Rational Expectation, Bubbles and Foreign Exchange Market*

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I. Introduction

Since a floating exchange rate system was introduced for the major world currencies in the spring of 1973, exchange rates have been quite volatile. This has posed a difficult problem to the management of monetary policy and economic policy in general. In response to these real world developments, numerous attempts to theoretically formalize exchange rate determination have been made, including monetary models, overshooting models, and portfolio balance models. However, empirically, none of these models performs adequately. As to the exchange rate theory today, there exists wide variety of opinions and no definite consensus ever reached.

Recently, the phrase "the deviation of exchange rates from the economic fundamentals" has been widely heard. This reflects the inability of current models to explain exchange rate fluctuations, and has resulted in attention being focused on models which attempt to explain exchange rate movements through expectation factors which have no direct connection with these "economic fundamentals". "Bubble" is one typical example of a theory which directly treats such kind of

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expectations. This paper will investigate bubbles and their policy implications largely in regard to the foreign exchange market.

The word "bubbles" or "speculative bubbles" first came to be associated with economic phenomena in 1720. At that time, England, and then France, suffered financial confusion related to two instances of large scale speculation and the collapse of stock prices. (In England, stock speculation involved the South Sea Company, while in France it involved the Mississippi Company.) These two cases were labeled the South Sea Bubble and the Mississippi Bubble, respectively. Since that time, the rise and fall in asset prices associated with speculation which causes assets to deviate from their fundamental price has been called a bubble.

As explained previously, the word "bubbles" characterizes a situation in which an asset price deviates from its fundamental value. The occurrence of such events may include cases based on irrational factors. (For instance, speculation arising from panic or frenzy.) Using such a wide interpretation, however, makes it prohibitively difficult to analyze the phenomena of bubbles. Thus, the topic of this paper is not bubbles in this general sense but rather the bubbles which are consistent with the hypothesis of rational expectations. The policy implications of such bubbles are still quite large, and their importance cannot be overlooked.[1]

In the first part of this paper, the general structure of bubble is introduced. Then it is applied to the foreign exchange market, and finally its policy implications are discussed.

II. Asset Prices and Bubbles

1. Bubbles and a Simple Rational Expectations Model

This section introduces the required framework for a rational bubble.

First, the relationship between bubbles and asset prices corresponding to economic fundamentals in a simple rational expectations asset pricing model is explained, and basic characteristics of bubbles are presented. Also, stochastic characteristics of a bubble and some possible hypotheses concerning expansion paths of bubbles are explained.

1. For example, concerning the role of policy authorities and bubble-type speculation, Kindleberger(1978) stated, "I contend that markets work well on the whole, and can normally be relied upon to decide the allocation of resources and, within limits, the distribution of income, ... but that occasionally markets will be overwhelmed and need help".

Let us consider a model where the price of an asset X at time t, x(t), is determined by the economic fundamentals at time t, z(t), and the expected value of the asset X at time t+1. That is,

$$x(t) = a\,E_t x(t+1) + b\,z(t), \quad 0<a<1 \qquad (1)$$

where $E_t x(t+1)$ is the expected value of $x(t+1)$ given information at time t.

Rational expectation models usually assume that the current price x(t) reflects only current and future expected values of economic fundamentals. Consequently, the relationship in (1) is expected to hold at time t+1 and t+2 as well.

$$E_t x(t+1) = a E_t x(t+2) + b E_t z(t+1) \qquad (2)$$

$$E_t x(t+2) = a E_t x(t+3) + b E_t z(t+2) \qquad (3)$$

Inserting (2) into (1) gives

$$\begin{aligned} x(t) &= a\{ a E_t x(t+2) + b E_t z(t+1)\} \\ &\quad + b\,z(t) \\ &= a^2 E_t x(t+2) + b\{ a E_t z(t+1) \\ &\quad + z(t)\} \end{aligned} \qquad (4)$$

Similarly, substituting (3) into (4) gives

$$\begin{aligned} x(t) &= a^3 E_t x(t+3) + b\{ z(t) \\ &\quad + a E_t z(t+1) + a^2 E_t z(t+2)\} \end{aligned} \qquad (5)$$

Repeating this procedure for the asset price until time t+n−1, the following equation is obtained

$$x(t) = a^n E_t x(t+n) + b \sum_{i=0}^{n-1} a^i E_t z(t+i) \qquad (6)$$

Since $0<a<1$ [2], if $E_t x(t+n)$ is finite, $\lim_{n\to\infty} a^n E_t x(t+n) = 0$. Consequently, (1) becomes

$$x(t) = b \sum_{i=0}^{\infty} a^i E_t z(t+i) \quad . \qquad (7)$$

2. If $a>1$, fundamentals are given increasingly important weight as time passes. Solutions of normal rational expectations models explode and lose meaning.

Thus x(t) can be written as a weighted average of current and future values of economic fundamentals. (Hereafter, the rational expectations price which reflects these economic fundamentals will be called the fundamentals price and will be denoted by a bar over the price variable $\overline{x}(t)$.)

Solution (7) was derived by assuming that at each time in the future asset prices are in close conformity with these fundamentals prices. If this is not assumed, solution (7) is no longer unique, although it remains a solution to equation (1). This is because equation (1) itself does not assume the exclusion of possible deviation through speculation of asset prices from economic fundamentals. Concerning this point, the following explanation can be made. The homogeneous equation of the original simple rational expectations asset pricing model (1) is

$$c(t) = a E_t c(t+1) \tag{1'}$$

The solution of (1') corresponds to the "bubble term" in the context of model (1). The general solution of (1) is the sum of the particular solution corresponding to the fundamentals price

$$\overline{x}(t) = b \sum_{i=0}^{\infty} a^i E_t z(t+i) \quad . \tag{7'}$$

and the solution c(t) to the homogeneous equation (1'). That is,

$$x(t) = \overline{x}(t) + c(t) \quad . \tag{8}$$

"Bubble term" of the solution of (8) can be interpreted as the speculative expectations which can lead to asset prices deviating from their fundamentals prices.

This relationship between "bubble term" and speculative expectation can be generalized to the case where the asset pricing model has a more complicated structure. In such a case, amongst solutions of homogeneous equations corresponding to higher order difference(or differential) equations that show the asset pricing mechanism, bubbles correspond to those solutions which are unstable.

Since these homogeneous equations replace the economic fundamentals term in the original model with zero, bubbles have no direct link with the economic fundamentals and can be regarded as factors which should be added to the fundamentals price.

Solutions of homogeneous equations corresponding to bubbles satisfy the original rational expectations model but are difficult to treat in that: (1) these solutions are explosive (unstable), and; (2) a homogeneous equation corresponding to a bubble cannot be solved uniquely (indeterminacy). As a result, such solutions are usually excluded from the set of solutions a priori, and the fundamentals price (7') is used as the unique solution of the model.

In the case where there are expectations that an asset price will deviate from its fundamentals price at time t+1 principally because of speculation, there must also be expectations that this kind of speculation will continue at time t+2. In other words, investors must be uncertain as to the timing this speculation will end. Assume, for instance, that investors know with certainty that speculation will end at time n (n>t) and that the asset price will conform to the fundamentals price at that time. Hence, the sepculation must terminate at time n—1, and the asset price at time n—1 as well must conform to the fundamentals price. In the same manner, the speculation must terminate at time n—2. Thus, if the timing when speculation ends is known with certainty, rational investors will halt speculation at time t, and the asset price at time t will be expected to conform to the fundamentals price. Consequently, in the case of an asset whose price is known with certainty at a specified future date (government bonds with a fixed date of redemption, etc.), rational bubbles cannot be formed.[3]

2. Classification of Bubbles

Solutions to the formula for the growth of rational bubbles have numerous possible stochastic specifications. It may be possible to use these different specifications to explain fluctuations in stock prices, gold prices or hyperinflation. Three typical solutions are: (1) a stochastic bubble which does not include an innovation; (2) a stochastic bubble which does include an innovation; and (3) a deterministic bubble.[4]

A. Stochastic Bubbles without a Regenerating Mechanism

In this class of bubble, speculation in the next period is assumed to have a probability α of ceasing ($0< \alpha <1$). Hence the asset price at that time may conform to the fundamentals price, but there is also the probability $1- \alpha$ that speculative expectations will continue and the asset price will continue to deviate from the fundamentals price. Because (1') must hold, the bubble's expansion process may be written as

3. There is not yet sufficient research as to what extent bubbles in the asset market can have a rational existence. Tirole(1982) concludes that if the number of asset holders is finite, the existence of bubbles is consistent with myopic rationality but not consistent with strong rationality that assumes long-run dynamic optimization. Also, in the case where there are multiple substitutable assets, the general equilibrium problem of under what conditions bubbles can exist for other assets when the existence of a bubble for one asset has been ruled out has not been adequately solved. Blanchard and Watson(1982) have provided some suggestions concerning this question but the issue has not advanced beyond their research.

4. This paper limits discussion to these three relatively popular classes of bubbles. However, an infinite number of specifications are theoretically possible.

$$c(t+1) = \begin{cases} 0 & \text{with a probability } \alpha \\ \frac{1}{1-\alpha}\frac{1}{a}c(t) & \text{with a probability } 1-\alpha \end{cases} \quad (9)$$

In this specification, capital gains must cover the risk of the bubble collapsing if the bubble continues. Therefore, the greater is the risk of the bubble collapsing, the greater must be the capital gain based on the speculative rise of the asset price. However, this class of bubble is specified in such a way that once it collapses, it must always be zero in subsequent periods. In other words, once the bubble bursts, it cannot reappear.

B. Stochastic Bubbles with Regenerating Mechanisms

Blanchard and Watson (1982) provides insight into the expansion process of a more general bubble which includes a regenerating mechanism. Consider an innovation term in a bubble which corresponds to changes in speculative expectations which are unanticipated one time period ago. Let this innovation be u(t). By adding this term to the stochastic bubble, one can consider a speculation process which autonomously appears and disappears. In this case, the bubble's expansion-collapse process can be written as

$$c(t+1) = \begin{cases} u(t+1) & \text{with a probability } \alpha \\ \frac{1}{1-\alpha}\frac{1}{a}c(t) + u(t+1) & \text{with a probability } 1-\alpha \end{cases} \quad (10)$$

For simplification at present, assume that the probability of the bubble's collapse α is fixed, and that the innovation term u(t) on the speculative bubble is white noise. The case where α is an endogenous variable will be considered in Section III when a bubble is applied to the foreign exchange market.

C. Deterministic Bubbles

The simplest solution for (1') is given by

$$c(t+1) = \frac{1}{a}c(t), \quad 0 < a < 1 . \quad (11)$$

This case corresponds to investors being certain that the speculation which created the bubble at the first place will continue in the next period and beyond. Since the parameter a must lie between 0 and 1, the deviation of the asset price from its fundamentals price will grow exponentially and permanently.

This deterministic bubble is a special case of a stochastic bubble without a

Table 1 Some examples of Typical Bubbles

	Formula for bubble	Can it explain fluctuations in asset prices?	Does it possess a regenerating mechanism?	Examples of phenomena to which it is applied
Stochastic bubble	$c(t+1)=\begin{cases}\frac{1}{a(1-\alpha)}c(t) & (\text{probability } 1-\alpha)\\ 0 & (\text{probability } \alpha)\end{cases}$	Yes	No	Bubbles in markets for assets such as foreign exchange rate, stocks, gold
Stochastic bubble with an innovation	$c(t+1)=\begin{cases}\frac{1}{a(1-\alpha)}c(t)+u(t+1) & (\text{probability } 1-\alpha)\\ u(t+1) & (\text{probability } \alpha)\\ E_t u(t+j)=0,\ j=1,2\cdots & \end{cases}$	Yes	Yes	
Deterministic bubble	$c(t+1)=(1/a)^{t+1}\,c(0)$	No	No	Hyperinflation

regenerating mechanism. When there is absolutely no fear that speculation will halt (α =0), this latter class of bubble corresponds to the deterministic bubble. Since the expansion path of the deterministic bubble is oversimplified, this bubble is not adequately convincing.[5] However, because of its simplicity, this class of bubble is easy to use as a starting point for empirical research. In particular, this deterministic bubble resembles stochastic bubbles which occur in situations of large scale speculation where market participants have relatively great certainty regarding future course of speculative investment. (An example of such large scale speculation is the hyperinflation in Germany and Hungary following the First World War.) These deterministic bubbles therefore have practical application. Flood and Garber (1980) investigated empirically the question of whether or not self-fulfilling expectations, which can be approximated by a deterministic bubble, accelerated hyperinflation during the period of German hyperinflation. The results of this research do not adequately confirm the existence of a bubble during this hyperinflation period, but the methodology used provided a stimulus for other researchers. For instance, Burmeister and Wall (1982) obtained empirical results confirming the existence of a bubble during the hyperinflation period by adding a stochastic error term to the deterministic bubble.

This section has introduced the various expansion-collapse processes of bubbles which are consistent with various economic phenomena. A simple summary is presented in Table 1.

5. Concerning theoretical problems of deterministic bubbles, see Diba and Grossman(1983).

III. Bubbles and the Foreign Exchange Market

In the preceeding section, the salient features of bubbles were explained simply. Here, bubbles as an explanatory factor of exchange rate fluctuations will be considered. First, a mechanism for generating bubbles in the foreign exchange market will be investigated. Next, the characteristics of bubbles in conjunction with various popular models of exchange rate determination will be investigated.

1. The Generating Process of Bubbles

Here, let us consider the generating process of bubbles. As previously explained, bubbles are speculative phenomena based on self-fulfilling expectations of the price rise of a particular asset and are, in principle, not related to fundamentals. However, the concrete mechanism of generating such speculation has not been explained so far. There are various guesses as to this mechanism such as: (1) the occurrence of bubbles is promoted by some form of news concerning the economic fundamentals (Dornbusch[1982a], Diba and Grossman[1982], Isard[1978]); or (2) essentially irrelevant information becomes the catalyst for generating bubbles (Diba and Grossman[1982]). If these guesses are correct, the occurrence of bubbles is closely intertwined with so-called "peso problems" situations and "extraneous beliefs".[6] However, it is also possible to consider cases where bubbles arise through active speculation on the part of market participants in situations where no news exists about economic fundamentals or other factors.

Concerning this possibility of active speculation in foreign exchange markets, Komiya and Suda(1983) take a negative view. Because of the smaller share of individual groups in the foreign exchange market, they believe that it is probably impossible for any group to have a speculative impact on the market.[7] However, this

6. Brief explanations about "peso problem" and "extraneous beliefs" will be given in Appendix 1.

7. "In the case of exchange rate speculation, the share of individual groups in the market as a whole is extremely small. Transactions for 1981 in the Tokyo foreign exchange market, which is an interbank market, totaled 900 billion dollars (total of spot, forward, and swap transactions). The yearly transactions among foreign exchange banks and domestic customers were over 500 billion dollars. Excluding both foreign exchange banks whose speculative activities are strictly controlled by the restrictions governing the holding of foreign exchange and trading companies which are considered to basically avoid exchange risk, the greatest amount of yearly foreign exchange transactions of single company such as automobile maker, household electronics manufacturer, and petroleum company is estimated to be one billion dollars. In other words, in this 500 billion dollar market, no company is likely to have over 1% share of the market.... Even in daily transactions, the share of individual groups in this market is small in comparison to market volume. Thus, in the ordinary perception of speculators,

view is not necessarily widespread. In recent years, the activities of a group of investors in Chicago's International Monetary Market have received a great deal of attention. It is well known that the majority of the members of this group in Chicago's International Monetary Market seek speculative profits. The typical pattern of these investors is to cause single directional movements, either buying or selling a currency, promote either appreciation or depreciation of the currency, and then reap the profits. These activities are thought to have a large impact on the New York Market as well; hence the existence of influential speculators in foreign exchange markets is difficult to refute.[8] These speculative activities are not limited to the Chicago's International Monetary Market. Speculation not based on economic fundamentals by foreign exchange dealers is thought to have sometimes a large impact on the market. From the viewpoint of foreign exchange dealers, their "challenge" towards market participants can serve as a sufficient catalyst for creating bubbles.[9]

Another point which must be kept in mind concerning the creation of bubbles is speculative actions based on chart analysis. Speculative activities based on chart analysis often imply that when the exchange rate appreciates currency should be purchased and when it depreciates currency should be sold. When a signal appears in time series data on the exchange rate that the dollar will appreciate or depreciate, this class of speculative activity will amplify the appreciation or depreciation of the

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these individual groups cannot artificially force the yen dollar rate up to sell currency or force the market price down through intensive currency sales to purchase currency. That is, price manipulation in the foreign exchange market is not possible." (Komiya and Suda(1983) pp. 145-146)

8. In the simple rational expectations model hypothesized so far, it has been assumed that market participants have the same knowledge concerning the structure of the model and values of related variables. In such a model, a particular group of market participants cannot act as a catalyst for speculation because all market participants must begin speculation at the same time. Theoretically, in order for the model to approach reality, it is necessary to adopt the assumption of diversity in the information about the market known by market participants. Although theory under this assumption of diverse information is not well developed, Blanchard and Watson(1982) provide some insights into the issues involved.

9. From the middle of April to the beginning of May 1982, the yen dollar rate rose from 248 yen per dollar to 231 yen per dollar. Concerning this, Koguchi has said, "This movement in the exchange rate cannot be explained by any economic fundamentals. This is because the catalyst which forced the yen rate up by 16 yen in a short time was the purchase of yen through the sale of one billion dollars by a certain Middle East group. This one billion dollars was sold over the Easter holiday when the exchange markets were dull. The dollars were not immediately put on the exchange market but rather were split up and held by various New York banks. This news quickly spread to other Middle East speculators who also began to sell dollars. Next, the news reached fund managers in Europe and the U.S. who likewise sold

dollar. This kind of activity acts as a bridge between the appearance of a large scale bubble and news about economic fundamentals or small scale speculation. In other words, concerning a possible scenario for the appearance of a bubble, the following can be said: (1) The exchange rate deviates from its fundamentals rate because of "excessive market reaction" towards news of economic fundamentals or "dealers' challenge". (2) Speculative actions based on chart analysis which uses (1) as a signal cause the exchange rate to deviate more from its fundamentals value. (3) Through the continuation of this deviation from the fundamentals value, the belief that the deviation will continue in the future is confirmed amongst market participants in general, and this affects the exchange market in a self-fulfilling fashion.[10]

2. Bubbles and Theories of Exchange Rate Determination

In Section II, bubbles were examined in the context of a simple reduced form asset pricing model. In this section, the relationship between bubbles and structural models of exchange rate determination will be investigated. First, we will investigate Dornbusch(1982a,b) in detail which discussed the role of bubbles in exchange rate determination and subsequently problems with his view will be pointed out. Also, the characteristics of bubbles in the context of theoretical specifications of monetary models, overshooting models, and portfolio balance models of exchange rate determination will be discussed.

A. Bubbles in the Dornbusch Model

Dornbusch(1982a,b) emphasizes that bubble-like situtation can have a large

dollars and bought yen. When the yen rate began to appreciate, the speculative group of Chicago's International Monetary Market received the signal from chart analysis that it was time to sell dollars and they proceeded to do so. They were followed consecutively by banks around the world, exchange dealers, and exchange managers of firms. As a result of all this, the dollar yen rate moved radically from 247 yen per dollar to 231 yen per dollar. Now the question that remains is why this particular Middle East group decided to sell dollars at that time. The author has had opportunities to talk with fund managers of this Middle East group and ask them about their method of operations and why they decided to sell dollars in April 1982. They replied that they did not purchase yen because they thought that the Japanese economic performance would improve. Rather, they did so because they thought they would influence the exchange rate by selling dollars. They chose the Easter holiday when the exchange markets would be dull as the time to *challenge* the market." (Koguchi(1983) pp. 13-16)

10. This scenario assumes that in addition to rational market participants, some market participants act on the basis of "extraneous beliefs" (chart analysis). This is difficult to treat adequately in the simple theoretical model presented in Section II.

impact on the determination of exchange rates and furthermore that this class of bubbles has been rather important under the floating exchange rate system since its adoption in 1973. In Dornbusch's model, the bubble is directly influenced by interest rates, the one economic fundamentals variable in the model. This is incorrect. First, let us investigate this point.

a. Dornbusch's Interest Arbitrage Model

Dornbusch (1982a) assumes that the economic fundamentals are represented only by interest rates and that the principal factor in determining exchange rates is the interest parity condition. It is in this context that Dornbusch examines bubbles.

First, for simplification, it is assumed that domestic and foreign bonds are perfect substitutes. In other words, investors are assumed to be risk neutral.[11] In this case, the interest parity condition can be expressed as

$$\{\text{the interest differential}\} = \{\text{expected rate of change in the exchange rate}\} \tag{12}$$

Now, assuming that the exchange rate denominated by home currency unit is e(t) and that the nominal interest rate is i(t), this can be written as

$$i(t) - i^*(t) = e(t+1) - e(t) \tag{12'}$$

(Hereafter, all variables other than interest rates are in log form. * signifies foreign country variables).

Assume now that the exchange rate at time t deviates from its fundamentals rate because of speculation. In other words, market participants in general expect with a probability α that speculation will cease at time t+1 and the fundamentals rate will be realized, but with a probability $1-\alpha$ that speculation will continue and the exchange rate will deviate from its fundamentals rate by a factor c(t+1). As a result, at time t as well a deviation of c(t) occurs through speculation which expects a capital gain at time t+1. Assume that the fundamentals rate is $\bar{e}$ and that for simplification

11. If investors are risk averse, then they demand a risk premium for the holding of a risky asset. In order for an equilibrium with a bubble to be reached in such a case, the equation that {risk premium} + {return demanded by risk neutral investors} = {capital gain from the bubble} must hold. For this equality to hold, the rise in the asset price (capital gain) arising from speculation in the future must be greater than that when investors are risk neutral, which means that the bubble must be even more explosive. However, because a bubble is fundamentally explosive, the assumption of the existence of a risk premium does not change the basic characteristics of the bubble. That is, the assumption of substitutability between domestic and foreign assets does not influence the basic characteristics of the bubble.

this fundamentals rate at time t+1, $\bar{e}(t+1)$, is known at time t. Then, the following relationship holds among the exchange rate, the fundamentals rate and the bubble.[12]

$$e(t)=\bar{e}(t)+c(t) \tag{13}$$

$E_t e(t+1) = \alpha$ {the exchange rate that is realized when the the bubble bursts} $+ (1-\alpha)$ {the exchange rate that is realized when the bubble does not burst}

$$\begin{aligned} &= \alpha\bar{e}(t+1)+(1-\alpha)\{\bar{e}(t+1)+c(t+1)\} \\ &= \bar{e}(t+1)+(1-\alpha)c(t+1) \end{aligned} \tag{14}$$

Dornbusch, assuming that the fundamentals rate does not change between time t and time t+1, simplifies equation (14) and obtains

$$E_t e(t+1) = \bar{e}(t+1)+(1-\alpha)c(t+1) \tag{15}$$

Using equations (12), (13), and (15), the expansion process of the bubble can be written as

$$c(t+1)=\begin{cases} \dfrac{c(t)}{1-\alpha}+\dfrac{i(t)-i^*(t)}{1-\alpha} & \text{with a probability } 1-\alpha \\ 0 & \text{with a probability } \alpha \end{cases} \tag{16}$$

Concerning the equation equivalent to (16), Dornbusch(1982s) states;

> leaving aside interest differentials, the equation shows the fundamental problem of bubbles: the more *over*valued the exchange rate the more rapidly it is *app*reciating, the more *under*valued the more rapidly the rate is *de*preciating. Bubbles are not self-correcting except by a crash. Bubbles, while they last, involve the possibility of temporary, cumulative deviations from fundamentals. The presence of interest differentials introduces the possibility that the exchange rate can remain unchanged even though there is over or undervaluation. ...... The bubble will be larger the larger the interest differential and the probability of a crash. For example, a 20% probability of a crash and a five percentage point interest differential sustain a 25% overvaluation!.

12. In order to facilitate understanding, the mathematical exposition which follows differs somewhat from that in Dornbusch's original paper.

b. Problems with the Dornbusch Model

In reality, the offsetting of bubbles by interest rate differentials is incorrect. This is clear from the results of Section II which state that bubbles and economic fundamentals do not have any direct connection. However, let us illustrate it in the context of Dornbusch's model. Dropping his assumption that the fundamentals rate is fixed over time, using equations (12)-(14), and solving for the bubble at time t+1, we obtain

$$c(t+1)=\begin{cases}\frac{c(t)}{1-\alpha}+\frac{i(t)-i^*(t)-\bar{e}(t+1)+\bar{e}(t)}{1-\alpha} & \text{with a probability } 1-\alpha \\ 0 & \text{with a probability } \alpha\end{cases} \tag{17}$$

Further assuming that exogenous interest rates determine the exchange rate, the fundamentals rate itself should satisfy the interest parity condition. Namely,

$$i(t)-i^*(t)=\bar{e}(t+1)-\bar{e}(t) \quad . \tag{18}$$

Substituting (18) into (17), the following is obtained.

$$c(t+1)=\begin{cases}\frac{c(t)}{1-\alpha} & \text{with a probability } 1-\alpha \\ 0 & \text{with a probability } \alpha \quad .\end{cases} \tag{19}$$

Adding Dornbusch's assumption that $\bar{e}(t+1)=\bar{e}(t)$, from (18) it is obvious that $i(t)-i^*(t)$ must equal zero. In other words, in the situation where equation (16) holds, the interest differential must be zero. Equation (16) is reduced to equation (19), and the offsetting of bubbles by interest rate differentials does not occur.

Let us examine this result in the context of Section II. Comparing the interest arbitrage condition (12) with the rational expectations asset pricing model,

$$x(t) = aE_t x(t+1) + bz(t) \tag{1}$$

we note that in (12) $x(t)=e(t)$, $z(t)=i(t)-i^*(t)$, $a=1$, $b=-1$.[13] The relationship for the corresponding bubble must be

13. When a=1, without information concerning the expected value of the long-run equilibrium exchange rate, the exchange rate cannot be determined. Moreover, there is no guarantee that x(t) will converge to a finite value. For these reasons, the above model cannot be regarded as being complete. However, this has no effect on the discussion of bubbles which follows.

$$c(t) = E_t c(t+1) \tag{20}$$

(This is the case where a=1 in (1').)

Assuming that the probability that speculation will cease is α, equation (20) gives

$$c(t+1) = \begin{cases} \dfrac{c(t)}{1-\alpha} & \text{with a probability } 1-\alpha \\ 0 & \text{with a probability } \alpha \end{cases} \tag{19'}$$

The above equation verifies the earlier conclusion.

In contrast to Dornbusch (1982a), bubbles are not related to the economic fundamentals. Even in a more general structural model of exchange rate determination, it can be shown that bubbles are not directly influenced by economic fundamentals.

B. Bubbles and More General Exchange Rate Models

In the analysis in III.2.A., bubbles were investigated in the context of Dornbusch(1982a) where interest rates are exogenous and exchange rates are determined largely by interest rate arbitrage. In what follows, the characteristics of bubbles are compared in the context of more general rational expectations models such as small country monetary models, overshooting monetary models, and portfolio balance models.

a. Small Country Monetary Model

The analysis of the small country monetary model used here follows Mussa(1980). This model assumes that investors are risk neutral, and was completed by adding the assumptions of purchasing power parity (P.P.P.) and equilibrium in the money market to the condition of interest parity. This model can be expressed by the following three equations.

$$p(t) = e(t) + p^*(t) \qquad \text{(P.P.P.)} \tag{21}$$

$$i(t) - i^*(t) = E_t e(t+1) - e(t) \qquad \text{(interest parity condition)} \tag{22}$$

$$M(t) = p(t) + ay(t) - ki(t) \quad a>0,\ k>0 \qquad \text{(money market equilibrium condition)} \tag{23}$$

where p(t) is the price level
M(t) is exogenous money supply
y(t) is real income
k is the interest semi-elasticity

Using equations (21)-(23), the reduced form for the exchange rate is obtained.

$$e(t) = \frac{k}{1+k} E_t e(t+1) + \frac{1}{1+k} z(t) \tag{24}$$

Here $z(t) = M(t) - ay(t) - p^*(t) + ki^*(t)$. That is, z(t) combines information on exogenous economic fundamentals related to exchange rate determination. Equation (24) corresponds to the simple rational expectations asset pricing model

$$x(t) = aE_t x(t+1) + bz(t) \quad , \quad 0 < a < 1 \tag{1}$$

when $a = k/(1+k)$ and $b=1/(1+k)$. Since $k>0$, the condition $0<a<1$ is satisfied and the bubble presented in Section II can therefore be applied directly to equation (24). Since this model makes the strict assumption that P.P.P. holds at every instant in time, when a bubble influencing the nominal exchange rate appears, a corresponding bubble affecting the price level also appears. Hence, bubbles have absolutely no effect on the real exchange rate.

b. Overshooting Monetary Model

In the simple monetary model presented in a. above, the strict assumption that P.P.P. always holds is made. However, in reality, the exchange rate frequently deviates from P.P.P. greatly for a long period. Hence, the assumption that P.P.P. always holds is too strict. Let us investigate the more general monetary model. This model removes the assumption that P.P.P. always holds from the simple monetary model and includes delayed price adjustment and exchange rate overshooting.[14] The model in a. assumes that the price level is determined by the fulfillment of the P.P.P. condition, but the general monetary model assumes that the price level adjusts gradually towards the expected equilibrium level. Assume that the equilibrium price (denoted as $\tilde{p}(t)$) fulfills P.P.P.[15] Moreover, assume the following two factors

14. The overshooting monetary model presented here is based on Mussa(1983). For details concernig the rationality of the price adjustment rule and its theoretical derivation, see Mussa(1981b).

15. There is no general consensus concerning an equilibrium price level. The P.P.P. criterion is only one possibility. The P.P.P. hypothesis abstracts from real economic shocks and implicitly assumes that the real equilibrium exchange rate is constant.

concerning price adjustment from time t−1 to time t: (1) The deviation from the equilibrium price level at time t−1 is reduced by a factor of 100h%$(0<h<1)$at time t and; (2) Changes in the expected equilibrium price at time t−1 are fully reflected in the price at time t. By dropping the assumption of P.P.P. from the simple monetary model and adding the above specification for the equilibrium price and the price adjustment function, the revised model is represented by the following four equations.

$$\tilde{p}(t) = e(t) + p^*(t) \qquad \text{(equilibrium price level)} \tag{25}$$

$$p(t+1) - p(t) = h\{\tilde{p}(t) - p(t)\} + \{E_t\tilde{p}(t+1) - \tilde{p}(t)\} \qquad \text{(price adjustment rule)} \tag{26}$$

$$i(t) - i^*(t) = E_t e(t+1) - e(t) \qquad \text{(interest parity condition)} \tag{22}$$

$$M(t) = p(t) + ay(t) - k\,i(t) \qquad \text{(money market equilibrium condition)} \tag{23}$$

This model is a generalization of the simple monetary model. Also, this model can be viewed as a variant of overshooting model in Dornbusch (1976).[16]

When speculation that can be represented by stochastic bubbles occurs in an economy which resembles this overshooting model, the real exchange rate is dragged by the nominal exchange rate bubble because price adjustment is slower than exchange rate adjustment. At the moment speculation ends, the nominal exchange rate bubble collapses, but since the influence of the bubble on prices remains, the real exchange rate changes greatly in the opposite direction from that in which it had been moving.

c. Rational Expectations Portfolio Balance Model

Another recent popular theory of exchange rate determination is the portfolio balance theory. In this theory, exchange rates in the short run are determined by the relative prices of domestic and foreign currency denominated assets. Exchange rates determined in this fashion change the current account in the medium and long run. In the process, the net foreign asset balance changes and this feeds back to exchange rates. Two examples of theoretical models which use this approach are Mitsuhiro Fukao (1982) and Kyoji Fukao (1983). Mitsuhiro Fukao (1982) assumes regressive

16. For details regarding the derivation and dynamic characteristics of the solution of this model, see Appendix 2.

expectations where people think the exchange rate approaches its long run equilibrium level at a certain adjustment speed. Hence, his model is not suitable for an analysis of bubble since the latter assumes rational expectations. Here, let us consider bubbles in the context of Kyoji Fukao's rational expectations portfolio balance model.

In this two country model, it is assumed for investors that home currency denominated assets are riskless, while foreign currency assets bear the risk of exchange fluctuations. The total amount of exchange risk that foreign and domestic investors must hold is assumed to be approximated by own country's total assets accumulated through past current account surpluses. Using these assumptions, the following partial equilibrium model is constructed.[17]

Let Z(t) be real net holdings of foreign assets, r(t) and r*(t) be the real domestic and foreign interest rates respectively, and s(t) be the log of the real exchange rate (denominated by home currency). Now, the conditions for an equilibrium in the foreign exchange market can be written as

$$r(t) - r^*(t) = E_t s(t+1) - s(t) - Z(t)/b \quad . \tag{27}$$

This uncovered interest parity condition includes a risk premium term Z(t)/b. Real foreign currency denominated assets Z(t) change along with a disequilibrium in the real current account. The real current account is proportionate to the difference between the real exchange rate and the "equilibrium exchange rate" (the log of the real exchange rate which brings about *equilibrium in the current account written* as $\bar{s}(t)$) in this period. In equation form, this becomes

$$Z(t) = Z(t-1) + a(s(t) - \bar{s}(t)) \quad . \tag{28}$$

Solving the simultaneous equations (27) and (28) for the particular solution gives the real exchange rate as a weighted average of current and future values of the domestic and foreign interest rates, the exchange risk, and the equilibrium exchange rate which brings about equilibrium in the current account. However, the general solution for the real exchange rate includes unstable paths which correspond to speculative bubbles.[18] Through these bubbles, the exchange rate deviates from the economic fundamentals.

This section has investigated the characteristics of the three principal rational

17. See Kyoji Fukao(1983) for details regarding the rigorous formalization of this model and the derivation of solutions. The symbols in this paper differ somewhat from those in Fukao's paper.

18. A detailed explanation of this point is provided in Appendix 3.

Table 2 Comparison of the Characteristics of Three Exchange Rate Determination Models

	Monetary Model	Overshooting Monetary Model	Portfolio Balance Model
Presented originally by	Mussa (1980)	Mussa (1983)	Kyoji Fukao (1983)
Fundamentals	Money Supply Real Income Foreign Interest Rate Foreign Price Level	Money Supply Real Income Foreign Interest Rate Foreign Price Level	Cumulative Current Account Long Run Equilibrium Exchange Rate Foreign and Domestic Interest Rate Differential
Expectations Hypothesis	Rational	Rational	Rational
P.P.P.	Assumed	Not Assumed	Not Assumed
Attitude of Investors to risk	Risk Neutral	Risk Neutral	Risk Averse
Nominal Exchange Rate Bubble	Possible	Possible	Possible
Real Exchange Rate Bubble	Not Possible	Possible	Possible

expectation exchange rate determination models and their connection with bubbles. A summary of these results is presented in Table 2.

There are several important conclusions that can be drawn by comparing these three models. Except the simple monetary model, real exchange rates can be affected by bubbles. This random fluctuation in real exchange rates can have a large and long-term impact on the real economic system.[19] For instance, in the case where appreciation of the dollar caused by a bubble continues, US export industries will suffer greatly. As a result, marginal export industries will disappear. Even when the bubble collapses and the dollar returns to a normal level, it is probable that the export industries destroyed will not be able to rebuild themselves. When economies have characteristics that resemble overshooting monetary models or portfolio balance models, the appearance of a bubble is likely to have an impact on the real economic sector and give rise to serious problems. Thus, in situations where bubbles might appear, it is not desirable to believe a priori that the exchange rate is determined solely by economic fundamentals.

19. A theoretical analysis of the adjustment process of an economy in the face of such exogenous shocks is given in Aoki and Edward(1982) etc.

IV. Exchange Rate Fluctuations and the Effect of Monetary Policy with a Fixed Rule

In the previous section, the effects of various classes of expectation factors on exchange rate determination were investigated. Here, we will investigate the relationship between these expectations and monetary policy formulated according to known rules. Assume the simple rule for monetary policy that authorities attempt to stabilize the exchange rate by controlling the money supply.[20] The effect of this class of monetary policy in a bubble situation will be considered.

1. Modeling Monetary Policy with a Fixed Rule

There are many possible ways to model monetary policy rules which view the exchange rate as a medium-term target. However, here let us assume the simplest rule that authorities change the money supply in such a way as to offset fluctuations in the exchange rate. Although three representative exchange rate determination models were introduced in Section III, in what follows a slightly revised simple monetary model will be used to investigate the effect of this monetary policy.[21] Specifically, revisions in the model will entail treating the money supply M(t) not as an exogenous variable but rather one which the monetary authorities passively adjust in response to fluctuations in the exchange rate. M(t) thus becomes an endogenous variable with monetary authorities having the following response function.[22]

20. This paper does not deal with the pros and cons of monetary policy management which considers the exchange rate as a medium-term target or a monetary policy undertaken to stabilize the exchange rate. In reality, many problems concerning this issue should be noted. For instance, (1) it is possible that conflicts would arise between movements in the exchange rate and movements in prices and the unemployment rate which are the final targets of monetary policy, (2) depending on each country's degree of autonomy, the importance of the exchange rate differs, and (3) when monetary policy is conducted which views the exchange rate as a medium-term target, fluctuations in the exchange rate can bring about fluctuations in the interest rate or the money supply. That is, such monetary policy might be destabilizing. Concerning the management of monetary policy which views the exchange rate as a medium-term target, see Mussa(1981).

21. It is also possible to use an overshooting monetary model because such a model also includes the money supply. Although an overshooting model only complicates the analysis, the principal conclusions remain unchanged. For this reason, a simple monetary model will be used here.

22. This can be regarded as the change in the money supply corresponding to a "leaning against the wind" type of monetary policy which is not sterilized. It is possible to formalize "leaning against the wind" type of rules for controlling the money supply in a variety of ways. See Mussa(1980) and Canzoneri(1983) concerning this point.

$$M(t)-M(t-1)=-m\{e(t)-e(t-1)\},\ m>0 \tag{29}$$

Since all of the variables in this equation are in log form, the money supply is supposed to change by an amount of a fixed coefficient m times rate of change in the exchange rate.

When market participants are already aware of this monetary policy rule, this response of monetary authorities is reflected in the market, and the exchange rate is determined rationally. If bubbles do not occur, the exchange rate is determined as a weighted average of future expected values of economic fundamentals. That is,

$$\bar{e}(t)=\frac{1}{1+m+k}\sum_{s=0}^{\infty}\left(\frac{k}{1+m+k}\right)^{s}\times E_t w(t+s)+A \tag{30}$$

where $w(t)$ represents the exogenous economic fundamentals $(w(t)=ki^{*}(t)-p^{*}(t)-ay(t))$

A is a constant which depends on the initial values of the exchange rate and the money supply.

The future weight of the economic fundamentals or the "discount rate" of these future fundamentals, $k/(1+k+m)$, depends on parameter m. When m is large, i.e., when the money supply is expected to strongly respond to exchange rate fluctuations, the influence of the expected future values of the economic fundamentals on the current exchange rate diminishes. This means that market participants believe that the influence of changes in the economic fundamentals on the exchange rate will be lessened by offsetting money supply operations by the authorities.

2. Monetary Policy with Fixed Rule and Bubbles

If bubbles do exist, the expected expansion path of a bubble can be calculated from the reduced form equation for the exchange rate in the case where monetary policy rule is fixed as above. The following equation is obtained.

$$\{\text{expected value of the bubble at time } t+1\}=\frac{1+m+k}{k}\times\{\text{bubble at time } t\} \tag{31}$$

Thus, the more responsive is monetary policy (the larger is m), the more explosive will be the path of the bubble. If the magnitude of deviation from the fundamentals rate brought about by the expected speculation in the next period is fixed, the smaller the deviation in this period from the fundamentals rate caused by speculative expectations will be the more responsive is monetary policy. However, once a speculative bubble does appear, the more responsive monetary policy is, the faster it will grow. Hence, this class of monetary policy which attempts to offset exchange rate will not necessarily prove effective in lessening the impact of a bubble on the exchange rate.[23]

3. Extensions of the Bubble Model

So far in the analysis of the characteristics of bubbles and the effect of fixed monetary policy, the probability α of the bubble collapsing was assumed to be given. However, more realistically, it is possible to think of α not as a constant but as an endogenous variable affected by various factors. For instance, Blanchard and Watson(1982) suggest that the magnitude of α may be a function of the length of the bubble's continuation and/or the magnitude of the bubble, while Dornbusch(1982b) views the collapse of bubbles as arising from new information in the market about the economic fundamentals. Let us briefly consider these two cases where the probability of the bubble's collapse α is a function of (1) the magnitude of the bubble and (2) news concerning economic fundamentals. In the former case, the natural assumption is made that the probability of the bubble's collapse increases with the size of the bubble. The second case assumes that the probability of the bubble's collapse is increased by information concerning economic fundamentals reaching the market should such information be unfavorable to bubble-type speculation.

In these extended models where the probability of the bubble's collapse is an endogenous variable, the expected expansion path of the bubble is unaffected.[24] In the case where the probability of the bubble's collapse increases with the size of the bubble, the higher this probability is, the more explosive the bubble becomes and the quicker, on average, the collapse of the bubble is.

In the second case where the probability of the bubble's collapse is influenced by news about the economic fundamentals, there is a new policy implication. In such a

23. The conclusion that a "leaning against the wind" type of money supply control can destabilize the exchange rate when bubbles exist in the foreign exchange market is not unique to the model used here. Models used by Turnovski(1983) and Canzoneri(1983) give the same conclusion. It is likely that this is a common characteristic shared with rational expectations models that treat the money supply as an economic fundamentals.

24. In a monetary model, the expansion path of the bubble must satisfy the relationship that {the bubble at time t} =a× {the expected value of the bubble at time t+1}. When the magnitude of the bubble in the initial time period is determined, the expected expansion path from this time on depends only on the parameter a. Hence, it is not related to the economic fundamentals or the probability of the bubble's collapsing.

case, it is possible that policies which "surprise" the market can be effective in collapsing the bubble. For instance, in a situation where monetary authorities offset fluctuations in the exchange rate by conducting a "leaning against the wind" type of control of the money supply, assume that the yen depreciates as the result of the appearance of a bubble. If authorities tighten even more their control of the money supply (a stronger "leaning against the wind" stance) and market participants view this as a new policy rule, then this policy revision acts as news about economic fundamentals which are considered to be disadvantageous towards the bubble and strengthens the probability of the collapse of speculation concerning the yen depreciation. If this type of scenario occurs, appropriately revising policy rules to surprise the market will be desirable.[25]

V. Conclusions

This paper has investigated bubbles, centering on the application of such theory to the foreign exchange market. However, theoretical foundations of bubbles themselves are not yet complete in that: (1) an adequate explanation of the generating mechanism of bubble has not yet been presented; and (2) research in the case of the existence of multiple assets is not sufficiently developed. Moreover, empirical analysis has been extremely limited by the difficulty of theoretically specifying the expansion path of bubbles and by the difficulty of discriminating among three factors which affect the price of assets: (1) influence of economic fundamentals, (2) influence of bubbles, and (3) the influence of "extraneous beliefs" Indeed, empirical analyses so far reached can roughly be grouped into the following three categories: (1) Analyses which attempt to look for bubbles from characteristics of the distribution of asset price data (Blanchard and Watson[1982]); (2) Investigations of the applicability of rational expectations models, which a priori exclude the existence of bubbles, to asset prices. These analyses compare the actual distributions of asset price data to the expected distributions of asset prices based on the anticipated economic fundamentals (Singleton[1980], Huang[1981], etc.); (3) Attempts to directly estimate bubbles from data, assuming a priori an expansion path for the bubbles (Flood and Garber[1980], Burmeister and Wall [1982], etc.).

Analyses in the first two categories have difficulty in discriminating between the influence of "extraneous beliefs" etc. and the influence of bubbles, while an analysis in the third category is so far limited to empirical work on hyperinflation where the expansion process of the bubble can be approximated by a deterministic bubble. In such a class of empirical analysis of the foreign exchange market, it will be difficult to

25. For this proof in the context of a monetary model, see Appendix 4.

extract a pure bubble phenomenon because bubbles will likely be intertwined with "peso problems" and speculative activities based on chart analysis ("extraneous beliefs"). However, on the basis of present theoretical knowledge, it is clear that the effect of monetary policy which views the exchange rate as a medium-term target will differ depending on whether exchange rate fluctuations are caused by fundamentals or the appearance of bubbles. Thus, it is necessary to develop empirical study of bubbles in the future.

Appendix 1. Types of Expectations in Exchange Rate Determination

There have been various developments in exchange rate theory in the 1970's, centering around the asset approach to exchange rate determination. Some of the models developed were monetary models, overshooting models, and portfolio balance models. However, structural parameter estimates for the exchange rate functions of the above models obtained by using data for the 1970's are extremely unstable. Hence, the forecasting ability of these models is poorer than random walk models or the forward exchange rate.[26]

Reasons for the instability of parameters in these structural models have not been sufficiently clarified, but one possible explanation will be the importance of expectation factors which have not yet been adequately considered. The first of these expectation factors to receive attention was the impact of "news". More recently, bubbles have been in the limelight along with other expectation factors such as "peso problems" and "extraneous beliefs". Let us briefly survey expectation factors other than bubbles which are considered to affect exchange rates.[27]

A. The Theory of News

Amongst those expectation factors considered to affect exchange rate determination, the first to attract widespread attention was the role of news. Some of the economic fundamentals in this period which were previously forecasted are already reflected in the exchange rate and cannot serve as a principal factor which affects exchange rate. In contrast, economic fundamentals which were not anticipated can serve as a principal factor which shifts the exchange rate in this period. This unanticipated portion of the economic fundamentals is called "news". The news

26. See Meese and Rogolf(1983) for a detailed empirical comparison of these structural models.

27. The following papers treat these various expectations in detail. Concerning "news", see Frenkel(1980) and Dornbusch(1980). For "extraneous beliefs" see Dornbusch(1982a,b), Shiller(1981), and for "peso problems" refer to Krasker(1980), Dornbusch(1982a,b).

hypothesis has received relatively widespread acceptance, despite the fact that it cannot explain large, single direction shifts in asset prices in cases where significant news about economic fundamentals is not transmitted to the market. Also, changes in asset prices which can be explained by economic fundamentals are frequently much less than observed changes in asset prices. It is difficult to believe that large fluctuations in exchange rates can be perfectly explained only by this news hypothesis.[28] Unidirectional movements in the exchange rate are often explained by the "bandwagon effect". The bandwagon effect is interpreted frequently as the irrational "intoxication" in the market, but this type of market movement can also be explained rationally by introducing expectation factors such as bubbles or "peso problems" explained below.

B. "Peso Problems"

Krasker(1980) first argues that when there is a small probability of an event which would cause a large exchange rate fluctuation the standard test for the efficiency of the corresponding forward exchange market is not always valid. The word "peso problem" was first used in an analysis of the expectations effect on the market that the Mexican peso would depreciate in 1976. A typical example of this class of event which causes a large fluctuation in the exchange rate would be readjustment of exchange rates under a fixed exchange rate system. However, theoretically, this hypothesis can also be applied to large fluctuations of exchange rates under the floating rate system. For instance, Dornbusch(1982a) emphasizes the possibility of the current exchange rate being strongly influenced in cases where people as a group anticipate large changes in the government's economic policies. As an example, he cites the fall of the French franc in 1925-26 caused by the fear of imposing taxes on capital and increases in the money supply by the French government.

This example of "peso problems" places emphasis on the role of expectations concerning the actions of policy authorities. However, the "peso problems" hypothesis can be applied not only to those cases where confidence in policy authorities or the political system is shaken, but to a much wider class of situations; For instance, let us consider the case of a "dollar got stronger as the result of an emergency". More specifically, let this describe the appreciation of the dollar in reaction to news of the intensification of the Iran-Iraq war, which is hard to explain

28. Concerning empirical research on uniqueness and validity of rational expectations models which compare asset price movements and movements in values of economic fundamentals, see Shiller(1981) and Singleton(1980). Empirical research on the foreign exchange market has been conducted by Huang(1981) who concludes that compared with movements in economic fundamentals, exchange rate fluctuations have been volatile.

from the current fundamentals perspective. This phenomenon is explained by the large increase in the subjective expectations of investors concerning the realization of a scenario where the expanding Middle East war leads to the closing of the Hormuz Strait, and rising oil prices cause an enormous worsening in the economic fundamentals of Japan and Europe. This case is none other than one of "peso problems".

Consider this case in the context of the simple rational expectations model

$$x(t) = aE_t x(t+1) + bz(t) \tag{A1}$$

where x(t) is an exchange rate in terms of the home currency
z(t) is an exogenous variable representing the economic fundamentals

Assuming that bubble-like expectations not reflected in the economic fundamentals do not occur, the exchange rate is determined solely by these economic fundamentals. Namely,

$$x(t) = \overline{x}(t) = b\sum_{i=0}^{\infty} a^i E_t z(t+i) \tag{A2}$$

For simplification, assume that the values of these economic fundamentals follow only two paths.

$$z(t+i) = \begin{cases} \bar{z} & \text{with a probability } 1-\beta \\ \bar{z} + z^* & \text{with a probability } \beta \end{cases} \tag{A3}$$

Here $\bar{z}$ is the normal value of the economic fundamentals when the Hormuz Strait is not closed and z^* represents the worsening in these economic fundamentals when the Hormuz Strait is closed. In this case, if the Hormuz Strait will not be closed, $\beta = 0$ and the exchange rate x(t) becomes

$$x(t) = b\bar{z}/(1-a) \tag{A4}$$

Including the possibility that the Hormuz Strait may be closed, this becomes

$$x(t) = b\bar{z}/(1-a) + ab\beta z^*/(1-a) \tag{A5}$$

Here, if the possibility that the Hormuz Strait will be closed becomes larger ($\beta\uparrow$) or the expected negative effects of closing this strait increase ($z^*\uparrow$) , then the exchange rate deviation from the ex post observed values of the economic fundamentals increases. As clearly shown by this formalization of the preceding scenario, the "peso problems" hypothesis attempts to understand exchange rate fluctuations based solely on expectations concerning economic fundamentals. A clear line is thus drawn between this hypothesis and bubbles which consider expectations independent of the economic fundamentals.

C. Extraneous Beliefs

"News" and "peso problems" assume the rationality of market participants, but "extraneous beliefs", considering that the hypothesis of rationality might be overly strong, treats the case where market participants possess incorrect expectations. "Extraneous beliefs" was formally presented by Shiller(1981) as one hypothesis for explaining fluctuations in stock prices. However, this hypothesis can be applied exactly to fluctuations in exchange rates. In this case, market participants do not have sufficient knowledge as to the economic structure of exchange rate determination, and not being able to discriminate between relevant and irrelevant information, they form expectations using irrelevant variables as to exchange rate determination.

Let us illustrate this in conformity with the previous simple rational expectations model. Assuming that the true reduced form for exchange rate determination is given by equation (1), market participants in general include irrelevant variables q(t) which are not part of the economic fundamentals in the exchange rate model. Hence, they form expectations based on the equation

$$x(t) = a E_t x(t+1) + b z(t) + cq(t) \tag{A6}$$

Dornbusch(1982a) stated that market participants have some difficulty in perceiving this mistake when autocorrelation of the irrelevant variables is high and that in such a case the exchange rate will deviate greatly from its fundamentals rate. He also showed, using a simple model, that the variance of the exchange rate will also be larger. Moreover, Dornbusch(1982b) emphasized the possibility that market participants do not concentrate on a fixed set of variables but simply shift their attention from one irrelevant variable to the other consecutively. That is, market participants have a sort of "fashion" sense. It is thought that this hypothesis grasps movements in actual exchange rates, but Dornbusch provides no explanation as to how specific irrelevant variables become part of the model or why the "fashion" changes and why investors change their attention from one irrelevant variable to another consecutively. The present level of the "extraneous beliefs" hypothesis is no more than pointing out the vague possibility of arising such beliefs.

Appendix 2. Overshooting Monetary Models

A. Deriving solutions

Blanchard and Kahn(1980) explains general methods of the solution of linear rational expectations models. Here, methods of solution using operators will be employed.

First, define the forward difference operator D and the forward operator F as follows. For the variable z(t),

$$D z(t) = z(t+1) - z(t) \tag{A7}$$

$$F z(t) = z(t+1) \quad . \tag{A7'}$$

Using (A7')repeatedly gives $F^n z(t) = z(t+n)$. Also, the relationship that $D = F - 1$ holds.

Using the difference operator to rewrite the overshooting model gives the following four equations.

$$\tilde{p}(t) = e(t) + p^*(t) \qquad \text{(equilibrium price equation)} \tag{A8}$$

$$Dp(t) = h\{\tilde{p}(t) - p(t)\} + DE_t\tilde{p}(t) \qquad \text{(price adjustment function)} \tag{A9}$$

$$i(t) - i^*(t) = DE_t e(t) \qquad \text{(interest parity condition)} \tag{A10}$$

$$M(t) = p(t) + ay(t) - ki(t) \qquad \text{(money market equilibrium condition)} \tag{A11}$$

Substituting the price adjustment function into the equilibrium price equation gives

$$Dp(t) = h\{e(t) + p^*(t) - p(t)\} + DE_t e(t) + DE_t p^*(t) \tag{A12}$$

Substituting (A10) into money market equilibrium condition (A11) gives

$$DE_t e(t) = \frac{1}{k}(p(t) - g(t)) - DE_t p^*(t) \tag{A13}$$

where $g(t) = M(t) - ay(t) + br^*(t)$, and $r^*(t) = i^*(t) - DE_t p^*(t)$ which shows the expected real yeild on foreign bonds.

Rewriting (A12) and (A13) in matrix notation to get,

$$\begin{bmatrix} -h-D & h+D \\ \frac{1}{k} & -D \end{bmatrix} \begin{bmatrix} p(t) \\ E_t e(t) \end{bmatrix} = \begin{bmatrix} -hE_t p^*(t) - DE_t p^*(t) \\ \frac{1}{k} E_t g(t) + DE_t p^*(t) \end{bmatrix} \quad . \tag{A14}$$

Now we can derive the particular solution which gives the exchange rate as well as the price level when the exchange rate does not deviate from its fundamentals rate.

First, formally solving (A14) for the price level, the following is obtained

$$p = \frac{\begin{vmatrix} -hE_t p^* - DE_t p^* & h+D \\ \frac{1}{k} E_t g + DE_t p^* & -D \end{vmatrix}}{\begin{vmatrix} -h-D & h+D \\ \frac{1}{k} & -D \end{vmatrix}} \quad . \tag{A15}$$

Since the difference operator can be treated as a scalar, simplifying (A15) gives

$$p = \frac{-E_t g/k}{(D-1/k)} \quad . \tag{A16}$$

Rewriting (A16) using F and letting that $\theta = k/(1+k)$, this becomes

$$p = \frac{-E_t g/k}{(F-1-1/k)} = \frac{-E_t g/k}{(F-\frac{1}{\theta})} \quad . \tag{A17}$$

Expanding the operator fraction as an infinite series gives

$$\frac{-1}{F-1/\theta} = l_0 + l_1 F + l_2 F^2 + \cdots \quad . \tag{A18}$$

Thus, from (A18) the following must always hold

$$(F-\frac{1}{\theta})(l_0 + l_1 F + l_2 F^2 + \cdots) = -1 \quad . \tag{A19}$$

Expanding the left side, it is clear that $l_0 = \theta$, $l_1 = \theta^2$, $l_2 = \theta^3 \cdots$ must obtain.

Using this result in (A17), the expected value of prices at time $s\,(s \geq t)$ can be written as

$$\begin{aligned} E_t\, p(s) &= \frac{-E_t\, g/k}{(F-1/\theta)} \\ &= \frac{\theta}{k}\sum_{j=0}^{\infty}\theta^j F^j E_t\, g \\ &= \frac{1}{1+k}\sum_{j=0}^{\infty}\theta^j E_t\, g(s+j) \\ &\equiv G(s) \quad . \end{aligned} \tag{A20}$$

Similarly, the expected value of the exchange rate at time s can be written as

$$E_t\, e(s) = G(s) - E_t p^*(s) \quad . \tag{A21}$$

Since the characteristic roots of (A14) are $\lambda_1 = 1/\theta > 1$, $\lambda_2 = 1-h < 1$, the general solution at time $s \geq t$ which includes the possibility of deviation from the fundamentals rate is given by

$$E_t\, p(s) = G(s) + a(t)(1-h)^{s-t} + c(t)(\tfrac{1}{\theta})^{s-t} \tag{A22}$$

$$E_t\, e(s) = G(s) - E_t\, p^*(s) + a_e(t)(1-h)^{s-t} + c_e(t)(\tfrac{1}{\theta})^{s-t} \tag{A23}$$

where $a(t)$, $c(t)$, $a_e(t)$, and $c_e(t)$ are constants representing initial values of the initial period t.

Since

$$DE_t\, e(t) = \frac{1}{k}(p(t) - g(t)) - DE_t p^*(t) \tag{A13}$$

always holds, $a_e(t) = \big(1/(-hk)\big)a(t)$, $c_e = c(t)$, and equations (A22) and (A23) become

$$E_t\, p(s) = G(s) + a(t)(1-h)^{s-t} + c(t)(\tfrac{1}{\theta})^{s-t} \tag{A24}$$

$$E_t\, e(s) = G(s) - E_t p^*(s) - \frac{1}{hk}\, a(t)\,(1-t)^{s-t} + c(t)(\tfrac{1}{\theta})^{s-t} \quad . \tag{A25}$$

Also, p(t) is a predetermined variable whose value is given at time t (the value of p(t) is determined at time t−1). Since the equation $p(t) = G(t) + a(t) + c(t)$ holds, this can be solved for a(t) and the results substituted into (A24). Hence the final solution of the model becomes

$$E_t p(s) = G(s) + (p(t) - G(t) - c(t))(1-h)^{s-t} + c(t)\left(\frac{1}{\theta}\right)^{s-t} \tag{A26}$$

$$E_t e(s) = G(s) - E_t p^*(s) - \frac{1}{hk}(p(t) - G(t) - c(t))(1-h)^{s-t} + c(t)\left(\frac{1}{\theta}\right)^{s-t} \tag{A27}$$

Interpreting these results in the context of (A27) which gives the exchange rate function, $G(s) - E_t p^*(s)$ is the long-run equilibrium exchange rate, $-\frac{1}{hk}(p(t) - G(t) - c(t))(1-h)^{s-t}$ is the overshooting term, and $c(t)\left(\frac{1}{\theta}\right)^{s-t}$ is the expected value of the bubble.

B. Overshooting in the model

In the case where a bubble does not exist at time t, equations (A26) and (A27) become

$$E_t p(s) = G(s) + (p(t) - G(t))(1-h)^{s-t} \tag{A28}$$

$$E_t e(s) = G(s) - E_t p^*(s) - \frac{1}{hk}(p(t) - G(t))(1-h)^{s-t} \tag{A29}$$

For simplification, assume

$$E_{t-1} g(s) = M - ay + br^* = \bar{g},\ E_{t-1} p^*(s) = p^* \quad \text{for } s = t-1,\ t,\ \cdots\cdots \tag{A30}$$

In this case, the following holds.

$$G(s) = \frac{1}{1+k} \sum_{j=0}^{\infty} \theta^j E_{t-1} g(s+j) = \bar{g} \tag{A31}$$

As a result, if the economy is in equilibrium at time t—1 with P.P.P. holding, we obtain

$$p(t-1) = \bar{g} \tag{A32}$$

$$e(t-1) = \bar{g} - p^* \tag{A33}$$

Assume that at time t expectations of a permanent increase in the money supply $\triangle M$ are created. Since p(t) is a predetermined variable, it does not change from its equilibrium rate at time t—1, and the new equilibrium value at time t is given by

$$p(t)=\overline{g} \tag{A34}$$

$$e(t)=\overline{g}-p^{*}+(1+\frac{1}{hk})\triangle M \tag{A35}$$

From this point, the price level and the exchange rate follow the expected dynamic path given by

$$E_t p(s)=\overline{g}+\triangle M-\triangle M(1-h)^{s-t} \tag{A36}$$

$$E_t e(s)=\overline{g}-p^{*}+\triangle M+\frac{1}{hk}(1-h)^{s-t}\triangle M \tag{A37}$$

as the economy gradually approaches the new long-run equilibrium expressed by the following.

$$\overline{p}=\overline{g}+\triangle M \tag{A38}$$

$$\overline{e}=\overline{g}+\triangle M-p^{*} \tag{A39}$$

A comparison of this new equilibrium with that equilibrium which held at time t—1 can be written as follows.

$$\overline{e}-e(t-1)=\triangle M \tag{A40}$$

$$\overline{p}-p(t-1)=\triangle M \tag{A41}$$

$$p(t)-p(t-1)=0 \qquad \text{(price level unchanged at time t)} \tag{A42}$$

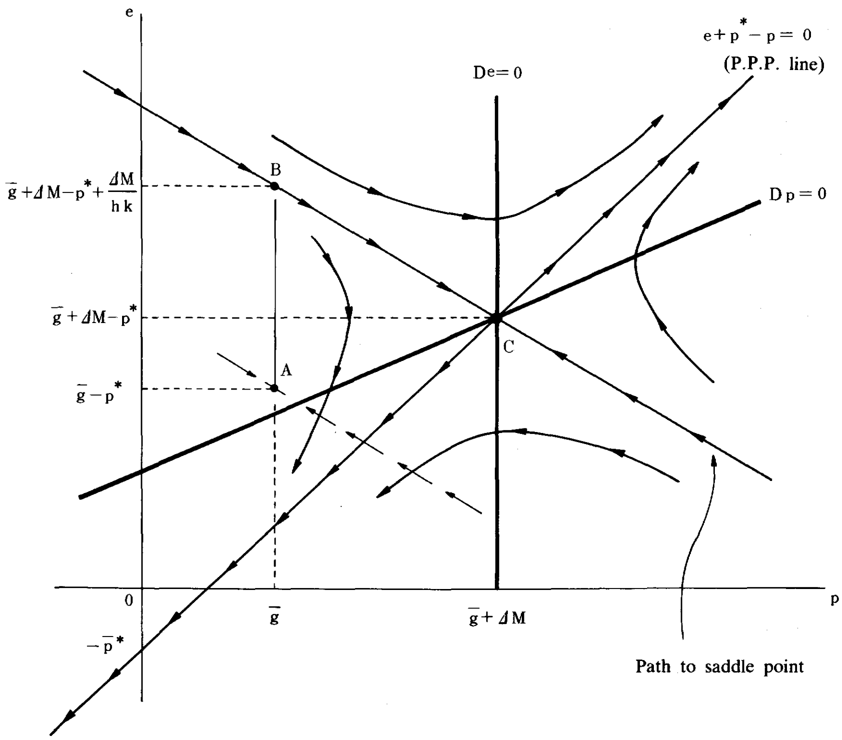
$$e(t)-e(t-1)=(1+\frac{1}{hk})\triangle M \quad \text{(overshooting of the exchange rate at time t is given by } \Delta M/hk) \tag{A43}$$

Remembering that k is the semi-elasticity of the interest rate and h is a parameter showing the adjustment speed of prices, it is clear that the dynamic characteristics of this model are similar to those of Dornbusch's overshooting model(1976).

C. Bubbles in the Model

As shown by the phase diagram in Figure 1, in this model if the economy moves away from the saddle path because of the existence of a bubble, the economy nevertheless is still gradually expected to approach the P.P.P. line. This corresponds

Figure 1. Phase Diagram of the Overshooting Monetary Model



to the expected value of the real exchange rate derived from equations (A26) and (A27)

$$E_t\,(e(s)+p^*(s)-p(s)) = \frac{1+hk}{-hk}\,\{p(t)-G(t)-c(t)\}\,(1-h)^{s-t} \qquad \text{(A44)}$$

approaching zero.

However, the real exchange rate, which is influenced by a delayed price adjustment, is influenced by bubbles in the following fashion: Assuming a stochastic bubble; (1) As long as the exchange rate bubble does not collapse, the bubble ex post grows faster than the expected value of the bubble ex ante. The bubble which is involved with the price level adjustment consists only of ex ante expected values. As a result, the adjustment in prices is slower than the exchange rate adjustment and the real exchange rate is pulled by the nominal exchange rate; (2) In the event that the exchange rate bubble collapses, the corresponding price bubble adjusts thereafter at the fixed rate h. Hence, at this point the real exchange rate changes greatly.

Appendix 3. Bubbles and Rational Expectations Portfolio Balance Model

The partial equilibrium model of the foreign exchange market used in the main body of the paper is given by

$$r^*(t)+E_t s(t+1)-s(t)=r(t)+Z(t)/b \tag{A45}$$

$$Z(t)=Z(t-1)+a(s(t)-\bar{s}(t)) \tag{A46}$$

If the expected path of the long-run equilibrium exchange rate, $\bar{s}(t)$, and the foreign and domestic interest rates, $r(t)$ and $r^*(t)$, all of which are exogenous variables, are known, this model can be solved rationally. Given this information, the following difference equation concerning future exchange rates can be obtained from equations (A45) and (A46).

$$E_t s(t+j+1)-\left(\frac{a}{b}+2\right) E_t s(t+j)+E_t s(t+j-1)=E_t z(t+j) \tag{A47}$$

Here $z(t+j)$ is a linear combination of the exogenous economic fundamentals. The particular solution of this difference equation is given by

$$\lambda^2-(a/b+2)\lambda+1=0 \tag{A48}$$

Letting the smaller characteristic roots of this equation be represented by λ and the larger by λ^{-1}, the following inequality obtains.

$$0<\lambda<1<\lambda^{-1} \tag{A49}$$

Thus, the general solution of the expected path of the real exchange rate becomes

$$E_t s(t+j)=(\text{expected fundamentals rate})+c(t)(\lambda^{-1})^j \tag{A50}$$

$c(t)(\lambda^{-1})^j$ is the bubble term. For a rigorous derivation of this general solution, see Kyoji Fukao(1983).

Appendix 4. The Effect of Monetary Policy when the Probability of the Bubble's Collapse is an Endogenous Variable

A. Endogenizing the Probability of Bubble's Collapse

When the probability of the bubble's collapse is a function of news concerning economic fundamentals and the magnitude of the bubble, the probability of the bubble collapsing at time t+1 can be written as

$$\alpha(t+1)=\alpha(c^2(t),\ I(t+1)) \qquad . \qquad \text{(A51)}$$

Here $I(t+1)=\bar{e}(t+1)-E_t\bar{e}(t+1)$ ($\bar{e}(t+1)$ is the fundamentals rate at time $t+1$). The difference between the actual exchange rate and the expected value of the fundamentals rate reflects news delivered to the market about economic fundamentals. The assumptions about this function stated in the main body of the paper can be summarized as follows.

$$\frac{\partial \alpha(t+1)}{\partial c^2(t)} > 0$$

$$\frac{\partial \alpha(t+1)}{\partial I(t+1)} < 0, \quad \text{if} \quad c(t) > 0$$

$$\frac{\partial \alpha(t+1)}{\partial I(t+1)} > 0, \quad \text{if} \quad c(t) < 0$$

B. The Effect of Monetary Policy

Assume that the "leaning against the wind" type of the money supply control rule is represented as

$$M(t)-M(t-1)=-m\{e(t)-e(t-1)\} \qquad m>0 \qquad . \qquad \text{(A52)}$$

Further assume that the fundamentals exchange rate in the monetary model is represented by $\bar{e}_m(t)$. In this case, as briefly explained in the main body of the paper, the solution to this exchange rate becomes

$$\bar{e}_m(t)=\frac{1}{1+m+k}\sum_{s=0}^{\infty}\left(\frac{k}{1+m+k}\right)^s E_t w(t+s)+A \qquad \text{(A53)}$$

where $w(t)=ki^*(t)-p^*(t)-ay(t)$ and $A=M(t-1)+me(t-1)$.

For simplification assume that w(t) follows a random walk and that $E_t w(t+s)=q$ for $s=0$, 1, 2, …… . Hence, (A53) becomes

$$\bar{e}_m(t)=\frac{q+A}{1+m+k}\ \frac{1}{1-\dfrac{k}{1+m+k}}=\frac{q+A}{1+m} \tag{A54}$$

If it is announced that policy rules from time t will change from m to m′ ($m<m'$) and market participants believe this announcement, the new fundamentals rate is given by

$$\bar{e}_{m'}(t)=\frac{q+A'}{1+m'} \tag{A55}$$

$$A'=M(t-1)+m'e(t-1)$$

In this case, news concerning economic fundamentals at time t becomes

$$\begin{aligned} I(t) &= \bar{e}(t)-E_{t-1}\bar{e}(t) \\ &= \bar{e}_{m'}(t)-\bar{e}_m(t) \\ &= \frac{(q+A')(1+m)-(q+A)(1+m')}{(1+m)(1+m')} \\ &= \frac{Q(m-m')}{(1+m)(1+m')} \qquad (A62) \end{aligned} \tag{A56}$$

where $Q=q+M(t-1)-e(t-1)$

Thus, if $Q>0$, $I(t)<0$. When a yen depreciation bubble occurs($c(t)>0$), an announcement of a stricter "leaning against the wind" monetary policy ($m\to m'$, $m<m'$) can be expected to increase the probability of the bubbles' collapse.

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