A New Framework for Measuring the Credit Risk of a Portfolio: The "ExVaR" Model

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This paper proposes a new framework for the quantitative evaluation of the credit risk of a portfolio by extending the concept of value at risk. In practice, the risk evaluation period is set individually for each transaction in the portfolio and a simulation is carried out on the movements of default probabilities, interest rates, and collateral asset prices as well as on the realization of defaults of counterparties. The result fixes the cash flow along the simulated path and leads to the present value of the total cash flows. By repeating this procedure many times, we obtain the probability distribution of the present value, by which we can evaluate the price and the risk of the portfolio. This framework enables us comprehensively and objectively to measure the risk taking into account the diversification/concentration effect, the collateral effect, and the correlation between credit risk factors and market risk factors. After presenting the methodology, the paper calculates the risk of hypothetical test portfolios. They are used to discuss the applicability of the framework to practical uses.

Key words: Value at risk; Credit risk; Risk integration; Default probability; Diversification/concentration; Collateral effect

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I. Introduction

In the evolution of quantitative approaches for the risk management of financial institutions, the major focus has until recently been on the treatment of market risk. As a result, market participants seem to have reached a consensus on the effectiveness of Value at Risk[1] (hereafter, VaR) as the aggregate measure of the market risk of a trading portfolio, although there remain a number of issues concerning the treatment of banking accounts. With respect to credit risk, however, the methods in practice have been mostly qualitative rather than quantitative. It is only recently that the quantitative analyses have begun to attract attention.[2] One of the points at issue is that the characteristics of the credit markets vary widely among countries and types of transaction. Hence, it seems that market participants have no common approach for measuring the credit risk of a portfolio.

In this paper, we propose a new framework to quantify the credit risk of a portfolio. We extend the concept of VaR so that it can apply to credit risk measurement. We calculate the risk, by Monte Carlo simulation, of a number of test portfolios, some of which include secured transactions and/or derivative products. In constructing the framework, we are particularly conscious of the existence of illiquid loans and real estate collateral, both of which are important features of the middle market in a number of countries, including Japan. Although some building blocks of the model may not be specific enough to implement at this stage, the framework itself will be effective for the future development of innovations in risk management.

This paper is organized as follows. After clarifying the definition of credit risk and the basics of credit risk analysis in Chapter II., we present, in Chapter III., the concept and the details of the proposed risk measurement model, "the ExVaR model." As a pilot study on the model, we calculate and compare the risk amounts of various hypothetical test portfolios in chapters IV. and V. In particular, Chapter IV. investigates the characteristics of the risk measure defined in this paper. Chapter V. considers the applicability and the limits of the model in practical use, where the issues include a review of credit risk management during the collapse of the "bubble" in Japan in the 1990s, and the setting of interest rate standards for a new loan. Finally, Chapter VI. summarizes the results of the paper as well as the remaining tasks.

II. Basic Framework of the Quantitative Analysis of Credit Risk

A. Definition of Credit Risk

The phrase *credit risk* can be interpreted in various ways. Section II. A. clarifies the definition adopted in this paper. In addition, the relationship with other definitions of this phrase is briefly outlined.

1. See, for example, Mori, Ohsawa, and Shimizu (1996) for the basics of the VaR.
2. Examples of comprehensive materials on credit risk management includes Backman et al. (1995), Nishida (1995), and Sekino and Sugimoto (1993).

1. Type I credit risk and type II credit risk

In this paper, we define **credit risk** as *the possible decrease in the present value of future cash flows from financial transactions, which results from both counterparties' defaults and the increased possibility of future defaults.*[3] A default means a counterparty's breaking of the original contract. To clarify the meaning of credit risk, we can divide it into two parts—type I and type II credit risk. **Type I credit risk** is defined as *the possible decrease in the present value of future cash flows from financial transactions, which results from counterparties' defaults*. It does not take into consideration any future change of the default probability, but assumes the current default probability to be constant, and focuses on whether the default occurs given the probability. No default means no realization of type I credit risk. When we assume a single transaction in a single period as an example, the probability distribution of the cash flow is discrete, composed of only two states (i.e., default and non-default). With respect to a portfolio including many transactions and counterparties, the probability distribution is almost continuous, where we can measure the maximum loss under a certain confidence level, as is the case with the VaR of market risk. In the case when the default characteristics of the individual transactions in a portfolio can be recognized as independent of each other, the probability distribution of the cash flows approaches a normal distribution curve, according to the central limit theorem, as the number of transactions gets larger and the portfolio becomes more diversified. Moreover, in a completely diversified portfolio, type I credit risk is zero, since the standard deviation of the normal distribution is infinitely small. In this sense, type I credit risk is avoidable by complete diversification. In reality, however, one should not assume complete diversification *a priori*, since there are some correlations among individual transactions and a portfolio held by a bank often has some degree of credit concentration.

On the other hand, **type II credit risk** is defined as *the remaining risk that is calculated by subtracting type I credit risk from total credit risk*. In other words, it results from the possibility of an increase in the counterparties' default probabilities in the future. As the default probability of the counterparty increases, the expected present value of the cash flows from the transaction decreases, since not only does the expected value of the default loss increase but also the replacement cost in case of closing it before maturity decreases due to the increased credit risk premium.

There are various approaches to defining and analyzing credit risk in the finance industry. Some of them are found only to deal with either type I credit risk or type II credit risk. However, each type of credit risk can be significant for many portfolios. Hence, this paper proposes a framework for evaluating both of them simultaneously.

2. Other approaches to a definition of credit risk

The definition of credit risk shown in Section II. A. 1. is based on the concept that the credit risk should be measured as the future decrease in the value of a financial

3. This paper focuses on the defaultability of counterparties as the source of the credit risk. In reality, some derivative products, such as an option on a private company's debenture, include credit risk from the defaultability of the third party, which in the example quoted is the issuer of the debenture. This type of risk is not discussed in this paper.

transaction from its current value, where the value always reflects the default probability of a counterparty perceived at that time. On the other hand, credit risk can be defined in different ways. In a typical example, it is defined as the difference between a transaction's current value, which reflects defaultability, and the hypothetical current value, which assumes the free default. This approach can be seen, for example, in the concept of a risk asset in the calculation of BIS capital adequacy requirements for internationally active banks. A risk asset roughly corresponds to the expected future loss since it is calculated as an asset's value times the multiplier reflecting the defaultability of the counterparty. That way of thinking is quite different from the one proposed in this paper.

For the credit risk of derivative products, the concept of **credit risk exposure** is often used. This can be divided into two parts: current exposure, which is defined as replacement cost, and potential future exposure, which is defined as the potential increase in replacement cost. Credit risk exposure means the value that is exposed to credit risk, and it does not include any information on the default probability of a counterparty. Roughly speaking, we can see that the future loss corresponds to the credit risk exposure multiplied by the default probability. In this paper, the credit risk exposure is not separately evaluated. Instead, it is taken into account in quantifying the credit risk amount by simultaneously simulating the market rate movement, the default probability movement, and the realization of default.

B. Directions of Quantitative Analyses of Credit Risk

Quantitative analyses of credit risk include two important areas. This section briefly explains their contents and aims in order to clarify the purpose of the research in this paper.

The first area is the evaluation of the creditworthiness of individual counterparties in a quantitative and objective way. In Section II. A. as well as in later sections, it is implicitly assumed that we have sufficient information on default probabilities. However, the accurate evaluation of the creditworthiness of counterparties is in practice an important starting point for the risk estimation process. In addition, it is the basis for pricing financial transactions. Traditionally, financial institutions have analyzed the creditworthiness of firms by inspection. They have often classified firms by creditworthiness or by credit ratings, without calculating specific default probabilities. One approach to advance the situation is to estimate the default probability corresponding to each credit rating. Moreover, it is effective to make use of more quantitative and objective procedures for credit analysis together with relatively qualitative and subjective procedures such as traditional inspection. Examples of the former procedures include (1) default forecasting models, which are based on linear discrimination analysis,[4] Probit/Logit models,[5] or neural network analysis; (2) estimation of the implied default probability of the debenture issuer from the spread in the

4. See, for example, Altman (1971, 1983) for analyses of the U.S. data and Goto (1989) for analyses of Japanese data.
5. See, for example, Boyes, Hoffman, and Low (1989) and Johnsen and Melicher (1994) for application to the U.S. corporations.

market;[6] and (3) application of option pricing theory to the value of a firm.[7] These analyses are included in the first area. We should note that these issues can be dealt with by the varied approaches shown above. Hence, they should be answered by each financial institution selecting its own methodology to fit the situation, rather than studied in a unified framework. This paper does not discuss them further.

The second area is the calculation of credit risk of a portfolio given the counterparties' default probabilities and the process of change in the future. This is analogous to the calculation of market risk, as a VaR, given the dynamic process of the term structure of interest rates. We should note, however, that the concept of VaR needs to be extended for credit risk evaluation, as explained in Chapter III. This paper focuses on the second area.

III. A Model for Calculating the Integrated Risk Measure, ExVaR

Our purpose is to measure integrated risk, which includes both credit and market risk of a portfolio. We have prepared a pilot model for the purpose. Chapter III. explains the concept, assumptions, and specifications of the model.

A. Definition of the ExVaR

The traditional VaR for measuring the market risk of a trading portfolio is defined as the maximum decrease in portfolio value during a defined holding period at a specified confidence level (e.g., 99 percent). This can be calculated by methods such as the variance-covariance method, historical simulation, or Monte Carlo simulation. However, VaR cannot be used to evaluate the credit risk of financial transactions since we must not neglect cash flows during the risk evaluation period, which can be years long in the case of illiquid banking portfolios. To deal with the problem, we recognize as risk the uncertainty of cash flows during the risk evaluation period, instead of the uncertainty of market values at the end of the holding period. The cash flows include

[1] interest income/expense;
[2] claims that can be recovered at a counterparty's default, if any; and
[3] assumed cash flows on closing the transaction at the end of the risk evaluation period with no preceding default.

These are determined by a path of the market rates and default probabilities of counterparties. In this paper, we consider all of [1] to [3], while the traditional VaR focuses only on [3]. For the exact evaluation of the time value of cash flows coming on different dates, we assume that all the cash flows are reinvested into short-term riskless assets and rolled over to a fixed date—for example, five years ahead in this paper—and the resulting total return is discounted back to the present value.

We use the Monte Carlo method to carry out a simulation that determines the path of market rates, default events, and finally the present value of the resulting cash

6. See, for example, Wu and Yu (1996) as a recent study.
7. See Merton (1974) for the basics of the theory.

flows. We obtain a large number of present values of cash flows by repeating the simulation. The *expected value of these present values* (hereafter, **ExPV**) can be interpreted as the approximate current value of the portfolio.[8] We can also define risk, including both credit and market risk, as *the ExPV minus the 99 percentile minimum (x percentile minimum in general) of the present values*. We call it the **ExVaR**, standing for the extended value at risk. Again, the ExVaR is an extended version of the VaR in that the former is based on the uncertainty of the future cash flows from the portfolio, while the latter is based on the uncertainty of the future value of the portfolio.

B. Assumptions of the ExVaR Model

Section III. B. explains the individual assumptions for the ExVaR calculation.

1. Default probability

This paper assumes that (1) we have already got a system to determine the credit rating accurately; (2) that the output of the system can be applied to risk measurement; and (3) that we have estimated the default probability corresponding to each credit rating based on the historical default data.

We then need information on the future dynamics of the default probability. We assume *a priori* a lognormal stochastic process from among the alternative models.[9] This is described in mathematical form thus:

$$\begin{aligned} dh_i(t) &= h_i(t + dt) - h_i(t) \\ &= h_i(t)\mu_j(t)dt + h_i(t)\sigma_j(t)dz_j(t) \end{aligned}$$

where $dh_i(t)$ is a change in $h_i(t)$, the default probability, during a small period from t to $t + dt$, and i and j are the indexes denoting an individual counterparty and its type of industry, respectively.[10] $\mu_j(t)$ and $\sigma_j(t)$ are the parameters denoting the trend and volatility of the default probability process. Both of them are set by type of industry. $dz_j(t)$ denotes a standard Brownian process to represent the movement of creditworthiness of firms or the industry j. It can be written that $dz_j(t) = \varepsilon_j(t)\sqrt{dt}$, where $\varepsilon_j(t)$ is a random variable subject to the standard normal distribution.[11]

8. The ExPV is the expected value of discounted cash flows under a real probability measure. On the other hand, the theoretically accurate price can be calculated as the expected value of discounted cash flows under an equivalent martingale measure. The martingale measure is interpreted economically as the modified probability that reflects the risk preference of market participants. If the market participants are risk-neutral, the ExPV is completely equal to the theoretical price since the real probability is exactly the same as the equivalent martingale measure.

   There is a great deal of research on pricing of financial transactions taking into account the risk preference of market participants. See, for example, Madan and Unal (1993), Duffie and Singleton (1994), Duffie, Schroder, and Skiadas (1993), Duffie and Huang (1994), and Jarrow and Turnbull (1995). However, the practical use of such research does not seem to be common yet. Hence, in this paper, the price of a portfolio is approximated by the ExPV under a real probability measure.
9. See Grenadier and Hall (1995) as an example of research where the lognormal stochastic process is applied to the default probabilities in pricing a financial transaction.
10. It is not appropriate to apply the lognormal process to $h_i(t)$ itself since the default probability is defined within the range (0, 1). The problem is removed by adding the constraint that $h_i(t) \leqq 1$. The algorithm in the pilot model for this paper has the same effect as the above condition since the model interprets the case $h_i(t)>1$ as $h_i(t)=1$ where a default certainly occurs.
11. Correlations between risk factors are considered in the set of standard normal variables $\varepsilon_j(t)$. As shown later in Table 4, there are correlations between such risk factors as default probabilities of different types of industry, interest rates, an equity price, and a real estate price.

This model reflects the idea that grouping the homogeneous counterparties in terms of the default probability process can be achieved by classifying them on the basis of the type of industry. Therefore, parameter settings and random number generation are done by type of industry, not by counterparty, and the calculation burden is markedly reduced. It should be noted that only one attribute of counterparties—type of industry—may be insufficient to grasp the characteristics of their default probability change. Other than the type of industry, such attributes as credit rating, region (or the country), or the size of the counterparties can also be effective for grouping. Although we need detailed empirical study of the issue before implementing the risk measurement system, we focus *a priori* on the industry type factor in describing the ExVaR framework. Empirical analysis is also required to estimate the parameters $\mu_j(t)$ and $\sigma_j(t)$. In this paper, we estimate them from the historical data, assuming that the trend is always zero and volatility is constant with regard to the time. (The result is shown in Table 4 later.) For more accurate and forward-looking analysis, we should make adjustments for the effect of macroeconomic fluctuations due to the business cycle. In our calculation, the time interval, dt, is set to one month and a period of up to five years is considered.

2. Credit enhancement (collateral effect in particular)

In Japan and other countries, a number of loans have some credit-enhancing features, typically a collateral or a guarantee. We should evaluate the effect appropriately since it has a large influence on both the value and the risk of the transaction. In this paper, we focus on the treatment of collateral—mainly real estate and equity collateral—although many other forms of credit enhancement are actually available.

In collateral loans, a creditor is expected to collect the minimum amount of the following three items in case of a default of a debtor (when the recovery rate on the secured asset is 100 percent):

[1] fair value (replacement cost) of the transaction at the time of default;
[2] fair value of the collateral (minus the amount of preemption of others, if any); and
[3] collateral limit (*kyokudo-gaku*).

Thus, we need to model the future dynamics of collateral value in addition to that of the market rates underlying the transaction. In this paper, we do not discuss how to construct the model but assume *a priori* a lognormal stochastic process for prices of real estate and the equity of collateral. This assumption seems appropriate for the equities but may not hold good for real estate. Moreover, the dynamics of each real estate, or each equity, differ individually. However, we assume, for simplification, that all types of collateral have a uniform rate of return, which is equal to that of the benchmark—the Land Price Indexes of Urban Districts (all urban districts, average), published by Japan Real Estate Institute, for real estate and the Nikkei 225 Stock Average for equities. In addition, we also posit a third collateral class, other collateral. In this paper, we assume the deposits are dominant in other collateral and set the price volatility of this category to zero.

3. Recovery rates

The treatment of recovery rates influences the estimation of credit risk as much as that of the credit enhancement effect. Many approaches to this issue are possible, from a simple method of setting a specific constant as "the average recovery rate" or "the recovery rate of each asset class" to a more statistical method of modeling the future recovery rate with a stochastic process.

In our model, two kinds of recovery rates are to be input—the **secured asset recovery rate** and the **unsecured asset recovery rate**. The former means the expected rate of collected cash to the fair value of collateral in the defaulted secured asset, and the latter means the expected rate of collected cash to the pre-default fair value of the unsecured asset. The secured asset recovery rate is ideally equal to one, but it is actually less than one due to the existence of negotiation costs with subordinated collateral holders or the decrease of time value until completion to liquidate the collateral. In this paper, we assume *a priori* that the rate is equal to 0.9. The unsecured asset recovery rate reflects the situation where some portion of the unsecured asset could be recovered in some cases. In this paper, we conservatively set the rate equal to zero.

4. Interest rates[12]

Future yield curve movements are simulated by a combination of the Monte Carlo method and a factor model in which three major vectors[13] are adopted as factors from among the principal components of monthly yield curve data during the past eight years. This approach has the advantage that it facilitates interpretation of change in the yield curve. There are some alternative approaches, such as simulation with a multivariate normal distribution applied to the set of interest rates corresponding to specific periods.

5. Integration of credit and market risk (correlation between default probabilities and market rates)

Changes in the default probability of counterparties and changes in market rates, such as interest rates, are not in general independent of each other. For example, in a recession period, default probabilities tend to increase while interest rates tend to decrease, reflecting monetary expansion. Thus, there is a negative correlation between the two variables.[14] We take the correlations into account by jointly estimating the credit and market risks. In this paper, we calculate the correlation matrix of the default probabilities of each industry and the random factors corresponding to the three major principal components of yield curve movements, and use the matrix to simulate variables in the future.

6. Time horizon of the risk evaluation

We set the risk evaluation period, during which the cash flow from a transaction is analyzed and its uncertainty is the source of the risk, for each transaction in the portfolio. The concept of this period is similar to that of the so-called holding period—typically one day—in the traditional VaR for market risk. In the ExVaR calculation, however, the period is a time horizon of the risk evaluation, rather than

12. Although this paper deals with the interest rates in a single currency—Japanese yen—it is easy to extend the analysis to multicurrency portfolios.
13. The three principal components of the yield curve movement can be interpreted, in turn, as (1) the parallel shift; (2) the change of the slope; and (3) the change of the curvature of the yield curve. Many empirical studies report that these three components can explain more than 99 percent of total yield curve movement.
14. See, for example, Duffee (1994, 1995), who reports an empirical study using the data in the United States.

the period during which a portfolio is held. Therefore, we call it the **risk evaluation period** in this paper.

We define the risk evaluation period of a transaction to be *the estimated period required to complete the liquidation of a transaction after deciding to do so*. For example, one day would be acceptable as the period for highly liquid bonds. On the other hand, the time to maturity would be appropriate as the period for loans which cannot readily be liquidated. This definition is natural in terms of the meaning of risk.[15] We should note again that the risk evaluation period is set according to transaction, not portfolio. In chapters IV. and V., we set the period *a priori* to be the time to maturity of each transaction for loans and swaps and one year for all debentures.

7. Diversification/concentration effect

The effect of investment diversification/concentration on credit risk is also important. In the ExVaR framework, the effect is automatically taken into account since the information on individual transactions are input and dealt with. We test the effect in Section IV. B.

C. Specifications of the ExVaR Model

The structure of our pilot model for calculating the ExVaR is shown in Figure 1. The information input in five files is processed along the flow chart. A single simulation corresponds to the procedures within the dotted line of Figure 1, outputting a present value of cash flows from the portfolio given simulated market information. A histogram showing the distribution of the present values can be obtained by repeating the simulation many times. After the calculation of the ExPV (expected value of the present values), ExVaR can be derived as the ExPV minus the minimum present value with a 99 percentile confidence level, for example. Each file shown in Figure 1 is explained below.

[Input File 1]

The file stores the data on the individual financial transactions. An example is shown in Table 1, including the information such as counterparty names (or indexes), transaction categories (or indexes), principal, interest rates, time to maturity, collateral categories (or indexes), collateral values, collateral limits, and preemption amounts. In the case of a plain interest rate swap (as opposed to a traditional loan), the principal and the interest rate are interpreted as the notional amount and the fixed interest rate,[16] respectively. In addition, the file also stores each transaction's risk evaluation period, which is determined in accordance with the liquidity characteristics.

15. Apart from the notion, adopted here, that the risk evaluation period is set individually according to the liquidity of each transaction, there is another way of thinking, which is that the unique period of risk evaluation is applied to all transactions.

16. Where a spread is put on the floating rate side of the interest rate swap, the fixed-rate information to be input is the notional fixed rate minus the spread.

Figure 1 Structure of the ExVaR Model

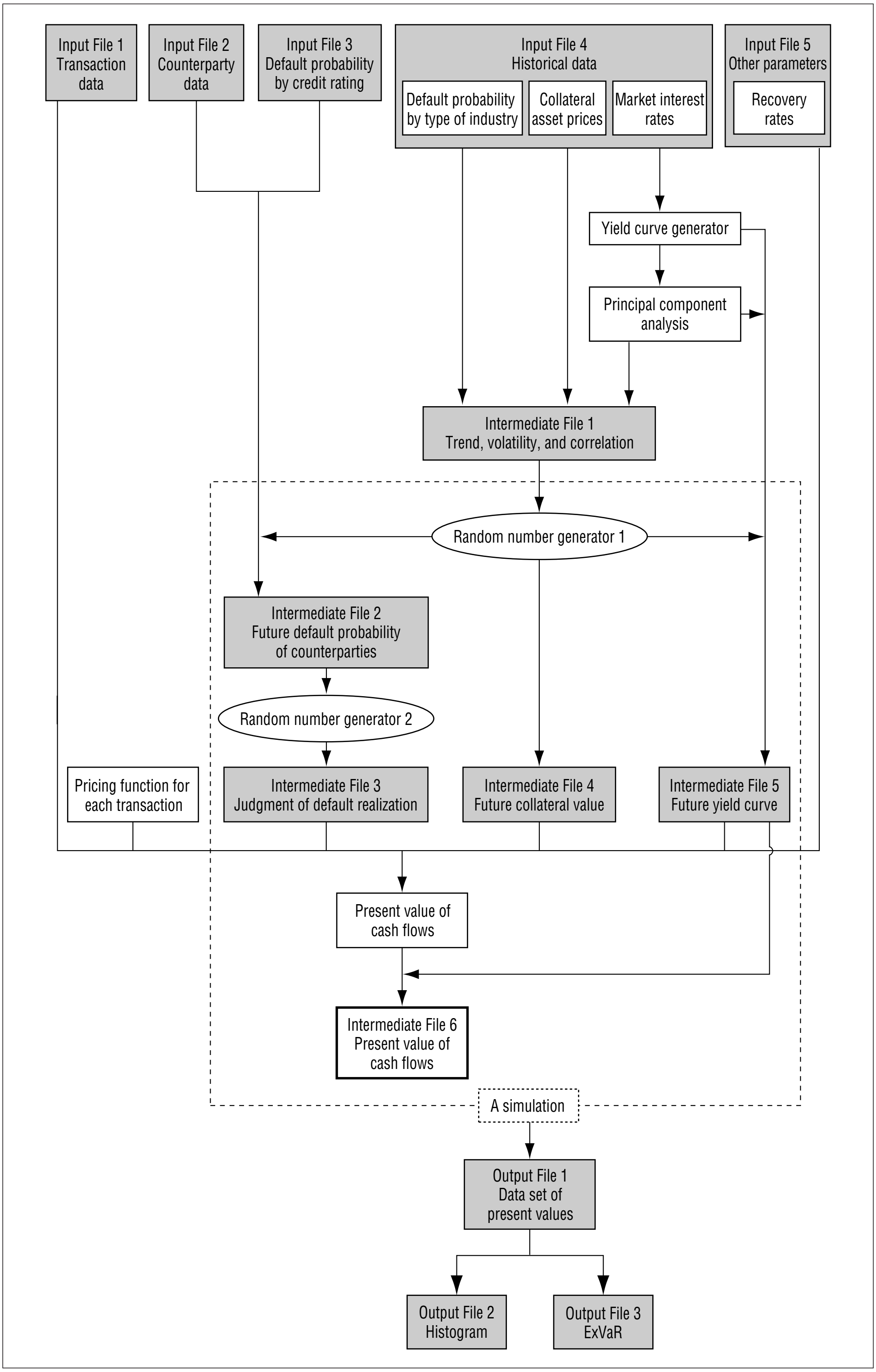


Table 1 An Example of Input File 1

Transaction code	Counterparty name (index)	Transaction category (index*)	Principal (¥ billions)	Interest rate (percent per annum)	Time to maturity (year)	Risk evaluation period (year)	Collateral category (index**)	Collateral value (¥ billions)	Collateral limit (¥ billions)	Preemption amount (¥ billions)
1	1	1	0.5	4.0	3.0	3.0	0	0	0	0
2	1	2	1.0	3.7	5.0	1.0	0	0	0	0
3	2	1	1.2	6.0	3.0	3.0	1	2.0	1.2	0.5
4	2	3	1.2	5.0	3.0	3.0	0	0	0	0
5	3	1	1.0	4.5	2.0	2.0	1	1.3	1.0	0
6	3	4	2.0	5.1	3.0	3.0	0	0	0	0
7	4	1	1.5	5.0	4.0	4.0	1	1.5	1.5	0
8	4	3	1.5	4.9	4.0	4.0	0	0	0	0
9	5	1	0.5	6.0	2.0	2.0	2	0.6	0.5	0
10	5	4	1.0	5.0	5.0	5.0	0	0	0	0
11	6	1	1.5	3.5	5.0	5.0	0	0	0	0
12	6	2	1.0	3.4	3.0	1.0	0	0	0	0
13	7	1	0.8	4.5	2.0	2.0	1	0.7	0.8	0
14	7	3	0.8	4.8	2.0	2.0	0	0	0	0
15	8	1	0.6	6.5	3.0	3.0	2	0.7	0.6	0
16	8	4	1.0	5.3	2.0	2.0	0	0	0	0
17	9	1	1.4	5.0	5.0	5.0	1	2.0	1.4	0.5
18	9	3	1.4	5.0	5.0	5.0	0	0	0	0
19	10	1	1.0	5.0	4.0	4.0	1	1.2	1.0	0
20	10	4	1.5	5.0	3.0	3.0	0	0	0	0
:	:	:	:	:	:	:	:	:	:	:

*1 = Loan (no amortization), 2 = Corporate bond (fixed coupon), 3 = Interest swap (pay-fix, plain), 4 = Interest swap (receive-fix, plain)
**0 = Non-collateral, 1 = Real estate collateral, 2 = Equity collateral, 3 = Other collateral (e.g., deposit)

[Input File 2]

The file stores data on counterparties. An example is shown in Table 2, including the counterparties' credit ratings and attributes by which we classify firms into homogeneous groups. In this paper, only the type of industry is used as an attribute.

Table 2 An Example of Input File 2

Counterparty index	Counterparty name	Credit rating	Type of industry (index)	Size (index)	Region (index)
1	Firm XXX	3	1	—	—
2	Firm YYY	6	1	—	—
3	Firm ZZZ	4	2	—	—
4	Firm	5	3	—	—
5	Firm	6	3	—	—
6	Firm	2	3	—	—
7	Firm	4	4	—	—
8	Firm	7	6	—	—
9	Firm	5	6	—	—
10	Firm	5	8	—	—

[Input File 3]
The file stores estimated default probabilities corresponding to each credit rating. An example is shown in Table 3. One of the common approaches for the estimation is to analyze the historical data of defaults by ratings and to derive the average default rate in each rating.

Table 3 An Example of Input File 3

Credit rating	Average default rate (percent per annum)
1	0.01
2	0.1
3	0.5
4	1.0
5	2.0
6	3.0
7	4.0
8	5.0
9	10.0
10	30.0

[Input File 4]
The file stores historical time-series data on the average default probability by type of industry, collateral prices (or the price indexes), and market interest rates. In this paper, the average default probability is roughly estimated as the ratio of the suspension of business transactions with banks (Federation of Bankers Associations of Japan), by type of industry, divided by the estimated number of corporations found in the *Corporate Business Statistics Quarterly* (Ministry of Finance of Japan).

[Input File 5]
The file stores all the required parameters that are not included in Input Files 1 to 4. In the current model, only the secured and unsecured asset recovery rates are included in the file. Additional information can be stored here if the model is modified.

[Intermediate File 1]
The file stores estimated statistics, such as the trend, volatility, and correlation of the historical time-series data in Input File 4. An example is shown in Table 4. In deriving these, it is assumed that the default probabilities and collateral asset prices are subject to a lognormal process while market interest rates are analyzed in accordance with a factor model with three principal components (PC) as the factors. The three principal components are shown in Figure 2.

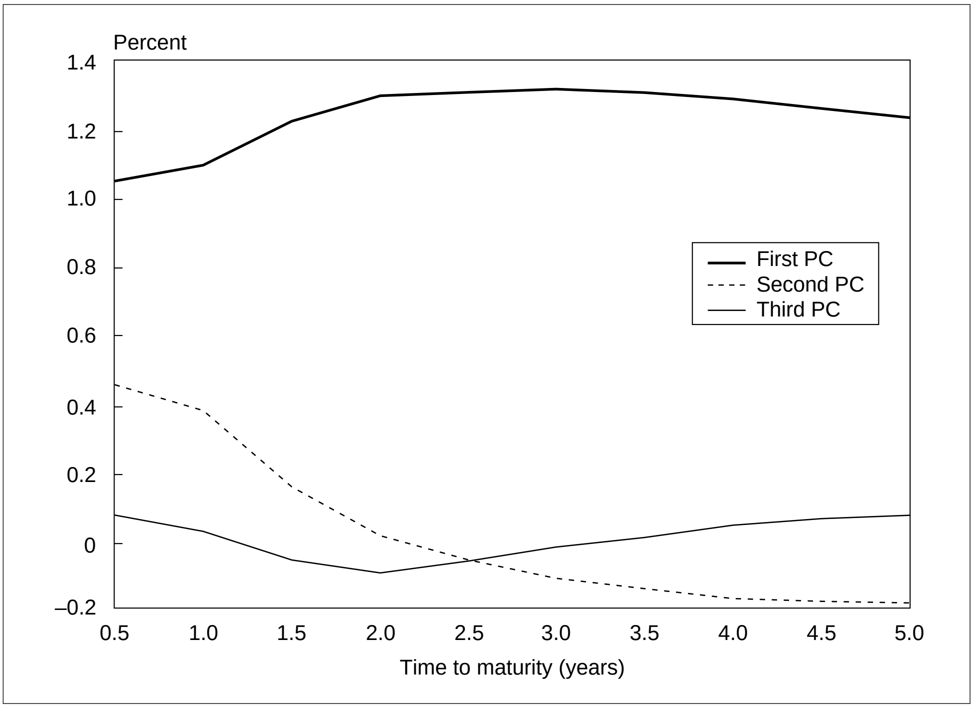
In calculating the ExVaR of test portfolios in chapters IV. and V., the data in Table 4 are basically applied.

Table 4 An Example of Intermediate File 1

	Construction	Wholesaling	Manufacturing	Transportation and communications	Real estate	Services	Retailing	Others	Land price	Nikkei	IR movement	IR movement	IR movement
	Def. prob.	Def. prob.	Def. prob.	Def. prob.	Def. prob.	Def. prob.	Def. prob.	Def. prob.	Movement	Movement	First PC	Second PC	Third PC
Trend	—	—	—	—	—	—	—	—	0.050	0.086	—	—	—
Standard deviation	0.386	0.312	0.323	0.442	0.498	0.386	0.332	0.550	0.039	0.211	—	—	—

Correlations	Construction	Wholesaling	Manufacturing	Transportation and communications	Real estate	Services	Retailing	Others	Land price	Nikkei	IR movement	IR movement	IR movement
Construction	1.000	0.767	0.812	0.569	0.713	0.773	0.807	0.508	–0.048	–0.137	–0.271	–0.018	0.241
Wholesaling	0.767	1.000	0.669	0.588	0.597	0.743	0.694	0.533	–0.122	–0.242	–0.452	0.012	0.159
Manufacturing	0.812	0.669	1.000	0.603	0.758	0.684	0.764	0.382	–0.128	–0.283	–0.065	0.133	–0.036
Transportation and communications	0.569	0.588	0.603	1.000	0.572	0.547	0.509	0.396	–0.138	–0.175	–0.077	–0.284	0.062
Real estate	0.713	0.597	0.758	0.572	1.000	0.625	0.708	0.388	0.019	–0.147	–0.247	0.006	0.044
Services	0.773	0.743	0.684	0.547	0.625	1.000	0.726	0.508	–0.045	–0.216	–0.305	–0.030	0.324
Retailing	0.807	0.694	0.764	0.509	0.708	0.726	1.000	0.490	–0.035	–0.189	–0.126	0.039	0.081
Others	0.508	0.533	0.382	0.396	0.388	0.508	0.490	1.000	–0.062	0.098	–0.416	–0.006	–0.054
Land price	–0.048	–0.122	–0.128	–0.138	0.019	–0.045	–0.035	–0.062	1.000	0.251	–0.070	–0.025	0.512
Nikkei	–0.137	–0.242	–0.283	–0.175	–0.147	–0.216	–0.189	0.098	0.251	1.000	–0.095	–0.185	0.188
IR First PC	–0.271	–0.452	–0.065	–0.077	–0.247	–0.305	–0.126	–0.416	–0.070	–0.095	1.000	–0.152	–0.153
IR Second PC	–0.018	0.012	0.133	–0.284	0.006	–0.030	0.039	–0.006	–0.025	–0.185	–0.152	1.000	0.065
IR Third PC	0.241	0.159	–0.036	0.062	0.044	0.324	0.081	–0.054	0.512	0.188	–0.153	0.065	1.000

Figure 2 Three Principal Components of Interest Rate Movements



[Intermediate Files 2, 4, 5 and Random Number Generator 1]

Random Number Generator 1 outputs multivariate normal random numbers,[17] of which the correlation is given by the result in Intermediate File 1. They are applied to the stochastic model of each variable to simulate its movement. For example, the current default probability of a firm is given by Input Files 2 and 3. Starting from the initial probability, its future path evolves in accordance with both the generated random numbers and the stochastic process of the industry to which the firm belongs. An example is shown in Table 5. With the same procedures, collateral asset prices and the yield curve are simulated from initial period into the future. Examples are shown in Table 6 and Figure 3, respectively.

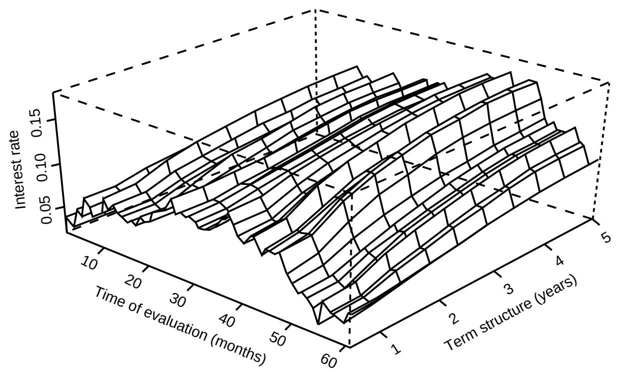
Table 5 An Example of Intermediate File 2

Counterparty (index)	1 month	2 months	3 months	· · · · · ·	58 months	59 months	60 months
1	3.00	3.08	3.15	· · · · · ·	4.62	4.51	4.55
2	2.00	1.88	1.75	· · · · · ·	2.13	2.30	2.28
3	0.10	0.11	0.10	· · · · · ·	0.15	0.17	0.17
· ·	· ·	· ·	· ·	· · · ·	· ·	· ·	· ·
98	0.50	0.52	0.51	· · · · · ·	0.42	0.40	0.38
99	5.00	5.12	5.30	· · · · · ·	4.87	4.75	4.82
100	0.10	0.09	0.11	· · · · · ·	0.10	0.11	0.12

Table 6 An Example of Intermediate File 4

Collateral asset categories	1 month	2 months	3 months	· · · · · ·	58 months	59 months	60 months
Land Price Indexes	93.15	92.91	92.67	· · · · · ·	95.12	95.24	95.37
Nikkei 225 Stock Average	19,723	19,688	19,756	· · · · · ·	21,020	21,154	21,254

Figure 3 An Example of Intermediate File 5



17. Multivariate normal random numbers are generated in the pilot model by multiplying the Cholesky matrix with a vector composed of the standard normal random numbers that are generated by applying the Box-Müller method to uniform random numbers. In general, Monte Carlo simulations can be more effective by using (1) low discrepancy sequences, such as Sobol or Faure, instead of uniform random numbers; and (2) variance reduction procedures, such as the antithetic variable method.

[Intermediate File 3 and Random Number Generator 2]

In a simulation, after determining the default probability paths, we need to judge whether the firm defaults or not. There is no direct relationship, but only an indirect relationship, between default probability magnitude and the realization of default. In other words, a default may be realized by a firm with a very small default probability while a default may not be realized by a firm with a very large default probability.

Random Number Generator 2 outputs uniform random numbers between 0 and 1. These are compared with the future default probability of a firm month by month. Where the random number is less than the default probability, this is interpreted as the realization of default in the month concerned. The result of judgment is stored in Intermediate File 3, of which an example is shown in Table 7.

Table 7 An Example of Intermediate File 3

■ 0 = Non-Default, 1 = Default

Counterparty (index)	1 month	2 months	3 months	· · · · · ·	58 months	59 months	60 months
1	0	0	0	· · · · · ·	0	0	0
2	0	0	0	· · · · · ·	0	0	0
3	0	0	0	· · · · · ·	1	—	—
· ·	· ·	· ·	· ·	· · · ·	· ·	· ·	· ·
98	0	0	0	· · · · · ·	0	0	0
99	0	0	1	· · · · · ·	—	—	—
100	0	0	0	· · · · · ·	0	0	0

[Intermediate File 6]

In a simulation, the realized cash flows can be determined given (1) the default/non-default information in Intermediate File 3; (2) the collateral values in Intermediate File 4; and (3) the transactions' payoff based on the market rates in Intermediate File 5. The cash flows are assumed to be reinvested in riskless assets until a specific date (five years later, in this paper) and the value discounted back to the present. The present value is stored in Intermediate File 6. An example is shown in Table 8.

Table 8 An Example of Simulated Cash Flows and Present Value

¥ billions

Transaction (code)	0.5 year	1.0 year	1.5 years	· · · · · ·	4.0 years	4.5 years	5.0 years
1 (loan, non-default)	0.025	0.025	0.025	· · · · · ·	0.025	0.025	1.025
2 (swap, non-default)	–0.002	–0.002	–0.001	· · · · · ·	0.002	0.001	0.002
3 (loan, default, secured)	0.030	0.030	0.927	· · · · · ·	0.000	0.000	0.000
4 (loan, default, unsecured)	0.030	0.030	0.000	· · · · · ·	0.000	0.000	0.000
· ·	· ·	· ·	· ·	· · · ·	· ·	· ·	· ·
Total cash flow	3.057	3.041	2.968	· · · · · ·	2.554	2.544	98.521

⬇

Total present value	106.733

[Output File 1]

The present value of cash flows is calculated many times by repeating the simulation.

The result in Intermediate File 6 is stored every time in Output File 1. In this paper, the number of simulations is set between 10,000 and 100,000.

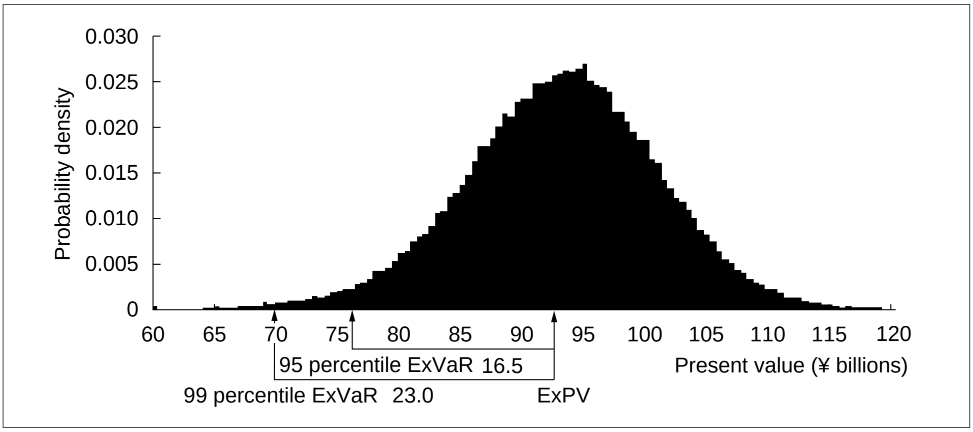
[Output File 2]

The file contains a histogram made from the data in Output File 1. Three examples are shown below.

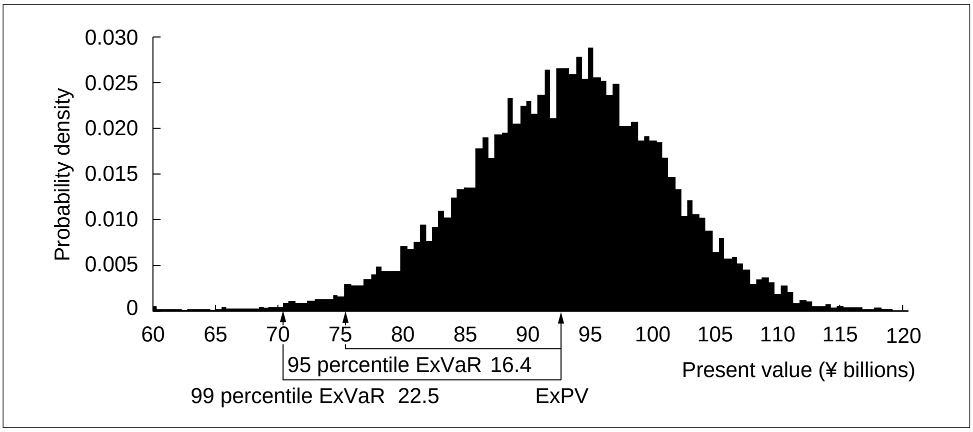
Figure 4 [1] shows the result of 100,000 simulations involving a portfolio composed of 100 loans. Figure 4 [2] shows the result of 10,000 simulations of the same portfolio. Figure 4 [3] shows the result of 10,000 simulations of a portfolio composed of 100 interest rate swaps (plain pay-fix rate). These results have three implications, as follows:

[1] The histogram of 100,000 simulations has a very smooth shape.
[2] The histogram of 10,000 simulations also has a shape smooth enough to permit analysis of the risk amount.
[3] The shapes of histograms are not necessarily symmetrical.

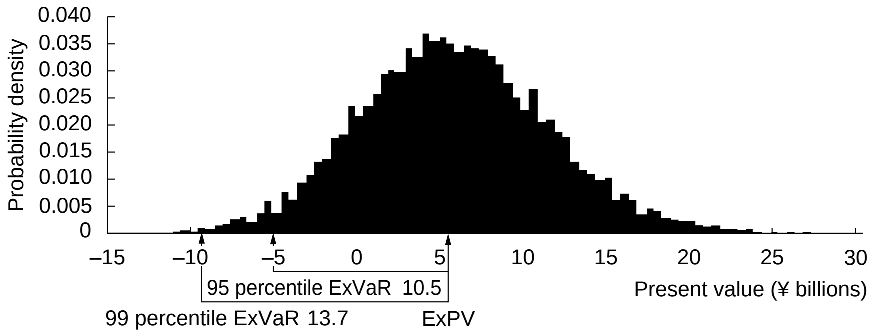
Figure 4 An Example of a Histogram of the Present Value of Cash Flows
[1] 100,000 Simulations for a Loan Portfolio



[2] 10,000 Simulations for a Loan Portfolio



[3] 10,000 Simulations for a Swap Portfolio



[Output File 3]
The file contains the ExPV, which is interpreted as the approximate value of the portfolio, and the ExVaR, which is calculated from the data in Output File 1. In this paper, ExVaR is calculated with a 99 percentile and/or 95 percentile confidence level. Figure 4 [1], [2], and [3] shows the ExPV and the ExVaR of each portfolio.

IV. Calculation of the ExVaR for Test Portfolios

The ExVaR model is constructed so that it can be effective with regard to the following four points that cannot be omitted from a proper evaluation of credit risk.

[1] Integration of on-balance and off-balance sheet transactions, with the risk evaluation period set in accordance with each transaction's liquidity.
[2] Effect of diversification/concentration in terms of counterparties and attributes of counterparties.
[3] Integrated evaluation of credit and market risk.
[4] Effect of collateral.

In this chapter, we assume a number of hypothetical test portfolios, and calculate and compare their ExPV and ExVaR to investigate how the above four points are reflected in the results. This analysis not only certifies the effectiveness of the ExVaR framework but also suggests that the above four features need to be taken into account in whatever models one uses to evaluate credit risk.

Specifications of test portfolios used in chapters IV. and V. are shown in the list in the Appendix. The number of simulations is usually 10,000, except in cases which require more accuracy, where we use 100,000 simulations.

A. Integration of On-Balance and Off-Balance Sheet Transactions with the Risk Evaluation Period Set Based on the Transaction's Liquidity

Six test portfolios (#1-1 to #1-6) are set out below and their risk amounts evaluated. The analysis, serving as a simple example of an ExVaR calculation, describes two aspects: the integration of on-balance and off-balance sheet transactions, and the setting of the risk evaluation period according to the liquidity of the transaction.

1. Integrated risk evaluation of on-balance and off-balance sheet transactions

First, we set up six test portfolios as follows. Each portfolio has one or two transactions with 100 counterparties. Conditions other than the type of transaction are basically the same for all six portfolios so that the difference of risk amounts can be attributed to the difference in the different types of transaction. That is, every counterparty belongs to the same industry (industry #2 here[18]) and has the same default probability (3 percent per annum, which corresponds to credit rating #6 in the paper). Looking at the individual portfolio, the portfolio #1-1 is composed of 100 loans, each of which is a transaction with each counterparty. Every loan has the same conditions: the principal is ¥1 billion, the interest rate is 5 percent per annum (payable semiannually), the time to maturity is five years (and the risk evaluation period is accordingly five years), and there is no collateral set. Portfolio #1-2 is composed of 100 debentures, each of which is issued by each counterparty. Every debenture has the same conditions: the principal is ¥1 billion (with no sinking provision), the coupon rate is 5 percent per annum (payable semiannually), the time to maturity is five years (but the risk evaluation period is one year, reflecting the liquidity), and there is no collateral set. Portfolio #1-3 is composed of 100 plain interest rate swaps (paying fixed rates), each of which is a transaction with each counterparty. Every swap has the same conditions: the notional amount is ¥1 billion, the fixed interest rate is 5 percent per annum (payable semiannually), the time to maturity is five years (and the risk evaluation period is accordingly five years), and there is no collateral set. Portfolios #1-4, #1-5, and #1-6 are composed of 100 loans and debentures, 100 loans and swaps, and 100 debentures and swaps, respectively, while the other conditions are the same as those in portfolios #1-1, #1-2, and #1-3.

The calculated ExVaR of these portfolios is shown in Table 9.

Table 9 ExPV and ExVaR of Test Portfolios #1-1 to #1-6

Portfolio	Portfolio composition			ExPV (¥ billions)	95 percentile ExVaR (¥ billions)	99 percentile ExVaR (¥ billions)
	100 loans	100 debentures	100 swaps			
#1-1	✓			93.0	16.4	22.5
#1-2		✓		88.6	11.7	15.8
#1-3			✓	5.3	10.5	13.7
#1-4	✓	✓		181.6	25.2	34.8
#1-5	✓		✓	98.0	23.1	30.0
#1-6		✓	✓	93.7	18.7	24.1

The first implication from the results is that the integrated evaluation is supposed to measure the risk of a portfolio, including both on-balance and off-balance sheet transactions, rather than sum up the risks of separate sub-portfolios. For example, the 99 percentile ExVaR of portfolio #1-5, which is composed of loans and swaps, is ¥30 billion while the summed ExVaR of portfolios #1-1 and #1-3, which are composed of loans and swaps, respectively, is ¥36.2 billion. It means that the latter ExVaR shows too conservative a value, which is 20 percent more than the former ExVaR. There is the same tendency in a combination of loans and debentures or of debentures and swaps.

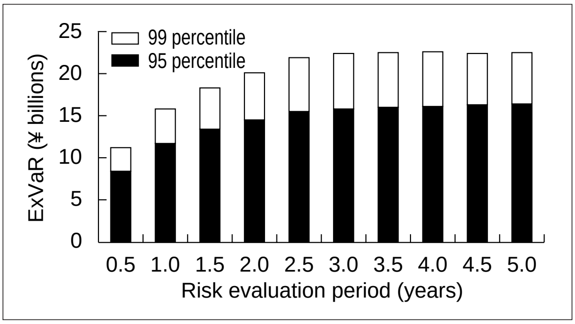
18. In Chapter III., eight types of industry are specified as an example of classification. In Chapter IV., types of industry are denoted by the index numbers, 1 to 8, for convenience of description.

2. The effect of risk evaluation period

The second implication from the above results is that the ExVaR is largely influenced by the risk evaluation period. For example, the 99 percentile ExVaR of portfolios #1-1 and #1-2 is ¥22.5 billion and ¥15.8 billion, respectively. This large difference is caused solely by the difference in risk evaluation period: five years for all transactions in #1-1 and one year for all transactions in #1-2. Although the direction of the difference can be easily imagined without any calculation, the degree of difference can be found only after quantitative analysis. To set the risk evaluation period accurately is especially important for liquid transactions, since some discretion is unavoidable in determining the period of such transactions. Figure 5 shows the ExVaR of portfolio #1-2 against the risk evaluation period, which is moved from half a year to five years for every transaction in it. This suggests that the increase in risk accompanied by lengthening the period is slower than in the $\sqrt{T}$ effect[19] for the traditional VaR.

Figure 5 ExVaR and the Risk Evaluation Period

Risk evaluation period (years)	95 percentile ExVaR (¥ billions)	99 percentile ExVaR (¥ billions)
0.5	8.4	11.2
1.0	11.7	15.8
1.5	13.4	18.3
2.0	14.5	20.1
2.5	15.5	21.9
3.0	15.8	22.4
3.5	16.0	22.5
4.0	16.1	22.6
4.5	16.3	22.4
5.0	16.4	22.5



B. The Effect of Diversification/Concentration

One of the most important points in credit risk management is the measurement and control of the effect of credit diversification/concentration. This reflects the fact that there are currently very few tools for directly hedging credit risk,[20] while the market risk can be flexibly hedged by various transactions. Therefore, many people suggest that one should diversify the credit portfolio to reduce the risk effectively. However, financial institutions also find some advantage in concentrating their credit portfolios. For example, it is efficient to investigate and monitor counterparties that specialize in a certain industry or geometric area. Such a trade-off is well known as the credit paradox. To optimize the risk profile of a portfolio with the trade-off considered, we need to measure the risk amount that properly reflects the diversification/concentration effect. In Section IV. B., we set up some test portfolios and calculate their ExVaR for comparison in order to investigate whether the ExVaR framework satisfies this prerequisite.

19. It is known that the VaR for the market risk is proportional to the square root of the risk evaluation period (i.e., the holding period) in the case of measuring the risk of the assets of which the price is linear to the risk factor.
20. For example, one may short the equity of a firm to hedge the default risk of a loan to the firm. Although the hedge can be effective to some degree, it is not more than a rough cross-hedge in that the appropriate hedge ratio is difficult to determine. One may also use credit derivatives to completely hedge the default risk of a loan. However, it seems that the market for credit derivatives has not yet become mature enough to be used flexibly.

It is convenient to classify the diversification/concentration effect into the following two elements:

[1] Diversification/concentration in terms of counterparties.

[2] Diversification/concentration in terms of attributes (types of industry) of counterparties.

Element [1] is the effect that the risk of a portfolio composed of a large number of small loans is less than that of a portfolio composed of a small number of large loans even if the total credits of both are the same. On the other hand, element [2] is the effect that the risk of a portfolio composed of loans to counterparties in a large number of industries is less than that of a portfolio composed of loans to counterparties in a small number of industries when all the other conditions affecting the two portfolios are the same.

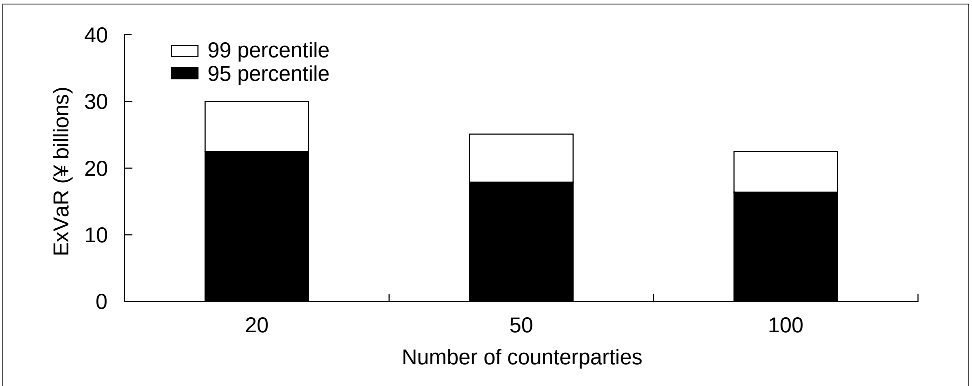
We see the two effects in turn below, as well as the effect of the volatility of default probabilities on the risk amount.

1. Diversification/concentration in terms of counterparties

The three portfolios used here are composed of 100, 50, and 20 loans, respectively, each of which has a principal of ¥1 billion, ¥2 billion, and ¥5 billion, respectively, and is given to different counterparties with the same default probability in the same industry (#2). Looking at the results shown in Figure 6, we see that the ExVaR decreases as the number of counterparties in a portfolio increases. This phenomenon reflects the effect of diversification/concentration in terms of counterparties.

Figure 6 Number of Counterparties (Diversification) and ExVaR

Portfolio	Number of counterparties	Principal (¥ billions)	ExPV (¥ billions)	95 percentile ExVaR (¥ billions)	99 percentile ExVaR (¥ billions)
#2-1	100	1.0	93.0	16.4	22.5
#2-2	50	2.0	93.0	17.9	25.1
#2-3	20	5.0	93.4	22.5	30.0

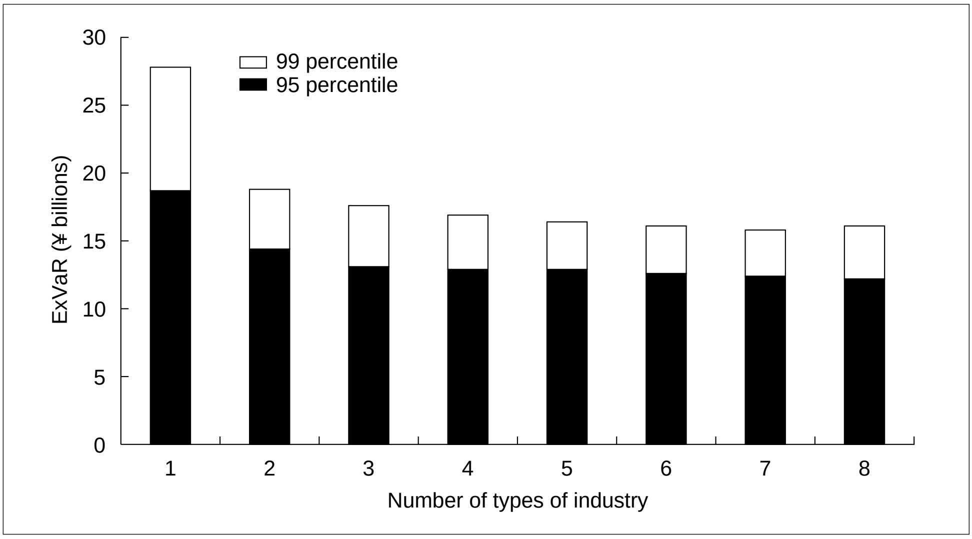


2. Diversification/concentration in terms of attributes (types of industry) of counterparties

The eight portfolios used here are composed of 100 loans with a principal of ¥10 billion, each of which is given to 100 counterparties, where the number of types of industry of counterparties is 1, 2, . . . 8 in the respective portfolios. Looking at the results shown in Figure 7, we see that the ExVaR decreases as the number of types of industry of counterparties in a portfolio increases. This reflects the effect of diversification/concentration in terms of type of industry of counterparties.

Figure 7 Number of Types of Industry (Diversification) and ExVaR

Portfolio	Number of types of industry	Number of counterparties in an industry	Principal of loan (¥ billions)	ExPV (¥ billions)	95 percentile ExVaR (¥ billions)	99 percentile ExVaR (¥ billions)
#2-4	1	100	1.0	93.3	18.7	27.8
#2-5	2	50	1.0	92.9	14.4	18.8
#2-6	3	33, 34	1.0	92.7	13.1	17.6
#2-7	4	25	1.0	92.8	12.9	16.9
#2-8	5	20	1.0	92.9	12.9	16.4
#2-9	6	16, 17	1.0	92.8	12.6	16.1
#2-10	7	14, 15	1.0	92.7	12.4	15.8
#2-11	8	12, 13	1.0	92.9	12.2	16.1



3. The effect of the volatility of default probabilities on risk

We compare the ExVaR using the historical volatility (HV) of default probabilities with the ExVaR using the doubled HV. Looking at the results shown in Table 10, we see that (1) the ExVaR increases as the volatility increases; and (2) the degree of increase is moderate when the portfolio is diversified in terms of type of industry of counterparties.

Table 10 Volatility of Default Probabilities and ExVaR

Portfolio	Number of types of industry	Number of counterparties in an industry	Principal of loan (¥ billions)	95 percentile ExVaR (¥ billions)		99 percentile ExVaR (¥ billions)	
				HV × 1	HV × 2	HV × 1	HV × 2
#2-1	1	100	1.0	16.4	30.1	22.5	49.7
#2-5	2	50	1.0	14.4	23.2	18.8	33.8
#2-7	4	25	1.0	12.9	17.7	16.9	23.9
#2-11	8	12, 13	1.0	12.2	14.7	16.1	19.5

C. Integration/Separation of Credit and Market Risk Evaluation

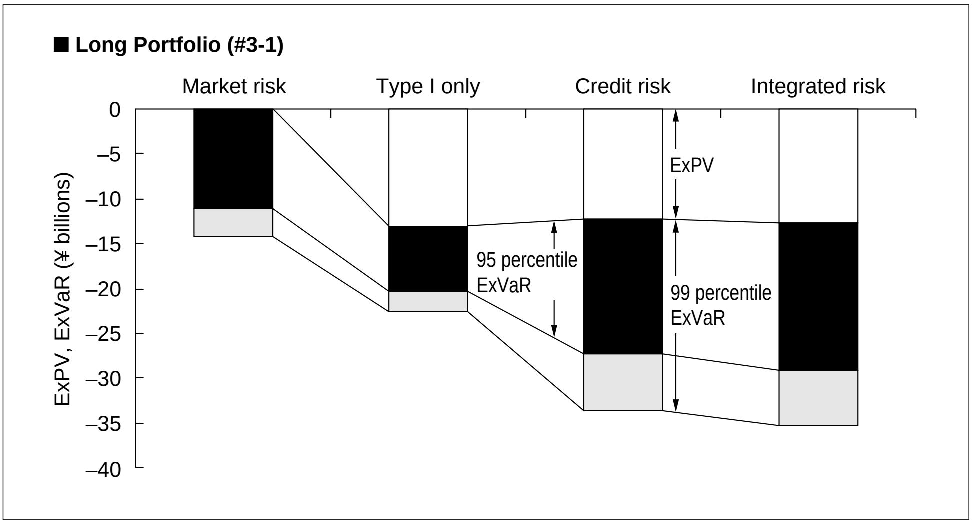
All the ExVaR outputs thus far include both credit risk (type I and II) and market risk. Such a treatment is desired for integrating the overall risk in a portfolio. However, there may be some demands for separate evaluation of credit risk and/or market risk. Thus, in this section, we calculate the risks separately for a loan portfolio and for a swap portfolio. To isolate credit risk, simulations are carried out on the hypothetical condition that the principal component vectors representing yield curve volatility are all zero. To isolate market risk, simulations are executed on the hypothetical condition that the default probabilities of counterparties are all zero and the volatilities of default probabilities are also zero. These conditions are summarized in Table 11. The results of the ExVaR calculation are shown in Figure 8.

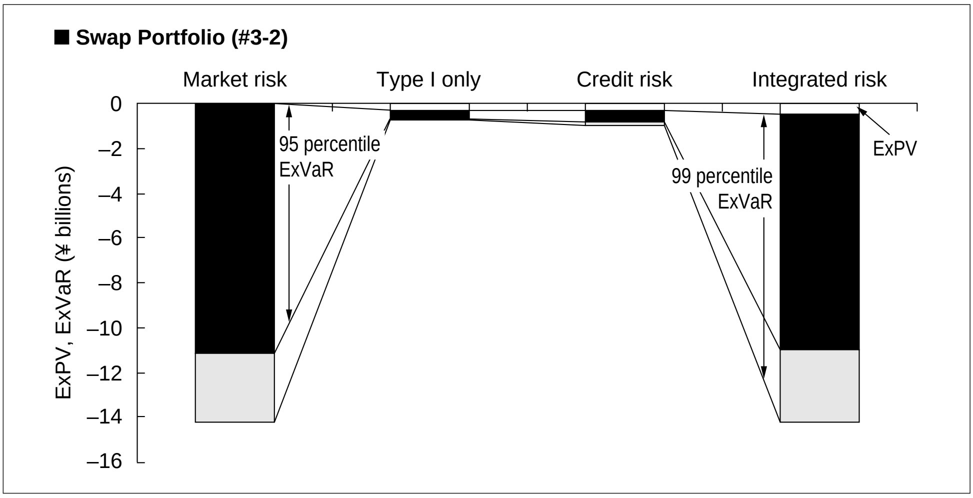
Table 11 Risks and Risk Factors

Risk factor	Credit risk		Market risk	Integrated risk
	Type I only			
Default probability (DP)	Yes	Yes	No	Yes
Volatility of DP	No	Yes	No	Yes
Yield curve (YC)	Yes	Yes	Yes	Yes
Volatility of YC	No	No	Yes	Yes

Figure 8 Separate Calculation of Credit and Market Risks

Portfolio	Transaction category	Risk factor	ExPV (¥ billions)	95 percentile ExVaR (¥ billions)	99 percentile ExVaR (¥ billions)
#3-1	Loan	Market risk	105.8	11.2	14.3
		Credit risk	93.4	14.9	21.3
		Type I only	92.7	7.1	9.4
		Integrated risk	93.0	16.4	22.5
#3-2	Swap	Market risk	5.8	11.2	14.3
		Credit risk	5.4	0.4	0.6
		Type I only	5.4	0.3	0.4
		Integrated risk	5.3	10.5	13.7





In accordance with the definitions in Section III. A., each risk in a loan portfolio and a swap portfolio can be described by the above figure.

Looking first at the risk profile of the loan portfolio, we see a distinct difference between total credit risk (type I and type II) and type I credit risk alone. As shown in Section II. A., type I credit risk is caused by the incompleteness of diversification of the portfolio. In other words, an ideally diversified portfolio does not have such risk. Hence, the type I credit risk observed here results from the fact that the number of transactions in a portfolio is finite (i.e., 100), and this is interpreted as concentration risk. On the other hand, type II credit risk is caused by the uncertainty of future default probability changes, which are independent of the diversification/concentration effect. Total credit risk is type I plus type II credit risk. The integrated risk simultaneously evaluates total credit risk and market risk, which is generally different from the simple sum of the individually evaluated total of credit risk and market risk.

On the other hand, looking at the risk profile of a swap portfolio, whose transactions have the same notional amount as the principal of the above loan, we see that the credit risk is much smaller than that of the loan portfolio. This reflects the fact that the expected future value of a swap position is far smaller than its notional amount. We also see that the integrated risk is slightly smaller than the individually evaluated interest rate risk. Although this feature is not common to every swap portfolio, such a case is probable. To understand the phenomenon intuitively, we can think of the possibility that a certain interest rate path could raise a large loss near maturity in case of non-default, but the interest rate risk could not be realized when the transaction is closed before maturity due to the default of the counterparty.

D. The Effect of Collateral

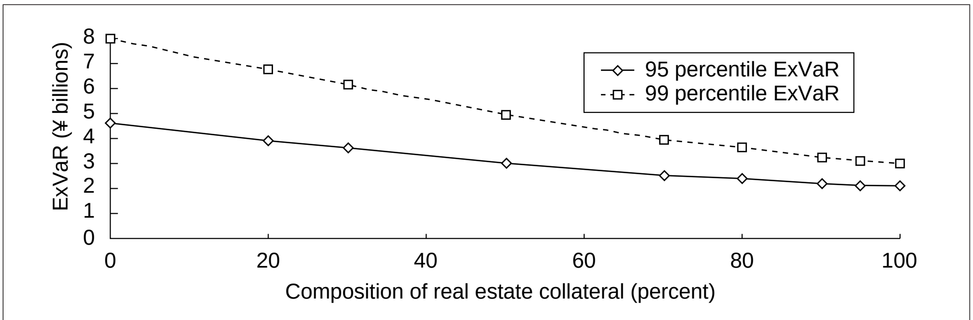
All the portfolios we have discussed thus far are composed only of unsecured transactions. In this section, we investigate the effect of collateral on price (ExPV) and risk (ExVaR). To focus on the collateral effect, we only calculate credit risk in this section, excluding interest rate risk.

1. Risk evaluation by type of collateral asset

We consider real estate and equity as the collateral assets in this paper, assuming that they vary subject to lognormal stochastic processes with a correlation between them. We set up 10 test portfolios composed of 100 loans, each of which has either real estate collateral or equity collateral. The only difference between the portfolios is the ratio of the two types of collateral loans. We calculate the ExPV and ExVaR of a portfolio composed only of real estate collateral loans, a portfolio composed only of equity collateral loans, portfolios composed of both real estate collateral loans and equity collateral loans, and a portfolio composed only of unsecured loans. The results are shown in Figure 9.

Figure 9 Risk Amount by Type of Collateral Asset

Portfolio	Non-collateral (number of loans)	Real estate collateral (number of loans)	Equity collateral (number of loans)	ExPV (¥ billions)	95 percentile ExVaR (¥ billions)	99 percentile ExVaR (¥ billions)
#4-1	100	—	—	93.0	14.9	21.3
#4-2	—	0	100	103.7	4.6	7.9
#4-3	—	20	80	103.7	3.9	6.7
#4-4	—	30	70	103.8	3.6	6.1
#4-5	—	50	50	103.8	3.0	4.9
#4-6	—	70	30	103.9	2.5	3.9
#4-7	—	80	20	103.9	2.4	3.6
#4-8	—	90	10	104.0	2.2	3.2
#4-9	—	95	5	104.0	2.1	3.1
#4-10	—	100	0	104.0	2.1	3.0



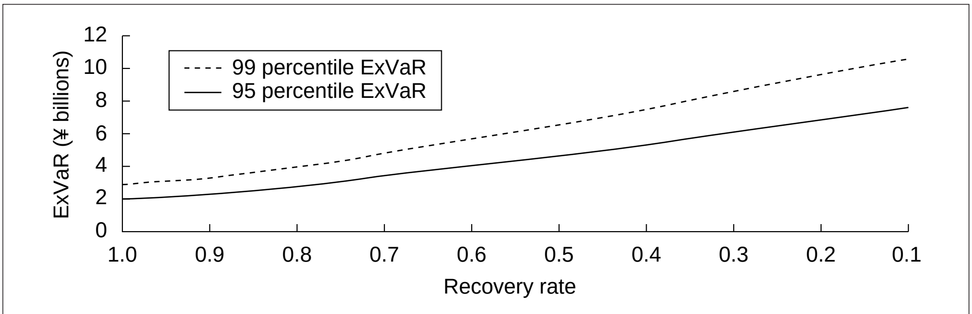
Looking at the results, we see that the ExPV of a portfolio composed of collateral loans is larger than that of a portfolio composed of non-collateral loans. This is natural since the credit enhancement effect of collateral increases the value of loans (in other words, it decreases the credit risk premiums). We also see that the ExVaR of a portfolio composed of collateral loans is much smaller than that of a portfolio composed of non-collateral loans and that the ExVaR of a portfolio composed of real estate collateral loans is smaller than that of a portfolio composed of equity collateral loans. The latter reflects the fact that the volatility of real estate is smaller than that of equities as shown in Table 4 before. In addition, we observe, in Figure 9, slight convexity in the curve that represents the ExVaR against the ratio of the two types of collateral loans in a portfolio. This can be interpreted as the effect of diversification in collateral assets.

2. The effect of recovery rates

All the risks have been calculated thus far with the unsecured and secured asset recovery rates fixed at 0.0 and 0.9, respectively, for all transactions. In reality, the appropriate recovery rates depend on the features of individual transactions. Although a strict treatment that includes such a dependency may be desired for exact risk measurement, the use of constant rates for similar transactions, as in this paper, seems sufficient for an approximate risk estimate. In this case, we need to know how sensitive the resulting risk is to changes in the recovery rate settings. Figure 10 shows the sensitivity of the ExPV and ExVaR to the secured asset recovery rate. It suggests a large influence of the recovery rate on the estimation. For example, 99 percentile ExVaR increases by approximately 50 percent (from ¥3.1 billion to ¥4.8 billion) as the recovery rate is lowered from 0.9 to 0.7.

Figure 10 Secured Asset Recovery Rate and ExVaR

Recovery rate	ExPV (¥ billions)	95 percentile ExVaR (¥ billions)	99 percentile ExVaR (¥ billions)
1.0	51.4	2.0	2.9
0.9	50.6	2.2	3.1
0.8	50.6	2.7	3.8
0.7	49.5	3.4	4.8
0.6	49.0	4.0	5.6
0.5	48.4	4.6	6.5
0.4	47.9	5.3	7.5
0.3	47.4	6.0	8.5
0.2	46.9	6.8	9.6
0.1	46.5	7.6	10.6



V. Practical Applications

In Chapter IV., we confirmed the basic characteristics of the ExVaR by calculating it for a number of portfolios under various conditions. In this chapter, we investigate the applicability of the ExVaR for practical use in the risk management of financial institutions.

A. Scenario Analysis and Stress Test on Movement of Real Estate Prices

In this section, we study, from a technical point of view, whether the deterioration of Japanese financial institutions' assets after the collapse of the "bubble" could have been reduced if credit risk monitoring by such a measure as the ExVaR had prevailed before. The issue is also interpreted as a verification of the ExVaR's ability to forecast the degree of possible deterioration of future assets. For simplification, we focus on real estate among the items influenced by the "bubble." We analyze the influence of the decrease in real estate prices on the risk of loans with real estate collateral and on the risk of loans to the real estate industry.

First, four hypothetical portfolios are set up as follows. Each of them is composed of 100 loans with a real estate collateral. The portfolios differ in two ways: (1) the types of industry of counterparties included are diversified or concentrated only in the real estate industry; and (2) the current value of the collateral of each loan is lower[21] or higher than the principal of the loan. The details of the four portfolios are shown in the Appendix. Next, as shown in Table 12, five scenarios are assumed showing a stochastic process (i.e., trend and volatility) of the real estate collateral price and of the default probability for the real estate industry. The ExVaR with scenario #2, in which all the parameters are based on the historical data, corresponds to the common approach to evaluating risk by the use of the past information. On the other hand, the ExVaR with other scenarios (#1, #3, #4, and #5), which includes some subjective factors, is interpreted as so-called scenario analysis, or as a stress test if the hypothesis is extreme.

Figure 11 shows the ExPV and 99 percentile ExVaR (only the credit risk) for the four portfolios under the five scenarios. In the bar diagram, the ExVaR is shown by the length of the bar, of which the upper end and the lower end correspond to the

Table 12 Scenarios to Be Analyzed

	Land price		Default probability of real estate industry		Characteristics of scenario
	Trend (per annum)	Volatility (per annum)	Trend (per annum)	Volatility (per annum)	
Scenario #1	0.20	0.04*	—	0.50*	Bull market in real estate (RE) prices
Scenario #2	0.05*	0.04*	—	0.50*	Neutral market in RE prices
Scenario #3	–0.10	0.20	—	0.50*	Bear market in RE prices
Scenario #4	–0.10	0.20	0.20	1.00	Bear market in RE prices; depression in RE industry
Scenario #5	–0.10	0.20	0.50	1.00	Bear market in RE prices; serious depression in RE industry

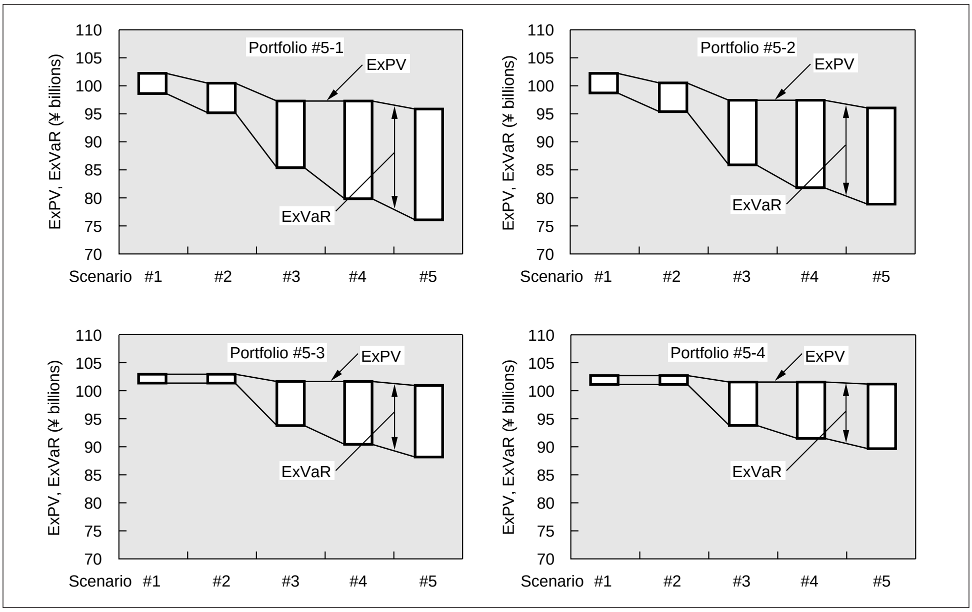
Note: The asterisk (*) indicates the value to be estimated from historical data.

Figure 11 Results of the Scenario Analysis

¥ billions

Portfolio	Characteristic of portfolio	Scenario #1		Scenario #2		Scenario #3		Scenario #4		Scenario #5	
		ExPV	ExVaR	ExPV	ExVaR	ExPV	ExVaR	ExPV	ExVaR	ExPV	ExVaR
#5-1	Concentrated; insufficient collateral	102.1	3.2	100.6	5.2	97.3	11.4	97.3	16.8	96.0	19.2
#5-2	Diversified; insufficient collateral	102.1	3.1	100.5	5.1	97.2	11.3	97.1	15.3	96.0	17.2
#5-3	Concentrated; collateral rule observed	103.0	1.9	103.0	1.9	101.7	8.0	101.6	11.1	101.0	12.9
#5-4	Diversified; collateral rule observed	103.0	1.9	103.0	1.9	101.7	8.0	101.6	10.2	101.1	11.5

21. Actually, it is said that, during the "bubble" period in Japan, some financial institutions made real estate collateral loans of which the principal was larger than the value of the real estate collateral, expecting a future increase in collateral value.



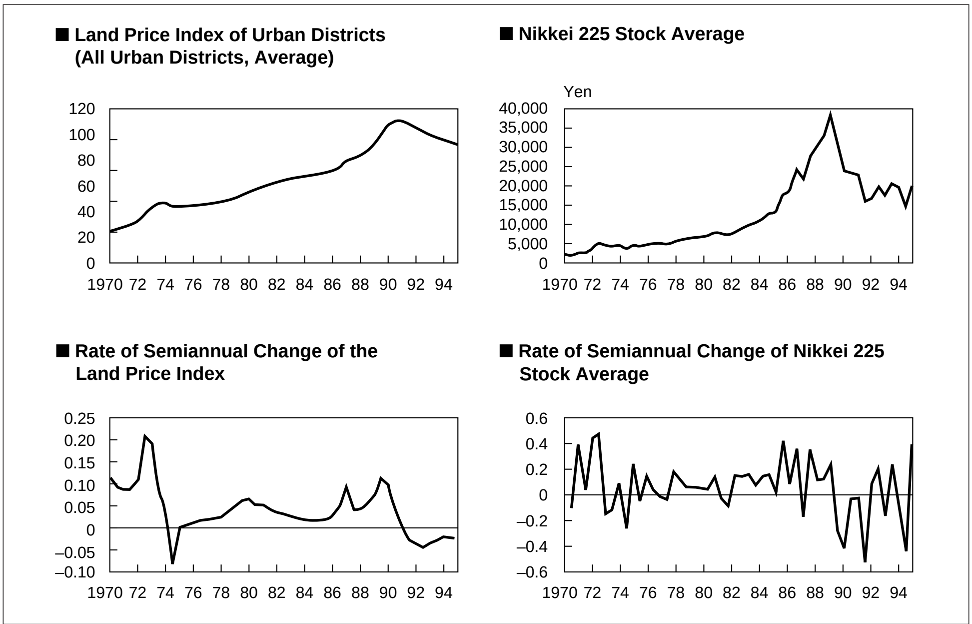
ExPV and the 99 percentile minimum present value, respectively. Looking at the result for scenario #2 first, the portfolios (#5-3 and #5-4) with the collateral rule observed[22] have relatively small risk (¥1.9 billion), disregarding the factor of diversification/concentration. The portfolios (#5-1 and #5-2) with collateral of less value have relatively large risk (approximately ¥5 billion) in comparison with portfolios #5-3 and #5-4, but the absolute level of risk can be recognized as not too large, since the risk is limited within about 5 percent of the ExPV.[23] Such an analysis based only on historical data ends up with an optimistic result.

Next, the ExVaR is analyzed for the subjective scenarios (#1, #3, #4, and #5). Scenario #3 puts a negative trend on the process of the real estate price and sets the volatility higher than in the past. With this scenario, the ExVaR of portfolios #5-1 and #5-2 are almost double the ExVaR with scenario #2 and the ExVaR of portfolios #5-3 and #5-4 are almost four times the ExVaR with scenario #2. Such results expose the weakness of loan management that depends excessively on real estate collateral. Moreover, scenarios #4 and #5 allow the default probability of the real estate industry to be higher, assuming a depression in the industry, in addition to the same decrease in real estate prices as with scenario #3. With these scenarios, the ExVaR outputs for all five portfolios are higher than those in any former scenario. On the absolute level of risk, the ExVaR outputs are also evaluated as extremely high. For example, even the ExVaR outputs for portfolios #5-3 and #5-4, which have conservative collateral, are larger than 10 percent of the ExPV of the respective portfolios.

22. The collateral rule here is assumed to prescribe that the principal of a loan should not be more than the collateral value times the fixed multiplier, which is set at 0.8 in this paper.
23. The portfolio of which the ExVaR is about 5 percent of the value (ExPV) can be recognized as satisfactory in terms of the risk/value ratio if we assume, for example, that the BIS capital adequacy requirement implies a potential loss of 8 percent of the asset value in a financial institution.

Looking just at the above results, it seems that the ExVaR which is mechanically calculated based on the upward-sloping real estate historical prices could hardly have given us any warning of the possible impact of the collapse of the "bubble." This temporary implication results partly from the inappropriate assumption that the real estate price follows a lognormal stochastic process. The historical data in Figure 12 show that the usual lognormal process does not fit the movement of the real estate index sufficiently in two ways: (1) the price movement in reality has been very smooth along the trend; and (2) the price experienced a structural change, that is, the sudden change of the trend at the collapse of the "bubble." Hence, applying the lognormal process to real estate price data could lead to the underestimation of the risk of a portfolio that is sensitive to the real estate price, since the risk of structural change cannot be taken into account and the volatility of the price is estimated as relatively small.[24] To overcome the problem, it is necessary to describe the real estate price movement more accurately by means of a model with a "bubble"-generating mechanism in it. This issue remains to be resolved. It would be interesting to study whether the improved model for real estate prices enables the ExVaR to forecast objectively the risk of the "bubble" collapse even from the historical data alone. The important implication from the above analysis is that, if one had intended to investigate the effect of a possible stress in the real estate market, one could have realized the impact by use of the ExVaR. Generally speaking, since the ExVaR measures risk in normal situations, it is desirable to complement the ExVaR analysis with such tools as a stress test, as is the case with the traditional VaR for market risk.

Figure 12 Real Estate Price Index, Stock Price Index, and Their Rate of Return



24. The volatility of the real estate price (the Land Price Indexes of Urban Districts) is estimated to be 3.9 percent, which is much smaller than that of the stock price (Nikkei 225 Stock Average), which is estimated to be 21.1 percent.

B. Theoretical Spread Required in a New Transaction

In this section, a procedure is shown for the theoretical setting of the interest rate standard, which may be called "interest rate guideline" or "transfer rate," for a new loan. The required risk premium is calculated in accordance with the marginal increase in the ExVaR and is taken into account in determining the interest rate standard.

First, it should be noted again that credit risk is largely influenced by the so-called portfolio effect, such as the diversification effect. Hence, the marginal effect of a new loan on the existing portfolio should be evaluated, instead of the independent effect of the single loan, in measuring the ExPV or the ExVaR.

Let us assume the case, as an example of the theoretical determination of the interest rate standard, that a new loan is added to an existing portfolio (#6-1) composed of 99 identical loans, where a new loan has the same conditions except for the counterparty's credit rating and collateral. The issue is to determine the interest rate to be set for the new loan. Since the initial cash outflow for the new loan is ¥1 billion, the principal, we should find the interest rate at which the loan marginally increases the value of the portfolio by exactly ¥1 billion. Figure 13 shows the solution, as the risk-unadjusted interest rate standard, calculating the ExPV in the range of 3 percent to 10 percent interest rates and plotting them in diagrams. The resulting rates are effective only on the assumption that the financial institution is risk-neutral.

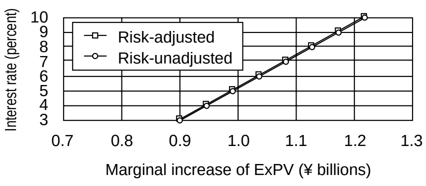
Since the actual financial institution is risk-averse, however, the risk premium should be added to the risk-unadjusted interest rate standard. This paper considers an approach for the determination of the risk premium as follows. It is assumed that the financial institution recognizes the risk of a portfolio by the ExVaR with the specific confidence level that the financial institution thinks is optimal according to its own risk preference. For example, the more risk-averse it is, the higher confidence level it intends to measure the ExVaR with. The financial institution needs additional capital, of which the amount is equal to the marginal increase in the ExVaR, for a new loan to maintain the soundness of its business. The cost of capital is generally more expensive than that of other liabilities. We adopt a method, from among alternatives, to estimate the cost of capital, which we define as the notional cost minus the risk-free rate, as the market risk premium multiplied by the equity beta of the financial institution.[25] The idea is based on the capital asset pricing model (CAPM). In this paper, we assume *a priori* that the market risk premium is 6.0 percent, the equity beta is 1.0, and, as a result, the cost of capital (i.e., the required rate of return on capital) is 6.0 percent. This means that the annual cost of maintaining capital sufficient for the risk is given as 6.0 percent of the ExVaR. Hence, the required spread as a risk premium to a new loan is 6.0 percent of the marginal ExVaR divided by the principal of the loan. By adding this spread to the risk-unadjusted interest rate standard, we calculate the risk-adjusted interest rate standard, which is also shown in Figure 13. This can be interpreted as the minimum requirement for the interest rate of a new loan so that the loan's expected loss and risk are compensated with the interest income. In this sense, a financial institution can substitute the interest rate standard for the traditional transfer rate between the ALM section and the branches.

25. See, for example, Copeland, Koller, and Murrin (1990) on the method of estimating the cost of capital.

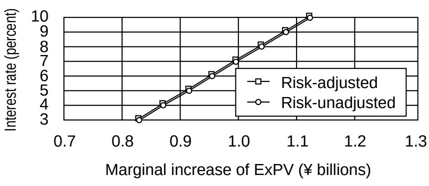
Figure 13 An Example of Calculations of the Interest Rate Standard

Existing portfolio		A new loan			Marginal increase of ExPV (¥ billions)	99 percentile ExVaR		Interest rate standard (percent)	
	Contents	Credit rating (default probability)	Collateral	Interest rate (percent)		Marginal increase (¥ billions)	Cost of capital (¥ billions)	Risk-unadjusted	Risk-adjusted
#6-1	99 loans to industry #2; credit rating: 6 (default probability 3 percent); interest rate: 5 percent	Rating: 4 (1 percent)	None	3	0.90	0.13	0.008	5.18	5.26
				4	0.95	0.13	0.008		
				5	0.99	0.13	0.008		
				6	1.04	0.13	0.008		
				7	1.08	0.13	0.008		
				8	1.13	0.13	0.008		
				9	1.17	0.13	0.008		
				10	1.22	0.13	0.008		
		Rating: 6 (3 percent)	None	3	0.83	0.17	0.010	7.13	7.27
				4	0.87	0.18	0.011		
				5	0.91	0.19	0.011		
				6	0.95	0.21	0.012		
				7	0.99	0.23	0.014		
				8	1.04	0.23	0.014		
				9	1.08	0.23	0.014		
				10	1.12	0.24	0.014		
		Rating: 8 (5 percent)	None	3	0.77	0.41	0.024	8.66	8.94
				4	0.81	0.43	0.026		
				5	0.85	0.44	0.026		
				6	0.89	0.44	0.026		
				7	0.93	0.45	0.027		
				8	0.97	0.46	0.028		
				9	1.01	0.47	0.028		
				10	1.05	0.48	0.029		
		Rating: 6 (3 percent)	Real estate	3	0.94	0.17	0.010	4.34	4.43
				4	0.98	0.16	0.010		
				5	1.03	0.16	0.010		
				6	1.07	0.15	0.009		
				7	1.11	0.15	0.009		
				8	1.16	0.15	0.009		
				9	1.20	0.15	0.009		
				10	1.24	0.14	0.009		

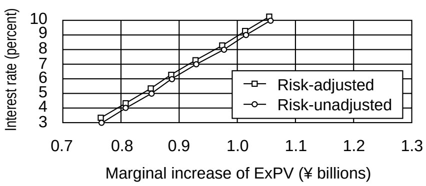
■ **Rating 4, Non-Collateral**



■ **Rating 6, Non-Collateral**



■ **Rating 8, Non-Collateral**



■ **Rating 6, Collateralized**

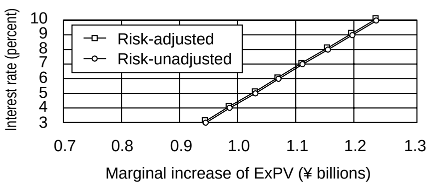


Figure 13 assumes an existing portfolio composed of 99 identical loans and considers the addition of a new loan to the portfolio. As a new loan, four types of loan are set up that differ from each other in credit rating or the presence of collateral. Looking at the results, we see that the interest rate standard for the loan with collateral is lower than that for the loan without collateral. We also find the distinct characteristic that the worse the credit rating of a new loan, the higher the interest rate standard for the loan. While these tendencies may be obvious without any calculation for most financial institutions, few financial institutions appear to analyze the issue for their operation as quantitatively and logically as proposed here. We admit that real business is so complex that such a theory is not always effective. Still, we should emphasize the importance of using some logic as the basis for practical operations.

C. Applicability of ExVaR to the Asset Allocation Strategy in Financial Institutions

Such an index as the ratio of the realized, or the expected, rate of return to the risk of a portfolio can be effective for a financial institution in managing its capital allocation strategy. A typical example of such an index is the so-called risk-adjusted return on capital (RAROC). If a universal bank, whose business ranges from retail banking to investment banking, adopts this type of performance index, it needs to evaluate the risk of different types of business on the same scale. For this purpose, the ExVaR can function effectively, since its coverage of type of risk is broader—including credit risk (type I and II) and market risk—than the traditional VaR, which is only for market risk. For example, the ExVaR can provide meaningful information for a financial institution to pursue an optimal business structure by comparing the risk-adjusted performance of retail banking with that of investment banking.

This performance index, which is applicable to different types of risk on the same scale, not only realizes the mutual comparison of these risks but also accurately measures the total risk held in a financial institution. This enables the management to evaluate clearly the soundness of the business and to take up a business opportunity effectively (in other words, appropriate risk taking). Moreover, improvement of disclosure is also expected from the standpoint of equity holders and investors.

VI. Conclusion

This paper proposed a new framework, which we call the ExVaR, for the integrated evaluation of both credit and market risk. Through the calculation of the risk of hypothetical test portfolios, basic characteristics of the ExVaR were shown and its applicability was discussed.

In the conclusion, the results of this paper and the remaining issues are summarized.

A. Results of the Paper

[1] The ExVaR evaluates both the credit and market risks of a portfolio on the same scale, taking into account the effect of the liquidity of individual transactions in

the portfolio. As a result, the applicability of the ExVaR is broader than that of the traditional VaR.

[2] The ExVaR quantitatively reflects the effects of diversification/concentration and collateral, both of which are unavoidable issues in the measurement of credit risk.

[3] The framework of the ExVaR can generally be applied to any type of financial transaction, including derivative products.

[4] The limits of the ExVaR are discussed and the need for scenario analyses or stress tests is suggested.

B. Remaining Issues

[1] Implementation of the ExVaR to a real large-sized portfolio held by a financial institution should be realized. There are such issues as the effective sampling or grouping of transactions, which substitutes for the input of the data for individual transactions, and enables computation with less of a burden than the normal Monte Carlo simulation.

[2] The default probability movement model should be improved. For example, factor analysis with macroeconomic variables or a model including lagged variables, such as ARIMA, could modify the simple mechanism of the lognormal process assumed in the paper.

[3] The effective attributes by which to define homogeneous firms with respect to default probability movements should be studied empirically and theoretically.

[4] The real estate price movement model should be improved over and above the simple lognormal process assumed in the paper.

[5] Individual issues such as the effect of guarantees, the treatment of subordinated assets, and the estimation of recovery rates should be tackled.

[6] The method of pricing defaultable assets should be improved by implementing some theoretical pricing models, rather than by an approximation with the expected value of discounted future cash flows from the asset.

The ExVaR is a flexible and comprehensive framework for risk measurement. This means that one needs to investigate carefully how to place the individual building blocks of the model for implementation. On the other hand, there is no single optimal method of measuring the risk that is applicable to every case, since the method should be different as the purposes of measurement or the management strategy of the institution differ. Hence, it is important to interpret the concept of the ExVaR as a framework and to make settings that are appropriate to the situation.

Appendix: List of Test Portfolios Used for the ExVaR Calculation

IV. A. Integration of On-Balance and Off-Balance Sheet Transactions with the Risk Evaluation Period Set Based on the Transaction's Liquidity

Portfolio #1-1

Transaction	Number of counterparties	Ind. #I	II	III	IV	V	VI	VII	VIII	Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral	Value	Limit	Preempt.
Loan	100		100							3	1.0	5	5	5	None			

Portfolio #1-2

Transaction	Number of counterparties	Ind. #I	II	III	IV	V	VI	VII	VIII	Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral	Value	Limit	Preempt.
Debenture	100		100							3	1.0	5	5	1	None			

Portfolio #1-3

Transaction	Number of counterparties	Ind. #I	II	III	IV	V	VI	VII	VIII	Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral	Value	Limit	Preempt.
Swap (pay-fix)	100		100							3	1.0	5	5	5	None			

Portfolio #1-4

Transaction	Number of counterparties	Ind. #I	II	III	IV	V	VI	VII	VIII	Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral	Value	Limit	Preempt.
Loan	100		100							3	1.0	5	5	5	None			
Debenture	100		100							3	1.0	5	5	1	None			

Portfolio #1-5

Transaction	Number of counterparties	Ind. #I	II	III	IV	V	VI	VII	VIII	Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral	Value	Limit	Preempt.
Loan	100		100							3	1.0	5	5	5	None			
Swap (pay-fix)	100		100							3	1.0	5	5	5	None			

Portfolio #1-6

Transaction	Number of counterparties	Ind. #I	II	III	IV	V	VI	VII	VIII	Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral	Value	Limit	Preempt.
Debenture	100		100							3	1.0	5	5	1	None			
Swap (pay-fix)	100		100							3	1.0	5	5	5	None			

IV. B. The Effect of Diversification/Concentration

Portfolio #2-1

Transaction	Number of counterparties	Type I	II	III	IV	V	VI	VII	VIII	Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral	Value	Limit	Preempt.
Loan	100		100							3	1.0	5	5	5	None			

Portfolio #2-2

Transaction	Number of counterparties	Type I	II	III	IV	V	VI	VII	VIII	Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral	Value	Limit	Preempt.
Loan	50		50							3	2.0	5	5	5	None			

Portfolio #2-3

Transaction	Number of counterparties	Type I	II	III	IV	V	VI	VII	VIII	Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral	Value	Limit	Preempt.
Loan	20		20							3	5.0	5	5	5	None			

Portfolio #2-4

Transaction	Number of counterparties	Type I	II	III	IV	V	VI	VII	VIII	Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral	Value	Limit	Preempt.
Loan	100	100								3	1.0	5	5	5	None			

Portfolio #2-5

Transaction	Number of counterparties	Type I	II	III	IV	V	VI	VII	VIII	Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral	Value	Limit	Preempt.
Loan	100	50	50							3	1.0	5	5	5	None			

Portfolio #2-6

Transaction	Number of counterparties	Type I	II	III	IV	V	VI	VII	VIII	Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral	Value	Limit	Preempt.
Loan	100	33	33	34						3	1.0	5	5	5	None			

Portfolio #2-7

Transaction	Number of counterparties	Type I	II	III	IV	V	VI	VII	VIII	Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral	Value	Limit	Preempt.
Loan	100	25	25	25	25					3	1.0	5	5	5	None			

Portfolio #2-8

Transaction	Number of counterparties	Type I	II	III	IV	V	VI	VII	VIII	Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral	Value	Limit	Preempt.
Loan	100	20	20	20	20	20				3	1.0	5	5	5	None			

Portfolio #2-9

Transaction	Number of counterparties	Type I	II	III	IV	V	VI	VII	VIII	Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral	Value	Limit	Preempt.
Loan	100	16	16	17	17	17	17			3	1.0	5	5	5	None			

Portfolio #2-10

Transaction	Number of counterparties	Type I	II	III	IV	V	VI	VII	VIII	Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral	Value	Limit	Preempt.
Loan	100	14	14	14	14	14	15	15		3	1.0	5	5	5	None			

Portfolio #2-11

Transaction	Number of counterparties	Type I	II	III	IV	V	VI	VII	VIII	Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral	Value	Limit	Preempt.
Loan	100	12	12	12	12	13	13	13	13	3	1.0	5	5	5	None			

IV. C. Integration/Separation of Credit and Market Risk Evaluation

Portfolio #3-1

Transaction	Number of counterparties	Type I	II	III	IV	V	VI	VII	VIII	Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral	Value	Limit	Preempt.
Loan	100		100							3	1.0	5	5	5	None			

Portfolio #3-2

Transaction	Number of counterparties	Type I	II	III	IV	V	VI	VII	VIII	Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral	Value	Limit	Preempt.
Swap (pay-fix)	100		100							3	1.0	5	5	5	None			

IV. D. The Effect of Collateral

Portfolio #4-1

Transaction	Number of counterparties	Type I	II	III	IV	V	VI	VII	VIII	Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral	Value	Limit	Preempt.
Loan	100		100							3	1.0	5	5	5	None			

Portfolio #4-2

Transaction	Number of counterparties	Type I	II	III	IV	V	VI	VII	VIII	Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral	Value	Limit	Preempt.
Loan	100		100							3	1.0	5	5	5	RE	1.0	1.0	0

Portfolio #4-3

Transaction	Number of counterparties	Type I	II	III	IV	V	VI	VII	VIII	Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral	Value	Limit	Preempt.
Loan	100		100							3	1.0	5	5	5	Equity	1.0	1.0	0

Portfolio #4-4

Transaction	Number of counterparties	Type I	II	III	IV	V	VI	VII	VIII	Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral	Value	Limit	Preempt.
Loan	50		50							3	1.0	5	5	5	RE	1.0	1.0	0
Loan	50		50							3	1.0	5	5	5	Equity	1.0	1.0	0

Portfolio #4-5

Transaction	Number of counterparties	Type I	II	III	IV	V	VI	VII	VIII	Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral	Value	Limit	Preempt.
Loan	80		80							3	1.0	5	5	5	RE	1.0	1.0	0
Loan	20		20							3	1.0	5	5	5	Equity	1.0	1.0	0

Portfolio #4-6

Transaction	Number of counterparties	Type I	II	III	IV	V	VI	VII	VIII	Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral	Value	Limit	Preempt.
Loan	50		50							3	1.0	5	5	5	RE	1.0	1.0	0

V. A. Scenario Analysis and Stress Test on Movement of Real Estate Prices

Portfolio #5-1

Transaction	Number of counterparties									Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral			
		Type I	II	III	IV	V	VI	VII	VIII							Value	Limit	Preempt.
Loan	50					50				3	1.0	5	5	5	RE	0.8	1.0	0
Loan	50					50				3	1.0	5	5	5	RE	1.0	1.0	0.5

Portfolio #5-2

Transaction	Number of counterparties									Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral			
		Type I	II	III	IV	V	VI	VII	VIII							Value	Limit	Preempt.
Loan	50	10	10	10	10	10				3	1.0	5	5	5	RE	0.8	1.0	0
Loan	50	10	10	10	10	10				3	1.0	5	5	5	RE	1.0	1.0	0.5

Portfolio #5-3

Transaction	Number of counterparties									Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral			
		Type I	II	III	IV	V	VI	VII	VIII							Value	Limit	Preempt.
Loan	100					100				3	1.0	5	5	5	RE	1.2	1.0	0

Portfolio #5-4

Transaction	Number of counterparties									Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral			
		Type I	II	III	IV	V	VI	VII	VIII							Value	Limit	Preempt.
Loan	100	20	20	20	20	20				3	1.0	5	5	5	RE	1.2	1.0	0

V. B. Theoretical Spread Required in a New Transaction

Portfolio #6-1

Transaction	Number of counterparties									Default probability (percent)	Principal (¥ billions)	Interest (percent)	Time to maturity (years)	Holding period (years)	Collateral			
		Type I	II	III	IV	V	VI	VII	VIII							Value	Limit	Preempt.
Loan	99		99							3	1.0	5	5	5	None			

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