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Atsutoshi Mori, Makoto Ohsawa, and Tokiko Shimizu

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ABSTRACT

This paper gives an overview of the value-at-risk concept which is increasingly used for measuring market risk of portfolio. First, it explains the detailed method of calculating value at risk by using a pilot model developed by the authors. Second, practical and theoretical issues which need to attract special attention when calculating value at risk are examined. Finally, the paper presents the results of risk/return simulation in order to identify factors which could affect the performance of the value at risk model.

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** Research Division 1, Institute for Monetary and Economic Studies, Bank of Japan

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Atsutoshi Mori, Makoto Ohsawa, and Tokiko Shimizu
Institute for Monetary and Economic Studies, Bank of Japan

Preface

Derivative market has been expanding conspicuously since the latter half of 1980s. As a result, major national markets become closely integrated, financial transactions are more efficiently executed, and the structure and function of international financial markets have changed dramatically.

The technological evolution in the financial industry, symbolized by the rapid expansion of derivative instruments, provides us with new tools that could be exploited to unbundle risks as well as to reduce the cost of risk management. Thus comes a stream of new products with more elaborate risk structures. At the same time, the development in financial engineering uncovered the existence of very complicated and yet unrecognized risks associated with more traditional financial instruments. This finding underscores the need for financial intermediaries to manage risks not only of off-balance-sheet derivative transactions independently but also of the entire portfolio including traditional on-balance-sheet instruments as a whole.

The performance of portfolio risk management has become one of the determinant factors for profitability of any financial intermediaries. Participants in the financial market assess the ability of enterprises to manage risk exposures more seriously than ever as the practice of public disclosure of financial information are gaining wider acceptance. All these developments contribute to the fact that market participants are now well aware of the importance of portfolio risk management[1].

In the meantime the techniques used by market participants to assess and manage risks are constantly being developed. *Maturity ladder and duration* are traditional methods that have been used over the last ten years to measure market risk inherent in a portfolio. Quite recently, however, the method of measuring market risk exposures employing the concept of *value at risk*, gained considerable popularity. According to a survey by G30 published in December 1994, about a half of the major dealers operating internationally are already implementing risk

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[1] For example, see "Derivatives: Practices and Principles", The Group of Thirty, July 1993.

management system based on value-at-risk, and the additional thirty percent indicates to follow suit by the end of 1995[2].

Value-at-risk, representing the amount of risk inherent in a portfolio, is presented as the possible maximum loss the portfolio could experience over a given holding period with certain likelihood. Compared with the other methods, it has three advantages: first, value at risk summarizes the amount of risk embedded in a portfolio as a single number taking account of correlation between different types of risks; second, the assessment of the risk exposures by the method would be more objective because value at risk not only presents the amount of losses but also complements itself with the associated likelihood; third, the method enables senior management of enterprises to assess the magnitude of risks involved since the risk shown as the size of potential monetary losses can be easily compared with the portfolio's expected return and with the firm's own capital. Value at risk could also be used to estimate credit risk exposures, though it has thus far mainly been a tool for measuring market risk. Thus value at risk is sufficiently flexible for assessing various types of risk exposures. It is also true, however, that value at risk has shortcomings to be improved in the future.

This paper is structured as follows: In chapter I we will explain the detailed method to calculate value at risk to fully understand the theory and market practice of risk management. In explaining it, concrete example of value-at-risk calculation is presented by using a pilot model developed by Institute for Monetary and Economic Studies of the Bank of Japan. Chapter II examines practical and theoretical issues which need to attract special attention when calculating value-at-risk. In the final chapter, we will show simulation studies where value at risk is compared with subsequent actual losses and discuss their implications.

I. Calculation of value-at-risk

A. Model

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[2] "Derivatives: Practices and Principles Follow-up Surveys of Industry Practice", The Group of Thirty, December 1994.

The value at risk calculation model can be seen as a system to estimate the potential losses a portfolio could experience with a certain likelihood over a given holding period. Value at risk calculation entails the following three steps:

- To input the information on environmental changes which affect firm's portfolio value;
- To assess the sensitivity of the firm's portfolio;
- To estimate the losses a portfolio could experience with a certain likelihood under the expected changes.

information on environmental changes → value-at-risk calculation model → estimated loss of a portfolio

the sensitivity of the firm's portfolio → value-at-risk calculation model

It is important to clarify the purpose of calculating value at risk prior to specifying a model. The degree of sophistication of the model mainly depends on how the estimation results are utilized. Risk evaluation at trading sections may require highly specialized and sophisticated models, while firms' senior management assessing firm-wide risk management performance may be perfectly content with more generalized and simplified models. Furthermore the structure of models and information fed into it should be defined depending on whether the relevant observation period over which value at risk is calculated is characterized as "normal or stressful". In this paper we calculate the value at risk of a firm on the assumption that the firm's senior management wants to assess the risk exposures of its entire portfolio over the period at which normal market conditions prevail.

1. To input the information on environmental changes which affect the firms' portfolio value;

Before we create possible scenarios regarding future changes in underlying prices, interest rates, and foreign exchange rates, it is necessary to identify *risk factors* that could affect the value of a portfolio, and to specify the method to evaluate its volatility:

How to identify *risk factors* ?

We term the factors that could affect the value of a portfolio *risk factors*. They could vary depending on the structure of the portfolio or the types of risks we deal with. For example, if we seek to observe price fluctuation risks in financial, capital and commodity markets, the risk factors are prices in those markets such as interest rates, exchange rates, stock prices, and commodity prices. If we are to measure credit risk triggered by the deterioration in counterparts' creditworthiness, risk factors would be variables representing their credit standing such as the default probability or factors affecting it.

How to create scenarios on future price movements ?

Most common market practice at present is to simply rely on information extracted from historical data to create scenarios of future price movements. There are several established methods for doing this as follows: *matrix method* to be discussed later (sometimes referred as *variance-covariance method*), *historical simulation method*[3], and *Monte Carlo simulation method*[4]. These methods all subscribe to the assumption that the general pattern of past price fluctuations more or less dictates the future price changes, while the assumed

[3] Historical simulation method first derives an empirical distribution of the changes experienced by a portfolio over a holding period prior to the time of calculation. Value-at-risk is then calculated as the maximum losses in the distribution.

[4] Monte Carlo simulation method first computes the variances and covariances of the changes experienced by a portfolio over a holding period prior to the time of calculation. Then a series of pseudo random variables are generated, assuming that they follow the very distribution based on which these variances and covariances are computed. Finally value-at-risk is calculated as the maximum losses in the pseudo random variables.

pattern of future price fluctuations can vary from one method to the other. Occasionally, more elaborate statistical models are employed to address the problem that the information simply derived from historical data does not necessarily generate reasonably accurate estimates of future price movement.

2. To capture the sensitivity of the firm's portfolio to environmental changes

Before estimating the losses a portfolio could incur under the expected environmental changes, we need to know the sensitivity of a portfolio to these changes, in other words, how much gain or loss would be materialized if each risk factor changes by the unit amount. The sensitivity per se, sometimes referred as *basis-point-value*, has been traditionally used as one of the indicators for market risk. To calculate the sensitivity we proceed as follows: first we increase each risk factor by the unit amount, e.g. one basis point or one yen, and observe how much gain or loss would be materialized based on to the evaluation formula for each component of the portfolio. Then we aggregate these potential gains or losses across various components of the portfolio to arrive at the sensitivity of portfolio as a whole.

3. To estimate the losses a portfolio could experience under the expected changes

In the historical simulation and Monte Carlo simulation methods, gains or losses in a portfolio are calculated based on the scenario regarding future changes in market prices. Then either the biggest loss or the bottom one percentile is taken to be the value at risk of the portfolio (See Chart 1).

Under the matrix or variance-covariance method, we assume each risk factor to change by the amount of its past estimated standard deviation[5]. We then compute changes in the value of portfolio by using sensitivities of the portfolio to each risk factor[6]. If there are multiple risk factors, we take into account

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[5] It is usually called the volatility of a risk factor.

[6] In the matrix or variance-covariance method, changes in underlying prices are assumed to follow the normal distribution. Some consider the assumption to be

correlation between them according to the formulae listed in Chart 2.

Finally it is required to decide how high a likelihood associated with the losses would be admissible as a measurement of market risk. We first compute the standard deviation of changes in the value of portfolio resulting from the expected changes in risk factors. We then multiply the standard deviation in accordance with a specific confidence level regarded appropriate. For example, if we think that the likelihood of actual losses exceeding the value at loss should stay under 1%, or the value at risk should be calculated with a confidence level of 99%, then we multiply the standard deviation by 2.33. We obtain the multiplier by assuming the changes in the value of portfolio will follow the normal distribution[7] (See Chart 3). Given the assumption of normality with the structure of the portfolio constant, we multiply annualized value-at-risk amount by the square root of n/250 to obtain the value at risk over n business days[8].

B. Calculation procedure of value-at-risk

In this chapter we explain how to calculate value-at-risk in line with calculation procedure of a pilot model developed by the Institute for Monetary and Economic Studies of the Bank of Japan.

1. The calculation procedure

(1) The information on environmental changes which affect the firm's portfolio value.

<u>Risk factor</u>

The pilot model calculates risk exposures to Japanese yen interest rate risk inherent in the portfolio of financial instruments. We identify as the risk factors

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fairly strong because most financial data are known to follow fatter tailed distributions than the Gaussian.

[7] Theoretically the changes in the value of a portfolio calculated here is not normally distributed because it is a linear function of the changes in the risk factors, which are assumed to be log-normally distributed.

[8] We assume that there are two hundred fifty business days per year. See footnote 22 as a reason why we multiply the daily value-at-risk by $\sqrt{n}$.

zero-coupon-yields[9] of yen interest rates with nine maturity (1, 3, 6 month(s), 1, 2, 3, 5, 7, and 10 year(s))[10].

The volatility of risk factors

The model adopts the matrix method and assume that each risk factor is expected to change by the amount of estimated volatility. We also assume that the risk factors preserve the estimated correlation.

(2) Assessment of the sensitivity of the firm's portfolio

The model includes the formulae for seven financial instruments. They are interest rate futures, FRA[11], interest rate swaps, interest rate future options, cap/floor, and swaptions. For option-related instruments, we first calculate its current price by using Black formula[12]. We then change the risk factors independently by one basis point to see how prices of the instruments change accordingly. Each change in the price is the delta value of the option-related instrument. Finally we multiply by the delta values the changes in underlying prices, interest rates, or exchange rates. As for the other instruments, we move

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9 Zero-coupon yield represents compounded market interest rate of a zero-coupon bond. Zero-coupon yield of t years, r_t, is expressed by

$r_t = m \ln(1+R_t/m)$

where R_t is the compounded interest rate of a t-year zero-coupon bond accrued m-times a year.

10 We could choose as our risk factor forward rate in place of zero-coupon yield. Zero-coupon yield closely resembles the spot rate, which is used frequently, and is therefore intuitively appealing. Forward rate, on the other hand, is found to be easier to treat as a factor of the yield curve.

11 FRA (Forward Rate Agreement) is a contract to exchange the difference between the contract rate and the observed market rate on the specified day multipled by the notional amount.

12 There are many variations of option pricing formulae. They have strengths as well as weaknesses. Although some believe Black-Scholes formula has limited successes in pricing interest-related options, we employ the formula for simplicity. Future prices are the underlyings in Black formula, while Black-Sholes formula employs spot prices as the underlying prices.

nine risk factors independently by one basis point to obtain prices of the instruments. Finally we aggregate all the price changes across to obtain the sensitivity[13].

(3) Estimation of the losses a portfolio could experience under the expected changes

We calculate the standard deviation of the change in the value of the portfolio by using the matrix method. Equipped with the result, we calculate value-at-risk of the portfolio over a period of one week with the confidence level of 95%.

2. The model specification

The model is composed three input files, three calculation routines, and three output files (See Chart 4). Their outlines are as follows.

<u>Input file 1</u>

Inputs the terms of contract for the non-option financial instruments. (See Chart 5).

Tool 1: interest rate swaps

Tool 2: interest rate futures, and FRA

<u>Input file 2</u>

Inputs the terms of contract for the option-related instruments.

Tool 3: interest rate future options

Tool 4: cap/floor

Tool 5: swaptions

<u>Input file 3</u>

Inputs the past market interest rates.

[13] In our model we exclude curvature risk associated with options. Curvature risk arising from options can be estimated by Greek Letter, scenario, and Monte Carlo simulation methods. Greek Letter method adds gamma and vega risks to delta(linear) risk.

Calculation routine 1

Calculates daily cash-flow forward starting from the pre-specified date by using the terms of contract for the non-option instruments in Input file 1 along with the current yield curve (See Chart 6).

Calculation routine 2

Based on necessary market interest rates from Input file 3, we draw zero-coupon yield curve. From the yield curve, we calculate daily discount factors to estimate present value of the portfolio[14] (See Chart 7).

Calculation routine 3

Calculates historical volatility and correlation matrix of the nine risk factors, after converting the historical data in Input-file 3 into zero-coupon yield.

Output file 1

With the discount factors obtained from Routine 2, we discount the cash-flow data for the non-option instruments in Routine 1 to obtain the present values. We then aggregate the present values across the non-option instruments to obtain the present value of the non-option portion of the portfolio, denoted by PV. We recalculate the present values of each of the non-option instruments denoted by $PV_{(+1bp)}$ which is obtained by increasing each of the nine risk factors independently by one basis point. To obtain sensitivity against each one of the risk factors denoted by δ, we take differences between the original and recalculated present values, or $\delta = PV_{(+1bp)} - PV$.

Output file 2

Employing Black formula, we calculate the present price of the option-related instruments in Input file 2. We then obtain the sensitivity similarly as in the case

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[14] Zero-coupon yield (r_t) is related to discount factor (DF_t), which discounts cash-flow into present value, as follows.

$DF_t = \exp(-r_t\, t)$.

of the non-option instruments explained above. The required short term interest rates come from the yield curve in Routine 2. The volatility is obtained from the market data in Input file 2.

Output file 3

Calculates standard deviation of the changes in the value of the portfolio, denoted by σ, by using the sensitivity in Output file 1 and 2, as well as the volatility and correlation matrix in Routine 3. We then calculate value at risk with a confidence level of 95% over a two-week period by multiplying σ by 1.64 times $\sqrt{10/250}$[15] (see Chart 2 above).

3. An example: value at risk calculated on November 30th, 1994

We calculated value at risk for a hypothetical portfolio with three financial instruments. The firm holds:

FRA

Notional amount:	250 million yen
Interest rate:	2.450%
Buyer or Seller:	Seller
Period:	95/1/15--95/4/15

IRS

Notional amount:	500 million yen
Floating rate	6 month Libor + 0.25%
Fixed rate	3.8%
Payer or Receiver:	Fixed rate payer
Period:	93/12/20--95.4.15
Interest Payment Cycle:	6 month

Swaption

[15] We assumed that two weeks have ten business days.

Notional amount: **500 million yen**

Right to Pay Fixed

Fixed Rate **3.8%**

Buyer or Writer: **Writer**

Period: **95/3/25--99.3.25**

Striking Rate: **4.06%**

Interest Payment Cycle: 6 month

Its cash-flow for the non-option instruments is in Chart 8. The yield curves are shown in Charts 9 and 10. Its estimated volatility is tabulated in Chart 11. The correlation matrix is calculated as in chart 12.

Chart 13 depicts the sensitivities against the nine risk factors of the three financial instruments and of the portfolio as a whole. The graph is referred as *delta map*, and is being used in trading floor as a tool to show to which factor(s) a portfolio is most sensitive. As for our hypothetical portfolio, it turns out that the interest rate swap is highly sensitive to the two-year zero-coupon yield, because all cash-flows of the swap, which will be matured at December 20, 1996, is most affected by the movement of two-year yield. And so is the swaption, the maturity of which is four years and four months, to the five-year zero-coupon yield. Hence the overall portfolio is highly sensitive to these two risk factors.

The value-at-risk of the entire portfolio over a two-week horizon with a confidence level of 95% is ¥1.62 billion, which would be about a quarter of ¥5.91 billion of equivalent with the simple sum of value-at-risk of three instruments and about forty percent of ¥4.01 billion obtained on the assumption that the risk factors are not corelated with each other (see Chart 14). Such a reduction in the value-at-risk for the entire portfolio is due to the fact that (i) the delta values for the same risk factor turn out to be in opposite signs for some of the instruments and this reduces the sensitivity of the portfolio as a whole and (ii) the correlation between the risk factors contriubtes to reducing the risk.

The value-at-risk of the whole portfolio for a one-day holding period is ¥510 million as calculated ¥1.62 billion times the square root of one-tenth. It is ¥2.3 billion when assuming a confidence level of 99% (obtained as 16.2 times 2.33/1.64) (see Chart 2).

II. Examination of the model

It is obvious that there are number of methods to calculate value-at-risk. Even if we do not dispute the legitimacy of the matrix method, discrepancies in the value-at-risk of a portfolio could still arise from the differences in methods to calculate volatility and correlation coefficient, let alone from model parameters such as confidence level or holding period.

In the following, we will theoretically examine the methods for estimating volatility of and correlation between risk factors because it is hard to predict a priori the relationship between the assumption made to obtain them and the effects of the assumptions on the level of value-at-risk[16].

A. On the method to estimate volatility of risk factor

The volatility of risk factor significantly influence the value-at-risk. Therefore senior manages of enterprises have to ask themselves the following question. "What would be the most appropriate method to estimate volatility if we want to appropriately assess the risk of our portfolio?" Obviously the estimated volatility should be able to predict the future volatility as accurately as possible while due consideration should be made to economic cost of the calculation. Its calculation cost should be reasonable as well.

Methods to estimate volatility broadly fall into two categories, one using historical volatility calculated from the past market data, and the other employing specially designed models[17]. We examine mainly historical volatility method in the following.

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[16] Confidence level and holding period obviously affect value-at-risk. However, we shall not look into their effects on value-at-risk because we could easily observe the effects by adjusting them expost.

[17] Model-based methods include ARCH/GARCH and stochastic volatility model. The former utilizes past price changes to estimate future volatility; the latter postulates that volatility is a product of pure probabilistic disturbances.

Parameters and factors affecting the estimated historical volatility include the length of observation period, weights given to observations, the treatment of missing data, and observation intervals. The criteria below may be helpful when we make decisions on the parameters.

- Is sufficient information taken into account to predict the future changes in volatility?
- Is the information on a unique event in the past which is highly unlikely to repeat in the future excluded?
- Is the information appropriately weighted?

1. Length of observation period

The length of observation period affects volatility significantly. Chart 15 clearly shows how the volatility varies with the lengths of observation period of three months to three years. How much of the data is practically available sometimes decides the length. Supposing that it not be a problem, we still have a few issues to be examined beforehand if we were to decide the length appropriate for calculating value-at-risk.

First, we have to keep in mind that the amount of information simply measured by the length of observed period does not necessarily determine the precision of estimation. In other words, long observed period does not necessarily guarantee that the information gained is useful for estimating future price changes. For example, if the market experienced structural changes, be it institutional or otherwise, the data observed prior to the change might not be useful in estimating future fluctuations.

Before deciding on the issue of appropriate length, we may have to examine every single unit period more carefully. This could be done as follows. First we decide how long the adequate unit period is if it were to be long enough to observe a pattern. Then we decide if the market pattern may reiterate itself in future, may have some time trend, or may be an outlier.

Yen interest rate volatility during the 1993 - 1994 period shown in Chart 16, for example, reveals that the volatility of medium-to-long term interest rates increased starting from late 1993 through early 1994. If the increase was deemed only temporary after careful analysis, the volatility before that period could be used[18].

2. How to give weights to observations

How to give weights to data is another important issue to consider in using historical data. For example, we could give more weights to the most recent data, hoping to obtain estimated volatility with contemporaneous attributes. While this method has unintended advantage in that it can smooth out the fluctuation in the data, it would inflate the value-at-risk excessively should a large shock hit at the time of calculation. Hence if some kind of weighting scheme is used, it is imperative to check the appropriateness of the underlying idea of the scheme.

3. How to treat missing data

It is known that missing data present in time-series data could influence estimation result. In complementing missing data we need to keep in mind two general principles, namely (i) avoiding; to discretion and (ii) having potential price movement preserved as much as possible.

There are following three methods widely used in the market to address the problems related to missing data.

Omitting missing data

This method satisfies two aforementioned principles if we accept the hypothesis that volatility is mostly caused by trading[19]. There are, however, contradicting research results which show that the volatility of two consecutive business days with a holiday in-between, e.g. Friday and Monday, is higher than

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[18] It is very difficult to determine which period we choose it to estimate future volatility. The problem is characteristic to any estimation methods of risks based on historical data. In this context analytical models for explaining future interest rates such as term structure models used for option pricing, might be used at the same time to at least partly overcome the problem.

[19] K. French and R. Roll, "Stock return variances: the arrival of information and the reaction of traders", *Journal of Financial Economics,* Vol.17(September 1986), pp.5---26.

that with no holiday in-between, e.g. Thursday and Friday. If this is true, we ought not to completely leave out missing data in light of the second principle. Moreover, some adjustment is obviously needed whenever contemporaneous correlation between different markets is being studied, because the two sequences of data are not completely synchronized.

Complementing with a preceding or succeeding datum

One way to make up a missing datum is to use one immediately preceding it. This may not meet to the second principle even though it may conform to the first. A possible solution to this may be to impute an average of the preceding and succeeding data[20].

Complementing with other market data

One way to manufacture a missing datum is to use a datum of the other market which is open on the day. This can be a good idea in that it uses available market information to its full extent. However, it might contradict to the first principle. Therefore, a stable correlation between the above two markets should be established before we follow this method.

4. How to deal with outliers

A characteristic of an outlier needs to be carefully examined in deciding whether we include it in the analysis. We include an outlier if the similar fluctuation is likely to occur in future and vice versa. In cases where we evaluate the performance of portfolio during an episode of market stress -- so-called *stress testing* --, we need to focus on excessive price changes or outliers in the past to predict the amount of risk at stake. Cases like this suggest that information included in an outlier, if closely scrutinized, could be used to improve risk management.

5. Observation interval

[20] Dummy variables incorporating holiday effects or substitution with Kalman filter may be effective as well.

We need to pay attention to the interval of data to estimate volatility. Let a daily volatility be σ_1, and n-day volatility σ_n, then the relation between the two would be $\sigma_n = \sqrt{n} \times \sigma_1$ in theory[21]. In other words, a volatility estimated over any n-day period, if properly adjusted by squire root of n, should stay constant. However, Chart 17 exhibits that the adjusted volatility vary considerably depending on the observation intervals.

In determine observation intervals appropriate for accurately estimation the amount of risk further consideration should be made about whether they are consistent with the holding period of the portfolio. The holding period, at least conceptually, should reflect not only how transactions of relevant instruments are being carried out but also how much liquidity the markets have. The volatility should also be calculated over the same interval. In practice however, it is both too complicated and unrealistic to decide an appropriate holding period for each financial instrument and measure the volatility accordingly. Although theoretically at least the requirement of strict consistency demands calculation of intraday volatility, it is not easy to obtain such data in retrospect. In any event, since this issue is also relevant in evaluating liquidity risk of a portfolio, we should seek to develop feasible solutions to reconcile these two diverging point of view.

6. How to sample data

When we adopt the historical volatility method, we need to reconcile two frequently cited contradicting demands. On one hand we need to obtain sufficient number of data. On the other hand we wish to avoid autocorrelation between the number of data. One of the following three methods are routinely used as a sampling method (see Chart 18).

Moving-window / Overlapping Method

The method calculates volatility from the subsamples chosen by the daily

[21] If daily interest rates $X_1,\ldots,X_n$ are independent and identically distributed with variance $V(X_i) = \sigma^2$, then

$$V(X_1 + \cdot\cdot\cdot + X_n) = V(X_1) + \cdot\cdot\cdot + V(X_n) = n\sigma^2.$$

shifting window. For example, if we took the window width of five days, the first and second subsamples would share four consecutive days. The method makes an efficient use of limited amount of data. However, it introduces a large amount of autocorrelation to the subsamples from which the volatility is computed.

Box-car / Non-overlapping Method

The method is sometimes used as a way to alleviate autocorrelation associated with the moving-window method, for data of its subsamples do not overlap. One problem is, however, that a considerably large sample is necessary to obtain sufficient number of subsamples. In addition, it is sometimes criticized that the method could be discretionary in determining on which day of the week we should start collecting subsamples because the volatility based on data starting from Monday with constant window width is likely to be different from that based on data starting from Tuesday.

Bootstrap Method

The method is one of the nonparametric subsampling methods for statistical estimation. There are two merits: it requires no parametric distributional assumption; it can generate ample subsamples even from a small original sample. Nevertheless, theoretical investigation is necessary to decide if independence assumed for the original sample, a critical assumption to many, could seriously jeopardize the outcome.

If we were to calculate volatility with the method, we proceed as follows. First we take a sample from original data. The number of data in a single sample corresponds to the window width in the two preceding methods. Then we take subsamples of the same size randomly with replacement from the sample. Finally we calculate volatility for each subsample and the volatility gives the required variability.

7. Prediction methods employing implied volatility

Implied volatility is reportedly used from time to time when value-at-risk is calculated at trading desks. In cases where option market is not sufficiently developed, however, we may have to question its reliability. Even if highly reliable implied volatility data exist, it is very difficult to obtain from the market reliable implied covariance data between risk factors. Historical data must be substituted for the component.

B. On the method to estimate correlation between risk factors

Another important factor that may affect value-at-risk calculation is correlation between risk factors. It is important to decide how to interpret it and to what extent it is incorporate in calculation.

Again we have to ask ourselves the following question. "What would be the most appropriate method to estimate correlation between the risk factors if we as, a firm's senior manager, want to accurately assess the risk of our portfolio?" Appropriateness here pertains not only to adequately averting risk but also to pursuing more efficient interval capital allocation equipped with accurate knowledge on the amount of risk exposures. From the latter viewpoint, if there is sufficient rationale behind it, taking into account correlation between risk factors may be preferable.

1. Premises in taking correlation into consideration

Judging whether we should take into account correlation between risk factors is boiled down to the question whether we ought to allow the total amount of risk reduced when aggregating the amount of risk associated with each risk factor. Here we have to consider issues on (i) stability of the estimated correlation and (ii) consistency of judgment on the management of the portfolio.

We ought to investigate stability of the estimated correlation thoroughly if we were to use obtained value-at-risk as an indicator of the firm's risk management[22]. Strictly examining the stability, however, may not be practical because doing so would be technically demanding as well as costly. Therefore the market practitioners sometimes have to resort to the simple sum of value-at-risk for each

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[22] A statistical test of stability involves comparison between the relevant correlation converted to a random variable with mean zero and unit standard deviation by the so-called Z-transformation and a bench mark correlation.

risk factor as an indicator of risk exposures, partly become the correlation coefficient is regard to fluctuate significantly at times and thus be unreliable.

As for the consistency of judgment, we should check whether arbitraging section exists in the department responsible for managing the relevant portfolio, or evaluate if the relevant financial instruments are hedged against each other. If the trading positions are consistent with correlation within the relevant portfolio, that correlation should be reflected on value-at-risk calculation. However, some argue that operationally feasible correlation should reflect rather the prevailing market liquidity, than the consistency of judgment. If so, risk evaluation office may wish to present two distinct numbers, one being the simple sum of risks calculated at individual business unit separately, and the other incorporating correlation to the calculation reflecting the consistency of the judgment.

We generally conclude that it is preferable from the viewpoint of efficient capital allocation to estimate the amount of risk with correlation between the relevant risk factors taken into account if we consider the correlation sufficiently stable. 1) Even if correlation is stable, 2) discounting the amount of risk with historical correlation would not comfort to reasonable conservatism, 3) in cases where proper coordination is not implemented amongst the operations of the relevant portfolio, 4) since we are not able to react to the situation where the presumed correlation disappears. The following table describes these relations. Depending on to what extent the judgment is consistent or coordinated, we would take various measures, from simple aggregation to partial discount, if we belong to the left bottom cell cases in medium shade marked as "correlation not to be considered".

Premises on which correlation is considered

	stable correlation	unstable correlation
consistent judgment	correlation to be considered	correlation not to be considered
inconsistent judgment	correlation not to be considered	correlation not to be considered

2. Scope of correlation

Keeping stability of correlation and consistency of judgment in mind, we ought to decide scope of correlation required for risk calculation.

(1) Correlation between broadly-defined market risk factors (See chart 19)

Correlation between broadly defined market risk factors such as stock prices, interest rates, and exchange rates, is not generally considered when calculating risk. That is, a prevailing method is to simply aggregate each risk amount for relevant risk factor, reflecting only correlation within each of the risk factors such as that between long and short term rates within an interest risk factor. This stems from the fact that in many cases financial instruments associated with those factors tend to be dealt independently and correlation between the factors tends to be less stable than those, for example, between interest rates in the same currency. However, the correlation may be considered to utilize the resources more efficiently if we predict the correlation continues to be stable.

(2) Correlation between interest rates with different maturities denominated in the same currency

The yen interest rates with the maturity of more than three years show high correlation among them generally exceeding 80% see Chart 12. Then the next question is what the appropriate number of risk factors to be considered is.

Two different points of view exist as to how we identify risk factors in an interest rate in one currency, e.g. yen swap rate. Some argue that we consider as our risk factors all the points on yield curve that can change in future. The other claim that we regard as our risk factors only the major factors affecting the relevant markets.

According to the former view, each portion of yield curve over one-month interval should generate corresponding forward or zero coupon rate. Then we recognize these rates as a whole to be our risk factors and calculate their correlation matrix. While the method requires to set up a huge matrix involving all the risk factors upon calculation of value-at-risk, it is not necessary to extract the diagonal components because they tend to be close to unity.

(3) Correlation between different types of interest rates in the same currency

Again two different views exist as to how we identify risk factors in various interest rates in the same currency. Some argue that we regard as our risk factors only the main type of interest rates. The remaining *basis risk* interest rates will be added to the amount of risk. The other claim that we consider as our risk factors every type of interest rates that possibly changes. If we adopt the former point of view, then we recognize various national government's treasury bond rates in the national currencies as the risk factors for value-at-risk calculation. We then calculate basis risk using the spread between the other various interest rates and the government's bond rate.

(4) Correlation between market risk factors and credit risk factors

At present it seems that market risks stemming from price changes in the underlying financial instruments and credit risks arising from changes in counterparty's creditworthiness are being estimated separately. This is because most financial intermediaries have separate credit administration and market risk management sections, and because it is in many respects costly to examine stability of the correlation between the two risks. However, we would be able to obtain the combined risk. Nevertheless, it is theoretically possible to consider correlation between market prices and default probabilities to evaluate risks associated with both risk factors in a more integrated fashion.

3. Stability of correlation

We need to ascertain fully the stability of correlation between risk factors before we decide to what extent correlation should be taken into account and to what degree correlation should reduce the amount of risk. The past two year history of correlation between the yen interest rates reveals that they have been unstable, especially between the short term rates and between the short and medium-to-long term rates (see Chart 20)[23].

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[23] This so-called correlation risk, resulting from the instability of correlation, has been gradually recognized by the market due to increasing transaction of a type of financial instruments called *correlation products*, pay-off of which is determined by multiple underlying asset prices.

Since correlation movements significantly affect value-at-risk, we have to reasonably estimate correlation between risk factors. One possible method to correlation estimation is obtaining implied-correlation from the hybrid derivative products market, which is so called *correlation products* based on multiple underlying asset prices. However some kind of theoretical value or historical data is still required, because the data observed in the market is not reliable at present.[24].

In deciding how we should consider correlation in risk calculation it is worth thinking over what types of correlation are assumed in pricing financial instruments[25]. Any pricing model actually used has many assumptions, for example, on short and long term interest rate changes. Correlation assumed in the model, at least in principle, should be the one used for risk management. In other words, we need to explore whether a financial intermediary using a option-pricing model could employ the model's assumption on changes in interest rates when calculating value-at-risk. It is possible, however, that the same model cannot be realistically used for the pricing and risk management purposes because the precision and speeds required differ considerably between the two purposes. Therefore, where consistency is called for, we may have to arrange a separate evaluation model for risk management consistent to some extent with the framework of the pricing model.

III. Risk/return simulation

Value-at-risk is one of the indicators of risk that reflects each financial intermediary's basic ideas on risk management, such as on what the main risk factors affecting the value of a portfolio are, or how future market price changes can be predicted. Thus we need to establish in advance a policy on how we approach to managing the risk in order to utilize value-at-risk effectively. At the same time, we

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[24] We could predict the correlation by employing multi-parameter ARCH/GARCH or stochastic volatility model as we could for volatility. Some argue, however, that these models are not realistic because they require substantial parameter estimation work.
[25] Similar problem arises when we deal with volatility in risk management.

need to examine relationship between the calculated amount of risk and the actual losses. By closely investigating the risk estimated in advance and the actual changes in the value of portfolio, we would be able to modify our policy to a more realistic one.

In the following, we shall expand the model used to calculate value-at-risk in the previous chapter and discuss simulation studies which compare the risk expected in advance and the actual losses.

A. Framework of the simulation study

We designed the simulation study to verify the reliability of the model. Equipped with hypothetical portfolios and actual historical data, we ran simulation to compare value-at-risk calculated by a model in chapter I with the actual losses[26].

When calculating value-at-risk, we rely on the following assumptions: portfolio mix, volatility of risk factors and correlation between them remain constant during the holding period, and observed daily data of price movements in risk factors is treated as statistically independent. Moreover, we don't take into account any trends of the portfolio value movement and non-linear risks by employing the matrix method. By comparing the calculated value-at-risk with the actual changes of portfolio value it is possible to check if the assumptions are appropriate. Since we assume that the composition of the portfolio will not change during the observed period, the assumption we are really checking is the one on the market prices.

We would like to keep in mind that the method has some limitations in evaluating the reliability of value-at-risk. First value-at-risk calculated with a confidence level of 99% is the minimum amount of money a portfolio, *on average*, could lose in once out of one hundred times. Since changes in market prices at consecutive time are not necessarily independent, value-at-risk calculated with a confidence level of 99% is not equivalent to the amount of losses the portfolio could

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[26] The BIS Fisher report introduced this kind of simulation studies as a tool of public disclosure, which shows a performance of risk management ("Public Disclosure of Market and Credit Risks by Financial Intermediaries", BIS Euro-currency Standing Committee, September 1994).

experience in once out of one hundred consecutive historical time points. Second when assessing the confidence level, *on average* part of the value-at-risk statement gets in the way. The losses exceeding value-at-risk could occur more than two out of one hundred days, even if value-at-risk is calculated with a confidence level of 99%. The probability of this happening reaches as high as 26%[27]. If we admit a simulation result that the losses exceeding value-at-risk occur as frequently as three out of one hundred days in order to avoid type I error, the probability of error where we overlook the calculation with a lower confidence level than 99% could become higher[28]

Framework of simulation study

<u>The relevant portfolio</u>

We assume a portfolio of forty instruments related to yen interest rates. The instruments are interest rate futures, FRA, interest rate swaps, and swaptions (See chart 21).

<u>Simulation period</u>

We measure the value-at-risk and the actual losses of the relevant portfolio for the two-year period between 1993 and 1994. Contract terms of the instruments and the remaining period of each instruments are assumed to be constant during the period.

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[27] Error of rejecting a hypothesis even if it is true is called type I error in statistics. In this case rejecting the hypothesis that the value-at-risk is calculated with a confidence level of 99% could be type I error.

[28] Denote by H_0 a null hypothesis and by H_A the alternative. Suppose we hypothesize that

H_0: The value-at-risk is calculated with a confidence level of 99%,

H_A: The value-at-risk is calculated with a confidence level of 95%.

If we do not reject the null hypothesis even if the losses exceeding the value-at-risk register three out of one hundred days, the probability of type II error reaches as high as 25%, though that of type I error is only 2%.

Calculation of value-at-risk

Confidence level: 95% (1.64 σ)

Holding period: 2 weeks (10 business days)

Calculation method for volatility and correlation

Frequency: both daily and semiannually

Data observation period: period of one year preceding the time of calculation

Weighting of data: simple average

Treatment of missing data: take average between preceding and succeeding data.

Treatment of outliers: including all outliers

Observation interval: 2 weeks (10 business days)

Data sampling method: Moving-Window method

Correlation: Either estimating from historical data, assuming no-correlation, or summing up of risks for each risk factor

Calculation of actual losses

Frequency: daily

Data observation interval: Calculating portfolio values at two week interval. A decline in the value is recognized as realized loss.

B. Results

1. In cases where volatility and correlation coefficient are recalculated daily

The simulation study revealed that the losses exceeding corresponding value-at-risk occurred on fifty one business days in two-year period or 474 business days (See chart 22). Frequency of the event approaching 10% far exceeded the expected frequency of 5% when we made the assumptions on market prices[29].

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[29] The results of this simulation study doesn't reduce usefulness of value-at-risk as a risk measurement. The loss exceeded estimated value-at-risk only once during 1994, 237 days. As for 1993, the number of the days when actual losses exceeded value-at-risk must be smaller than that in the above case if we assume the portfolio adjustments

Scrutinizing the period between August and November of 1993, when the losses exceeding the value-at-risk appeared in cluster revealed that correlation between risk factors, especially that between one-month and longer-term interest rates, during the period was significantly lower than the historical correlation at the end of August 1993, which we used for our value-at-risk calculation (See Chart 23). Moreover, risk factors fluctuated more wildly than the historical volatility, which we used upon the value-at-risk calculation(See Chart 24). On top of that, we can point out that sustained decline in interest rates, not postulated in our calculation, was observed during this period. When there was divergence from the premises, both in terms of volatility and correlation, the actual losses exceeding corresponding value-at-risk were not totally unexpected.

2. In cases where volatility and value-at-risk are recalculated semiannually

This simulation study revealed that the losses exceeded corresponding value-at-risk on fifty three out of 474 business days (See Chapter 25). Though this frequency of the event was practically the same as that registered in the case above, which is fifty one business days, closer examination revealed that the first half of 1994 experienced the event more frequently (six times) than the case above (once). This could be attributed to the fact that it was not possible to reflect the dramatic increase in risk factor volatility, begun at the end of 1993 and lasted during the first half of 1994 (see Chart 16 above), to the amount of risk because both volatility and correlation were recalculated only semiannually.

3. In cases where historical correlation between the risk factors is not considered

We also performed a simulation study without using the historical correlation coefficient. These cases involved either no correlation, which amounts to all the non-diagonal components of correlation matrix being zero, or simple aggregation of the amount of risk across risk factors, which comes down to the non-diagonal components being either plus or minus one. Out of 474 days, the actual losses surpassed corresponding value-at-risk 16 days when no correlation was assumed (See chart 26), and one day for simple aggregation (See chart 27). Frequency of the

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during the period.

event was significantly lower than that in the cases with historical correlation (53 times). Naturally the level of value-at-risk was one and a half times higher for the non-correlation case, and two to four times higher for the total correlation case.

C. Implications

The risk/return simulation studies indicate the following.

1) Credibility of value-at-risk depends on the accuracy in volatility and correlation estimation method employed in the calculation procedure. If unexpected movements in volatility and correlation are not captured sufficiently, credibility of value at risk could be easily impaired.

2) Frequent recalculation of volatility and correlation coefficient would make it possible to reflect the substantive changes in the market to the amount of risk in a timely fashion. If such market condition persists, we can to some extent prevent the losses from exceeding the estimated value at risk.

3) Value at risk of a portfolio based on the assumption of either non-correlation or total positive or negative correlation between risk factors could keep the losses from exceeding corresponding value at risk. The amount of risk involved, however, might be judged overstated compared with the actual losses.

When value-at-risk is used as a tool for risk management, we need to evaluate thoroughly whether the assumptions on the market changes are appropriate and realistic. Value-at-risk is only one of the many measurements created in the process of improving risk management. We expect market participants to provide us with answers to the theoretical and practical issues we discussed, and to explore alternative approaches to rational and practical risk management.

Conclusion

When we use value-at-risk as one of the measurements of risk, we need to think thoroughly over whether our policy in terms of managing risks is appropriate, how accurate the machinery is to calculate it, and how we should evaluate it.

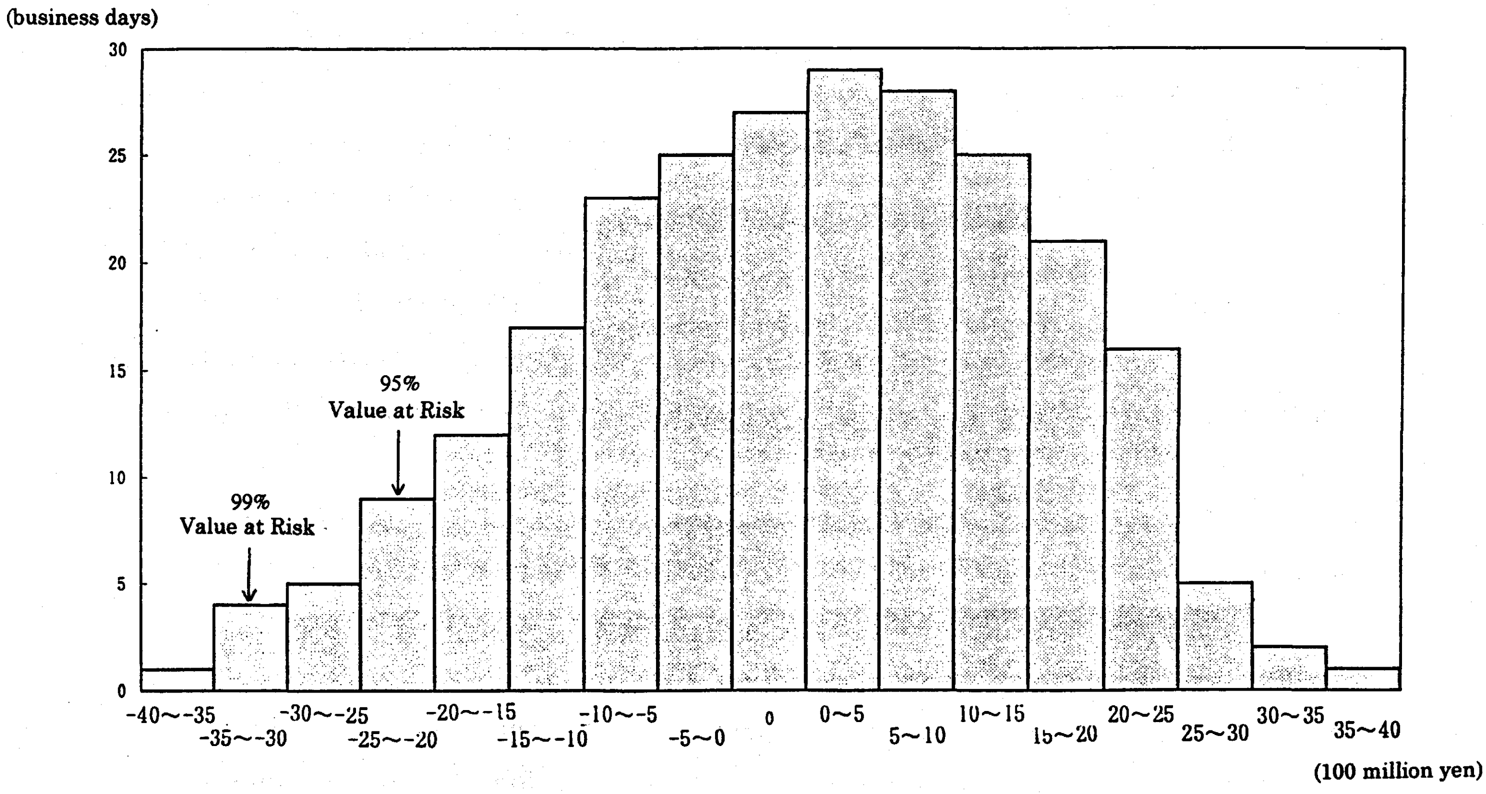
First establishing a sound policy on how we approach to managing risk is a prerequisite for calculating value-at-risk. Sound risk management, with or without such measurements as value-at-risk, entails ability to spot hidden vulnerability of a portfolio to financial risk. This ability depends more on the instincts, experiences, and judgment on where to look for rather than the tools with which to look for.

As more market participants recognize the importance of risk management, we have to be more responsible for the rationale behind how we reach that particular number, particularly when we expand the role of value at risk to include a portfolio of the entire firm, and when we allocate capital and make the public disclosure of financial information based on it. We have to be able to convince market participants at least of how precisely we estimate future changes of risk factors as well as of how we expect the correlation between risk factors to change.

Finally we need to reach a consensus on how we assess and utilize value at risk and other measures more effectively. We expect to have more on the methods to demonstrate the reliability of value-at-risk. The evolution of trading, symbolized by the rapid expansion of sophisticated financial instruments such as derivatives will encourage market participants to explore better risk management. Value at risk is one of the indicators to quantify the amount of portfolio risk and it is only a product of market at present with limits because of strong assumption employed in it. However, we believe that the concepts of value at risk is sufficiently broad as a system to evaluate the value of a portfolio. We also think that it will be frequently used as one of effective tools to calculate the amount of various risks. We strongly hope that the interested parties conduct researches on the several problems associated with value-at-risk we have discussed so far and develop more advanced risk management.

(Chart 1)

Histogram of daily losses and gains (an example)



Value-at-risk calculation with the matrix method

$$\text{VAR (Value-at-risk)} = \phi \times \sqrt{\tau} \times \sigma_p$$

where

ϕ : Confidence interval (Probability of losses not exceeding VAR, a multiple of the standard deviation)
τ : Holding period of relevant portfolio (year)
σ_p : Standard deviation of changes in the value of a portfolio (per year)

σ_p is obtained as follows.

(with single risk factor)

$$\sigma_p = \delta \times \sigma$$

δ : Portfolio's sensitivity to the risk factor
σ : Volatility of the risk factor (standard deviation of past changes)

(with two risk factor)

$$\Delta P = \Delta r_1 \delta_1 + \Delta r_2 \delta_2$$
$$\sigma_P^2 = \delta^2{}_1\sigma^2{}_1 + \delta^2{}_2\sigma^2{}_2 + 2\rho_{12}\delta_1\delta_2\sigma_1\sigma_2$$

ΔP : Amount of changes in the value of a portfolio
Δr_i : Range of fluctuation of risk factor i
δ_i : Sensitivity of the portfolio to risk factor i
σ_i : Volatility of risk factor i
ρ_{ij} : Correlation coefficient between risk factors i and j

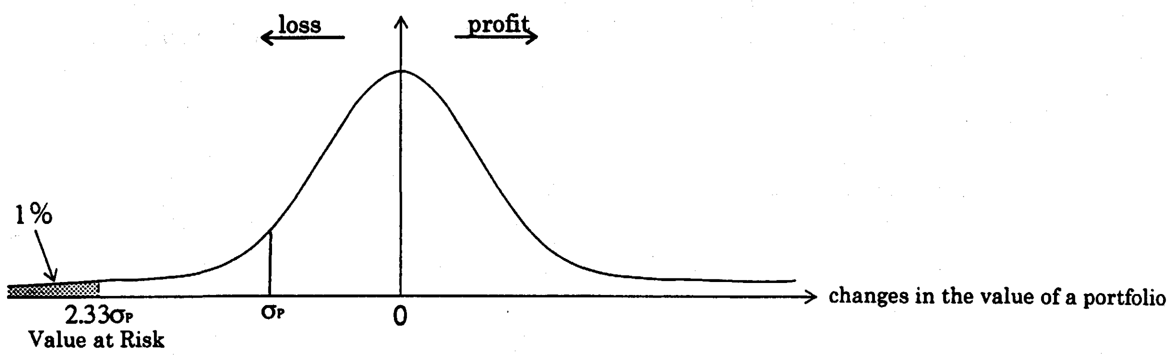
(with n risk factors)

$$\sigma_p^2 = \sum_{i=1}^{n} \delta_i^2\sigma_i^2 + 2\sum\sum \rho_{ij}\delta_i\delta_j\sigma_i\sigma_j$$
$$= \begin{bmatrix} \delta_1\sigma_1 & \delta_2\sigma_2 & \cdots & \delta_n\sigma_n \end{bmatrix} \begin{bmatrix} 1.0 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & 1.0 & \cdots & \rho_{2n} \\ \vdots & & \ddots & \\ \rho_{n1} & \cdots & & 1.0 \end{bmatrix} \begin{bmatrix} \delta_1\sigma_1 \\ \delta_2\sigma_2 \\ \vdots \\ \delta_n\sigma_n \end{bmatrix}$$

$$\text{VAR} = \phi \times \sqrt{\tau} \times \sqrt{\begin{bmatrix} \delta_1\sigma_1 & \delta_2\sigma_2 & \cdots & \delta_n\sigma_n \end{bmatrix} \begin{bmatrix} 1.0 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & 1.0 & \cdots & \rho_{2n} \\ \vdots & & \ddots & \\ \rho_{n1} & \cdots & & 1.0 \end{bmatrix} \begin{bmatrix} \delta_1\sigma_1 \\ \delta_2\sigma_2 \\ \vdots \\ \delta_n\sigma_n \end{bmatrix}}$$

(Chart 3)

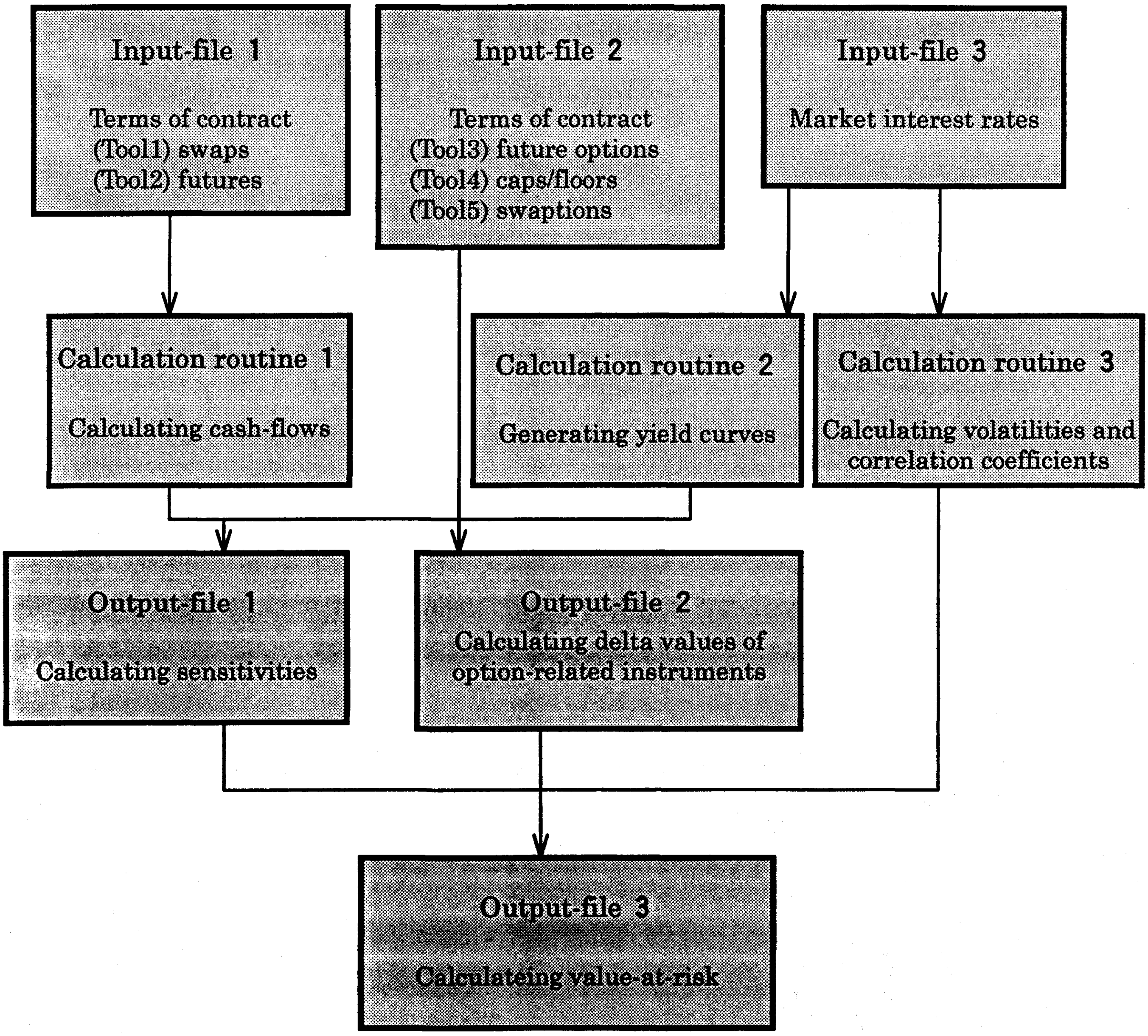
Distribution of changes in the value of a portfolio



The abscissas represent changes in the value of a portfolio (gain in positive, loss in negative) and the ordinates represent probability density of gains or losses indicated on the abscissas. The area between abscissas and the curve is the probability of the event. The shaded area indicates the losses with at most 1% likelihood and the right-end figure in the range is value-at-risk with a confidence level of 1%.

(Chart 4)

Structure of value-at-risk calculation model



Input data on terms of contract for financial instruments

Tool 1 Interest rate swaps:
- amount of capital
- first day of transaction
- last day of transaction
- rate receivable
- payment interval
- rate payable
- payment interval

Tool 3 Interest rate future options:
- call/put
- selling/buying
- amount
- maturity
- exercise price

Tool 2 Interest rate futures:
- selling/buying
- amount
- maturity

Tool 4 Caps/floors:
- cap/floor
- selling/buying
- amount of capital
- cap/floor rate
- interval of interest payment
- first day of transaction
- last day of transaction

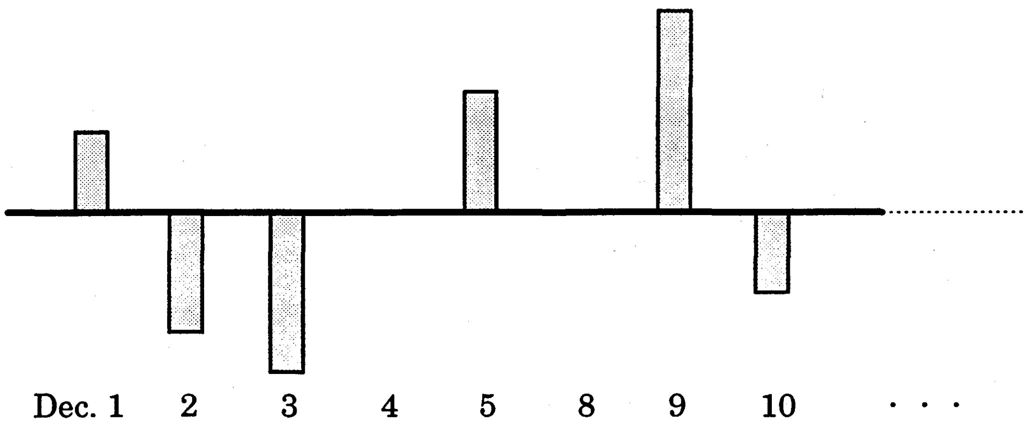
Tool 2' F R A s:
- amount of capital
- selling/buying
- first day of transaction
- last day of transaction
- contracted interest rate

Tool 5 Swaptions:
- fixed rate receivable/
  fixed rate payable
- ooption period
- selling/buying
- amount of capital
- first day of swap
- last day of swap
- rate receivable
- payment interval
- rate payable
- payment interval

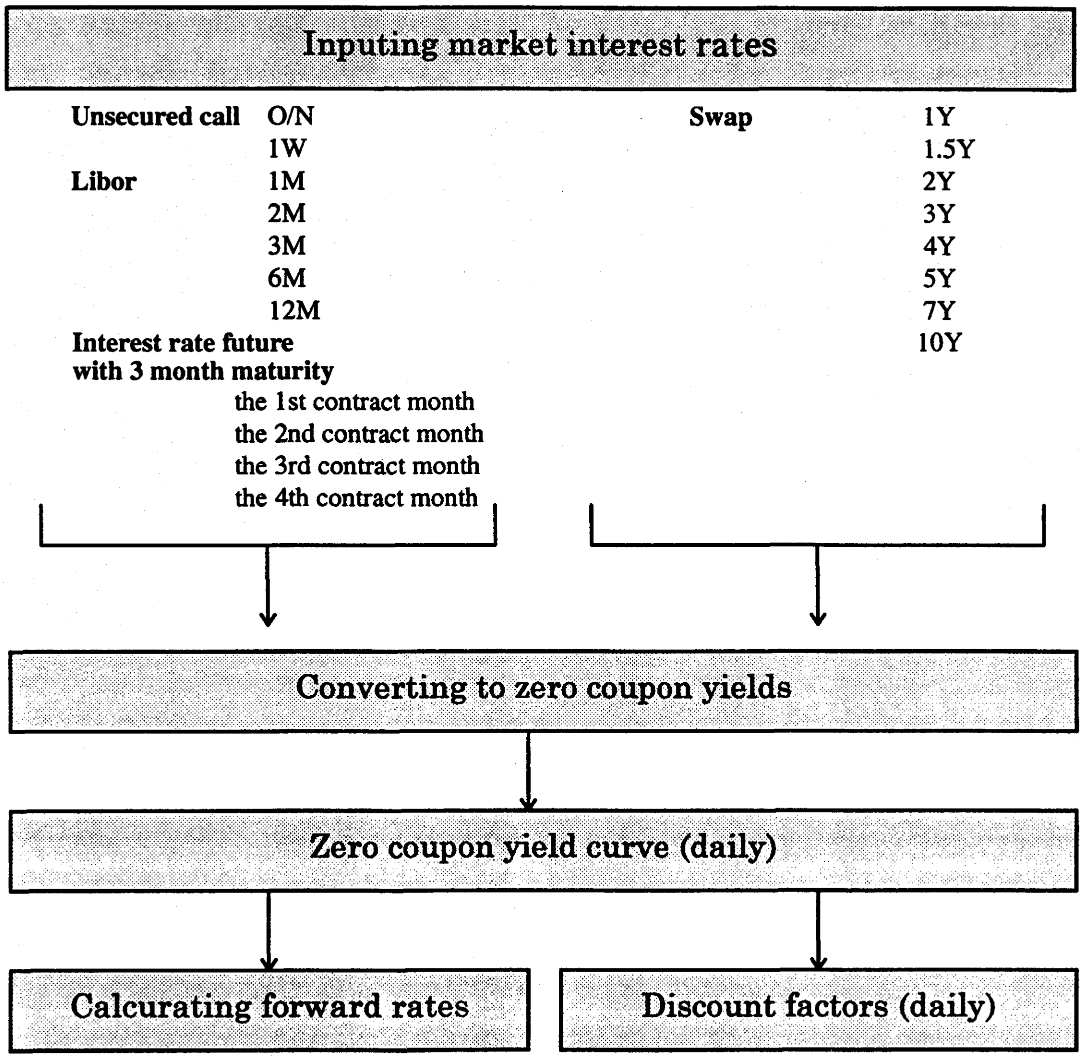
(Chart 6)

Calculating cash-flows

Calculate future cash-flows according to the terms contracts.



Drawing yield curve



(Chart 8)

Cash-flow data

(100 million yen)

date	Cash-flow	transaction number
15 Jan, 95	$\dfrac{25\times(0.0245-r_{3M(95/1)})\times 3/12}{1+r_{3M(95/1)}\times 3/12}$	①
20 Jun, 95	$50\times(r_{6M(94/12)}+0.0025-0.038)\times 6/12$	②
20 Dec, 95	$50\times(r_{6M(95/6)}+0.0025-0.038)\times 6/12$	②
20 Jun, 96	$50\times(r_{6M(95/12)}+0.0025-0.038)\times 6/12$	②
20 Dec, 96	$50\times(r_{6M(96/1)}+0.0025-0.038)\times 6/12$	②

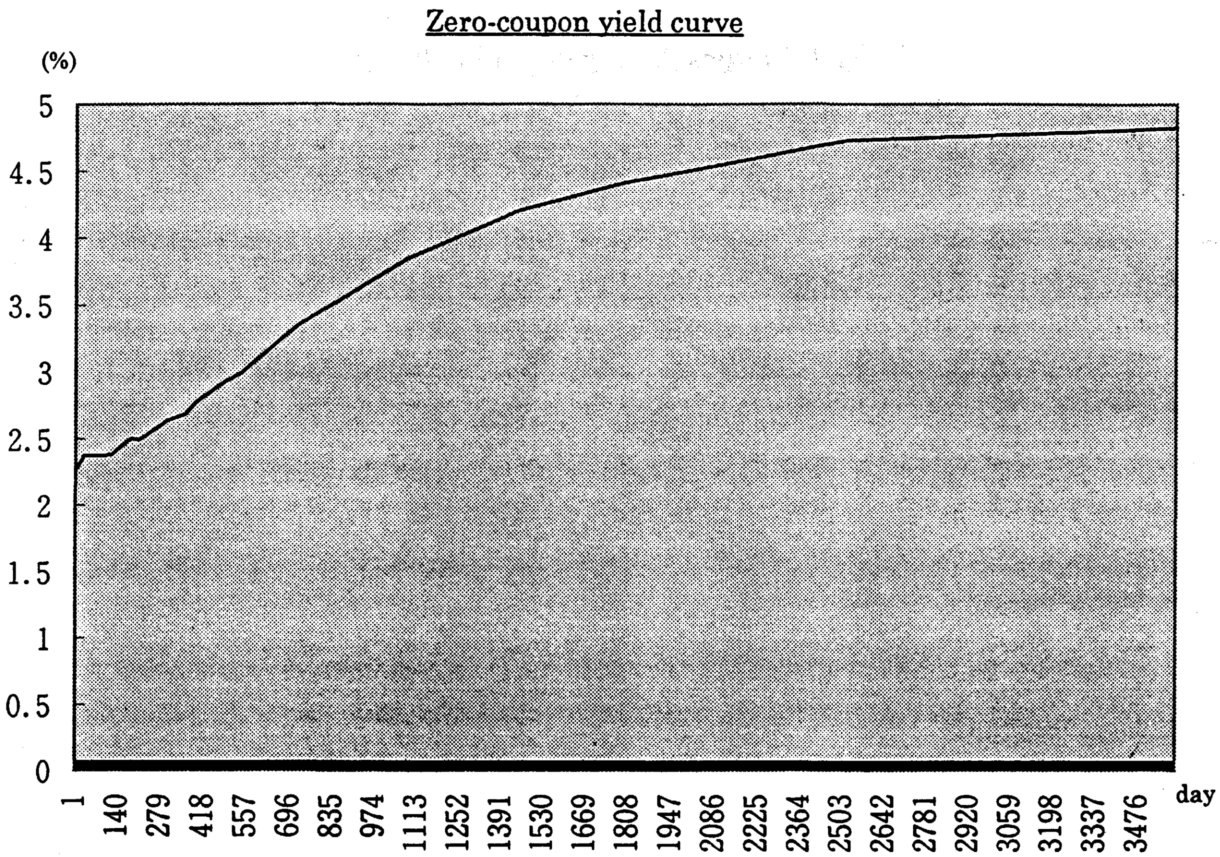
$r_{iM(t)}$: i-month interest rate at point t

(Chart 9)

Market interest rates (Nov.30, 1994)

unsecured call O/N	2.28125%
unsecured call 1 week	2.3125%
1 month Libor	2.3776%
2 month Libor	2.3776%
3 month Libor	2.3802%
6 month Libor	2.5156%
12 month Libor	2.7526%
1 year swap rate	offer 2.7 % – bid 2.64%
1.5 year swap rate	offer 3.03% – bid 2.99%
2 year swap rate	offer 3.37% – bid 3.35%
3 year swap rate	offer 3.87% – bid 3.85%
4 year swap rate	offer 4.22% – bid 4.18%
5 year swap rate	offer 4.42% – bid 4.3 %
7 year swap rate	offer 4.71% – bid 4.69%
10 year swap rate	offer 4.82% – bid 4.78%
interest rate future 94/12 contract	2.38%
interest rate future 95/3 contract	2.59%
interest rate future 95/6 contract	2.86%
interest rate future 95/9 contract	3.18%
interest rate future 95/12 contract	3.54%

(Chart 10)



Zero-coupon yield curve
(%)
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
1
140
279
418
557
696
835
974
1113
1252
1391
1530
1669
1808
1947
2086
2225
2364
2503
2642
2781
2920
3059
3198
3337
3476
day

(Chart 11)

Risk factors' historical volatilities

Observed period : 3 years (from Dec. 91 to Nov. 94)

Window width for calculating volatilities : 2 weeks (moving-windows method*note)

(%)

	1-month interest rate	3-month interest rate	6-month interest rate	12-month interest rate	2-year interest rate	3-year interest rate	5-year interest rate	7-year interest rate	10-year interest rate
σ	25.9	20.3	21.1	26.9	26.5	26.0	24.1	21.7	19.2

*note : See Chart 18

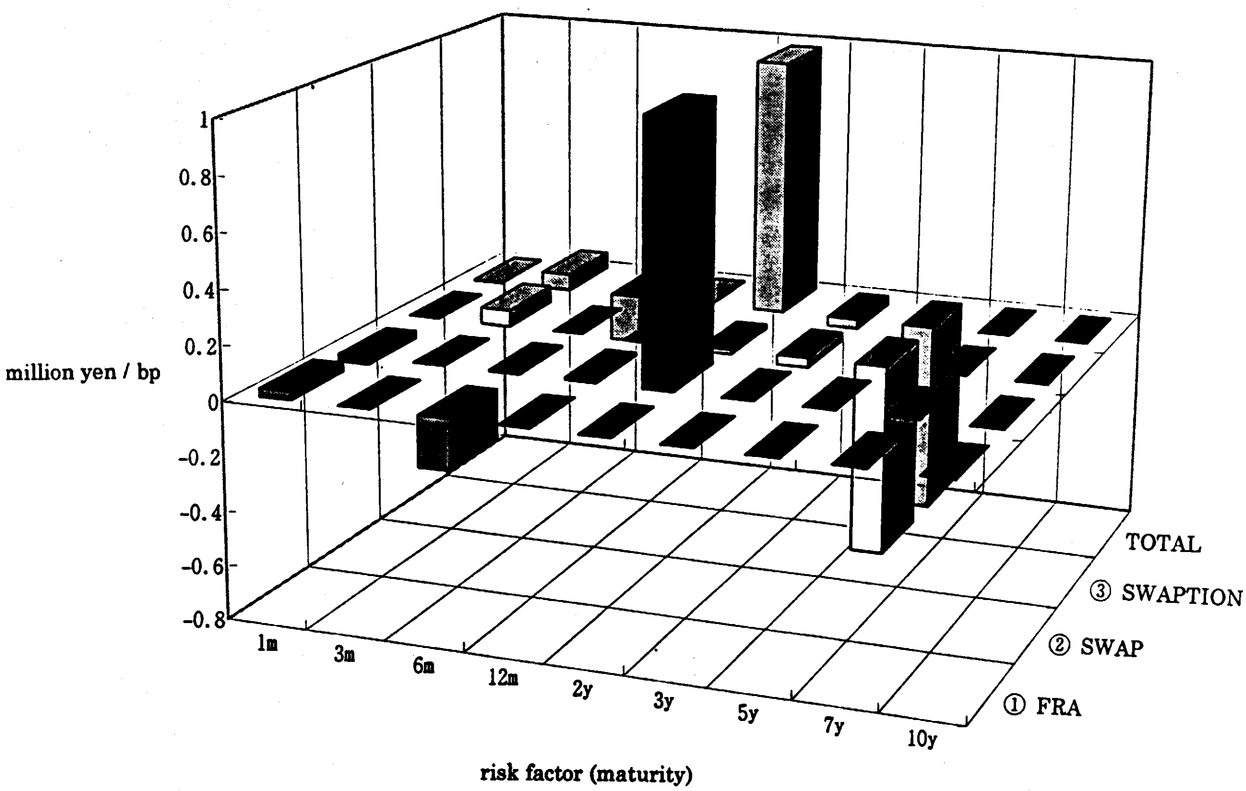
Correlation matrix between risk factors

Observed period : 3 years (from Dec. 91 to Nov. 94)
Window width for calculating volatilities : 2 weeks (moving-windows method)

	1m	*3m*	*6m*	*12m*	*2y*	*3y*	*5y*	*7y*	*10y*
1m	1.000	0.639	0.495	0.404	0.243	0.286	0.284	0.198	0.177
3m	0.639	1.000	0.789	0.625	0.459	0.500	0.531	0.467	0.425
6m	0.495	0.789	1.000	0.725	0.615	0.649	0.643	0.553	0.497
12m	0.404	0.625	0.725	1.000	0.734	0.762	0.727	0.619	0.570
2y	0.243	0.459	0.615	0.734	1.000	0.895	0.843	0.750	0.728
3y	0.286	0.500	0.649	0.762	0.895	1.000	0.957	0.858	0.808
5y	0.284	0.531	0.643	0.727	0.843	0.957	1.000	0.931	0.875
7y	0.198	0.467	0.553	0.619	0.750	0.858	0.931	1.000	0.940
10y	0.177	0.425	0.497	0.570	0.728	0.808	0.875	0.940	1.000

(Chart 13)

<u>Delta map of the portfolio</u>



(Chart 14)

Value-at-risk

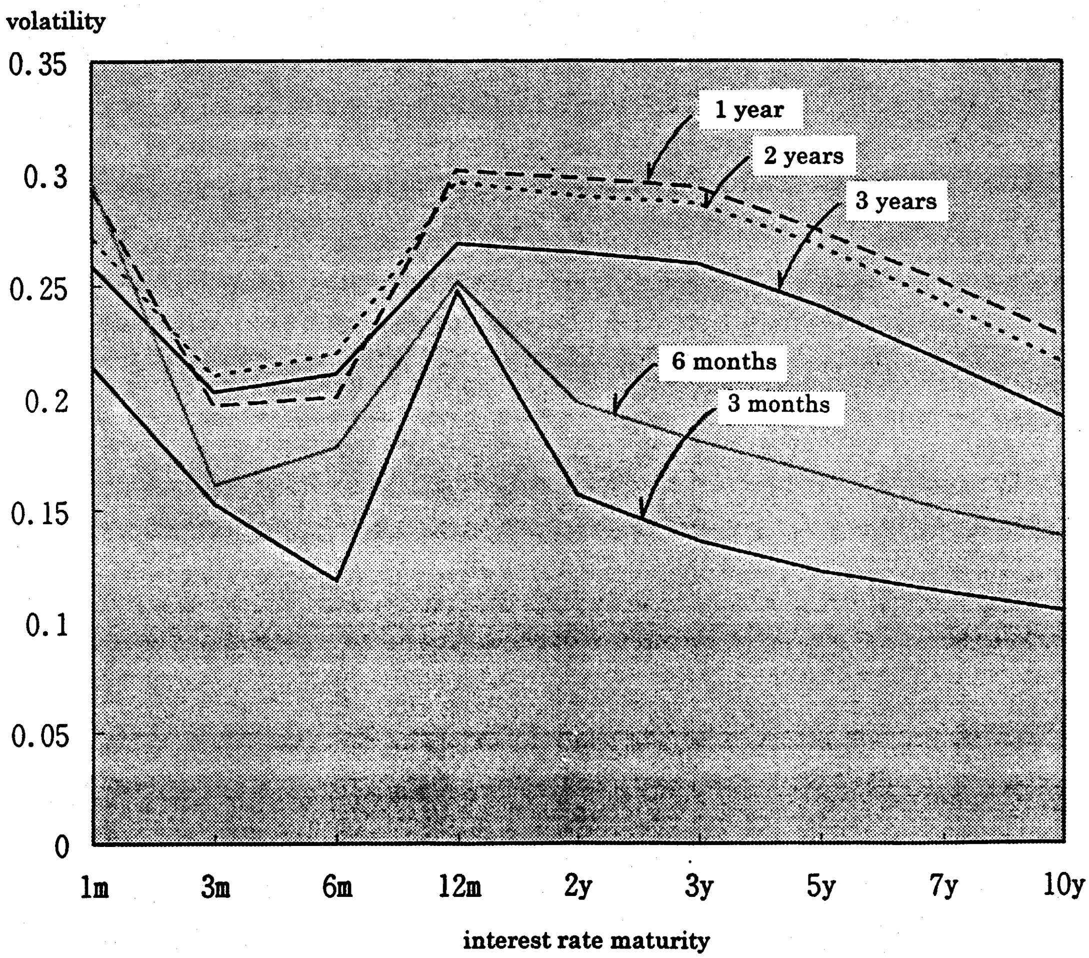
Confidence level : 95% (1.64 σ)
Holding period : 2 weeks (10 business days)

(10 million yen)

	amount of cacpital	VAR
Instrument 1 FRA	2,500	3.0
Instrument 2 Interest rate swap	5,000	31.6
Instrument 3 Swaption	5,000	24.5
Entire portfolio	12,500	16.2
(for reference) ①+②+③		59.1
$\sqrt{①^2+②^2+③^2}$		40.1

(Chart 15)

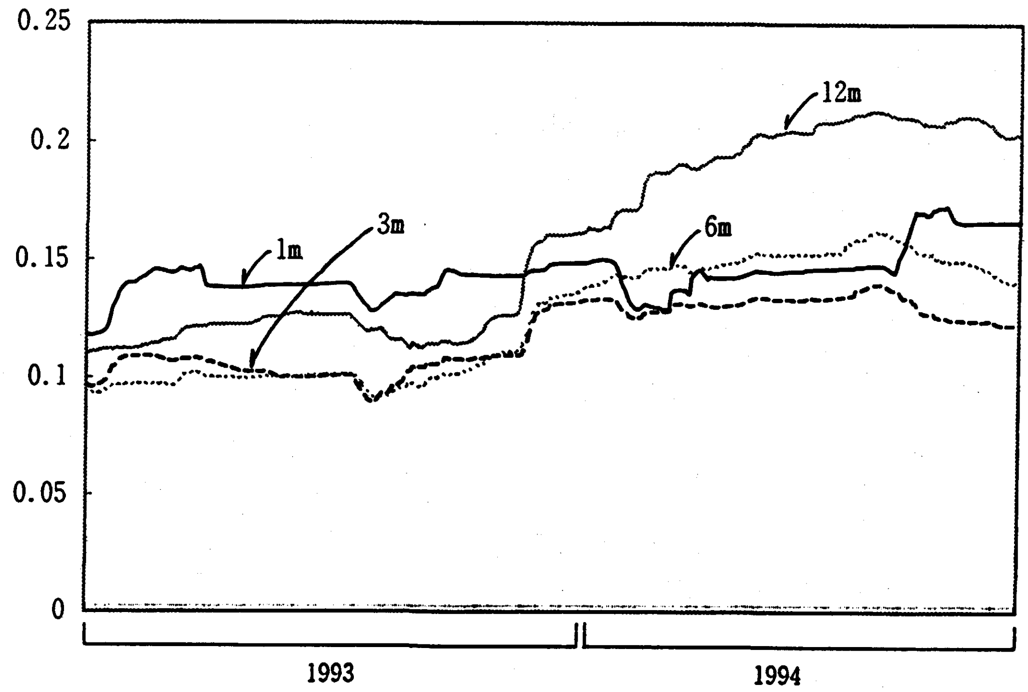
Volatility curves based on various observed period



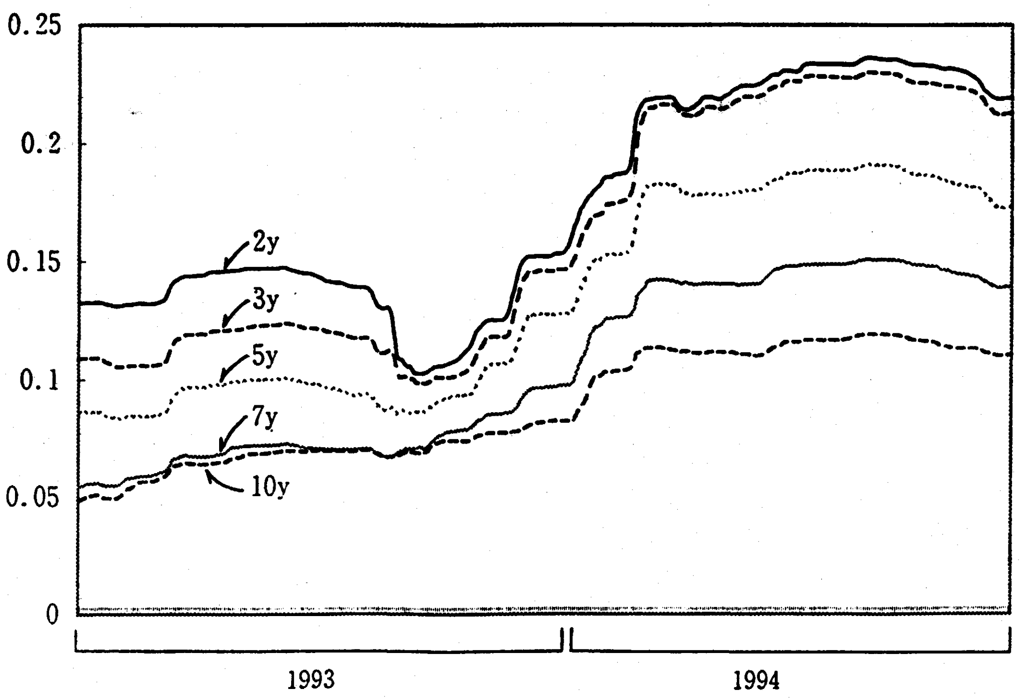
(Note) : Observed period is calculated backwards beginning November 1994.

(Chart 16)

Short term interest rates volatilities



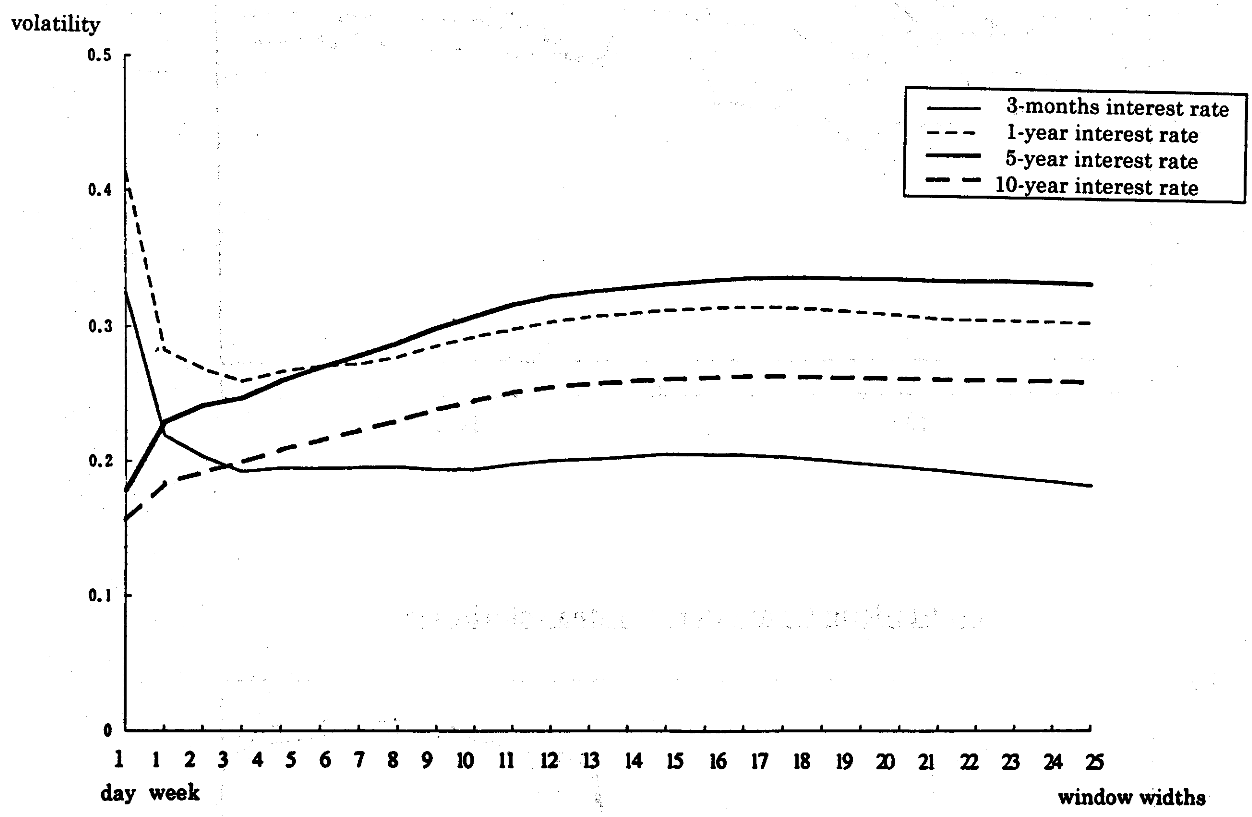
Mid-to-long term interest rates volatilities



(Note): Observed period is calculated backwards beginning November 1994.

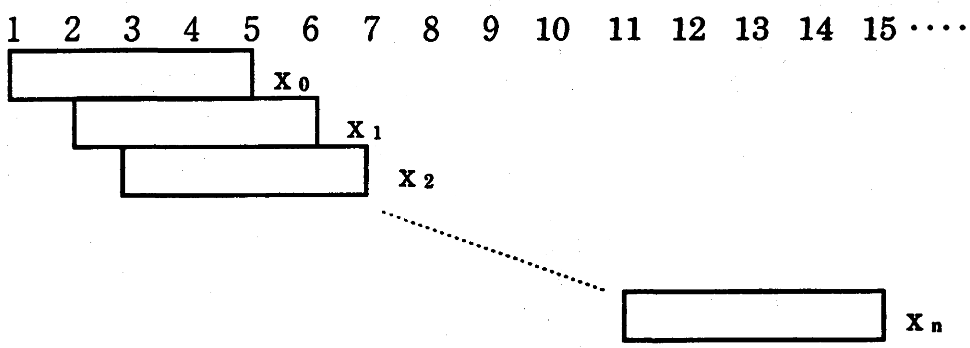
(Chart 17)

Window widths for calculating volatilities



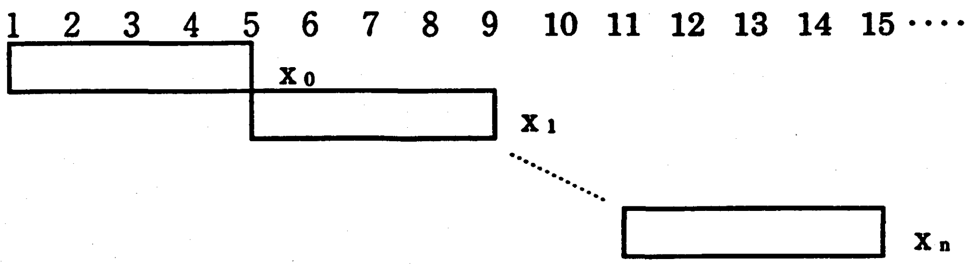
Subsampling methods for calculating volatility

(Moving-windows / Overlapping method)

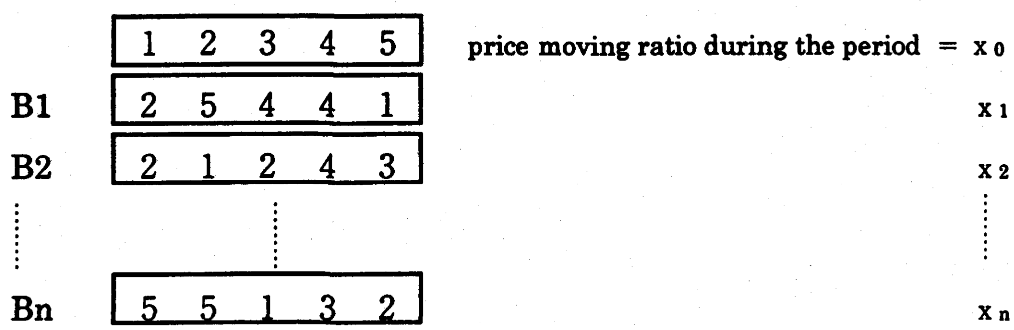


x_i: i-th subsample
volatility: standard deviation of x_i

(Box-car / Non-overlapping method)



(Bootstrap method)



Randomly draw a sample out of five data with replacement. Then repeat the sampling n times (B_1 to B_n) to obtain subsamples x_1 to x_n.

(Chart 19)

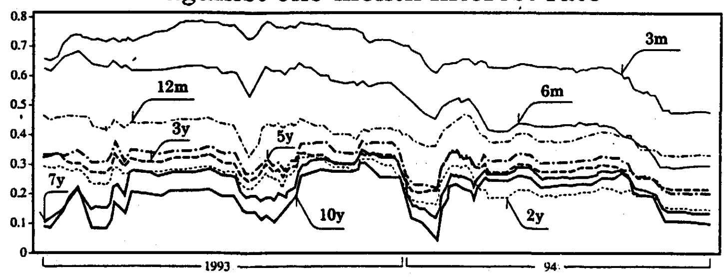
Correlation matrix of interest rates, stock prices and exchange rates

Observed period: 3 years (from January 1992 to December 1994)
Window width of calculating correlation: 2 weeks (moving-windows method)

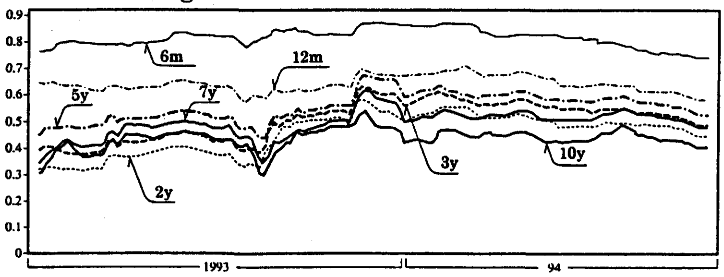
	3M Libor	*2Y Swap*	*10Y Swap*	*10Y JGB*	*Nikkei*	*¥/$*
3M Libor	1.000	0.446	0.437	0.389	0.097	0.122
2Y Swap	0.446	1.000	0.771	0.716	0.443	0.100
10Y Swap	0.437	0.771	1.000	0.906	0.388	0.125
10Y JGB	0.389	0.716	0.906	1.000	0.415	0.099
Nikkei	0.097	0.443	0.388	0.415	1.000	0.032
¥/$	0.122	0.100	0.125	0.099	0.032	1.000

Correlation between interest rates

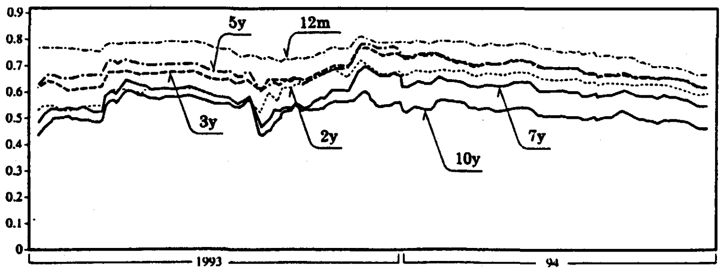
against one-month interest rate



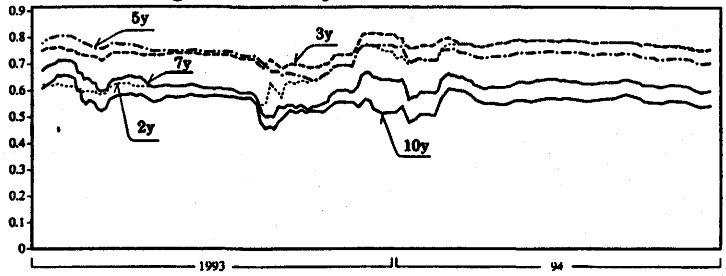
against 3-months interest rate



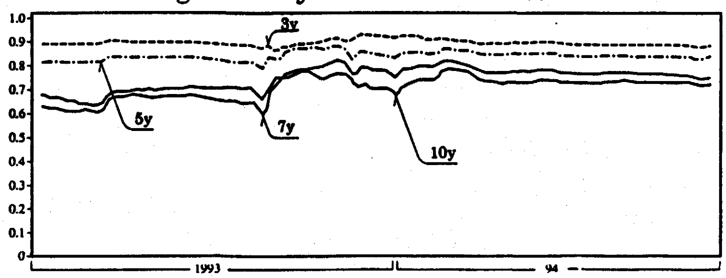
against 6-months interest rate

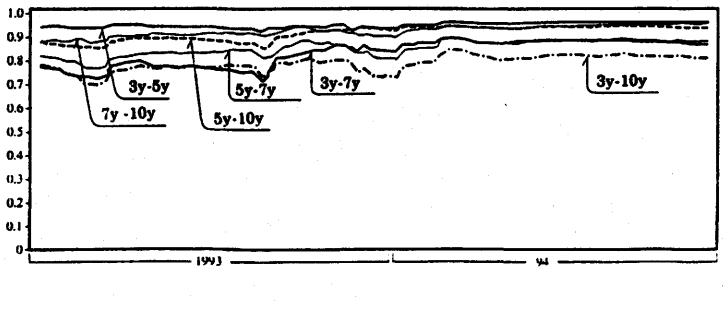


against one-year interest rate



against 2-years interest rate





(Chart 21)

Portfolio to be simulated

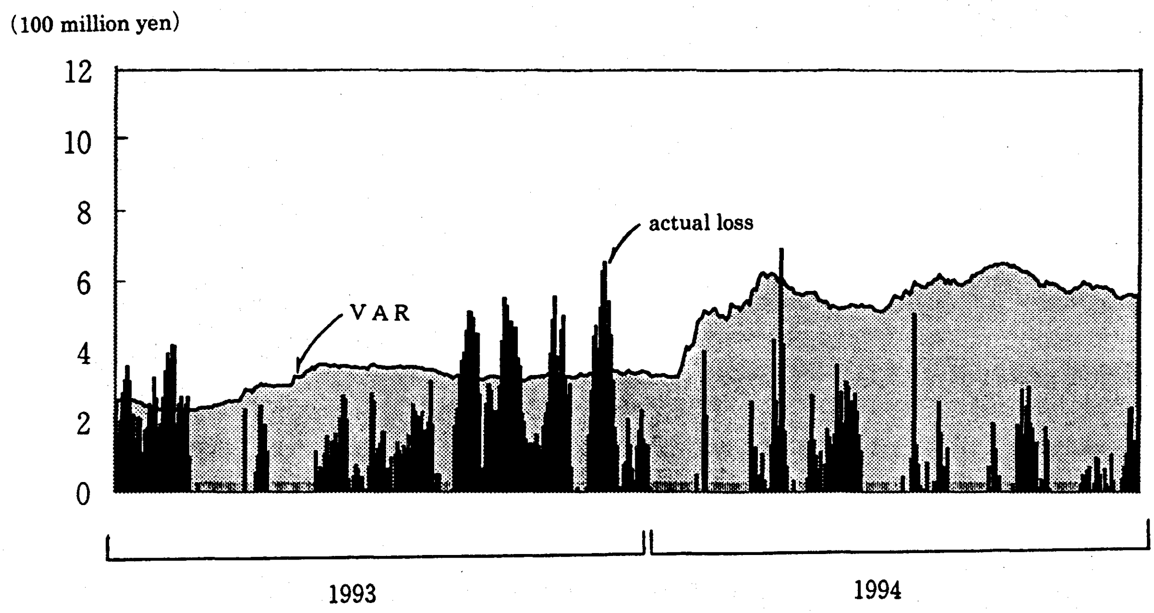
(100 million yen)

	number of transactions	national amounts	average notional amount per transaction
Interest rate future	8	688	86
FRA	4	90	22.5
Interest rate swap	22	1,965	89.3
Swaption	6	675	112.5
Total	40	3,418	85.5

(Chart 22)

Risk / return simulation

Historical volatility and historical correlation observed for one year preceding to the time of calculation and calculated daily.



(Chart 23)

<u>Correlation between risk factors from August to November 1993</u>

Observation period: from August to November 1993

	1M	*3M*	*6M*	*12M*	*2Y*	*3Y*	*5Y*	*7Y*	*10Y*
1M	1.000	0.469	0.233	0.179	-0.133	-0.065	0.073	0.088	0.100
3M	0.469	1.000	0.783	0.599	0.209	0.441	0.547	0.451	0.253
6M	0.233	0.783	1.000	0.749	0.384	0.566	0.604	0.449	0.170
12M	0.179	0.599	0.749	1.000	0.637	0.727	0.638	0.478	0.189
2Y	-0.133	0.209	0.384	0.637	1.000	0.856	0.596	0.494	0.277
3Y	-0.065	0.441	0.566	0.727	0.856	1.000	0.850	0.732	0.478
5Y	0.073	0.547	0.604	0.638	0.596	0.850	1.000	0.929	0.677
7Y	0.088	0.451	0.449	0.478	0.494	0.732	0.929	1.000	0.790
10Y	0.100	0.253	0.170	0.189	0.277	0.478	0.677	0.790	1.000

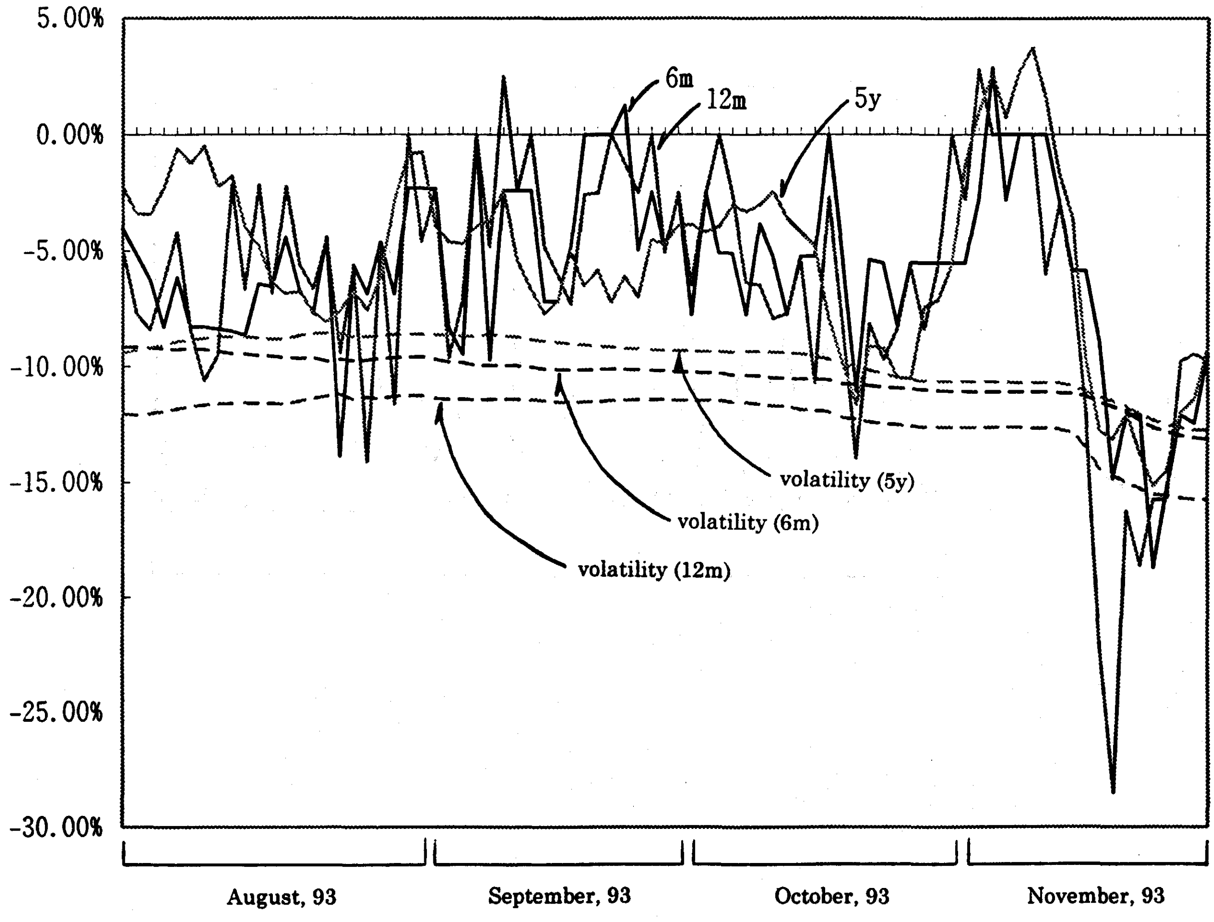
<u>Correlation between risk factors at the end of August 1993</u>

Observation period: from September 1992 to August 1993

	1M	*3M*	*6M*	*12M*	*2Y*	*3Y*	*5Y*	*7Y*	*10Y*
1M	1.000	0.770	0.615	0.429	0.239	0.263	0.287	0.198	0.143
3M	0.770	1.000	0.843	0.615	0.460	0.483	0.514	0.424	0.396
6M	0.615	0.843	1.000	0.719	0.590	0.634	0.630	0.533	0.507
12M	0.429	0.615	0.719	1.000	0.607	0.686	0.660	0.538	0.507
2Y	0.239	0.460	0.590	0.607	1.000	0.885	0.856	0.763	0.761
3Y	0.263	0.483	0.634	0.686	0.885	1.000	0.942	0.817	0.799
5Y	0.287	0.514	0.630	0.660	0.856	0.942	1.000	0.907	0.861
7Y	0.198	0.424	0.533	0.538	0.763	0.817	0.907	1.000	0.929
10Y	0.143	0.396	0.507	0.507	0.761	0.799	0.861	0.929	1.000

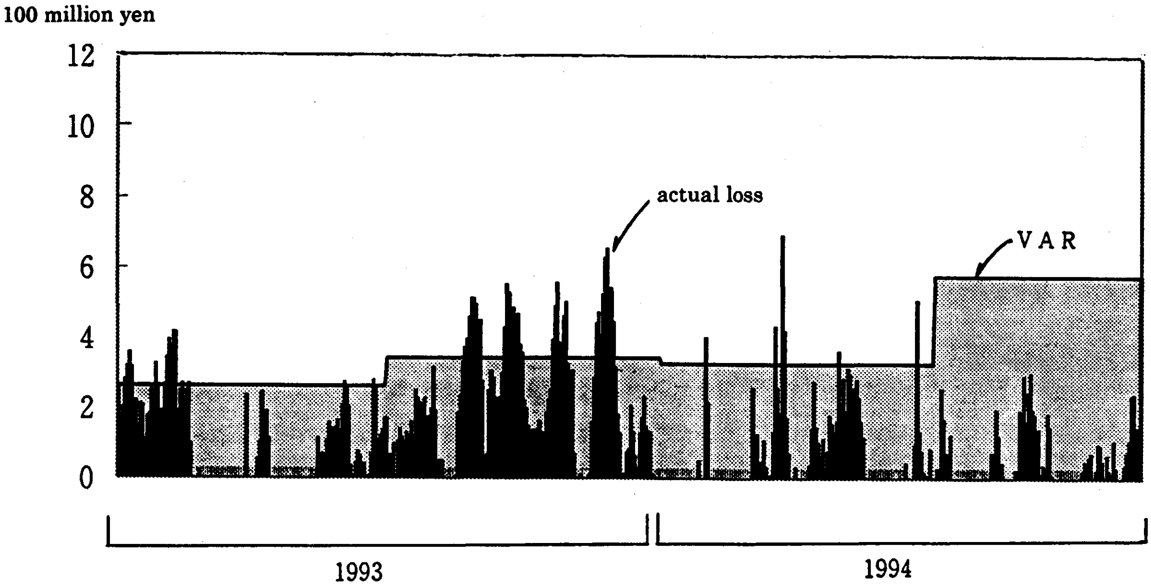
(Chart 24)

Changes in risk factors and volatilities upon VAR calculation



Risk / return simulation

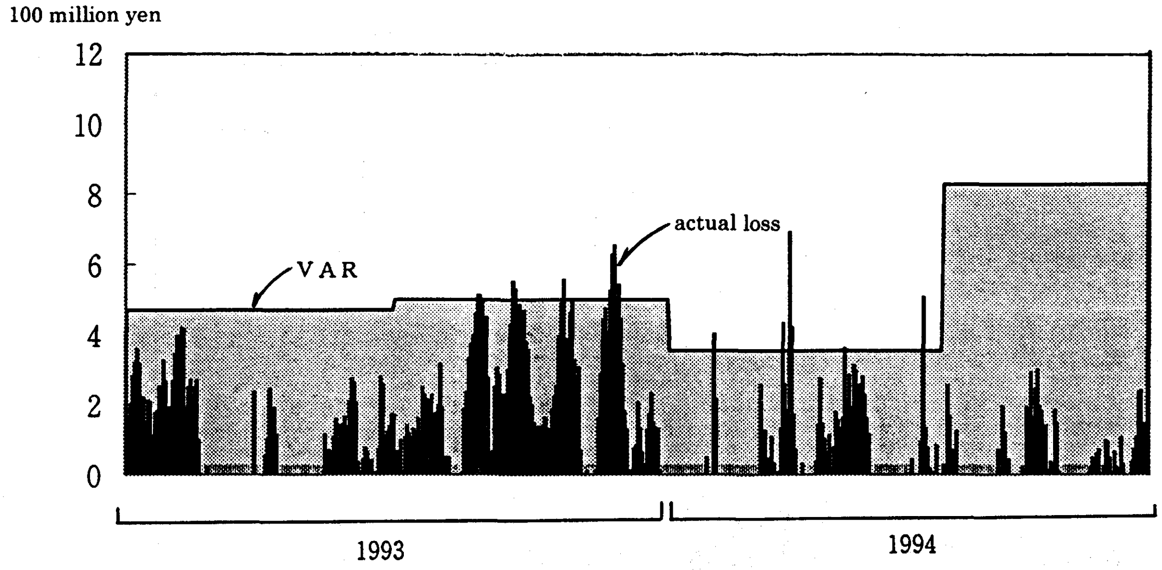
Historical volatility and historical correlation observed for one year preceding to the time of calculation and calculated semiannually



(Chart 26)

Risk / return simulation

Historical volatility observed for one year preceding to the time of calculation and calculated semiannually. No correlation is assumed.



(Chart 27)

Risk / return simulation

Historical volatility observed for one year preceding to the time of calculation and calculated semiannually. Simple addition of risk amount of each risk factor (Correlation coefficient ±1)