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ABSTRACT

The purpose of this paper is to address some of the important issues in conducting stress testing in individual financial institutions. The first part of the paper proposes alternative calculation methods of "Value at Risk" to correct its shortcomings and shows how effective those methods are in measuring more accurately porofolio risks under turbulent market circumstances. The second part of the paper argues that the most important components of stress testing are (i) to detect and quantify "hidden hazards" which attract less attention in day-to-day risk management, (ii) to put in place an appropriate safeguards against them, and (iii) to prepare contingency plans for any stressful situation in order to minimize realized losses. We conclude that the approriate way of disclosing the result of stress testing can enhance the effort of individual market participants to conduct more meaningful stress testing.

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A Framework for More Effective Stress Testing

by

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I. Introduction

The technique of portfolio risk management in financial institutions has been evolving remarkably over the past few years both in terms of theory and practice. Among other things "value at risk" (hereafter "VAR") has taken root as a most prevailing concept for measuring market risk inherent in portfolios.[2] VAR is utilized by the senior management of leading institutions to identify capital at risk and eventually derive appropriate position limits allocated to each trading or business unit. In the proposed regulation by the Basel Committee on Banking Supervision[3], proprietary risk management models ("internal models") based on the VAR concept can be, under certain conditions, recognized as a legitimate measurement of market risk incurred by trading portfolios, which minimum capital charges are based on.

While VAR is an objective and statistically sophisticated method of capturing comprehensive market risk profile of portfolio, it is often pointed out that there are several technical shortcomings which cannot be overlooked when striving to measure risks more accurately. For instance, arbitrary assumptions such as lognormal distribution of changes in underlying risk factors and stable correlation between them are often regarded as too naïve, in particular when market prices exhibit very volatile movement. Furthermore, in the process of deriving VAR, some simplification is often made to avoid computational burden (e.g. ignoring nonlinear risk inherent in options) which, in turn, renders it difficult for risk managers or senior management to identify those risks should they turn out to be too significant to ignore.

Another argument about the limitations of VAR is that it is not a useful indicator of risks existing in extremely stressful market conditions such as Black Monday in 1987 and the ERM crisis in 1992. To address this issue, some leading institutions started to conduct so-called "stress testing" separately from VAR calculation, where potential losses incurred under hypothetical stress situations may be derived. However, the methodology of stress testing is

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2 Although this paper focuses mainly on market risk, it should be noted that VAR is such a flexible concept that any quantifiable risks, such as credit and liquidity risk, can be measured within the same framework.

3 Bank for International Settlement (1995).

by no means uniform, and the role of stress testing in resource and capital allocation within financial institutions seems still very limited compared with that of VAR.

In this paper we attempt to explore how to address the remaining issues related to market risk management that were described above. In Chapter II we propose alternative calculation methods of VAR to correct its shortcomings and to show how effective those methods are in measuring more accurately portfolio risks under turbulent market circumstances. In Chapter III we argue that the most important component of stress testing is to detect (and if possible, quantify) "hidden hazards" which attract less attention in day-to-day risk management, but can be abruptly revealed under extraordinary market conditions. In our view stress testing does not end with mathematical calculation of potential risks incurred during stressful events: the more important purpose of stress testing is to put in place an appropriate security system to protect against hidden hazards and to prepare contingency plans for any stressful situation in order to minimize realized losses. In the concluding chapter we will touch on the appropriate way of disclosing the result of stress testing so that the effort of individual market participants, who are conducting more meaningful stress testing, can be enhanced.

II. Alternative methods of VAR calculation to capture market risks more accurately

It is widely believed among leading financial institutions that VAR is not providing any meaningful information on potential losses arising from very large fluctuation of market prices observed during so-called “market stress”. Therefore, the stress testing that many institutions are currently conducting to grasp the magnitude of quantitative effects of market stress is conducted not within the framework of VAR but by assuming certain types of stress scenarios. Those scenarios include not only price scenarios such as sudden jumps in market prices but also political and natural disaster scenarios, usually designed to take into account past stressful events. Although it is widely recognized that senior management and risk managers can identify the many often overlooked pitfalls in portfolios in the process of designing scenarios and scrutinizing their possible consequence on the return/loss profile [the importance of this risk management process will be discussed in more detail in the next chapter], it is very difficult to translate calculated risk amounts into specific actions to minimize risks, partly because the plausibility of risk amounts derived from the exercise depends crucially on the likelihood of preset scenarios. This is partly the reason why the quantitative results of stress testing is not seriously taken into account in allocating capital and other resources to each business unit or in establishing position limits to it.

Therefore, our approach in this paper in obtaining quantitative measurement of risk under stressful circumstances is different from the approach market participants are taking and pursuing. We try to preserve the objective and stochastic nature of VAR as much as possible and attempt to see how ordinary calculation method of VAR can be modified to capture risk profile of portfolio in turbulent market conditions. Accordingly, we choose two recent turbulent quarters, the first quarter of 1994 and that of 1995, which are hereafter termed “Period I” and “Period II”, respectively. Backtesting was conducted over the period for a hypothetical portfolio by applying various modified concepts of VAR.

From Chart 1 it is intuitively clear that the two periods are manifested by very rough market developments. Charts 2-4 present some of the statistical indicators on the periods to demonstrate that the volatilities of risk factors are high and erratic and that correlations between them are unstable. Our hypothetical portfolio used for the backtesting is described in detail in Chart 5, which is well diversified on Nikkei 225, Japanese Government Bond futures, and US Government Note futures, including several option positions. The structure of the simplified portfolio can be regarded as reflecting some of the common features of a Japanese bank's portfolio.

Chart 6 shows the result of backtesting where VAR is calculated by variance/covariance method assuming lognormal distribution of price changes with a confidence interval of 99 %. Volatilities and correlations are derived from historical market data over the preceding one-year period and reestimated on a daily basis. The variance/covariance method is widely used among leading institutions, but in calculating VAR, nonlinear risk of options beyond delta is usually neglected for sheer convenience.[4] The results of backtesting demonstrate that the actual losses incurred by the portfolio exceed VAR 4 times during Period I and 5 times during Period II out of a total of 61 observation points, which corresponds to the confidence interval of 93.4 % and 91.8 %, respectively. This contrasts with the predetermined confidence interval of 99 %. In order to illustrate the effects of not incorporating nonlinear risk, we next conducted a Monte Carlo simulation to arrive at VAR, where all the option positions were revalued by the Black-Scholes formula. In this case (Chart 7), the occasions where the actual losses surpassed VAR were reduced from 4 times to 2 and from 5 times to 4 during Period I and II, respectively, out of a total of 61 observation points, which is still excessive given the predetermined confidence interval of 99 %.

It is often pointed out that VAR calculation relies on relatively strong assumptions, such as lognormal distribution of price changes and stable correlation between risk factors, and it is well-known that such assumptions are not satisfied very often. Firstly, to take account of kurtosis of price distribution we conducted two alternative simulations: (i) a Monte Carlo simulation assuming multivariate t-distribution of price changes and (ii) a nonparametric historical simulation (Detailed procedures of the simulation assuming multivariate t-distribution are explained in the Appendix). As a heavy-tail distribution, we arbitrarily chose a t-distribution in this paper for analytical convenience, whose degree of freedom is set so that it describes actual market data most properly. However, we could have used a more general distribution which would have been not only leptokurtic but also

[4] A more comprehensive discussion on the characteristics and shortcomings of variance/covariance method may be found in Mori, Ohsawa and Shimizu (1995).

skewed, though the parameter estimation of density function would have been computationally much more difficult.[5] The empirical distribution of changes in risk factors obtained by bootstrap is shown in Charts 8 and 9, and, as expected, exhibits a deviation from the normal distribution. We also tried to preserve correlations between financial variables in the simulations, which were modified later on to observe the effects of correlation breakdown.

The results of both of the simulations are shown in Charts 10 and 11. In the case of the simulation assuming multivariate t-distribution, the occasions where the actual losses surpassed VAR were reduced further from twice to once during Period I and from 4 times to twice during Period II, which endorses the effectiveness of this method. By conducting the nonparametric historical simulation we obtained the same result in terms of the coverage of actual losses, while the level of VAR is on average somewhat lower than that obtained by the simulation assuming multivariate t-distribution.

Secondly, we sought to modify assumed stability of correlations in obtaining VAR. Admitting that a great deal of efforts including some of the papers presented for this research conference are being made to develop statistical models that will capture the dynamics of variance/covariance matrix of risk factors, we assumed here, again for sheer simplicity, a worst case scenario where correlations are either +1 or -1. We compared three simulations that assumed (i) the lognormal distribution, (ii) a t-distribution, and (iii) an empirical distribution obtained nonparametrically, the results of which are presented in Charts 12-14. While VAR obtained from the simulation assuming the lognormal distribution does not cover one and two actual losses during Period I and II --the same result as the simulation assuming a multivariate t-distribution preserving correlations--, respectively, the simulation assuming a t-distribution and the nonparametric simulation successfully improved the performance of

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[5] Kariya, et al. (1995) showed that the distribution of market returns of the Japanese markets is not only leptokurtic but also skewed. Nagahara (1995) adopts the type VII and IV family of the Pearson System to express daily returns of stock prices, which forms a broad class of distributions including t-distribution and Cauchy distribution. Coppes (1995) found by adopting Paretian distribution to various exchange rates that though daily changes clearly have heavy-tail features, the distributions of monthly changes may be derived from the normal distribution.

VAR model by further reducing the number of such occasions from two to one for Period II. In the latter case the occasion (41st day of Period II) where Nikkei 225 declined by 3.9 percent per day was also tracked by the model. Therefore, our tentative conclusion is that the simulation taking account of kurtosis of distributions of price changes as well as the instability of correlations could improve the performance of VAR model in terms of the coverage of actual losses. At the same time the limitation of VAR model became clear by recognizing that even after some modification of VAR models it is not able to cover the extreme market volatility as observed on the 21st day of Period I and the 17th day of Period II, when Nikkei 225 moved over one-day period by 7.6 percent and 5.8 percent, respectively.

III. Toward more meaningful stress testing -- Detection of hidden hazards, security system, and contingency plans --

In the previous chapter we tried to modify VAR to capture the risks in turbulent market situations. Although the modification of VAR models appears to correct some of the deficiencies, it goes without saying that extreme tail events, such as the stock market crash in 1987 and the collapse of ERM cannot be described in the VAR framework, as also indicated in the simplified simulations in Chapter II. Therefore, leading financial institutions that considerably expose themselves to market risks are conducting scenario-based stress testing, which, as discussed in Chapter II, has a different setting than VAR.

In our view, the most important purpose of stress testing regarding those extreme events may be summarized as follows: (i) to identify and, if possible quantify, **hidden hazards** which tend to be overlooked in the day-to-day operation but which may turn out to be crucial in stressful situations, (ii) to put in place a **security system** to protect against the hidden hazards, and (iii) to prepare **contingency plans** necessary for traders and risk managers to be able to confidently respond to abrupt stressful events. In other words, the magnitude of losses incurred under market stress crucially depends on *ex ante* preparation ((i) and (ii) above) and *ex post* corrective actions ((iii)). Therefore, the *ex ante* calculation of risk of losses which neither takes into consideration hidden hazards nor assumes corrective actions carries little information by itself.

In the following sections we try to raise some concrete issues which may be very important in conducting exercises from (i) to (iii). The list of issues and the solutions to them presented below, however, are not exhaustive, and individual institutions need to identify their own issues and find solutions to them.

1. Detection and quantification of hidden hazards

Needless to say, the "hidden hazards" that are important to be detected in stress testing vary from one institution to the other. However, there are a few commonly important risks that come to mind easily. First, it is generally perceived that **liquidity risk** is experienced most painfully under market stress. As a matter of fact, liquidity risk comes first to the fore not only under market stress but also in any kind of crisis situation --for instance, a big earthquake in Tokyo would not destroy the Japan's financial system as a whole, but the temporary standstill of financial activity could lead in the first place to difficulties with timely settlement anywhere in the world.

Market stress tends to dry up liquidity in the derivatives markets. This renders it difficult to rebalance positions, which in turn gives rise to greater market risks. There are also cases where a large number of positions (such as options) which suddenly change from positive-valued transactions to negative-valued is simulatneously settled. In this case liquidity risk could translate into funding risk.

Although many market participants admit that liquidity (or funding) risk is the most perilous, at least in the short run, there seems to have been no tangible success in accurately measuring liquidity risk. For instance, our knowledge is still very limited as to how liquidity risk is actually expressed. Liquidity shortage tends to increase the risk of losses both by removing the opportunity to hedge and by increasing transaction costs when hedging is still feasible in some way or other. Moreover, studies on the source of liquidity shortage have yet to appear. Large price shocks per se could clearly trigger liquidity drain. However, other factors such as uncertainty over the credit standing of counterparties as well as certain pricing formulae of complex derivatives transactions (e.g. mortgage-backed securities whose prices are dependent on prepayment behavior of home owners which is not necessarily rational and hence very difficult to predict) could be also very important, which makes the task of analyzing the source of liquidity shortage very complicated. Once these issues are solved, one will be in a position to quantify liquidity risk and integrate it into the market/credit risk management system (Although the adjustment in the holding period of VAR is one possible way of taking account of liquidity, it appears to be only a very small first step forward).

The second notable hazard sometimes ignored in the ordinary market situation is the **non-linear risk of options**. Although front offices and individual traders usually reprice the entire range of positions every second so that higher order of price risks in options are automatically taken into consideration, the middle offices or risk management sections occasionally ignore them deliberately in order to reduce the calculation burden of portfolio risks. However, there are occasions in turbulent situations where higher order of risks such as the third derivatives of the asset value with respect to the price of the underlying assets (sometimes called "speed") does matter, which makes even relatively prudent risk management watching beyond delta (such as negative gamma) pointless (An example is given in Chart 15).[6]

The third aspect which is often neglected (mainly due to technical difficulty) is the **sudden change in the creditworthiness of counterparties**. While there is no doubt that the quantitative analysis of credit risk of counterparties is becoming increasingly sophisticated, discrete changes or sudden deterioration in credit risk in the face of market stress seems to have been outside the scope of analyses. However, given that credit standing is by no means stable for many participants [such as those in emerging markets where derivatives transactions become increasingly popular], the risk needs to be taken into consideration in some way.

2. Security system for protecting against hazards

After identifying the hidden hazards discussed above, prudent risk management needs to decide what kind of security system should be put in place against individual hazards. Since the hazards are very damaging once unveiled, it is crucial to implement the security system in advance and to contain the risk to a level which can be regarded as manageable by senior management.

To protect against liquidity risk, it is imperative to constantly monitor the total level of and locational distribution of liquidity readily available within the company. Careful monitoring and execution might be warranted to prevent the settlement dates and trigger/strike prices of option transactions from concentrating within a narrow range. Reserving alternative ways of hedging (for instance, hedging through different markets from

[6] Estrella (1995) presents a refined theoretical argument about the possibility that the Taylor approximation of the Black-Scholes formula could err by unacceptable degrees. Although his theoretical argument seems accurate, the risk of Taylor approximation might be somewhat exaggerated, since his numerical example about convergence condition assumes no change in portfolio over a long period. Although convergence condition is satisfied only in the range of two standard deviations under an assumption of one-year holding period and 35 % annualized volatility, the range widened considerably to 31.3 standard deviations if the holding period is assumed to be one day. This indicates that the use of the Taylor approximation for the *daily* risk management is not so devastating as he suggests.

under the normal circumstances) might be also worthwhile.

In order to prepare for non-linear risks of options it is necessary to establish a good reporting system where every single option-related position is uncovered not necessarily on a daily basis but certainly quite frequently and regularly. There are cases where some nonlinear risk is completely ignored because it is hidden in complex financial transactions. Far-out-of-the-money positions also tend to be discounted in daily risk management.

Another important issue is how to establish an early warning system within firms, which could signal senior management that a stressful situation was approaching. This is not to say that market participants could easily predict market stress well in advance, but rather that it is important to monitor regularly and on a continuous basis any kind of factors likely to indicate stressful consequences such as changes in market liquidity and credit risk of counterparties.

3. Contingency plans

Although an *ex ante* security system for protecting against hidden hazards is very important, it would not be sufficient to appropriately respond to a genuine crisis situation, mainly since market stress is characterized as a multifold intertwining or chain transmission of various shocks that is usually very difficult to predict beforehand. Thus it is very important to design a contingency plan that would enable traders and risk managers to cope with the crisis in a coordinated fashion.

There seem to be several issues which should be addressed in preparing a contingency plan for stressful situations. One formidable issue is how loss limits can be appropriately implemented. Loss limits are believed to help contain possible trading losses to a certain level. This is indeed an effective and useful strategy, because individual dealers tend to behave as if (i) their utility function was dependent upon gains and losses as opposed to final wealth in a neoclassical setting and (ii) the utility function was concave over gain and convex over losses, which are intertemporarily not rational, as the prospect theory of Kahneman and Tversky shows. If so, dealers faced with increasing losses of their positions tend to gamble further without trying to close the position, which usually end up with further losses.[7]

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[7] Needless to say, the setting of loss limits is only half of the story, and it counts how to

But it seems that this is not the end of the story. In market stress, loss limits triggered across the market could on the contrary accentuate stress by reducing market liquidity. This resembles the function of portfolio insurance observed during the stock market crash in 1987, which can be interpreted as a typical example of coordination failure because of the lack of market-wide information on the side of individual market participants. Hence, automatic execution of loss limits in market stress might not be the best possible action. Although it is almost impossible for individual players to predict the general market reaction under stressful circumstances with any degree of accuracy, some coordinated efforts among them to establish a better information system might be worth considering as a means toward this end.

It is also crucial to consider ways of designing decision-making processes within a firm under market stress. Under stressful events traders tend to become myopic and incapable of making reasonable decisions. At the same time, senior management may also not be in a position to deliver appropriate and timely instructions to individual units. Therefore, a sort of "special risk management squad" delegating senior management in an emergency situation may be better suited to coordinating actions across units and make speedy executions.

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implement them. Especially in stressful situations actual losses of dealers' positions tend to easily exceed preassigned loss limits in a very short period of time. Thus frequent monitoring of the difference between accumulated losses and loss limits is imperative to make loss limits effective in practice.

IV. Conclusion -- Incentive mechanism for voluntary stress testing by individual financial institutions

We have thus far discussed how we can make stress testing more effective and meaningful. Although the alternative framework we described in this paper is obviously just one possible option, we hope that it could serve as a starting point for further discussion.

Even if we all might agree on the importance of academically tackling the issues related to stress testing laid out in the paper, we should not be too complacent with the willingness of market participants to conduct stress testing and actually act based on the results. Market stress might seem to be regarded as a rare event --so rare a case that it may not happen while a person in charge holds his/her position, which gives less incentive to the management to prepare for it at significant expense. Under market stress one might feel that what each participant can do is very limited, since a chain reaction to large shocks is unavoidable and uncontrollable. Thus, there is always a perception that risk management under stressful conditions is in short supply.

How can we induce market participants to conduct meaningful stress testing? One of the effective ways of doing this is, of course, through market discipline, namely the disclosure of the result of stress testing. It seems, however, that the disclosure of possible risk of losses in a stressful situation (which is certainly much larger than VAR) is likely not to have the effects of obtaining market credibility but of frightening investors. Therefore, there is no incentive to disclose it.

In order to promote incentive to disclose, some positive aspect needs to be promoted. For instance, it might not be a bad idea that after showing risk of losses one could explain what kind of security system and contingency plans discussed above have been put in place. In this context, back testing can provide strong evidence of the functioning of security systems and contingency plans. For instance, if realized losses under stressful conditions are less than *ex ante* risk of losses, one can demonstrate the concrete actions taken that contributed to this. In summary, while the disclosure of risk of losses in stressful situations is not meaningful, the disclosure of the internal procedure or system for stress testing would give investors high quality information about the level of risk management craftsmanship within individual companies and thereby help reduce both the uncertainty and risk premiums being charged to them.

Appendix: VAR simulation assuming multivariate t-distributions

To choose the appropriate degree of freedom of a t-distribution we compared the market data with random numbers generated according to specific degrees of freedom in terms of two different criteria: (i) kurtosis and (ii) confidence intervals measured by standard deviations. The observation period of market data is 1Q/1994-1Q/1995. The high kurtosis of market data shown in Chart 16 suggests heavy-tail distribution of all the risk factors during this period, although the levels of kurtosis vary widely. From the confidence intervals shown in Chart 17, we concluded that actual market data is represented by the degree of freedom, either 6 or 7. For sheer simplicity we chose the degree of 7 for all the risk factors to run the simulations.

Let $\begin{pmatrix} X_{11} \\ \vdots \\ X_{41} \end{pmatrix} \ldots\ldots \begin{pmatrix} X_{17} \\ \vdots \\ X_{47} \end{pmatrix}$ be seven independent vectors drawn from $N\left(0, \begin{pmatrix} 1 & & \mathbf{O} \\ & \ddots & \\ \mathbf{O} & & 1 \end{pmatrix}\right)$

and $z_1, \ldots, z_4$ be four independent numbers drawn from $N(0,1)$, where x_{ij} is i-th risk factor and $x_{1j}, \ldots, x_{4j}$ are mutually independent. If we define

$$Q_i = \left(z_i \,/ \left[\left(\sum_{j=1}^{7} X_{ij}^2 \right) / \, 7 \right]^{\frac{1}{2}} \right),$$

then Q_i follows t-distribution with the seven degree of freedom and $Q_1, \ldots, Q_4$ are mutually independent. Next let C be a variance/covariance matrix estimated by using historical market data, and $C^{\frac{1}{2}}$ be Cholesky decomposition of C. If we define

$$Y = \begin{pmatrix} Y_1 \\ \vdots \\ Y_4 \end{pmatrix} = C^{\frac{1}{2}} \begin{pmatrix} Q_1 \\ \vdots \\ Q_4 \end{pmatrix},$$

then Y follows multivariate student t-distribution with seven degrees of freedom.

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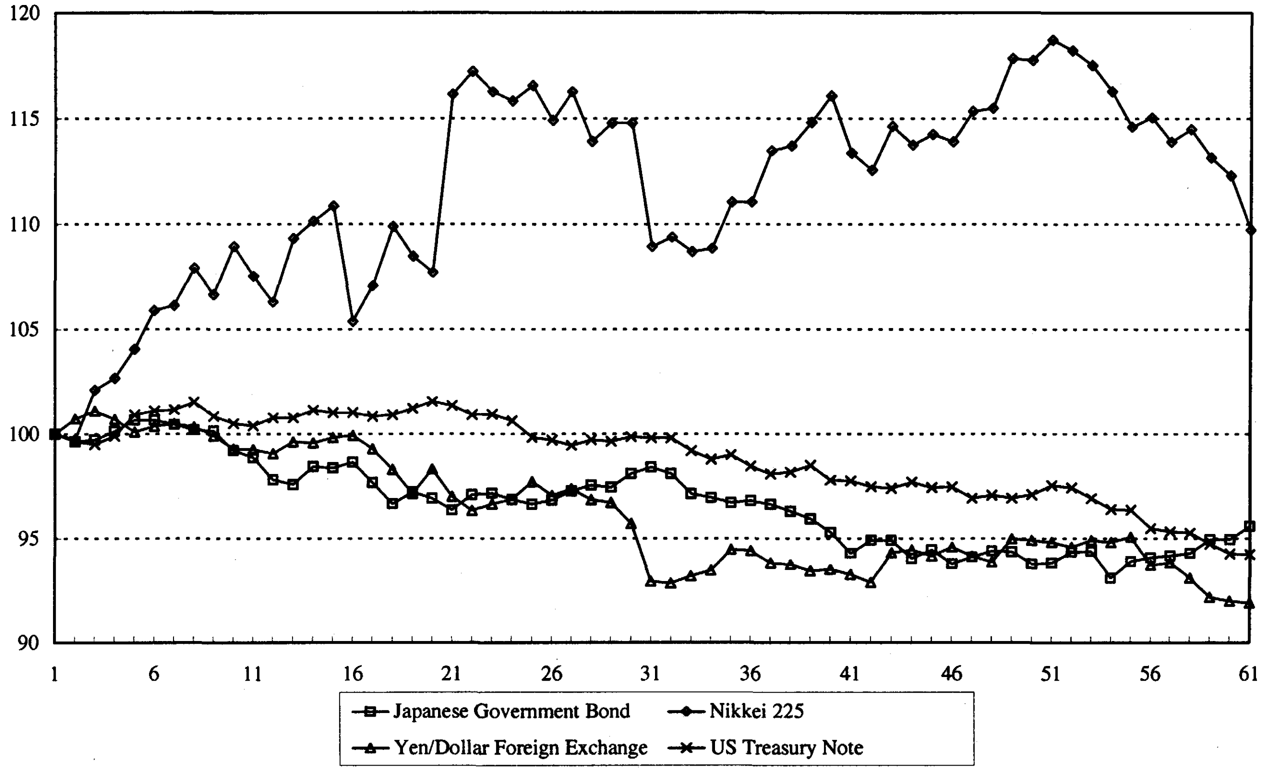
Chart 1. Developments in risk factors

All numbers are indexed as the first day's rates set at 100.

Period I (January 1994 - March 1994)



Period II (January 1995 - March 1995)

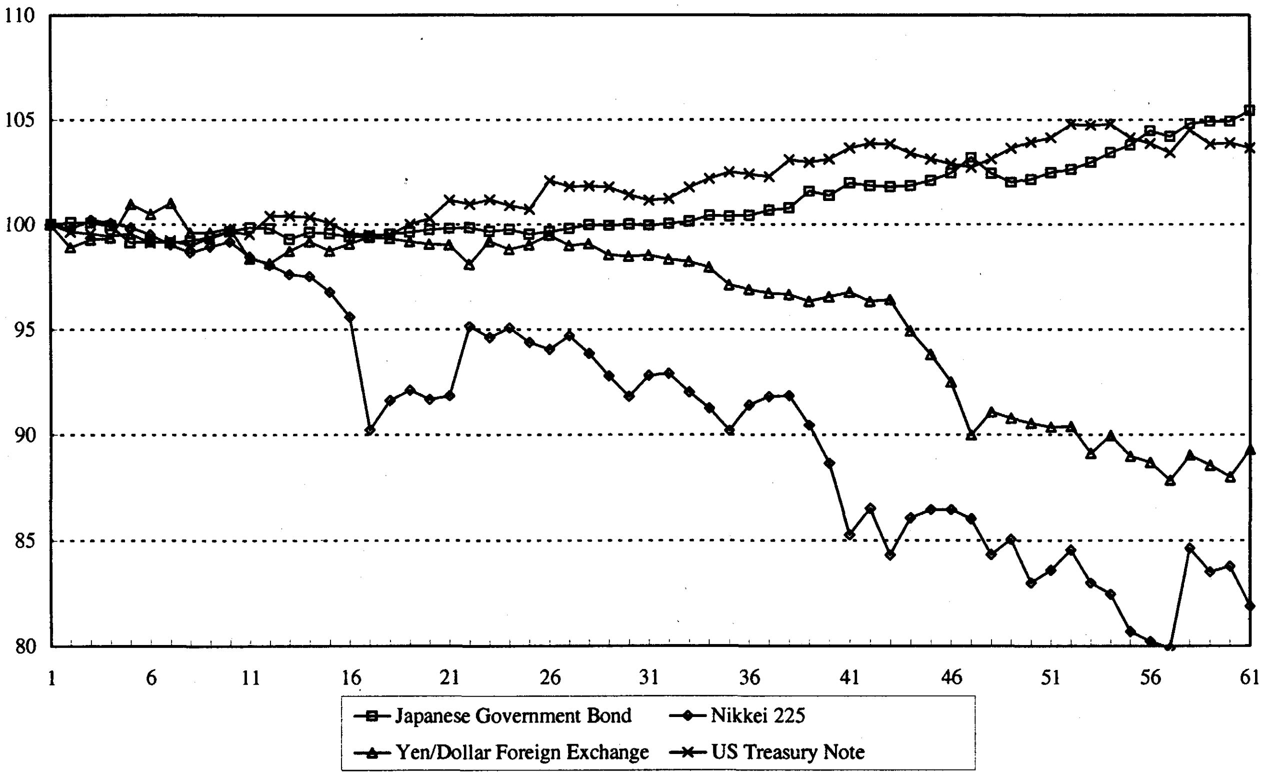
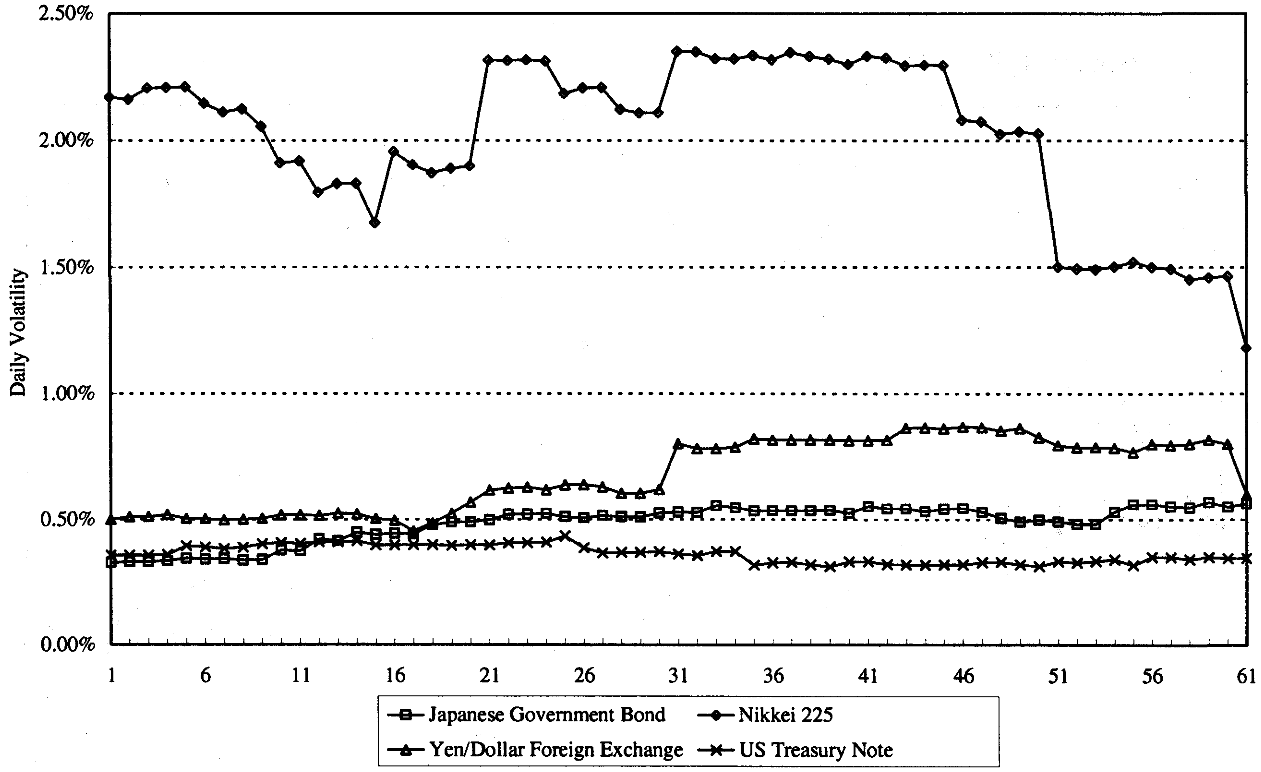


Chart 2. Daily Volatility (observation period: 30 preceding days)

Period I (January 1994 - March 1994)



Period II (January 1995 - March 1995)

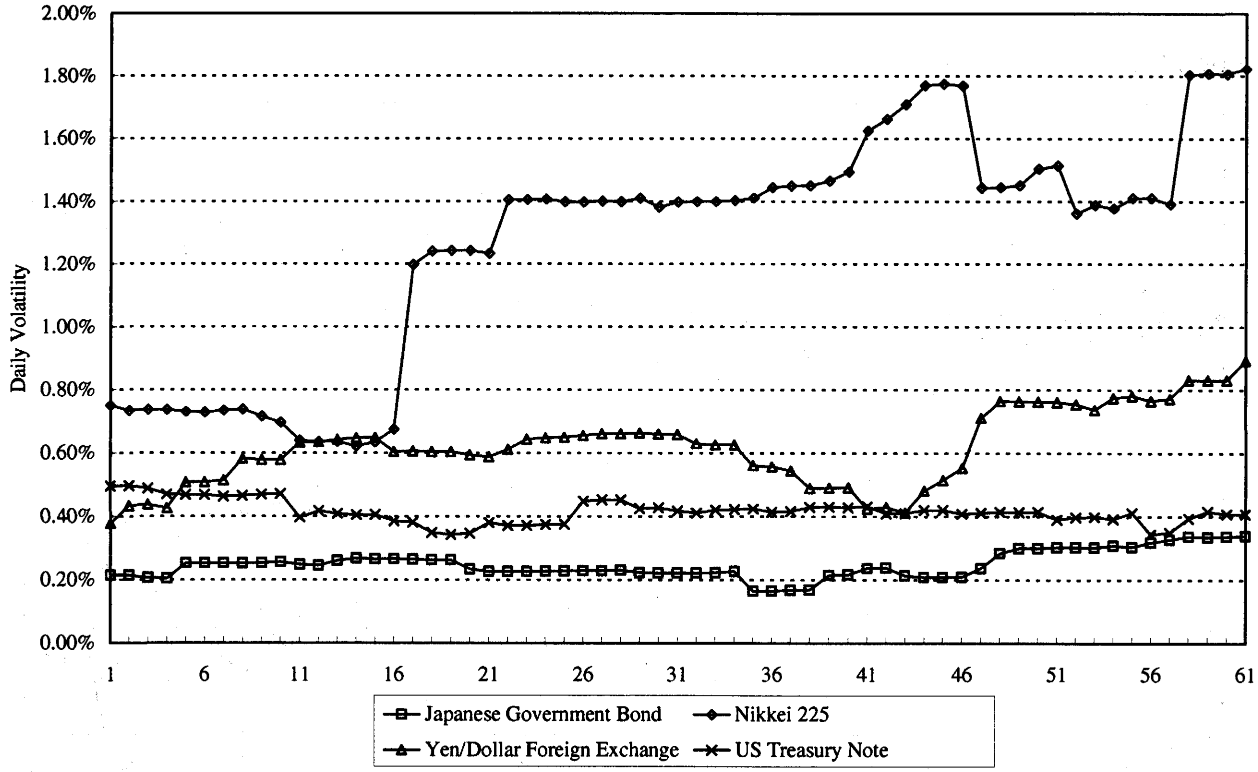
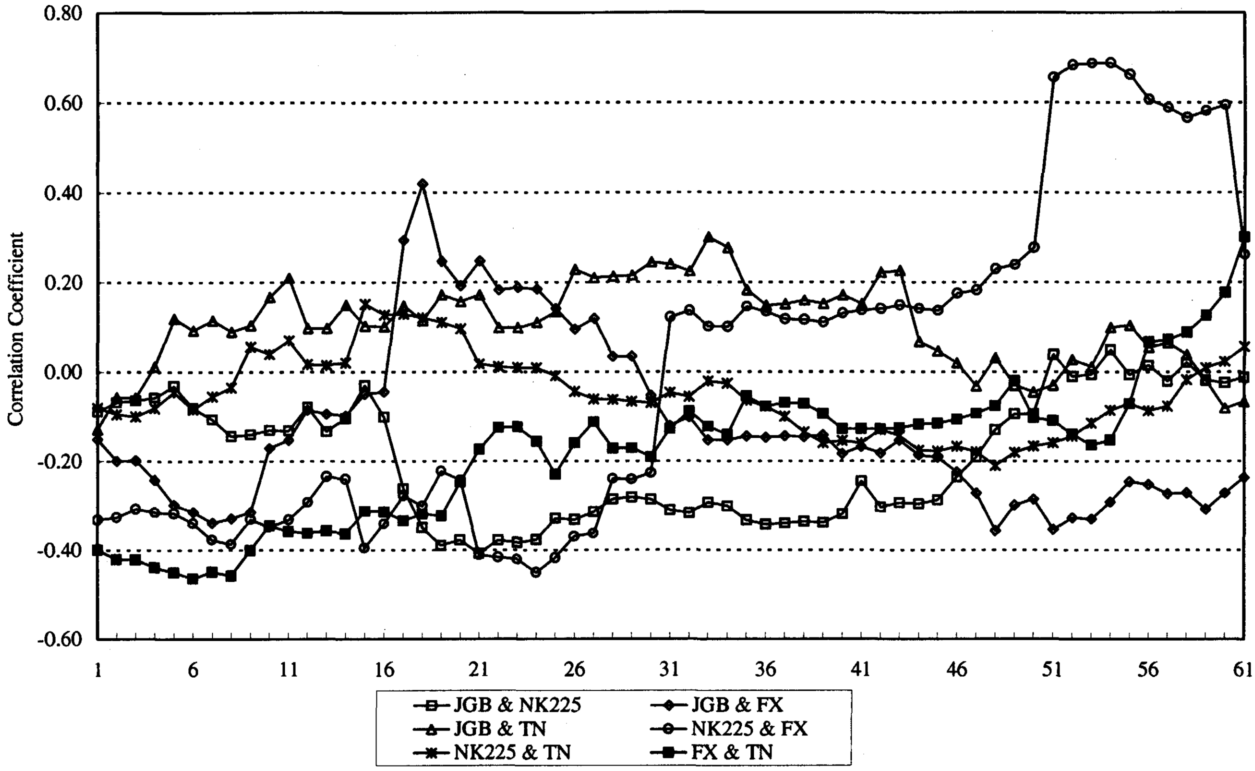


Chart 3. Correlation coefficient (observation period: 30 preceding days)

Period I (January 1994 - March 1994)



Period II (January 1995 - March 1995)

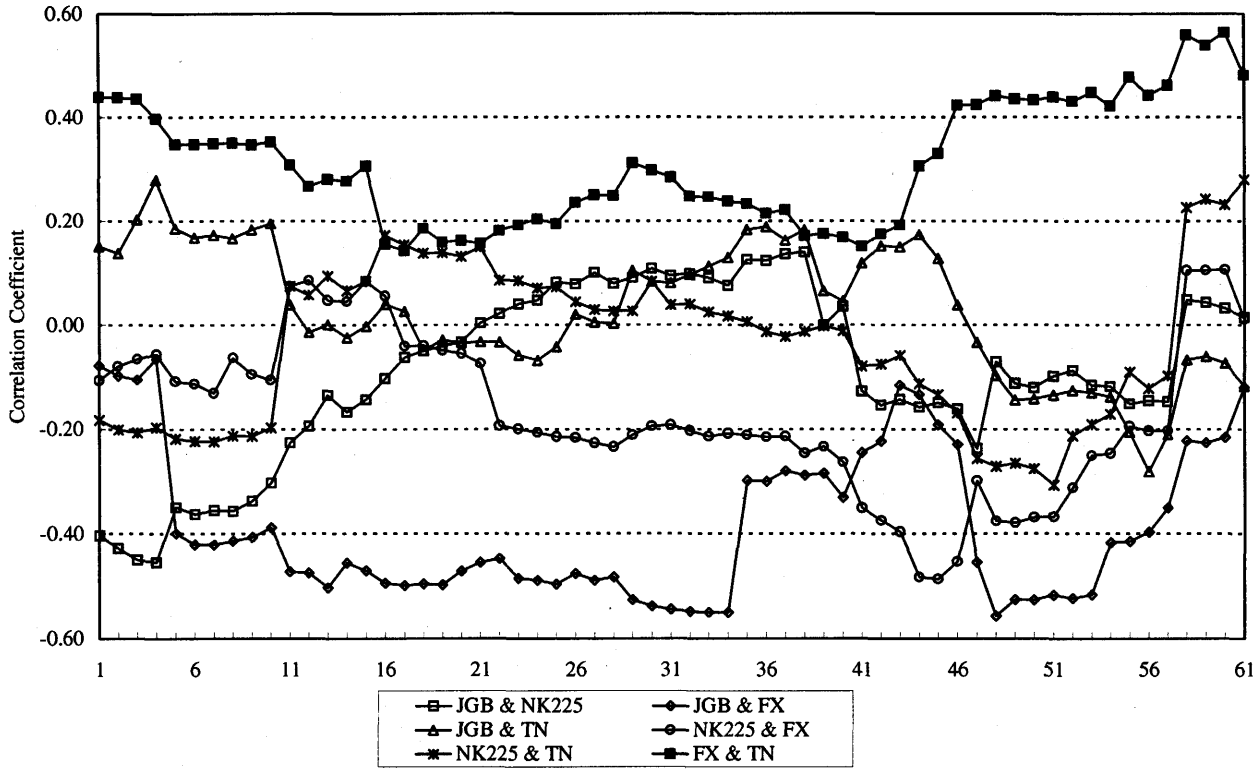


Chart 4. Statistical indicators of volatility

(1) Numbers of days out of 61 observations when the price change over the previous day exceeds 95 % confidence interval (=1.645 σ)

	JGB futures	Nikkei 225	¥/$ rate	US TN($) 1/	US TN(¥) 2/
Period I	22 (12/10)	9 (4/5)	5 (3/2)	6 (5/1)	6 (4/2)
Period II	5 (2/3)	10 (8/2)	14 (9/5)	5 (0/5)	10 (6/4)

1/ US treasury note futures measured in the US dollars.

2/ US treasury note futures measured in the Japanese yen.

3/ Figures in parenthesis denotes numbers of the days when the upward/downward changes (↑, ↓) exceed 95 % confidence level.

(2) Means (μ) and standard deviations (σ)

		JGB futures	Nikkei 225	¥/$ rate	US TN($)	US TN(¥)
Period I	μ	0.0325%	0.0028%	-0.0444%	0.0201%	-0.0243%
	σ	0.2986%	1.3099%	0.7613%	0.3638%	0.8119%
Period Ii	μ	-0.0320%	0.0615%	-0.0274%	-0.0513%	-0.0787%
	σ	0.4030%	1.2365%	0.6154%	0.4713%	0.8677%

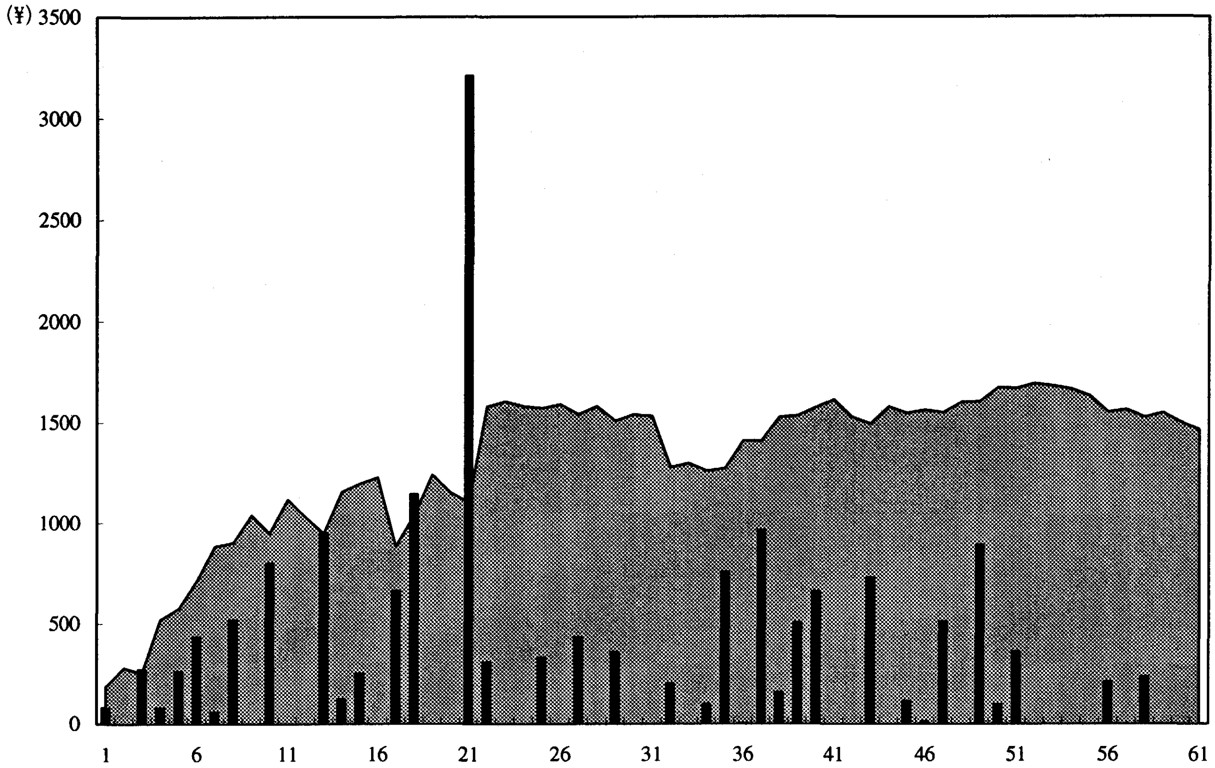
Chart 5. Components of hypothetical portfolio for simulation

Products	Positions*	Market Conditions
JGB Futures	Long 1.2 units	
JGB Futures Option (call)	Short 2.0 units	Strike Rate: Spot At-the-Money on the first day of the period Volatility: 5.0% (annualized) Discount Rate: 2.0% (annualized) Maturity: 2 months (fixed over the simulation period)
Nikkei 225 Futures	Long 2.0 units	delta-hedged by the Nikkei 225 option position described below.
Nikkei 225 Option (call)	Short 4.0 units	Strike Rate: Spot At-the-Money on the first day of the period Volatility: 20.0% (annualized) Discount Rate: 2.0% (annualized) Maturity: 2 months (fixed over the simulation period)
US T-Note Futures	Long 0.2 units	

* One position unit corresponds to 20,000 Japanese yen in the simulation.
The hypothetical portfolio described above is assumed to be constant over the simulation period.

Chart 6. Result of backtesting: Variance-Covariance method

Period I (January 1994 - March 1994)



Period II (January 1995 - March 1995)

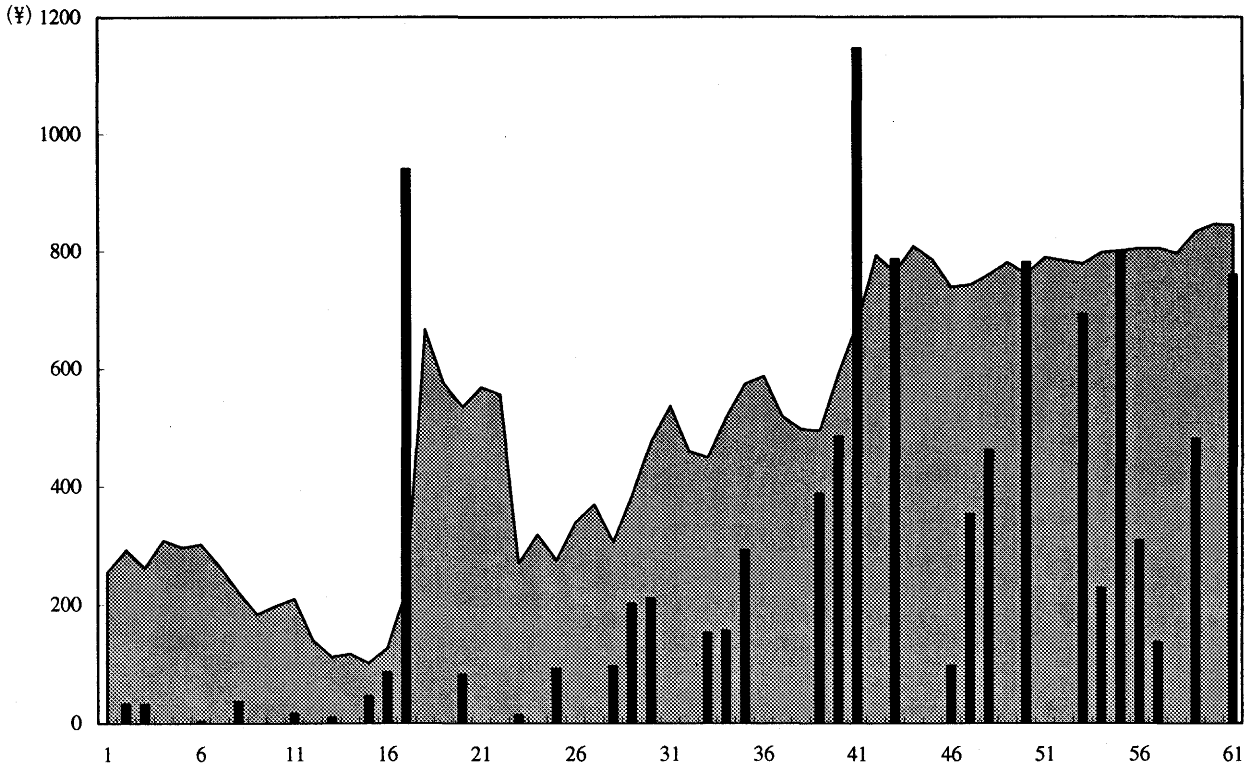
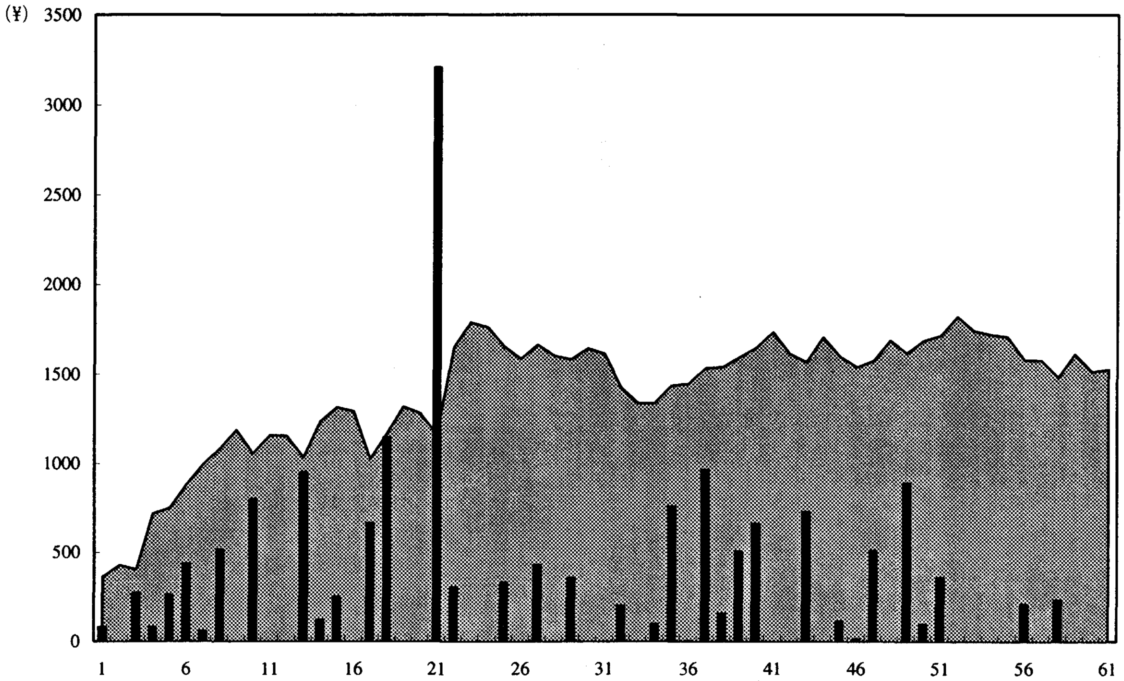


Chart 7. Result of backtesting: Monte Carlo simulation

Period I (January 1994 - March 1994)



Period II (January 1995 - March 1995)

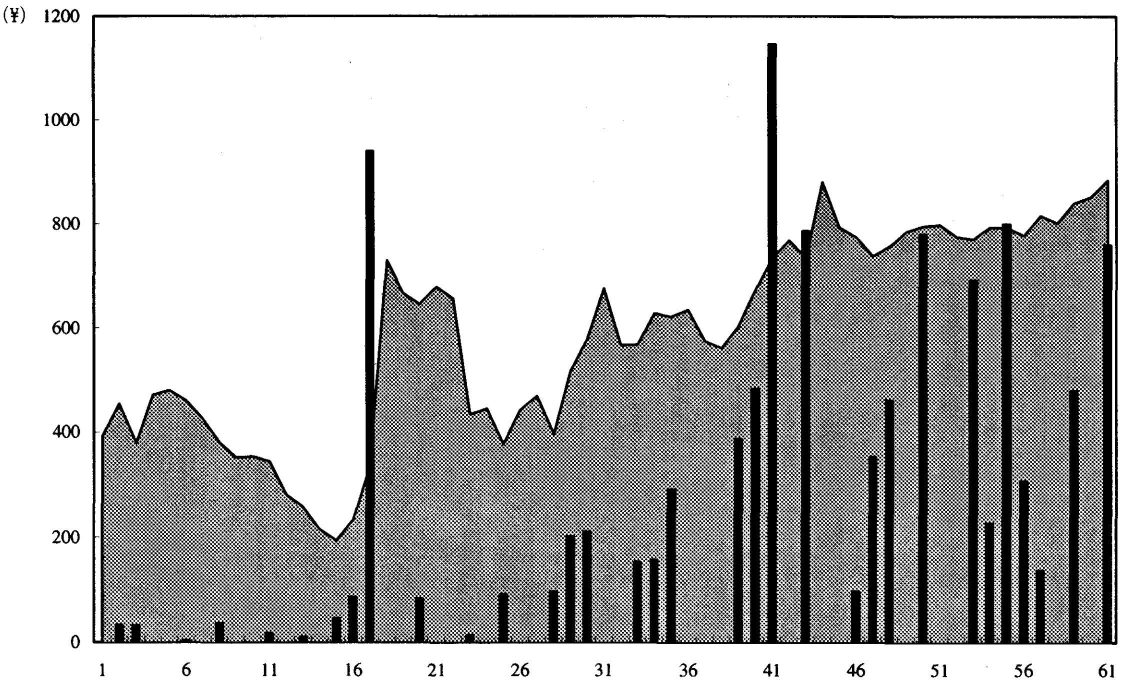
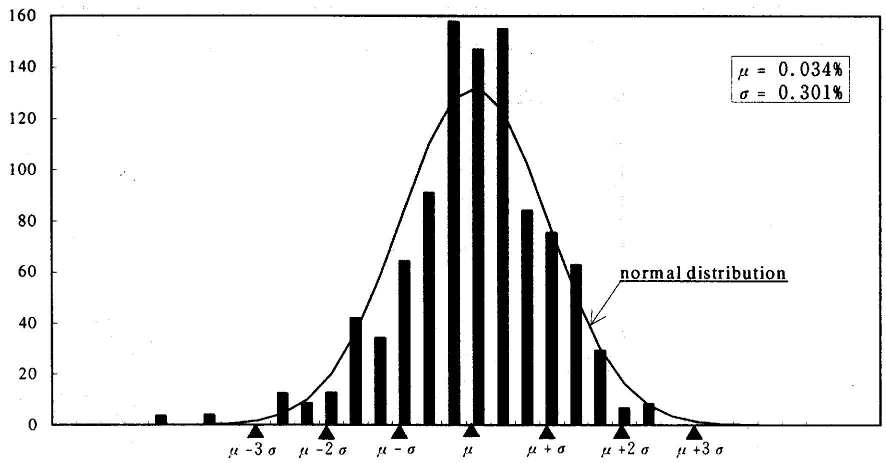
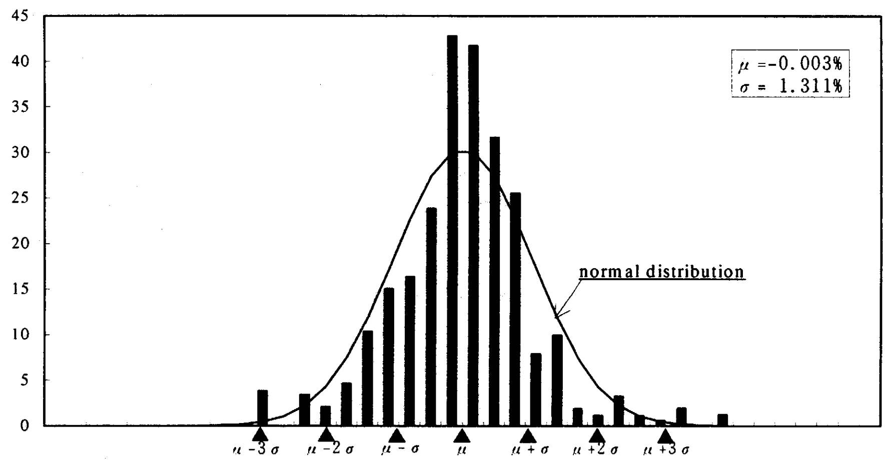


Chart 8. Probability distribution of daily changes obtained by bootstrap (as of the first day of Period I)

Japanese Government Bond futures



Nikkei 225



US Government Note futures (Yen-basis)

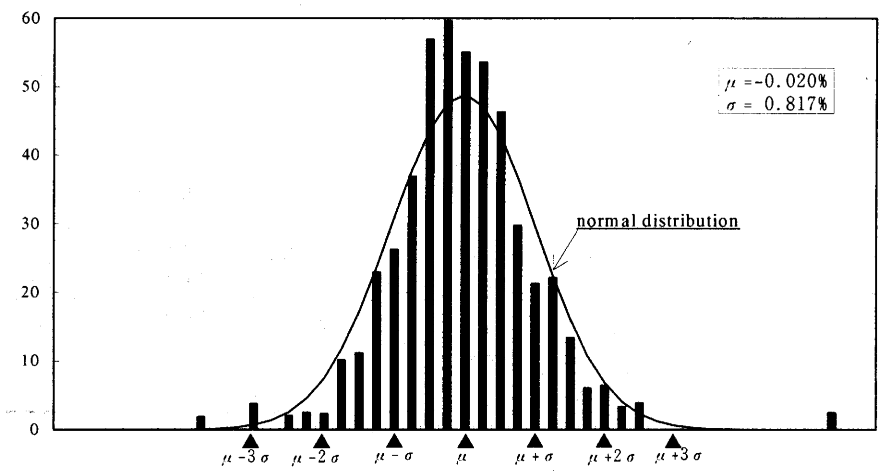
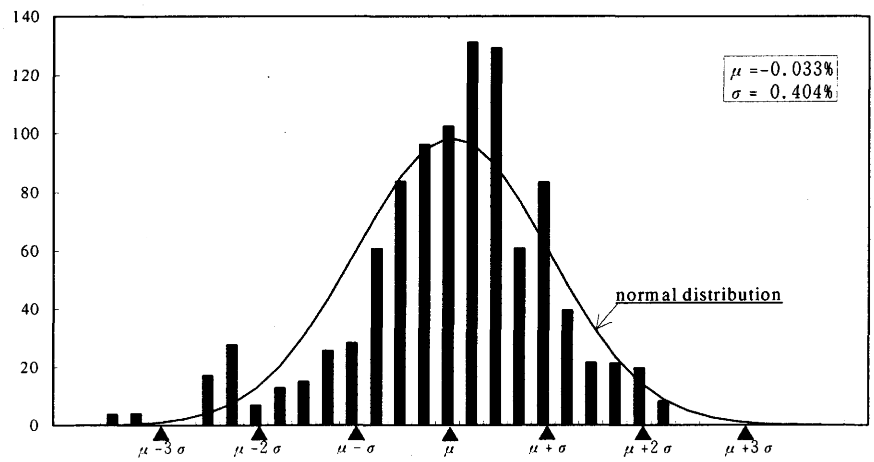
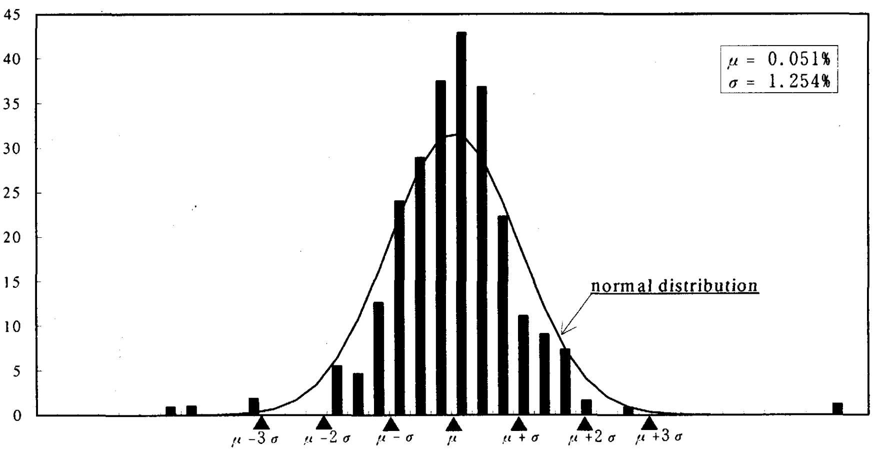


Chart 9. Probability distribution of daily changes obtained by bootstrap (as of the first day of Period II)

Japanese Government Bond futures



Nikkei 225



US Government Note futures (Yen-basis)

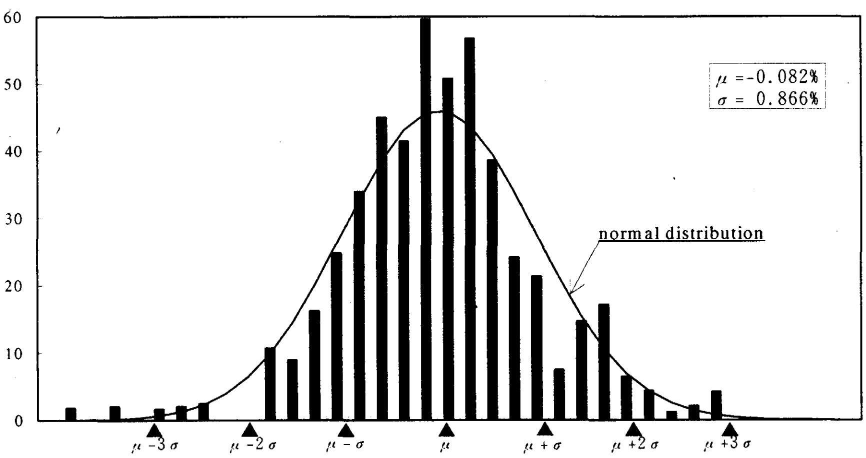
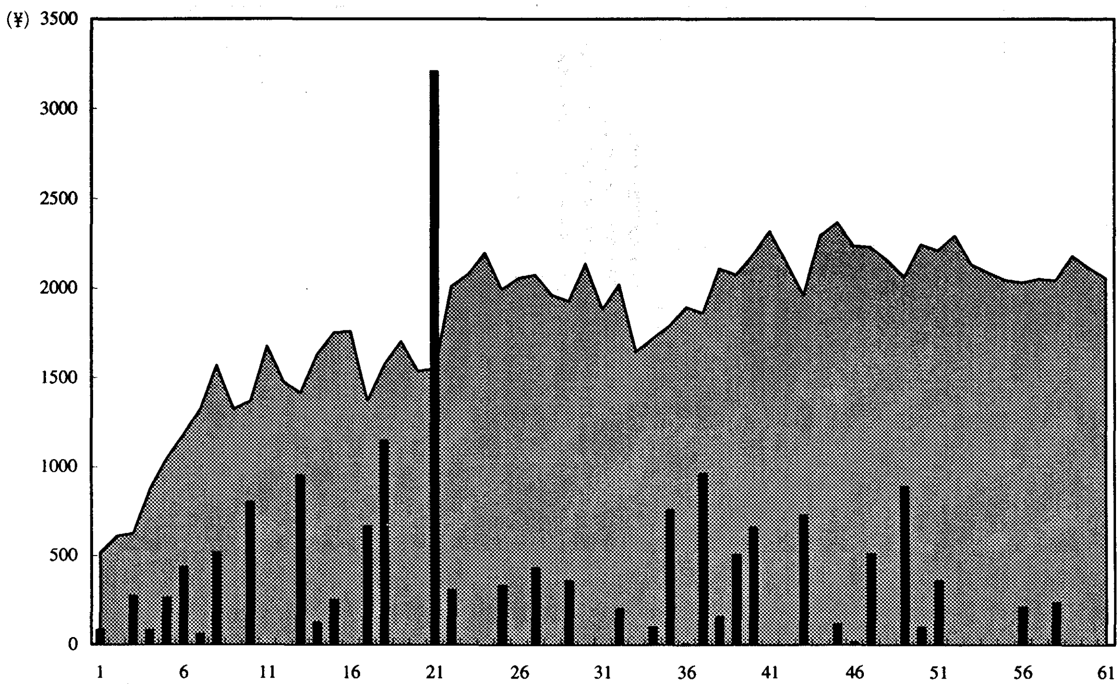


Chart 10. Result of backtesting: Monte Carlo simulation assuming a t-distribution

Period I (January 1994 - March 1994)



Period II (January 1995 - March 1995)

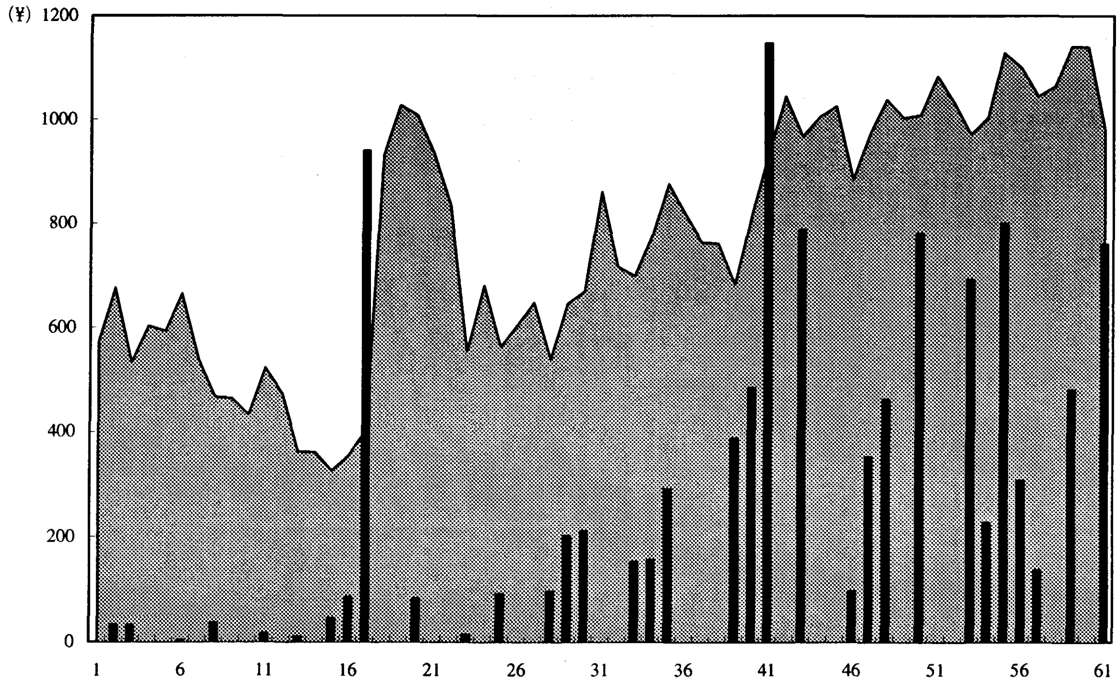
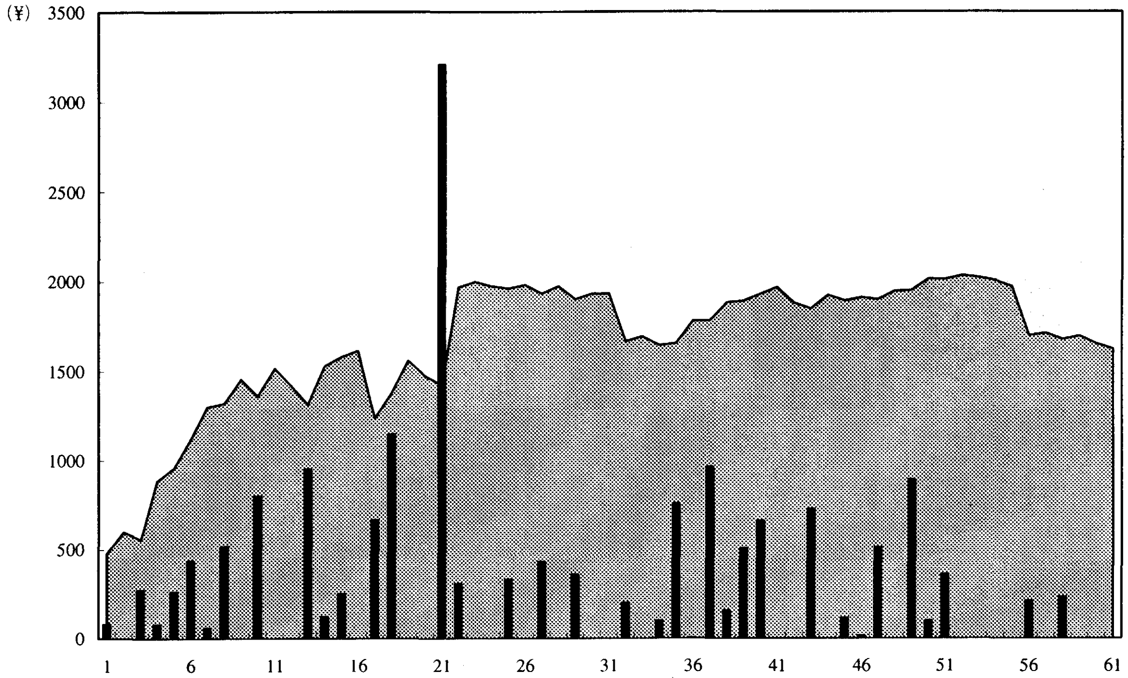


Chart 11. Result of backtesting: Nonparametric historical simulation

Period I (January 1994 - March 1994)



Period II (January 1995 - March 1995)

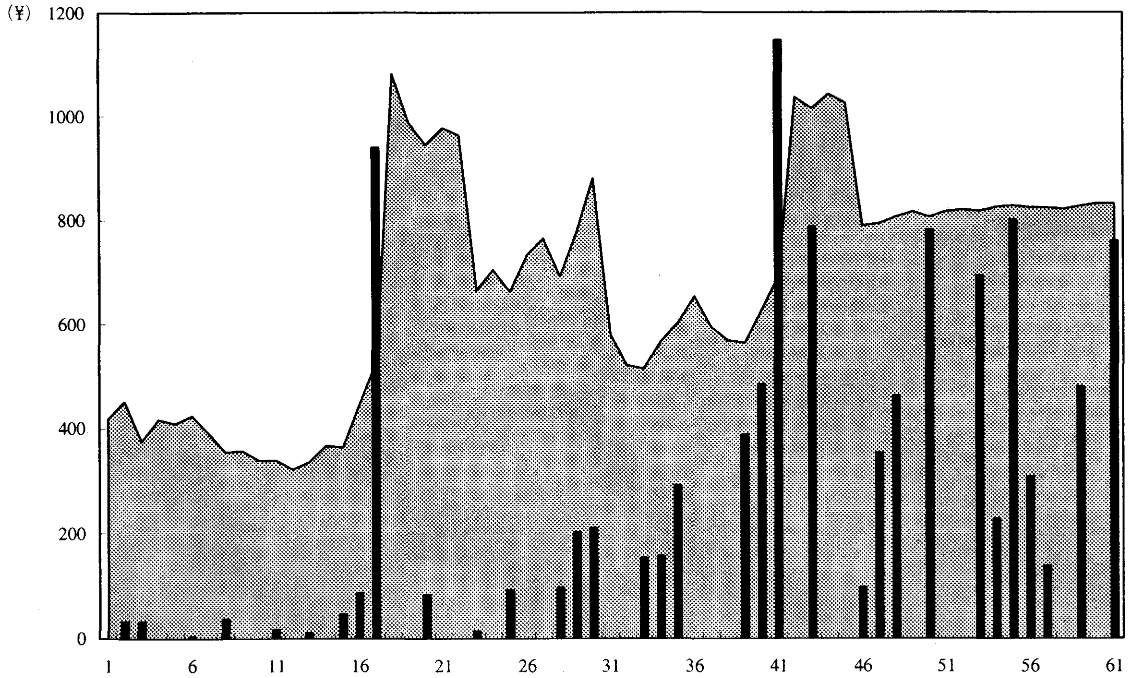
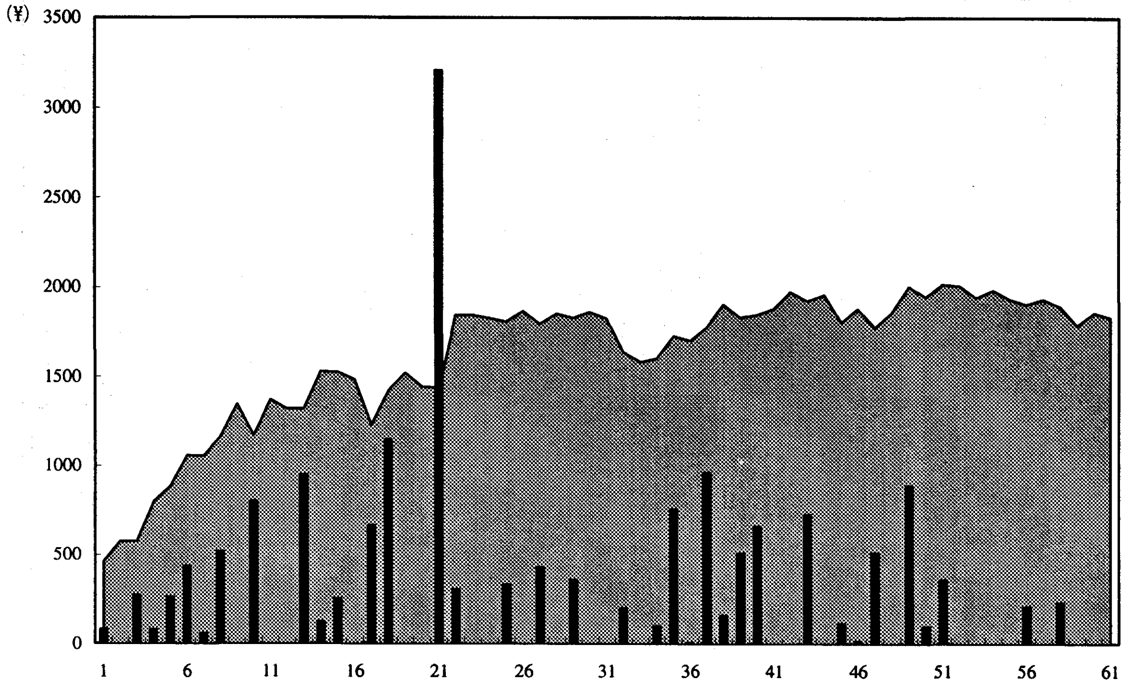


Chart 12. Result of backtesting: Monte Carlo simulation assuming worst correlations

Period I (January 1994 - March 1994)



Period II (January 1995 - March 1995)

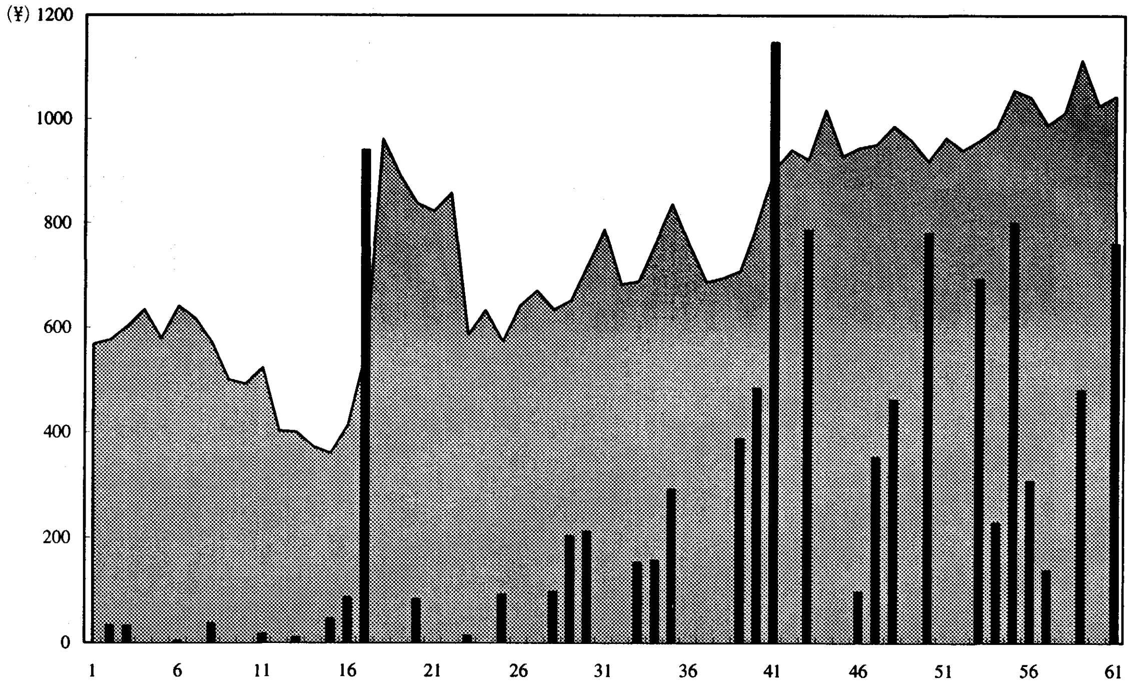
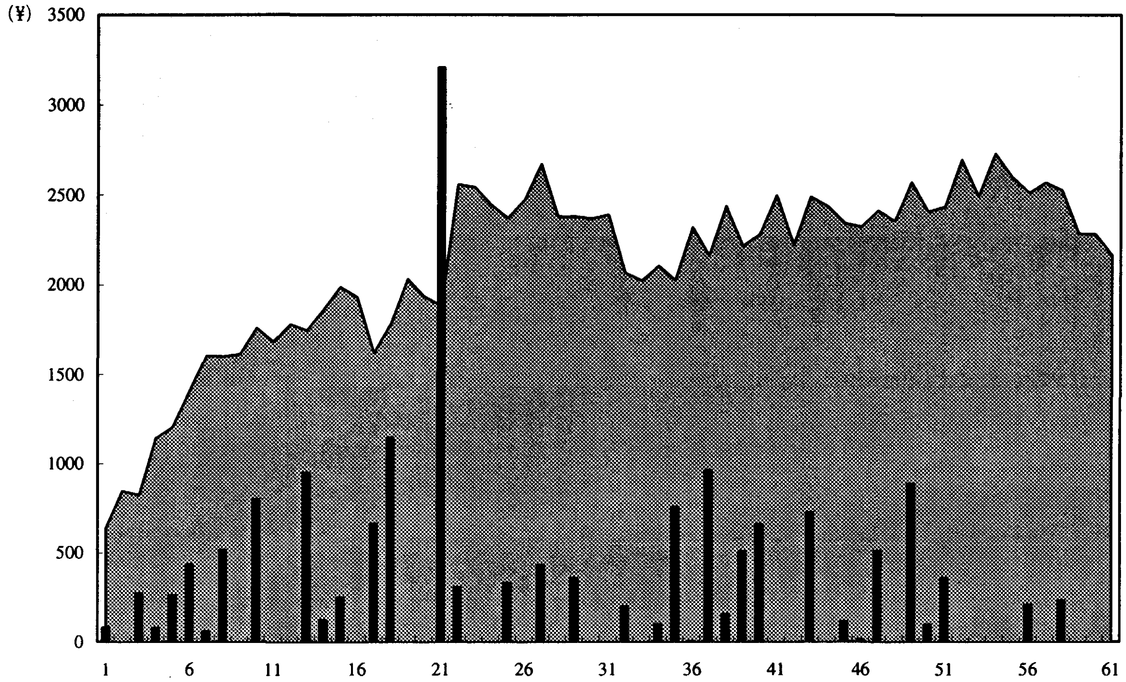


Chart 13. Result of backtesting: Monte Carlo simulation assuming a t-distribution and worst correlations

Period I (January 1994 - March 1994)



Period II (January 1995 - March 1995)

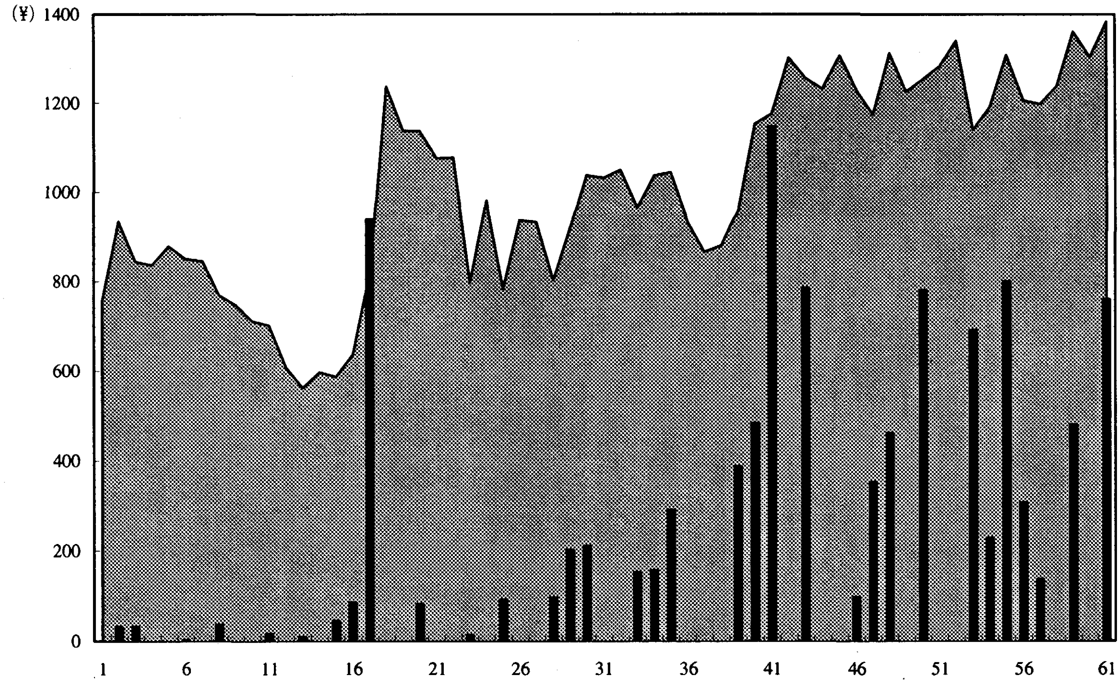
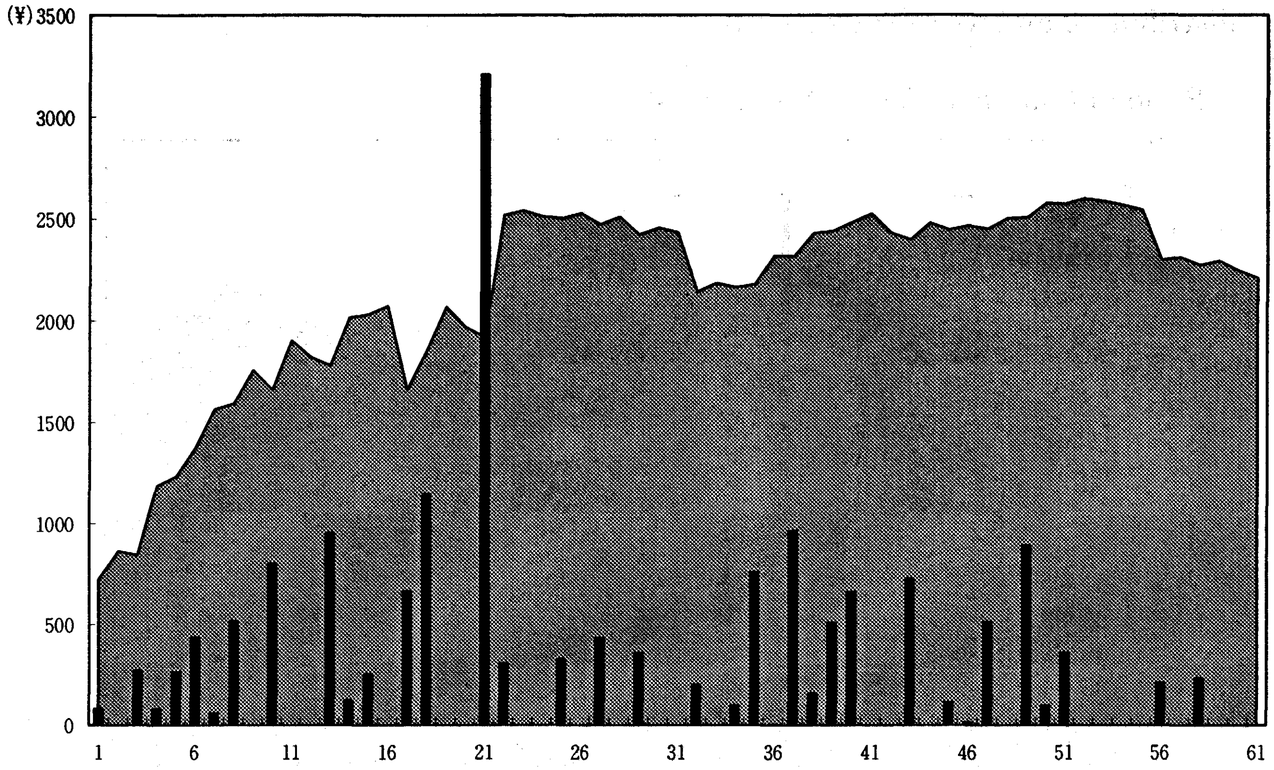


Chart 14. Result of backtesting: Nonparametric simulation assuming worst correlations

Period I (January 1994 - March 1994)



Period II (January 1995 - March 1995)

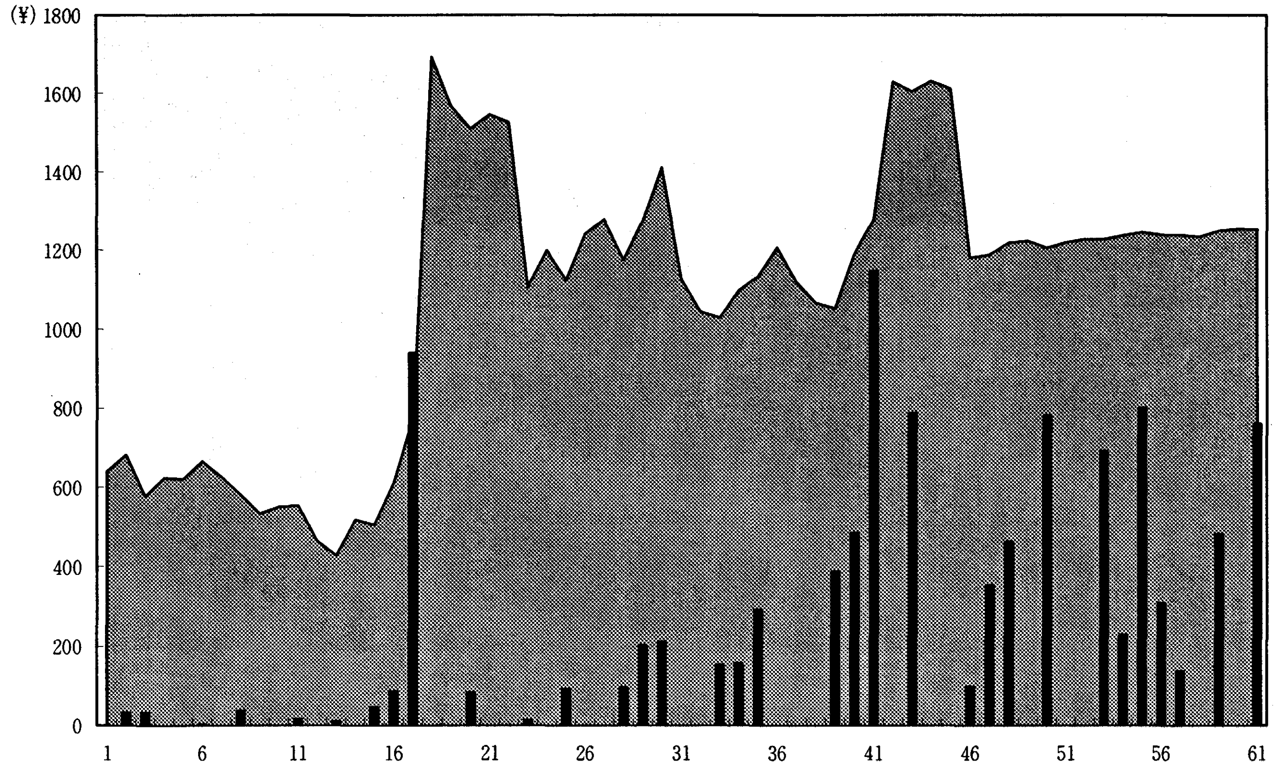
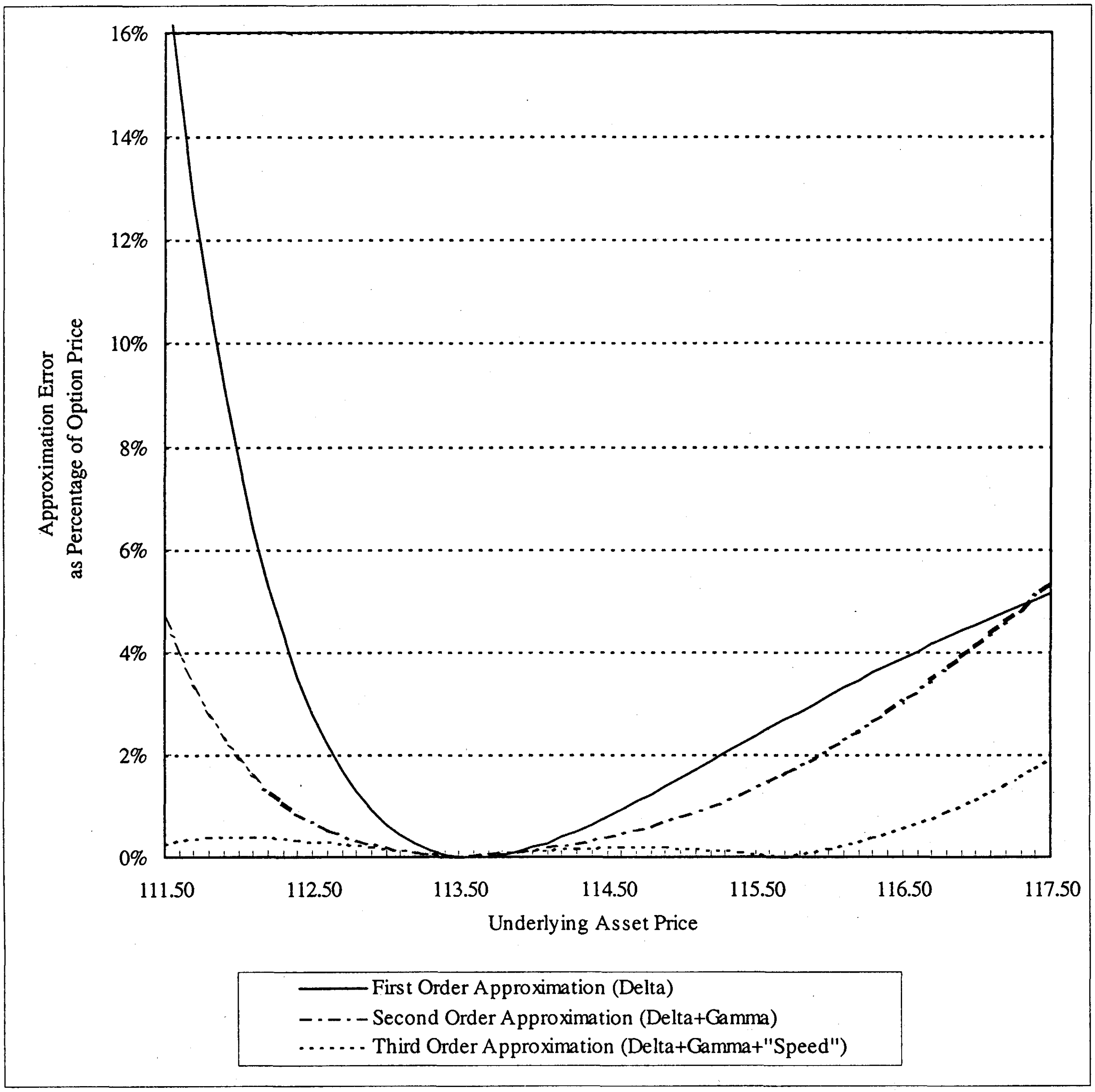


Chart 15. Delta, Gamma, and "Speed" Risks of Option Position —— An Example



Market conditions applied in this example are summarized below.

Trading Position:	Long Call Option	
Underlying Asset:	Cash bond with 4.6% coupon	
Option Condition:	Strike Price	= 110.00
	Price Volatility	= 5.0% (annual)
	Discount Rate	= 1.0% (annual)
	Time to Maturity	= 0.2 year

Chart 16. Kurtosis of distributions of changes in financial variables

(1) Kurtosis of t-distributions

degrees of freedom of t-distributions	standard deviations	kurtosis
5	1.29099	9.0
6	1.22474	6.0
7	1.18322	5.0
8	1.15470	4.5
9	1.13389	4.2
10	1.11803	4.0
∞ 1/	1.0	3.0

1/ normal distribution.

(2) Kurtosis of market data calculated over the preceding two-week period.

data	standard deviations	kurtosis
JGB futures	0.00277	7.40978
Nikkei 225	0.00790	4.88272
US T-note futures	0.00357	6.55272
¥/$ exchange rate	0.00556	9.92348

Chart 17. Confidence intervals

(1) Confidence intervals of t-distributions measured by the standard deviations

degree of freedom of t-distribution	95 percentile	99 percentile
6	1.943 σ	3.143 σ
7	1.895 σ	2.998 σ
8	1.860 σ	2.896 σ
9	1.833 σ	2.821 σ
10	1.812 σ	2.764 σ
∞	1.64 σ	2.33 σ

(2) Confidence intervals of distributions of market data

data	95 percentile	99 percentile
JGB futures	1.571 σ	2.579 σ
Nikkei 225	1.827 σ	3.103 σ
US T-note futures	1.485 σ	3.381 σ
¥/$ exchange rate	1.514 σ	2.550 σ