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FOREIGN EXCHANGE NETTING AND SYSTEMIC RISK *

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Abstract

As a natural consequence of a major development in financial activity since the introduction of flexible exchange rates in early 1970's, it is felt that payment systems have become a potential source of serious financial crises. Payment system is a set of arrangements made for the purpose of discharging obligations assumed by economic agents in their economic transactions. There are two basic ingredients of payments systems: they are settlement arrangements, and netting arrangements. Netting is an offsetting of a similar type of financial obligations, and only the net difference is settled. Foreign exchange transactions account for a large share of all payment flows in major financial centers.

The purpose of this paper is to introduce a formal model of foreign exchange contracts netting, and present an analysis of multilateral and bilateral netting from the view point of credit risk reduction. In particular, we are interested in comparing these two different forms of netting arrangements with respect to inherent systemic risks involved. The point of our present paper is to show that when more than two banks defaults, indirect loss sharing of participants could harm the participants to multilateral netting beyond the potential risk level of bilateral netting arrangements. The concept of systemic credit exposure is used for this purpose.

Keywords: Payment system; multilateral netting; bilateral netting; systemic risk; credit risk; foreign exchange.

JEL classification numbers: G15,F31,F33,D40.

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1 Introduction

Since the introduction of flexible exchange rates in early 1970's we have seen a major development in financial activity. The introduction of new financial instruments, deregulation of activities of different types of financial institutions, remarkable advances of computer and communication technologies, the growing internationalization of financial markets and volatile movements of assets prices including foreign exchange rates have all contributed to an unprecedented growth of financial activity. It has induced conspicuous increases both in the volume and the value of payment flows in major industrial countries as well as across national borders.

As a natural consequence of this development, it is keenly felt that payment systems have become a potential source of serious financial crises. For this reason, the central banks of the Group of Ten countries have studying the nature of risks and possible policy problems concerning payment and settlement systems. See, for example, the Angell Report [1] in which the nature of various risks and policy problems under different netting systems are discussed and the Lamfalussy Report [2] in which the minimum standards for the design and operation of cross-border and multi-currency netting and settlement schemes are proposed.

Payment system is a set of arrangements made for the purpose of discharging obligations assumed by economic agents in their economic transactions. There are two basic ingredients of payments systems: they are settlement (or fund transfer) arrangements, and netting (or clearing) arrangements. Netting is an offsetting of a similar type of financial obligations, and only the net difference is settled. Thus arrangements for the binding netting of financial obligations provide a service that is a very close substitute for the function of money as a medium of payment. Netting between two parties is called bilateral netting, and netting among multiple parties is called multilateral netting. In the latter case netting is also referred to as clearing.

Foreign exchange transactions account for a large share of all payment flows in major financial centers. Because financial institutions have strong incentives to lower credit risk and payment flows, there have been active movements toward structural innovations in interbank clearing and settlement procedures and a number of proposals have been presented to establish multilateral foreign exchange netting arrangements in recent years. As a mater of fact, the first foreign exchange multilateral netting scheme, ECHO, started its operation in Europe last August.

In this paper we would like to introduce a formal model of foreign exchange contracts netting, and present an analysis of multilateral and bilateral netting from the view point of credit risk reduction. In particular, we are interested in comparing these two different forms of netting arrangements with respect to inherent systemic risks involved. In our analysis we will introduce a comparative statics type argument in intrinsically stochastic environments. The approach might be termed as a comparative contingencies. By looking at an event, given by a possible finite sequence of defaulting banks, we shall compare credit risks of a bank between the two different netting arrangements.

In multilateral netting arrangements in general, a clearing house will become the ultimate counterparty to every participant to the arrangements. Thus, one could argue that credit risk of each participant must be judged in the light of the financial resources of the clearing house. In fact, the Angell Report explicitly states this point. Okina [11] emphasizes it in his discussion of prudential rules in multilateral netting. In our analysis of multilateral netting in this paper, however, we have not followed this suggestion in order to simplify our analysis. Instead, in our model all the losses that the clearing house will incur due to failures of participating banks will be distributed among surviving banks in proportion to their profit levels with respect to defaulters. It is our view that if one wants to introduce collateral and margins in a model of multilateral netting arrangements, then participants' credit exposure to the clearing house must be taken into account in considering their credit risks.

The paper is organized as follows. After a brief introduction to the literature in the sequel, we present in Section 2 a formal model of foreign exchange contracts netting based upon individual transactions and contracts of financial institutions. In Section 3 the concepts of bilateral exposures and multilaterally netted bilateral exposures are formally defined and their comparison is made in Proposition 2 which is a basis of a casual argument stating that if netting is done among many, then credit risk reduction is more effective than in the case of netting between two parties. The point of our present paper, however, is to show that this is not the whole story. When more than two banks defaults, indirect loss sharing of participants could harm the participants to multilateral netting beyond the potential risk level of bilateral netting arrangements. In Section 4 systemic risk is analyzed following a very simple scenario of two defaulting banks, and some of its implications are discussed. Although this section takes up a very simple case of systemic risks, results obtained here can be extended to general cases. In fact, it is done in Section 5 which treats a general case of an arbitrarily given sequence of defaulters. By analyzing general properties of loss share coefficients, we deduce cases in which multilateral netting gives higher systemic credit exposures than the sum of bilateral credit exposures. Finally, in Section 6 we summarize some of the results obtained in this paper.

In the literature there have been very few studies that present formal models in discussing payment systems. It appears that serious studies on payment systems have just begun. Schoenmaker [13], and Rochet and Tirole [12] present a formal model of interbank settlement system. Eisenberg [6], Chakravorti [5], and Schoenmaker [14] present an analysis of systemic risks in settlement systems. Humphrey [9] gives a simulation study of systemic risk. We are not aware of formal models in which different netting arrangements are analyzed. However, the Angell Report [1] gives a very nice discussion and a verbal analysis of risk structures in different netting schemes. Borio and Van den Bergh [4] also gives an extensive verbal discussion of payment system in general.

2 A Model of Foreign Exchange Contracts Netting

We shall consider a finite set I of financial institutions or *banks* that engage in cross-border or off-shore foreign exchange transactions over discrete points in time that may be referred to as *dates* also. A *foreign exchange contract or obligation* specifies a contractual duty to deliver a defined (foreign) currency in exchange for another (foreign) currency between two institutions on an agreed date or *value date.* The words — transactions, contracts, and obligations — are used synonymously in this paper. Institutions that engage in a financial contract are called *parties.* The opposite party to a financial contract is a *counterparty.* They are counterparties to each other. In this section we will build up a model of foreign exchange contracts netting based upon individual transactions of financial institutions.

2.1 Individual Transactions and Contracts

Given a set of financial institutions I, typical elements of I will be written as

$$h, i, j \in I.$$

The number of financial institutions, $\sharp I$, to be considered for the purpose of our analysis is assumed to be at least four so that $4 \leq \sharp I < \infty$.

Now, let us consider foreign exchange transactions between banks $i, j \in I$ at time t_1, and collect all the transactions that take place on the same date t_1 with identical value date $t_2 (> t_1)$. Denote by

$$J_{ij}(t_1, t_2)$$

the set of all such transactions or contracts. With this notation above, we have

$$J_{ij}(t_1, t_2) = J_{ji}(t_1, t_2)$$

for any i and j and $J_{ij}(t_1, t_2) = \emptyset$ if $i = j$. For convenience, a contract $k \in J_{ij}(t_1, t_2)$ in this paper represents a "matched pair" of promises to deliver foreign currencies to counterparties. A *foreign exchange contract* $k \in J_{ij}(t_1, t_2)$ at t_1 obligates bank i to pay bank j at time t_2 the amount $y_{jik}(t_1, t_2, c')$ of currency c' and receives the amount $y_{ijk}(t_1, t_2, c)$ in currency c for some pair of currencies c and c'. For notational convenience we put $y_{ijk}(t_1, t_2, c'') = 0$ if the contract k does not involve an exchange of currency c''. In the notation $y_{ijk}(t_1, t_2, c)$, suffix i refers to a party, j to its counterparty, and k to a contract. If $y_{ijk}(t_1, t_2, c)$ is positive, then it means that party i is to receive the amount $y_{ijk}(t_1, t_2, c)$ from its counterparty j, and the counterparty is to pay the amount $-y_{jik}(t_1, t_2, c)$ to the party i. If $y_{ijk}(t_1, t_2, c)$ is negative, then i is to pay to j the amount $-y_{ijk}(t_1, t_2, c)$, and j is to receive the amount $y_{jik}(t_1, t_2, c)$ from i. Thus, for any contract

$k \in J_{ij}(t_1, t_2)$, we have

$$y_{ijk}(t_1, t_2, c) + y_{jik}(t_1, t_2, c) = 0 \tag{1}$$

which states that the amount to be received or paid ($= y_{ijk}(t_1, t_2, c)$) by a party is equal to the amount to be paid or received ($= -y_{jik}(t_1, t_2, k)$) by its counterparty respectively. For expositional convenience we will use the following mathematical notation: for any real number v

$$\begin{aligned} v^+ &\equiv \max\{v, 0\} \\ v^- &\equiv \max\{-v, 0\} \\ v &= v^+ - v^-. \end{aligned}$$

In these notations $y_{ijk}(t_1, t_2, c)^+$ and $y_{jik}(t_1, t_2, c)^+$ represent the receipts, and $y_{ijk}(t_1, t_2, c)^-$ and $y_{jik}(t_1, t_2, c)^-$ the payments of i and j respectively concerning a contract $k \in J_{ij}(t_1, t_2)$. In order to avoid misunderstandings as regards to our notation, let us record the following obvious relations:

$$\begin{aligned} y_{ijk}(t_1, t_2, c)^+ &= y_{jik}(t_1, t_2, c)^- \\ y_{ijk}(t_1, t_2, c)^- &= y_{jik}(t_1, t_2, c)^+ \end{aligned}$$

which simply state that a receipt of a party is a payment of its counterparty and vice versa.

We assume that there is a finite set of (foreign) currencies traded in the markets. It is denoted by C. We assume $2 \leq \sharp C < \infty$. It is also assumed that there is a *"base currency"*, denoted by b , that is used to express values of all other currencies or that is the standard of value among different currencies.

2.2 Bilateral Netting of Contracts

Netting is an offsetting of receipts and payments to be made for a similar type of financial contracts. *Bilateral netting* is a netting between two parties. It is considered to be a most natural form of netting in foreign exchange transactions as large number of "matched trades" with same currency, same value date and same counterparties exist.

Consider two banks i and j and the set of contracts $J_{ij}(t_1, t_2)$ between them. Without a netting of contracts, banks i and j face payments of

$$\sum_{k \in J_{ij}(t_1, t_2)} y_{ijk}(t_1, t_2, c)^- \quad \text{and}$$
$$\sum_{k \in J_{ji}(t_1, t_2)} y_{jik}(t_1, t_2, c)^-$$

respectively at time t_2 as a result of contracts in $J_{ij}(t_1, t_2) = J_{ji}(t_1, t_2)$.

In order to discuss several distinct forms of netting that can be applied to conventional foreign exchange contracts, we shall define, for any t_1, t_2, c with $t_1 < t_2$,

$$y_{ij}(t_1, t_2, c) \equiv \sum_{k \in J_{ij}(t_1, t_2)} y_{ijk}(t_1, t_2, c).$$

Here, summing the amounts $y_{ijk}(t_1, t_2, c)$ of payments and/or receipts instead of the amounts $y_{ijk}(t_1, t_2, c)^-$ of payments and the amounts $y_{ijk}(t_1, t_2, c)^+$ of receipts separately over all the contracts in $J_{ij}(t_1, t_2)$ means that the various payments $y_{ijk}(t_1, t_2, c)^-$ are offset against various receipts $y_{ijk}(t_1, t_2, c)^+$ for $k \in J_{ij}(t_1, t_2)$ to arrive at one net amount $y_{ij}(t_1, t_2, c)$. It therefore represents the amount of currency c that bank i is to receive from bank j at time t_2 as a result of foreign exchange transactions taking place between them at time t_1 under a bilateral netting arrangement between the two parties. As before a positive $y_{ij}(t_1, t_2, c)$ means a net receipt and a negative $y_{ij}(t_1, t_2, c)$ a net payment on the part of bank i. With this notation, it follows from the equality in (1) that we have for each t_1, t_2, c

$$y_{ij}(t_1, t_2, c) + y_{ji}(t_1, t_2, c) = 0, \tag{2}$$

which means that the amount that i is to receive from j is equal to the amount that j is to pay to i for each currency c under bilateral netting.

For notational convenience, if there are no transactions in $J_{ij}(t_1, t_2)$ for which $y_{ijk}(t_1, t_2, c) \neq 0$, then we put

$$y_{ij}(t_1, t_2, c) = 0. \tag{3}$$

Thus, if $J_{ij}(t_1, t_2) = \emptyset$, we have

$$y_{ij}(t_1, t_2, c) = 0$$

for all $c \in C$, and in particular

$$y_{ii}(t_1, t_2, c) = 0$$

for all $c \in C$.

We assume that all the forward foreign exchange contracts take place during a definite time span that is given by time periods between T_1 and T_2. Let us define for each t, t_2, c with $t \leq t_2$

$$x_{ij}(t, t_2, c) \equiv \sum_{t_0 \leq t_1 \leq t} y_{ij}(t_1, t_2, c),$$

where t_0 is the date at which an initial transaction of currency c with value at t_2 took place between i and j. As foreign exchange contracts between parties i and j with same currency and same value date accumulate as time elapses toward the value date, the amount that i is to pay to or receive from j in currency c at time t_2 changes. This net amount as of time t is expressed by $x_{ij}(t, t_2, c)$. Before we proceed to a description of types of bilateral netting, we like to check two basic equalities relating to $x_{ij}(t, t_2, c)$.

First, it is immediate from the definition of $x_{ij}(t, t_2, c)$ that it is the "accumulated" bilateral position at time t of bank i with respect to j for contracts in currency c with value at t_2, that is, for any t, t_2, c with $t < t_2$, we have

$$x_{ij}(t, t_2, c) = x_{ij}(t-1, t_2, c) + y_{ij}(t, t_2, c).$$

Secondly, we have again a fundamental "mirror image" equality for any t, t_2, c with $t < t_2$

$$x_{ij}(t, t_2, c) + x_{ji}(t, t_2, c) = 0 \tag{4}$$

because

$$\begin{aligned}
& x_{ij}(t, t_2, c) + x_{ji}(t, t_2, c) \\
= \; & \sum_{t_0 \le t_1 \le t} y_{ij}(t_1, t_2, c) + \sum_{t_0 \le t_1 \le t} y_{ji}(t_1, t_2, c) \\
= \; & \sum_{t_0 \le t_1 \le t} \{ y_{ij}(t_1, t_2, c) + y_{ji}(t_1, t_2, c) \} \\
= \; & 0
\end{aligned}$$

where the last equality follows immediately from (2).

2.3 Different Forms of Bilateral Netting

Position netting or *payment netting* is a form of offset under which two parties informally make one net payment between themselves for each currency and value date. But there is no change in their contractual obligations. Thus, in this form of netting the amount of payment in currency c due for bank i at time t_2 is $x_{ij}(t, t_2, c)^-$ (if this is equal to 0, then $x_{ji}(t, t_2, c)^- > 0$). But the contractual obligation requires bank i to pay j the amount $\sum_{t_1 \le t} \sum_k x_{ijk}(t_1, t_2, c)^-$.

Netting by novation or *obligation netting* between two parties refers to the replacement of contracts between them for delivery of a specified currency on the same date by one single net amount for that date in such a way that original contractual obligations are satisfied and discharged. Thus, under netting by novation between banks i and j, bank i is obligated to pay the amount $x_{ij}(t, t_2, c)^-$ or bank j is obligated to pay $x_{ji}(t, t_2, c)^-$ rather than the amount $\sum_{t_1 \le t} \sum_k x_{ijk}(t_1, t_2, c)^-$ and $\sum_{t_1 \le t} \sum_k x_{jik}(t_1, t_2, c)^-$ respectively.

Under the netting by novation between banks i and j, the amount $x_{ij}(t, t_2, c)$ which is identical to $-x_{ji}(t, t_2, c)$ represents a "novated" (i.e., new) contract for the net amount as of time t. In this sense $x_{ij}(t, t_2, c)$ (and $x_{ji}(t, t_2, c)$) expresses the bilateral novation process through time t between parties i and j with respect to currency c and value date t_2.

Under the bilateral netting, whether it is position netting or netting by novation, bilaterally netted bilateral settlement position in currency c of bank i with respect to j at time t is given by the amount $x_{ij}(t,t,c)$.

In *close-out netting* all the outstanding contracts are replaced by current contracts with same value dates and same currencies to compute their present market values. That is, they are all marked to market. Using these present market values, payments and receipts are offset to arrive at one single net amount in the base currency which represents the bilateral position between two parties in close-out netting. This procedure is equivalent to computing the current "replacement costs" of outstanding contracts between the parties. This type of netting arrangement is invoked in case of a default of a counterparty.

In our comparison of bilateral netting with multilateral netting in this paper, we will assume bilateral netting to be netting by novation for clearing and settlement and to be close-out netting in the events of defaults by participants. However, since we will be mainly concerned with credit risk in a narrow sense (see Section 3 below), we will be uniquely dealing with close-out netting.

2.4 Market Value of Forward Transactions

Let $q(t, t_2, c)$ denote forward exchange rates of currencies $c \in C$ quoted in terms of the base currency b at time t with value at time t_2. Now, for a foreign exchange transaction $k \in J_{ij}(t_1, t_2)$ which took place at date t_1, there is a pair of currencies (c,c') for which we have $y_{ijk}(t_1,t_2,c) \neq 0$, $y_{ijk}(t_1,t_2,c') \neq 0$ and $y_{ijk}(t_1,t_2,c'') = 0$ for $c'' \neq c, c'$. For the pair (c,c') one must have

$$q(t_1,t_2,c)y_{ijk}(t_1,t_2,c) + q(t_1,t_2,c')y_{ijk}(t_1,t_2,c') = 0$$

meaning that one unit of currency c is to be exchanged for

$$\frac{q(t_1,t_2,c)}{q(t_1,t_2,c')} \left(= -\frac{y_{ijk}(t_1,t_2,c')}{y_{ijk}(t_1,t_2,c)} \right)$$

units of currency c' at time t_2 which is the market forward exchange rate at time t_1. It thus follows that

$$\sum_{c \in C} q(t_1,t_2,c)y_{ijk}(t_1,t_2,c) = 0.$$

This simply says that, at origination, the value of an at-market foreign exchange contract is zero. Since this equality holds for any contract $k \in J_{ij}(t_1,t_2)$, one must have

$$\sum_{c \in C} q(t_1,t_2,c)y_{ij}(t_1,t_2,c)$$

$$\begin{aligned} &= \sum_{k\in J_{ij}(t_1,t_2)} \sum_{c\in C} q(t_1,t_2,c) y_{ijk}(t_1,t_2,c) \\ &= 0. \end{aligned}$$

However, for $t_1 < t \leq t_2$, one may have

$$\sum_{c\in C} q(t,t_2,c) y_{ij}(t_1,t_2,c) \neq 0$$

due to fluctuations in forward exchange rates $q(t,t_2,c)$. If this amount is negative, bank i's forward book shows a loss concerning the transactions at t_1 with value at t_2. The amount $\sum_c q(t,t_2,c)y_{ij}(t_1,t_2,c)$ represents the *mark-to-market value* or *net present value* at time t of the transactions at time t_1 between banks i and j with value at time t_2. We denote this value by

$$v_{ij}(t_1,t_2)(t) \equiv \sum_{c\in C} q(t,t_2,c) y_{ij}(t_1,t_2,c).$$

We now calculate the *mark-to-market value* or *net present value* of the forward book of bank i with respect to bank j as of time t which will be written as $v_{ij}(t)$. It is given by

$$v_{ij}(t) \equiv \sum_{\substack{t_1\leq t \\ t<t_2\leq T_2}} v_{ij}(t_1,t_2)(t)$$

for each t. Note that by our notational convention in (3) we have $v_{ij}(t) = 0$ if $i = j$ or if $J_{ij}(t_1,t_2) = \emptyset$ for all $t_1 \leq t$.

It will become convenient to rewrite this expression for the computation of net present values of the forward book.

$$\begin{aligned} &\sum_{\substack{t_1\leq t \\ t<t_2\leq T_2}} v_{ij}(t_1,t_2)(t) \\ &\quad = \sum_{t_2}\sum_{t_1}\sum_{c} q(t,t_2,c) y_{ij}(t_1,t_2,c) \\ &\quad = \sum_{t_2}\sum_{c}\sum_{t_1} q(t,t_2,c) y_{ij}(t_1,t_2,c) \\ &\quad = \sum_{t_2}\sum_{c} q(t,t_2,c) \sum_{t_1} y_{ij}(t_1,t_2,c) \\ &\quad = \sum_{t_2}\sum_{c} q(t,t_2,c) x_{ij}(t,t_2,c). \end{aligned}$$

It thus follows that one has

$$v_{ij}(t) = \sum_{t<t_2\leq T_2} \sum_{c\in C} q(t,t_2,c) x_{ij}(t,t_2,c).$$

$v_{ij}(t)$ is the *bilateral position* of bank i with bank j at time t concerning i's forward book[1]. The equality above shows that the bilateral position can also be calculated on the basis of the accumulated bilateral forward position in each currency for each value date, i.e. $x_{ij}(t, t_2, c)$, instead of going back to all the original contracts. We note that

$$\begin{aligned} v_{ij}(t) &= \sum_{t_2}\sum_{c} q(t, t_2, c) x_{ij}(t, t_2, c) \\ &= -\sum_{t_2}\sum_{c} q(t, t_2, c) x_{ji}(t, t_2, c) \\ &= -v_{ji}(t) \end{aligned}$$

so that we have

$$v_{ij}(t) + v_{ji}(t) = 0. \tag{5}$$

Since $v_{ij}(t)$ is the net present value of all forward transactions of i with j evaluated by forward prices at time t, the equality (5) is interpreted to say that in forward transactions both two parties cannot win at the same time — a party or its counterparty must loose between the two.

We record the above two properties as:

Fact 1 [Mark-to-Market Value] *The mark-to-market value of the forward book of a bank with respect to one of its counterparties can be calculated on the basis of its accumulated bilateral forward position in each currency with separate value dates, i.e.*

$$v_{ij}(t) = \sum_{t<t_2\leq T_2}\sum_{c\in C} q(t, t_2, c) x_{ij}(t, t_2, c).$$

Fact 2 [Zero-Sum Property] *Profits and losses arising from forward foreign exchange transactions have the zero-sum property, that is, for any banks i and j and for any $T_1 \leq t \leq T_2$ one has*

$$v_{ij}(t) + v_{ji}(t) = 0.$$

2.5 Multilateral Netting by Novation and Substitution

In netting by novation and substitution a contract between two parties is amended in such a way that a third party, i.e. a central clearing agent, is interposed as a central counterparty, to each of the two parties, and two new (= "novated") contracts, where the central clearing agent is substituted for the counterparty in the original contract, are created. The original contract

[1]For concreteness, in calculating the positions of forward books at time t, value dates start from $t+1$, *i.e.* the first business day from the date t, in this paper. This could be $t+n$ with some $n = 1, 2, \ldots$ depending upon institutional rules of clearing. For example, in case of the rule of the ECHO, one has $n = 2$.

between the two parties is then satisfied and discharged. It can be regarded as a prototypical multilateral netting arrangement where financial contracts among a given group of financial institutions, composed of more than three institutions, are netted.

More specifically, there is a central clearing agent – a clearing house – and a number of participating banks to the netting arrangement. Participants submit their foreign exchange contracts between any two of the participants to the clearing house. The clearing house is substituted as the central counterparty to each party for a foreign exchange contract submitted by a pair of participating banks. The obligations between the participants are discharged as a result of binding multilateral netting among the participants. At each point in time the clearing house maintains a current and new (or novated) net position for each currency and for each value date with respect to each participant. These net positions determine the amounts due to each bank from the clearing house, or vice versa, for each currency and for each value date. But once a default of participating banks occurs, close-out netting takes place.

3 Risk Assessment

Financial contracts are subject to different types of risk during the course of their lives. First, two types of risk can be distinguished: credit risk in broad sense and market risk. *Credit risk in broad sense* arises from the possibility of default by counterparties whereas *market risk* arises from the possibility of adverse movement in market variables such as foreign exchange rates. Market risks or *foreign exchange* risks can be hedged by individual financial institutions. Credit risks, however, cannot be fully hedged by individual banks alone. It is here that institutional arrangements and regulations need to play an important role.

Credit risk in broad sense may again be divided into two types: settlement risk and credit risk in a narrow sense. *Settlement risk* refers to a possibility of a counterparty's failure to make settlements during the course of a business day whereas *credit risk in a narrow sense* refers to a possibility of a counterparty's failure to honor its forward obligations.

Since there have been several studies in recent years concerning settlement risks (for example, see Rochet and Tirole [12] and Schoenmaker [13],[14]), we would like to focus on credit risk in a narrow sense.

The cost to a bank of a counterparty's default on a forward contract is the cost of replacing the cash flows specified by the contract. Thus, a default-induced loss will occur only if the mark-to-market value of a contract rise to a positive value. One is thus led to define actual credit exposure using the bilateral position of individual banks.

3.1 Bilateral Credit Exposures

We first consider bilateral netting by novation between two banks i and j. The amount $x_{ij}(t,t,c)$, $c \in C$, exhibits the bilateral settlement position of bank i with respect to bank j in the sense that it represents the amount of currency $c \in C$ that bank i is to receive from bank j at time t. Potential loss of bank i may arise during the course of business day t from bank j's failure to make settlements. It will be called the *bilateral settlement exposure* in currency $c \in C$ of bank i with respect to j.

Let us denote the bilateral settlement exposure at time t by $\xi_{ij}(t,c)$. Then, by definition we have

$$\xi_{ij}(t,c) \equiv x_{ij}(t,t,c)^+$$

for i, $j \in I$ and $c \in C$. Note that settlement exposures at time t derive from contracts that are due to be settled on that day. Thus, it is natural for exposures to be defined currency by currency.

But in the event of a default of a counterparty, there is another potential loss. It arises from the couterparty's failure to honor its forward obligations. When forward obligations are dishonored, it is natural to assume that each bank replaces them by similar forward obligations in current markets to resume its forward positions in each currency. Thus, the idea of close-out netting, which is invoked in case of a default, is to compute replacement costs of dishonored forward obligations. Loss arising from the failure of a counterparty to honor its forward obligations will be termed as *actual exposure* of a party with respect to its counterparty. It may also be called *mark-to-market exposure* or *net present value exposure*. Following the notation of our model, the *(actual) bilateral exposure* at time t of bank i with respect to bank j, denoted by $\varepsilon_{ij}(t)$, is thus defined by

$$\begin{aligned} \varepsilon_{ij}(t) &\equiv v_{ij}(t)^+ \\ &= \left(\sum_{t<t_2\leq T_2} \sum_c q(t,t_2,c)x_{ij}(t,t_2,c) \right)^+ \end{aligned}$$

3.2 Multilateral Credit Exposures

3.2.1 Notional Bilateral Position

Let the set of financial institutions I represent the group of participating banks to an arrangement of multilateral netting by novation and substitution for clearing and settlement, and that of multilateral close-out netting in case of a default. The clearing house will be denoted by the letter H. Define for each $i \in I$ and t, t_2, c

$$\begin{aligned} x_{iH}(t,t_2,c) &\equiv \sum_{j\in I} x_{ij}(t,t_2,c), \\ x_{Hi}(t,t_2,c) &\equiv \sum_{j\in I} x_{ji}(t,t_2,c). \end{aligned}$$

Other notations with respect to H are introduced similarly. For example, for $i \in I$ and t,

$$\begin{aligned} v_{iH}(t) &\equiv \sum_{j\in I} v_{ij}(t), \\ v_{Hi}(t) &\equiv \sum_{j\in I} v_{ji}(t). \end{aligned}$$

In essence suffix H indicates that sum is taken over all participating banks to netting and clearing arrangements.

Under a multilateral netting arrangement by novation and substitution, *multilateral settlement position* in currency c at time t is given by

$$\begin{aligned} x_{iH}(t,t,c) &= \sum_{j\in I} x_{ij}(t,t,c) \\ &= \sum_{j\in I} \sum_{t_1 \le t} \sum_{k\in J_{ij}(t_1,t)} y_{ijk}(t_1,t,c). \end{aligned}$$

for each participating bank $i \in I$. Here, for every contract $k \in J_{ij}(t_1,t)$, $t_1 \le t$, the clearing house is substituted for the counterparty j so that the bilateral settlement position $x_{ij}(t,t,c)$ of i with respect to j becomes only a part of settlement position of i with respect to the clearing house H. Taking the sum of $x_{ij}(t,t,c)$'s over $j \in I$ means totalling all those parts of bilateral settlement positions of counterparties of bank i for which the clearing house is substituted. Thus, the *multilateral* settlement position of i given by $x_{iH}(t,t,c)$ is the *bilateral* settlement position of i with respect to the clearing house after substitutions and novations of obligations are effected.

We are now concerned with the amount of forward book positions of bank i that is exposed to a default risk. If bank $j(\neq i)$ will default at time t, potential loss of bank i is *not* necessarily given by the amount $v_{iH}(t)^+$ *nor* $v_{ij}(t)^+$. It is because original contracts of bank i with bank j are replaced by those with the clearing house. Hence, possible losses that each bank must face in multilateral netting depend upon how losses are allocated among participating banks in netting arrangements. Without going into details of various loss allocation rules, we would like to make one basic assumption on a loss allocation rule which seems to us general enough to be adopted in this context. (See Subsection 3.2.3 below.)

Bilateral position under a multilateral netting arrangement will be called *notional bilateral position.* It is *notional* because all the counterparties in the contracts presented for netting are substituted by the clearing house and bilateral positions between two participating banks no longer exist. The *notional bilateral forward position* of bank i with bank j at time t is

$$v_{ij}(t) = \sum_{t<t_2\leq T_2} \sum_{c\in C} q(t,t_2,c)x_{ij}(t,t_2,c).$$

The *multilateral (forward) position* of bank i at time t is

$$v_{iH}(t) = \sum_{j\in I} v_{ij}(t).$$

The amount $v_{iH}(t)$ is the bilateral position of bank i with respect to the clearing house at time t under close-out netting. We say bank i at time t has a *multilateral profit position* if $v_{iH}(t) > 0$ and a *multilateral loss position* if $v_{iH}(t) < 0$.

3.2.2 Foreign Exchange Risk Hedging and Clearing Efficiency

Let us introduce a few more concepts before going into definitions of forward book credit exposures. We say that bank i's forward position is *perfectly hedged* or that bank i's *foreign exchange risk* is *perfectly hedged* at time t if

$$(\forall c \in C, \forall t_2 \leq T_2) \quad \sum_{j\in I} x_{ij}(t,t_2,c) = 0.$$

If bank i's forward position is perfectly hedged, then, for all $T_1 \leq t \leq T_2$, we have

$$\begin{aligned} v_{iH}(t) &= \sum_{j\in I} v_{ij}(t) \\ &= \sum_{j\in I} \sum_{t<t_2\leq T_2} \sum_{c\in C} q(t,t_2,c)x_{ij}(t,t_2,c) \\ &= \sum_{t<t_2\leq T_2} \sum_{c\in C} q(t,t_2,c) \sum_{j\in I} x_{ij}(t,t_2,c) \\ &= 0 \end{aligned}$$

so that for all t

$$v_{iH}(t) = 0.$$

That is, if the forward position of a bank is perfectly hedged, then its forward position with the clearing house is nil. From a traditional banker's point of view it represents an ideal situation for foreign exchange transactions and, moreover, it will correspond to the most ideal case for contracts netting efficiency as all the credits exactly offset all the debits in each currency at every due date.

Fact 3 [Clearing Efficiency] *If a participating bank's forward position is perfectly hedged, then no settlements are needed at due dates as credits and debits are fully matched.*

3.2.3 Bilateral Default Risk under Multilateral Netting

Even if bank i's foreign exchange risk is perfectly hedged so that $v_{iH}(t) = 0$, it faces default risks of counterparties. Assume that one of the counterparties, say bank j, of forward contracts that bank i had, defaulted at time t. Then, a general rule of the clearing house is to allocate default induced losses among the participants, called *concerned* participants or *concerned* banks, who have contracts with the defaulting bank maturing at time t. Losses are allocated pro rata to the profit levels of the concerned participants. By assuming this loss allocation rule, we define *multilaterally netted bilateral exposure or indirect bilateral exposure* of bank i with respect to bank j at time t, denoted by $\eta_{ij}(t)$, to be

$$\eta_{ij}(t) \equiv \left(\frac{v_{ij}(t)^+}{\sum_{h \in I} v_{hj}(t)^+} \right) v_{Hj}(t)^+.$$

It simply says that the loss $v_{Hj}(t)^+$ of the clearing house caused by the default of bank j is allocated among those participants i having notional profit position with bank j, i.e., $v_{ij}(t)^+ > 0$, according to the ratio

$$\left(\frac{v_{ij}(t)^+}{\sum_{h \in I} v_{hj}(t)^+} \right)$$

of its profit level to the total of profits made by concerned banks in transaction with the defaulting bank j.

Now, assume bank j's forward position is not perfectly hedged so that at time t

$$v_{jH}(t) \neq 0$$

and assume that the net present value of its forward books show a great deal of losses, a typical situation for a defaulting bank. Then, the *actual credit exposure of the clearing house with respect to bank* j at time t is

$$\begin{aligned}
v_{Hj}(t)^+ &= v_{jH}(t)^- \\
&= \left(\sum_{i \in I} v_{ji}(t) \right)^- \\
&= \left(\sum_{i \in I} v_{ji}(t)^+ - \sum_{i \in I} v_{ji}(t)^- \right)^- \\
&= \max \left\{ 0, - \left(\sum_{i \in I} v_{ji}(t)^+ - \sum_{i \in I} v_{ji}(t)^- \right) \right\} \\
&= max \left\{ 0, - \left(\sum_{i \in I} v_{ij}(t)^- - \sum_{i \in I} v_{ij}(t)^+ \right) \right\} \\
&= \max \left\{ 0, \sum_{i \in I} v_{ij}(t)^+ - \sum_{i \in I} v_{ij}(t)^- \right\}. \qquad (6)
\end{aligned}$$

We thus obtain

$$\begin{aligned}
\sum_{i\in I} v_{ij}(t)^+ - v_{Hj}(t)^+ &= \sum_{i\in I} v_{ij}(t)^+ - \max\left\{0, \sum_{i\in I} v_{ij}(t)^+ - \sum_{i\in I} v_{ij}(t)^-\right\} \\
&= \min\left\{\sum_{i\in I} v_{ij}(t)^+, \sum_{i\in I} v_{ij}(t)^-\right\}. \qquad (7)
\end{aligned}$$

Since we assumed $v_{jH}(t) < 0$ so that

$$\sum_{i\in I} v_{ji}(t)^+ < \sum_{i\in I} v_{ji}(t)^-,$$

we obtain

$$v_{Hj}(t)^+ = \sum_{i\in I} v_{ij}(t)^+ - \sum_{i\in I} v_{ij}(t)^-.$$

The above equality shows that the total sum of the actual (forward book) exposures of participating banks with respect to bank j, *i.e.*, $\sum_{i\in I} v_{ij}(t)^+$, is reduced by the amount, $\sum_{i\in I} v_{ij}(t)^-$, of the losses of the forward books of the participating banks with respect to bank j. Let us record this fact.

Proposition 1 [Credit Exposure of the Clearing House] *Suppose that bank j has a multilateral loss position at time t. Then, the sum of the bilateral credit exposures of other participants with respect to the bank j at time t exceeds the actual credit exposure of the clearing house with respect to j by the amount of the sum of losses that the participants are making in transactions with j, i.e.*

$$\sum_{i\in I} v_{ij}(t)^+ - v_{Hj}(t)^+ = \sum_{i\in I} v_{ij}(t)^-. \qquad (8)$$

One can immediately compare direct bilateral exposure with indirect bilateral exposure (*i.e.*, multilaterally netted bilateral exposure) using the equality (8). In fact,

$$\begin{aligned}
\varepsilon_{ij}(t) - \eta_{ij}(t) &= v_{ij}(t)^+ - \left(\frac{v_{ij}(t)^+}{\sum_{h\in I} v_{hj}(t)^+}\right) v_{Hj}(t)^+ \\
&= \frac{v_{ij}(t)^+}{\sum_{h\in I} v_{hj}(t)^+}\left(\sum_{h\in I} v_{hj}(t)^+ - v_{Hj}(t)^+\right) \\
&= \frac{v_{ij}(t)^+}{\sum_{h\in I} v_{hj}(t)^+}\left(\sum_{h\in I} v_{hj}(t)^-\right) \qquad (9)
\end{aligned}$$

where the third equality follows from the equation (8). Hence, we obtain the following:

Proposition 2 [Bilateral Exposure vs. Multilaterally Netted Bilateral Exposure] *The difference between direct bilateral exposure and indirect bilateral exposure of a bank i with respect to any other bank $j \neq i$ having a multilateral loss position at time t is exactly equal to the sum of losses of individual banks in transaction with the bank j multiplied by the proportion of bank i's profits to the total profits of all the participating banks in transactions with the bank j, i.e.*

$$\frac{v_{ij}(t)^+}{\sum_{h\in I} v_{hj}(t)^+}\left(\sum_{h\in I} v_{hj}(t)^-\right).$$

Let us introduce a notion which indicates the extremity of market risk. We say that the *foreign exchange risk* of *bank i at time t* is *extreme* if for all $h \neq i$

$$v_{ih}(t)^+ = 0.$$

and for some $h \neq i$

$$v_{ih}(t)^- \neq 0.$$

One may decompose the reduction of total credit exposures in (8) into that of individual banks. We define (apparent) *bilateral benefits of multilateral netting*, denoted by $\beta_{ij}(t)$,[2] as the residual of direct bilateral exposures over indirect bilateral exposures, that is,

$$\begin{aligned}\beta_{ij}(t) &\equiv \varepsilon_{ij}(t) - \eta_{ij}(t)\\ &= v_{ij}(t)^+ - \left(\frac{v_{ij}(t)^+}{\sum_{h\in I} v_{hj}(t)^+}\right) v_{Hj}(t)^+.\end{aligned}$$

It follows from the definition of the bilateral benefits of multilateral netting and the equation (9) that

$$\begin{aligned}\beta_{ij}(t) &= \frac{v_{ij}(t)^+}{\sum_{h\in I} v_{hj}(t)^+}\left(\sum_{h\in I} v_{hj}(t)^-\right)\\ &= \frac{v_{ij}(t)^+}{\sum_{h\in I} v_{hj}(t)^+}\left(\sum_{h\in I} v_{jh}(t)^+\right).\end{aligned}$$

Thus, if the foreign exchange risk of bank j is not extreme, there will be positive (apparent) bilateral benefits (*i.e.*, $\beta_{ij}(t) > 0$) of multilateral netting on the part of concerned participant i, having a positive bilateral exposure, *i.e.*, $v_{ij}(t)^+ > 0$. But if the foreign exchange risk of bank j is extreme, then the above equation for $\beta_{ij}(t)$ shows that $\beta_{ij}(t) = 0$, that is, there are no bilateral benefits of multilateral netting over bilateral netting. Thus one obtains the following proposition.

[2]One should not be misled by the term benefit. This need not be an actual benefit when one considers a possibility of systemic risk.

Proposition 3 [Bilateral Benefits/No–Benefits of Multilateral Netting] *Suppose that bank j is making a loss on its forward book at time t. If its foreign exchange risk is not extreme, then for any concerned participant having a positive (notional) bilateral credit exposure, the bilateral benefit of multilateral netting is positive. But if bank j's foreign exchange risk is extreme, then there will be no benefits of multilateral netting.*

The implication of this proposition is that as the loss-making bank's foreign exchange risk increases the bilateral benefit of multilateral netting declines [3].[4] It has a policy implication as well: in order for a multilateral netting system to have increased efficiency for reducing credit risks, extreme foreign exchange contract positions should be controlled.

4 Systemic Risk Assessment — A Simple Case

The failure of one participant in a multilateral netting system to meet its required obligations when due may cause other participants unable to meet their obligations which in turn could trigger a chain reaction of defaults among participants. The risk of this type is known as *systemic risk*. The credit exposure of a participant faced with a possibility of systemic risk will be analyzed in this and the following section.

4.1 A Scenario of Systemic Risk

The approach followed here is to consider a "scenario" of a possible chain reaction of defaults in multilateral netting arrangements and to find actual credit exposures of individual participants in the given scenario of bank defaults.

[3] Of course, admittedly, we are very loose here in the statement of an increasing foreign exchange risk. It is not entirely impossible to give a formal statement of this fact but we do not think it is worth while to further complicate our setting of the model.

[4] Financial contracts netting systems provide de facto payment systems for financial contracts in the sense that a financial contract is used for a partial or entire discharge of another financial obligation. This means that a netting system makes financial contracts means of payment or medium of exchange. It may be called "quasi-money". However, this quasi-money is not supplied by any of the monetary authorities, *i.e.* central banks. Thus, it raises the traditional issue of whether and how the supply of money can be controlled by the supervisory authorities.

In a multilateral netting system the maximal amount of money being supplied corresponds to the case where all the participating banks do not have any foreign exchange risks so that for each $i \in I$

$$v_{iH}(t) = 0.$$

Then, no additional settlements will be due and the amount of money $m(t)$ "created" at settlement time t would be

$$\sum_{i,j \in I} x_{ij}(t, t, c)^{+}$$

for each currency $c \in C$.

In this subsection we describe a scenario of a most simple systemic risk. A typical scenario is as follows:

> There is an original defaulter $j_1 \in I$ that fails to meet its obligations at time t. Bank j_1 faces extreme foreign exchange risk at time t and its mark-to-market value of its forward positions shows a big loss by the amount $L_1 > 0$. This situation is described as
>
> $$\begin{aligned} v_{j_1H}(t)^- &= v_{Hj_1}(t)^+ = L_1 > 0, \\ v_{j_1H}(t)^+ &= 0. \end{aligned}$$
>
> Due to the big loss suffered by j_1, it fails to meet its obligations at time t. The bank j_1 is the original defaulter.
>
> Assume that all the other participants can meet their obligations at time t. But according to the assumed multilateral netting arrangement of this paper the loss-share rule becomes effective once there is a defaulter. The remaining participants must make additional settlement payments following the general loss-share rule of the netting arrangement. Now, assume, for the purpose of making the very nature of systemic risk more transparent, there is a participant $j_2 \neq j_1$ that has *perfectly hedged* positions so that it faces no foreign exchange risk at time t, *i.e.*
>
> $$v_{j_2H}(t) = 0,$$
>
> but has a huge profit position with respect to the original defaulter j_1 so that
>
> $$\begin{aligned} v_{j_2j_1}(t)^+ &= \pi_2 \\ (\quad v_{j_2j_1}(t)^- &= 0 \quad). \end{aligned}$$

To sum up, the situation is such that there are at least two defaulting participants, one initial and the other subsequent. The initial defaulter, in a sense, forced itself to default by taking a huge foreign exchange risk whereas the second and subsequent defaulter had perfectly hedged foreign exchange positions. But it has a huge bilateral profit position with the original defaulting bank.

4.2 Systemic Credit Exposure in a Simple Case

The possibility of a chain reaction of defaults among participants to a multilateral netting scheme clearly depends upon multilateral credit exposures of participants. We will compare below the credit exposures of participating banks under bilateral netting system with those under multilateral netting system by following the above scenario.

Let us introduce further notation to express credit exposures under multilateral netting system with possibility of multiple defaults. Given a finite sequence of banks (in our present case a sequence of two banks) $\{j_n\}_{n=1,2}$, the *systemic credit exposure* at time t of bank i with respect to the sequence, written $\sigma_{ij_1j_2}(t)$, is defined by

$$\sigma_{ij_1j_2}(t) \equiv \left(r_{ij_1}(t) + r^2_{ij_1}(t)r_{j_2j_1}(t)\right) v_{Hj_1}(t)^+ + r^2_{ij_2}(t)v_{Hj_2}(t)^+$$

where $r_{ij}(t)$ and $r^2_{ij_n}(t)$, $n = 1, 2$ are given by

$$r_{ij}(t) \equiv \frac{v_{ij}(t)^+}{\sum_{h\in I} v_{hj}(t)^+} \left(= \frac{v_{ij}(t)^+}{\sum_{h\neq j} v_{hj}(t)^+}\right), \quad r^2_{ij_n}(t) \equiv \frac{v_{ij_n}(t)^+}{\sum_{h\neq j_1,j_2} v_{hj_n}(t)^+}.$$

The *benefit of multilateral netting* of bank i with respect to the given sequence $\{j_n\}_{n=1,2}$ of banks is defined by

$$\beta_{ij_1j_2}(t) \equiv \varepsilon_{ij_1}(t) + \varepsilon_{ij_2}(t) - \sigma_{ij_1j_2}(t).$$

This can be rewritten as follows:

$$\begin{aligned} \beta_{ij_1j_2}(t) &= (\varepsilon_{ij_1}(t) - \eta_{ij_1}(t)) + (\varepsilon_{ij_2}(t) - \rho_2(t)\eta_{ij_2}(t)) \\ &\quad - r^2_{ij_1}(t)r_{j_2j_1}(t)v_{Hj_1}(t)^+ \end{aligned} \tag{10}$$

with $\rho_2(t)$ defined by

$$\rho_2(t) \equiv \frac{\sum_{h\in I} v_{hj_2}(t)^+}{\sum_{h\neq j_1,j_2} v_{hj_2}(t)^+}.$$

It follows that we have

$$\beta_{ij_1j_2}(t) = \beta_{ij_1}(t) + \beta_{ij_2}(t) - \left[(\rho_2(t) - 1)\,\eta_{ij_2}(t) + r^2_{ij_1}(t)r_{j_2j_1}(t)v_{Hj_1}(t)^+\right].$$

Since $\rho_2(t) > 1$ when $v_{j_2j_1}(t)^+ > 0$, one obtains

$$\beta_{ij_1j_2}(t) < \beta_{ij_1}(t) + \beta_{ij_2}(t).$$

We would like to record this property as follows:

Proposition 4 [Strictly Decreasing Benefits of Multilateral Netting] *Given a possibility of defaults by a sequence of two banks with the original defaulter having a multilateral loss position and the second and subsequent defaulter having a bilateral profit position with the original defaulter, it is always the case that the benefit of multilateral netting of a concerned participant is strictly less than the sum of bilateral benefits of multilateral netting, i.e.*

$$\beta_{ij_1j_2}(t) < \beta_{ij_1}(t) + \beta_{ij_2}(t).$$

If we apply the equality (9) to $j = j_1$, we get

$$\varepsilon_{ij_1}(t) - \eta_{ij_1}(t) = r_{ij_1}(t) \sum_{h \in I} v_{hj_1}(t)^-.$$

Since we have $v_{j_2 H}(t) = 0$ so that

$$\sum_{h \in I} v_{hj_2}(t)^+ = \sum_{h \in I} v_{hj_2}(t)^-, \quad v_{Hj_2}(t)^+ = 0,$$

it follows that

$$\begin{aligned} \varepsilon_{ij_2}(t) - \rho_2(t)\eta_{ij_2}(t) &= r_{ij_2}(t) \left(\sum_{h \in I} v_{hj_2}(t)^+ - \rho_2(t) v_{Hj_2}(t)^+ \right) \\ &= r_{ij_2}(t) \sum_{h \in I} v_{hj_2}(t)^-. \end{aligned}$$

Using these values of $\varepsilon_{ij_1}(t) - \eta_{ij_1}(t)$ and $\varepsilon_{ij_2}(t) - \rho_2(t)\eta_{ij_2}(t)$ along with the assumed property $v_{j_2 j_1}(t)^- = 0$ and (10), we can rewrite the expression for $\beta_{ij_1 j_2}(t)$ as below:

$$\beta_{ij_1 j_2}(t) = r_{ij_1}(t) \sum_{h \in I} v_{hj_1}(t)^- + r_{ij_2}(t) \sum_{h \in I} v_{hj_2}(t)^- - r_{ij_1}^2(t) r_{j_2 j_1}(t) v_{j_1 H}(t)^-.$$

In order to make the nature of systemic exposures transparent we consider the systemic exposure of bank i which is a concerned participant with respect to the initial defaulter bank j_1 but *not* with respect to the secondary defaulter bank j_2. Denoting the set of concerned participants with respect to bank j_n by J_n, for $i \in J_1 \backslash J_2$, we have

$$\beta_{ij_1 j_2}(t) = r_{ij_1}(t) \sum_{h \in I} v_{hj_1}(t)^- - r_{ij_1}^2(t) r_{j_2 j_1}(t) v_{j_1 H}(t)^-.$$

Thus, if we put

$$\begin{aligned} L_1 &= v_{j_1 H}(t)^-, & l &= \textstyle\sum_{h \in I} v_{hj_1}(t)^-, \\ \Pi &= \textstyle\sum_{h \in I} v_{hj_1}(t)^+, & \pi_2 &= v_{j_2 j_1}(t)^+, \end{aligned}$$

we have

$$\beta_{ij_1 j_2}(t) = r_{ij_1} L_1 \left(\frac{l}{L_1} - \rho_2(t) \frac{\pi_2}{\Pi}\right)$$

where L_1 is the net loss of defaulting bank j_1, l the total of losses of banks from contracts with j_1, Π the total of profits of banks from contracts with j_1, and π_2 the profit of the secondary defaulter j_2 from contracts with j_1. Thus,

$$\beta_{ij_1 j_2}(t) < 0 \Longleftrightarrow \frac{l}{L_1} < \rho_2(t) \frac{\pi_2}{\Pi}.$$

Since the loss L_1 suffered by the initial defaulting bank j_1 is assumed to be much larger than the total l of losses suffered by the other banks in transactions with the bank j_1 and since the profit π_2 of the secondary defaulter j_2 in transactions with the initial defaulting bank j_1 is a relatively large component of the total Π of the profits made by banks in their transactions with the bank j_1, it follows that we have

$$\beta_{ij_1j_2}(t) < 0$$

as $\rho_2(t) \geq 1$. In other words the systemic credit exposure of a concerned participant with the initial defaulter but not with the secondary defaulter under a multilateral netting system is greater than its bilateral credit exposure under a bilateral netting arrangement.

One could give a general statement of this fact as below:

Proposition 5 [Systemic Credit Exposures] *Consider a possibility of a chain reaction of defaults at time t given by a sequence of defaulting banks $\{j_n\}_{n=1,2}$. Assume that the initial defaulting bank j_1 faced with high foreign exchange risk suffers a huge mark-to-market loss by an amount L_1 and that the secondary defaulting bank j_2 with or without the perfect hedged position has a large profit amount π_2 arising from transactions with the initial defaulting bank. Then, if the proportion of the secondary defaulter's profit to the total of profits made by participants in forward transactions with the initial defaulting bank j_1 is greater than the proportion of the total forward book losses suffered by the participants in transactions with the initial defaulter j_1 to the large amount of the forward book loss suffered by the initial defaulter j_1 due to its huge foreign exchange risk, then for any participating bank having a net profit position with the initial defaulter but not with the secondary defaulter, it is always the case that its systemic credit exposure under a multilateral netting arrangement is greater than the sum of bilateral credit exposures with respect to the defaulting banks under a bilateral obligation netting arrangement.*

With the cost of increased complexity we can arrive at expressions for systemic credit exposures in a more general case where a given finite sequence of defaulting banks $\{j_n\}_{n=1,\ldots,N}$ has any length N, $1 \leq N \leq \sharp I - 1$. We will discuss it in the next section and extend Proposition 4 and Proposition 5 to a geneal case. But note that the simple cases in this section illustrate the essence of the nature concerning the stability of netting arrangements.

5 Systemic Risk Assessment — A General Case

We now turn to a general case of systemic risk and consider a possibility of chain reactions of multiple defaults of participating banks to a multilateral netting system. If a finite sequence of defaulting banks $\{j_n\}_{n=1,\ldots,N}$ at time t is given, a bank $j \in \{j_n\}_{n=1,\ldots,N}$ is called a *defaulter* or a *defaulting* bank and a bank $j \notin \{j_n\}_{n=1,\ldots,N}$ is called a *survivor* or a *surviving bank.* Given a defaulting bank j, a surviving bank i is called *positively concerned* if

$$v_{ij}(t)^+ > 0,$$

and is called *negatively concerned* if

$$v_{ij}(t)^- > 0.$$

5.1 Default Sequence and Loss Share Property

Let a finite sequence of banks $\{j_n\}_{n=1,\ldots,N}$ with $1 \le N \le \sharp I - 1$ is a possible set of defaulters at time t. For simplicity we assume that all the defaults at time t occurs in a time stream matching the order in the sequence $\{j_n\}_n$ during the date t but before the beginning of the next date $t+1$.

We first discuss some of the basic properties of the loss share rule for a given sequence of defaulting banks at time t. We then proceed to describe systemic credit exposures in general.

Now, our assumed loss share rule is to distribute the losses of the clearing house, due to a default of a participant, among positively concerned surviving banks according to the ratio of their notional bilateral profit levels with respect to the defaulter at each round of default. Let us first define

$$r_{ij_n}^m(t) \equiv \frac{v_{ij_n}(t)^+}{\sum_{h \ne j_1,\ldots,j_m} v_{hj_n}(t)^+}$$

for $i \ne j_1, \ldots, j_m$ and $n \le m \le N$. $r_{ij_n}^m(t)$ represents the proportion of the loss, $v_{Hj_n}(t)^+$, caused by the n-th defaulter j_n at time t, which the bank i has to share *immediately after* the m-th default. Coefficients, $r_{ij_n}^m(t)$'s may be called *direct bilateral loss share* coefficients at m-th round. We will need notation to express proportions of losses shared by subsequent defaulters for their preceding defaulters.

Given a finite sequence of banks $\{j_n\}_{n=1,\ldots,N}$ at time t, define for $m = 2, \ldots, N, n = 1, \ldots, N-1, m \ge n+1$

$$r_{mn}(t) \equiv \sum_{s=n}^{m-1} r_{j_m j_n}^s(t) r_{sn}(t) \tag{11}$$

with a convention that

$$r_{nn}(t) \equiv 1$$

for $n = 1, \ldots, N-1$. In particular, we have

$$\begin{aligned} r_{21}(t) &= r_{j_2 j_1}^1(t) \\ r_{31}(t) &= r_{j_3 j_1}^1(t) + r_{j_3 j_1}^2 r_{21}(t) \\ &\vdots \\ r_{N1}(t) &= r_{j_N j_1}^1(t) + r_{j_N j_1}^2(t) r_{21}(t) + \ldots + r_{j_N j_1}^{N-1}(t) r_{(N-1)1}. \end{aligned}$$

In this notation, $r_{21}(t)$ is the proportion of the loss of the initial defaulter at time t which must be shared by the second defaulter, and is equal to the proportion of its notional bilateral profit among all the positively concerned survivors after the initial default. Similarly, $r_{31}(t)$ is the total sum of the proportions of the loss of the initial defaulter at time t which must be shared by the third defaulter by the time of its own default, and is equal to the sum of the proportion of the loss which the third defaulter must share immediately after the initial default plus the proportion of the proportional loss share of the second defaulter with respect to the loss caused by the initial defaulter, which the third defaulter must share immediately after the second default. In general, $r_{mn}(t)$ is the total sum of the direct and indirect proportional loss shares of the m-th defaulter for the loss of the n-th defaulter accumulated as the sequential defaults continued up to the $(m-1)$-st round. In this sense, coefficients $r_{mn}(t)$'s may be called *total bilateral loss share coefficients* among defaulters.

We can prove the following basic property satisfied by above loss share coefficients.

Proposition 6 [Property of Loss Share Coefficients] *For any $n = 1, \dots, N$, the following equality holds:*

$$\sum_{s=n}^{k} r^s_{j_m j_n}(t) r_{sn}(t) = r^k_{j_m j_n}(t) \tag{12}$$

for $m = n+1, \dots, N$ and $k = n, \dots, m-1$.

Proof: We first prove the equation (12) for $n = 1$. For $k = 1$, (12) is true for any $m = 2, \dots, N$ as both the left-hand side and the right-hand side of the equation (12) are trivially given by $r^1_{j_m j_1}(t)$. By induction argument, assume that the equation (12) is true for all $m = 2, \dots, N$ up to $k-1$, we shall show that (12) is true for all $m = 2, \dots, N$ for k. Indeed, for any $m = 2, \dots, N$, we have

$$\begin{aligned}
&\sum_{s=1}^{k} r^s_{j_m j_1}(t) r_{s1}(t) \\
&= \sum_{s=1}^{k-1} r^s_{j_m j_1}(t) r_{s1}(t) + r^k_{j_m j_1}(t) \sum_{s=1}^{k-1} r^s_{j_k j_1}(t) r_{s1}(t) \\
&= r^{k-1}_{j_m j_1}(t) + r^k_{j_m j_1}(t) r^{k-1}_{j_k j_1}(t) \\
&= \frac{v_{j_m j_1}(t)^+}{\sum_{h \neq j_1, \dots, j_{k-1}} v_{h j_1}(t)^+} + \left(\frac{v_{j_m j_1}(t)^+}{\sum_{h \neq j_1, \dots, j_k} v_{h j_1}(t)^+} \right) \left(\frac{v_{j_k j_1}(t)^+}{\sum_{h \neq j_1, \dots, j_{k-1}} v_{h j_1}(t)^+} \right) \\
&= \frac{v_{j_m j_1}(t)^+}{\sum_{h \neq j_1, \dots, j_{k-1}} v_{h j_1}(t)^+} \left(\frac{\sum_{h \neq j_1, \dots, j_k} v_{h j_1}(t)^+ + v_{j_k j_1}(t)^+}{\sum_{h \neq j_1, \dots, j_k} v_{h j_1}(t)^+} \right)
\end{aligned}$$

$$
\begin{aligned}
&= \frac{v_{j_m j_1}(t)^+}{\sum_{h \neq j_1,\ldots,j_k} v_{hj_1}(t)^+} \\
&= r^k_{j_m j_1}(t)^+
\end{aligned}
$$

where the second equality follows from the induction hypothesis. This completes the proof of the equation (12) for all $m = 2, \ldots, N$ and $k = 1, \ldots, m-1$ for the case of $n = 1$.

By the induction hypothesis, assume that the equation (12) is true for all $m = n+1, \ldots, N$ and $k = n, \ldots, m-1$ up to the case of $n-1$. We shall show (12) for all $m = n+1, \ldots, N$ and $k = n, \ldots, m-1$ for the case of n. We use again an induction argument on k.

If $k = n$, then for all $m = n+1, \ldots, N$ both the left-hand side and the right-hand side of (12) are equal to $r^n_{j_m j_n}(t)$. Assume it is true up to $k-1$. Then, we have

$$
\begin{aligned}
&\sum_{s=n}^{k} r^s_{j_m j_n}(t) r_{sn}(t) \\
&= \sum_{s=n}^{k-1} r^s_{j_m j_n}(t) r_{sn}(t) + r^k_{j_m j_n}(t) \sum_{s=n}^{k-1} r^s_{j_k j_n}(t) r_{sn}(t) \\
&= r^{k-1}_{j_m j_n}(t) + r^k_{j_m j_n}(t) r^{k-1}_{j_k j_n}(t) \\
&= \frac{v_{j_m j_n}(t)^+}{\sum_{h \neq j_1,\ldots,j_{k-1}} v_{hj_1}(t)^+} + \left(\frac{v_{j_m j_n}(t)^+}{\sum_{h \neq j_1,\ldots,j_k} v_{hj_n}(t)^+} \right) \left(\frac{v_{j_k j_n}(t)^+}{\sum_{h \neq j_1,\ldots,j_{k-1}} v_{hj_n}(t)^+} \right) \\
&= \frac{v_{j_m j_n}(t)^+}{\sum_{h \neq j_1,\ldots,j_{k-1}} v_{hj_n}(t)^+} \left(\frac{\sum_{h \neq j_1,\ldots,j_k} v_{hj_n}(t)^+ + v_{j_k j_n}(t)^+}{\sum_{h \neq j_1,\ldots,j_k} v_{hj_n}(t)^+} \right) \\
&= \frac{v_{j_m j_n}(t)^+}{\sum_{h \neq j_1,\ldots,j_k} v_{hj_n}(t)^+} \\
&= r^k_{j_m j_n}(t)^+
\end{aligned}
$$

where the second equality follows from the induction hypothesis. This completes the steps in our induction arguments of a proof of the proposition. ■

When the n-th bank j_n in the sequence $\{j_n\}_{n=1,\ldots,N}$ defaults at the n-th round, the bank j_m, which itself defaults at a later m-th round, must share the burden of the loss of the clearing house due to the default of the bank j_n. The proportional burden of the bank j_m from this default is $r^n_{j_m j_n}(t)$. But at the next round of defaults, the $(n+1)$-st bank j_{n+1} in the sequence defaults. It brings in additional burdens of loss shares to surviving banks in two ways. A direct additional burden of loss share might come from the loss of the clearing house in case the new defaulter j_{n+1} has a multilateral loss position. A more important *indirect* additional burden of loss share comes from an increase in the proportions of loss share for losses caused by each of its preceding defaulters. Thus, in this manner the proportional burden of the bank j_m for loss caused by a preceding defaulter j_n accumulates as additional banks default prior to the default of the bank j_m itself. Now, what the above Proposition 6 says is that at k-th round of default this accumulated proportional burden of the bank j_m for loss caused by its preceding defaulter j_n simply amounts to the *ratio of its notional bilateral profit level* with respect to the bank j_n among all the positively concerned survivors at the k-th round ($n < k < m$).

As an immediate corollary to Proposition 6 one obtains

$$\begin{aligned} r_{mn}(t) &= \sum_{s=n}^{m-1} r^s_{j_m j_n}(t) r_{sn}(t) \\ &= r^{m-1}_{j_m j_n}(t) \ . \end{aligned}$$

We record it as the following proposition.

Proposition 7 [Loss Share Coefficients and Direct-Indirect Loss Share] *Given two defaulters j_m and j_n with j_n preceding j_m in the default sequence at time t, the total sum of proportional shares of loss, due to the default of j_n, that the bank j_m has to share directly or indirectly, is exactly equal to the direct proportional loss share of j_m excluding the first $m-1$ defaulters, that is,*

$$r_{mn}(t) = r^{m-1}_{j_m j_n}(t) \ .$$

We are now ready to show the similar property as in Proposition 6 for an arbitrary participating bank to multilateral netting arrangements.

Proposition 8 [Loss Share Coefficients in General] *For any participating bank $i \in I$ to the multilateral netting, the direct loss share coefficients at the m-th round of defaults with respect to the loss of the clearing house due to the n-th defaulter ($n \leq m$), $r^m_{ij_n}(t)$, is exactly equal to its accumulated direct and indirect loss shares up to the m-th round of defaults, that is, for any $i \neq j_1, \ldots, j_n, n = 1, \ldots, N$ or for any $i = j_{n'}$, with $n' \geq m \geq n$, $n' \neq n$, one has*

$$r^m_{ij_n}(t) = \sum_{s=n}^{m} r^s_{ij_n}(t) r_{sn}(t) \tag{13}$$

for $m = n, \ldots, N$.

Proof: By Proposition 6 it is sufficient to prove (13) for $i \neq j_1, \ldots, j_N$. We use the result of Proposition 7 and an induction argument on m exactly alike the one in Proposition 6.

Let $i \neq j_1, \ldots, j_N$ be given arbitrarily. Given any $n = 1, \ldots, N$, if $m = n$, both the left-hand side and the right-hand side of (13) are given by $r^n_{ij_n}(t)$.

Assume, by induction, the equality (13) is true up to $m-1$ for any $n = 1, \dots, N$. Then, we have

$$\begin{aligned}
&\sum_{s=n}^{m} r^{s}_{ij_n}(t) r_{sn}(t) \\
&= \sum_{s=n}^{m-1} r^{s}_{ij_n}(t) r_{sn}(t) + r^{m}_{ij_n}(t) r_{mn}(t) \\
&= r^{m-1}_{ij_n}(t) + r^{m}_{ij_n}(t) r^{m-1}_{j_m j_n}(t) \\
&= \frac{v_{ij_n}(t)^+}{\sum_{h \neq j_1,\dots,j_{m-1}} v_{hj_n}(t)^+} + \left(\frac{v_{ij_n}(t)^+}{\sum_{h \neq j_1,\dots,j_m} v_{hj_n}(t)^+} \right) \left(\frac{v_{j_m j_n}(t)^+}{\sum_{h \neq j_1,\dots,j_{m-1}} v_{hj_{m-1}}(t)^+} \right) \\
&= \frac{v_{ij_n}(t)^+}{\sum_{h \neq j_1,\dots,j_{m-1}} v_{hj_n}(t)^+} \left(\frac{\sum_{h \neq j_1,\dots,j_m} v_{hj_n}(t)^+ + v_{j_m j_n}(t)^+}{\sum_{h \neq j_1,\dots,j_m} v_{hj_n}(t)^+} \right) \\
&= r^{m}_{ij_n}(t)
\end{aligned}$$

where the second equality is a consequence of the induction hypothesis and Proposition 7. This completes the proof. ∎

Let us introduce further notation to express surviving banks' total direct and indirect loss shares. For each $i \neq j_1, \dots, j_N$ and for each $n = 1, \dots, N$, define

$$l_{in}(t) \equiv \sum_{m=n}^{N} r^{m}_{ij_n}(t) r_{mn}(t).$$

$l_{in}(t)$ is the total sum of proportional shares by bank i of the loss caused by the n-th defaulter, accumulated at the end of the default sequence $\{j_n\}_{n=1,\dots,N}$ at time t. Note the similarity as well as the difference between $l_{in}(t)$ and $r_{mn}(t)$. $l_{in}(t)$ is the total bilateral loss share of a surviving bank i with respect to the loss of the n-th defaulter whereas $r_{mn}(t)$ is the total bilateral proportional loss share of the m-th defaulter with respect to the loss of the n-th defaulter. As an immediate corollary to Proposition 8, we obtain

$$\begin{aligned}
l_{in}(t) &= \sum_{m=n}^{N} r^{m}_{ij_n}(t) r_{mn}(t) \\
&= r^{N}_{ij_n}(t).
\end{aligned}$$

Proposition 9 [Survivors' Total Loss Share Coefficients] *For any surviving bank i, the total sum of direct and indirect proportional shares of the loss of the clearing house due to the n-th defaulting bank is identical to the direct proportional share of the loss of the clearing house due to the n-th defaulter when all the N defaulters are excluded in the calculation of its proportional share, that is,*

$$l_{in}(t) = r^{N}_{ij_n}(t).$$

5.2 General Systemic Credit Exposure

Given a finite sequence of banks $\{j_n\}_{n=1,\ldots,N}$ at time t with $1 \leq N \leq \sharp I - 1$, the *systemic credit exposure* of bank i at time t with respect to the sequence $\{j_n\}_{n=1,\ldots,N}$, written $\sigma_{ij_1\ldots j_N}(t)$, is defined by

$$\sigma_{ij_1\ldots j_N}(t) \equiv \sum_{n=1}^{N} \left(\sum_{m=n}^{N} r_{ij_n}^{m}(t) r_{mn}(t) \right) v_{Hj_n}(t)^{+} \tag{14}$$

for $i \neq j_1, \ldots, j_N$ and $n \leq m \leq N$.

In order to understand the meaning of the definition given by (14), let us consider a sequence of three defaulting banks $\{j_n\}_{n=1,2,3}$ at time t. Then, we have

$$\begin{aligned} \sigma_{ij_1j_2j_3}(t) \quad &\equiv \left(r_{ij_1}^{1}(t) + r_{ij_1}^{2}(t) r_{21}(t) + r_{ij_1}^{3}(t) r_{31}(t) \right) v_{Hj_1}(t)^{+} \\ &+ \left(r_{ij_2}^{2}(t) + r_{ij_2}^{3}(t) r_{32}(t) \right) v_{Hj_2}(t)^{+} + r_{ij_3}^{3}(t) v_{Hj_3}(t)^{+}. \end{aligned}$$

The meaning of the last term $r_{ij_3}^{3}(t) v_{Hj_3}(t)^{+}$ is straightforward. It is the bank i's direct share of the loss of the clearing house due to the third defaulting bank j_3. The second term consists of two parts. The first part $r_{ij_2}^{2}(t) v_{Hj_2}(t)^{+}$ is comparable to the last term except for the defaulting bank. It is the bank i's direct share of the loss of the clearing house caused by the second defaulting bank j_2. The second part $r_{ij_2}^{3}(t) r_{32}(t) v_{Hj_2}(t)^{+}$ represents the bank i's indirect share of the loss that the third defaulting bank j_3 had to share for the loss caused by the second defaulting bank. The first term is composed of three parts. The first part $r_{ij_1}^{1}(t) v_{Hj_1}(t)^{+}$ is again straightforward. It is the bank i's direct share of the loss of the clearing house due to the failure of the first bank j_1. The second part $r_{ij_1}^{2}(t) r_{21}(t) v_{Hj_1}(t)^{+}$ is the bank i's indirect share of the loss that the second defaulting bank j_2 had to share for the loss caused by the first defaulting bank j_1. Similarly, the third part $r_{ij_1}^{3}(t) r_{31}(t) v_{Hj_1}(t)^{+}$ represents the bank i's indirect share of the loss that the third defaulting bank j_3 had to share for the loss caused by the first defaulting bank j_1.

Thus, in general, in the expression (14), n-th term is composed of $N - n+1$ parts. The first part $r_{ij_n}^{n}(t) v_{Hj_n}(t)^{+}$ represents the bank i's direct share of the loss of the clearing house due to the default of n-th bank. The m-th part $r_{ij_n}^{m}(t) r_{mn}(t) v_{Hj_n}(t)^{+}$, $n < m \leq N$, represents the bank i's indirect share of the loss that the m-th defaulting bank j_m had to share for the loss caused by the n-th defaulting bank j_n.

With the interpretation of each term in the expression (14) as above, the systemic credit exposure of a survived bank i represents the sum of its share of losses that are directly or indirectly related to its forward book profits position with respect to defaulting banks. In order to see that a participant may have to share a part of the losses that are *indirectly* related to its profit position, let us take for example the term $r_{ij_1}^{3}(t) r_{31}(t) v_{Hj_1}(t)^{+}$ in a sequence of three-bank defaults. It represents the part of the losses of the initial defaulting bank j_1 which came to be shared by the surviving bank i because it was originally due to be shared by bank j_3 that subsequently defaulted.

Let us rewrite the expression of systemic credit exposure given by (14) using the properties of loss share coefficients derived in the previous section. By Proposition 9 we obtain

$$\begin{aligned}\sigma_{ij_1\cdots j_N}(t) &= \sum_{n=1}^{N}\left(\sum_{m=n}^{N} r_{ij_n}^{m}(t)r_{mn}(t)\right)v_{Hj_n}(t)^+\\ &= \sum_{n=1}^{N} l_{in}(t)v_{Hj_n}(t)^+\\ &= \sum_{n=1}^{N} r_{ij_n}^{N}(t)v_{Hj_n}(t)^+\\ &= \sum_{n=1}^{N}\left(\frac{v_{ij_n}(t)^+}{\sum_{h\neq j_1,\ldots,j_N} v_{hj_n}(t)^+}\right)v_{Hj_n}(t)^+.\end{aligned}$$

The systemic credit exposure of a surviving bank is defined to be the total of sequentially accumulated proportional shares of losses of the defaulters, that the survivor has to share in proportion to its notional bilateral profit level with respect to each defaulter. It can be calculated in two mutually equivalent ways: One natural way is to calculate the amount

$$\sum_{n=1}^{N}\left(\sum_{m=n}^{N} r_{ij_n}^{m}(t)r_{mn}(t)\right)v_{Hj_n}(t)^+$$

as in the definition (14) which, at each round of an additional default, one proportionally divides the total direct and indirect loss shares of the defaulter among the positively concerned survivors at that round. Second way is to calculate the amount

$$\sum_{n=1}^{N}\left(\frac{v_{ij_n}(t)^+}{\sum_{h\neq j_1,\ldots,j_N} v_{hj_n}(t)^+}\right)v_{Hj_n}(t)^+$$

which, at each round of default, simply divides the loss of the clearing house due to the multilateral loss position of the defaulter of that round among the positively concerned survivors according to their notional bilateral profit level with respect to the defaulter of that round. Let us record this as follows:

Proposition 10 [Calculation of Systemic Credit Exposure] *The systemic credit exposure of a surviving bank i can be calculated in the following two equivalent ways:*

$$\sigma_{ij_1,\ldots,j_N}(t) = \sum_{n=1}^{N}\left(\sum_{m=n}^{N} r_{ij_n}(t)r_{mn}(t)\right)v_{Hj_n}(t)^+$$

$$= \sum_{n=1}^{N} r_{ij_n}^{N}(t) v_{Hj_n}(t)^{+}.$$

5.3 Benefit Function

We are ready to define the *benefit of multilateral netting* of bank i with respect to a given finite sequence $\{j_n\}_{n=1,\dots,N}$ of banks at time t. It is defined by

$$\beta_{ij_1\dots j_N}(t) \equiv \sum_{n=1}^{N} \varepsilon_{ij_n}(t) - \sigma_{ij_1\dots j_N}(t) \tag{15}$$

The benefit of multilateral netting is thus the difference between the sum of notional bilateral credit exposures and the systemic credit exposures with respect to a given sequence of banks. So to speak, our approach is that of the *comparison by contingencies* in assessing the possible benefit of multilateral netting *vis-à-vis* bilateral netting.

It is convenient to rewrite (15) as below:

$$\begin{aligned}
&\beta_{ij_1\dots j_N}(t) \\
&= \left[\varepsilon_{ij_1}(t) - \left(r_{ij_1}^{1}(t) + r_{ij_1}^{2}(t) r_{21}(t) + \dots + r_{ij_1}^{N}(t) r_{N1}(t)\right) v_{Hj_1}(t)^{+}\right] + \dots \\
&\quad + \left[\varepsilon_{ij_n}(t) - \left(r_{ij_n}^{n}(t) + r_{ij_n}^{n+1}(t) r_{(n+1)n}(t) + \dots + r_{ij_n}^{N}(t) r_{Nn}(t)\right) v_{Hj_n}(t)^{+}\right] + \dots \\
&\quad + \varepsilon_{ij_N}(t) - r_{ij_N}^{N}(t) v_{Hj_N}(t)^{+} \\
&= \sum_{n=1}^{N}\left(\varepsilon_{ij_n}(t) - \rho_n(t)\eta_{ij_n}(t)\right) - \sum_{n=1}^{N-1}\left(\sum_{m=1}^{N-n} r_{ij_n}^{n+m}(t) r_{(n+m)n}(t)\right) v_{Hj_n}(t)^{+}
\end{aligned} \tag{16}$$

where

$$\rho_1(t) \equiv 1 \text{ and } \rho_n(t) \equiv \frac{\sum_{h\neq j_1} v_{hj_n}^{+}(t)}{\sum_{h\neq j_1,\dots,j_n} v_{hj_n}^{+}(t)} \text{ for } n \geq 2 .$$

When there is a possibility of multiple defaults, a simple sum of bilateral benefits of multilateral netting usually overstates benefits of multilateral netting. This fact was pointed out in Proposion 4 in a most simple case of systemic risk. We shall proceed to show that the statement is true in a general case also.

It follows from (16) that

$$\begin{aligned}
&\beta_{ij_1\dots j_N}(t) \\
&= \sum_{n=1}^{N}\left(\varepsilon_{ij_n}(t) - \eta_{ij_n}(t)\right) - \Big[\sum_{n=1}^{N}(\rho_n(t) - 1)\eta_{ij_n}(t) \\
&\qquad + \sum_{n=1}^{N-1}\left(\sum_{m=1}^{N-n} r_{ij_n}^{n+m}(t) r_{(n+m)n}(t)\right) v_{Hj_n}(t)^{+}\Big]
\end{aligned}$$

$$
\begin{aligned}
= & \sum_{n=1}^{N} \beta_{ij_n}(t) - \Big[\sum_{n=1}^{N}(\rho_n(t)-1)\eta_{ij_n}(t) \\
& + \sum_{n=1}^{N-1}\left(\sum_{m=1}^{N-n} r_{ij_n}^{n+m}(t) r_{(n+m)n}(t)\right) v_{Hj_n}(t)^+\Big].
\end{aligned}
$$

Therefore, we obtain

$$\beta_{ij_1}\cdots j_N(t) < \sum_{n=1}^{N} \beta_{ij_n}(t)$$

if one of the following conditions hold:

- $(\exists j_n) v_{j_n j_{n-1}}(t)^+ > 0$ and $v_{ij_n}(t)^+ > 0$, or
- $(\exists(n,m)$ with $n < m) v_{Hj_n}(t)^+ > 0, v_{j_m j_n}(t)^+ > 0$, and $v_{ij_n}(t)^+ > 0$.

In the former instance one has

$$\eta_{ij_n}(t) > 0 \text{ and } \rho_n(t) > 1$$

for j_n, and in the latter instance one has

$$r_{ij_n}^{m}(t) r_{mn}(t) v_{Hj_n}(t)^+ > 0$$

so that in each of these instances a positive amount must be subtracted from $\sum_{n=1}^{N} \beta_{ij_n}(t)$ to arrive at the amount $\beta_{ij_1\cdots j_N}(t)$. We thus obtain the following proposition.

Proposition 11 [General Decreasing Benefits of Multilateral Netting] *Suppose that there is a possibility of multiple defaults among participants to a multilateral netting system with the initial defaulting bank having a multilateral loss position. Then, for any participating bank i to the netting system, the sum of bilateral benefits of multilateral netting always overstates actual benefits of multilateral netting, i.e.* $\beta_{ij_1\cdots j_n}(t) < \sum_{n=1}^{N} \beta_{ij_n}(t)$, *if, in a sequence of possible multiple defaulters at time t,*

- *there is a subsequent defaulter with whom the participant i is positively concerned and who is positively concerned with the immediately preceding defaulter, or*
- *there are at least two defaulters with the preceding defaulter having a multilateral loss position such that the participant i is positively concerned with the preceding defaulter and that the succeeding defaulter is positively concerned with the preceding defaulter.*

In comparing merits of multilateral netting with those of bilateral netting from a view point of credit risk reduction one might be tempted to compare multilaterally netted bilateral exposures with (actual) bilateral credit exposures. However, if one does compare bilateral netting with multilateral netting using the sum of bilateral benefits of multilateral netting in the similar spirits as above, then Proposition 11 above warns us that in possible events of multiple bank failures one overestimates the benefits of multilateral netting because the actual benefit of multilateral netting is strictly less than the sum of bilateral benefits of multilateral netting due to a possibility of an indirect sharing of losses of defaulting banks.

One should probably stress the fact that there are possibilities of indirect sharing of losses in face of a chain reaction of multiple defaults that could reduce the attractiveness of multilateral netting system over that of bilateral netting system. Actual situation for multilateral netting may be worse in the sense that a reversal of relative attractiveness of netting between multilateral and bilateral netting may occur when one allows for a consideration of multiple defaults. In order to show that there are possibilities of negative benefits of multilateral netting over bilateral netting, we wish to proceed to exhibit conditions under which we would have

$$\beta_{ij_1\cdots j_N}(t) < 0$$

in general.

For this purpose it will become convenient to rewrite the equation (15) again using Proposition 10 as below:

$$\begin{aligned}
&\beta_{ij_1,\dots,j_N}(t) \\
&= \sum_{n=1}^{N} \left(\varepsilon_{ij_n}(t) - r^N_{ij_n}(t) v_{Hj_n}(t)^+\right) \\
&= \sum_{n=1}^{N} \left(v^+_{ij_n}(t) - \left(\frac{v_{ij_n}(t)^+}{\sum_{h\neq j_1,\dots,j_N} v_{hj_n}(t)^+}\right) v_{Hj_n}(t)^+\right) \\
&= \sum_{n=1}^{N} \left(\frac{v_{ij_n}(t)^+}{\sum_{h\neq j_1,\dots,j_N} v_{hj_n}(t)^+}\right) \left(\sum_{h\neq j_1,\dots,j_N} v_{hj_n}(t)^+ - v_{Hj_n}(t)^+\right).
\end{aligned}$$

The last term of the last equality above can be rewritten using the equality (6)

$$\begin{aligned}
&\sum_{h\neq j_1,\dots,j_N} v_{hj_n}(t)^+ - v_{Hj_n}(t)^+ \\
&= \sum_{h\neq j_1,\dots,j_N} v_{hj_n}(t)^+ - \max\left\{\sum_{h\in I} v_{hj_n}(t)^+ - \sum_{h\in I} v_{hj_n}(t)^-, 0\right\}
\end{aligned}$$

$$= \min\left\{\sum_{h\in I} v_{hj_n}(t)^- - \sum_{h=j_1,\dots,j_N} v_{hj_n}(t)^+, \sum_{h\neq j_1,\dots,j_N} v_{hj_n}(t)^+\right\}.$$

Hence, we obtain

$$\begin{aligned}
&\beta_{ij_1\dots j_N}(t)\\
&= \sum_{n=1}^{N}\left(\frac{v_{ij_n}(t)^+}{\sum_{h\neq j_1,\dots,j_N} v_{hj_n}(t)^+}\right)\times\\
&\quad\left(\min\left\{\sum_{h\in I} v_{hj_n}(t)^- - \sum_{h\neq j_1,\dots,j_N} v_{hj_n}(t)^+, \sum_{h\neq j_1,\dots,j_N} v_{hj_n}(t)^+\right\}\right) \quad (17)
\end{aligned}$$

We shall come back to this expression after introducing a couple of concepts concerning hedging in the next subsection.

5.4 Comparison of Credit Risks under Multilateral and Bilateral Netting

Recall that bank i has a *perfectly hedged* position at time t if

$$(\forall c \in C, \forall t_2 \leq T_2) \quad \sum_{j\in I} x_{ij}(t, t_2, c) = 0.$$

If bank i has a perfectly hedged position at time t, then

$$v_{iH}(t) = 0.$$

Even if a bank has a perfectly hedged position, it cannot avoid default risk of a counterparty. This motivates us to introduce a further concept of perfect hedging. Let us say that bank i has a *pairwise perfectly hedged* position with respect to bank $j \neq i$ at time t if

$$(\forall c \in C \text{ and } \forall t_2 \leq T_2) \; x_{ij}(t, t_2, c) = 0. \quad (18)$$

A perfect pairwise hedging with respect to all $j \neq i$ clearly implies a perfect hedging. But note that if a party has a pairwise perfectly hedged position with respect to its counterparty, then it will not face default risk of its counterparty. Since our focus is the reduction of credit risk rather than that of market or foreign exchange risk in comparing different netting arrangements, we shall analyze in this subsection participants who have a perfectly hedged position but *not* a pairwise perfectly hedged position with an initial defaulter for, otherwise, it will not face credit risk. However, in order to minimize the complexity of analysis we analyze cases of a chain of bank defaults where a participant has pairwise perfectly hedged positions with secondary and subsequent defaulters.

A scenario of systemic risk we consider in this subsection is as follows:

- The original defaulter $j_1 \in I$ fails to meet its obligations at time t. The bank j_1 faces foreign exchange risk at time t and its multilateral position shows a huge loss by an amount $L_1 > 0$ so that
$$v_{Hj_1}(t)^+ = v_{j_1H}(t)^- = L_1 > 0 \ .$$
- Bank i has a perfectly hedged forward position so that
$$v_{iH}(t) = 0,$$
but it has bilateral profit position with the initial defaulter j_1. The bank i, however, has pairwise perfectly hedged positions with the remaining defaulting banks $j_2, \ldots, j_N$.

Then, we have
$$v_{ij_n}(t)^+ = 0 \text{ for } n = 2, \ldots, N \ .$$
To ease our notation, put
$$\begin{aligned} \pi &\equiv v_{ij_1}(t)^+ > 0 \ , \\ l &\equiv \sum_{h\in I} v_{hj_1}(t)^-, \\ \Pi &\equiv \sum_{h\in I} v_{hj_1}(t)^+ \ , \text{ and} \\ \delta &\equiv \frac{\sum_{h=j_2,\ldots,j_N} v_{hj_1}(t)^+}{\sum_{h\in I} v_{hj_1}(t)^+} \ . \end{aligned}$$
π is the notional bilateral profit of the bank i with respect to the initial defaulter j_1 and Π is the total sum of notional bilateral profits of participants with respect to the defaulter j_1. l, on the other hand, is the total sum of notional bilateral losses of participants with respect to j_1. We have
$$L_1 = \Pi - l$$
and since we are assuming that the initial defaulter j_1 is making a huge loss, put
$$\Pi = Ml$$
for some "large" number M. Then,
$$L_1 = (M-1)l$$
Now, it follows from the equality (17) that we have
$$\begin{aligned} &\beta_{ij_1\cdots j_N}(t) \\ &= r^N_{ij_1}(t) \min\left\{ \sum_{h\in I} v_{hj_1}(t)^- - \sum_{h=j_1,\ldots,j_N} v_{hj_1}(t)^+, \sum_{h\neq j_1,\ldots,j_N} v_{hj_1}(t)^+ \right\} \\ &= r^N_{ij_1}(t) \left(\sum_{h\in I} v_{hj_1}(t)^- - \sum_{j_2,\ldots,j_N} v_{hj_1}(t)^+ \right) \qquad (19) \end{aligned}$$

where the first and second equalities follow from the assumed properties that i has pairwise perfectly hedged positions with respect to banks $j_2, \ldots, j_N$ so that $v_{ij_n}(t)^+ = 0$ for $n = 2, \ldots, N$, and that

$$v_{Hj_1}(t)^+ = \sum_{h \in I} v_{hj_1}(t)^+ - \sum_{h \in I} v_{hj_1}(t)^- > 0.$$

Using simplified notation, one obtains

$$\beta_{ij_1 \ldots j_N}(t) = r^N_{ij_1}(t)(l - \delta\Pi) \ . \tag{20}$$

Since $\delta\Pi = \delta M l$ and since i is positively concerned with j_1 so that $r^N_{ij_1}(t) > 0$, we have

$$\beta_{ij_1 \ldots j_N}(t) < 0 \Longleftrightarrow \delta M > 1 \ . \tag{21}$$

This gives a necessary and sufficient condition under which one has

$$\beta_{ij_1 \ldots j_N}(t) < 0 \ .$$

Recall that M indicates that the total of the notional bilateral losses of the initial defaulter with its counterparties is M times the total of its profits with its counterparties. Thus, the systemic credit exposure under a multilateral netting system is always greater than the sum of bilateral credit exposures with respect to the possible sequence of defaulting banks if and only if M times the proportion of the sum of notional bilateral profits of all the remaining defaulters to the total of notional bilateral profits with the initial defaulter is greater than 1, that is, $\delta M > 1$; in other words, if and only if M is large enough so that its inverse is strictly less than the proportion of the sum of notional bilateral profits of the remaining defaulters to the total of notional bilateral profits of all the participants with respect to the initial defaulter. This condition will be met if the total of bilateral losses of the initial defaulter is very large relative to its profits at time t.

Since one has

$$\begin{aligned} \delta = \frac{\sum_{h=j_2,\ldots,j_N} v_{hj_1}(t)^+}{\sum_{h \in I} v_{hj_1}(t)^+} &> \frac{1}{M} = \frac{\sum_{h \in I} v_{hj_1}(t)^-}{\sum_{h \in I} v_{hj_1}(t)^+} \\ \Longleftrightarrow \quad \sum_{h=j_2,\ldots,j_N} v_{hj_1}(t)^+ &> \sum_{h \in I} v_{hj_1}(t)^- \ , \end{aligned}$$

still another way of stating the condition above is that the total of notional bilateral profits of the remaining defaulters, with respect to the initial defaulter, is strictly greater than the total of notional bilateral losses of all the participants with respect to the initial defaulter.

We thus obtain the following proposition which we regard as the main result of the present paper:

Proposition 12 [Multilateral vs. Bilateral Netting] *Consider a possible arbitrary finite sequence of defaulting banks* $\{j_n\}_{n=1,\dots,N}$, $1 < N < \sharp I$ *at time* t*. Assume that the initial defaulting bank* j_1 *faced with high foreign exchange risk suffers a huge mark-to-market gross loss by an amount* L_1 *which is* M *times the amount of the sum of mark-to-market notional bilateral losses of other participants with respect to the initial defaulter* j_1*.*

Consider any participating bank that has a net bilateral profit position with the initial defaulter. And assume that at time t *it has a multilateral perfectly hedged position and pairwise perfectly hedged positions with respect to each of the defaulting banks except for the initial defaulter. Then, its systemic credit exposure under multilateral netting is strictly greater than the sum of bilateral credit exposures with respect to each of the banks along the sequence of possible defaulters under bilateral netting if and only if the total of notional bilateral profits of the remaining defaulters, with respect to the initial defaulter, is strictly greater than the total of notional bilateral losses of all the participants with respect to the initial defaulter, or differently put, if and only if* M *is large enough so that its inverse is strictly less than the proportion of the sum of notional bilateral profits of the remaining defaulters to the total of notional bilateral profits of all the participants with respect to the initial defaulter.*

5.5 Loss Share by Unconcerned Participants and Pairwise Perfect Hedging

We have pursued the comparison of credit risk under netting arrangements between bilateral and multilateral netting with a basic assumption that all the losses incurred by the clearing house due to defaults of participants are shared by positively concerned participants only. Our intention was to free our analysis from risk factors of the clearing house and unconcerned participants. In a general multilateral netting arrangement, however, there are three further elements that are built into a system. First is the imposition of a part of losses incurred by the clearing house to itself. Second is a requirement of collaterals and/or margins on participating banks. Third is the imposition of loss sharing on participating banks regardless of whether they are concerned or not.

Admittedly, it is important to incorporate the role of the clearing house in reducing credit risk of participants as a management institution that shares a part of the losses induced by defaulting participants, but it seems to us that it will produce a considerable complication of our analysis in that it will surely add the default risk of the clearing house itself to the credit risk of each participant. It means that one needs to incorporate credit exposure to the clearing house in calculation of credit exposures of each participant. We did not go into this aspect of credit exposures in this paper.

Collaterals and margins certainly reduce credit risk of participants of any netting arrangements. Thus, it will not make a fair comparison between multilateral and bilateral netting if we introduce requirements of collaterals and/or margins in a multilateral netting arrangement only. Hence, we altogether left out these requirements in the present paper.

We wish to introduce in this subsection the third element pointed out above — that is, loss sharing by unconcerned participants of netting arrangement. We shall describe a general loss share rule of unconcerned participants by a given set of functions $\{g_i : R_+ \to R\}_{i \in I}$ that satisfy

- $g_i(L) > 0$ for any $L > 0$ and $i \in I$
- $\sum_{i \in I} g_i(L) < L$ for any $L > 0$

where L is the total losses of the clearing house.

Given a finite sequence of possible defaulters $\{j_n\}_{n=1,\dots,N}$ at time t, the total sum of losses caused by defaulters in $\{j_n\}_n$ is denoted by

$$L(t) \equiv \sum_{n=1}^{N} v_{Hj_n}(t)^+ .$$

If a set of functions $\{g_i\}_{i \in I}$ describe the part of loss sharing among participants, which is not necessarily related to their notional bilateral profit positions with the defaulters, then out of $L(t)$ the amount

$$\sum_{i \in I} g_i (L(t)) \ \left(< L(t)\right)$$

is shared by the participants without any regard to their profit positions. Let us assume that the remaining part of the losses are allocated among survived concerned participants pro rata to their notional bilateral profit levels with respect to defaulters. Then, the loss of the clearing house due to the defaulting bank j_n is

$$v_{Hj_n}(t) - \frac{v_{Hj_n}(t)^+}{\sum_{n=1}^{N} v_{Hj_n}(t)^+} L^\star(t)$$

where

$$L^\star(t) \equiv L(t) - \sum_{i \in I} g_i (L(t)) .$$

Let us write

$$v^\star_{Hj_n}(t)^+ \equiv v_{Hj_n}(t)^+ - \frac{v_{Hj_n}(t)^+}{\sum_{n=1}^{N} v_{Hj_n}(t)^+} L^\star(t) .$$

Then, $v^\star_{Hj_n}(t)^+$ represents the part of the loss due to the failure of the bank j_n, which is shared among the positively concerned participants. The *systemic credit exposure* at time t of bank $i \neq j_1, \dots, j_N$ with respect to the given sequence $\{j_n\}_{n=1,\ldots,N}$, written $\sigma^{\star}_{ij_1\ldots j_n}(t)$, is defined as in (14) with $v_{Hj_n}(t)^+$ replaced by $v^{\star}_{Hj_n}(t)^+$ and an independent loss sharing part g_i added, that is

$$\sigma^{\star}_{ij_1\ldots j_N}(t) \equiv \left(\sum_{n=1}^{N}\sum_{m=n}^{N} r^{m}_{ij_n}(t) r_{mn}(t)\right) v^{\star}_{Hj_n}(t)^+ + g_i\left(L(t)\right). \tag{22}$$

The benefit function (15) is redefined accordingly using the above (22).

$$\beta^{\star}_{ij_1\ldots j_N}(t) \equiv \sum_{n=1}^{N} \varepsilon_{ij_n}(t) - \sigma^{\star}_{ij_1\ldots j_N}(t). \tag{23}$$

Now, it is intuitively clear that a pairwise perfect hedging gives the most efficient case of bilateral netting. However, given Fact (3) which states that perfect hedging implies the clearing efficiency, it would first seem natural to expect that in case of a pairwise perfect hedging both bilateral and multilateral netting reduce credit exposures of a participant to nil. This expectation, however, is false. Contrary to the expectation, the case of a pairwise perfect hedging exemplifies the very nature of a weakness inherent in multilateral netting. It is this: in face of a pairwise perfect hedging with respect to each and every possible defaulter, bilateral credit exposures are reduced to nil whereas systemic credit exposures remain due to indirect loss sharing under multilateral netting.

We shall check the validity of the above statement below. Indeed, given a finite sequence $\{j_n\}_{n=1,\ldots,N}$ of banks at time t and a bank $i \notin \{j_n\}_{n=1,\ldots,N}$, assume that the bank i had a pairwise perfectly hedged position with respect to bank j. Then, for any t, one has

$$\begin{aligned} v_{ij}(t) &= \sum_{t<t_2\leq T_2}\sum_{c\in C} q(t,t_2,c)x_{ij}(t,t_2,c) \\ &= 0. \end{aligned}$$

by (18). Thus, if the bank i has a pairwise perfectly hedged position with respect to every $j_n, n = 1,\ldots,N$, then

$$(\forall n)(\forall t)\ \varepsilon_{ij_n}(t) = v_{ij_n}(t)^+ = 0.$$

It therefore follows from the definition (23) that we have

$$\beta^{\star}_{ij_1\ldots j_N}(t) = -\sigma^{\star}_{ij_1\ldots j_N}(t).$$

Thus, this represents a worst possible case for multilateral netting if one has

$$\sigma^{\star}_{ij_1\ldots j_N}(t) > 0, \tag{24}$$

in that entire bilateral credit exposures vanish whereas the full systemic credit exposures remain in multilateral netting. But, since $r^m_{ij_n}(t) = 0$ for all n and $m \geq n$, it follows that

$$\begin{aligned}
&\sigma^\star_{ij_1 \dots j_N}(t) \\
&= \left(\sum_{n=1}^{N} \sum_{m=n}^{N} r^m_{ij_n}(t) r_{mn}(t) \right) v^\star_{Hj_n}(t) + g_i\left(L(t)\right) \\
&= g_i\left(L(t)\right) \ > \ 0
\end{aligned}$$

when $L(t) > 0$. Hence, we obtain (24). Let us summarize this result in the following:

Proposition 13 [Indirect Loss Share of Multilateral Netting] *Consider a multilateral netting system with a class of indirect loss share functions $\{g_i\}_{i \in I}$ and compare it with bilateral netting. Given a possibility of a finite sequence of defaulting banks $\{j_n\}_{n=1,\dots,N}$, $1 < N < \sharp I$, at time t such that the initial defaulter has a multilateral loss position, assume that a bank $i \neq j_1, \dots, j_N$ had a pairwise perfectly hedged position with each of the banks $j_1, \dots, j_N$. Then,*

1. *The sum of bilateral credit exposures of bank i with respect to banks $j_1, \dots, j_N$ is always zero.*

2. *Systemic credit exposure at time t of bank i with respect to the sequence $\{j_n\}_{n=1,\dots,N}$ is strictly positive.*

3. *Benefit of multilateral netting at time t of bank i with respect to $\{j_n\}_{n=1,\dots,N}$ is strictly negative.*

The implication of this proposition is that a bank may avoid default risk in bilateral netting by maintaning the level of credit risk as well as market risk "very low", but it may not be able to avoid the default risk in a multilateral netting in cases where loss sharing by unconcerned participants are required.

6 Concluding Remarks

In this paper we introduced a formal model of foreign exchange contracts netting, and presented an analysis of multilateral and bilateral netting from the stand point of credit risk reduction. In particular, we compared the two different forms of netting arrangements with respect to inherent systemic risks involved. We would like to summarize some of the basic results of the paper and consider implications for institutional arrangements and their appropriate regulation.

1. If, in a very natural setting of a sequence of defaults, a chain reaction of which begins with a participating bank having a huge loss due to its extreme foreign exchange risk taken by it, then for any participating bank that is a positively concerned participant with respect to the original defaulter is exposed to an increased credit risk in multilateral netting system than in bilateral netting system provided that there is a subsequent defaulter that has a huge profit position relative to the total profits with respect to the original defaulter. (Proposition 5 and Proposition 12)

2. The sum of bilateral benefits to a participant of multilateral netting usually overstates its actual benefits if a systemic credit risk is to be taken account. (Proposition 4 and Proposition 11)

3. Consider multilateral netting arrangements which require loss sharing by unconcerned participants as well. If, in a sequence of multiple defaults, a participant has a pairwise perfectly hedged position with each defaulter, then it is exposed to credit risk under multilateral netting even if it is *not* exposed to credit risk under bilateral netting. (Proposition 13)

4. For those banks that are willing to take large foreign exchange risks, there will be not much gain in efficiency using a multilateral netting system. Their incentives to join the multilateral netting system must be motivated reasons other than efficiency and reductions of transaction costs. One conceivable "motivation" of this might be related to moral hazards and some sort of adverse selection which we did not discuss in this paper. (Proposition 3)

5. For efficient operation of a multilateral netting system, controlling and regulating foreign exchange risks of participating banks enhance efficiency of the netting system. This implies desirability of setting prudent limits on each participant's mark-to-market value position with respect to the clearing house. (Fact 3)

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