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Extracting Actuality from Judgement: A New Index for the Business Cycle

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We have developed an index for the business cycle that statistically extracts the actual state of the economy from the opinions of businessmen about their own business conditions. It is free from the common problem of conventional indexes that their values depend on the macroeconomic variables selected, which tends to be arbitrary. Our index is calculated using Bank of Japan TANKAN data. Empirical results show that the troughs and peaks projected by our index seem to have systematic relationships with business cycles that have been identified ex post by Japan's Economic Planning Agency.

I. Introduction

Economists are currently debating whether Japan's prolonged recession, which started in April 1991, has come to an end. It is not unusual, however, for economists to disagree, one of the main reasons being the absence of a commonly accepted definition of the business cycle—in discussing the cycle, some focus on GNP and its components, while others focus on the sentiment of businessmen and consumers.

Although the business cycle is generally recognized as representing the aggregated movement of macroeconomic variables, economists can still disagree on its actual state depending on which variables they regard as the most important. The significance of this problem is not sufficiently understood in public discussion of the business cycle, which is very often both an economically and politically important issue.

There have been several attempts to construct a numerical index representing the state of the business cycle: The diffusion index (DI) and the composite index (CI), published monthly by Japan's Economic Planning Agency, are two such examples. However, these suffer from several common problems such that they are not constructed based on any objective statistical criteria, and that their values critically depend on which

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macroeconomic variables are selected.

To overcome such problems, some attempts have been made in recent years to derive a numerical index based on objective statistical principles. A new index of coincident and leading economic indicators by Stock and Watson (1989) is such an index. The Stock and Watson (SW) index attempts to extract a common factor that is hidden in multiple time series data. The value of the SW index can differ significantly depending on the variables selected.

This paper adopts a different approach. Instead of measuring the co-movement of macroeconomic variables, it attempts to determine the actual state of the economy from businessmen's views on current and future economic conditions with respect to their firms. To this end, it utilizes *Bank of Japan TANKAN* ("*BOJ TANKAN*") data, which is based on a regular opinion survey of businessmen's opinions.

We use a variable that reflects businessmen's judgements about the state of the economy, instead of more conventional macroeconomic variables like the index of industrial production, because the business cycle is vividly captured and interpreted by businessmen, which is reflected on their opinions of current and future business conditions. Therefore, our approach is a more direct way of getting information about the current state of the economy and predicting where it is heading. Using an index that extracts the actual state of the economy from businessmen's judgements, the paper attempts to statistically predict the turning points of the business cycle.

The rest of the paper is organized as follows: Section II discusses the problems of conventional indexes of the business cycle with special reference to the DI and the CI published by Japan's Economic Planning Agency. It also discusses the SW index. Section III presents our method to construct an index that extracts the actual state of the economy from businessmen's judgements reported in *BOJ TANKAN* data. It then presents the results of our empirical analyses. It also comments on the current state of the Japanese economy and where it is heading. Section IV concludes the paper.

II. Main Business Cycle Indexes and Their Shortcomings

The DI and CI, published monthly by Japan's Economic Planning Agency, consist of three indicators: A leading indicator (13 series), a coincident indicator (11 series), and a lagging indicator (8 series).[1] The DI represents the ratio (expressed as a percentage) of

[1]Each indicator consists of the following series.

The Leading Indicator

(1) final demand inventory index (reverse cycle)
(2) index of raw material inventory (manufacturing, reverse cycle)
(3) new job openings (excludes new graduates)
(4) real machinery orders (private sector demand excluding shipbuilding and electricity)
(5) floor space of new construction (commercial)

the series that increased in value during the latest three months. The coincident DI is considered to be an important indicator of troughs and peaks: Its 50% line is frequently used for identifying the turning point of a business cycle. The official reference dates, however, are determined by the Economic Planning Agency on the basis of much broader information including other main economic indicators and professional opinions in addition to the coincident indicator. The CI also uses the same data series as the DI but, unlike the DI which only considers the direction of changes in the series, the CI takes into account degree of change. The CI is therefore considered to be useful in measuring the speed of the business cycle.

Although these business cycle indexes are widely used, it is also recognized that they suffer from various defects. First, they are not derived from any objective statistical principles. Second, their numerical values critically depend on the macroeconomic variables selected for inclusion and which tends to be arbitrary. Third, indexes are not consistent because as data series become less relevant they are often replaced by new series.[2] These indexes, widely used in Japan, were adopted from NBER indexes de-

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(6) floor space of new housing construction
(7) number of months of construction orders on hand
(8) consumer durable shipping index
(9) Nikkei commodity index (overall)
(10) money supply (M_2+CDs)
(11) profit index (manufacturing)
(12) investment index (manufacturing)
(13) projected business condition for small and medium firms (all industries)

The Coincident Indicator

(1) index of industrial production
(2) index of raw material consumption (manufacturing)
(3) electricity consumption
(4) index of operating ratio (manufacturing)
(5) index of labor input (manufacturing)
(6) shipping index of investment goods (excludes transportation vehicles)
(7) department store sales
(8) index of commercial sales (wholesale)
(9) current profits (all industries)
(10) sales of small and medium-sized firms (manufacturing)
(11) active openings (excludes new graduates)

The Lagging Indicator

(1) final demand inventory index
(2) index of raw material inventories (manufacturing)
(3) employment index of regular workers (manufacturing)
(4) real investment of corporations
(5) household consumption expenditures (nationwide, workers)
(6) corporate tax revenue
(7) unemployment ratio (reverse cycle)
(8) average contracted interest rate on loans and discounts (all banks)

[2]Kanoh (1990) tries to overcome these DI defects by using the survey data.

veloped in the United States. The above-mentioned defects have already been recognized in the United States.

Stock and Watson (1989) proposed a more reliable index of the business cycle that measures the state of the economy using a time-series model. The business cycle is generally defined as a co-movement of macroeconomic variables. Thus, Stock and Watson suggested decomposing total variations of macroeconomic variables into a common component and unique (idiosyncratic) components. The common component is thought to be driven by a single unobservable dynamic variable, which may be called the state of the economy. This suggests the following formulation:

$$X_{it} = \beta_i + \gamma_i(L)C_t + u_{it} \tag{1}$$

where X_{it} is a macroeconomic variable; β_i is an unknown parameter; and $\gamma_i(L)$ represents the lag polynomial:

$$\gamma(L) = \gamma_{i1}L + \gamma_{i2}L^2 + \cdots + \gamma_{ip}L^p$$
$$L^jX_t = X_{t-j}$$

In equation (1), $\gamma_i(L)\,C_t$ and u_{it} represent the common and unique components of macroeconomic variables to X_{it}. The common component is determined by unobservable factor C_t, which is called the "state of the economy." Unique factor u_{it} may have an autoregressive structure:

$$u_{it} = \Psi_i(L)\varepsilon_{\mathrm{it}} \tag{2}$$

Now let X_t represent a vector of M macroeconomic variables and u_t a vector of the corresponding unique components. Then, the SW model may be written as follows:

$$\begin{aligned} X_t &= \beta + \gamma(L)C_t + u_t \\ &= \beta + \gamma(L)C_t + \Psi(L)\varepsilon_t \end{aligned} \tag{3}$$

where $X_t=(X_{1t}, X_{2t}, \ldots, X_{Mt})'$, $\beta=(\beta_1, \beta_2, \ldots, \beta_M)'$, and $\epsilon_t=(\epsilon_{1t}, \epsilon_{2t}, \ldots, \epsilon_{Mt})'$ are M-dimensional vectors; C_t is a scalar; and $\gamma(L)$ and $\Psi(L)$ are M-dimensional vectors of lag polynomials:

$$\gamma(L) = (\gamma_1(L), \gamma_2(L), \ldots, \gamma_M(L))'$$
$$\Psi(L) = (\Psi_1(L), \Psi_2(L), \ldots, \Psi_M(L))'$$

We assume that ϵ_{it}'s are mutually and serially uncorrelated. Variable C_t, which represents the state of the economy, is assumed to have an autoregressive structure:

$$\begin{aligned} C_t &= \phi_1 C_{t-1} + \phi_2 C_{t-2} + \phi_3 C_{t-3} + \ldots + \phi_q C_{t-q} + \zeta_t \\ &= \phi(L)\,C_t + \zeta_t \end{aligned} \tag{4}$$

where ζ_t is a disturbance term with a normal distribution and uncorrelated with ϵ_{it}'s. Equations (3) and (4) constitute a dynamic factor model that Stock and Watson (1989) utilized in measuring the state of the economy.

The algorithm of a Kalman filter can be applied in estimating equations (3) and (4), taking equation (3) as an observational equation and equation (4) as a state equation (Appendix A). However, since these equations contain a large number of unknown parameters, their estimation generally requires the maximum likelihood method.[3]

For macroeconomic variables, Stock and Watson (1989) used four series from the U.S. economy: Industrial production, real personal income less transfer payments, real manufacturing and trade sales, and employee-hours in non-agricultural establishments. For Japan, Okusa (1992) applied and estimated the SW model using analogous Japanese data.

The SW index may be viewed as an attempt to extract a common factor from macroeconomic variables by extending the methodology of the principal component and factor analysis to multiple time-series data. Since principal component analysis constructs a linear combination of a given set of variables that explains variations in a suitable way, results can differ according to which variables are selected. Therefore, unless there is some objective criterion for selecting appropriate variables, simply defining the state of the economy as a set of macroeconomic variables will disregard an unresolved problem.

Thus, we do not attempt to measure the state of the economy using macroeconomic variables in this paper. Instead, we attempt to extract the actual state of the economy that must exist behind businessmen's views. If such judgements reflect an overall assessment of actual business conditions, then our index (that statistically extracts the actual state of the economy from such judgements) will be a more appropriate measure of the overall state of the economy than conventional indexes. Even if their judgements suffer from lags and other idiosyncratic tendencies, we can still extract "actuality" from "judgement" as long as their judgements are *systematically* based on the actual state of the economy. Thus, in the next section, we shall develop an index of the business cycle that statistically extracts "actuality" from "judgement" while utilizing the model developed by Stock and Watson (1989).

III. The Business Cycle Index Based on "*BOJ TANKAN*" Data

A. Construction of the Business Cycle Index Using "*BOJ TANKAN*" Data

This paper makes use of "*BOJ TANKAN*" data, which are based on Bank of Japan survey of corporate short-term business forecasts. To grasp changing business conditions, each quarter, Bank of Japan surveys the opinions of business leaders. The survey is conducted every February, May, August, and November, covering some 700 major companies nationwide.[4] It should be noted that the survey concerns itself with judge-

[3] *See* Okusa (1992) for a concrete algorithm.

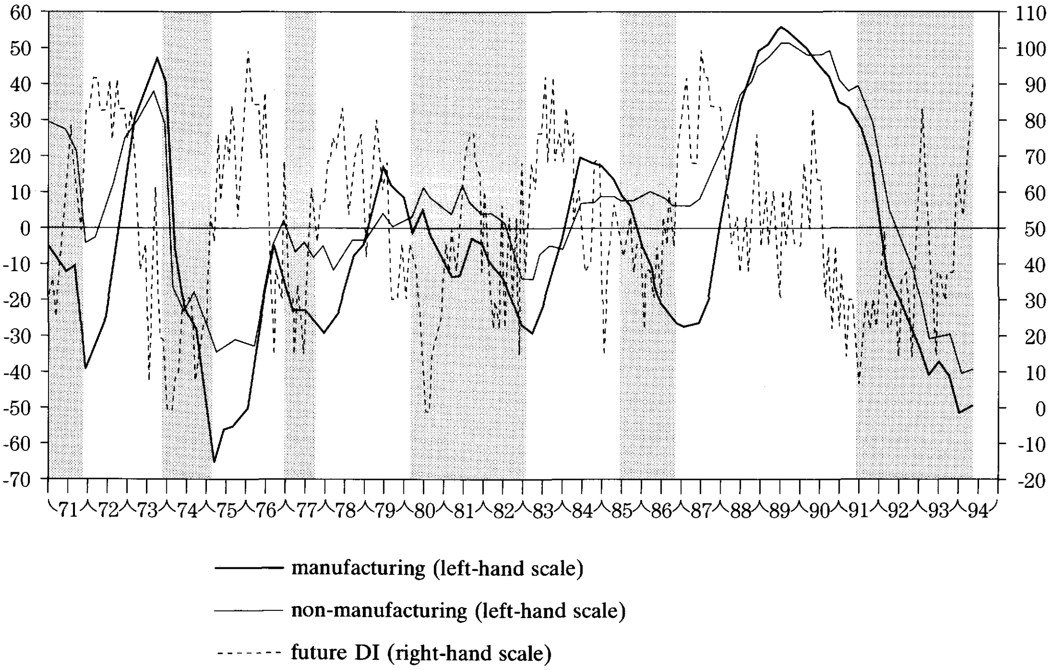
[4] These companies are selected among large non-financial corporations listed on the Tokyo Stock Exchange

ments of representative companies regarding the business conditions with respect to the firms, not economic conditions in Japan as a whole. Each company is asked to select one out of the three answers — "good," "not so good," or "bad" — regarding the "current" condition and expected "future" condition of the industry to which it belongs. The survey is conducted by Bank of Japan and published as "*BOJ TANKAN*" based index (DI).[5]

Bank of Japan considers the DI an important source of information about the state of the Japanese economy and frequently uses it to determine up-to-date economic conditions of Japanese industries and to forecast their future direction. Figures 1 and 2 show the "future" and "current" DI's, respectively, of the manufacturing and non-manufacturing industries — along with the leading and coincident DIs of Japan's Economic Planning Agency discussed in the previous section.

These figures show the following characteristics. First, in relation to the peaks and troughs of the business cycle that are determined *ex post* by the Economic Planning Agency, business judgements tend to precede actual economic downturns and to lag after the economy actually turns upward. These characteristics are evident in both manufactur-

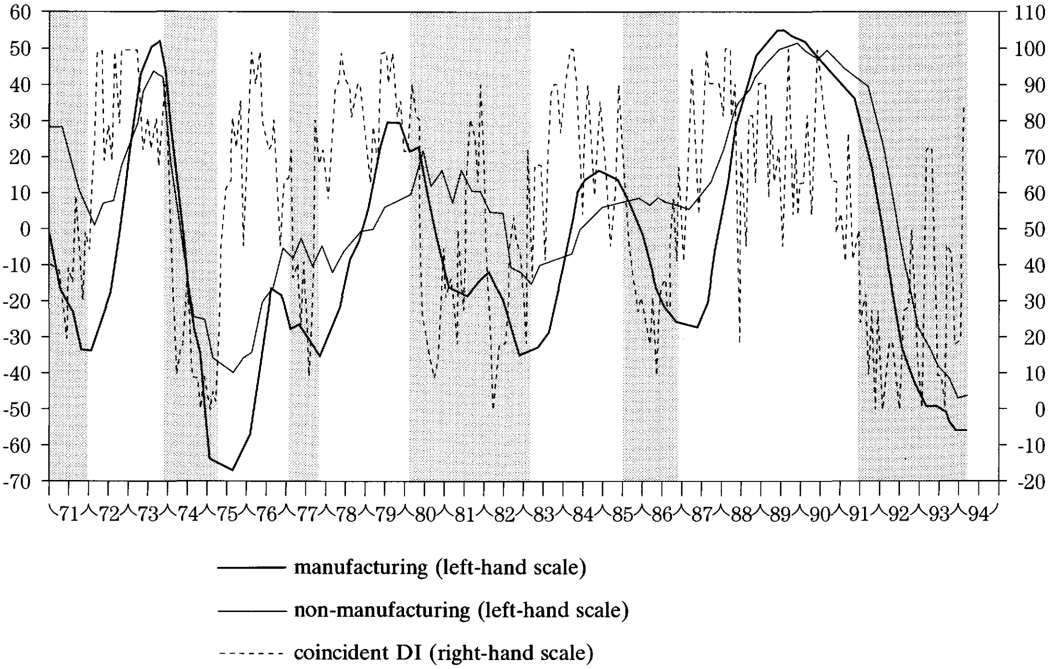
Figure 1
"Future" DI



with capital of more than ¥1 billion, and which are considered to be representative of their industries. *See* Annex for a more detailed explanation.

[5]DI= (the percentage of "good" in total responses)− (the percentage of "bad" in total responses).

Figure 2
"Current" DI



Note: Shaded area indicates period of economic downturn as determined by Japan's Economic Planning Agency.

ing and non-manufacturing industries. If we look at them more carefully, we find that actual troughs of the business cycle tend to come before the troughs of the "current" DI, but they seem to match relatively well those of the "future" DI. This being the case, it has been pointed out that "*BOJ TANKAN*" data is likely to reflect the "sentiment" of businessmen, and thus tends to reflect their pessimistic inclination.

These characteristics notwithstanding, if such problems can be statistically treated, we should be able to extract common information about the state of the business cycle. Indeed, we should be able to construct a more reliable index from DI data because it exhibits more smooth movement than individual macroeconomic variables.

Such an index can be constructed as follows. Suppose that, in the i-th data (*e.g.* the "current" DI of manufacturing industry) of the "*BOJ TANKAN*" survey we have obtained $r_{it} = (r_{1it}, r_{2it}, r_{3it})$, where r_{1it} is the number of companies that have answered "good," r_{2it} "not so good," r_{3it} "bad," from N_{it} companies. In this case, we can estimate the probability of a company choosing "good" (p_{1it}) as r_{1it}/N_{it}, and that of a company choosing "not so good" (p_{2it}) as r_{2it}/N_{it}. However, the data we are dealing with consists of qualitative rather than quantitative data, and further, the categories are of ordinal nature view. Thus, we need to effect the following data transformation:

$$X_{1it} = \log\left[r_{1it}/(r_{2it} + r_{3it})\right]$$
$$X_{2it} = \log\left[(r_{1it} + r_{2it})/r_{3it}\right] \tag{5}$$

where X_{1it} represents the log-odds of a company choosing "good," and similarly X_{2it} represents the log-odds of a company choosing "good" or "not so good." Note that $X_{1it} \leqq X_{2it}$ always holds. We can obtain the average $\xi_{it} = (\xi_{1it}, \xi_{2it})'$ of $X_{it} = (X_{1it}, X_{2it})'$ as (Appendix B):

$$\xi_{1it} = \log\left[p_{1it}/(p_{2it} + p_{3it})\right]$$
$$\xi_{2it} = \log\left[(p_{1it} + p_{2it})/p_{3it}\right] \tag{6}$$

and its variance-covariance matrix Σ_i as (Appendix B):

$$\Sigma_i = \begin{pmatrix} 1/(Np_{1it}) + 1/N(p_{2it}+p_{3it}) & 1/N(p_{1it}+p_{2it})\,(p_{2it}+p_{3it}) \\ 1/N(p_{1it}+p_{2it})\,(p_{2it}+p_{3it}) & 1/N(p_{1it}+p_{2it}) + 1/(Np_{3it}) \end{pmatrix}$$

This Σ_i can be estimated from the data as:

$$\hat{\Sigma}_i = \begin{pmatrix} 1/(r_{1it}) + 1/(r_{2it}+r_{3it}) & N/(r_{1it}+r_{2it})\,(r_{2it}+r_{3it}) \\ N/(r_{1it}+r_{2it})\,(r_{2it}+r_{3it}) & 1/(r_{1it}+r_{2it}) + 1/(r_{3it}) \end{pmatrix}$$

Note that X_{1it} and X_{2it} are not mutually independent.

We assume that the average $\xi_{it} = (\xi_{1it}, \xi_{2it})'$ of $X_{it} = (X_{1it}, X_{2it})'$ can be expressed as a function of the state of the economy (C_t) as:

$$X_{1it} = \beta_{i1} + \gamma_i(L)\,C_t + u_{1it}$$
$$X_{2it} = \beta_{i2} + \gamma_i(L)\,C_t + u_{2it} \tag{7}$$

Here we have assumed for simplicity that lag polynomial $\gamma_i(L)$ of C_t is common to both equations, and only coefficients β_{i1} and β_{i2} are different (Appendix C). Thus, X_{it} can be approximated as $X_{it} \sim N(\xi_{it}, \Sigma_i)$.

Now suppose that there are M survey samples in each period. Then, for each of such samples, we have to construct two equations and then estimate a system of 2M equations, taking into account their variance-covariance structure. In general, we assume independence between the i-th and j-th error terms, that is, between (X_{1it}, X_{2it}) and (X_{1jt}, X_{2jt}). Thus, the 2M×2M dimensional variance-covariance matrix of u_t will be given as follows:

$$\Sigma = \begin{pmatrix} \Sigma_1 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & \Sigma_2 & 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & \cdot & \cdot & \Sigma_M \end{pmatrix}$$

In comparison with the SW model, we assume a simpler serial correlation structure $\Psi_i(L)=0$ and, instead, a more complex cross-sectional variance-covariance structure for u_t. This is because what we have is "qualitative" sample data and therefore there is not much sense in imposing a complex serial correlation structure on the error terms.

B. Empirical Results

In our empirical study, we have used the survey data on current and future industrial conditions under the heading of "*Handan Chosa Gyokyo* (Survey on Business Judgement and Industrial Conditions)" in "*BOJ TANKAN*" data. Covered were 400 representative manufacturing and 250 representative non-manufacturing companies. This data has been log-transformed according to equation (5) and X_{it} in the final data set was constructed as:

$$X_{it} = \begin{bmatrix} X_{11t} \\ X_{21t} \\ X_{12t} \\ X_{22t} \\ X_{13t} \\ X_{23t} \\ X_{14t} \\ X_{24t} \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{current judgement} \\ \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{future judgement} \\ \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{current judgement} \\ \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{future judgement} \end{matrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} \text{manufacturing.} \\ \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} \text{non-manufacturing} \end{matrix}$$

To construct a model, we need to determine the lag structures of equations (3) and (4). Here we have assumed for simplicity that $\gamma(L)$ has the same degree of lags for all i's $(1, 2, \ldots, 4)$. We have also decided to include in the model all terms of $\phi(L)$ and $\gamma(L)$ with the number of lags up to the maximum degree.

For the error terms, we have introduced four parameters σ_1^2, σ_2^2, σ_3^2, and σ_4^2 into the variance-covariance matrix such that:

$$\Sigma = \begin{pmatrix} \sigma_1^2\Sigma_1 & 0 & 0 & 0 \\ 0 & \sigma_2^2\Sigma_2 & 0 & 0 \\ 0 & 0 & \sigma_3^2\Sigma_3 & 0 \\ 0 & 0 & 0 & \sigma_4^2\Sigma_4 \end{pmatrix}$$

This is done to handle the over-dispersion problem that variance of a binomial or multinominal distribution often becomes greater than that set by a model when model specification is insufficient. If our model specification is sufficient, then those parameters should have estimated values that are close to one. We also normalized the variance of ζ_t, the error term of C_t, to be one. This is necessary for model identification.[6]

Under those specifications, we have applied Akaike's Information Criterion (AIC) and Bayesian Information Criterion (BIC), and have selected a model that minimizes those criteria. Here we have obtained a minimum value of the likelihood function from

[6]This can be understood as follows: Suppose the standard deviation of ζ doubles. Then, if ϕ_i's similarly doubles while β_i's and γ_i's all halve, then the relationship between the model and the data will be unaffected. Therefore, to determine all the parameters, we need to fix one of them.

iterative calculations by the scoring method.[7] Table 1 shows the estimated AIC and BIC values for several models. By either criterion, a model with $p=2$ and $q=4$ will be selected, where p is the dimension of $\gamma(L)$ and q is the dimension of $\phi(L)$. The result was the same when the lag structure of the judgement about the future state of the economy was shifted backward by one lag relative to the judgement about the current state.

Table 2 shows the estimated coefficients of the above model. We can read from these results the following characteristics:

(1) $p=2$ suggests that judgements are influenced by the current and preceding state of the economy. The coefficient (γ_{i2}) of C_{t-1} is greater than that (γ_{i1}) of C_t for judgement about the current state, while the reverse is true for judgement about the future state. This corresponds to the fact that judgement about the future state precedes judgement about the current state. We also find that judgement lags reality.

(2) The coefficients for manufacturing are greater than those of non-manufacturing. This implies that the influence of business reality on judgement is greater in the manufacturing sector than in the non-manufacturing sector.

(3) C_t has an autoregressive structure with relatively many lags (q=4). Moreover, all solutions of the characteristic equation for the autoregressive process of C_t are imaginary. This implies that the movements of C_t have a cyclical and a relatively complex structure.

Table 1
Estimation Results

q \ p	1	2	3
1	−536.18	−565.26	−574.00
	−535.83	−564.69	−573.20
2	−589.45	−595.46	−590.72
	−588.99	−594.77	−589.81
3	−595.93	−596.63	−592.87
	−595.36	−596.36	−591.85
4	−595.34	−597.27	−596.45
	−594.66	−596.83	−595.31
5	−591.52	−593.88	−592.56
	−590.72	−592.86	−591.31

(above : AIC, below : BIC)

[7]The calculations were effected by a SAS program, using a HITAC M680 computer. The average CPU computing time was about six hours.

Table 2
Estimation Results (Parameter Values)
$(p, q) = (2, 4)$

β_{11}	-1.284835	γ_{11}	0.0107737	γ_{12}	0.0120676	ϕ_1	3.4386187	σ_1^2	1.694047
β_{21}	1.3824363		(0.0172642)		(0.0188326)		(0.1334236)		(0.0888133)
β_{12}	-1.318292	γ_{21}	0.0193493	γ_{22}	0.0017573	ϕ_2	-4.679545	σ_2^2	2.081692
β_{22}	1.5977187		(0.0157105)		(0.0168973)		(0.3614731)		(0.1067380)
β_{13}	-1.027615	γ_{31}	0.0055984	γ_{32}	0.0087284	ϕ_3	2.977319	σ_3^2	3.1909692
β_{23}	1.8378636		(0.0115352)		(0.0125461)		(0.3495013)		(0.1517868)
β_{14}	-1.067595	γ_{41}	0.0095219	γ_{42}	0.003584	ϕ_4	-0.745829	σ_4^2	3.5479173
β_{24}	1.9940852		(0.0108593)		(0.0116247)		(0.1207321)		(0.1683944)

Note: Figures in ( ) are standard errors.

(4) σ^2 for manufacturing is larger than that for non-manufacturing. And σ^2 for judgement about the future state is larger than that for judgement about the current state. These reflect the explanatory power of each model.

Figure 3 shows the estimated state of the economy (C_t) from the first quarter of 1966 to the third quarter of 1993. From this figure, we can read the following characteristics: (1) C_t is smoother than the coincident indicator; (2) the troughs of C_t and the business cycle coincide, and therefore we can identify troughs without looking for a cross of leading and coincident DI's; (3) C_t does not exhibit a pessimistic tendency that was apparent in the DI based on "*BOJ TANKAN*" data.

Next, let us look at projected values of C_{t+k} (k=1, 2, ...) based on equation (4) as shown in Figure 4. According to the projection based on "*BOJ TANKAN*" data up to August 1993, the state of the economy worsens after autumn of 1993 after a brief plateau and hits a second bottom in the first quarter of 1995. As this projection is based on data up to August 1993, it does not take into consideration the effects of the seventh official discount rate reduction in autumn 1993 nor the announcement effect of news of the 'New Package of Economic Measures' in 1994.

It would be an interesting exercise to project the effects of income tax reduction and deregulation that would presumably affect both business reality and judgement. Thus, we have performed a similar projection for the second and third quarters of 1994 with the additional data of the fourth quarter of 1993 and the first quarter of 1994. In this projection, we have retained the same parameter values for γ, ϕ and σ in Table 2 that were used in the first projection.

We have compared the first and second projections for the period after the fourth quarter of 1994 (Figure 4). The new projection shows that the economy will recover more quickly from the current recession compared with the first projection that was based on data up to August 1993. This suggests that business judgement about the state of the

Figure 3
Estimated State of the Economy (C_t)

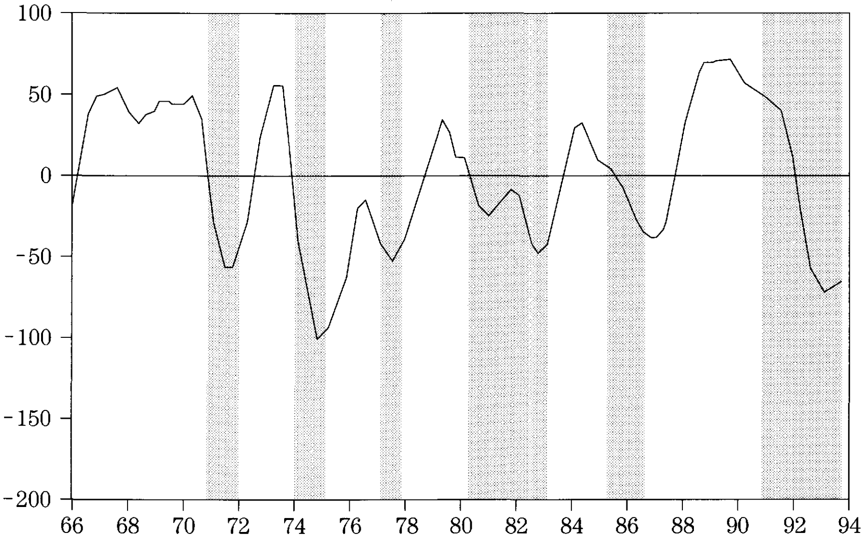
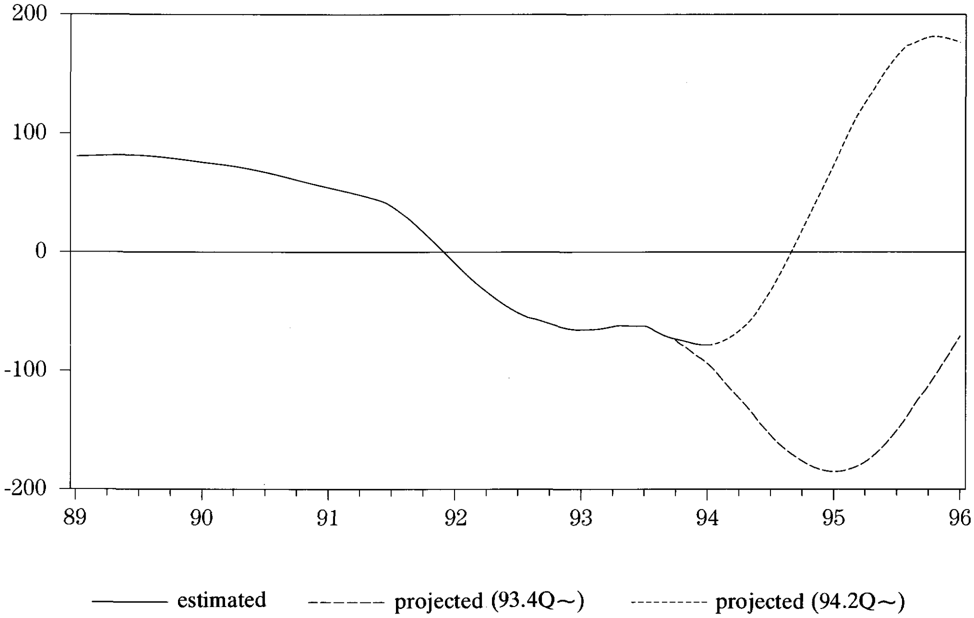


Figure 4
Comparison of Two Projections



economy has significantly improved from late 1993 to early 1994.

IV. Concluding Remarks

This paper has been an attempt to construct a statistical index that captures the "state of the economy," which has never been defined unambiguously. We adopted a new approach that differs from the previous approach of Stock and Watson (1988,1989) and that of Okusa (1992) who applied the methodology of Stock and Watson to Japanese data. Our approach has been to extract statistically a common factor (C_t) from business judgement survey data about the "state of the economy."

The common factor (C_t) extracted from businessmen's judgement of the state of the economy seems to have systematic relationships with business cycle reference dates identified *ex post* by Japan's Economic Planning Agency. Furthermore, C_t based on the latest data exhibits a marked upturn around the first quarter of 1994.

This paper has attempted to extract statistically a common factor from the survey data, and from which a numerical index has been constructed. However, further studies are necessary to evaluate its usefulness as an index for the state of the economy. While this paper has mainly focused on developing the statistical theory for the index, improving its practical application should be a topic of future research.

Appendix A.

Equations (3) and (4) are developed as follows:

$$
\begin{bmatrix} X_{11} \\ X_{21} \\ \cdot \\ \cdot \\ \cdot \\ X_{1M} \\ X_{2M} \end{bmatrix}
=
\left[\begin{array}{ccccc:cccc:c}
1 & & & & & \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1p} & \\
 & 1 & 0 & & & \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1p} & \\
 & & \cdot & & & \cdot & \cdot & & \cdot & \\
 & & & \cdot & & \cdot & \cdot & & \cdot & 0 \\
 & 0 & & \cdot & & \cdot & \cdot & & \cdot & \\
 & & & & 1 & \gamma_{M1} & \gamma_{M2} & \cdots & \gamma_{Mp} & \\
 & & & & 1 & \gamma_{M1} & \gamma_{M2} & \cdots & \gamma_{Mp} &
\end{array}\right]
\begin{bmatrix} \beta_{11} \\ \beta_{21} \\ \cdot \\ \cdot \\ \beta_{1M} \\ \beta_{2M} \\ C_t \\ C_{t-1} \\ \cdot \\ \cdot \\ C_{t-q-1} \end{bmatrix}
+
\begin{bmatrix} u_{11} \\ u_{21} \\ \cdot \\ \cdot \\ \cdot \\ u_{1M} \\ u_{2M} \end{bmatrix}
$$

$$
\begin{bmatrix} \beta_{11} \\ \beta_{21} \\ \cdot \\ \cdot \\ \beta_{1M} \\ \beta_{2M} \\ C_t \\ C_{t-1} \\ \cdot \\ \cdot \\ C_{t-q+1} \end{bmatrix}
=
\left[\begin{array}{ccccc:ccccc}
1 & & & & & & & & & \\
 & 1 & 0 & & & & & & & \\
 & & \cdot & & & & & 0 & & \\
 & & & \cdot & & & & & & \\
0 & & & 1 & & & & & & \\
 & & & & 1 & & & & & \\
\hdashline
 & & & & & \phi_1 & \phi_2 & \cdot & \cdot & \phi_q \\
 & & & & & 1 & 0 & \cdot & \cdot & 0 \\
 & & 0 & & & \cdot & \cdot & & & \cdot \\
 & & & & & \cdot & & \cdot & & \cdot \\
 & & & & & 0 & \cdot & \cdot & 1 & 0
\end{array}\right]
\begin{bmatrix} \beta_{11} \\ \beta_{21} \\ \cdot \\ \cdot \\ \beta_{1M} \\ \beta_{2M} \\ C_{t-1} \\ C_{t-2} \\ \cdot \\ \cdot \\ C_{t-q} \end{bmatrix}
+
\begin{bmatrix} 0 \\ \cdot \\ \cdot \\ 0 \\ \varepsilon_{11} \\ \varepsilon_{21} \\ \cdot \\ \cdot \\ \varepsilon_{1M} \\ \varepsilon_{2M} \end{bmatrix}
$$

Appendix B.

This appendix shows how to derive each element of variance-covariance matrix Σ, using X_{1it} and X_{2it} as an example. The covariance of $X_{it}=(X_{1it}, X_{2it})$ is given by:

$$\mathrm{Cov}\,(X_{1it}, X_{2it})=\mathrm{Cov}\,(\log[r_{1it}/r_{2it}+r_{3it}], \log[r_{1it}+r_{2it}/r_{3it}])$$

the LHS of which can be further developed as follows:

$$
\begin{aligned}
&\mathrm{Cov}\,(\log[r_{1it}/r_{2it}+r_{3it}], \log[r_{1it}+r_{2it}/r_{3it}]) \\
&= \mathrm{Cov}\,(\log[r_{1it}], \log[r_{1it}+r_{2it}])-\mathrm{Cov}\,(\log[r_{1it}], \log[r_{3it}]) \\
&\quad - \mathrm{Cov}\,(\log[r_{2it}+r_{3it}], \log[r_{1it}+r_{2it}])+\mathrm{Cov}\,(\log[r_{2it}+r_{3it}], \log[r_{3it}]) \ldots\ldots\ldots (*)
\end{aligned}
$$

Next, we develop each term of equation $(*)$, using the following propositions (1)-(3):
<Propositions>

(1) $V(f(x)) = f'(E(x))^2V(x)$
(2) $\mathrm{Cov}\,(f(x_1), g(x_2)) = f'(E(x_1))\, g'(E(x_2))\, \mathrm{Cov}(x_1, x_2)$
(3) $\mathrm{Cov}\,(x_1, x_2) = -Np_{x_1}p_{x_2}$

(1) The First Term:

$$
\begin{aligned}
&\mathrm{Cov}\,(\log[r_{1it}], \log[r_{1it}+r_{2it}]) \\
&\quad = 1/E\,(r_{1it}) \cdot 1/E\,(r_{1it}+r_{2it}) \cdot \mathrm{Cov}\,(r_{1it}, r_{1it}+r_{2it}) \\
&\quad = p_{3it}/N\,(p_{1it}+p_{2it})
\end{aligned}
$$

(2) The Second Term:

$$
\begin{aligned}
&\mathrm{Cov}\,(\log[r_{1it}], \log[r_{3it}]) \\
&\quad = 1/E(r_{1it}) \cdot 1/E(r_{3it}) \cdot \mathrm{Cov}\,(r_{1it}, r_{3it}) \\
&\quad = -1/N
\end{aligned}
$$

(3) The Third Term:

$$\begin{aligned}&\mathrm{Cov}\,(\log\,[r_{2it}+r_{3it}],\ \log\,[r_{1it}+r_{2it}])\\&\quad= 1/E\,(r_{1it})\cdot 1/E\,(r_{1it}+r_{2it})\cdot \mathrm{Cov}\,(r_{1it},\ r_{1it}+r_{2it})\\&\quad= -p_{1it}p_{3it}/N\,(p_{1it}+p_{2it})\,(p_{2it}+p_{3it})\end{aligned}$$

(4) The Fourth Term:

$$\begin{aligned}&\mathrm{Cov}\,(\log\,[r_{2it}+r_{3it}],\ \log\,[r_{3it}])\\&\quad= 1/E\,(r_{2it}+r_{3it})\cdot 1/E(r_{3it}\cdot \mathrm{Cov}\,(r_{2it}+r_{3it},\ r_{3it}))\\&\quad=p_{1it}/N\,(p_{2it}+p_{3it})\end{aligned}$$

From (1) – (4), we obtain:
The RHS of (∗)

$$\begin{aligned}&= p_{3it}/N(p_{1it}+p_{2it})+1/\mathrm{N}+\ p_{1it}p_{3it}/N\,(p_{1it}+p_{2it})\,(p_{2it}+p_{3it})+p_{1it}/N\,(p_{2it}+p_{3it})\\&= (p_{1it}+p_{2it}+p_{3it})^2/N\,(p_{1it}+p_{2it})\,(p_{2it}+p_{3it})\\&= 1/N\,(p_{1it}+p_{2it})\,(p_{2it}+p_{3it})\end{aligned}$$

Thus, substituting r_{it}/N into p_{it}, we obtain:

$$\hat{\mathrm{Cov}}\,(X_{1it},\ X_{2it}) = N/(r_{1it}+r_{2it})\,(r_{2it}+r_{3it})$$

Similarly, we can obtain the variance of X_{1it} and X_{2it}. Finally, we can obtain the variance-covariance matrix of $X_{it}=(X_{1it},\ \mathrm{X}_{2it})$ as follows:

$$\hat{\Sigma}_i = \begin{pmatrix} 1/(r_{1it})+1/(r_{2it}+r_{3it}) & N/(r_{1it}+r_{2it})\,(r_{2it}+r_{3it}) \\ N/(r_{1it}+r_{2it})\,(r_{2it}+r_{3it}) & 1/(r_{1it}+r_{2it})+1/(r_{3it}) \end{pmatrix}$$

Appendix C.

This appendix presents a model that describes how sample companies respond to a questionnaire regarding their opinion about the state of the economy.

Let C_t represent the state of the economy, which is, of course, an unknown variable. Let us suppose that a company (say j) responds to the questionnaire according to the following rule:

If $C_t > S_{1j}$, then the state is good or will be good .... (1)
If $S_{1j} > C_t > S_{2j}$, then the state will not change .... (2)
If $S_{2j} > C_t$, then the state is bad or will be bad .... (3)

where S_{1j} and S_{2j} represent critical values of j.

Now suppose that S_{1j} and S_{2j} are different among companies and they are normally distributed as $N(\xi_1,\ \sigma^2)$ and $N(\xi_2,\ \sigma^2)$. Then, the probability of company j to choose (1) is given by:

$$Pr\,[C_t > S_{1j}] = Pr[C_t-\xi_1)/\sigma > Z] = \Phi\,(\beta_1+\gamma C_t)$$

where $\beta_1=(\xi_1/\sigma)$ and$\gamma=(-1/\sigma)$ represent unknown parameters, Z a random variable that has a standard normal distribution, and Φ its distribution function.

Similarly, the probabilities of company j to choose (2) and (3) are given, respectively, as:

$$\begin{aligned} Pr\,[S_{1j} > C_t > S_{2j}] &= Pr[Z > (C_t-\xi_1)/\sigma] - Pr\,[(C_t-\xi_1)/\sigma > Z] \\ &= \Phi\,(\beta_2+\gamma C_t) - \Phi(\beta_1+\gamma C_t) \end{aligned}$$

$$\begin{aligned} Pr\,[S_{2j} > C_t] &= Pr\,[(C_t-\xi_1)/\sigma > Z] \\ &= 1-\Phi\,(\beta_2+\gamma C_t) \end{aligned}$$

This model is called a multivariate Probit model. Note that, to reduce the number of parameters, we have introduced a common parameter $\gamma = (-1/\sigma)$ in the two Φ's in the model.

On the other hand, the logistic distribution function is given by:

$$\Psi\,(x) = \exp\,(x)/\{\,1 + \exp\,(x)\}$$

This logistic distribution function $\Psi\,(x)$ takes values very close to those of normal distribution function $\Phi\,(x)$ while it is often much easier to deal with in calculation. For example, we can use the following simple relationship for $\Psi(\beta + \gamma C_t)$:

$$\log\,[\Psi(\beta+\gamma C_t)/\{1-\Psi(\beta + \gamma C_t)\}] = \beta + \gamma C_t$$

Thus, we deliberate the so-called multivariate Logit model, which replaces Φ by Ψ in the multivariate Probit model.

Annex

Outline of the Short-term Economic Survey
of Enterprises in Japan ("*TANKAN*")
Compiled by Bank of Japan

The Short-term Economic Survey of Enterprises in Japan (so-called "*Tankan*") was initiated in 1957 and has been compiled quarterly in February, May, August and November by the Research and Statistics Department of Bank of Japan. The survey covers more than 700 principal enterprises nationwide (banking and insurance companies excluded), most of which are capitalized at over ¥1 billion. The sales value of the principal enterprises covered by the survey represents 72% of the total sales value of all Japanese enterprises capitalized at ¥1 billion or more, and 30% of the total sales of all Japanese enterprises capitalized at ¥10 million or more (as of fiscal 1992).

The survey consists of both quantitative and qualitative data. Quantitative data comprises real economic activity (production, sales, fixed investments, inventories, exports, number of employees, etc.) and financial conditions (profits, borrowing, etc.). Qualitative data, the "Judgement Survey," takes the form of diffusion indexes (DIs) that reflects entrepreneurs' views on present and future business conditions with respect to a wide range of items including inventory level; production capacity; financial position; lending attitude of financial institutions; changes in interest rates, output prices, etc.

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