The Joint and Several Effects of Liquidity Constraints, Financing Constraints, and Financial Intermediation on the Welfare Cost of Inflation

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This paper examines two features of modern economies that are often overlooked when formally considering the welfare costs of inflation. The first is the short-term financing requirements of firms, and the second is the joint roles played by banks in providing valued liquidity services to households and in acting as financial intermediaries. Measured welfare losses of moderate inflation are seen to become quite large when firms finance their working capital expenses by issuing short-term debt, with estimates of those losses ranging to over 450 percent higher than is the case when these financing requirements are ignored. Banks are seen to mitigate substantially the welfare costs of inflation by lessening the distortions in household decisions, and by intermediating a larger share of short-term loans to firms as inflation increases.

Key words: Financing constraints; Financial intermediation;
Welfare costs of inflation

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I. Introduction

In 1997, *per capita* holdings of M1 assets in Japan totaled ¥1.5 million. Of this total, ¥369,000 consisted of currency in circulation, with the balance representing demand deposits.[1] These assets were held principally for the liquidity services that they provided in facilitating transactions. While bank deposits did provide some interest income, the rate of interest was generally very low. For example, from 1971–96 the average interest rate paid on ordinary deposits was 1.43 percent, while the inflation rate averaged 4.3 percent, implying a negative real return. Consequently, transactions carried out with these liquid monetary assets were subjected to inflation taxes. Given the high volume of *per capita* holdings of M1 assets in Japan, the avoidance of inflation taxes could result in a misallocation of resources sufficient to induce significant welfare losses.

Inflation may also adversely affect credit conditions when debt obligations are denominated in currency units. The inflation premium in nominal market interest rates raises the cost of borrowing, and deters activities that these funds are used to finance. Most firms finance a significant portion of their working capital expenses with short-term debt. High inflation raises their financing costs, thereby requiring higher productivities both from their marginal unit of capital, thus retarding investment, and from their marginal worker, thereby reducing employment. As a consequence, overall economic activity declines, and this can result in a substantial increase in the welfare costs of inflation.

Commercial banks that raise funds by offering valued liquidity services in the form of demand deposit accounts can mitigate these welfare costs somewhat by inter-mediating the loans from households to firms, where the latter are used to finance working capital expenses. However, the extent to which banks can intermediate these loans is limited by the demand for their deposit offerings. For this reason, bank-intermediated loans can coexist with direct placement of private paper in the capital markets, even in the absence of private information that would induce scale economies in monitoring as described by Diamond (1984).

This paper examines the interaction between liquidity constraints associated with household transactions and financing constraints associated with working capital requirements of firms. The models are calibrated to the Japanese economy. It is found that in the absence of financing constraints, the welfare costs of moderate inflation are significant, but not large, much in line with the results obtained by Cooley and Hansen (1989, 1991) for the U.S. economy. However, in the absence of bank intermediation, these costs effectively double when the firms' wage bills are financed by bond issues, more than triple when gross investment is financed by bond issues, and increase by more than fourfold when all working capital expenses are financed with bonds, rendering these costs quite large. The introduction of a bank that

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1. Humphrey, Pulley, and Versala (1996) document the high usage of currency in the payments system in Japan relative to the United States and Europe. By way of comparison, *per capita* currency holdings in the United States in 1996 totaled US$1,524, or ¥191,000 (using an exchange rate of ¥125/US$1). However, approximately two-thirds of U.S. currency is estimated by Porter and Judson (1995) to be held outside the United States, suggesting that this difference in currency usage is even more pronounced than these numbers imply.

provides deposits to households and uses the proceeds to intermediate a portion of the household loans to firms by purchasing a share of the firms' bond issues is then seen to mitigate these losses significantly. In the case where firms fully finance their working capital expenses with bonds, bank intermediation lowers the measured welfare costs by 15–20 percent. These results suggest that banks can play an important role in alleviating the adverse economic effects that fully anticipated inflation has on the economy.

In the next chapter, a general model is developed that can be parameterized to obtain all of the cases described above. These models are calibrated in Chapter III, and the results are presented in Chapter IV. The final chapter presents the conclusions.

II. The Theoretical Model

This chapter develops a general equilibrium, representative agent model in which banks provide liquidity services through deposit account offerings, and after meeting reserve requirements, use the proceeds to purchase a portion of the short-term bond issues of firms. The remaining bonds are purchased by households, which use the deposit accounts to purchase a subset of their consumption goods. Firms' working capital expenses are financed with the revenues raised from issuing bonds. For ease of exposition, the model is structured such that parameterizations can be selected that produce any one of the following five versions of the general model: (A) a simple cash-in-advance model with no bank and no financing constraints on firms; (B) model (A), where firms issue one-period bonds to finance their wage bill; (C) model (A), where firms issue one-period bonds to finance gross investment; (D) model (A), where firms finance all of their working capital, i.e., their wage bill and gross investment, by issuing bonds; and (E) the general case described above, which is model (D), with banks providing deposit accounts and intermediating a portion of the loans from households to firms.

A. Household Sector
The representative household makes its consumption/savings decision by selecting optimally a consumption bundle and a short-term asset portfolio allocation between money, bank deposits, and bonds. It is assumed to be the residual claimant to *per capita* shares of period profits for firms and banks.[2] The household's consumption purchases are subject to liquidity constraints, where a portion of the consumption goods is purchased with money, and the remainder is purchased with bank deposits. It also makes a labor/leisure choice.

The household maximizes lifetime utility, with period utility, u, derived from leisure, l_t, and two consumption goods, where the latter are distinguished by the means of payment needed to acquire them. Money is used to purchase c_{1t}, referred to as the "cash good," and bank deposits are used to acquire c_{2t}, referred to as the "deposit good."

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2. Since this paper is not concerned with asset pricing *per se*, the equity markets are not modeled.

$$\max_{\{c_{1t},\ c_{2t},\ n_t^s,\ l_t,\ M_{t+1}^d,\ X_{t+1}^d,\ B_{t+1}^d\}} \sum_{t=0}^{\infty} \beta^t u(c_{1t},\ c_{2t},\ l_t;\ \phi_1),\ \beta \in (0,\ 1),\ \phi_1 \in \{0,\ 1\}. \tag{1}$$

The household's choice set includes optimal sequences for the consumption goods, leisure, labor supply, n_t^s, and the next period's holdings of money, M_{t+1}^d, deposits, X_{t+1}^d, and bonds, B_{t+1}^d. The household's subjective discount factor is given by β, and ϕ_1 is an indicator variable that is one in model (E), where households have a positive demand for bank deposits, and zero otherwise, when consumption of the "deposit good" is also zero.

The optimization in equation (1) takes initial asset holdings, $(M_0^d,\ X_0^d,\ B_0^d)$, as given, and is subject to four constraints. The first is the budget constraint:

$$P_t(c_{1t} + \phi_1 c_{2t}) + M_{t+1}^d + \phi_1 X_{t+1}^d + \phi_2 B_{t+1}^{db}$$
$$\leq W_t n_t^s + M_t^d + (1 - \phi_1) J_t + (1 + r_{xt}) \phi_1 X_t^d \tag{2}$$
$$+ (1 + r_{bt}) \phi_2 B_t^{db} + \textstyle\prod_t^f + \phi_1 \textstyle\prod_t^{cb},\ \phi_1 \phi_2 \in \{0,\ 1\},$$

where P_t is the money price of goods, W_t is the money wage, J_t is a lump-sum, *per capita* monetary transfer from the government, and r_{xt} and r_{bt} are deposit and bond rates, respectively. Note that in models (A)–(D), there is no bank (and no bank profits), and monetary injections are direct transfers to households rather than reserves injections to the banks, as is the case in model (E). This implies that the deposit good and deposit balances are zero, and $J_t > 0$. In model (A), there are no financing constraints, implying B^{db} is zero. This is modeled by setting ϕ_2 to zero for model (A), and to one otherwise. The household therefore allocates its nominal labor income, $W_t n_t^s$, beginning-of-period post-transfer nominal money balances, $M_t^d + (1 - \phi_1) J_t$, initial deposit balances and interest income on deposits, $(1 + r_{xt}) \phi_1 X_t^d$, and the principal plus interest on previous bond investments, $(1 + r_{bt}) \phi_2 B_t^{db}$, to nominal consumption $P_t(c_{1t} + \phi_1 c_{2t})$ and its asset portfolio positions, $M_{t+1}^d,\ \phi_1 X_{t+1}^d,\ \phi_2 B_{t+1}^{db}$, which are carried over to the next period.

The household faces two liquidity constraints. The first is that nominal consumption purchases of the cash good are constrained by the household's initial money balances, which include the monetary transfer in models (A)–(D).

$$P_t c_{1t} \leq M_t^d + (1 - \phi_1) J_t,\ \phi_1 \in \{0,\ 1\}. \tag{3}$$

The second liquidity constraint limits nominal consumption purchases of the deposit good by the stock of the household's deposit balances carried over from the previous period.[3]

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3. While the cash-in-advance constraint, equation (3), is well entrenched in the literature, the "deposit-in-advance" constraint, equation (4), is less common. Recent examples of its use are Edwards and Vegh (1997) and Hartley (1998). These two constraints, together with a choice of a utility function for which the cash and deposit goods are imperfect substitutes, allow Inada conditions to ensure positive demands for both money and bank deposits.

$$P_t c_{2t} \le X_t^d, \tag{4}$$

where equation (4) is binding only in model (E).

Lastly, the household faces a time constraint on its labor/leisure decision.

$$n_t^s + l_t \le 1. \tag{5}$$

B. Recursive Representation of the Household's Optimization Problem

To ensure stationarity, nominal variables in the model are normalized throughout by the nominal money supply, M_t, where the gross growth rate of the nominal money supply is given by $G_{t+1} = M_{t+1}/M_t$. Define $p_t = P_t/M_t$, $m_t^d = M_t^d/M_t$, $x_t^d = X_t^d/M_t$, $b_t^{db} = B_t^{db}/M_t$, $w_t = W_t/M_t$, $j_t = J_t/M_t$, $\pi_t^{cb} = \prod_t^{cb}/M_t$, and $\pi_t^f = \prod_t^f/M_t$. Then, the household problem becomes

$$\max_{\{c_{1t}, c_{2t}, n_t^s, l_t, m_{t+1}^d, x_{t+1}^d, b_{t+1}^d\}} \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, l_t; \phi_1), \tag{6}$$

given m_0^d, x_0^d, and b_0^{db}, subject to

$$p_t(c_{1t} + \phi_1 c_{2t}) + G_{t+1}(m_{t+1}^d + \phi_1 x_{t+1}^d + \phi_2 b_{t+1}^{db}) \le$$
$$w_t n_t^s + m_t^d + (1 - \phi_1) j_t + (1 + r_{xt}) \phi_1 x_t^d \tag{7}$$
$$+ (1 + r_{bt}) \phi_2 b_t^{db} + \pi_t^f, + \phi_1 \pi_t^{cb}.$$

$$p_t c_{1t} \le m_t^d + (1 - \phi_1) j_t. \tag{8}$$

$$p_t c_{2t} \le x_t^d. \tag{9}$$

$$n_t^s + l_t \le 1. \tag{10}$$

The household's problem can now be set up as a dynamic programming problem. Dropping the time subscripts and using the prime (′) notation to denote the next period's values, define the value function as $v(s^h)$, where the household's state vector is given by $s^h = [m^d, x^d, b^{db}, S]$, where S is the aggregate state vector, defined below. Given the initial state, s_0^h, the Bellman equation for this problem becomes

$$v(s^h) = \sup_{\lambda^h(s^h) \in \Gamma^h(s^h)} [u(c_1, c_2, l; \phi_1) + \beta v(s^{h\prime})], \tag{11}$$

where $\lambda^h(s^h) = [c_1(s^h), c_2(s^h), n^s(s^h), l(s^h), m^d(s^h), x^d(s^h), b^{db}(s^h)]$ is the vector of household decision rules drawn from the feasible set of correspondences $\Gamma^h(s^h)$ defined by equations (7)–(10).

Using the envelope conditions, the first-order conditions yield the following set of Euler equations.

$$\beta u'_{c_1}(w/p'G') = u_l, \tag{12}$$

$$\beta u'_{c_2}(w/p'G') + \beta u'_l(r'_x w/w'G') = u_l, \tag{13}$$

$$\beta u'_l(1 + r'_b)(w/w'G') = u_l, \tag{14}$$

where $G' = M'/M$. Equations (12)–(14) have the interpretation of equating the marginal cost of forgoing a unit of leisure in exchange for the marginal benefits of a unit of labor, where the additional labor income is saved as money in equation (12), deposits in equation (13), and bonds in equation (14). In equation (12), the additional labor income purchases $(w/p'G')$ units of the cash good in the next period, each of which has a present value of $\beta u'_{c1}$. The left-hand side of equation (13) displays two benefits from placing additional labor income on deposit. First, it permits consumption of the deposit good to rise during the next period by $(w/p'G')$ units, each of which has a present value of $\beta u'_{c2}$. Second, it earns interest income that can be used to reduce labor supply, and hence increase leisure, by $(r'_x w/w'G')$ units, each of which has a present value of $\beta u'_l$. The left-hand side of equation (14) indicates that the additional labor income which is used to purchase one-period bonds in the amount w, which have a gross return of $(1 + r'_b)$, enables leisure to be increased (or labor supply to be reduced) during the next period by $(1 + r'_b)(w/w'G')$ units, each of which has a present value of $\beta u'_l$. Note that in model (A), firms do not issue bonds, implying that there is no bond market, and that equation (14) is therefore dropped from the model. Also note that in models (A)–(D), there is no bank, and hence deposits are zero. In this case, equation (13) is dropped from the model.

C. Firm Sector
The firm sector is assumed to be perfectly competitive, and comprised of a large number of identical firms, which for simplicity is set equal to the number of households.[4] Ignoring agency costs, the representative firm is assumed to act in the interest of its owners. Its objective is to maximize the present discounted value of the stream of dividends, or period profits.

$$\max_{\{k_{t+1},\ n_t^d,\ B_t^s\}} \sum_{t=0}^{\infty} \beta^{t+1}(u_{c_{1_{t+1}}}/P_{t+1})\Pi_t^f, \tag{15}$$

with period profits given by

$$\Pi_t^f = P_t F(k_t,\ n_t^d) - \phi_2(1 + r_{bt})B_t^s - \phi_3 P_t[k_{t+1} + (1 - \delta)k_t]$$
$$- \phi_4 W_t n_t^d, \ \phi_2,\ \phi_3,\ \phi_4 \in \{0, 1\}. \tag{16}$$

Note that nominal profits, Π_t^f, are paid out as dividends each period in currency that households must hold one period before spending them, say, for cash goods, due

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4. The reason for not modeling the firm sector by a single large firm is to enable a cleaner presentation of the dynamic programming problem with separate firm and aggregate state variables.

to the liquidity constraint, equation (3). In the interim, prices may change such that each unit of currency received in period t buys $1/P_{t+1}$ units of consumption goods in period $t+1$, which are, in turn, valued at date $t+1$ at the marginal utility $u_{c_{t+1}}$, and the total is discounted back one period at the rate given by the discount factor, β, to determine its present value.

The firm chooses sequences for its gross investment, or the next period's capital stock, k_{t+1}, employment, n_t^d, and the nominal supply of one-period bonds. The firm's revenues equal nominal sales, which are represented by $P_t F(k_t, n_t^d)$, with output given by the production function, $F: \Re_{++}^2 \to \Re_+$, which is continuous and twice-differentiable, with $F_1, F_2, F_{12} > 0$ and $F_{11}, F_{22} < 0$. The firm's expenses depend on the financing requirements.

$$(1 - \phi_3) P_t [k_{t+1} - (1 - \delta) k_t] + (1 - \phi_4) W_t n_t^d \leq \phi_2 B_t^s. \tag{17}$$

In model (A), there is no financing constraint, $\phi_2 = 0$, and both gross investment and the wage bill are financed out of current revenues, which corresponds to setting the indicator variables $\phi_3 = \phi_4 = 1$. In model (B), the wage bill is financed with bond issues, and gross investment is financed with current period revenues, or $\phi_2 = \phi_3 = 1$ and $\phi_4 = 0$. In model (C), gross investment is financed with bonds, and the wage bill is financed out of current revenues, or $\phi_2 = \phi_4 = 1$ and $\phi_3 = 0$. In models (D) and (E), the firm's entire working capital expenses, i.e., both its gross investment and the wage bill, are financed with bonds, implying $\phi_2 = 1$ and $\phi_3 = \phi_4 = 0$. In models (B)–(E), the firm must retire the bonds issued in the current period by using current period revenues to pay the principal and interest, based on the current period interest rate, r_{bt}.

Given the firm's stock of capital, k_0, and the outstanding stock of one-period bonds, B_0^s, the firm chooses its demand for labor, or the sequence $\{n_t^d\}$, makes its gross investment decision by choosing the next period's capital stock, or the sequence $\{k_{t+1}\}$, and the quantity of bonds needed for financing its working capital, given by the sequence $\{B_t^s\}$.

D. Recursive Representation of the Firm's Optimization Problem

To normalize the nominal variables by M_t, define $b_t^s = B_t^s / M_t$, and the firm's problem becomes

$$\max_{\{k_{t+1}, n_t^d, b_t^s\}} \sum_{t=0}^{\infty} \beta^{t+1} (u_{c_{t+1}} / p_{t+1}) (G_{t+1}) \pi_t^f. \tag{18}$$

Given k_0 and b_0^s, the firm's normalized period profits are given by

$$\pi_t^f = p_t F(k_t, n_t^d) - \phi_2 (1 + r_{bt}) b_t^s - \phi_3 p_t [k_{t+1} - (1 - \delta) k_t] - \phi_4 w_t n_t^d, \tag{19}$$

subject to the financing constraint

$$(1 - \phi_3) p_t [k_{t+1} - (1 - \delta) k_t] + (1 - \phi_4) w n_t^d \leq \phi_2 b_t^s. \tag{20}$$

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For a recursive representation of the firm's problem, using the prime ($'$) notation, define the firm's value function as $V(s^f)$, where the firm's state vector is given by $s^f = [k, b^s, S]$. Bellman's equation becomes

$$V(s^f) = \sup_{\lambda^f(s^f) \in \Gamma^f(s^f)} [R + \beta V(s^{f'})], \qquad (21)$$

where the one-period return function is $R = \beta(u'_{c_1}/p')(1/G')\pi^f_t$ and the vector of the firm's decision rules, denoted $\lambda^f(s^f) = [k'(s^f), n^d(s^f), b^s(s^f)]$, is drawn from the feasible set of correspondences $\Gamma^f(s^f)$ defined by the financing constraint, equation (20).

The Euler equations for the firm's problem depend on the method of financing its working capital. For model (A), all working capital is financed out of current revenues, $\phi_2 = 0$ and $\phi_3 = \phi_4 = 1$, and the Euler equations become

$$\beta^2(u''_{c_1}/p'')(1/G'')p'[F'_k + (1 - \delta)] = \beta(u'_{c_1}/p')(1/G')p. \qquad (22)$$

$$pF_n = w. \qquad (23)$$

Equations (22) and (23) are the efficiency conditions on the capital investment and labor decisions of the firm. As an example, the utility loss to the household of forgoing current period profits sufficient to purchase one unit of capital at the normalized price of p, is given by (after discounting) the right-hand side of equation (22). The left-hand side is the net benefit to the household of this investment, which consists of two terms. The first is the discounted value of additional output made possible in the next period, and the second is the discounted value of the additional units of undepreciated capital stock remaining (or the amount by which the firm could reduce output in the next period and shift revenues toward increasing its dividend payout). At the margin, the firm is indifferent between making the investment and raising dividends, implying that equation (22) holds with equality. Equation (23) has a similar interpretation, albeit this is not a dynamic choice and both costs and benefits of employing the marginal unit of labor have the same discount factor that appears on both sides of equation (23), and thereby cancels. The firm equates the marginal revenue product of labor in the current period, given on the left-hand side, to the marginal factor cost, or wage rate on the right-hand side.

For model (B), the firm finances its gross investment out of current revenues (retained earnings), $\phi_3 = 1$, and finances its wage bill by issuing one-period bonds, $\phi_2 = 1$ and $\phi_4 = 0$. In this case, the Euler equation on the capital investment decision, equation (22), remains unchanged; however, the employment decision now reflects the costs of financing. Equation (23) is thus replaced by equation (24).

$$pF_n = (1 + r_b)w. \qquad (24)$$

The right-hand side of equation (24) reflects the present discounted value of the cost of employing one unit of labor, where w is the (normalized) price of labor, each unit of which is financed by bonds that mature one period hence. Note that the value of the claim against the firm that is represented by the bond incorporates a one-period

discount, since the currency that the firm uses to refinance the bond is received by the household but cannot be used for another period due to the liquidity constraint. The left-hand side of equation (24) is the additional dividends that the firm can pay out in the current period due to the increase in revenues from the higher output associated with the increase in employment. The costs and benefits must be equal at the margin, implying that equation (24) holds with equality.

For model (C), the firm finances its wage bill out of current revenues, $\phi_4 = 1$, and finances its gross investment from funds raised by issuing one-period bonds, $\phi_2 = 1$ and $\phi_3 = 0$. In this case, the optimal employment decision is identical to model (A), and given by the Euler equation (23). The gross investment decision must reflect the financing costs, and appears as equation (25):

$$\beta^2(u''_{c_1}/p'')(1/G'')p'[F'_k + (1 + r'_b)(1 - \delta)] = \beta(u'_{c_1}/p')(1/G')(1 + r_b)p. \quad (25)$$

The right-hand side of equation (25) has an interpretation that is analogous to that given for the right-hand side of equation (24), where the firm is financing the marginal unit of capital investment at the normalized price of p. The left-hand side of equation (25) gives the total present value of the benefits to the marginal investment unit, with the first term reflecting the increase in potential dividend payout in the next period due to the greater output and revenues. The second term corresponds to the value of the additional undepreciated capital stock that enables the firm in the next period to reduce its investment, along with the financing costs. The two-period discount reflects the fact that the additional capital stock reduces the next period's financing requirements.

In models (D) and (E), both the wage bill and gross investment are financed by issuing one-period bonds. In these cases, the appropriate Euler equations for capital investment and employment must reflect these financing costs, and are thus given by equations (24) and (25), respectively.

E. Commercial Banking Sector

In model (E), the commercial bank, standing in for a perfectly competitive industry, is introduced to provide valued liquidity services in the form of demand deposit accounts, and to intermediate loans between households and firms. Like firms, the bank is owned by households and, in the absence of agency costs, its objective is to maximize the present discounted value of the stream of dividends, or nominal period profits *per capita*, $\prod_t^{cb}$.

$$\max_{\{Z_{t+1},\, B^{db}_{t+1},\, X_{t+1}\}} \sum_{t=0}^{\infty} \beta^{t+1}(u_{c_{1t+1}}/P_{t+1})\prod_t^{cb}, \quad (26)$$

where Z_t is the bank's *per capita* reserves, B_t^{db} is the *per capita* stock of bonds purchased from the firm by the bank, and X_t is its *per capita* stock of deposits. The bank profits are given by its net cash flow:

$$\Pi_t^{cb} = (1 + r_{bt})B_t^{db} + Z_t - (1 + r_{xt})X_t - \xi X_t, \quad \xi > 0, \tag{27}$$

with cash inflow equal to the principal plus interest received on bonds, $(1 + r_{bt})B_t^{db}$, plus its reserve holdings Z_t less the principal and interest paid on deposits, $(1 + r_{xt})X_t$, less its cost of servicing deposits ξX_t, with ξ the cost per currency unit of deposits.

The bank takes the initial balance sheet as given, or Z_0, B_0^{db}, X_0, and performs the maximization in equation (26) by choosing optimal sequences $\{Z_{t+1}\}$, $\{B_{t+1}^{db}\}$, $\{X_{t+1}\}$, subject to its reserve requirements and balance-sheet constraints:

$$Z_t = \zeta X_t, \quad \zeta \in (0, 1), \tag{28}$$

$$Z_t + B_t^{db} \leq X_t, \tag{29}$$

where ζ is the reserve ratio applied to deposits.

The simplifying assumption that the bank pays out its entire net cash flow each period as dividends renders the bank's problem a static one-period optimization. This problem can be written in prime ($'$) notation with normalized variables by defining $z = Z/M$, $b^{db} = B^{db}/M$, and $x = X/M$.

$$\max_{z', \, b^{db'}, \, x'} \pi^{cb'}, \tag{30}$$

where

$$\pi^{cb'} = (1 + r_b')b^{bd'} + z' - (1 + r_x')x' - \xi x', \tag{31}$$

subject to

$$z' = \zeta x', \tag{32}$$

$$z' = b^{db'} \leq x'. \tag{33}$$

The first-order condition to this problem is

$$(1 + r_x') = (1 - \zeta)(1 + r_b') + \zeta - \xi, \tag{34}$$

which establishes the spread between the loan (or bond) rate and the deposit rate that ensures bank profits are dissipated through competition.

III. Equilibrium

This chapter defines the equilibrium and identifies the set of equations that must be solved to obtain the steady-state values for each of the five models described above.

A. Defining Equilibrium

Define the aggregate state vector as S = $[K, b, z, x]$, where K and b are the aggregate *per capita* stocks of capital and bonds. Then, aggregate decision rules corresponding to the decision rules of the representative household and representative firm can be defined as $\Lambda^h(S) = [C_1(S), C_2(S), N^s(S), L(S), \hat{m}^{d'}(S), \hat{x}^{d'}(S), \hat{b}^{db'}(S)]$ and $\Lambda^f(S) = [K'(S), N(S), b(S)]$, respectively, where C_1, C_2, N^s, L, $\hat{m}^{d'}$, $\hat{x}^{d'}$, $\hat{b}^{db'}$, K', N, and b are aggregate *per capita* variables. To construct the equilibrium, a state-dependent monetary policy rule needs to be specified. Since this paper is interested in the effects of steady-state inflation on welfare under various financing constraints on firms' working capital expenses, the gross monetary growth rate is taken as a constant across states, or $G'(S) = G > \beta$. The inequality ensures that money will be valued in equilibrium. Note that for model (E), where monetary policy is conducted through reserves injections into the banking system, the monetary rule implies that G also equals the gross growth rate of bank reserves.

A *recursive competitive equilibrium* is defined by the value functions $v(s^h)$ and $V(s^f)$, the household decision rules, $\lambda^h(s^h)$, the firm's decision rules, $\lambda^f(s^f)$, the corresponding aggregate decision rules, $\Lambda^h(S)$ and $\Lambda^f(S)$, the pricing functions, $p(S)$, $w(S)$, $r_b(S)$, and $r_x(S)$, and the policy rule, $G'(S)$, that satisfy

   (i) household optimization, equations (12)–(14);
   (ii) firm optimization, equations (22)–(23) for model (A), equations (22) and (24) for model (B), equations (23) and (25) for model (C), and equations (24)–(25) for models (D) and (E);
   (iii) commercial bank optimization, equation (34) in model (E);
   (iv) liquidity constraints, equation (8) and in model (E), equation (9);
   (v) the time resource constraint, equation (10);
   (vi) the firm's financing constraint, equation (20) in models (B)–(E);
   (vii) the bank's reserve requirements, equation (32) in model (E);
   (viii) the bank's balance-sheet constraint, equation (33) in model (E);
   (ix) aggregate consistency conditions, or $\lambda^h(s^h) = \Lambda^h(S)$ and $\lambda^f(s^f) = \Lambda^f(S)$, $\forall S$; and
   (x) equilibrium conditions in the goods, labor, money, bond (in models [B]–[E]), and deposit (in model [E]) markets: $C_1(S) + C_2(S) + K'(S) - (1 - \delta)K = F(K, N(S))$, $N^s(S) = N(S)$, $\hat{m}^d = 1$, $\hat{b}^{db} + b^{db} = b$, and $\hat{x}^d = x$.

B. Steady-State Equilibria

Each of the models described above has been rendered stationary by normalizing the nominal variables by the nominal money supply. This implies that the steady-state equilibria will be characterized by constants for the consumption bundle, labor, leisure, and normalized asset stocks. The equilibria are then found as the solutions to the following sets of equations, when modified by toggling appropriately the indicator variables, $\phi_1 - \phi_4$.

1. From the household sector

$$pC_1 = 1. \tag{35}$$

$$pC_2 = x \text{ (for model [E]).} \tag{36}$$

$$N + L = 1. \tag{37}$$

$$\beta(u_{C_1}/p) = G(u_L/w). \tag{38}$$

$$\beta[r_x(u_L/w) + (u_{C_2}/p)] = G(u_L/w) \text{ (for model [E]).} \tag{39}$$

$$\beta(1 + r_b) = G \text{ (for models [B]–[E]).} \tag{40}$$

2. From the firm sector

$$wN = b \text{ (for model [B]).} \tag{41}$$

$$p\delta K = b \text{ (for model [C]).} \tag{42}$$

$$p\delta K + wN = b \text{ (for models [D]–[E]).} \tag{43}$$

$$\beta[F_K + (1 - \delta)] = 1 \text{ (for models [A] and [B]).} \tag{44}$$

$$pF_N = w \text{ (for models [A] and [C]).} \tag{45}$$

$$pF_N = (1 + r_b)w \text{ (for models [B], [D], and [E]).} \tag{46}$$

$$\beta[F_K + (1 + r_b)(1 - \delta)] = (1 + r_b) \text{ (for models [C], [D], and [E]).} \tag{47}$$

3. From the banking sector

$$z = \zeta x \text{ (for model [E]).} \tag{48}$$

$$z + b^{db} = x \text{ (for model [E]).} \tag{49}$$

$$(1 + r_x) = (1 - \zeta)(1 + r_b) + \zeta - \xi \text{ (for model [E]).} \tag{50}$$

4. From equilibrium conditions

$$C_1 + \delta K = F(K, N) \text{ (for models [A]–[D]).} \tag{51}$$

$$C_1 + C_2 + \delta K = F(K, N) \text{ (for model [E]).} \tag{52}$$

$$\hat{b}^{db} + b^{db} = b \text{ (for model [E]).} \tag{53}$$

Note that in all models, the cash-in-advance constraint, equation (35), coupled with the definition of G as the gross growth rate of the money supply, implies that the steady-state inflation rate is also equal to G. That is, from equation (35), $p = 1/C$ = constant in the steady-state. This implies that the money price of goods and the nominal money supply are growing at the same rate, i.e., at the gross inflation rate, $P'/P = G$.

IV. Calibration and Computation of the Steady-State

To perform the calibration, the economies studied are specialized with common preferences and technology. The utility function is assumed to be loglinear, with the indicator variable $\phi_1 = 1$ for model (E), and zero otherwise.

$$u(C_1, C_2, L; \phi_1) = \ln C_1 + \eta\phi_1 \ln C_2 + \gamma \ln L, \quad \eta, \phi_1 > 0, \quad \phi_1 \in \{0, 1\}. \quad (54)$$

The marginal utilities of consumption of the cash and deposit goods and of leisure are thus given by $u_{C_1} = 1/C_1$, $u_{C_2} = \eta\phi_1/C_2$, and $u_L = \gamma/L$, respectively.

A Cobb-Douglas production function is assumed for all models.

$$F(K, N) = K^\alpha N^{1-\alpha}, \alpha \in (0, 1). \quad (55)$$

The marginal products of capital and labor are respectively $F_K = \alpha K^{1-\alpha}N^{1-\alpha}$ and $F_N = (1 - \alpha) K^\alpha N^{-\alpha}$.

In each of the models (A)–(D), there are four parameters to be determined: β, γ, δ, and α, along with the exogenously set policy variable, G. To determine these values, five conditions must be imposed from the data. First, α is taken as capital's share of income, and computed from the Annual Report on National Accounts (ARNAs) as the sample (quarterly) average over the period from January 1970 to January 1996 to be 0.3916.[5] Second, the investment-output ratio is fixed at 0.2114, which is the quarterly average over the same sample period, where investment is defined as gross private domestic investment. Third, the steady-state value of N is set to 0.336, which corresponds to a 40-hour workweek. Fourth, the policy parameter G is set equal to 1.010752, which is the average quarterly gross inflation rate over the sample period.[6] Finally, for consistency, equation (40) is used to determine a value for the discount factor β that was used throughout all five models. Given G, it is found by setting r_b to 0.0117419. This value is obtained by computing the average quarterly interest rate paid on ordinary deposits over the (monthly) sample period from February 1971 to January 1996, and adding to that figure the average spread of the

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5. Following Cooley and Prescott (1995), a share of income to the self-employed attributed to labor is imputed from the data to be identical to that for publicly held firms. The following equation is used to determine α in each period based on the items in the ARNAs. These values are then averaged over the sample. Compensation of employees [item 3.3] + $(1 - \alpha)$ (entrepreneurial income [item 3] + indirect taxes and subsidies [item 5] + income of public enterprises [item 3.2]) = $(1 - \alpha)$ GNP.

6. This inflation rate is based on the monthly Consumer Price Index. Source: *Monthly Report on the Consumer Price Index*, Statistics Bureau, Management and Coordination Agency.

(backward three-month moving-average) of the three-month commercial paper rate (primary market) over the deposit rate for the period from January 1989 to November 1996, which is the only period for which these data are available.[7] The average quarterly deposit rate was 0.0035668 and the average spread was 0.008175.

For model (E), there are three additional parameters: η, ζ, and ξ. The three additional restrictions from the data that are used in the calibration of the model are (1) the average deposit rate, r_x, is computed directly as described above; (2) the average currency-deposit ratio over the sample period, where deposits are taken as the non-currency components of M1, is set equal to M/X or equivalently $1/x$; and (3) the average ratio of bank reserves to total deposits (as defined above) over the (monthly) sample period from January 1970 to August 1997 is set equal to the reserve ratio, or $\zeta = 0.041063$.[8]

The above data restrictions enable the models to be solved in the steady-state under the "benchmark" inflation policy, G, which corresponds to the average policy over the period from January 1970 to August 1997. Model (A) consists of the six equations (35), (37)–(38), (44)–(45), and (51), and six endogenous variables: C_1, N, L, K, p, and w. Model (B) is comprised of the eight equations (35), (37)–(38), (40)–(41), (44), (46), and (51), the six endogenous variables of model (A), and b and r_b. Model (C) consists of the eight equations (35), (37)–(38), (40), (42), (45), (47), and (51), and the same set of endogenous variables as in model (B). Model (D) is made up of the eight equations (35), (37)–(38), (40), (43), (46)–(47), and (51) in the same set of eight endogenous variables as in models (B) and (C). Model (E) is comprised of the 14 equations (35)–(40), (43), (46)–(50), and (52)–(53), with the following six endogenous variables added to the list from models (B), (C), and (D): C_2, b^{db}, b^{db}, z, x, and r_x. The steady-state values for each of these models are displayed in Table 1.

Table 1 Benchmark Settings

Benchmark gross inflation rate
$G = 1.010752$
Parameters
$\beta = 0.999021613$, for all models
$\alpha = 0.03916$, for all models
$\delta = 0.0011489$, for models (A) and (B)   0.0017860, for models (C), (D), and (E)

$\gamma = 1.50692$, for models (A) and (C)
     1.49436, for models (B) and (D)
     6.20527, for model (E)

$\eta = 3.15503$, for model (E)
$\zeta = 0.041063$, for model (E)
$\xi = 0.007693$, for model (E)

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7. In choosing to fit the interest rates to the data, the calibration yields artificially low values for the depreciation rate and the capital/output ratio. Had the model been calibrated to the latter, the bond rate would have been unrealistically high. Source: *Economic Statistics Monthly*, Research and Statistics Department, Bank of Japan.

8. Source: *Economic Statistics Monthly*, Research and Statistics Department, Bank of Japan.

Steady-state values					
Variables	Model (A)	Model (B)	Model (C)	Model (D)	Model (E)
Output, y	9.641	9.641	9.483	9.483	9.483
Consumption, C	7.603	7.603	7.479	7.479	7.479
Consumption, C_1					1.795
Consumption, C_2					5.684
Investment	2.038	2.038	2.005	2.005	2.005
Capital, K	1,773.869	1,773.869	1,700.933	1,700.933	1,700.933
Employment, N	0.336	0.336	0.336	0.336	0.336
Real bonds, b/p		5.797	2.005	7.707	7.707
Household share of bonds, b^{dh}/b					0.293
Bank share of bonds, b^{db}/b					0.707

From Table 1, it is noteworthy that the steady-state values for the five models are nearly identical, with the exception of the size of the bond market. Comparing the quantity of real bonds that the firm must issue to meet its financing requirements in models (B)–(D), it is evident that under these calibrations the firm's working capital expenses comprise roughly three-quarters of its wage bill. Also note that the steady-state equilibria of models (D) and (E) are identical in all respects other than the fact that in model (E) the household finances only 30 percent of the firm's working capital requirements directly. The bank provides the balance of the firm's financing with funds that it raises by attracting household savings into its bank deposit offerings.

V. Quantifying the Welfare Cost of Inflation

In this chapter, the welfare costs of moderate inflation rates are quantified for each of the five models. The procedure follows Cooley and Hansen (1989), where the deep parameters of the model determined in the calibration are held fixed, and the inflation policy is altered over a range of values for the steady-state gross inflation rate, G. The new steady-states are determined under the alternative policies, and welfare losses are measured by comparing lifetime utility under an alternative policy with that under the benchmark steady-state. Specifically, the welfare loss associated with an increase in G above the benchmark inflation is measured as the percent increase in period consumption that the household would require under the higher inflation regime to be indifferent between it and the benchmark policy.

The purposes of conducting these exercises are twofold. First, the exercises demonstrate how important financing constraints can be in determining the welfare costs of inflation. As discussed below, their effect is very pronounced. Second, comparisons between models (D) and (E) illustrate how significant reductions in the welfare costs of inflation can be realized when banks intermediate the loans between households and firms while providing valued liquidity services to households in the form of demand deposit accounts.

A. Financing Constraints and the Welfare Costs of Inflation

Model (A) is the standard cash-in-advance economy that has been analyzed extensively in the literature. One feature of the model is that fully anticipated inflation acts as a tax on consumption purchases. As the inflation tax increases, households seek to reduce their consumption expenditures, and they partially offset the accompanying utility loss by increasing leisure time. This additional leisure comes at the expense of labor, and employment and hence output falls. The magnitude of these effects has been documented by Cooley and Hansen (1989) in a model calibrated to fit the U.S. postwar economy.[9] They find the measured welfare costs of this inflation tax distortion to be significant, but not excessive for moderate inflations. For example, increasing the inflation rate from 0 percent to 10 percent resulted in a welfare loss of a 0.376 percent reduction in period consumption. Similarly, relatively low costs are found for model (A) when calibrated to the Japanese economy. As seen in Table 2, column 2, an increase in the inflation rate from the benchmark value of 4.3 percent to 10 percent results in an estimated loss of welfare that corresponds to a 0.226 percent reduction in period consumption.[10] From Table 3, column 2, this welfare loss corresponds to a reduction of 0.92 percent in investment (from 2.0380 to 2.0191) and a decline of 0.93 percent in both employment (from 0.3360 to 0.3329) and output (from 9.6406 to 9.5512).

These welfare costs of inflation increase considerably when financing constraints are imposed on the working capital expenses of the firm. As previously discussed, higher inflation results in a higher bond rate that raises the firm's financing costs. This further retards investment and reduces employment, as output declines. The magnitude of these losses varies with the incidence of the "tax." This is illustrated in Table 2 by the figures reported in the bottom row of columns 3, 4, and 5. When the firm finances its wage bill with one-period bonds and its gross investment out of current period sales revenues, model (B) (column 3), an increase in the inflation rate

Table 2 Welfare Costs of Inflation

Welfare losses measured as percent reduction in period consumption					
Inflation rate [100 percent × (G − 1)]	Model (A)	Model (B)	Model (C)	Model (D)	Model (E)
3.00	−0.0505	−0.1042	−0.1847	−0.2387	−0.2006
4.30	0	0	0	0	0
5.00	0.0273	0.0569	0.0997	0.1295	0.1085
6.00	0.0666	0.1392	0.2427	0.3164	0.2644
7.00	0.1062	0.2228	0.3863	0.5049	0.4212
8.00	0.1460	0.3075	0.5305	0.6953	0.5790
9.00	0.1861	0.3936	0.6752	0.8875	0.7376
10.00	0.2264	0.4807	0.8205	1.0814	0.8872

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9. Their model differs from model (A), in that they include labor indivisibilities as described in Rogerson (1988) and Hansen (1985).

10. The welfare loss is computed by obtaining steady-state values for consumption, and leisure under the benchmark case, denoted $\bar{c}_1$, $\bar{c}_2$, $\bar{l}$, and again under the alternative inflation policy, denoted $\tilde{c}_1$, $\tilde{c}_2$, $\tilde{l}$, and then solving the following equation for ϕ_{ch}: $\sum_{t=0}^{\infty} \beta^t u(\bar{c}_1, \bar{c}_2, \bar{l}) = \sum_{t=0}^{\infty} \beta^t u[(1 + \phi_{ch})\tilde{c}_1, (1 + \phi_{ch})\tilde{c}_2, \tilde{l})]$. The welfare costs reported in Table 2 are then given by 100 percent times ϕ_{ch}.

Table 3 Steady-State Effects of Inflation

Steady-state values at the benchmark 4.3 percent and 10 percent inflation					
Variables	Model (A)	Model (B)	Model (C)	Model (D)	Model (E)
Output					
4.30	9.6405	9.6406	9.4834	9.4834	9.4834
10.00	9.5511	9.4622	9.2881	9.2014	9.2646
Consumption					
4.30	7.6025	7.6025	7.4786	7.4786	7.4786
10.00	7.5320	7.4619	7.3579	7.2833	7.3333
Consumption, cash good					
4.30					1.7951
10.00					1.7423
Consumption, deposit good					
4.30					5.6835
10.00					5.5910
Investment					
4.30	2.0380	2.0380	2.0048	2.0048	2.0048
10.00	2.0191	2.0003	1.9362	1.9181	1.9313
Employment					
4.30	0.3360	0.3360	0.3360	0.3360	0.3360
10.00	0.3329	0.3298	0.3321	0.3290	0.3312
Real bonds					
4.30		5.7972	2.0048	7.7075	7.7075
10.00		5.6109	1.9362	7.3744	7.4250
Household share of bonds					
4.30					0.2929
10.00					0.2779
Bank share of bonds					
4.30					0.7071
10.00					0.7221

from the benchmark 4.3 percent to 10 percent results in a more than doubling of the welfare losses relative to the case of no financing constraints, model (A) (column 2), i.e., from 0.226 percent to 0.481 percent. The corresponding percentage declines in investment (from 2.0380 to 2.0003), employment (from 0.3360 to 0.3298), and output (from 9.6406 to 9.4622) are also seen essentially to double to 1.85 percent. Alternatively, when gross investment is financed with bond issues, while the wage bill is financed out of current sales revenues, model (C), the welfare losses increase to a 0.821 percent reduction in period consumption, which is more than 3.5 times that of model (A). The additional distortion in the firm's factor employment decisions is reflected quantitatively in Table 3, where production is seen to become more labor intensive, with investment declining by 3.42 percent (from 2.0048 to 1.9362), while employment falls by 1.16 percent (from 0.3360 to 0.3321), and output drops by 2.06 percent (from 9.4834 to 9.2881). In this case, even though investment represents only about one-quarter of the firm's working capital expenses, the effective tax on capital has a more adverse effect on welfare than when this effective tax applies only to labor. As expected, applying the financing constraint to both the wage

bill and gross investment, model (D), further increases the welfare losses. They are seen in column 5 to rise to the sizable figure of a 1.081 percent loss in period consumption, which is more than 4.75 times the losses computed for model (A), when no financing constraints apply. Investment falls by 4.33 percent, employment declines by 2.08 percent, and output is reduced by 3.01 percent.

B. Financial Intermediation and the Welfare Costs of Financing Working Capital

When the firm's financing requirements expand, the size of the bond market grows accordingly. Again noting that roughly three-quarters of the firm's working capital expenses go to pay the wage bill, it is evident that the bond market would be about three times larger in model (B) than in model (C), and about four times larger in model (D) than in model (C). These figures are borne out in Table 3. In each of these cases, there was no financial intermediary, that is, lending was direct from households to firms. In model (E), a bank is introduced through which a portion of these loans is intermediated. The extent to which banks are able to provide financing to firms is limited by household demand for demand deposit account offerings. This demand for bank deposits arises out of both the pecuniary return and the liquidity services that they yield. Because the liquidity services are valued by households, the deposit rate can be held below the bond rate, even when these liquidity services are costly for the bank to provide. In this case, households can respond to an increase in the inflation rate by relying more on interest-bearing bank deposits for their transactions and less on cash, thereby reducing the distortionary effects of the inflation tax, and the welfare losses are lower than they would otherwise be in the absence of the bank.

Refer to Table 2, columns 5 and 6. By introducing the bank into the model, the welfare losses associated with an increase in the inflation rate from the benchmark 4.3 percent to 10 percent are seen to fall by 17.96 percent (from a 1.0814 percent loss in period consumption to 0.8872 percent). This sharp decline is accompanied by smaller reductions in investment of 3.67 percent (from 2.0048 to 1.9313) versus 4.33 percent, in employment of 1.43 percent (from 0.3360 to 0.3312) versus 2.08 percent, and in output of 2.31 percent (from 9.4834 to 9.2646) versus 3.01 percent. Note that the financing requirements of the firm are higher under the 10 percent inflation regime when banks intermediate a portion of the loans. That is, with an increase in the inflation rate to 10 percent from 4.3 percent, the size of the bond market (in real terms) declines by 3.67 percent in model (E), versus 4.33 percent in model (D). This result is simply due to the fact that output and hence the working capital expenses of the firm are not as adversely affected by inflation in model (E). Moreover, note that the shift in the short-term liquid asset holdings of households lowers the currency-deposit ratio, and that this portfolio adjustment of households results in an increase in the share of loans to firms that are intermediated by the bank, which rises from 70.71 percent to 72.21 percent, or by about 2.1 percent. Therefore, a byproduct of inflation is a tendency for the size of the banking sector to expand.[11]

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11. This result is consistent with those of Ireland (1994) and Marquis and Reffett (1994), whereby an increase in inflation induces a shift of resources into the financial services sector and generates welfare losses.

VI. Conclusions

Per capita holdings of highly liquid monetary assets that carry a negative real return are very high in Japan. Inflation lowers this return and serves to tax transactions for which these assets are used. While individuals can avoid these taxes by actively managing their asset portfolios, the economy as a whole cannot. This suggests that high inflation may distort resource allocations sufficiently to induce significant welfare losses. The results in the previous chapter suggest that these welfare losses can become quite large for even moderate rates of inflation, when firms use short-term debt instruments to finance their working capital expenses.

However, when banks offer valued liquidity services in the form of interest-bearing demand deposit accounts, households are able to shield themselves somewhat from inflation, and the accompanying distortions in resource allocations are reduced. As inflation increases, households rely more on bank deposits and less on currency for transaction purposes, and the size of the banking sector relative to the size of the economy (or real deposits/output) expands, as banks intermediate a larger share of working capital loans to firms. These results underscore the important role that financial intermediation can play in affecting the welfare consequences of inflation, and they suggest that extensions of this analysis to private information economies— in which banks perform roles as delegated monitors of risky short-term debt, or as efficient players in risk-transferring financial markets with high participation costs—would be a fruitful avenue for future research.

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