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The EaR Model and the Expanded VaR Model: An Application to Bond Portfolios

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Abstract:

In this paper, as a technique to measure the market risk of banking accounts, we construct models for the statistical measurement of the movement of period profit or loss. In doing so, we present two frameworks, namely, the earning-at-risk (EaR) model, which captures the period profit or loss in terms of the realized profit or loss according to the currently used cost method, and the expanded value-at-risk (VaR) method which also includes the movement of market valuation as part of the period profit or loss.

We then examine the usefulness and limitations of both models by performing simulations on bond portfolios. It is shown that the EaR model is limited by the fact that it is not capable of comparing strategies from the standpoint of risk versus return. When using the EaR model as a risk measurement tool, it is essential to conduct long-term simulations.

Key Words: earning-at-risk (EaR); value-at-risk (VaR); period profit and loss; cost method; market value method

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1. Introduction

The market risk of banking accounts held by Japanese banks is considerably high. However, there is at present no definitive answer as to how to define the risk, how to measure the risk, and what actions to take in relation to it.

As to the market risk of trading accounts, it is well known that the value-at-risk (VaR) method, which is a present-value-based probabilistic risk evaluation method, is increasingly becoming a standard for risk measurement. On the other hand, there is no consensus as to how to measure the risk of banking accounts. There is no consensus even for such a fundamental question as whether the risk should be measured in present value terms (as in the case of the VaR model) or in terms of realized period profit or loss in the light of the special characteristics of banking accounts. From the former standpoint, expanding the concept of VaR is suggested; from the latter standpoint, earning-at-risk (EaR) is suggested as a direction for future research. However, both are still at the developmental stage, so that neither is ready for practical application.

In this paper, we will thus construct a simple EaR model as well as a simple expanded VaR model, by focusing our attention on bond portfolios, whose market risk is relatively great in comparison to the other components of banking accounts. We will then clarify the usefulness and limitations of these models by performing simulations.

The paper is organized as follows. In Section 2, we will construct a simple EaR model and a simple expanded VaR model. In Section 3, we will perform simulations based on imaginary bond portfolios, in order to clarify the usefulness and limitations of both models. Finally, in Section 4, we will consider some remaining issues relating to the market risk management of banking accounts.

At the outset, the main conclusions of the paper may be summarized as follows:

1. Both models, as constructed in the paper, are superior to the so-called scenario method.
2. The EaR model yields a smaller measurement of risk than the expanded VaR model.
3. The EaR model displays a larger change in the shape of the distribution of profits corresponding to a change in investment strategy.
4. The EaR model is not capable of comparing strategies in terms of risk versus return.

2. Construction of Models

2-1. Trading Accounts and Banking Accounts

The difference between the trading account and the banking account is discussed in detail by Kiyama, et al. (1996). Here, we will only summarize the risk characteristics of different types of financial products held in banking accounts, such as deposits and loans, bonds and stocks. The risk characteristics of different financial products held by Japanese banks are shown in Diagram 1. The characteristics of bond portfolios, which are the focus of this paper, are that, although the liquidity of bonds themselves is high, the actual controllability of risk[1] is low because a certain minimum balance must be maintained to satisfy the collateral requirement[2] .

Diagram 1. Risk Characteristics by Category

	banking accounts			trading accounts
	deposits and loans	bonds	stocks	
liquidity	#	&[3]	%	&
identification of positions	# prepayment[4] rollover[5]	&	&	&

[1] Needless to say, because of the availability of various hedging instruments, it is theoretically possible to transform the long position to a short position. However, this is difficult in practice because of various restrictions on futures outstanding (e.g., on the type of futures instruments) and the use of the cost method, which presumes that bonds are held until maturity and not subject to dealing tilted bond portfolios.

[2] There are various collateral requirements, such as those associated with exchange settlement, transactions in short-term funds and deposits at organized settlement systems.

[3] In practice, there are bonds with extremely low liquidity, such as privately placed corporate bonds. However, these bonds are ignored because their share in the bond portfolios of city banks is small.

[4] This problem occurs in those financial products which allow a cancellation of contracts before maturity, such as housing loans and time deposits. Although it is possible to express this mathematically as an option which the customer maintains the right to exercise, the difficulty lies in the fact that one cannot adequately explain the exercise of an option by economic rationality alone.

identification of basis risk	# policy-determined interest rates[6]	%[7]	#	%
risk control	%[8]	# collateral	# cross-holding	&

*notations: &...easy ,%...relatively easy ,#...difficult

2-2. EaR and Expanded VaR

As stated before, the most distinctive characteristic of banking accounts lies in their low liquidity (low actual risk controllability). For this reason, the holding period of banking accounts becomes long, making it important to think of period profit or loss. Thus, we will attempt to construct a model to measure the movement of period profit or loss as the risk. In so doing, we define the EaR model as a model in which, according to the currently used cost method, only the realized profit or loss is treated as the period profit or loss, and the Expanded VaR model as a model in which the movement of market value is also included in the period profit or loss. On the basis of the simple cash flow depicted in Diagram 2, they can be expressed in terms of the following

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[5] In the retail business of deposits and loans, there exist some financial products which are highly likely to be renewed at maturity. That is to say, for these products, actual maturity is uncertain.

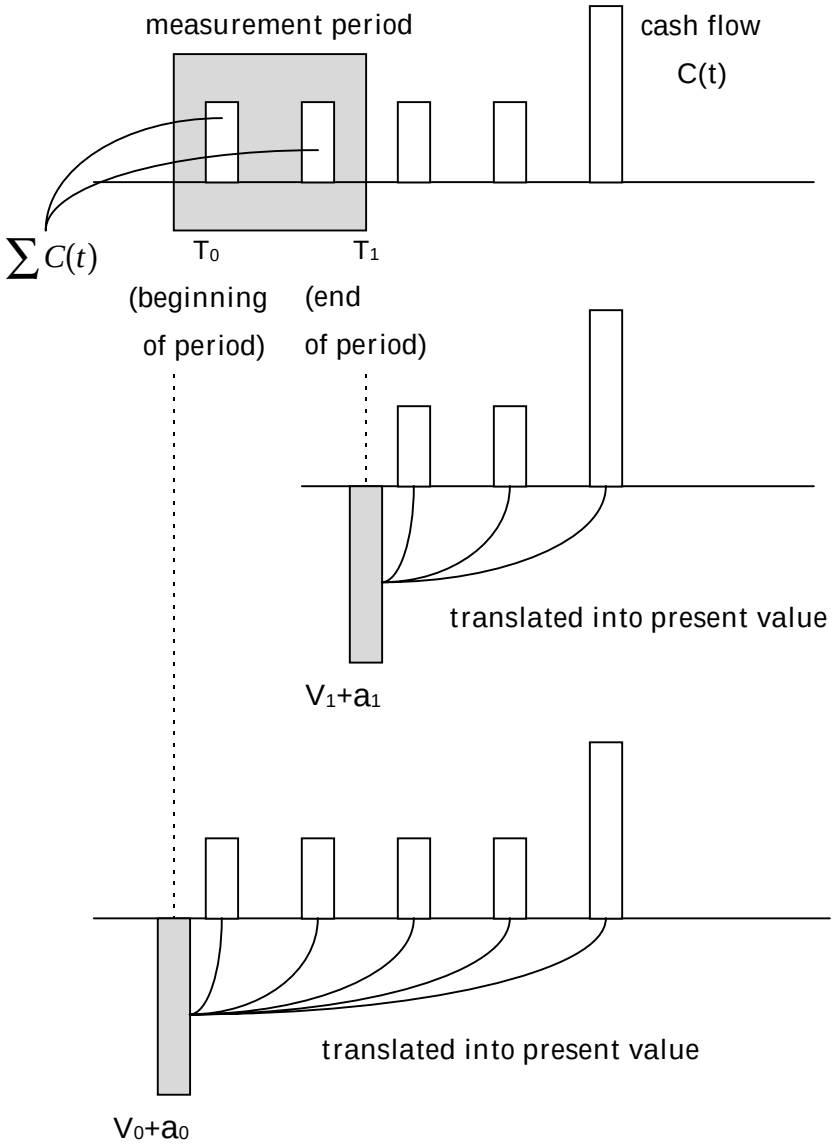
[6] For some financial products held in banking accounts, the interest rates such as the short-term prime and long-term prime lending rates, are determined by reference to policy-determined interest rates. Unlike financial products whose interest rates are determined by reference to market rates, these interest rates are changed only when the reference interest rates are changed by a certain amount (i.e., changes are step-wise). Thus, a different model specification is required.

[7] The basis risk (individual risk) of bonds is also a troublesome problem. However, we are here assuming a portfolio consisting primarily of Japanese government bonds, so that we have minimized it in a relative sense.

[8] The situation must be different from bank to bank. However, the general practice is believed to be that the interest rate risk of deposits and loans is concentrated in the ALM section, which in turn identifies the daily flow of deposits and loans, and controls the risk by using interest rate futures, bond futures and swaps.

equations[9].

Diagram 2. An Example of Cash Flow



<the definition of notations>

C(t): cash flow B: book value (acquisition cost)

: market value (excluding accrued interest)

r(t): market interest rate a:accrued interest T: time

* subscript 0 refers to the beginning of the period; subscript 1 refers to the end of the period.

[9] For details, see Merton and Ono (1996).

EaR = realized period cash flow[10]

(period profit or loss) + accrued interest

$$= \sum_{T_1 \geq t \geq T_0} C(t) + (a_1 - a_0)$$

Expanded VaR = realized period cash flow

(period profit or loss) + market value at the end of the period

- market value at the beginning of the period

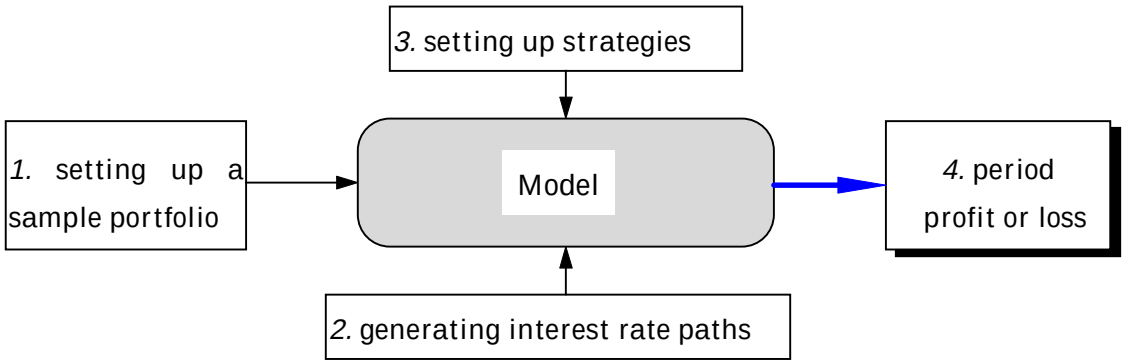
$$= \sum_{T_1 \geq t \geq T_0} C(t) + (V_1 + a_1) - (V_0 + a_0)$$

Now, needless to say, if we add up all of the period profits or losses from the beginning of a transaction to the end, EaR (the cost method) and expanded VaR (the market value method) will both yield the same profits, $\sum C(t) - B$. In view of this fact, some argue that the difference between the two methods only concerns the question of allocation, namely, when to realize the profit or loss, and that risk can appropriately be managed by the market value method, which is an extension of VaR. On the other hand, as long as bank's income statements are made in terms of realized profits or losses, others argue that risk should be managed in terms of realized period profits or losses according to the cost method. These points will be examined by performing simulation exercises in Section 3.

2-3. An Outline of the Models

In this paper, we construct an EaR model and an expanded VaR model, which both measure the movement of profit or loss during the subsequent 5-year period as the risk. In calculating the risk, we use an approach based on Monte Carlo simulations. Both models have the same basis structure, the only difference being in the methodology to calculate the period profit or loss (see Diagram 3).

Diagram 3. The Basic Structure of the Models



[10] Only at maturity, -B is to be added.

1. setting up a sample portfolio (initial values)[11]

We construct a representative sample bond portfolio of Japanese city banks, a portfolio which is financed by short-term funds and invested in bonds. The size of the portfolio is set equal to 2 trillion yen, which was the average balance held by city banks for the September 1996 period.

2. generating interest rate paths[12]

From the historical data on Japanese government bonds and 3-month CDs, numbers are generated randomly according to the multivariate normal distribution in order to generate paths of interest rates.

3. setting up strategies (investment policy)

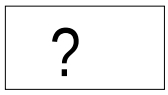
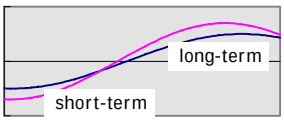
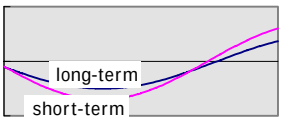
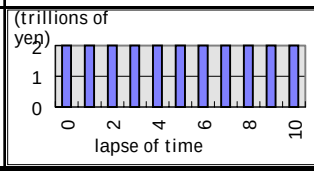
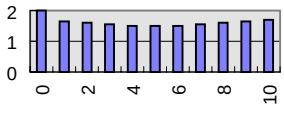
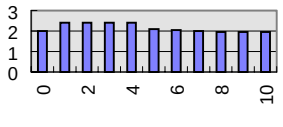
The following three strategies will be set up according to different scenarios (or interest rate expectations)."Strategy A" keeps the same balance regardless of the interest rate expectation. "Strategy B" reduces the balance in expectation of a future rise in interest rates. "Strategy C" increases the balance in expectation of a decline in interest rates for the time being. Diagram 4 depicts the interest rate expectation and the corresponding balance of bonds associated with each strategy. Appendix 3 shows how the ladder changes.

Simulation is performed for 10 semi-annual periods for 5 years. For each period, strategy-driven transactions are made right in the middle. The transactions prices are calculated by the interest rate generated in *2.* above. Underwriting of (10-year) bonds is also made in the middle of each period. The coupon rate is set equal to the interest rate on 10-year bonds, as generated in *2.*.

[11] For details, see Appendix 1.

[12] For details, see Appendix 2.

Diagram 4. The Scenarios, Interest Rate Expectations, and the Period Balances of the Three Strategies

	Strategy A	Strategy B	Strategy C
Scenario	Interest rate expectations are difficult to form. A certain balance (2 trillion yen) is to be held constantly.	With current interest rates being at the bottom, a rise in interest rates is expected in the future. The balance is to be reduced for now, and will be restored after a certain period of time (in 4 to 5 years).	Structural adjustment is still under way, so that a continued decline in interest rates is expected for the time being. The balance is to be increased for now, and is planned to be reduced after a certain period of time (in 2 to 3 years).
Interest rate expectations			
Changes in the balance			

4. Calculation of period profit or loss

Based on the idea in (2-2) of the previous section, we calculate the period profit or loss in the following manner.

Expanded VaR:period profit or loss = funds profit + redemption profit or loss + sales gain or loss + change in valuation gain or loss

EaR :period profit or loss = funds profit + redemption profit or loss + sales gain or loss

The variance in this period profit or loss is recognized as the risk.

5. Other assumptions

(i) Accounting treatment is made by the cost method. No amortization[13] is made; (ii) No consideration is made of the securities transactions tax or the reserves for government bond price fluctuations[14] ; (iii) Bonds are entirely redeemed at maturity.

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[13] This refers to the practice of realizing a certain amount of the acquisition value of "over-par" bonds as an accounting profit or loss in every period until maturity.

[14] This is a system whereby taxable reserves are maintained corresponding to 2 percent of the outstanding value of Japanese government bonds as well as 2 percent of the increase in the balance during each period, in preparation for a future loss.

3. Simulation Results

3-1 Comparison of the EaR Model and the Expanded VaR Model

The simulation results are shown in Diagram 5 and Diagrams 6 and 7. From these, the following points can be made.

Diagram 5. The Time-series Profiles of Expected Profits and the Measurement of Risk[15] (Strategy A)[16]

(in hundreds of millions of yen)

		period 1	period 2	period 3	period 4	period 5	period 6	period 7	period 8	period 9	period 10
expected profits	expanded VaR	253	260	267	251	238	266	257	296	288	293
	EaR	389	345	345	330	344	324	299	254	254	242
risk measurement 1	expanded VaR	-723	-775	-873	-923	-952	-962	-1,047	-989	-981	-957
	EaR	-18	-56	-88	-113	-156	-179	-188	-202	-225	-277
risk measurement 2	expanded VaR	-873	-949	-949	-1,068	-1,114	-1,220	-1,418	-1,159	-1,046	-1,131
	EaR	-24	-96	-158	-202	-310	-423	-404	-444	-438	-473

[15] "Risk measurement 1" is the 99 percent confidence interval (2.33 σ) calculated by assuming that the distribution of period profits or losses is normal. "Risk measurement 2" is the 99th percentile value obtained by sequentially ordering period profits or losses from the largest to the smallest. Because risk measurement 2 is larger than risk measurement 1, we know that the distribution is more skewed with, fat tail on the higher interest rate side, than the normal distribution. Presumably, this reflects the impact of the bias which is generated when the interest rate paths are formed, with the rate of return measured in terms of the compound interest rate as the risk factor (i.e.,given the same interest rate volatility, the tail of the interest rate path is extended in the higher interest rate range because a higher level of interest rates leads to a greater fluctuation of interest rates). That is to say, long-term simulations require caution. In addition, in order to consider the possible impact caused by the limited number of simulations (i.e., small sample size), we have also compared the computation results across different sample sizes. According to the following table, the impact of small sample size appears to be small.

		first period		10th period	
		sample of 400	sample of 500	sample of 400	sample of 500
risk measurement 1	expanded VaR	-734	-723	-988	-957
	EaR	-18	-18	-286	-277
risk measurement 2	expanded VaR	-913	-873	-1,202	-1,131
	EaR	-23	-24	-487	-473

(in hundreds of millions of yen)

[16] Strictly speaking, it is necessary to compare future values by transforming them first into present values. However, we have not made the present value transformation because our emphasis here is on grasping overall tendencies.

Diagram 6. The Distribution of Period Profits or Losses over the 5-year period

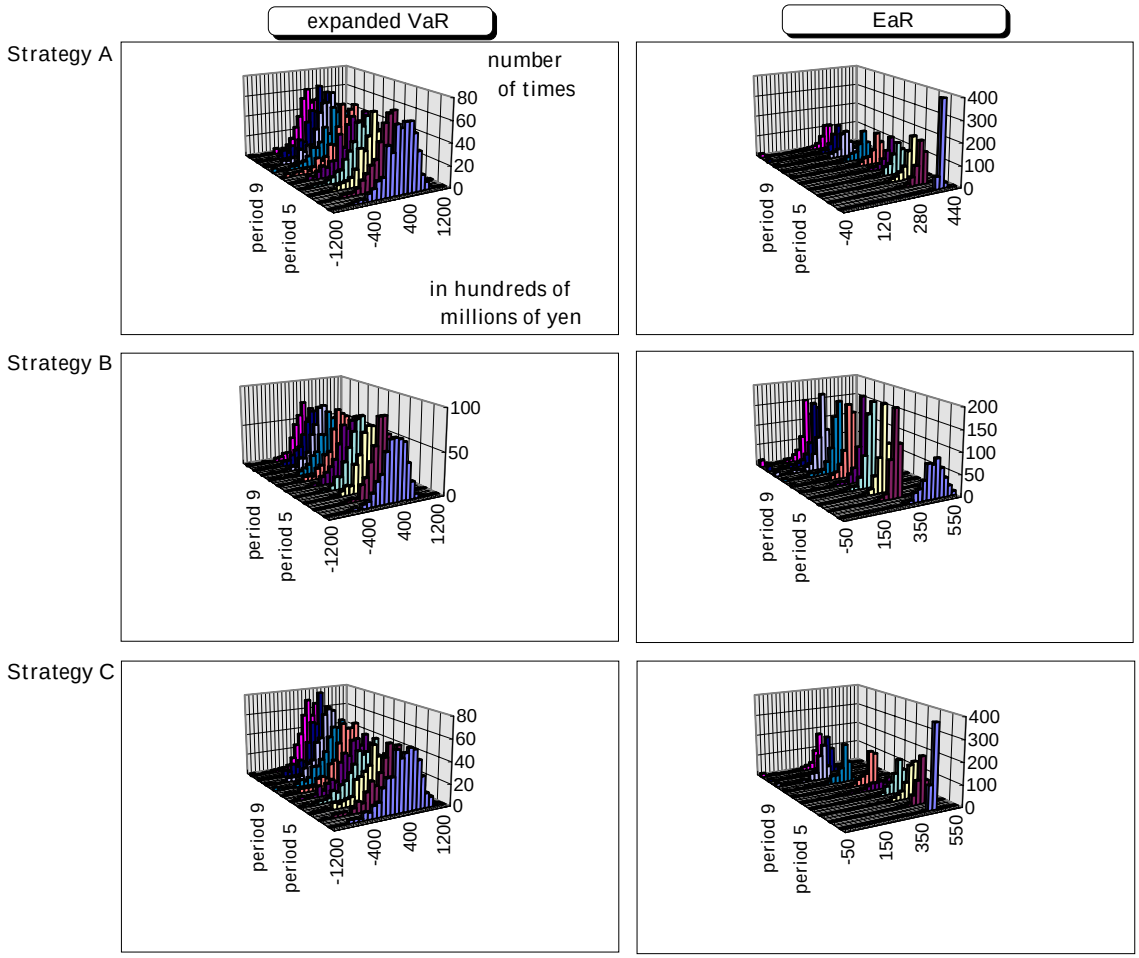
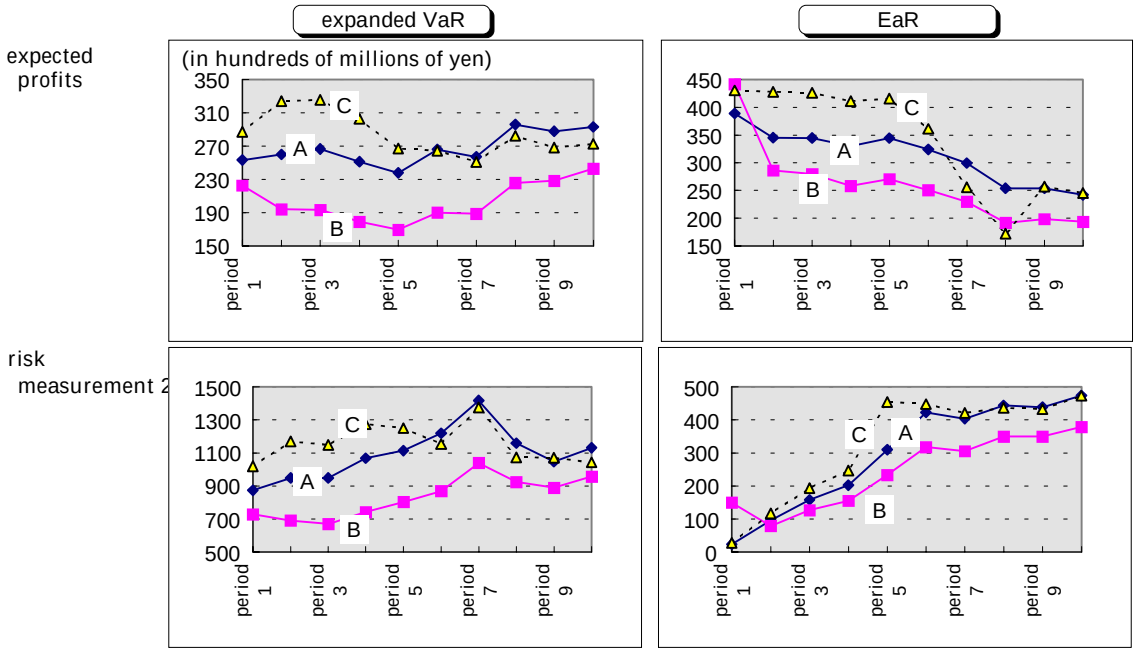


Diagram 7. Time-series Movements of Expected Profits and the Measurement of Risk



(i) The amount of risk calculated from the EaR model is extremely small compared with that calculated by the expanded VaR model. This tendency becomes even more pronounced for a more immediate future period. In particular, for the first and second periods, if we measure period profit or loss in terms of realized profit or loss (on the basis of the EaR model), it appears that stable profits can be expected. Conversely, the amount of risk calculated by the expanded VaR is excessive relative to the level of bank profits[17] (see Diagram 5).

(ii) In terms of time-series transition, although the amount of risk tends to increase with time both for the expanded VaR model and the EaR model, this tendency is more pronounced for the EaR model (see Diagram 6 and the bottom right-hand column of Diagram 7; note how the shape of the distribution of profits or losses changes with time in Diagram 6). This is a natural result in view of the fact that uncertainty increases with the passage of time.

(iii) Looking at the time-series transition of expected profits (Diagram 5), the EaR-based profits decline with the passage of time, while the expanded VaR-based profits appear roughly constant. This is due to the condition of the initial portfolio, attributable to the fact that the unrealized losses become realized when high-coupon and high-book value bonds with an initial remaining maturity of 4-5 years are redeemed (for detailed characteristics of the bonds, see Appendix 1). This indicates that, in the case of the expanded VaR model which is based on market value, current actions (e.g., the purchase of long-term bonds considerably above par) are vividly reflected in the amount of risk, whereas they are not sufficiently reflected until redemption (in the distant future) in the case of the EaR model. That is to say, EaR requires a longer-term simulation.

3-2 Comparison by Strategy

Looking at the time-series transition of the distribution of period profits or losses (Diagram 6), the EaR model shows a larger change of the shape of the distribution depending on the strategy. The risk is relatively larger during a period when there are sales operations (i.e., the first period in strategy B, and the fifth period

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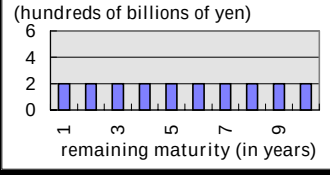
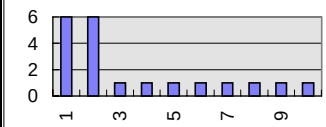
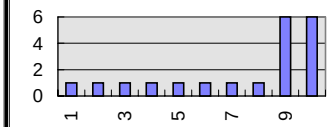
[17] For the September 1996 period, the average net operating profits of city banks were 119.2 billion yen. Of course, the reported case is based on the sample portfolio, and changes in valuation gains or losses in banking accounts are not recognized as profit or loss in income statements, making it meaningless to compare these numbers. This is intended only as an indication of the magnitude of the impact.

in strategy C), such that the distribution becomes more skewed. From the point of view of utilizing this for the selection of a strategy, the limitations of the EaR model become clear.

In order to compare strategies from the point of view of risk versus return, we have formulated a risk/return diagram by adding three strategies (Diagram 8). The upper column of Diagram 9 is based on the average values from the most recent 2-year period, while the bottom column is based on the average values from the entire 5-year period. From these, the following points can be made:

Diagram 8. Additional 3 Strategies

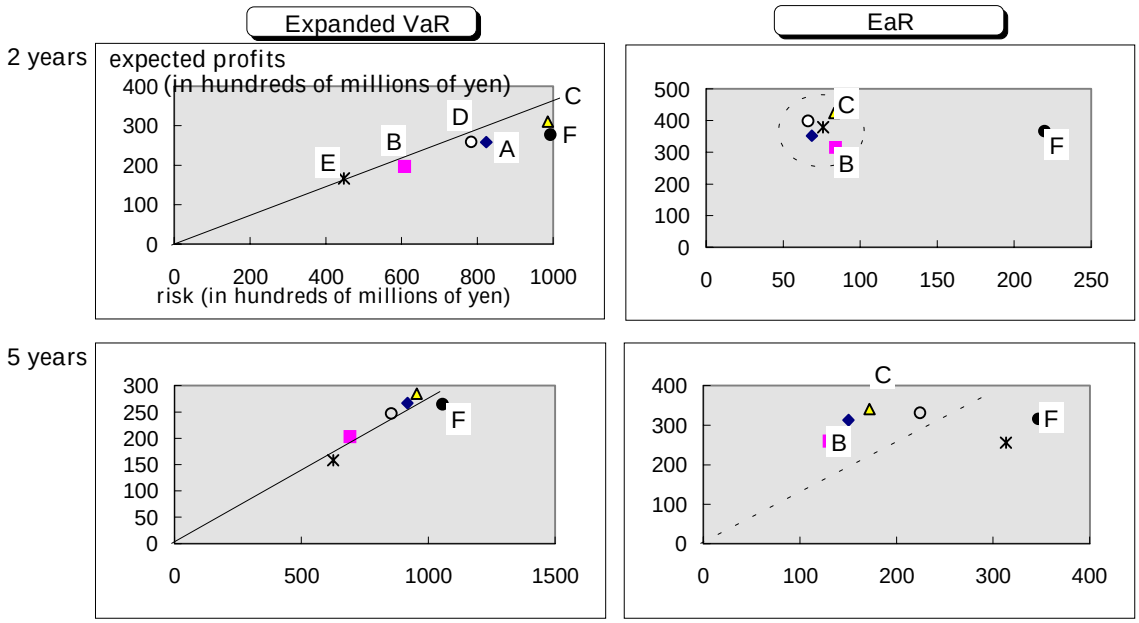
	Strategy D	Strategy E	Strategy F
Specific content	To hold all zones equally. During the period, this ladder is to be maintained. That is to say, the bonds being redeemed are to be entirely replaced by newly-issued 10-year bonds.	Concentrated in short-term bonds. During the period, this ladder is to be maintained. that is to say, the ones being redeemed are to be replaced by existing 2-year bonds and newly-issued 10-year bonds.	Concentrated in long-term bonds. During the period, this ladder is to be maintained. that is to say, newly-issued 10-year bonds are not only to be subscribed to but also to be purchased in the secondary market; in addition, bonds are to be sold when the remaining maturity becomes shorter than 8 years.
ladder			

(i) In the case of the expanded VaR model, regardless of the length of the sample period (2 or 5 years), the tradeoff between risk and return (i.e., a positively sloped straight line) is visible. In the case of the EaR model, for the longer period (the right hand bottom column), such a relationship is visible,but for the shorter period (the right hand upper column), this tendency is not visible, which is problematic in the selection of a strategy. For example, for the first and second periods alone, Strategy C, which increases the balance of bonds, appears to be an ideal strategy in that the expected profits are high while risk is kept low (Diagrams 7 and 9). That is to say, from a myopic management strategy focused solely on the realized profits or losses in the near future, this suggests a bias towards increasing the balance of bonds under present conditions in which, like the initial conditions of the above simulation, the spread between short term and long term interest rates is large, such that profits can be earned by increasing the volume.

(ii) In terms of any indicator, Strategy F which is concentrated in long-term bonds is unfavorable. In the case of city bank portfolios, it is frequently the case that the long-term zone is flexibly adjusted because of high liquidity and the ease with

which the position can be hedged by futures transactions. However, unless the outlook for interest rates is appropriate, there is a possibility that a return which is commensurate with the risk is not being realized.

Diagram 9. Risk vs. Return Diagram



3-3. Summary

(The characteristics of the EaR model in comparison with the expanded VaR model)

(i) The amount of risk calculated by the EaR model is extremely small compared with that obtained from the expanded VaR model. However, the amount of risk increases with the passage of time, and the difference (in terms of ratio) tends to diminish. According to the simulation results, the bond portfolio appears to be a "safe" investment in the short run on the basis of the EaR model. On the other hand, on the basis of the expanded VaR model, it can be pointed out that the current volume contains a considerable risk relative to the level of profits.

(ii) In the case of the EaR model, the shape of the distribution of profits changes substantially depending on the strategy. For a given single period, the risk of bond selling operations appears larger than the risk of bond purchasing operations. The movement of sales price associated with market fluctuations is recognized as the risk for that period, while the effect of reduced exposure caused by sales can only gradually be realized through the smaller movement of funds profits. On the other hand, in the case of purchases, the effect is realized through the gradual increase in the variance of funds profits as well as the valuation loss or gain at redemption. That is to

say, the impact of current actions is not vividly reflected in the amount of risk; risk is affected only after a long time lapse.

(iii) In the case of the EaR model, it is not possible to compare strategies in terms of risk versus return. As stated above, risk can be properly reflected only through long-term simulation, allowing different strategies to be compared in terms of risk versus return.

(The usefulness of models)

(iv) The simulation-based model which has been constructed in this paper is superior to the scenario-based model for the following reason. In thinking about the process of making decisions about bond portfolio investment strategies, one realistic methodology is to make trial estimations of profits realizable under several (subjective) interest rate scenarios and then to formulate a strategy (i.e., the so-called scenario method). In this case, although the risk management section must formulate interest rate risk scenarios in order to measure the risk of the chosen strategy (the impact on the management of the worst case), it is difficult to formulate objective interest rate scenarios. In this regard, the present model, which is based on Monte Carlo simulations, is useful because it is capable of statistically measuring the risk without formulating interest rate scenarios.

4. Conclusion

In this paper, we have constructed the EaR model and the expanded VaR model as a tool for measuring the market risk of banking accounts, and clarified the characteristics of each model by performing simulations on bond portfolios.

The results of the study reported here are two-fold. First, it has demonstrated the frameworks of the EaR model and the expanded VaR model as tools for measuring the market risk of banking accounts. Second, by performing simulations on bond portfolios, it has clarified the limitations of the EaR model as a risk measurement tool. The fundamental problem of the EaR model is that it is not capable of comparing strategies in the short run in terms of risk versus return. Use of the EaR model has been shown to be essential for performing long-term simulations.

Finally, in view of the above results, let us consider the problems inherent in city bank bond portfolios. According to the simulation results, the desirable investment strategy which is derived from the EaR model is similar to the actual pattern of city bank behavior. This probably reflects the fact that the management of bond portfolios by city banks gives emphasis on period profits or losses based on the

cost method (e.g., a bias towards accumulating balances in pursuit of funds profits when the spread between short term and long term interest rates is large; a strong tendency to avoid losses at redemption and to prefer current profits).

Bond portfolios exert a significant impact on bank's financial statements. Thus, it is to some extent unavoidable that bond portfolios are more constrained by financial accounting considerations than trading accounts. However, as stated before, a management strategy based on the acquisition cost method (the EaR model) has a fundamental problem, in that it cannot select a strategy from the point of view of risk versus return. It is thus necessary to recognize the possibility that the chosen investment strategy is not efficient.

For example, if interest rates rise in the future, there is no guarantee that the usefulness of long-term holding, which has been the basis for using the cost method, will remain valid.[18] Moreover, with the enhancement of disclosure in recent years, the various problems of the current system may again be placed under close scrutiny, along with a question such as "is it possible to continue to maintain large unrealized capital losses even for the long-term holding portion?" Eventually, bond portfolios will shift toward a more ideal pattern in which they are valued at market price, they are managed so as to maximize return in terms of market value, and risk is valued at market price. At that time, fundamental questions will be asked again as to the propriety of the market value method[19] and the role and appropriate size of bond portfolios.

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[18] The average coupon rate was 6.26 percent during the March 1990 period when the interest rate spread was negative (i.e., funds profits were negative). In contrast, the average coupon rate for the March 1996 period was 4.99 percent, suggesting the possibility that the negative interest rate spread will continue for a longer period this time (according to the composition of coupon rates in Appendix 1, there is even a possibility that the average coupon rate will decline further in the future). See Appendix 4 for the effectiveness of long-term holding.

[19] See Appendix 4.

Appendices

Appendix 1. Sample Portfolio (initial values)

The sample portfolio is composed of 20 hypothetical bonds[20] whose remaining maturity is sequentially set every half year. The initial portfolio is set to have a barbell-shaped position[21] amounting to 2 trillion yen, as shown in Diagram 10. By using the interest rates prevailing on August 16, 1996, the portfolio has an average current yield of 4.44 percent, an unrealized loss at redemption of 58.5 billion yen, and a valuation gain of 75 billion yen. It is assumed that the entire borrowing is made with 3-month CDs[22] .

Diagram 10. The Initial Portfolio

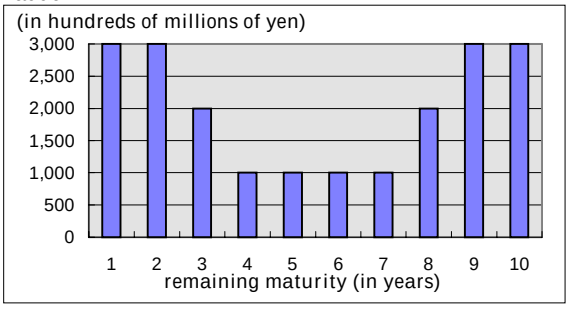
Detailed characteristics of the bonds

	remaining maturity (in years)	coupon rate (in percent)	face value (in hundreds of millions of yen)	unit book value (in yen)
1	0.25	5	1,500	101
2	0.75	4.9	1,500	103
3	1.25	4.8	1,500	102
4	1.75	5	1,500	102
5	2.25	4.8	1,000	100
6	2.75	4.9	1,000	101
7	3.25	6.4	500	105
8	3.75	7.3	500	112
9	4.25	6.4	500	110
10	4.75	6.3	500	110
11	5.25	5.5	500	107
12	5.75	5	500	105
13	6.25	4.2	500	105
14	6.75	4.4	500	105
15	7.25	3.7	1,000	99
16	7.75	4.6	1,000	105
17	8.25	4.4	1,500	105
18	8.75	3.3	1,500	102
19	9.25	3.2	1,500	101
20	9.75	3.1	1,500	100

Aggregate data

face value	20,000	(in hundreds of millions of yen)
book value	20,585	(in hundreds of millions of yen)
unrealized loss at redemption	-585	(in hundreds of millions of yen)
valuation gain	750	(in hundreds of millions of yen)
average current yield	4.44	(in percent)

ladder



Appendix 2. Generating interest rate paths

Diagram 11 shows the statistical data concerning the rate of return of the 3-

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[20] These are set by reference to the Japanese government bonds existing on August 16, 1996. Options and futures are not considered.

[21] Generally, this is considered to be a conservative investment strategy. Because interim financial statement do not disclose information on the ladders, we have decided to use the barbell-shaped portfolio for convenience.

[22] In calculating the funds profits, we use the 3-month moving average of the 3-month CD rates (on the assumption that, in view of availability, borrowing is made at succeeding points in time).

month CD rates and the (compound) generic interest rates[23] , generated from the (weekly) Japanese government bond yields for January 1994 to August 1996. Based on these data, we have generated random numbers according to the multivariate normal distribution[24] , and obtained the monthly[25] series of the 3-month CD rates and 1-year to 10-year interest rates for 10 years (for a total of 11 x 120). We have treated these as one path of interest rates, and generated 500 such paths[26] . For intermediate interest rates such as the 1.5-year interest rate, we have obtained them by linear interpolation.

Diagram 11. The Means, Standard Deviations and Correlation Coefficients of Interest Rates

	3-month CD	1-year	2-year	3-year	4-year	5-year	6-year	7-year	8-year	9-year	10-year
interest rate level	0.42	0.69	1.15	1.58	2.07	2.43	2.70	2.97	3.14	3.23	3.32
mean	-0.0102	-0.0067	-0.0032	-0.0013	-0.0004	0.0002	0.0001	-0.0001	-0.0002	-0.0003	-0.0005
standard deviation	0.282	0.244	0.202	0.154	0.121	0.102	0.088	0.080	0.069	0.063	0.059

	3-month CD	1-year	2-year	3-year	4-year	5-year	6-year	7-year	8-year	9-year	10-year
3-month CD	1.000										
1-year	0.924	1.000									
2-year	0.768	0.860	1.000								
3-year	0.674	0.802	0.947	1.000							
4-year	0.589	0.739	0.847	0.946	1.000						
5-year	0.491	0.646	0.766	0.893	0.980	1.000					
6-year	0.415	0.570	0.689	0.833	0.932	0.978	1.000				
7-year	0.345	0.507	0.629	0.780	0.888	0.942	0.980	1.000			
8-year	0.332	0.492	0.624	0.773	0.871	0.927	0.968	0.987	1.000		
9-year	0.367	0.515	0.617	0.768	0.866	0.917	0.956	0.973	0.986	1.000	
10-year	0.420	0.552	0.635	0.763	0.859	0.907	0.939	0.938	0.948	0.948	1.000

[23] In the case of bonds, because remaining maturity changes (becomes smaller) every day, bonds corresponding to the fixed remaining maturities of one year, 2 years, etc. do not always exist. Here, the yield estimated for each fixed remaining maturity (e.g., one year, two years, etc.) from individual yields is called the generic interest rate. Yen-denominated bonds are significantly affected by coupon characteristics because of the propensity to prefer current receipts (i.e., preference for bonds priced around 100 yen over those priced over par), making it necessary to exercise caution in time-series analysis. This point, however, is not taken into consideration in this paper.

[24] In this paper, random numbers are generated according to the multivariate normal distribution by taking a Choleski-decomposition of the correlation matrix and multiplying it with a vector of normally distributed random numbers.

[25] The transformation of weekly data into monthly data was made by simply multiplying it by the square root of T ($\sqrt{T}$).

[26] In order to construct a exact and precise model, we need much more than 500 paths. The emphasis of our analysis here, however, is on how differences in strategies and initial portfolios affect the computation results, with a view to identifying the limitations of the models.

Appendix 3 Movement of the Ladder by Strategy (Diagram 12)

*Units in hundreds of billion yen (vertical scale)

Appendix 4 The Acquisition Cost Method vs. the Market Value Method

Here, we will not get involved in the accounting problem of " acquisition cost method vs. market value method".[27] Instead, we will consider so-called "management volatility theory", which is one of the views opposing market value accounting. The theory asserts that market value accounting increases the volatility of profits, damaging the stability of management.

Diagram 13 compares profits or losses between the cost method (EaR) and the market value method (expanded VaR), when the purchased bonds are held until maturity (with no sale or purchase in the interim) for the preceding 20 periods for 10 years.[28] Looking at the cumulative profits measured in terms of EaR, we find that net

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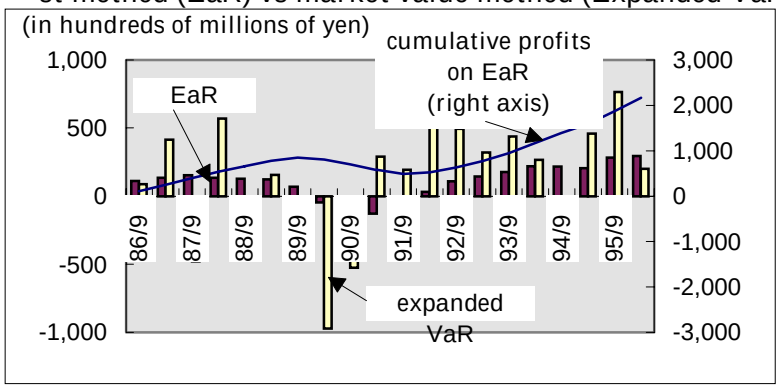
[27] For details, see Daigo (1995) and Okina (1993).

[28] The underwriting of bonds is set equal to 15 billion yen par bond. If bonds are issued

positive profits have eventually resulted from having ignored the interim valuation losses and from having held the bonds for a long period of time. It is difficult to know how to reconcile the market value method with the portfolio theory, which states that, even for those assets which are highly volatile in the short run, the risk becomes small over a long investment horizon.

Diagram 13. ^^st method (EaR) vs market value method (Expanded VaR)



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every month, this means that there is a constant holding of bonds amounting to 1.8 trillion yen (15 billion x 12 months x 10 years). The market value of each issue of bonds was simply calculated by using the simple interest rate obtained by the linear interpolation of the 3-month CD rate and the (10-year) futures yield.