Accelerated Investment and Credit Risk under a Low Interest Rate Environment: A Real Options Approach

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Empirical studies have found that a low interest rate environment accelerates firms' investment and debt financing, leading to subsequent balance-sheet problems in many countries in recent years. We examine the mechanism whereby firms' debt financing and investment become more accelerated and the credit risk rises under a low interest rate environment from the perspective of a real options model. We find that firms tend to increase debt financing and investment not only under strong expectations of continued low interest rates but also when there are expectations of future interest rate increases, and such behavior causes higher credit risk. We also find that when future interest rate rises are expected, the investment decisions vary depending on how firms incorporate the possibility of future interest rises. Specifically, myopic firms make "last-minute investments" based on concerns over future interest rate hikes, and this behavior increases their credit risk. In contrast, economically rational firms choose to decrease their investments, carefully considering the likelihood of future interest rate hikes.

Keywords: Low interest rate environment; Investment behavior; Credit risk; Real options model; Corporate finance; Time-inconsistent discount rate; Behavioral economics

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I. Introduction

Empirical studies have found that a low interest rate environment accelerates firms' investment and debt financing, leading to subsequent balance-sheet problems in many countries in recent years.[1] This paper seeks to analyze the mechanism whereby firms' debt financing and investment become more accelerated and credit risk rises under a low interest rate environment from a theoretical perspective.

This paper addresses two issues. The first is to confirm whether expectations of continued low interest rates[2] have the effect of accelerating firms' investments and increasing their credit risk. The second is to confirm whether firms make last-minute investments[3] while the debt financing costs are still favorable when there are expectations of a future increase in interest rates, and whether such behavior increases credit risk.

We analyze these issues using a real options framework. Specifically, we extend the model in Sundaresan and Wang (2007), which considers the relation between investment and credit risk. Former real options models, such as the model in Dixit and Pindyck (1994), give a clear answer to the question of when investments should be made. But they do not address the question of how much debt should be issued to make these investments. Sundaresan and Wang (2007) answer this question by incorporating corporate finance theory as represented by Leland (1994), enabling examination of the relationship between investment and capital structure.[4]

However, Sundaresan and Wang (2007) assume that interest rates are fixed into the future. So while their model can be used for examinations under continuous low interest rates, it cannot be used to consider the impact under expectations that interest rates will rise in the future. To address this problem, we extend the model in Sundaresan and Wang (2007) for analysis under a variable interest rate model using the technique developed by Grenadier and Wang (2007).

We show that in such models myopic firms concerned over the possibility of future interest rate hikes may rush to make last-minute investments. Specifically, we analyze the behavior of both myopic firms and economically rational firms under a rising interest rate environment to see the differences in their debt financing and investment behavior. The former make investment decisions only considering the impact of interest rate hikes until the point in time when they raise funds and make investment. In contrast, the latter also consider the possibility of interest rate hikes after they raise funds and make investment. The former are myopic in the sense that they fail to consider the possibility that debt financing costs may increase if they need to refinance in the future.

1. For example, see Okina, Shirakawa, and Shiratsuka (2001) and Hoshi (2001) regarding increased investment during the Japanese bubble economy period.
2. In this paper, "interest rates" refers to risk-free interest rates, and we distinguish this from "interest payments," which are the interest rates that companies actually pay on borrowings (the interest rates reflecting credit risk). Also, in this paper the term "low interest rates" refers to risk-free rates that are around the same level as the economic growth rate and the corporate profit growth rate. Low interest rates would be around zero under the recent low growth rate environment in Japan, and around 6–8 percent during high-growth periods such as during the Japanese bubble economy.
3. The meaning of "last-minute investments" as used in this paper is explained in Section III.C.
4. This line of research was continued in many subsequent papers including Mauer and Sarkar (2005), Lyandres and Zhdanov (2006a, b), Zhdanov (2007, 2008), Nishihara and Shibata (2009), Yagi *et al.* (2008), and Egami (2009).

This phenomenon has been noted in the field of behavioral economics,[5] for example, managers who change posts every few years tend to pay little heed to cost increases after the ends of their terms of office.

In this paper we divide firms' investment behavior into the two categories described above. The fundamental difference between the two lies in whether the discount rate used for the investment decision is set consistently or inconsistently before and after the investment. In other words, the behavior of economically rational firms meets the requirements of standard finance theory, without arbitrage between short-term and long-term interest rates. It incorporates all possibilities of future interest rate increases into decisions at the investment time, so the discount rate does not change over time. In this case, the firms make investment decisions using a uniform discount rate whereby debt financing costs remain unchanged not only while the present managers are in office, but also thereafter. In this sense, this may be considered "time-consistent" investment behavior.

In contrast, myopic firms do not consider cost increases under subsequent refinancing when they make investment decisions. Therefore, their decisions are made under inconsistent settings that accept changes in the discount rate over time. In such cases, the present managers accept future cost increases from refinancing after their terms of office that lower equity and firm value. Because the managers make investment decisions emphasizing performance during their own terms of office, the discount rates do not meet the above-mentioned non-arbitrage condition, but rather accept lower short-term and higher long-term discount rates. The optimal investment decision under these settings is called "time-inconsistent" investment behavior,[6] and this has become the subject of behavioral economics research in recent years.

We find the following results. First, we find that under a continuous low interest rate environment firms invest even if the investment profits are low, because the low interest rate lifts their equity value and makes their funding conditions advantageous. But this results in a high ratio of debt to profits. Moreover, this behavior increases the credit risk, that is, the more investments are made under a low interest rate environment, the shorter the time interval from investment to bankruptcy.

Second, when future interest rate rises are expected, the investment decisions vary depending on how firms incorporate this possibility of future interest rises. Specifically, myopic firms rush to make last-minute investments based on concerns over future interest rate hikes, and this behavior increases their credit risk. In contrast, economically rational firms choose to decrease their investments, carefully considering the likelihood of future interest rate hikes.

The rest of this paper is organized as follows. In Section II, we consider investment and debt financing under continuous low interest rates based on Sundaresan and Wang (2007). In Section III, we extend their paper using the technique in Grenadier and Wang

5. Prior research on this discount rate issue includes Laibson (1997), Harris and Laibson (2003), and Grenadier and Wang (2007).

6. In this example, "time-inconsistent" is interpreted as making investment decisions emphasizing performance while the managers are still in office, while "time-consistent" is interpreted as making investment decisions that consistently maximize firm value both during and after their terms of office, and may be seen as a type of agency problem.

(2007) to consider the case when there are expectations of future interest rate increases. Section IV summarizes the conclusions.

II. Investment and Debt Financing Model under Continuous Low Interest Rates

In this section, we examine a firm's investment and debt financing decision under continuous low interest rates based on the model in Sundaresan and Wang (2007).

A. Model Specifications

Assume that a firm is contemplating a new investment. The investment requires an initial expense (sunk cost), and the investment profit X_t is gained at the end of each period. X_t is stochastic over time, and follows a geometric Brownian motion.[7,8]

$$dX_t = \mu X_t\, dt + \sigma X_t\, dW_t, \quad X_0 = x. \tag{1}$$

The conventional real options model (Figure 1) determines the optimal investment time comparing the investment profit with the sunk cost. Because the future profit is stochastic and may decline after the investment is made, there is a risk that the sunk cost may not be recovered. For this reason, the firm does not invest[9] even if the discounted present value of the investment profit exceeds the sunk cost. The investment is only made when it will generate sufficient profit.

The conventional real options model does not specify whether the funds for the sunk cost are raised from equity or debt. In a world without taxes or bankruptcy costs, the funding costs are constant whether the funds are raised from equity or debt (or some combination of the two), and do not affect the investment decision (Modigliani and Miller [1958]).

Sundaresan and Wang (2007) assume a more realistic world where taxes and bankruptcy costs exist, and devise a model where the sunk cost is financed from equity and debt (Figure 2). In this case, interest payments b are incurred as a cost after the firm invests and the firm goes bankrupt if it cannot cover the interest payments. At the

7. Here, X_t is defined under a risk-neutral measure. Thus, μ is the growth rate after adjusting for risk, that is, after deducting the risk premium from the real growth rate. For this reason, we must assume that μ is less than the risk-free rate r. This is a necessary condition for the discounted present value of future profit and the equity value to be finite values. Thus, if $r > \mu$, then the integral $\int_{[0,\infty)} e^{-rt}(X_0 e^{\mu t})\, dt$ converges to $X_0/(r-\mu)$, but if $r \leq \mu$ then the same integral diverges.

8. This paper assumes, based on Sundaresan and Wang (2007), that pretax profit (earnings before interest and taxes, or EBIT) follows a geometric Brownian motion. Other papers, however, adopt a different definition whereby a firm's sales follow a geometric Brownian motion and profit is defined as sales minus operating costs (Mella-Barral and Perraudin [1997] and Shibata and Yamada [2009]). While this latter definition is more realistic in that it can result in negative profits, it is known that operating costs have almost no influence on investment decisions and simply change the level of the investment profits. Considering this point, we adopt the former definition where a negative profit can result after subtracting interest payments.

   In addition, while this paper constructs a model that posits X_t as profits, X_t can also be read as operating cash flow. In this case, corporate bankruptcy results not from excessive debt but from a funds shortage.

9. Under classical investment theory before the appearance of the real options model, that is, using the net present value method, the optimal investment takes place at the point where the discounted present value of the investment exceeds the sunk cost.

Figure 1 Investment Decision under the Conventional Real Options Model

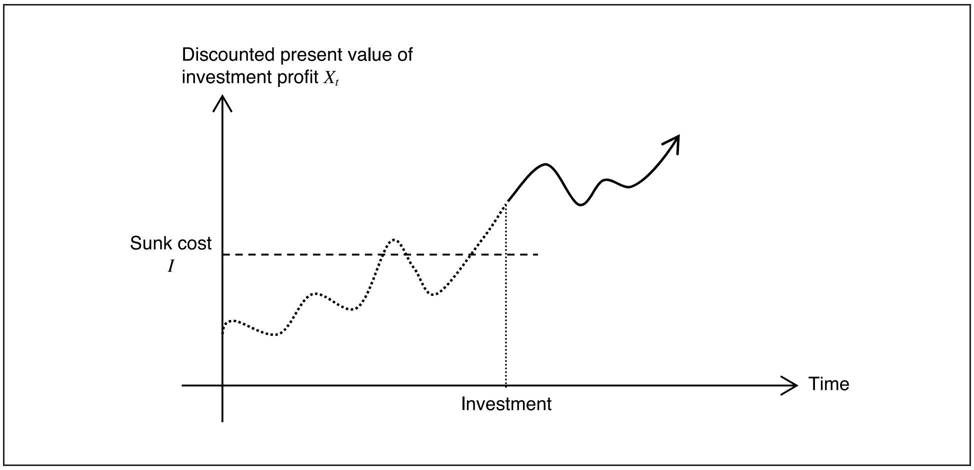
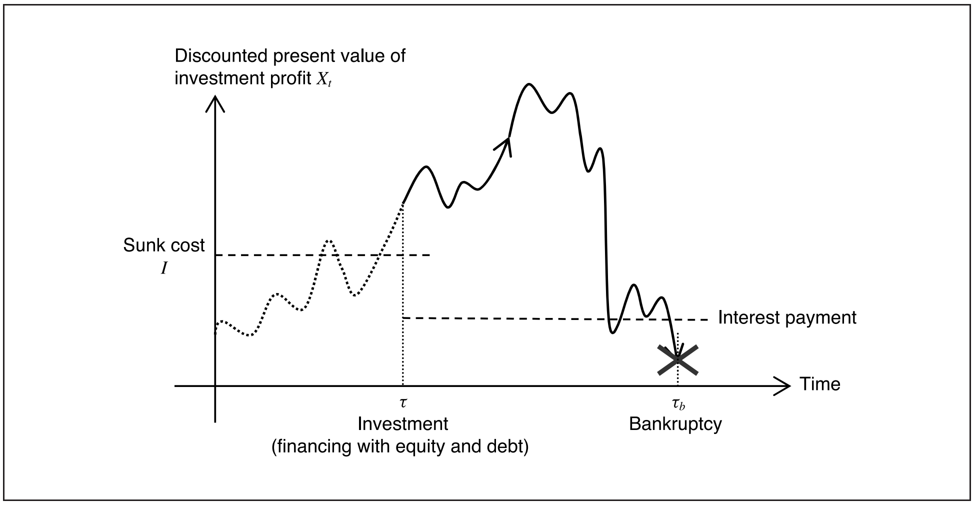


Figure 2 Investment Decision under a Real Options Model Considering Debt Financing

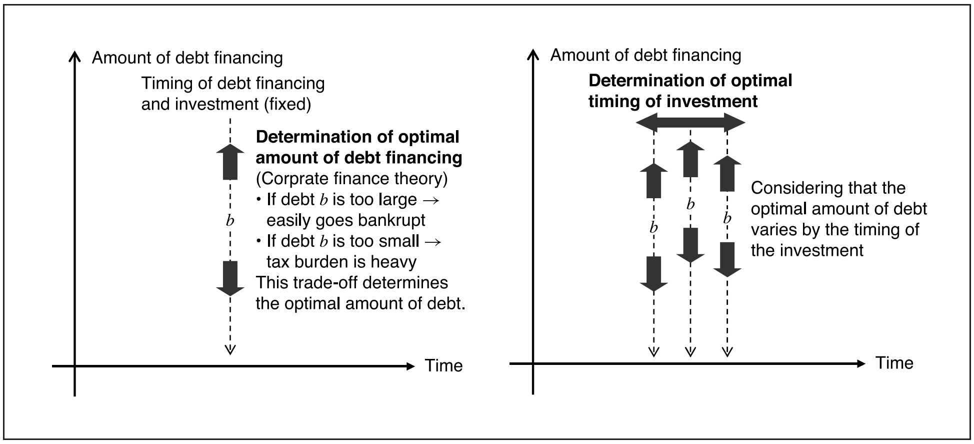


bankruptcy, the remaining firm value belongs to the debt holders, but a revaluation loss, which is named the bankruptcy cost, is incurred in the liquidation.

Because this model, unlike the conventional real options model, incorporates the possibility of bankruptcy after the firm invests, the firm considers this possibility and simultaneously determines (1) the optimal investment time and (2) the optimal capital structure, that is, the optimal amount of debt.[10] This paper now considers how

10. The bankruptcy is determined by the equity holders to maximize the equity value. (It may also be determined through negotiations between the equity holders and the debt holder, but this is ignored here since it is not the main focus.) In this model, however, the investment decisions are made by the firm, and in this context the firm determines the investment timing and debt amount assuming the optimal bankruptcy behavior by the equity holders, rather than having the equity holders determine bankruptcy.

Figure 3 Determination of Investment Time and Debt Financing



this investment and debt financing behavior is affected by the present interest rate environment and future interest rate levels.

The details of the model are explained in Section II.B, but first we present an outline of the mechanism for determining the optimal investment time and the optimal amount of debt.

First, we fix the investment time temporally (Figure 3, left-hand chart). This investment time may not be optimal. It is optimized later on. Once the investment time is set, the model reverts to deciding the amount of debt, so we apply standard corporate finance theory. Because (1) the bankruptcy costs increase if the debt is too large and (2) the tax benefits decline if the debt is too small, the firm considers this trade-off and decides the amount of debt to maximize the firm value as the sum of the equity value and the debt value.

It is important to note that the optimal debt amount changes depending on the investment time (Figure 3, right-hand chart), which is fixed temporally. The firm determines the optimal investment time paying attention to this factor. The optimal investment time is determined comparing the sunk cost I and the firm value with the optimal capital structure. Through this process, the firm simultaneously decides the optimal investment time and the amount of debt.

B. Determination of Optimal Investment Time and Amount of Debt

As noted above, we first fix the investment time t temporally and then determine the debt amount at that time. In this process, we consider the trade-off between bankruptcy costs and tax benefits, following standard corporate finance theory.[11]

First, we define the amount of the firm's interest payments each period as b and set the amount of debt as its discounted present value $B \equiv b/r$, where r is the risk-free rate. When the interest payments are large, this accelerates firm bankruptcy and

11. See, for example, Leland (1994) and Goldstein, Ju, and Leland (2001).

increases bankruptcy costs. The optimal time of bankruptcy τ_b is determined as the equity value is maximized, as follows:

$$E(X_t) = \max_{\tau_b \in F_t} \mathrm{E}_t\left[\int_t^{\tau_b} e^{-r(s-t)}(1-\tau_{ax})(X_s - b)\, ds\right]. \tag{2}$$

In this equation, τ_{ax} is the tax rate and the expectation E_t is defined under the risk-neutral measure. F_t is the filtration generated by the Brownian motion concerning X_t. Each period the equity holders receive profits $(1-\tau_{ax})(X_t - b)$, deducting taxes and interest payments b from the investment profit X_t. Bankruptcy is determined by equity holders when this profit turns negative and there is no possibility of recovering the equity value. Thereafter, the equity value becomes zero.

The debt value is expressed as shown in equation (3) using the optimized bankruptcy timing τ_b^* from equation (2):

$$D(X_t) = \mathrm{E}_t\left[\int_t^{\tau_b^*} e^{-r(s-t)} b\, ds + e^{-r(\tau_b^* - t)}(1-\alpha)W_b(X_{\tau_b^*})\right]. \tag{3}$$

Prior to bankruptcy, the debt holders receive interest payments, which are expressed by the first term, and upon bankruptcy they receive the residual value, which is defined by the second term, where α is the bankruptcy cost, that is, the revaluation loss upon bankruptcy, and $W_b(X_{\tau_b^*})$ expresses the firm value prior to revaluation at bankruptcy.

$$W_b(X_{\tau_b^*}) = \mathrm{E}_{\tau_b^*}\left[\int_{\tau_b^*}^{\infty} e^{-r(s-\tau_b)}(1-\tau_{ax})X_s\, ds\right]. \tag{4}$$

The firm value $W(x)$ is defined as the sum of the equity value $E(x)$ and the debt value $D(x)$.

$$\begin{aligned} W(X_t) &= E(X_t) + D(X_t) \\ &= W_a(X_t) + TB(X_t, b) - BC(X_t, b), \end{aligned} \tag{5}$$

where

$$W_a(X_t) = \mathrm{E}_t\left[\int_t^{\infty} e^{-r(s-t)}(1-\tau_{ax})X_s\, ds\right] \tag{6}$$

is the firm value in the case that the funds are all raised from equity, and $TB(X_t, b)$ is the tax benefit.

$$TB(X_t, b) = \tau_{ax}\mathrm{E}_t\left[\int_t^{\tau_b^*} e^{-r(s-t)} b\, ds\right]. \tag{7}$$

$BC(X_t, b)$ is the bankruptcy cost.

$$BC(X_t, b) = \alpha\mathrm{E}_t\left[e^{-r(\tau_b^* - t)}(1-\tau_{ax})W_a(X_{\tau_b^*})\right]. \tag{8}$$

The firm decides the optimal amount of debt to maximize $W(X_t)$, that is, the firm determines the amount of interest payments b to minimize the funding costs. It is important to note that the optimal amount of debt depends on the temporarily fixed investment timing t.

Next, we determine the optimal investment timing using the firm value $W(X_t)$ with the optimal capital structure. The firm decides the optimal investment time τ to maximize the option value to invest as follows:[12]

$$V(X_t) = \max_{\tau \in F_t} \mathrm{E}_t[e^{-r(\tau - t)}(W(X_\tau) - I)]. \tag{9}$$

The optimal capital structure, which depends on the investment time, is also determined at the same time.

C. Model Solution

There is an analytical solution to the above optimization problem (see Appendices 1 and 2 for the details). First, we derive the equity value as follows:

$$E(x) = (1 - \tau_{ax})\left(\left(\frac{x}{r-\mu} - \frac{b}{r}\right) - \left(\frac{x_b}{r-\mu} - \frac{b}{r}\right)\left(\frac{x}{x_b}\right)^{\gamma}\right), \quad x > x_b, \tag{10}$$

where

$$x_b = \frac{\gamma}{\gamma - 1}\frac{b}{r}(r - \mu) \tag{11}$$

expresses the bankruptcy threshold, and the firm goes bankrupt when $x < x_b$. The equity value becomes zero after bankruptcy, where γ is the negative root of the following characteristic equation.

$$\frac{1}{2}\sigma^2\gamma(\gamma - 1) + \mu\gamma - r = 0. \tag{12}$$

The debt value is then derived as follows:

$$D(x) = \begin{cases} \dfrac{b}{r} - \left(\dfrac{b}{r} - W_b(x_b)\right)\left(\dfrac{x}{x_b}\right)^{\gamma}, & x > x_b, \\ W_b(x_b) \equiv (1 - \alpha)\dfrac{(1 - \tau_{ax})x_b}{r - \mu}, & x \leq x_b, \end{cases} \tag{13}$$

where $W_b(x_b)$ is the residual value of the firm after revaluation at the time of bankruptcy x_b. In equation (13), the term $b/r - W_b(x_b)$ represents the loss given default,

12. This paper determines the investment time that maximizes the firm value, and this is also the same as the time that maximizes the equity value. That is because the firm value minus the sunk cost can be rewritten as $W(x) - I = E(x) - I - D(x)$), and the right half of this equation can be interpreted as the equity value minus the funds that the equity holders must raise $(I - D(x))$, and this is maximized.

the term $(x/x_b)^\gamma$ indicates the probability of default[13] and their product $(b/r - W_b(x_b)) \cdot (x/x_b)^\gamma$ expresses the expected loss.

The firm value $W(x)$ is defined by the sum of the equity value and the debt value, so it can be calculated analytically as $W(x) = E(x) + D(x)$. The option value to invest $V(x)$ can also being solved analytically using this $W(x)$ as follows:

$$V(x) = \begin{cases} (W(x_I) - I)\left(\dfrac{x}{x_I}\right)^\beta, & x < x_I, \\ W(x) - I, & x \geq x_I, \end{cases} \tag{14}$$

where

$$x_I = \psi \frac{\beta}{\beta - I} \frac{(r - \mu)}{1 - \tau_{ax}} I \tag{15}$$

is the investment threshold, and the firm invests when the profit exceeds x_I. β is the positive root of the characteristic equation,

$$\frac{1}{2}\sigma^2 \beta(\beta - 1) + \mu\beta - r = 0, \tag{16}$$

and ψ and h are constants.

$$\psi \equiv \left(1 + h^{\frac{1}{\gamma}} \left(\frac{\tau_{ax}}{1 - \tau_{ax}}\right)\right)^{-1} \leq 1, \tag{17}$$

$$h \equiv 1 - \gamma\left(1 - \alpha + \frac{\alpha}{\tau_{ax}}\right) \geq 1. \tag{18}$$

The optimal debt amount is derived using h, τ_{ax}, and ψ as follows:

$$b^* = h^{\frac{1}{\gamma}} \frac{\gamma - 1}{\gamma} \frac{\beta}{\beta - 1} \frac{r\psi}{1 - \tau_{ax}} I. \tag{19}$$

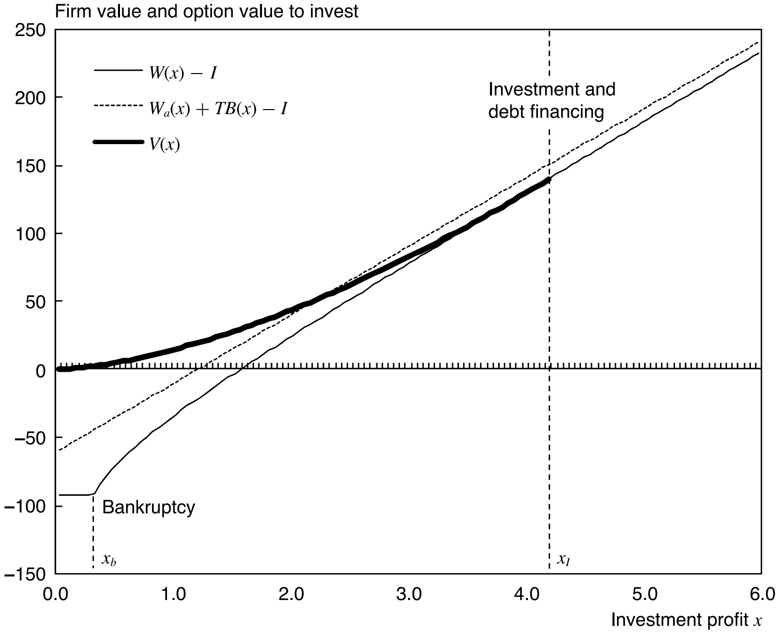
Figure 4 shows the firm value $W(x)$, the bankruptcy threshold x_b, the investment threshold x_I, and the option value to invest $V(x)$ derived above.[14]

In this figure, the thin solid line expresses the net investment value defined by $W(x) - I$. This value becomes asymptotic to the value $W_a(x) + TB(x) - I$ expressed by the dotted line as the investment profit increases (as $x \to \infty$). This dotted line shows "the ideal conditions" with no bankruptcy and with only tax benefits. The net investment value $W(x) - I$ is less than the value under the ideal conditions by the amount of the expected loss. The net investment value separates from the ideal dotted line as the value of x declines, and the firm goes bankrupt when $x < x_b$.

13. For details, see Shibata and Yamada (2009).

14. The parameters are set at $\mu = 0$ percent, $\sigma = 15$ percent, $I = 100$, $\alpha = 50$ percent, $\tau_{ax} = 50$ percent, and $r = 1$ percent.

Figure 4 Investment Decision Considering the Possibility of Bankruptcy



The thick solid line expresses the option value to invest $V(x)$. It becomes equal to the thin solid line $W(x) - I$ in the area $x \geq x_I$ where the firm has already invested. In contrast, $x < x_I$ is the area where the firm does not invest and the firm has the option to invest and gain the net investment value $W(x) - I$ in the future. It is important to note that here the firm does not invest at the stage when the net present value $W(x) - I$ becomes positive. This is because there exists the risk that the profit may decline in the future, making it impossible to recover the sunk cost. The firm takes this risk into consideration and waits until the investment generates sufficient profit x_I. At this x_I, $V(x)$ smoothly paste to $W(x) - I$.

D. Comparative Statics and Implications

Next we consider how the bankruptcy threshold and the investment threshold change according to the risk-free rate. We also consider how the firm's probability of default (PD), expected loss (EL), and other credit risk indices change.

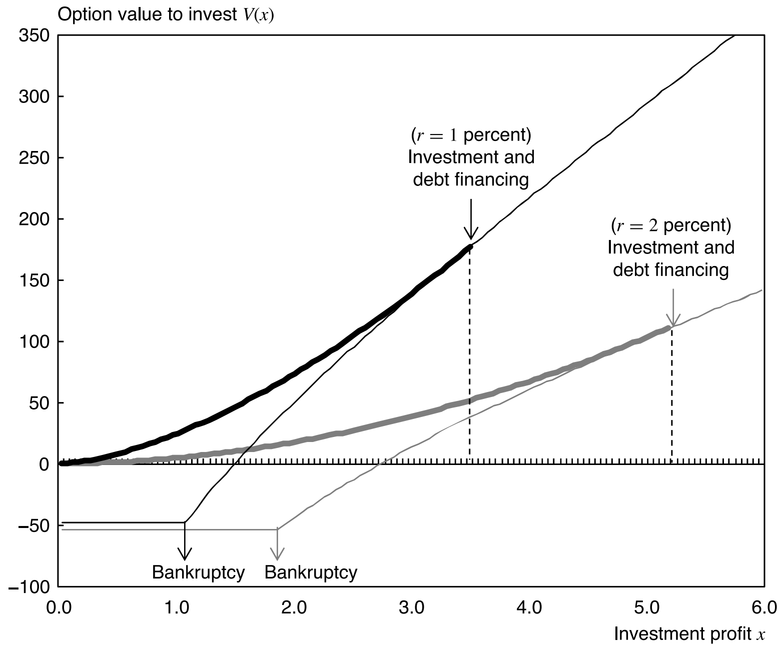
Figure 5 shows the solutions with the risk-free rate at 1 percent and at 2 percent.[15]

15. The parameters are set at $\mu = 0$ percent, $\sigma = 15$ percent, $I = 100$, $\alpha = 30$ percent, $\tau_{ax} = 30$ percent, $r =$ 1 percent, and these are used for the subsequent comparative statics.

When the corporate profit growth rate μ (the growth rate after deducting the risk premium) is set at 0 percent in this manner, as explained in Footnote 2, a "low interest rate" is one where the risk-free rate is at the same level as μ, that is, in the neighborhood of 0 percent. For this reason, the risk-free rate approaches 0 percent in the comparative statics as well.

Even if μ is set at a high level, a decline in the risk-free rate r to the level of μ has the same effect as a decline in r to nearby 0 percent under the parameter settings in this paper. This decline promotes firms' debt

Figure 5 Investment and Bankruptcy Decisions under a Low Interest Rate Environment



In Figure 5, we can see that the investment threshold is lower, that is, the investment time is earlier, under a low interest rate (1 percent). We also see that the difference between the investment threshold and bankruptcy threshold is smaller under a low interest rate environment. In other words, the time interval from investment to bankruptcy grows shorter under a low interest rate environment. This is because low interest rates have a lot of influence on the investment timing but do not have much influence on the bankruptcy timing.

Table 1 shows how the probability of default (PD) and the expected loss (EL) change according to the risk-free rate. We can see that the lower the interest rate, the earlier the firm invests, even if the investment profits (1) are low. We also see that the ratio of debt to profits (2)/(1) rises as the interest rate declines. In the extreme case ($r = 0.1$ percent, 0.5 percent), the investment profits are smaller than the interest payments ($x_I < b$). This suggests some firms make speculative investment decisions seeking capital gains. For this reason, the PD and EL rise sharply under a low interest rate environment. In other words, firms tend to choose investments with high credit risk (PD, EL) under such an environment.

financing and investment and heightens credit risk. This result can also be confirmed by the derivation of the investment threshold in equation (15) and the bankruptcy threshold in equation (11). Looking at the equations, firms' debt financing and investment behavior depend not on the level of the risk-free rate r, but rather on the differential between μ and r, demonstrating that the ratio between μ and r is an important determining factor.

Table 1 Probability of Default (PD) and Expected Loss (EL) under Low Interest Rates

Risk-free rate r (percent)	Investment timing (profit x at time of investment) (1)	Amount of debt (interest payments b) (2)	(2)/(1)	PD (percent)	EL (percent)
5.0	9.6	6.9	0.71	26	18
2.0	5.2	3.9	0.75	39	30
1.0	3.5	2.9	0.84	51	42
0.5	2.6	2.7	1.04	64	56
0.1	1.7	4.6	2.68	88	84

III. Debt Financing and Investment Model under Rising Interest Rates

In this section, we extend the Sundaresan and Wang (2007) model presented in Section II and construct a model that incorporates changes in future interest rates.

A. Model Specifications

In this section, the risk-free rate r set as a constant in Section II varies following a Poisson process. In other words, we assume that at time τ_i the risk-free rate jumps from r_{i-1} to r_i. The frequency of this change is given by equation (20):

$$P(\tau_{i+1} \in (t, t + dt] \mid \tau_i < t < \tau_{i+1}) = \lambda_i \, dt. \tag{20}$$

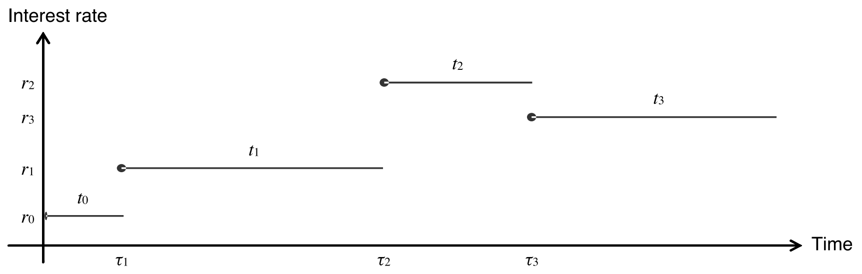
In other words, the instantaneous probability of the next jump following the i-th jump at time t ($\tau_i < t < \tau_{i+1}$) is expressed by λ_i (Figure 6).[16]

B. Optimal Investment Problem

First, we consider the case where the debt financing is not taken into account for simplification. When there are expectations of rising interest rates, the investment decision varies depending on how the firm incorporates the possibility of future interest rate increases. In this paper, we consider two types of firms. The first is myopic firms that consider this possibility partially in their investment decisions, and the second is economically rational firms that do so perfectly. Myopic firms make their investment decisions considering only the interest rate increases until they raise funds and invest. In contrast, economically rational firms consider that interest rates may rise not only before but also after they raise funds and invest. The former are myopic in the sense that they fail, at the time they make investments, to consider the possibility of higher debt financing costs if they need to refinance in the future.

16. Because the main focus of this paper is to consider firms' debt financing and investment under changes in interest rates, we only consider the changes in interest rates, and assume that the investment profit growth rate (μ) remains constant. On the other hand, considerations of how much interest rates should change when profit growth rates change would require the construction of a model whereby interest rates and profit growth rates change simultaneously or in correlation, but such an inquiry lies outside the subject of this paper.

Figure 6 Interest Rate Changes Following a Poisson Process



1. Myopic firms: time-inconsistent discount rate

Assume there are expectations that interest rates will increase from r_0 to r_1, and that the interest rate will increase at time τ_1.[17] τ_1 is a random variable given by the Poisson process defined in Section III.A. The optimal investment problem when interest rates will rise in the future is expressed by the following equation:

$$V(X_t) = \max_{\tau \in F_t} \mathrm{E}_t \Bigg[e^{-r_1(\tau-\tau_1)-r_0(\tau_1-t)} \left(\int_\tau^\infty e^{-r_1(s-\tau)} X_s\, ds - I \right) 1_{\{\tau > \tau_1\}} + e^{-r_0(\tau-t)} \left(\int_\tau^\infty e^{-r_0(s-\tau)} X_s\, ds - I \right) 1_{\{\tau \le \tau_1\}} \Bigg]. \quad (21)$$

The first term shows the value if the firm invests after the interest rate rise, and the second term shows the value if the firm invests before the interest rate rise. With the first term, the interest rate has already been increased when the firm invests, so the rate r_1 is used for the discount rate. With the second term, the interest rate has not yet been increased when the firm invests, so r_0 is used for the discount rate.

2. Economically rational firms: time-consistent discount rate

In contrast, economically rational firms raise funds considering the possibility of changes in interest rates both before and after these firms raise funds and invest. The investment problem of these firms is formulated as follows:

$$V(X_t) = \max_{\tau \in F_t} \mathrm{E}_t \Bigg[e^{-r_1(\tau-\tau_1)-r_0(\tau_1-t)} \left(\int_\tau^\infty e^{-r_1(s-\tau)} X_s\, ds - I \right) 1_{\{\tau > \tau_1\}} + e^{-r_0(\tau-t)} \left(\int_\tau^{\tau_1} e^{-r_0(s-\tau)} X_s\, ds + \int_{\tau_1}^\infty e^{-r_1(s-\tau)} X_s\, ds - I \right) 1_{\{\tau \le \tau_1\}} \Bigg]. \quad (22)$$

17. For simplification of the argument developed here, this paper only considers the case of a single jump in the interest rate. Multiple jumps can be considered as essentially repetitions of a single jump (see Grenadier and Wang [2007]). It is also possible to consider interest rate drops, but this paper limits its considerations to interest rate increases.

The difference from equation (21) is in the second term. Since these firms consider the possibility of additional interest rate increases after the investment, r_1 is used as the discount rate after the time τ_1. On the other hand, the first term remains unchanged since there is no possibility of interest rate increases after the investment.

The correct investment decisions are derived from the optimization problem for economically rational firms. This optimization problem meets the requirements of standard financial theory, without arbitrage between short-term and long-term interest rates. This type of discount rate is referred to as a "time-consistent" discount rate, and takes the form of an exponential function to time. In contrast, the discount rate in the optimization problem for myopic firms is referred to as a "time-inconsistent" discount rate, as introduced in behavioral economics in recent years. It is characterized by a weak discount rate in the short term and a strong discount rate in the long term. This discount rate's function resembles a hyperbolic function, so it is referred to as a quasi-hyperbolic discount rate.[18]

Even with the same future interest rate expectations, the investment decisions differ depending on how the discount rate is recognized by different firms. Because both perceptions may actually occur,[19] we now examine them both and compare the results. Hereafter in this paper, myopic and economically rational firms are referred to as time-inconsistent and time-consistent firms, respectively.

C. Model Solution and Implications

First we consider the time-consistent investment decision. Figure 7 presents the investment threshold and the option value to invest under this case.[20] For comparison, in addition to the solution under expectations of an interest rate increase (0.5 percent → 1 percent), we also show the solution with constant interest rates at 0.5 percent and at 1 percent. Because time-consistent firms consider all possibilities of interest rate increases at the time they invest, their discount rate is higher than under a constant interest rate of 0.5 percent. For this reason, the value after investment expressed by the dotted line is lower than that under the case with a constant low interest rate of 0.5 percent. Consequently, the solution shows that time-consistent firms wait to invest until the investment profit x is sufficiently high (see Appendix 4 for the details).

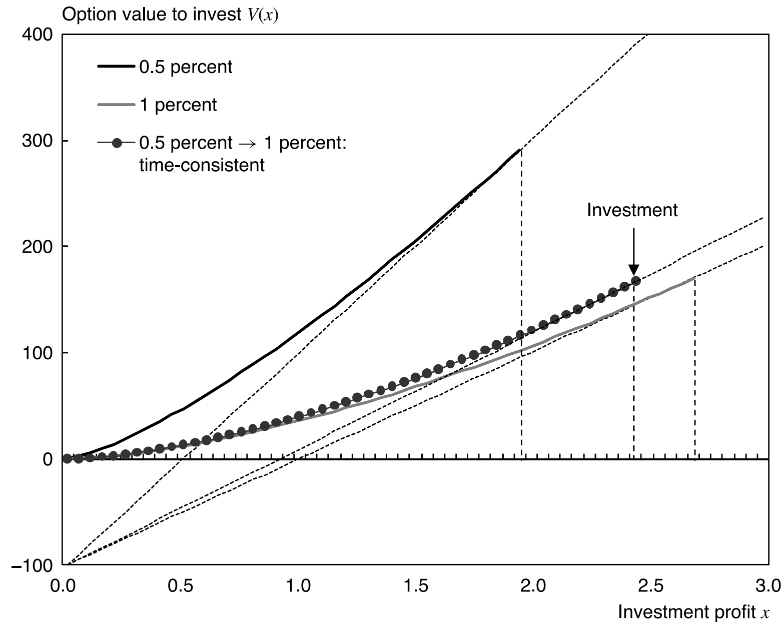
The results for time-inconsistent firms (Figure 8) are exactly the opposite. They invest earlier, that is, invest at a lower profit level, than when the interest rate remains low at a constant 0.5 percent. Under expectations of a future interest rate increase, time-inconsistent firms consider only interest rate hikes until they make their investment

18. Hyperbolic discount rates are representative examples of myopic behavior. Hyperbolic discount rates increase over time because they use hyperbolic functions as discount rates rather than conventional exponential functions. In other words, they are expressions with a relatively weak short-term discount and a relatively strong long-term discount. However, as the discount rate changes continuously over time, the intertemporal optimization problem becomes complex. For this reason, Laibson (1997) and others consider a quasi-hyperbolic discount rate, which changes discretely, and apply the discount rate for the intertemporal optimization problems (Harris and Laibson [2003], etc.). In this paper, we follow the method in Grenadier and Wang (2007), which introduces the quasi-hyperbolic discount rate to the real options field.

19. In general, it is said that time-inconsistent discount rates easily emerge under the following conditions: (1) when managers change every few years and easily overemphasize current performance; (2) when making long-term investments; and (3) when investments have a first-mover advantage.

20. In addition to the parameters used in Figure 5 (see Footnote 16), Figures 6–10 also adopt an interest rate increase probability of $\lambda = 5$ percent.

Figure 7 Time-Consistent Investment Decision



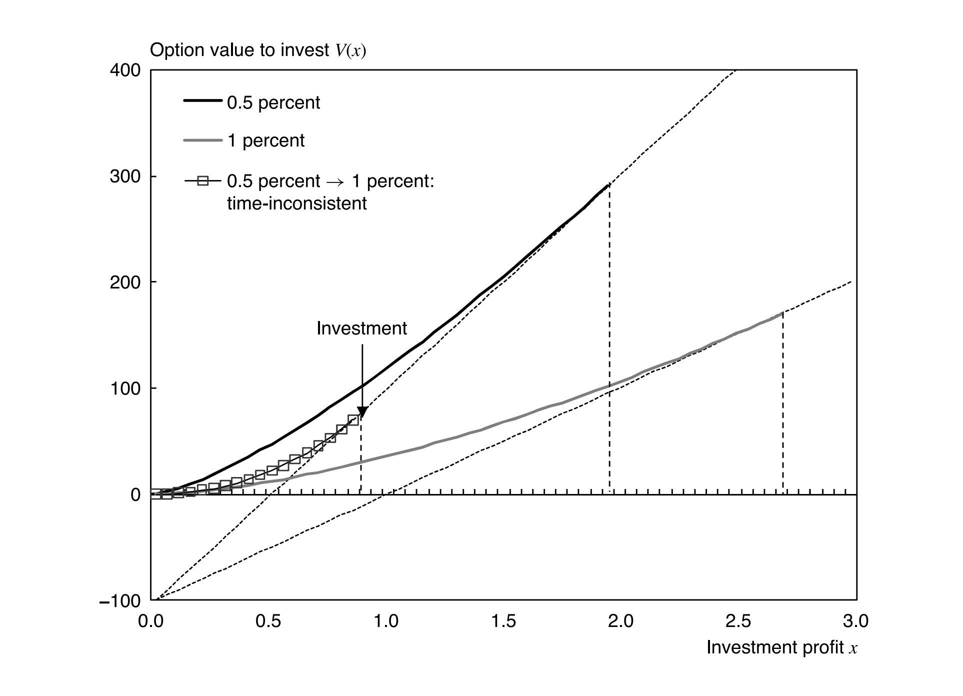
decisions. Therefore, they consider the funding costs to be lower if they invest before the interest rate rises, and they evaluate the option value to invest as higher. In contrast, they evaluate the option value to invest as lower if they make the investment after the interest rate rises because the funding costs increase. Consequently, compared with the case under a constant low interest rate of 0.5 percent, when there are expectations of interest rate rises time-inconsistent firms have the incentive to invest earlier because of the perception that the option value will decrease if they wait until the interest rate rises. These types of rushed investments are considered "last-minute investments," which are caused by excessive concerns that the option value will decline if the firms wait.

The above scenario can be directly understood by comparing the two option values to invest in Figure 8. The thick solid line shows the perceived option value to invest when low interest rates continue, and the line connecting the plotted squares shows the perceived option value to invest under expectations of a future interest rate increase. In the latter case, the firms believe that if interest rates increase before they invest, the option value to invest will decline to the value shown by the solid gray line. Therefore, they evaluate the option value as lower as the expectations of the interest rate increase (see Appendix 3 for the details).

D. Case Considering Debt Financing

We now consider the model where firms' debt financing and bankruptcy are taken into account.

Figure 8 Time-Inconsistent Investment Decision



First, we consider the time-inconsistent firms. In this case, the optimal investment problem is expressed by the following equation:

$$V(X_t) = \max_{\tau \in F_t} \mathrm{E}_t\big[e^{-r_1(\tau-\tau_1)-r_0(\tau_1-t)}(W^{[1]}(X_\tau) - I)1_{\{\tau>\tau_1\}} + e^{-r_0(\tau-t)}(W^{[0]}(X_\tau) - I)1_{\{\tau\le\tau_1\}}\big]. \tag{23}$$

The first term shows the value when the firm issues debt and invests after the interest rate increase, and the second term shows the value when the firm issues debt and invests prior to the interest rate increase. $W^{[i]}(X_t)$ $(i = 0, 1)$ express the firm value under r_0 and r_1, respectively.

$$W^{[i]}(X_t) = E^{[i]}(X_t) + D^{[i]}(X_t), \tag{24}$$

where $E^{[i]}(X_t)$ and $D^{[i]}(X_t)$ are the equity value and the debt value when the risk-free rate is r_1 or r_2.

$$E^{[i]}(X_t) = \max_{\tau_b^{[i]} \in F_t} \mathrm{E}_t\left[\int_t^{\tau_b^{[i]}} e^{-r_i(s-t)}(1-\tau_{ax})(X_s - b^{[i]})\,ds\right], \tag{25}$$

$$D^{[i]}(X_t) = \mathrm{E}_t\left[\int_t^{\tau_b^{[i]}} e^{-r_i(s-t)}b^{[i]}\,ds + e^{-r_i(\tau_b^{[i]}-t)}(1-\alpha)W_b^{[i]}\big(X_{\tau_b^{[i]}}\big)\right]. \tag{26}$$

In the time-inconsistent case, if the firm issues debt and invests prior to the interest rate increase, the risk-free rate is perceived to remain constant at r_0, determining the amount of debt, the firm's bankruptcy, and the investment timing.

For time-consistent firms, the optimal investment problem is expressed as follows:

$$V(X_t) = \max_{\tau \in F_t} \mathrm{E}_t \Big[e^{-r_1(\tau-\tau_1)-r_0(\tau_1-t)}(W^{[1]}(X_\tau) - I)1_{\{\tau>\tau_1\}} \\ + e^{-r_0(\tau-t)}(W^{[0\to 1]}(X_\tau) - I)1_{\{\tau \le \tau_1\}} \Big], \tag{27}$$

where $W^{[0\to 1]}(X_t)$ expresses the firm value incorporating the interest rate increase ($r_0 \to r_1$).

$$W^{[0\to 1]}(X_t) = E^{[0\to 1]}(X_t) + D^{[0\to 1]}(X_t). \tag{28}$$

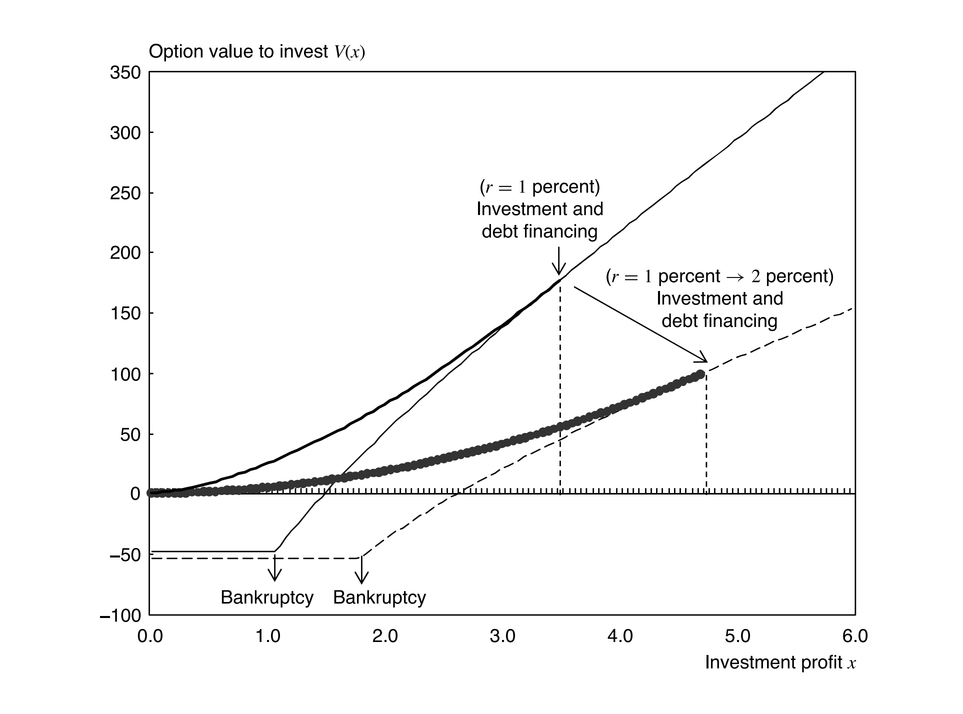
$E^{[0\to 1]}(X_t)$ is the equity value incorporating the interest rate increase ($r_0 \to r_1$).

$$E^{[0\to 1]}(X_t) \\ = \max_{\tau_b^{[\lambda]} \in F_t} \mathrm{E}_t \Bigg[\left(\int_t^{\tau_1} e^{-r_0(s-t)}(1-\tau_{ax})(X_s - b^{[\lambda]})\, ds \right) 1_{\{\tau_b^{[\lambda]} \le \tau_1\}} \\ + \Bigg(\int_t^{\tau_1} e^{-r_0(s-t)}(1-\tau_{ax})(X_s - b^{[\lambda]})\, ds \\ + e^{-r_0(\tau_1-t)} \int_{\tau_1}^{\tau_b^{[\lambda]}} e^{-r_1(s-t)}(1-\tau_{ax})(X_s - b^{[\lambda]})\, ds \Bigg) 1_{\{\tau_b^{[\lambda]} > \tau_1\}} \Bigg]. \tag{29}$$

The first term is the equity value when bankruptcy occurs before the interest rate increase, and the second term is the equity value when bankruptcy occurs after the interest rate increase. Similarly $D^{[0\to 1]}(X_t)$ is the debt value incorporating the interest rate increase ($r_0 \to r_1$), with the first term indicating bankruptcy before the interest rate increase and the second term bankruptcy after the interest rate increase.

$$D^{[0\to 1]}(X_t) \\ = \mathrm{E}_t \Bigg[\left(\int_t^{\tau_b^{[\lambda]}} e^{-r_0(s-t)} b^{[\lambda]}\, ds + e^{-r_0(\tau_b^{[\lambda]}-t)}(1-\alpha) W_a^{[0]}\big(X_{\tau_b^{[\lambda]}}\big) \right) 1_{\{\tau_b^{[\lambda]} \le \tau_1\}} \\ + \Bigg(\int_t^{\tau_1} e^{-r_0(s-t)} b^{[\lambda]}\, ds + e^{-r_0(\tau_1-t)} \int_{\tau_1}^{\tau_b^{[\lambda]}} e^{-r_1(s-\tau_1)} b^{[\lambda]}\, ds \\ + e^{-r_1(\tau_b^{[\lambda]}-\tau_1)}(1-\alpha) W_a^{[1]}\big(X_{\tau_b^{[\lambda]}}\big) \Bigg) 1_{\{\tau_b^{[\lambda]} > \tau_1\}} \Bigg]. \tag{30}$$

Figure 9 Investment and Bankruptcy: Time-Consistent Firms

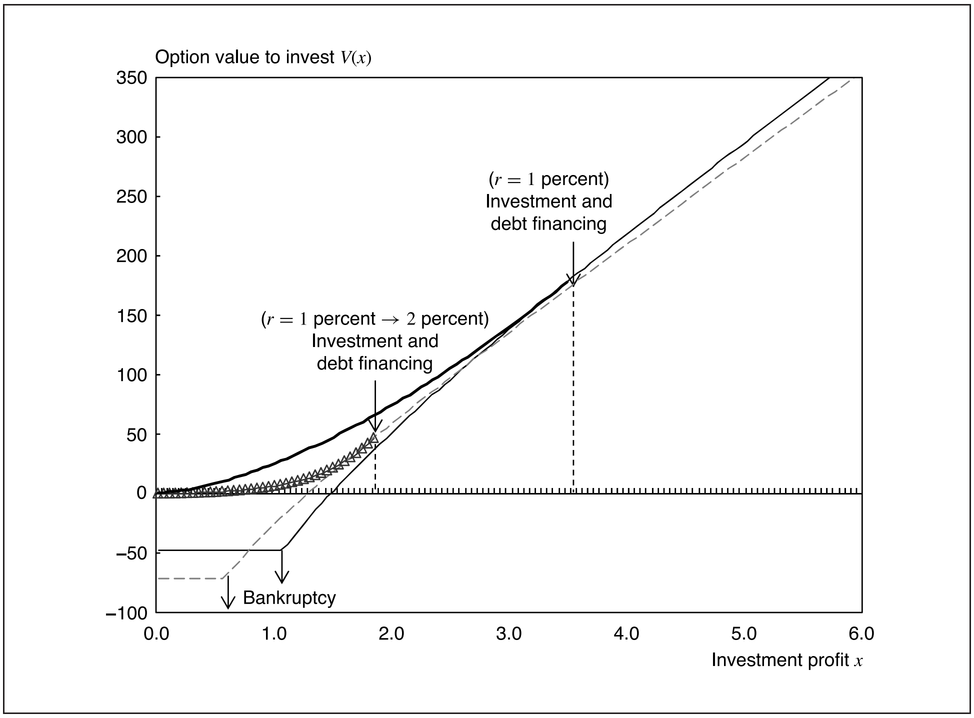


As explained above, the time-consistent firms determine the debt financing and bankruptcy incorporating all future interest rate increases into their decisions.

Figure 9 presents the solutions to the above optimization problems as a graph. Because time-consistent firms consider subsequent interest rates increase perfectly at the time they invest, the investment is made later and the time interval from investment to bankruptcy is longer. Consequently, the credit risk of investment is comparatively lower (see Appendix 6 for the details).

In contrast, because time-inconsistent firms make last-minute investments as examined in the previous section, the investment timing is comparatively early and the time interval from investment to bankruptcy is relatively short (Figure 10). This suggests that time-inconsistent firms choose high-credit-risk, last-minute investments (see Appendix 5 for the details).

Figure 10 Investment and Bankruptcy: Time-Inconsistent Firms



IV. Conclusion

In this paper, we have examined the mechanism whereby firms' debt financing and investment become more accelerated and credit risk rises under a low interest rate environment from a theoretical perspective using a real options model. We find that firms tend to increase debt financing and investment, and credit risk rises not only under strong expectations of continued low interest rates but also when there are expectations of future interest rate hikes. The main results and their implications are as follows.

First, we find that under a continuous low interest rate environment firms invest and take on debt even if the investment profits are comparatively low, because the low interest rate lifts their equity value and makes funding conditions advantageous. We also find that under a low interest rate environment the more investments are made, the higher the credit risk and the shorter the time interval from investment to bankruptcy. For these reasons, credit risk must be managed with greater prudence under a low interest rate environment.

Next, we find that when there are expectations of future interest rate increases, investment decisions vary depending on how firms incorporate this possibility of future interest rate rises. The results indicate that time-inconsistent firms, which incorporate this possibility in a short-sighted manner, rush to make last-minute investments. In contrast, time-consistent firms, which incorporate the possibility in an economically rational manner, actually decrease investments, carefully considering the likelihood of future interest rate hikes. These findings indicate the importance of accurately grasping the standards that firms follow in their investment decisions when we analyze the economic effects of interest rate increases.

Finally, we note some future avenues of research. This paper assumes that the investment profit growth rate remains constant, because we focus our analysis on firms' debt financing and investment under changes in the interest rate environment. The question of how much interest rates should change when profit growth rates change would require another type of model whereby interest rates and profit growth rates change simultaneously or in correlation with each other. Extension to this type of model remains for future investigation.

In addition, our analysis is limited to debt financing and investment concerning new investments of firms. Therefore, we do not consider the debt position of the firms prior to making investments, or how the debt position influences the investment decision. In reality, the investment decisions of firms with excessive debts may well differ, as well as their reactions to changes in the interest rate environment. Extension to a model that would facilitate these types of analyses also remains for future investigation.

APPENDIX 1: OPTIMAL DEBT FINANCING PROBLEM

In Appendices 1 and 2, we explain the debt financing and investment problem when interest rates remain constant following Sundaresan and Wang (2007). In Appendix 1, we derive the equity value $E(X_t)$, the debt value $D(X_t)$, the firm value $W(X_t)$, and the optimal amount of debt. In Appendix 2, we derive the optimal investment time and investment value $V(X_t)$ based on the results of Appendix 1.

First, we transform the optimization problem of equity value in equation (2) into a bankruptcy decision problem at each time t.

$$E(X_t) = \max\left\{\max_{\tau_b \in F_{t+dt}} \mathrm{E}_t\left[\int_t^{\tau_b} e^{-r(s-t)}(1-\tau_{ax})(X_t - b)\,ds\right], 0\right\}. \tag{A.1}$$

The first term of the equation expresses the value when the firm does not go bankrupt at time t as ($\tau_b \in F_{t+dt}$), and the second term expresses the value as zero when the firm goes bankrupt. Dividing the first term into the present investment profit $(1-\tau_{ax})(X_t-b)$ and the future equity value $E(X_{t+dt})$, the relationship between $E(X_t)$ and $E(X_{t+dt})$ can be derived as follows (Bellman equation).

$$E(X_t) = e^{-rdt}\max\{(1-\tau_{ax})(X_t - b)\,dt + \mathrm{E}_t[E(X_{t+dt})], 0\}. \tag{A.2}$$

The equation when the firm does not go bankrupt,

$$E(X_t) = e^{-rdt}((1-\tau_{ax})(X_t - b)\,dt + \mathrm{E}_t[E(X_{t+dt})]), \tag{A.3}$$

can be transformed into the following stochastic differential equation:

$$rE(X_t)\,dt = (1-\tau_{ax})(X_t - b)\,dt + \mathrm{E}_t[dE(X_t)]. \tag{A.4}$$

Here we apply Ito's formula to the term $dE(X_t)$ to derive the differential equation that satisfies $E(X_t)$,

$$\frac{1}{2}\sigma^2 x^2 E''(x) + \mu x E'(x) - rE(x) + (1-\tau_{ax})(x-b) = 0, \tag{A.5}$$

and the boundary conditions are as follows:

$$\begin{cases} E(x_b) = 0, \\ E'(x_b) = 0, \\ E(x \to \infty) \to (1-\tau_{ax})(x/(r-\mu) - b/r). \end{cases} \tag{A.6}$$

The first and second conditions are to determine the optimal bankruptcy threshold x_b, which not only requires that the equity value becomes zero at x_b, but also that it smoothly connects to zero. For this reason, these are referred to as the value matching condition and the smooth pasting condition, respectively. The third condition requires that when the profit grows large the equity value approaches the discounted present value of the profit. This is a condition to exclude bubble solutions. Equation (A.5) is known as a Euler differential equation, and the solution generally takes the following form:

$$E(x) = (1-\tau_{ax})\left(\frac{x}{r-\mu} - \frac{b}{r}\right) + Bx^{\beta} + Cx^{\gamma}, \tag{A.7}$$

where β is the positive constant defined by equation (16) and γ is the negative constant defined by equation (12). Substituting equation (A.7) into the boundary conditions (A.6), we obtain three equations with B, C, and x_b as unknown variables, which can be solved as follows:

$$\begin{cases} B = 0, \\ C = -(1-\tau_{ax})\left(\dfrac{x_b}{r-\mu} - \dfrac{b}{r}\right)\left(\dfrac{1}{x_b}\right)^{\gamma}, \\ x_b = \dfrac{\gamma}{\gamma-1}\dfrac{b}{r}(r-\mu). \end{cases} \tag{A.8}$$

Substituting these into equation (A.7), the equity value can be derived as shown in equation (A.9):

$$E(x) = (1-\tau_{ax})\left(\left(\frac{x}{r-\mu} - \frac{b}{r}\right) - \left(\frac{x_b}{r-\mu} - \frac{b}{r}\right)\left(\frac{x}{x_b}\right)^{\gamma}\right), \quad x > x_b. \tag{A.9}$$

In the same manner, the debt value in equation (3) can be transformed into the following Bellman equation, clarifying the relation between $D(X_t)$ and $D(X_{t+dt})$:

$$D(X_t) = e^{-rdt}(b\,dt + \mathrm{E}_t[D(X_{t+dt})]), \quad t < \tau_b. \tag{A.10}$$

Applying Ito's formula to equation (A.10), the differential equation that satisfies $D(X_t)$ becomes as follows:

$$\frac{1}{2}\sigma^2 x^2 D''(x) + \mu x D'(x) - rD(x) + b = 0, \tag{A.11}$$

$$\text{boundary conditions: } \begin{cases} D(x \to \infty) \to b/r, \\ D(x_b) = W_b(x_b). \end{cases} \tag{A.12}$$

This equation has an analytical solution:

$$D(x) = \begin{cases} \dfrac{b}{r} - \left(\dfrac{b}{r} - W_b(x_b)\right)\left(\dfrac{x}{x_b}\right)^{\gamma}, & x > x_b, \\ W_b(x_b) \equiv (1-\alpha)\dfrac{(1-\tau_{ax})x_b}{r-\mu}, & x \le x_b. \end{cases} \tag{A.13}$$

$W(x)$ is then derived as the sum of $E(x)$ and $D(x)$.

$$W(x) = \begin{cases} (1-\tau_{ax})\dfrac{x}{r-\mu} + \tau_{ax}\left(1 - \left(\dfrac{x}{x_b}\right)^{\gamma}\right)\dfrac{b}{r} \\ -\alpha\left((1-\tau_{ax})\dfrac{x_b}{r-\mu}\right)\left(\dfrac{x}{x_b}\right)^{\gamma}, & x > x_b, \\ W_b(x_b) \equiv (1-\alpha)(1-\tau_{ax})\dfrac{x_b}{r-\mu}, & x \le x_b. \end{cases} \tag{A.14}$$

The optimal amount of debt can be solved by differentiating this firm value by b:

$$b = h^{\frac{1}{\gamma}}\frac{\gamma-1}{\gamma}x, \tag{A.15}$$

where h is the constant defined by equation (18). Note that at this time the amount of debt is dependent on the profit x at the time of investment. The firm value with the optimal capital structure is derived by substituting equation (A.15) into equation (A.14).

$$W(x) = \psi^{-1}\frac{(1-\tau_{ax})x}{r-\mu}, \quad x > x_b, \tag{A.16}$$

where ψ is the constant larger than one defined by equation (17). The parts of $W(x)$ other than ψ express the firm value when the financing is all from equity. Having the value of ψ greater than one means the firm value has been increased by optimizing the capital structure.

APPENDIX 2: OPTIMAL INVESTMENT PROBLEM CONSIDERING DEBT FINANCING

In Appendix 2, we seek the optimal investment time and the option value to invest $V(X_t)$ by using the optimal debt amount (A.15) and the firm value $W(x)$ (A.16) derived in Appendix 1.

The optimization problem equation (9) reverts to the investment decision problem at each time t.

$$V(X_t) = \max\left\{W(X_t) - I, \max_{\tau \in F_{t+dt}} \mathrm{E}_t[e^{-rdt}(W(X_\tau) - I)]\right\}. \tag{A.17}$$

The first term of the equation shows the value when the investment is made at time t, and the second term shows the value when the firm does not invest ($\tau \in F_{t+dt}$). Because the value when the firm does not invest equals the future option value to invest $e^{-rdt}V(X_{t+dt})$, the relationship between $V(X_t)$ and $V(X_{t+dt})$ can be derived as follows (Bellman equation):

$$V(X_t) = \max\{W(X_t) - I, e^{-rdt}\mathrm{E}_t[V(X_{t+dt})]\}. \tag{A.18}$$

The equation when the firm does not invest,

$$V(X_t) = e^{-rdt}\mathrm{E}_t[V(X_{t+dt})], \tag{A.19}$$

can be transformed into the following stochastic differential equation:

$$rV(X_t)\,dt = \mathrm{E}_t[dV(X_{t+dt})]. \tag{A.20}$$

We then apply Ito's formula to the term $dV(X_t)$, to derive the differential equation that satisfies $V(X_t)$,

$$\frac{1}{2}\sigma^2 x^2 V''(x) + \mu x V'(x) - rV(x) = 0, \tag{A.21}$$

and the boundary conditions are as follows:

$$\begin{cases} V(x_I) = W(x_I) - I, \\ V'(x_I) = W'(x_I). \end{cases} \tag{A.22}$$

These are the conditions to determine the optimal investment threshold x_I, and they are also the value matching condition and the smooth pasting condition for $V(X_t)$ to smoothly connect with $W(x_I) - I$ at x_I. Equation (A.21) is also a Euler differential equation, and the solution generally takes the following form:

$$V(x) = Bx^\beta + Cx^\gamma, \tag{A.23}$$

where β is the positive constant defined by equation (16) and γ is the negative constant defined by equation (12). Substituting equation (A.23) into the boundary conditions (A.22) and giving the condition $V(x) < \infty$, we obtain three equations with B, C, and x_I as unknown variables, which can be solved as follows:

$$\begin{cases} B = (W(x_I) - I)\left(\dfrac{1}{x_I}\right)^{\beta}, \\ C = 0, \\ x_I = \psi \dfrac{\beta}{\beta - 1} \dfrac{(r - \mu)}{1 - \tau_{ax}} I, \end{cases} \tag{A.24}$$

where ψ is the constant defined by equation (17), and x_I expresses the optimal investment threshold. Substituting these into equation (A.23), the option value to invest can be derived as follows:

$$V(x) = \begin{cases} (W(x_I) - I)\left(\dfrac{x}{x_I}\right)^{\beta}, & x < x_I, \\ W(x) - I, & x \geq x_I. \end{cases} \tag{A.25}$$

Finally, substituting the investment threshold x_I for equation (A.15) gives the optimal debt amount at the optimal investment timing as shown in the following equation:

$$b^* = h^{\frac{1}{\gamma}} \frac{\gamma - 1}{\gamma} \frac{\beta}{\beta - 1} \frac{r\psi}{1 - \tau_{ax}} I. \tag{A.26}$$

APPENDIX 3: OPTIMAL INVESTMENT PROBLEM FOR TIME-INCONSISTENT FIRMS

The optimal investment problem (21) can be transformed into the following Bellman equation, and this clarifies the relation between $V(X_t)$ and $V(X_{t+dt})$:

$$V(X_t) = \max\{X_t/(r_0 - \mu) - I, e^{-r_0 dt}\mathrm{E}_t[V(X_{t+dt}) + \lambda[V_1(X_{t+dt}) - V(X_{t+dt})]]\}, \tag{A.27}$$

where $V_1(X_t)$ expresses the option value to invest after the interest rate increase ($r_0 \to r_1$).

$$V_1(X_t) = \max_{\tau \in F_t} \mathrm{E}_t\left[\int_{\tau}^{\infty} e^{-r_1(s-\tau)} X_s \, ds - I\right]. \tag{A.28}$$

We now solve the optimal investment problem after the interest rate increase and then use this solution to solve the optimal investment problem for the case incorporating the possibility of future interest rate increases.

By applying Ito's formula to equation (A.28), we derive the differential equation that satisfies $V_1(X_t)$:

$$\frac{1}{2}\sigma^2 x^2 V_1''(x) + \mu x V_1'(x) - r_1 V_1(x) = 0, \tag{A.29}$$

$$\text{boundary conditions: } \begin{cases} V_1\big(x_I^{(1)}\big) = x_I^{(1)}/(r_1-\mu) - I, \\ V_1'\big(x_I^{(1)}\big) = 1/(r_1-\mu), \end{cases} \tag{A.30}$$

where the boundary conditions (A.30) determine the optimal investment threshold $x_I^{(1)}$. These are the value matching conditions and smooth pasting conditions for $V_1(x)$ to smoothly connect with $x_I^{(1)}/(r_1-\mu) - I$ at $x_I^{(1)}$.

This equation has an analytical solution:

$$V_1(x) = \begin{cases} \left(\dfrac{x_I^{(1)}}{r_1-\mu} - I\right)\left(\dfrac{x}{x_I^{(1)}}\right)^{\beta_1}, & x < x_I^{(1)}, \\ \dfrac{x}{r_1-\mu} - I, & x \geq x_I^{(1)}. \end{cases} \tag{A.31}$$

The optimal investment threshold $x_I^{(1)}$ can also be solved from the boundary conditions (A.30) as follows:

$$x_I^{(1)} = \frac{\beta_1}{\beta_1 - I}(r_1-\mu)I, \tag{A.32}$$

where β_1 is the positive root of the characteristic equation to solve the optimization problem.

$$\frac{1}{2}\sigma^2\beta_1(\beta_1-1) + \mu\beta_1 - r_1 = 0. \tag{A.33}$$

The above solution is now used to solve equation (A.27). By applying Ito's formula to equation (A.27), we derive the differential equation that satisfies $V(X_t)$.

$$\frac{1}{2}\sigma^2 x^2 V''(x) + \mu x V'(x) - r_0 V(x) + \lambda[V_1(x) - V(x)] = 0, \tag{A.34}$$

$$\text{boundary conditions: } \begin{cases} V\big(x_I^{(\lambda)}\big) = x_I^{(\lambda)}/(r_0-\mu) - I, \\ V'\big(x_I^{(\lambda)}\big) = 1/(r_0-\mu). \end{cases} \tag{A.35}$$

This equation has an analytical solution:

$$V(x) = \begin{cases} \delta \cdot V_1(x) - \left(\delta \cdot V_1\left(x_I^{(\lambda)}\right) - \left(\dfrac{x_I^{(\lambda)}}{r_0 - \mu} - I\right)\right)\left(\dfrac{x}{x_I^{(\lambda)}}\right)^{\beta_\lambda}, & x < x_I^{(\lambda)}, \\ \dfrac{x}{r_0 - \mu} - I, & x \geq x_I^{(\lambda)}, \end{cases} \tag{A.36}$$

where δ expresses the additional discount rate from the interest rate increase ($r_0 \to r_1$).

$$\delta = 1/(1 - (r_1 - r_0)T), \quad T = 1/\lambda. \tag{A.37}$$

The optimal investment threshold $x_I^{(\lambda)}$ can also be solved numerically as the root of the following equation:

$$(\beta_\lambda - \beta_1)\delta\left(\frac{x_I^{(1)}}{r_1 - \mu} - I\right)\left(\frac{x_I^{(\lambda)}}{x_I^{(1)}}\right)^{\beta_1} = (\beta_\lambda - 1)\frac{x_I^{(\lambda)}}{r_0 - \mu} - \beta_\lambda I, \tag{A.38}$$

where β_λ is the solution of the characteristic equation to solve the optimization problem:

$$\frac{1}{2}\sigma^2\beta_\lambda(\beta_\lambda - 1) + \mu\beta_\lambda - (r_0 + \lambda) = 0. \tag{A.39}$$

APPENDIX 4: OPTIMAL INVESTMENT PROBLEM FOR TIME-CONSISTENT FIRMS

The optimal investment problem (22) can be transformed into the following Bellman equation, and this clarifies the relation between $V(X_t)$ and $V(X_{t+dt})$:

$$V(X_t) = \max\{W_\lambda(X_t) - I, e^{-r_0 dt}\mathrm{E}_t[V(X_{t+dt}) + \lambda[V_1(X_{t+dt}) - V(X_{t+dt})]]\}, \tag{A.40}$$

where $V_1(X_t)$ expresses the option value to invest after the interest rate increase ($r_0 \to r_1$) defined by equation (A.28). $W_\lambda(X_t)$ is the discounted present value incorporating future interest rate increases.

$$W_\lambda(X_t) = \mathrm{E}_t\left[\int_t^{\tau_1} e^{-r_0(s-t)}X_s\,ds + \int_{\tau_1}^{\infty} e^{-r_1(s-t)}X_s\,ds\right]. \tag{A.41}$$

We first derive $W_\lambda(X_t)$ to solve the optimal investment problem (A.40).

By applying Ito's formula to equation (A.41), we derive the differential equation that satisfies $W_\lambda(X_t)$:

$$\frac{1}{2}\sigma^2 x^2 W_\lambda''(x) + \mu x W_\lambda'(x) - r_0 W_\lambda(x) + \lambda[W_\lambda^1(x) - W_\lambda(x)] = 0, \tag{A.42}$$

where $W_\lambda^1(x)$ is the discounted present value incorporating future interest rate increases.

$$W_\lambda^1(x) = \mathrm{E}_t\left[\int_t^\infty e^{-r_1(s-t)} X_s\, ds\right] = \frac{x}{r_1 - \mu}. \tag{A.43}$$

This equation has an analytical solution:

$$W_\lambda(x) = \frac{x}{r_\lambda - \mu}. \tag{A.44}$$

Here r_λ is the long-term interest rate that considers future interest rate increases:

$$r_\lambda = \mu + \frac{(r_0 + \lambda) - \mu}{(r_1 + \lambda) - \mu}(r_1 - \mu). \tag{A.45}$$

Note that the natural results hold for the interest rate increase probability λ.

$$\lim_{\lambda \to 0} r_\lambda = r_0, \quad \lim_{\lambda \to \infty} r_\lambda = r_1. \tag{A.46}$$

We now use the above solution to solve equation (A.40). Applying Ito's rule to equation (A.40), we derive the differential equation that satisfies $V(X_t)$:

$$\frac{1}{2}\sigma^2 x^2 V''(x) + \mu x V'(x) - r_0 V(x) + \lambda[V_1(x) - V(x)] = 0, \tag{A.47}$$

$$\text{boundary conditions: } \begin{cases} V\big(x_I^{(\lambda)}\big) = W_\lambda\big(x_I^{(\lambda)}\big) - I, \\ V'\big(x_I^{(\lambda)}\big) = W_\lambda'\big(x_I^{(\lambda)}\big). \end{cases} \tag{A.48}$$

This equation has an analytical solution:

$$V(x) = \begin{cases} \delta \cdot V_1(x) - \Big(\delta \cdot V_1\big(x_I^{(\lambda)}\big) - \big(W_\lambda\big(x_I^{(\lambda)}\big) - I\big)\Big)\left(\dfrac{x}{x_I^{(\lambda)}}\right)^{\beta_\lambda}, & x < x_I^{(\lambda)}, \\ W_\lambda(x) - I, & x \geq x_I^{(\lambda)}, \end{cases} \tag{A.49}$$

where δ expresses the additional discount rate (A.37) from the interest rate increase ($r_0 \to r_1$).

The optimal investment threshold $x_I^{(\lambda)}$ can also be solved numerically as the root of the following equation:

$$(\beta_\lambda - \beta_1)\delta\left(\frac{x_I^{(1)}}{r_1 - \mu} - I\right)\left(\frac{x_I^{(\lambda)}}{x_I^{(1)}}\right)^{\beta_1} = (\beta_\lambda - 1)W_\lambda\big(x_I^{(\lambda)}\big) - \beta_\lambda I, \tag{A.50}$$

where β_λ is the constant defined by equation (A.39).

APPENDIX 5: DEBT FINANCING AND INVESTMENT PROBLEM FOR TIME-INCONSISTENT FIRMS

The optimal debt financing and investment problem (23) can be transformed into the following Bellman equation, and this clarifies the relation between $V(X_t)$ and $V(X_{t+dt})$:

$$V(X_t) = \max\{W^{[0]}(X_t) - I, e^{-r_0 dt}\mathrm{E}_t[V(X_{t+dt}) + \lambda[V^{[1]}(X_{t+dt}) - V(X_{t+dt})]]\}, \tag{A.51}$$

where $V_1(X_t)$ expresses the option value to invest after the interest rate increase ($r_0 \to r_1$).

$$V^{[1]}(X_t) = \max_{\tau \in F_t} \mathrm{E}_t[e^{-r_1(\tau - t)}(W^{[1]}(X_\tau) - I)]. \tag{A.52}$$

We now solve the debt financing and investment problem after the interest rate increase (A.52), and then use this solution to solve the debt financing and investment problem for the case incorporating the possibility of future increases in the interest rate (A.51).

Because equation (A.52) is the debt financing and investment problem with a fixed interest rate (r_1), we can apply an analytical solution derived in Section II (Appendix 1). First, we derive the equity value $E^{[1]}(x)$ and the debt value $D^{[1]}(x)$ that comprise the firm value $W^{[1]}(x)$ as follows:

$$E^{[1]}(x) = (1 - \tau_{ax})\left(\left(\frac{x}{r_1 - \mu} - \frac{b^{[1]}}{r_1}\right) - \left(\frac{x_b^{[1]}}{r_1 - \mu} - \frac{b^{[1]}}{r_1}\right)\left(\frac{x}{x_b^{[1]}}\right)^{\gamma_1}\right), \quad x > x_b^{[1]}, \tag{A.53}$$

where

$$x_b^{[1]} = \frac{\gamma_1}{\gamma_1 - 1}\frac{b^{[1]}}{r_1}(r_1 - \mu) \tag{A.54}$$

expresses the bankruptcy threshold, and the firm goes bankrupt when $x < x_b^{[1]}$. The stock price after bankruptcy is zero. Meanwhile γ is the negative root of the following characteristic equation to solve the optimization problem:

$$\frac{1}{2}\sigma^2\gamma_1(\gamma_1 - 1) + \mu\gamma_1 - r_1 = 0. \tag{A.55}$$

The debt value is then derived as follows:

$$D^{[1]}(x) = \begin{cases} \dfrac{b^{[1]}}{r_1} - \left(\dfrac{b^{[1]}}{r_1} - W_b^{[1]}\big(x_b^{[1]}\big)\right)\left(\dfrac{x}{x_b^{[1]}}\right)^{\gamma_1}, & x > x_b^{[1]}, \\ W_b^{[1]}\big(x_b^{[1]}\big) \equiv (1 - \alpha)\dfrac{(1 - \tau_{ax})x_b^{[1]}}{r_1 - \mu}, & x \le x_b^{[1]}, \end{cases} \tag{A.56}$$

where $W_b^{[1]}\big(x_b^{[1]}\big)$ is the residual value (revised valuation) of the firm at the time of bankruptcy $x_b^{[1]}$. $W^{[1]}(x)$ is derived as the sum of $E^{[1]}(x)$ and $D^{[1]}(x)$.

$$W^{[1]}(x) = \begin{cases} \dfrac{(1-\tau_{ax})x}{r_1-\mu} + \left(1-\left(\dfrac{x}{x_b^{[1]}}\right)^{\gamma_1}\right)\dfrac{\tau_{ax}b^{[1]}}{r_1} \\ -\left((1-\tau_{ax})\dfrac{\alpha x_b^{[1]}}{r_1-\mu}\right)\left(\dfrac{x}{x_b^{[1]}}\right)^{\gamma_1}, \quad x > x_b^{[1]}, \\ W_b^{[1]}\big(x_b^{[1]}\big) \equiv (1-\alpha)\dfrac{(1-\tau_{ax})x_b^{[1]}}{r_1-\mu}, \quad x \le x_b^{[1]}. \end{cases} \tag{A.57}$$

We then seek the optimal amount of debt by differentiating this firm value by $b^{[1]}$.

$$b^{[1]} = h^{[1]\frac{1}{\gamma_1}}\frac{\gamma_1-1}{\gamma_1}x, \tag{A.58}$$

where $h^{[1]}$ is a constant.

$$h^{[1]} \equiv 1-\gamma_1\left(1-\alpha+\frac{\alpha}{\tau_{ax}}\right) \ge 1. \tag{A.59}$$

Note that the optimal amount of debt depends on the profit x at the time of investment. The firm value with optimal capital structure is derived by substituting equation (A.58) into equation (A.57).

$$W^{[1]}(x) = \psi^{[1]^{-1}}\frac{(1-\tau_{ax})x}{r_1-\mu}, \quad x > x_b^{[1]}, \tag{A.60}$$

where $\psi^{[1]}$ is a constant.

$$\psi^{[1]} \equiv \left(1+h^{[1]\frac{1}{\gamma_1}}\left(\frac{\tau_{ax}}{1-\tau_{ax}}\right)\right)^{-1} \le 1. \tag{A.61}$$

The option value to invest $V^{[1]}(x)$ can be solved analytically using this $\psi^{[1]}$.

$$V^{[1]}(x) = \begin{cases} \big(W^{[1]}\big(x_I^{[1]}\big)-I\big)\left(\dfrac{x}{x_I^{[1]}}\right)^{\beta_1}, & x < x_I^{[1]}, \\ W^{[1]}(x)-I, & x \ge x_I^{[1]}, \end{cases} \tag{A.62}$$

where $x_I^{[1]}$ is the investment threshold.

$$x_I^{[1]} = \psi^{[1]}\frac{\beta_1}{\beta_1-I}\frac{(r_1-\mu)}{1-\tau_{ax}}I. \tag{A.63}$$

β_1 is the constant defined by equation (A.33). The optimal debt amount at the optimal investment timing can also be solved by substituting the investment threshold $x_I^{[1]}$ for x in equation (A.58).

$$b^{[1]} = h^{[1]\frac{1}{\gamma_1}} \frac{\gamma_1 - 1}{\gamma_1} \frac{\beta_1}{\beta_1 - 1} \frac{r_1 \psi^{[1]}}{1 - \tau_{ax}} I. \tag{A.64}$$

Next, we solve equation (A.51) using the above solution. We apply Ito's rule to equation (A.51) to derive the differential equation that satisfies $V(X_t)$.

$$\frac{1}{2}\sigma^2 x^2 V''(x) + \mu x V'(x) - r_0 V(x) + \lambda[V^{[1]}(x) - V(x)] = 0, \tag{A.65}$$

$$\text{boundary conditions: } \begin{cases} V\left(x_I^{[\lambda]}\right) = W^{[0]}\left(x_I^{[\lambda]}\right) - I, \\ V'\left(x_I^{[\lambda]}\right) = W'^{[0]}\left(x_I^{[\lambda]}\right), \end{cases} \tag{A.66}$$

where $W^{[0]}(x)$ is the firm value when the interest rate is fixed at r_0, and this is solved in exactly the same manner as $W^{[1]}(x)$ (it is equal to equation [A.60], substituting the subscript 0 for the subscript 1).

This equation has an analytical solution:

$$V(x) = \begin{cases} \delta \cdot V^{[1]}(x) - \left(\delta \cdot V^{[1]}\left(x_I^{[\lambda]}\right) - \left(W^{[0]}\left(x_I^{[\lambda]}\right) - I\right)\right)\left(\dfrac{x}{x_I^{[\lambda]}}\right)^{\beta_\lambda}, & x < x_I^{[\lambda]}, \\ W^{[0]}(x) - I, & x \geq x_I^{[\lambda]}, \end{cases} \tag{A.67}$$

where δ expresses the additional discount rate (A.37) from the interest rate increase ($r_0 \to r_1$). The optimal investment threshold $x_I^{[\lambda]}$ can also be solved numerically as the root of the following equation:

$$(\beta_\lambda - \beta_1)\delta\left(W^{[1]}\left(x_I^{[1]}\right) - I\right)\left(\frac{x_I^{[\lambda]}}{x_I^{[1]}}\right)^{\beta_1} = (\beta_\lambda - 1)W^{[0]}(x_I^{\lambda}) - \beta_\lambda I, \tag{A.68}$$

where β_λ is the constant defined by equation (A.39).

APPENDIX 6: DEBT FINANCING AND INVESTMENT PROBLEM FOR TIME-CONSISTENT FIRMS

The optimal debt financing and investment problem (27) can be transformed into the following Bellman equation, and this clarifies the relation between $V(X_t)$ and $V(X_{t+dt})$:

$$V(X_t) = \max\{W^{[0\to 1]}(X_t) - I, e^{-r_0 dt}\mathrm{E}_t[V(X_{t+dt}) + \lambda[V^{[1]}(X_{t+dt}) - V(X_{t+dt})]]\}, \tag{A.69}$$

where $V^{[1]}(X_t)$ expresses the option value to invest after the interest rate increase ($r_0 \to r_1$) defined by equation (A.52). $W^{[0\to 1]}(X_t)$ is the firm value incorporating the interest rate increase ($r_0 \to r_1$) defined by equations (28), (29), and (30). Hereafter, we first derive $W^{[0\to 1]}(X_t)$ to solve the debt financing and investment problem (A.69).

Because $W^{[0\to 1]}(X_t)$ is defined as the sum of $E^{[0\to 1]}(X_t)$ and $D^{[0\to 1]}(X_t)$, we first solve these problems. We apply Ito's rule to equation (29) to derive the differential equation that satisfies $E^{[0\to 1]}(X_t)$.

$$\frac{1}{2}\sigma^2 x^2 E^{[0\to 1]''}(x) + \mu x E^{[0\to 1]'}(x) + (1-\tau_{ax})(x_t - b) - r_0 E^{[0\to 1]}(x) + \lambda[E^{[1]}(x) - E^{[0\to 1]}(x)] = 0, \tag{A.70}$$

where $E^{[1]}(X_t)$ is the equity value after the interest rate increase (A.53). This equation has an analytical solution:

$$E^{[0\to 1]}(x) = (1-\tau_{ax})\left(\left(\frac{x}{r_\lambda-\mu} - \frac{b^{[\lambda]}}{r_\lambda}\right) - \left(\frac{x_b^{[\lambda]}}{r_\lambda-\mu} - \frac{b^{[\lambda]}}{r_\lambda}\right)\left(\frac{x}{x_b^{[\lambda]}}\right)^{\gamma_1}\right), \quad x > x_b^{[\lambda]}, \tag{A.71}$$

where

$$x_b^{[\lambda]} = \frac{\gamma_\lambda}{\gamma_\lambda - 1}\frac{b^{[\lambda]}}{r_\lambda}(r_\lambda - \mu) \tag{A.72}$$

expresses the bankruptcy threshold, and the firm goes bankrupt when $x < x_b^{[\lambda]}$. The stock price after bankruptcy is zero. Meanwhile γ_λ is the negative root of the following characteristic equation to solve the optimization problem:

$$\frac{1}{2}\sigma^2\gamma_\lambda(\gamma_\lambda - 1) + \mu\gamma_\lambda - r_\lambda = 0. \tag{A.73}$$

The debt value $D^{[0\to 1]}(X_t)$ is derived in the same manner, as follows:

$$D^{[0\to 1]}(x) = \begin{cases} \dfrac{b^{[\lambda]}}{r_\lambda} - \left(\dfrac{b^{[\lambda]}}{r_\lambda} - W_b^{[\lambda]}\big(x_b^{[\lambda]}\big)\right)\left(\dfrac{x}{x_b^{[\lambda]}}\right)^{\gamma_\lambda}, & x > x_b^{[\lambda]}, \\ W_b^{[\lambda]}\big(x_b^{[\lambda]}\big) \equiv (1-\alpha)\dfrac{(1-\tau_{ax})x_b^{[\lambda]}}{r_\lambda - \mu}, & x \le x_b^{[\lambda]}, \end{cases} \tag{A.74}$$

where, $W_b^{[\lambda]}\big(x_b^{[\lambda]}\big)$ is the residual value (revised valuation) of the firm at the time of bankruptcy $x_b^{[\lambda]}$. $W^{[0\to 1]}(X_t)$ is derived as the sum of $E^{[0\to 1]}(X_t)$ and $D^{[0\to 1]}(X_t)$, and the optimal amount of debt is also derived as follows:

$$b^{[\lambda]} = h^{[\lambda]\frac{1}{\gamma_\lambda}}\frac{\gamma_\lambda - 1}{\gamma_\lambda}x. \tag{A.75}$$

Note that the optimal amount of debt is dependent on the profit x at the time of investment.

Next, we solve equation (A.69) using the above solution $W^{[0\to 1]}(X_t)$. We apply Ito's rule to equation (A.69) to derive the differential equation that satisfies $V(X_t)$.

$$\frac{1}{2}\sigma^2 x^2 V''(x) + \mu x V'(x) - r_0 V(x) + \lambda[V^{[1]}(x) - V(x)] = 0, \tag{A.76}$$

$$\text{boundary conditions: } \begin{cases} V\left(x_I^{[\lambda]}\right) = W^{[0\to 1]}\left(x_I^{[\lambda]}\right) - I, \\ V'\left(x_I^{[\lambda]}\right) = W'^{[0\to 1]}\left(x_I^{[\lambda]}\right). \end{cases} \tag{A.77}$$

This equation has an analytical solution.

$$V(x) = \begin{cases} \delta \cdot V^{[1]}(x) - \left(\delta \cdot V^{[1]}\left(x_I^{[\lambda]}\right) - \left(W^{[0\to 1]}\left(x_I^{[\lambda]}\right) - I\right)\right)\left(\dfrac{x}{x_I^{[\lambda]}}\right)^{\beta_\lambda}, & x < x_I^{[\lambda]}, \\ W^{[0\to 1]}(x) - I, & x \geq x_I^{[\lambda]}, \end{cases} \tag{A.78}$$

where $V^{[1]}(X_t)$ expresses the option value to invest after the interest rate increase ($r_0 \to r_1$) derived in equation (A.62), and δ expresses the additional discount rate (A.37) from the interest rate increase ($r_0 \to r_1$). The optimal investment threshold $x_I^{[\lambda]}$ can be solved numerically as the root of the following equation:

$$(\beta_\lambda - \beta_1)\delta\left(W^{[1]}\left(x_I^{[1]}\right) - I\right)\left(\frac{x_I^{[\lambda]}}{x_I^{[1]}}\right)^{\beta_1} = (\beta_\lambda - 1)W^{[0\to 1]}(x_I^{\lambda}) - \beta_\lambda I, \tag{A.79}$$

where β_λ is the constant defined by equation (A.39).

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