Extracting Market Expectations from Option Prices: Case Studies in Japanese Option Markets

Hisashi Nakamura and Shigenori Shiratsuka

This paper focuses on the recently developing financial derivatives markets, and examines the usefulness of option prices as an information variable for monetary policy implementation. A set of option prices provides us with information on the entire probability distribution of the future values of underlying assets. Such information enables us to examine the development of market expectations. The paper estimates a time series of implied probability distributions from daily option prices on stock price index and long-term government bond futures in Japan. The estimation is done for a sample of daily closing prices for the following three periods: (1) the period of a collapsing "bubble" in the stock market in 1989–90; (2) the period of serious stock market slump in 1992–94; and (3) the period of increasing anxiety in the market about a possible deflationary spiral in 1995.

Key words: Option prices; Implied probability distribution; Market expectations; Monetary policy; Information variables

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I. Introduction

In this paper, we focus on the recently developing financial derivatives markets in Japan and examine the usefulness of option trading prices as an information variable for monetary policy implementation. By estimating the entire probability distribution of future asset prices from a set of option prices and tracking the changes in that distribution over time, we examine the development of market expectations and how monetary policy has affected that development.

In general, asset prices reflect market participants' expectations about the future. During the "bubble" era, Japan experienced large fluctuations both in asset prices and in the real economy, and market expectations seem to have played an important role behind the scenes. For example, market expectations became bullish during the emergence and expansion of the asset price bubble and remained almost unchanged even when interest rates were revised upward. In such circumstances, a substantial rise in interest rates is required in order to change market expectations. To put it differently, the effects of tight monetary policy, even accompanied by quite high interest rates, do not appear significant until market expectations change. Once such expectations have been revised, downward pressure on asset prices becomes extremely large because the revision of expectations intensifies the original effects of higher interest rates through the expansion of risk premiums.

We examine how market expectations concerning underlying asset prices are formed and how they change, using daily price data on the option markets in Japan. Specifically, we estimate the entire implied probability distribution of future values of underlying assets from a set of option prices with the same time-to-maturity, but with different exercise prices. This method enables us to obtain information about the dispersion of market expectations concerning asset price fluctuations, as well as about market participants' beliefs about the direction of market price changes and the probability of an extreme outcome in the market.

We study the practical usefulness of this methodology through case studies involving Japanese option markets. Given the data constraints, three periods have been selected: (1) the period of the collapsing "bubble"[1] from the second half of 1989 to 1990; (2) the period of serious stock market slump from 1992 to early 1994; and (3) the period of increasing anxiety in the market about the deflationary spiral in 1995. In each period, we consider on a time-series basis (1) how market expectations of stock prices and long-term interest rates changed; (2) how and when the relationships between such market expectations and economic indicators changed; and (3) how monetary policy implementation affected such market expectations.

This paper is composed as follows. In Chapter II, we present the basic framework for examining the development of market expectations. We discuss the estimation methodology of implied probability distributions, based on the limitation in readily available data in Japanese option markets. In addition, we conceptually study how we

1. It is interesting to examine how market expectations changed during the initial period of the "bubble" era. However, due to the fact that Nikkei 225 option trading started in June 1989 and Japanese government bond (JGB) futures option trading started in May 1990, this paper must limit its scope of analysis to the period after the "bubble" burst.

can interpret the changes in market expectations from the movements of summary statistics of the estimated implied probability distributions over time. In Chapter III, we conduct case studies for various episodes in Japanese financial markets from 1989 to 1995 to examine the practical usefulness of our methodology for extracting market expectations. We estimate the summary statistics of the implied probability distribution on a daily basis, and examine the changes in market expectations captured in estimation results, with reference to monetary policy implementation. In Chapter IV, we provide a conclusion with some policy implications. In the appendices, we show the specific calculation methods used for the implied probability distributions, as well as the basic framework of financial option trading in Japan.

II. Basic Framework for Analyzing Changes in Market Expectations

Since the 1970s, methods of extracting market expectations concerning future asset prices by using derivatives trading prices have been developed and are improving.[2] In this chapter, we first summarize various methodologies for extracting market expectations from data for financial markets. We also point out some data problems in conducting empirical analysis in the Japanese option markets. Then, we discuss which method is appropriate for application to Japanese option markets. Finally, we examine the interpretation of the estimates and the bias inherent in a discrete approximation.

A. Method for Extracting Market Expectations

We summarize the methodology used to extract market expectations from option prices based on past studies.

1. The method used for extracting market expectations

There are three methods used to analyze market expectations concerning future asset prices based on financial market data.

The first method is to estimate the implied forward rates from swap rates or government bond yields, and observe expected interest rates by maturity. The second method is, by using option price information, to observe the implied volatility estimated by the Black-Scholes Model (Black and Scholes [1973]), and to evaluate the dispersion of expectations concerning future asset prices. The third method also uses option price information, although it goes as far as deriving the entire risk-neutral implied probability distribution of future prices of the underlying assets.

By estimating the entire expectation distribution, the third method enables us to examine market expectations concerning future outcomes in detail. In other words, we can obtain information about the dispersion of market expectations concerning asset price fluctuations, as well as about market participants' beliefs about the direction of market price changes and the probability of an extreme outcome in the market. This method is

2. A comprehensive survey of these methods can be found in Neuhaus (1995), Söderlind and Svensson (1997), Bahra (1996, 1997), and Oda and Yoshiba (1998).

deemed useful for close monitoring of the impact of monetary policy on market expectations. In the following, we will discuss rather extensively the specifics of how to estimate the implied probability distribution.

2. Estimation methods for the implied probability distribution

The methods used to estimate the implied probability distribution fall into two categories according to whether or not they *ex ante* assume a specific distribution function.

Models that *ex ante* assume a specific distribution function include a model which assumes underlying asset prices follow a jump probability distribution process[3] (Malz [1996]; Bates [1991, 1996, 1997]), and a model which assumes a probability density function of underlying asset prices that follows a mixture of some lognormal distribution (Melick and Thomas [1997]; Bahra [1996]; Söderlind and Svensson [1997]; Söderlind [1998]). There are various other possible models that assume different probability distribution patterns. However, in the actual analysis, it is impossible to recognize *ex ante* the probability distribution of the underlying asset prices, and thus these models inherit the deficiency of not being able to preclude arbitrariness from the estimation process.

Other less-constrained models, in which a specific distribution pattern is not assumed *ex ante*, include a method of directly estimating the probability distribution of option price data by using finite difference methods[4] (Breeden and Litzenberger [1978]; Neuhaus [1995]), a method that utilizes a smoothing curve called the spline function (Oda and Yoshiba [1998]), and a method that applies kernel regression (Aït-Sahalia and Lo [1998]).

B. Application to Japanese Markets

Next, based on the features of Japanese option markets, we examine which estimation method to apply in our study. We use the Nikkei 225 option (a European-type option) and the Japanese government bond (JGB) futures option (an American-type option).[5] The underlying assets of those options are the Nikkei 225 Stock Average

3. It is not necessarily logical to assume that asset prices such as stock prices change continuously, since social and economic information emerges intermittently. In finance theory, a jump probability distribution process is often used to cope with discontinuous price changes. In detail, there are two types in a jump distribution process: a pure jump probability distribution process and a diffusion probability distribution process. A pure jump probability distribution process assumes that price changes follow a discrete Poisson process. A diffusion probability distribution process assumes that price changes follow a weighted average of the Wiener process (continual price changes) and the Poisson process (intermittent price changes).
4. Finite difference methods convert differential equations into a set of difference equations and the difference equations are solved iteratively, and are widely used to solve the partial differential that represents the derivatives price. To illustrate the approach, we consider a first-order differentiable continuous function $f(S)$. A number of equally spaced stock prices are chosen as $S_1, S_2, \ldots, S_k, \ldots$, and the width of the space as ΔS. The first-order differential $\partial f/\partial S|_{S=S_k}$ at point S_k can be approximated as a *forward difference approximation* $\{f(S_{k+1}) - f(S_k)\}/\Delta S$, a *backward difference approximation* $\{f(S_k) - f(S_{k-1})\}/\Delta S$, and a *central difference approximation* $\{f(S_{k+1}) - f(S_{k-1})\}/2\Delta S$. Among these, this paper adopts the central difference approximation, which is the one with a more symmetrical approximation. For details on the finite difference methods, see Chapter 15 of Hull (1997).
5. We apply the calculation method for the European-type option explained in Appendix 1 to the JGB futures option that is the American-type option. In general, calculation of the American-type option is more complicated than that of the European one, since buyers can arbitrarily exercise the option. However, Chen and Scott (1993) showed that the American-type interest rate futures option can be priced as the European-type option if sellers submit margins and the margins are marked to market every day. Since the JGB futures option applies a similar margin system, we believe that there should be no problem in treating it as the European-type option.

(hereafter, Nikkei 225) and the 10-year JGB futures, respectively (see Appendix 2 for the outline of each transaction).[6]

1. Limitations of available data for Japanese option markets

Some cautions are in order concerning the empirical analysis of Japanese option data. First, only a small number of opened strike prices are available. Since there is very small trading volume for the in-the-money (ITM) strike prices, reliability of price information is not entirely satisfactory.

Second, there is a limitation involved in using closing price data. Namely, using closing price data will not guarantee that the option premiums of different strike prices were traded at the same time. If there are substantial time differences in the traded times of closing prices across different strike prices, arbitrage may not function thoroughly. In this case, the probability density of the implied probability distribution will inherit certain biases.

Third, it has been pointed out that, during the transitional period of most actively traded contract months, trading prices will not fully reflect market expectations concerning future outcomes. As a result, the most actively traded contract month might not be the contract month with the largest trading volume.

2. Strategy to deal with the data problems

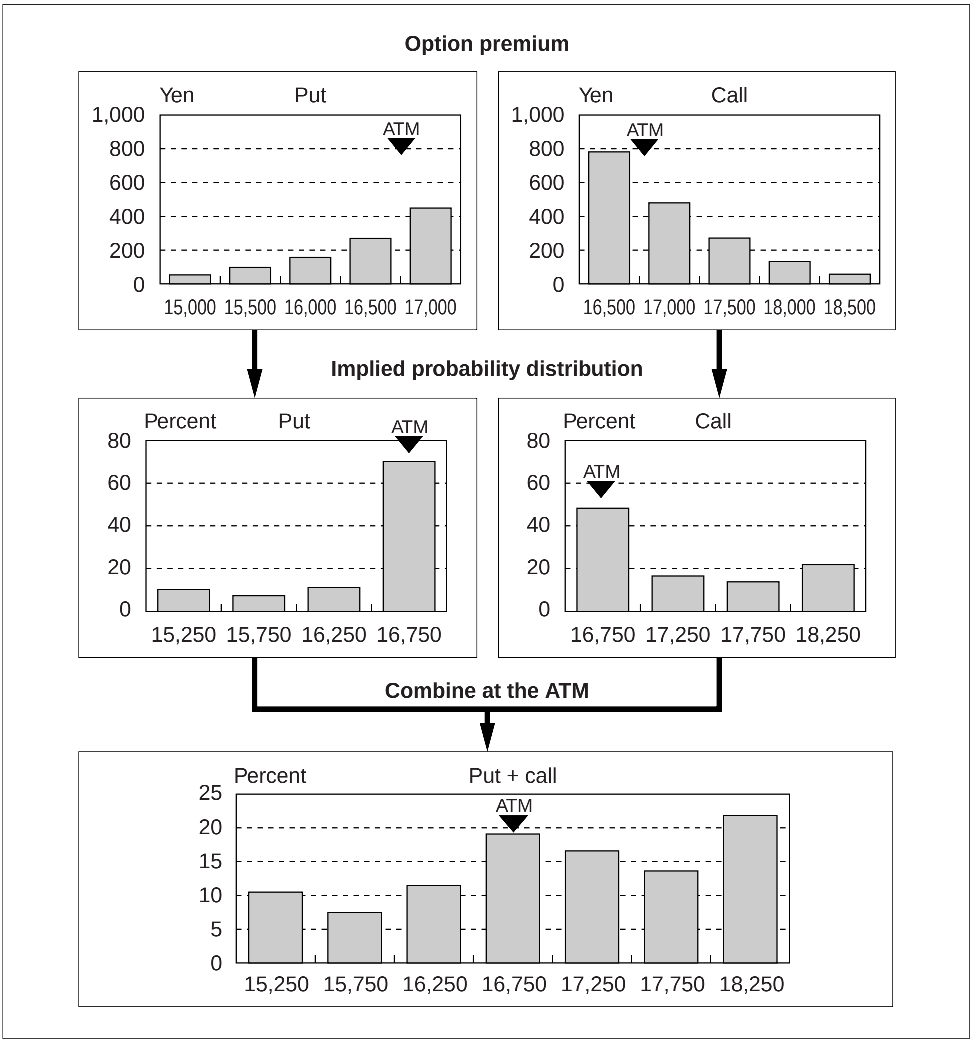
One possible strategy for overcoming the problems mentioned above is to impose some restrictions to guarantee that the results give a reasonable probability distribution, such as assuming a mixture of some lognormal, and a fitted spline function. Another strategy is to apply a simple discrete approximation method to carefully sorted data. We have used the latter strategy to estimate a time series of implied probability distributions with the following devised data treatment.[7]

First, we have applied a simple discrete approximation method. It is difficult to get a robust estimation result when we apply a complicated methodology to a small sample. Considering the lesser observations across the different strike prices in the Japanese option markets, advanced methodology is not necessarily required. In addition, a simple methodology may even give rise to clearer policy implications.

Second, we have used price data regarding both put and call options that are at-the-money (ATM) and out-of-the-money (OTM). We first calculate two probability distributions from the option premium for call and put options separately, then combine two probability distributions at the ATM strike price to form the complete probability distribution (see Figure 1 for the illustration of the estimation procedure). In this sense, our methodology has the merit of effectively utilizing trading price information by making the ATM strike price the boundary and using both put and call options on the OTM side. Therefore, there is an advantage in estimating the implied probability distribution using limited quotations of option premiums across different strike prices, such as Japanese option markets.

6. A Euroyen interest rate futures option has also been traded in Japan since July 1991, although due to the short overlapping period for the purpose of our analysis and the rather low trading volume, we judged that the credibility of price data on the Euroyen interest rate futures option markets is questionable and thus excluded it from our study.
7. Of course, it should be noted that a bias might be introduced, even though we apply a simple method to carefully sorted data, especially when the expiration date of the option approaches and the number of traded strike prices in the markets becomes smaller. Further research is required on the robustness of estimation results across the different methods for estimating implied probability distributions.

Figure 1 Estimation Method for Implied Probability Distributions



Third, we have excluded mispriced observations, such as those which result in a negative probability density, in estimating the complete probability distribution. As previously mentioned, using closing price data does not guarantee that the option premiums of different strike prices were traded at the same time, and arbitrage may not function thoroughly.

Fourth, we have excluded from our analysis as much as possible the data of the final week of each contract month. As a result, the most actively traded contract month might not be the contract month with the largest trading volume.

C. Framework for Interpretation of Changes in Market Expectations

In this paper, we examine the changes in market expectations by using a time series of four summary statistics or moments (mean, standard deviation, skewness, and excess kurtosis) of the estimated implied probability distributions. Then, after defining the four summary statistics, we discuss how we can interpret changes in market expectations from changes in these statistics. In addition, we discuss the possibility of biases induced by the discrete approximation method, which we employ in this paper.

1. Summary statistics and shape of distribution

We employ a time series of summary statistics of estimated implied probability distributions to investigate the changes in market expectations. Specifically, we obtain mean, standard deviation (Stdv), skewness (Skew), and excess kurtosis (Ex-Kurt) from the histogram of implied probability distribution.

Since stock and bond prices are both positive values, even when market expectations belong to the same population corresponding to the same probability space, the expectation distribution of future assets will follow a lognormal distribution. In other words, the distribution is expected to be skewed to the right. However, it is not convenient to use a lognormal distribution as a benchmark for evaluating the size of the above summary statistics. Therefore, we calculate these summary statistics by using the strike price converted into a logarithm, and employ a normal distribution as the benchmark for evaluating the four summary statistics. As shown in Appendix 1, if we use $p(K_i)$ obtained by the formulations (A.14) and (A.15), the four summary statistics can be shown as follows:

$$\text{Mean} = \sum \frac{\ln(K_i) + \ln(K_{i+1})}{2} p(K_i) \tag{1}$$

$$\text{Stdv} = \sqrt{\sum \left(\frac{\ln(K_i) + \ln(K_{i+1})}{2} - \text{Mean} \right)^2 p(K_i)} \tag{2}$$

$$\text{Skew} = \sum \left(\frac{\ln(K_i) + \ln(K_{i+1})}{2} - \text{Mean} \right)^3 p(K_i) \Big/ \text{Stdv}^3 \tag{3}$$

$$\text{Ex-Kurt} = \sum \left(\frac{\ln(K_i) + \ln(K_{i+1})}{2} - \text{Mean} \right)^4 p(K_i) \Big/ \text{Stdv}^4 - 3. \tag{4}$$

While the mean of the estimated risk-neutral implied probability shifts in parallel with the size of risk premium compared with the true probability distribution, the risk premium itself is difficult to estimate.[8] Thus, in the following, we focus on changes of moments higher than the first, i.e., mean, and examine their movements over time.[9]

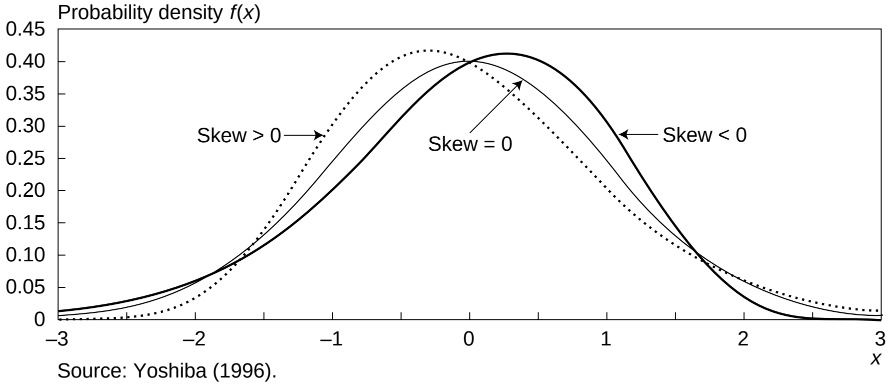
8. The model used in this paper assumes a risk-neutral world, while market participants in the actual market are not necessarily risk-neutral. In addition, it is likely that the risk preferences of market participants change over time. In that case, the (risk-neutral) implied probability distribution, estimated under the assumption of risk-neutral valuation, will differ from the true probability distribution. However, Cox and Ross (1976) claimed that, compared with the true probability distribution, the risk-neutral implied probability distribution shifts in parallel with the size of the risk premium, and thus will not affect moments higher than the second.

9. As Bates (1991) and others have pointed out, it is generally known that the standard deviation decreases as the maturity date of the contract is approached. Therefore, we try to control the impact of changes in time-to-maturity on the estimated time series of the standard deviation by multiplying the square root of (360/time-to-maturity) to obtain the annual rate.

The changes in market expectations captured by the movements of each summary statistic can be summarized as follows. From the standard deviation, we can see the dispersion of distributions, that is, how wide the market expectations are, which cannot be distinguished by only observing the mean.

Skewness is a statistic that shows the extent of asymmetry of the distribution: it is zero in the case of a symmetric distribution such as the normal distribution, positive when distribution is skewed to the right, and negative when skewed to the left (Figure 2).

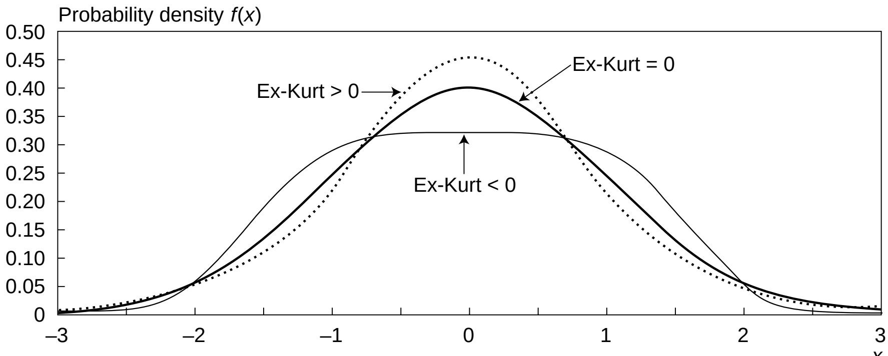
Figure 2 Skewness



Source: Yoshiba (1996).

Excess kurtosis is a statistic that shows the degree of sharpness of a probability density near its center and the likelihood of extreme outcomes: it is also zero in the case of a normal distribution. When positive, probability density at the center of distribution is high, and thus the shape of the distribution becomes sharp at the center and also fat-tailed. When negative, the distribution becomes flat at the center and light-tailed (Figure 3).

Figure 3 Excess Kurtosis



Source: Yoshiba (1996).

It might be worth explaining in detail what kind of information about changes in market expectations can be captured by the shift in the excess kurtosis. As the excess kurtosis increases, (1) the extent of sharpness at the center will increase; and (2) the degree of fatness at the tail will increase. If we compare such changes intuitively with an economic agent's expectation formation process, two interpretations are possible: (1) an agent is expecting that asset prices are not likely to change from the center of distribution; or (2) an agent is expecting that asset prices are likely to change substantially, namely, to take outlier values. Therefore, it is not clear whether we can draw any implications for economic agents' expectations by just observing the changes in the excess kurtosis.

In order to decide which of the above two interpretations we should take, we need to combine excess kurtosis with standard deviation, a measure that represents the dispersion of the whole distribution. Specifically, when there is no change or decrease in the standard deviation, an increase in the excess kurtosis will sharpen the probability density near its center, and can be interpreted as an increase in economic agents' confidence in the current market level. On the contrary, when there is an increase in the standard deviation, a rise in the excess kurtosis will make the tails of distribution fatter, and can be regarded as indicating a higher possibility of extreme outcomes. Such interpretations are summarized in Table 1.

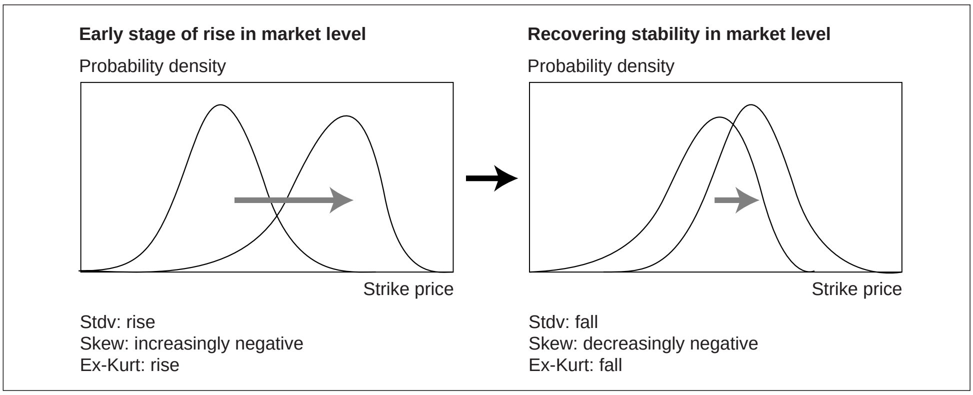
Table 1 Interpretation of Fluctuation in Standard Deviation and Excess Kurtosis

		Excess kurtosis	
		Rise	Fall
Standard deviation	Rise	Larger risks on price changes + stronger expectation of an extreme outcome	Larger risks on price changes + weaker confidence concerning the current price level
	Fall	Smaller risks on price changes + stronger confidence concerning the current price level	Smaller risks on price changes + weaker expectation of an extreme outcome

2. Interpretation of changes in summary statistics

When the market level fluctuates, how do the above summary statistics react, and through them, what kinds of changes in market participants' expectations can be grasped? Figure 4 shows an image of typical changes in probability distribution when the

Figure 4 Changes in Distribution of Market Expectations during the Rise in Market Level



market level is rising. As shown in the left part of the figure, the standard deviation and the excess kurtosis rise and the skewness shifts in a negative direction. Subsequently, after a short time and with gradually recovering stability in the market, both the standard deviation and the excess kurtosis decline, the extent of negative shift of the skewness shrinks, and the distribution will approach a normal distribution.[10]

We interpret the changes in implied probability distribution as capturing the changes in the expectations of a representative investor in the market.[11] Namely, a representative investor's expectation of stock price fluctuations will follow a normal distribution in the long run, although it will exhibit stronger confidence in either a rise or a fall in the short run reflecting factors such as the current phase of the business cycle. For example, in the early phase of a market rise, the representative investor will expect a further improvement in companies' performances and thus expect the market to rise for the time being, resulting in a swelling of the right side of the expectation distribution. Consequently, the distribution will be skewed to the left, and the skewness becomes negative. In addition, since the risk of price fluctuations increases, the standard deviation increases, the fatness of the left tail increases relatively, and the excess kurtosis rises.

In summary, for short-term changes in the market, skewness will change in the direction opposite to that of the market level, while the standard deviation and the excess kurtosis increase. The changes of these three summary statistics show how market expectations have responded to external shocks, and thus tell us the magnitude of any unexpected shocks that hit the market. In addition, by examining the length of the adjustment period needed to regain the original position, we can see to what extent market expectations have been smoothly adjusted.

3. Biases from discrete approximation

We have so far discussed how to interpret each summary statistic by assuming a continuous probability distribution. However, the actual analytical method, which we employ in this paper discretely, approximates a continuous distribution based on an idea called the finite difference method with discretely observed option prices. Thus, there is a possibility that discrete approximation may introduce errors into each summary statistic. We simulate the errors of the discrete approximation against the continuous standardized normal distribution, when the center of the distribution is shifted.[12]

Figure 5 is an illustration that shows the extent of errors introduced when the center of the distribution is shifted by 0.4 times one standard deviation from the original center located in the center of the interval in the histogram. The normal distribution shifts from the dark to the light solid line, and the discrete approximations of the distribution before and after the shift are shown as histograms of the dark solid line and of the shadowed area, respectively.[13]

10. When the market level is falling, the skewness will shift in a positive direction, while the standard deviation and the excess kurtosis generally change as they do in the case of a market rise. In the following, therefore, we can equally discuss the case of a market fall by changing the sign of skewness.

11. In interpreting the changes in implied probability distribution from the viewpoint of economic theory, we owe much to our discussion with Takeo Hoshi.

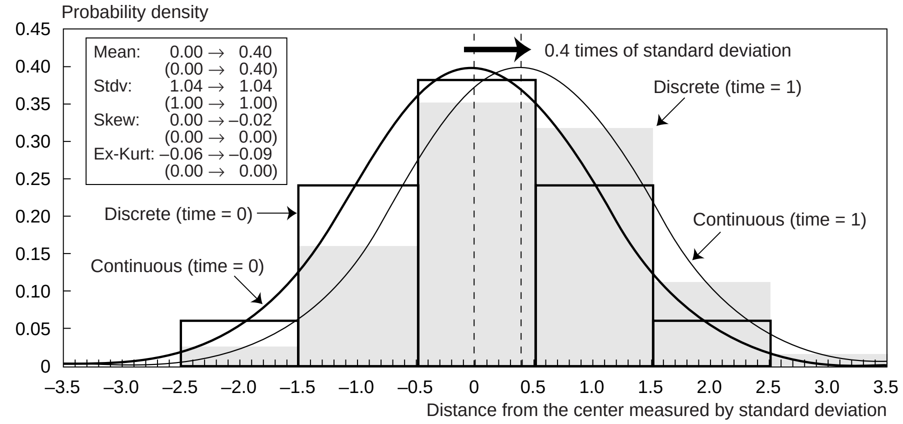
12. We owe much to our discussions with Genshiro Kitagawa, Yuichi Nagahara, and Seisho Sato in dealing with errors in discrete approximation.

13. Since the actual fluctuation of asset prices is, in general, asymmetric and fat-tailed, we often focus on excess kurtosis that shows the fatness of the tails in distribution. However, taking account of the fact that errors of discrete approximation are largest in excess kurtosis in our analytical framework, this summary statistic requires wide interpretation.

In this case, although the original probability distribution is a symmetric normal distribution, the discretely approximated distribution is skewed to the left because the center of the original distribution has shifted to the right from the center of the interval in the histogram. When we see the errors in each summary statistic shown in the box in Figure 5, errors of the mean and the standard deviation are small and do not increase that much as the center of the distribution shifts. In contrast, errors in the skewness and excess kurtosis will increase slightly as the center shifts.

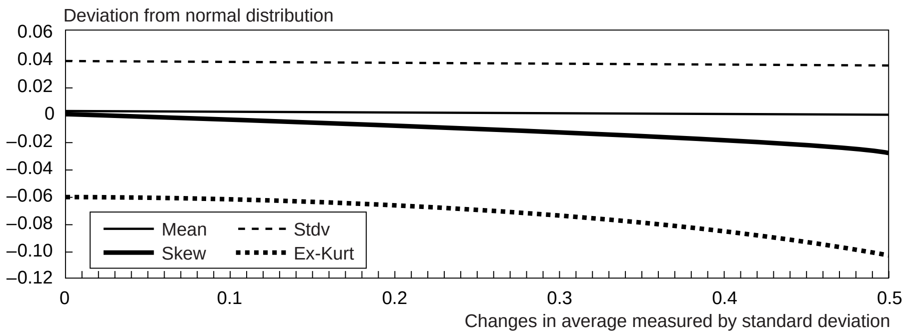
Our next simulation, shown in Figure 6, examines to what extent the approximated error changes according to the shift in the center of distribution.[14] Errors in mean are almost negligible: in the standard deviation they are about 0.04 points,

Figure 5 Bias in Discrete Approximation



Notes: 1. This chart shows errors in the discrete approximation of standard normal distribution, when the center of the distribution is shifted from the center by 0.4 times one standard deviation.
2. Figures in the box show the impact of shift in the center of distribution on summary statistics. Figures in parentheses indicate summary statistics in original standard normal distributions.

Figure 6 Simulation of the Impact of Discrete Approximation



Note: This chart shows dispersion of various summary statistics of the discrete approximation by the interval of one standard deviation from those of standard normal deviation, as the center of the distribution moves.

14. This simulation continues to assume the intervals of the discrete approximation as one standard deviation, and depicts how each summary statistic disperses from that of a standard normal distribution when the center of the distribution moves by an interval of 0.01 times one standard deviation.

although they are stable and do not increase as the original distribution shifts away from the center; errors in skewness increase as the original distribution shifts away from the center, although the size is only 0.04 points at the largest. In the meantime, errors in excess kurtosis increase from 0.06 to 0.10 points, and when the original distribution shifts from the center more than 0.25 times one standard deviation, the error starts to increase by a larger amount.

So what are the effects of such discrete approximation errors on the actual estimation? In the actual option transaction, intervals of strike prices are set constant regardless of the price level of the underlying assets or time-to-maturity. Therefore, effects of the above errors based on the standard deviation will change as stock prices and times-to-maturity change.

In order to see this, we have estimated a matrix (Table 2) showing what size of stock price volatility (i.e., the annualized standard deviation) corresponds to half of the strike price interval (¥250) under different combinations of stock price (rows) and time-to-maturity (columns). When the stock price level is more than ¥28,000, and as long as the time-to-maturity of the option is more than five days, stock price volatility will not exceed 0.25 times one standard deviation,[15] the level at which effects of errors in discrete approximation start to increase. However, when the stock price falls to the level of ¥24,000, stock price volatility exceeds 0.25 times one standard deviation even with a time-to-maturity of more than 20 days, and thus the effects of errors may become substantial.[16]

Table 2 Impact of Changes in Stock Price Level and Time-to-Maturity to Approximation Errors

Stock price (yen)	Time-to-maturity (days)								
	5	10	15	20	25	30	35	40	45
14,000	15.2	10.7	8.7	7.6	6.8	6.2	5.7	5.4	5.1
16,000	13.3	9.4	7.7	6.6	5.9	5.4	5.0	4.7	4.4
18,000	11.8	8.3	6.8	5.9	5.3	4.8	4.5	4.2	3.9
20,000	10.6	7.5	6.1	5.3	4.7	4.3	4.0	3.8	3.5
22,000	9.6	6.8	5.6	4.8	4.3	3.9	3.6	3.4	3.2
24,000	8.8	6.3	5.1	4.4	4.0	3.6	3.3	3.1	2.9
26,000	8.2	5.8	4.7	4.1	3.6	3.3	3.1	2.9	2.7
28,000	7.6	5.4	4.4	3.8	3.4	3.1	2.9	2.7	2.5
30,000	7.1	5.0	4.1	3.5	3.2	2.9	2.7	2.5	2.4
32,000	6.6	4.7	3.8	3.3	3.0	2.7	2.5	2.3	2.2
34,000	6.2	4.4	3.6	3.1	2.8	2.5	2.4	2.2	2.1
36,000	5.9	4.2	3.4	2.9	2.6	2.4	2.2	2.1	2.0
38,000	5.6	3.9	3.2	2.8	2.5	2.3	2.1	2.0	1.9
40,000	5.3	3.8	3.1	2.7	2.4	2.2	2.0	1.9	1.8

Notes: 1. Figures in this table indicate how large volatility (i.e., annualized standard deviation) of stock prices corresponds to half of the interval between adjacent exercise prices, that is, ¥250.
2. Shaded figures are those that exceed the mean daily volatility (annualized) of stock prices (from 1989 to 1996) multiplied by 0.25 ($22.3 \times 0.25 = 5.566$).

15. In calculating the standard deviation, which serves as the evaluation criterion, we used daily volatility (annualized) of the Nikkei 225 during 1989–96.
16. In 1989, when Nikkei 225 option trading started, the Nikkei 225 was over ¥30,000, and thus might not have introduced any large errors even with ¥500 strike price intervals. However, because of the subsequent fall in stock prices, intervals of strike prices have become relatively wide. The estimate in this paper can be interpreted to imply the necessity of reviewing the strike price intervals.

An often-used approach to analyze changes in market participants' expectations is to compare the distribution shapes themselves before and after certain events, because these are easy to understand. However, based on the above simulation, it should be noted that there is a risk of misreading the discrete approximation errors as precise indicators of changes in market expectations, since this approach depends on option prices at a specific point in time. In addition, as previously mentioned, option trading in the Japanese markets features only a few strike prices and the market is still not deep enough.[17] Therefore, if we interpret the changes in implied probability distribution by relying on the distribution shapes at a specific point in time, changes explained in this way are subject to disturbing factors such as temporary mispricing.

Based on the above discussion, this paper tracks a time series of summary statistics (the standard deviation, skewness, and excess kurtosis) of implied probability distributions to examine the changes in financial market participants' expectations.[18] In addition, taking into account the fact that the intervals of the strike prices are relatively large and thus likely to introduce errors by discrete approximation, we observe the general trend of changes in each summary statistic which is shown by the distribution shape, and focus on examining the relationship between asset price movements and changes in distribution shape.

III. Development of Financial Markets and Market Expectations

We will now apply the method of analysis introduced in the previous chapter to periods in which financial markets have experienced large fluctuations, and examine how market participants' expectations have changed in the course of those events. Specifically, we will consider three periods: (1) the second half of 1989 to 1990, the period of the collapse of the "bubble" in the stock market; (2) 1992 to the beginning of 1994, the period of record-low stock prices; and (3) 1995, the period of increasing anxiety in the market about a possible deflationary spiral.

A. Overview

Instead of proceeding directly with a case study of each period, we first summarize the development of financial markets—short- and long-term interest rates, foreign exchange rates, and stock prices—as well as the condition of the Japanese economy and the responses of monetary policy. We also try to draw a rough image of how the implied probability distribution extracted from the Nikkei 225 options has changed over time.

17. While normally there are about five traded strike prices in the Nikkei 225 options, the S&P 500 stock price index options have some 30 prices.
18. There are various ways of examining the time-series changes in estimated implied probability distribution. For example, Söderlind (1998) tried to evaluate the market expectations for British monetary policy during the European currency crisis in 1992, and instead of analyzing several statistics that show the distribution shape, used a 90 percent confidence interval around a risk-neutral expected value.

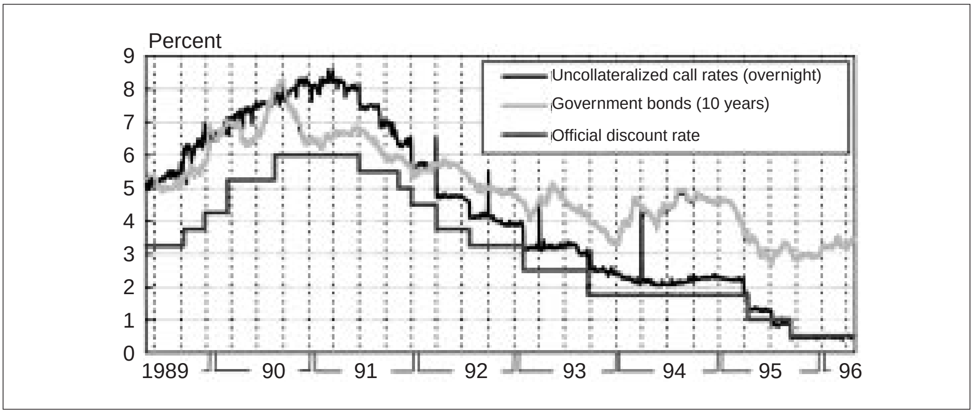
1. Development of financial markets and monetary policy

Based on Figure 7, we first review the development of financial markets and monetary policy from the second half of 1989 to 1995.

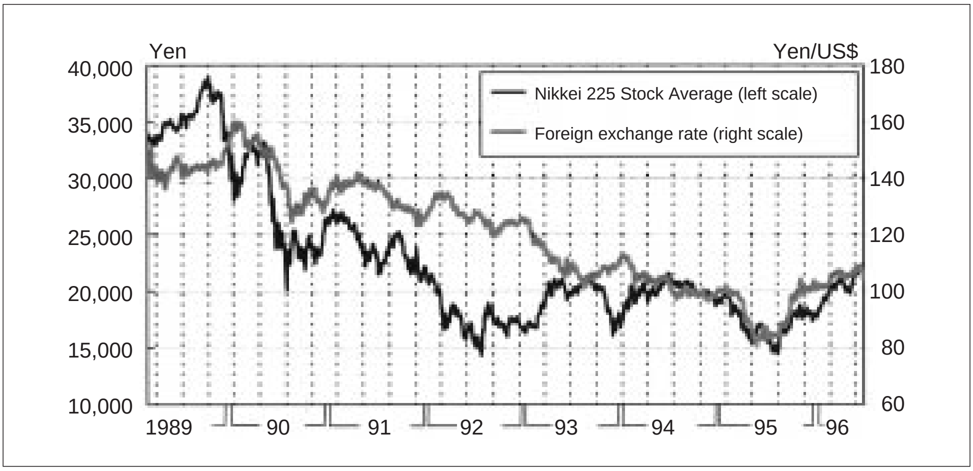
In the period of the collapsing "bubble," the Bank of Japan initiated a preemptive tightening of monetary policy on May 31, 1989, followed by a second tightening in mid-October and a third on December 25, four days before the Nikkei 225 reached its peak. Even after the stock price peaked, this tight policy stance was maintained until mid-1991, when the discount rate was lowered from 6.0 percent to 5.5 percent on July 1 to end the two-year tightening period. In the meantime, the uncollateralized overnight call rate moved roughly in parallel with the discount rate, while the 10-year government bond yield sometimes moved in the opposite direction to that of

Figure 7 Development of Financial and Foreign Exchange Markets

[1] Long- and Short-Term Interest Rates



[2] Foreign Exchange Rates and Stock Prices



Source: Bank of Japan, *Economic Statistics Monthly.*

the call rate. With regard to the stock prices, after reaching its peak at ¥38,915 in December 1989, the Nikkei 225 tumbled during the period from February to May 1990. After a temporary rebound, it fell again from the latter half of 1990 to a level below ¥20,000. Subsequently, the Nikkei 225 rallied to ¥26,000, although it dropped to some ¥22,000 in the second half of 1991. The Economic Planning Agency reported that the boom in the Japanese economy had peaked out in April 1991.

An easy monetary policy continued toward mid-1992, when stock prices bottomed. During the period from late 1991 to mid-1992, the Bank of Japan cut the official discount rate four times, from 5.5 percent to 3.25 percent, and market interest rates declined, reflecting the cuts in the discount rate and the deceleration of the economic expansion. The uncollateralized overnight call rate dropped below 5 percent in April 1992 and long-term interest rates declined accordingly. In the meantime, the stock market dropped substantially, reflecting the downward revisions of companies' performances toward early April 1992. It suffered a further decline due to increasing uncertainty about the future of the economy, and in August it fell below ¥15,000 for the first time in the post-"bubble" period. The Nikkei 225 showed a slight recovery with the planning of the Emergency Economic Policy Package at end-August, moved around the ¥17,000 level in early 1993, and rose rapidly to above ¥20,000 in March, when an additional fiscal stimulus package was confirmed.

Due to the prolonged recession, the easy monetary policy continued in 1993, with the official discount rate cut to 2.5 percent in April and to 1.75 percent in September. Short-term interest rates consistently declined, while long-term interest rates temporarily rose in response to favorable economic indicators in the spring, although they then declined sharply until the beginning of 1994 due to the yen's appreciation and weak prospects for the economy. Reflecting optimistic views on the economy, stock prices rose in the spring of 1993, fell during October and November, and recovered slightly thereafter. In the foreign exchange market, the yen rapidly appreciated toward the summer of 1993.

In 1995, anxiety about deflation intensified. In the foreign exchange market, the yen started to appreciate rapidly after February 1995, reflecting the disturbances such as the Mexican Peso Crisis, and the appreciation accelerated to temporarily break the ¥80 level against the U.S. dollar in mid-April. The market's forecast about the future of the Japanese economy weakened, and the Nikkei 225 fell below ¥15,000 in mid-June 1995. Under these circumstances, the Bank of Japan continued its easy monetary policy by announcing at end-March that it would let the uncollateralized overnight call rate decline, on average, to a level slightly below that of the official discount rate, and lowered the official discount rate from 1.75 percent to 1.0 percent in April 1995. Subsequently, short-term interest rates further declined to a record-low level reflecting a policy of additional easing in July (when the uncollateralized overnight call rate was allowed to decline further) and September (when the official discount rate was cut from 1.0 percent to 0.5 percent). Long-term interest rates also remained at low levels. In the foreign exchange market, after July, policy coordination among leading countries led the yen to modify its overshoot to around the ¥100 level against the U.S. dollar. The Nikkei 225 also rebounded after July.

2. Movements of expectations in the stock market

Now we turn to the changes in market expectations during this period by observing the changes in summary statistics about the implied probability distribution estimated from option prices (Figure 8).

We can see that the standard deviation increased when stock prices dropped sharply. This increase in standard deviation is notable in four phases: (1) from end-1989, when the Nikkei 225 reached its peak, to the fall of 1990, when it fell below ¥20,000; (2) from mid-1991 to mid-1992, when the Nikkei 225 fell below ¥15,000 for the first time in the post-"bubble" period; (3) from the second half of 1993 to early 1994; and (4) from the beginning to the middle of 1995, when the yen rapidly appreciated and anxiety about a possible deflationary spiral increased. In addition, the level of the standard deviation increased after end-1989, which seems to imply that market participants became more conscious of the risk of price volatility after the stock market entered its adjustment phase.

The skewness shifted toward a negative value when stock prices were rising, and toward a positive value when stock prices were falling. For example, when we look in detail at the second half of 1989, there were three phases (the second half of July, the second half of September, and from the second half of November to end-December) in which the rise of the Nikkei 225 accelerated. The skewness shifted from slightly positive to negative in all three cases. During the period around October 1991 when stock prices tumbled, the skewness shifted back to a positive value. This relationship between the market fluctuation and skewness is consistent with our hypothesis shown in the previous chapter that the adjustment speed of participants' confidence on the future price will become asymmetric between the upper and lower directions of the current market level according to the phases of market fluctuation. The excess kurtosis was negative on average, although the extent of negativity shrank or sometimes turned positive under large price fluctuations and also showed a jump in the case of an extreme price change.

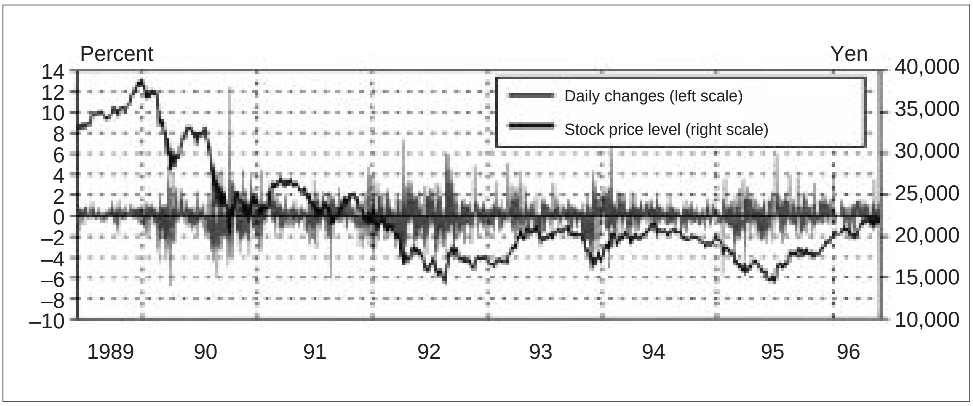
Table 3 calculates the coefficients of correlation between the daily changes of stock prices and the summary statistics for implied probability distributions. The table confirms the above observations on the movements of summary statistics in accordance with fluctuations in market levels. The skewness showed a relatively high negative correlation with the changes in stock prices. The standard deviation and excess kurtosis indicated a positive correlation with the absolute value of daily changes in stock prices.

Table 4 compares the summary statistics for implied probability distributions with those for the distribution of daily changes in stock prices. Over the sample periods, the mean was almost identical and the standard deviation showed a similar tendency, while the skewness and excess kurtosis were quite different. The skewness was positive in many periods in the time-series data, while, except for 1992, it was negative in the implied probability distribution. The excess kurtosis was consistently positive and formed a fat-tail distribution in the time-series data, while it was negative in the implied probability distribution.

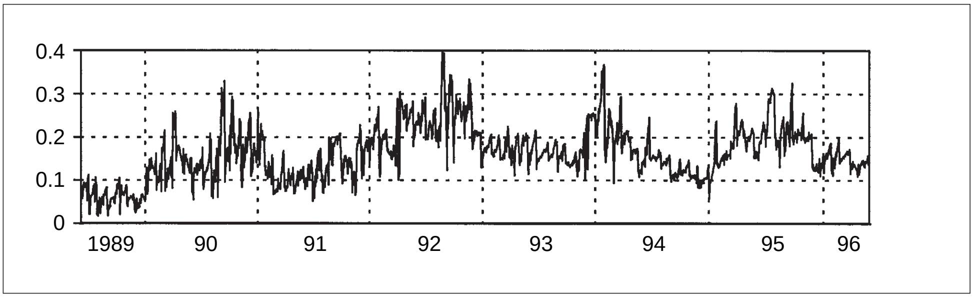
In the following sections, we analyze the changes in financial market expectations by examining the time series of the summary statistics of implied probability

Figure 8 Stock Prices and Market Expectations (Overview)

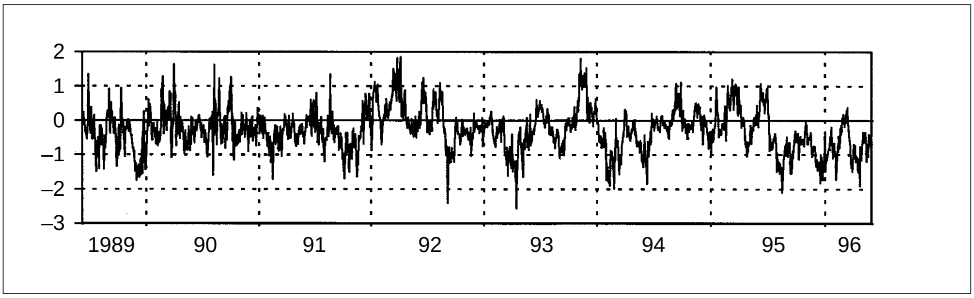
[1] Nikkei 225 Stock Average



[2] Standard Deviation



[3] Skewness



[4] Excess Kurtosis

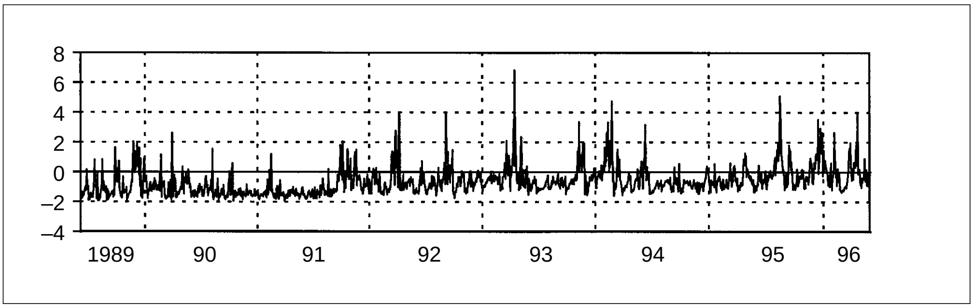


Table 3 Correlation between Market Level Changes and Summary Fluctuations

	Mean	Stdv	Skew	Ex-Kurt
Full sample	0.775	0.227	–0.347	0.081
1989	0.544	0.102	–0.408	0.113
1990	0.663	0.113	–0.532	0.160
1991	0.725	0.008	–0.435	0.146
1992	0.801	0.143	–0.347	0.182
1993	0.849	0.078	–0.350	0.301
1994	0.871	0.406	–0.234	0.167
1995	0.894	0.072	–0.309	0.011
1996	0.813	–0.126	–0.297	0.017

Notes: 1. Figures in the table are the coefficients of correlation between daily changes of the Nikkei 225 (closing price) and summary statistics. With respect to standard deviation and excess kurtosis, we take absolute values for the daily changes of the Nikkei 225.
2. The sample period for 1989 is the second half of the year and that for 1996 is the first half of the year.

Table 4 Comparison of Summary Statistics: Distribution of Daily Changes and Implied Probability Distribution

	Daily changes				Implied probability distribution			
	Mean	Stdv	Skew	Ex-Kurt	Mean	Stdv	Skew	Ex-Kurt
Full sample	–5.663	23.197	0.443	5.348	–5.753	16.411	–0.274	–0.603
1989	31.377	8.321	–0.346	1.323	31.206	6.285	–0.452	–0.866
1990	–39.233	32.447	0.829	5.751	–38.632	15.359	–0.151	–1.140
1991	–3.700	21.066	–0.065	2.508	–2.386	13.311	–0.352	–1.046
1992	–25.286	29.785	0.436	1.064	–25.185	23.188	0.051	–0.569
1993	2.956	20.505	0.252	1.813	3.234	17.362	–0.203	–0.362
1994	13.406	17.753	0.867	8.910	10.410	16.227	–0.345	–0.453
1995	0.739	22.503	0.122	2.304	0.348	19.230	–0.362	–0.126
1996	28.048	14.657	0.698	2.897	29.271	14.918	–0.713	–0.073

Notes: 1. Mean (daily changes) and standard deviation are annualized.
2. The sample period for 1989 is the second half of the year and that for 1996 is the first half of the year.

distributions for the following three periods: (1) the second half of 1989 to the first half of 1991, the period of the collapse of the "bubble" in the stock market; (2) 1992 to the beginning of 1994, the period of serious stock market slump; and (3) 1995, the period of increasing anxiety in the market about a possible deflationary spiral.

B. During the Collapse of the "Bubble"

First, with respect to the period of the collapsing "bubble," we examine the three periods that show striking movement from the viewpoint of stock price and interest rate fluctuation as well as monetary policy operation: (1) from the introduction of the Nikkei 225 options to December 29, 1989, when stock prices peaked; (2) from the peak of stock prices to April 1990, when stock prices tumbled below ¥30,000; and (3) from June to October 1990, when the Nikkei 225 rapidly fell after a short rebound to a level temporarily below ¥20,000. The points at issue here will be how strongly confidence changed and how monetary policy responses affected such changes.

1. Prior to the peak of the Nikkei 225 (June–December 1989)

First, we will take up the period ranging from the introduction of the Nikkei 225 options to December 29, 1989, a day when stock prices peaked.

Figure 9 shows the changes in stock price expectations during this period. As mentioned, in the second half of 1989 toward the peak of the Nikkei 225, there were three phases (the second half of July, the second half of September, and from the second half of November to end-December) in which the rising pace of the Nikkei 225 accelerated. In each period, the skewness declined from around zero to a negative value, the standard deviation remained almost constant, and the excess kurtosis moved up and down substantially when stock prices soared.

When we examine these periods with reference to monetary policy up to the rise in the official discount rate from 3.25 percent to 3.75 percent on October 11, the standard deviation and skewness both increased, and the excess kurtosis also increased while showing volatile ups and downs. From such changes in the shape of the distribution, we conjecture that, while the forecast for the official discount rate rise prevailed, the risk of stock price changes also intensified. Therefore, a view that the rise in the official discount rate was already incorporated in the market sentiment at that time is not consistent with the movement of the estimated implied probability distribution.

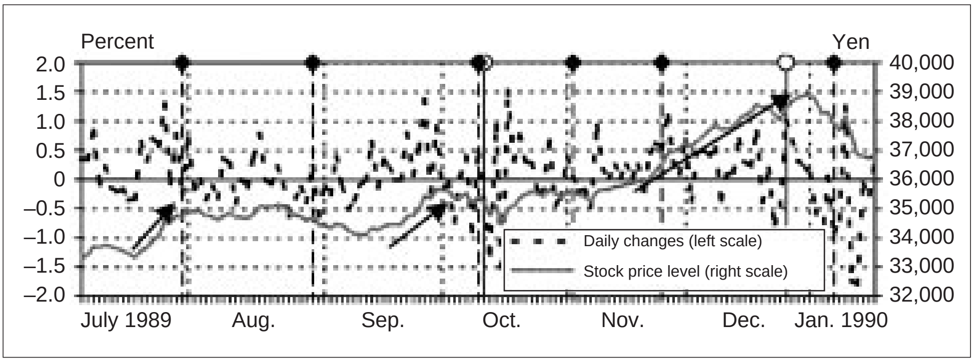
If we focus on the movement of the standard deviation, we can see that there was a large jump at the time of shifts in the most actively traded contract months after July 1989. In particular, at the time of shift from the October contract to the November contract in 1989 (shown in Figure 9 as a vertical dashed line), not only was the magnitude of the jump large but the shift itself was delayed until immediately before the exercise date. This movement in the standard deviation can be interpreted as indicating that while the forecast for the official discount rate rise prevailed, the risk of short-term stock price changes became large, and thus resulted in a delay in the shift of the most actively traded contract months and a jump in the standard deviation.[19]

During the rise of the Nikkei 225 to its peak, from mid-November to end-December, the skewness initially declined, although it started to increase in mid-December, reducing the extent of negativity. The standard deviation gradually declined, while the excess kurtosis showed an increasing trend with up and down movements. Such changes in expectation distribution can be interpreted to imply that confidence in future stock price rises intensified, reflecting the rise in the stock market. This rise in the stock market was longer in duration and larger in magnitude than the other two phases. The potential risk of a stock price fall might have been increasing during this period, although recognition of such risk seems to have diminished in accordance with the actual rise in the stock market.[20]

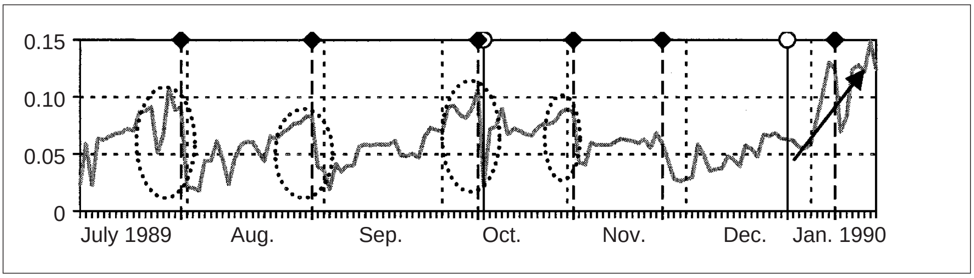
19. With respect to the interpretation of the large jumps in the standard deviation at the time of shifts in the most actively traded contract months, it should be noted that the accuracy of the estimation results was not necessarily high when the expiration date of the option approached.
20. Some commentators have pointed out that massive quantities of mutual funds subscribed to by individual investors were set up during this period.

Figure 9 Market Expectations of the Nikkei 225 (from July 1989 to January 1990)

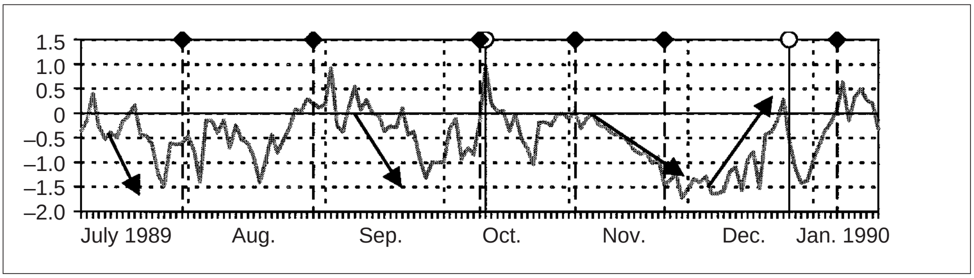
[1] Nikkei 225 Stock Average



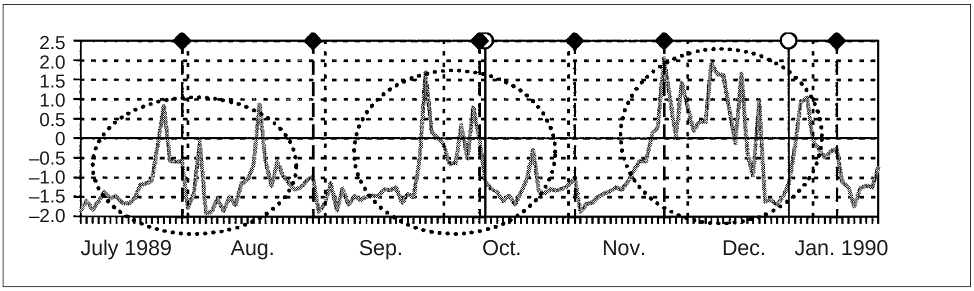
[2] Standard Deviation



[3] Skewness



[4] Excess Kurtosis



Note: Each vertical line with a circle at the top shows the date when the official discount rate was changed (October 11, 1989: 3.25 percent → 3.75 percent; December 25, 1989: 3.75 percent → 4.25 percent), and each vertical dashed line with a diamond at the top shows the date when the most actively traded contract months were changed.

Finally, during the period from the beginning of 1990, when stock prices peaked and started to fall, the skewness and standard deviation increased substantially, implying that market participants became more cautious regarding the downside risk of the stock market. In fact, it was reported that uncertainty in the market spread rapidly after the rise in the official discount rate.

2. After the peak of the Nikkei 225 (December 1989–May 1990)

Next, we consider the period from December 1989 to May 1990, when stock prices peaked and fell sharply to below ¥30,000 (Figure 10). During this period, the Nikkei 225 fell by some ¥10,000, and there were roughly five declining phases.

The skewness, in general, was slightly positive from mid-February, reflecting the declining trend of the market, while showing an increase when stock prices fell. The standard deviation gradually increased, and excess kurtosis exhibited a volatile movement, jumping in a positive direction when stock prices declined. Such a movement in the summary statistics is consistent with the typical patterns summarized in the previous section.[21]

However, what was notable in this period was the large fluctuation and long adjustment period of the skewness when the market level fell. During this period, once stock prices fell, the increase in the skewness was quite large, and, even after stock prices regained their stability, it took a long time until the increased skewness returned to the level prior to the stock price fall. Such movements of the skewness imply the possibility that, even after stock prices entered the declining phase, bullish sentiment in the market did not retreat, and thus resulted in the delayed adjustment of market expectations during the price fall.[22]

Figure 11 shows graphically how market expectations developed during this period. Two cases are assumed in which the market fell as time elapsed from 0 to 1, and further to 2. It seems that, even after stock prices began to fall, market participants' bullish sentiments did not easily diminish. This made the implied probability distribution more skewed to the right compared with the case of other stock price falls, resulting in more time required for a resumption of the symmetric shape.

According to press reports, market participants began to expect a rebound of stock prices in late April. In addition to their underlying sentiment that the stock price would bottom out at ¥28,000, they welcomed the effects of the yen's appreciation, because it would restrain inflation fears and thus contain the rise in interest rates. Indeed, when we look at the probability distribution, rises in the standard deviation and excess kurtosis seem to have come to a halt, and market expectations might have been focused on a regaining of stability.

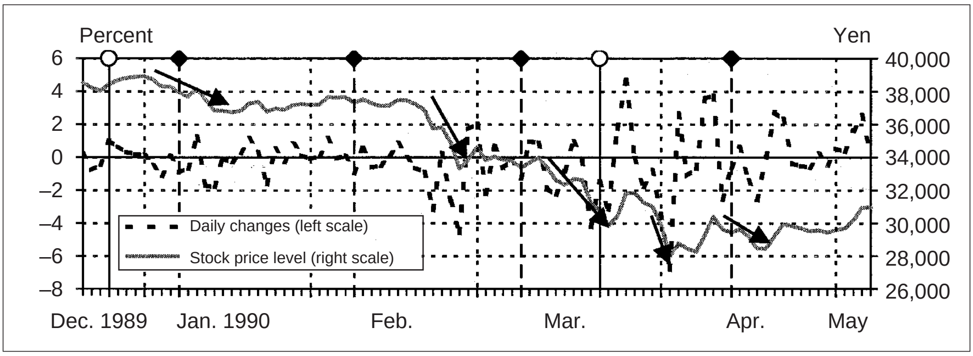
3. Further tightening: the Nikkei 225 falls below ¥20,000 (July–November 1990)

The third period we consider is that from July to November 1990. Although the Nikkei 225 showed a slight recovery in May, it fell sharply again after July, and

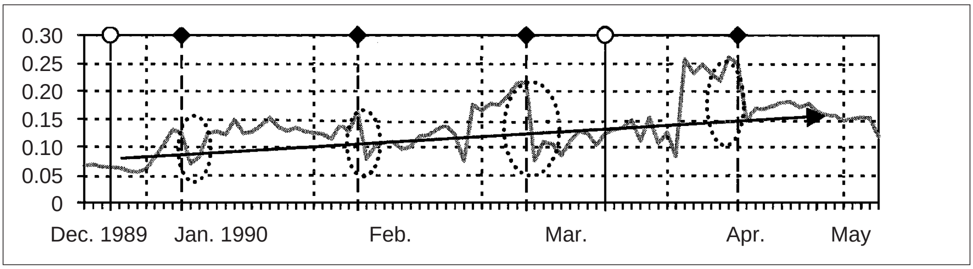
21. Monetary policy was further tightened on March 20 through a rise in the official discount rate from 4.25 percent to 5.25 percent, although it did not affect the overall development of expectations since such tightening seemed to have already been incorporated in market expectations.
22. Some people hold the view that, during this period, economic conditions were adversely affected by the progressive depreciation of the yen and the rise in U.S. interest rates, and thus confused the market's sense of direction. In the course of the market's seeking a bottom, cautiousness about future prospects might have intensified.

Figure 10 Market Expectations of the Nikkei 225 (from December 1989 to May 1990)

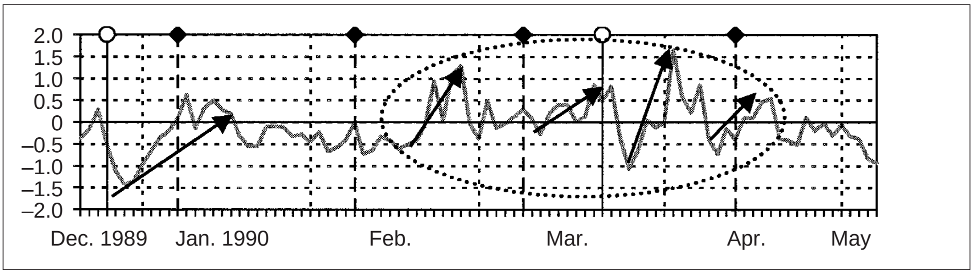
[1] Nikkei 225 Stock Average



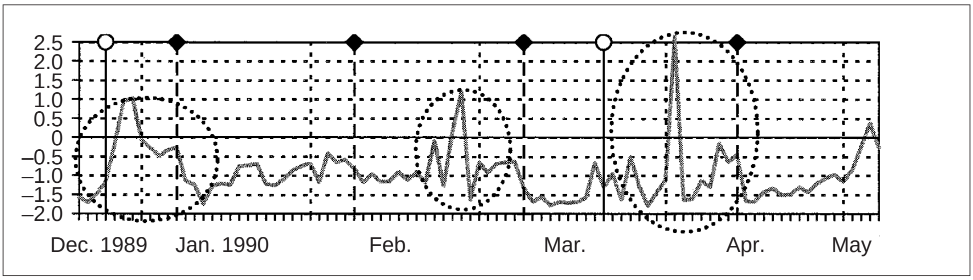
[2] Standard Deviation



[3] Skewness

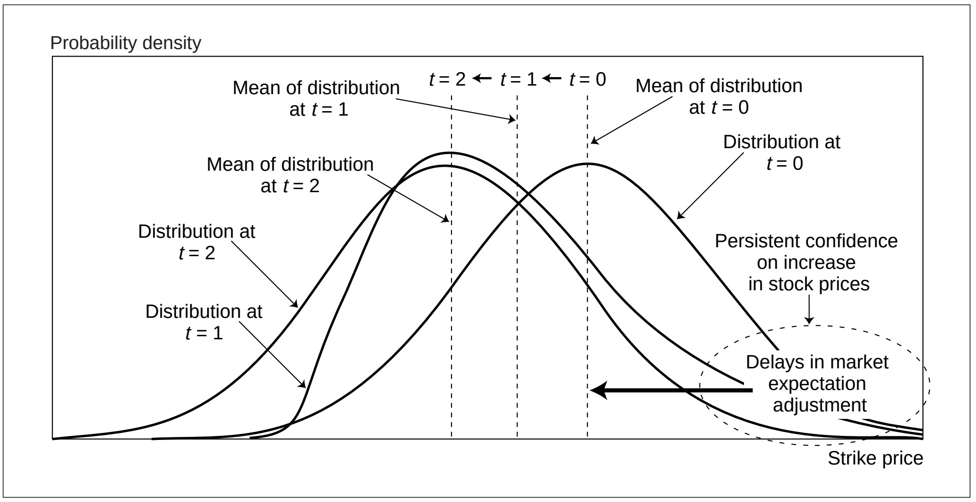


[4] Excess Kurtosis



Note: Each vertical line with a circle at the top shows the date when the official discount rate was changed (December 25, 1989: 3.75 percent → 4.25 percent; March 20, 1990: 4.25 percent → 5.25 percent), and each vertical dashed line with a diamond at the top shows the date when the most actively traded contract months were changed.

Figure 11 Market Expectations after Reaching the Historically Highest Price in the Stock Markets



temporarily fell below ¥20,000 in October.[23] When we look at the development of stock price expectations during this period (Figure 12), the standard deviation generally rose, and the skewness rose when stock prices fell.

Particularly interesting is the large jump in standard deviation before and after the shift of the most actively traded contract month during the period from July to September. Specifically, the shift from September options to October options was delayed until September 11, three days before the exercise date, and the jump in standard deviation was quite large. On August 30, owing to the boost in oil prices caused by the Iraqi invasion of Kuwait, the official discount rate was raised for the fifth time during the "bubble" era, from 5.25 percent to 6.0 percent, and there seems to have been increasing uncertainty in the market about the future even one month ahead.

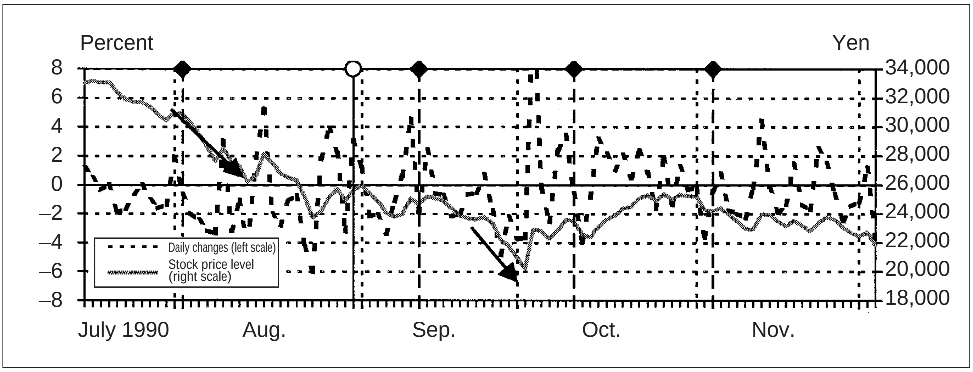
Prior to a sharp drop in stock prices from end-September to early October, the skewness rapidly increased from mid- to late September while stock prices declined moderately. This implies the possibility that there had been a delay in adjusting stock price expectations downward, and it is possible to interpret this to mean that, as a result of bullish sentiment having been largely revised, stock prices fell sharply from end-September to early October.

Let's examine how expectations derived from the 10-year JGB futures option developed during the same period (Figure 13). The standard deviation increased toward end-August, when the official discount rate was raised, although it showed a large drop when the most actively traded contract month shifted. This implies that the timing of the official discount rate rise was approaching, and the risk of price fluctuations in the immediate future was quite large. In addition, when declining bond prices bounced back from end-September (because of the decline in interest rates), the standard deviation also showed a rapid rise, implying an increasing possibility of price fluctuations.

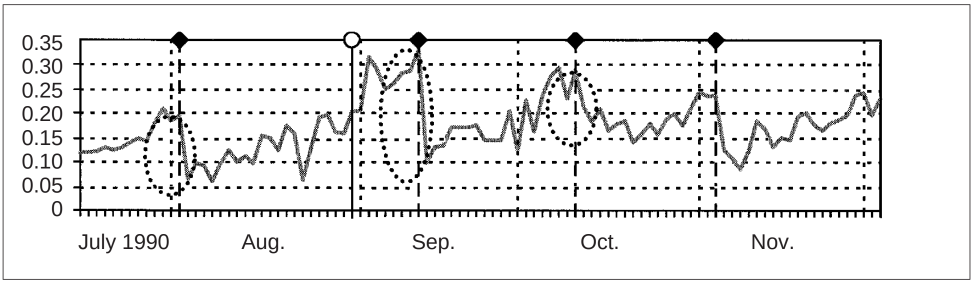
23. During this period, the Nikkei 225 (closing price) hit bottom at ¥20,221.86 on October 1, although it momentarily fell below ¥20,000 and recorded ¥19,781.70 during the trading day.

Figure 12 Market Expectations of the Nikkei 225 (from July to November 1990)

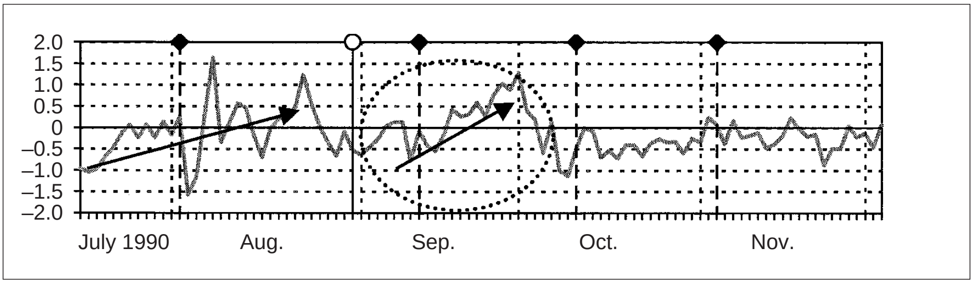
[1] Nikkei 225 Stock Average



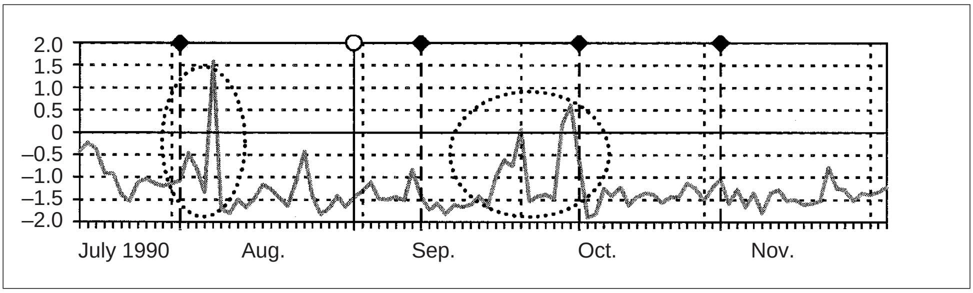
[2] Standard Deviation



[3] Skewness



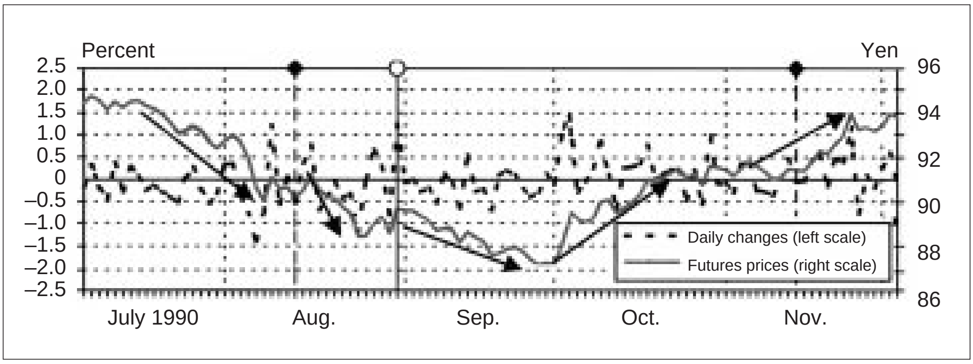
[4] Excess Kurtosis



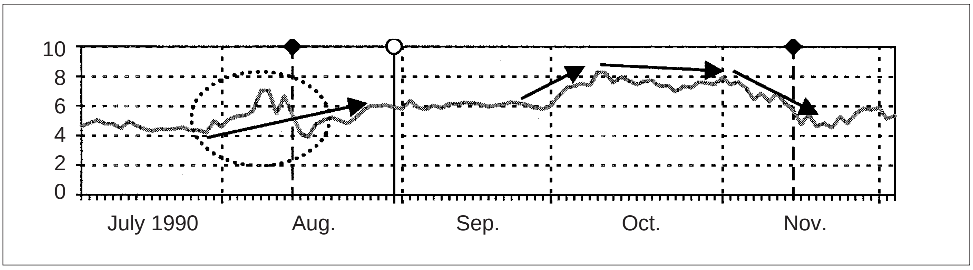
Note: Each vertical line with a circle at the top shows the date when the official discount rate was changed (August 30, 1990: 5.25 percent → 6.0 percent), and each vertical dashed line with a diamond at the top shows the date when the most actively traded contract months were changed.

Figure 13 Market Expectations of the JGB Futures (from July to November 1990)

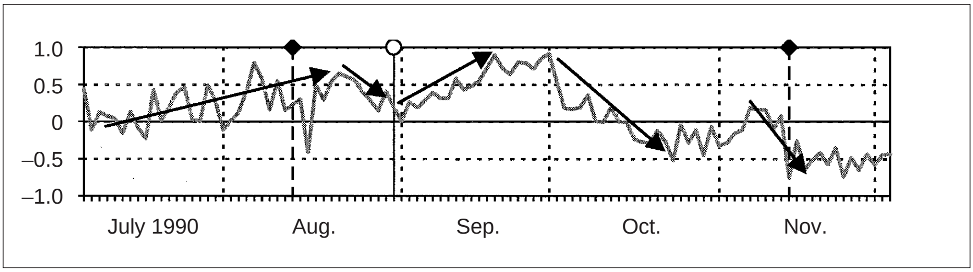
[1] JGB Futures



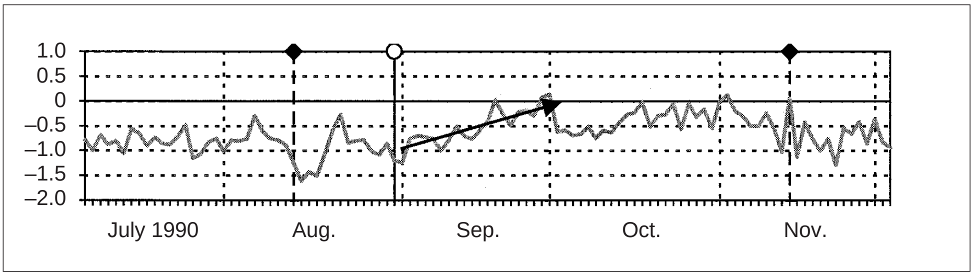
[2] Standard Deviation



[3] Skewness



[4] Excess Kurtosis



Note: Each vertical line with a circle at the top shows the date when the official discount rate was changed (August 30, 1990: 5.25 percent → 6.0 percent), and each vertical dashed line with a diamond at the top shows the date when the most actively traded contract months were changed.

Subsequently, the standard deviation declined after the beginning of November. In the meantime, the skewness moved in an opposite direction in accordance with the rise and fall of bond futures prices.

C. The Period of Serious Stock Market Slump

Next, we will examine the period of serious stock market slump, from 1992 to the beginning of 1994, by dividing the period into two parts: (1) from March to October 1992, which includes the point in time when the Nikkei 225 fell below ¥15,000 for the first time after the collapse of the asset price bubble; and (2) from October 1993 to March 1994, when stock prices collapsed again.

1. The fall of the Nikkei 225 below ¥15,000 for the first time after the collapse of the asset price bubble (March–October 1992)

First, we examine the movement of market expectations before and after August 18, 1992, the day on which the Nikkei 225 fell below ¥15,000 for the first time since the asset price bubble collapsed. Figure 14 shows a time series of summary statistics of implied probability distributions.

The standard deviation was on an increasing trend from mid-1991 and had reached a rather high level by March 1992 (see Figure 8). It further increased when stock prices tumbled (from late March to early April, in mid-June, and in August), showing that market participants had become increasingly cautious concerning the risk of price changes. Skewness moved in the opposite direction to stock price changes, and, on average, increased its level from mid-June to August, implying that downward adjustment of stock prices expectations was delayed. Excess kurtosis showed large fluctuations, which suggests the emergence of large disturbances in the stock market. In particular, from the large fluctuations in late March to early April (when stock prices were falling), and in September (which saw a strong rebound after the bottoming out in August), we can observe that expectations became volatile during these periods.[24]

In the JGB futures market (Figure 15), prices rose (yields fell) from mid-June, and expectations of a fall in interest rates increased in the market. Prior to the cut in the official discount rate in July, the standard deviation declined slightly, while the excess kurtosis, although volatile, increased to around zero. This movement implies that confidence in the level of long-term interest rates intensified at that time, and that the market had already incorporated a cut in the official discount rate.

2. Another fall in stock prices (October 1993–March 1994)

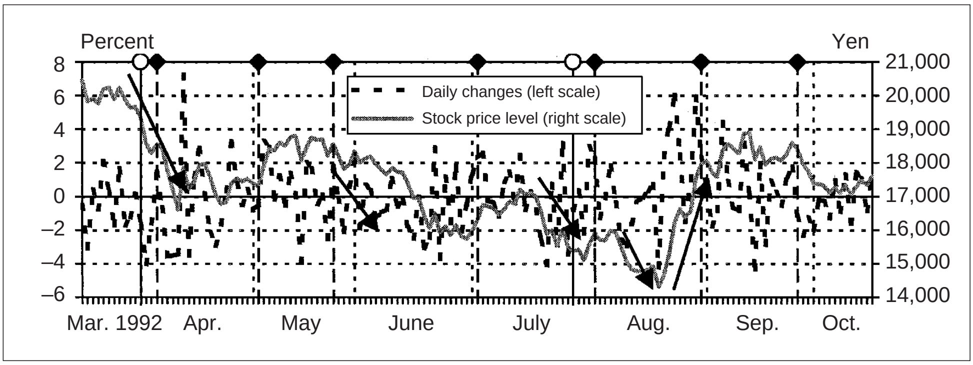
Next, we analyze the period from October 1993 to March 1994, when stock prices fell again. Figure 16 shows the changes in stock price expectations.

During the period from late October to early December, when stock prices tumbled, the standard deviation increased and the excess kurtosis rose with large swings, suggesting the increased risk of stock price fluctuations. Skewness also increased, implying that there was a delay in adjusting expectations to the decline in market level.

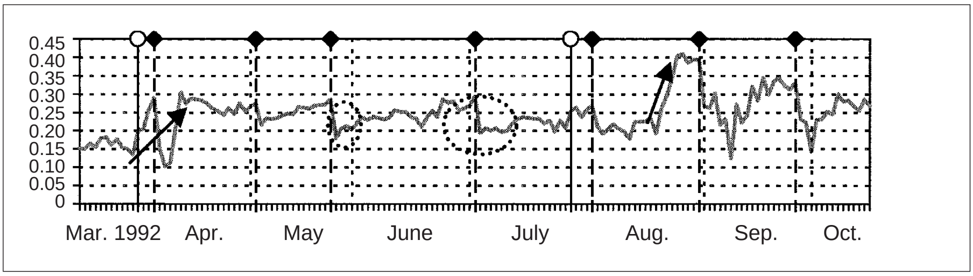
24. In addition, a jump of standard deviation at the time of contract shift during the stock price bottom can also be interpreted to show the increase in uncertainty about future stock prices.

Figure 14 Market Expectations of the Nikkei 225 (from March to October 1992)

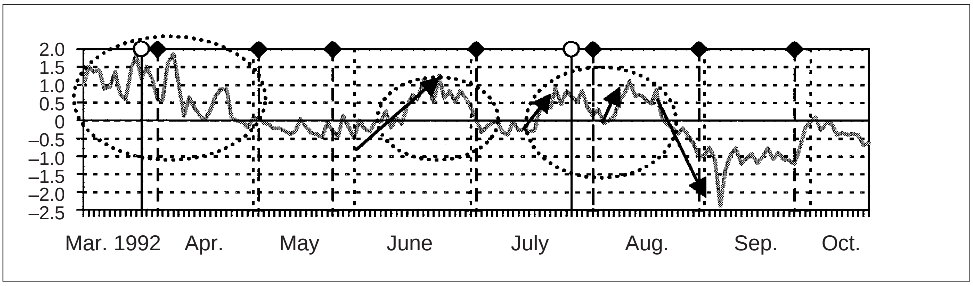
[1] Nikkei 225 Stock Average



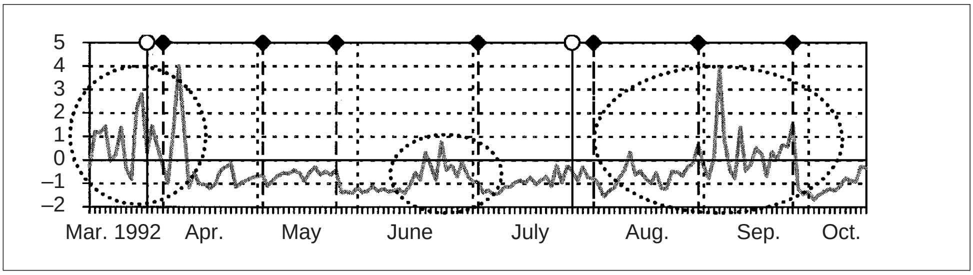
[2] Standard Deviation



[3] Skewness



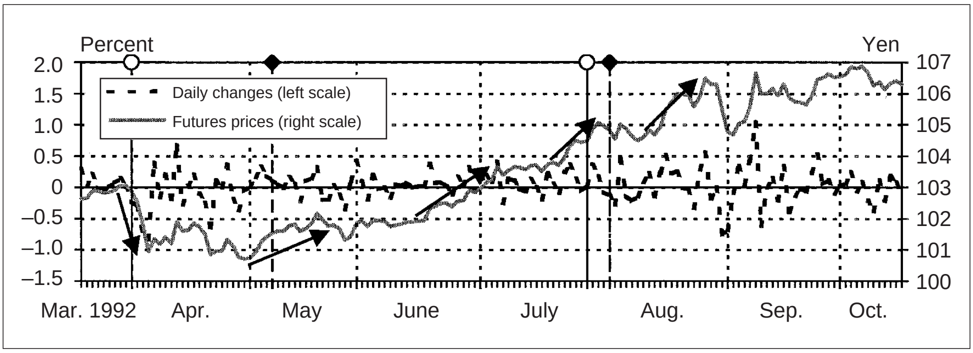
[4] Excess Kurtosis



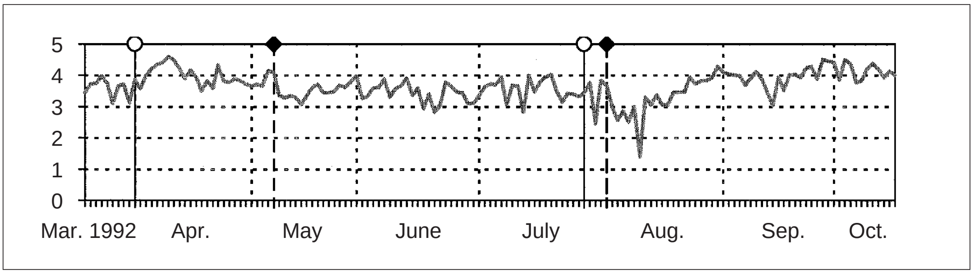
Note: Each vertical line with a circle at the top shows the date when the official discount rate was changed (April 1, 1992: 4.5 percent → 3.75 percent; July 27, 1992: 3.75 percent → 3.25 percent), and each vertical dashed line with a diamond at the top shows the date when the most actively traded contract months were changed.

Figure 15 Market Expectations of the JGB Futures (from March to October 1992)

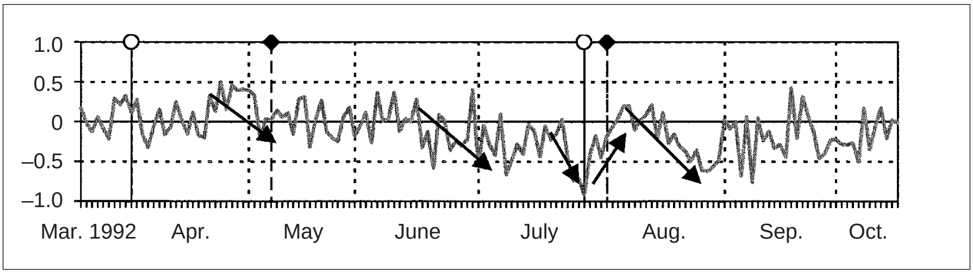
[1] JGB Futures



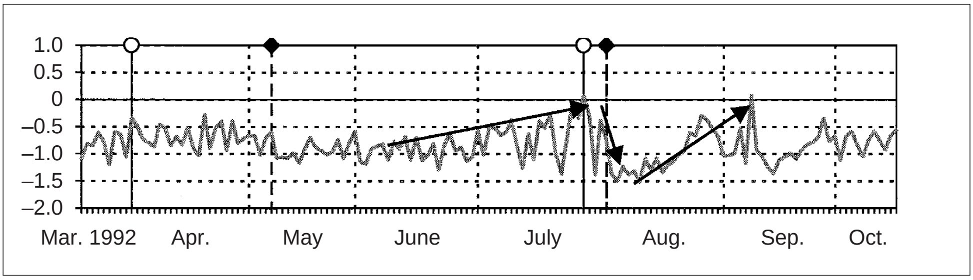
[2] Standard Deviation



[3] Skewness



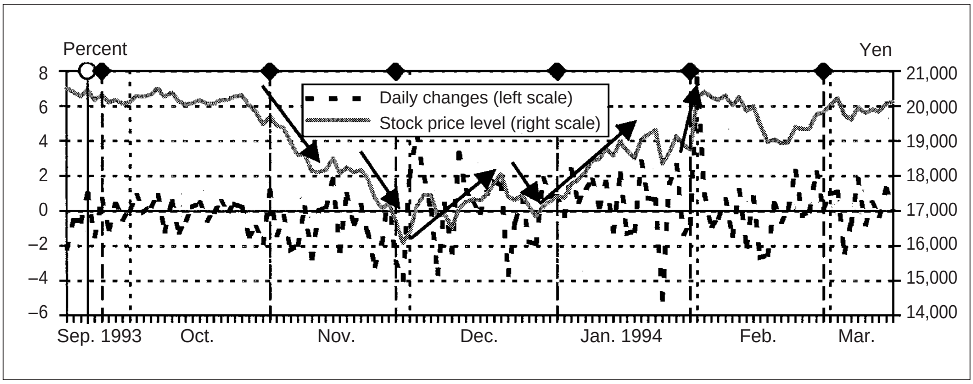
[4] Excess Kurtosis



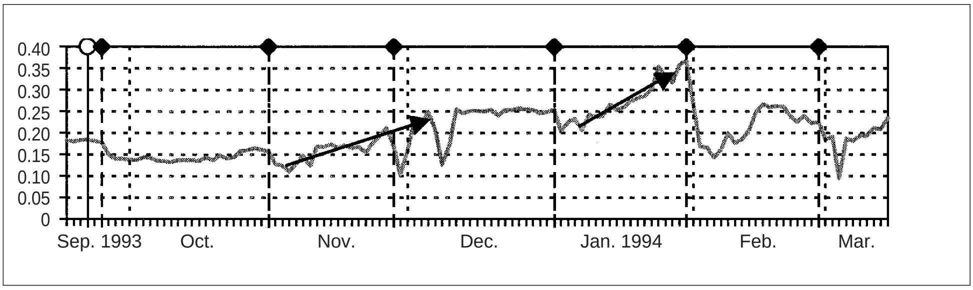
Note: Each vertical line with a circle at the top shows the date when the official discount rate was changed (April 1, 1992: 4.5 percent → 3.75 percent; July 27, 1992: 3.75 percent → 3.25 percent), and each vertical dashed line with a diamond at the top shows the date when the most actively traded contract months were changed.

Figure 16 Market Expectations of the Nikkei 225 (from September 1993 to March 1994)

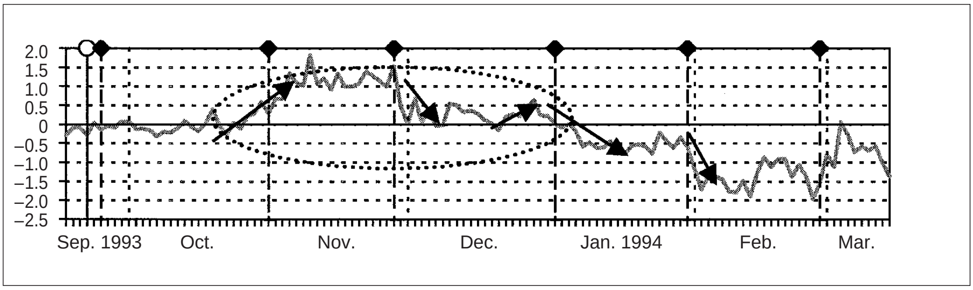
[1] Nikkei 225 Stock Average



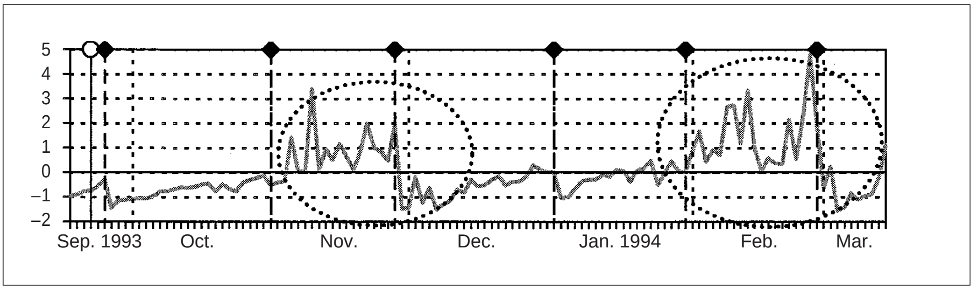
[2] Standard Deviation



[3] Skewness



[4] Excess Kurtosis



Note: Each vertical line with a circle at the top shows the date when the official discount rate was changed (September 21, 1993: 2.5 percent → 1.75 percent), and each vertical dashed line with a diamond at the top shows the date when the most actively traded contract months were changed.

In January 1994, stock prices recovered, although the standard deviation also rose rapidly, suggesting an increase in price fluctuation risk. In addition, since there was a large jump in the standard deviation at the time of changes in the most actively traded contract months, uncertainty concerning the sustainability of the stock price recovery seems to have been strong. In fact, when price recovery came to a halt in February, the standard deviation increased sharply and the excess kurtosis became highly volatile, suggesting that the anxiety about the stock price fall had spread in the stock market.

Developments in the JGB futures market are shown in Figure 17. Between late November 1993 and early February 1994, when bond prices rose and yields fell, the skewness remained negative while moving up and down according to changes in bond prices. This implies that downward risk of bond prices was large and confidence in price rises was not necessarily strong. Moreover, the standard deviation showed a distinctive movement, substantially increasing in January 1994 and falling rapidly in February. At this time, in addition to diminishing expectations of an early cut in the official discount rate due to the recovery in the stock prices, the concern that an extra issue of government bonds and the resumption of bond sales by the government might worsen supply and demand conditions in the market seems to have led to increased expectations of a falling market.

D. Anxiety about a Possible Deflationary Spiral

Finally, we will consider the period from December 1994 to September 1995, when the yen appreciated rapidly and anxiety about a possible deflationary spiral intensified. Specifically, we examine the relationship between market expectations and monetary policy operations for the sub-periods of (1) December 1994 to May 1995, when the yen appreciated rapidly; and (2) April to September 1995, when the Nikkei 225 temporarily fell below ¥15,000.

1. Rapid appreciation of the yen (December 1994–May 1995)

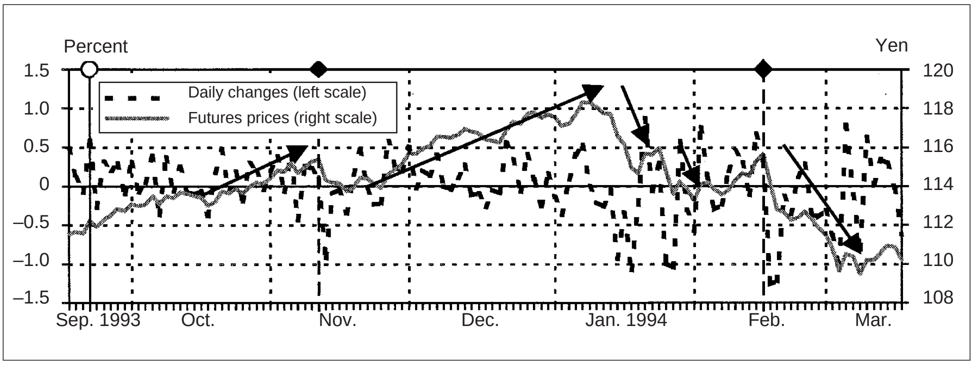
First we focus on the period from December 1994 to May 1995. During this period, the yen gradually appreciated from January onward, then accelerated rapidly after March to break the ¥80 level against the U.S. dollar. The stock market was in a declining trend from January onward, and after undergoing alternate phases of sudden and gradual declines, the Nikkei 225 fell close to ¥15,000 early in April.

Figure 18 shows changes in stock price expectations during this phase. During the first decline in late January 1995, immediately after the Great Hanshin-Awaji Earthquake,[25] all three summary statistics (the standard deviation, skewness, and excess kurtosis) substantially increased. Subsequently, however, the skewness dropped back to the level prior to the increase in accordance with changes in the most actively traded contract months, and the increase of the skewness and excess kurtosis disappeared in a relatively short period of time. Therefore, the impact of the stock price fall seems to have been smoothly absorbed by the market as a temporary shock.

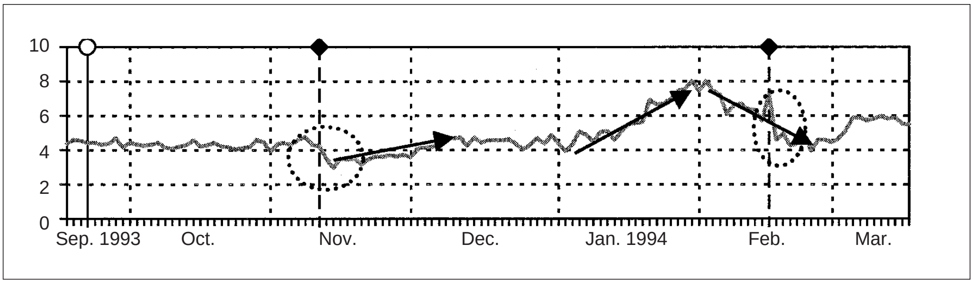
25. The Great Hanshin-Awaji Earthquake was the worst natural disaster in 70 years in Japan. More than 5,000 people died, and an additional two million people, including foreign residents, were significantly affected.

Figure 17 Market Expectations of the JGB Futures (from September 1993 to March 1994)

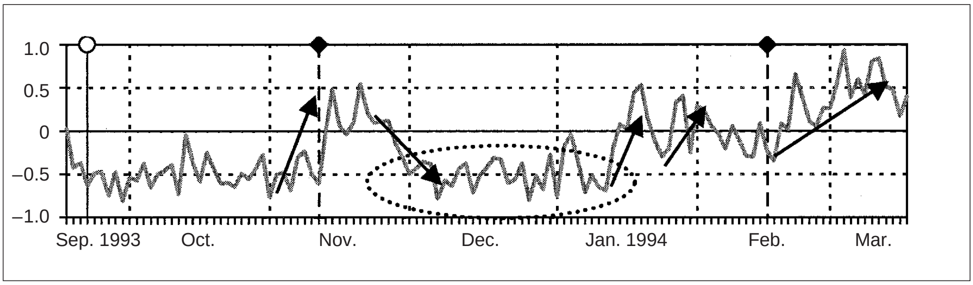
[1] JGB Futures



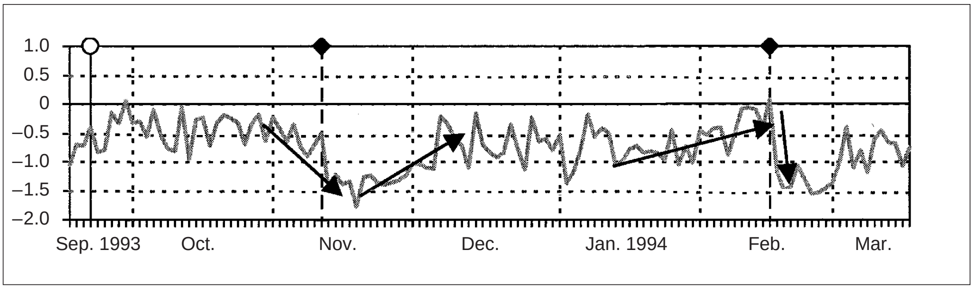
[2] Standard Deviation



[3] Skewness



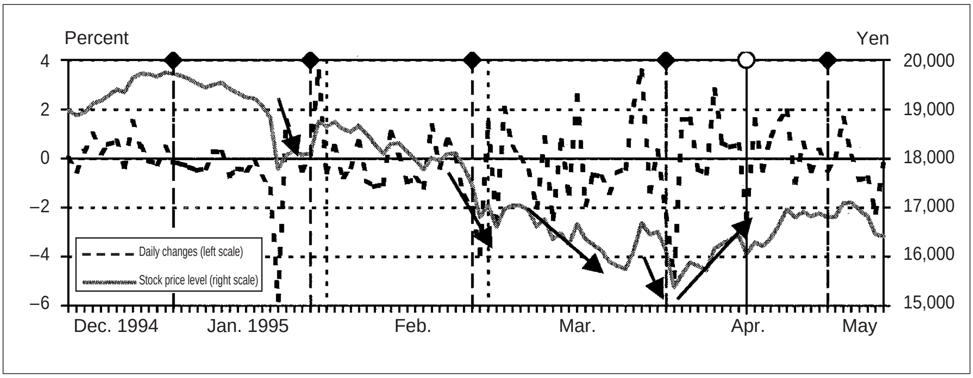
[4] Excess Kurtosis



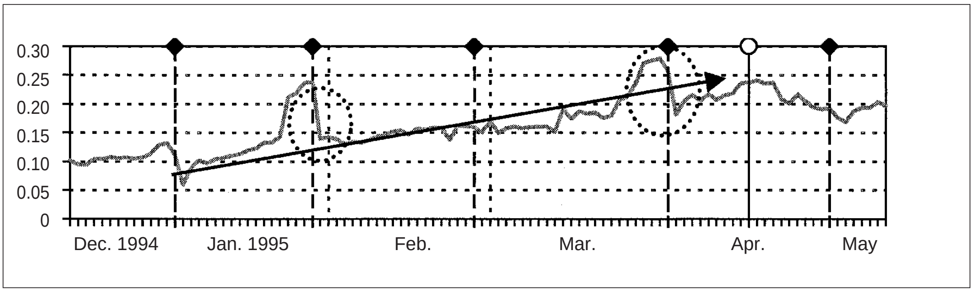
Note: Each vertical line with a circle at the top shows the date when the official discount rate was changed (September 21, 1993: 2.5 percent → 1.75 percent), and each vertical dashed line with a diamond at the top shows the date when the most actively traded contract months were changed.

Figure 18 Market Expectations of the Nikkei 225 (from December 1994 to May 1995)

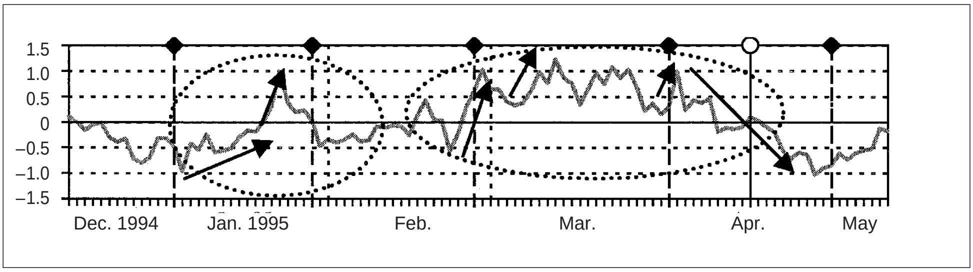
[1] Nikkei 225 Stock Average



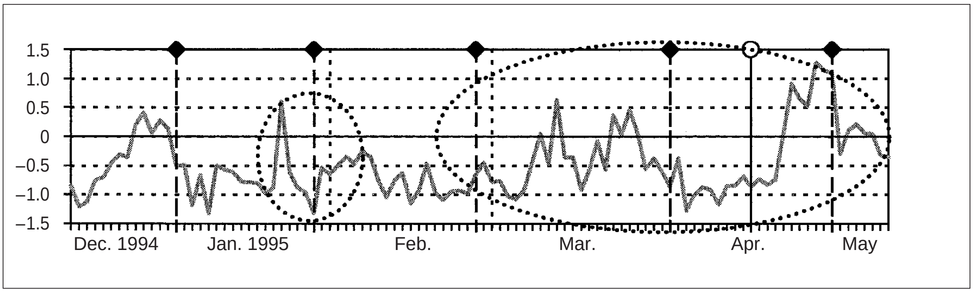
[2] Standard Deviation



[3] Skewness



[4] Excess Kurtosis



Note: Each vertical line with a circle at the top shows the date when the official discount rate was changed (April 14, 1995: 1.75 percent → 1.0 percent), and each vertical dashed line with a diamond at the top shows the date when the most actively traded contract months were changed.

In contrast, during the stock price fall from late February to early April, the skewness, while showing volatile movement opposite to the changes in the stock prices, remained consistently positive. The standard deviation increased, and the excess kurtosis exhibited volatile movements. Such movements in the summary statistics imply that downward adjustments of stock price expectations lagged behind, while the risk of a price fall was increasing.

In the meantime, from late February onward, long-term interest rates declined and bond prices increased. When we look at the expectation distribution of the JGB futures prices (Figure 19), the standard deviation increased substantially from mid-February to mid-April, while the skewness gradually declined in accordance with the rise in bond prices. These movements suggest that, from late February onward, market participants started to incorporate expectations of a cut in the official discount rate, which occurred in April.

2. The fall of the Nikkei 225 below ¥15,000 (April–September 1995)

Next, we take up the period from April to September 1995, when the Nikkei 225 fell below ¥15,000. During this period, the yen slowed its pace of appreciation and turned to depreciation from July onward, and the Nikkei 225 fell below ¥15,000 twice (in mid-June and from end-June to the beginning of July).

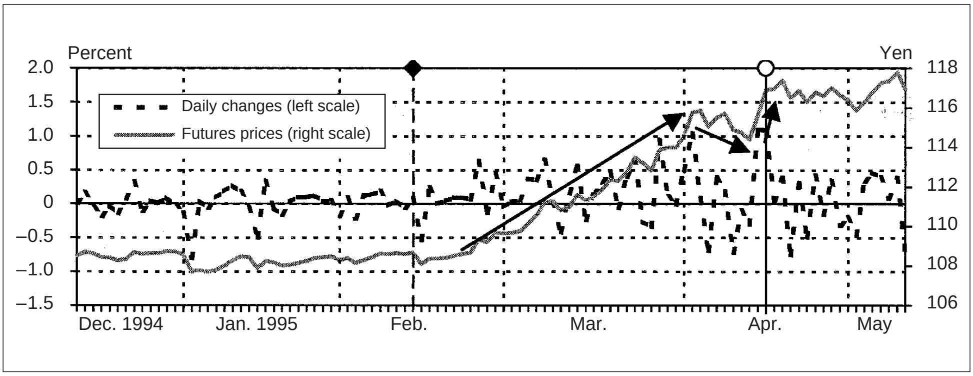
The changes in market expectations during this phase of falling stock prices (Figure 20) were that the skewness increased along with the price fall in mid-June and maintained this increased level, while the standard deviation rose. The excess kurtosis exhibited unstable movements, although its swing remained small. These movements in the summary statistics suggest that there was a delayed adjustment to the fall in stock prices, and due to the differing prospects about the effects of the economic stimulus package, market expectations seem to have diversified.

In addition, the summary statistics exhibited irregular movements from mid-July to September. Unique to this period were the large positive value of the excess kurtosis and the large jump in the standard deviation when the most actively traded contract months changed. Such changes imply that, even in the course of stock price recovery after July, market participants remained well aware of a downside risk in stock prices.

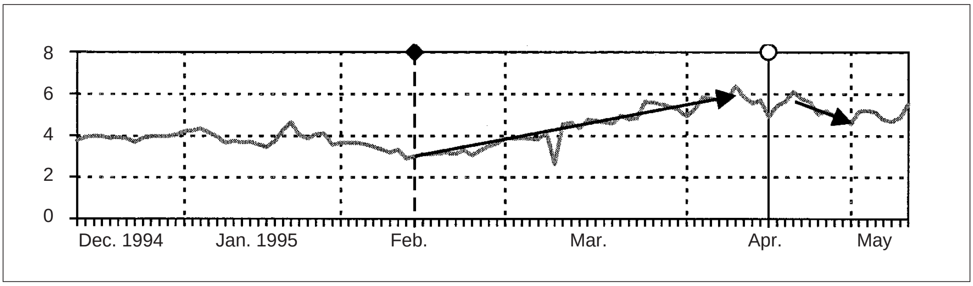
While a consensus prevailed concerning the economic slowdown that occurred from May to July, the JGB futures prices rose and, from mid-July onward, they turned down, then rose again from mid-August onward. The probability distribution of the JGB futures prices (Figure 21) tells us that, around the turning point of the market in early July, the standard deviation increased and the excess kurtosis also increased with some volatility, which implies that there was a perceived increase in the risk of large price fluctuations. In addition, from mid-August to early September, when the official discount rate was raised, the standard deviation rose, which implies that market participants were gradually absorbing the forthcoming rate raise. However, there was a large jump in the standard deviation in mid-August, when the most actively traded contract months changed, which suggests that there might have been a large shift in market views around this time.

Figure 19 Market Expectations of the JGB Futures (from December 1994 to May 1995)

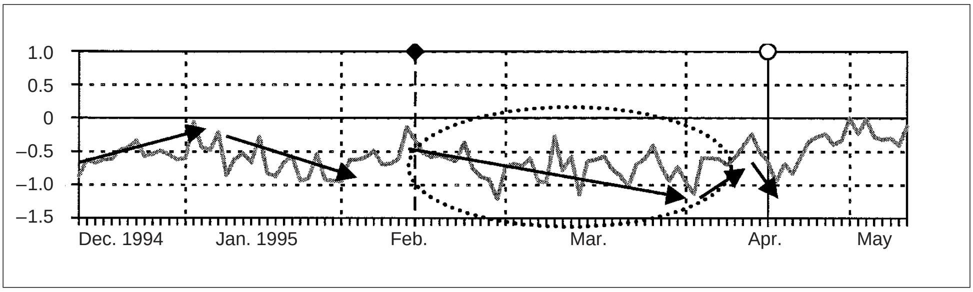
[1] JGB Futures



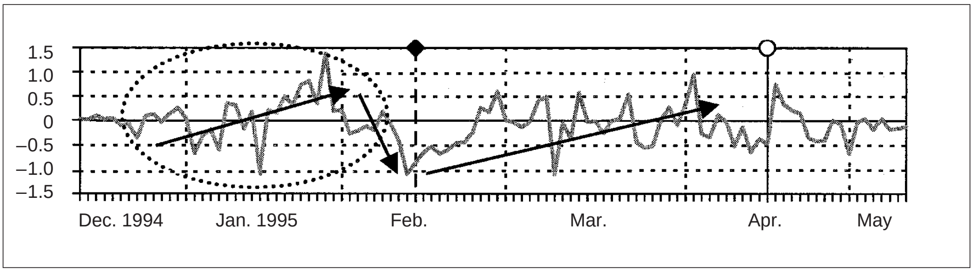
[2] Standard Deviation



[3] Skewness



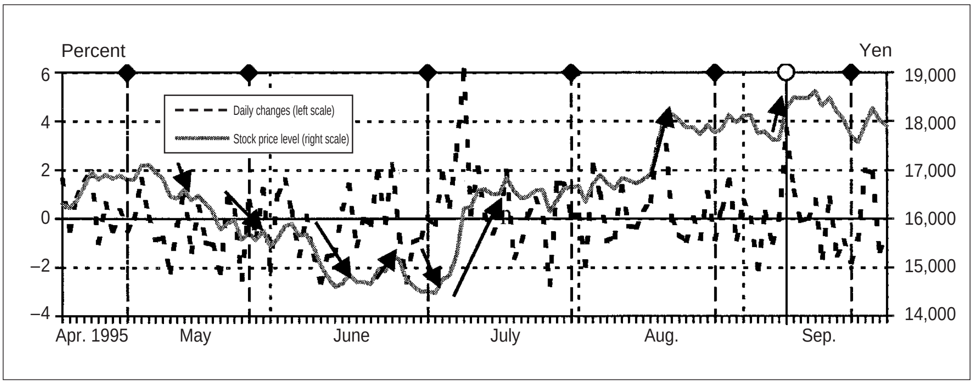
[4] Excess Kurtosis



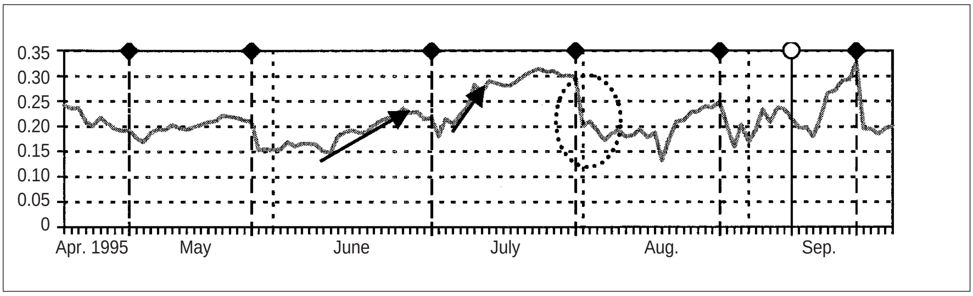
Note: Each vertical line with a circle at the top shows the date when the official discount rate was changed (April 14, 1995: 1.75 percent → 1.0 percent), and each vertical dashed line with a diamond at the top shows the date when the most actively traded contract months were changed.

Figure 20 Market Expectations of the Nikkei 225 (from April to September 1995)

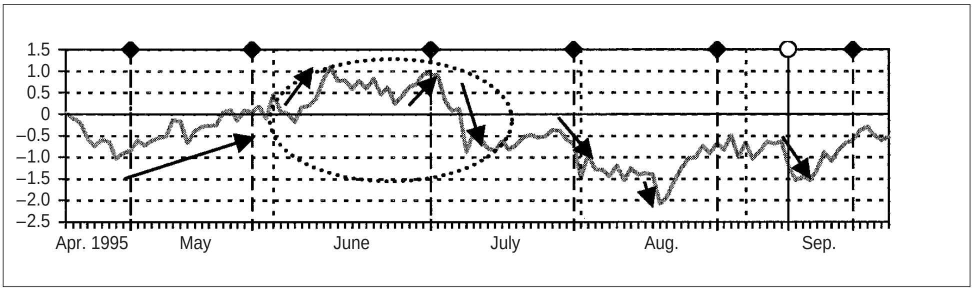
[1] Nikkei 225 Stock Average



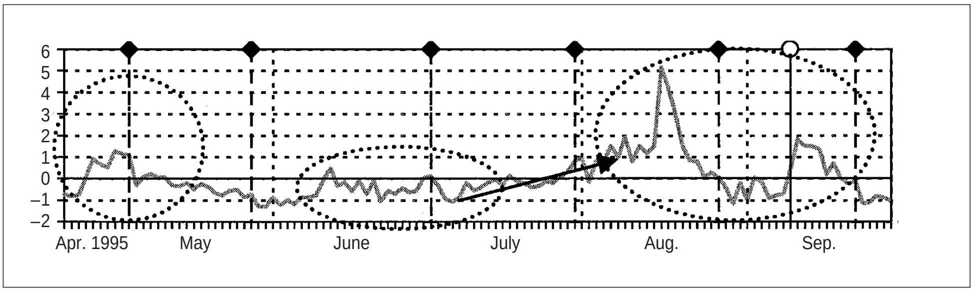
[2] Standard Deviation



[3] Skewness



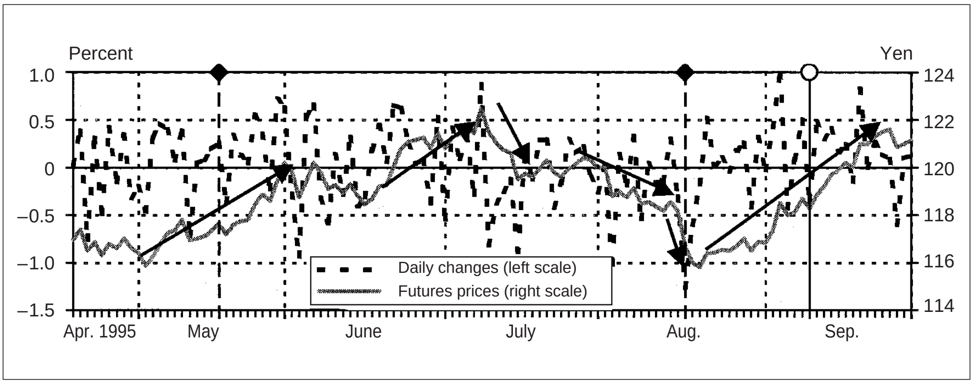
[4] Excess Kurtosis



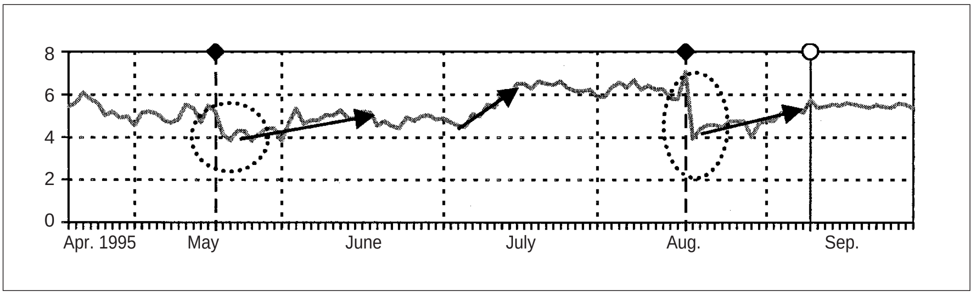
Note: Each vertical line with a circle at the top shows the date when the official discount rate was changed (September 8, 1995: 1.0 percent → 0.5 percent), and each vertical dashed line with a diamond at the top shows the date when the most actively traded contract months were changed.

Figure 21 Market Expectations of the JGB Futures (from April to September 1995)

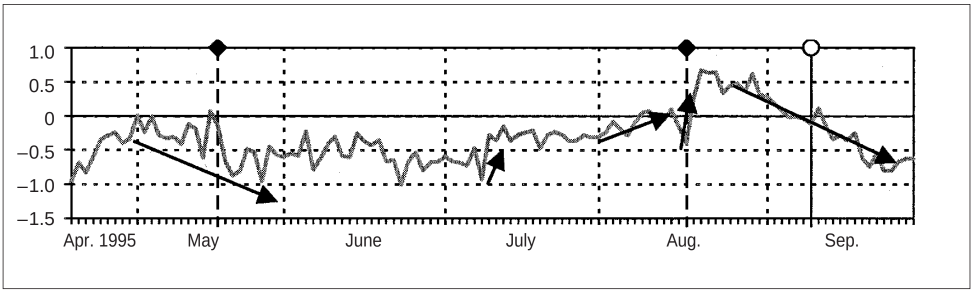
[1] JGB Futures



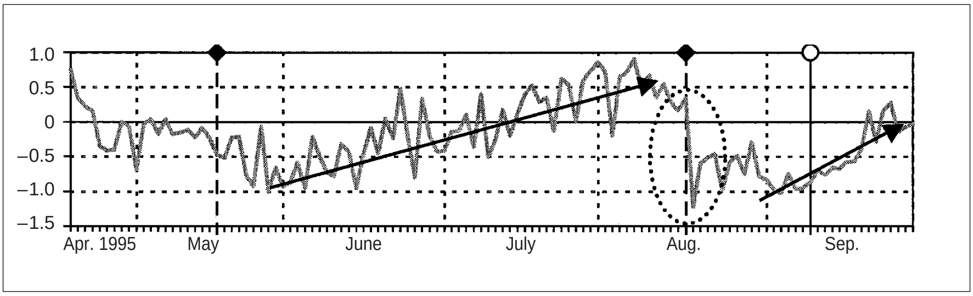
[2] Standard Deviation



[3] Skewness



[4] Excess Kurtosis



Note: Each vertical line with a circle at the top shows the date when the official discount rate was changed (September 8, 1995: 1.0 percent → 0.5 percent), and each vertical dashed line with a diamond at the top shows the date when the most actively traded contract months were changed.

IV. Conclusion

In this paper, we have estimated the time series of implied probability distributions for the periods of the collapsing “bubble” (1989–90), serious stock market slump (1992–94), and increasing anxiety about a deflationary spiral (1995). Based on these estimates, we considered (1) how market expectations of future stock prices and long-term interest rates changed; (2) how and when the relationships between such market expectations and economic indicators changed; and (3) how monetary policy implementation affected such market expectations.

In summary, market participants’ expectations are too diverse and informative to be captured merely by using a single summary statistic, i.e., the mean, because the same mean value implies different market expectations and policy implications, depending on the shape of probability distribution of expected outcome. In particular, since market participants’ confidence in stock prices differs substantially depending on timing, we can expect to capture more market information, both qualitatively and quantitatively, by carefully examining the changes in market participants’ expectations that lie behind stock price fluctuations. The results presented in this paper suggest that the implied probability distribution extracted from option prices will provide useful information for the conduct of monetary policy operations.

This paper found typical patterns linking large fluctuations of asset prices and the changes in the shape of implied probability distribution extracted from option prices by tracking the time-series movements of summary statistics. It also considered how to interpret such time-series movements in probability distribution as changes in market expectations, and discussed possible policy implications. However, studies on how to make use of the information extracted from option prices in policy judgments have only just begun.[26] For example, we have dealt with the effects of asset price fluctuations on market expectations, while issues such as the effects of specific patterns of changes in probability distribution on market fluctuations will be worth examining for analyses of market liquidity and stress situations.

In addition, as Woodford (1994) and Bernanke and Woodford (1997) pointed out in considering the role of asset prices in monetary policy operations, if a central bank tries to conduct monetary policy based on a certain feedback rule concerning asset prices, bilateral causality will emerge between asset prices and market participants’ expectations concerning future monetary policy, and thus may lead to a destabilization of the economy.

26. Methods of extracting expectations concerning future asset prices from option price information have already been put in practice by many central banks around the world. Implied forward rates are used in most G-7 countries. However, as regards the utilization of option price information, many countries, at most, evaluate the dispersion of expectations concerning future asset prices by using implied volatility estimated by the Black-Scholes formula. Countries that actively utilize risk-neutral implied probability distributions in monetary policy judgment are still quite limited in number.

APPENDIX 1: METHODOLOGY FOR ESTIMATING IMPLIED PROBABILITY DISTRIBUTIONS

In this appendix, we examine two types of estimation methods for implied probability distributions: one method is to estimate a probability density function, and the other is to estimate a cumulative distribution function. In this paper, we apply the latter estimation method.

A. Estimation of Probability Density Functions

First, we explain the method for estimating a probability density function directly. Supposing a risk-neutral market player, the price of a European-type call option (C) is given by

$$\begin{aligned} C &= \exp[-r(T-t)]E[\max(0, F_T - K)] \\ &= \exp[-r(T-t)]\int_{-\infty}^{+\infty} w(F_T)\max(0, F_T - K)dF_T, \end{aligned} \tag{A.1}$$

where F_T, K, $w(F_T)$, and r denote the price of an underlying asset on the expiration date ($t = T$) of the option, the strike price of the option, a risk-free interest rate, and a probability density function for the value F_T, respectively.

In option trading, an investor is required to maintain funds in a margin account when he or she writes an option. This is because the investor's broker and the exchange want to be satisfied that the investor will not default if the option is exercised. Although the size of the required margin depends on circumstances, the margin calculation is repeated every day. When the calculation indicates that a significantly greater margin is required, a margin call will be made. Thus, the pricing formula for a call option (A.1) can be simplified as follows:

$$C = \int_{-\infty}^{+\infty} w(F_T)\max(0, F_T - K)dF_T. \tag{A.2}$$

In addition, if we let C_K and C_{KK} be the first and second derivatives of the call option price, then the call option price satisfies the following conditions:

$$C_K = -\int_{K}^{+\infty} w(F_T)dF_T, \tag{A.3}$$

$$C_{KK} = w(K). \tag{A.4}$$

Similarly, if we let P, P_K, and C_{KK} be the European-type put option price, and the first and second derivatives of the put option price, then we have

$$P = \int_{-\infty}^{+\infty} w(F_T)\max(0, K - F_T)dF_T, \tag{A.5}$$

$$P_K = \int_{-\infty}^{K} w(F_T)dF_T, \tag{A.6}$$

$$P_{KK} = w(K). \tag{A.7}$$

In the practical application of this approach, a problem arises: there are only a finite number of strike prices. This implies that equations (A.3), (A.4), (A.6), and (A.7) do not hold strictly in the observed market prices, because they assume that the variable C is continuous in K. Thus we apply the second-order finite difference method, as Breeden and Litzenberger (1978) proposed, to equation (A.4), and obtain $w(K_i)$, the probability density function of the strike price K_i as follows:

$$w(K_i) \approx \frac{C_{i-1} - 2C_i + C_{i+1}}{(K_{i+1} - K_{i-1})^2}. \quad \text{(A.8)}$$

If we let $p(K^*)$ be the probability density over $[K_i - 0.5\kappa, K_i + 0.5\kappa]$, where κ is some positive value, then we have

$$p(K_i^*) \approx w(K_i) \cdot \kappa, \quad \text{(A.9)}$$

which yields

$$p(K_i^*) \approx \frac{C_{i-1} - 2C_i + C_{i+1}}{K_{i+1} - K_{i-1}}, \quad \text{(A.10)}$$

where $\kappa = K_{i+1} - K_i (i = 1, 2, 3, \ldots)$. This implies that we can estimate the probability density for a strike price from three call option prices.

Similarly, in the case of a put option, by applying the second-order finite difference method to equation (A.7), we have

$$p(K_i^*) \approx \frac{P_{i+1} - 2P_i + P_{i-1}}{K_{i+1} - K_{i-1}}. \quad \text{(A.11)}$$

However, this approach has a disadvantage: if the range of observable strike prices is not wide, the sum of the probabilities will be smaller than unity. Moreover, it cannot distinguish how much of the missing probability is attributable to the upper or lower edge of the probability distribution. This suggests the impossibility of estimating the skewness of the distribution of expected asset price changes.

B. Estimation of Cumulative Distribution Functions

Next, we examine the other approach, which estimates not the probability density function but rather the cumulative distribution function or implied probability distribution from the option prices (Neuhaus [1995]). This approach focuses on the information content of call (put) option premiums that contain payoff probabilities above (below) the respective strike price. In this case, the relative frequency is calculated as the difference of the cumulative probabilities for the two adjacent strike prices.

In the call option, from equation (A.3), we have

$$C_K = -\int_K^{+\infty} w(F_T)dF_T = -p(F_T \geq K). \quad \text{(A.12)}$$

This equation implies that the first-order derivative of the call option price contains information on the cumulative distribution function. By applying the first-order finite difference method, we obtain

$$p(F_T \geq K_i) \approx \frac{C_{i-1} - C_{i+1}}{K_{i+1} - K_{i-1}}, \tag{A.13}$$

yielding the probability as

$$p(K_i) = p(F_T \geq K_i) - p(F_T \geq K_{i+1}) \approx \frac{C_{i-1} - C_{i+1}}{K_{i+1} - K_{i-1}} - \frac{C_i - C_{i+2}}{K_{i+2} - K_i}. \tag{A.14}$$

Analogously, from equation (A.6), we have the probability from the put option prices as

$$p(K_i) = p(F_T \leq K_{i+1}) - p(F_T \leq K_i) \approx \frac{P_{i+2} - P_i}{K_{i+2} - K_i} - \frac{P_{i+1} - P_{i-1}}{K_{i+1} - K_{i-1}}. \tag{A.15}$$

Here, we estimate the probability for the class between two adjacent strike prices, although Breeden and Litzenberger (1978) calculate the probability density over the constant interval around the respective strike price K_i^*.

With this estimation method, we can derive the upper part of a probability distribution above the lowest strike price of a call option, and the lower part of a probability distribution below the highest strike price of a put option. Thus, we can eliminate the missing probability and obtain the complete probability distribution by combining these two probability distributions. Since the objective of this paper is to estimate the implied probability distribution to examine the development of market expectations, we employ not the method of Breeden and Litzenberger (1978) but that of Neuhaus (1995).

It is often pointed out that ITM options tend to be priced incorrectly. Therefore, we employ the following strategy to estimate a complete probability distribution: we first calculate two probability distributions from the option premium for OTM call and put options separately, then combine these probability distributions to the complete probability distribution.[27] In other words, we use the probability distribution derived from OTM put options in the lower range from the ATM strike price, and that derived from OTM call options in the upper range.

27. Needless to say, even though they are OTM options, deep-OTM options tend to be priced incorrectly. Therefore, we excluded the observation in estimating the complete probability distribution, if the calculated relative frequency turned negative.

APPENDIX 2: BASIC FRAMEWORK OF OPTION TRADING IN JAPAN

In this appendix, we summarize the basic trading framework of the Nikkei 225 option and the JGB futures option (see the Appendix Table for an outline of each transaction).

The Nikkei 225 option (a European-type option) began trading in June 1989. The contract months are four consecutive near-term expiration months, and five strike prices are set ¥1,000 and ¥500 above and below the strike price closest to the central price, which is initially set as the closing price of the Nikkei 225 on the business day before the first day of trading. The last trading day is the business day before the second Friday of each expiration month, and the option can be exercised on the business day following the last trading day.

The JGB futures option (an American-type option) began trading in May 1990. The contract months are the two closest months to March, June, September, and

Appendix Table Trading Mechanism of Financial Options in Japan

	Nikkei 225 option	JGB futures option
Date launched	June 12, 1989	May 11, 1990
Trading exchanges	Osaka Securities Exchange	Tokyo Stock Exchange
Underlying assets	Nikkei 225 stock price index	Ten-year JGB futures
Trading unit	Nikkei 225 multiplied by ¥1,000	JGB futures of ¥100 million in face value
Contract month	Four consecutive near-term expiration months	Two months closest to March, June, September, and December (quarterly months) plus one or two serial months (non-quarterly months: since November 20, 1997)
Expiration date	The business day before the second Friday of each expiration month	The last business day of the month prior to the delivery month of the underlying JGB futures
Trading period	Four months	Six months
Strike prices	Five strike prices are initially opened with ¥500 intervals, i.e., 2 OTMs, 1 ATM, and 2 ITMs. Additional strike prices will be available according to the movement of the underlying Nikkei 225 Stock Average.	Seven strike prices are initially opened with ¥1 intervals, i.e., 3 OTMs, 1 ATM, and 3 ITMs. Additional strike prices will be available according to the underlying JGB futures' price movement.
Delivery date	European-type option (the business day following the last trading day)	American-type option (a buyer of an option may exercise the option at any time prior to expiration). All in-the-money options at the end of the expiration date will be automatically exercised, unless otherwise instructed.
Quotation	¥5 in the case of premiums at or below ¥1,000, and ¥10 in the case of premiums above ¥1,000	1/100 points per 100 points (¥10,000 per contract)
Daily price limit	±¥1,000 (previous day's closing price of Nikkei 225 is less than 20,000); ±¥1,500 (20,000–less than 30,000); ±¥2,000 (30,000–less than 40,000); ±¥2,500 (40,000 or more)	Two points upward or downward (¥2 million per contract)

Sources: For the Nikkei 225 options, Osaka Securities Exchange (http://www.ose.or.jp); for options on 10-year JGB futures, Tokyo Stock Exchange (http://www.tse.or.jp/).

December,[28] and seven strike prices are set with three ¥1 intervals above and below the central price, which is set based on the closing price of the previous business day. The last trading day is the last business day of the month prior to the delivery month of the underlying JGB futures, and the option can be exercised at any time during the six-month trading period.

28. It should be noted that, in addition to the four contract months mentioned above, short-term option trading that starts from the 20th of the month prior to the last trading month has taken place since November 20, 1997.

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