# Indexed Bonds and Monetary Policy: The Real Interest Rate and the Expected Rate of Inflation

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*This paper presents a method for deriving the real interest rate and the expected rate of inflation from the market information contained in indexed government bonds. It also discusses the implications and potential use for monetary policy of the information derived about the real interest rate and the expected rate of inflation.*

*In theory, the real interest rate represents the marginal product of capital or the discount rate used in intertemporal market exchanges. Therefore, it acts to signal conditions in the real economy. The expected rate of inflation represents the average expectation of market participants about future inflation. Therefore, it affects the economic decisions of market participants. It contains information about the judgment of market players, which is useful as a leading indicator of the future price level.*

*This paper uses data on U.K. government-indexed bonds. It shows that the derived real interest rate and the expected rate of inflation provide very useful information for monetary policy. This paper also shows that the Fisher equation and the rational expectations hypothesis are incompatible, and that the expected rate of inflation obtained from the Fisher equation is far more stable than the realized rate of inflation and the expected rate of inflation obtained from the rational expectations hypothesis.*

Key words: Indexed bond; Real interest rate; Expected rate of inflation

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# I. Introduction

Indexed bonds are designed to ensure the real value of both principal and interest by linking them to the general price level. This paper presents a method for deriving the real interest rate and the expected rate of inflation from the market information contained in indexed government bonds. It also discusses the implications and potential use of the information derived about the real interest rate and the expected rate of inflation for monetary policy.

The real interest rate represents the marginal product of capital or the discount rate used in intertemporal market exchanges. Therefore, it acts as a signal of the conditions in the real economy. The expected rate of inflation represents the average expectation of market participants about future inflation. Therefore, it affects the economic decisions of market participants. It contains information about the judgment of market players, which is useful as a leading indicator of the future price level. Both the real interest rate and the expected rate of inflation provide information that would be useful for monetary policy that seeks to achieve and maintain macroeconomic and price stability.

At present, Japan has no market for indexed government bonds. However, if the government were to issue indexed bonds and they were traded freely in the market, such bonds would provide valuable information about the real interest rate and the expected rate of inflation on a daily basis. In view of the speedy availability and high objectivity of the data, indexed bonds would provide very useful information for monetary policy. In fact, many countries such as the United Kingdom, Canada, Sweden, and Israel have already created indexed government bond markets. Their experiences show that the issuance of indexed government bonds creates no problems in the financial market and causes no financial losses in the government budget. Moreover, Federal Reserve Board Chairman Alan Greenspan has said on several occasions that inflation-sensitive indexed bonds would be very beneficial for monetary policy, and the U.S. Treasury Department began issuing indexed U.S. Treasury bonds in January 1997.

This paper is organized as follows: Section II discusses the Fisher equation, which is employed to decompose the nominal interest rate into the real interest rate and the expected rate of inflation. In particular, it points out problems that would arise in using the Fisher equation in empirical studies, and shows that if we were to use the real interest rate obtained from indexed bonds, we could mitigate those problems. Section III explains the method for deriving theoretically the real interest rate and the expected rate of inflation. It emphasizes the importance of paying careful attention to the tax system, risk measure, and term structure of interest rates. Section IV calculates the interest rate and the expected rate of inflation from the data on the U.K. government-indexed bonds. Section V studies the implications of the derived real interest rate and the expected rate of inflation for monetary policy. In particular, it shows that the real interest rate does not follow a random walk and is stable, that inflation systematically affects the real interest rate, and that the expected rate of inflation leads actual inflation. Furthermore, the statistical test of the Fisher equation indicates that it is incompatible with the rational expectations hypothesis.

Section VI summarizes the main findings of this paper and suggests the direction for future research.

## II. The Usefulness and Limitations of the Fisher Equation

Irving Fisher decomposed the nominal interest rate into the real interest rate and the expected rate of inflation. He suggested that a rise in the expected rate of inflation leads to an increase in the nominal interest rate. The Fisher equation is usually given as follows:

$$R_t = r_t + \pi_t^e \tag{1}$$

where $R_t$ represents the nominal interest rate, $r_t$ the real interest rate, and $\pi_t^e$ the expected rate of inflation. The Fisher equation simply states that the nominal interest rate can be decomposed into two elements. It is equivalent to the idea that one can transform a nominal variable into a real variable using a price index as the deflator. Therefore, the Fisher equation is not a behavioral equation.

Although many empirical studies exist concerning the price level and the interest rate, no definite conclusion has been reached on the question of whether or not the Fisher equation holds in the actual economy. This is mainly because the right-hand side variables in equation (1) are not directly observable in the market, and therefore the test of the Fisher equation must usually assume the hypothesis of a constant real interest rate and the hypothesis of rational expectations. In other words, it is testing the joint hypothesis of (1) a constant real interest rate; (2) the rational expectations about inflation; and (3) the Fisher equation. As a result, if the test rejects the joint hypothesis, one cannot be sure which of the three hypotheses has actually been rejected.

To analyze this point further, let us assume that the Fisher equation holds and the nominal interest rate is stable. In this case, if the expected rate of inflation is changing, then it follows that the real interest rate is also changing negatively with respect to the expected rate of inflation. On the other hand, if the real interest rate is stable in the short run, as it reflects the marginal productivity of capital (according to neo-classical growth theory), then it follows that the expected rate of inflation is stable.

In short, when we attempt to distinguish between the real interest rate and the expected rate of inflation, neither of which is directly observable in the market, we must make an extra assumption about the behavior of one variable and then infer the behavior of the other variable. In this case, as we have seen, the behavior of the inferred variable can differ completely depending on the assumption we make.

In other words, all the empirical studies have made an arbitrary assumption about one unobservable variable and derived the behavior of the other unobservable variable, and then tested the Fisher equation. This is essentially a methodological tautology. We should have derived all the variables from market data and then tested the Fisher equation. To do so, we should have looked at equation (1) from a different perspective: that is, as an arbitrage condition between a nominal bond (which trades with the nominal interest rate) and an indexed bond (which trades with the real

interest rate).[1] Although previously this interpretation did not make sense because no indexed bond markets existed, many countries have begun to issue indexed government bonds, and it has become possible to use the real interest rate evaluated in the market. As a result, at least two of the three variables in the Fisher equation have become available. This makes it possible to reduce the degree of arbitrariness from that which existed in previous empirical studies.

## III. A Theoretical Derivation of the Real Interest Rate and the Expected Rate of Inflation

This paper proceeds under the assumption that arbitrage takes place between nominal bonds and indexed bonds. The basic strategy is that we can derive the expected rate of inflation from arbitrage condition (1) after obtaining the nominal and real interest rates from nominal and indexed bonds.

### A. Estimation of the Real and Nominal Interest Rates

In general, there are many forms of indexed bonds. In this section, I take the United Kingdom as an example to examine the characteristics of indexed bonds. The main characteristics of such bonds are that they ensure the maintenance of the real value (measured by a standard base-year index) of both principal and interest. The deflator that is used for this purpose is the price index eight months before actual interest payments become due. As interest is paid every six months, the price index that is employed for the next interest payment is already given. This system makes it possible for bondholders to sell at the correct discount even if the trade is conducted in between the interest payments. It implies that the most recent price index is assumed to be that of two months earlier. For example, even if the trade is conducted immediately after an interest payment, the next interest payment, which is due six months later, is already determined because the deflator that will be used is the price index eight months before the next interest payment and two months before the most recent payment. The gap between the real value and actual payments widens as the price index employed becomes older. Therefore, the deflator should be the most recently available price index, which turns out to be the price index of two months earlier.[2]

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1. In this case, we can regard the Fisher equation as a behavioral equation because investors now face a portfolio choice between the two kinds of government bonds. In a simple case, we can derive arbitrage equation (1) in the bond market from the condition that the present values of one unit each of the nominal bonds and the indexed bonds are equal when they are held for infinite periods. In this case, if investors are risk-averse, a risk premium for inflation should exist. It will be affected by changes in the expected rate of inflation and in the maturity. However, the risk premium is often ignored in Fisher equation (1) because it involves many difficult problems such as the degree of risk aversion and the estimation of inflation risk and because it is considered to be small relative to the real interest rate and the expected rate of inflation. This paper also ignores the risk premium. Nevertheless, it should be kept in mind that the risk premium will be important in the pricing of the indexed bonds in the issue market. In general, the risk premium will make the interest rate of nominal bonds higher than that of indexed bonds so that the government can save interest payments by issuing indexed bonds.
2. The newest price index may become available within a month. However, in the United Kingdom, it takes two months before the newest price index can be confirmed, as it is often revised during the first two-month period. Therefore, the price index of two months earlier is regarded as the most recently available price index in practice.

We can derive the real and nominal interest rates (yields) by solving the price equations for the nominal and indexed bonds with the same maturity and the same period to maturity under the assumption that the discounted present values of their market prices are identical. To be more specific: first, we obtain the nominal interest rate (yield) from the relationship between the discounted present value of the nominal bond and its market price. Second, we obtain the real interest rate from the indexed bond price equation, into which the obtained nominal interest rate is substituted. The solution method we employ is the numerical method, which seeks to determine the optimal value by changing the interest rate gradually while satisfying the equality between the market price and the discounted present value. Third, we can obtain the expected rate of inflation by substituting these real and nominal interest rates into the Fisher equation (Appendix). The above formulation and method are known to have several problems. Let me discuss these problems in turn.

### B. Taxes

Capital gains on the principal are tax-free for both nominal and indexed bonds under the U.K. tax system. However, interest income is subject to income tax, up to the maximum rate of 40 percent. This applies to both nominal and indexed bonds. Therefore, it appears that the tax system treats nominal and indexed bonds on an equal basis. However, in fact, the tax system treats indexed bonds more favorably than nominal bonds because indexed bonds generally have larger capital gains due to inflation, which are tax-free. In particular, indexed bonds provide high-income households with an effective means of tax savings. As Bootle (1991) has argued, the break-even inflation rate that is derived from the indexed bonds and nominal bonds with a similar period to maturity has an inverse relationship with the tax rate.[3] The higher the tax rate, the lower the break-even inflation rate. In practice, there are two methods of calculating the average effective tax rate. The first uses the tax revenues from each kind of government bond and their outstanding value. The second estimates the average effective tax rate by minimizing the sum of the squared residuals in the equation, which regresses the after-tax effective yield on the average duration of the investment principal. This is the method used by Woodward (1990), who found that the effective tax rate changed every year, and that the rate of change was about 25 percent on average. Deacon and Derry (1994a) considered the tax rates of 0 percent, 25 percent, and 40 percent.

This paper uses a general model that assumes taxes on the yields of both indexed and nominal bonds (Appendix). However, in the following empirical studies, I ignore the effects of taxes and present the empirical results only for the case of a 0 percent tax rate. An important point to note is that this assumption of a 0 percent tax rate leads to an overestimation of the expected rate of inflation by the amount of tax savings because the break-even rate of inflation becomes the expected rate of inflation.[4]

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3. The break-even inflation rate is the rate at which indexed bonds and nominal bonds become indifferent for investors according to the Fisher equation. It is lower than the expected rate of inflation by the amount of tax savings.

4. According to Deacon and Derry (1994a), the estimated expected rate of inflation differs some 1.5 percent between the cases of a 0 percent tax rate and a 25 percent tax rate.

## C. Duration

I have already discussed the method of comparing the nominal and indexed bonds with the same maturity and the same period to maturity. Another method is to compare those with the same duration. The duration is the weighted average of the periods before the cash payments are received, where the weights are the present values of the cash flows. It indicates the average period required to recover the invested principal.

The requirement to match the duration for indexed and nominal bonds arises from the following considerations: if we look at only the period to maturity as the standard of comparison for individual bonds, a difference in the pattern of cash flows will be ignored. That would be an imperfect comparison between indexed and nominal bonds. Therefore, we need a standard of comparison that takes into consideration both the coupon rate of a bond and the effective interest rate. In this respect, the duration is a better measure than the period to maturity.

Deacon and Derry (1994a) have shown, however, that the estimated expected rate of inflation becomes unstable and exhibits jumps because the maturities diverge if nominal and indexed bonds with the same duration are compared. Bootle (1991) has also argued against choosing duration as the standard of comparison because nominal and indexed bonds have very different risk characteristics. Consequently, in this paper, I do not choose the duration but the period to maturity as the standard of comparison.

## D. The Term Structure of Interest Rates

The expected rate of inflation obtained from equations (A.1) and (A.2) in the Appendix is a point estimate that represents the annual average of the expected future rates of inflation. Therefore, it does not capture the movement of the expected rate of inflation in the future. However, the changes in the expected rate of inflation, in addition to the point estimate, contain useful information for monetary policy. This information about long-term changes in the expected rate of inflation can be obtained from the estimated term structure (the yield curve) of nominal and real interest rates.

The term structure of interest rate changes reflects expectations about the future rates of interest and inflation. Therefore, it is often difficult to estimate the term structure of interest rates, and there is no established method to estimate it. The first problem is that government bonds are not issued in a continuous and smooth manner, so the periods to maturity tend to have uneven breaks.[5] The second problem is whether we should estimate the yield curve or the discount function for coupons to find the term structure of interest rates. The third problem is that differences in the tax system may affect the risk premium. The fourth problem is that the price index used as the deflator has a lag of eight months, which may introduce some bias into the estimated rates of real interest and inflation. Finally, the number of indexed bonds traded in the market is limited to 13 items

5. The general approach of finance theory has been to build a model of the term structure of interest rates under the assumption that the interest rates follow a stochastic process (Cox, Ingersoll, and Ross, 1985). For example, Brown and Schaefer (1994) have applied the Cox, Ingersoll, and Ross model to the study of indexed bonds. However, from the viewpoint of empirical researchers, the unique movements of interest rates of various bonds do not follow any known stochastic process. Instead, they are likely to be influenced by a series of local shocks to the markets. Therefore, it is important to analyze market-specific information. In other words, we should not ignore market-specific shocks and excessively smooth the yield curve, as is often done when working with a pure theory of finance.

at present (November 1994), which creates the problem of insufficient degree of freedom and weakens the reliability of market prices.

Recent studies have adopted three different estimation methods: the first is the method adopted by McCulloch (1975), which estimates a "discount function" for coupons to find the shape of the yield curve. The second method is to estimate a "par yield curve" based on the coupons that would be obtained if held to full maturity. The third is the method employed by Nelson and Siegel (1987) and Svensson (1994), which estimates an "implied forward rate curve" from the spot rates of bonds with different maturities.[6] These methods use various techniques to improve the fit of the model. However, the more complicated the method, the more difficult it is to apply. In view of this trade-off, Deacon and Derry (1994a,b) recommend the method of Nelson and Siegel (1987) and Svensson (1994), which is relatively simple but produces a reasonably good fit.

The estimation of the term structure of interest rates itself has become a major area for research, and its improvement is indispensable for better monetary policy. In this paper, however, I will not go into fuller detail than the above discussion on the methodology of its estimation.

## IV. An Empirical Analysis Using U.K. Government-Indexed Bonds

Among the various indexed bonds issued in many countries, I will look at the U.K. government-indexed bonds known as "index-linked gilts" in the present study. This is because they are issued in high volume (outstanding issues accounted for 15 percent of U.K. government bonds in 1994) and they are actively traded daily. As a result, we can expect that their market prices have less bias. Moreover, they are most frequently used in studies, like the present one, that attempt to estimate the real interest rate from indexed bonds (for example, Brown and Schaefer [1994] and Woodward [1990]).

In the present study, I have used the end-of-month data for U.K. government-indexed bonds (index-linked gilts) from January 1983 to September 1994.[7] I have selected six items from the five-year bonds (that is, the bonds with five years to maturity) and four items from the 10-year bonds (the bonds with 10 years to maturity).[8]

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6. For example, the annual interest rate four years from now can be calculated as follows: 5 × (the spot rate of government bonds with the maturity of five years) – 4 × (the spot rate of government bonds with the maturity of four years). Svensson (1994) and Kuroda (1982) estimated the term structure of the expected rates of inflation under the assumption of a constant real interest rate. Deacon and Derry (1994a) used a more sophisticated method by estimating the real interest rate as the difference between the nominal forward rate and the implied forward rate.
7. The data are listed in the government bond column in the *Financial Times* on a daily basis.
8. The five-year indexed bonds used in the present study are as follows (the name of each bond represents the year/month of its maturity): 1988/3 bonds (coupon rate = 2%, the period = 1983/1–1984/6), 1990/1 bonds (2%, 1984/7–1986/12), 1992/3 bonds (2%, 1987/1–1988/9), 1994/5 bonds (2%, 1988/9–1990/6), 1996/9 bonds (2%, 1990/7–1993/3), 1998/4 bonds (4.625%, 1993/4–1994/9). The five-year nominal bonds of the same periods are as follows: 1988/10 bonds (9.5%), 1990/1 bonds (13%), 1992/2 bonds (10%), 1994/3 bonds (14.5%), 1996/5 bonds (15.25%), 1998/9 bonds (15.5%). The 10-year indexed bonds are as follows: 1996/9 bonds (2%, 1983/1–1989/9), 2001/9 bonds (2.5%, 1989/10–1992/6), 2003/5 bonds (2.5%, 1992/7–1993/9), 2004/10 bonds (4.375%, 1993/10–1994/9). The 10-year nominal bonds with the same period are as follows: 1996/5 bonds (15.25%), 2002/8 bonds (9.75%), 2003/9 bonds (10%), 2003/9 bonds (10%).

I have paired nominal and indexed bonds with the same maturity and the same period to maturity, and estimated the nominal and real interest rates as well as the expected rate of inflation, using the method discussed in the previous section. However, when retail prices change substantially during the eight months before maturity, the prices of indexed bonds may fall suddenly and the real interest rate may rise sharply. To avoid this problem, I have not employed real interest rates near maturity. Furthermore, because the calculation is complicated by the fact that the interest payment does not necessarily start six months after their issuance, I have estimated the interest rates only after the first interest payment. In short, I have chosen the more stable interest rates in the middle period excluding the periods immediately after issuance and immediately before maturity. Finally, I have used the retail price index (RPI) as the deflator.[9]

Figures 1 and 2 show the real interest rates in the government bond market for five- and 10-year bonds. Both figures exhibit a very stable movement of the real interest rate for indexed bonds. This result is in sharp contrast to the movement of the *ex post* real interest rate obtained as the nominal interest rate minus the realized rate of retail price inflation, which has been frequently used in previous empirical studies and in policy discussions concerning monetary policy. More specifically, previous studies have indicated that the real interest rate often falls below zero during periods of inflation and rises above the nominal interest rate during the period of deflation. However, the real interest rate that is implicit in the market for indexed bonds shows remarkable stability over a period of 10 years, despite much greater movement at the retail price level. I will discuss the implications of this finding in Section V.

**Figure 1  Real Interest Rates in the Government Bond Market (Five-Year Bonds)**
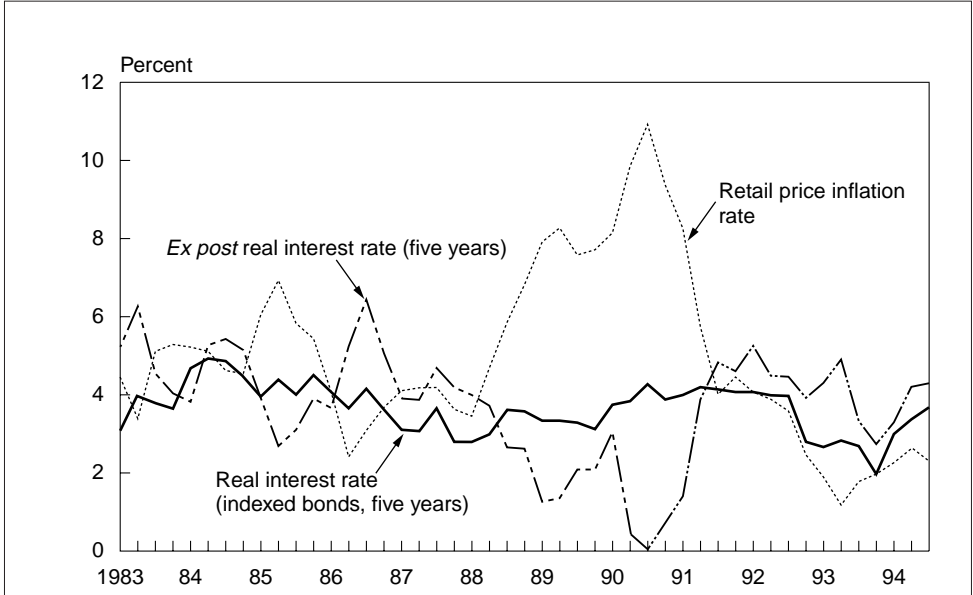


9. It should be noted that when there is a need for adjustment in the consumption basket because of changes in the consumption pattern, the Bank of England makes an adjustment between the old and new price indexes.

**Figure 2  Real Interest Rates in the Government Bond Market (10-Year Bonds)**

Percent

Ex post real interest rate (10 years)

Retail price inflation rate

Real interest rate
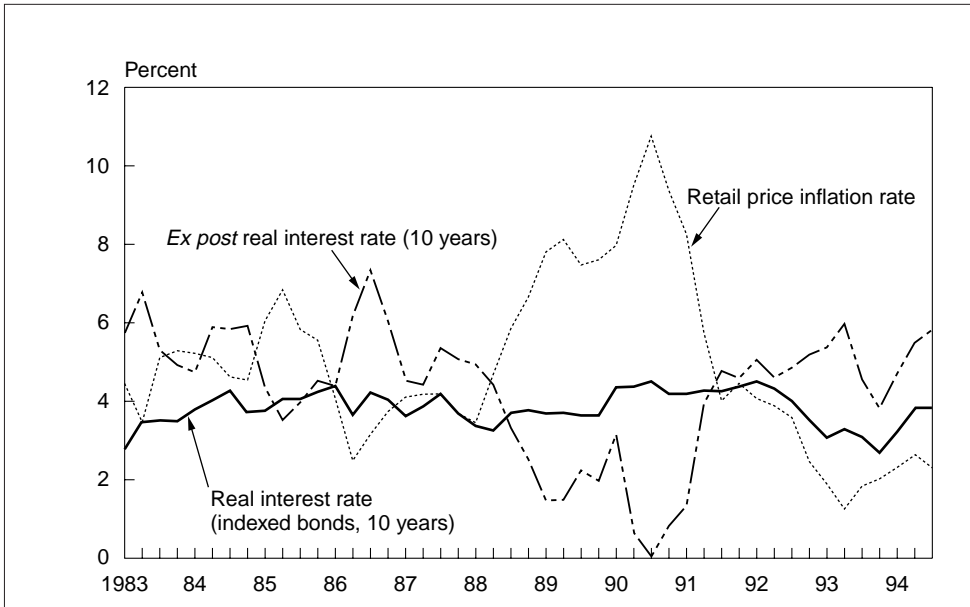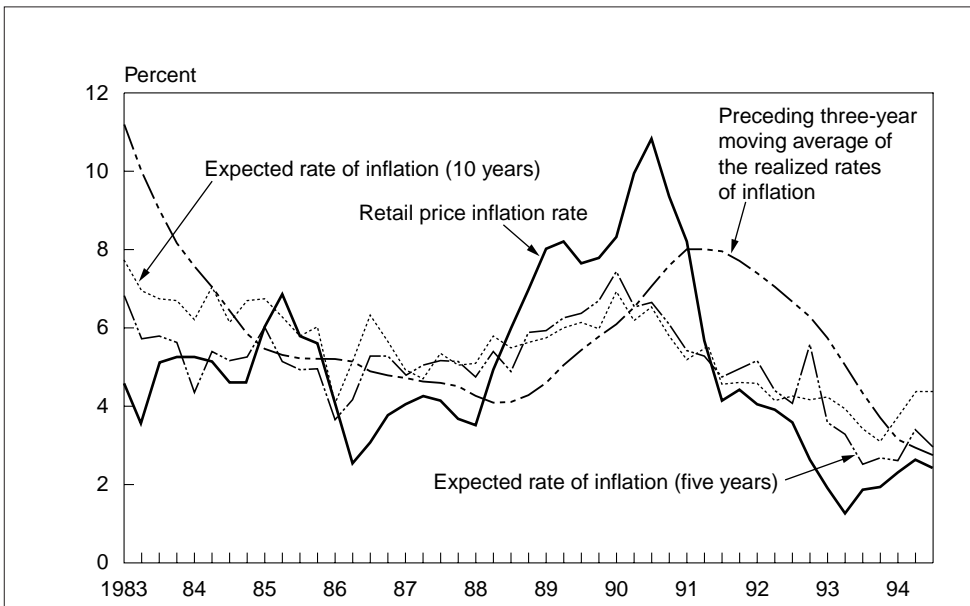(indexed bonds, 10 years)

Figure 3 shows the expected rate of inflation obtained from the real and nominal interest rates implicit in the indexed bonds, ignoring the effects of taxes. It again shows the relative stability of the expected rate of inflation compared to the *ex post* rate of retail price inflation. One possible reason for this stability is that the time

**Figure 3  Expected Rate of Inflation**

Percent

Expected rate of inflation (10 years)

Retail price inflation rate

Preceding three-year moving average of the realized rates of inflation

Expected rate of inflation (five years)

**9**

horizon of participants in the market for government bonds extends to all the years before maturity, and therefore the expected rate of inflation represents the average rate of inflation during this long period. To throw some light on this point, I have also presented the preceding three-year moving average of the realized rates of inflation in Figure 3. Given that the expected rate of inflation can differ greatly from the actual inflation rate in the past, I would like to emphasize the following point: that is, although the preceding three-year average is about as stable as the expected rate of inflation, its peaks and troughs lag substantially those of the expected rate of inflation. Therefore, we cannot take the moving average of the realized inflation as a good proxy for the expected rate of inflation. Moreover, as discussed in footnote 6, Deacon and Derry (1994a) estimated changes in the expected rate of inflation from the term structure of interest rates. They found that the expected rate of inflation would not change very much and would remain stable over the next 1–20 years. This suggests that the expected rate of inflation will remain fairly stable until new information becomes available, and it does not suggest that changes in future inflation are expected to be small. In other words, the expected rate of inflation remains stable without specific information about some future events such as a scheduled change in the value-added tax.

The expected rate of inflation obtained from the Fisher equation represents the average of expectations of market participants, but at the same time, as I have mentioned earlier, it also represents the "break-even inflation rate" at which the nominal and indexed bonds become indifferent. Therefore, if actual inflation is higher than expected inflation (as in the period from mid-1988 to early 1991 in Figure 3), then it will be profitable to hold indexed bonds in terms of interest income. Moreover, as the expected rate of inflation is obtained under the assumption of a zero tax rate, it is likely to be overestimated. If so, the period during which indexed bonds brought benefits historically would have been further extended. Conversely, if expected inflation is higher than actual inflation, then it will be profitable to hold nominal bonds. As these examples indicate, the difference between the expected and actual rates of inflation represents the difference in their interest incomes. It then follows that the greater the gap between the expected and actual rates of inflation, the greater the opportunity for gains from arbitrage between nominal and indexed bonds. To put it another way, if arbitrage were working, then the gap between the actual and expected rates of inflation would have been smaller.

## V. Implications for Monetary Policy

In Section IV, I derived the real interest rate and the expected rate of inflation from data on U.K. government-indexed bonds. In this section, employing statistical analysis of those data, I would like to discuss the implications for monetary policy.

### A. Stability of the Real Interest Rate
First, I would like to emphasize the stability of the real interest rate obtained from the market prices of indexed bonds. The main characteristics of the real interest rate obtained from indexed bonds and the *ex post* real interest rate, which is most often

used, are evident from Table 1, which lists the basic statistics of the main variables. Although their real interest rates are grouped in the 3.7 percent–4.3 percent range, irrespective of the period of maturity, the standard deviation of real interest rates obtained from indexed bonds is far lower than that of the *ex post* real interest rates. In particular, the difference between the minimum and maximum values for the indexed-bond real interest rate is only two to three percentage points, while the difference for the *ex post* real interest rate is some seven percentage points.

Moreover, according to the χ-squared test for normality, the distribution of the indexed-bond real interest rate is closer to the normal distribution than that of the *ex post* real interest rate. According to the standard deviations, the expected rate of inflation obtained from the Fisher equation is more stable than actual retail price inflation. In short, the expected rate of inflation has a smaller minimum-maximum difference and its distribution is closer to normality than actual retail price inflation.

**Table 1  Basic Statistics of the Main Variables**

■ 1983/I to 1994/III

|  | Indexed-bond real interest rate (five years) | Indexed-bond real interest rate (10 years) | *Ex post* real interest rate (five years) | *Ex post* real interest rate (10 years) | Nominal interest rate (five years) | Nominal interest rate (10 years) | Expected rate of inflation (five years) | Expected rate of inflation (10 years) | Retail price inflation rate (RPI) |
|---|---|---|---|---|---|---|---|---|---|
| Mean | 3.673 | 3.816 | 3.724 | 4.324 | 8.694 | 9.294 | 5.035 | 5.449 | 4.970 |
| S.D. | 0.641 | 0.457 | 1.468 | 1.684 | 1.492 | 1.233 | 1.144 | 1.082 | 2.242 |
| Min. | 1.980 | 2.650 | 0.060 | 0.170 | 4.670 | 5.750 | 2.460 | 3.080 | 1.220 |
| Max. | 4.960 | 4.530 | 6.510 | 7.490 | 11.190 | 11.290 | 7.440 | 7.840 | 10.890 |
| χ-squared test | 0.449 | 2.928 | 4.234 | 8.279 | 10.691 | 5.450 | 2.252 | 0.703 | 5.074 |

## B. Factors That Affect the Real Interest Rate

Fama (1975) has assumed that the real interest rate is constant. However, the real interest rate obtained from indexed bonds exhibits variations. Nevertheless, if the variations reflect random variations around a constant, it will be difficult to reject statistically the hypothesis of a constant real interest rate. This section studies the factors that cause changes in the real interest rate, using simple statistics.

Table 2 shows that the indexed-bond real interest rate has a positive correlation with the nominal interest rate and the inflation rate. This contrasts sharply with the *ex post* real interest rate of nominal bonds, which has a negative correlation with the nominal interest rate and the inflation rate. Moreover, the indexed-bond real interest rate and the *ex post* real interest rate of nominal bonds have a very low and unstable correlation, which changes the sign between five-year and 10-year bonds. This implies that the *ex post* real interest rate of nominal bonds, which has often been used as a proxy for the real interest rate in many empirical studies, can be very misleading if the real interest rate obtained from the market prices of indexed bonds is correct.

If a rise in the nominal interest rate corresponds one-to-one to a rise in the inflation rate, the real interest rate will be constant. In this case, a change in the interest

rate initiated by monetary policy would be neutral for the real economy. However, if a positive correlation exists between the real interest rate and the inflation rate, then this suggests that monetary policy will affect the real economy.[10]

Table 3 presents statistics on the distribution of the residuals that represent deviations of the indexed-bond real interest rates from the average. Although the

**Table 2  Correlation Matrix**

|  | $_5ir$ | $_{10}ir$ | $_5nr$ | $_{10}nr$ | $_5R$ | $_{10}R$ | $_5\pi^e$ | $_{10}\pi^e$ | $\pi$ |
|---|---|---|---|---|---|---|---|---|---|
| $_5ir$ | 1.0000 | | | | | | | | |
| $_{10}ir$ | 0.7553 | 1.0000 | | | | | | | |
| $_5nr$ | 0.1517 | −0.0866 | 1.0000 | | | | | | |
| $_{10}nr$ | 0.0464 | −0.2112 | 0.9694 | 1.0000 | | | | | |
| $_5R$ | 0.6765 | 0.5748 | −0.1464 | −0.3077 | 1.0000 | | | | |
| $_{10}R$ | 0.7016 | 0.5106 | −0.0441 | −0.1611 | 0.9646 | 1.0000 | | | |
| $_5\pi^e$ | 0.2646 | 0.3022 | −0.2768 | −0.4130 | 0.8655 | 0.8132 | 1.0000 | | |
| $_{10}\pi^e$ | 0.4760 | 0.1486 | −0.0111 | −0.0913 | 0.8132 | 0.9261 | 0.8026 | 1.0000 | |
| $\pi$ | 0.3510 | 0.4394 | −0.7525 | −0.8399 | 0.7616 | 0.6710 | 0.7575 | 0.5779 | 1.0000 |

Note: $_5ir$ = indexed-bond real interest rate (five years)  $_{10}ir$ = indexed-bond real interest rate (10 years)
$_5nr$ = *ex post* real interest rate (five years)  $_{10}nr$ = *ex post* real interest rate (10 years)
$_5R$ = nominal interest rate (five years)  $_{10}R$ = nominal interest rate (10 years)
$_5\pi^e$ = expected rate of inflation (five years)  $_{10}\pi^e$ = expected rate of inflation (10 years)
$\pi$ = retail price inflation rate

**Table 3  Statistics on the Distribution of the Residuals**

■ 1983/I to 1994/III

| Residuals | Indexed-bond real interest rate (five years) minus mean (3.673) | Indexed-bond real interest rate (10 years) minus mean (3.816) |
|---|---|---|
| Mean | 0.000 | 0.000 |
| S.D. | 0.641 | 0.457 |
| Min. | −1.693 | −1.166 |
| Max. | 1.287 | 0.714 |
| $\chi$-squared test | 0.449 | 2.928 |
| Autocorrelation | | |
| Lag 1 | 0.717 | 0.690 |
| Lag 2 | 0.550 | 0.444 |
| Lag 3 | 0.453 | 0.341 |
| Lag 4 | 0.315 | 0.129 |

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10. It is important to note, however, that the real interest rate is obtained from indexed bonds, and may not reflect the marginal productivity of capital in the real economy. This point has many important policy implications, and requires a separate study. Some of the relevant studies are Summers (1983), which uses a small macroeconomic model to investigate the relationship between interest rates and the real economy; Yoshikawa (1984), which compares the proposition of classical economics that monetary policy cannot affect the real interest rate and the Keynesian proposition that it can; and Shiller (1980), which finds some empirical evidence for the claim that the Federal Reserve can control the real interest rate.

basic statistics on the residuals seem to suggest a normal distribution (in particular, for the five-year indexed bonds), they exhibit a high level of autocorrelation and serial correlation. Therefore, they do not support the hypothesis of a random distribution. That is, they reject the hypothesis of a constant real interest rate.

Next, Table 4 shows the results of unit-root tests on the indexed-bond real interest rate. They statistically reject the unit-root hypothesis for some of the lags. Even in the other cases that are not rejected, their statistics have large values. Therefore, in view of the low power of the augmented Dickey-Fuller (ADF) test for a small sample case, they do not support the unit-root hypothesis. On the other hand, figures 1 and 2 suggest that the indexed-bond real interest rate follows a stationary stochastic process. Therefore, we may conclude that the indexed-bond real interest rate does not follow a random walk, and that it is affected by some structural factors while maintaining stationarity.[11]

**Table 4  Results of Unit-Root Tests on the Indexed-Bond Real Interest Rate**

■ 1985/II to 1994/III

|  | Augmented Dickey-Fuller (ADF) Test | |
|---|---|---|
|  | Indexed-bond real interest rate (five years) | Indexed-bond real interest rate (10 years) |
| Lag 0 | −2.6117* | −2.4847 |
| Lag 1 | −2.2429 | −2.4335 |
| Lag 2 | −2.0161 | −1.9186 |
| Lag 3 | −2.0200 | −2.4928 |
| Lag 4 | −2.4489 | −2.3828 |
| Lag 5 | −2.5538 | −2.4758 |
| Lag 6 | −2.1871 | −3.2606** |
| Lag 7 | −2.2877 | −2.8381* |
| Lag 8 | −2.7016* | −3.5745** |

Note: Both estimates include constant variables.
   * indicates 10 percent level of significance ≒ −2.60, ** indicates 5 percent level of significance ≒ −2.94, and *** indicates 1 percent level of significance ≒ −3.612.

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11. Andrade and Clare (1994) assume that the real interest rate follows a random walk with a trend. Chapman and Ogaki (1993) and Rose (1988) report that the *ex post* U.S. real interest rate follows a nonstationary process. In contrast, Neusser (1991) reports that the *ex post* real interest rates of six industrial countries including the United States follow a stationary process. As these examples show, the stationarity of the real interest rate is not a well-confirmed assumption. However, the purpose here is to make transparent the difference between the *ex post* real interest rate, which has been used most frequently, and the *ex ante* real interest rate, which is discussed in the present paper. The two real interest rates would be identical only if the expected rate of inflation and the realized rate of inflation were identical, except for stochastic errors. According to Figure 3, however, they are quite different, and therefore the *ex post* and *ex ante* real interest rates are also quite different.

## C. A Model of the Real Interest Rate

In order to examine this issue further, I have tested the Granger causality between the indexed-bond real interest rate and the inflation rate. Table 5 shows the results, which indicate the absence of the Granger causality between the two variables.

The absence of the Granger causality suggests that one can estimate a single-equation regression of the real interest rate, treating the inflation rate as an exogenous variable.[12] It is generally believed that inflation is endogenously determined by supply and demand conditions in the goods market and by macroeconomic variables such as money supply, and that the indexed-bond real interest rate does not influence the market determination of inflation. Therefore, this paper will report the estimation results of a single-equation regression model of the indexed-bond real interest rate, treating inflation as an exogenous variable. The estimation uses the quarterly data from 1984/I to 1994/III. It uses the period from 1993/IV to 1994/III as the range of extrapolation. After the standard model selection procedure, the ordinary least squares (OLS) method has produced the following estimation results:[13]

Indexed-bond real interest rate (five years) = $_5ir$

$$_5ir_t = 0.8956 \,_5ir_{t-1} + 0.0669\pi_t \qquad\qquad (2)$$
$$\quad (20.611) \qquad\quad (2.321)$$

$R^2 = 0.988$, s = 0.419, Durbin's h = $-0.422$
Normality $\chi^2(2) = 1.043$, RESET $F(1, 36) = 1.8345$
Forecast $\chi^2(4) = 12.361$, Forecast Chow $F(4, 37) = 3.043$.

**Table 5  Result of the Granger Causality Test between the Indexed-Bond Real Interest Rate and the Inflation Rate**

■ 1983/IV to 1994/III, F-test

| Dependent variables | Autoregressive lag | Additional dependent variables | Distributive lag | $F(4, 36)$ |
|---|---|---|---|---|
| Indexed-bond real interest rate (five years) | 1 to 3 | Inflation rate | 0 to 3 | 1.0795 |
| Inflation rate | 1 to 3 | Indexed-bond real interest rate (five years) | 0 to 3 | 1.1766 |
| Indexed-bond real interest rate (10 years) | 1 to 3 | Inflation rate | 0 to 3 | 2.4886 |
| Inflation rate | 1 to 3 | Indexed-bond real interest rate (10 years) | 0 to 3 | 1.0895 |

Note: $F(4, 36)$: 5 percent level of significance $\doteqdot$ 2.61.

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12. Engle, Hendry, and Richard (1983) show that the absence of the Granger causality is a necessary condition for exogeneity. Of course, this does not mean that no statistical inference is possible.
13. I have used the model selection procedure and the test statistics presented in Doornik and Hendry (1994).

Indexed-bond real interest rate (10 years) = $_{10}ir$

$$_{10}ir_t = 1.5672 + 0.4996\,_{10}ir_{t-1} + 0.070\pi_{t-3} \qquad (3)$$
$$\qquad\quad (3.196) \qquad (3.671) \qquad\quad (2.721)$$

$R^2 = 0.506$, s = 0.294, Durbin's h = 0.9477
Normality $\chi^2(2) = 1.5463$, RESET $F(1, 36) = 0.7918$
Forecast $\chi^2(4) = 8.156$, Forecast Chow $F(4, 36) = 2.010$.

where t-statistics are in parentheses and $\pi_t$ represents the inflation rate.

The most interesting finding is that the past inflation rates ($\pi_t$ and $\pi_{t-3}$) have a significant effect on the current indexed-bond real interest rate. The interest rate structure of government bonds has two characteristics: (1) it is determined as a general equilibrium of all financial markets; and (2) it is determined by the term structure of time-series interest rates and cross-section interest rates. Unfortunately, there is no general model of interest rate determination that incorporates the two characteristics consistently and simultaneously. Nevertheless, indexed bonds are designed to ensure the real value of both the principal and interest after adjustment for inflation, and therefore the expected rate of inflation should affect their demand and supply. Other things being equal, as the expected rate of inflation rises, the relative demand for indexed bonds should increase and the real interest rate of indexed bonds should decline. Therefore, at least from a partial equilibrium point of view, there should be a negative correlation between the inflation rate and the real interest rate. In fact, Mundell (1963) and Mishkin (1981) have found a negative correlation between the two. This is in sharp contrast to the finding of this paper.[14]

To examine further the relationship between the real interest rate and inflation, it becomes necessary to define the rate of inflation more precisely. The rate of inflation used in this paper is the annual rate of change in the RPI during the past year. However, the rate of inflation that is relevant in the indexed bond market is the annual average rate of inflation during the period of the bonds' maturity. They are quite different: one is a realized rate, while the other is an expected rate. Therefore, the information they contain is also different. In the next section, I will examine the relationship between the expected rate of inflation and the realized rate of inflation.

### D. The Expected Rate of Inflation as a Leading Indicator

In the previous section, I suggested that the expected rate of inflation is a leading indicator of the realized rate of inflation. In practice, however, the past realized inflation contains the most information about the future course of inflation. In fact, as we saw in Figure 3, the expected rate of inflation derived from the indexed bonds has often failed to predict future inflation, particularly when unexpected inflation occurs.

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14. This positive correlation may be related to the Gibson Paradox, which Keynes named after A. H. Gibson, who found a positive correlation between the price level and the nominal interest rate. However, it goes beyond the simple Gibson Paradox and indicates a positive correlation between the inflation rate and the real interest rate, which has more important implications for monetary policy.

Therefore, we should test the informational utility of the expected rate of inflation derived from indexed bonds by checking if it increases the predictive accuracy of the autoregressive model of realized inflation. Here I have checked the leads and lags of the expected rate of inflation using the Granger causality test. Table 6 presents the results, which indicate that the expected rate of inflation leads the realized rate of inflation in the sense of Granger causality.

In the following, using various tests, I select the forecasting models of inflation that contain the autoregressive process of realized inflation and the expected rate of inflation.[15] The regression employs quarterly data from 1983/IV to 1994/II. It uses the period from 1993/III to 1994/II as the range of extrapolation. After the standard model selection procedure, the OLS method produced the following regression with the expected rate of inflation for five years:

$$\pi_{t+1} = 1.3146\pi_t - 0.56138\pi_{t-1} + 0.2520{}_5\pi_t^e \qquad (4)$$
$$\quad (9.361) \qquad (-4.158) \qquad (2.885)$$

$R^2 = 0.983$, s $= 0.7769$
Normality $\chi^2(2) = 2.6002$, RESET $F(1, 37) = 1.8362$
Forecast $\chi^2(4) = 0.9421$, Forecast Chow $F(4, 38) = 0.2271$

where ${}_5\pi_t^e$ represents the expected rate of inflation derived from five-year indexed government bonds. If we impose the restriction of eliminating ${}_5\pi_t^e$ from equation (4), then it produces $F(1, 36) = 8.2313$ and worsens the fit of the model at the 1 percent level of significance.

**Table 6  Result of the Granger Causality Test between the Leads and Lags of the Expected Rate of Inflation**

■ 1983/III to 1994/IV, F-test

| Dependent variables | Autoregressive lag | Additional dependent variables | Distributive lag | $F(3, 39)$ |
|---|---|---|---|---|
| Realized rate of inflation | 1 to 2 | Expected rate of inflation (five years) | 0 to 2 | 6.5183** |
| Expected rate of inflation (five years) | 1 to 2 | Realized rate of inflation | 0 to 2 | 3.0737* |
| Realized rate of inflation | 1 to 2 | Expected rate of inflation (10 years) | 0 to 2 | 4.6865** |
| Expected rate of inflation (10 years) | 1 to 2 | Realized rate of inflation | 0 to 2 | 1.9538 |

Note: $F(3, 39)$: ** indicates 1 percent level of significance ≒ 4.31, and * indicates 5 percent level of significance ≒ 2.84.

15. Here I have again used the procedure and the statistics presented in Doornik and Hendry (1994).

The selected model that contains the expected rate of inflation for 10 years has a slightly different specification:

$$\pi_{t+1} = 1.1818\pi_t - 0.3641\pi_{t-1} + 0.2520\,_{10}\pi_t^e - 0.3576\,_{10}\pi_{t-3}^e \qquad (5)$$
$$\quad\;\;(7.204)\qquad\;\;(-2.209)\qquad\;\;(2.571)\qquad\;\;(-1.785)$$

$R^2 = 0.984$, s = 0.7693
Normality $\chi^2(2) = 0.7693$, RESET $F(1, 34) = 0.2267$
Forecast $\chi^2(4) = 2.3844$, Forecast Chow $F(4, 35) = 0.5296$

where $_{10}\pi_t^e$ represents the expected rate of inflation derived from 10-year indexed government bonds. If we impose the restriction of eliminating $_{10}\pi_t^e$, $_{10}\pi_{t-3}^e$ from equation (5), then it produces $F(2, 35) = 5.1013$ and worsens the fit of the model at the 5 percent level of significance.

These regressions show that the expected rate of inflation contains some valuable information that significantly improves the prediction of future inflation. Equations (4) and (5) explain the same variable ($\pi_{t+1}$) with slightly different combinations of explanatory variables, and therefore they form a non-nested model. We can compare the two models (4) and (5) using an encompassing test. The test results are presented in Table 7. They indicate that neither model can encompass the other, but that equation (5) is marginally superior to equation (4). Therefore, I have plotted the realized and predicted values based on equation (5), as shown in Figure 4.

The predicted values show a very good fit of the model, and the error terms exhibit a low serial correlation. The expected rates of inflation derived from the 10- and five-year indexed bonds are very similar. Nevertheless, as Table 1 indicates, the expected rate of inflation derived from the 10-year indexed bonds has a smaller standard deviation and a distribution closer to the normal distribution. Therefore, it appears to be a better predictor of future inflation.

It is known that the turning point of the prediction will lag one period when an autoregressive model is used for prediction. It is especially worthy of comment that the turning points of the prediction match very well the peaks and troughs of the

**Table 7  Encompassing Test for the Expected Rate of Inflation Models**

■ 1983/IV to 1993/II

| $H_0$: equation (5) encompasses equation (4) | Test methods | $H_0$: equation (4) encompasses equation (5) |
|:---:|:---:|:---:|
| −1.2203 | Cox ~ $N(0, 1)$ | −1.5489 |
| 1.1220 | Ericsson IV ~ $N(0, 1)$ | 1.4266 |
| 1.0711 | Sargan ~ $\chi^2$ | 2.7327 |
| 1.0734 | Joint model ~ $F(1, 34)$ | 1.3964 |

Note: Refer to Doornik and Hendry (1994) for statistics. This table can be read so that, when a statistic is closer to zero, the selected model has a higher probability of encompassing the alternative model, and when a statistic is significantly higher or lower than zero, the selected model does not encompass the alternative.

**Figure 4  Realized and Predicted Values Based on Equation (5)**
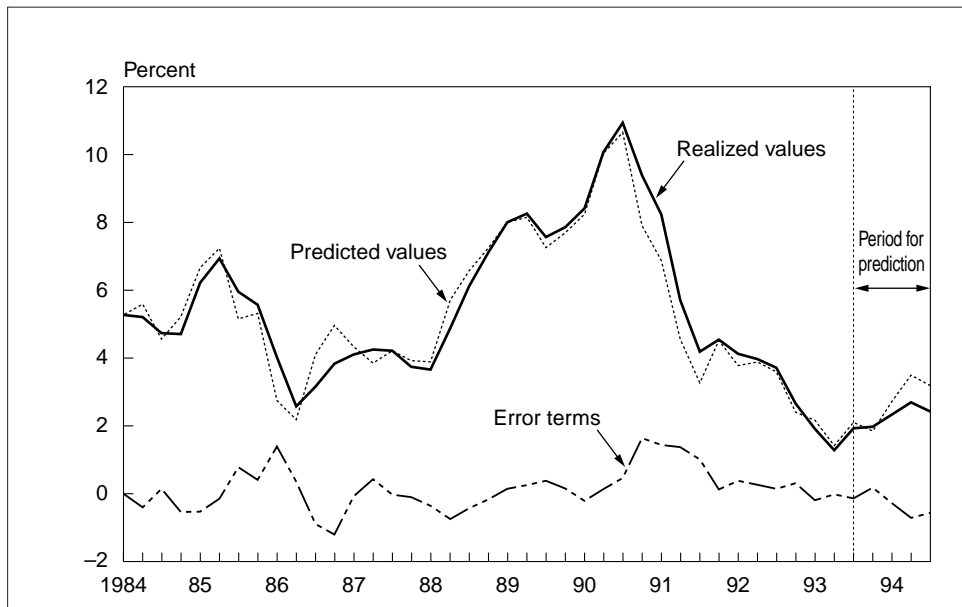


actual inflation time-series if we add the information from the expected rate of infla-
tion. The ability to predict the turning points of inflation accurately is especially
important for making monetary policy decisions. Therefore, the expected rate of
inflation derived from indexed bonds provides the monetary authorities with very
valuable information.

### E. The Fisher Equation and the Rational Expectations Hypothesis

This paper has used market information for indexed government bonds to obtain
nominal and real interest rates, and then derived the expected rate of inflation from
the Fisher equation. The paper has found that the real interest rate is not constant
and its movement is affected by inflation and its own past values. It has also found
that the expected rate of inflation contains valuable information for predicting future
inflation. In other words, we could gain useful information about future inflation,
the real interest rate, and its movement from the Fisher equation.

Nevertheless, we should explicitly verify the validity of the Fisher equation by
employing statistical tests. To do so, let us test the Fisher equation using the same
nominal and real interest rates derived from indexed bonds, but with the expected
rate of inflation obtained from different sources. The most frequently used hypothe-
sis of expectations about inflation is the hypothesis of rational expectations. This is
the hypothesis that the expected rate of inflation is equal to the realized rate of infla-
tion except for stochastic errors. This hypothesis can be formally modeled as follows:

$$\pi_t^e = \pi_{t+1} + \varepsilon_t \tag{6}$$

where $\varepsilon_t$ represents an error term, which is independent from $\pi_{t+1}$. Substituting this into equation (1), we obtain the following equation:

$$R_t = r_t + \pi_{t+1} + \varepsilon_t. \tag{7}$$

Using the nominal interest rate for bonds, the indexed-bond real interest rate, and the realized inflation rate in the next period, we can test the unitary coefficients for explanatory variables, the absence of serial correlations, and the normality of error terms, and thereby test the joint hypothesis of the Fisher equation and the rational expectations hypothesis. Now we can generalize equation (7) as follows:

$$R_t = \alpha_t + \beta\pi_{t+1} + \varepsilon_t. \tag{8}$$

Then we can test for $\alpha = \beta = 1$, $\varepsilon_t \approx N(0, \sigma^2)$, and $\text{cov}(\varepsilon_t, \varepsilon_{t-j}) = 0$, $\forall j$.[16] Table 8 presents the test results. They reject the hypothesis of $\alpha = \beta = 1$ at the 1 percent level of significance. The D.W. statistics indicate the presence of a strong serial correlation. Although the normality of error terms is not rejected, the RESET F-test suggests misspecification.[17] These results indicate that the joint hypothesis of the rational expectations and the Fisher equation does not hold.

In summary, I have obtained the nominal and real interest rates from the U.K. government nominal and indexed bonds, and tested the Fisher equation under the assumption of rational expectations. The test results have been negative. It means either that (1) the Fisher equation does not hold; or (2) the rational expectations hypothesis does not hold; or (3) that both do not hold. Therefore, if the Fisher equation holds, it means that the rational expectations hypothesis does not hold, and

**Table 8  Test for the Fisher Equation Hypothesis**

■ 1983/IV to 1994/II

| | $\alpha$ | $\beta$ | Statistic | | | |
|---|---|---|---|---|---|---|
| | t-statistic ($\alpha = 0$) | t-statistic ($\beta = 0$) | t-statistic ($\alpha = 1$) | t-statistic ($\beta = 1$) | Normality $\chi^2(2)$ | D.W. |
| Indexed bond (five years) | 1.8033 (26.753)** | 0.4006 (8.955)** | 11.990** | −13.397** | 0.385 | 1.100 |
| | $R^2 = 0.995$, $\sigma = 0.650$,  RESET $F(1, 36) = 51.769$** | | | | | |
| Indexed bond (10 years) | 2.0437 (25.098)** | 0.2691 (4.814)** | 12.817** | −13.076** | 0.094 | 0.715 |
| | $R^2 = 0.994$, $\sigma = 0.783$,  RESET $F(1, 36) = 18.174$** | | | | | |

Note: ** indicates 1 percent level of significance [t-statistic ≒ 2.704, $F(1, 36)$ ≒ 7.31].

16. If the expected rate of inflation contains measurement errors, it will create a correlation between explanatory variables and error terms (known as the error-in-variables problem). This introduces a bias in the OLS estimation. If the interest rate of nominal bonds and the real interest rate of indexed bonds contain no estimation error, the bias is downward and depends on the relative size of the standard deviation of estimation errors and the true standard deviation of the expected rates of inflation. That is, $\beta$ tends to be estimated as less than one, and the hypothesis test may suffer from bias. However, assuming that the standard deviation of the estimation errors is substantially less than that of the true deviation, I will ignore this problem.
17. See Doornik and Hendry (1994) for the RESET F-test.

vice versa. The conclusion is that the Fisher equation and the rational expectations hypothesis are incompatible.

This conclusion is basically consistent with the findings of Summers (1983), Mishkin (1990), Yamada (1991), and the Bank of Japan (1994). However, the following point should be emphasized: that is, they assumed a constant real interest rate, in addition to the assumptions made in this paper, tested the Fisher equation, and obtained negative results. This paper has already rejected the hypothesis of a constant real interest rate in figures 3 and 4. Therefore, at least, part of the reason their tests produced negative results may be due to the assumption of a constant real interest rate.[18] However, this paper rejected the Fisher equation under the rational expectations hypothesis even if we allow for a variable real interest rate. Moreover, although the previous studies have assumed rational expectations, they have not discussed carefully the implications of the rational expectations hypothesis in the testing of the Fisher equation. In contrast, this paper has improved on previous test results by obtaining the real interest rate from market information, and has identified the reason for the rejection of the Fisher equation as the incompatibility of the Fisher equation and the rational expectations hypothesis.

Now it is possible to test whether the Fisher equation or the rational expectations hypothesis fails to hold. This leads to the following question: which is closer to the true expectations of economic agents—those obtained under the rational expectations hypothesis or those obtained from the Fisher equation? However, it is impossible to answer this question because the true expectations are not directly observable. In practice, we have little choice but to examine the compatibility of expectations obtained under various assumptions with the observed economic data. This paper has shown that the Fisher equation obtained as an arbitrage condition and the rational expectations hypothesis are incompatible. The conclusion at present is that we have no choice but to check the consistency of assumed expectations with the observed economic data in general.[19]

## VI. Conclusion

The real interest rate and the expected rate of inflation are not directly observable in the market. This paper has presented a method of deriving the real interest rate and the expected rate of inflation from the data on the U.K. government-indexed bonds.

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18. Fama and Gibbons (1982) used an estimation method that allowed the possibility of a variable real interest rate. Andrade and Clare (1994) used a Karman filter to estimate the Fisher equation under the assumption that the real interest rate changes gradually. They concluded that the hypothesis of a constant real interest rate could not be rejected.
19. Tirole (1982) has proved theoretically that speculative behavior and rational expectations are incompatible. This conclusion is known as the no-trade theorem under rational expectations. Tirole started with the definition that speculative transactions are a process of transforming the risk of changing asset prices. Then he argued that if investors have the same prior about the asset price distribution, they would not trade because they understand that the market price reflects all the information possessed by the investors and therefore leaves no unexploited opportunities in the market. It is often suggested that for this conclusion to be consistent with active trades in the government bond market, it is natural to assume, for example, bounded rationality, which produces slowly changing expectations. Sargent (1993) surveys the applications of bounded rationality to finance and macroeconomics.

It has shown that the derived real interest rate and the expected rate of inflation provide very useful information for monetary policy. The empirical analysis in this paper has shown that the Fisher equation and the rational expectations hypothesis are incompatible, and that the expected rate of inflation obtained from the Fisher equation is far more stable than the realized rate of inflation and the expected rate of inflation obtained from the rational expectations hypothesis.

As this paper has shown for the case of indexed bonds, new financial products can reveal information that has not been directly observable in the market. The value of indexed bonds as a new financial product has been already proved in United Kingdom, Canada, Sweden, and Israel. And even if a government does not issue indexed bonds, private-sector companies may start issuing indexed corporate bonds in the near future. In view of these new financial developments, the monetary authorities and economic researchers should investigate the potential use of information that can be obtained from new financial products. The information that would be useful for monetary policy is not only realized inflation, but the expected rate of inflation, which would reveal the judgments of economic agents that lie behind their actions.

Because Japan has no market for indexed bonds at present, one possible way to obtain the real interest rate is to use data on the marginal product of capital and consumers' time preferences. As I have pointed out, the real interest rate obtained from indexed bonds reflects the conditions of the financial markets, and may not reflect the true conditions of the real economy. Therefore, one promising future research project is to obtain the real interest rate from the production and utility functions, which should more accurately reveal the conditions of the real economy. Combining this real interest rate with the nominal interest rate obtained from financial markets, we may be able to discover more fundamental information about the real economy such as the expected rate of inflation, the degree of risk aversion, and the time preferences of economic agents.

Finally, for the fiscal authorities, the important practical question is how much they can save in interest payments if they issue indexed bonds. This is a separate issue, however, and belongs to the topic of indexed-bond issuance. I will leave it as a topic for future research.

## Appendix: The Basic Structure of an Indexed Bond and Its Discounted Present Value

Figure 1 for the Appendix shows the relationship between the interest payments of indexed bonds and the price index. Keeping this relationship in mind, we can express the discounted present value of an indexed bond as follows:

$$PV_t = \frac{cou\cdot(1-\tau)\cdot p2/p1}{(1+R_t)^m} + \frac{cou\cdot(1-\tau)\cdot p3/p1\cdot(1+\pi_t^e)^g}{(1+R_t)^{m+1}}$$

$$+ \frac{cou\cdot(1-\tau)\cdot p3/p1\cdot(1+\pi_t^e)^{g+1}}{(1+R_t)^{m+2}} \cdots + \frac{cou\cdot(1-\tau)\cdot p3/p1\cdot(1+\pi_t^e)^{g+n-1}}{(1+R_t)^{m+n}}$$

$$+ \frac{prin\cdot p3/p1\cdot(1+\pi_t^e)^{g+n-1}}{(1+R_t)^{m+n}}$$

where $cou$ = the coupon rate (a half year), $prin$ = the principal, $p1$ = the price index in the base year, $p2$ = the price index used as a deflator in the second interest payment, $p3$ = the most recent price index (two months before), $m$ = the number of months between the present and the next interest payment divided by the unit of six months,[20] $g$ = the number of months between the most recent price index and the price index used for the second interest payment from now divided by the unit of six months,[21] $n$ = the number of interest payments left before maturity, and $\tau$ = the tax rate on the interest.

Next, let us rewrite Fisher equation (1) as follows:

$$1 + R_t = (1 + r_t)\cdot(1 + \pi_t^e). \tag{1'}$$

Substituting this equation (1′) into the above equation of the discounted present value, we can obtain the discounted present value of the indexed bond as follows:

$$PV_t = \frac{cou\cdot(1-\tau)\cdot p2/p1}{(1+R_t)^m} + \frac{p3/p1}{(1+r_t)^{g-1}}\left\{ cou\cdot(1-\tau)\cdot\left[ \frac{1-\dfrac{1}{(1+r_t)^n}}{r_t} \right] + \frac{prin}{(1+r_t)^n} \right\}$$

$$\tag{A.1}$$

where $g = m + 1$.[22]

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20. For example, if the next interest payment is three months from now, then $m = 3/6 = 0.5$.
21. For example, if the period between the month of the most recent price index and the month of the second interest payment from now is nine months, then $g = 9/6 = 1.5$.
22. The present formulation of the discounted present value follows that of Woodward (1990). However, they differ slightly in detail, and the method of discounting for nominal bonds differs completely.

Similarly, we can derive the discounted present value of nominal bonds as follows:

$$PV_t = \frac{cou \cdot (1 - \tau)}{(1 + R_t)^m} + \left\{ cou \cdot (1 - \tau) \cdot \left[ \frac{1 - \dfrac{1}{(1 + R_t)^n}}{R_t} \right] + \frac{prin}{(1 + R_t)^n} \right\}. \qquad (A.2)$$

Simultaneously solving equations (A.1) and (A.2), we can obtain the real and nominal interest rates.

**Figure 1 for the Appendix   Relationship between the Interest Payments of Indexed Bonds and the Price Index**



Note: Each interval in the above scale represents one month.
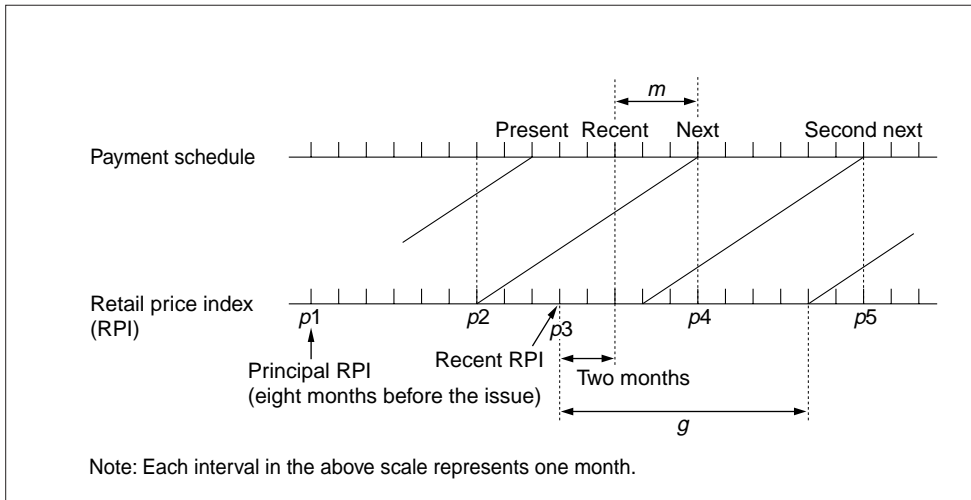
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