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INTRADAY PRICE VOLATILITY AND TRADING VOLUME: A CASE OF THE JAPANESE GOVERNMENT BOND FUTURES

Toshiaki Watanabe

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ABSTRACT

This paper examines the relation between price volatility and trading volume using intraday data on the Japanese Government Bond (JGB) Futures contracts. Based on the mixture-of-distributions hypothesis, we first set up a model in which price volatility and log volume are jointly determined by a single latent common factor. Using a quasi-maximum likelihood procedure via the Kalman filter, the model is then fitted to data. We find that the common factor is not persistent, and that there exist highly persistent idiosyncratic noises, providing evidence for misspecification of the mixture-of-distribution hypothesis. In addition, we find evidence of bi-directional causality as well as simultaneous causality between volatility and volume which cannot be explained by our common factor model although our common factor model fits data better than a VAR model presented by Watanabe (1993, Chapter 4). The presence of a significant causality from volume to volatility suggests that high-frequency trading volume data may provide useful information for financial risk management.

* Associate Professor, Faculty of Economics, Tokyo Metropolitan University

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Toshiaki Watanabe
Tokyo Metropolitan University

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This paper examines the relation between price volatility and trading volume using intraday data on the Japanese Government Bond (JGB) Futures contract. Based on the mixture-of-distributions hypothesis , we first set up a model in which price volatility and log volume are jointly determined by a single latent common factor. Using a quasi-maximum likelihood procedure via the Kalman filter, the model is then fitted to data. We find that the common factor is not persistent, and that there exist higly persistent idiosyncratic noises, providing evidence for misspecification of the mixture-of-distributions hypothesis. In addition, we find evidence of bidirectional causality as well as simultaneous causality between volatility and volume which cannot be explained by our common factor model although our common factor model fits data better than a VAR model presented by Watanabe (1993, Chapter 4). The presence of a significant causality from volume to volatility suggests that high-frequency trading volume data may provide useful information for financial risk management.

*Associate Professor, Faculty of Economics, Tokyo Metropolitan University, 1-1 Minami Ohsawa, Hachioji, Tokyo, Japan, 192-03.

1 Introduction

This paper examines the relation between price volatility and trading volume using intraday Japanese Government Bond (JGB) Futures data.

There is now considerable evidence that price volatility in financial markets changes randomly over time. An understanding of the properties of volatility dynamics is, hence, important for financial risk management. The last twenty years have seen a surge of interest in modelling changing volatility. Besides, studies of the joint dynamics of volatility and other variables are important. It is now well known that high price volatility is associated with high trading volume. The October crash in 1987 is the best known example of this phenomenon. Karpoff (1987) reviews previous studies on the return-volume relation in various financial markets, in which he cites 18 studies that document this phenomenon.

In considering this phenomenon, we cannot neglect the *mixture-of distributions hypothesis* proposed by Clark (1973) and developed by Epps and Epps (1976), Tauchen and Pitts (1983), and Andersen (1993). The basic idea of this hypothesis is that the amount of information that flows into the market changes randomly over time. Changes in asset prices are usually prompted by the arrival of new information. If the amount of information arrivals changes randomly over time, so does daily return volatility. A well known phenomenon of a high degree of volatility persistence can be explained by serial correlations in the amount of information flow. The amount of information flow may also influence the trading volume. As the amount of information that flows into the market increase, traders' expectations spread, and hence the larger is the trading volume. This hypothesis is also consistent with a well known phenomenon of comovements between volatility and volume.

Based on this idea, Lamoureux and Lastrapes (1990) have entered the current trading volume directly into the GARCH variance equation. Using the daily data for 20 common stocks in the New York Stock Exchange, they have found that variance persistence vanishes and the conditional normality of returns is not rejected when volume is included as an explanatory variable in the variance equation. Their results give strong support for the mixture-of-distributions hypothesis. Locke and Sayers (1993) applied this approach to minute-by-minute data on the S&P 500 Index Futures.[1]In contrast with the results of Lamoureux and Lastrapes (1990), they find significant variance persistence even after controlling for volume. Their approach, however, has a certain drawback. If return volatility and trading volume are jointly determined, their results may be subject to the simultaneity bias.

This paper takes to a different approach. We first set up a model in which volatility and volume are jointly determined by a single latent common factor such as the amount of information flow. Then, we discuss a quasi-maximum likelihood procedure based on the Kalman filter to estimate our model.

The data we use are 5-minute returns and trading volume for the Japanese Government Bond (JGB) Futures contract from March 3 to May 31 in 1995. Since these data show well known W-shaped intraday patterns and the means and standard deviations vary across intraday time periods, we adjust data by subtracting the mean and dividing standard deviation. Using the quasi-maximum likelihood method, our model is fitted to these adjusted data. For a comparison with our model, we also estimate an alternative model, presented by Watanabe (1993, Chapter 4), in which the log volatility and the log volume are specified as a vector autoregressive (VAR) process.

Our empirical findings are:

(1) While the common factor is not persistent, there exist persistent idiosyncratic noises. This means that high persistence of volatility cannot be explained by the common factor and casts some doubt on the mixture of distributions hypothesis.

(2) The latent common factor model proposed in this paper fits data better than the VAR model of Watanabe (1993 Chapter 4).

(3) However, there exists a bidirectional causality as well as a simultaneous relation between volatility and volume, which cannot be explained by the common factor model. The presence of a significant causality from volume to volatility suggests that high-frequency trading volume data may provide useful information for risk financial risk management.

[1]They have also tried other variables such as the number of floor transactions, the number of price changes, and executed order imbalance as proxies for the number of information arrivals.

The remainder of this paper proceeds as follows; Section 2 reviews the mixture-of-distributions hypothesis and introduces our common factor model. Section 3 explains the estimation method for our model. Section 4 describes our data. Section 5 fits our model to data and summarizes estimation results. Conclusions are given in Section 6.

2 The Model

2.1 The Mixture of Distributions Hypothesis

We begin with a brief review of the mixture-of-distributions hypothesis proposed by Clark (1973).

Changes in asset prices are usually prompted by the arrival of new information, and each change can be modelled as a random variable. Let R_{it} denote the ith intraday increment in the logarithm of asset price in day t. The basic idea of the mixture-of-distributions hypothesis is that the amount of information that arrives at the market during a certain time interval changes randomly over time. If the amount of information that flows into the market during day t is I_t, the change in the logarithm of asset price on day t, that is, the asset return on day t, is represented by

$$R_t = \sum_{i=1}^{I_t} \epsilon_{it}. \tag{1}$$

Suppose further that ϵ_{it} is i.i.d. with mean 0 and finite variance σ^2. If I_t is sufficiently large, applying the Central Limit Theorem to equation (1) yields that the distribution of R_t conditional on I_t is approximately normal with mean 0 and variance $\sigma^2 I_t$. Hence, we may rewrite equation (1) as:

$$R_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim NID(0,1). \tag{2}$$

where σ_t^2 represents the vaiance of R_t conditional on I_t, which we call return volatility, i.e.,

$$\sigma_t^2 = \sigma^2 I_t. \tag{3}$$

If I_t changes randomly over time, so does the asset return volatility and hence R_t is drawn from a mixture of distributions. As is well known, volatility shocks persist over time. If we assume that I_t is autocorrelated, the resulting model can give rise to this persistence. For instance, suppose that the logarithm of I_t follows an AR(1) process, i.e.,

$$ln(I_t) = a' + bln(I_{t-1}) + w_t, \quad w_t \sim NID(0, \sigma_w^2), \tag{4}$$

Combining equation (3) with (4), we have

$$h_t^2 = a + bh_{t-1}^2 + w_t, \quad w_t \sim NID(0, \sigma_w^2), \tag{5}$$

where $h_t \equiv ln(\sigma_t^2)$ and $a = a' + (1-b)ln(\sigma^2)$.

The model that consists of equations (2) and (5) is usually called the *stochastc volatility* model.

The amount of information I_t may also influence the trading volume. The rationale is that the larger the amount of information that flows into the market, the more do the traders' expectations spread and hence the larger is the trading volume. If so, the mixture of distributions hypothesis is also consistent with a well known phenomenon of a comovement between volatility and volume.

Lamoureux and Lastrapes (1990) investigates this hypothesis using the GARCH model. They enter the current trading volume directly into the GARCH variance equation in the following way.

$$\sigma_t^2 = \alpha_0 + \alpha_1 \sigma_{t-1}^2 \epsilon_{t-1}^2 + \alpha_2 \sigma_{t-1}^2 + \alpha_3 V_t. \tag{6}$$

where V_t represents trading volume.

Using the data for 20 common stocks in the NYSE, they have found that: (1) α_3 is statistically significant, (2) α_1 and α_2 are small and statistically insignificant, and (3) the conditional normality for ϵ_t is not rejected. These results give a strong support for the mixture-of-distributions hypothesis.

Locke and Sayers (1993) applied this approach to minute-by-minute data on the S&P 500 Index Futures. In contrast to the results of Lamoureux and Lastrapes (1990) who used daily data, they find significant variance persistence after controlling for volume.

Their approach, however, has a certain drawback. As Lamoureux and Lastrapes (1990) have mentioned in their paper, their results are subject to simultaneity bias if return volatility and trading volume are jointly determined.[2]

[2] Tauchen and Pitts (1983) and Andersen (1993) have presented a theoretical model

2.2 Latent Common Factor Model

This paper reconsiders Lamoureux and Lastrapes' (1990) and Locke and Sayers' (1994) results using a different approach. We set up a model in which volatility and volume are jointly determined by a single latent common factor such as the number of information arrivals. In particular, we assume that the logarithm of σ_t^2, denoted by h_t, and the logarithm of volume, denoted by LV_t , consist of a common factor and an idiosyncratic noise, i.e.,

$$h_t = c_1 + z_1 x_{1,t} + x_{2,t}, \tag{7}$$

$$LV_t = c_2 + z_2 x_{1,t} + x_{3,t}, \tag{8}$$

where c_1, c_2, z_1, and z_2 are constant, $x_{1,t}$ is the common factor that affects both volatility and volume, and $x_{2,t}$ and $x_{3,t}$ are idiosyncratic noises. While the common factor and idiosyncratic noises are assumed to be mutually independent and have no lead and lagged relations, we allow for serial correlations in the common factor and idiosyncratic noises. Specifically, we model the common factor and two idiosyncratic noises as AR(1) processes, i.e.,

$$\begin{aligned}
x_{1,t} &= b_{11}x_{1,t-1} + w_{1,t}, \quad w_{1,t} \sim NID(0,1), & (9)\\
x_{2,t} &= b_{22}x_{2,t-1} + w_{2,t}, \quad w_{2,t} \sim NID(0,\sigma_{22}), & (10)\\
x_{3,t} &= b_{33}x_{3,t-1} + w_{3,t}, \quad w_{3,t} \sim NID(0,\sigma_{33}), & (11)
\end{aligned}$$

where $w_{1,t}, w_{2,t}$, and $w_{3,t}$ are mutually and serially independent.

Notice that the variance of $w_{1,t}$ is normalized to unity and that the Lamoureux and Lastrapes (1990) approach is valid only if $\sigma_{33} = 0$.

3 Methodology

In this chapter, we explain how to estimate the parameters in our model that consists of equations (2) and (7)-(11). As will be shown below, our model can be transformed into a linear state space form. Thus, we adopt the quasi-maximum likelihood method via the Kalman filter employed by Nelson

in which volatility and volume are a joint random function of information flows. For the estimatiom methods for their models, see also Lamoureux and Lastrapes (1994) and Watanabe (1995).

(1988), Watanabe (1993, Chapter 4), Harvey, Ruiz, and Shephard (1994), and Ruiz (1994). Specifically, we first transform our model into a linear state space form. Since the observation noise in the resulting state space form is not normally distributed, we approximate the density for the observation noise with a normal distribution with the same mean and variance. Under this approximation, executing the Kalman filter to the state space form produces the quasi-likelihood, which is maximized to obtain the estimates of the model parameters.

3.1 State Space Representation

We first transform our model into a linear state space form. Squaring both sides of equation (2) and then taking the log yields:

$$\begin{aligned} ln(R_t^2) &= h_t + ln(\epsilon_t^2), \\ &= c_1 + z_1 x_{1,t} + x_{2,t} + ln(\epsilon_t^2), \end{aligned} \tag{12}$$

where the mean and the variance of $ln(\epsilon_t^2)$ are known to be approximately -1.27 and 4.935 respectively. (See Abramovitz and Stegun (1970).)

Equation (12) may be rewritten as:

$$ln(R_t^2) = c_1 - 1.27 + z_1 x_{1,t} + x_{2,t} + v_t, \tag{13}$$

where $v_t = ln(\epsilon_t^2) + 1.27$, and the mean and the variance of v_t are 0 and 4.935 respectively.

We represent equation (13) with the log volume equation (8) as:

$$\boldsymbol{y}_t = \boldsymbol{c} + \boldsymbol{Z}\boldsymbol{x}_t + \boldsymbol{v}_t, \tag{14}$$

where

$$\boldsymbol{y}_t \equiv \begin{bmatrix} ln(R_t^2) \\ LV_t \end{bmatrix}, \quad \boldsymbol{c} \equiv \begin{bmatrix} c_1 - 1.27 \\ c_2 \end{bmatrix}, \quad \boldsymbol{Z} \equiv \begin{bmatrix} z_1 & 1 & 0 \\ z_2 & 0 & 1 \end{bmatrix},$$

$$\boldsymbol{x}_t \equiv \begin{bmatrix} x_{1,t} \\ x_{2,t} \\ x_{3,t} \end{bmatrix}, \quad \boldsymbol{v}_t \equiv \begin{bmatrix} v_t \\ 0 \end{bmatrix},$$

and $\boldsymbol{v}_t$ is non-normal but i.i.d. with mean $\mathbf{o}$ and covariance matrix $\boldsymbol{H}$ given by

$$\boldsymbol{H} \equiv \begin{bmatrix} \sigma_v^2 & 0 \\ 0 & 0 \end{bmatrix}.$$

Equations (9)-(11) are represented by

$$\boldsymbol{x}_t = \boldsymbol{B}\boldsymbol{x}_{t-1} + \boldsymbol{w}_t, \tag{15}$$

where

$$\boldsymbol{B} \equiv \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix}, \quad \boldsymbol{w}_t \equiv \begin{bmatrix} w_{1,t} \\ w_{2,t} \\ w_{3,t} \end{bmatrix},$$

and $\boldsymbol{w}_t$ is normal with mean $\mathbf{o}$ and covariance matrix $\boldsymbol{\Sigma}$ given by

$$\boldsymbol{\Sigma} \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix}.$$

Equations (14) and (15) constitute the familiar linear state space form with $\boldsymbol{x}_t$ as a state vector.

For convenience, let us remove constant term $\boldsymbol{c}$ from equation (14). Then, our model is:

$$\begin{aligned} \boldsymbol{y}_t^* &= \boldsymbol{Z}\boldsymbol{x}_t + \boldsymbol{v}_t, & (16) \\ \boldsymbol{x}_t &= \boldsymbol{B}\boldsymbol{x}_{t-1} + \boldsymbol{w}_t, & (17) \end{aligned}$$

where $\boldsymbol{y}_t^* \equiv \boldsymbol{y}_t - \boldsymbol{c}$.

$E(\boldsymbol{y}_t)(= \boldsymbol{c})$ can be estimated consistently by the sample mean of $\boldsymbol{y}_t$ (See Priestley (1981)), and the sample mean is a quasi-maximum likelihood (QML) estimator, that will be explained below, of $E(\boldsymbol{y}_t)$ uncorrelated with the QML estimator of the stochastic part of the model (See Harvey (1989, pp.201-202)). In what follows, we simply use the sample mean of $\boldsymbol{y}_t$ for $E(\boldsymbol{y}_t)$ and treat $E(\boldsymbol{y}_t)$ as known.

3.2 Kalman Filter

Since the model described in equations (16) and (17) is in the familiar linear state space form, we can apply the Kalman filter to it. The Kalman filter is the algorithm that provides the estimator of the state vector in a linear state space model. [3] If the noise terms in the state space model are normally distributed, the Kalman filter provides the minimum mean square estimator (MMSE). Notice that the observation error in our model, $\boldsymbol{v}_t$, is not normally distributed. In such a case, the Kalman filter does not generally provide the MMSE but still provides the estimator that minimizes the mean square error within a class of linear estimators, that is, the minimum mean square linear estimator (MMSLE).

In our model, the equations of the Kalman filter are:

<u>One-Step-Ahead Prediction:</u>

$$\boldsymbol{x}_{t|t-1} = \boldsymbol{B}\boldsymbol{x}_{t-1|t-1}, \tag{18}$$

$$\boldsymbol{P}_{t|t-1} = \boldsymbol{B}\boldsymbol{P}_{t-1|t-1}\boldsymbol{B}' + \boldsymbol{\Sigma}; \tag{19}$$

<u>Updating:</u>

$$\boldsymbol{x}_{t|t} = \boldsymbol{x}_{t|t-1} + \boldsymbol{P}_{t|t-1}\boldsymbol{Z}'\boldsymbol{F}_t^{-1}\boldsymbol{\nu}_t, \tag{20}$$

$$\boldsymbol{P}_{t|t} = \boldsymbol{P}_{t|t-1} - \boldsymbol{P}_{t|t-1}\boldsymbol{Z}'\boldsymbol{F}_t^{-1}\boldsymbol{Z}\boldsymbol{P}_{t|t-1}, \tag{21}$$

where $\boldsymbol{x}_{t|t-1}$ is the MMSLE of $\boldsymbol{x}_t$ given information at $t-1$, $\boldsymbol{x}_{t|t}$ is the MMSLE of $\boldsymbol{x}_t$ given information at t, $\boldsymbol{P}_{t|t-1}$ is the covariance matrix of $(\boldsymbol{x}_t - \boldsymbol{x}_{t|t-1})$, $\boldsymbol{P}_{t|t}$ is the covariance matrix of $(\boldsymbol{x}_t - \boldsymbol{x}_{t|t})$, $\boldsymbol{\nu}_t$ is the prediction error vector defined by:

$$\boldsymbol{\nu}_t \equiv \boldsymbol{y}_t^* - \boldsymbol{Z}\boldsymbol{x}_{t|t-1}, \tag{22}$$

and $\boldsymbol{F}_t$ is its covariance matrix given by:

$$\boldsymbol{F}_t = \boldsymbol{Z}\boldsymbol{P}_{t|t-1}\boldsymbol{Z}' + \boldsymbol{H}. \tag{23}$$

Once the initial values of $\boldsymbol{x}_{t|t-1}$ and $\boldsymbol{P}_{t|t-1}$ are given, equations (20)-(25) can be solved recursively. As usual, we use the unconditional mean and variance of the state vector for those initial values, i.e.,

$$\begin{aligned} \boldsymbol{x}_{1|0} &= E(\boldsymbol{x}_t), \\ &= \mathbf{o}, \qquad (24) \\ \mathrm{vec}(\boldsymbol{P}_{1|0}) &= \mathrm{vec}(Var(\boldsymbol{x}_t)), \\ &= [\boldsymbol{I} - \boldsymbol{B} \otimes \boldsymbol{B}]^{-1}\mathrm{vec}(\boldsymbol{Sigma}), \qquad (25) \end{aligned}$$

[3]For a detailed discussion of the Kalman filter, the reader is referred to Harvey (1989).

where $\otimes$ is the Kronecker product, while the vec($\cdot$) operator indicates that the columns of the matrix are being stacked one upon the other.

3.3 Quasi-Likelihood Function

If the error terms were normally distributed for all t, then the prediction error $\boldsymbol{\nu}_t$ would be normally and independently distributed with mean 0 and variance $\boldsymbol{F}_t$, so that the log likelihood function would be given by:

$$lnL = -Tln(2\pi) - \frac{1}{2}\sum_{t=1}^{T} ln|\boldsymbol{F}_t| - \frac{1}{2}\sum_{t=1}^{T} \boldsymbol{\nu}_t' \boldsymbol{F}_t^{-1} \boldsymbol{\nu}_t, \tag{26}$$

where T is the number of observations.

We choose the model parameters so that this function is maximized. Unfortunately, our observation error, $\boldsymbol{v}_t$, is non-normal, so that equation (26) is not the true likelihood function and is, hence, called the ***quasi-likelihood function.*** The asymptotic properties of the quasi-maximum likelihood estimator are, however, well known. (See Dansmuir (1979) and Ruiz (1994).)

4 Data Description

Our primary data set consists of 5-minute returns and trading volume for the Japanese Government Bond (JGB) Futures contracts that expire in March and June 1995. The sample period is from March 3 to May 5 in 1995. Each intraday data is comprised of 24 values during 9:00-11:00 and 30 values during 12:30-15:00. During this period, the most heavily traded is the contract that expires in March 1995. All price data in this project are for that contract. The trading volume is the sum for the contract that expires in March 1995 and the one that expires in June 1995. Let returns (%) hereafter be defined as $R_t = 100ln(P_t/P_{t-1})$, where P_t = futures price at time t. Overnight returns and the lunch break returns are not 5-minute returns, and we therefore delete those data in the subsequent analysis, so that the first data for the mornig session is the return and volume from 9:00 to 9:05 and the first data for the afternoon session is from 12:30 to 12:35. We also omit the data when trading volume is zero. Since the the number of obsevations which are ommitted is only 4, this omission is of little consequence. Total sample after these corrections is 3234 observations. The return and log volume series are respectively plotted in Figure 1.A and 1.B.

Descriptive statistics for returns and volume are reported in Table 1.A. The sample mean of the 5-minute return of 0.003 % is indistinguishable from zero at standard significance levels given the sample standard deviation of 0.063%. The null hypothesis of normality is rejected both for returns and for trading volume at the 1% significance level. The sample skewness of 1.70 for returns and 0.288 for trading volume and the sample kurtosis of 25.0 for returns and 1.16 for trading volume are both highly statistically significant. The positive kurtosis coefficients are indicative of leptokurtic distributions. At the same time, the maximum of 0.867% for returns and 9.67 for volume and the minimum of -0.537% for returns and 1.61 for volume do not suggest the presence of sharp discontinuities in the series. The Ljung-Box Q statistics, constructed for lags of 6 and 12 days, test for serial correlation in the return and the log volume series. These statistics are not large enough to reject the null hypothesis of strict white noise for returns at 1 % while they are large enough for the log volume. The conclusion must be that the log volume series is serially correlated. Although not presented here, we conduct augmented Dicky-Fuller tests for the log volume series, which reject the unit root null hypothesis at 1% significance level.

Table 1.B presents correlation coefficients between return per se and trading volume and between absolute return and trading volume both on a contemporaneous basis and on a lead and lagged basis. On one hand, correlation cofficients between return per se and volume are low. There is no apparent relationship between return per se and trading volume. On the other hand, there exists a remarkable correlation between the absolute returns and the trading volume.

Some authors have noted intraday pattern in both the mean and variance of price movements. Wood, MacInish, and Ord (1985), Jain and Joh (1988), and Lockwood and Linn (1990) document a U-shaped intraday pattern in mean and standard deviations of the U.S. stocks, while Chang, Fukuda, Rhee, and Takano (1993) document a W-shaped pattern in means and standard deviations of price changes in Japanese stock market. In order to evaluate the intraday seasonality of the JGB Futures returns and volume, Figure 2 plots the average sample mean for each 5-minute interval. At the opening for the morning session (t=1), extremely large values are observed for the means and the standard deviations of returns and log volume. At the closing for the morning session (t=24), the standard deviation of returns and the mean of log volume have relatively large values. At the opening for the afternoon session (t=25), only the standard deviation of returns has a large value. At the closing for the afternoon session (t=54), the standard deviation of returns and the mean of log volume have large values. W-shaped patterns are observed for the standard deviation of returns and the mean of log volume.

Since the means and standard deviations vary across intraday time periods, we adjust data by subtracting the mean and dividing standard deviation.[4] The adjusted returns and log volume are plotted in Figure 3. Summary statistics and correlation coefficients for the adjusted returns and log volume are reported in Table 2. Adjusted returns and log volume have much smaller kurtosis than unadjusted returns and log volume. Correlation coefficients for adjusted data are also smaller than those for unadjusted data. These results show that the leptokurtosis and the comovement of returns and log volume are partly attributed to intraday seasonality. In contrast to the unadjusted returns, LB(6) and LB(12) are large enough to reject the null hypothesis that adjusted returns are serially uncorrelated at 1%. However, R_t in our model is assumed to be serially uncorrelated. Hence, we remove the serial correlation in the adjusted return series using a first order autoregression. (This lag length was selected by the Schwarz (1978) criterion.[5]) We use the residuals obtained from this regression for R_t. Because LV_t^* is allowed to be serially correlated, we use the adjusted log volume after subtracting the sample mean for LV_t^*.

5 Estimation

5.1 Stochastc Volatility Model

Before estimating our model, it may help to estimate a simple stochastic volatility (SV) model, which is given by equations (2) and (5). Following the method described in section 2, we can represent it in the following state space form.

$$y_t^* = h_t^* + v_t, \quad \epsilon_t \equiv exp(v_t/2) \sim NID(0,1), \tag{27}$$

[4]Andersen and Bollerslev (1994) propose a method of removing intraday seasonality based on the Fourier flexible form.

[5]The Akaike (1974) criterion led to long lags.

$$h_t^* = bh_{t-1}^* + w_t, \quad w_t \sim NID(0, \sigma_w^2), \tag{28}$$

where

$$\begin{aligned} y_t^* &\equiv ln(R_t^2) - E(ln(R_t^2)), \\ &= ln(R_t^2) + 1.27 - a/(1-b), && (29) \\ h_t^* &\equiv h_t - E(h_t), \\ &= h_t - a/(1-b), && (30) \\ v_t &= ln(\epsilon_t^2). && (31) \end{aligned}$$

Table 3.A reports the estimated coefficients and standard errors. The estimated value of b is 0.983, which indicates a high degree of persistence in volatility.

So far, we have assumed that ϵ_t is normally distributed. In the last section, normality of returns is rejected. In particular, they have a more leptokurtic distribution. Following Watanabe (1993 Chapter 4), Harvey, Ruiz, and Shephard (1994), and Ruiz (1994), we also estimate the SV model under the alternative assumption that ϵ_t follows a Student-t distribution, which is more leptokurtic than the normal. Although not presented here, we find that normality cannot be rejected at the 10% significance level and hence the leotokurtosis of 5-minute JGB Futures returns can fully be explained by changing volatility. Hence, all analyses below will be conducted under the normality assumption of ϵ_t.

Next, let us enter the log trading volume into the variance equation (30) in the following way.

$$h_t^* = bh_{t-1}^* + cLV_t^* + w_t, \quad w_t \sim N(0, \sigma_w^2). \tag{32}$$

The results are shown in Table 3.B. The estimated value of c is 0.027 with a standard error of 0.002. The LR statistic for the null hypothesis that $c = 0$ is 14.6. Hence, the trading volume is significantly positive. However, the estimated value of b of 0.974 still shows a high persistence of volatility. This shows that inclusion of trading volume does not eliminate serial dependence of return volatility at all. This result is consistent with Locke and Sayers (1993), who used intraday data, and inconsistent with Lamoureux and Lastrapes (1990), who used daily data.

5.2 Latent Common Factor Model

Now, we estimate our latent common factor model. The results are reported in Table 4. Let us first examine the persistence of the common factor and two idiosyncratic noises. On one hand, the estimated value of b_{11} is 0.294, which implies that the common factor is not so persistent. On the other hand, the estimated values of b_{22} and b_{33} are respectively 0.991 and 0.980, which indicates substantial persistence in two idiosyncratic noises.

Next, let us test whether two idiosyncratic noises are constant. Since under the null hypothesis, σ_{22} and σ_{33} are on the admissible parameter space, the distribution of the LR statistic is given by

$$LR \sim \frac{1}{2}\chi_0^2 + \frac{1}{2}\chi_1^2, \tag{33}$$

where χ_0^2 is a degenerate distribution with all its mass at the origin. The size of the LR test can therefore be set appropriately by using 2α, rather than α, significance point of a χ_1^2 distribution for a test of size α. (See Havey (1989), 5.1.2.) The LR statistic for the null hypothesis that $\sigma_{22} = 0$ is 175.1 and that for the null hypothesis that $\sigma_{33} = 0$ is 144.2. The critical value for $\alpha = 5\%$ is 2.71. Therefore, we reject the constancy of idiosyncratic noises at the 5% significance level.

The results that the common factor is not persistent and that there exist idiosyncratic noises which have high persistence provide evidence for misspecification of the mixture of distributions model. It is consistent with Locke and Sayers (1993) who used intraday data and is inconsistent with Lamoureux and Lastrapes (1990) who used daily data.

5.3 A Comparison with the VAR Model

For a comparison with our latent common factor model, we also analyze an alternative model proposed by Watanabe (1993 Chapter 4), in which the vector $\boldsymbol{x}_t$ that consists of the log volatility, h_t, and the log volume LV_t is specified as an AR(1) process, i.e.,

$$\boldsymbol{x}_t = \boldsymbol{A} + \boldsymbol{B}\boldsymbol{x}_{t-1} + \boldsymbol{w}_t, \quad \boldsymbol{w}_t \sim NID(\mathrm{o}, \boldsymbol{\Sigma}), \tag{34}$$

where

$$\boldsymbol{x}_t \equiv \begin{bmatrix} h_t \\ LV_t \end{bmatrix}, \quad \boldsymbol{A} \equiv \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \quad \boldsymbol{B} \equiv \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix},$$

$$\boldsymbol{w}_t \equiv \begin{bmatrix} w_{1t} \\ w_{2t} \end{bmatrix}, \quad \text{and} \quad \boldsymbol{\Sigma} \equiv \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}.$$

This model can also be estimated using the quasi-maximum likelihood procedure via the Kalman filter. (See Watanabe (1993 Chapter 4) for the detail.) The results are reported in Table 5. Although this model is not based on any theoretical arguments, it is useful to analyze dynamic relations as well as a simultaneous relation between volatility and volume.

In this model, the simultaneous relation between volatility and volume is obtained from the covariance between w_{1t} and w_{2t}, that is, $\sigma_{12}(=\sigma_{21})$. The estimated value of σ_{12} is 0.405. The LR test statistic for the null hypothesis that $\sigma_{12} = \sigma_{21} = 0$ is 220.2. This result provides strong evidence for a simultaneously positive relation between volatility and trading volume.

Next, let us examine the dynamic relation between volatility and volume. On one hand, the estimated value of b_{21} is 0.686. The LR statistic for the null hypothesis that $b_{21} = 0$ is 42.3. Therefore, the conclusion must be that there is a significantly positive relation between the previous volatility and present volume. On the other hand, the estimated value of b_{12} is -0.551. The LR statistic for the null hypothesis that $b_{12} = 0$ is 18.5. This result suggests that the effect of the previous volume on present volatility is also significant.

Finally, let us check which model fits data better between the latent common factor model and the VAR model. While the likelihood of the VAR model is -11369.5, that of the latent common factor model is -11210.0. Since these two models have the same number of unknown parameters, the conclusion must be that the common factor model fits data better than the VAR model.

5.4 Specification Tests

In our latent common factor model, we have assumed that the common factor and two idiosyncratic noises are mutually independent and have no lead and lagged relations. Now, let us test whether these assumptions hold true. To do so, we rewrite the latent common factor model as follows:

$$\begin{bmatrix} y_t^* \\ LV_t^* \end{bmatrix} = \begin{bmatrix} z_1 & 1 & 0 \\ z_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{1,t} \\ x_{2,t} \\ x_{3,t} \end{bmatrix} + \begin{bmatrix} v_t \\ 0 \end{bmatrix}, \tag{35}$$

$$\epsilon_t \equiv exp(v_t/2) \sim N(0,1)$$

$$\begin{bmatrix} x_{1,t} \\ x_{2,t} \\ x_{3,t} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} x_{1,t-1} \\ x_{2,t-1} \\ x_{3,t-1} \end{bmatrix} + \begin{bmatrix} w_{1,t} \\ w_{2,t} \\ w_{3,t} \end{bmatrix}, \tag{36}$$

$$\begin{bmatrix} w_{1t} \\ w_{2t} \\ w_{3t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \right)$$

where we assumed that the following parameters are equal to zero.

$$b_{12}, b_{13}, b_{21}, b_{23}, b_{31}, b_{32}, \sigma_{12}(=\sigma_{21}), \sigma_{13}(=\sigma_{31}), \sigma_{23}(=\sigma_{32})$$

To test whether each of these parameters is zero, we conduct Lagrange Multiplier (LM) tests. Table 6 reports LM test statistic for each parameter. The parameters that are statistically significant at 5 % significance level are b_{23} and $\sigma_{23}(=\sigma_{32})$. At 10 % significance level, b_{21} is also significant. σ_{23} represents a simultaneous relation between two idiosyncratic noises. b_{23} is a dynamic relation from the idiosyncratic noise of trading volume to that of volatility and b_{21} is a dynamic relation from the idiosyncratic noise of volatility to the common factor. The VAR model provides evidence for a bidirectional causality as well as a simultaneous relation between volatility and volume.

The results of LM tests show that none of these causal relations can fully be explained by the latent common factor model and that there is still some room for improving the simple common factor model presented in this paper by introducing these causal relations. Of course, theoretical arguments are also required to explain the dynamic relation between volatility and volume. The causality from volatility to volume can easily be explained by the presence of liquidity traders and feedback traders who rebalance their portfolio after large price changes. It is, however, difficult to expalin the causality from volume to volatility. If there certainly exists the causality from volume to volatility, intraday trading volume may provide useful information for financial risk management. Unfortunately, most of the previous work on the return-volume relation have focused on the contemporaneous relations and have not analyzed the lead and lagged relations extensively. (See Karpoff (1978).) Exceptions are Rogalski (1978), Jain and Joh (1988), and Gallant, Rossi, and Tauchen (1992). None of these authors have found a strong evidence of causality from volume to volatility.

6 Conclusions

This paper analyzed the relation between return volatility and trading volume using intraday data from the JGB Futures using the latent common factor model. Although we obtained some interesting results, further research is required to confirm our results. Our results are consistent with Locke and Sayers (1993) who used intraday data and is inconsistent with Lamoureux and Lastrapes (1990) who used daily data. It is, hence, an interesting problem whether our results are specific to intraday data. Although not presented here, we conducted the same analysis using 10-minute and 15-minute time intervals and we did not find any major changes in the above results. We also conduct the same analysis using the unadjusted data and did not find any major changes either. It is worthwhile to conduct the same analysis using data with other frequencies such as daily data.

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TABLE 1

Unadjusted 5-Minute Returns and Log Volume for the JGB Futures

A. Summary Statistics

Sample Period: March 3-May 31 1995
Number of Observations: 3234

Statistics	Returns (%)	Log Volume
Mean	0.003	6.52
St.Dev.	0.063	0.848
Skewness†	1.70	0.288
Kurtosis††	25.0	1.16
Maximum	0.867	9.67
Minimum	-0.537	1.61
LB(6)‡	15.35	2186.0
LB(12)‡‡	19.77	2416.4

B. Correlation Coefficients

(R, LV)	(R, LV_{-1})	(R_{-1}, LV)
0.097	0.080	0.048
$(\|R\|, LV)$	$(\|R\|, LV_{-1})$	$(\|R_{-1}\|, LV)$
0.493	0.321	0.279

†Skewness=0 for a normal (Gaussian) distribution. The standard errors of this statistic is $\sqrt{6/T} = 0.043$, where T =sample size.

††Kurtosis=0 for a normal (Gaussian) distribution. The standard errors of this statistic is $\sqrt{24/T} = 0.086$.

‡The critical values for LB(6) are: 10.64 (10%), 12.59 (5%), 16.81 (1%).

‡‡The critical values for LB(12) are: 18.55 (10%), 21.03 (5%), 26.22 (1%).

TABLE 2

Adjusted 5-Minute Returns and Log Volume for the JGB Futures

A. Summary Statistics

Statistics	Returns (%)	Log Volume
Mean	-0.002	-0.001
St.Dev.	0.989	0.991
Skewness†	-0.095	-0.203
Kurtosis††	2.53	0.38
Maximum	6.87	3.27
Minimum	-5.61	-5.63
LB(6)‡	22.7	1592.5
LB(12)‡‡	37.2	2156.4

B. Correlation Coefficients

(R, LV)	(R, LV_{-1})	(R_{-1}, LV)
0.017	0.002	0.003
$(\|R\|, LV)$	$(\|R\|, LV_{-1})$	$(\|R_{-1}\|, LV)$
0.392	0.170	0.254

†Skewness=0 for a normal (Gaussian) distribution. The standard errors of this statistic is $\sqrt{6/T} = 0.043$, where T =sample size.

††Kurtosis=0 for a normal (Gaussian) distribution. The standard errors of this statistic is $\sqrt{24/T} = 0.086$.

‡The critical values for LB(6) are: 10.64 (10%), 12.59 (5%), 16.81 (1%).

‡‡The critical values for LB(12) are: 18.55 (10%), 21.03 (5%), 26.22 (1%).

TABLE 3.A
Stochastic Volatility Model without Volume:

$$y_t^* = h_t^* + v_t, \quad \epsilon_t \equiv exp(v_t/2) \sim N(0,1)$$

$$h_t^* = bh_{t-1}^* + cLV_t + w_t, \quad w_t \sim N(0, \sigma_w^2)$$

Parameter	Coefficient	Standard Error
b	0.983	0.022
σ_w^2	0.022	0.00002
Log-likelihood	-7192.2	

TABLE 3.B
Stochastic Volatility Model with Volume:

$$y_t^* = h_t^* + v_t, \quad \epsilon_t \equiv exp(v_t/2) \sim N(0,1)$$

$$h_t^* = bh_{t-1}^* + cLV_t + w_t, \quad w_t \sim N(0, \sigma_w^2)$$

Parameter	Coefficient	Standard Error
b	0.974	0.002
c	0.027	0.002
σ_w^2	0.149	0.00001
Log-likelihood	-7184.9	
LR(1)* for H_0: $c = 0$	14.6	

*The critical values are: 2.71 (10%), 3.84 (5%), 6.64 (1%).

TABLE 4
Latent Common Factor Model:

$$\begin{bmatrix} y_t^* \\ LV_t^* \end{bmatrix} = \begin{bmatrix} z_1 & 1 & 0 \\ z_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{1,t} \\ x_{2,t} \\ x_{3,t} \end{bmatrix} + \begin{bmatrix} v_t \\ 0 \end{bmatrix},$$

$$\epsilon_t \equiv exp(v_t/2) \sim N(0,1)$$

$$\begin{bmatrix} x_{1,t} \\ x_{2,t} \\ x_{3,t} \end{bmatrix} = \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \begin{bmatrix} x_{1,t-1} \\ x_{2,t-1} \\ x_{3,t-1} \end{bmatrix} + \begin{bmatrix} w_{1,t} \\ w_{2,t} \\ w_{3,t} \end{bmatrix},$$

$$\begin{bmatrix} w_{1,t} \\ w_{2,t} \\ w_{3,t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix} \right)$$

Parameter	Coefficient	Standard Error
z_1	0.599	0.0005
z_2	0.843	0.0013
b_{11}	0.294	0.0076
b_{22}	0.991	0.0011
b_{33}	0.980	0.0011
σ_{22}	0.009	9.619e-06
σ_{33}	0.008	1.273e-05
Log-likelihood	-11210.0	
LR(1)* for H_0: $\sigma_{22} = 0$	175.1	
LR(1)* for H_0: $\sigma_{33} = 0$	144.2	

*The critical value is: 2.71 (5%).

TABLE 5

VAR Model:

$$\begin{bmatrix} y_t^* \\ LV_t^* \end{bmatrix} = \begin{bmatrix} h_t^* \\ LV_t^* \end{bmatrix} + \begin{bmatrix} v_t \\ 0 \end{bmatrix},$$

$$\epsilon_t \equiv exp(v_t/2) \sim N(0,1)$$

$$\begin{bmatrix} h_t^* \\ LV_t^* \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} h_{t-1}^* \\ LV_{t-1}^* \end{bmatrix} + \begin{bmatrix} w_{1t} \\ w_{2t} \end{bmatrix},$$

$$\begin{bmatrix} w_{1t} \\ w_{2t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} \right)$$

Parameter	Coefficient	Standard Error
b_{11}	1.33	0.008
b_{22}	-0.077	0.004
b_{12}	-0.551	0.002
b_{21}	0.686	0.006
σ_{11}	0.240	0.005
σ_{22}	0.722	0.017
$\sigma_{12}(=\sigma_{21})$	0.405	0.010
Log-likelihood	-11369.5	
LR(1)* for H_0: $b_{12}=0$	18.5	
LR(1)* for H_0: $b_{21}=0$	42.3	
LR(1)* for H_0: $\sigma_{12}=\sigma_{21}=0$	220.2	

*The critical values are: 2.71 (10%), 3.84 (5%), 6.64 (1%).

TABLE 6

Specification Tests for the Common Factor Model Based on LM Statistics

$$\begin{bmatrix} y_t^* \\ LV_t^* \end{bmatrix} = \begin{bmatrix} z_1 & 1 & 0 \\ z_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{1,t} \\ x_{2,t} \\ x_{3,t} \end{bmatrix} + \begin{bmatrix} v_t \\ 0 \end{bmatrix},$$

$$\epsilon_t \equiv exp(v_t/2) \sim N(0,1)$$

$$\begin{bmatrix} x_{1,t} \\ x_{2,t} \\ x_{3,t} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} x_{1,t-1} \\ x_{2,t-1} \\ x_{3,t-1} \end{bmatrix} + \begin{bmatrix} w_{1,t} \\ w_{2,t} \\ w_{3,t} \end{bmatrix},$$

$$\begin{bmatrix} w_{1,t} \\ w_{2,t} \\ w_{3,t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \right)$$

Null Hypothesis	LM Statistics
$b_{12} = 0$	0.272
$b_{13} = 0$	0.111
$b_{21} = 0$	2.81
$b_{23} = 0$	22.5
$b_{31} = 0$	0.021
$b_{32} = 0$	2.41
$\sigma_{12} = \sigma_{21} = 0$	0.458
$\sigma_{13} = \sigma_{31} = 0$	1.96
$\sigma_{23} = \sigma_{32} = 0$	14.0

FIGURE 1.A
JGB Futures Returns

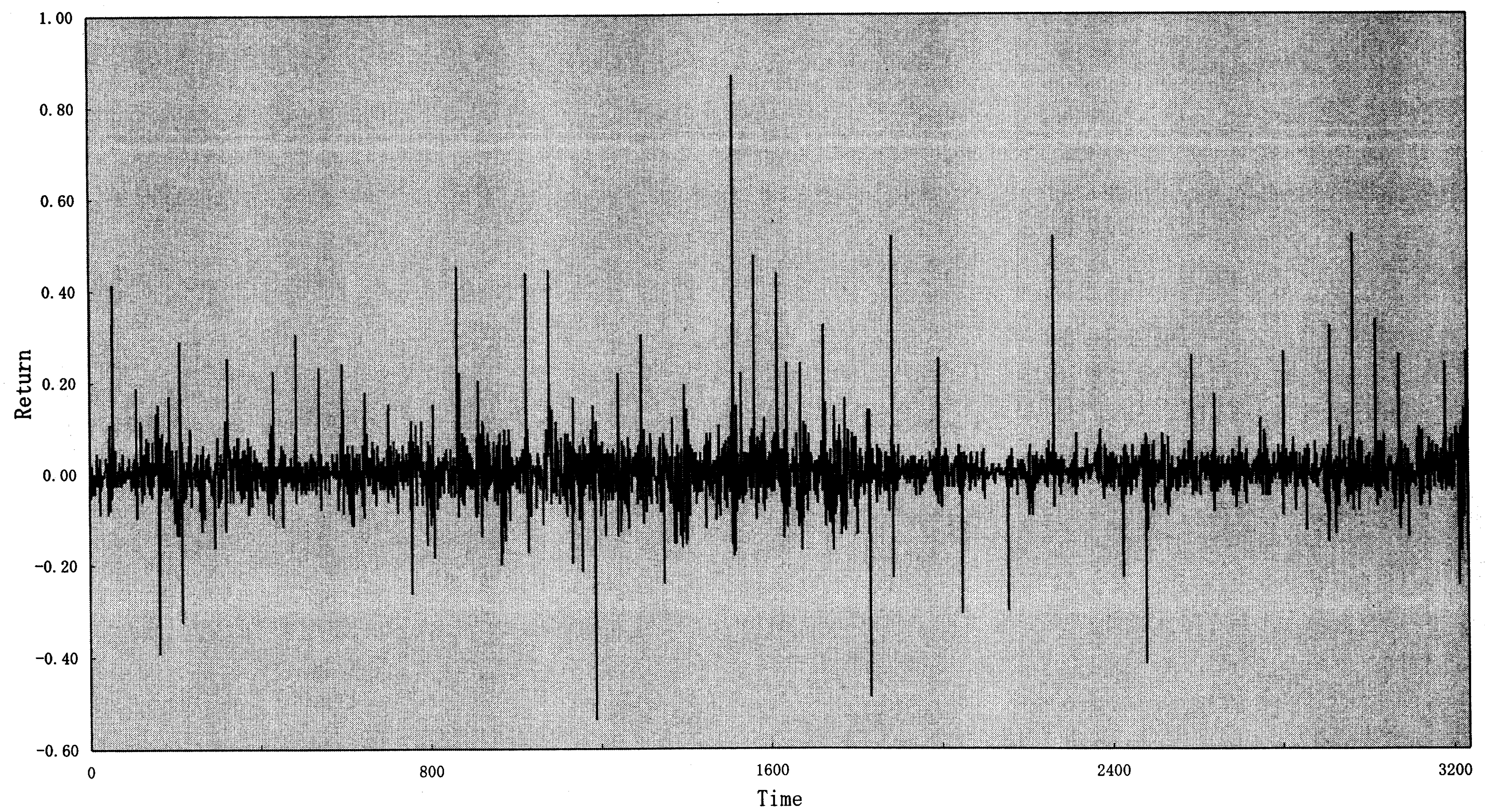

FIGURE 1.B
JGB Futures Log Trading Volume

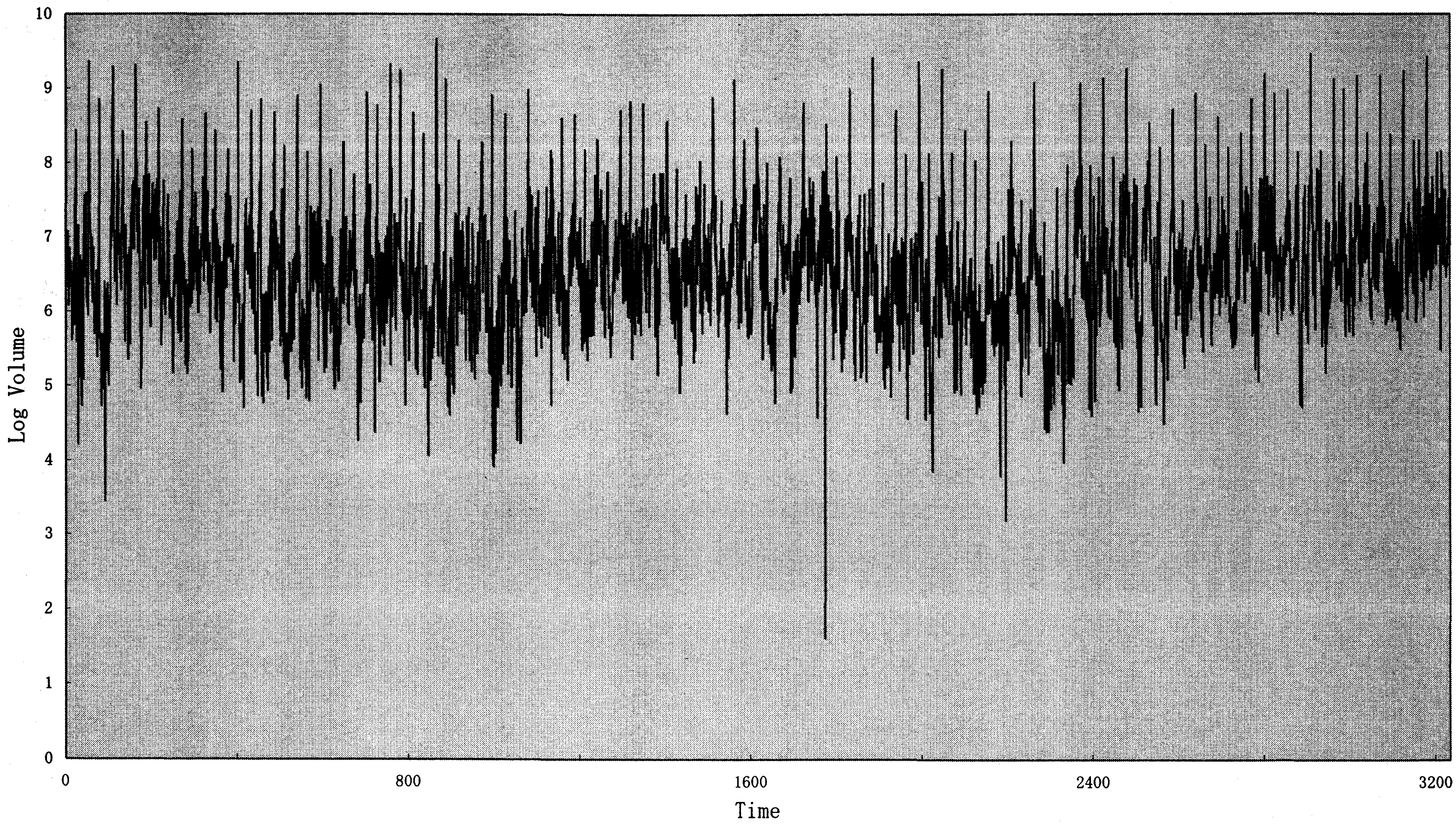

FIGURE 2.A
Mean of Intraday Returns

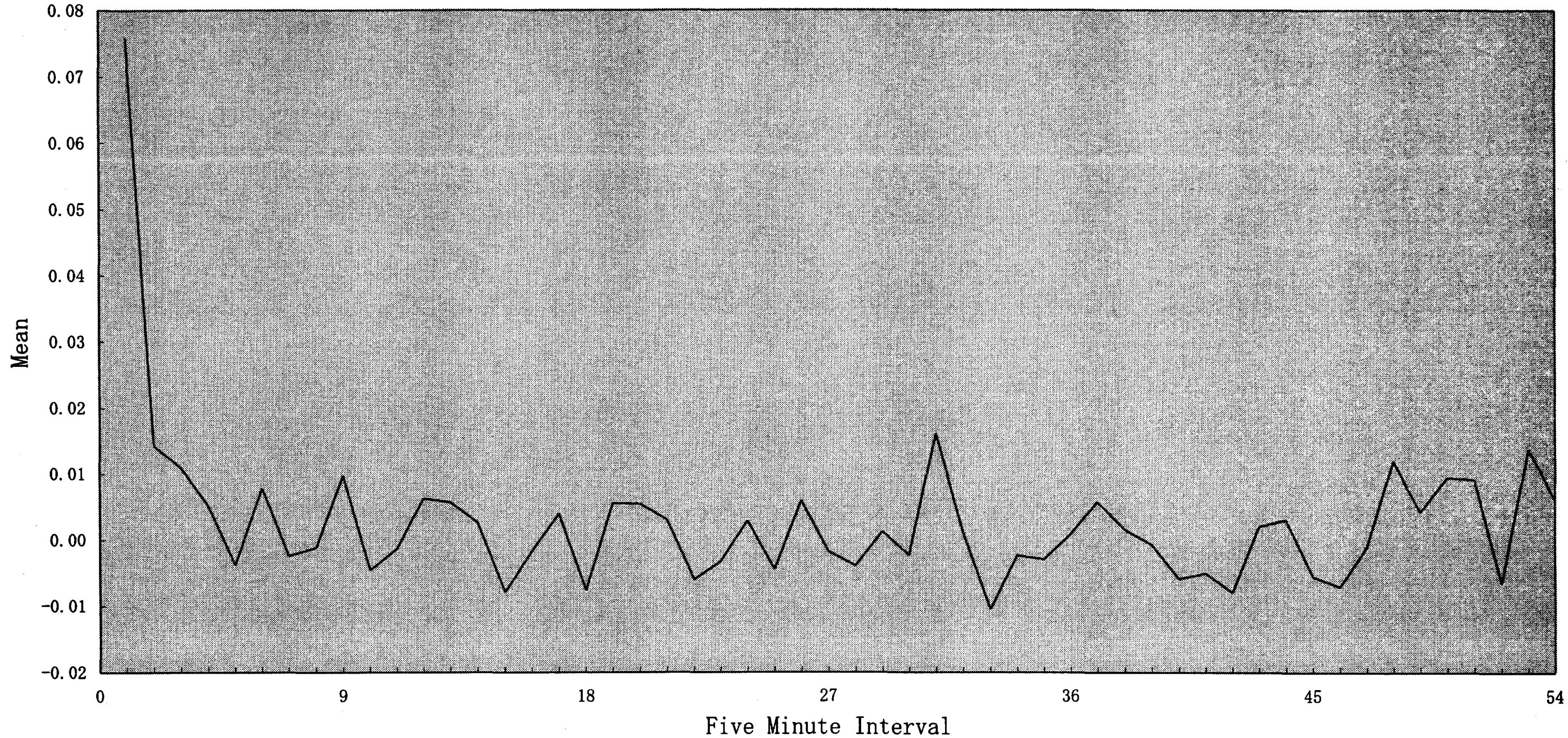

FIGURE 2.B
Std. Dev. of Intraday Returns

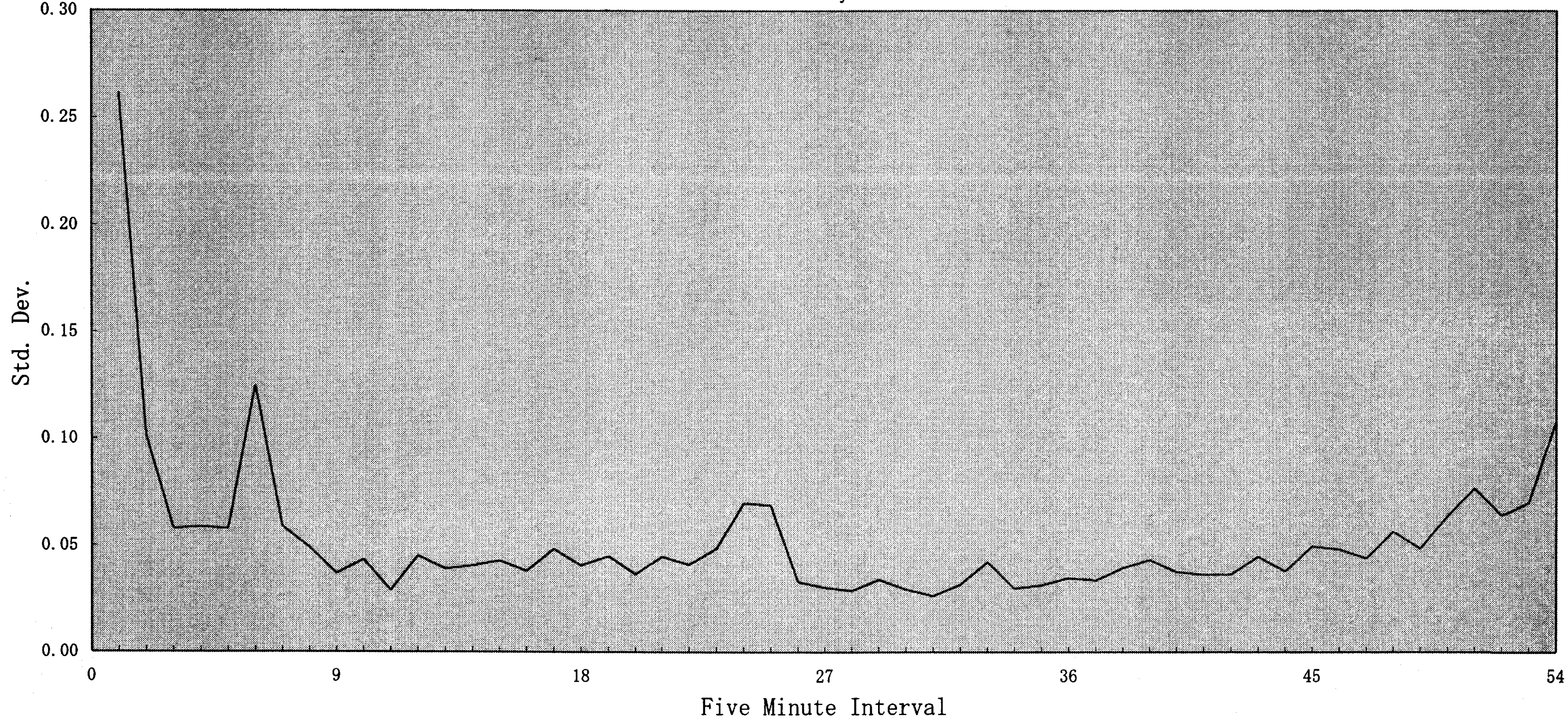

FIGURE 3.A
Mean of Intraday Log Trading Volume

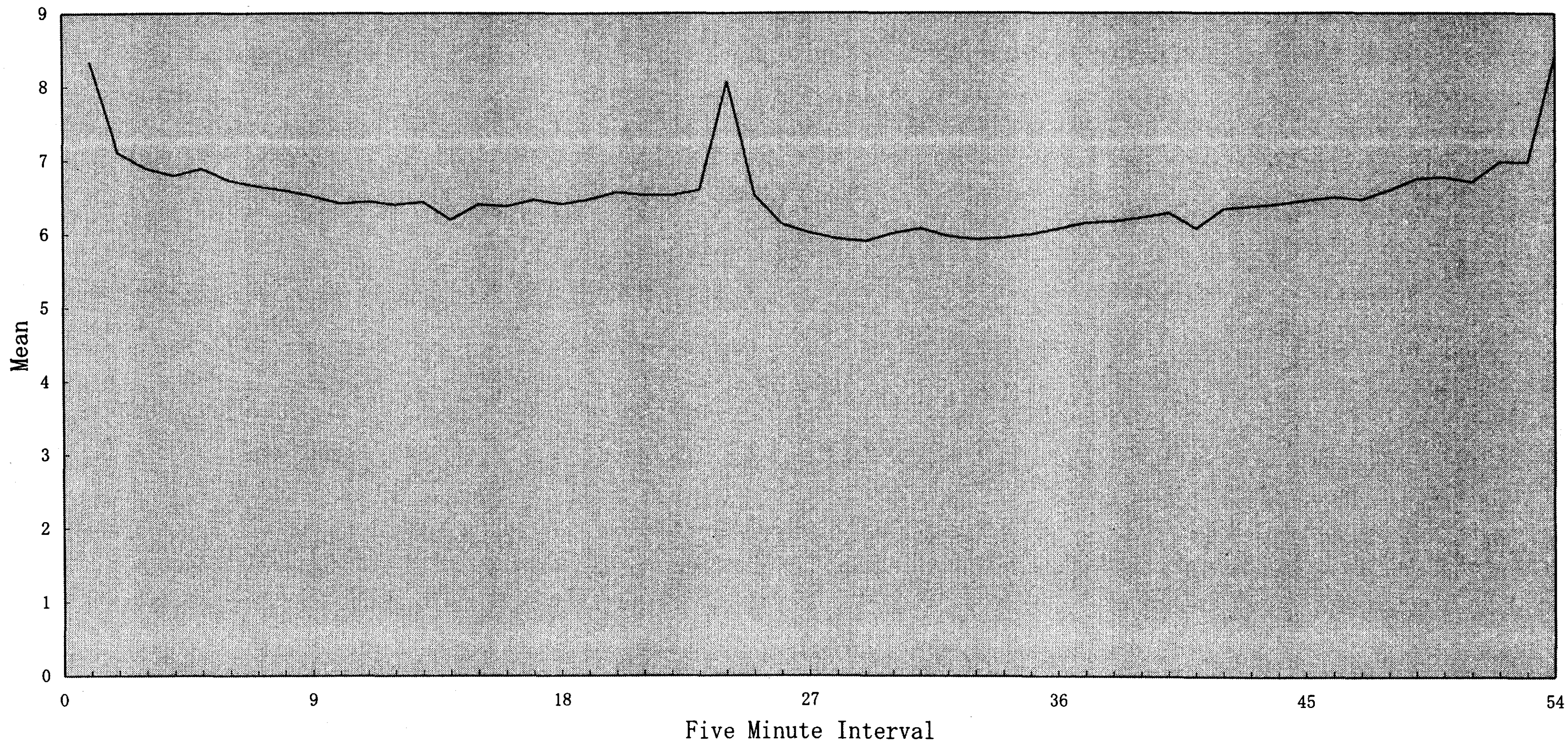

FIGURE 3.B
Std. Dev. of Intraday Log Trading Volume

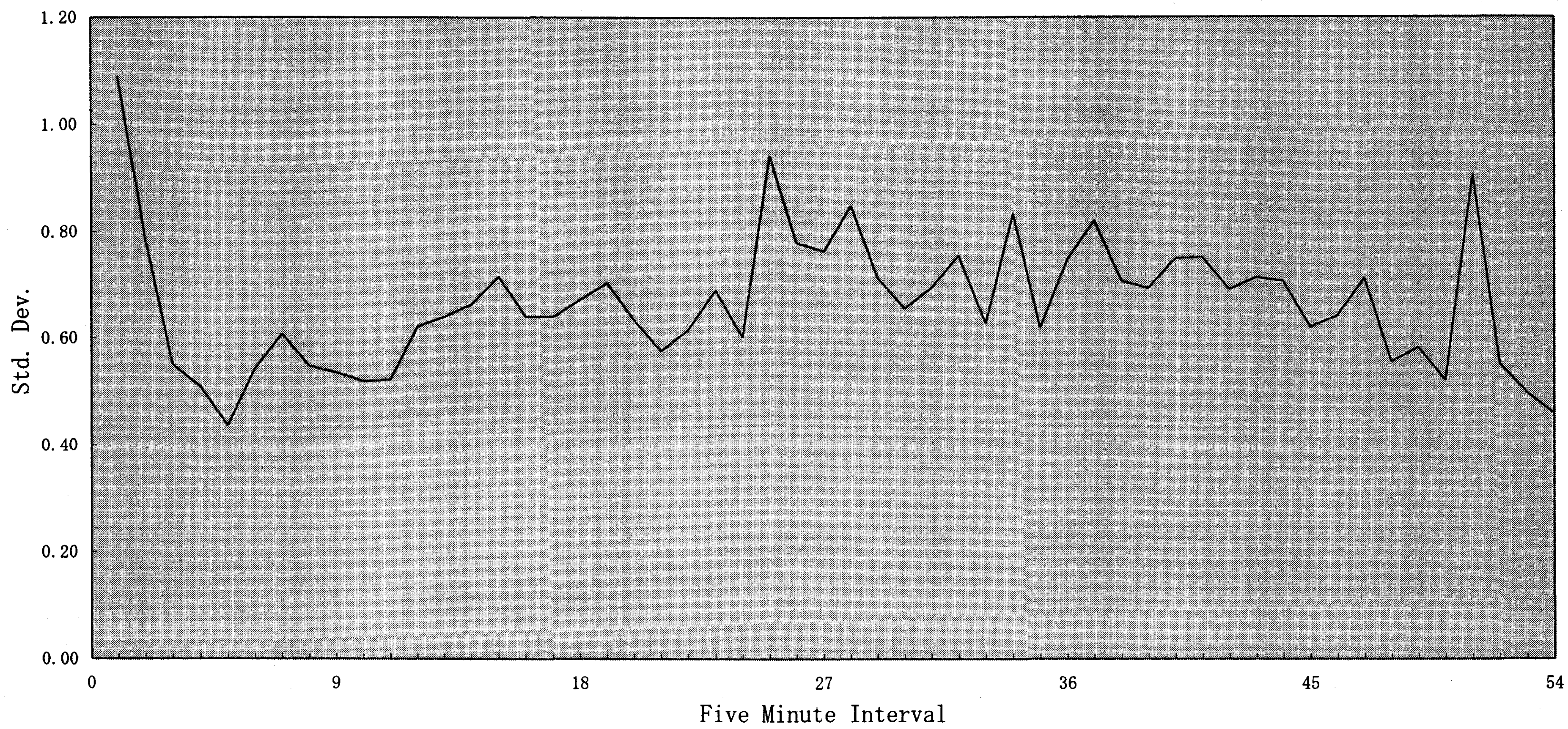

FIGURE 4.A
JGB Futures Standardized Returns

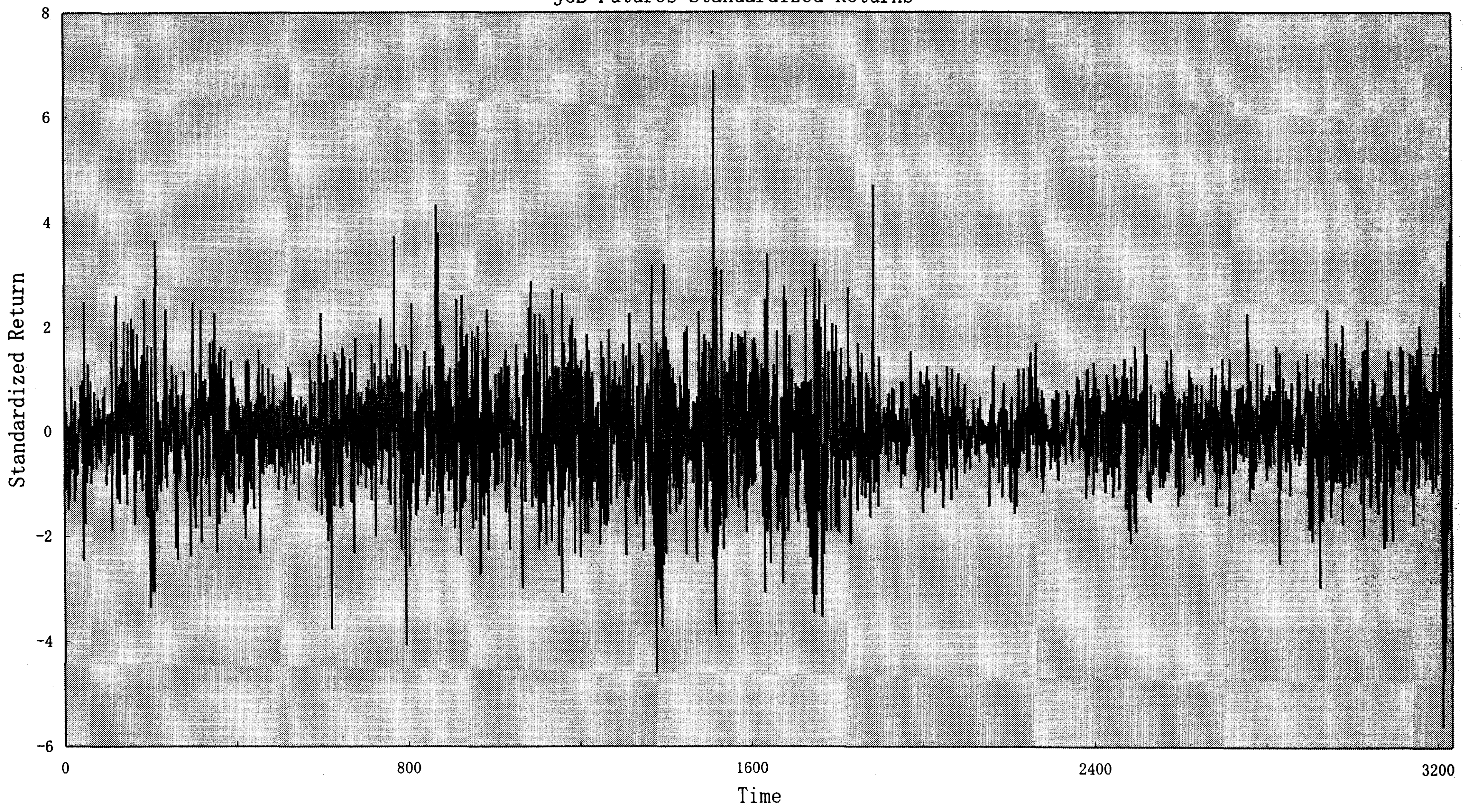

FIGURE 4.B
JGB Futures Standardized Log Trading Volume