination of long-term interest rates. This latter hypothesis can be further divided into two

Table 1 Classification of Expectations Theory

- Expectations Theory
  - Unbiased Expectations Theory
  - Risk Premium Hypothesis
    - Constant Risk Premium Hypothesis
    - Time-varying Risk Premium Hypothesis

hypotheses: (a) the constant risk premium hypothesis, such as the liquidity premium hypothesis of Hicks (1939); and (b) the time-varying risk premium hypothesis with the risk premium that changes over time as well as terms to maturity. There exist a variety of risk premia such as a liquidity premium that compensates for the risk incurred by purchases of long-term bonds, and, a term premium created by the mismatching of the planned investment period and the actual maturity of bonds purchased. In this paper, the risk premium refers to all these varying risk premia.

Expectations theory, therefore, can be classified as in the Table 1 depending on the assumed significance of the expectations factors and the time series behavior of the risk premium. In what follows, I will focus on the most general hypothesis, the time-varying risk premium hypothesis.

In the time-varying risk premium hypothesis, as a result of the interest rate arbitrage, the expected period holding rate of return of an n-period bond at time t (that is, coupon income plus expected capital gains or losses) equals the sum of the short-term interest rate corresponding to their holding period and the time-varying risk premium:

$$E_t[H_{n,t}]=R_{1,t}+\phi_{n,t}, \quad n=2,3,\cdots,N \tag{1}$$

where

$$H_{n,t}=\frac{P_{n-1,t+1}-P_{n,t}+C\cdot M_v}{P_{n,t}} \tag{2}$$

$H_{n,t}$: period holding rate of return of an n-period bond
$R_{1,t}$: short-term interest rate
$\phi_{n,t}$: risk premium

$P_{n,t}$: price of an n-period bond
C : coupon rate
M_v : redemption price .

$E_t[\ ]$ indicates $E_t[\cdot|\Omega_t]$, market expectations formed rationally on the basis of available information at time t, Ω_t.

Based on the time-varying risk premium hypothesis shown in equation (1), the unbiased expectations theory and constant risk premium hypothesis will be examined. Since the risk premium is assumed to be zero in unbiased expectations theory, we have:

$$E_t[H_{n,t}]=R_{1,t}, \quad n=2,3,\cdots,N. \tag{1)'$$

That is, in unbiased expectations theory, the expected period holding rate of return of an n-period bond is equal to the actual short-term interest rate regardless of its maturity. It should be noted here that the risk-neutrality of investors in holding bonds must be in fact a prerequisite for unbiased expectations theory to hold. On the other hand, in the constant risk premium hypothesis, it is assumed that risk premium does not change over time ($\phi_{n,t}=\phi_n$).

$$E_t[H_{n,t}]=R_{1,t}+\phi_n, \quad n=2,3,\cdots, N \tag{1)''$$

It is believed that the longer the maturity of a bond is, the larger is the risk premium, thus the longer the maturity, the higher becomes the expected period holding rate of return.[1]

$$E_t[H_{N,t}]>E_t[H_{N-1,t}]>\cdots>E_t[H_{1,t}]=R_{1,t} \tag{3}$$

Of course there is no need to mention that the constant risk premium hypothesis assumes that investors are risk-averse.

Nextly, we will formulate the term structure equation, based on the time-varying risk premium hypothesis. Letting the yield on an n-period bond be $R_{n,t}$, the price of such a bond is given by:

1. As Modigliani and Sutch (1966) pointed out, it should be noted that, if investors had a preferred habitat of holding bonds with a specified maturity, the risk premium could not be described by an increasing function of maturities; that is, the risk premium can be represented by a convex function whose bottom falls to such a specified maturity.

$$P_{n,t}=\sum_{j=1}^{n}\frac{1}{(1+R_{n,t})^{j}}C\cdot M_{v}+\frac{1}{(1+R_{n,t})^{n}}M_{v} \tag{4}$$

which is rewritten as:

$$P_{n,t}=\frac{C\cdot M_{v}}{R_{n,t}}+\frac{1}{(1+R_{n,t})^{n}}(M_{v}-\frac{C\cdot M_{v}}{R_{n,t}}). \tag{4$'$}$$

As Shiller (1979) pointed out, using equation (4)′, one can arrive at a linearized version of the expected period holding rate of return by taking a Taylor expansion truncated after the linear term.[2]

$$R_{n,t}=\gamma_{n}E_{t}[R_{n-1,\ t+1}]+(1-\gamma_{n})(R_{1,t}+\phi_{n,t}) \tag{5}$$

where

$$\gamma_{n}=\gamma(1-\gamma^{n-1})/(1-\gamma)$$
$$\gamma=1/(1+\bar{R})$$

$\bar{R}$: average yield of an n-period bond
γ : discount rate implied by the average yield of an n-period bond

Equation (5) is a first order difference equation and its recursive solution provides a well-known "term structure equation," which shows the long-term interest rate as the sum of the expected short-term interest rates and risk premium.[3]

$$R_{n,t}=\frac{1-\gamma}{1-\gamma^{n}}\sum_{k=0}^{n-1}\gamma^{k}E_{t}[R_{1,t+k}]+\theta_{n,t}\ ,\ 0<\gamma<1\ ,\ \theta_{n,t}=\frac{1-\gamma}{1-\gamma^{n}}\sum_{k=0}^{n-1}\gamma^{k}E_{t}[\phi_{n-k,t+k}] \tag{6}$$

That is, the yield on an n-period bond can be represented as the sum of the weighted average (whose total weight equals one) of a sequence of expected short-term interest rates, and risk premium, $\theta_{n,t}$. This risk premium is the weighted average of risk premia $\phi_{n-k,\ t+k}$ $k=0,..\ n-1$, which is assumed to arise between time t to time $t+n-1$ by a consecutive holding of such an n-period bond, and, in general, changes over

2. This expected period holding rate of return gives an appropriate approximation around the par value provided that the yield on bonds and coupon become equal. However, when the bond price deviates significantly from the par value, the approximation error is likely to become large.

3. For the derivation of the term structure equation, see Shiller (1979).

time.

B. Risk Premium and Capital Asset Pricing Model

In the former sub-section II. A., expectations theory was summarized and the term structure equation was derived. In this sub-section, I will analyze the characteristics of the risk premium by using the capital asset pricing model (CAPM, hereafter), and then show that the risk premium of an n-period bond can be considered as a multiplication of the average risk of capital assets markets and the relative degree of risk, well-known as a beta coefficient, incurred by holding such a bond.

CAPM, developed by Sharpe (1964), Lintner (1965) etc., is a theory of risky asset holding and explores cross-sectional relationships among yields on various assets with different risks.[4] The basic concept of this model is that the yield on a risky asset, such as a bond, can be described as the sum of the yield on a safe asset plus the relative risk accompanying the holding of such an asset. That is;

$$E_t[R_{i,t}]=R_{f,t}+\beta_{i,t}\{E_t[R_{m,t}]-R_{f,t}\} \tag{7}$$

where

$R_{i,t}$: yield on risky asset i
$R_{f,t}$: yield on safe asset f
$R_{m,t}$: average yield in capital assets markets
$\beta_{i,t}$: relative risk of risky asset i.

Comparing this equation with the above-mentioned risk premium hypothesis shown in (1), we notice that $R_{i,t}$ corresponds to $H_{n,t}$ and $R_{f,t}$ to $R_{1,t}$ in this order; as a result, the risk premium, $\phi_{n,t}$, is given by:

$$\phi_{n,t}=\beta_{i,t}\{E_t[R_{m,t}]-R_{f,t}\}. \tag{8}$$

Thus, by applying CAPM, we can describe the risk premium, $\phi_{n,t}$, as a multiplication of the average risk of capital assets markets (shown by the difference between the average expected rate of return in capital assets markets and the yield on a safe asset), and the relative degree of risk incurred by the holding of an n-period risky asset. Denoting the relative degree of risk as $\beta_{n,t}$, and if $\beta_{n,t} > 1$, then the risk of holding an n-period bond is greater than the average risk of capital assets markets. On the contrary, if $\beta_{n,t} < 1$, we arrive at a risk which is smaller than average.

4. For details of CAPM, see Fama (1976).

III. Optimizing Behavior of Investors and Expectations Theory

In Section II, it was shown that long-term interest rates are determined by expected short-term interest rates and the time-varying risk premium and that, in CAPM, the risk premium is described as a multiplication of the average risk of capital assets markets and the relative degree of risk. However, we note that such an argument can basically only be true in an equilibrium market, and that the argument disregards how such underlying factors as investors' attitudes toward risks and economic environments, interact with expectations theory. In this section, I will re-examine expectations theory by analyzing the optimizing behavior of investors in a stochastic environment, and the sufficient conditions for it to hold will be derived.

A. Optimizing Behavior of Investors

In order to analyze the behavior of investors, we assume:

(i) There exist nonstorable goods and n kinds of assets with different coupons in an economy. The investors' income Y_t is stochastically given, and the state of this economy at time t is characterized by this income.

(ii) Investors try to maximize their life-time expected utility by predicting future states of the economy on the basis of information currently available.

(iii) The market is competitive, and there are no taxes, transaction costs or other forms of friction.

Then, the optimization problem of investors in such an economy can be stated as follows.[5]

$$\max_{\{x_t\}\{A_j(t)\}} \quad E_t\left[\sum_{t=0}^{\infty} \delta^t u(x_t)\right]$$

subject to

$$P(t)x_t+\sum_{j=1}^{n} P_j(t)A_j(t)=P(t)Y_t+\sum_{j=1}^{n}(P_j(t)+C_j)A_j(t-1)$$

$$t=0,\cdots,\infty \quad 0<\delta<1,\ u'>0,\ u''<0,\ \text{given } A_j(-1)$$

where

δ : time preference rate
x_t : consumption
$P(t)$: price-level
$A_j(t)$: amounts purchased of asset j

5. For such arguments, see Lucas (1978), and Donaldson and Mehra (1984).

$P_j(t)$: price of asset j
C_j : coupon of asset j
Y_t : investors' income other than assets.

That is, investors, having a stochastically drawn income, attempt to allocate their total income to consumption and asset holdings so as to maximize their expected utility. The first order necessary condition for this problem can be given as:[6]

$$P_j(t)\frac{u'(x_t)}{P(t)} = E_t\left[\frac{\delta u'(x_{t+1})}{P(t+1)}\{P_j(t+1)+C_j\}\right],\quad j=1,\cdots,n. \tag{9}$$

This means that the utility lost by forgoing consumption to purchase an asset j at time t, at the margin, should be equal to the expected utility to be gained at time t+1 by selling that asset and consuming its proceeds.

We will consider how the price of asset j is determined, using this first order condition of equation (9). At first, since $u'(x_t)/P(t)$ is known at time t, we can divide both sides of equation (9) by $u'(x_t)/P(t)$, which gives:

$$P_j(t)=E_t[\Phi(t+1)\{P_j(t+1)+C_j\}],\qquad j=1,\cdots,n$$

where (10)

$$\Phi(t+1)=\frac{\delta u'(x_{t+1})}{u'(x_t)}\cdot\frac{P(t)}{P(t+1)}.$$

Thus, alternatively, the price of asset j is determined so as to be equal to the expected discount value of the proceeds gained by purchasing asset j at time t and selling it at time t+1, discounted by one period random discount factor $\Phi(t+1)$. By definition, the one period random discount factor $\Phi(t+1)$ is a combination of the intertemporal marginal rate of substitution between time t consumption and time t+1 consumption, and the expected rate of inflation, provided that the time preference rate of investors remains constant. In particular, if utility function $u(x_t)$ is a class of a constant relative risk aversion, $u'(x_t) = x_t^{-\alpha}$, then $u'(x_{t+1})/u'(x_t)$ depends on the (expected) rate of change of consumption accompanying intertemporal change of an economic state and the degree of relative risk aversion. That is, $u'(x_{t+1})/u'(x_t) = (x_t/x_{t+1})^{\alpha}$. Thus the price of asset j can be determined by: (a) expectations of the future economy given by the (expected) rate of change in consumption; (b) investors' degree of risk aversion;

6. Though it is not explicitly treated here, such an optimization problem can be solved by dynamic programming as was conducted by Lucas (1978).

and (c) expected rate of inflation. As is clear from the definition of the one period random discount factor $\Phi(t+1)$, the expected rate of inflation in (c) and the price of an asset j, $P_j(t)$, are negatively correlated. Although the relationship betwen (a), (b) and $P_j(t)$ is somewhat complicated, it can be summarized as follows:

(i) When investors are risk-neutral ($\alpha = 0$)
In this case, since $u'(x_{t+1})/u'(x_t) = 1$ regardless of expected future states of the economy, the one period random discount factor is degenerated into $\delta P_{(t)}/P_{(t+1)}$. Thus, the price of asset j, $P_j(t)$, is equal to the expected discount value of the proceed at time t+1 obtained by selling that asset, discounted by the time preference rate times the inverse of the expected rate of inflation.

(ii) When investors are risk-averse ($\alpha > 0$)
The implication for the price movement depends on the investors' expectations regarding the future states of the economy. When investors anticipate the economy to follow an upward trend ($x_t/x_{t+1} < 1$), the expected marginal rate of substitution will be smaller than one; thus, if the expected rate of inflation is assumed to be given, the more risk-averse the investors are, the smaller value $\Phi(t+1)$ takes and the lower the price of asset j becomes. On the contrary, when the economy is deemed to follow a downward trend ($x_t/x_{t+1} > 1$), $u'(x_{t+1})/u'(x_t)$ becomes larger than one, and the more risk-averse investors are, the higher the price of asset j becomes.

B. Bond Prices and Yield on Bonds

We will shed light on how the bond price, especially a fixed-coupon bond price, is determined in terms of asset pricing theory. The price of an n-period bond, $P_{n,t}$, which has a fixed redemption price (M_v) and a coupon rate (C), is given by substituting $P_j(t+n)=M_v$, $C_j=C \cdot M_v$ into equation (10). Solving this result from time t+n to time t gives:

$$P_{n,t}=E_t\left[\sum_{k=1}^{n}\prod_{v=1}^{k}\Phi(t+v)\right]C\cdot M_v+E_t\left[\prod_{v=1}^{n}\Phi(t+v)\right]M_v \tag{11}$$

or

$$P_{n,t}=E_t\left[\sum_{k=1}^{n}S^{(k)}(t)\right]C\cdot M_v+E_t\left[S^{(n)}(t)\right]M_v$$

where (11)'

$$S^{(k)}(t)=\frac{\delta^k u'(x_{t+k})}{u'(x_t)}\cdot\frac{P(t)}{P(t+k)}\ .$$

Therefore, the price of an n-period bond equals the expected discount value of total coupon payments plus the redemption price, divided by a sequence of the one period random discount factor. As was already mentioned, we notice that the price of an n-period bond is determined by the following three factors: (a) expected future states of the economy; (b) investors' degree of risk aversion; and (c) expected rate of inflation.

On the other hand, the relationship between interest rates and the one period random discount factor is obtained by substituting (4)′ into (11)′ and rewriting the results with $(1+R_{n,t})^n \doteqdot 1+nR_{n,t}$

$$R_{n,t}=\frac{1}{n}\left\{\frac{1+nC}{E_t[\sum_{k=1}^{n} S^{(k)}(t)]\,C+E_t[S^{(n)}(t)]}-1\right\} \tag{12}$$

This indicates that $R_{n,t}$ is negatively correlated with the random discount factor, and the relationship of these two factors can be summarized as follows:

Firstly, as is clear from the definition of the random discount factor $S^{(k)}(t)$, ceteris paribus, the higher the expected rate of inflation, the higher the yield on bonds becomes.

Secondly, if an economy is anticipated to follow an upward trend such as during an upswing, the bond price falls and the yield increases. And provided that the expected rate of inflation is assumed to be positive, we generally have, from equation (12), a positively sloped yield curve, since the random discount factor $S^{(k)}(t)$ is a decreasing function of maturity, k. On the contrary, when an economy is assumed to follow a downward trend, the yield declines, and we have, in general, a negatively sloped yield curve.

Furthermore, the shape of the yield curve is also affected by the risk premium as will be discussed later. The relationship between the yield curve and risk premium can be summarized in the following manner; the larger investors' degree of risk aversion is and the more difficult it is to predict the expected rate of inflation, the greater the risk premium becomes. As a result, the yield curve shifts upward by the magnitude of risk premium.

C. Sufficient Conditions for Expectations Theory to Hold

In the former sub-section III. B., we looked into the price determination mechanism of coupon-bonds. In this sub-section, by comparing this result with the definition of expectations theory in Section II, we will derive the appropriate conditions for both unbiased expectations theory and the constant risk premium hypothesis to hold.

Since $P_j(t)$ is known at time t, both sides of equation (10) can be divided by $P_j(t)$, which gives:

$$E_t[\Phi(t+1)\cdot H'_{j,t}]=1\ ,\quad j=1,\cdots,n \tag{13}$$

where

$$H'_{j,t}=\frac{P_j(t+1)+C_j}{P_j(t)}\ .$$

Thus, applying Jensen's law to this equation, we can express the expected period holding rate of return of asset j, $H'_{j,t}$, (or $H_{n,t}+1$ in equation (1)) as:

$$E_t[H'_{j,t}]=\frac{1-cov_t(H'_{j,t}\ ,\ \Phi(t+1))}{E_t[\Phi(t+1)]}\ . \tag{14}$$

On the other hand, since the transactions of short-term instruments such as gensaki have a nonstochastic rate of return ($H'_{j,t}=1+R_{1,t}$), the short-term interest rate is given as:

$$1+R_{1,t}=\frac{1}{E_t[\Phi(t+1)]}\ . \tag{15}$$

Thus, from equations (14) and (15), we arrive at the conclusion that the expected period holding rate of return of asset j, $E_t[H'_{j,t}]$, should be equal to the sum of $1+R_{1,t}$ and $-cov_t[H'_{j,t},\ \Phi(t+1)]/E_t[\Phi(t+1)]$. With equation (1) defining expectations theory in mind, we observe that risk premium $\phi_{j,t}$ is determined as a combination of $H'_{j,t}$, $\Phi(t+1)$ and time t, and that it can be changed over time. That is:

$$\phi_{j,t}=-\frac{cov_t(H'_{j,t},\Phi(t+1))}{E_t[\Phi(t+1)]}\ ,\qquad j=1,\cdots,\ n\ . \tag{16}$$

From the above analysis, we can identify the characteristics of expectations theory and the appropriate conditions for it to hold:

(i) Equation (16) shows that the risk premium generally changes over time. Both the constant risk premium hypothesis ($\phi_{j,t}$ = constant) and unbiased expectations theory ($\phi_{j,t}=0$) are considered as special cases of the time-varying risk premium hypothesis. In addition, as was already mentioned, the risk premium is a combination of the expected future states of the economy, investors' degree of risk aversion and expected rate of inflation, and is determined by the interaction of these factors.

(ii) For unbiased expectations theory to hold ($\phi_{j,t}=0$), the covariance between $H'_{j,t}$ and $\Phi(t+1)$ must be serially independent. This requires, for example, two conditions to be satisfied: (a) that investors are risk-neutral; and (b) that price-level movements can be perfectly predicted, with no inflation risk. Although these two conditions are too strict to be fulfilled in general, as long as investors are nearly-risk-neutral ($\alpha = 0$) and price-level movements can be, for the most part, predicted, the risk premium becomes close to zero ($\phi_{j,t} \fallingdotseq 0$); thus unbiased expectations theory can approximately hold. Furthermore, when time-series fluctuations of the risk premium are relatively small, this risk premium can be regarded as time-invariant and the constant risk premium hypothesis can be said to be approximately valid.

IV. Empirical Studies in Japan

We have seen so far that neither unbiased expectations theory nor the constant risk premium hypothesis, sufficiently hold in general. In this section, we will examine empirically whether such expectations theories can actually hold in Japan.

A. Yields on Government Bonds and the Gensaki Rate

We will at first examine tranditional expectations theories using data regarding yields on government bonds and the gensaki rate in Japan. For testing procedures, a variety of methods are proposed and applied, such as the variance bonds test of Shiller (1979) and Singleton (1980), and the maximum likelihood ratio test of Sargent (1979) and Hansen and Sargent (1981). In this paper, we employed a method used by Mankiw and Summers (1984) and Shiller, Campbell and Shoenholtz (1983); the relationship between the expected change of the long-term interest rate implied by the yield curve and its actual change is empirically studied.

Specifically, assuming that the risk premium is constant and that bonds are perpetual on a simple arithmetic of the term structure equation of (6), we have:

$$R_{n,t+1}-R_{n,t}=\frac{1-\gamma}{\gamma}\theta_n+\frac{1-\gamma}{\gamma}(R_{n,t}-\frac{1}{1-\gamma^n}R_{1,t})+\frac{1-\gamma}{\gamma(1-\gamma^n)}v_{t+1} \tag{17}$$

where

$$v_{t+1}=\sum_{k=1}^{n-1}\gamma^k\{E_{t+1}[R_{1,t+k}]-E_t[R_{1,t+k}]\}.$$

From equation (17), we find that if traditional expectations theories hold, the long-term interest rate should change from time t to time t+1 by the magnitude of a long- and short-term interest rate spread ($R_{n,t}-R_{1,t}$) times a constant $(1-\gamma)/\gamma=\bar{R}$ (average yield on the long-term interest rate) plus expectation errors of v_{t+1} that arises from newly acquired information at time t+1. Since expectations are formed rationally at time t, v_{t+1} is uncorrelated with the known variables at time t, and can be regarded as a white noise. Or put it in another way, the orthogonalization conditions are fulfilled in equation (17). Thus we can estimate equation (17) by OLS.

Hence, we can examine traditional expectations theories empirically by estimating the following regression equation by OLS,

$$R_{n,t+1} - R_{n,t} = \alpha + \beta(R_{n,t} - R_{1,t}) + u_{t+1}, \quad \alpha < 0, \ \beta > 0 \tag{18}$$

and testing whether the estimated parameter $\hat{\alpha}$, $\hat{\beta}$ fulfills the theoretical value implied by expectations theory. Since $(1-\gamma)/\gamma = .01$,[7] we test the following two null hypotheses:

(i) H_1: $\hat{\alpha} = 0$, $\hat{\beta} = .01$ (unbiased expectations theory)

(ii) H_2: $\hat{\beta} = .01$ (constant risk premium hypothesis).

If both hypotheses (i) and (ii) should be rejected, then either the assumption that the risk premium is time-invariant, or that the expectations theory as represented by equation (6), would be rejected. It should be noted, however, that with the method employed in this paper we cannot distinguish which has been rejected.

The estimation periods for this test are from April 1977 to June 1984 ("the whole period") and from October 1981 to June 1984 ("the recent period"). The reasons for choosing these two periods are as follows:

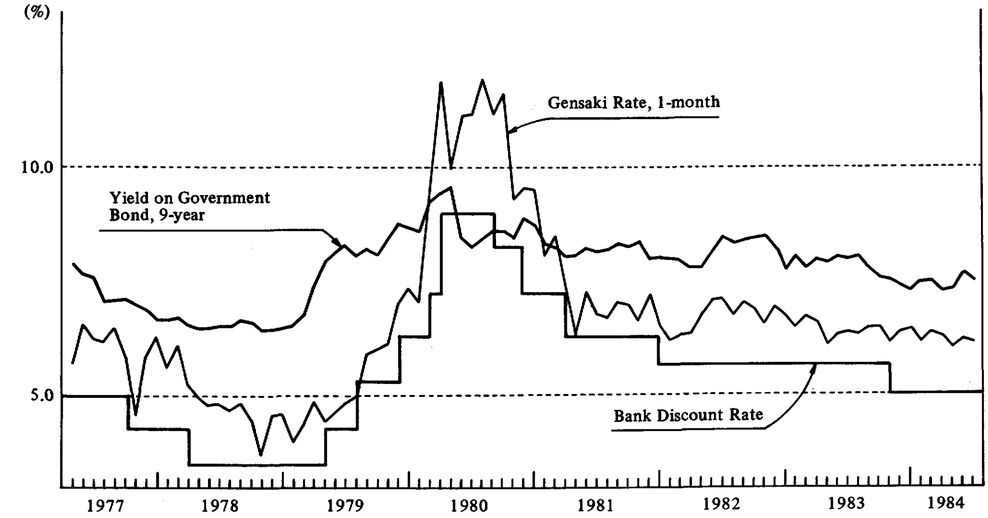
April 1977 was selected as the starting time of "the whole period" since this was the time when restrictions on the sale of government bonds by underwriting syndicates was eased considerably (so-called liquidation of government bonds); thereafter, yields on government bonds showed smoother fluctuations than before reflecting the supply and demand conditions of the market. "The recent period" after October 1981 was then taken up, because, after mid-1981, the arbitrage between the long- and short-term interest rates could be smoothly carried out as the liberalization of the money market have been sufficiently implemented.

As is clearly shown in Figure 1, the spread between long- and short-term rates underwent significant change around mid-1981; that is, while the long- and short-term interest rate spread has been fluctuating by only 1 to 2% since mid-1981, they had shown great volatility prior to 1981, especially from 1979 to 1980. Therefore, it is rather dubious if the mentioned period (1977 to mid-1981) can be taken up as a sample period in testing expectations theories, and this is the reason for having conducted separately a test on "the recent period."

7. An average yield on government bonds in the sample period is around 7.5% for each maturity. By transforming it into a monthly rate, we get .6%; therefore, $\bar{R} = 1/1.006 = .99$.

Figure 1 Recent Developments of Interest Rates in Japan

A. Yield on Government Bond and the Gensaki Rate



B. Long- and Short-term Interest Rate Spread and Change of Long Rates

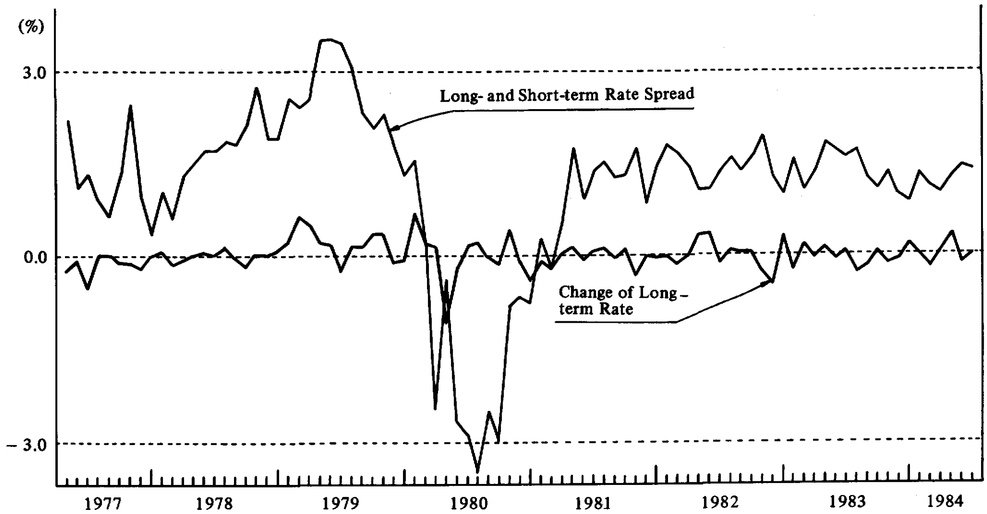


Table 2 Regression Results and Test Statistic of Term Structure of Interest Rates between Government Bonds Yield and the Gensaki Rate[1]

A. April 1977 to June 1984[2]

Dependent variable	Estimated Parameters[3]		D.W.	F-statistic[4]	
	α	β	SSE	H_1	H_2
9-year	−.0281 (.0345)	.0213 (.0197)	1.805 .248	.334	.328
8-year	−.0154 (.0390)	.0095 (.0226)	1.966 .269	.152	.000
7-year	.0149 (.0394)	−.0196 (.0224)	1.848 .278	1.079	1.747
6-year	−.0054 (.0386)	−.0035 (.0221)	1.680 .282	.407	.374
5-year	−.0054 (.0394)	−.0010 (.0248)	1.779 .299	.215	.197
4-year	.0221 (.0506)	−.0030 (.0340)	1.800 .366	.110	.147
3-year	.0016 (.0604)	−.0023 (.0442)	1.602 .403	.049	.077
2-year	−.0523 (.0662)	−.0504 (.0549)	1.553 .399	1.718	1.210
1-year	−.0616 (.0931)	−.0185 (.1112)	1.916 .345	1.229	.066

Table 2 shows the OLS estimates and test statistics by maturity. We can conclude:

Firstly, only the yields on bonds with eight and nine years fulfill the sign condition of $\hat{\beta}>0$ in the whole period; the sign of $\hat{\beta}$ coefficient is negative for other maturities; and it is negative in all maturities in the recent period. Furthermore, although the estimate of $\hat{\alpha}$, corresponding to the risk premium, is almost fulfilling the negative sign condition in the whole period, it turned positive for yields of any maturities in the recent period.

B. October 1981 to June 1984

Dependent variable	Estimated Parameters[3]		D.W.	F-statistic[4]	
	α	β	SSE	H_1	H_2
9-year	.3615 (.1532)	−.2840 (.1104)	2.066 .185	4.229*	7.097*
8-year	.4445 (.1178)	−.3793 (.0900)	2.081 .157	11.127*	18.714*
7-year	.2138 (.1226)	−.2086 (.0906)	1.824 .230	4.262*	5.824*
6-year	.1550 (.1180)	−.1560 (.0811)	1.825 .207	4.262*	4.190*
5-year	.2510 (.1420)	−.2220 (.1076)	1.413 .227	2.921*	4.649*
4-year	.3273 (.1405)	−.3497 (.1304)	1.803 .248	4.261*	7.607*
3-year	.1886 (.1054)	−.2680 (.1140)	1.807 .259	3.443*	5.946*
2-year	.1004 (.0850)	−.2045 (.1030)	1.895 .247	2.908	4.342*
1-year	.0780 (.0766)	−.2001 (.1120)	2.080 .246	2.228	3.052

Notes: 1. Yield on government bonds and the gensaki rate are end-of-month data.
2. For 4-year yield April 1978 to June 1984, for 3-year yield April 1979 to June 1984, for 2-year yield April 1980 to June 1984, for 1-year yield December 1980 to June 1984, respectively.
3. ( ) indicates a standard error.
4. * indicates that the hypothesis is rejected at the 5% level.

Secondly, regarding the magnitude of parameter $\hat{\beta}$, only the yields on eight and nine years in the whole period are close to the theoretical value; for others, they are deviating downward. Especially in the recent period, this downward deviation is obvious; for example, the yield on nine years is -.2840, far below the theoretical value of .01.

Thirdly, we examined the validity of expectations theory by testing hypotheses (i) and (ii). Neither of these hypotheses was rejected in the entire period. This implies that both unbiased expectations theory and the constant risk premium hypothesis are valid. However, choosing the entire period as a testing sample is rather problematic for already mentioned reasons. We therefore tested the hypothesis in the recent period, where the hypotheses (i) and (ii) were rejected for almost any maturities.

B. Between Gensaki Rates

We next examine, by using gensaki rates, whether the relationship between interest rates with shorter maturities can be explained by a traditional framework of expectations theory.

Letting the interest rate of a one-term asset be $R_{1,t}$, and that of a two-term asset, $R_{2,t}$, the holding rate of the return of a two-term asset becomes equal to the sum of "rolling-over" of one-term assets and the risk premium, which is given as:

$$R_{2,t} = \theta + \lambda R_{1,t} + (1-\lambda) E_t [R_{1,t+1}], \ 0<\lambda<1 . \tag{19}$$

And if we presume that investors' expectations are formed rationally, then $E_t[R_{1,t+1}]$ is:

$$E_t [R_{1,t+1}] = R_{1,t+1} + \varepsilon_{t+1} \tag{20}$$

(ϵ_{t+1} is an error term). Substituting (20) for (19) and arranging it give:

$$R_{1,t+1} - R_{2,t} = -\frac{\theta}{1-\lambda} + \frac{\lambda}{1-\lambda}(R_{2,t} - R_{1,t}) - \varepsilon_{t+1} . \tag{21}$$

Furthermore, for the interest rate of a three-term asset of $R_{3,t}$, we also get:

$$R_{2,t+1} - R_{3,t} = -\frac{\theta'}{1-\lambda'} + \frac{\lambda'}{1-\lambda'}(R_{3,t} - R_{1,t}) - \varepsilon'_{t+1} . \tag{22}$$

Since rational expectations are presumed, both (21) and (22) fulfill the orthogonalization conditions, thus making use of OLS possible. That is, we have estimated on the basis of OLS:

$$R_{1,t+1} - R_{2,t} = \alpha' + \beta'(R_{2,t} - R_{1,t}) + \varepsilon'_{t+1}, \ \alpha'<0, \ \beta'>0 \tag{23}$$

$$R_{2,t+1} - R_{3,t} = \alpha'' + \beta''(R_{3,t} - R_{1,t}) + \varepsilon''_{t+1}, \ \alpha''<0, \ \beta''>0 \tag{24}$$

and then by testing whether estimated parameters fulfill the theoretical values derived from expectations theory, we can conduct an empirical investigation on the validity of expectations theory. To put it in more concrete terms, since $\lambda = 1/2$, $\lambda' = 1/3$, we tested the following two null hypotheses.

(iii) H_3: $\alpha' = 0$ and $\lambda' = 1$, or $\alpha'' = 0$ and $\lambda'' = .5$ (unbiased expectations theory)

(iv) H_4: $\lambda' = 1$, or $\lambda'' = .5$ (constant risk premium hypothesis)

Table 3 shows the estimated results and test statistic. The estimated parameters of both regression equations of (23) and (24) have right signs in both the whole and

Table 3 Regression Results and Test Statistic of Term Structure of Interest Rates between Gensaki Rates

A. Between 2-month and 1-month gensaki rates

Sample period	Estimated Parameters[1]		D.W.	F-statistic[2]	
	α'	β'	SSE	H_3	H_4
April 1977 – June 1984	−.2480	.5220	2.388	12.435*	2.326
	(.0865)	(.3134)	.644		
October 1981 – June 1984	−.1774	.9968	1.804	8.536*	.000
	(.0390)	(.0226)	.269		

B. Between 3-month and 2-month gensaki rates

Sample period	Estimated Parameters[1]		D.W.	F-statistic[2]	
	α''	β''	SSE	H_3	H_4
April 1977 – June 1984	−.1913	.3096	2.306	8.294*	1.259
	(.0780)	(.1687)	.581		
October 1981 – June 1984	−.1367	.4171	1.783	7.338*	.058
	(.0620)	(.3443)	.223		

Notes: 1. ( ) indicates a standard error.
2. * indicates that the hypothesis is rejected at the 5% level.

recent periods. Moreover, in the recent period, the estimated value of $\beta'$and β'' proved almost in unity with the theoretical value. We also tested hypotheses (iii) and (iv). While hypothesis (iii) was rejected in both the whole and recent periods, hypothesis (iv) was not rejected.

C. Seemingly Unrelated Regression

So far we have examined expectations theory in relationship to yields on government bonds and to the gensaki rate by maturities. However, in reality, investors who try to maximize their present discount value of the asset they hold, transfer their assets actively among bonds and money market instruments of different maturities, by watching all money and capital market developments. Thus, in general, there exists a correlation for expected errors of different maturities. If we take it into consideration, it is more desirable and appropriate to examine expectations theory within a multi-equation context by using a method of seemingly unrelated regression or GLS. Using data on government bonds with a maturity of nine through one year and a two-month gensaki rate, we arrived at regression equation of (17) within a multi-equation context by seemingly unrelated regression, and jointly tested the null hypotheses of (i) and (iii), or of (ii) and (iv).

Table 4 shows the result of the seemingly unrelated regression and chi-square distribution for the later period χ (q) where q is the number of restrictions imposed. In our case, q = 10 for the constant risk premium hypothesis and q = 20 for the unbiased expectations theory, respectively. We found that the standard error of a coefficient becomes smaller and the joint null hypotheses are significantly rejected.

D. Summary of Empirical Analysis and its Interpretation

In the previous sub-sections, we examined empirically traditional expectations theories in Japan by using yields on government bonds and gensaki rates. The result of the test may be summarized as follows.

Expectations theory can be supported between yields on government bonds and gensaki rates through the whole period, while in the recent period, neither unbiased expectations theory nor the constant risk premium hypothesis can hold, which implies the existence of the risk premium that changes over time. On the other hand, as for the relationship between gensaki rates, unbiased expectations theory does not hold in either the whole or recent period while the constant risk premium hypothesis does.

The following two factors may well be attributable to the rejection of traditional expectations theories between yields on government bonds and the gensaki rate.

The first is the impact of the volatile fluctuations of the U.S. long-term interest

Table 4 Seemingly Unrelated Estimation Results Chi-square Distribution[1]

Dependent variable	Estimated Parameters[2]		D.W.
	α	β	SSE
9-year	.3387 (.0603)	−.2673 (.0380)	2.077 .185
8-year	.3859 (.0716)	−.3332 (.0470)	1.847 .231
7-year	.1430 (.0716)	−.1532 (.0470)	1.847 .232
6-year	.0931 (.0503)	−.1113 (.0261)	1.852 .208
5-year	.2163 (.0641)	−.1947 (.0406)	1.428 .227
4-year	.1779 (.0704)	−.2039 (.0552)	1.990 .253
3-year	.0956 (.0582)	−.1567 (.0458)	2.148 .263
2-year	.0682 (.0519)	−.1593 (.0434)	1.906 .248
1-year	.0591 (.0569)	−.1669 (.0688)	1.874 .249
2-month	−.1783 (.0589)	1.0072 (.5056)	1.892 .247
			Chi-square distribution
Unbiased Expectations Theory (q=20)			149.7197*
Constant Risk Premium Hypothesis (q=10)			115,7256*

Notes: 1. Sample period is October 1981 to June 1984.
2. ( ) indicates a standard error.
3. * indicates that the hypothesis is rejected at the 5% level.

rates. In recent years, the yield on government bonds in Japan is increasingly becoming volatile reflecting the wide fluctuation of U.S. interest rates. There are increasing risks of capital losses owing to the unanticipated U.S. interest rates movement. Under this environment, risk-averse investors seek more risk premia to compensate for possible capital loss in choosing their portfolio. This causes the variation of the risk premium over time and makes the relationship between the long-term rates and expected short-term rates more ambiguous. In fact, the estimated parameter $\hat{\beta}$ is biased in a downward manner in the recent period. Such a biased estimate would be considered to be the result of a simple application of OLS to equation (20), provided that the risk premium is constant in spite of the possible over-time variation of the risk premium. The estimated parameter $\hat{\beta}$ is biased in a downward manner as:

$$\hat{\beta} = \beta - \alpha \cdot \frac{\mathrm{cov}\,(R_{n,t} - R_{1,t}\,,\ \theta_t)}{\mathrm{var}\,(R_{n,t} - R_{1,t})}\,. \tag{17}$$

Secondly, we can point out the length of the time horizon in the decision-making of investors. In traditional expectations theory, it is assumed that investors make their decisions on a portfolio by anticipating the overall future movements of short-term rates up until their planned investment period. However, reflecting the volatility of long-term interest rates in Japan, which is caused by U.S. interest rates, it is now becoming increasingly difficult to predict future movements of interest rates in Japan. Thus, investors' expectations might come to be more myopic than expectations theory assumes, and they overreact to the temporal shock accompanying the U.S. interest rate fluctuations.

V. Conclusion

In this paper, we re-examined expectations theory by explicitly considering the optimizing behavior of investors in a stochastic environment. It was shown that the time-varying risk premium is important for the determination of the term structure of interest rates from both the theoretical and empirical perspectives. This result indicates the importance of the time-varying risk premium when discussing expectations theory. Extensive efforts are now being made to modify expectations theory by incorporating the time-varying risk premium such as that by Bodie, Kane and McDonald (1983), Campbell and Shiller (1984). However, since the risk premium is not observable, we have not yet reached a consensus on how to measure it empirically. More elaborate works are awaited along this line for a better understanding of the term structure of interest rates.

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