How Can We Extract a Fundamental Trend from an Economic Time-Series?

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This paper attempts to extract a fundamental trend, which we call a "trend-cycle component," from an economic time-series. The "trend-cycle component" consists of a medium-term business cycle component and a long-term trend component. The objective is to eliminate the short-term irregular and seasonal variations that hide a fundamental trend in an economic time-series.

We test five different time-series methods. Among them, the Henderson moving average (which is incorporated in an X-12-ARIMA seasonal adjustment program), the Band-Pass filter (which utilizes a Fourier transformation), and the DECOMP are found to be effective in extracting a "trend-cycle component" with a cyclical period longer than 1.5 years. However, no method is found to be effective in extracting a "long-term trend component" with a cyclical period longer than that of a medium-term business cycle. Although the HP filter is somewhat successful, it still contains a component with a cyclical period of about three years that corresponds to a business cycle.

These methods are useful for forecasting a wide variety of economic variables because they reveal a fundamental trend in the time series. In addition, statistical programs are available for easy application. They have, however, a few shortcomings. First, it is often difficult to provide a meaningful economic interpretation of the revealed characteristics of the "trend-cycle component." Second, the addition of new data can change the estimation results. In particular, an extracted component around the end of a sample period is likely to be revised with new data. Special caution is in order, therefore, in interpreting the estimation results and forecasting the time series when the data exhibit large variations. In this case, comparing the results of different methods provides a useful way to assess the reliability of an extracted "trend-cycle component."

Key words: Fundamental trend; Time-series analysis; X-12-ARIMA; DECOMP; HP filter; Fourier transformation; Beveridge and Nelson decomposition

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I. Introduction

An economic time-series consists of multiple components corresponding to short-term irregular and seasonal variations, a medium-term business cycle, and a long-term trend movement. Most macroeconomic analysis is concerned with a medium-term business cycle and a long-term trend movement. However, these fundamental movements are hidden in the original economic data because various irregular and seasonal variations are dominant in the data. Therefore, it is often difficult to read directly from the original data the fundamental movement of an economic variable under study.

Economists have tried various methods of overcoming this problem. A good example is the method of seasonal adjustment. This method aims at extracting the fundamental movement of an economic variable by eliminating seasonal variations from the original time-series data. However, in many cases, seasonal adjustment alone is not capable of extracting the fundamental movement of an economic variable. For example, it is difficult to gauge the exact direction of a business cycle by looking at the month-to-month change in the seasonally adjusted price and industrial production indexes, since they include a lot of volatile variations caused by temporary shocks.[1] For this reason, if we want to gauge the direction of a business cycle, we need to find a method that eliminates short-term irregular variations and extracts a fundamental movement that corresponds to the business cycle.

We may be also interested in a long-term theoretical relationship between economic variables.[2] In this case, what we want to extract is not a medium-term business cycle component, but a long-term trend component with a longer cyclical period. Moreover, this long-term trend component may be useful when one wants to obtain a stationary stochastic process, by eliminating the extracted component from the original time-series data. However, we cannot read such a long-term trend component directly from the original data or the seasonally adjusted data. Nor can we assume a straight line or a specific curve as representing a long-term trend component. We need some kind of objective method that enables us to extract a long-term trend component.

Economists have utilized many statistical methods to extract such medium- and long-term components from economic time-series data. These methods can be categorized into two groups. The first group attempts to extract the components, using the characteristics of the time-series data. The second group attempts to extract the components by using cross-sectional data, in an attempt to break down the original time-series data.

1. An alternative way is to look at the year-to-year change of a time-series variable on a monthly basis. However, it is difficult to identify the turning point of a time-series variable when a year-to-year change is used. Kimura (1995), for example, argues that a year-to-year change has several shortcomings. He points out that, first, the value of a year-to-year change depends on the month-to-month variation in the previous year. Second, a year-to-year change can give wrong information about the turning point of the trend.
2. In this paper, we include a deterministic linear trend and a nonstationary stochastic trend in the concept of a "long-term trend component." Although they do not actually possess a cyclical component, we here interpret them as having an "infinite cycle." Thus, they belong to the group of components with a "long cycle." We must also recognize that a long-term trend component will not be detected if its cyclical period is longer than that of the data.

In practice, economists have mainly developed and used time-series methods for their research, because such methods possess several advantages. First, they do not require additional data other than an original time-series. In contrast, cross-sectional methods require more detailed data on the sub-components of an original time-series. In the case of a price index, for example, the latter methods require data on sub-items in the price index basket. This requirement means extra work, and it is sometimes impossible to meet that requirement because of the scarcity of the sub-items of data. Second, time-series methods do not require special assumptions concerning the characteristics of changes in the components of an original time-series. On the other hand, when using cross-sectional methods one must specify the characteristics of changes in each component, which often requires an additional analysis.

Nevertheless, time-series methods have a few shortcomings. First, a change in the sample period due to the introduction of new data often causes a shift in the extracted component. As time-series methods smooth out the original variations in the light of time-series characteristics, it cannot avoid this problem of instability of an extracted component, particularly around the end of a sample period. Second, time-series methods do not always provide a proper economic interpretation of the movement of an extracted component.

This paper attempts to extract and analyze a medium- and long-term cyclical component from the following macroeconomic variables: the price level (CPI), money supply (M2+CDs), and real output (index of industrial production [IIP]). We test the following five time-series methods: the Henderson moving average (which is incorporated in the X-12-ARIMA seasonal adjustment program), the Band-Pass filter (which utilizes a Fourier transformation), the HP filter (Hodrick and Prescott [1980]), the DECOMP, and the Beveridge and Nelson decomposition.

We define a "trend-cycle component" as a combination of a medium-term business cycle component and a long-term trend component in an economic time-series. The "trend-cycle component" is meant to capture the fundamental movement of an economic time-series, which excludes irregular and seasonal variations. Further eliminating a middle-frequency component that corresponds to a business cycle from the "trend-cycle component," we can define a "long-term trend component" as the remaining low-frequency component. Finally, we may obtain a "medium-term business cycle component" as the difference between the "trend-cycle component" and the "long-term trend component." We have adopted these definitions because there is no statistical method by which we can directly extract a "medium-term business cycle component."

The main findings of this paper may be summarized as follows: the Henderson moving average, the Band-Pass filter, and the DECOMP are effective methods for extracting a "trend-cycle component" with a cyclical period longer than 1.5 years. Therefore, they provide us with a useful means for analyzing a trend movement in the economic time-series. However, they are not effective methods for decomposing the "trend-cycle component" into the medium-term business cycle component and the long-term trend component. For the purpose of extracting a long-term trend component, the HP filter may be a somewhat more effective method.

The extracted component obtained by the filter, however, seems to contain part of a middle-frequency component that corresponds to a business cycle.

Time-series methods are known to have the shortcoming that an extracted component tends to be revised as new data are added to a sample period. Therefore, we have conducted a stability test on the three methods (the Henderson moving average, the Band-Pass Filter, and the DECOMP methods) that are found to be effective in extracting the "trend-cycle component." We have found that the components extracted by each method are likely to be revised around the end of a sample period with an addition of new data, particularly when the original data have highly volatile variations. In such a case, there tend to be large differences between the components extracted by each method. Therefore, by comparing the results of different methods, we can assess the reliability of the extracted "trend-cycle component" and the likelihood of a future revision.

We may conclude that time-series methods provide us with an objective and easy way to extract a "trend-cycle component" from an economic time-series. We must be aware, however, that time-series methods also have several limitations. First, it is often difficult to provide a proper economic interpretation of the movement of an extracted component. Second, the results are often unstable with respect to the addition of new data. Therefore, we may need to supplement the results of time-series methods by utilizing a cross-sectional analysis and economic theory. As long as we recognize these limitations, time-series methods provide us with an effective means of analyzing the direction of many important economic time-series.

The rest of the paper is organized as follows: Chapter II discusses methods for extracting a fundamental trend from an economic time-series. These are divided into two groups. The first group comprises time-series methods, and the second cross-sectional methods. After discussing the characteristics of both methods, we focus on the details of the time-series methods used in this paper. Chapter III discusses the characteristics of macroeconomic variables, to which we apply the time-series methods. Then, using graphs and spectral analysis, we examine the characteristics of an extracted "trend-cycle component." Chapter IV tests the stability of an extracted "trend-cycle component" with respect to the addition of new data, in particular, around the end of a sample period. Chapter V evaluates the performance of different methods and the stability of their results. Chapter VI concludes this paper with a summary of our findings and unresolved issues.

II. The Purpose and Methods of Extracting a "Trend-Cycle Component"

A. The Concept of a "Trend-Cycle Component" and Its Objective

An economic variable change is driven by various factors. It may change in response to exogenous shocks and also to endogenous forces. More often than not, it changes under the influence of both endogenous and exogenous factors. To study the mechanism of an economic change, it is often useful to decompose the movement of an economic variable into several components, and then analyze each component separately. We will use the cyclical period of each component as the yardstick for such decomposition.

Decomposition by cyclical period has several advantages. First, it can easily eliminate short-term variations with a short cyclical period, which may arise from, for example, seasonal factors and measurement errors in an economic time-series. In economic analysis, it is important to separate a fundamental business cycle and trend movement from such short-term variations. Without using the cyclical period as a yardstick, it will be impossible to achieve such separation. Second, decomposition by cyclical period helps us gauge the time scale of an economic cycle. The impact of an economic change depends not only on its magnitude, but also on its duration. Decomposition by cyclical period, therefore, provides us with a lot of useful information.

We must, however, be aware of its limitations. In the past, there were several occasions on which the impact of a one-time shock lasted for a relatively long period of time. We experienced a sharp rise in the oil price twice during the 1970s, for example, but the rises themselves did not last long. However, they brought about a rise in the expected rate of inflation in many countries for a long time thereafter. There are investment models in which a single technology shock can have a lasting positive effect on future investment and production because it improves the cash flow of a company.[3] In the case in which a short-term variation is interconnected with a longer-term movement,[4] there is a possibility that decomposition by cyclical period could fail to capture a component corresponding to either the cause or the effect of an economic change.

In this paper, we will decompose an economic time-series by cyclical period into the three components: a long-term trend component, a medium-term business-cycle component, and a short-term component. That is, we assume the following equation:

an economic time-series = a long-term trend component
+ a medium-term business cycle component + a short-term component.

The medium-term component corresponds to a business cycle with a cyclical period of a few years. The short-term component corresponds to high-frequency variations such as seasonal changes and measurement errors. The long-term component corresponds to low-frequency movement, which does not belong to the short- and medium-term components, and includes a deterministic as well as a nonstationary stochastic trend. Because the Japanese business cycles have widely different cyclical periods, it is not straightforward to specify a critical value of the cyclical period that distinguishes between the short-, medium-, and long-term components. In this paper, we adopt a definition in which the short-term component has a cyclical period of less than one or 1.5 years, and the long-term component longer than five or

3. See Kiyotaki and Moore (1995). Chapter 4 of Saito (1996) presents a concise summary of investment models.
4. Ghysels (1988) and Miron (1996) argue that seasonal variations, which are often eliminated in economic analysis, are closely interconnected with the medium-term business cycle.

six years. The medium-term component is composed of movements with a cyclical period between the two.

In practice, however, economic analysis normally decomposes economic data into two components: (1) a trend-cycle component (the medium-term and long-term components), which excludes measurement errors, small irregular shocks, and seasonal variations; and (2) a long-term trend component, which excludes the medium-term business-cycle component in addition to the short-term component.

The first component is used, for example, when the aim is to analyze and forecast the direction of a business cycle or to study the medium-term relationship between economic variables. Ideally, one wants to separate the business-cycle component from the long-term trend component. However, since it is difficult to do so in practice (see below), the first difference of a trend-cycle component is often used instead, as the business-cycle component.

On the other hand, the second component is used when the aim is to analyze and forecast a path of the long-term trend movement, which is independent from the business cycle, or to study the long-term relationship between economic variables. It is customary, for example, to define the GDP gap as the difference between the actual GDP path and the long-term trend path of GDP, which is a proxy of an equilibrium (potential) GDP.[5] Another possibility is to extract the short- and medium-term components as a stationary time-series by eliminating the long-term trend from the data. No reliable method is yet available, however, for extracting a long-term trend capable of performing this function. Yet, another possibility is to distinguish between different kinds of shocks that affect an economic variable by decomposing its time series into the long-term trend component and all the other components.[6]

B. Time-Series Methods and Cross-Sectional Methods

There are two kinds of methods of extracting a trend-cycle component from economic data: time-series methods and cross-sectional methods. Cross-sectional methods decompose an economic variable into several sub-components, and judge whether it exhibits a trend or an irregular change by comparing the corresponding changes in the sub-components. Shiratsuka (1997), for example, decomposes the CPI into sub-components and calculates the price change of each component. Then he calculates a price index that captures a trend movement in the price index (the "underlying inflation trend") by excluding the sub-components with unusually large changes (outliers). Basically, he is assuming that the unusual variations found in the sub-components represent transitory changes, and therefore they should not be included in the "underlying inflation trend."[7]

5. See Haltmaier (1996), for example.
6. The most common practice is to distinguish between permanent and transitory shocks. For example, the real business-cycle theory views a permanent shock as a supply shock originating from a technological innovation, and a transitory shock as a demand shock originating from monetary and fiscal policies. There are two methods of distinguishing between permanent and transitory shocks: one applies to a single variable, and the other to multiple variables. In this paper, we only use the single variable method of Beveridge and Nelson (1981). See Blanchard and Quah (1989) for a multivariate method.
7. Shiratsuka (1997) does not impose any periodic conditions on the "underlying inflation trend." Therefore, this "underlying inflation trend" is not necessarily the same as the "trend-cycle component" defined in this paper.

On the other hand, a time-series method uses the characteristics of an economic time-series variable, and uses those characteristics to decompose the variable into components such as the trend-cycle component. Time-series methods can be categorized into two types by the difference in their basic approaches.

The first type mainly focuses on the period and smoothness of variations of a time series. Eliminating and extracting components according to a preset parameter (a cyclical period), one decomposes the data into a trend-cycle component and a transitory component. In other words, one extracts a smooth component from the data and defines it as a trend-cycle component. The rest is regarded as a transitory component. Three examples of this type are the Henderson moving average, the Band-Pass filter, and the HP filter. The Henderson moving average smooths a time series by taking its moving average and extracts a long- or medium-cycle component from the data. The Band-Pass filter extracts a component with a specified cyclical period, by using the power spectrum.[8] The HP filter can be categorized in the first type since it extracts a trend by specifying a parameter that defines the smoothness of a time series (Hodrick and Prescott [1980]).

The second type specifies a time-series model that has a trend-cycle component and a transitory component. It extracts a trend-cycle component that fits the model best in light of an objective criterion such as the AIC. Three examples of this type are the method of fitting a linear (or nonlinear) deterministic trend, the Beveridge and Nelson method, and the DECOMP. The method of fitting a deterministic trend is the simplest one.[9] The Beveridge and Nelson method decomposes a time series into a stochastic trend, which represents a trend-cycle component, and the remainder into a transitory component. The DECOMP assumes and tries to estimate the best model by using a state space model that explicitly incorporates a stochastic trend, a stationary AR factor, a seasonal component, and white noise.

C. The Time-Series Method of Extracting a Trend-Cycle Component

In this paper, we use five time-series methods to extract a trend-cycle component. Since there is a lot of readily available software, these methods are frequently used in economic analysis. Among them, the Henderson moving average, the Band-Pass filter, and the HP filter belong to the extraction method that specifies the length of the periodic (smoothness) parameter, as explained in the previous section. The Henderson moving average is incorporated in an X-12-ARIMA seasonal adjustment program. The Band-Pass filter uses a Fourier transformation.[10] The HP filter uses a method proposed by Hodrick and Prescott (1980). The Beveridge and Nelson method and the DECOMP belong to the method that specifies a time-series model which endogenously decomposes a time series into several components. In the following section, we briefly explain these methods (for the Band-Pass filter and the DECOMP, see Appendix 2 for more details).

8. Using a spectrum with a specified cyclical period is a common practice in engineering fields. This approach, however, does not have a particular name in economic literature. In this paper, to distinguish it from other approaches, we call it the Band-Pass filter.
9. The problem of this approach is that the selection of a deterministic trend becomes arbitrary and it can greatly affect the result. In particular, it is said that when the data contain a stochastic trend, this approach may misinterpret part of the stochastic trend as a short-term cyclical component.
10. See Appendix 1 for the Fourier transformation.

1. The Henderson moving average that is incorporated in an X-12-ARIMA seasonal adjustment program

The X-12-ARIMA is a seasonal adjustment program, which uses a moving average. In the seasonal adjustment process, it extracts a trend-cycle component that has a short cyclical period from an original time-series by eliminating irregular variations. Suppose the original time-series consists of a trend-cycle component (TC_t), a seasonal component (S_t), and an irregular component (I_t) (subscript t represents time). In this X-12-ARIMA method, we regard TC_t as a trend-cycle component, which we defined in our previous chapter (a "long-term trend component" itself cannot be derived by this method). Each component can be obtained as follows:

① Obtain a temporary TC_t by calculating the 12-month moving average of the original data, which tries to even out and therefore eliminate S_t and I_t from the original time-series.
② Obtain a temporary $S_t + I_t$ by subtracting the temporary TC_t(①) from the original data.
③ Obtain a temporary S_t by calculating the moving average for several years of the temporary $S_t + I_t$(②), which evens out I_t.
④ Obtain a temporary $TC_t + I_t$ by subtracting the S_t(③) from the original data.
⑤ Obtain a revised TC_t by calculating the Henderson moving average of the temporary $TC_t + I_t$(④), which evens out I_t.
⑥ Repeat steps ②–⑤ and obtain the final value of (TC_t, S_t, I_t).

The cyclical period of an extracted trend-cycle component depends on the order of the Henderson moving average.[11] In the case of monthly data, either the 9th, 13th, or 23rd order of the moving average is normally employed. Furthermore, in order to obtain the mid-point moving average around the end of a sample period, we add the forecast values of the ARIMA to the original time-series. As a result, the reliability of the extracted trend-cycle component around the end of the original time-series data depends on the accuracy of the ARIMA forecast values.

2. The Band-Pass filter using the Fourier transformation

The Band-Pass filter refers to the filtering methods that extract a component with a cyclical period which lies within a specified band. Therefore, there are many possible variations in terms of this type of filtering. In this paper, we use a filtering method that utilizes the Fourier transformation. The steps are as follows:

(1) Obtain a sample periodogram for each frequency by a Fourier transformation of the original data.
(2) Keep the sample periodograms within a specified periodic band, while replacing the rest with zero.
(3) Reconstruct the time series by applying the inverse Fourier transformation to the new periodograms.

An advantage of this method is that there is no need to make seasonal adjustments. This is so because eliminating all the components with a cyclical period of less than a year automatically eliminates all seasonal variations. In this paper, we try to

11. The Henderson moving average is one of weighted moving-average methods, whose weights are predetermined in advance. For more details, see Bureau of the Census (1995).

extract a trend-cycle component (a long-term trend component plus a medium-term business-cycle component) by eliminating all the components with a cyclical period of less than a specified value. Since this method assumes the stationarity of the data, we apply the Fourier transformation to the first-differenced time-series data.[12]

3. The Hodrick and Prescott filter (HP filter)

The HP filter aims to extract a trend component that closely follows actual values with a given degree of smoothness. It assumes that a time series consists of two components: a growth component (g_t) and a cyclical component (c_t). Then it calculates the value of g_t that solves the following minimization problem:

$$\underset{\{g_t\}_{t=-1}^{T}}{\mathrm{MIN}} \left\{ \sum_{t=1}^{T} c_t^2 + \lambda \sum_{t=1}^{T} \left[(g_t - g_{t-1}) - (g_{t-1} - g_{t-2}) \right]^2 \right\},$$

where λ is a parameter that determines the relative weight given to the two terms. The greater its value, the smoother and closer the growth component is to a straight line. The smaller its value, the closer it is to actual variations. Therefore, the cyclical period of each component depends on the value of λ. Generally, analysts set λ at 1600 for quarterly data and 14400 for monthly data in order to extract a long-term trend component, which has a longer cycle than that of a business cycle.

4. DECOMP

DECOMP is a seasonal adjustment program that uses a state space model (Gersch and Kitagawa [1984] and Kitagawa [1986]). It assumes that a time series consists of the following five components: a trend component (T_t), a stationary AR component (V_t), a seasonal component (S_t), a trading day component (D_t), and white noise (ε_t). The trend component (T_t) is determined by the following m-th order stochastic difference equation:

$$(1 - B)^m T_t = \nu_{1t}, \text{ where } \nu_{1t} \sim N(0, \tau_1^2),$$

where B is a lag operator, which is defined by $BT_t = T_{t-1}$. DECOMP projects this stochastic difference equation on a state space model, and then estimates each component using the information square-root filter.[13] Although the analyst uses the information that the trend component is smooth, he does not decide the value of a

12. Because the Fourier representation assumes stationarity, it cannot be applied to a nonstationary time-series. Otherwise, the nonstationary component will have a long-cycle sample periodogram, and its power spectrum will have a very large value. Moreover, the error size of the long-cycle component will not decline with an increase in the sample size, because the estimated spectrum under the Fourier transformation is inconsistent. Therefore, it is necessary to repeat taking a difference in the time series until the spectrum distribution becomes smooth and the time series becomes stationary. On the other hand, the time series with more than second-order difference cannot be reconstructed after the Band-Pass filter is applied. In view of this property, the order of difference in this paper is the first order, although some of the time series applied in this paper such as M2+CDs become stationary only after taking their second-order differences. Also, in view of the loss of information about a trend due to taking a difference, extracting a long-term trend component with a very long cycle should be avoided.
13. The information square-root filter is somewhat similar to the Kalman filter. The filter uses the square-root decomposition instead of a variance-covariance matrix in the Kalman filter algorithm, which seeks a smoothed distribution of a time series. Its purpose is to overcome the problem of numerical instability when there is a nonstationary factor involved in a model, such as a seasonal adjustment model.

parameter that defines the degree of smoothness (such as the order of a stochastic difference equation). Instead, he chooses the value of each parameter that minimizes the Akaike information criterion (AIC), which indicates the fit of the model.[14]

The estimated trend component should correspond to what we have called the "long-term trend component" because it is an $I(m)$ process (where m is a positive integer), which contains no drift term. However, there is a possibility that it also contains a business-cycle component, and therefore it corresponds to what we have called the "trend-cycle component." This is because DECOMP uses the AIC to determine the overall fit of the model with an AR component. Therefore, we do not know beforehand whether the estimated component corresponds to the "long-term trend component" or the "trend-cycle component."

5. The Beveridge and Nelson decomposition

The Beveridge and Nelson (1981) method decomposes a time series into a permanent component and a cyclical component when it contains both deterministic and stochastic trends. It defines the movement that can be explained by the deterministic and stochastic trends as the permanent component, and the rest as the cyclical component. It expresses a time series as an ARIMA model, and then replaces this model by the MA(∞) representation. Using this MA(∞) representation, one can write the $(t+s)$ period value of the time series that is expected at period t as follows:

$$E_t p_{t+s} = a_0 s + p_t + \left(\sum_{i=1}^{s}\beta_i\right)\varepsilon_t + \left(\sum_{i=2}^{s+1}\beta_i\right)\varepsilon_{t-1} + \left(\sum_{i=3}^{s+2}\beta_i\right)\varepsilon_{t-2} + \cdots,$$

where p represents the time series, a_0 a constant, and ε a stochastic term. Subscript t represents time. On the right-hand side of the equation, the terms other than the first term

$$\left[p_t + \left(\sum_{i=1}^{s}\beta_i\right)\varepsilon_t + \left(\sum_{i=2}^{s+1}\beta_i\right)\varepsilon_{t-1} + \left(\sum_{i=3}^{s+2}\beta_i\right)\varepsilon_{t-2} + \cdots\right],$$

(where $s \rightarrow \infty$) represent those variations that affect the expected values. Therefore, they are called the permanent component (μ_t). The cyclical component is then obtained by deducting the permanent component from the original time-series. Since the permanent component is an $I(1)$ process, it conceptually corresponds to what we have called the "long-term trend component."

III. The Characteristics of a "Trend-Cycle Component" and a "Long-Term Trend Component"

A. Main Characteristics of an Economic Time-Series

In this paper, we analyze the consumer price index (CPI; All Japan [General]), money

14. Using the AIC to determine the best fit of a model is essentially the same idea as minimizing the loss function in the HP filter. The only difference is whether a parameter is determined endogenously within a model. Using DECOMP, a parameter value can be objectively determined by the AIC, while analysts have to select it rather arbitrarily when using the HP filter (see Ishiguro [1986]).

supply (M2+CDs), and the index of industrial production (IIP).[15] The sample period is from January 1976, when the impact of the first oil crisis had largely dissipated, to March 1997. First, we study the characteristics of the three economic time-series, using graphs, autocorrelation, spectral analysis, and a stationarity test.[16]

1. Graphs and autocorrelation

First, we examine the graphical movement of each time series. They all have an upward trend in the logarithm graphs (Figure 1), which suggests that these time series may have either stochastic or deterministic trends. To eliminate such trends, we take a first difference of logarithm of each time series (Figure 2). The graphs show that they all have short-term high-frequency cycles. The standard deviation of the first difference is very large for IIP, which exhibits a volatile movement. But it is small for CPI and M2+CDs (Table 1).[17] There is no apparent trend shift in the first difference of CPI and IIP; but a major trend shift exists in the first difference of M2+CDs around 1990.

Next, we calculate autocorrelation and examine the smoothness of the time-series variations. Autocorrelation in the logarithm form maintains a high level even for long lags (Figure 3). This indicates that each time series may not be stationary around its average. Autocorrelation in the first difference of logarithm falls sharply in line with a lag length for IIP and CPI, while it remains relatively high for M2+CDs (Figure 4). This is because CPI and IIP contain high-frequency cycles while a change in M2+CDs is relatively smooth.

2. Spectral analysis (for more details, see Appendix 1)

Spectral analysis reveals the cyclical structure of a time series. By using a Fourier transformation on the first difference of logarithm of each time series, we obtain its power spectrum distribution. The first difference is necessary because we need a stationary time-series for spectral analysis. There is a possibility that some time series require the second or more difference for the stationarity. However, using a second or more difference will cause a problem in that it eliminates important information about the rate of change of an economic variable. Therefore, we use only the first difference in this paper. The results (Figure 5) show that, in the case of M2+CDs, a long-cycle (low-frequency) "long-term trend component" is strong, while that of a short-cycle (high-frequency) component is weak. In the case of IIP, not only long- but also short-cycle components are strong, confirming its high volatility. In the case of CPI, a long-cycle component is relatively weak in comparison with those of M2+CDs and IIP, while a short-cycle component is relatively strong, although it is weaker than that of IIP.

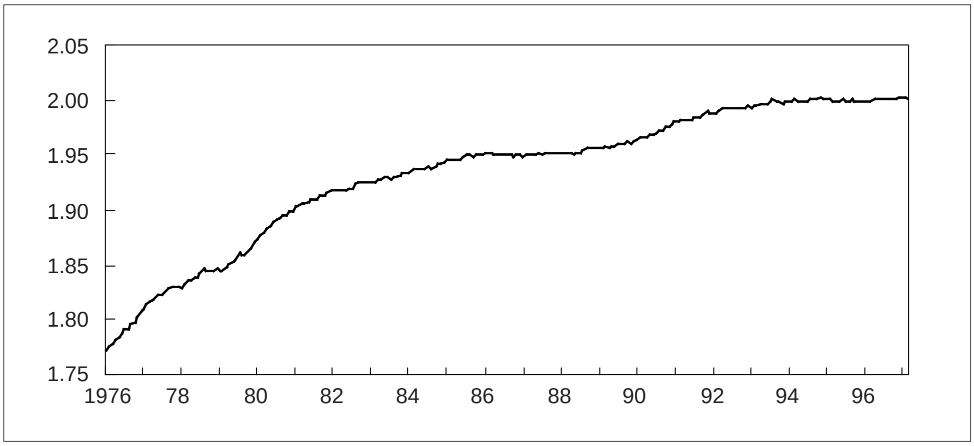
15. IIP covers only the manufacturing industry, whose GDP share has been declining over time in Japan. Therefore, it may not be the most representative index for capturing an overall production movement in the Japanese economy as a whole. However, in order to use monthly data for all the variables in this paper, we have decided to use IIP. When we experimented with the GDP quarterly data in the spectral analysis and the stationarity test ($I(1)$), the results were essentially the same as those of IIP. The difference between the GDP and IIP time series, therefore, may be limited to a level of the growth rate.

16. In Section III.A, to capture the main characteristics of the time series, we have employed the seasonally adjusted data using an X-12-ARIMA program.

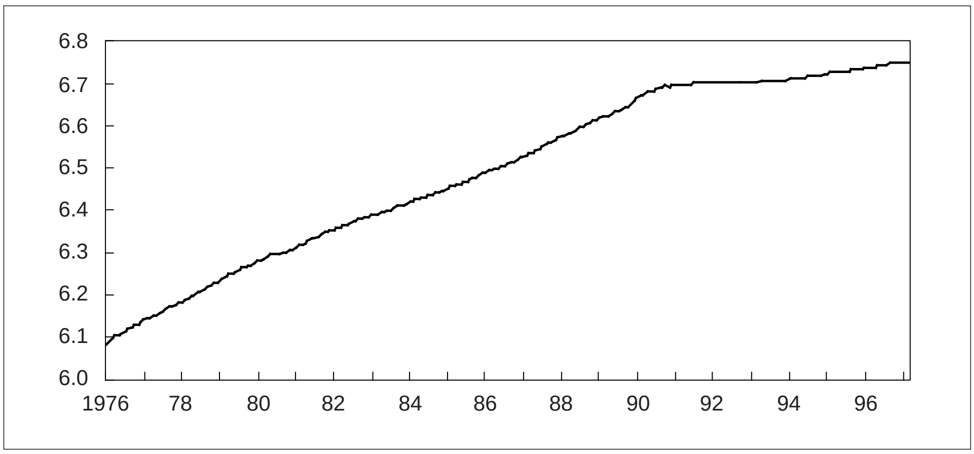
17. According to our results, the presence of a trend in the growth rate of M2+CDs cannot be rejected. In such a case, it may not be appropriate to compare volatility by using the standard deviation around its average. In order to evaluate the result more accurately, we must use another method to judge volatility.

Figure 1 The Graphical Movements of Each Time Series (Logarithmic Level, Seasonally Adjusted)

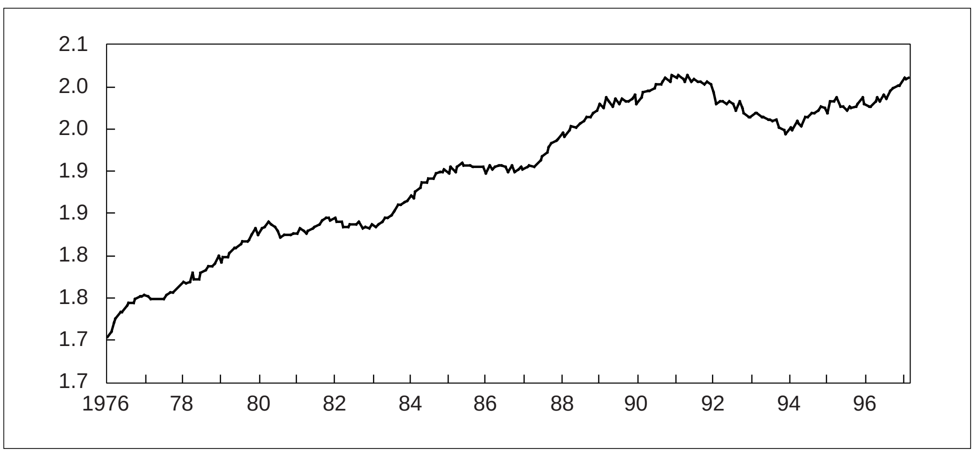
[1] CPI



[2] M2+CDs



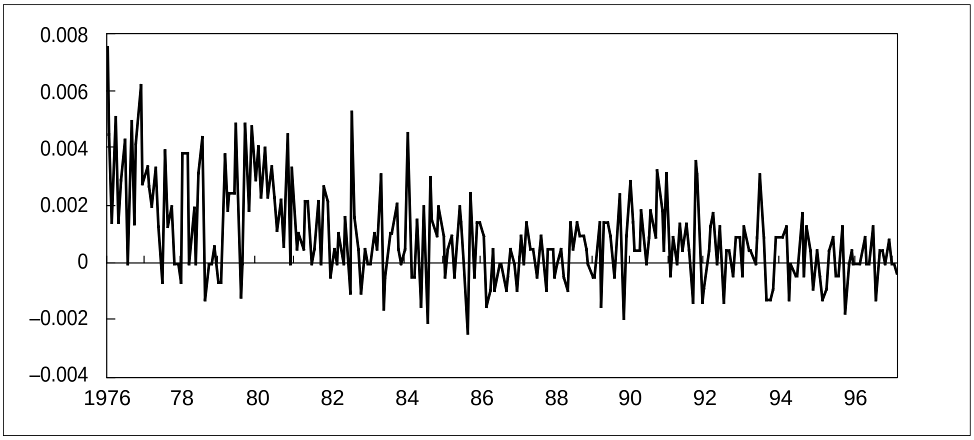
[3] IIP



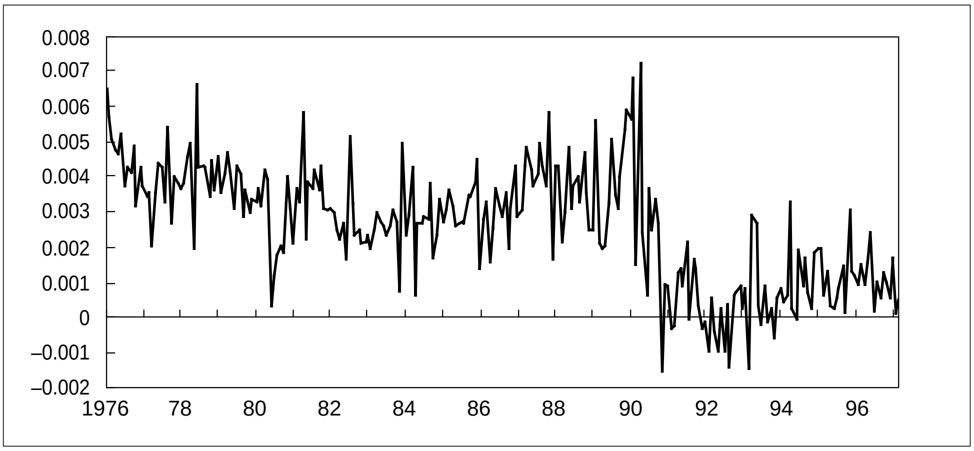
Note: Seasonal adjustment was done using the X-12-ARIMA method.

Figure 2 The Graphical Movements of Each Time Series (First Difference in Logarithm, Seasonally Adjusted)

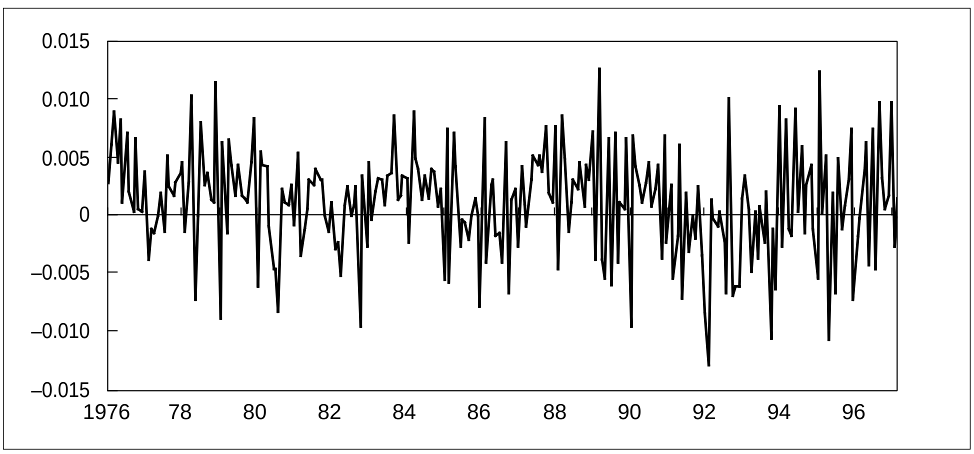
[1] CPI



[2] M2+CDs



[3] IIP

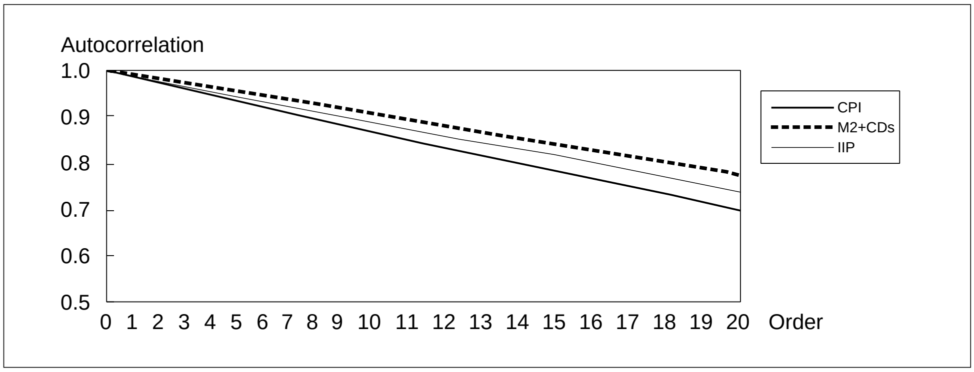


Note: Seasonal adjustment was done using the X-12-ARIMA method.

Table 1 Mean and Standard Deviation of the First Difference in Each Series (January 1975–March 1997)

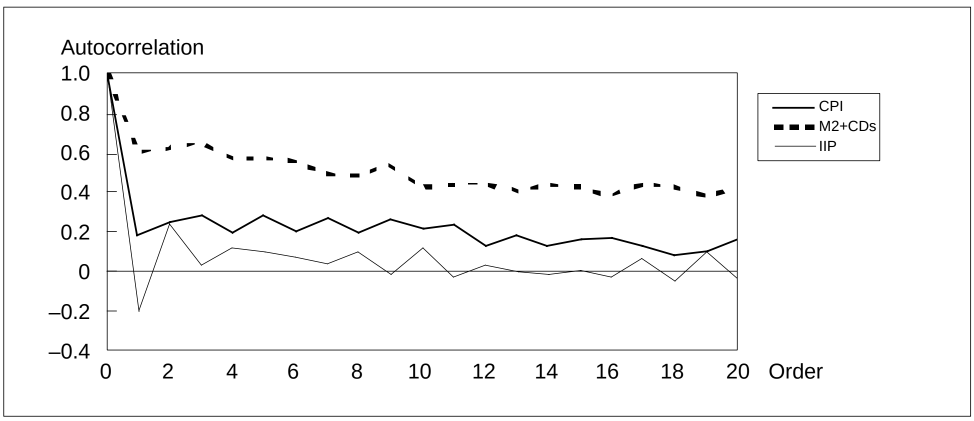
	Mean	Standard deviation
CPI	0.000934	0.001630
M2+CDs	0.002626	0.001695
IIP	0.001205	0.004572

Figure 3 Autocorrelation of Each Time Series (Logarithmic Level, Seasonally Adjusted)



Note: Seasonal adjustment was done using the X-12-ARIMA method.

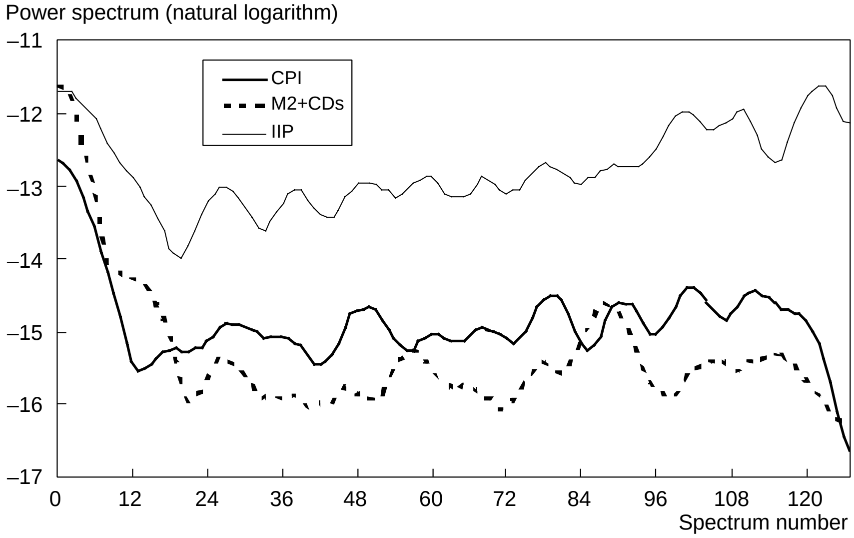
Figure 4 Autocorrelation of Each Time Series (First Difference in Logarithm, Seasonally Adjusted)



Note: Seasonal adjustment was done using the X-12-ARIMA method.

To sum up, CPI, M2+CDs, and IIP all have a trend and, furthermore, the first difference of M2+CDs seems to have a trend. A volatile short-cycle component is strong in IIP and also to a lesser extent in CPI. A long-cycle component is stronger in M2+CDs than IIP and CPI.

Figure 5 The Power Spectra of Each Time Series (First Difference in Logarithm, Seasonally Adjusted, Power Spectrum)



Spectrum number	0	··· 12	··· 24	···36	··48	··60	··72	·84	·96	108	127
Period/months	∞	··· 21	··· 10	··· 7	··· 5	···4	·3.5	··3	2.6	2.4	2

Notes: 1. See Appendix 1 for the details of the power spectrum.
2. Seasonal adjustment was done using X-12-ARIMA.

3. Stationarity test

It is important to test the stationarity of an economic time-series because the validity of econometric analysis often depends on whether it is stationary or nonstationary. For this purpose, we employ the Augmented Dickey-Fuller (ADF) test. The ADF test estimates the following equations:

$$\Delta P_t = \gamma P_{t-1} + \sum_{i=2}^{n} \beta_i \Delta P_{t-i+1} + \varepsilon_t \qquad (1)$$

$$\Delta P_t = \alpha_0 + \gamma P_{t-1} + \sum_{i=2}^{n} \beta_i \Delta P_{t-i+1} + \varepsilon_t \qquad (2)$$

$$\Delta P_t = \alpha_0 + \gamma P_{t-1} + \alpha_1 t + \sum_{i=2}^{n} \beta_i \Delta P_{t-i+1} + \varepsilon_t, \qquad (3)$$

where P represents the original time-series, Δ the first difference, and ε an error term. Subscript t represents time. Equation (1) is a pure random-walk model. Equation (2) adds a drift term (α_0) to equation (1). And equation (3) further adds a linear time trend ($\alpha_1 t$) to equation (2). The ADF test suggests that a time series has a unit root if γ is not statistically significantly different from zero, and it is stationary if γ is significantly different from zero ($\gamma < 0$).[18] Table 2 presents the results of the ADF test on the seasonally adjusted CPI, M2+CDs, and IIP.[19] They indicate that CPI is trend-stationary. M2+CDs is $I(2)$. And IIP is $I(1)$, that is, a unit root with a drift term.[20]

Table 2 Unit Root Test (ADF Test) (January 1975–March 1997)

Monthly	(1) Without drift and trend	(2) With drift	(3) With drift and trend
CPI~$I(0)$	×	○*	○
M2+CD~$I(0)$	×	×	×
M2+CD~$I(1)$	×	×	×
M2+CD~$I(2)$	○	○	○
IIP~$I(0)$	×	×	×
IIP~$I(1)$	○	○*	○

Notes: 1. $I(\ )$ refers to the number of order in difference. For example, $I(1)$ stands for a result that was obtained by applying a "first difference" of original time-series data to this test.
×: when null hypothesis (H_0: $\gamma = 0$) has not been rejected.
○: when null hypothesis (H_0: $\gamma = 0$) has been rejected and alternative hypothesis (H_1: $\gamma < 0$) has been accepted.
2. Asterisk indicates when alternative hypothesis (H_1: $\gamma < 0$) has been accepted, and either the drift term of equation (2) or the drift and trend term of equation (3) are significant at the 1 percent level.

B. The Characteristics of Extracted Trend-Cycle Components and Long-Term Trend Components

Now we apply five different methods of extraction discussed in Section II.C to the above macroeconomic variables, and then compare their results. At the same time, we examine whether and how the time-series characteristics of each variable found in the previous section affect those results. In the following analysis, we use the original time-series without seasonal adjustments, unless noted otherwise. In comparing the cyclical structures of the time series, we perform spectral analysis on their first difference.

1. The Henderson moving average

The Henderson moving average is a method that is incorporated in an X-12-ARIMA seasonal adjustment program to extract a trend-cycle component. In the program dealing with monthly data, the number of terms in the Henderson moving average is automatically selected among 9, 13, and 23. However, depending on the number of terms used in the calculation of the moving average, the characteristics of an extracted component can be significantly different.

18. The ADF test is the most popular unit root test because it only requires applying the least squares estimation method to equations. However, one weakness of the ADF test is that its power (i.e., the possibility which precisely denies the hypothesis when it is wrong) declines sharply when the true value is close to unity.

19. In the present test, we have seasonally adjusted the time-series data using the X-12-ARIMA program. However, Ghysels and Perron (1993) argue that using seasonally adjusted data for the unit root test creates a bias in the estimation of coefficients, and therefore that its power of test declines. Consequently, we should interpret the results of our unit root test with some caution.

20. The test assumes that there is no change in the trend and the drift term. Therefore, if there is a structural change during a sample period, the results may differ (see Soejima [1995]). Considering this possibility, we have tested M2+CDs, breaking the sample period into several sub-periods. The results show that M2+CDs is $I(1)$ before 1990, and is $I(0)$ between 1982 and 1990, suggesting that we should be cautious in interpreting the test results.

In the case of CPI, the moving average with 23 terms exhibits a smooth movement while those with 9 and 13 terms include short-term variations with a cyclical period of a few months to less than a year (Figure 6). Spectral analysis on the first difference of the logarithm (Figure 7 [2]) shows that the moving average with 23 terms retains components with a cyclical period longer than a year, while eliminating most components with a shorter cyclical period. It suggests that the moving average with 23 terms is better suited to the task of extracting a trend-cycle component. On the other hand, the moving averages with 9 and 13 terms fail to extract a clear long-term component because they contain components with a cyclical period of 6–8 months. This finding also applies to the case of M2+CDs and IIP. They likewise have a smooth moving average with 23 terms, extracting a long-term component, while those with 9 and 13 terms are affected by components with a shorter cyclical period (figures 8 and 9).

Next we compare the extracted components of each time series in terms of the extraction ratio (Table 3).[21] In the case of the moving average with 23 terms, the cyclical period of components with an extraction ratio of less than 50 percent is 12 months for CPI and M2+CDs, and 14 months for IIP. Similar results are obtained for the moving average with 9 and 13 terms. The cyclical period with the ratios of less than 50 percent for IIP is seven months for the moving average with 9 terms, and 12 months for that with 13 terms. It is a longer cyclical period than CPI and M2+CDs in both cases. These results show slight variations in the cyclical structure of time series among the three economic variables. However, these variations seem to be very small to be concerned with in practice. In conclusion, the moving average with 23 terms is appropriate for studying the business cycle.[22]

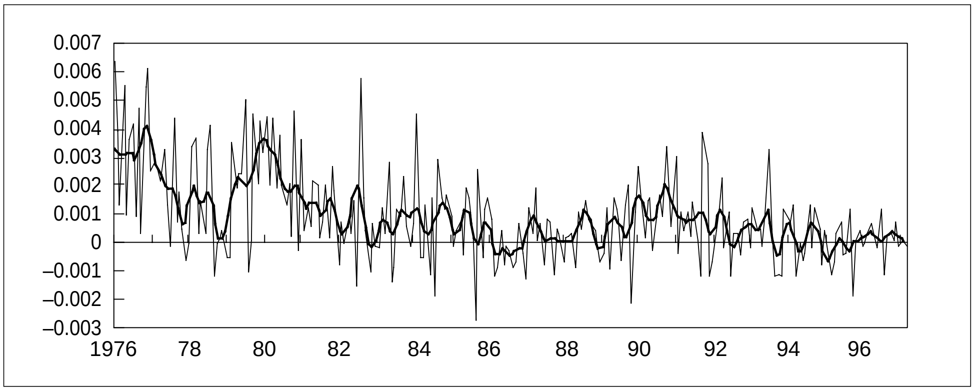
2. The Band-Pass filter

The Band-Pass filter uses a Fourier and inverse Fourier transformation to extract a cyclical component from a time series. It is particularly useful if our aim is to extract a pre-specified component. Figure 10-1 shows the results of the Band-Pass filter applied to the first difference of logarithm of CPI. When a component with a cyclical period longer than six months or one year is extracted by this method, there still exist short-term variations because it contains some seasonal factors. However, when a component with a cyclical period longer than 1.5 years is extracted, the filter eliminates seasonal variations and exhibits a smooth movement, which corresponds to the trend-cycle component.[23] In this way, the Band-Pass filter can extract a trend-cycle component without seasonal adjustments. Basically, it can also extract a long-term trend component with a longer cyclical period, although the length of a time series and an estimated error in the spectrum may limit the range of the band we can choose.

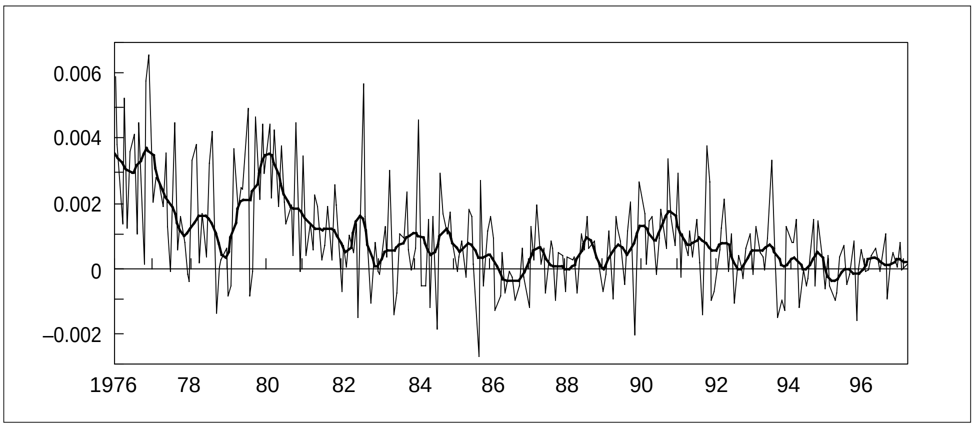
21. The extraction ratio compares the size of an extracted component after filtering to the size before filtering for each cyclical period. In the case of the Henderson moving average, a long-term component is extracted by taking the moving average to a seasonally adjusted time series (by the X-12-ARIMA program). As a result, the extraction ratio is calculated as the square root of (power spectrum of the trend-cycle component)/(power spectrum of the seasonally adjusted time series).
22. In this paper, we have concluded that the moving average with 23 terms is the best among the three cases (9, 13, 23 terms) that are automatically selected. However, an option of the X-12-ARIMA program enables us to choose any odd number of terms. Therefore, there may be some number of terms that outperforms the case of 23 terms.
23. An application of the Band-Pass filter to M2+CDs and IIP has produced similar results (figures 10-2 and 10-3).

Figure 6 Trend-Cycle Components of CPI Extracted by Henderson Moving Average (First Difference in Logarithm)

[1] Moving average: 9 terms



[2] Moving average: 13 terms



[3] Moving average: 23 terms

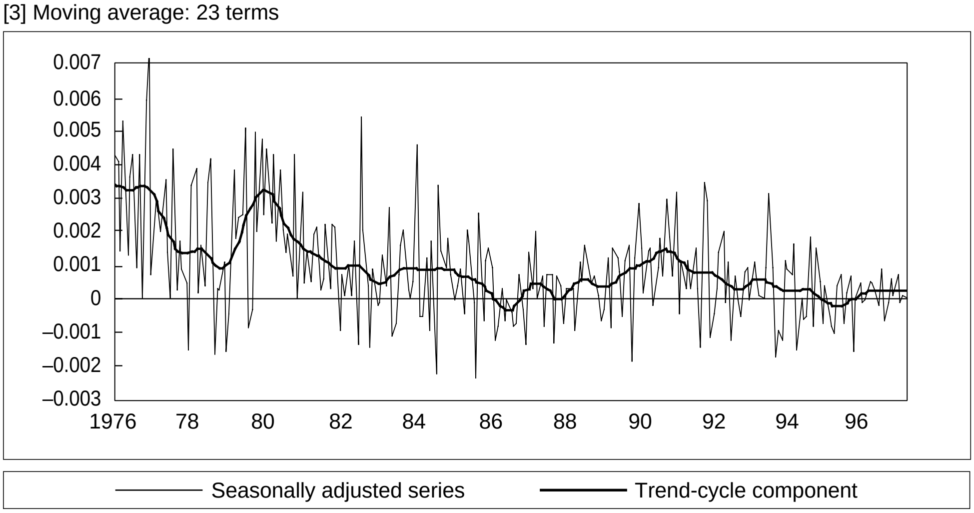
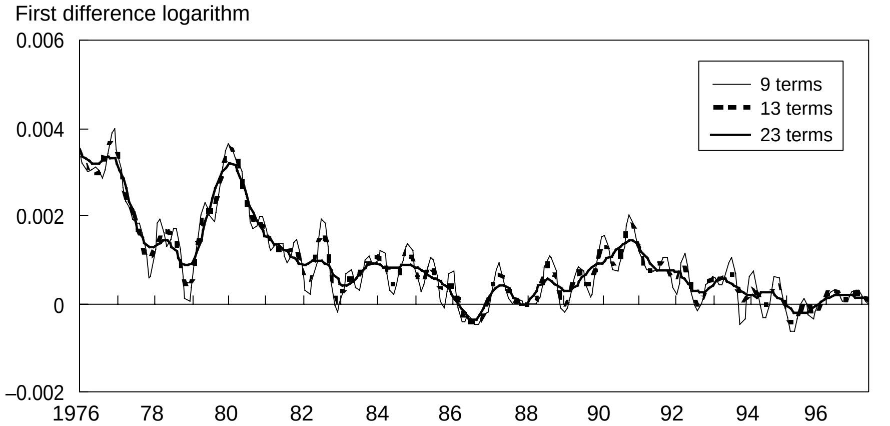
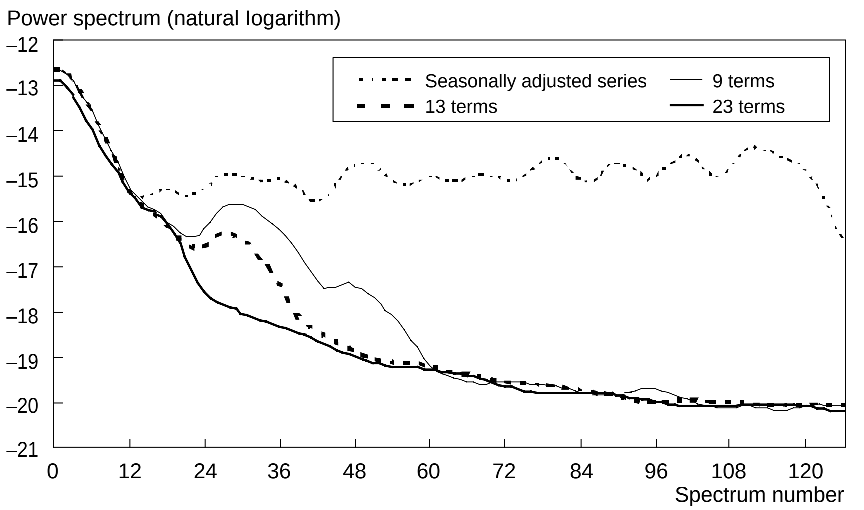


Figure 7 Trend-Cycle Components of CPI Extracted by Henderson Moving Average: Differences by Term

[1] Trend-cycle component of CPI (9, 13, 23 terms)



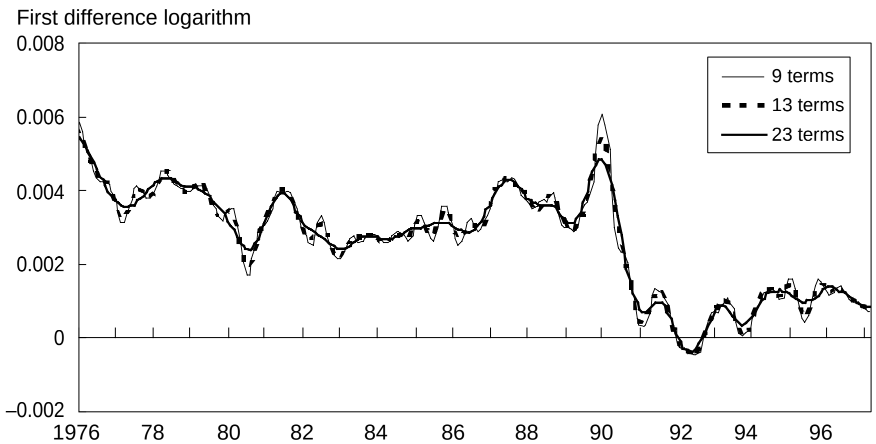
[2] Power spectra of trend-cycle components (CPI)



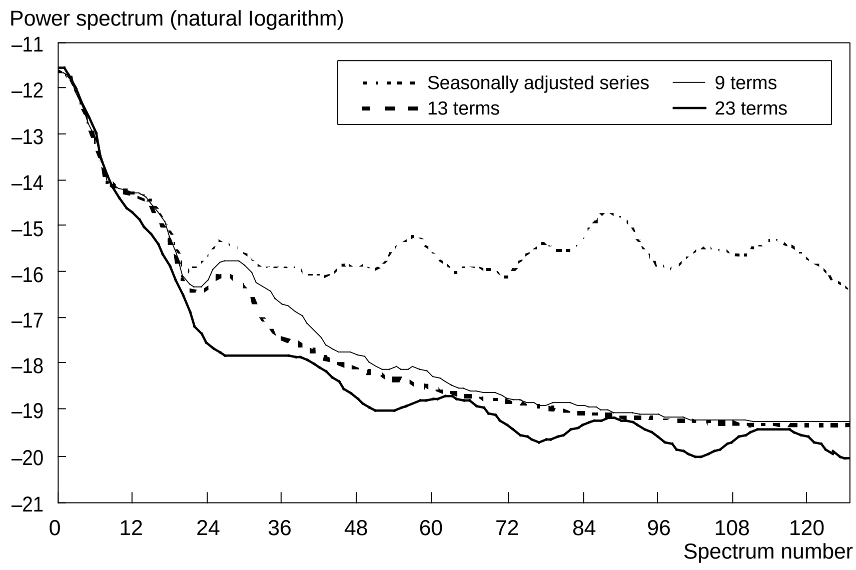
Spectrum number	0	··· 12	··· 24	··· 36	··· 48	··· 60	·· 72	· 84	· 96	108	127
Period/months	∞	··· 21	··· 10	··· 7	··· 5	··· 4	· 3.5	·· 3	2.6	2.4	2

Figure 8 Trend-Cycle Components of M2+CDs Extracted by Henderson Moving Average: Differences by Term

[1] Trend-cycle component of M2+CDs (9, 13, 23 terms)



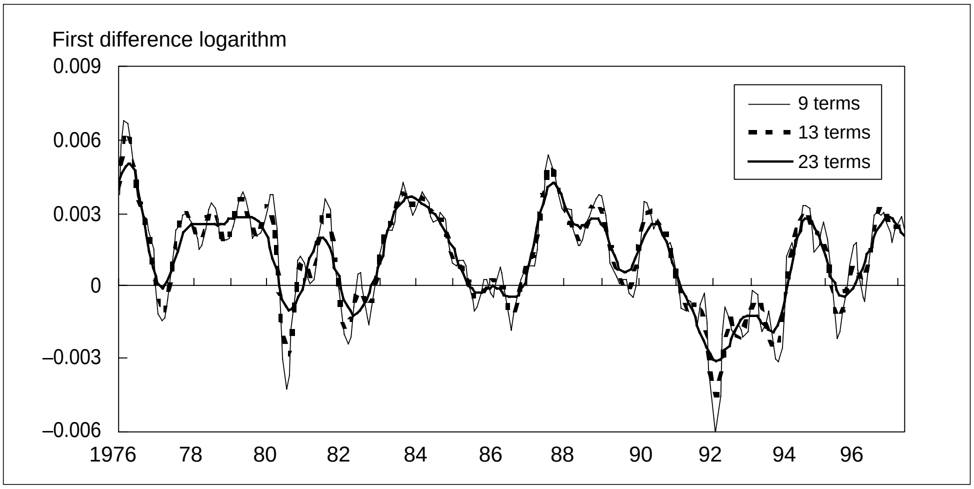
[2] Power spectra of trend-cycle components (M2+CDs)



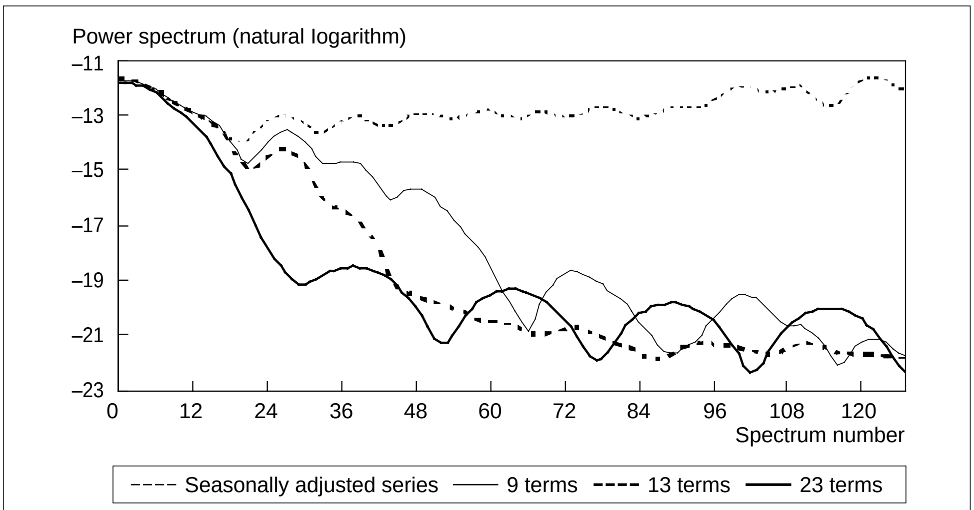
Spectrum number	0	12	24	36	48	60	72	84	96	108	127
Period/months	∞	21	10	7	5	4	3.5	3	2.6	2.4	2

Figure 9 Trend-Cycle Components of IIP Extracted by Henderson Moving Average: Differences by Term

[1] Trend-cycle component of IIP (9, 13, 23 terms)



[2] Power spectra of trend-cycle components (IIP)



Spectrum number	0	12	24	36	48	60	72	84	96	108	127
Period/months	∞	21	10	7	5	4	3.5	3	2.6	2.4	2

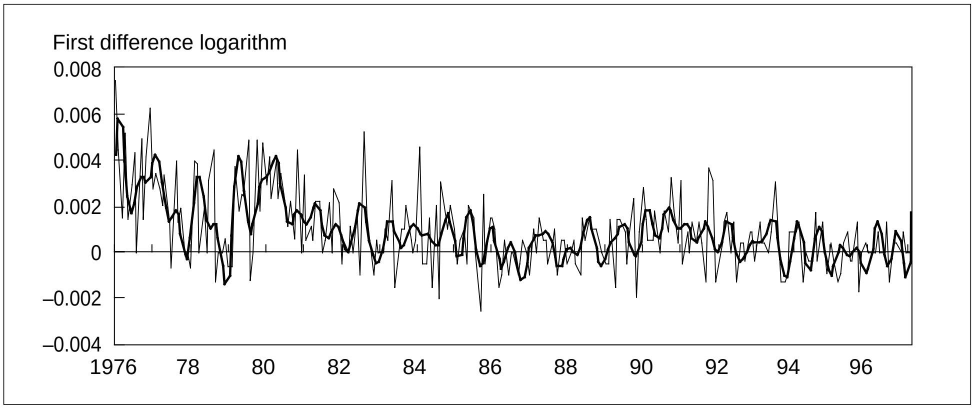
Table 3 Comparison between the Extracted Components of Each Time Series in Terms of the Extraction Ratio of Less than 50 Percent

	9 terms	13 terms	23 terms
CPI	7 months	9 months	12 months
M2+CDs	6 months	8 months	12 months
IIP	7 months	12 months	14 months

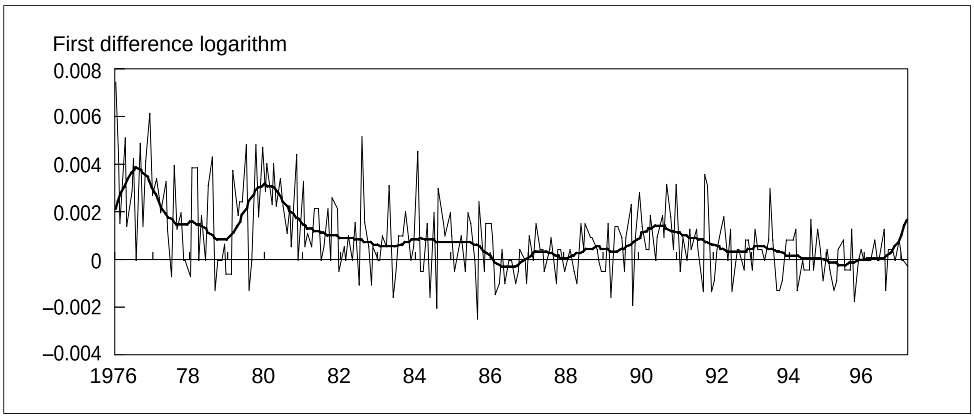
Note: The extraction ratio is calculated by a square root of (power spectrum of trend-cycle component/power spectrum of seasonally adjusted series).

Figure 10-1 Trend-Cycle Components of CPI Extracted by Band-Pass Filter (First Difference in Logarithm)

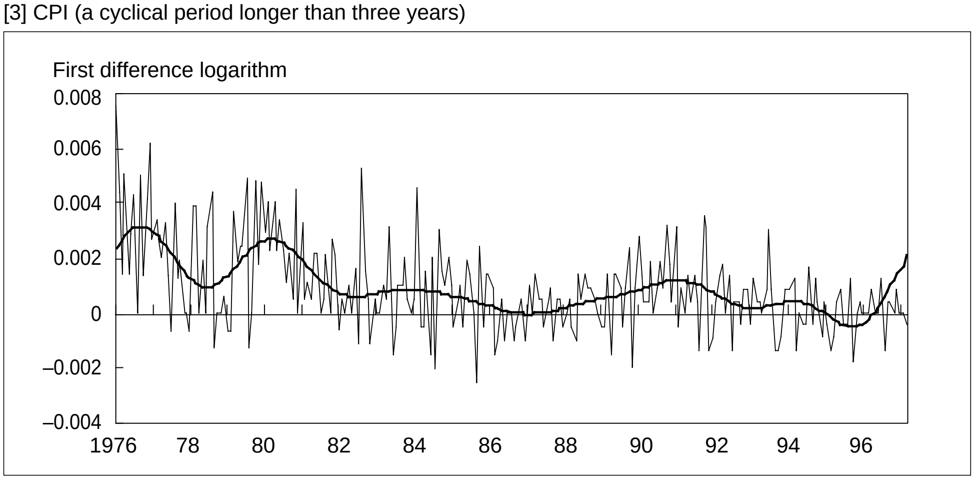
[1] CPI (a cyclical period longer than six months)



[2] CPI (a cyclical period longer than 1.5 years)



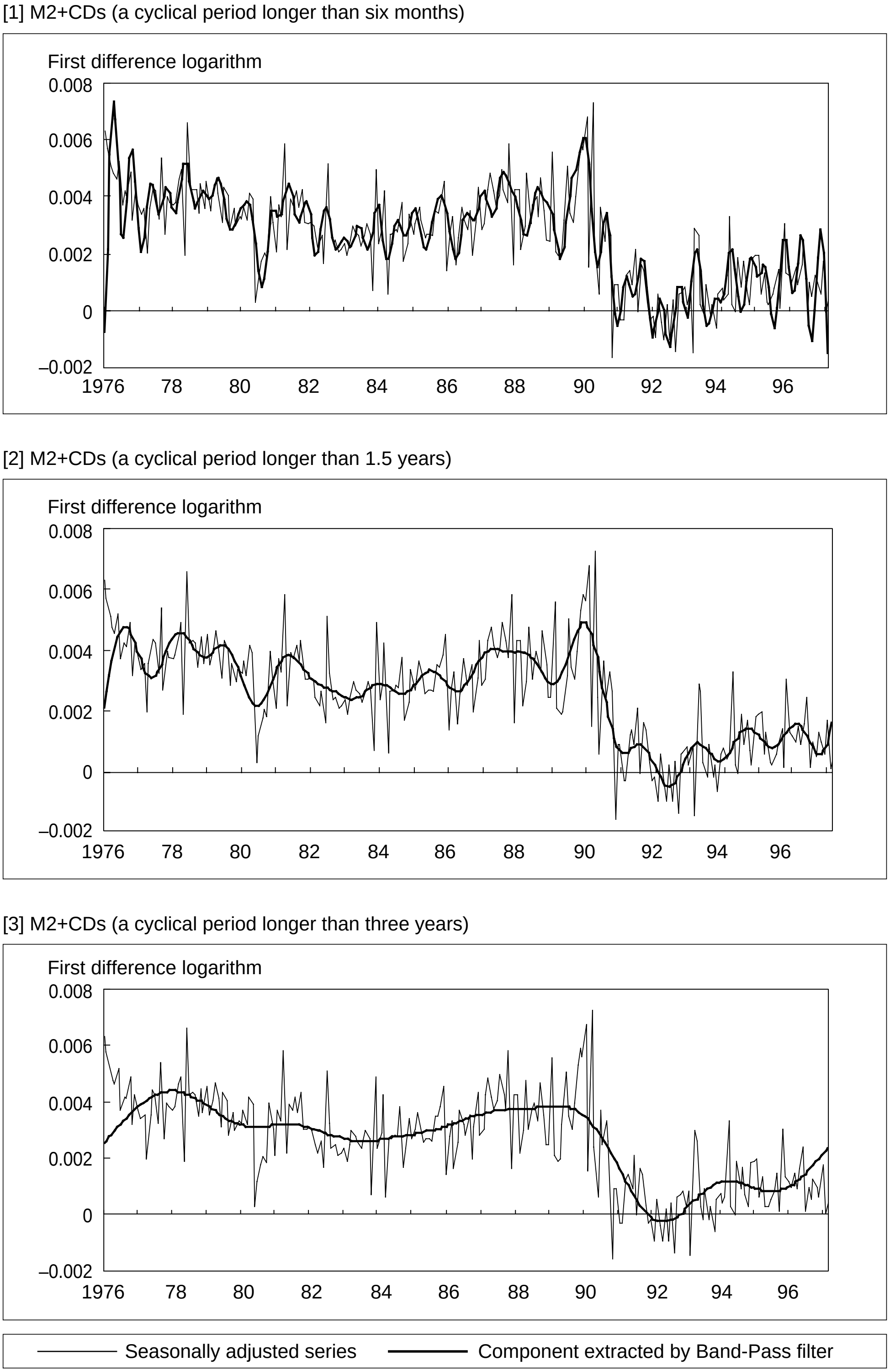
[3] CPI (a cyclical period longer than three years)



Seasonally adjusted series — Component extracted by Band-Pass filter

Note: Seasonal adjustment was done using X-12-ARIMA.

Figure 10-2 Trend-Cycle Components of M2+CDs Extracted by Band-Pass Filter



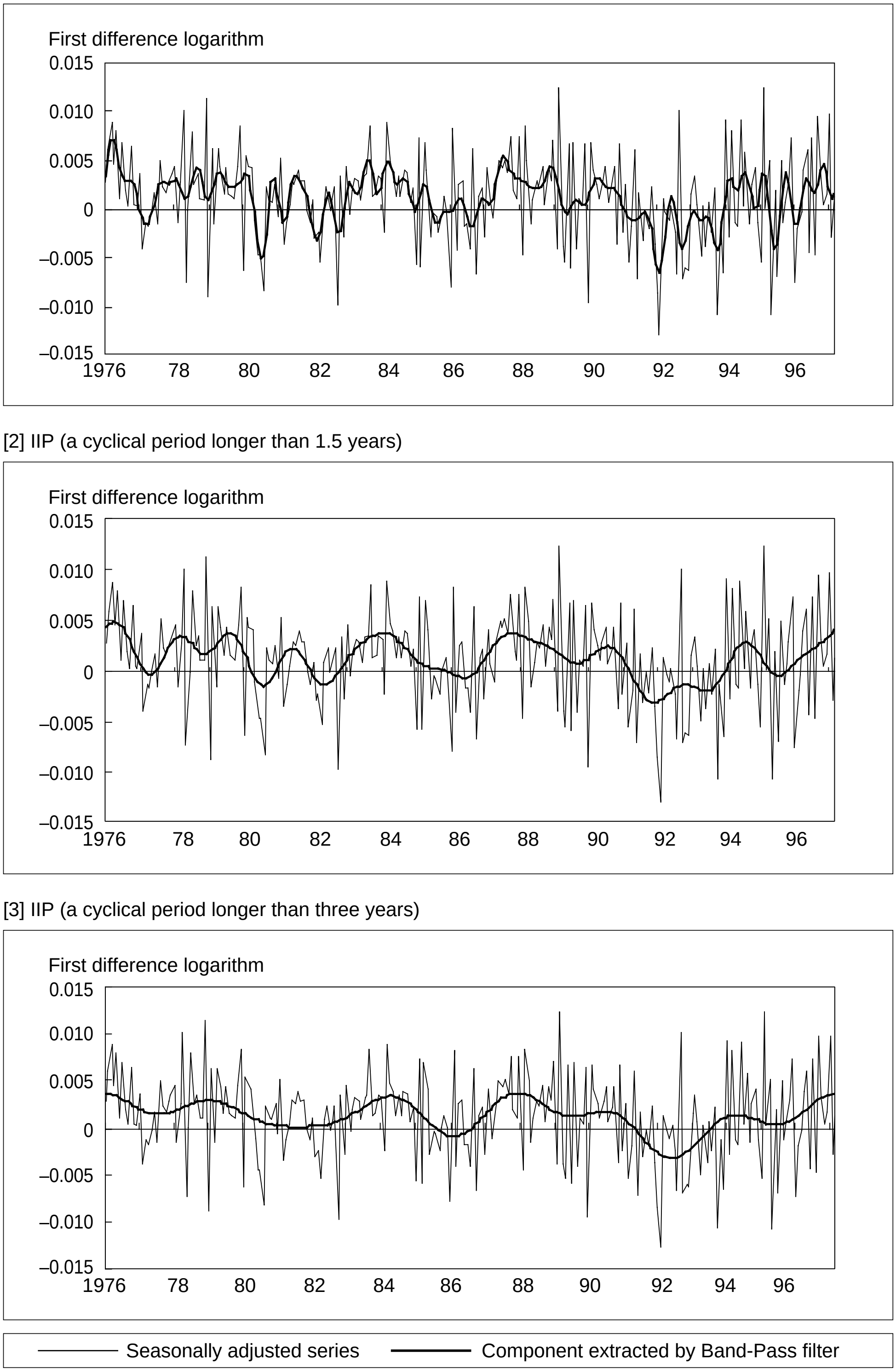
Note: Seasonal adjustment was done using X-12-ARIMA.

Figure 10-3 Trend-Cycle Components of IIP Extracted by Band-Pass Filter

[1] IIP (a cyclical period longer than six months)

[2] IIP (a cyclical period longer than 1.5 years)

[3] IIP (a cyclical period longer than three years)



Note: Seasonal adjustment was done using X-12-ARIMA.

However, around the beginning and the end of a sample period, a large divergence often emerges between the extracted trend-cycle component and the original time-series (seasonally adjusted). Why does this large divergence exist? In the Band-Pass filter, a time series is expressed in the form of the following Fourier representation:[24]

$$p_t = T^{-1/2}\left\{\alpha_0 + \sqrt{2}\sum_{j-1}^{n}(\alpha_j \cos\lambda_j t + \beta_j \sin\lambda_j t)\right\} \quad \text{(where } T \text{ = odd number)}$$

$$p_t = T^{-1/2}\left\{\alpha_0 + \sqrt{2}\sum_{j-1}^{n}(\alpha_j \cos\lambda_j t + \beta_j \sin\lambda_j t) + \alpha_{n+1}(-1)^t\right\}$$

(where T = even number).

Because the Fourier representation takes the form of a periodic function with the sample period as a multiple of its cyclical period, it must have the same value at both ends of the sample period. In the above equations, $\sin \lambda_j t = 0$ and $\cos \lambda_j t = 1$ hold at both ends of the sample period. As a result, p_t becomes the summation of (α_j) at both ends of the sample period. As Figure 10 shows, the value of Fourier representation at both ends of the sample period takes an average value of the two ends of the original time-series. Because these two end-values are usually different, the value obtained by the Fourier representation tends to diverge from the original time-series at both ends of the sample period.

We have examined the range of divergence in the case of CPI when the Band-Pass filter is applied. When the extracted cycle is longer than 1.5 years, the range of divergence is about six months at both ends of the sample period. When it is longer than three years, the range is about a year at both ends of the sample period. One way to correct this divergence is to add to the original data one-year, out-of-sample forecasts of an ARIMA model, and then apply the Band-Pass filter to those extended data. This produces a smooth trend-cycle component, which is consistent with the movement of the original data (Figure 11).[25]

Judging from these results, we must conclude that the Band-Pass filter is not an appropriate method for extracting a "long-term trend component" because it involves an unreliable estimation around the end of a sample period. Instead, it is an appropriate method for extracting a "trend-cycle component," which is a combination of a long-term trend and a medium-term business-cycle component. This can be achieved by extracting a component with a cyclical period longer than 1.5 years.

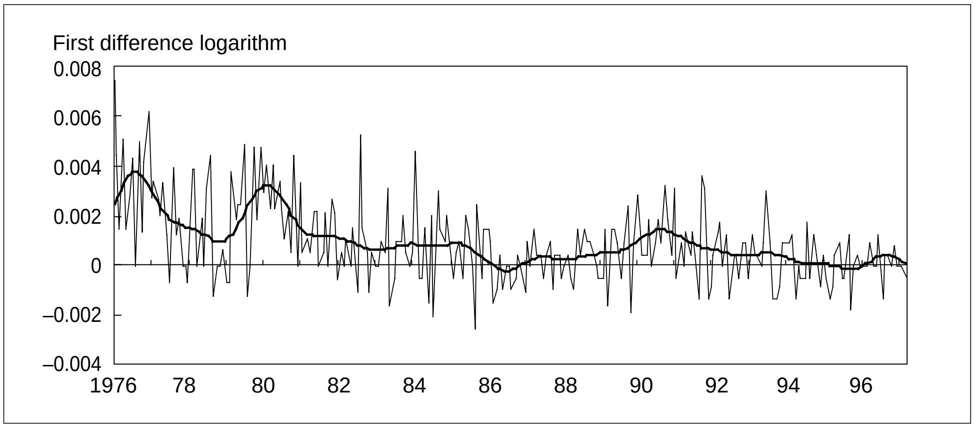
3. The Hodrick and Prescott filter

The Hodrick and Prescott (HP) filter is generally regarded as a good method to extract what we have called a "long-term trend component" in this paper. However, our analysis below shows that it is not necessarily successful in extracting a "long-term trend component" because the component extracted by this method (the growth component) contains a medium-term business-cycle factor.

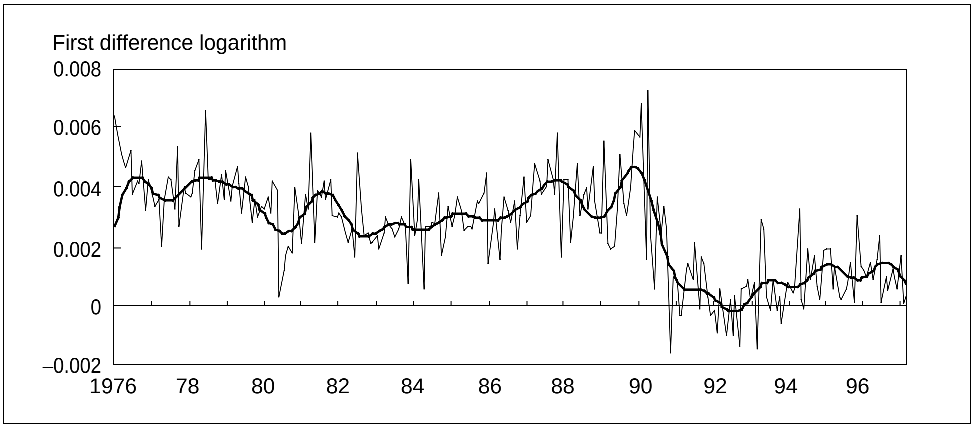
24. See Appendix 1 for the Fourier representation.
25. This method also produces the same results for M2+CDs and IIP (Figure 11).

Figure 11 Trend-Cycle Components Extracted by Band-Pass Filter (With One Year Out-of-Sample Forecast by ARIMA)

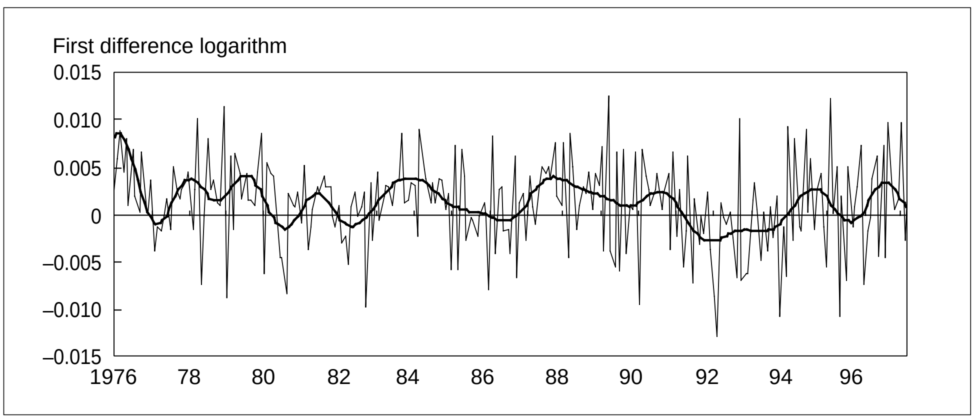
[1] CPI (a cyclical period longer than 1.5 years)



[2] M2+CDs (a cyclical period longer than 1.5 years)



[3] IIP (a cyclical period longer than 1.5 years)



Seasonally adjusted series — Component extracted by Band-Pass filter

Note: Seasonal adjustment was done using X-12-ARIMA.

The HP filter (with λ = 14400) produces a smoother component for the monthly time-series of CPI, M2+CDs, and IIP (Figure 12) than the Henderson moving average (with 23 terms) and the Band-Pass filter (with a cyclical period longer than 1.5 years). Spectral analysis (using the first difference of logarithm) shows that the extracted growth component contains many components with a cyclical period of 3–5 years (spectrum number between 4 to 7) (Figure 13). To check this point in detail, we have compared the spectra of the growth and cyclical components (Table 4). IIP has a greater spectrum for the growth component with a cyclical period longer than five years (spectrum number 4 and less), while M2+CDs and CPI have a greater spectrum for the growth component with a cyclical period longer than three years (spectrum number 7 and less). Therefore, the extracted growth components of M2+CDs and CPI contain a shorter cyclical component, which corresponds to a business cycle.[26]

Why does the extracted cyclical component differ among the economic variables? The HP filter extracts a growth component so as to make the cyclical component small while maintaining a given degree of smoothness. When an economic time-series is volatile like IIP, there is a trade-off between improving the smoothness of the growth component and reducing the business-cycle component. Therefore, we try to maintain the smoothness in the growth component by allowing some increase in the business-cycle component. As a result, the growth component of IIP tends to contain only a longer cyclical period than the other two variables. In applying the HP filter, we must be aware that the more volatile a time series, the longer the cyclical period of the extracted growth component.

4. The DECOMP method

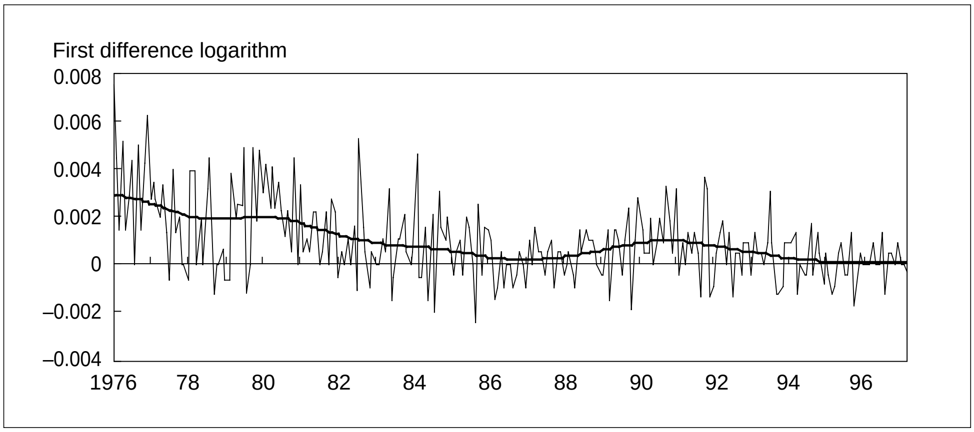
The extraction results of DECOMP applied to the monthly time-series of CPI, M2+CDs, and IIP are presented in Figure 14. In all the cases, the first difference of logarithm of the trend component (which corresponds to the "trend-cycle component" in our definition) exhibits a smooth movement, which is similar to that of the Henderson moving average (with 23 terms) and the Band-Pass filter (with a cyclical period longer than 1.5 years). Spectral analysis shows that the trend component is extracted as a component with a cyclical period longer than 1.5 years, and the other component with a shorter cyclical period is represented in the AR and white noise terms. The figure shows that DECOMP makes a clear-cut decomposition into the trend and other components. Therefore, the extracted trend component seems to correspond to what we have called the "trend-cycle component" (that is, a combination of a long-term trend component and a medium-term business-cycle component). As discussed in Section II.C, an $I(m)$ process with $m \geq 1$ seems to constitute a long-term trend. However, since DECOMP optimizes the fit of a model with an AR term, it is possible that a business-cycle component also enters into the process of extracting a trend component.

A further examination reveals that the boundary between the trend component and the AR component differs among the three economic time-series. The cyclical period at which the power spectrums of the trend and the AR component intersect

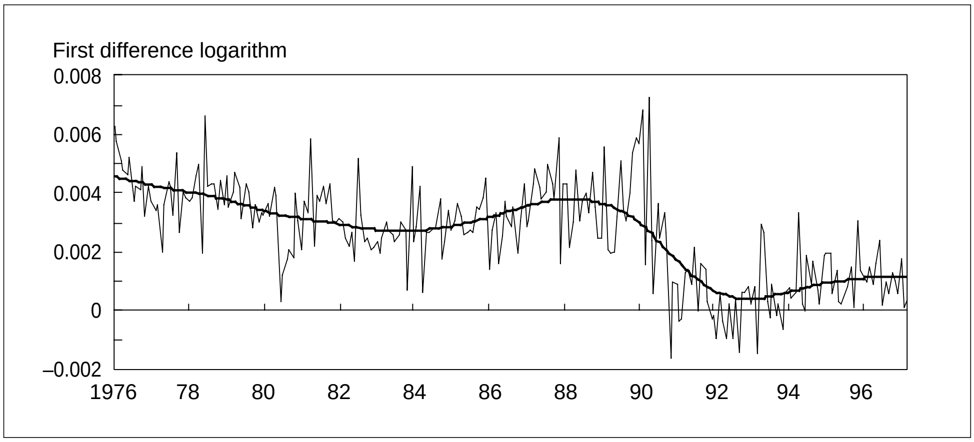
26. The same results hold for the quarterly data of CPI and M2+CDs. In the analysis of the HP filter with λ =1600, the power spectrum of the growth component has a greater value for the component with a cyclical period longer than three years. It indicates that the extracted growth component contains a business-cycle factor (Table 4).

Figure 12 Growth (Trend-Cycle) Components Extracted by HP Filter (λ = 14400)

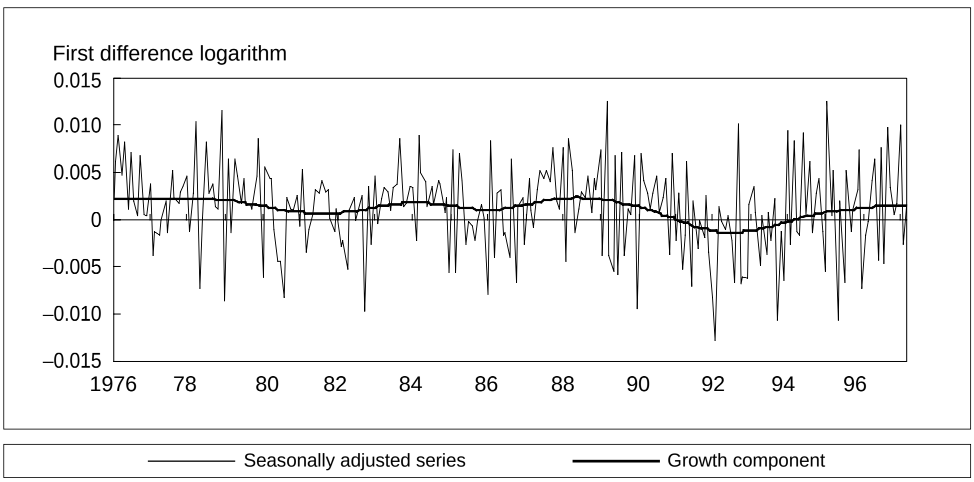
[1] CPI



[2] M2+CDs



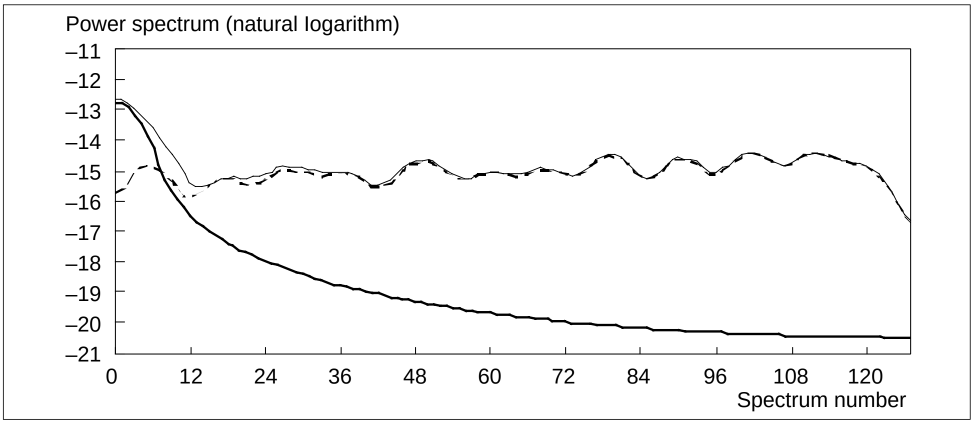
[3] IIP



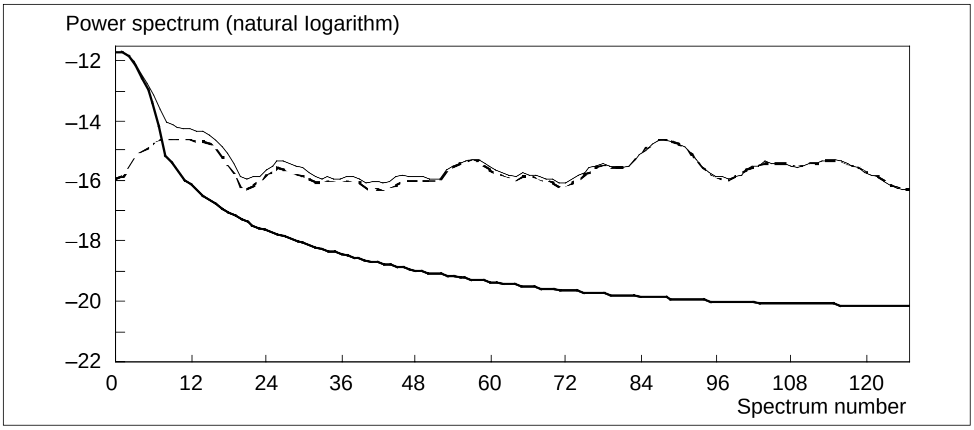
Note: Seasonal adjustment was done using X-12-ARIMA.

Figure 13 The Power Spectra of Components Extracted by HP Filter

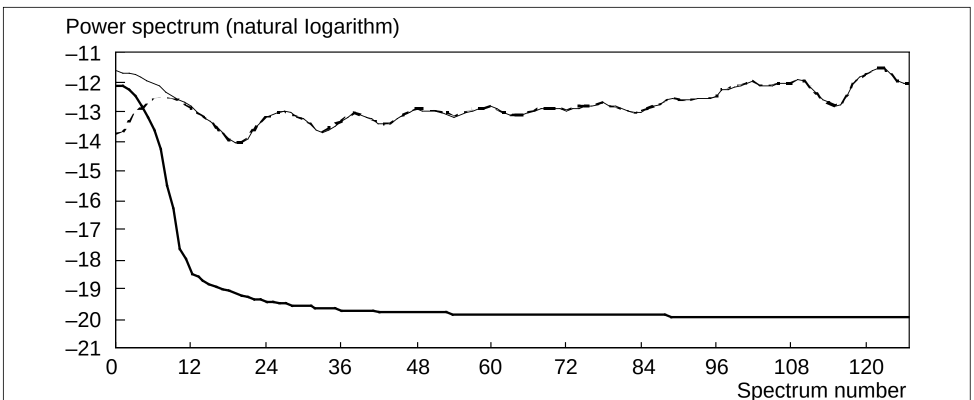
[1] CPI



[2] M2+CDs



[3] IIP



Spectrum number	0	··· 12	··· 24	··· 36	··· 48	··· 60	··· 72	··· 84	··· 96	··· 108	··· 127
Period/months	∞	··· 21	··· 10	··· 7	··· 5	··· 4	··· 3.5	··· 3	··· 2.6	··· 2.4	··· 2

—— Seasonally adjusted series —— Growth component ---- Cyclical component

Note: Seasonal adjustment was done using X-12-ARIMA.

Table 4 The Power Spectra of the Growth and Cyclical Components Extracted by HP Filter (A Comparison)

	Monthly		Quarterly		Note	
					Monthly	Quarterly
	Spectrum number	Period	Spectrum number	Period	Unit root test	
CPI (original)	7	3 years	6	3.5 years		
CPI (seasonally adjusted)	8	2.65 years	12	1.8 years	$I(0)$ + d.	$I(0)$ + d.
M2+CDs (original)	8	2.65 years	7	3 years		
M2+CDs (seasonally adjusted)	8	2.65 years	7	3 years	$I(2)$	$I(2)$
IIP (original)	4	5.3 years	—	—		
IIP (seasonally adjusted)	5	4.25 years	—	—	$I(1)$ + d. + t.	
Real GDP (original)	—	—	N.A.	N.A.		
Real GDP (seasonally adjusted)	—	—	9	2.4 years		$I(1)$ + d. + t.

Note: Seasonal adjustment was done using X-12-ARIMA.
d.: with drift term
t. : with time trend
N.A.: not available

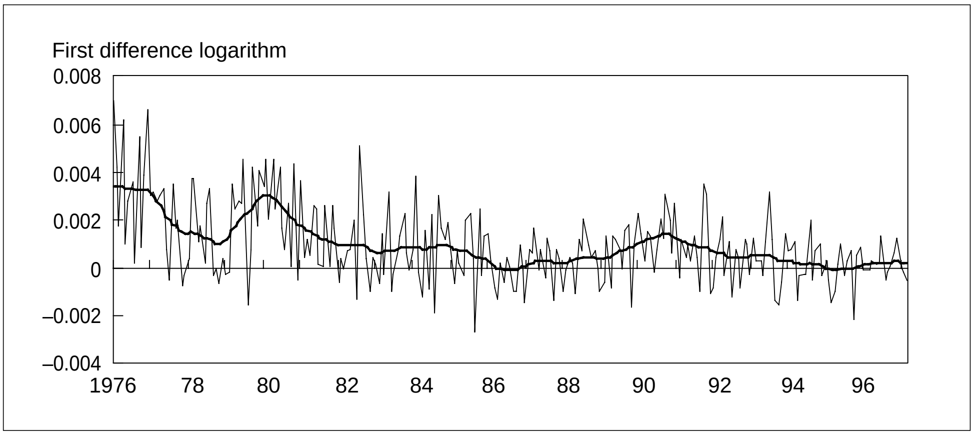
is 1.5 years for CPI, 1.3 years for M2+CDs, and 1.9 years for IIP (Figure 15). The reason for these differences may be as follows. M2+CDs exhibits relatively smooth variations because it has a strong trend component and a weak cyclical component. Therefore, in DECOMP, the stochastic difference equation that is supposed to capture a trend movement captures part of the short-term variations to minimize the AIC. In contrast, IIP is dominated by strong short-term variations, since the model keeps the AIC from increasing. Therefore, the stochastic difference equation clearly recognizes and eliminates short-term variations, which are instead captured by the AR equation. As a result, the trend component of M2+CDs contains more short-term components than that of IIP.[27]

For all three time series, the selected degree of the stochastic difference equation is 2, which suggests that the trend component can be approximated by an $I(2)$ process. This appears to be inconsistent with the result of the stationarity test in Section III.A

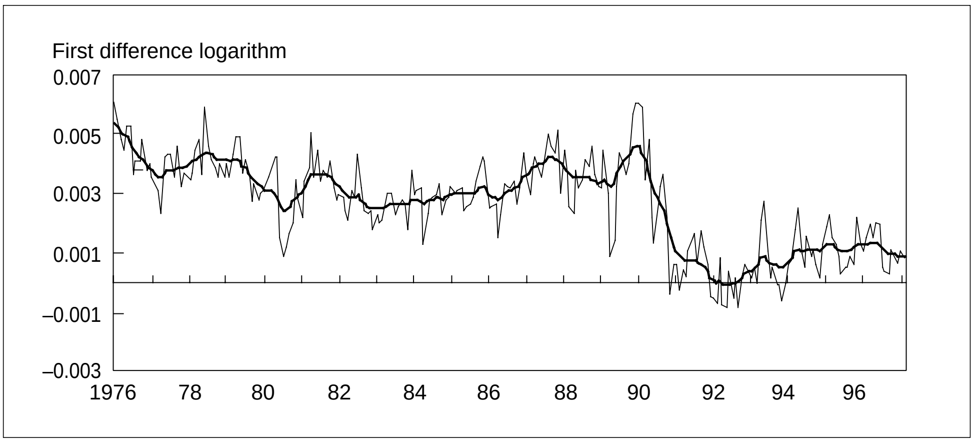
27. The cyclical structure of the extracted components is relatively stable. This is because, among the three time series, DECOMP uses the same degree of 2 in the stochastic difference equation and the AR equation. In fact, according to Kitagawa (1997), most time-series analyses fit best at the degree of 2 in the stochastic difference equation. However, the degree of the AR equation should change according to the volatility of a time series. In general, when a higher degree of the AR equation is selected by the AIC, the AR contains a longer cyclical component, making the cyclical period of the extracted trend component longer, and the AR also contains seasonal variations. This makes it inappropriate to use a high degree (≥4) of the AR equation. Even if the AIC selects a high degree for the AR equation, it is often advisable to choose a lower degree of the second best by checking its graphs. In the present analysis, the AIC selects the degree of 6 for the AR equation in the case of M2+CDs. However, its graph indicates that the AR produces strong short-term cyclical variations that do not exist in the original time-series data. Since this is clearly inappropriate, we have selected the second-best degree according to the AIC, which turns out to be 2 for the AR equation. A shortcoming of the DECOMP method, therefore, is the difficulty of choosing an appropriate degree of the AR equation.

Figure 14 Trend (Trend-Cycle) Components Extracted by DECOMP

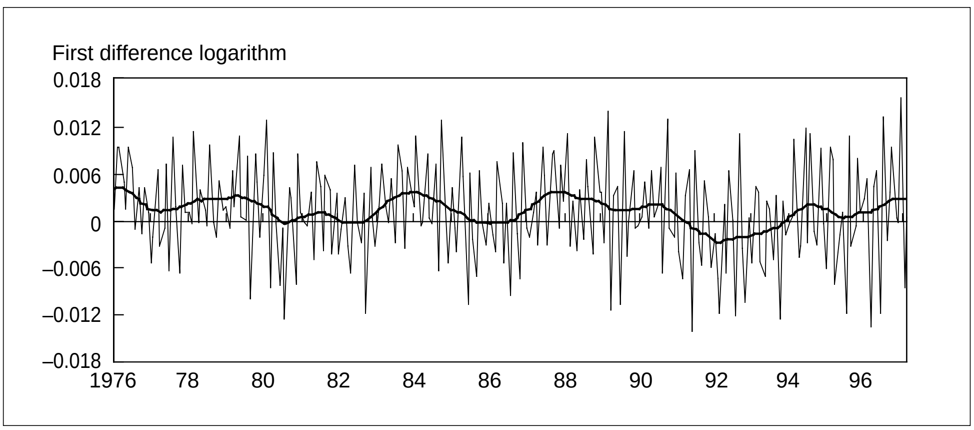
[1] CPI



[2] M2+CDs



[3] IIP

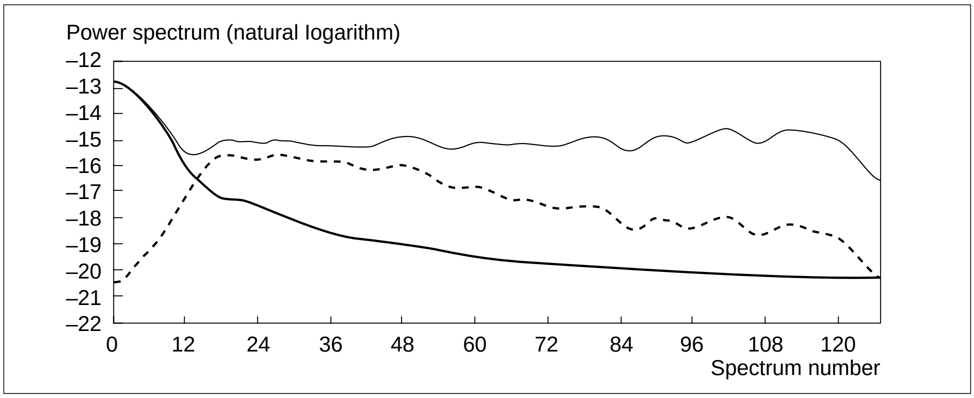


Seasonally adjusted series — Trend component

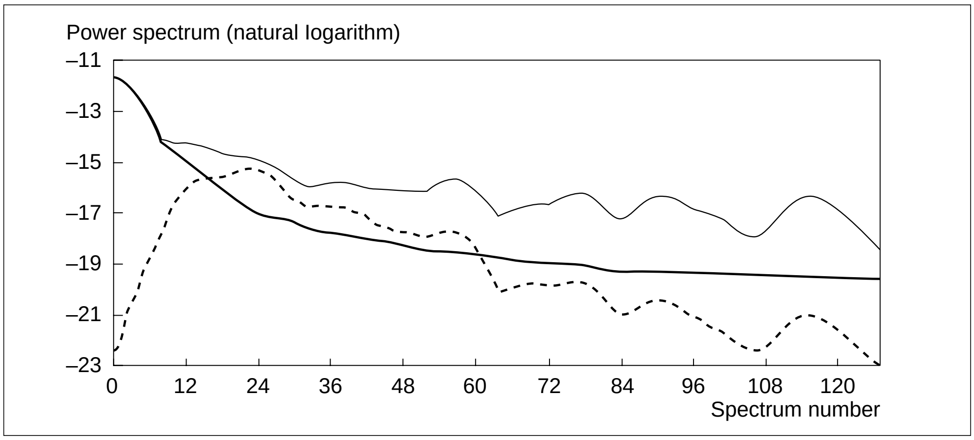
Note: Seasonal adjustment was done using X-12-ARIMA.

Figure 15 The Power Spectra of Components Extracted by DECOMP

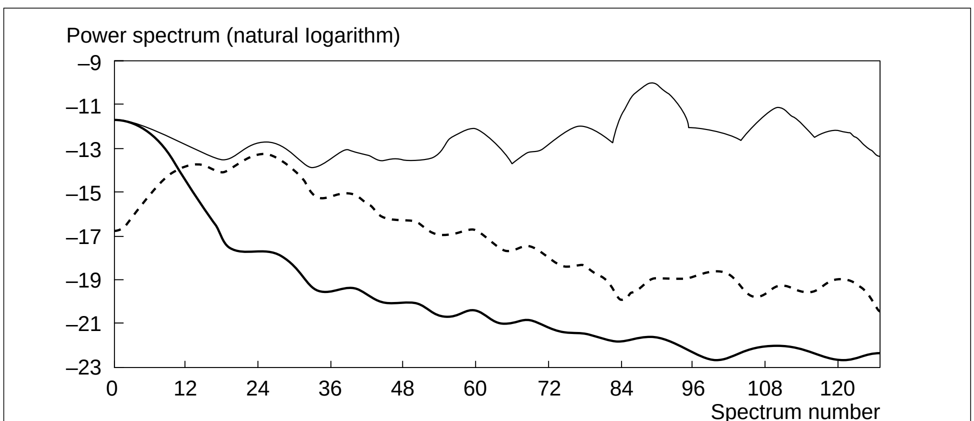
[1] CPI



[2] M2+CDs



[3] IIP



Spectrum number	0	12	24	36	48	60	72	84	96	108	127
Period/months	∞	21	10	7	5	4	3.5	3	2.6	2.4	2

Seasonally adjusted series — Trend component — AR component

Note: Seasonal adjustment was done using X-12-ARIMA.

in that CPI is an $I(0)$ process with a drift, M2+CDs an $I(2)$ process without a drift, and IIP an $I(1)$ process with a drift. This apparent inconsistency arises because DECOMP assumes the trend component to be a process without a drift. Therefore, when a drift is not taken into account, DECOMP recognizes CPI and IIP as an $I(2)$ process rather than an $I(1)$ process.

5. The Beveridge and Nelson decomposition[28]

The Beveridge and Nelson method decomposes a time series into a permanent component and a cyclical component. Figure 16 shows the application of the Beveridge and Nelson decomposition to CPI, in which the results are presented in a logarithm level form.[29] The permanent component virtually overlaps the original time-series because it definitely dominates the cyclical component. Until 1981, immediately after the second oil crisis, the cyclical component is positive and relatively large, but after 1981 the permanent component dominates the movement. The cyclical component becomes negative around 1987 and after 1994, when the rate of inflation was near zero. As such, the cyclical component has a serial correlation, implying that some temporary shocks pushed down the CPI in those periods. M2+CDs also has a similar serial correlation in the cyclical component. The cyclical component is positive during the period up to 1990, when its growth was high. In particular, the cyclical component is strong in 1976–78 and in 1990, yet it turned negative after 1991. On the other hand, IIP is different from CPI and M2+CDs. The cyclical component of IIP exhibits irregular variations and has no serial correlation. This suggests that the IIP movement is caused mainly by permanent shocks.

The results of spectral analysis indicate that a component with a cyclical period longer than two years is mostly incorporated in the permanent component, while a shorter cyclical component comprises both permanent and cyclical components (Figure 17). This is because the model divides a time series into an $I(1)$ process and the other component. In view of these results, the Beveridge and Nelson decomposition is therefore not an appropriate method when we want to extract a specific cyclical component.

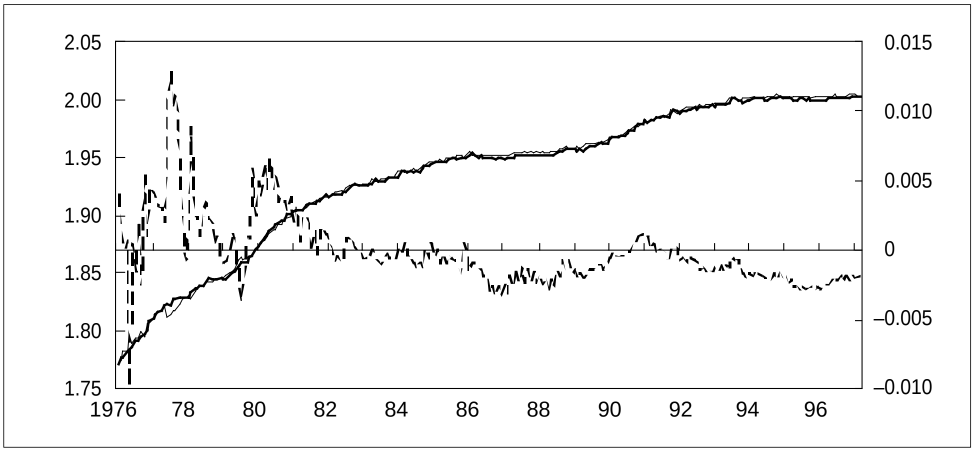
IV. The Stability of an Extracted Trend-Cycle Component and a Long-Term Trend Component

This chapter examines the stability of an extracted trend-cycle component with respect to a change in the sample period. When we are studying and forecasting a business cycle, we are mostly interested in the fundamental movement around the end of the sample period. Therefore, it is desirable for an extracted trend-cycle

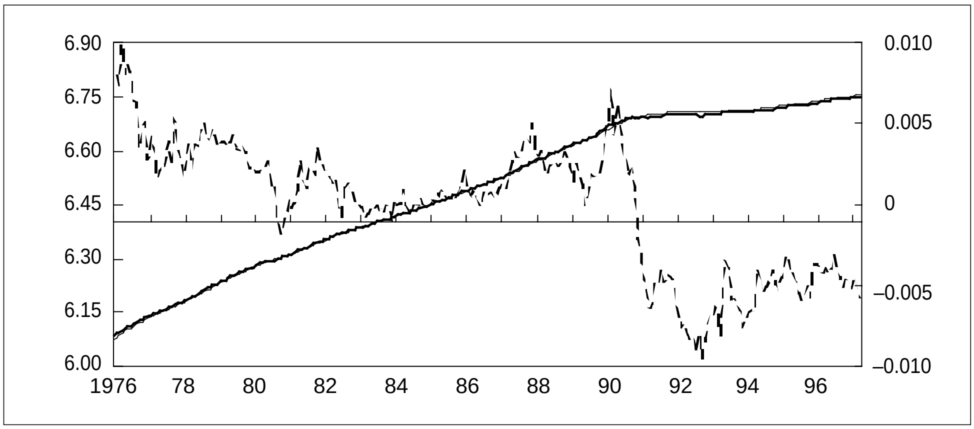
28. The ARIMA model selected in the Beveridge and Nelson decomposition is as follows: CPI (3, 1,3), M2+CDs (1, 1, 2), and IIP (2, 1, 1).
29. This method assumes that a time series is a nonstationary process with a stochastic trend. However, the stationarity test has indicated that CPI is a stationary process with a deterministic trend. Although the ARIMA model can be still estimated in this case, we should be cautious in interpreting the results.

Figure 16 Decomposed Components by Beveridge and Nelson Method (Logarithmic Level)

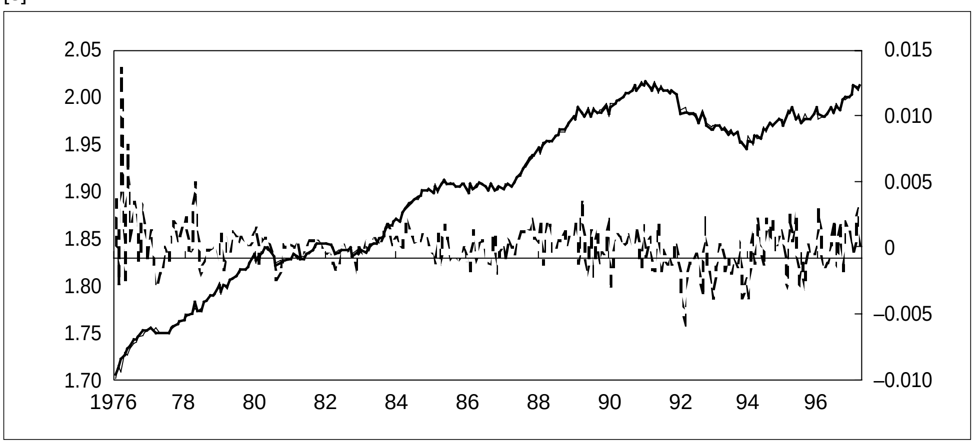
[1] CPI



[2] M2+CDs



[3] IIP

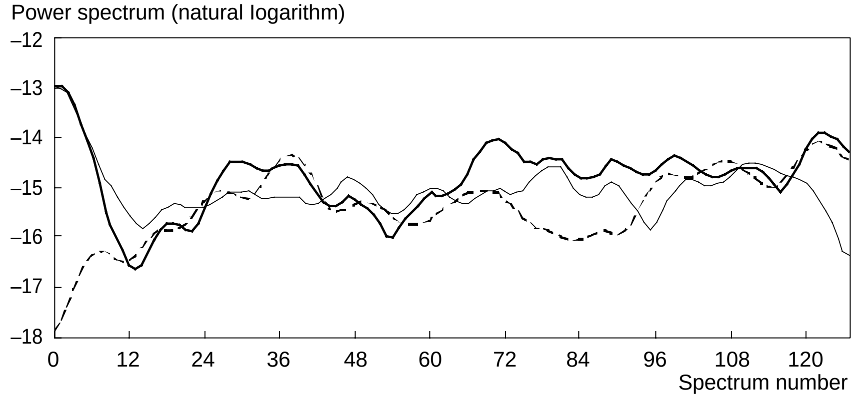


Seasonally adjusted series (left scale) Permanent component (left scale) Cyclical component (right scale)

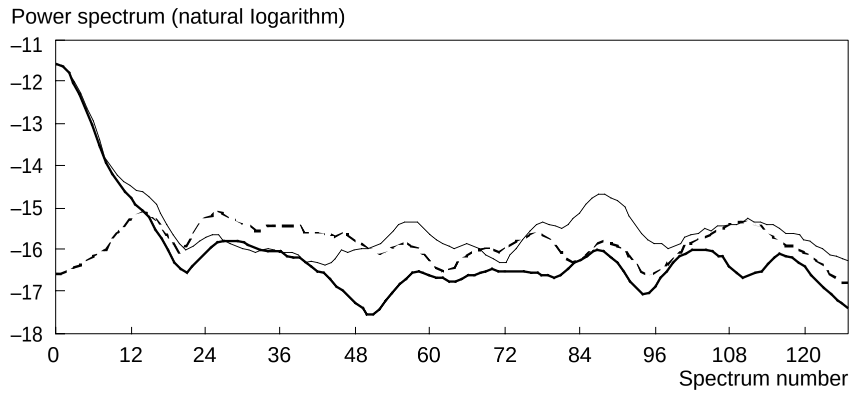
Note: Seasonal adjustment was done using X-12-ARIMA.

Figure 17 The Power Spectra of Components Extracted by the Beveridge and Nelson Method

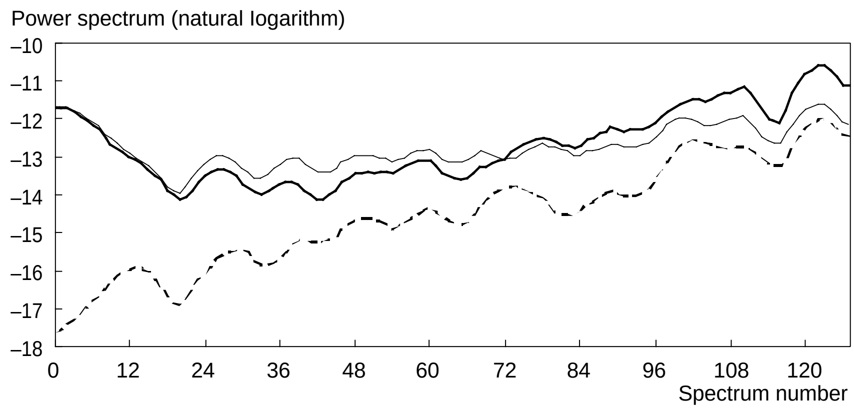
[1] CPI



[2] M2+CDs



[3] IIP



Spectrum number	0	··· 12	··· 24	···36	··48	··60	·72	·84	96	108	127
Period/months	∞	··· 21	··· 10	··· 7	··· 5	··· 4	·3.5	··3	2.6	2.4	2

—— Seasonally adjusted series —— Permanent component - - - - Cyclical component

Note: Seasonal adjustment was done using X-12-ARIMA.

component to remain stable when new data are added to the time series.[30] However, in the time-series analysis, it is unavoidable that an addition of new data changes the extracted component somewhat. Moreover, a larger change in the extracted component tends to occur around the end of the sample period. Therefore, for practical purposes, it is important to gauge the extent of the range and the magnitude of a potential change in the extracted trend-cycle component.

In the following sections, we qualitatively analyze how a change in the sample period affects the extracted trend-cycle component. We will use the monthly time-series data of CPI. The base sample period is from January 1976 to March 1986. We extend the sample period by one year until March 1997, and check how the extracted component is affected by changes in the sample period.

1. The Henderson moving average

Since this method uses a mid-point moving average, it needs some forecast data. An ARIMA model is used to generate the necessary forecasts. As actual data become available to replace the forecasts, an extracted trend-cycle component is likely to be revised.[31]

In our analysis, the number of terms in the moving average is 23. The results show that the revision of an extracted component is limited to one year at the end of a sample period (Figure 18). The magnitude of the revision, however, differs from year to year. There are few revisions when the variation of a time series is moderate. However, a large revision is made when the rate of growth of CPI dropped sharply in March 1986 and March 1994, as well as when it accelerated in March 1990. In particular, in 1986 and 1994, even the direction of change is revised. This implies that the extracted component lacked reliability in those years.[32]

We may conclude that the reliability of the trend-cycle component falls during one year at the end of a sample period, particularly when the variation of a time series is large. It does not fall, however, in the other part of the sample period.

2. The Band-Pass filter

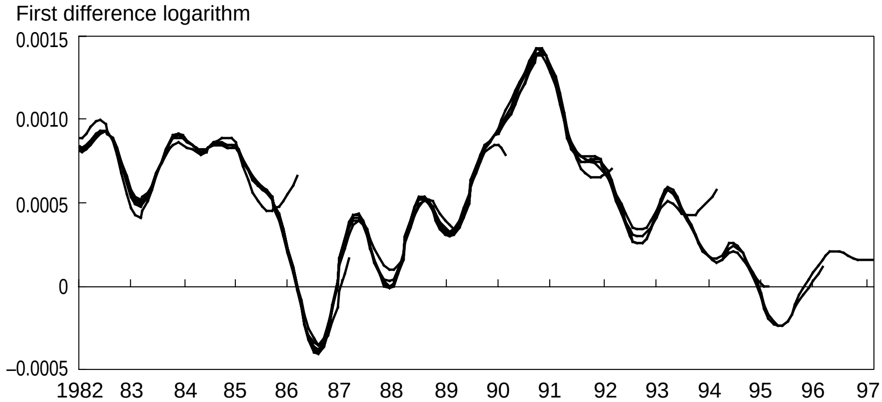
We specify a cyclical period longer than 1.5 years for the Band-Pass filter. As discussed earlier, the extracted trend-cycle component becomes inappropriate at both ends of a sample period for about six months. Therefore, we add one-year,

30. Here we are assuming that an extracted component converges quickly to the true component with an addition of new data so that stability implies a high reliability of the extracted component. It is possible that when the convergence speed is slow, the stability of an extracted component may reflect the stability of some inappropriate component. Nevertheless, since we do not know the true component that is unknown, it is only practical to assume that the extracted component obtained from all the available data is the closest one to the true component. Therefore, studying the graphs should help us judge whether an extracted component with partial data is converging to the true value or staying around a wrong value. This, of course, is only a practical approach and therefore subject to limitations, particularly when a difference between the true value and the wrong value is small.

31. This is the same as seasonal adjustment. Forecasts of an ARIMA model are reliable in the case of a seasonal component because they do not change with an addition of new data. In contrast, they are less reliable in the case of a trend-cycle component.

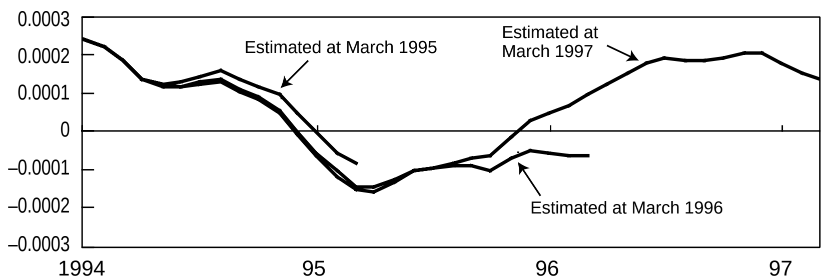
32. The lack of reliability in an extracted trend-cycle component may be due to the large number of terms (23) in the Henderson moving average, which makes it heavily dependent on forecasts. However, reducing the number of terms will increase short-term cyclical factors (approximately a six-month cycle in the present case) contained in the extracted trend-cycle component, making it more difficult to judge the direction of a business cycle. We have experimented with the CPI data, using 13 terms in the Henderson moving average. Even in this case, the extracted trend-cycle component is subject to a major revision in 1986 and 1994.

Figure 18 Magnitude of the Revision: X-12-ARIMA (CPI)



Note: Figures 18 to 22 show all the results obtained by the estimation when we add one year to the end of the sample period in sequence. When there is little revision, we consider the estimation as "highly stable."

[Reference]

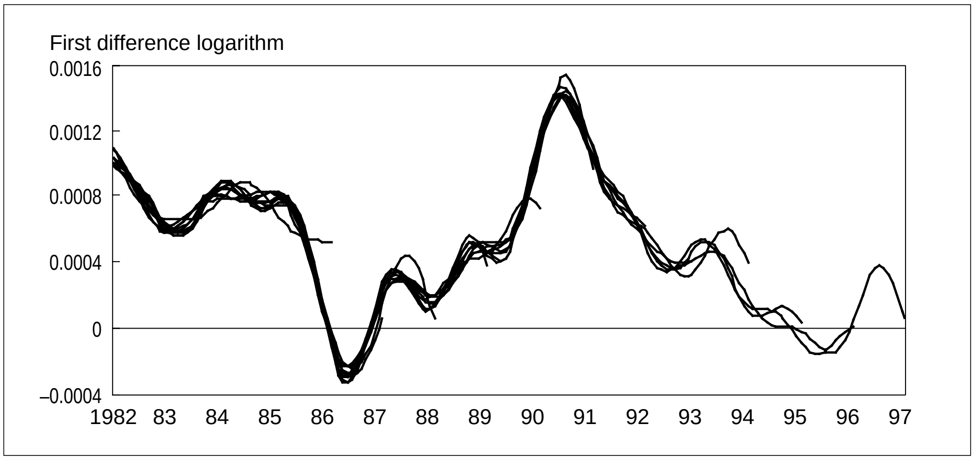


out-of-sample forecasts by an ARIMA model to the time-series data. This is the same procedure as that we have used in the Henderson moving average.

We find that the extracted component is revised over an extended period by an addition of new data (Figure 19). This is because estimated coefficients in the Fourier representation change with an addition of data. An extracted component is somewhat unstable, although the magnitude of revision is relatively small.

The revision of an extracted component around the end of a sample period is relatively small. In particular, in 1986 and 1994, when variation in the data is large, the revision is smaller in the Band-Pass filter than in the Henderson moving average. Furthermore, the direction of change is correctly predicted in the Band-Pass filter from the start. This is due to the fact that how a trend-cycle component is extracted depends on the structure of the spectrum calculated over a whole sample period. As a result, the extracted component does not depend heavily on the accuracy of a forecast by an ARIMA model.

Figure 19 Magnitude of the Revision: Band-Pass Filter (CPI)



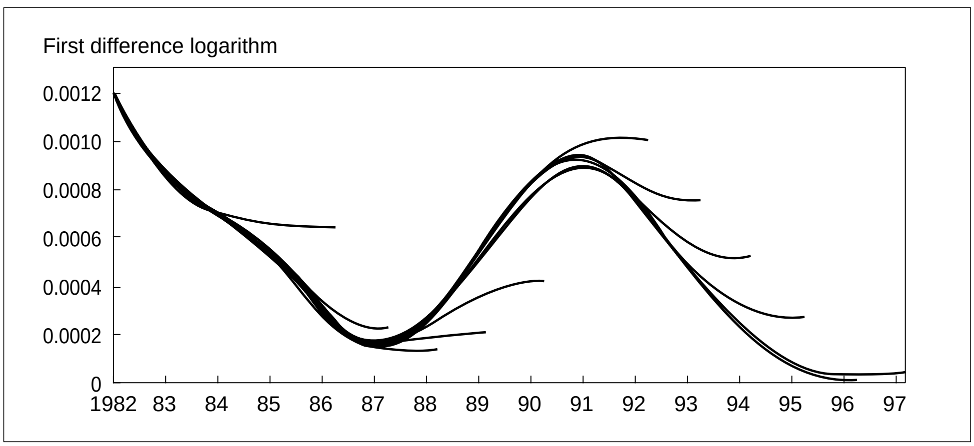
From the above analysis, we may conclude that although the Band-Pass filter is somewhat inferior in stability to the Henderson moving average, it is more reliable in the direction of change of the extracted trend-cycle component when variation in the time series is large.

3. The Hodrick and Prescott filter

The growth component extracted by the HP filter is sensitive to a change in the sample period (Figure 20). Two years at the end of a sample period are highly likely to be revised later on. The HP filter estimates the growth component around the end of a sample period by minimizing its variation. Therefore, the growth component tends to be revised upward when the time series is accelerating, while it tends to be revised downward when it is decelerating.

The extracted growth component should not be identified as the "long-term trend component" because the component contains a cyclical period of three to five years. These results suggest that the extracted growth component does not have a

Figure 20 Magnitude of the Revision: HP Filter (CPI)



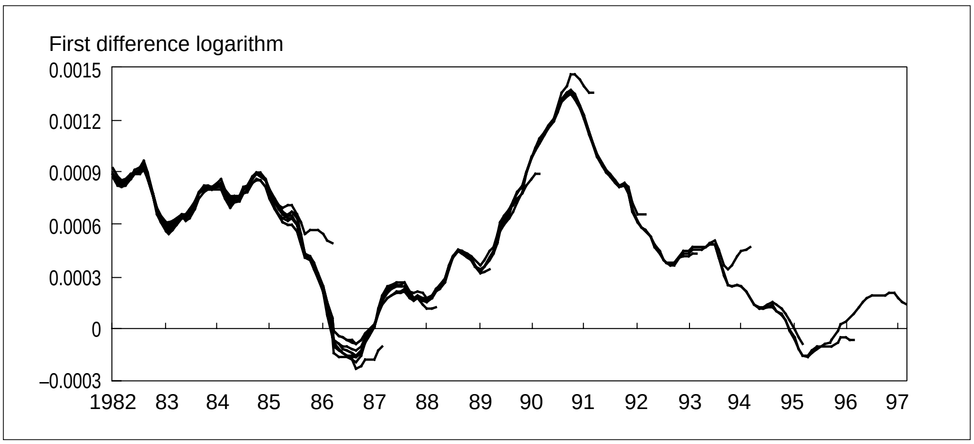
high degree of reliability because the component during the two years at the end of a sample period needs to be continuously revised.[33] We may conclude therefore that the HP filter is not desirable if we are interested in the fundamental movement around the end of time-series data.

4. DECOMP

A trend component extracted by DECOMP is relatively stable with respect to an extension of the sample period by one year. In the case of CPI, it is stable in all periods except for 1985–87 and 1994, when the pattern of variations in CPI has changed sharply (Figure 21). The direction of change, moreover, is correct even in the unstable period of 1985–87. Although it is slightly inferior to the Henderson moving average in overall stability, DECOMP seems to perform better than the moving average in that the direction of change is reliable and revisions are small even when time-series variations are large.

This is because DECOMP optimizes the smoothness of a trend component utilizing the whole data so that the impact of new data is smaller than that in the Henderson moving average. We may conclude therefore that DECOMP offers a promising method of studying the direction of a business cycle.

Figure 21 Magnitude of the Revision: DECOMP (CPI)

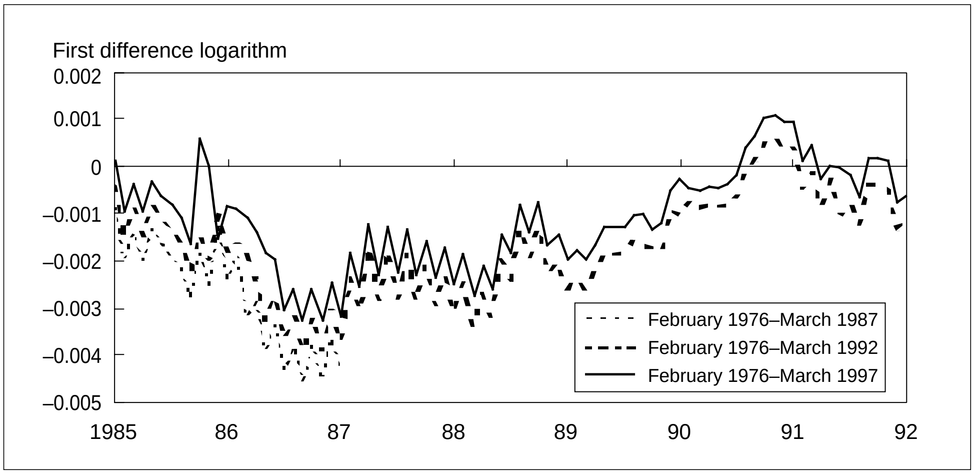


5. The Beveridge and Nelson decomposition

Finally, we examine how an extension of a sample period affects each component extracted by the Beveridge and Nelson decomposition (Figure 22).[34] In the case of CPI, it revises the permanent component downward while revising the cyclical

33. Figure 20 shows that the end of a growth component extracted by the HP filter is estimated to take an average value of the rates of change in all the time-series data, making it a smooth curve around the end of a sample period. This is because the impact of the sum of squares of cyclical components in the loss function is relatively small around the end of a sample period, compared to those in other periods, and therefore the extracted component tends to be selected so as to minimize the sum of squares of the second differences of the growth component.
34. An addition of data for one year causes such a small change in the component that the original and revised components overlap closely. Therefore, we have presented three cases in Figure 22, each with an addition of five years of new data.

Figure 22 Magnitude of the Revision: Beveridge and Nelson Method (CPI)



component upward by the same amount. This is because an addition of new data lowers the forecast of CPI generated by an ARIMA model, and thereby revises the permanent component downward. The figure shows a gradual change in the ARIMA forecast caused by an addition of new data. This implies that the forecast value by the ARIMA model will change gradually by an addition of new data.

V. Conclusion

This paper has attempted to extract a trend-cycle component and a long-term trend component from economic data, using five different time-series methods. The five methods can be divided into two groups. The first group requires specifying parameter values on frequency or smoothness in advance. The second group automatically selects parameter values so as to optimize the fit of an extracted component. The first group comprises the Henderson moving average, the Band-Pass filter, and the HP filter. The second group comprises the DECOMP and the Beveridge and Nelson decomposition. The results of our analysis using monthly time-series data for CPI, M2+CDs, and IIP are summarized in Table 5.

In order to eliminate short-term variations associated with measurement errors and irregular movements, and to extract a trend-cycle component (a long-term trend component and a business-cycle component), in which the cyclical period corresponds to more than 1.5 years, the Henderson moving average (with 23 terms), the Band-Pass filter, and DECOMP have turned out to be effective methods. In fact, their extracted components are almost identical in shape and value (Figure 23). In judging the peaks and bottoms of an economic time-series *ex post*, it would not make any difference whichever method is used.[35]

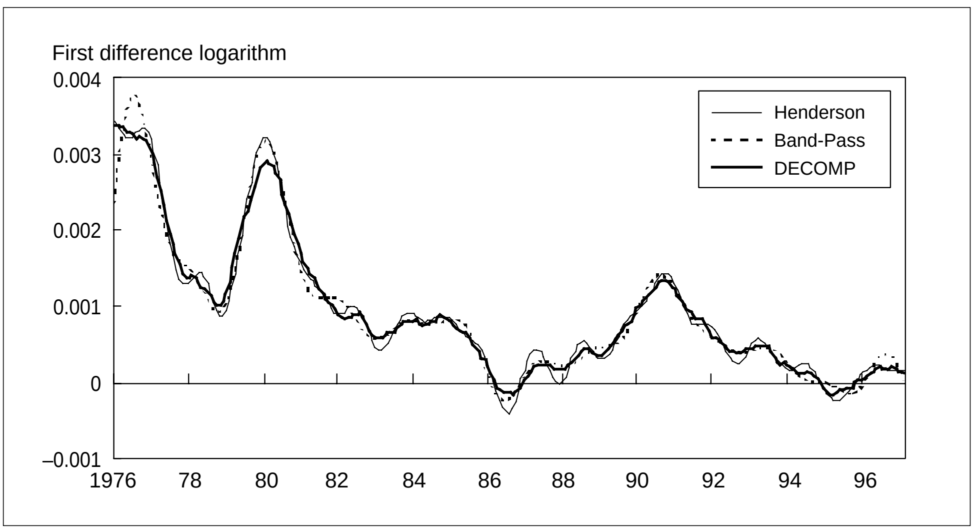
35. Nevertheless, judgment at each point of time may not be identical because *ex post* revisions around the end of a sample period are possible later on. This issue is left as a topic for future research.

Table 5 Summary Comparison of Five Methods

	Characteristics	Extracted component	Periodic structure	Stability of component	Ease of application	Overall evaluation
Henderson moving average (23 terms)	Extracting a component by choosing the number of terms that determine the smoothness	A long-term trend component and a business-cycle component	Some differences among time series, but no problem in practice	Relatively stable, but revisions around the end of a sample period (one year)	Easy (using the X-12-ARIMA seasonal adjustment program)	Good for business-cycle analysis; the number of terms in moving average should be 23
Band-Pass filter (longer than 1.5 years)	Extracting a component within a specified frequency band	A long-term trend component and a business-cycle component	No difference among time series, no problem in practice	Unstable over a sample period, but revisions are relatively small	Relatively difficult (using Fourier and inverse Fourier commands)	Good for business-cycle analysis; somewhat weak in stability
HP filter (monthly data with λ = 14400)	Choosing a parameter value of λ to determine the smoothness	A long-term trend component (mixed with part of a business-cycle component)	Difference among time series	Unstable for two years at the end of a sample period	Easy (using RATS/GAUSS)	Capable of extracting a long-term trend component, but it contains a business-cycle factor; unstable in periodic structure; adjustment in λ is necessary
DECOMP	Endogenously extracting a component (model-specific)	A long-term trend component and a business-cycle component	No difference among time series, no problem in practice	Relatively stable, but revisions around the end of a sample period (one year)	Easy (program attached in home page: http://www.ism.ac.jp/~sato/)	Good for business-cycle analysis, but be careful about selecting the order of the AR
Beveridge-Nelson decomposition	Endogenously extracting a component (model-specific)	A component that corresponds to a permanent shock	Unable to extract a specified component	Relatively unstable over a sample period	Easy (using RATS/GAUSS)	Capable of extracting a permanent component and a cyclical component

Note: A long-term trend component refers to a component with a cyclical period longer than five (or six) years, and a business-cycle component, one to five (or one to six) years.

Figure 23 Comparison between Trend-Cycle Components Extracted by Henderson Moving Average, Band-Pass Filter, and DECOMP



However, there are significant differences between the three methods when it comes to the stability of an extracted trend-cycle component. In the sample period other than its end years, the Henderson moving average produces the most stable component. Because the other two methods revise their trend-cycle components using the whole time-series data, there is a possibility of continuous revisions in theory. In fact, in the case of CPI, the Band-Pass filter produces frequent revisions although DECOMP produces few. In this respect, the Band-Pass filter is inferior to DECOMP.

Around the end of a sample period, for all the methods, an extracted trend-cycle component becomes unreliable, especially when time-series variations are large. In particular, the Henderson moving average tends to extract an incorrect component in both the direction and the level because it depends on the accuracy of forecasts by an ARIMA model. In this situation, DECOMP and the Band-Pass filter tend to extract a component that has at least a correct direction. Although it is difficult to draw a clear conclusion from the above results, it seems possible to reduce the risk of making a directional error in the trend-cycle component by employing several methods at the same time.

From this perspective, we may conclude that the parallel use of the Henderson moving average and DECOMP is appropriate since their performances are similar and the software programs are readily available.[36] In the application of the Henderson moving average, the X-12-ARIMA seasonal adjustment program automatically selects 9 or 13 for the number of terms in the moving average. This is inappropriate for gauging the direction of a business cycle because the extracted component contains short-term variations with a cyclical period of six to eight months. Therefore, it is desirable to use an option in the program that enables us to select 23 as the number of terms in the moving average.

36. As discussed in Section II.A, when components are extracted by their cyclical period, they may have some correlation. To check this point, we have estimated the correlation coefficient for each component extracted by the Henderson moving average and DECOMP. The results show that the correlation coefficients of all components are essentially zero in the case of the Henderson moving average, and the correlation coefficients of a trend component are also essentially zero in the case of DECOMP. Therefore, their trend-cycle components are independent of other components. On the other hand, the AR component in DECOMP that represents variations with shorter cycles has a high correlation with white noise. This suggests that decomposition between the AR component and white noise may not be properly done in the DECOMP (see table below).

Correlation between Decomposed Components (CPI)

[1] X-12-ARIMA

	Trend-cycle	Seasonal	Irregular
Trend-cycle	1.00000	—	—
Seasonal	0.01364	1.00000	—
Irregular	0.04224	–0.05969	1.00000

[2] DECOMP

	Trend	AR	Seasonal	White noise
Trend	1.00000	—	—	—
AR	0.01076	1.00000	—	—
Seasonal	–0.02202	0.00172	1.00000	—
White noise	–0.00566	0.46092	0.04609	1.00000

None of the five methods has been successful in extracting only the long-term trend component in a way that clearly separates it from the medium-term business-cycle component. The HP filter seems to perform best in this task among the five methods. However, a growth component extracted by the HP filter still contains a substantial number of business-cycle factors. Moreover, the cyclical period of an extracted component differs in accordance with the characteristics of an economic time-series under study. We could reduce the number of business-cycle elements by selecting an appropriate parameter value for λ. When we select its value, it is convenient to use spectral analysis to check the cyclical distribution of the extracted components. When sufficient data are available, the Band-Pass filter can be a useful method for extracting a long-term trend component.

Next, we summarize the effects of the time-series characteristics of an economic variable on the cyclical structure of an extracted component. We have studied the characteristics of several economic time-series, such as smoothness and stationarity in Section III.A. Among them, the smoothness (the weight of short-term factors) of a time series has a strong effect on the cyclical structure of an extracted component. This effect is particularly dominant in the case of the HP filter. It also explains a variety of cyclical structures of an extracted component by the Henderson moving average and DECOMP. On the other hand, the cyclical structure of an extracted component does not depend on whether the economic time-series is stationary or nonstationary.

Finally, we mention some unresolved issues. The time-series methods discussed in this paper have several shortcomings. First of all, it is sometimes difficult to provide a meaningful economic interpretation of the movement of an extracted trend-cycle component or a long-term trend component, and to identify the factors that determine it. This problem may be addressed by multivariate time-series analysis and cross-sectional micro data. In this respect, it would be useful, for example, to estimate the causality among economic variables both computed by the original series and by the extracted components, and examine the differences. Another idea would be to compare the underlying trend component extracted by the limited influence estimator in Bryan and Cecchetti (1994) and Shiratsuka (1997) and the trend-cycle component extracted by the times-series methods.

Second, in the period of large variations in an economic time-series, an extracted trend-cycle component around the end of a sample period tends to be revised frequently with new data. It becomes important, therefore, to know how precisely the extracted trend-cycle component can predict the direction of a business cycle at each point in time. Applying the Henderson moving average and DECOMP to actual economic time-series and evaluating their predictive accuracy at a time of economic significance is an important topic for future research.

APPENDIX 1: SPECTRAL ANALYSIS WITH THE FOURIER TRANSFORMATION

There are two kinds of time-series analysis: one operates in a time domain, the other in a frequency domain. The latter is called spectral analysis. It decomposes a time series into a sum of multiple components with different frequencies, and analyzes the characteristics of the time series by examining each component in the frequency domain. This appendix explains the Fourier transformation, which forms the basis for spectral analysis.[37]

A. Fourier Transformation

Suppose there are T sample data (p_1, p_2, . . . p_T), which are generated by a stationary stochastic process $\{p_t\}$. In theory, all the data can be described as a linear combination of T independent explanatory variables. In the Fourier analysis, explanatory variables take the form of constant and periodic functions, which correspond to a frequency in $[0, \pi]$.[38] First, define number n that depends on whether T is even or odd as follows:

$$n = (T)/2 \qquad \text{(where } T = \text{even number)} \qquad \text{(A.1)}$$
$$n = (T-1)/2 \qquad \text{(where } T = \text{odd number).} \qquad \text{(A.2)}$$

Next, define n frequencies in an equal distance as follows:

$$\lambda_j = 2\pi j/T \qquad (j = 1, 2, \ldots, n). \qquad \text{(A.3)}$$

For each frequency λ_j, let a pair of $\sin\lambda_j t$ and $\cos\lambda_j t$ constitute an explanatory variable. Then p_t can be expressed as follows:

$$p_t = T^{-1/2}\left\{\alpha_0 + \sqrt{2}\sum_{j-1}^{n}(\alpha_j \cos\lambda_j t + \beta_j \sin\lambda_j t)\right\}$$
$$\text{(where } T = \text{odd number)} \qquad \text{(A.4)}$$

$$p_t = T^{-1/2}\left\{\alpha_0 + \sqrt{2}\sum_{j-1}^{n}(\alpha_j \cos\lambda_j t + \beta_j \sin\lambda_j t) + \alpha_{n+1}(-1)^t\right\}$$
$$\text{(where } T = \text{even number).} \qquad \text{(A.5)}$$

These two equations are called a Fourier representation. α_j and β_j in equations (A.4) and (A.5) can be expressed as follows:[39]

$$\alpha_j = (2/T)^{1/2}\sum_{t=1}^{T} p_t \cos\lambda_j t \qquad \text{(A.6)}$$

$$\beta_j = (2/T)^{1/2}\sum_{t=1}^{T} p_t \sin\lambda_j t. \qquad \text{(A.7)}$$

37. This appendix is based on Harvey (1981) and Yamamoto (1988).
38. The Fourier transformation attempts to approximate an original series by a linear combination of trigonometric functions.
39. α and β are derived from a relationship between trigonometric functions and imaginary numbers (Euler's formula).

An estimator of p_t in the Fourier representation is known to be inconsistent. To make it consistent, we apply a technique called a "lag window," which smoothes λ_j defined in equation (A.3).[40] Then $(\alpha_j^2 + \beta_j^2)$ can be expressed as follows:

$$PS_j = \alpha_j^2 + \beta_j^2 = (2/T)\left[\left(\sum_{t=1}^{T} p_t \cos\lambda_j t\right)^2 + \left(\sum_{t=1}^{T} p_t \sin\lambda_j t\right)^2\right]. \qquad \text{(A.8)}$$

This is called the power spectrum (PS_j).

B. How to Read a Spectral Analysis Graph

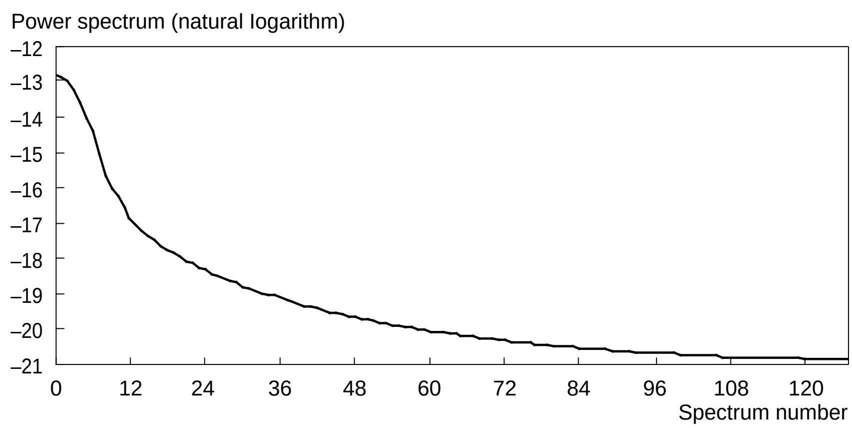
We explain a spectral analysis graph using the example of our actual sample period. In this paper, we have used monthly time-series data from January 1976–March 1997 with a sample size of 255. In this case, $n = 127$ ($j = 1, 2, \ldots n$) by equation (A.2) above. Next, from equation (A.3), we obtain λ_j by multiplying j by 2π and then divide it by T (255).

Using the following relationship between frequency and cycle:

$$\text{(frequency)} \times \text{(cyclical period)} = 2\pi,$$

we can obtain the cyclical period.[41] Appendix Figure 1 shows these concepts in a graph of spectral analysis. In the graph, the horizontal axis measures frequency. The closer it is to zero, the lower the frequency (the longer the cyclical period). The first term ($j = 0$) is a constant and represents a trend component with an infinite cyclical period. The vertical axis represents the logarithm of a power spectrum.

Appendix Figure 1 Fourier Transformation



	0	⋯	1	⋯	3	⋯	10	⋯	50	⋯	100	⋯	127
λ (cycle)	0	⋯	π/127	⋯	3π/127	⋯	10π/127	⋯	50π/127	⋯	100π/127	⋯	π
Period	∞	⋯	255	⋯	85	⋯	25	⋯	5	⋯	2.5	⋯	2
Months			21.3 years		7.1 years		2.1 years		5 months		2.5 months		2 months

40. All lag windows in this paper employ $0.75\sqrt{T}$ where T is the number of samples. The software we have used is RATS Version 4.2, and the window is Tent Window.
41. The same method applies to quarterly data. With the same sample period, we have $T = 85$ and $n = 42$. From those, we can calculate the period and λ_j. In the case of quarterly data, the shortest cycle is two quarters, and therefore the period becomes a half year.

APPENDIX 2: TIME-SERIES METHODS OF EXTRACTING A TREND-CYCLE COMPONENT AND A LONG-TERM TREND COMPONENT

A. The Band-Pass Filter

The Band-Pass filter is a method of extracting a component whose cyclical period falls within a "band" specified by an analyst. In this paper, we first calculate the spectrum by using the Fourier transformation.[42] Next, we eliminate cyclical components outside the specified band. Then, applying the inverse Fourier transformation to the processed periodograms, we can extract a specified cyclical component.

1. Essentials of the Band-Pass filter

First, applying the Fourier transformation to a time series, we obtain a sample periodogram for each frequency. Then, keeping intact the sample periodograms corresponding to a specified band, we replace all the other periodograms by zeros. That is, we set zero into α_j and β_j (j = the spectral number) in equations (A.6) and (A.7) in Appendix 1:

$$\alpha_j = (2/T)^{1/2} \sum_{t=1}^{T} p_t \cos\lambda_j t = 0 \qquad \text{(A.6)*}$$

$$\beta_j = (2/T)^{1/2} \sum_{t=1}^{T} p_t \sin\lambda_j t = 0. \qquad \text{(A.7)*}$$

Finally, applying the inverse Fourier transformation to those new periodograms, we reconstruct a time series that contains only the components with cyclical periods that belong to the specified band.

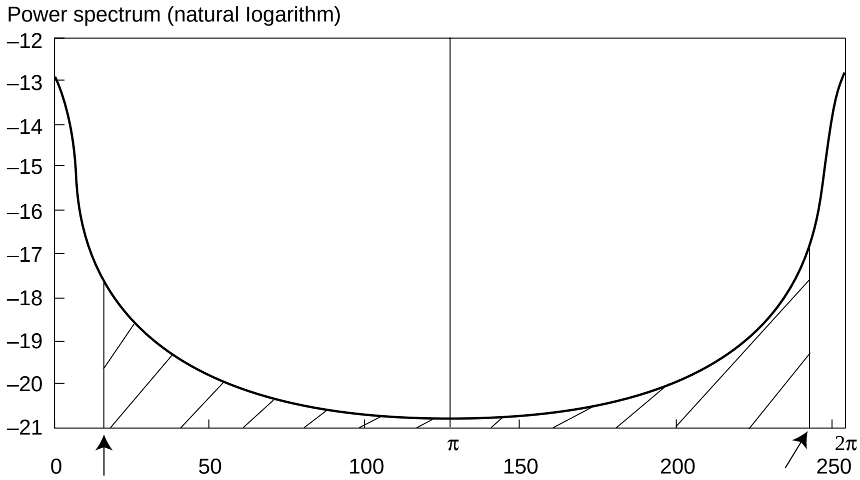
We must recognize that the power spectrum is symmetrical at the mid-point π because equation (A.3) in Appendix 1 ($\lambda_j = 2\pi j/T$) is defined over the range $[0, \pi]$.[43] Therefore, when we replace periodograms in a specified band in $[0, \pi]$ by zeros, we must do the same in the corresponding band in $[\pi, 2\pi]$. For the case of our sample period (January 1976–March 1997 with 255 samples), Appendix Table 1 and Appendix Figure 2 show points at which the periodogram should be separated for extracting a specified cyclical component. In Appendix Figure 2, the arrow indicates the point at which the periodogram should be separated. The shadow indicates the area in which $\alpha_j = \beta_j = 0$ should hold.

Appendix Table 1 The Periodogram That Should Be Separated

	$[0, \pi]$	$[\pi, 2\pi]$
More than three years	$j \leq 7$	$j \geq 249$
More than two years	$j \leq 10$	$j \geq 246$
More than 1.5 years	$j \leq 14$	$j \geq 242$
More than one year	$j \leq 21$	$j \geq 235$
More than nine months	$j \leq 28$	$j \geq 228$
More than six months	$j \leq 41$	$j \geq 215$

42. See Appendix 1 for the Fourier transformation.
43. Because the power spectrum is symmetric in $[0, \pi]$ and $[\pi, 2\pi]$, it is sufficient for spectral analysis to focus only on $[0, \pi]$. This is the reason why discussions on the Fourier transformation in Appendix 1 and most textbooks are limited to $[0, \pi]$.

Appendix Figure 2



The Band-Pass filter has several advantages over other methods. First, it is relatively easy to implement and only takes a short time to calculate. Second, since it can extract a component within a specified band, specifying the cyclical period to be over one year, we can apply the method without performing seasonal adjustments in advance.

2. Shortcomings

(1) The sample periodogram assumes that an original time series is stationary. It is necessary therefore to convert a time series into a stationary series by taking its differences before we apply a Fourier transformation. Normally, we take its first difference because recovering necessary information becomes difficult for a higher-order difference. However, there is a possibility that the first differences of some economic time-series remain nonstationary.

(2) As already discussed in Appendix 1, the estimated periodogram is known to be inconsistent. Therefore, when a long-term component is stronger than a short-term component, the long-term component does not necessarily represent the true spectrum. One must be particularly cautious in interpreting the results when extracting a long-term trend component.

B. DECOMP

DECOMP is a seasonal adjustment program, which uses a state space model (Gersch and Kitagawa [1984] and Kitagawa [1986]).[44] It decomposes a time series into a trend component, a stationary AR component, a seasonal component, and a trading day component.[45]

44. For more details on a state space model, see Kitagawa (1986) and Hiromatsu and Naniwa (1993). This section is essentially based on Kitagawa (1986).
45. The name DECOMP comes from DECOMPOSITION.

1. Essentials of the DECOMP

DECOMP decomposes a time series into the following components:

$$p_t = T_t + V_t + S_t + D_t + \varepsilon_t, \tag{A.9}$$

where p represents a time series, T a trend component, V a stationary AR component, S a seasonal component, D a trading day component, and ε white noise. Subscript t represents time. Now we explain each component in more detail.

a) A trend component (*T*)

The trend component (T_t) is assumed to follow the m-th order stochastic difference equation:

$$(1 - B)^m T_t = \nu_{1t} \qquad \text{where } \nu_{1t} \sim N(0, \tau_1^2), \tag{A.10}$$

where B is a lag operator defined by $BT_t = T_{t-1}$.[46]

b) A stationary AR component (*V*)

The stationary AR component (V_t) is assumed to take the form of an n-th order autoregressive model:

$$V_t = \sum_{i=1}^{n} a_i V_{t-i} + \nu_{2t} \qquad \text{where } \nu_{2t} \sim N(0, \tau_2^2). \tag{A.11}$$

This V represents short-term local variations, which can be ignored in the long term, while the trend component captures a long-term trend movement.

c) A seasonal component (*S*)

A seasonal component with the cyclical period of q (S_t) satisfies the following approximation:[47]

$$(1 - B^q)S_t = 0. \tag{A.12}$$

From this, we can derive the following equation:[48]

$$\sum_{i=0}^{q-1} B^i S_t = 0. \tag{A.13}$$

Finally, allowing variations through time, we obtain the following model of the seasonal component:

$$\sum_{i=0}^{q-1} B^i S_t = \nu_{3t} \qquad \nu_{3t} \sim N(0, \tau_3^2). \tag{A.14}$$

d) A trading day component (*D*)

The trading day component (D) represents the effect of differences in the number of trading days in each month. Assuming that the sum of trading day components in a

46. Kitagawa (1986) states that the order m normally takes a value of 1, 2, or 3.
47. The value of period q is 4 for quarterly series and 12 for monthly series. Equation (A.12) implies that taking a first difference between January this year and January last year will eliminate a seasonal factor.
48. This implies that the sum of seasonal components over one cycle is equal to zero.

week is equal to zero, we have the following relationship:

$$\sum_{i=1}^{7} \beta_{it} = 0. \tag{A.15}$$

Letting D_{it}* represent the number of i-th day of the week in t-th month, the trading day component D_t can be written as follows:

$$D_t = \sum_{i=1}^{7} \beta_{it} D_{it}^* = \sum_{i-1}^{6} \beta_{it}(D_{it}^* - D_{7t}^*) \equiv \sum_{i=1}^{6} \beta_{it} D_{it}. \qquad \text{(A.16)[49]}$$

DECOMP represents these components in a state space model, and extracts a trend-cycle component through the filter of the square root of an information matrix.

The characteristics of the DECOMP method are as follows. First, it formalizes a trend as a stochastic process and estimates a trend using a formal model. Second, it determines the parameter values of the model by an objective statistical method such as the AIC, in order to minimize the arbitrariness of the analyst. Third, it formalizes the components other than the trend in a stochastic model.

2. Shortcomings

(1) DECOMP assumes that error terms in all the components are white noise. In practice, however, they may take a form of MA. In this case, extra errors may enter into the extracted components if DECOMP assumes white noise.[50] In theory, increasing the order of the AR component will absorb MA terms. In practice, however, it may destabilize the estimation results of the DECOMP. Therefore the potential problem of a non-white noise remains unresolved in the DECOMP method.

(2) DECOMP is known to produce an excessive seasonal adjustment called "seasonal dip" because it imposes strict conditions on the seasonal adjustment.[51] In this case, such a "seasonal dip" may affect the extraction results of other components.

(3) We have listed the objectivity of model specification as one of the main characteristics of the DECOMP method. However, as pointed out by Hiromatsu and Naniwa (1993), this automatic model specification has one problem: the possibility that estimated components of two different models can be very different even if the performance of the models is almost identical (that is, the absolute value of the AIC is less than one) by the AIC. Therefore, it becomes difficult to decide which extracted component is closer to the true trend component. In such case, one should use other methods, such as examining the graphs, in addition to the AIC.[52]

49. For simplicity, DECOMP assumes that the trading day component does not change over time, ignoring white noise.
50. See Yoshikawa (1992).
51. See Kimura (1996a, b) for details. Kawasaki and Sato (1997) compare the performance of seasonal adjustments using X-12-ARIMA and the DECOMP.
52. See Ishiguro, Kitagawa, and Sakamoto (1983) on this point, including the characteristics of the AIC.

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