Empirical Tests of "Bubbles" in the Foreign Exchange Market*

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I. Introduction

Since a floating exchange rate system was adopted for the major currencies in the spring of 1973, the exchange rates have repeatedly shown substantial volatility. This has posed a major problem in recent years in the conduct of the monetary policy. However, various theoretical models concerning exchange rate determination have not been able to explain such movements fully and, as a result, "the deviation of exchange rates from the economic fundamentals" has become a major issue in international economies. This has brought the empirical importance of speculative bubbles in the foreign exchange market to the fore.

"The Bubbles" have been observed on numerous occasions in economic history. These include many major events which gave a serious blow to the economy of one country and also to the world economy as a whole. The most well-known examples are the Dutch Tulip Mania in 1636-37, the South Sea Bubble in England in 1720 and the Great Crash of the U.S. stock market in 1929.

This paper examines the existence of bubbles based on rational expectation in the foreign exchange market. In particular, we focus on the following two questions.

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1) Have bubbles *constantly* played an important role as the source of exchange rate fluctuations? If not,
2) have bubbles played a major role for a considerable period of time as an *exceptional* factor for exchange rate fluctuations?

A clue to empirical tests to answer these questions can be found in the distribution of the exchange rate change. If speculative expectations for an increase (or decrease) in asset prices are developed by a bubble[1], asset prices will rise (or fall), increasing the expectations for a further increase in asset prices and accelerating the rise (or fall) in asset prices. This process, however, cannot go on forever. It will inevitably collapse when the view, that the deviation of asset prices from economic fundamentals cannot increase or decrease any further, prevails on the market. At this point asset prices plummet (or jump).

As can be seen from the above formulation, the bubble changes asset prices in a specific direction as long as the process of its expansion continues, and when collapsing, creates huge price changes in the opposite direction. This implies that if the price formation of a financial asset is affected by bubbles, the rate of change in the market price of that asset should have peculiar distribution characteristics such as, 1) a high serial correlation and 2) fat tails (a high excess kurtosis).

In conducting empirical tests of the bubble, the present paper employs two alternative assumptions on fundamentals. One is a random walk and the other is a rational expectation portfolio balance model. The reason for using the random walk hypothesis may require some explanation. One practical reason for choosing this assumption is that a bubble is often considered to be a short-run phenomenon; therefore, it is desirable to use weekly data rather than monthly or quarterly data in the estimation. Since weekly data for fundamental time series other than the interest rate are not available, the random walk assumption becomes a convenient specification. However, the main reason for adopting this assumption is that there is strong support, both theoretically and empirically, for the argument that the exchange rate follows random walk pattern in the short-run.

The procedure for an empirical test of the bubble adopted for the present paper is as follows:

Under the random walk assumption, the following two tests are conducted. First, a tails test is employed to determine whether the distribution of exchange rate changes has fat tails due to the presence of speculative bubbles. Second, a runs test is used under the hypothesis that the presence of bubbles would produce serial correlation in the exchange rate changes.

Next, in an attempt to discriminate bubbles from overshooting, two facts are

1. Refer to Okina (1984) for the mechanism of generating bubbles.

used: 1) the movements in the forward premium should also reflect the bursting of large bubbles; and 2) the movement in the forward premium and runs will not be independent. The next diagnostic check therefore conducts a chi-square test using weekly data. Furthermore, based on a portfolio balance model, a variance bound test is conducted to check whether the actual volatility of the exchange rate is significantly larger than the theoretical value.

The above tests concern the constant existence of bubbles in the foreign exchange market. The next subject is whether the bubble played a major role as an exceptional factor for exchange rate fluctuations for a considerably long period of time. In order to answer this question, it is appropriate to construct an explicit model of exchange rate determination because, in this case, trends in fundamental factors will have to be taken into consideration.

Based on the rational expectation portfolio balance model, a monthly regression equation, which explains the real exchange rate with fundamentals, is estimated. Based on this, time series characteristics of estimation errors (residuals) are examined to ascertain if the deviation of the exchange rate from fundamentals has a property which would indicate the existence of a speculative bubble.[2]

Further, for the period for which the existence of a bubble is suggested by such an analysis, a stochastic bubble is directly estimated by using dummy variables.

However, in a simple estimation by using a dummy variable, factors other than the bubble, such as important "news" and the collapse of "extraneous beliefs", which are not considered in the model, cannot be distinguished from the bubble. Therefore the following two checks were added.

1) In order to remove the influence of the irrational bandwagon effect, the serial correlation is adjusted.

2) Ad hoc dummies representing other factors such as "news" are included in the equation to be compared with the bubble dummy regarding to their explanatory power.

Finally, formal analysis were supplemented by examining the anecdotal evidence as to how the market movements were perceived by market observers during the period when the existence of a bubble was suspected. This is because it is extremely difficult to identify bubbles in the exchange rate solely by econometric techniques.

The results of empirical tests and their implications may be summarized as follows:

1) Although it cannot be ruled out that bubbles occur frequently in the exchange market, they are short-run phenomena and, when viewed on a monthly basis, they do

2. See Appendix 1 for the theory of speculative bubbles.

not seem to be the major factor in exchange rate fluctuations (most bubbles collapse within a short period of time).

2) At the same time, however, it cannot be denied that there were specific periods when bubbles lasted for exceptionally long time and seemed to affect the exchange rate significantly. Such possibility was strongest in the case of the depreciation of the yen during the period from April to October 1982. Both econometric analysis and anecdotal evidence provide circumstantial evidence for the bubble nature of the yen depreciation during this period.

II. Diagnostic Checks of Bubbles Based on a Random Walk Model

1. Random Walk Model

Based on a random walk model, this section is intended to test whether bubbles can be regarded as the main factor for exchange rate fluctuations. The tails test with attention focused on the excess kurtosis, and the runs test with attention focused on the serial correlation of the rate of the exchange rate change are employed.

Simple random walk hypothesis[3] concerning the exchange rate may be described as below.

$$e(t) = e(t-1) + v(t) \tag{1}$$

where e(t) is the natural logarithm of the exchange rate denominated by home currency unit and v(t) the disturbance term.

Here no serial correlation is assumed for v(t) but the expected value does not necessarily have to be zero (when the expected value is not zero, it means that there is a trend, either appreciation or depreciation, for the currency concerned).

An alternative hypothesis may be formulated by adding a bubble term to this simple random walk hypothesis: the actual exchange rate is assumed to be determined by a random walk for the fundamentals rate $\bar{e}(t)$, with the added influence of a speculative bubble.

$$\left\{\begin{array}{ll} \bar{e}(t) = \bar{e}(t-1) + v(t) & (2) \\ c(t) = \begin{cases} \dfrac{\rho}{1-\pi} c(t-1) + u(t), \ \rho \geqq 1 & \text{probability} \quad 1-\pi \\ u(t) & \text{probability} \quad \pi \end{cases} & (3) \\ e(t) = \bar{e}(t) + c(t) & (4) \end{array}\right.$$

3. See Appendix 2 for the random walk hypothesis.

where c(t) is the size of the bubble, ρ the structural parameter determining dynamic path of the bubble and u(t) the disturbance term.

Equations (2) ~ (4) form the exchange rate determination model in this case.

2. Tails Test

If bubbles exist in the foreign exchange market, one would expect to observe large outliers in a distribution of exchange rate changes which correspond to the time at which the bubble bursts. One type of diagnostic checks, the tails test, provides a means of detecting the presence of such outliers (see Blanchard and Watson 1982). The existence of a bursting bubble in the foreign exchange market will tend to generate a relatively small depreciation (or appreciation) while the bubble exists, followed by a relatively large appreciation (or depreciation) at the time when the bubble bursts. In other words, the burst of a bubble will tend to produce large outliers in the change of market price causing the distribution of e(t) – e(t-1) to have fat tails, that is the excess kurtosis will be high. However, there is no a priori reason to believe that a bubble process will affect the skewness of the distribution. A test for the existence of a bursting bubble in the foreign exchange market should therefore compare the observed value of excess kurtosis with that produced by a normal distribution. For, if the random walk hypothesis is valid and if the disturbance term v(t) follows a normal distribution, the excess kurtosis will be zero. Therefore, it should be possible to examine the existence of a bubble by testing whether the excess kurtosis significantly exceeds zero[4]

$$(\quad \mathrm{Krto} = \frac{\frac{1}{n}\sum_{i=1}^{n}(X(i)-\bar{X})^4}{S^4} - 3 \tag{5}$$

where X(i) represents a stochastic variable, $\bar{X}$ the average of X(i) and S the standard deviation of X(i) $< \; = \sqrt{\frac{1}{n}\sum (X(i)-\bar{X})^2} \; >$).

Table 1 shows the excess kurtosis and skewness of weekly, five-weekly and ten-weekly data on exchange rate changes of five major currencies against the U.S. dollar for the period from early May 1973 to the end of 1980.[5]

4. The excess kurtosis of the distribution can be large not only in those cases where a speculative bubble exists but also in those cases where the excess kurtosis of the distribution of the innovation concerning fundamentals is large. See Blanchard and Watson (1982).

5. The reason for giving the value of skewness here is that since the assumptions concerning bubbles and fundamentals here do not affect the skewness, there is a possibility that the

Table 1 Distribution Characteristics of Rates of Change of the Exchange Rates of Major Currencies: Early May 1979 – End of 1980

1) Weekly (401 weeks)

	S.D.	Skew.	Krto.
Canada	0.0050	–0.7011	5.1580
France	0.0122	–0.3323	4.1609
West Germany	0.0129	–0.1711	4.5379
Japan	0.0119	–0.3018	5.0783
U.K.	0.0108	–0.0463	1.4726

2) Five-weekly (81 weeks)

	S.D.	Skew.	Krto.
Canada	0.0125	–0.0130	0.1739
France	0.0298	–0.0968	0.6418
West Germany	0.0352	–0.1612	0.3843
Japan	0.0308	0.7444	1.4965
U.K.	0.0278	–0.0856	–0.0692

3) Ten-weekly (41 weeks)

	S.D.	Skew.	Krto.
Canada	0.0179	0.2060	–0.6030
France	0.0474	0.1757	0.9983
West Germany	0.0534	0.2825	1.1180
Japan	0.0468	0.4422	–0.3244
U.K.	0.0437	–0.2422	–0.5822

Standards for testing the deviation from a normal distribution

	Skewness			Kurtosis		
	n = 40	n = 100	n = 400	n = 40	n = 100	n = 400
5%	±0.59	±0.39	±0.20	1.06	0.77	0.41
1%	±0.87	±0.57	±0.29	2.04	1.39	0.67

Memo: Second week of 1978 – 18th week of 1983 (35th week of 1982 for Japan): 278 weeks, 243 weeks for Japan

	S.D.	Skew.	Krto.
Canada	0.0056	–0.1537	1.5285
France	0.0161	–0.2888	3.5918
West Germany	0.0158	0.0321	2.1971
Japan	0.0155	0.1362	0.8155
U.K.	0.0148	–0.6601	2.8851

		Skew.	Krto.
n = 250	5%	±0.25	0.52
	1%	±0.36	0.87

Notes: S.D.: standard diviation
Skew: skewness (in the case of a normal distribution, 0)
Krto.: excess kurtosis (in the case of a normal distribution, 0)
(rates of the exchange rate change is approximated by logarithmic differentials)

Source: Harris Bank

According to Table 1, the excess kurtosis showed the following characteristics.
1) Significantly positive for the weekly data.
2) Though they are positive for the five-weekly data with the exception of the U.K. sterling, they are not very significant.
3) The signs are mixed for the ten-weekly data and their significance is small in general.[6]

These test results suggest that although there is a possibility of bubbles occurring frequently on the market, they are normally short-lived, and long-lived bubbles are unlikely to occur often.

3. Runs Test[7]

The runs test is a test for the randomness of the data which does not rely on the strong assumption that the disturbance v(t) of the fundamentals rate follows normal distribution.

The results of runs test on the exchange rate changes of five major currencies against the U.S. dollar during the period from early May 1973 to the end of 1980 are given in Table 2, which shows the following results.
1) With the exception of the French franc, the weekly data show a significant positive serial correlation consistent with the existence of bubbles.
2) The five-weekly data invariably suggest a positive but not very significant correlation.
3) The ten-weekly data show both positive and negative correlations without significance.

Those results are consistent with those obtained by tails tests which support the constant occurrence of short-lived bubbles.

III. Empirical Tests of Bubbles Based on the Efficient Market Hypothesis

The results of the tails test and the runs test are consistent with the existence of

premise for the analysis might not be satisfied if the skewness deviates from a normal distribution (significantly deviating from zero). Such problem, however, did not arise in practice as the skewness in general did not show any significant deviation from zero.

6. These results are consistent with those in the financial literature in which the distributions of the rates of daily returns on common stocks are shown to have more kurtosis than those of the monthly returns which are closer to the normal distribution. For these points, see Mandelbrot (1963), Blattberg and Gonedes (1974), Fama (1976), etc.

7. For more details on runs test, see Appendix 3.

Table 2 Runs in the Rates of Change of Major Currencies: Early May 1973 – End of 1980

1) Weekly (401 weeks)

	Obs.	Exp.	S.D.	Ratio.
Canada	173	200.5	10.0	−2.79**
France	187	200.5	10.0	−1.37
West Germany	174	200.5	10.0	−2.70**
Japan	163	200.5	10.0	−3.61**
U.K.	175	200.5	10.0	−2.58*

2) Five-weekly (81 weeks)

	Obs.	Exp.	S.D.	Ratio.
Canada	38	40.5	4.47	−0.63
France	34	40.5	4.47	−1.56
West Germany	40	40.5	4.47	−0.14
Japan	39	40.5	4.47	−0.25
U.K.	37	40.5	4.47	−0.88

3) Ten-weekly (41 weeks)

	Obs.	Exp.	S.D.	Ratio.
Canada	24	20.5	3.16	1.04
France	20	20.5	3.16	−0.31
West Germany	22	20.5	3.16	0.34
Japan	22	20.5	3.16	0.60
U.K.	18	20.5	3.16	−0.73

Memo: Second week of 1978 – 18th week of 1983 (35th week of 1982 for Japan): 278 weeks, 243 weeks for Japan

	Obs.	Exp.	S.D.	Ratio.
Canada	131	139.5	8.29	−0.95
France	132	139.5	8.29	−0.88
West Germany	123	139.5	8.29	−1.96*
Japan	116	122.0	7.75	−0.76
U.K.	135	139.5	8.29	−0.54

Notes: Number of runs obtained by dividing data into two groups by mean rate of appreciation/depreciation
Obs.: observed number of runs
Exp.: expected number of runs
S.D.: standard deviation of the number of runs
Ratio = (Obs.-Exp.)/S.D. (= test statistics)
* denotes significant at the 5% level.
** denotes significant at the 1% level.

Source: Harris Bank

bubbles for a relatively short period of time. The above conclusion is based on the strong assumption that distribution for innovations in the fundamentals are not leptokurtic. Instead of such an assumption, a risk neutral efficient market[8] is assumed here to test the speculative bubbles.

1. Discrimination from Overshooting

Among the important structural models of the exchange rate determination, the overshooting model can also produce a leptokurtic distribution and a positive correlation (a small number of runs) in the rate of change in the exchange rate.

That is, if the innovations of fundamentals have a leptokurtic distribution,[9] it implies that when unexpected news concerning fundamentals is released on the market, the exchange rate will show marked appreciation (depreciation) immediately, overshooting from the new long-term equilibrium level, gradually depreciate (appreciate) toward a new long-term equilibrium level. This process will create a distribution of exchange rate changes with characteristics similar to those observed when a bubble exists.

However, the dynamic path of the exchange rate change markedly differs in two cases. That is, in the case of the overshooting model, a long "run" is observed following an outlier in the rates of change of the exchange rate, whereas, in the case of a bubble, marked depreciation (appreciation) of the exchange rate due to the collapse of the speculative bubble is preceded by a run of appreciation (depreciation),[10] as shown in Figures 1 and 2.

Table 3 shows, as an example, the relationship between outliers and runs in the

8. For the sake of clarity, argument here proceeds based on the hypothesis that market participants are risk neutral. However, even if they are risk averse, exactly the same analysis can be applied by assuming a situation in which the forward rate is an unbiased estimate of the spot rate with random risk premiums.

9. In the event of the innovation of fundamentals not being leptokurtic, it is possible that the news offset each other and even if the true structure of the exchange rate determination can be approximated by the overshooting model, the exchange rate may move at random.

10. Strictly, however, there is a possibility that such a dynamic path as shown in Figure 1 can be produced by factors other than bubbles such as expectations on the future value of fundamentals being betrayed at a specific point in time. Let us assume, for instance, that there is a U.S. presidential candidate commiting himself to a tight monetary policy, whose success is ensured. In this case, as the election comes nearer, it means that the tight monetary policy is approaching and the dollar will gradually appreciate as the exchange rate reflects it. In the event of this candidate being defeated contrary to expectations, the exchange rate will jump down. This implies that when there is a dynamic path produced by fundamental factors as shown in Figure 1, the major news (disappointment) must have reached the market at the point t_1. There was a

Figure 1 Dynamic Path of a Bubble

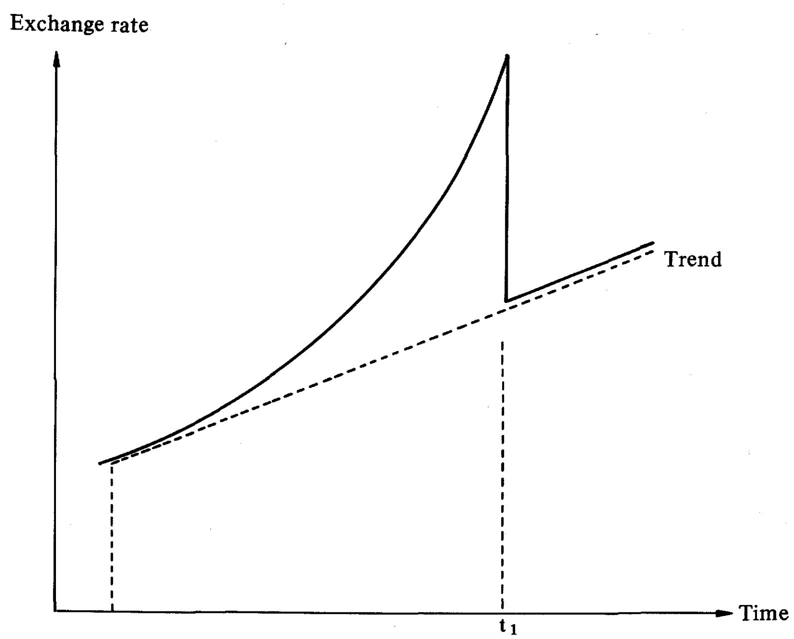


Figure 2 Dynamic Path of Overshooting

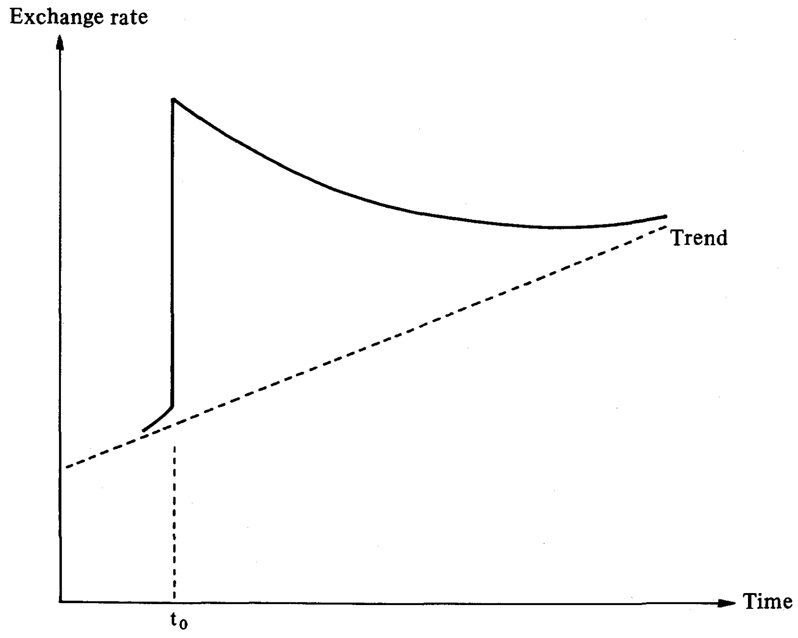


Table 3 Outliers and Runs of Exchange Rates: Movements of Weekly Data around the First Week of November 1978

	Prior to the first week of November	The first week of November (slump of the yen and European currencies) ↓	After the first week of November
French franc	− + − + − + + + + +	−	+ − − − + + + + − −
German mark	− + − + + + + + + +	−	+ − − − + + + + − −
Japanese yen	− + − + + − + + + +	−	− − + − + + + − − −
U.K. sterling	− + − + + + + − + +	−	− − + − + + + + − −

Notes: +: $\{e(t) - e(t-1)\} < \{e(t) - e(t-1)$ average of the period from early May to the end of December 1980$\}$

−: $\{e(t) - e(t-1)\} > \{e(t) - e(t-1)$ average of the period from early May to the end of December 1980$\}$

$e(t)$: Natural logarithmic values of the exchange rates of various currencies against the U.S. dollar on a yen basis.

Source: Harris Bank

exchange rates of European currencies and the yen against the U.S. dollar from August 1978 to January 1979, a period during which the exchange rates violently fluctuated. Substantial depreciation of these major currencies against the U.S. dollar was observed during the first week of November[11] and the movements of runs around that time indicate that these currencies had invariably shown an upward trend prior to the substantial depreciation but followed individual trends thereafter. This clearly suggests that they are closer to the dynamic path of a bubble rather than that of overshooting.

2. Independence of Forward Premiums and Runs

This section is concerned with a joint test of the efficient market hypothesis and

possibility of such news in the case of November 1978 (correction of expectations on fundamentals with President Carter's announcement of a policy to defend the dollar) but not in the case of the upsurge in yen in November 1982. For this point, see Section VI.

11. The reason for excluding the Canadian dollar from the runs test was that the exchange rate of the Canadian dollar against the U.S. dollar showed only minor fluctuations in November 1978.

the bubble hypothesis, bearing identification with the overshooting model in mind.

Assuming a risk neutral efficient market,[12] the forward exchange rate at the period t for the period j, f(t,j), becomes

$$f(t,j) = E_t\, e(t+j). \tag{6}$$

Accordingly, the forward premium for one period will be:

$$\begin{aligned} f(t,1) - e(t) &= E_t e(t+1) - e(t) \\ &= E_t(\bar{e}(t+1) - \bar{e}(t)) + (E_t c(t+1) - c(t)). \end{aligned} \tag{7}$$

Since the bubble has a specific expected explosive path,[13] it can be transformed as below.

$$\begin{aligned} &E_t c(t+1) - c(t) = (\rho - 1) c(t), \quad \rho > 1 \\ &f(t,1) - e(t) = E_t(\bar{e}(t+1) - \bar{e}(t)) + (\rho - 1) c(t) \end{aligned} \tag{7}'$$

Also, the rate of change for forward premiums can be expressed as below.

$$\begin{aligned} (f(t,1) - e(t)) - (f(t-1,1) - e(t-1)) = &\{(E_t \bar{e}(t+1) - \bar{e}(t)) \\ &- (E_{t-1} \bar{e}(t) - \bar{e}(t-1))\} + (\rho - 1)(c(t) - c(t-1)) \end{aligned} \tag{8}$$

$$= \{\text{ effect of the expected rate of change in the fundamentals }\} + \{\text{ effect of changes in the bubble }\}$$

If the change in the exchange rate from the period t−1 to the period t was mainly brought about by a speculative bubble, it should be possible to ignore the movements of the expected rate of change in the fundamentals. Therefore, the change in the spot exchange rate and that in forward premium should be of the same direction. And, if bursting bubbles cause large exchange rate changes, they tend to create runs of exchange rate changes with signs opposite to those of the period prior to the "bursting".

Hence, if exchange rate dynamics are really affected by the existence of bursting bubbles, one should in fact expect a high correlation between the test for a forward premium and the test on runs. A chi-square test can be used to test the independence of these two diagnostic check results.

12. This statement is strictly valid only when prices are non-stochastic. Empirically, however, the neglect of this factor may be of little significance. Those points are shown in Frenkel and Razin (1980).

13. See Appendix 1 for this point.

Table 4 shows the distribution of the magnitude of weekly exchange rate changes in five countries during the flexible exchange rate period (from May 1973 to March 1983). A chi-square test for the existence of bursting bubbles is conducted on a sample of 85 weeks for which the exchange rate changes by 3% or more (85 weeks corresponds to 3.3% of the 2575 time-series cross-section pooled data). An exchange rate changes of 3% or more was chosen as a criterion for selecting a sample of weeks because weeks for which the exchange rate changes more than 4% are too small in numbers to test the presence of bursting bubbles (they are only 1.3% of the data which implies that large burstings are observed at most only once in 80 weeks), while a criterion for exchange rate changes of 1% or 2% is too broad to identify those outliers as bursting bubbles.

First step of chi-square test is classifying data according to two attributes. The classification is based on the following two criteria:

Table 4 Distribution of Rates of Change in the Exchange Rate

	Canadian dollar	French franc	German mark	Japanese yen	U.K. sterling	Pooled total
0.0% – 0.999%	488 (94.8)	343 (66.6)	326 (63.3)	352 (68.3)	337 (65.4)	1846 (71.6)
1.0% – 1.999%	25 (4.9)	110 (21.4)	119 (23.1)	106 (20.6)	124 (24.0)	484 (18.8)
2.0% – 2.999%	1 (0.2)	36 (7.0)	49 (9.5)	38 (7.4)	36 (7.0)	160 (6.2)
3.0% – 3.999%	1 (0.2)	15 (2.9)	10 (1.9)	13 (2.5)	14 (2.7)	53 (2.0)
4.0% – 4.999%	0	5 (1.0)	5 (1.0)	3 (0.6)	2 (0.4)	15 (0.6)
5.0% – 5.999%	0	4 (0.8)	4 (0.8)	3 (0.6)	1 (0.2)	12 (0.5)
6.0% – 6.999%	0	1 (0.2)	2 (0.4)	0	0	3 (0.1)
7.0% up	0	1 (0.2)	0	0	1 (0.2)	2 (0.1)

Note: The figures in brackets denote the shares of the rates of change in various currencies or in the pooled total. Sampling period: first week of May 1973 – third week of March 1983.

Source: Harris Bank

1) Classification by the relationship between the forward premium and the exchange rate
   - The forward premium and the exchange rate of the period t change in the same direction (consistent with a bubble).
   - The forward premium and the exchange rate of the period t change in the opposite directions (not consistent with a bubble).

2) Classification by signs of exchange rate changes
   - An outlier of rate of change in the exchange rate of the period t and rate of change in the exchange rate preceding it are of different signs (consistent with a bubble).
   - An outlier of rate of change in the exchange rate of the period t and rate of change in the exchange rate preceding it are of the same sign (not consistent with a bubble).

When classified by these two criteria, the data belong to either one of the four categories shown in Table 5. The null hypothesis is that there is no relationship between attributes based on the two classifications (independent) and the variables under this null hypothesis follows a chi-square distribution with one degree of freedom.[14]

$$X = \frac{(F(11)\,F(22) - F(12)\,F(21))^2 \times N}{F(1.)\,F(2.)\,F(.1)\,(F.2)} \tag{9}$$

When $X > 3.84$ the null hypothesis is rejected at a 5% level. Table 6 summarizes chi-square values obtained by actual data of 85 weeks. The results shown by Table 6 indicate that the null hypothesis cannot be rejected and that it is difficult to assume that these 85 outliers were mainly due to the bursting of bubbles.

However, it is perhaps necessary to keep in mind that this interpretation of the results bases itself on the assumption of risk neutral efficient market hypothesis, which is not undisputed the validity.[15]

14. Strictly speaking, this result is valid when $F(ij) > 5$ for all i, j. If $F(ij) < 5$, Yates' adjustment should be made as below.

$$X = \frac{\min\{F(11)\,F(22) - F(12)\,F(21) \pm \frac{N}{2}\}^2 \times N}{F(1.)\,F(2.)\,F(.1)\,F(.2)}$$

See Kunizawa (1966).

15. See Appendix 4 for detailed examination on this point.

Table 5 Fourfold Table for X^2 Test (1)

		Classification by the relationship between forward premiums and the exchange rate		
		Consistent with bubbles	Not consistent with bubbles	Total
Classification by signs for rates of change in the exchange rate	Consistent with bubbles	F (11)	F (12)	F (1.)
	Not consistent with bubbles	F (21)	F (22)	F (2.)
	Total	F (.1)	F (.2)	N

Notes: F(ij): number of weeks belonging to i, j categories.
N: total number of data (85 in this case).

Table 6 Fourfold Table for X^2 Test (2)

1. Consistency examined over 1 week			
F (11) = 16	F (12) = 22	F (1.) = 38	
F (21) = 20	F (22) = 27	F (2.) = 47	
F (.1) = 36	F (.2) = 49	N = 85	X = 0.00 < 3.84
2. Consistency examined over 2 weeks			
F (11) = 6	F (12) = 17	F (1.) = 23	
F (21) = 19	F (22) = 43	F (2.) = 62	
F (.1) = 25	F (.2) = 60	N = 85	X = 0.17 < 3.84
3. Consistency examined over 3 weeks			
F (11) = 3	F (12) = 15	F (1.) = 18	
F (21) = 21	F (22) = 46	F (2.) = 67	
F (.1) = 24	F (.2) = 61	N = 85	X = 0.87 < 3.84

Notes: "Consistency examined over n weeks" means to classify the various data by the following criteria:

1) In the case of the criterion for signs of rates of change in the exchange rate, whether a run consistent with a bubble was formed for consecutive n weeks prior to the occurrence of an outlier.
2) In the case of the criterion for forward premiums, whether the change in forward premium is consistent with the collapse of a bubble is judged by using the average premium for n weeks prior to an outlier and for n weeks following an outlier.

Source: Harris Bank

IV. Variance Bound Test Based on a Rational Expectation Portfolio Balance Model

1. Rational Expectation Portfolio Balance Model

So far the analysis has been conducted without specifying what the fundamentals of the exchange rate are. In contrast, in this section it is explicitly assumed that the fundamental exchange rate can be described by a rational expectation portfolio balance model.[16]

This model consists of two countries. The home currency-denominated-assets are assumed to be safe; foreign currency-denominated-assets, however, are risky because of exchange rate risk. Since an investor is assumed to be risk averse, he demands risk premia for holding the latter. It can be assumed that they are proportional to the stock of net foreign currency denominated assets as approximated by the cumulative sum of current accounts.

Assuming $Z(t)$ to be real net foreign assets, $r(t)$ and $r^*(t)$ real domestic and foreign interest rates and $s(t)$ the real exchange rate in logarithm, the equilibrium condition for the foreign exchange market can be expressed in an interest parity condition as below.

$$r(t) - r^*(t) = E_t s(t+1) - s(t) - Z(t)/b \tag{10}$$

Since real net foreign assets $Z(t)$ is assumed to be the cumulative sum of current accounts and it can be assumed that the current account balance in real terms can be adjusted in proportion to the difference between the real exchange rate for the present and an equilibrium real exchange rate (expressed as $\bar{s}(t)$, which balances current accounts), the following formula can be obtained.

$$Z(t) = Z(t-1) + a\,(s(t) - \bar{s}(t)) \tag{11}$$

A partial equilibrium model consisting of the above two equations can be solved rationally, if domestic and foreign real interest rates, $r(t)$ and $r^*(t)$, which are assumed to be exogenous variables, and the expected path for the long-term equilibrium real exchange rate $\bar{s}(t)$ are given. This model can be used as the basis for the empirical analysis of bubbles below.[17]

16. The rational expectation portfolio balance model used in this paper was theoretically formulated by Fukao (1983). See Fukao (1983) for the details of the model.

17. However, unlike the world of models, these variables can be endogenous variables in reality.

2. Variance Bound Test

Based on the above model, the variance bound test can be applied to test the existence of bubbles. This is a test to examine if the fluctuations in actual asset prices are too volatile compared with the theoretical values of variance when asset prices are determined by a rational expectation model.[18]

Let us consider the case in which asset price at time t, X(t), can be expressd as a weighted average of expected fundamentals as below, as the rational expectation solution of an asset price model.[19]

$$X(t) = h\sum_{s=0}^{\infty} k^{s} E_t W(t+s),\ 0<k<1 \tag{12}$$

The asset price at time t which corresponds to the perfect foresight, $X^{PF}(t)$, can be expressed as the following:

$$X^{PF}(t) = h\sum_{s=0}^{\infty} k^{s} W(t+s). \tag{13}$$

In this case, as long as the theoretical model without bubbles is correct, the variance of $X^{PF}(t)$ should be larger than that of the present asset price. Thus,

$$\mathrm{var}(X^{PF}(t)) \geqq \mathrm{var}(X(t)). \tag{14}$$

Conversely, if var(X(t)) is larger than the $\mathrm{var}(X^{PF}(t))$ led from a theoretical model, it can be concluded that the exchange rate fluctuations are subject to the effect of factors other than the model such as bubbles.

Huang (1981) applied the variance bound test to a rational expectation model for exchange rate determination. Using a two-country monetary model, Huang (1981) conducted a variance bound test on the exchange rates of the mark and the U.K. sterling against the U.S. dollar and demonstrated that variances of actual exchange rate are far more volatile than the theoretical variance based on the rational expectation monetary model.

However, since the monetary model employed by Huang was based on a strong

Therefore, when we estimate the model this possibility is taken into consideration.

18. See Shiller (1978), Le Roy and Porter (1981) and Singleton (1980) for the details of the variance bound test.

19. Here, k and W are treated as scalars for the sake of simplicity. However, they can be generalized as vectors. See Singleton (1980) for this point. The argument hereafter is mainly based on Singleton (1980).

assumption that P.P.P. (purchasing power parity) holds continuously, the violation of variance bound was to some extent anticipated.

In this paper, a variance bound test on actual exchange rate fluctuations will be conducted on the model which has already been explained.

Firstly, from simultaneous equations (10) and (11), we have

$$E_t s(t+j+1) - (\frac{a}{b}+2) E_t s(t+j) + E_t s(t+j-1) = E_t w(t+j) \tag{15}$$

where $w(t + j)$ is a linear combination of all the exogenous fundamentals in the model. This difference equation has the characteristic equation below.

$$q^2 - (\frac{a}{b}+2) q + 1 = 0 \tag{16}$$

Assuming the smaller root of this equation to be λ, the other can be expressed as λ^{-1} and we obtain $0 < \lambda < 1 < \lambda^{-1}$. In this case, the real exchange rate can be expressed as below.[20]

$$\begin{aligned} s(t) = {} & (1-\lambda) \sum_{i=0}^{\infty} \lambda^i \bar{s}(t+i) + \lambda \sum_{i=0}^{\infty} \lambda^i (r^*(t+i) - r(t+i)) \\ & - (1-\lambda) \frac{1}{a} Z(t-1) \end{aligned} \tag{17}$$

The equation (17) can be transformed as:

$$\begin{aligned} s(t) = {} & \sum_{i=0}^{\infty} \lambda^i \{(1-\lambda) \bar{s}(t+i) + \lambda (r^*(t+i) - r(t+i))\} \\ & - \frac{1-\lambda}{a} Z(t-1). \end{aligned} \tag{18}$$

This implies that the solution takes the form of a weighted average of expected fundamentals, which provides the basis for a variance bound test. Here, $\bar{s}(t + i)$, a and λ are not directly observable. However, following Fukao (1983), assuming the equilibrium real rate to be constant, and using the estimated parameters of the regression equation which will be explained in Section V (Table 8), it is possible to derive the theoretical value of variance under perfect foresight ($var(X^{PF}(t))$).

Table 7 shows the results of the variance bound test based on the procedure as explained above. According to the results, it cannot be said that the variance of the actual real exchange rate is excessive compared with that based on the rational expectation portfolio balance model. This suggests that, for the monthly data for this period (February 1974 - December 1980), bubbles did not have dominant influence on real exchange rates.

20. See Fukao (1983) for derivation of the equation (17). The equation (17) corresponds to the equation (17) used by Fukao (1983), though notation is different.

Table 7 Results of Variance Bound Test

	Variance
Variance of first differences of actual real exchange rates (var (Ds(t))	0.00102
Variance of first differences of real exchange rates under perfect foresight (var ($Ds^{PF}(t)$)	0.00200

(Calculation period: February 1974 – December 1980)

Notes: 1) Here $\lambda = 0.92$, $a = 13.3$ and $\bar{s} = 6.56$ are assumed.
See Table 8 for the details of the regression equation which provides the basis for this calculation and the data.
2) As for the weighted average of the difference between domestic and foreign interest rates and the real exchange rate, the ratio of the weight for 25 months ($\sum_{i=0}^{24} \lambda^i$) to the total weight ($\sum_{i=0}^{\infty} \lambda^i$) is 87.6%. This implies that the assumption of perfect foresight for two years has an about 90% precision level of perfect foresight for the indefinite future.
3) The reason that the first difference for the real exchange rate is used for calculating variance bound is that it is necessary to secure stationarity.

V. Residual Tests of Bubbles Based on a Rational Expectation Portfolio Balance Model

1. Estimation of the Rational Expectation Portfolio Balance Model

The above results suggest that the role of bubbles in exchange rate fluctuations is mainly of a short-run nature and is of minor influence on the medium- and long-run fluctuations of the exchange rate. However, the above conclusion merely states that medium- and long-run bubbles do not occur frequently; it does not reject the possibility of the existence of a long-lived bubble as a relatively rare phenomenon. In this section we examine such a possibility and, for that purpose, estimate the exchange rate equation as an analytical tool (on a monthly basis).

Firstly, in order to estimate the partial equilibrium model with actual data, the following assumptions are made.[21]

1) Only Japan and the U.S. are considered (The rest of the world is ignored).

21. See Fukao (1983) for the details of these assumptions.

2) Assets in foreign currencies held by the government sector are ignored.
3) Following the pure expectation theory concerning the term structure of the interest rate, anticipation for short-term interest rates in the future is assumed to be reflected in long-term nominal interest rates.
4) The deregulation of the inflow of short-term capital in early 1979 can be expressed as a once for all shift in the structural parameter.
5) The expected rate of inflation can be approximated by an ex-post inflation rate one year after the present period.
6) Japan's net foreign assets can be approximated by cumulative balance of current accounts (a three-month lead for Z(t) to consider exchange rate risks on a contract basis).
7) The future equilibrium real exchange rate is constant.

The equation (18) can be rewritten under the assumptions 1) ~ 7) as below.

$$s(t) = \alpha_0 + \alpha_1 (r(t) - r^*(t)) + \alpha_1' \, \mathrm{DUM} \, (r(t) - r^*(t)) + \alpha_2 Z(t) + \alpha_2' \, \mathrm{DUM} \, Z(t) \quad (19)$$

Here, while $r^*(t + i) - r(t + i)$ in the equation (17) denotes a differential in short-term real interest rates, $r^*(t) - r(t)$ in the equation (19) denotes a differential in long-term real interest rates. This corresponds to the assumption that long-term nominal interest rates reflect the present and expected future values of short-term nominal interest rates in accordance with the pure expectations theory of term structure.

DUM denotes a dummy for the deregulation of the inflow of short-term capital, taking the value of 0 before March 1979 and 1 from April 1979 onward. Table 8 summarizes the results of the regression by using the monthly data and the details of the data used.

1) Compared with the results obtained by Fukao (1983) with the quarterly data, the serial correlation of residuals is higher and the coefficient of determination is lower. However, the coefficients have appropriate sign and are significant.
2) TSLS was also conducted in addition to the OLS, taking into consideration the endogeneity of the variables. However, no significant difference was observed between OLS results and TSLS results (accordingly, mainly the OLS will be used in this paper).

2. Residual Analysis by Kalman Filtering

If bubbles exist, they will be reflected in the exchange rate equation as below.

$$s(t) = \alpha_0 + \alpha_1 (r(t) - r^*(t)) + \alpha_2 Z(t) + c(t) \quad (20)$$

$$E_t c\,(t+1) = \rho\, c(t), \quad \rho > 1 \tag{21}$$

During the process of bubble expansion, the autocorrelation coefficient exceeds 1. However, it becomes a large negative value when the bubble collapses. Accordingly, in order to test the existence of a stochastic bubble from the autocorrelation coefficient of the estimation residual, it is necessary to estimate the autocorrelation coefficient by a time varying parameter technique. A representative technique of such varying parameter estimation is the Kalman filtering.

In this paper, therefore, an autocorrelation coefficient of the residual u(t) of the portfolio balance model shown in Table 8[22] was estimated by Kalman filtering.

$$\begin{cases} u(t) = a_t u\,(t-1) + \varepsilon\,(t) & (22) \\ \text{subject to } a_t = T a_{t-1} + v\,(t) & (23) \end{cases}$$

Here, ε (t) and v(t) are assumed to be serially uncorrelated, mutually independent disturbances. T is an unknown parameter ($T = 1.0$ is usually assumed). In the present analysis, alternative three values, i.e., $T = 0.99$, $T = 1.0$ and $T = 1.01$ are tried to estimate a_t. Since the results of Kalman filtering are sometimes sensitive to the selection of the sample period, several estimations were conducted by changing the starting period of estimation. Figures 3–9[23] show the movements of a_t thus estimated. The conclusions obtained from the estimation results can be summarized as below.

1) In the case of $T = 1.01$, a_t generally exceeds 1 with a possibility of the formula being too advantageous for the bubble hypothesis.

2) In the cases of $T = 1.0$ and $T = 0.99$, the values of a_t are reasonably placed between 0.4 and 1.2 regardless of the estimation period, suggesting that these specifications may be acceptable.

3) The locus of a_t values are considerably affected by the selection of sample

22. The dynamic path of a bubble can be expressed by the equations system consisting of (22) and (23) during the process of its expansion. Of course, it does not mean that the estimated autocorrelation coefficient at the time of its emergence in the market is over 1. It is necessary for the bubble to last for a while before the estimated parameter exceeds 1. Also, Kalman filtering technique is not capable of tracing the drastic fluctuations when the bubble collapses. This formulation is, therefore, only a rough approximation of the bubble process.

23. In each case, data of the first 35 months were used to calculate the initial value, thus the first parameter plotted is for the 36th month. Also, since there was no marked difference between the OLS and TSLS parameters, the OLS residual was used to obtain the autocorrelation coefficients of autoregressive equation of residuals. Figure 9 was added to show that similar results can be obtained with the TSLS residual.

Table 8 Estimation Results of the Rational Expectaton Portfolio Balance Model

Dependent variable: real yen/dollar exchange (home currency unit, logarithm) with the sample period from January 1974 to December 1982.

	Constant term	Real interest rate differential		Real exchange rate risk		$\bar{R}^2$ D.W.
		January (February TSLS) 1974 – December 1982	Dummy for April 1979 – December 1982	January (February TSLS) 1974 – December 1982	Dummy for April 1979 – December 1982	
OLS	6.55976 (293.9)	–0.53937 (2.3)	0.46447 (1.4)	–0.00151 •(8.2)	0.00091 (4.2)	0.5165 0.2210
TSLS	6.56952 (290.0)	–0.56439 (2.3)	0.73668 (2.1)	–0.00163 (8.7)	0.00096 (4.3)	0.5055 0.2269

Notes: 1) Real exchange rate (logarithm) = log (nominal exchange rate in yen) + log (U.S. Manufactured Goods Wholesale Price Index) – log (Japan Manufactured Goods Wholesale Price Index).

2) Real interest rate differential = real interest rate in Japan – real interest rate in U.S. Real interest rate = log {longest term government bond interest rate (annual compound interest rate) + 1} – log {Manufactured Goods Wholesale Price Index (t + 12)} + log {Manufactured Goods Wholesale Price Index (t)}

3) Real exchange risk: the cumulative balance of current accounts (unit: $1 million with a lead of 3 months) based on Japan's net foreign assets outstanding in 1977 divided by the U.S. Manufactured Goods Wholesale Price Index.

4) Dummies for the interest rate differential and the real exchange risk correspond to structural changes due to the deregulation of the inflow of short-term capital in early 1979.

5) The result of TSLS estimation was obtained by using lagged values of the interest rates differential and the real exchange risk (and each dummy for the deregulation of the inflow of short-term capital) as the instrumental variable to estimate endogenous variables.

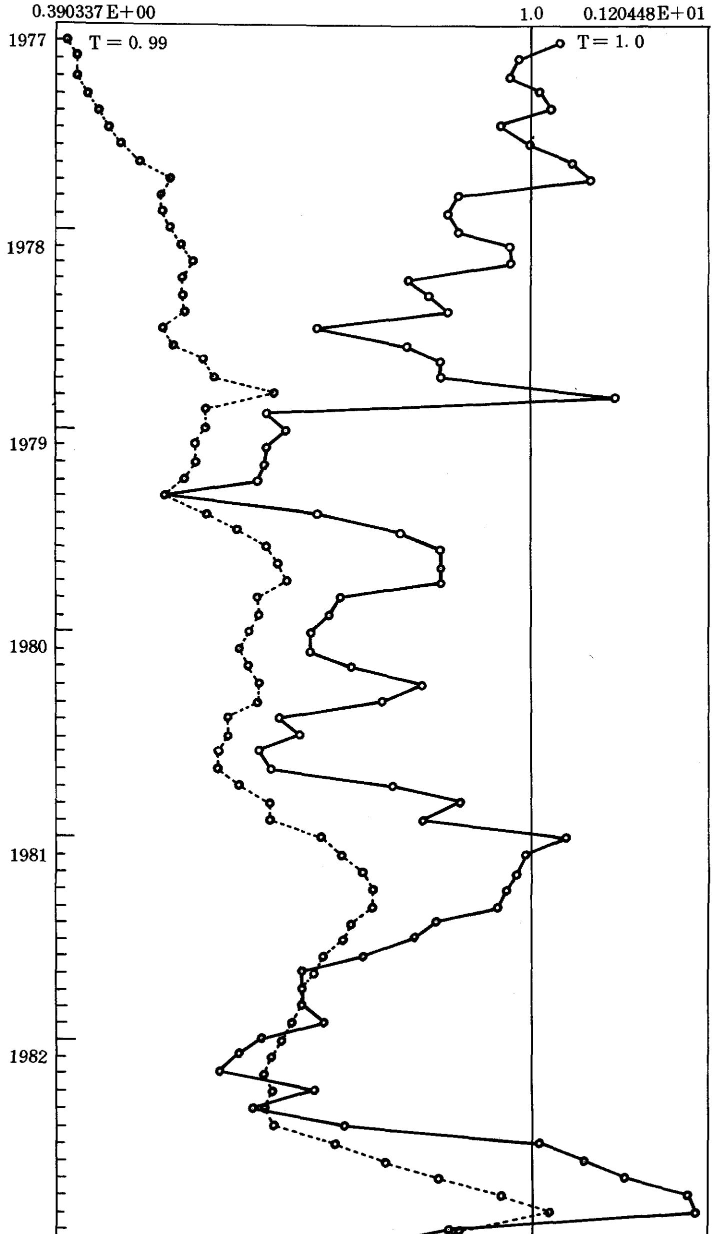
6) The values in brackets are the t-value of coefficients and $\bar{R}^2$ is the coefficient of determination (degree of freedom adjusted).

period. In general, they are volatile when T = 0.99 and stable when T = 1.0.

4) In all cases, a_t increases largely for the period from April to October 1982, exceeding 1 at a peak of October 1982 in many cases. While a_t shows a considerable rise for October 1978, it is rather short-lived and not as significant as is for the period from April to October 1982.

Judging from these results, if a speculative bubble ever existed in the past, it was likely to be in the latter half of 1978 and during the period from April to October 1982. Especially, there are strong signs which indicate the existence of a rather long-lived bubble during the latter period.

Figure 3 Movements of Autocorrelation Coefficient a_t of OLS Residual (1)
(Sample Period: February 1974-December 1982)



Note: The case of T=1.01 was omitted as it exceeded 1 by a considerable margin at times (same with Figs. 4 and 5).

Figure 4 Movements of Autocorrelation Coefficient a_t of OLS Residual (2)
(Sample Period: February 1975-December 1982)

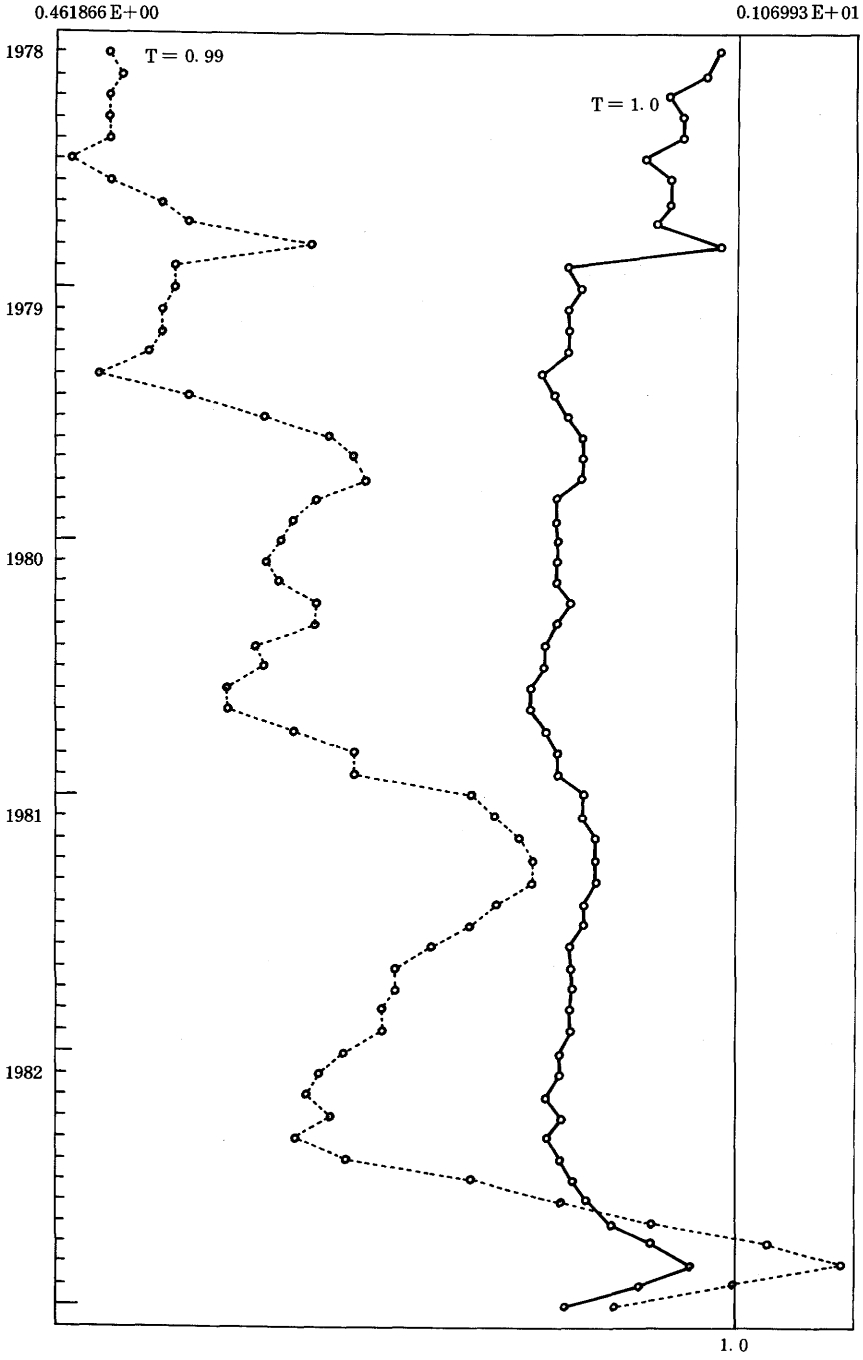


Figure 5 Movements of Autocorrelation Coefficient a_t of OLS Residual (3)

(Sample Period: February 1976-December 1982)

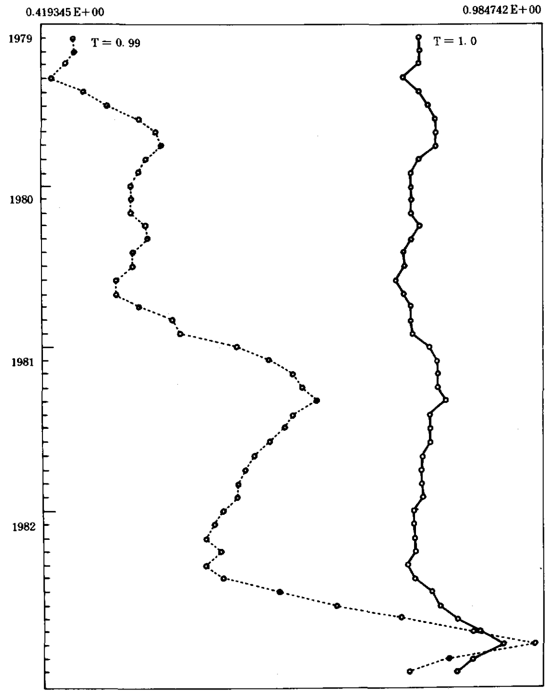


Figure 6 Movements of Autocorrelation Coefficient a_t of OLS Residual (4)

(Sample Period: February 1977-December 1982)

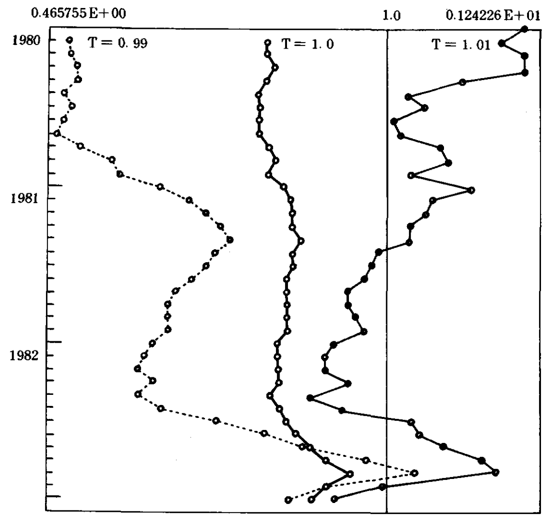


Figure 7 Movements of Autocorrelation Coefficient a_t of OLS Residual (5)
(Sample Period: February 1978-December 1982)

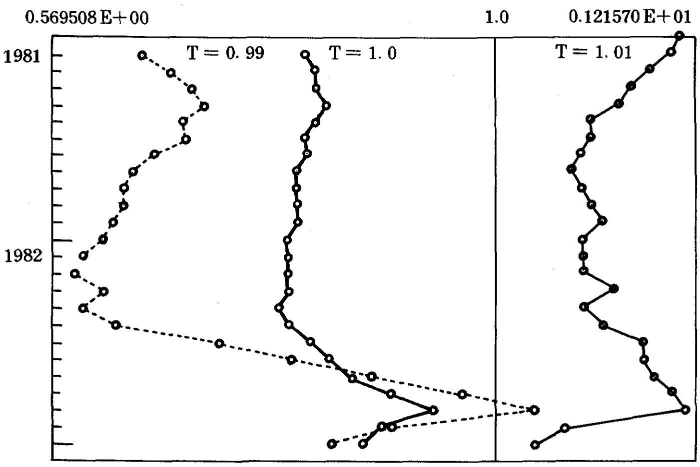


Figure 8 Movements of Autocorrelation Coefficient a_t of OLS Residual (6)
(Sample Period: February 1979-December 1982)

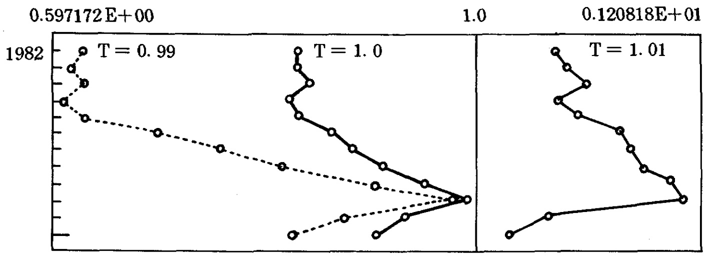
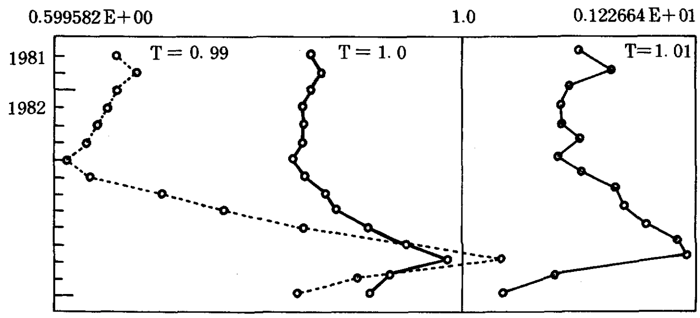


Figure 9 Movements of Autocorrelation Coefficient a_t of TSLS Residual
(Sample Period: April 1979-December 1982)



VI. Direct Estimation of the Bubble

1. Estimation Procedure and Results

In this section the exchange rate function with a simple stochastic bubble is estimated by introducing dummy variables for the period from April to October 1982 when the existence of a bubble is strongly suspected.

The specification of simple stochastic bubble employed is:

$$c(t+1) = \begin{cases} \left(\dfrac{\lambda^{-1}}{1-\pi}\right) c(t) & \text{probability } 1-\pi \\ 0 & \text{probability } \pi . \end{cases} \tag{24}$$

(where π is the probability for the collapse of speculation)

The portfolio balance model (the equation (24)) implies the following relationship between the parameter α_1 and λ.

$$\alpha_1 = \frac{\lambda}{12(1-\lambda)} \tag{25}$$

Thus the value of λ can be obtained from the parameter in the model, which can then be used to construct dummy variables.[24]

The actual estimation procedure can be summarized as in Figure 10 and the results obtained by this procedure are given in Table 9.

As Table 9 clearly shows, it seems that the appreciation of the dollar during the period from April to October 1982 can be explained best when a stochastic bubble with around 10% subjective probability for the collapse of speculation for each period is assumed. Figure 11 (p.29) presents the results of estimation when the subjective probability for the collapse was assumed to be 9%. It traces the process of dollar appreciation during the period from April to October 1982 extremely well.

However, the collapse of a bubble is not as instant as assumed by a simple stochastic bubble specification and the large margin of error for the upsurging of the yen suggests the possibility that the formulation of the process of a simple stochastic bubble is still inadequate.

2. Comparison of Bubble Dummies and Ad Hoc Dummies

One possible criticism of such direct estimation is the argument that it is self-

24. See Fukao (1983) for the details of the relationship betweeen α_0, α_1 and λ.

Figure 10 Estimation Procedure of Stochastic Bubble by Using Dummy Variable

Estimation of the parameter with a model without bubble (results of OLS in Table 8).

↓

Trial calculation of the λ from the structural parameter α_1.

↓

Calculation of the values of dummy variables (Bubb) as below by using the trial value of λ^{-1} and the assumed value of the subjective probability of the collapse of the bubble (π).

$$
\text{Bubb} = \begin{cases}
0 & \cdots\cdots\cdots \text{ February 1974 – March 1982} \\
1 & \cdots\cdots\cdots \text{ April 1982} \\
(1-\pi)^{-1}\lambda^{-1} & \cdots\cdots\cdots \text{ May 1982} \\
(1-\pi)^{-2}\lambda^{-2} & \cdots\cdots\cdots \text{ June 1982} \\
(1-\pi)^{-3}\lambda^{-3} & \cdots\cdots\cdots \text{ July 1982} \\
(1-\pi)^{-4}\lambda^{-4} & \cdots\cdots\cdots \text{ August 1982} \\
(1-\pi)^{-5}\lambda^{-5} & \cdots\cdots\cdots \text{ September 1982} \\
(1-\pi)^{-6}\lambda^{-6} & \cdots\cdots\cdots \text{ October 1982} \\
0 & \cdots\cdots\cdots \text{ November and December 1982}
\end{cases}
$$

↓

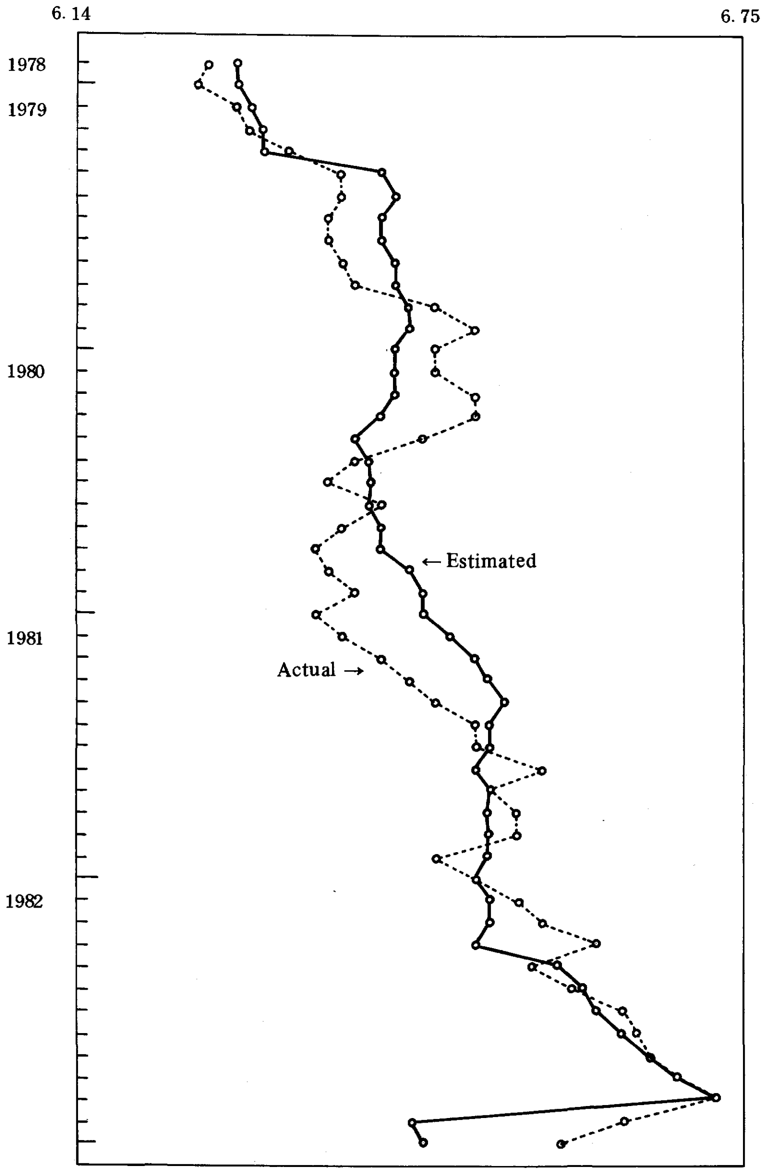
New trial values of α_1 and λ are obtained from the results of estimation by using a bubble dummy and Bubb values are then divided by λ^{-1} to ascertain if a posterior π does not deviate from the assumed value of π.

Table 9 Results of Estimation with Bubble Dummy Added

Assumed probability of the collapse of the bubble	The bubble during April – October 1982	Constant term	Real interest rate differential		Real exchange risk		$\bar{R}^2$ D.W.
			January 1974 – December 1982	Dummy after April 1979	January 1974 – December 1982	Dummy after April 1979	
(Case 1) 2.0%	0.126 (7.2)	6.558 (359.9)	–0.529 (2.7)	0.316 (1.1)	–0.015 (10.0)	0.006 (3.1)	0.678 0.41
(Case 2) 6.0%	0.113 (7.3)	6.559 (360.9)	–0.531 (2.7)	0.327 (1.2)	–0.015 (10.0)	0.006 (3.2)	0.679 0.41
(Case 3) 9.0%	0.097 (7.3)	6.559 (361.4)	–0.533 (2.8)	0.343 (1.2)	–0.015 (10.1)	0.006 (3.2)	0.680 0.42
(Case 4) 12.0%	0.084 (7.3)	6.559 (361.2)	–0.535 (2.8)	0.358 (1.3)	–0.015 (10.1)	0.006 (3.2)	0.680 0.42
(Case 5) 16.0%	0.066 (7.2)	6.560 (359.9)	–0.538 (2.8)	0.380 (1.4)	–0.015 (10.0)	0.006 (3.3)	0.671 0.43

Note: The basic structure and data for the estimated equation are the same as those for Table 8.
Estimation method: OLS.

Figure 11 Results of Estimation with Bubble Dummy



Notes: 1) The subjective probability for the collapse of speculative bubble is assumed to be 9% for each period.
2) The sample period is from January 1974 to December 1982 with plotting for the period from November 1978 onward.

evident that if we add dummy variables for the period which the explanatory power of fundamentals is not sufficient, its superficial fitness is bound to improve. One answer to this can be obtained by comparing the case of adding a dummy approximating a stochastic bubble with that of adding an ad hoc dummy expressing the specific independent factors for each period.

From such a point of view the dummy variable approximating a stochastic bubble and that approximating the "news" of each period (the result of estimation incorporating the latter is given in Table 10) were compared by an F test. As shown in Table 11, when the "news" of each period was approximated by the ad hoc dummies, the F-value reached 7.3, 2.6 times the 1% significance level, suggesting an improvement in fitness of the estimated equation. However, the F-value concerning the dummy variable approximating a stochastic bubble was 53.8, 7.8 times the 1% significance level. Judging from these results, the stochastic bubble dummy seems to be superior to the ad hoc dummies in terms of explanatory power.

Next, both the stochastic bubble dummy and the ad hoc dummies were added to the same equation. To avoid excessive multi-collinearity, the ad hoc dummies are added only for the three months, i.e., August, September and October 1982, for which they are highly significant (the t-values of the coefficients exceeding 3). As shown in Table 12, while the stochastic bubble dummy continues to be significant, the ad hoc dummies completely lose their significance. This strongly suggests the higher explanatory power of the stochastic bubble dummy compared with the ad hoc dummies.

3. Effects of Change in Sample Period

The second criticism which may be raised against the results under Section VI.1 is that estimation is made by assuming the stability of the model. In other words, if a change in sample period results in a major change in α_1, or λ^{-1}, the above argument will lose much of its persuasiveness. In order to ascertain this point, the estimation period was shortened so that the results of estimation would become more sensitive to changes in data, and the estimation period was also moved to examine the changes in α_1, λ^{-1} and coefficients for dummy (Table 13, P.33). The following observations were thus made.

1) The structural parameters such as α_0 and α_1, etc., are stable (in particular, α_1 showed a minimum of 0.89 and a maximum of 0.92 and corresponding λ^{-1} in either case was approximately 1.09).

2) The bubble dummy coefficient was stable between 0.081 and 0.100).

It can thus be concluded that the results of estimation conducted up to Section VI.2 are rather robust with regard to the selection of the sample period.

Table 10 Results of Estimation with Ad Hoc Dummies Added for April-October 1983

		Coefficient	
Constant term		6.559	(350.8)
Real interest rate differential	For the whole period	–0.535	(–2.7)
	Dummy from April 1979 onward	0.343	(1.2)
Real exchange risk	For the whole period	–0.00151	(–9.8)
	Dummy from April 1979 onward	0.00059	(3.1)
Independent dummies	April 1982 dummy	0.072	(1.1)
	May 1982 dummy	0.110	(1.7)
	June 1982 dummy	0.163	(2.5)
	July 1982 dummy	0.177	(2.7)
	August 1982 dummy	0.196	(3.0)
	September 1982 dummy	0.243	(3.7)
	October 1982 dummy	0.287	(4.3)
$\bar{R}^2$/D.W.		0.661/0.39	

Note: The basic structure and data for the estimated equation are the same as those for Table 8.
The figures in brackets are t-values.

Table 11 F-Test on Stochastic Bubble Dummy and Ad Hoc Dummies

	Stochastic bubble dummy	Ad hoc dummy
Distribution followed	F distribution of degree of freedom (1,102)	F distribution of degree of freedom (7,96)
F-value	53.8	7.3

(Memo)
Level of Significance of F Distribution

Degree of freedom	1% significance	5% significance
(1,100)	6.90	3.94
(1,125)	6.84	3.92
(7,100)	2.82	2.10
(7,125)	2.79	2.08

Table 12 Estimation with Both Stochastic Bubble and Ad Hoc Dummies

		Coefficient	
Constant term		6.559	(355.6)
Real interest rate differential	For the whole period	–0.533	(–2.7)
	Dummy from April 1979 onward	0.339	(1.2)
Real exchange risk	For the whole period	–0.00151	(–9.9)
	Dummy from April 1979 onward	0.00059	(3.1)
Stochastic bubble dummy*		0.106	(3.9)
Independent dummies for each period	August 1982 dummy	0.006	(0.1)
	September 1982 dummy	0.022	(0.3)
	October 1982 dummy	0.030	(0.3)
R^2/D.W.		0.670/0.42	

* Corresponding Case 3 in Table 9.

Note: The basic structure and data for the estimated equation are the same as those for Table 8. The figures in brackets are t-values.

4. Adjustment of Serial Correlation of Errors

Another problem concerning the results of estimation is the low D.W. ratio. As has been examined in Section IV, the existence of a bubble is reflected in the model in the form of serial correlation of residuals. It is therefore apparent that a mechanical adjustment of serial correlation of residuals may also remove the influence of the bubble. However, since it is extremely difficult to distinguish the serial correlation of residuals due to a bubble from that due to other reasons, the generalized least squares method (GLS) using the Cochran-Orcutt method was mechanically applied (Table 14).

One possible justification of this estimation procedure is that since the bubble which we are concerned with is an exceptional phenomenon, the adjustment of serial correlation for the whole period may not affect the significance of the bubble dummy so much. In fact, as shown in Table 14, the coefficient of the stochastic bubble dummy is still significant even if we adjust the serial correlation by using GLS.[25]

25. However, the coefficient of the dummy variable for the deregulation of the inflow of short-term capital concerning real interest rate differentials does not satisfy expected sign.

Table 13. Changes in Estimation Period and Corresponding Changes in Structural Parameter

Estimation period	Constant term (α_0)	Real interest rate differential (α_1)	Real exchange risk (α_2)	Bubble dummy*
March 1977 – March 1982	6.56	– 0.90	– 0.106	–
April 1977 – April 1982	6.57	– 0.90	– 0.110	0.081
May 1977 – May 1982	6.56	– 0.90	– 0.103	0.090
June 1977 – June 1982	6.57	– 0.90	– 0.106	0.100
July 1977 – July 1982	6.56	– 0.90	– 0.106	0.098
August 1977 – August 1982	6.56	– 0.90	– 0.107	0.093
September 1977 – September 1982	6.56	– 0.91	– 0.106	0.089
October 1977 – October 1982	6.56	– 0.92	– 0.104	0.085
November 1977 – November 1982	6.56	– 0.90	– 0.095	0.082
December 1977 – December 1982	6.56	– 0.89	– 0.087	0.081

* Corresponding to Case 4 in Table 9.

Note: α_1 and α_2 are net coefficient values adding the value of the dummy for the deregulation of the inflow of short-term capital.

Table 14 Direct Estimation of Stochastic Bubble with GLS

		Coefficient	
Constant term		6.576	(110.0)
Real interest rate differential	For the whole period	–0.77	(–1.5)
	Dummy from April 1979 onward	–0.19	(–0.3)
Real exchange risk	For the whole period	–0.00119	(–2.3)
	Dummy from April 1979 onward	0.00029	(1.3)
Dummy for stochastic bubble*		0.031	(2.4)
Estimated value of serial correlation		0.911	
$\bar{R}^2$/D.W.		0.136/1.95	
($\bar{R}^2$ with serial correlation of residual adjusted)		0.917	

* Corresponding to Case 3 in Table 9.

Note: The basic structure and data for the estimated equation are the same as those for Table 8.

The figures in brackets are t-values.

VII. Anecdotal Evidence – How the Appreciation of the Dollar in the Latter Half of 1982 Was Perceived

The empirical analysis conducted so far points to the possibility that the appreciation of the dollar during the period from April to October 1982 was reflecting a bubble based on self-fulfilling expectations. At the same time, however, news of the "betrayed expectation", can produce a similar dynamic path in the exchange rate. Therefore, let us supplement the empirical analysis by reviewing how the appreciation of the dollar during the above period was actually received in economic circles.

The prevailing view until around April (when the yen stood at around ¥240 against the dollar) was that it was due to the interest rates differential. The newspapers also gave prominence to such a viewpoint in March and April, for example, a survey conducted by the Fuji Bank concluding that "the depreciation of the yen is primarily due to the difference in long-term interest rates"; the results of a simulation study conducted by the Economic Planning Agency with a world economy model concluding that "the 5.5% interest rates differential between U.S. and Japan results in the depreciation of the yen by 10%" (the Nihon Keizai, March 4, 1982 and the Yomiuri, April 8, 1982).

However, in June when the yen fell below ¥250 against the dollar, interest rates could no longer explain the overall appreciation of the dollar and some argued that "there is no reason for the depreciation of the yen at this stage and we can only conclude that the present market is intoxicated with the appreciation of the dollar"(the Nihon Keizai, June 23, 1982). In early August when the yen fell to ¥259 against the dollar, the "interest rate differential" view was replaced by the view that "the dollar appreciates because the expectation that the dollar to appreciate has taken root" (the Asahi, August 6, 1982).

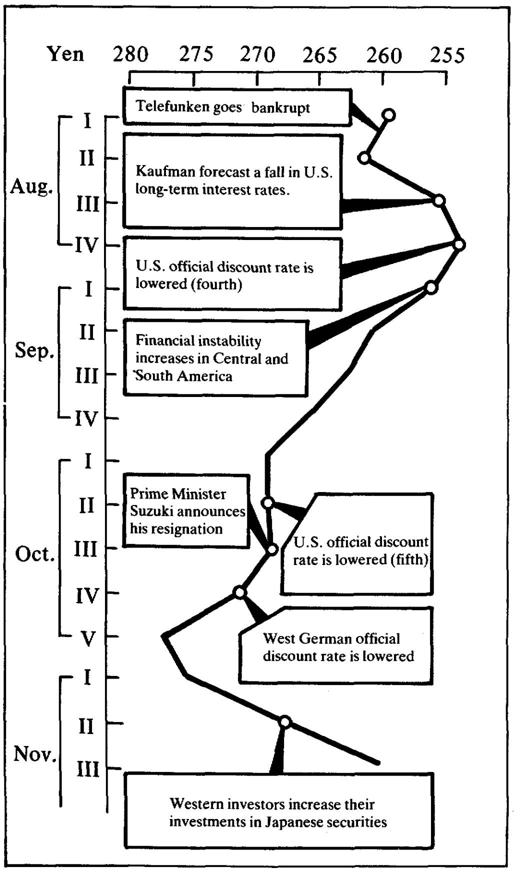
At the end of September when the exchange rate declined further to below ¥270, the Nihon Keizai carried an article entitled "Industries Expect the Weak Yen to Stay" and predicted that the weak yen would continue for the time being. The following view expressed by trading firms was no other than that of a bubble itself: "It is a matter of time for the yen to fall below ¥270. The present market has an impetus. However, it does not mean that the present high dollar rate will last forever. The exchange rate of ¥240 is appropriate for the yen in view of the fundamentals of the Japanese economy. The focal point for the time being is when the yen will find a chance for making an upsurge" (the Nihon Keizai, September 29, 1982).

When the yen rate approached ¥280 to the dollar at the end of October, "increasing pessimism about the future of the yen" was reported (the Nihon Keizai, October 26, 1982). In the meantime, chart analysis designed to forecast the future from the past movements of the exchange rate drew attention, inviting the comment: "It is like a beauty contest. If everyone believes in one direction, the market may move in that direction" (the Nihon Keizai, September 29, 1982).

However, the yen then abruptly turned upward and recovered to ¥259 against the dollar on November 29. It is worthy to note that there was a lack of "news" at that time, which would explain the upsurge of the yen. In fact, the figure carried by the Nihon Keizai of November 20, showing the movements of the yen-dollar rate and listing major "news" during the period covered (Figure 12), failed to show any news which brought about a change in the exchange rate.

The above developments imply that the appreciation of the dollar/depreciation of the yen and subsequent bouncing cannot be explained fully by a scheme such as the expectations on specific fundamentals and their modification (betrayed expectations). Judging from such anecdotal evidence, it can be concluded that the depreciation of the yen during the period from April to October 1982 strongly suggests a bubble phenomenon.

Figure 12 Movements of the Yen Rate and the "News"
(at a week's end basis on the Tokyo exchange market; $1 = Yen)



VIII. Conclusion

The results of empirical tests so far made may be summarized as below.

1) Analysis based on a random walk model, analysis assuming an efficient market and the variance bound test assuming a portfolio balance model are invariably based on strong assumptions and none of them can provide decisive conclusion by itself. However, all the results obtained show that it is difficult to assume that speculative bubbles frequently occurred in the foreign exchange market for considerably long periods of time. If they did occur frequently in the foreign exchange market, they were presumably of a short-run nature.

2) The results of supplemental analysis based on a portfolio balance model are coordinate with the above conclusion. However, they also show that the appreciation of the dollar during the period from April to October 1982 can be interpreted as an exceptionally long speculative bubble.[26] This conclusion is by large supported by the results of direct estimation of a bubble by using dummy variables. Such results are also consistent with the anecdotal evidence of this period.

Such results of empirical tests have the following policy implications. There seems to be a possibility of a substantial deviation of the exchange rate from fundamentals for a considerably long period, which is caused solely by speculative expectations. Such speculative expectations are of a self-fulfilling nature and it is necessary for the authorities to show a firm stance so as to change the market expectations in such a case.[27]

Appendix 1. On the Theoretical Framework of Speculative Bubbles

This section is intended to explain the theoretical framework for rational bubbles to the extent necessary for testing the existence of bubbles on the foreign exchange market.[28]

26. In the past there was hardly any empirical study of bubbles on the foreign exchange market. However, Woo has recently published a discussion paper (Woo, 1984). Though Woo's approach is different from the present author's, the results are completely coordinate with those obtained in this paper in that bubbles lasting for about five months can exist as rare phenomena. However, the movements of the yen-dollar rate in 1982 were not included in the estimation period.

27. See Okina (1984) for this point.

28. See Okina (1984) for the details of the theory of speculative bubbles and its application to the

A bubble on the asset market denotes the deviation of asset prices from fundamentals due to self-fulfilling speculation. Let us consider a simple rational expectation model.

$$x(t) = aE_t x(t + 1) + bZ(t), \quad 0 < a < 1 \tag{A1}$$

Here, x(t) is the price of the asset X at t, $E_t x\,(t + 1)$ denotes the expected value of the asset price at time $t + 1$ given the information available at time t and Z(t) is "a composite fundamental" at time t.

The asset price reflecting only fundamentals (to be expressed as the fundamentals price, $\bar{x}(t)$) can be expressed as below:

$$\bar{x}(t) = b\sum_{i=0}^{\infty} a^i E_t Z(t + i). \tag{A2}$$

If the bubble at time t is expressed as c(t):

$$x(t) = \bar{x}(t) + c(t). \tag{A3}$$

And a rational bubble has to satisfy the following relation:

$$c(t) = aE_t c(t + 1). \tag{A4}$$

(A4) corresponds to a homogeneous solution of the difference equation (A1).

In general, a rational bubble corresponds to an unstable root of the homogeneous solutions to the difference (differential) equation concerning the determination of asset prices. Various stochastic formulations are possible for solutions to such rational bubbles. The following stochastic bubble mechanism can be considered as a representative formulation.

$$c(t + 1) = \begin{cases} u(t+1) & \text{probability} \quad \pi \\ \dfrac{1}{1-\pi}\dfrac{1}{a}c(t) + u(t+1) & \text{probability} \quad 1 - \pi \end{cases} \tag{A5}$$

where π denotes the probability of the collapse of self-fulfilling expectations during each period, $u(t + 1)$ a disturbance term corresponding to unexpected changes in speculative expectations ($E_t u(t + 1) = 0$).[29]

foreign exchange market.

29. Strictly speaking, there is a possibility of π being variable. This possibility is ignored in this paper.

However, this formulation is too complicated to test empirically. Thus, a simpler stochastic model such as shown below is considered, though it is theoretically rather insufficient.

$$c(t+1) = \begin{cases} 0 & \text{probability} \quad \pi \\ \dfrac{1}{1-\pi}\dfrac{1}{a}c(t) & \text{probability} \quad 1-\pi \end{cases} \qquad \text{(A6)}$$

Direct estimation of bubbles in this paper is made according to this formulation (This corresponds to the assumption that $u(t+1)$ in (A5) is negligible).

Appendix 2. On the Random Walk Hypothesis

With regard to the exchange rate movements, the following empirical facts are well-known: 1) the spot exchange rate fluctuates roughly according to a random walk on a short-run; and 2) no structural model outperforms the random walk model. As regards these points, Mizoguchi and Kariya (1983) take the standpoint that fundamental factors for exchange rate determination are too complex to be described with a simple structural model. The random walk hypothesis for asset prices (exchange rate) is also supported by the standpoint of the efficient market hypothesis. That is, of those factors which affect asset prices, those which are predictable are all incorporated into asset prices in advance by market participants. In this case, only unanticipated factors affect asset prices. It is thus assumed that this is reflected by asset prices which approximately follow a random walk on a short-run basis. See Mussa (1979) and Mizoguchi and Kariya (1983) for the point that the spot rate approximately follows a random walk. Also see Meese and Rogoff (1983) for the comparison of structural and random walk models with regard to forecasting performance.

Appendix 3. On a Runs Test

A runs test is inherently a test for randomness or the absence of serial correlation. The advantage of this test is that in contrast to the common Durbin-Watson test for serial correlation, a runs test is distribution free.

In the Durbin-Watson test, the so-called d statistics is defined as:

$$d = \sum_{t=2}^{n} (e(t)-e(t-1))^2 / \sum_{t=1}^{n} e(t)^2 \tag{A7}$$

$$\text{where } e(t) = \beta e(t-1) + u(t) \tag{A8}$$

and u(t) is assumed to be distributed as N(0, var(u)). A low value of d indicates the presence of serial correlation in the series given the assumption that the random error term u(t) is normally distributed. If, however, the serial correlation is due to a bursting bubble, a large outlier will dramatically increase the value of d when the bubble bursts, even though d may be small while bubble grows. Use of the Durbin-Watson test under a bubble-related leptokurtic distribution will therefore give misleading results.

The runs test does not rely on the normality assumption when testing for randomness in the data. Take for example the coin tossing sequence HHHTTHTTTT which consists of R $=$ 4 runs. For a given sample number, a small R indicates clustering of similar observations while a value of R near one-half the sample size indicates regular randomness. The distribution of the number of runs can be approximated by a normal distribution for a large sample under the null hypothesis of randomness. Assuming the number of runs r, and the number of data n($>$ 40), Z approximately follows a normal distribution.

$$Z = \frac{\left| r - (\frac{n}{2}+1) \right|}{\sqrt{\frac{n(n-2)}{4(n-1)}}} \tag{A9}$$

Runs are obtained not only from dichotomous data (such as coin tossing) but also from measured data that can be divided into two groups by some value such as the median or the mean. Since a run is a sequence of realizations of a variable which belongs to the same "side", the increment of the bubble causes observations of the exchange rate to occur on the same "side", while the bubble grows and then to reverse when the bubble bursts. Therefore, the existence of bubbles in the market causes a run to be longer and the total number of runs to be smaller on average than in the no-bubble case. The power of the runs test is also dependent upon the value of π . For a formal discussion, see Blanchard and Watson (1982).

Appendix 4. On a Chi-square Test Using the Efficient Market Hypothesis

In this paper the importance of bubbles was examined by a chi-square test based on the efficient market hypothesis. The results obtained were disadvantageous for bubbles. However, it is possible that such results were due to the efficient market hypothesis which is tested with the existence of bubbles as a joint hypothesis. While it is not easy to examine this point, if we assume a simple stochastic bubble, the following procedure can be employed.

It can be assumed that since a simple stochastic bubble is based on an instant collapse, even when daily data are used instead of weekly data, there is a tendency for outliers to be detected as in the case of weekly data. Table A-1 shows the ratios between weekly rates of change in the exchange rate and the maximum value of the daily rate of change during the week concerned,[30] classifying them by runs and forward premiums to show if they are consistent with bubbles.

The results given in Table A-1 show that when the criterion based on runs is

Table A-1 Ratios between Maximum Daily Rates of Change and Weekly Rates of Change Classified by Signs for Rates of Change and Future Premiums

	Consistent with bubbles	Not consistent with bubbles
Criterion based on rates of change in the exchange rate	63%	53%
Criterion based on the relationship between future premiums and the exchange rate	55%	59%

Notes: 1) If the maximum daily rate of change exceeds the weekly rate of change, it is replaced by 100% in calculating the average (to modify the influence of a single extraordinary observation).
2) In either case, consistency was examined on a weekly basis.

Sources: Weekly data: Harris Bank.
Daily data: International Finance Statistics.

30. Assuming the daily exchange rate to be S(t), the movements of the exchange rate in one week can be expressed as below.

Time	Day of week	Exchange rate
t_0	Friday	$S(t_0)$
t_2	Monday	$S(t_1)$
t_2	Tuesday	$S(t_2)$
t_3	Wednesday	$S(t_3)$
t_4	Thursday	$S(t_4)$
t_5	Friday	$S(t_5)$

satisfied, the rates of change in daily data tend to be high and are consistent with stochastic bubbles. In contrast, even when the criterion based on forward premiums is satisfied, the rates of change in daily data are in fact low and are not consistent. Therefore, judging from this, we suspect that there is a possibility that the rejection of stochastic bubbles by chi-square test may be due to the risk neutral efficient market hypothesis.[31]

In addition, contrary to the efficient market hypothesis, there is another possibility, observed in the weekly bulletin of the Harris Bank, that the forward rate does not reflect the expected exchange rate. For instance, the issue of April 4, 1980, commented that although there was "widespread speculation in the market that mark will shortly ease to the 2.00 level", the forward rate failed to reflect this "widespread speculation." (Table A-3).

This observation seems to suggest that it is risky at the present stage to derive a strong conclusion from the joint test of bubbles by using the efficient market hypothesis.

Table A-3 Future Rates of Mark as of April 4, 1980

Spot.	1.9700 Mark/Dollar
One-month future	1.9527
Three-month future	1.9253
Six-month future	1.8900
Twelve-month future	1.8481

Source: Harris Bank.

In this case, "the ratio between the weekly exchange rate changes and the maximum daily rate of change during the week concerned" means the following:

$$\frac{\underset{i=1,\cdots,5}{\text{Max}} \{S(t_i) - S(t_{i-1})\}}{S(t_5) - S(t_0)}.$$

31. Strictly speaking, there are normally two problems in using daily data. Firstly, such factors as readjustment of EMS, (see Table A-2). The second is the "day of the week effect", i.e., the rate of change in the exchange rate tends to be high on a specific day of the week compared with other days (see McFarland, Pettit and Sung 1982, for the details of the day of the week effect of daily data). Of these two problems, the former can be ignored for the present purpose because of the small number of such adjustments included in the sample period and the day of the week effect can be ignored because it is relatively small in the present analysis.

Table A-2 Events which may Directly Affect the Exchange Rate

January 19 (Saturday) 1974
French franc drops out of EEC snake.

March 5 (Friday) 1976
French franc drops out of EEC snake.

June 8 (Tuesday) 1976
5 billion dollar standby credit to U.K. is announced.

November 1 (Friday) 1978
President Carter, U.S. Treasury and Fed. announed measures to stabilize and strengthen dollar.

March 23 (Monday) 1981
Italian lira devalued 6% in EMS.

October 4 (Sunday) 1981
In an EMS realignment, mark and guilder revalued by 5.5% and French franc and lira devalued by 3%.

June 14 (Monday) 1982
In an EMS realignment, French franc devalued by 5.75%.

Source: R. Solomon (1982)

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