Effects of Measurement Error on the Output Gap in Japan

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Potential output is the largest amount of products that can be produced by fully utilizing available labor and capital stock; the output gap is defined as the discrepancy between actual and potential output. If data on production factors contain measurement errors, total factor productivity (TFP) cannot be estimated accurately from the Solow residual (i.e., the portion of output that is not attributable to labor and capital inputs). This may give rise to distortions in the estimation of potential output and the output gap.

The primary purpose of this paper is to discuss theoretically how measurement errors and quality changes in production factors affect estimates of potential output and the output gap. The main results are (1) that effects of quality changes in production factors can be left in the Solow residual for correct estimation of potential output and the output gap, but (2) that measurement errors in utilization of capital stock and labor should be removed. Estimation of Japan's output gap, in particular, may be distorted by the absence of data on capacity utilization in non-manufacturing sectors.

To resolve this problem, we consider two definitions of output gap and compare their performance. The first definition (the conventional output gap) assumes capacity utilization to be 100 percent in non-manufacturing sectors. Then we fit a certain trend to the Solow residual and define the trend as TFP and the regression residual as the measurement error of capacity utilization in non-manufacturing sectors. The second definition (the new output gap) uses data on electricity consumption to directly estimate capacity utilization in non-manufacturing sectors. In this case, we can take the Solow residual to be TFP.

Next, we compare the performance of the two definitions of output gap in terms of their consistency with the reference dates of business cycle *and with various DIs in* Short-Term Economic Survey of Enterprises in Japan *published by the Bank of Japan, including the business conditions DI. We show that the new output gap is superior to the conventional output gap. Furthermore, when the new output gap is used in a Phillips curve, estimates of parameters are more stable than when we use the conventional output gap. These results suggest that the new output gap is a suitable measure of slackness in the Japanese economy.*

Key words: Productivity; Output gap; Measurement errors

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I. Introduction

Each economy can produce a certain amount of products during a certain period by fully utilizing the available labor force and capital stock. *Potential output* captures this concept in terms of real gross domestic product (GDP).[1] The discrepancy between actual output and potential output is called the *output gap*, a concept that captures excess demand or supply in an economy. Since the output gap expresses slackness of economic activity, it is an important measure of economic welfare. Furthermore, since the output gap measures pressure on prices, it provides useful information for the implementation of monetary policy. Inflationary pressure is put on prices (e.g., consumer prices) when the output gap shrinks during a boom. On the other hand, deflationary pressure emerges when the output gap expands during a recession.

Since the output gap is unobservable, it needs to be estimated. Among the various methods proposed to estimate the output gap, we introduce the *production-function approach*, which is based on an estimated aggregate production function.[2] This method estimates the output gap in three steps. First, we consider a production function with three factors: capital, labor, and total factor productivity (TFP). Second, potential output is calculated by substituting the entire amount of labor and capital into the previous production function. Finally, the output gap is obtained as a deviation rate of actual output from the potential output.[3] This is the classical and standard way of estimating the output gap, as used in *Economic Survey of Japan* (by the Economic Planning Agency [2000], or EPA).

Since measurement errors creep into data on capital and labor, estimation of the output gap may be distorted. Since TFP is unobservable as mentioned, it is extracted from the Solow residual, which is the remainder after contributions of capital and labor are subtracted from realized output.[4] If there is no measurement error, TFP coincides with the Solow residual; otherwise, it departs from the Solow residual. Erroneous estimation of TFP may affect estimates of potential output and the output gap.

There are two types of measurement errors: measurement errors in factor utilization and quality changes in production factors. For the former, it is well known that in Japan there are no data on capacity utilization in non-manufacturing sectors. For this reason, in estimating the output gap, capacity utilization in non-manufacturing sectors is often assumed to be 100 percent.[5] Suppose true capacity utilization declines in non-manufacturing sectors. Measured capacity utilization is untouched since it is constant at 100 percent; instead, the Solow residual decreases. If one mistakes the decrease in the Solow residual for a decline in TFP, potential

1. One definition of potential output is an upper boundary of an economy's producible real GDP; another definition is an average of an economy's real GDP (e.g., Giorno *et.al.* [1995], Congressional Budget Office [1995] for the United States, and Economic Planning Agency [2000] for Japan).
2. For existing estimates of Japan's output gap, see Economic Planning Agency (2000) and Bayoumi (2000).
3. In estimating the output gap, TFP is untouched. The reason is that since TFP reflects productivity of a whole economy, a reduction in output due to a decline in TFP does not imply an increase in the slackness of economic activity.
4. For research on variations in TFP, see Jorgenson and Griliches (1967), Denison (1967), and Kendrick and Grossman (1980). Hulten (2000) is a recent survey of the TFP literature.
5. For instance, EPA (2000) assumes that capacity utilization in non-manufacturing sectors is constant.

output is underestimated and so is the output gap. Therefore, to correctly estimate the output gap, measurement errors should be removed from Solow residuals.

An example of quality change in production factors is the argument that the Solow residual may have declined due to rapid depreciation of capital stock in the latter half of the 1990s. In this case, it is wrong to think of the decrease in the Solow residual as a decline in TFP. Nonetheless, depreciation of capital stock is similar to a decline in TFP, since both mean a reduction in production capability. This implies that whether depreciation of capital stock is taken to be a decline in TFP or a reduction in capital amount, estimates of potential output and the output gap are free from distortion. Consequently, to correctly estimate potential output and the output gap, effects of quality change can be left in the Solow residual.

We take two approaches to prevent measurement error in capacity utilization in non-manufacturing sectors from distorting estimates of the output gap. The first approach assumes that capacity utilization moves together with the business cycle. After we regress the Solow residual along a certain trend, we think of the fitted trend as TFP and of the regression residual as the measurement error in capacity utilization. In this paper, we call the output gap thus calculated the *conventional output gap*. This approach, however, suffers from a risk. Even when capital stock is rapidly outdated or true TFP varies with large structural changes in an economy, a deviation in the Solow residual from its trend is mistaken for a variation in capacity utilization in non-manufacturing sectors. This gives rise to a distortion in the estimation of the output gap.

The second approach incorporates direct estimates of capacity utilization in non-manufacturing sectors to prevent measurement errors from creeping into the Solow residual. In this paper, we use data on electricity consumption to provide an estimate of capacity utilization in non-manufacturing sectors. If the estimate is accurate, the Solow residual coincides with TFP. As a result, TFP can be obtained without identifying a trend in the Solow residual. We call the output gap thus calculated the *new output gap*. This approach has a risk, however. Errors created when estimating capacity utilization in non-manufacturing sectors are reflected in measurements of productivity. Since factor utilization is directly related to the output gap, such estimates require great accuracy.

We have no direct ways, even *ex post*, to evaluate the conventional and new output gap. Hence, we resort to the following practical criteria to compare the performances of the two definitions. First, we check the consistency of the output gap with the *reference dates of business cycle* (released by the Cabinet Office) and also examine the leads-and-lags relationships with various DIs (e.g., the business conditions DI) in *Short-Term Economic Survey of Enterprises in Japan* (the *Tankan*, published by the Bank of Japan). A key finding is that the new output gap is more consistent with the *reference dates of business cycle* and *Tankan* DIs than is the conventional output gap. In particular, the new output gap successfully traces the recovery of the Japanese economy that started from 1999. Second, we estimate Phillips curves and evaluate the performance of the output gap focusing on the stability of the parameters and the accuracy of predictions. A key finding is that the new output gap works better especially when considering parameter stability.

In this paper, we discuss various issues in estimating the output gap when using the production-function approach. In Section II, we summarize theoretically how measurement errors in data affect estimates of potential output and the output gap. We conclude that choice of an estimation method depends on whether there are measurement errors in factor utilization or quality changes in production factors. In Section III, we explain the conventional procedure for estimating the output gap and summarize its defects. In Section IV, we devise another method to estimate the output gap and clarify its differences from the conventional measure. Comparing the conventional and the new output gap, we find that the two series have moved quite differently in recent years. This suggests that it is very important to investigate sources of measurement error when estimating the output gap. In Section V, we evaluate the conventional and the new ways to measure the output gap with a focus on consistency with various business cycle indicators and its usefulness when estimating Phillips curves. In Section VI, we investigate how revision and accumulation of GDP statistics affect estimation of the output gap.[6] In Appendix 1, we list source data used in estimating the output gap. In Appendix 2, we justify use of a Cobb-Douglas function for the Japanese aggregate production function. In Appendix 3, we discuss our choice of a trend in the Solow residual during the asset-bubble period. In Appendix 4, we discuss the use of *market value of capital stock* that is calculated by applying depreciation rates that are consistent with prices in secondhand markets instead of gross capital stock.[7] In Appendix 5, we consider an ideal definition of labor share to calculate contributions of capital and labor.

II. Measurement Error and the Output Gap

In this section, we explain how the output gap is estimated with an aggregate production function and investigate effects of measurement error on the estimation theoretically. First, we present a basic process for estimating the output gap. Next, we analyze how the output gap is affected by measurement errors in capacity utilization, quality changes in capital stock, labor hoarding, quality changes in labor force, measurement errors in a labor share, and revision of GDP statistics. See Table 1 for detailed results of the analysis.

A. Basic Procedure for Estimating the Output Gap

In this paper, we assume a Cobb-Douglas production function that uses capital and labor as production factors. (See Appendix 2 for justification of the use of a Cobb-Douglas production function.) That is,

$$Y = A \cdot L^{\alpha} \cdot (\gamma \cdot K)^{1-\alpha},$$

6. For effects of revision of GDP statistics on the output gap in the United States, see Orphanides and van Norden (1999).
7. We can estimate depreciation rates of capital from secondhand-market prices. A market value of capital stock is calculated, based on these depreciation rates. See Masuda (2000) for details.

Table 1 Effects of Measurement Error

Measurement error	Effects on TFP	Effects on potential output	Effects on output gap
Capacity utilization is overestimated (underestimated).	TFP is underestimated (overestimated).	Potential output is underestimated (overestimated) by underestimation (overestimation) of TFP.	Output gap is underestimated (overestimated) by underestimation (overestimation) of potential output.
True TFP is a linear trend.	True TFP can be extracted by fitting a linear trend to the Solow residual.	Potential output is estimated accurately, since TFP is extracted accurately.	Output gap is estimated accurately since potential output is accurate.
Capital stock is overestimated (underestimated).	TFP is underestimated (overestimated).	Potential output is estimated accurately, since overestimation (underestimation) effects of capital stock are netted out by underestimation (overestimation) effects of TFP.	Output gap is estimated accurately since potential output is accurate.
True TFP is a linear trend.	True TFP can be extracted by fitting a linear trend to the Solow residual.	Potential output is overestimated (underestimated), since a decrease (increase) in the Solow residual from its linear trend is taken to be a decrease (increase) in capacity utilization.	Output gap is overestimated (underestimated) by overestimation (underestimation) of potential output.
Labor utilization is overestimated (underestimated). (e.g., labor hoarding)	TFP is underestimated (overestimated).	Potential output is underestimated (overestimated) by underestimation (overestimation) of TFP.	Output gap is underestimated (overestimated) by underestimation (overestimation) of potential output.
True TFP is a linear trend.	True TFP can be extracted by fitting a linear trend to the Solow residual.	Potential output is estimated accurately, since TFP is extracted accurately.	Output gap is estimated accurately since potential output is accurate.
Labor input is overestimated (underestimated). (e.g., deterioration of labor quality)	TFP is underestimated (overestimated).	Potential output gap is estimated accurately, since overestimation (underestimation) effects of maximum labor input are netted out by underestimation (overestimation) effects of TFP.	Output gap is estimated accurately since potential output is accurate.
True TFP is a linear trend.	True TFP can be extracted by fitting a linear trend to the Solow residual.	Potential output is overestimated (underestimated), since a decrease (increase) in the Solow residual from its linear trend is taken to be a decrease (increase) in capacity utilization.	Output gap is overestimated (underestimated) by overestimation (underestimation) of potential output.
The share of labor is overestimated (underestimated).	TFP is estimated with error. The Solow residual deviates from true TFP due to an upward trend of capital intensity.	Potential output is estimated with error if production factors are underutilized.	Output gap is estimated with error if potential output is estimated with error.
True TFP is a linear trend.	When capital intensity has an upward trend, true TFP cannot be extracted even by fitting a linear trend to the Solow residual.	Potential output is estimated accurately by taking a linear trend of the Solow residual (a sum of trends of true TFP and capital intensity) to be TFP.	Output gap is estimated accurately since potential output is accurate.
GDP statistics are revised downward (upward).	TFP is overestimated (underestimated) by a difference between preliminary and final estimates of GDP.	Potential output is overestimated (underestimated) by overestimation (underestimation) of TFP.	Output gap is estimated accurately since both potential output and actual output are overestimated (underestimated).
A linear trend has to be fitted to the Solow residual. A difference between preliminary and final estimates has zero mean.	TFP estimated from preliminary GDP coincides with that estimated from final GDP.	Potential output estimated from preliminary GDP coincides with that estimated from final GDP.	Output gap estimated from preliminary GDP underestimates (overestimates) that estimated from final GDP.

Note: Overestimation (underestimation) of the output gap means that its absolute value is overestimated (underestimated).

where Y is real GDP, A is TFP, L is labor inputs, K is capital stock, and γ is capacity utilization. The α is labor elasticity of production and coincides with a labor share if factor markets are competitive. Taking the logarithm of both sides of the equation, we obtain

$$\ln Y = \ln A + \alpha \ln L + (1 - \alpha)\ln(\gamma \cdot K). \tag{1}$$

The first term on the right-hand side of the equation is the contribution of TFP, the second is that of labor, and the third is that of capital. The remainder after the second and third terms are subtracted from the left-hand side of the equation is called the *Solow residual*, which coincides with TFP if Y, L, K, γ, and α are measured accurately.

Potential output (Y^*) obtains by replacing L with its maximum level (L^*) and γ with its maximum level (100 percent), while TFP is taken as given.

$$\ln Y^* = \ln A + \alpha \ln L^* + (1 - \alpha)\ln K.$$

Output gap (G) is defined as a discrepancy between actual and potential output. That is,

$$G = (Y - Y^*)/Y^* \cong \ln Y - \ln Y^* = \alpha(\ln L - \ln L^*) + (1 - \alpha)\ln\gamma.$$

Note that the output gap always takes a negative value. Its absolute value decreases as contributions of labor and capital approach their maximum levels. (Below, the output gap refers to its absolute value.) In this section, for ease of exposition, we use the log-approximation of the output gap in the third position, whereas in later sections we use its fractional expression in the second position.

B. Measurement Errors in Capacity Utilization

1. Effects of taking the Solow residual to be TFP

Data on Y, L, K, γ, and α are often subject to measurement error. Here we first discuss the effects of measurement errors in capacity utilization on estimation of the output gap. Suppose that capacity utilization is given by $\bar{\gamma}$ with measurement error. Then the Solow residual ($\bar{A}$) is

$$\ln\bar{A} = \ln Y - \alpha \ln L - (1 - \alpha)\ln(\bar{\gamma} \cdot K).$$

Substituting equation (1), we obtain

$$\ln\bar{A} = \ln A + (1 - \alpha)(\ln\gamma - \ln\bar{\gamma}). \tag{2}$$

If capacity utilization contains positive measurement errors ($\bar{\gamma} > \gamma$), we underestimate TFP if we take the Solow residual to be TFP.

Based on this Solow residual, we can derive the potential output as

$$\bar{Y}^* = \ln\bar{A} + \alpha \ln L^* + (1-\alpha)\ln K = Y^* + (1-\alpha)(\ln\gamma - \ln\bar{\gamma}).$$

It is observed that potential output is underestimated by the underestimation of TFP. Furthermore, the output gap is given by

$$\begin{aligned}\bar{G} &= \{\ln\bar{A} + \alpha\ln L + (1-\alpha)\ln(\bar{\gamma}\cdot K)\} - \{\ln\bar{A} + \alpha\ln L^* + (1-\alpha)\ln K\} \\ &= \alpha(\ln L - \ln L^*) + (1-\alpha)\ln\bar{\gamma} = G + (1-\alpha)(\ln\bar{\gamma} - \ln\gamma).\end{aligned}$$

Therefore, if capacity utilization is overestimated ($\bar{\gamma} > \gamma$), the estimated output gap (in absolute value) is smaller than the real output gap. This result is almost trivial when we remember that overestimation of capacity utilization means overestimation of contribution of capital services.

2. TFP following a linear trend

Even when a contribution of capital services creeps into the Solow residual, we can accurately estimate the output gap with knowledge of the behavior of true TFP.[8] For instance, suppose that true TFP grows at a constant rate. Then

$$\ln A = \beta_1 + \beta_2 \cdot t, \tag{3}$$

where we assume that behavior of TFP is governed by technological progress, which follows a stable growth path; on the other hand, contributions of labor and capital show cyclical behavior. Substituting equation (3) into equation (2), we obtain

$$\ln\bar{A} = \beta_1 + \beta_2 \cdot t + (1-\alpha)(\ln\gamma - \ln\bar{\gamma}).$$

If $(\ln\gamma - \ln\bar{\gamma})$ has zero mean, we can estimate $\ln A$ by regressing $\ln\bar{A}$ on time trend t.[9] Based on this TFP, we can estimate the output gap as follows.

$$\begin{aligned}\bar{G} &= \{\ln\bar{A} + \alpha\ln L + (1-\alpha)\ln(\bar{\gamma}\cdot K)\} - \{\ln A + \alpha\ln L^* + (1-\alpha)\ln K\} \\ &= \alpha(\ln L - \ln L^*) + (1-\alpha)\ln\gamma = G.\end{aligned}$$

Thus, as far as $(\ln\gamma - \ln\bar{\gamma})$ has zero mean and TFP follows a linear trend, we can accurately estimate TFP and thus the output gap. These favorable results are attributed to correctly identifying both the overestimation of the capital contribution and the underestimation of the TFP contribution.

8. There is no direct way of observing true TFP. Modeling TFP requires concrete knowledge of driving forces behind TFP. It is often assumed that TFP moves together with technological progress and its growth rate is almost constant. In this case, TFP is modeled as a linear trend. TFP moves, however, due to reasons other than just technological progress. Thus, the assumption of a linear trend is always subject to specification error.
9. When data on capital utilization have an upward bias, $(1-\alpha)(\ln\gamma - \ln\bar{\gamma})$ has a negative mean. Thus, when regressing $\ln\bar{A}$ on time trend t by using ordinary least squares, we underestimate β_1. This leads to an underestimation of potential output and thus to an underestimation of the output gap. Although this decreases the absolute level of the output gap, the variations are unaffected.

3. TFP not following a linear trend

Next, we assume that TFP does not follow a linear trend. That is,

$$\ln A = \lambda_1 + \lambda_2 \cdot t + M,$$

where M represents a remainder that cannot be explained by a linear trend and can be time-variant. Substituting this into equation (2), we obtain

$$\ln A = \lambda_1 + \lambda_2 \cdot t + M + (1 - \alpha)(\ln\gamma - \ln\bar{\gamma}).$$

Hence, if regressing $\ln\bar{A}$ on time trend t, we extract the wrong TFP ($\ln\tilde{A}$). That is,

$$\ln\tilde{A} = \lambda_1 + \lambda_2 \cdot t.$$

Based on this incorrect TFP, we obtain the output gap.

$$\begin{aligned}\tilde{G} &= \{\ln\bar{A} + \alpha\ln L + (1 - \alpha)\ln(\bar{\gamma}\cdot K)\} - \{\ln\tilde{A} + \alpha\ln L^* + (1 - \alpha)\ln K\}\\ &= \alpha(\ln L - \ln L^*) + (1 - \alpha)\ln\gamma + M = G + M.\end{aligned}$$

Suppose true TFP falls below a linear trend by M ($M < 0$). In this case, a linear trend overestimates TFP, which leads to overestimation of potential output by the same amount. Thus, the estimated output gap is wider (in absolute value) than the true output gap. Put differently, when true TFP falls below a linear trend, we misunderstand that data miss declines in capacity utilization, and overestimate the output gap.

C. Quality Changes in Capital Stock

Next, we assume that statistics capture quality changes in capital stock insufficiently and then investigate those effects on estimates of the output gap.[10] (1) Capital stock is overestimated when data on capital stock take into consideration *scrap* but not *depreciation*.[11] (2) Capital stock is overestimated if capital stock is deteriorated and outdated. And (3) capital stock is underestimated if R&D is not counted as investment.

Suppose that capital stock is given by $\bar{K}$ without taking quality changes into consideration. Then the Solow residual is

$$\ln\bar{A} = \ln Y - \alpha\ln L - (1 - \alpha)\ln(\gamma\cdot\bar{K}).$$

Substituting equation (1), we obtain

$$\ln\bar{A} = \ln A + (1 - \alpha)(\ln K - \ln\bar{K}).$$

10. When technological progress is entirely embodied in capital stock, the assumption that TFP progresses at a constant rate is meaningless. In this case, technological progress induces a decline in prices of capital goods, and thereby capital stock in the real term increases.

11. See Masuda (2000) for basic concepts and properties of capital stock data in Japan.

Suppose capital stock is overestimated ($\bar{K} > K$). By taking the Solow residual to be TFP, we underestimate TFP.

Based on this Solow residual, we can estimate potential output as

$$\bar{Y}^* = \ln\bar{A} + \alpha\ln L^* + (1-\alpha)\ln\bar{K} = Y^*.$$

Thus, potential output is estimated accurately even if TFP is wrongly estimated. This is because overestimation of production capability due to the overestimation of capital stock is netted out precisely by the underestimation of production capability due to the underestimation of TFP.

Since potential output is estimated accurately, so is the output gap. That is,

$$\bar{G} = \{\ln\bar{A} + \alpha\ln L + (1-\alpha)\ln(\gamma\cdot\bar{K})\} - \{\ln\bar{A} + \alpha\ln L^* + (1-\alpha)\ln\bar{K}\}$$
$$= \alpha(\ln L - \ln L^*) + (1-\alpha)\ln\gamma = G.$$

As before, if TFP follows a linear trend, it may be extracted accurately by regressing the Solow residual on a time trend. Quality change in capital stock, however, is irreversible. Once quality is lost, its effect lasts for a relatively long time and the Solow residual is likely to depart from a linear trend. As discussed before, when the Solow residual deviates from its linear trend it is mistaken for declines in capacity utilization. Thus, if we fit a linear trend to the Solow residual, potential output is overestimated by the deviation and the output gap is overestimated (in absolute value) as well.

D. Labor Hoarding

At the beginning of a recession, companies often avoid big layoffs. Furthermore, regular working hours are not reduced significantly. Unreported overtime working hours are reduced and labor hoarding emerges, however. Thus, reported labor input overestimates true labor inputs.

Suppose that labor input is reported to be $\bar{L}$ with measurement errors. Then the Solow residual ($\bar{A}$) is

$$\ln\bar{A} = \ln Y - \alpha\ln\bar{L} - (1-\alpha)\ln(\gamma\cdot K).$$

Substituting equation (1), we obtain

$$\ln\bar{A} = \ln A + \alpha(\ln L - \ln\bar{L}).$$

Suppose that the labor input contains positive measurement errors ($\bar{L} > L$). Then TFP is underestimated if the Solow residual is taken to be TFP.

Based on this Solow residual, we estimate potential output as

$$\bar{Y}^* = \ln\bar{A} + \alpha\ln L^* + (1-\alpha)\ln K = Y^* + \alpha(\ln L - \ln\bar{L}).$$

That is, potential output is underestimated by the underestimation of TFP. The output gap is estimated as

$$\bar{G} = \{\ln\bar{A} + \alpha\ln\bar{L} + (1-\alpha)\ln(\gamma\cdot K)\} - \{\ln\bar{A} + \alpha\ln L^* + (1-\alpha)\ln K\}$$
$$= \alpha(\ln\bar{L} - \ln L^*) + (1-\alpha)\ln\gamma = G + \alpha(\ln\bar{L} - \ln L).$$

Thus, if labor input is overestimated ($\bar{L} > L$), the output gap is underestimated (in absolute value). Note that overestimation of labor input implies overestimation of labor contribution. Thus, trivially the output gap is underestimated by that amount.

Suppose that TFP follows a linear trend. Then, by regressing the Solow residual on a time trend, the decline in the true labor amount is separated from movements of TFP. This allows us to correctly estimate TFP, potential output, and the output gap. On the other hand, unless TFP follows a linear trend, all estimates suffer from error to the extent that TFP departs from its linear trend.

Labor hoarding is treated in the same way as measurement errors in capacity utilization. Let η be a ratio of actual labor input to total labor input available in an economy (i.e., $L = \eta\cdot L^*$). We interpret this as labor-force utilization. Labor hoarding is an overestimation of η.

E. Quality Changes in the Labor Force

As educational levels advance, the same amount of labor input produces a larger output. When unemployment lasts for a long time, labor skills are lost and output is reduced. These are examples of variations in the quality of the labor force.

Consider deterioration of labor quality. Suppose that labor input is wrongly reported to be $\bar{L}$, since it is not measured in efficiency units. The Solow residual ($\bar{A}$) is

$$\ln\bar{A} = \ln Y - \alpha\ln\bar{L} - (1-\alpha)\ln(\gamma\cdot K).$$

Substituting equation (1), we obtain

$$\ln\bar{A} = \ln A + \alpha(\ln L - \ln\bar{L}).$$

When deterioration occurs in the labor force ($\bar{L} > L$), TFP is underestimated if we take the Solow residual to be TFP.

Next, we estimate potential output, based on this Solow residual. Note that when labor deteriorates in quality, the maximum amount of labor services decreases simultaneously. Let the maximum amount of labor be L^* in efficiency units and $\bar{L}^*$ in hours. Then we obtain the relationships $L = \eta\cdot L^*$ and $\bar{L} = \eta\cdot\bar{L}^*$, where η is labor-force utilization as previously defined. Therefore, potential output is

$$\bar{Y}^* = \ln\bar{A} + \alpha\ln\bar{L}^* + (1-\alpha)\ln K = Y^*,$$

where labor-force utilization is untouched even if labor quality varies. Thus, potential output is estimated accurately. This is because the overestimation of production capability due to the overestimation of labor is netted out by the underestimation of production capability due to the underestimation of TFP. Since potential output is estimated correctly, the output gap is also estimated accurately. That is,

$$\bar{G} = \{\ln\bar{A} + \alpha\ln\bar{L} + (1-\alpha)\ln(\gamma\cdot K)\} - \{\ln\bar{A} + \alpha\ln\bar{L}^* + (1-\alpha)\ln K\}$$
$$= \alpha(\ln\bar{L} - \ln\bar{L}^*) + (1-\alpha)\ln\gamma = G.$$

Here again, even when labor input experiences qualitative change, labor-force utilization is unchanged.

If $(\ln L - \ln\bar{L})$ averages zero and TFP follows a linear trend, we can estimate TFP correctly by regressing the Solow residual on a time trend. But as is the case with quality changes in capital stock, when TFP drops below a linear trend, the deviation is taken wrongly to be a decline in capacity utilization; potential output is overestimated by that amount and the output gap is overestimated as well.

F. Measurement Errors in Labor Share

Next, we investigate the effects of measurement errors in the labor share on estimates of the output gap. Suppose that a wrong labor share is given by $\bar{\alpha}$. The Solow residual $(\bar{A})$ is

$$\ln\bar{A} = \ln Y - \bar{\alpha}\ln L - (1-\bar{\alpha})\ln(\gamma\cdot K).$$

Substituting equation (1), we obtain

$$\ln\bar{A} = \ln A + (\bar{\alpha}-\alpha)\ln(K/L^*) - (\bar{\alpha}-\alpha)(\ln L - \ln L^*) + (\bar{\alpha}-\alpha)\ln\gamma. \quad (4)$$

When the labor share contains measurement errors, TFP also suffers from measurement errors if we take the Solow residual to be TFP. Moreover, in many countries including Japan, the capital equipment ratio (K/L^*) follows an upward trend. In this case, over time, the Solow residual departs from TFP. The direction of the departure varies, however.[12]

By taking the Solow residual to be TFP, we estimate potential output as

$$\bar{Y}^* = \ln\bar{A} + \bar{\alpha}\ln L^* + (1-\bar{\alpha})\ln K = Y^* - (\bar{\alpha}-\alpha)(\ln L - \ln L^*) + (\bar{\alpha}-\alpha)\ln\gamma.$$

Therefore, unless both capital and labor are fully utilized, potential output is wrongly estimated. In this case, the output gap is given by

$$\bar{G} = \{\ln\bar{A} + \bar{\alpha}\ln L + (1-\bar{\alpha})\ln(\gamma\cdot K)\} - \{\ln\bar{A} + \bar{\alpha}\ln L^* + (1-\bar{\alpha})\ln K\}$$
$$= \bar{\alpha}(\ln L - \ln L^*) + (1-\bar{\alpha})\ln\gamma = G + (\bar{\alpha}-\alpha)(\ln L - \ln L^*) - (\bar{\alpha}-\alpha)\ln\gamma.$$

Thus, if potential output is underestimated, the output gap is underestimated (in absolute value) as well.

12. The direction of the departure of the Solow residual from TFP depends on the units used to measure capital stock and working hours. We thank Masakazu Inada (Research and Statistics Department, the Bank of Japan) for drawing our attention to this important point.

If TFP follows a linear trend that grows at a constant rate, we have

$$\ln A = \beta_1 + \beta_2 \cdot t.$$

It should be noted that the capital equipment ratio also has a trend as discussed above. That is,

$$\ln(K/L^*) = \delta_1 + \delta_2 \cdot t.$$

Substituting these into equation (4), we obtain

$$\ln\bar{A} = \{\beta_1 + (\bar{\alpha} - \alpha)\delta_1\} + \{\beta_2 + (\bar{\alpha} - \alpha)\delta_2\} \cdot t - (\bar{\alpha} - \alpha)(\ln L - \ln L^*) + (\bar{\alpha} - \alpha)\ln\gamma.$$

Therefore, regressing $\ln\bar{A}$ on time trend t, we obtain a combined trend of A and $\ln(K/L^*)$, but cannot extract TFP separately.[13] That is, we obtain

$$\ln\tilde{A} = \{\beta_1 + (\bar{\alpha} - \alpha)\delta_1\} + \{\beta_2 + (\bar{\alpha} - \alpha)\delta_2\} \cdot t.$$

Based on this wrong TFP, we can estimate potential output as

$$\tilde{Y}^* = \ln\tilde{A} + \bar{\alpha}\ln L^* + (1 - \bar{\alpha})\ln K = Y^*.$$

Thus, even though TFP cannot be estimated separately, we can estimate potential output accurately. We can also estimate the output gap accurately

$$\begin{aligned}\bar{G} &= \{\ln\bar{A} + \bar{\alpha}\ln L + (1 - \bar{\alpha})\ln(\gamma \cdot K)\} - \{\ln\tilde{A} + \bar{\alpha}\ln L^* + (1 - \bar{\alpha})\ln K\} \\ &= -(\bar{\alpha} - \alpha)(\ln L - \ln L^*) + (\bar{\alpha} - \alpha)\ln\gamma + \bar{\alpha}(\ln L - \ln L^*) + (1 - \bar{\alpha})\ln\gamma = G.\end{aligned}$$

G. Revision of GDP Statistics

Finally, we discuss effects of revisions of GDP statistics on estimation of the output gap. We revise GDP estimates four times. Thus, there are five different figures for the same-period GDP: the *first preliminary quarterly estimate* (the *first QE*), the *second preliminary quarterly estimate* (the *second QE*), *final estimate*, *annual revision*, and *benchmark revision*. Since most source statistics are replaced when the second QE is revised for the final estimate, large discrepancies may occur between the output gap that is based on the preliminary GDP estimate and that based on the final estimate. Let the preliminary GDP estimate be given by $\bar{Y}$. Then the Solow residual ($\bar{A}$) is

$$\ln\bar{A} = \ln\bar{Y} - \alpha\ln L - (1 - \alpha)\ln(\gamma \cdot K).$$

When the final GDP estimate is published, we find

$$\ln A = \ln Y - \alpha\ln L - (1 - \alpha)\ln(\gamma \cdot K).$$

13. If labor utilization $\eta = L/L^*$ is equal to capacity utilization γ on average, $-(\bar{\alpha} - \alpha)(\ln L - \ln L^*) + (\bar{\alpha} - \alpha)\ln\gamma$ moves around zero and thus is almost equal to the regression residual obtained by regressing the Solow residual on a linear trend.

Then

$$\ln\bar{A} = \ln A + (\ln\bar{Y} - \ln Y).$$

This implies that the Solow residual absorbs the entire discrepancy that exists between the preliminary GDP estimate and the final estimate (the *preliminary-final-estimate discrepancy*). Suppose that the preliminary-final-estimate discrepancy is positive. By taking the Solow residual to be TFP, we overestimate TFP by the same amount.

When we estimate potential output based on the Solow residual that is obtained from the preliminary GDP estimate, we overestimate potential output by the preliminary-final-estimate discrepancy. That is,

$$\bar{Y}^* = \ln\bar{A} + \alpha\ln L^* + (1-\alpha)\ln K = Y^* + (\ln\bar{Y} - \ln Y).$$

The output gap is given by

$$\begin{aligned}\bar{G} &= \{\ln\bar{A} + \alpha\ln L + (1-\alpha)\ln(\gamma\cdot K)\} - \{\ln\bar{A} + \alpha\ln L^* + (1-\alpha)\ln K\}\\ &= \{\ln A + \alpha\ln L + (1-\alpha)\ln(\gamma\cdot K)\} - \{\ln A + \alpha\ln L^* + (1-\alpha)\ln K\} = G.\end{aligned}$$

This shows that the preliminary-final-estimate discrepancy causes no distortion when estimating the output gap.

Revision of GDP statistics causes a problem in estimating the output gap when we have to regress the Solow residual on a time trend to estimate TFP because of measurement errors in factor utilization. To separate TFP from the Solow residual, we regress $\ln\bar{A}$ on a time trend with publication of the preliminary GDP estimate, while we regress $\ln A$ with publication of the final estimate. If the preliminary-final-estimate discrepancy ($\ln\bar{Y} - \ln Y$) has zero mean, the two estimated trends coincide. Now suppose that the preliminary-final-estimate discrepancy ($\ln\bar{Y} - \ln Y$) is positive during a certain period. The regression residual of $\ln\bar{A}$ is larger by the preliminary-final-estimate discrepancy than the regression residual of $\ln A$. This implies that capacity utilization was overestimated before revision, and thus the output gap is underestimated by the same amount (in absolute value). As mentioned, the comments on the output gap are all in absolute value.

H. Summary

If statistics contain measurement errors, two kinds of errors may be committed. First, if we take the Solow residual to be TFP while overestimating factor utilization, we may think of it as declines in production capability. In addition, we may be wrong in choosing the appropriate trend to estimate TFP. In particular, we may mistake quality changes in production factors for changes in factor utilization. See Table 1 for a more comprehensive list of effects on TFP, potential output, and the output gap. Note again that comments regarding the output gap are all in absolute value.

(1) Declines in capacity utilization or labor hoarding mean that an economy suffers from slackness. If input data are subject to overestimation, we underestimate TFP, potential output, and the output gap by taking the Solow residual to be TFP.

(2) If TFP follows a linear trend, we can estimate TFP correctly by regressing the Solow residual on a time trend. Thus, we can obtain the correct figures for potential output and the output gap.
(3) If TFP falls below a linear trend, we overestimate TFP by the amount of the regression residual obtained in fitting a linear trend to the Solow residual. Thus, potential output is overestimated, which leads to overestimation of the output gap.

On the other hand,

(4) When quality changes in capital stock or in the labor force, the potential as well as the actual size of an economy shrinks. If we use the Solow residual as TFP without taking into account quality deterioration, TFP is underestimated. Nonetheless, potential output and the output gap are estimated accurately.
(5) If TFP follows a linear trend, we can estimate TFP correctly by regressing the Solow residual on a linear trend. Nevertheless, we will mistake quality declines in production factors for decreases in capacity utilization by the amount of the regression residual, and thus we overestimate potential output and the output gap.

Furthermore,

(6) If the labor share contains measurement errors, there will be estimation errors in TFP, potential output, and the output gap. In particular, when the capital equipment ratio grows on an upward trend, the Solow residual departs from TFP.
(7) If TFP follows a linear trend, we extract a combined trend for TFP and the capital equipment ratio. Nonetheless, we can accurately estimate potential output and the output gap.

Finally,

(8) Suppose GDP statistics are revised downward (a positive preliminary-final-estimate discrepancy). This implies that by taking the Solow residual to be TFP, the TFP and potential output estimated from preliminary GDP are greater than those estimated from final GDP by the amount of the revision.
(9) Revision of GDP statistics affects the output gap only when we regress the Solow residual on a linear trend. In this case, the output gap estimated from preliminary GDP is smaller than the output gap estimated from final GDP by the amount of the preliminary-final-estimate discrepancy.

III. Conventional Output Gap

In this section, we present estimates of the output gap that are based on the discussion in the previous section. In Japan, there are no statistics that capture capacity utilization in non-manufacturing sectors. Thus, usually, capacity utilization in non-manufacturing sectors is fixed at 100 percent. In this case, it is obvious that the effects of capacity utilization in non-manufacturing sectors creep into the Solow residual. Thus, to estimate TFP we usually fit a linear trend to the Solow residual. Then we estimate potential output as real GDP by assuming full utilization of labor and capital, with TFP as a given. The output gap is the deviation rate of actual output from potential output. This is the classic standard method to estimate the output gap, and in this paper we call the output gap thus estimated the conventional output gap. See Appendix 1 for a list of source data that are used and generated in this paper.

A. Deriving the Solow Residual

We assume the following Cobb-Douglas type of aggregate production function:[14]

$$Y_t = \bar{A}_t \cdot L_t^{\alpha} \cdot (\bar{\gamma}_t \cdot K_{t-1})^{1-\alpha},$$

where Y_t is real GDP, $\bar{A}_t$ is the Solow residual, K_{t-1} is capital stock at the end of a previous quarter, L_t is working hours, $\bar{\gamma}_t$ is capital utilization, and α is labor share. Note that we denote the Solow residual by $\bar{A}$ (not by A) and capacity utilization by $\bar{\gamma}$ (not by γ). This emphasizes that capacity utilization contains measurement errors, and thus the Solow residual does not coincide with TFP.

In estimating TFP, we need aggregate data on working hours, capital stock, capacity utilization, and labor share. We separate capital stock in manufacturing sectors from that in non-manufacturing sectors. Capacity utilization in manufacturing sectors can be obtained from *Indices of Operating Ratio (IOR)* with the historical peak normalized at 100 percent. There are no data on capacity utilization in non-manufacturing sectors like *IOR*. Hence, we assume that capacity utilization in non-manufacturing sectors is always 100 percent. That is,

$$\bar{\gamma}_t \cdot K_{t-1} = \gamma m_t \cdot KM_{t-1} + KN_{t-1},$$

where KM_{t-1} is capital stock in manufacturing sectors at the end of a previous quarter, KN_{t-1} is that in non-manufacturing sectors, and γm_t is capacity utilization in manufacturing sectors.

Working hours are calculated by multiplying working hours per capita by the number of workers. Let H_t be working hours per capita and N_t be the number of workers. Then we have $L_t = H_t \cdot N_t$. We fix labor share at its sample mean (an average from the first quarter of 1975 to the latest final-estimate quarter).

B. Applying a Linear Trend

Taking logarithms of both sides of the above production function and rearranging the result, we obtain

$$\ln\bar{A}_t = \ln Y_t - \alpha \ln L_t - (1-\alpha)\ln(\bar{\gamma}_t \cdot K_{t-1}).$$

As discussed, capacity utilization in non-manufacturing sectors is always assumed to be 100 percent. This implies that TFP is underestimated, which in turn suggests that

14. A Cobb-Douglas production function has the following properties: (1) constant returns to scale and (2) unit elasticity of factor substitution. We show in Appendix 2 that both properties are satisfied by applying a CES production function to Japan's aggregate data.

it is problematic to take the Solow residual ($\ln\bar{A}_t$) to be TFP. That is why we remove the effects of capacity utilization from the Solow residual by regressing it linearly. In doing so, we assume that Japan's TFP grew at a relatively high rate during the asset-bubble period, and thus the trend experienced kinks before and after that period. That is,

$$\ln\bar{A}_t = \beta_0 + \beta_1 \cdot t + \beta_2 \cdot \tau_t + \varepsilon_t,$$

where τ_t is the asset-bubble trend from the first quarter of 1985 to the fourth quarter of 1991 and is added to a full-sample trend. In this case, TFP is extracted as $\ln A_t = \beta_0 + \beta_1 \cdot t + \beta_2 \cdot \tau_t$.[15]

C. Estimation of Potential Output

To obtain potential output, we assume full utilization of existing capital stock and full input of available working hours. That is,

$$\ln Y_t^* = \ln A_t + \alpha \ln L_t^* + (1 - \alpha)\ln K_{t-1},$$

where Y_t^* is potential output and L_t^* is the maximum working hours. Implicitly, we raise capacity utilization in manufacturing sectors γm_t to 100 percent. Capacity utilization is fixed at 100 percent in non-manufacturing sectors.

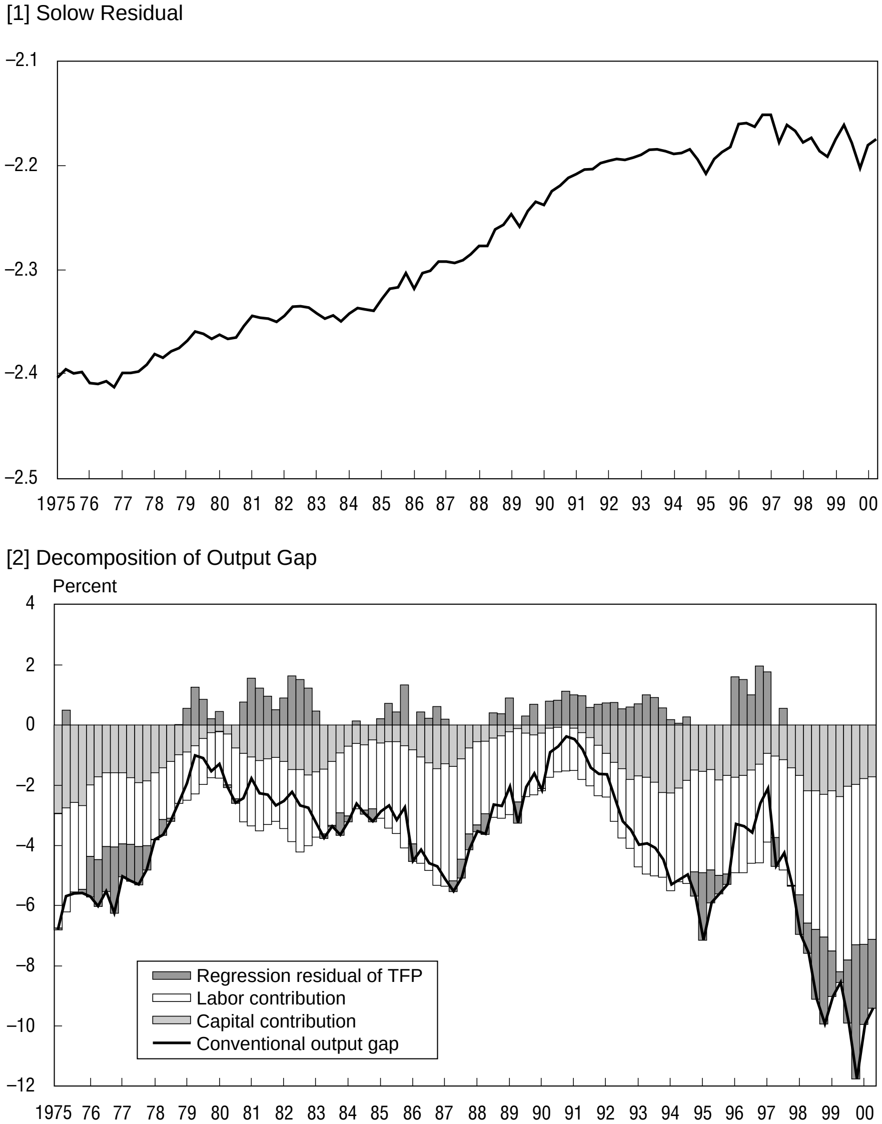
The maximum working hours are calculated as follows. First, note that the maximum working hours are given by multiplying the maximum working hours per capita by the maximum number of workers. The maximum working hours per capita are the sum of maximum scheduled working hours and maximum non-scheduled working hours per capita. The maximum scheduled working hours per capita are given by a linear trend with some kinks, taking into consideration legal restrictions on working hours. The maximum non-scheduled working hours per capita are given by the historical high. The maximum number of workers is calculated for two groups separately: the 15-to-64-year-old population and the over-64-year-old population. Participation rates for both groups are assumed to grow on trends that form ceilings of the historical time series.

D. Estimation of the Conventional Output Gap

The output gap is the rate of deviation of actual output from potential output and is given by $G_t = (Y_t - Y_t^*)/Y_t^*$. Figure 1 presents the series for the conventional output gap and its decomposition. Some comments are in order. First, since we define potential output as the largest product, the output gap always takes on a negative value. Note that by definition, our output gap is wider than that defined as the deviation of actual output from output that is obtained by the average utilization of capital and labor or output when inflation is stable.

15. The end of sample used to estimate the trend is matched with the final-estimate quarter of capital stock.

Figure 1 Conventional Output Gap



Sources: Cabinet Office, *National Accounts, Gross Capital Stock of Private Enterprises*, etc.

Second, the conventional output gap was smallest at the end of the asset-bubble period in 1991. It shrank again during the short-lived boom in 1996. Toward the end of the 1990s, the conventional output gap continued to expand rapidly and came close to –12 percent in the fourth quarter of 1999.

Third, the behavior of the conventional output gap in the latter half of 1999 was not necessarily consistent with the Bank of Japan's *Tankan*. The *Tankan* reported that business conditions began to improve and reversed in 1998 and that excess production capacity and employment diminished significantly in 1999 (Figure 2). In addition, it was determined in June 2000 that Japan's business cycle had reached its bottom at April 1999. Nevertheless, the conventional output gap continued to expand thereafter.

Thus, the conventional output gap is inconsistent with companies' perceptions of supply-demand conditions. This inconsistency is attributable to the way the conventional output gap is estimated, i.e., fitting a linear trend to the Solow residual due to the assumption of full capacity utilization in non-manufacturing sectors. In Japan, capital stock deteriorated drastically during the recent long-lasting economic slowdown. As production technology was renewed, the capital stock linked to old technology became rapidly outdated. Companies required new skills, but existing labor skills were old-fashioned. For these reasons, production capability was likely to decline further than the statistics reveal. If we fit a linear trend to the Solow residual, these declines in production capability are captured as regression residuals and are eventually thought of as declines in factor utilization. This leads to an overestimation of the output gap (in absolute value). This is observed in Figure 1's decomposition of the output gap, where the *regression residual of TFP* expanded rapidly in 1999.

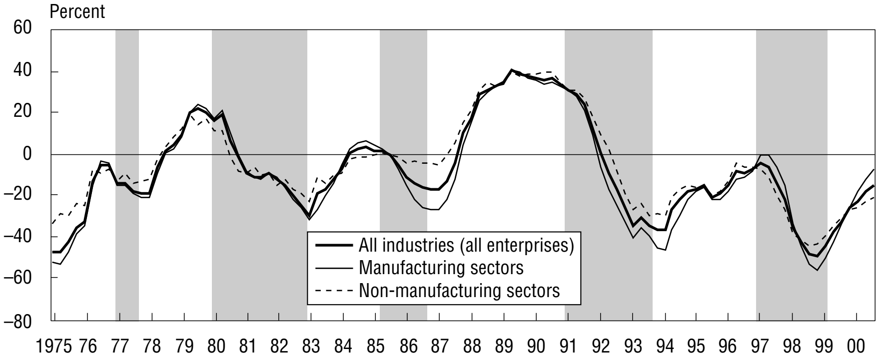
IV. New Output Gap

In estimating the conventional output gap, we fixed capacity utilization in non-manufacturing sectors at 100 percent. As a consequence, when capacity utilization actually declines in non-manufacturing sectors, we underestimate TFP and the output gap (in absolute value) if we take the Solow residual to be TFP. To avoid this error when calculating the conventional output gap, we estimate TFP by fitting a linear trend (with kinks) to the Solow residual. From this estimate of TFP, we calculate potential output and the output gap. However, the Solow residual also moves together with quality changes in capital and labor and with variations in true TFP, as well as changes in capacity utilization in non-manufacturing sectors. This gives rise to the possibility that fitting a linear trend to the Solow residual will produce inaccurate estimates of potential output and the output gap.

The theoretical discussion in Section II showed that if data on production capability contain measurement errors, we can estimate potential output and the output gap correctly by taking the Solow residual to be TFP. To do so, we have to find a way to directly estimate capacity utilization in non-manufacturing sectors. Then we can assume that the Solow residual will not be affected by capacity utilization rates and we can estimate the output gap by taking the Solow residual to be TFP. We call the output gap thus estimated the new output gap. Notice, however, that non-manufacturing sectors have a large share of capital stock. Moreover, capacity utilization affects the output gap directly. Therefore, we have to be careful when estimating capacity utilization in non-manufacturing sectors. Next, we introduce a method to estimate capacity utilization in non-manufacturing sectors that is based on consumption of commercial electric power and a production capacity indicator.

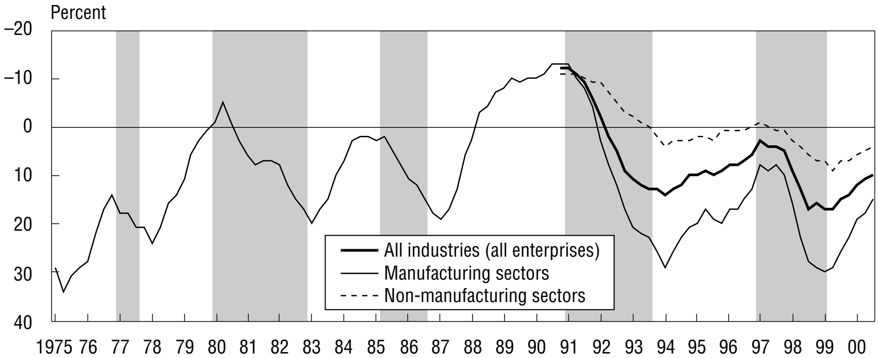
Figure 2 Supply-Demand Balance in *Tankan*

[1] Business Conditions DI



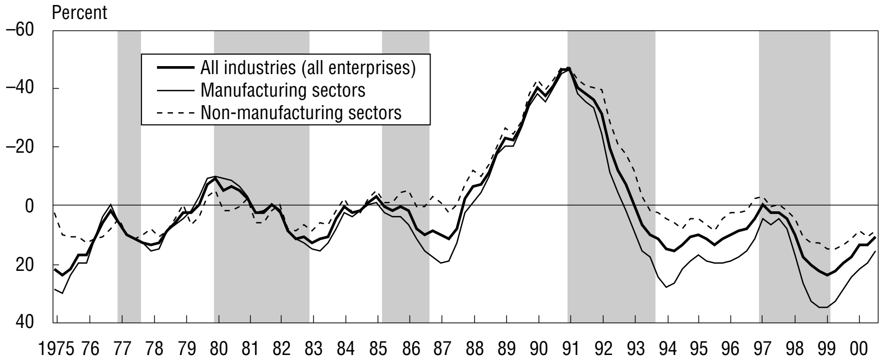
Note: "Favorable" – "unfavorable." Shadows indicate recession.

[2] Production Capacity DI



Note: "Excessive capacity" – "insufficient capacity." Shadows indicate recession.

[3] Employment Conditions DI



Note: "Excessive employment" – "insufficient employment." Shadows indicate recession.
Source: Bank of Japan, *Short-Term Economic Survey of Enterprises in Japan*.

A. Estimation of Capacity Utilization in Non-Manufacturing Sectors

Here we introduce two approaches to estimate capacity utilization in non-manufacturing sectors. In the first approach, we make use of a ratio of *electricity consumption* to *contracted electric power* in non-manufacturing sectors (*electric power units*). In the second, we use only a portion of variations in electric power units explained by a production capacity indicator of non-manufacturing sectors.

1. Estimation by electric power units

In an electric power sector, electricity consumption (kilowatt-hours) divided by contracted electric power (kilowatts) is called electric power units and is thought to be a kind of capacity utilization.[16] In fact, the behavior of electric power units for *large industrial power*, which reflects most electricity consumption in manufacturing sectors, mimics that of IOR-based capacity utilization. The series of electric power units for *commercial power*, which reflects most electricity consumption in non-manufacturing sectors, has an upward trend through the first half of the 1990s and is almost flat thereafter (Figure 3 [1]).[17] This may reflect an upward trend in electricity-consumption hours due to an extension of business hours. This suggests that we can make a more accurate proxy of capacity utilization by dividing electric power units by maximum electricity-consumption hours. That is,

$$\frac{\text{(Electric power units)}}{\left(\begin{array}{c}\text{Maximum}\\ \text{electricity-consumption hours}\end{array}\right)} = \frac{\text{(Electricity consumption)}}{\begin{array}{c}\text{(Contracted electric power)} \times\\ \text{(Maximum electricity-consumption hours)}\end{array}}$$

$$= \frac{\begin{array}{c}\text{(Actual electric power)} \times\\ \text{(Actual electricity-consumption hours)}\end{array}}{\begin{array}{c}\text{(Contracted electric power)} \times\\ \text{(Maximum electricity-consumption hours)}\end{array}} = \text{(Capacity utilization)}.$$

However, we have no data on maximum electricity-consumption hours. Instead, we detrend the series of electric power units for commercial power in non-manufacturing sectors and define capacity utilization of non-manufacturing sectors by a series of regression residuals with the peak set at 100 percent. This is equivalent to assuming that maximum electricity-consumption hours have an upward trend. To begin with, we estimate the following equation.

$$\lambda = \kappa_1 + \kappa_2 \cdot t + \varepsilon,$$

where λ is electric power units for commercial power. Capacity utilization in non-manufacturing sectors is obtained by normalizing $\mu = \kappa_1 + \varepsilon$ with the peak at 100 percent. That is,

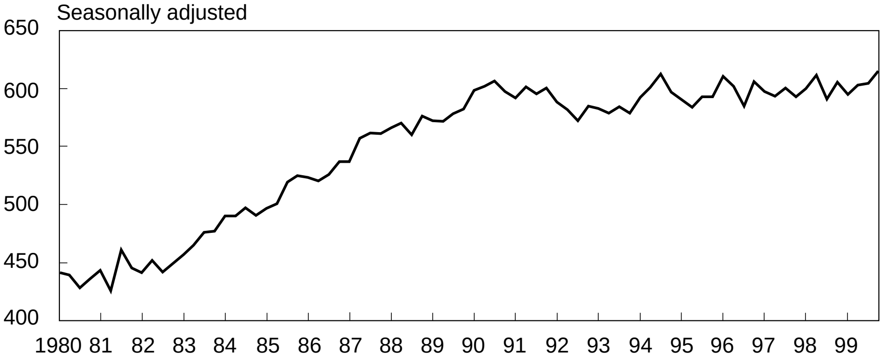
$$\gamma n = \mu / \max \mu,$$

16. In the past, contracted electric power is defined as total electric power necessary for all equipment. In fiscal 1989, the definition was changed to largest electric power ever realized. We made adjustments to the data to be able to stick to the first definition.
17. Customers of large industrial electric power include railway, newspaper, and telecommunications companies.

Figure 3 Capacity Utilization in Non-Manufacturing Sectors (I): Electricity Consumption Approach

[1] Electric Power Units for Commercial Power

Seasonally adjusted

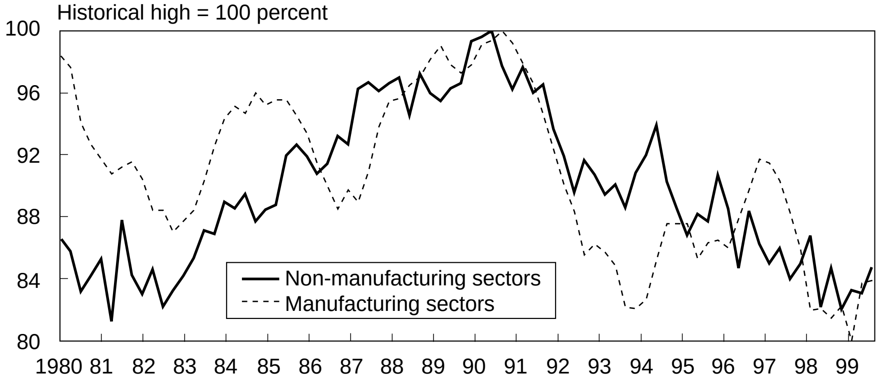


Notes: 1. Electric power units for commercial power = electricity consumption for commercial power/electric power contracted for commercial power.
2. Adjusted for measurement-unit change in fiscal 1989.
3. Seasonally adjusted excluding an outlier in 1993/III (irregular cold summer).

Sources: Ministry of Economy, Trade and Industry, *Basic Survey of Japanese Business Structure and Activities*; Federation of Electric Power Companies of Japan, *Electricity Demand*.

[2] Capacity Utilization in Non-Manufacturing Sectors from Electric Power Units for Commercial Power

Historical high = 100 percent



Estimation method: electric power units detrended and normalized with upper boundary at 100 percent.

Electric power units for commercial power: $\lambda = 2.33 \cdot t + \overbrace{455.2 + \varepsilon}^{\mu}$

Sample: 1980/I–1999/IV, estimation method: OLS

Capacity utilization in non-manufacturing sectors: $\gamma n = \mu / \max \mu$

where γn is capacity utilization in non-manufacturing sectors. Estimation results are given in Figure 3 [2]. For comparison, we also present IOR-based capacity utilization normalized with the peak at 100 percent (the fourth quarter of 1990). A glance at the figure shows that estimated capacity utilization in non-manufacturing sectors moves together with that of manufacturing sectors. The former, however, is more volatile than the latter. This suggests that electric power units for commercial power are disturbed by factors specific to electricity consumption and irrelevant to capacity utilization.

2. Estimation by production capacity BSI and electric power units

As mentioned, capacity utilization in non-manufacturing sectors estimated from electric power units for commercial power is more volatile than that in manufacturing sectors and may be disturbed by factors irrelevant to capacity utilization. Hence, we should extract the variations in electricity power units that are attributable to those in capacity utilization. To do this, we use production capacity BSI (Business Survey Index) reported in *Business Outlook Survey of the Ministry of Finance*. The BSI is a diffusion index of sufficiency of equipment (the share of firms that lack equipment net of the share of firms that have excess capacity). The BSI is similar to the production capacity DI reported in the *Tankan* by the Bank of Japan, but is more useful here since we can make use of a long sample that starts from the second quarter of 1983.[18] In Figure 4 [1], we present the production capacity BSI and the *Tankan*'s production capacity DI. We can see that the two show similar behavior, at least in the 1990s.

Here we regress electric power units for commercial power in non-manufacturing sectors both on a linear trend and on the production capacity BSI. Then we remove the linear trend and regression residuals to obtain capacity utilization in non-manufacturing sectors. We start with the following equation.

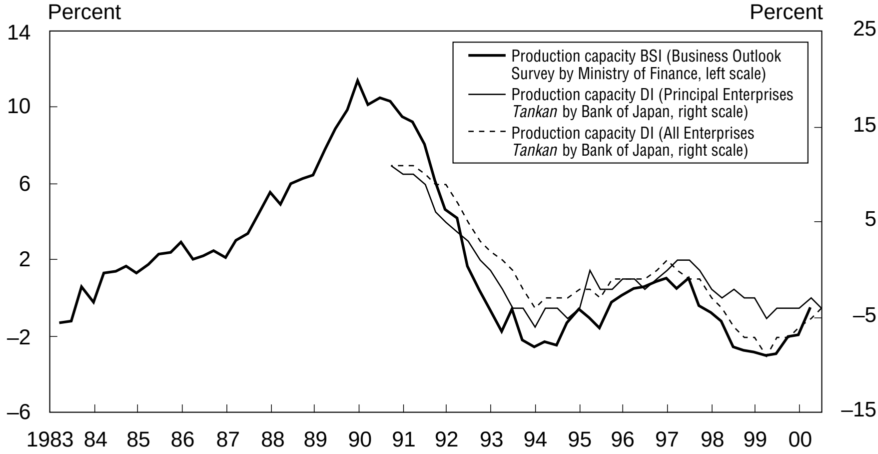
$$\lambda = \chi_1 + \chi_2 \cdot B + \chi_3 \cdot t + \varepsilon,$$

where B is production capacity BSI. Capacity utilization in non-manufacturing sectors is the remainder after extracting the trend and regression residuals, $\mu = \chi_1 + \chi_2 \cdot B$, normalized with the peak at 100 percent. As before, we present capacity utilization in manufacturing sectors and that in non-manufacturing sectors in Figure 4 [2]. We find that the two series show more similar behavior than when capacity utilization in non-manufacturing sectors was calculated only from electric power units for commercial power. Moreover, the large volatility previously observed in capacity utilization in non-manufacturing sectors is reduced and is now comparable to the volatility of capacity utilization in manufacturing sectors.

18. The *Tankan*'s production capacity DI of non-manufacturing sectors is available from 1990.

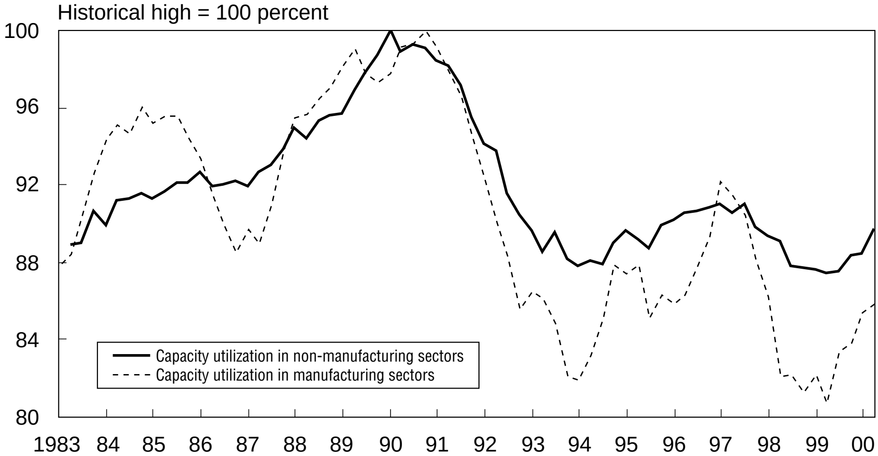
Figure 4 Capacity Utilization in Non-Manufacturing Sectors (II): Production Capacity BSI and Electric Power Units Approach

[1] Insufficient Capacity in Non-Manufacturing Sectors



Note: "Insufficient capacity" – "excessive capacity."

[2] Capacity Utilization in Non-Manufacturing Sectors from Production Capacity BSI and Electric Power Units for Commercial Power



Estimation method: electric power units detrended, net of noise, and normalized with upper boundary at 100 percent.

Electric power units for commercial power: $\lambda = \overbrace{487.3 + 4.72 \cdot B}^{\mu} + 2.14 \cdot t + \varepsilon$

Sample: 1983/II–1999/IV, estimation method: OLS

Capacity utilization in non-manufacturing sectors: $\gamma n = \mu / \max \mu$

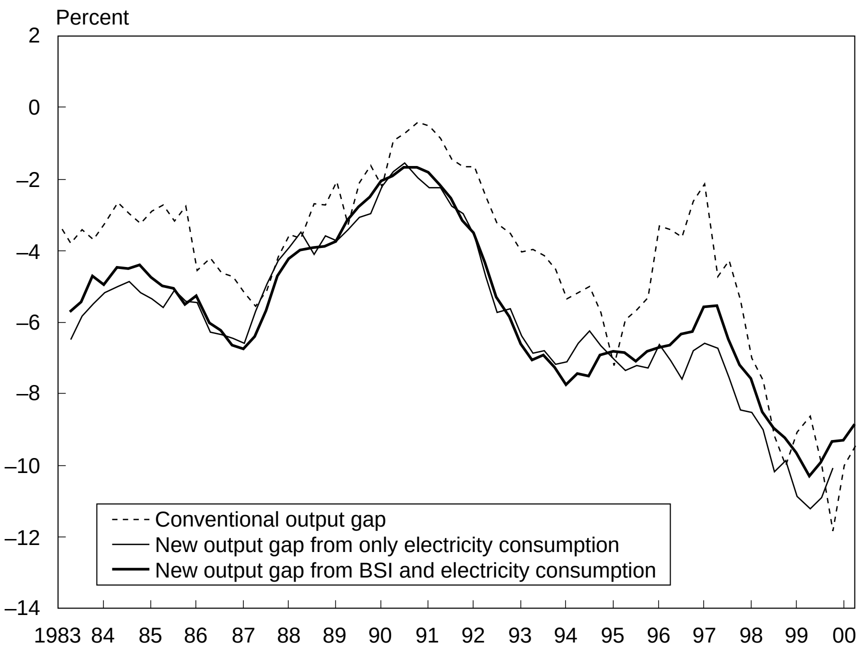
B. Estimation of the Output Gap

Now we have capacity utilization in non-manufacturing sectors. Therefore, the Solow residual is free from the effects of miscalculations of capacity utilization in non-manufacturing sectors. Consequently, we need not fit a linear trend to the Solow residual, and we can estimate the new output gap by taking the Solow residual to be TFP.[19] In Figure 5, we compare the new output gap with the conventional output gap.

The new output gap has the following properties. First, the new output gap is wider than the conventional output gap through the first half of the 1990s. This is because the conventional output gap assumes that capacity utilization in non-manufacturing sectors is always 100 percent, while the new output gap assumes that it is lower. As a result, the new output gap is wider than the conventional output gap.

The new and conventional output gap diverged from the latter half of 1998 to 1999. Especially, from 1999, the conventional output gap expanded rapidly, while the new output gap began to shrink. The behavior of the new output gap is consistent with other business cycle indicators, including *Tankan* DIs, which began to recover from the end of 1998, and the *reference dates of business cycle*, the recent bottom of which was April 1999.

Figure 5 Conventional and New Output Gap



Sources: Cabinet Office, *National Accounts*, *Gross Capital Stock of Private Enterprises, National Wealth Survey,* etc.

19. Note that in this case, as seen in the discussion in Section II, the output gap is calculated only from capacity utilization of labor and capital. Thus TFP is unnecessary to estimate the output gap.

V. Performance of the Output Gap

In this section, we compare the performances of the new and conventional output gap. As mentioned in the introduction, the main purpose of estimating the output gap is measuring social welfare and the pressure on prices. For these purposes, we evaluate the derived output gap from two perspectives. First, the output gap is useful as a measure of the aggregate demand-supply balance and as an indicator of business conditions. So we check the consistency of each output gap with the *reference dates of business cycle* and the Bank of Japan's *Tankan*. Second, the output gap is useful in measuring pressure on prices. So we use each output gap to estimate a Phillips curve and compare the goodness of fit, the stability of parameters, and the accuracy of projections. We will conclude that the new output gap has a better performance than the conventional output gap.

A. Consistency with Business Cycle Indicators

First, we check whether turning points in the output gap coincide with those of various business cycle indicators. In Figure 6 [1], we present peaks and bottoms of the *reference dates of business cycle* released by the Cabinet Office on the top row and turning points in the two series of output gaps in the two bottom rows. According to the figure, the turning points of the new output gap are closer to the peaks and bottoms of the *reference dates of business cycle* than the conventional output gap. In particular, the recent economic deterioration hit bottom in April 1999, which coincides with the bottom of the new output gap. On the other hand, the conventional output gap expanded during 1999 and is inconsistent with the *reference dates of business cycle*.

Next, we check whether turning points in the output gap coincide with those of various DIs reported in the Bank of Japan's *Tankan*. In Figure 6 [1], we present the turning points of three *Tankan* DIs (business conditions DI, production capacity DI, and employment conditions DI) in the middle rows. When we compare these with turning points in the two series of output gaps, we find that turning points in the conventional output gap do not coincide with those of the *Tankan* DIs in several cases. On the other hand, in most cases, the turning points in the new output gap coincide with turning points of at least one *Tankan* DI (shown by shadow). These suggest that with regard to turning points, the new output gap is more consistent with various business cycle indicators.

Furthermore, we calculate cross-correlation and evaluate how closely the output gap compares with other business cycle indicators. In Figure 6 [2], we plot cross-correlation curves between the output gap and *Tankan* DIs. A high peak of a curve at an origin implies high consistency of the output gap with a *Tankan* DI, while a low peak implies low consistency. When a peak is located to the left of an origin, the output gap moves behind the *Tankan* DI. Conversely, when a peak is located to the right, the output gap moves ahead. We can see that in general, the new output gap has a higher correlation with *Tankan* DIs. Furthermore, the conventional output gap tends to move behind *Tankan* DIs, while the new output gap moves ahead of the conventional output gap or simultaneously with *Tankan* DIs. For instance, the new output gap bottomed out in 1999, while the business conditions DI hit bottom at the end of 1998, and the employment

Figure 6 Correlation of Output Gap with Business Cycle Indicators

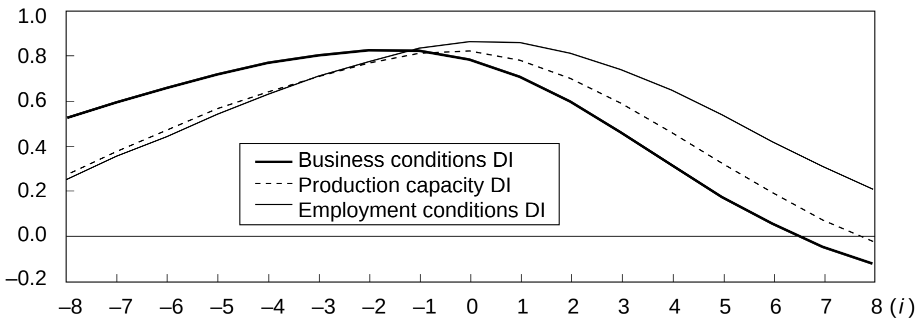
[1] Output Gap and Peaks and Bottoms of Business Cycle

	Bottom	Peak	Bottom	Peak	Bottom	Peak	Bottom
Reference dates of business cycle	83/I	85/II	86/IV	91/I	93/IV	97/I	99/II
Business conditions DI	83/I	84/IV	86/IV–87/I	89/II	93/IV–94/I	97/I	98/IV
Production capacity DI	83/I	84/III–85/II	87/I	90/III	94/I	97/I	98/III–99/II
Employment conditions DI	83/I	85/I	87/II	90/IV–91/I	94/II	97/I	99/I
Conventional output gap	—	84/II	87/II	90/IV	95/I	97/I	99/IV
New output gap	—	84/IV	87/I	90/III	94/I	97/II	99/II

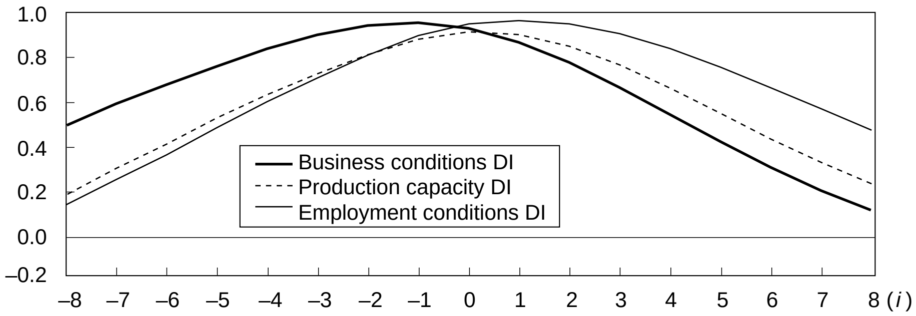
Notes: 1. Peaks and bottoms are defined around reference dates of the business cycle.
2. Shadow indicates that the peaks and bottoms of the output gap coincide with those of the DIs.
3. All Enterprises *Tankan* for business conditions DI, product capacity DI, and employment conditions DI.
4. Manufacturing industries through 1990/III and all industries thereafter for production capacity DI.

[2] Correlation of Output Gap and *Tankan* DIs

A. Conventional Output Gap



B. New Output Gap



Note: Correlation coefficient of Output Gap$_t$ and *Tankan* DI$_{t+i}$ (i = –8, –7, . . . 7, 8).

Sources: Cabinet Office, *Reference Dates of Business Cycle*; Bank of Japan, *Short-Term Economic Survey of Enterprises in Japan*.

conditions DI and the production capacity DI began to improve during 1999. In contrast, the conventional output gap remained large even in the latter half of 1999.

As seen above, the conventional output gap is less consistent with other business cycle indicators than the new output gap. A possible reason is that what the conventional output gap thinks of as variations in capacity utilization are really quality changes in capital and labor, which should not be reflected in the output gap. This is obvious from the decomposition of the conventional output gap (Figure 1). In fact, the rapid expansion of the conventional output gap at the ends of 1998 and 1999 is attributable to the expansion of the regression residual of TFP (deviations of the Solow residual from its linear trend). If we ignored the residual, the conventional output gap would have shrunk in 1999. This suggests that the recent reduction in the Solow residual is not due to a decline in capacity utilization in non-manufacturing sectors, but is attributed to a decline in productivity that results from a decline in true TFP, outdated capital stock, or deteriorated labor skills.

B. Estimation of Phillips Curves

Next, we use the conventional and new output gap to estimate Phillips curves for consumer prices and compare the goodness of fit, the stability of parameters, and the accuracy of prediction. In the estimation, we use the output gap of a previous quarter as well as that of a current quarter.

To begin, we estimate a Phillips curve by including a previous quarter's output gap as an explanatory variable. The sample starts at the third quarter of 1983 and ends at the first of 1998 (Figure 7 [1]). That is,

$$\pi_t = \alpha + \beta \cdot \pi_{t-1} + \gamma \cdot G_{t-1} + \delta \cdot m_{t\sim t-2},$$

where π is a quarter-to-quarter percent change in consumer prices per annum, and $m_{t\sim t-2}$ is a three-quarter backward moving average of quarter-to-quarter percent changes in import prices per annum.

According to the estimation result, a parameter on the new output gap is smaller than that obtained from the conventional output gap. This implies that a short-run Phillips curve is flatter when we use the new output gap than when we use the conventional output gap. In contrast, a parameter on the expected rate of inflation of consumer prices (quarter-to-quarter percent change per annum) is greater when we use the new output gap than when we use the conventional output gap. This implies that consumer prices are found to be stickier when we use the new output gap than when we use the conventional output gap. Note that whether we use the conventional output gap or the new output gap, the coefficient of determination is almost the same, implying that Phillips curves are equally fitted.

We predict future consumer price inflation by substituting the realized output gap and previously predicted consumer price inflation in the estimated Phillips curves from the second quarter of 1998 (Figure 7 [2]). We find that both Phillips curves underestimate actual inflation rates of consumer prices. We can say, however, that the underestimation is smaller when we use the new output gap than when we use the conventional output gap.

Figure 7 Phillips Curve (I)

[1] Estimation

$\pi_t = \alpha + \beta \cdot \pi_{t-1} + \gamma \cdot G_{t-1} + \delta \cdot m_{t\sim t-2}$

π_{t-1}: CPI (trend and cycle components) quarter-to-quarter percent change per annum

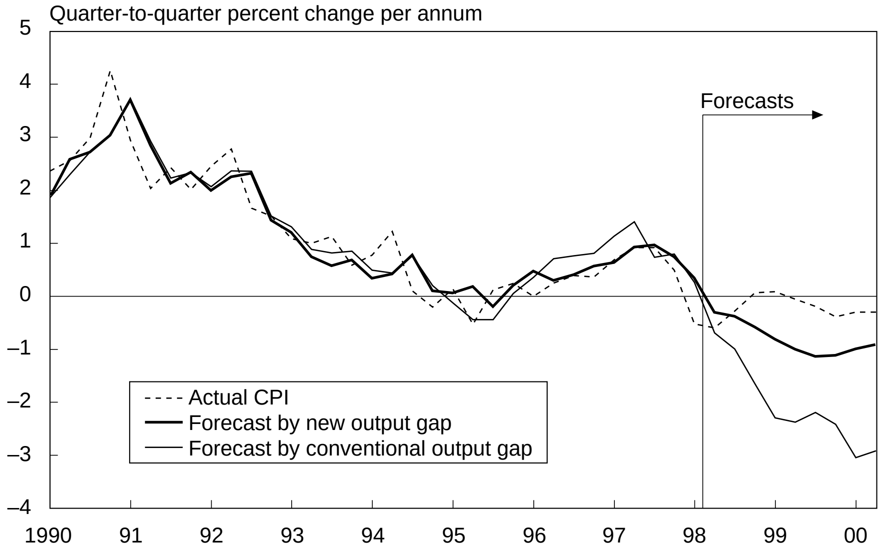
G_{t-1}: output gap in previous quarter

$m_{t\sim t-2}$: import prices (WPI, total average, yen basis) three-quarter backward moving average of quarter-to-quarter percent changes per annum

Explanatory variable	α	β	γ	δ	Adj-R^2	D.W.
Conventional output gap	1.473 (4.53)	0.563 (6.01)	0.270 (4.18)	0.010 (1.89)	0.772	1.66
New output gap from BSI and electricity consumption	1.440 (3.50)	0.616 (6.17)	0.190 (3.16)	0.009 (1.74)	0.746	1.84

Note: Sample: 1983/III–1998/I, t-values in parentheses.

[2] CPI Forecasts



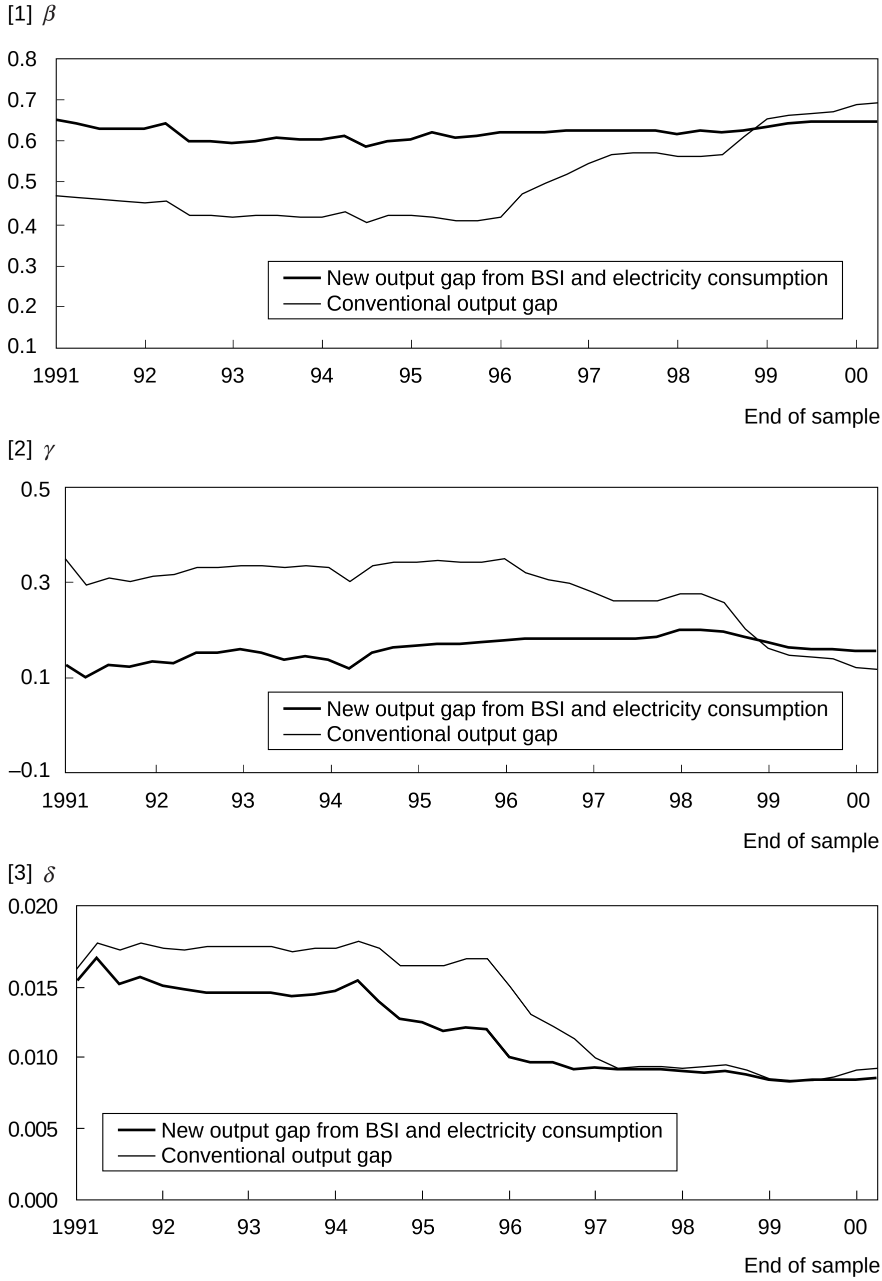
Sources: Cabinet Office, *National Accounts, Gross Capital Stock of Private Enterprises*, etc.; Ministry of Public Management, Home Affairs, Posts and Telecommunications, *Consumer Price Index*; Bank of Japan, *Wholesale Price Index*.

To see the stability of parameters, we estimate Phillips curves recursively. The start of sample is fixed at the third quarter of 1983 and the end shifts toward the current period (Figure 8). According to the results, parameters become rapidly unstable from 1998 when we use the conventional output gap (the thin line), whereas parameters are relatively stable when we use the new output gap (the thick line). Similar results are obtained when we use a current quarter's output gap as an explanatory variable to estimate Phillips curves (Figures 9 and 10).

The following reasons explain why the new output gap gives a better performance than the conventional output gap. Japan may have experienced considerable deterioration of factor quality from 1997 to 1998. The quality of capital stock, which

Figure 8 Stability of Phillips Curve (I)

Specification $\pi_t = \alpha + \beta \cdot \pi_{t-1} + \gamma \cdot G_{t-1} + \delta \cdot m_{t\sim t-2}$



Note: The start of sample is fixed at 1983/III, and the end shifts toward the current period.

Figure 9 Phillips Curve (II)

[1] Estimation

$$\pi_t = \alpha + \beta \cdot \pi_{t-1} + \gamma \cdot G_t + \delta \cdot m_t$$

π_{t-1}: CPI (trend and cycle components) quarter-to-quarter percent change per annum

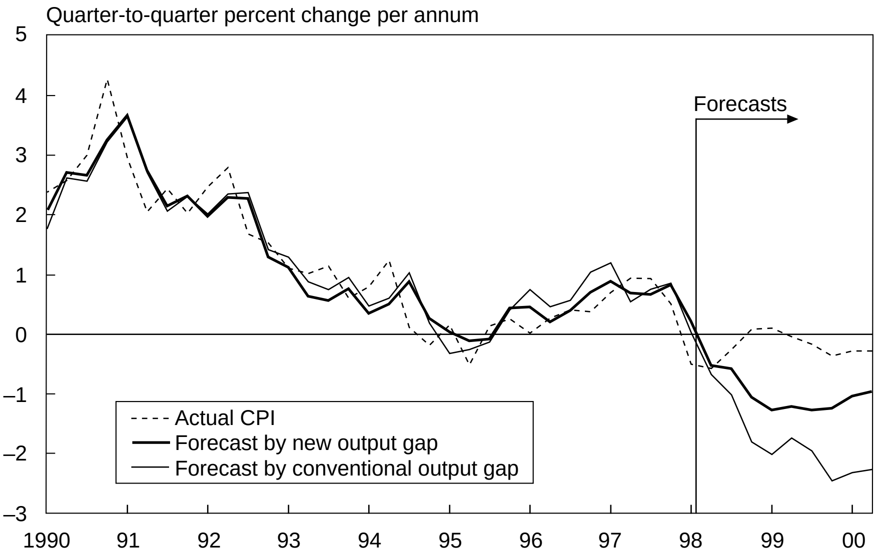
G_t: output gap in current quarter

m_t: import prices (WPI, total average, yen basis) quarter-to-quarter percent change per annum

Explanatory variable	α	β	γ	δ	Adj-R^2	D.W.
Conventional output gap	0.976 (3.14)	0.701 (7.56)	0.171 (2.83)	0.011 (2.93)	0.769	2.05
New output gap from BSI and electricity consumption	1.270 (3.66)	0.687 (7.93)	0.171 (3.39)	0.011 (2.93)	0.781	1.86

Note: Sample: 1983/II–1998/I, *t*-values in parentheses.

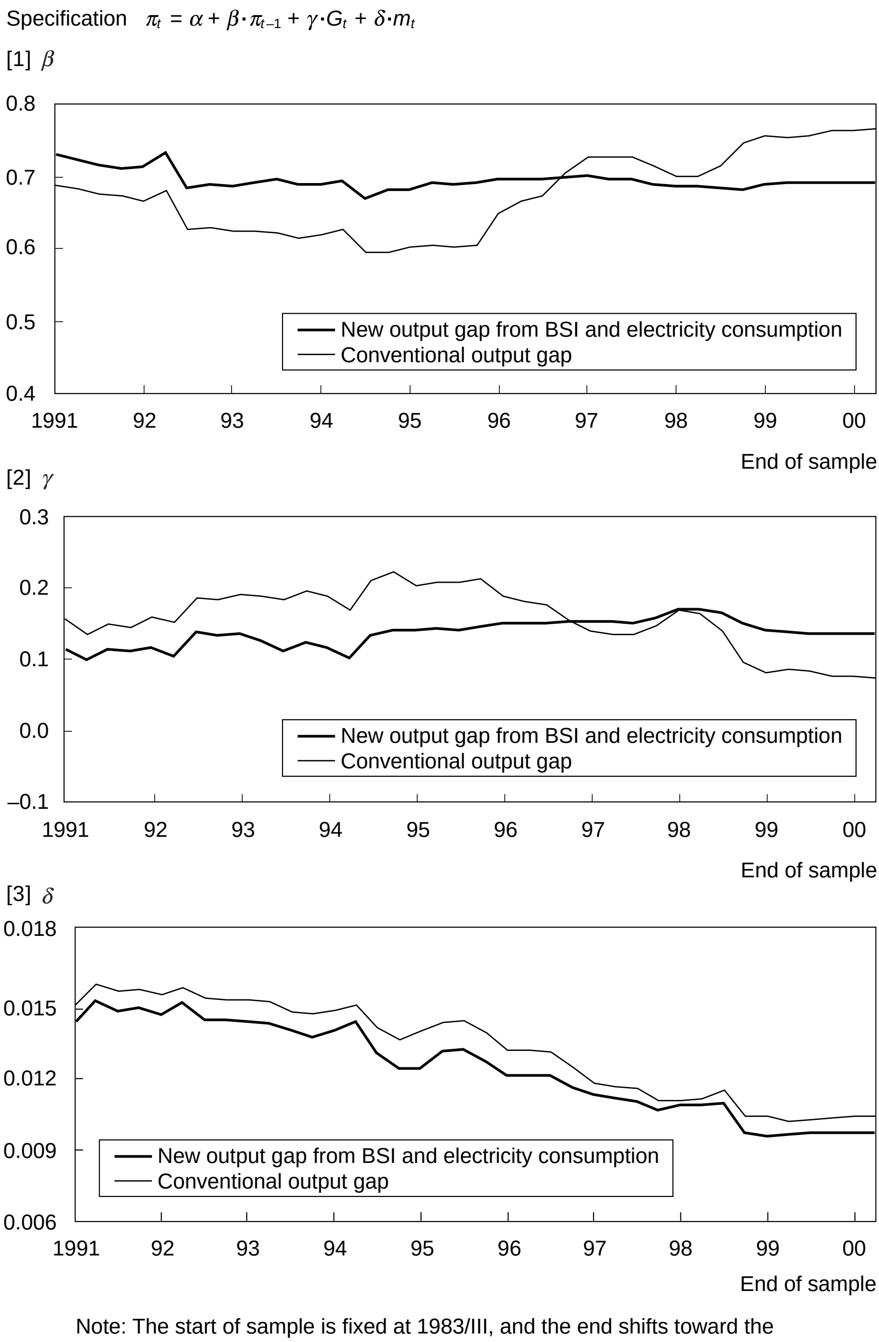
[2] CPI Forecasts



Sources: Cabinet Office, *National Accounts, Gross Capital Stock of Private Enterprises*, etc.; Ministry of Public Management, Home Affairs, Posts and Telecommunications, *Consumer Price Index*; Bank of Japan, *Wholesale Price Index*.

accumulated in the latter half of the 1980s during the asset-bubble period, may have deteriorated and rapidly became outdated while new fixed investment was restrained in the latter half of the 1990s. Furthermore, R&D investment may have been rapidly reduced and existing production know-how may have been outdated. The quality of the labor force may have deteriorated as labor skills were lost while there was a persistently high unemployment rate. True TFP also may have declined, as liquidation of production factors was delayed in spite of rapid structural changes in industrial organization and as productivity of social capital stock declined. These caused reductions in aggregate productivity. Thus, as discussed in Section II, we can estimate potential output and the output gap by treating reductions in the Solow residual as declines in TFP, as we did in estimating the new output gap.

Figure 10 Stability of Phillips Curve (II)



Note: The start of sample is fixed at 1983/III, and the end shifts toward the current period.

VI. Revision of GDP Statistics

In Japan, GDP statistics are revised four times: there are the first preliminary quarterly estimates (first QE), the second preliminary quarterly estimates (second QE), the final estimates, the annual revision, and the benchmark revision. As discussed in Section II, when we do not fit a linear trend to the Solow residual, the effects of revision of GDP statistics are absorbed by variations in the Solow residual and do not affect estimates of the output gap. Yet, when we estimate the conventional output gap, we assume capacity utilization in non-manufacturing sectors to be fixed at 100 percent and thus have to fit a linear trend to the Solow residual to extract TFP. In this case, the output gap is calculated wrongly by a preliminary-final-estimate discrepancy. In addition to the preliminary-final-estimate discrepancy, as data accumulate over time, estimated linear trends shift and thus the estimated output gap is changed. In this section, we analyze the effects of revision and accumulation of GDP statistics on estimation of the output gap.[20]

A. Four Figures for the Output Gap

Publication of GDP statistics is scheduled as follows. First preliminary quarterly estimates are published about two months and 10 days after a corresponding quarter ends. Second preliminary quarterly estimates are published one quarter after the first QE. A certain fiscal year's final estimates are published on December in the next fiscal year. Annual revisions are published one year after the final estimates. Benchmark revisions are published five years after.

To see the effects of revision and accumulation of GDP statistics on estimates of the output gap, we calculate and compare the following four series of output gaps. In this paper, we deal with the first QE, final estimates, annual revisions, and benchmark revisions (excluding second QE) and consider that data become more accurate in this order.[21] To make precise estimates, we would need first preliminary quarterly estimates, final estimates, annual revisions, and benchmark revisions for all necessary data, such as capital stock statistics. For simplicity, however, we ignore these revisions except for those of GDP statistics and use the latest figures for others.[22]

1. Real-time output gap

Real-time output gap is calculated from GDP statistics available in each estimation period. To construct a series of GDP available in each estimation period, we use benchmark revisions first, annual revisions second, final estimates third, and first preliminary quarterly estimates (first QE) fourth. Note that we made adjustments to eliminate the data gap that occurs due to benchmark revision.

20. We thank Naoko Hara (Research and Statistics Department, the Bank of Japan) for her empirical assistance in this section.
21. Before 1990, we have only preliminary quarterly estimates of gross national product (GNP). We converted GNP figures to GDP figures, using the final estimates of net factor incomes from abroad. That is, GNP = GDP + net factor incomes from abroad.
22. In fitting a linear trend, we assume an asset-bubble trend from the first quarter of 1985 to the fourth quarter of 1991 as well as a full-sample trend. Note that revision of GDP statistics affects measurement of a labor share. We, however, ignore the effect and use the latest estimates.

2. Quasi-real-time output gap

Quasi-real-time output gap is calculated under the assumption that in each estimation period, final estimates were known. To construct a series of GDP for the quasi-real-time output gap, we replace the first preliminary quarterly estimates with the final estimates in the series of GDP that was used for the real-time output gap. We keep the first preliminary quarter estimates for recent periods since the final estimates are not available.

3. Quasi-final output gap

The quasi-final output gap is calculated under the assumption that in each estimation period, the latest GDP estimates were known up to that period. To construct a series of GDP for the quasi-final output gap, we use 1990-based benchmark revisions first, 1990-based annual revisions second, 1990-based final estimates third, and 1990-based first preliminary quarterly estimates (first QE) fourth.

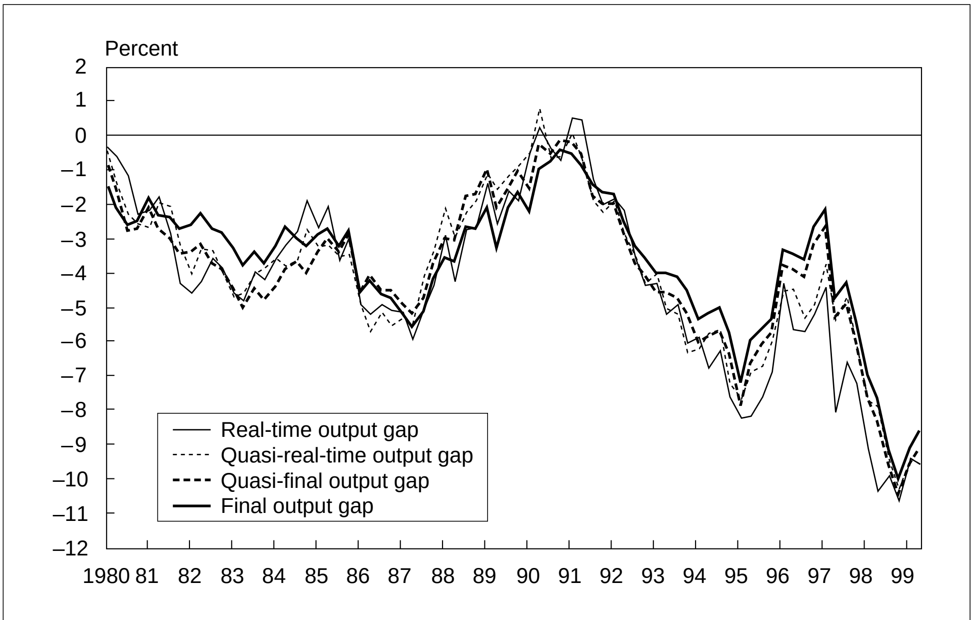
4. Final output gap

The final output gap is calculated under the assumption that in each estimation period, the whole series of the latest GDP estimates were known.

B. Effects of Revision and Accumulation of GDP Statistics

A difference between the real-time output gap and the quasi-real-time output gap reflects a preliminary-final-estimate discrepancy (Figure 11). A difference between the quasi-real-time output gap and the quasi-final output gap reflects an effect of benchmark revision (an effect of annual revision can be ignored). While both differences are basically attributed to revision of GDP statistics, a preliminary-final-estimate discrepancy has a greater effect than a benchmark-revision effect. This is

Figure 11 Effects of Revision and Accumulation of GDP Data: Conventional Output Gap



because source data used for preliminary estimates of GDP are substantially different from those used for final estimates of GDP.[23,24] Additionally, it should be noted in practical use that the output gap which is based on preliminary quarterly estimates of GDP is so volatile that we need time to evaluate its plausibility.

A difference between the quasi-final output gap and the final output gap reflects a pure effect of accumulation of GDP statistics. As we go back in time, the effects of data accumulation become larger than those of data revision. Yet as time passes, the effects of data revisions become larger. The effects of data accumulation become smaller since the data set for the quasi-final output gap gets closer to that for the final output gap.

VII. Conclusion

In this paper, we investigated the effects of measurement error in statistics on estimates of potential output and the output gap both from theoretical and empirical perspectives. Theoretical results can be summarized as follows: when there are measurement errors in factor utilization, estimation error occurs if we take the Solow residual to be TFP. In contrast, when there are quality changes in production factors, we can correctly estimate potential output and the output gap, even if we take the Solow residual to be TFP.

In Japan, there are no statistics on capacity utilization in non-manufacturing sectors. This is a big obstacle in estimating the output gap. To resolve this problem, we take two approaches. To estimate the conventional output gap, we first fixed capacity utilization in non-manufacturing sectors at 100 percent. Next, to obtain the output gap, we regressed the Solow residual on a time trend and thought of the trend as TFP and of the regression residual as measurement errors in capacity utilization. The defects of this approach are twofold. First, there is no guarantee that the estimated trend coincides with true TFP, and second, we mistake quality changes in production factors for variations in capacity utilization. Additionally, the conventional output gap is affected by revisions of GDP statistics from preliminary quarterly estimates to final estimates. To resolve these problems, we estimated the new output gap. Before we estimated the new output gap, we used electric power units for commercial power to estimate capacity utilization in non-manufacturing sectors. Then we could treat the Solow residual as TFP without regressing the Solow residual on a linear trend. We should note, however, that since factor utilization is closely related to the output gap, accuracy in estimates of capacity utilization is directly reflected in accuracy of an estimate of the output gap.

23. The first and second preliminary quarterly estimates are calculated from sampling data, such as *Monthly Report on the Family Income and Expenditure Survey* and *Financial Statements Statistics of Corporations by Industry, Quarterly*. On the other hand, the final estimates (or annual revision) are calculated by the commodity-flow method, and based on *Census of Manufactures, Census of Commerce, Current Survey of Commerce*, and *Summary Report on Trade of Japan*, etc.

24. Around 1996, the quasi-real-time output gap and quasi-final output gap diverged substantially. This is because GDP statistics were changed largely with the annual revision before benchmark revision.

When comparing the performance of the conventional and new output gap, we found that the latter is relatively consistent with various business cycle indicators. According to *Short-Term Economic Survey of Enterprises in Japan* by the Bank of Japan, the business conditions DI bottomed out at the end of 1998, and the production capacity DI and the employment conditions DI began to improve during 1999. According to the *reference dates of business cycle*, the bottom of the recent business cycle was April 1999. However, the conventional output gap expanded in the latter half of 1999. This inconsistency in the conventional output gap and other business cycle indicators results from the assumption that a deviation in the Solow residual from its linear trend is a decline in capacity utilization in non-manufacturing sectors. This suggests that we take the deviation to be a reduction in productivity due to a deterioration of production factors, such as capital stock. In fact, the new output gap, which takes these points into consideration, began to shrink from 1999, as various business cycle indicators show.

Furthermore, we compared the new output gap with the conventional output gap from the viewpoint of price projections through a Phillips curve. Whether we used the conventional output gap or the new output gap, the coefficient of determination was almost the same. Thus, there is no difference in the fit of a Phillips curve between the two series of output gaps. Next, we made dynamic forecasts of CPI based on the estimated Phillips curves.[25] According to the results, with either output gap, Phillips curves underestimate actual inflation rates in CPI. We can say, however, that the underestimation is smaller when we use the new output gap than when we use the conventional output gap. Finally, the parameters of a Phillips curve become unstable rapidly from 1998 when we use the conventional output gap, whereas they are stable when we use the new output gap.

As seen, the new output gap performs better than the conventional output gap. Nonetheless, we should be careful about using the new output gap, since its accuracy depends on how closely estimates of capacity utilization in non-manufacturing sectors obtained from electric power units reflect true capacity utilization.[26] Despite these caveats, the new output gap presented in this paper is useful for measuring slackness in the Japanese economy.

25. Dynamic forecasts are obtained by replacing expected inflation rates in an equation with predicted rates calculated from the estimated equation. We can check the performance of the estimates by comparing predicted values with real ones.

26. In addition, as discussed in Section II, utilization of labor force, or labor hoarding, affects the accuracy of estimates of the output gap.

APPENDIX 1: DATA DESCRIPTION[27]

A_t: Total factor productivity.

B_t: Production capacity BSI.

Note: Averaged three total production capacity BSIs (large, medium, small enterprises) weighted by shares of *other tangible assets* in *Financial Statements Statistics of Corporations by Industry, Quarterly*.

Sources: Ministry of Finance, *Business Outlook Survey of the Ministry of Finance, Financial Statements Statistics of Corporations by Industry, Quarterly*.

G_t: Output gap.

H_t: Working hours per capita.

Note: Total working hours in all industries at establishments with 30 or more regular employees (trend-cycle component).

Source: Ministry of Health, Labour and Welfare, *Monthly Labour Survey*.

H_t^*: Maximum working hours per capita.

Notes: The maximum number of working hours per capita is the sum of the maximum scheduled and unscheduled working hours. The maximum unscheduled working hours are given by the historical high. The maximum scheduled working hours are given by a linear trend, segmented as follows: (1) constant through 1987/IV, (2) decreasing during 1988/I–1993/IV, (3) constant during 1994/I–1997/I, (4) decreasing during 1997/II–1998/IV, and (5) constant from 1999/I.

Source: Ministry of Health, Labour and Welfare, *Monthly Labour Survey*.

K_t: Gross capital stock.

Note: All private enterprises (both incorporated and unincorporated enterprises) in all industries, adjusted for privatization of government enterprises.

Source: Cabinet Office, *Gross Capital Stock of Private Enterprises*.

KM_t: Capital stock in manufacturing sectors.

Note: Including construction in progress, adjusted for privatization of government enterprises.

Source: Cabinet Office, *Gross Capital Stock of Private Enterprises*.

KN_t: Capital stock in non-manufacturing sectors.

Notes: Including construction in progress adjusted for privatization of government enterprises.

Source: Cabinet Office, *Gross Capital Stock of Private Enterprises*.

L_t: Working hours.

Note: $L_t = H_t \cdot N_t$.

L_t^*: Maximum working hours.

Note: $L_t^* = H_t^* \cdot N_t^*$.

27. In Japan, *National Accounts* changed its definition from 68 SNA to 93 SNA on October 2000. When this paper was written, only 68 SNA was available.

N_t: Number of workers.
Note: All industries.
Source: Ministry of Public Management, Home Affairs, Posts and Telecommunications, *Labour Force Survey*.

N_t^*: Maximum number of workers.
Notes: The maximum number of workers is the sum of (1) the maximum number of workers 15–64 years old and (2) that of workers older than 64 years, as obtained below:
(1) Maximum number of workers 15–64 years old
We assume that the maximum labor participation rate rises along an upper boundary of the historical series. First, the labor participation rate is given by a ratio of workers to the population in the demographic group. Second, we regress the labor participation rate on a linear trend. The maximum labor participation rate is the sum of this linear trend and a maximum regression residual. Multiplying the population by this maximum rate, we obtain the maximum number of workers 15–64 years old.
(2) Workers more than 64 years old
Apply the procedure described above to obtain the maximum number of workers older than 64 years old.
Source: Ministry of Public Management, Home Affairs, Posts and Telecommunications, *Labour Force Survey*.

Y: Real gross domestic expenditure (real gross domestic product, GDP).
Note: First preliminary quarterly estimates, final estimates, etc.
Source: Cabinet Office, *National Accounts*.

Y^*: Potential output.

t: Full-sample linear time trend.

γm: Capacity utilization in manufacturing sectors.
Note: Index (1995 = 100, originally), normalized with 1990/IV = 100.
Source: Ministry of Economy, Trade and Industry, *Indices of Industrial Production*.

γn: Capacity utilization in non-manufacturing sectors.

α: Labor share.
Note: See Appendix 5.
Source: Cabinet Office, *National Accounts*.

λ: Electric power units for commercial power.
Note: Ratio of electricity consumption for commercial power to electric power contracted for commercial power. Adjusted for discontinuity of institutional change.
Source: Federation of Electric Power Companies of Japan, *Electricity Demand*.

τ: Asset-bubble trend.
Note: 0 through 1985/I, a linear trend during 1985/I–1991/IV, 28 from 1991/IV.

APPENDIX 2: CES PRODUCTION FUNCTION

In this paper, we use a Cobb-Douglas production function for an aggregate production function. A Cobb-Douglas production function assumes (1) unit elasticity of substitution between capital and labor; and (2) constant returns to scale. Here we start with a CES production function, which is a generalization of a Cobb-Douglas production function, and see how well it fits the Japanese economy.[28] Our analysis supports the use of the Cobb-Douglas production function.

A CES production function is given by

$$Y_t = A_t \cdot \{\delta(\gamma_t \cdot K_{t-1})^{-\rho} + (1-\delta)L_t^{-\rho}\}^{-\nu/\rho},$$

where ν is returns to scale. Elasticity of substitution σ is given by $1/(1+\rho)$. Log-transform the above equation, Taylor-expand to second order, and evaluate the result around $\rho = 0$. Then we obtain

$$\ln Y_t = \beta_1 + \beta_2 \cdot \tau_t + \beta_3 \cdot \{\ln(\gamma_t \cdot K_{t-1}) - \ln L_t\} + \beta_4 \cdot \ln L_t - \beta_5 \cdot \{\ln(\gamma_t \cdot K_{t-1}) - \ln L_t\}^2/2,$$

where we assume that TFP follows a linear trend during the asset-bubble period (a full-sample linear trend was found insignificant in the case of a CES production function). Furthermore, we have $\beta_3 = \nu\delta$, $\beta_4 = \nu(1-\delta)$, and $\beta_5 = \rho\nu\delta(1-\delta)$. Thus, we can calculate back parameters of a CES production function as follows.

$$\delta = \beta_3/(\beta_3 + \beta_4),\ \nu = \beta_3 + \beta_4, \text{ and } \rho = \beta_4(\beta_3 + \beta_4)/(\beta_3\beta_4).$$

The estimation results are presented in Appendix Table 1.

Appendix Table 1 Estimated Properties of a CES Production Function

	No full-sample trend	(C.f., including a full-sample trend)
Returns to scale	1.003	0.988
Elasticity of substitution	1.17	1.24

As shown in the table, both returns to scale and elasticity of substitution are close to 1. This implies that we can assume a Cobb-Douglas production function as the Japanese aggregate production function.

28. Giorno *et.al.* (1995) assumes a CES aggregate production function for the Japanese economy.

APPENDIX 3: CHOICE OF ASSET-BUBBLE TREND

In estimating the conventional output gap, we assume an asset-bubble trend (from the first quarter of 1985 to the fourth quarter of 1991) as well as a full-sample trend. Here we examine the choice of a sample period for the asset-bubble trend by a two-dimension grid search with regard to starting and ending quarters. We further assume that the asset bubble occurred between the first quarter of 1984 and the fourth quarter of 1993 and that it lasted for at least one year. A specification is given as follows.

$$\ln A_t = \beta_1 + \beta_2 \cdot t + \beta_3 \cdot \tau(s,e)_t,$$

where $\tau(s,e)$ is a linear trend that starts in period s and ends in period e. The top five alternatives in terms of log likelihood are given in Appendix Table 2.

Appendix Table 2 Best Selections of Asset-Bubble Trend

Trend period	Log likelihood
Case 1 (1985/I–1991/III)	330.8
Case 2 (1985/I–1991/II)	330.7
Conventional output gap (1985/I–1991/IV)	330.6
Case 3 (1985/I–1992/I)	330.3
Case 4 (1985/IV–1991/II)	330.2

Notes: 1. Sample period: 1975/I to 1998/II.
2. Estimation method: maximum likelihood with an AR(1) error term.

According to the table, it is likely that the asset bubble starts at the first quarter of 1985 and ends during 1991. Moreover, the choice of the asset-bubble period for the conventional output gap is not unreasonable, since it is ranked in the third position in terms of log likelihood.

APPENDIX 4: DETERIORATION OF CAPITAL STOCK

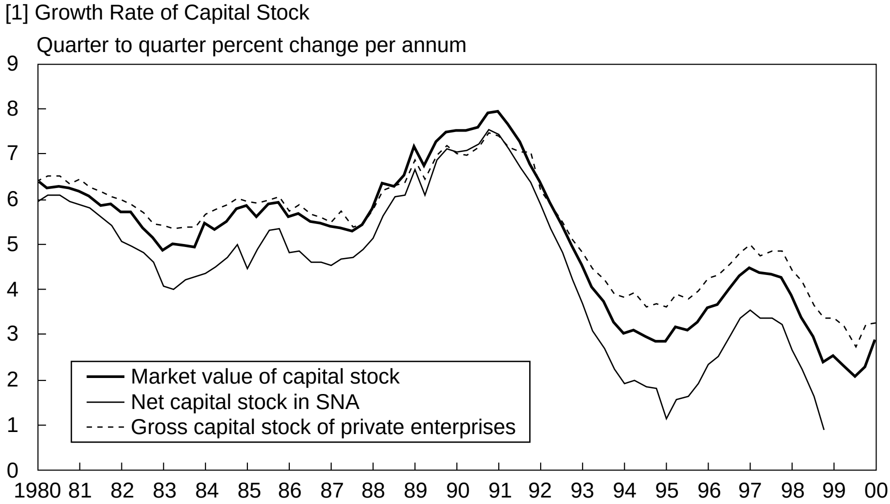
The *Gross Capital Stock of Private Enterprises* (by the Cabinet Office), which is used in estimating the output gap, reports *gross capital stock*, which takes only *scrap* into consideration. However, capital stock loses productivity as it is used. Therefore, to correctly evaluate productivity of capital stock, we have to see *net capital stock*, which takes into consideration *depreciation* as well as *scrap*. In this appendix, we estimate the output gap, based on a *market value of capital stock*, which uses depreciation rates calculated from prices in the secondhand market, instead of *Gross Capital Stock of Private Enterprises*, which is conventionally used to estimate the output gap.[29] According to the market value of capital stock, the difference between gross capital stock and net capital stock has grown in recent years (Appendix Figure 1 [1]). Note that as shown theoretically in Section II, quality changes in capital stock give rise to problems when estimating the conventional output gap, but do not affect estimates of the new output gap. Thus, it is enough to consider only the conventional output gap below.

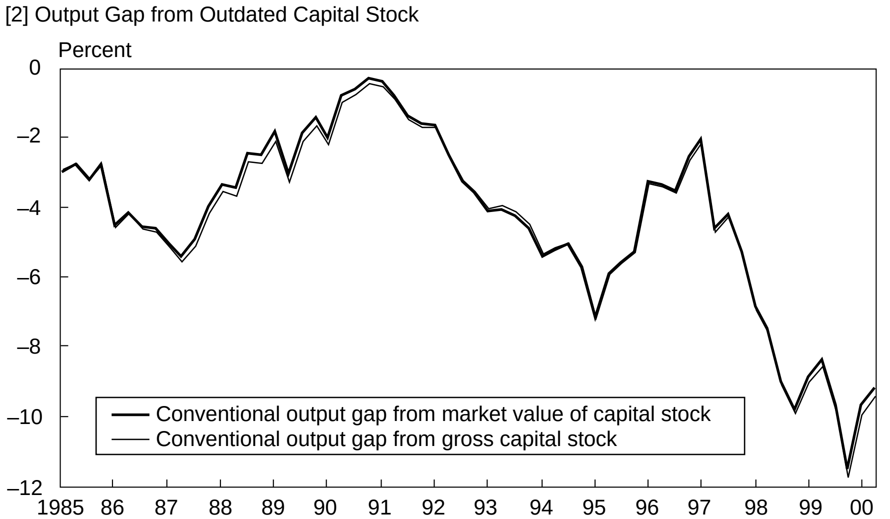
According to the estimation results, the conventional output gap is almost unaffected when *Gross Capital Stock of Private Enterprises* is replaced with the market value of capital stock (Appendix Figure 1 [2]). There are two reasons for this. First, changes in capital stock due to depreciation have relatively small effects in comparison to changes in capacity utilization. Second, depreciation of capital stock is partly absorbed by estimated TFP, or by the growth rate of the linear trend of the Solow residual, and thus has a small effect on the conventional output gap.

We do not claim that outdated capital stock has no effect on estimates of the conventional output gap. In particular, capital stock was more outdated than had been suggested by the market value of capital stock, as new fixed investment was restrained as a result of economic deterioration and the fact that industrial structure changed during the latter half of the 1990s. In this case, potential output and output gap (in absolute value) may be substantially overestimated.

29. See Masuda (2000) for concepts of capital stock, such as *scrap, depreciation, gross capital stock*, and *net capital stock* and also for a market value of capital stock.

Appendix Figure 1 Outdated Capital Stock





Sources: Cabinet Office, *National Accounts*, *Gross Capital Stock of Private Enterprises*, *National Wealth Survey*, etc.

APPENDIX 5: LABOR SHARE[30]

The α in equation (1) in Section II is just a parameter that determines labor elasticity of output. However, if capital and labor markets are competitive, α coincides with the labor share.[31] In this paper, we calibrate the value of α by a historical sample mean of the labor share (obtained from the first quarter of 1975 to the latest final estimates). However, there are various definitions for the labor share. Thus, a question remains as to which definition is best to estimate α used in the aggregate production function. This appendix discusses a labor share that is ideal for an aggregate production function. After the discussion, however, we see that definition of labor share has little effect on estimates of the output gap.

A. Aggregate Production Function and Labor Share

The left-hand side of an aggregate production function (Y) is gross domestic product and thus the denominator of labor share α should be gross domestic product. In particular, depreciation of capital stock should be taken into account because of its volume. Furthermore, there are two kinds of gross domestic product: factor price representation and market price representation, which adds net indirect taxes (indirect taxes net of subsidies) to factor price representation. We use the factor price representation, since income of the private sector does not include net indirect taxes. Thus, the denominator of labor share is gross domestic product net of net indirect taxes (*consumption of fixed capital* + *operating surplus* + *compensation of employees*). For the numerator of labor share, it is problematic that compensation of employees in *National Accounts* does not include compensation of employees in *unincorporated enterprises*, which includes their family workers. Taking these problems into consideration, the ideal labor share for an aggregate production function is given by

$$\text{Ideal labor share} = \frac{\begin{array}{c}\text{(Compensation of employees in unincorporated}\\ \text{enterprises) + (Compensation of employees)}\end{array}}{\begin{array}{c}\text{(Consumption of fixed capital) + (Operating surplus) +}\\ \text{(Compensation of employees)}\end{array}}.$$

However, *compensation of employees* in *unincorporated enterprises* is included in *households'* (*including unincorporated enterprises*) *operating surplus* in *National Accounts* and is not separated from the other parts. For this reason, we made alternative assumptions to create proxies of an ideal labor share defined as above. So far, we have used as α the *labor share* defined in *National Accounts*.

$$\text{Labor share 0} = \frac{\text{(Compensation of employees)}}{\text{(National income)}}.$$

30. We thank Masakazu Inada (Research and Statistics Department, the Bank of Japan) for his assistance in this section.

31. When a labor market is competitive, marginal products of labor are set equal to real wages. That is, $\partial Y/\partial L = \alpha \cdot A \cdot L^{\alpha-1} \cdot (\gamma \cdot K)^{1-\alpha} = w/p$, where w is a nominal wage and p is a price level. Multiplying the second and third terms by L and rearranging them, we have $\alpha = wL/pY$ = labor share.

This is a well-known definition of labor share, but it does not correspond to α since its denominator is national income.

[Alternative 1]

$$\text{Labor share 1} = \frac{\text{(Compensation of employees)}}{\begin{array}{c}\text{(Consumption of fixed capital) + (Operating surplus) +}\\ \text{(Compensation of employees) –}\\ \text{(Households' operating surplus)}\end{array}}.$$

While we do not count compensation of employees in unincorporated enterprises in the numerator, we subtract operating surplus of unincorporated enterprises from the denominator. This is based on the assumption that the labor share in unincorporated enterprises is equal to that in large firms. For precision, we should subtract consumption of fixed capital of unincorporated enterprises from the denominator, but we did not.

[Alternative 2]

$$\text{Labor share 2} = \frac{\begin{array}{c}\text{(Households' operating surplus) +}\\ \text{(Compensation of employees)}\end{array}}{\begin{array}{c}\text{(Consumption of fixed capital) + (Operating surplus) +}\\ \text{(Compensation of employees)}\end{array}}.$$

In this definition, all operating surplus of unincorporated enterprises is assumed to be compensation for labor input in unincorporated enterprises and thus is added to the numerator as employee income.

[Alternative 3]

$$\text{Labor share 3} = \frac{\begin{array}{c}\text{(Estimated compensation of employees in unincorporated}\\ \text{enterprises) + (Compensation of employees)}\end{array}}{\begin{array}{c}\text{(Consumption of fixed capital) + (Operating surplus) +}\\ \text{(Compensation of employees)}\end{array}}.$$

We estimate and add labor compensation for small business owners and their family workers to the numerator. Here we use *Salary in Private Sectors* (by the National Tax Administration in Japan) to calculate labor compensation for small business owners and their family workers as follows.

$$\begin{array}{l}\text{Compensation of employees}\\ \text{in unincorporated}\\ \text{enterprises}\end{array} = \frac{\begin{array}{c}\text{(Employee income}\\ \text{in private sector)}\end{array}}{\begin{array}{c}\text{(Number of employees}\\ \text{in private sector)}\end{array}} \times \frac{\begin{array}{c}\text{(Wage per worker}\\ \text{in small business)}\end{array}}{\begin{array}{c}\text{(Wage per worker}\\ \text{in entire sector)}\end{array}}$$

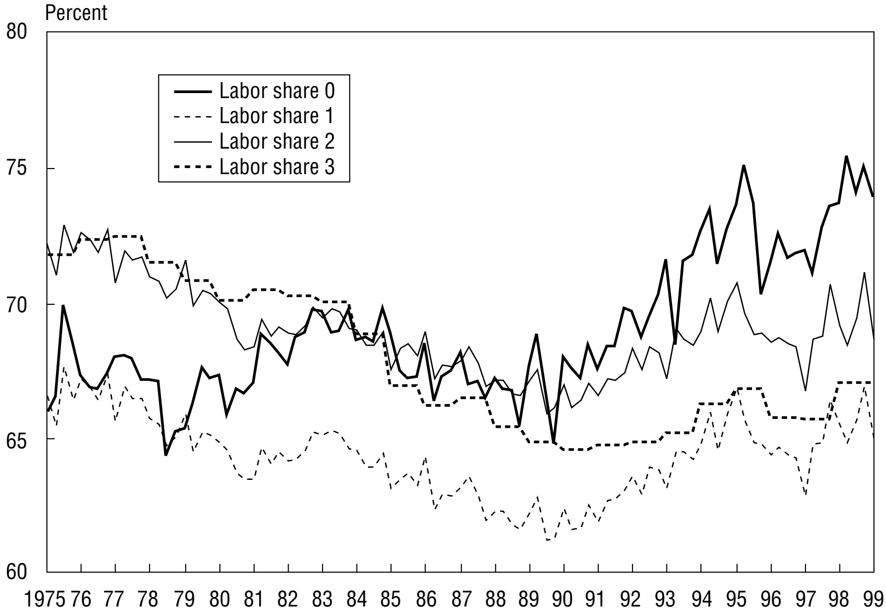
$$\times \text{(Number of workers in unincorporated enterprises).}$$

The first term on the right-hand side is employee income per capita in private companies except for small business. The second is the wage difference between wages for small business owners and their family workers and those for other workers. By multiplying these terms by the number of workers in unincorporated enterprises, we estimated compensation of employees in unincorporated enterprises in the whole economy. Only annual data are available for a labor share that uses this definition.

We plot the three series of labor share defined above as well as the labor share used in the text in Appendix Figure 2. According to the figure, the four series of labor share have moved with a maximum difference of 7 to 8 percent since 1975. Labor share 0, which is used in the text, has a clear upward trend. In contrast, labor share 3, which takes into consideration labor compensation in small business, has a clear downward trend. Labor share 2, in which operating surplus in small business is subtracted from the denominator, and labor share 1, in which operating surplus in small business is added to the numerator, are almost flat. We can say, however, that the two series seem to have bottomed out when the asset bubble hit its peak (from 1989 to 1990).

As shown in Section II, we can correctly estimate the output gap by regressing the Solow residual on a linear trend, even though there are measurement errors in the labor share. A problem occurs when we take the Solow residual to be TFP, as we did when estimating the new output gap. In this case, the estimation error in output gap is given by $(\bar{\alpha} - \alpha)(\ln L - \ln L^*) - (\bar{\alpha} - \alpha)\ln\gamma$. However, even if capacity utilization of the labor force and capital stock changes a little, the change is discounted more than 90 percent since the difference in labor share is 7 to 8 percent at most. For this reason, measurement errors in the labor share have little effect on estimates of the output gap.

Appendix Figure 2 Labor Share



Sources: National Tax Administration, *Salary in Private Sectors*; Cabinet Office, *National Accounts*.

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