

The Japanese Repo Market: Theory and Evidence

Naohiko Baba and Yasunari Inamura

Repurchase agreement (repo) transactions are widely used as a risk-free means of borrowing or lending funds and securities. Repo transactions can be categorized into (1) general collateral (GC) repos that borrow or lend funds, and (2) special collateral (SC) repos that borrow or lend specific securities. GC repo rates are priced at a level close to the risk-free interest rate, while SC repo rates are often priced far below the GC repo rates. This paper aims to examine the pricing mechanism of the Japanese repo market from both theoretical and empirical perspectives.

First, Duffie (1996) and Krishnamurthy (2001) show that (1) equilibrium in the repo market requires no-arbitrage profits from combining repo and cash bond transactions, (2) the equilibrium level of repo spreads between GC and SC repo rates is determined at the point where the supply and demand curves of the underlying bond issues intersect in the repo market, and (3) expected returns from future matched book trading are reflected in the cash prices of SC bond issues.

Second, the paper empirically examines the above theoretical implications using the data of repo rates and government bond prices in Japan. Our empirical results show that, regarding the on-the-run and the cheapest-to-deliver (CTD) issues, the above no-arbitrage condition is significantly satisfied.

Keywords: Repo; Government bond; No-arbitrage condition; Repo spread; On-the-run issues; Cheapest to deliver (CTD); Short sales

JEL Classification: G11, G12

Naohiko Baba: Financial Markets Department, Bank of Japan (E-mail: naohiko.baba@boj.or.jp)
Yasunari Inamura: Financial Markets Department (currently, Bank Examination and Surveillance Department), Bank of Japan (E-mail: yasunari.inamura@boj.or.jp)

.....
We thank Professor Makoto Saito (Hitotsubashi University) and two anonymous referees for their valuable comments and suggestions. We also benefited a great deal from discussions with the repo and bond dealers who participated in the research workshop on the repo and Japanese government bonds, which was held by the Financial Markets Department of the Bank of Japan. Any remaining errors are entirely our own.

I. Introduction

A repo transaction is a contract that exchanges securities with high creditworthiness, typically government bonds, for funds for a fixed period of time. The securities function as collateral for raising funds, and the funds function as collateral for borrowing securities. Thus, market participants recognize repo transactions as a risk-free means of raising or investing funds against securities as collateral.¹ Of these repo transactions, those for raising funds are called general collateral (GC) repos, while those for borrowing securities are called special collateral (SC) repos. GC repo transactions generate the linkage between the repo market and money markets including the interbank market, while SC repo transactions generate the linkage between the repo market and securities markets, typically the government bond market.

Regarding the linkage between the repo market and the interbank market, Griffiths and Winters (1997) found that in the United States, GC repo rates moved almost parallel with the uncollateralized federal funds (FF) rate. On the linkage between the repo market and the bond market, Jordan and Jordan (1997) empirically examined Duffie's (1996) theoretical insight that bond prices should reflect expected profits from future matched book trading, which takes advantage of repo spreads defined as the differences between GC and SC repo rates. In Japan, Shigemi *et al.* (2001) described the pricing of Japanese government bonds (JGBs) in the cash, futures, and repo markets after the stressful events that took place between 1998 and 1999.² This paper aims to examine the pricing mechanism of the Japanese repo market from both theoretical and empirical perspectives.

First, the paper theoretically reviews the pricing mechanism of the repo rates, focusing on the linkage between the repo market and the JGB market, following the above-mentioned studies. Theoretically, GC repo rates should be priced at a level close to the risk-free interest rate. Depending on the underlying bond issues, however, SC repo rates are often priced far below the GC repo rates, even though the issuers are the same or equally rated in terms of credit standing. A number of market participants recognize that repo spreads, defined as the differences between GC and SC repo rates, are closely related to the supply and demand balance of the underlying bond issues in the repo market.

Duffie (1996) and Krishnamurthy (2001) derived a mechanism by which repo rates can be priced differently depending on the underlying bond issues. The mechanism is summarized as follows: (1) equilibrium in the repo market requires no-arbitrage profits from trading that combines repo and cash bond transactions (no-arbitrage condition); (2) the equilibrium level of the repo spreads is determined

1. Legally, repo transactions in Japan differ from those in the United States and Europe. In Japan, they take the form of borrowing or lending of securities against funds as collateral, whereas in the United States and Europe, securities are sold or bought with an agreement to buy or sell back the same securities at a later date. Furthermore, in Japan, cross-border repo transactions have been inactive due partly to the withholding tax levied on the interest earned on repo transactions by nonresidents. Cross-border transactions, however, were expected to become more active following the introduction of a new U.S.-type repo system in April 2001. Also, the withholding tax on nonresidents was abolished in April 2002. See Appendix 1 for the scale of the Japanese repo market.

2. Shigemi *et al.* (2001) explained the mechanism by which a sudden change in the supply and demand balance of JGBs triggered by stressful events such as the Y2K problem brought about a deterioration in the arbitrage relationships between the cash, futures, and repo markets.

at the point where the supply and demand curves of the underlying bonds intersect in the repo market; and (3) expected returns from future matched book trading are reflected in the cash prices of SC bond issues.

Second, the paper empirically examines the above theoretical implications using the data of Japanese repo rates and the JGB prices. Our main finding is summarized as follows: when we regard the most recently issued (on-the-run) 10-year JGBs and the cheapest-to-delivery (CTD) issues as SC bond issues, the no-arbitrage condition is significantly satisfied between the repo spreads and the corresponding cash premiums, which are defined as the differences between the market prices of SC and GC bond issues.

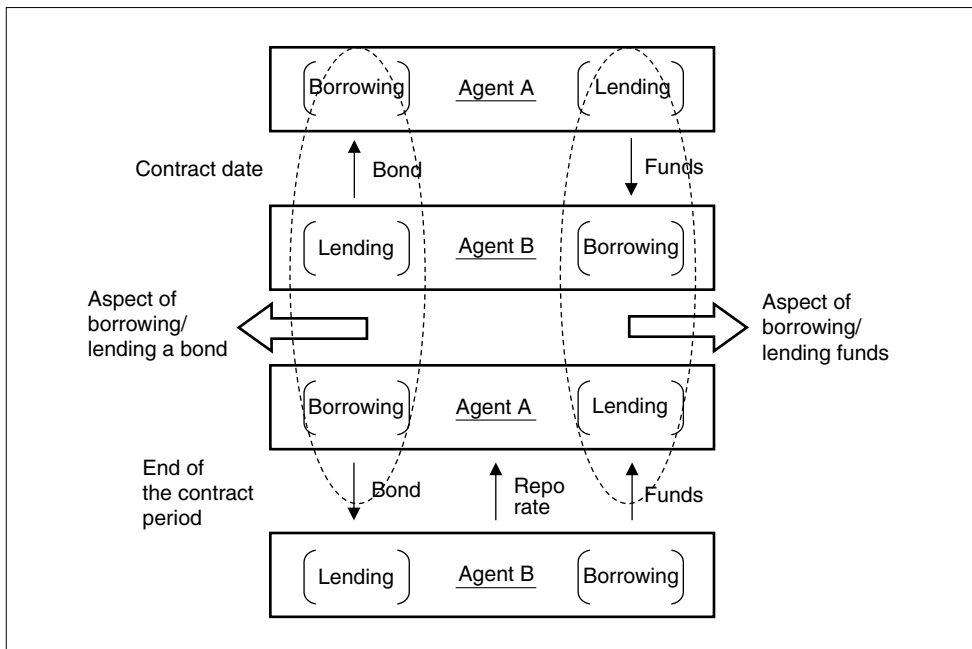
The rest of the paper is organized as follows. Section II presents an overview of the Japanese repo market, as well as the general framework of repo transactions. Section III reviews the mechanism by which repo rates can be priced differently depending on the underlying bond issues, based on Duffie (1996) and Krishnamurthy (2001). Section IV conducts a simple empirical analysis of the theoretical implications reviewed in Section III using the data of on-the-run 10-year JGBs and CTD issues. Section V concludes by referring to the limitations of the analysis in the paper.

II. Overview of Repo Transactions³

A. Two Aspects: Borrowing/Lending of Funds and Securities

As shown in Figure 1, a repo transaction starts when agent A exchanges his or her funds for a bond held by agent B at market price on the contract date. At the end of

Figure 1 General Framework of Repo Transactions



3. Repo Trading Research (2001) provides a concise review of repo transactions in Japan.

the contract period, agent A receives back the funds with interest, while the bond is returned to agent B. The interest rate thus paid on the funds is called the repo rate, which is defined as the “interest rate on the funds minus a securities lending fee.”

Figure 1 reveals two aspects of repo transactions. The first is the borrowing of funds against bonds as collateral. Suppose that agent A is an investor with surplus funds and agent B a bond dealer who needs to finance his or her bond portfolio. In this setup, the investor can lend funds in exchange for bonds with high creditworthiness as collateral. The bond dealer, on the other hand, can reduce funding costs by using his or her bond holdings as collateral, thereby curtailing the credit risk premium required by the investor.

The second is the borrowing of bonds against funds (cash) collateral. Suppose that agent A is a bond dealer who wants to build a short position and agent B an investor with an extensive bond portfolio. In this setup, the bond dealer can cover his or her short position through a repo transaction by borrowing the necessary bonds against cash collateral. The investor, on the other hand, can effectively reduce funding costs by lending the dealer the bonds in demand.

The former transactions for lending and borrowing funds are called GC repos, while the latter for lending and borrowing bonds are called SC repos. In GC repos, the underlying securities are not specified, while in SC repos, they are specified when the parties enter into a contract.

B. Risk-Free Characteristic of Repo Transactions

As stated above, in repo transactions, both bonds and funds function as collateral that ensures safety in the event of the default of either counterparty. In addition, “marking to market,” also called “margin calls,” is practiced to protect against bond price fluctuations during the contract period. If the lender of the bonds (borrower of funds) defaults while the bond price falls during the contract period, the borrower of the bonds (the lender of funds) cannot recover the full amount of his or her funds by selling the bonds put up as collateral. To avoid the loss, the system of margin calls enables traders to ask for additional provision of collateral when a shortfall occurs during the contract period, based on the calculation of the margins, that is, the shortage/excess of the value of the collateral. This kind of risk management method ensures a high degree of safety for repo transactions.⁴

C. Use of Repo Transactions

1. SC repos and short sales

Bond dealers can efficiently build short positions using repo transactions.⁵ The most typical way is to sell short a bond in the cash market while simultaneously borrowing the same bond through an SC repo for delivery of the bond. As illustrated in Figure 2 [1], the funds raised by selling short a bond can be used to cover the funds required for settling the SC repo transaction. In this way, short positions can be built without holding any initial capital or the specific bonds.⁶

4. See Repo Trading Research (2001) for details on various risk management methods for repo transactions.

5. In fact, Keane (1996) argues that many special repo transactions stem from the need to cover short sales.

6. For simplicity, we ignore the costs entailing the haircut for managing the risk of price fluctuations, the administrative costs, and the costs arising from differences in settlement timing.

At the end of the contract period, the bond dealer needs to buy the bond in the cash market to return it to the investor. The dealer can use the funds $P_1(1 + R)$ ($R =$ repo rate) paid by the investor to buy back the bond in the cash market (Figure 2 [2]).

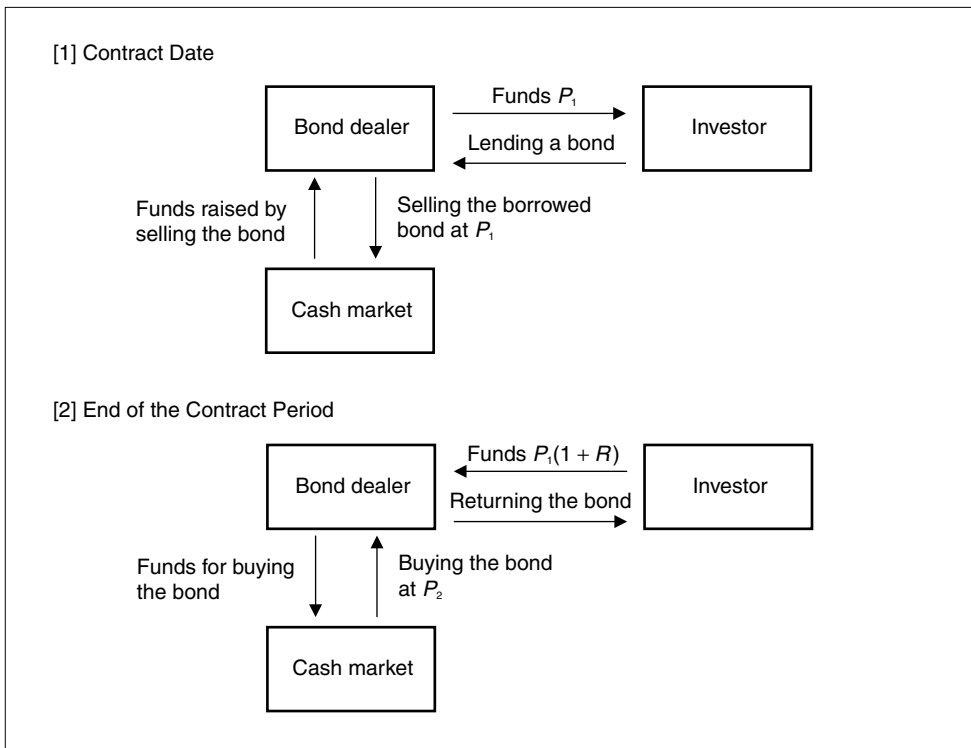
Why do bond dealers want to build short positions using repo transactions? The dealer's profit/loss from bond trading can be expressed as P_1 (selling price) $- P_2$ (buying price). At the same time, the dealer invests funds P_1 at the repo rate R and receives P_1R . Thus, the return for the dealer is expressed as $P_1 - P_2 + P_1R$. Note that the return is always positive when the bond price falls ($P_1 > P_2$) during the contract period as long as R takes a positive value.

In this way, when bond prices are expected to fall, speculative dealers have an incentive to build short positions. Risk-averse bond dealers also benefit from holding similar short positions, as they can hedge against losses from a possible fall in the prices of their bonds.

2. Matched book trading

Investors with rich bond portfolios can generate profits without being exposed to risks by taking advantage of various levels of repo rates. For example, the investors can lend (borrow) bonds (funds) at a low repo rate (a high securities lending fee), and lend (borrow) the funds (bonds) at a higher repo rate, yielding profits equivalent to the spread between the two repo rates. This kind of transaction is called "matched book" trading. While it is widely used in the United States as a means of reducing the funding costs of portfolios, it is still in limited use in Japan.

Figure 2 SC Repos and Short Sales



D. Repo Rates in Japan

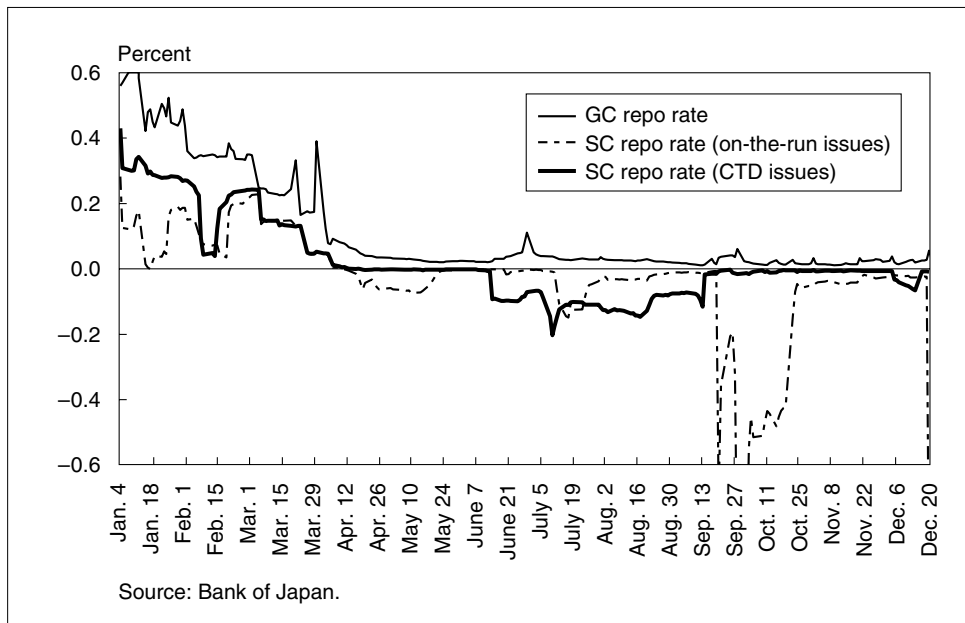
SC repo rates are mostly independent of, and lower than, GC repo rates. Figure 3 shows overnight GC and SC repo rates of the on-the-run issues of 10-year JGBs and the CTD issues.⁷ Evidently, there is a large difference between repo rates. SC repo rates often take negative values depending on the underlying bond issues. Here, we will give an intuitive explanation for this phenomenon despite the fact that the bond issues put up as collateral are of the same quality in terms of credit risk. A detailed examination will be made in Section III.

As stated earlier, the repo rate is defined as the interest rate minus a securities lending fee. All funds are equal in quality by definition. Thus, theoretically, the interest rate should equal the average rate for uncollateralized lending such as the Tokyo Interbank Offered Rate (TIBOR), regardless of whether it is a GC or SC repo rate. With regard to lending fees, since most issues can be used for GC repos, it is natural to think that fees are on the same level.

Suppose, however, that a certain issue cannot be substituted for others, resulting in having a unique supply/demand structure. If demand for this issue exceeds supply in the repo market, the market mechanism would work to raise the lending fee of this issue by (Δ lending fee) above that of other general issues. As a result, the repo rate for this issue would fall by (Δ lending fee) by definition, which leads to the following relationship: interest rate – (lending fee + Δ lending fee) = original repo rate – Δ lending fee. When (Δ lending fee) exceeds the original repo rate, the repo rate turns negative.

Similar to the GC repo rate, the SC repo rate is defined as the interest rate minus a securities lending fee for a specific issue. Thus, the repo spread is equivalent to “the

Figure 3 Repo Rates in 2001

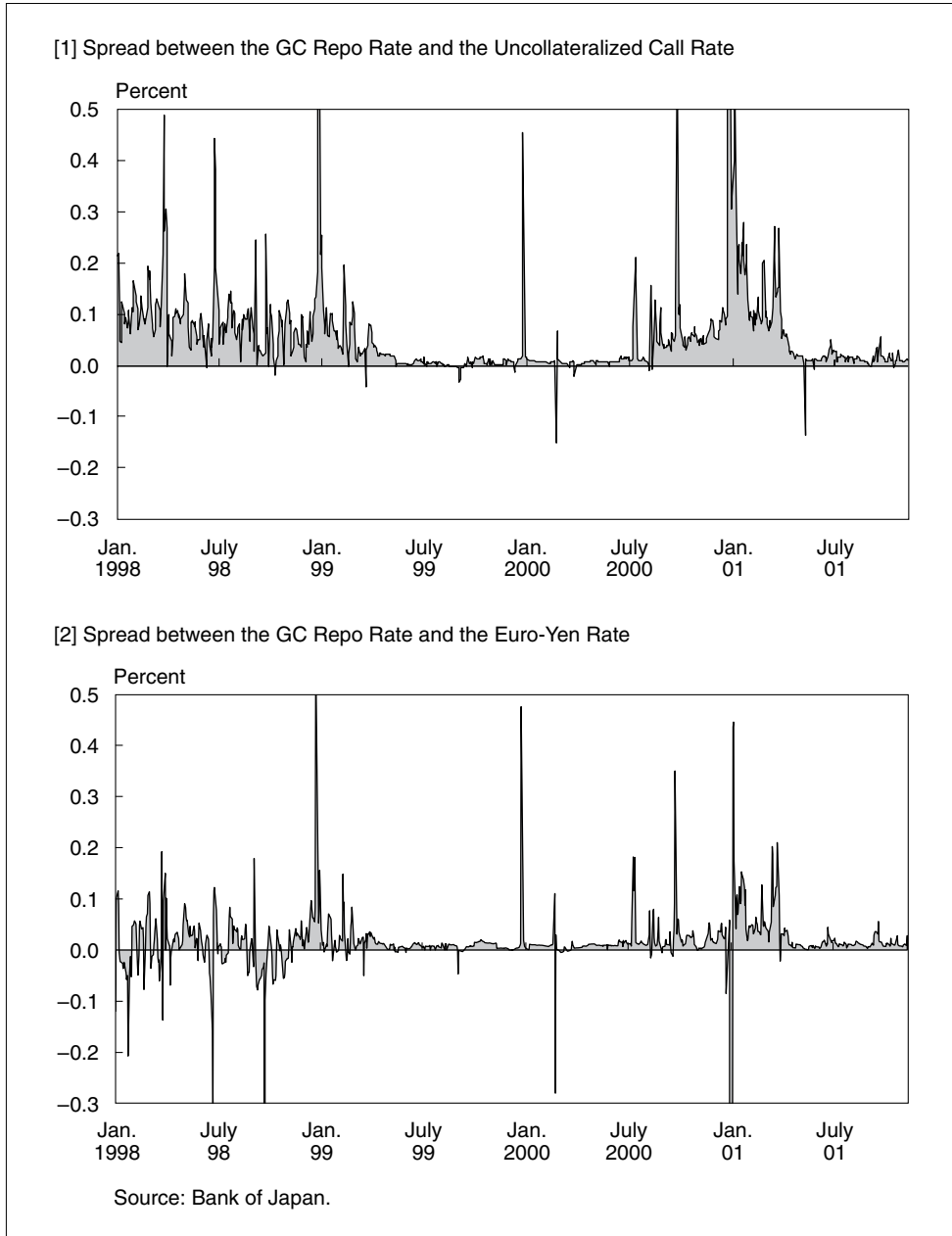


7. Settlement of overnight transactions is made two days after the contract date; this type of transaction is also known as a spot/next (S/N) transaction.

lending fee for a specific issue minus that for general issues,” which is equal to the above-mentioned (Δ lending fee). When the repo spread is positive, the lending fee for this specific issue becomes higher than that for general issues, reflecting a tight supply and demand condition for that issue.

On the other hand, Figure 4 shows the spreads between the overnight GC repo and major uncollateralized lending rates. Although GC repos have a risk-free characteristic,

Figure 4 Spreads between the GC Repo Rate and the Uncollateralized Rate



they are almost always higher than the uncollateralized call rate, also often staying above the euro-yen rate. This fact stands out when compared with the relationship between the GC repo and the FF rates in the United States. In the United States, the GC repo rate moves almost in parallel, a few basis points below the uncollateralized FF rate. For example, from January 1998 to August 2001, the spread between the GC repo and the uncollateralized call rate (FF rate for the United States) averaged 0.05 percent in Japan, compared with -0.07 percent in the United States.⁸

III. Theoretical Review

A. General Repo Rate

As shown in Figure 5, a general repo transaction can be thought of as the combined position of (1) a short sale of a bond at price P on the contract date and (2) a forward contract that receives $P \times (1 + \text{GC repo rate } R)$ at the end of the contract period.⁹

In equilibrium, the expected return from the above transaction, $E[F - P(1 + R)]$, should be zero, where E denotes the expectation operator. This leads to the following equation:

$$F = P(1 + R). \tag{1}$$

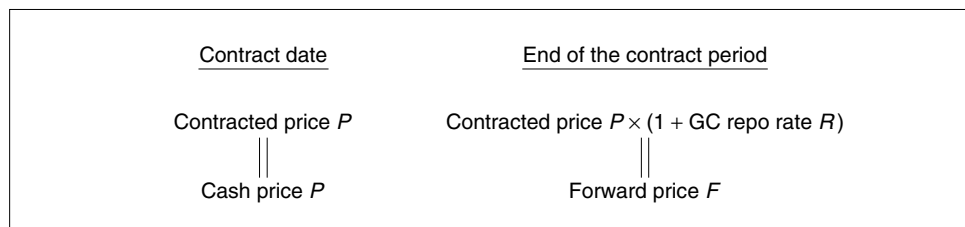
That is, the GC repo rate should be equivalent to the risk-free interest rate.¹⁰

B. Special Repo Rate

1. No-arbitrage condition

Now let us review how SC repo rates are priced following Duffie (1996) and Krishnamurthy (2001). The preceding discussion shows that the GC repo rate should be equal to the risk-free interest rate. On the other hand, as shown in Section II, the

Figure 5 Forward and GC Repo Transactions



8. At present, no consensus has been reached on why risk-free repo rates are higher than uncollateralized rates. We attempt to find an answer to this question in Appendix 2, focusing on the period after January 2001, when real-time gross settlement (RTGS) was introduced.

9. Strictly speaking, since repo transactions entail marking to market, they differ from forward transactions. Also, we disregard interest rate uncertainty for simplicity.

10. Equation (1) means that the period rate of return, computed from futures and cash prices, is equal to the GC repo rate. Market participants actually conduct arbitrage trading, which combines (1) the futures and cash sales/purchases of the CTD issues and (2) repo transactions. Repo Trading Research (2001) provides a simple example of this arbitrage trading. The rate of period return is also called the "implied repo rate." In a normal situation, the implied repo rates should be equal to the repo rates on the CTD issues as a result of arbitrage trading. In reality, however, this is not always the case because of inherent delivery risk when the CTD issue changes. See Shigemitsu *et al.* (2001) for details.

special bond issues have their own SC repo rates, which are lower than GC repo rates. For equilibrium to be reached in the presence of the repo spreads, the profit expected from arbitrage trading using repo spreads needs to be zero. As explained earlier, an arbitrage position can be built without holding any initial capital or bond issues by combining a repo contract with an outright cash bond transaction. Thus, examining the zero-profit condition from these transactions (no-arbitrage condition) will provide a clue to the level of equilibrium repo spreads.

For simplicity, we consider a two-period model without any costs incurred by (1) uncertainty over inventory arising from mismatching in the timing of buying and selling and (2) asymmetric information. We assume that settlement of transactions in bonds and funds takes place once a day. There are two bond issues, S and G , priced at P_S and P_G on date 1. The former is used as collateral for an SC repo and the latter for a GC repo. The respective repo rates are R_S and R_G . There are a large number of risk-neutral bond dealers in the market, who know for certain that the bond prices, $P_S > P_G$ on date 1, will converge to P_{con} on date 2.

To build an arbitrage position, a bond dealer will sell short the issue S and use the raised funds to borrow the same issue in the SC repo market. Simultaneously, the dealer will buy issue G to hedge against interest rate risk using the funds raised through a GC repo transaction. As a result, profit/loss π can be expressed as

$$\begin{aligned}\pi &= -[P_{con} - P_S] + [P_{con} - P_G] - P_G R_G + P_S R_S \\ &= P_S - P_G - P_G R_G + P_S R_S.\end{aligned}\quad (2)$$

The first and second terms of the right-hand side of equation (2) represent the capital gains on issues S and G . Equilibrium requires the profit from this arbitrage trading to be zero. Thus, the following no-arbitrage condition should hold:

$$\frac{P_S}{P_G} = \frac{1 + R_G}{1 + R_S}.\quad (3)$$

Equation (4) below is a logarithmic version of equation (3). The left-hand side of equation (4) represents the cash premium, which measures the extent to which S is evaluated higher than G , and the right-hand side represents the repo spread:

$$\frac{P_S - P_G}{P_G} = R_G - R_S.\quad (4)$$

Equation (4) shows that the profit from the repo spread is fully offset by the higher market price of issue S that was purchased to build the arbitrage position. It also implies that, in equilibrium, repo rates adjust themselves to eliminate any profits expected from the arbitrage position taking advantage of the difference in bond prices. The above argument suggests that the parity encompassing the repo and bond markets should hold, just like the parity linking the foreign exchange rate and interest rate differential between any two countries. Also, in equilibrium, the following condition (5) is always satisfied:

$$R_S \leq R_G. \tag{5}$$

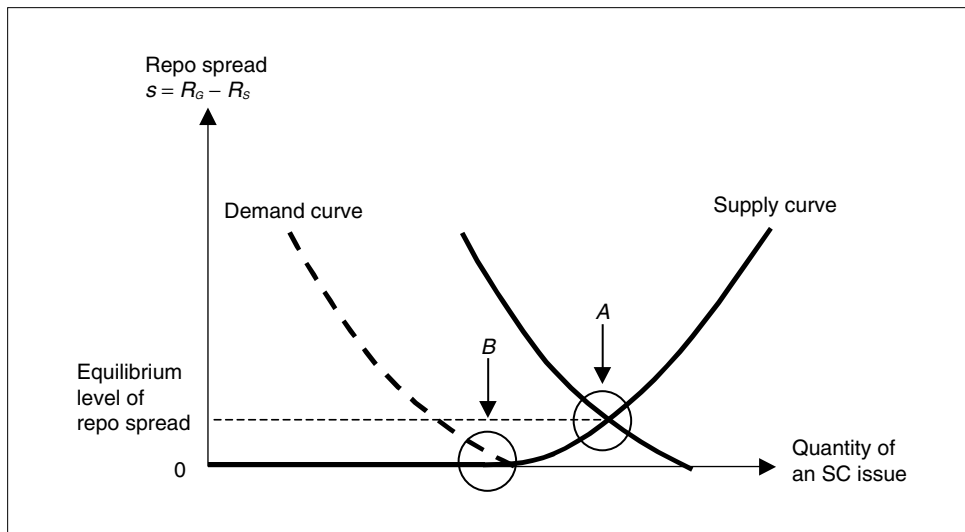
To prove condition (5), all we have to do is to prove that $R_S > R_G$ contradicts the notion of equilibrium. If $R_S > R_G$ holds, issue S borrowed through a repo transaction can be used in a GC repo transaction. This transaction enables a dealer to make arbitrage profits indefinitely without holding any initial capital, and thus equilibrium is never reached. Therefore, in equilibrium, condition (5) needs to be satisfied.¹¹

2. Deriving supply and demand curves

The above explanation enables us to understand that a repo spread and the no-arbitrage condition can coexist in the form of the parity between repo spread and cash premium. This no-arbitrage condition, however, alone cannot determine the equilibrium level of the repo spread and the cash premium. The equilibrium should be achieved at a level where demand and supply for the issue in the repo market meet. Let us take a look at Figure 6 to examine this point.

First, the supply curve is written as upward sloping. The rationale behind this is as follows. Let us assume a case where an investor with SC issues trades in the repo market. We assume for simplicity that the investor is a price taker. The investor can raise funds at a lower SC repo rate of R_S and at the same time invest them at a higher GC repo rate of R_G . Then, the investor's rate of return per issue will be equal to the repo spread, or the investor can lower portfolio-financing costs by the repo spread. Also, assuming that these transactions are cost-free, the investor with SC issues will

Figure 6 Determination of Repo Spread



11. Let us confirm that when condition (5) is satisfied in equilibrium, a positive arbitrage profit is not necessarily ensured. When trying to make a profit from the spread $R_G - R_S$, an issue borrowed through a GC repo cannot be put up as collateral in an SC repo. Thus, there is a need to purchase an SC issue in the cash market. In this case, even if a GC issue was sold short to hedge against interest rate risk, no-arbitrage condition (4) ensures a zero profit. If the position were unhedged, the transactions would no longer be arbitrage trading, but speculative trading, which does not guarantee a positive profit.

have an incentive to supply the entire portion of the SC issues to the repo market. In this case, the supply curve will level out at a certain point in the repo spread.¹²

In reality, however, transaction costs are incurred. In this case, not all SC issues will be supplied to the repo market. To elaborate on this point, let us examine a case where transaction costs are incurred and marginal cost increases with transaction volume. These transaction costs may include such costs as back-office operations and delivery costs. Under these circumstances, as long as the repo spread, that is, marginal profit, is larger than marginal cost, investors will supply SC issues to the SC repo market and will continue to do so until marginal cost rises to the level of the repo spread. Therefore, the supply curve should be upward sloping.

Next, let us turn to the shape of the demand curve. Bond dealers are the major agents borrowing collateral issues in the SC repo market. As shown in equation (4), in equilibrium the repo spread is fully offset by the cash premium. This means that, regardless of the size of the repo spread, bond dealers' net profits should be zero. Therefore, optimal behavior for dealers alone will not create a downward-sloping demand curve. As explained in Section II, the main reason for bond dealers to raise specific issues through SC repos is to cover their short positions in the cash market. In other words, behind dealers' demand for repos lie the investors who would like to buy the issues from dealers in the cash market. Since the no-arbitrage condition holds, these investors are paying an additional cost, equal to the repo spread, to acquire these specific issues.¹³

Why do investors place greater value on specific issues than on others? For example, life insurance companies tend to prefer issues with high current yields and coupons. Also, in the JGB futures market, fictitious issues with 10 years to maturity and a coupon rate of 6 percent are traded. The settlement price on the due date is determined by the futures price multiplied by the conversion factor (CF). The seller of the futures naturally prefers to deliver the CTD issues that will produce the smallest net basis when the above settlement price is subtracted from the contract price. Thus, there is an incentive for sellers of futures to hold the CTD issues in advance. Furthermore, investors generally like to trade issues with high market liquidity. Bank for International Settlements (1999a) defines a liquid market as "a market where participants can rapidly execute large-scale transactions with a small impact on prices." Factors causing differences in liquidity may include whether the issue is on the run, and whether the issue is being reopened, consolidating with others. In the JGB market, trading is heavily concentrated on the on-the-run issues and marketability soon becomes extremely thin as they are incorporated in investors' portfolios as "buy-and-hold" issues.

Now, let us assume that the benefit from holding a certain issue diminishes as the size of the holding increases: the larger the benefit stemming from an issue, the greater the decline in benefit from the addition of the same issue (diminishing

12. To be precise, the supply curve will be flat at a certain level of the repo spread, ending at the upper limit of supply of the SC issue.

13. The repo spread may reflect the convenience yield, which represents the convenience of holding actual commodities rather than futures positions in the same commodities. For example, if a commodity falls in short supply temporarily, it will be of great convenience to actually have the commodity in question. If the commodity is a raw material, there is great convenience in being able to continue production. See Hull (2000) for details.

marginal utility). If this is the case, a decrease in the repo spread means a decline in marginal cost relative to marginal utility, which will prompt investors to increase purchases. Similarly, since an increase in the repo spread means a rise in marginal cost, investors will reduce purchases to a level in line with marginal utility. As a result, the demand curve should be downward sloping.¹⁴

Under the supply and demand structure given above, the equilibrium point for an SC issue is given at point *A* in Figure 6, while that for a GC issue is at point *B*.

3. No-arbitrage condition in a multi-period setting

We have so far limited our attention to a two-period horizon. In reality, however, market participants operate over a multi-period horizon. Following Krishnamurthy (2001), we extend the no-arbitrage condition to a multi-period setting. As stated earlier, there is a close relationship between cash bond prices and SC repo rates. Also, investors with SC issues can raise funds at a low interest rate through SC repos, while at the same time they invest them in GC repos, earning returns equal to the repo spreads. Naturally, an investor who expects a certain issue to become special is likely to try to make profits by using such an issue in repo transactions. If a large number of market participants form similar expectations regarding the same issue, the present value of the expected repo spread will be reflected in the cash market price. Let us look at this point in more detail.

The assumptions are basically analogous to the two-period setting. Two issues exist. The two sets of cash prices and repo rates on date *t* are (1) $P_{S,t}$ and $R_{S,t}$ and (2) $P_{G,t}$ and $R_{G,t}$, respectively. We assume $P_{S,t} > P_{G,t}$. Market participants know that the prices of both issues will converge at P_{con} on date *T*.

Under this setting, a risk-neutral bond dealer builds an arbitrage position over the multi-period horizon. To be more specific, on date *t*, the dealer sells short the higher-priced issue and covers the position through an SC repo, while raising funds through a GC repo to buy a lower-priced issue to hedge against interest rate risk. Return π_t on date *t* + 1 can be expressed as

$$\pi_{t+1} = (P_{G,t+1} - P_{G,t}) - (P_{S,t+1} - P_{S,t}) - P_{G,t}R_{G,t} + P_{S,t}R_{S,t}. \quad (6)$$

Equilibrium would not be reached if this position continued to yield profits/losses. Therefore, under the condition that information on date *t* is shared by all market participants, equilibrium must be reached at the point where the expected return is zero, which is $E_t[\pi_{t+1}] = 0$. Hence,

$$E_t[(P_{G,t+1} - P_{G,t}) - (P_{S,t+1} - P_{S,t})] = P_{S,t}R_{S,t} - P_{G,t}R_{G,t}. \quad (7)$$

By assumption, market participants know that the prices of the two issues will converge on date *T*. Thus, equation (7) yields the following relationship (8) between present bond prices and repo rates by summing up from date 0 to *T* - 1:

$$P_{S,0} - P_{G,0} = \sum_{t=0}^{T-1} E_0[P_{G,t}R_{G,t} - P_{S,t}R_{S,t}]. \quad (8)$$

14. Duffie (1996) derives a downward-sloping demand curve by assuming the presence of risk-averse investors.

In equation (8), $P_{S,0} > P_{G,0}$ and $R_{S,t} < R_{G,t}$ hold. Thus, we reached the same conclusion as the two-period model that the repo spread is priced at a level that fully offsets the price differential between the underlying issues in the cash bond market. In addition, we can find another interesting insight from this equation. Equation (8) can be rewritten as follows:

$$P_{S,0} - P_{G,0} = P_{G,0}R_{G,0} - P_{S,0}R_{S,0} + \sum_{t=1}^{T-1} E_0[P_{G,t}R_{G,t} - P_{S,t}R_{S,t}]. \quad (9)$$

The left-hand side of equation (9) represents cash premium on date 0. The first term on the right-hand side represents the repo spread on date 0, which is valued in terms of return itself, not the rate of return. The second term denotes the sum of future repo spreads expected on date 0. Put differently, the expected repo spreads are reflected in the present cash premium. In the two-period model, issues were assumed to be special for just one period. However, when a certain issue is expected to remain special until some time in the future, a repo spread ensures that the no-arbitrage condition is satisfied over the multi-period horizon. That is, a cash premium could emerge when market participants expect an issue will be special because of the high likelihood that it will become the next CTD, for example.

The above discussion implies that the repo spread could provide a clue to future cash premium. If the term structure of the repo spread is estimated, the implied repo spread could be worked out backward by analogy with the pure expectations hypothesis. This line of logic suggests the possibility that the future cash premium of an SC issue may be estimated in advance from the implied repo spread. In reality, however, trading volume of term transactions in the Japanese repo market is so thin that the future cash premium thus computed might not be reliable.¹⁵ Furthermore, if we assume the presence of risk-averse market participants, the term structure of the repo spread would include a premium arising from uncertainty. In this case, the implied repo spread would lead to an overvaluation in future cash premium.¹⁶

4. Repo market under stress

Analysis so far has focused on a normal situation where the market is free from stress. By stress, we mean a situation where one or the other preconditions for the model describing a normal situation is not satisfied or where an unexpected event causes market participants to overreact in response to the event. Various mechanisms can cause stress, which makes it difficult to generalize. To illustrate the repo market under stress, we review a stressful event that occurred in Japan at the year-end of 1999, triggered by a rumor about repo transactions carried over to the following year. This episode will help us understand the limitations of the model describing a normal situation, once a stress occurs.

15. Shigemi *et al.* (2001) point out that compared with the U.S. market, term transactions, e.g., three-month repos, are not actively traded in the Japanese market. As a result, some market participants noted that in the formation of repo rates for long-term contracts, the cutoff rate for the Bank of Japan's repo operations acts as a benchmark for term instruments. See Bank for International Settlements (1999b) for the role of the repo market in monetary policy.

16. For example, Buraschi and Menini (2002) use the U.S. data to examine the term structure of repo spreads, rejecting the expectations hypothesis.

Shigemi *et al.* (2001) note that a rumor that “some public funds would not be engaged in repo transactions (lending bonds) over the year-end because of concerns over the Y2K problem” circulated in the repo and JGB markets around August 1999. This rumor fueled speculation that other government-related institutions would also withdraw from repo transactions over the same period. Bond dealers had built short positions on the assumption that large institutional investors such as public funds and life insurance companies would continue to lend bonds through repo transactions. Unsettled by the rumor, they rushed to close their positions, which triggered a precipitous fall in SC repo rates, thus rapidly expanding repo spreads.

According to Duffie (1996) and Krishnamurthy (2001), repo spreads are determined by the supply and demand balance of the SC issues in the repo market. In the above case, the rumor would cause the supply curve to shift leftward (Figure 6). Applying this logic, the following interpretation is possible. The speculation that the supply of the SC issues would be limited over the year-end created expectations among bond dealers that cash bond prices would shoot up. Thus, they immediately closed positions, triggering expansion in the repo spread.

Here, it should be noted that the expansion in the repo spread was possibly much greater than that predicted by Duffie (1996) and Krishnamurthy (2001). Shigemi *et al.* (2001) summarize the mechanism of the surge in the repo spread as follows: (1) the rapid expansion in basis prompted bond dealers, who had conducted arbitrage trading that combines cash and futures positions (short-basis trading), to cut losses by buying back cash bonds and selling futures; the resultant further expansion in basis caused a chain reaction by which implied repo rates fell further (a surge in lending fees), leading to a sharp rise in cash prices; and (2) some repo dealers, who had built short positions through repo transactions by lending cash bonds on a long-term basis, rapidly reversed the positions under loss-cut rules as losses expanded due to the sharp fall in repo rates.

IV. Empirical Analysis

As explained in Section III, no-arbitrage condition (4) should hold when the repo market is in equilibrium. In other words, when a repo spread exists, the price of the SC issue should be higher than other GC issues by the repo spread. When this relationship is extended to a multi-period setting, the bond price should reflect the repo spread expected to arise in the future. In this section, we try to empirically examine whether such a theoretical relationship holds in the Japanese repo market, basically following the methodology of Jordan and Jordan (1997).¹⁷

A. Data

In carrying out a precise analysis, we need to collect the data for the two following bond issues: GC and SC issues with the same coupon rate and remaining maturity.

.....
17. Jordan and Jordan (1997) found a significant relationship shown by equation (4) between the repo and Treasury securities markets in the United States.

In reality, however, we cannot find such a pair of issues, and thus need to estimate their theoretical prices. In this paper, we first estimated theoretical cash prices by employing the cubic spline function, and then proceeded to obtain cash premiums by comparing them with actual cash prices.^{18,19} To be more specific, we derived a discount factor from the price data of GC issues to compute the theoretical prices of SC issues.²⁰ The prices thus obtained effectively mean “the theoretical prices of the bond issues if they had not become special.” The difference between this theoretical price P_G and the actual market price P_S can be regarded as the cash premium. In this way, we computed the cash premiums of the on-the-run and CTD issues from the discount factors estimated from the data of 10-year JGBs, excluding the following issues: on-the-run, CTD, JGB issues with issue dates close to the CTD issues, and others that market participants reported were special.²¹ On-the-run and CTD issues are those after the Bank of Japan started collecting data on repo rates.²²

Table 1 shows (1) the average cash premiums on the on-the-run and the CTD issues and (2) the cash premiums on the corresponding GC issues. With a few exceptions,²³ we found that cash premiums on SC issues are larger than others.

Table 1 Estimated Cash Premiums (Average)

[1] On-the-Run Issues

	226th	227th	228th	229th	230th	229th*	231st	232nd	233rd	234th
Cash premium	0.33 (0.01)	0.20 (0.01)	0.11 (0.00)	0.18 (0.00)	0.02 (0.00)	-0.14 (0.00)	-0.05 (0.00)	0.03 (0.00)	0.09 (0.00)	0.19 (0.02)

[2] CTD Issues

	202nd	203rd	205th
Cash premium	0.24 (0.00)	0.12 (-0.01)	0.17 (-0.01)

- Notes: 1. Cash premium is defined as the difference between the market price of an SC issue and the theoretical price as expressed per ¥100 of par value.
 2. The charts in parentheses are the average cash premiums on the interest-bearing 10-year JGB GC issues during the same sample period, excluding the on-the-run, CTD, and other issues that market participants reported were special.
 3. The 229th issue is a reopened one.
 4. The sample period for the on-the-run issues is from the auction date to the last business date before the issuance of the next JGB issue. For the CTD issues, it is from the date when the issue became the CTD issue to the last business date before it lost the status.

18. In this paper, we employed the cubic spline function devised by McCulloch (1975). This function is widely used, since a simple linear regression can be employed for estimations. Given that the discount factor is estimated from different sets of price data of bonds with different remaining maturity, we used generalized least squares (GLS) on the assumption that the standard deviation of the error term of each issue is in proportion to the duration.

19. For JGB price data, we used the closing price as of 3:00 p.m. released by Japan Bond Trading Co. Ltd.

20. A discount factor can be understood as the price of a discount bond paying ¥1 at time-point t as a function of t .

21. The issues we excluded from our sample are the 196th, 199th, 202nd, 203rd, 205th, 209th, 210th, 221st, 225th, 226th, 227th, 228th, 229th (including the reopened issue), 230th, 231st, 232nd, 233rd, and 234th issues.

22. The Bank of Japan started collecting data on repo rates on December 18, 2000. We excluded the 225th issued on November 11, 2000 because repo spread data for the whole period during which the issue was the on-the-run issue are not available. Similarly, regarding the CTD issue, we used the data for issues for which repo spreads for the whole period are available.

23. The cash premium on the 229th issue was positive before it was reopened, but turned negative after it was reopened. According to the model in Section III, both cash premium and repo spread should fall when supply

Regarding the expected sum of repo spreads, we calculated equation (10) below using GC and SC repo rates reported by participants in the Bank of Japan's market operations using the methodology of Jordan and Jordan (1997):

$$RS_{it} \equiv \left[\sum_{\tau=t}^T (P_{G,\tau} R_{G,\tau} - P_{S,\tau} R_{S,\tau}) \right]_i \quad (10)$$

In this equation, i denotes the index for each issue, t the date of observation, and T the last day when the issue holds the status of the on-the-run or the CTD issue. R_G and R_S denote the GC and SC repo rates, respectively. This equation assumes that market participants expect the issues to cease to be special on the day they lose status as the on-the-run or the CTD issues.

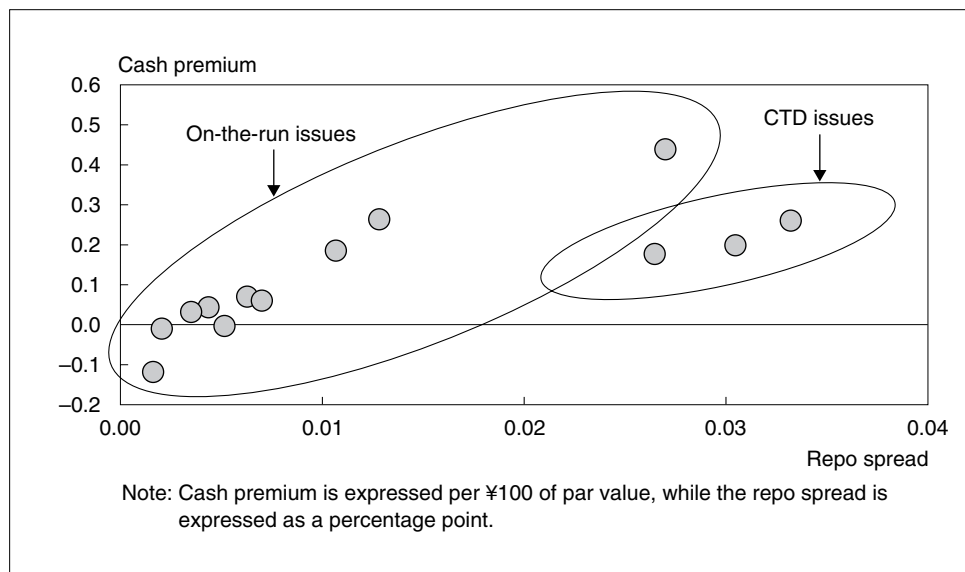
B. Relationship between Repo Spread and Cash Premium

Figure 7 depicts a cross-sectional relationship between (1) cash premiums²⁴ and (2) the expected repo spreads. There appears to be a positive correlation between them.

To statically examine this observation, we conduct a regression analysis using equation (11) below:

$$\Delta P_{i,t} = \alpha_i Dummy_i + \beta RS_{i,t}, \quad (11)$$

Figure 7 Relationship between Repo Spread and Cash Premium



increases due to reopening. The estimation result is consistent with the model in this regard. But, as shown in equation (5), in equilibrium, both cash premium and repo spread must be positive. Regarding this point, we had difficulty conducting a reliable analysis due to the limited data on reopened issues.

24. For the on-the-run issues, cash premiums were computed as of the issue date. For the CTD issues, they were computed as of their first trading day.

where $\Delta P_{i,t}$ denotes the cash premium on issue i on date t . Note that the right-hand side of equation (11) includes dummy variables, in addition to the expected repo spread, taking account of attributes specific to each issue.²⁵

If the model presented in Section III holds true, the no-arbitrage condition implies that (1) expected repo spreads and cash premiums are positively correlated significantly, and (2) the coefficient on the expected repo spreads does not significantly differ from one. To verify these hypotheses, we test the former based on the one-tailed t -test of null hypothesis $H_0: \beta = 0$ and alternative hypothesis $H_1: \beta > 0$, and the latter based on the two-tailed Wald test of null hypothesis $H_0: \beta = 1$ and $H_1: \beta \neq 1$.

Regression and hypothesis testing results are reported in Table 2. First, let us look at test (1). With regard to the on-the-run issues, the expected repo spread is

Table 2 Estimation Results

[1] On-the-Run Issues

Sample number: 191 (daily basis)

	Coefficient	t -value	p -value
Expected repo spread: β	3.878	1.938	0.054
Dummy 1 (α_1 : 226th)	0.272	6.748	0.000
Dummy 2 (α_2 : 227th)	0.173	5.211	0.000
Dummy 3 (α_3 : 228th)	0.096	3.386	0.001
Dummy 4 (α_4 : 229th)	0.167	6.139	0.000
Dummy 5 (α_5 : 230th)	0.008	0.272	0.786
Dummy 6 (α_6 : 229th [reopened])	-0.058	-2.129	0.035
Dummy 7 (α_7 : 231st)	-0.142	-5.346	0.000
Dummy 8 (α_8 : 232nd)	0.024	0.867	0.387
Dummy 9 (α_9 : 233rd)	0.090	3.457	0.001
Dummy 10 (α_{10} : 234th)	0.134	3.262	0.001
Adjusted R ²	0.544		

[2] CTD Issues

Sample number: 182 (daily basis)

	Coefficient	t -value	p -value
Expected repo spread: β	1.018	1.755	0.081
Dummy 1 (α_1 : 202nd)	0.226	22.21	0.000
Dummy 2 (α_2 : 203rd)	0.114	14.64	0.000
Dummy 3 (α_3 : 205th)	0.189	15.38	0.000
Adjusted R ²	0.439		

Note: The p -value of the coefficient on expected repo spread is based on the one-tailed t -test ($H_0: \beta = 0$, $H_1: \beta > 0$). The p -values of the coefficients on dummy variables are based on the two-tailed t -test.

[3] Wald Tests on the On-the-Run and the CTD Issues

$H_0: \beta = 1$, $H_1: \beta \neq 1$

	Wald statistics	p -values
On-the-run issues	2.070	0.150
CTD issues	0.001	0.975

25. By definition, the period during which an issue holds the status as either the on-the-run or the CTD issue does not overlap with others. For this reason, equation (11) has two suffixes, i and t . These, however, do not represent the panel data.

statistically significant at the 5 percent level, adopting the alternative hypothesis of test (1) above. At the same time, most dummy variables are significant at the 5 percent level, suggesting that individual attributes influence the level of cash premiums.

Similarly for the CTD issues, the expected repo spread is significant at the 10 percent level, adopting the alternative hypothesis of test (1). Also, all of the dummy variables are significant at the 5 percent level.

As for test (2), if the null hypothesis $H_0: \beta = 1$ holds true, the Wald statistics should follow the χ^2 distribution with freedom of degree one. The Wald statistics computed from the estimated coefficients of the on-the-run and CTD issues cannot significantly reject the null hypothesis. These results suggest that the no-arbitrage condition presented in Section III holds true statistically.

Here, note that the estimated β s on the expected repo spread greatly differ between the on-the-run and the CTD issues. There are two possible reasons for this. First, we assumed that the on-the-run issues are special for about one month, that is, until the next issue is issued. Thus, if the issue is special for more than one month, the estimated β might become overvalued to bridge the gap. Second, there seem to be large differences in the degree of uncertainty over the length of time during which the on-the-run and CTD issues are special. The multi-period models of Duffie (1996) and Krishnamurthy (2001) are based on the assumption that risk-neutral market participants have perfect foresight. If uncertainty is present while market participants are risk averse, however, a premium arising from uncertainty might be added to the cash premium. For the CTD issues, it is relatively easy to obtain information on the overall positions of futures contracts or implied repo rates, both of which can be used to forecast the length of time that the CTD issues are special. For the on-the-run issues, however, it is difficult to obtain robust implied repo rates due to (1) the unpredictability of supply and demand conditions because of the absence of a when-issued (WI) market²⁶ in Japan, and (2) thin market volume in futures compared with the CTD issues. Consequently, uncertainty about the length of time during which the on-the-run issue is special might be reflected in the cash premium.

V. Concluding Remarks

This paper has examined the Japanese repo market, which was launched as recently as April 1996. Despite its short history, the market now constitutes a main pillar of the Japanese money market, at least in terms of scale. After reviewing the theoretical mechanism by which repo rates are priced, we conducted a simple empirical analysis. Not many studies on overseas repo markets are available, let alone on the Japanese market. We thus hope that the paper will be of some help in activating discussions on the Japanese repo market among market participants, academics, and central bankers.

26. WI trading begins with the announcement of auction details, which include auction date, coupon rate, size of the issue, maturity date, interest payment date, bond code number, and whether the issue will be reopened or not. It ends on the auction or issue date. Settlement and delivery both take place on the issue date. Thus, we can regard it as a type of forward market.

Finally, we would like to add some words of caution. First, the paper disregards the effects of uncertainty. Second, it is based on the assumption of the Walrasian law of market clearing.

Regarding uncertainty, in real-life markets it is very difficult to determine when any specific issue becomes special and how long it maintains such status. In the United States, it is relatively easy to identify which issue becomes special since the repo spread tends to follow a predictable pattern. In Japan, in contrast, due partly to the fact that the JGB holdings are concentrated at a handful of institutions, some issues become special unexpectedly.²⁷ Furthermore, regarding the on-the-run JGB issues, the absence of a WI market implies that dealers must participate in auctions without much prior information on future demand conditions. This uncertainty may give rise to inventory costs, which might add a risk premium to the repo spread.

As for the market-clearing assumption, we should note that actual repo transactions are not conducted via open markets like stock exchanges, but on an over-the-counter basis, normally over the telephone between bond dealers, securities firms, banks, and life insurance companies. In other words, market participants need to search to locate appropriate counterparties. Furthermore, since transactions are negotiable between the parties concerned, the balance of bargaining power may influence terms and conditions.

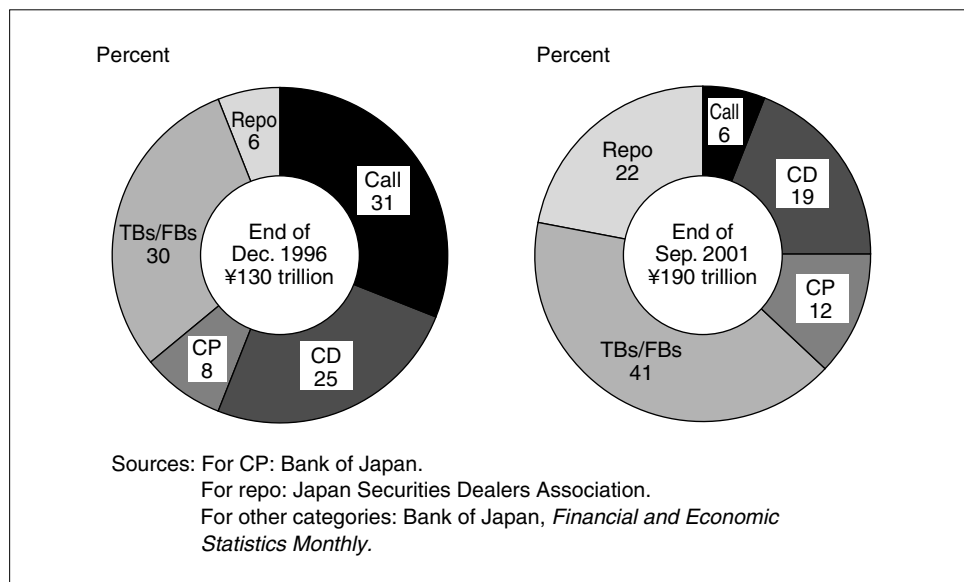
27. In U.S. markets, there is a close relationship between the auction cycle and repo spreads. For details, see Keane (1996).

APPENDIX 1: SCALE OF THE JAPANESE REPO MARKET

In Japan, there has been a financing method utilizing securities called a *gensaki* transaction. In *gensaki* transactions, as is the case with repo transactions, funds and securities are exchanged for a fixed period of time. However, market participants demanded further refinements of the *gensaki* transactions for the following reasons: (1) the *gensaki* transactions lacked an appropriate means of risk management against price fluctuations and default of counterparties during the contract period; and (2) the settlement of government bond transactions was shifting to the rolling method in 1996.^{28,29} Under these circumstances, the repo market was launched in April 1996 to provide investors with a market for investing surplus funds, and bond dealers such as securities firms with a market for financing their portfolios, which includes a means of covering short positions. Initially, as of the end of December 1996, the outstanding balance of the repo market stood at ¥8 trillion, which accounted for just 6 percent of the money market total (Appendix Figure 1). By the end of September 2001, however, the outstanding balance increased to ¥42 trillion, accounting for about 22 percent.³⁰

Appendix Figure 2 shows the scale of the repo markets in the United States, the United Kingdom, and Japan in terms of the outstanding balance itself as well as its percentage share to total government bonds outstanding. In the United States, whose repo market has a long history with a vast number of participants, the outstanding

Appendix Figure 1 Percentage Shares of the Short-Term Money Markets

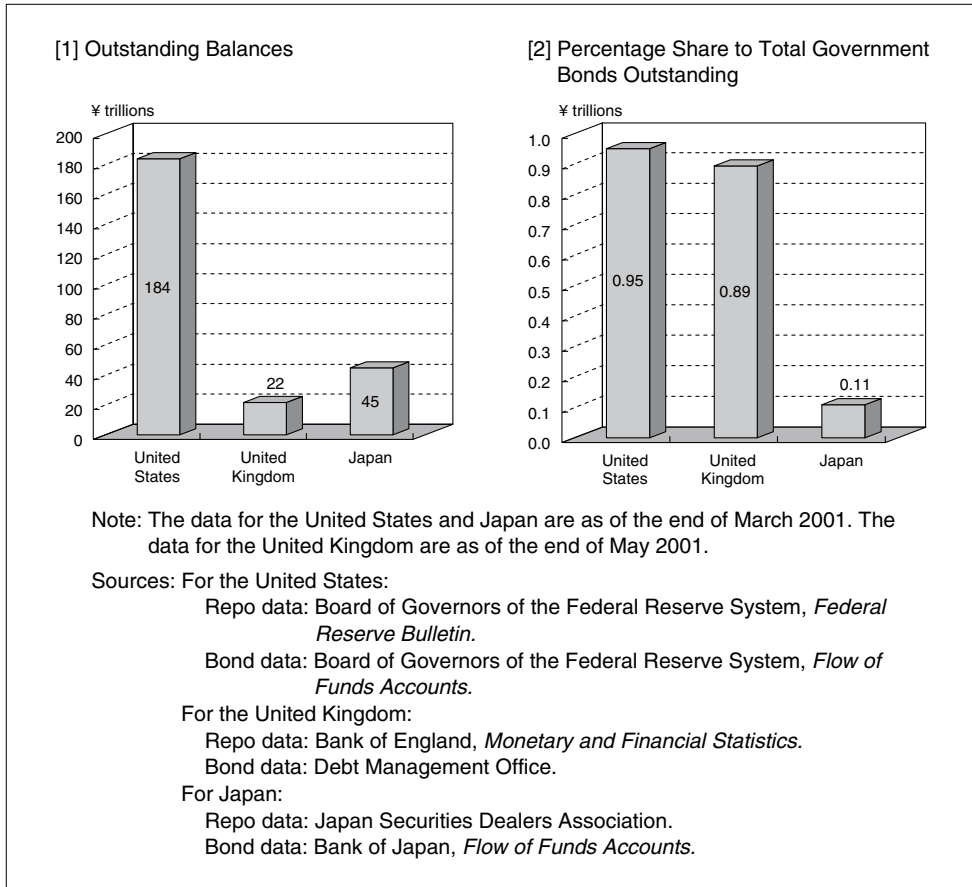


28. Under the rolling method, transactions are settled after a certain period of time from the contract date. Compared with the previous method, under which settlements were made on the 5th and 10th of each month, the rolling method is expected to reduce settlement risk by lowering the number of unsettled outstanding contracts.

29. For an outline and brief history of JGB borrowing/lending transactions, including the *gensaki* transactions, see Repo Trading Research (2001).

30. The outstanding balance of the repo market is calculated as the average of the amounts lent and borrowed in repo transactions.

Appendix Figure 2 Comparison of the Size of the Repo Markets



balance of the market stood at about ¥184 trillion (about US\$1,500 billion), which is more than four times the size of the Japanese market.³¹ The repo market in the United Kingdom, which was launched at roughly the same time as that in Japan, has an outstanding balance of about ¥22 trillion (about £116 billion at ¥190/£1), about half the size of the Japanese market.³² Whereas in both the United States and the United Kingdom, the outstanding balance of the repo market is about ¥900 billion per ¥1 trillion of government bonds, in Japan it is just ¥110 billion. This comparison may indicate that the Japanese repo market still has much room for further expansion.

31. In the United States, repo transactions started in 1918 when the Federal Reserve launched operations utilizing discounts on bankers' acceptances (BAs). The primary market for BA discounts was in London at that time. Given the high costs, however, efforts were being made to establish a market in the United States. Against this background, the Federal Reserve bought BA discounts with resale agreements, thereby providing funds at the official discount rate. These operations were the beginning of repo transactions. See Stigum (1989) for details.

32. In the United Kingdom, the volume of repo transactions utilizing U.S. Treasury bonds and German government bonds has increased since the launch of the Big Bang initiatives in 1986. Repo transactions utilizing U.K. government bonds, however, have been virtually limited to operations by the Bank of England. The U.K. repo market has been opened to all market participants since January 1996. See Repo Trading Research (2001) for details.

APPENDIX 2: EXPLAINING THE GAP BETWEEN THE GC REPO RATE AND UNCOLLATERALIZED INTEREST RATES AFTER THE INTRODUCTION OF RTGS

According to the simplest form of the no-arbitrage argument, when the GC repo and uncollateralized interest rates differ, profits can be made by raising funds at the lower of the two rates and investing them at the higher rate. In equilibrium, however, there would not be any arbitrage opportunities, and thus the rates should eventually converge. Furthermore, as the GC repo rate is a collateralized rate, it should theoretically be lower than any uncollateralized interest rates, which should include credit risk premiums.

As seen in Section II, however, the GC repo rate is generally higher than the uncollateralized interest rate. How can we explain this phenomenon? First, it has been pointed out that since the introduction of RTGS³³ in January 2001, settlement and collateral management costs for GC repo transactions have increased.³⁴ These costs can be regarded as invariable, at least in the short run. Figure 4 in Section II, however, shows that spreads between the GC repo and uncollateralized interest rates fluctuate over time. Thus, transaction costs alone do not seem to provide a satisfactory explanation for the movement of the spread.

Second, some have pointed out that the yield gap emerges because the length of time between contract date and settlement date differs between the GC repo and uncollateralized transactions. The settlement of repo transactions is usually conducted two days after the contract date, while uncollateralized call transactions are settled on the contract date.³⁵ This practice suggests that investment in the GC repo market against funding in the uncollateralized call market could generate the yield gap because of uncertainty over the availability of funds two days later. To examine this hypothesis, in Appendix Figure 3 we plotted the difference between (1) the spread between the GC repo rate and the uncollateralized overnight (O/N) call rate, and (2) the spread between the uncollateralized tomorrow/next (T/N) call rate and the uncollateralized O/N call rate, multiplied by two.³⁶ Here, the settlement date of the uncollateralized T/N call rate is the contract date plus one day. The figure shows that after the introduction of RTGS, the former is constantly above the latter. It suggests that the GC repo rate is higher than the uncollateralized call rate, even if the difference in the settlement timing is taken into account.

The above is an attempt to interpret the yield gap between the GC repo and the uncollateralized call rate in terms of the differences in transaction practices. Given that current monetary policy keeps the uncollateralized call rate at a record low, however, it might be natural that the two rates should not converge. Therefore, let us now consider why the GC repo and the uncollateralized rates other than the

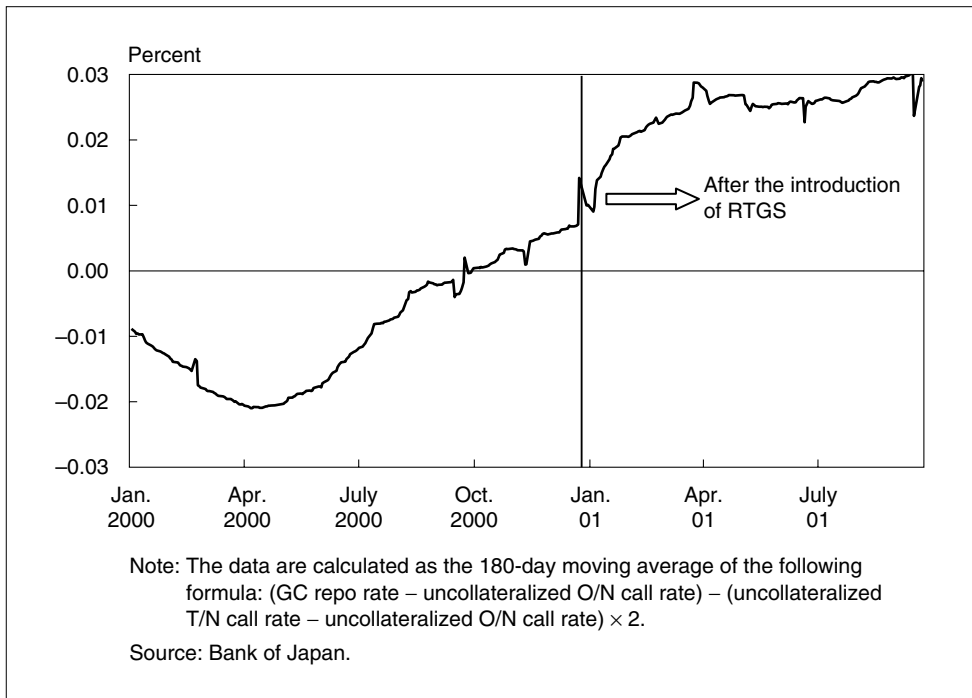
33. RTGS means the Bank of Japan's real-time gross settlement system for current account deposits and JGBs.

34. Compared with the previous designated-time net settlement, RTGS substantially reduces systemic risk. On the other hand, it increases workloads and collateral management costs, since it involves real-time settlement and management of outstanding balances, in addition to an increase in the amount of funds and JGBs required for settlement.

35. For details, see Bank of Japan (2002).

36. All are overnight rates.

Appendix Figure 3 Still-High Level of the GC Repo Rate after Adjustment for the Time Differences in Settlement



call rates, such as the euro-yen rate, do not converge. We attempt to examine the mechanism with special attention to the demand structure for funds in each market.

In the Japanese GC repo market, the main lenders are trust, city, long-term credit, and foreign banks, while main borrowers are securities firms. In the euro-yen market, on the other hand, both main lenders and borrowers are city, long-term credit, and foreign banks. These banks can borrow funds in the call market, where the call rate is held down by monetary policy, and lend them in both the repo and euro-yen markets. Put differently, city, long-term credit, and foreign banks function as main fund-lenders in both markets, interacting with different agents in need of funds in each market. Thus, these banks can build arbitrage positions using the repo, euro, and call markets, each of which has a distinctive fund demand structure.

Next, let us take a look at the fund demand structure in each market. The main borrowers in the euro-yen market, that is, city, long-term credit, and foreign banks, have access to various markets other than the euro-yen market. In contrast, the main borrowers in the repo market, securities firms, do not have any other major means to raise funds other than the repo market. Thus, it is extremely difficult for securities firms to search for more competitive funding means even if unfavorable rates are offered in the repo market. Their negotiating power vis-à-vis lenders is likely to be weak, which implies that the interest-rate elasticity of demand is low. Moreover, since the introduction of RTGS, the incentive to reduce settlement volume has led to an increase in direct deals, resulting in a tendency among market participants to conduct

repo transactions with a limited number of known counterparties.³⁷ The increase in direct deals has put securities firms in a more difficult position in finding competitive lenders, possibly weakening their power to negotiate further.

Given the above structure, let us now describe the optimal behavior for city, long-term credit, and foreign banks. What we employ here is a simple microeconomic tool. Let R_i denote the interest rate, Q_i quantity, MR_i marginal return ($i = 1$ for the repo market and $i = 2$ for the euro-yen market), and MC marginal cost.³⁸ We assume that city, long-term credit, and foreign banks are risk neutral and try to maximize profits in each period. Their optimal strategy, as expressed in Appendix Figure 4, would be to offer the rate that satisfies the following condition:

$$MC = MR_1 = MR_2.$$

If we let ϵ_1 and ϵ_2 ($\epsilon_1 < \epsilon_2$) denote the interest-rate elasticity of demand for funds in each market, then the marginal return can be expressed as follows:

$$MR_1 = R_1 \left(1 - \frac{1}{\epsilon_1}\right) \text{ and } MR_2 = R_2 \left(1 - \frac{1}{\epsilon_2}\right).$$

Now, we obtain the following optimal condition:

$$R_1 \left(1 - \frac{1}{\epsilon_1}\right) = R_2 \left(1 - \frac{1}{\epsilon_2}\right).$$

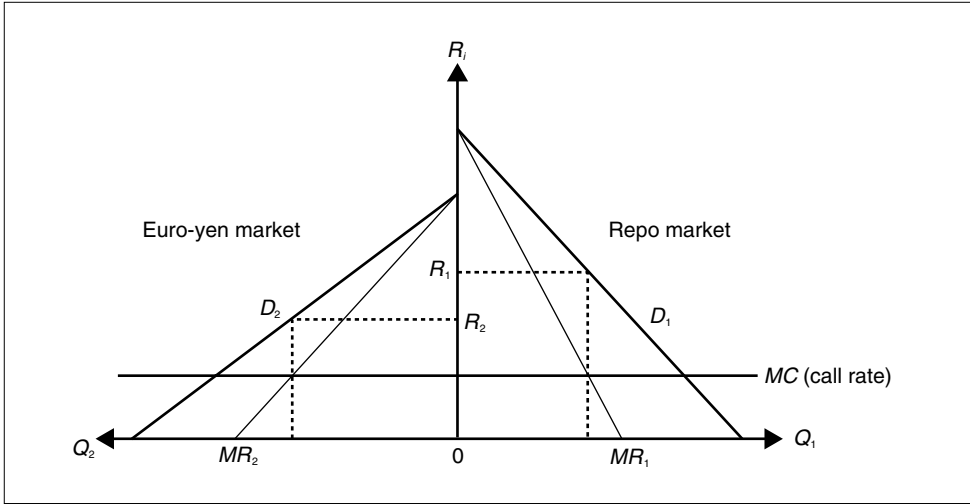
Thus, we find that $\epsilon_1 > \epsilon_2 \rightarrow R_1 < R_2$. This means that optimal behavior leads to the condition that the greater the interest-rate elasticity of demand for funds, the lower the rate that will be offered.

The above discussion provides a useful viewpoint on the question “Why don’t rates converge despite active arbitrage trading between the two markets?” The argument that interest rates converge as a result of active arbitrage trading stems from the assumption of perfect competition, which does not envisage lenders with market power. If such lenders exist, lending more funds to the repo market beyond a certain point in the name of arbitrage trading turns out to be sub-optimal in that it only leads to a decline in profits for them.

37. See Bank of Japan (2002).

38. Lenders (city, long-term credit, and foreign banks) are the same in both markets, and thus marginal cost is set at a level equivalent to the call rate.

Appendix Figure 4 Interest Rate Discrimination Strategy for a Lender of Funds



References

- Bank for International Settlements, "Market Liquidity: Further Findings and Selected Policy Implications," 1999a.
- , "Implications of Repo Markets for Central Banks," 1999b.
- Bank of Japan, Financial Markets Department, "RTGS-ka go no Kin'yu Shijo no Kino ni Kansuru Rebyu (A Review of the Financial Market Functioning after the Introduction of the RTGS System)," *Market Review*, 2002-J-1, Market Policies Group, Money and Capital Markets Division, February, 2002 (in Japanese).
- Buraschi, Andrea, and Davide Menini, "Liquidity Risk and Special Repos: How Well Do Forward Repo Spreads Price Future Specialness?" *Journal of Financial Economics*, 64 (2), 2002, pp. 243–284.
- Duffie, Darrell, "Special Repo Rates," *Journal of Finance*, 51 (2), 1996, pp. 493–526.
- Griffiths, Mark, and Drew Winters, "The Effect of Federal Reserve Accounting Rules on the Equilibrium Level of Overnight Repo Rates," *Journal of Business Finance and Accounting*, 24 (6), 1997, pp. 815–832.
- Hull, John, *Options, Futures, and Other Derivatives*, 4th edition, Prentice Hall, 2000.
- Jordan, Bradford, and Susan Jordan, "Special Repo Rates: An Empirical Analysis," *Journal of Finance*, 52 (5), 1997, pp. 2051–2072.
- Keane, Frank, "Repo Rate Patterns for New Treasury Notes," *Current Issues in Economics and Finance*, 2 (10), Federal Reserve Bank of New York, 1996.
- Krishnamurthy, Arvind, "The Bond/Old-Bond Special," mimeo, Northwestern University, 2001.
- McCulloch, Huston, "The Tax Adjusted Yield Curve," *Journal of Finance*, 30 (3), 1975, pp. 811–830.
- Repo Trading Research, *Saishin Repo Torihiki no Subete* (An Introduction to Practical Finance: The Latest Book on Repo Transactions), Nihonjitsugyo Shuppansha, Tokyo, 2001 (in Japanese).
- Shigemi, Yosuke, Sotaro Kato, Yutaka Soejima, and Tokiko Shimizu, "Market Participants' Behavior and Pricing Mechanisms in the JGB Markets: Analysis of Market Developments from the End of 1998 to 1999," Financial Markets Department Working Paper 01-E-1, Bank of Japan, 2001.
- Stigum, Marcia, *The Repo and Reverse Markets*, New York: Irwin, 1989.