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Innovation Choice, Product Life Cycles, and Optimal Trend Inflation

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Abstract

This study revisits the old and ongoing challenge of identifying the optimal trend inflation rate, using a novel model that incorporates a firm's innovation choices, product life cycles, and the interplay of the two factors. We construct an endogenous growth model with sticky prices, where firms have two options: to be an innovator or to be a follower. An innovator causes creative destruction, forcing all the incumbents to exit, and becomes a monopolist in its sector. A follower enters an existing sector by offering a product that is slightly different from the incumbent's products, inducing a product life cycle within the sector. Trend inflation impacts the firm's decision regarding which of the two options to choose by changing expected markups and profits. We show that the optimal trend inflation rate could exceed zero as it mitigates potential innovator losses upon the entry of followers, which in turn depresses the incentives for firms to be followers, promoting creative destruction and faster economic growth.

Keywords: Sticky prices; Optimal inflation; Product life cycle; Innovation; Productivity growth; Markups

JEL classification: E31, O31, O41

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1 Introduction

The discussion of an optimal inflation rate first appeared more than 50 years ago (Friedman (1969)). Since that time, various approaches to identifying the optimal rate, based on a variety of perspectives, have been presented.[1] In general, the optimal level of inflation proposed in the literature has risen from that first proposed by Friedman (1969), particularly following the financial crisis in which the central banks in major countries saw the effective lower bound (ELB). For example, Coibion et al. (2012) considered the gains from raising the steady state inflation rate so that hitting the effective lower bound becomes less likely, arguing that the optimal rate is positive. Indeed, as Diercks (2019) established, there has been no sign of convergence in the magnitude of the optimal inflation rate, which ranges from roughly -8% to around +8%.

In this paper, we revisit the problem of determining the optimal trend inflation rate by focusing on the interplay between the R&D activities of firms and the life cycle of products and provide a novel explanation of why the optimal level of inflation rate could exceed zero once these factors are taken into account. The literature of growth theory emphasizes creative destruction through R&D activities as a key driver of economic growth. While innovators initially set high prices to obtain monopolistic rents, these prices typically decline as followers enter the market. In the presence of price stickiness, trend inflation plays a crucial role in the product life cycle as it can mitigate potential innovator losses with the entry of followers. Thus, trend inflation can impact a firm's incentives to become an innovator, consequently influencing economic growth and overall welfare.

Specifically, we conduct a welfare analysis of trend inflation by constructing an endogenous growth model with sticky prices in which R&D activities, economic growth, and the life cycles of products interact with one another. The economy here comprises an infinite number of sectors, each of which consists of either one or two firms. In sectors with one firm, the firm is monopolist until another firm enters the sector. In sectors with two firms, the two firms compete monopolistically. All firms are subject

[1] The pioneering work of Khan et al. (2003), using a sticky price model with money, compared the costs of price adjustments, which increase as the trend inflation rate deviates farther from zero, to the costs of holding monetary balances, which increase as the trend inflation rate deviates from the opportunity cost of holding money. The authors argued that the optimal rate is negative but not as negative as that advocated by Friedman (1969) and is close to zero.

to nominal price friction and change their prices stochastically. Before entering a market, firms conduct R&D investments with an eye on two options regarding the nature of the product they create. When a firm creates a wholly new product, it becomes an innovator, causing creative destruction in the sector and forcing the other firms in the sector to exit, thus becoming a monopolist. When a firm succeeds in creating a product with only modest differences from the incumbent's product, it becomes a follower. Such a firm enters a sector with a single firm and competes with the incumbent. Firms spend more resources for innovating new products in an environment where the gains from becoming a follower are minor and the gains from being an innovator are large. Moreover, the gains from being a follower rely on the nature of the product life cycles, in particular the relative price of the two products when the incumbent and entrant begin to compete.

Using our model, we explore how trend inflation affects a firm's decisions. While in our model there are various channels through which the trend inflation rate affects the economy in different ways, our quantitative analysis shows that the optimal trend inflation rate could be positive. The key and novel channel that brings about this result is the effect of trend inflation on the relative price of a monopolist's products. It is notable that because firms are unable to adjust their prices flexibly, a positive trend inflation produces a decline in the relative prices of those products whose face-value prices are unadjusted. This decline in relative prices benefits innovators. Although they are unable to optimally reduce their prices at the time a follower enters the sector due to price stickiness, the decline in relative prices helps mitigate the losses caused by the follower's entry. Therefore, in the case of positive trend inflation, firms will allocate a larger portion of R&D resources to establish themselves as innovators, which leads to more creative destruction and faster economic growth. Admittedly, there are channels through which positive trend inflation adversely affects the economy, such as the channel involving price dispersions, as stressed in existing studies.[2] However, we show quantitatively that, under a reasonable set of parameter values, the dominant channels are the channels through which relative price changes induce innovation and economic growth, thus implying that the optimal trend inflation rate is positive.

[2] Oikawa and Ueda (2018) studied the implications of trend inflation with regard to R&D activities, focusing on this channel.

The remainder of the paper proceeds as follows: Section 2 reviews the pertinent literature, Section 3 describes the model, Section 4 provides a quantitative analysis of the optimal trend inflation rate and discusses the model's key properties, and Section 5 concludes the paper.

2 Literature Review

Importantly, our study follows on numerous studies dealing with the optimal trend inflation rate. A non-exhaustive list of these studies includes the pioneering work of Friedman (1969), Levy et al. (1997), King and Wolman (1999), Khan et al. (2003), Kim and Ruge-Murcia (2009), Schmitt-Grohé and Uribe (2012), Coibion et al. (2012), Oikawa and Ueda (2018), and Adam and Weber (2019, 2023).[3] [4] In particular, our paper is closely related to two lines of studies. The first appears in Adam and Weber (2019, 2023), where the focus was on the systematic trend of prices observed at the micro-level after product turnover, i.e., product life cycle. Motivated by the observation that the price levels of individual products are highest upon emergence, then decline in subsequent periods, the authors constructed a model in which a firm's productivity gradually rises over the product life cycle. They argued that the optimal trend inflation rate takes a positive value, so that the nominal marginal costs of a product, which reflect both a change in productivity level and the prices of production inputs, remain unchanged. Similar to Adam and Weber (2019, 2023), the firms in our model face price declines over the product life cycle. In contrast to their work, however, the product life cycles in our model are due to the entry of followers, which reduces the markup of the incumbent's product, as opposed to a change in productivity.[5] [6]

[3]For a survey on the optimal inflation rate, see also Schmitt-Grohé and Uribe (2010), Adam and Weber (2024), and Sugioka et al. (2024).

[4]See also, for example, Fuchi et al. (2008) and Mineyama et al. (2022), which assessed the optimal inflation rate in Japan using a DSGE model capable of addressing cases where multiple channels through which the trend inflation rate affects the economy act in concert, such as zero lower bound and the downward rigidity of nominal wages.

[5]See, for example, Fujiwara and Matsuyama (2022) for the impact of an increasing number of the market participants on the markup of firms in the market.

[6]As discussed in Melser and Syed (2016), there are a number of factors that could generate the product life cycle, including productivity studied in Adam and Weber (2019, 2023), the fashion effect, as examined in Ueda et al. (2019), and competition, as featured in our paper. The discussion in the current paper and the fashion effect featured in Bils (2009) are not mutually exclusive. Though the fashion effect may make a decline in price over the life cycle even sharper than that under the current model,

The second pertinent study, Oikawa and Ueda (2018), focused on the impact of the trend inflation rate on a firm's R&D activities using an endogenous growth model along the lines of Aghion and Howitt (1992). In this case, the authors constructed a model in which a firm's R&D investment leads to a higher growth of the economy and the firm's price adjustment costs reduce the expected return on its R&D investment. It was shown that the growth-maximizing inflation rate is the inverse of the economic growth rate, maintaining a zero growth rate for nominal wages so that firms do not need to alter their product prices.[7] Similar to Oikawa and Ueda (2018), our model explores how the trend inflation rate affects a firm's decision regarding R&D activities. The key difference is that, in our model, the firms have two options at entry, which is similar to the work of, for example, Akcigit and Kerr (2018) and Peters and Walsh (2022). Consequently, the level of trend inflation affects both the size of R&D investment and the composition of that investment. Our work differs from theirs in terms of the role of the interplay between product life cycle and R&D activities.

Our work is also related to two other literature strands. First, similar to Adam and Weber (2019, 2023), our work is built on recently expanding empirical studies on price dynamics over the product life cycle. Melser and Syed (2016) analyzed a large U.S. scanner dataset involving supermarket products and document a general tendency for prices to decline as items age. For Japan, studies such as Ueda et al. (2019) and Dong et al. (2019) identified life cycles qualitatively similar to those found for the U.S. data. Abe et al. (2016) documented similar characteristics of durable goods prices by analyzing electric appliances using the online shopping platform Kakaku.com.[8] Second, our work is related to recent studies that use a model with endogenous markups, contrasting with the prevailing standard model that assumes a markup varies exoge-

there are gains from a positive inflation rate so long as such a price decline prevails in the market. Bils (2009) argued "One is that durable models that exhibit faster quality improvement also obsolesce more rapidly, as they compete more directly with models that improve rapidly in quality. An alternative explanation is that part of the price increases for new models reflects valuation of fashion-type stylistic changes that depreciate over the model cycle."

[7]Miyakawa et al. (2022), an extension of Oikawa and Ueda (2018), suggest that setting the inflation rate so that the nominal wage growth rate is higher than 0% is optimal for encouraging an increase in the number of innovative firms, assuming that menu costs are paid at the time of entry.

[8]Our model omits quality improvements of products over their product life cycles. For the implications regarding this point, see, for example, Schmitt-Grohé and Uribe (2012), which studied the optimal trend inflation rate when measured inflation contains the systematic upward bias due to quality improvements of products.

nously, excluding variations due to nominal rigidities (Smets and Wouters (2007)). For example, Bilbiie et al. (2014) examined the role of inflation in shaping firm entry and markup levels, closely aligning with our study. Using a "General C.E.S. utility function" that explicitly incorporates consumers' preference for product variety, their model captured the impact of variety preferences on the inflation-markup relationship. They demonstrated that a positive inflation rate can enhance welfare by reducing steady-state markups and mitigating excessive entry. Also, Fujiwara and Matsuyama (2022) constructed a Homothetic Single Aggregator (HSA) New Keynesian sticky price model and explored the impact of entry and exit on the markup rate, showing that a higher concentration could lead to a rise in the markup rate. Edmond et al. (2015) studied the impact of international trade on markups and used Taiwanese producer-level data to show that empirically opening up to trade strongly increases competition and reduces markup distortion, as their theory suggests. Our work differs from theirs in terms of its focus on product life cycle and optimal trend inflation rate but shares some key elements regarding how competition among different products affects markups.[9]

3 Model

In our model, the economy comprises households, firms, and a central bank. Households have infinite lives, inelastically supply labor inputs to firms to produce earnings, and make consumption/saving decisions. There are an infinite number of sectors populated from 0 to 1; firms in these sectors produce goods either as a follower or as an incumbent innovator. Before entering a sector, firms invest in R&D. A firm's entry decision comes with two options: enter as an innovator or enter as a follower. The firm considers the size and proportion of the R&D activity required for each option, then chooses to conduct the R&D activity that maximizes its expected profits. Once entering

[9]From a different perspective, De Loecker and Eeckhout (2018) and Diez et al. (2018) estimated the time path of markups globally using international data. De Loecker and Eeckhout (2018) indicated that markups have been generally rising over the period from 1980 to 2016 in developed countries, including Japan, and that the level of markup is higher in the U.S. and Europe. On the other hand, Cabinet Office (2023) indicated that the markup set by Japanese firms has not been rising, contrasting with that of the U.S., and argued that "if [markup] is too low, it indicates a situation where the goods and services produced by the company are not priced commensurate with their value. Securing an appropriate markup is considered to be the key to creating a virtuous economic cycle in which companies increase their earning power and secure resources for wage increases and investment, leading to increased consumer spending and capital investment, which in turn leads to further sales growth for the company."

the market, the firm chooses the price that maximizes its profits under Bertrand competition, subject to Calvo-type nominal friction. The central bank chooses the trend inflation rate.

3.1 Household

The representative household has a preference given by

$$\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma} - 1}{1-\sigma}, \tag{1}$$

where C_t denotes consumption of the aggregate good. The budget constraint is

$$C_t + \frac{B_t}{P_t} = \frac{W_t}{P_t} N_t + \int_0^1 \frac{\Phi_{jt}}{P_t} dj + \frac{B_{t-1}}{P_t}(1 + i_{t-1}), \tag{2}$$

where N_t, P_t, B_t, i_t, W_t, and Φ_{jt} denote the aggregate labor supply, the aggregate price level, the nominal bond holdings, the nominal policy interest rate, the nominal wage rate, and the nominal profits of firm j at time t, respectively.

The maximization problem implies that.

$$\beta g_C^{-\sigma} = \frac{1}{1+r}, \tag{3}$$

where g_C represents the growth rate of consumption and r denotes the real interest rate.

3.2 Production

The economy consists of a continuum of sectors, $S \in [0, 1]$, where each sector S is composed of monopolistically competitive firms. The number of firms in each sector, $n(S)$, is assumed to be finite, $n(S) \in [1, \infty)$, as in Edmond et al. (2015). The number of firms in each sector is determined through creative destruction and the follow-up entries of firms, as specified in the next subsection. The final output Y_t is given by an aggregation of the sector-level outputs $y_t(S)$,

$$Y_t = \left[\int_0^1 y_t(S)^{\frac{\theta-1}{\theta}} dS \right]^{\frac{\theta}{\theta-1}}, \tag{4}$$

where θ is the price elasticity of substitution across sectors $S \in [0,1]$. In each sector S, output $y_t(S)$ is produced by aggregating each firm i's output $y_{it}(S)$ in sector S,

$$y_t(S) = \left[\sum_{i=1}^{n_t(S)} y_{it}(S)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}, \tag{5}$$

where $\gamma > \theta$ denotes the price elasticity of goods produced by firm i within a specific sector $S \in [0,1]$, and $n_t(S)$ is the number of firms operating in the sector. Each good $y_{it}(S)$ is produced using labor as the sole input,

$$y_{it}(S) = A_t(S) l_{it}(S). \tag{6}$$

We assume that the productivity $A_t(S)$ is at the same level within a sector.

Following Edmond et al. (2015), the aggregate price level P_t is defined as a function of each good's price in each sector, denoted by $p_t(S)$,

$$P_t \equiv \left[\int_0^1 p_t(S)^{1-\theta} dS \right]^{\frac{1}{1-\theta}}, \tag{7}$$

and the sector level price $p_t(S)$ is defined as,

$$p_t(S) \equiv \left[\sum_{i=1}^{n(S)} p_{it}(S)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}, \tag{8}$$

where $p_{it}(S)$ is the price of good produced by firm i in sector S. We then define the gross inflation rate as

$$\Pi_t \equiv P_t / P_{t-1}. \tag{9}$$

Given these price level definitions, cost minimization regarding the final output leads to the demand function for $y_{it}(S)$,

$$y_{it}(S) = \left(\frac{p_{it}(S)}{p_t(S)} \right)^{-\gamma} \left(\frac{p_t(S)}{P_t} \right)^{-\theta} Y_t. \tag{10}$$

Using equations (6), (10) and the definition of aggregate labor for production L_t:

$$L_t = \int_0^1 \sum_{i=1}^{n_t(S)} l_{it}(S) dS, \tag{11}$$

we can show that the aggregate labor productivity can be defined as

$$\frac{Y_t}{L_t} = \left\{ \int_0^1 \frac{1}{A_t(S)} \sum_{i=1}^{n_t(S)} \left(\frac{p_{it}(S)}{p_t(S)} \right)^{-\gamma} \left(\frac{p_t(S)}{P_t} \right)^{-\theta} dS \right\}^{-1}, \tag{12}$$

which implies that price dispersion negatively affects aggregate labor productivity.

In this setting, we can derive a closed-form solution for markups in the absence of price stickiness and the variation in productivity across sectors that we will introduce in the later sections. The solution is

$$\text{Markup}(S) = \frac{\left\{ \frac{1}{n(S)}\theta + \left[1 - \frac{1}{n(S)} \right] \gamma \right\}}{\left\{ \frac{1}{n(S)}\theta + \left[1 - \frac{1}{n(S)} \right] \gamma \right\} - 1}. \tag{13}$$

This equation, which corresponds to the same markup specification as Edmond et al. (2015), clearly shows that the markup decreases and is convex in $n(S)$ if $\theta < \gamma$. This reduction in markup clearly leads to a declining trend in the product life cycle of prices, which is an important aspect of this model. If there is price stickiness, we do not have a closed-form solution; however, the idea is the same as in this simple case. We characterize the general case of pricing in subsequent sections.

3.3 R&D Investment and Entry

Entrepreneurs have two avenues by which to enter a market: (i) they can engage in creative destruction by conducting R&D activities to create an innovative new product and, if successful, become an innovator, or (ii) they can engage in R&D activities to introduce a new variation of an existing product to a sector and, if successful, become a follower. Innovators who succeed in creative destruction drive out all existing varieties within the sector they enter, making the innovator the sole producer in that sector.[10]

[10] To maintain simplicity and avoid the case of limit pricing, we assume that all innovators engage in a two-stage price bidding game akin to Akcigit and Kerr (2018) and that they will be able to set prices

Before engaging in R&D investment to enter a market, entrepreneurs must first incur an entry cost c_E, which is measured in labor units. Upon payment of this entry cost, entrepreneurs can pursue both avenues by allocating non-zero resources to each type of R&D activity. The two types of R&D investments incur different costs, which are represented as convex functions of the probability of successful innovation as in previous studies. Then, entrepreneurs conduct R&D investment to maximize the value of entry V by choosing the success probability of becoming an innovator or a follower, σ_1 and σ_2,

$$V = \max_{\sigma_1,\sigma_2}(\sigma_1 V_1 - c_1\sigma_1^{\zeta} W) + (\sigma_2 V_2 - c_2\sigma_2^{\zeta} W), \tag{14}$$

where V_1 and V_2 represent the values of firms if they enter as innovators or followers, respectively. c_1 and c_2 are cost parameters, and ζ represents convexity of the R&D costs with respect to the success probability. The first-order conditions give

$$V_1 = \zeta c_1 \sigma_1^{\zeta-1} W, \tag{15}$$

$$V_2 = \zeta c_2 \sigma_2^{\zeta-1} W. \tag{16}$$

Due to the positive values for V_1 and V_2, as well as the convexity of R&D cost, these conditions imply that it is optimal to invest in both types of R&D.

The outcome of R&D activities is realized within the period in which the activities are conducted. We assume that when an innovative new product is introduced, creative destruction occurs in the sector and that, within the sector, all existing varieties are eliminated, making the innovator the sole producer in the sector. Accordingly, all incumbent firms face the risk of business closure in every period due to creative destruction by another innovator. Finally, the free entry condition implies

$$V = c_E W, \tag{17}$$

as monopolists. In the first stage, when a new firm enters a sector, both the existing firm (or firms) and the innovator are asked to pay a fee, set arbitrarily close to zero, if they wish to produce goods in the current period and beyond. In the second stage, all the firms that have paid the fee will announce their prices. Within the framework of this game, only the most productive firm will pay the fee in the first stage and participate in the second stage since the other firms will withdraw in the first stage, expecting that they will be unable to recover their fee in the second stage. Consequently, after entering the market, the entrant firm operates with monopoly pricing, as in Aghion and Howitt (1992), since the other firms will have abandoned the sector.

where c_E is an entry cost. When creative destruction succeeds at time t, productivity increases according to

$$A_t(S) = A_{t-1}(S) + \epsilon \bar{A}_{t-1}, \tag{18}$$

where ϵ is the step size of innovation and $\bar{A}$ represents the average productivity in the economy. On the other hand, when firms enter sector S as a follower, their productivity level is assumed to be the same as existing firms.

3.4 Aggregate Productivity Growth

Firm entries as innovators are associated with increases in productivity $A(S)$. Sectors experiencing creative destruction are chosen randomly, yielding a gap in productivity level between sectors. To ensure an invariant distribution of normalized productivity, $A(S)/\bar{A}$, across sectors, the following equation must hold for all $A/\bar{A} \in (0, \infty)$ on the balanced growth path (BGP):

$$\Phi\left(\frac{A}{\bar{A}}\right) = (1-\delta)\Phi\left(\frac{g_{\bar{A}}A}{\bar{A}}\right) + \delta\Phi\left(\frac{g_{\bar{A}}A}{\bar{A}} - \epsilon\right), \tag{19}$$

where $g_{\bar{A}} \equiv \bar{A}_t/\bar{A}_{t-1}$ is the aggregate productivity growth on the BGP and δ is the probability that creative destruction occurs. We can then characterize the balanced growth path as follows.

Proposition 1. Given the invariant distribution defined by (19), the aggregate productivity growth rate on the BGP is provided by $g_{\bar{A}} = 1 + \epsilon\delta$. Also, on the BGP, the output growth g_Y is equal to the aggregate productivity growth, i.e., $g_Y = g_{\bar{A}}$.

The proof of Proposition 1 is given in Appendix A. The proposition shows that the aggregate growth rate is equal to the rate of creative destruction δ times the step size ϵ, given that the source of economic growth is creative destruction in the economy.

3.5 Optimal Price Settings and Value Function

Following a standard new Keynesian model, we consider a time-dependent nominal friction, where firms cannot change their product prices with an exogenous probability

α. Similar to Adam and Weber (2019, 2023), we assume that the only exception is pricing at entry, i.e., the firm's initial pricing. Specifically, when firms enter the market either as an innovator or as a follower, they can set their prices freely for all periods.

For simplicity and tractability, we impose three assumptions regarding a firm's pricing behavior. First, we assume that the opportunity to change prices arises at the sector level. This implies that all firms within the same sector adjust their prices simultaneously when a price change becomes available and that no firms change their prices otherwise. Second, we only consider the case where $n(S)$ is either 1 or 2. This implies that a sector is either a monopoly market or a duopoly market. Because $n(S)$ does not exceed 2, sectors where $n(S)$ is already 2 do not face the further entry of other competitors going forward. On the other hand, sectors with $n(S) = 1$ could attract a follower in the current period and beyond with a period-by-period probability of entry λ. Third, we assume that firms are informed of the number of firms that prevail in their sector, including the potential of having a new entrant, when they choose their optimal prices.

Given these assumptions, firms that are able to adjust prices choose their optimal prices to maximize the discounted sum of their profits. Specifically, the maximization problem of profit along the BGP is formulated as,

$$\max_{p^*_{it}(S)} \sum_{j=0}^{\infty} \left(\frac{\alpha(1-\delta)}{\Pi(1+r)} \right)^j \left(p_{it}(S) y_{i,t+j}(S) - W_{t+j} l_{i,t+j}(S)\right) \tag{20}$$

subject to the demand function (10) and the production function (6). As in the standard new Keynesian model, firms choose the optimal price $p^*_{it}(S)$ to maximize the sum of their nominal profits discounted by the nominal interest rate, $\Pi(1+r)$, and the probability of not being able to change prices, α. Furthermore, unlike the standard new Keynesian model, the future profit is also discounted by the rate of survival from creative destruction $1-\delta$, given that all firms in the sector are driven out with probability δ once a new innovator arrives. Under the assumption that all sectors are either a monopoly or duopoly, i.e., $n(S) = 1$ or 2 for all $S \in [0,1]$, the optimization problem (20), and associated value functions, are separately characterized for the following three cases: (i) firms in a duopoly, (ii) a follower at entry, and (iii) firms in a monopoly. Below, we reformulate the optimization problems for each of these three cases.

(i) Duopoly When sector S is duopoly, two firms with imperfectly substitutable goods set their optimal prices under Bertrand competition. In this case, the optimization problem (20) is reformulated as,

$$v_1 \equiv \max_{p_{it}^*(S)} \sum_{j=0}^{\infty} \left(\frac{\alpha(1-\delta)}{\Pi(1+r)} \right)^j \left(p_{it}^*(S)^{1-\gamma} - p_{it}^*(S)^{-\gamma} \frac{g_W^j W_t}{A_t(S)} \right) \left(p_{it}^*(S)^{1-\gamma} + p_{-i,t}^*(S)^{1-\gamma} \right)^{\frac{\gamma-\theta}{1-\gamma}} P_{t+j}^{\theta} Y_{t+j}, \tag{21}$$

where $p_{-i,t}^*(S)$ is the optimal price set by the other firm. Assuming that all firms within the same sector simultaneously have the opportunity to adjust their prices, we can symmetrically characterize p^*. Here, g_W is the growth rate of nominal wages, which satisfies $g_W = \Pi g_{\bar{A}}$ on the BGP. The value function for firms in a duopoly is denoted by v_1. Note that the probability of a follower entering a sector, λ, does not appear in this case, as we assume that no new entries will occur in a duopoly.

(ii) A follower at entry Next, the optimization problem (20) is reformulated for a follower at entry as

$$v_2 \equiv \max_{p_{it}^*(S)} \sum_{j=0}^{\infty} \left(\frac{\alpha(1-\delta)}{\Pi(1+r)} \right)^j \left(p_{it}^*(S)^{1-\gamma} - p_{it}^*(S)^{-\gamma} \frac{g_W^j W_t}{A_t(S)} \right) \left(p_{it}^*(S)^{1-\gamma} + p_{-i,t-1}(S)^{1-\gamma} \right)^{\frac{\gamma-\theta}{1-\gamma}} P_{t+j}^{\theta} Y_{t+j}. \tag{22}$$

The only difference from the optimization problems in case (i) (i.e., duopoly) is that the other firm's price remains at the previous period's level, $p_{-i,t-1}(S)$, rather than at the optimized level. This reflects the fact that a follower can set its price freely at entry, whereas an incumbent firm cannot. The value function for a follower at entry is denoted by v_2.

(iii) Monopoly Finally, when an entrepreneurial firm successfully innovates, it becomes a monopolistic firm in sector S. In such a case, the firm sets its optimal price by considering not only future price change opportunities and the risk of creative destruction but also the probability of a follower entry, λ. Let $t+T$ be the period in which a follower enters sector S. Then, the optimization problem (20) is reformulated for a monopolistic firm as

$$v_3 \equiv \max_{p^*_{it}(S)} \sum_{T=1}^{\infty} \lambda(1-\lambda)^{T-1} \left\{ \sum_{j=0}^{T-1} \left(\frac{\alpha(1-\delta)}{\Pi(1+r)} \right)^j \left(p^*_{it}(S)^{1-\theta} - p^*_{it}(S)^{-\theta} \frac{g_W^j W_t}{A_t(S)} \right) P^{\theta}_{t+j} Y_{t+j} \right.$$
$$\left. + \sum_{j=T}^{\infty} \left(\frac{\alpha(1-\delta)}{\Pi(1+r)} \right)^j \left(p^*_{it}(S)^{1-\gamma} - p^*_{it}(S)^{-\gamma} \frac{g_W^j W_t}{A_t(S)} \right) \left(p^*_{it}(S)^{1-\gamma} + p^*_{-i,t+T}(S)^{1-\gamma} \right)^{\frac{\gamma-\theta}{1-\gamma}} P^{\theta}_{t+j} Y_{t+j} \right\}, \tag{23}$$

where the first and second term in the bracket represent the firm's profit before and after a follower enters sector S. The value function for firms in a monopoly is denoted by v_3. Here, the follower's optimal price at entry $p^*_{-i,t+T}$ is characterized by the follower's optimization problem in case (ii). Specifically, the monopolistic firm solves the optimization problem (23) subject to the follower's first order condition at entry,

$$p^*_{-i,t+T}(S)^{1-\gamma} \sum_{j=0}^{\infty} \left\{ (1-\theta) \left(\frac{\alpha(1-\delta)}{\Pi(1+r)} \right)^j p^*_{-i,t+T}(S) + \theta \left(\frac{\alpha(1-\delta) g_W}{\Pi(1+r)} \right)^j \frac{W_{t+T}}{A_{t+T}(S)} \right\} P^{\theta}_{t+j} Y_{t+j}$$
$$+ p^*_{i,t}(S)^{1-\gamma} \sum_{j=0}^{\infty} \left\{ (1-\gamma) \left(\frac{\alpha(1-\delta)}{\Pi(1+r)} \right)^j p^*_{-i,t+T}(S) + \gamma \left(\frac{\alpha(1-\delta) g_W}{\Pi(1+r)} \right)^j \frac{W_{t+T}}{A_{t+T}(S)} \right\} P^{\theta}_{t+j} Y_{t+j} = 0. \tag{24}$$

In all three cases, we are unable to derive closed-form solutions; therefore, we numerically compute the optimal price, $p^*_{it}(S)$, and the associated value functions for the three cases, v_1, v_2, and v_3. Note that on the BGP, the optimal price level grows at the same rate as nominal wages, $p^*_{i,t+1}(S) = g_W p^*_{it}(S)$, which implies that the equilibrium prices in existing sectors grow faster than the aggregate inflation rate, Π. This reflects the fact that the productivity level in existing sector S relative to the average productivity level, $A(S)/\bar{A}$, declines by $g_{\bar{A}}$ in every period, as the only source of productivity growth is creative destruction by innovators.

Given the value functions for firms in a duopoly v_1, a follower at entry v_2, and firms in a monopoly v_3, the values of being an innovator or a follower, denoted as V_1

and V_2 in (14), are respectively characterized by

$$V_{1,t} = v_{3,t} + (1-\alpha)\sum_{k=1}^{\infty}\left(\frac{(1-\delta)(1-\lambda)}{\Pi(1+r)}\right)^k v_{3,t+k}$$
$$+ (1-\alpha)\sum_{k=1}^{\infty}\left(\frac{1-\delta}{\Pi(1+r)}\right)^k \left(1-(1-\lambda)^k\right) v_{1,t+k}, \tag{25}$$
$$V_{2,t} = v_{2,t} + (1-\alpha)\sum_{k=1}^{\infty}\left(\frac{1-\delta}{\Pi(1+r)}\right)^k v_{1,t+k}. \tag{26}$$

Given that p_i^* and p_{-i}, P, and Y grow at the rates $g_W = g_{\bar{A}}\Pi$, Π, and $g_{\bar{A}}$, respectively, the growth rates of v_1, v_2 and v_3 are $g_{\bar{A}}^{2-\theta}\Pi$. Therefore, the nominal value of being an innovator or being a follower grows at the rate $\Pi g_{\bar{A}}^{2-\theta}$ on the BGP:

$$V_{1,t+1} = \Pi g_{\bar{A}}^{2-\theta} V_{1,t} \tag{27}$$
$$V_{2,t+1} = \Pi g_{\bar{A}}^{2-\theta} V_{2,t}. \tag{28}$$

3.6 Market Clearing Conditions

First, the labor market clearing condition implies that

$$N_t = L_t + m_t(c_E + c_1\sigma_{1t}^{\zeta} + c_2\sigma_{2t}^{\zeta}), \tag{29}$$

where the aggregate labor supply is the numeraire, and m_t is the number of entrepreneurs attempting to enter the market. The goods market clears simply with $Y_t = C_t$, since all produced goods are consumed by households.

We also need to balance entry and exit for both innovators and followers. For innovators,

$$m_t\sigma_{1t} = \delta_t, \tag{30}$$

and for followers,

$$m_t\sigma_{2t} = \frac{\delta_t\lambda_t(1-\delta_t)}{\delta_t + \lambda_t(1-\delta_t)}. \tag{31}$$

3.7 Equilibrium

Equilibrium is a state-contingent path for $\{(p_{it}(S), l_{it}(S))$ for $i = \{1, 2, \cdots n(S)\}$ and $S \in [0, 1], W_t, r_t, i_t, C_t, B_t, \sigma_{1t}, \sigma_{2t}\}_{t=0}^{\infty}$ such that the following hold:

1. a firm's choices $\{p_{it}(S), l_{it}(S)\}_{t=0}^{\infty}$ maximize profits for all $i = \{1, 2, \cdots n(S)\}$ and $S \in [0, 1]$, given the price adjustment frictions,

2. a household's choices $\{C_t, B_t\}_{t=0}^{\infty}$ maximize expected household utility,

3. the choice of entry as an innovator or as a follower maximize the entrepreneur's expected value of entry,

4. relative productivity $A_t(S)/\bar{A}$ follows the law of motion and forms an invariant distribution,

5. the markets for labor, goods and the number of firms clear,

where the initial values $B_{-1}(1 + i_{-1})$, $p_{i,-1}(S)$, and $A_{-1}(S)$ are given for $i = \{1, 2, \cdots n(S)\}$ and $S \in [0, 1]$.

4 What Does a High Trend Inflation Rate Bring about?

In this section, we show the quantitative implications of the model and discuss four channels that influence the determination of the optimal trend inflation rate.

4.1 Calibration

We calibrated model parameters to match the case of Japan. This involved assigning values to the model's parameters in two ways. Some parameters are directly determined based on available data or standard values. For the remaining parameters, we chose values that align with the model. Given the unique aspects of our model, namely (i) product life cycles, (ii) average markup and its variations, and (iii) economic growth, we used these as targets for our calibration. We conducted the calibration assuming a zero inflation rate, which corresponds to the average inflation rate observed in Japan from 2000 to 2020, or as otherwise noted.[11] Table 1 shows the cal-

[11]Since Adam and Weber (2019, 2023) did not address markup variations and endogenous growth, calibrating parameters to fit the product life cycle using observed life cycle data was sufficient for those

Parameter	Value	Target
β	0.997	Standard (annual: 0.99)
γ	17.1	SD of markup distribution: 12.6%
θ	3.98	Avg. markup: 20%
α	0.49	Higo and Saita (2007) (monthly: 0.79)
ϵ	0.0073	Average annual prod. growth: 0.263%
ζ	2.0	Standard
ϕ	2.0	Standard
c_1	1.0	Normalized
c_2	1.26	Higo and Saita (2007): Exit rate = 8.99%
c_E	0.62	Decline in price one year after introducing: 3.09%

Table 1: Calibration results

ibration results. There were 10 parameters to calibrate. Among them, for the R&D cost to be an innovator, $c_1 = 1$ serves as a normalization. The discount factor β is set to 0.997 as a standard value. The R&D cost function is assumed to be a quadratic function, i.e., $\zeta = 2$, as in previous studies; α is derived from previous empirical work pertinent to the Japanese economy, including Higo and Saita (2007). As for the other five parameters, we calibrated them using (i) the general tendency of changes in the prices for the first year after entry, (ii) the exit rate, (iii) the average markup, (iv) the variance of markups, and (v) the labor productivity growth.[12] To replicate the product price life cycle, which is associated with the first target, we referred to three previous studies: Abe et al. (2016) for durable goods, Dong et al. (2019) for non-durable goods, and Adam and Weber (2023) for services. Abe et al. (2016) computed the life cycle of electric appliances using the online shopping platform Kakaku.com between December 2012 and December 2015. We used the computation results for air conditioners, refrigerators, laundry, rice cookers, vacuum cleaners, microwave ovens, air purifiers, car navigation systems, TVs, printers, VCRs, laptop PCs, desktop PCs, and cameras and aggregated them using 2020-based consumer price index weights. We converted to relative terms using the CPI for durable goods in the corresponding periods. The dataset of Dong et al. (2019) was constructed using the POS scanner data of the Japanese Nikkei data, which includes transactions in supermarkets. The data

studies. However, unlike their work, our model does have implications for markups and growth, which necessitates additional calibration targets.

[12] γ, θ, ϵ, c_2, and c_E are jointly determined, meaning that the data targets shown in the table do not have a one-to-one correspondence with the parameters: however, the five moments play a crucial role in identifying their values. Please see Appendix B for the data description of the targets.

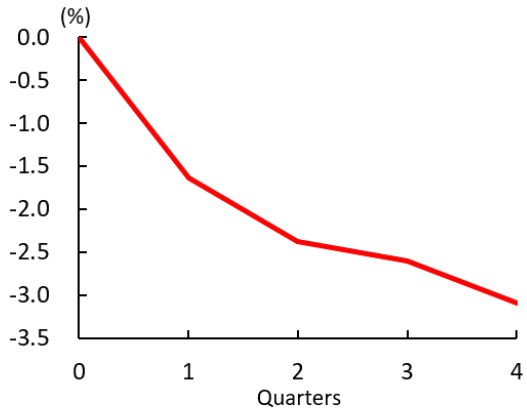

Figure 1: Product life cycle of prices up to fourth quarter taken from the data

contain information on prices and sales quantities for each product at each retail shop for every day from April 1988 to December 2017. We converted this to relative terms, as well. Finally, because we did not have data on the life cycle of services in Japan, we used the UK data shown in Table 5 of Adam and Weber (2023). The sample period extends from February 1996 to December 2016. We computed the average change in relative prices, corresponding to g_z/q_z in their model, using the ratio of the first column and fourth column of their table. Since value indicates only the average decline of prices over the life cycle periods, we assumed that the change is constant throughout the life cycle. We then aggregated the values for durable goods, non-durable goods, and services using 2020-based consumer price index weights. Figure 1 shows the product life cycle of prices derived from the data. Note that we used the decline in product price one year (four quarters) after the introduction of the product as a target for calibration.

4.2 Product Life Cycle of Key Variables for Innovators

Figure 2 shows the typical life cycle of the products produced by innovators for four key variables: relative price p_t/P_t, absolute price p_t, the gap between actual and optimal prices p_t/p_t^*, and real output y_t.[13] To highlight the impact of inflation, the cases in which the trend inflation rate is 2% and -2% are depicted.

[13]Note that there are an infinite number of sectors in the economy. Each sector differs in terms of when it last faces creative destruction, when firms in the sector change their prices the last time, and when a follower enters the sector, if one does enter. To showing the entire picture of the product life cycle, we took the average time path of the variables across sectors.

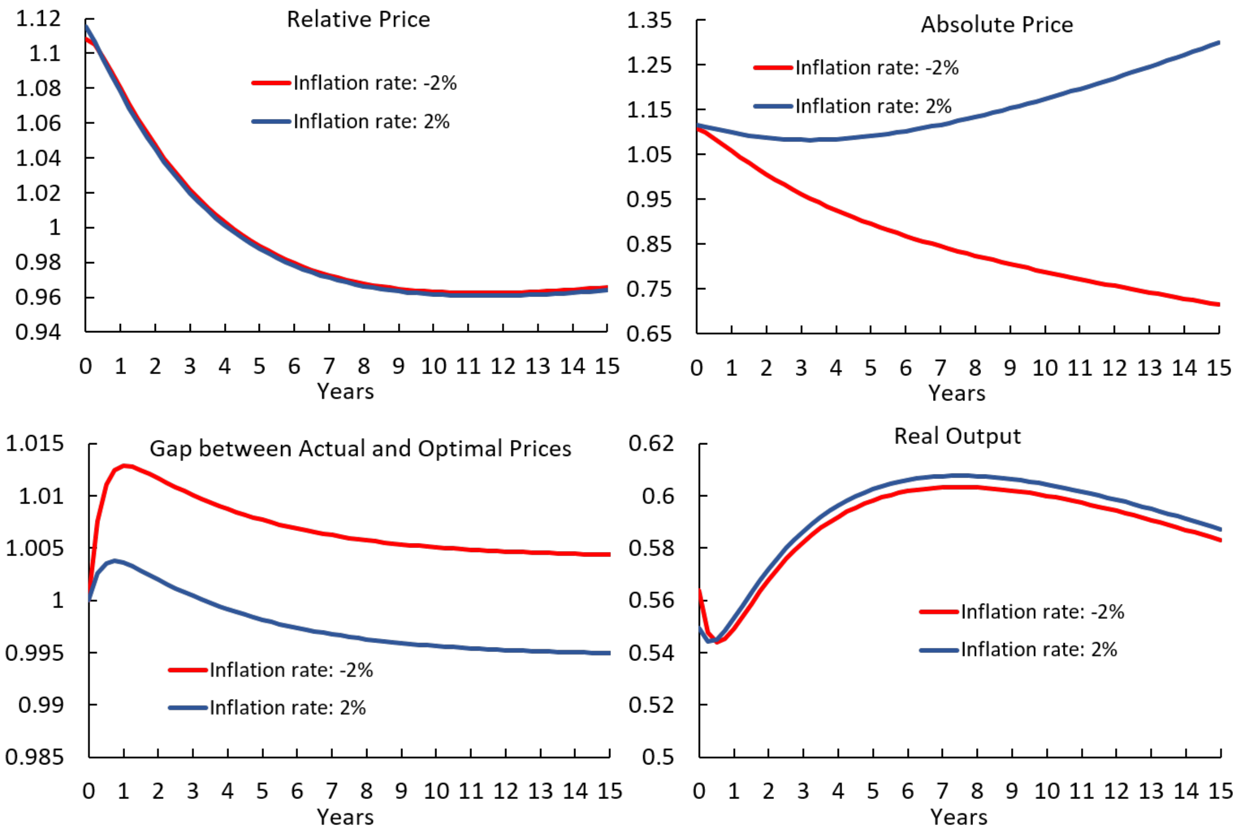

Figure 2: Life cycle of innovators' products

Note: The typical life cycle of the products produced by innovators for the key variables: relative price p_t/P_t (top-left), absolute price p_t (top-right), the gap between actual and optimal prices p_t/p_t^* (bottom-left), and real output y_t (bottom-right).

There are two driving forces behind the development of the product life cycle of the variables: the competition between innovators and followers and the trend inflation rate. The relative price of an innovator's goods (top-left graph in Figure 2) is high at the time the innovator enters the market with creative destruction. At this point, the innovators are monopolists, setting their product prices high so that their profits are maximized. These prices then start to decline for both cases of the trend inflation rate, consistent with the empirical observations noted in existing studies, including Adam and Weber (2023). These declines occur because of the entry of followers into the market. With followers in the market, the innovators reduce their relative prices whenever they can so as to stimulate demand and attract buyers who would otherwise seek out the products of followers. Approximately a decade after the entry of a follower, the downward pressure on the innovator's profits ceases, as opportunities to adjust prices arise and both innovators and followers set the same price.

The effects of the two forces are most clearly seen in the product life cycle of the absolute price (top-right graph in Figure 2). When the inflation rate is positive, it offsets the decline in the relative price and innovators enjoy a modest change in prices in absolute terms. In contrast, a negative inflation rate induces innovators to rapidly reduce the absolute prices. Therefore, as shown in the bottom-left graph in Figure 2, with nominal rigidity, the gap between the actual and optimal prices is close to 1 when the inflation rate is positive but clearly deviates from 1 when the inflation rate is negative, leading to losses.

The effects can also be seen in the dynamics of the output of the innovators (lower-right graph in Figure 2). Output is low in the periods immediately following the entry of an innovator since innovators behave as monopolists. Output increases in subsequent periods as followers enter the market and the market becomes a duopoly. Here, output is generally higher in an economy with a positive inflation rate compared to one with a negative inflation rate, due to the losses noted above.

4.3 Effect of Trend Inflation on Key Variables

The degree of competition between innovators and followers is also affected by the trend inflation rate. In this subsection, we examine how the trend inflation rate affects the market condition and the key variables.

4.3.1 Effects on the Firms' Profits

Figures 3 and 4 show the relationship between the trend inflation rate and the profits and relative prices of firms for four states that differ in terms of the competition condition in the market. The states include a monopoly condition, where only an innovator exists in the market, and a duopoly condition, where an innovator and a follower co-exist and compete in the market, with this latter state being further divided into two states, duopoly I and duopoly II, based on whether the two firms set different prices (duopoly I) or the same price (duopoly II). Note that in a duopoly market, the product prices of the two firms are different at the time a follower enters the market and remain so during a period when an existing innovator is unable to adjust its price (the duopoly I state). The prices then converge once the innovator has a chance to change

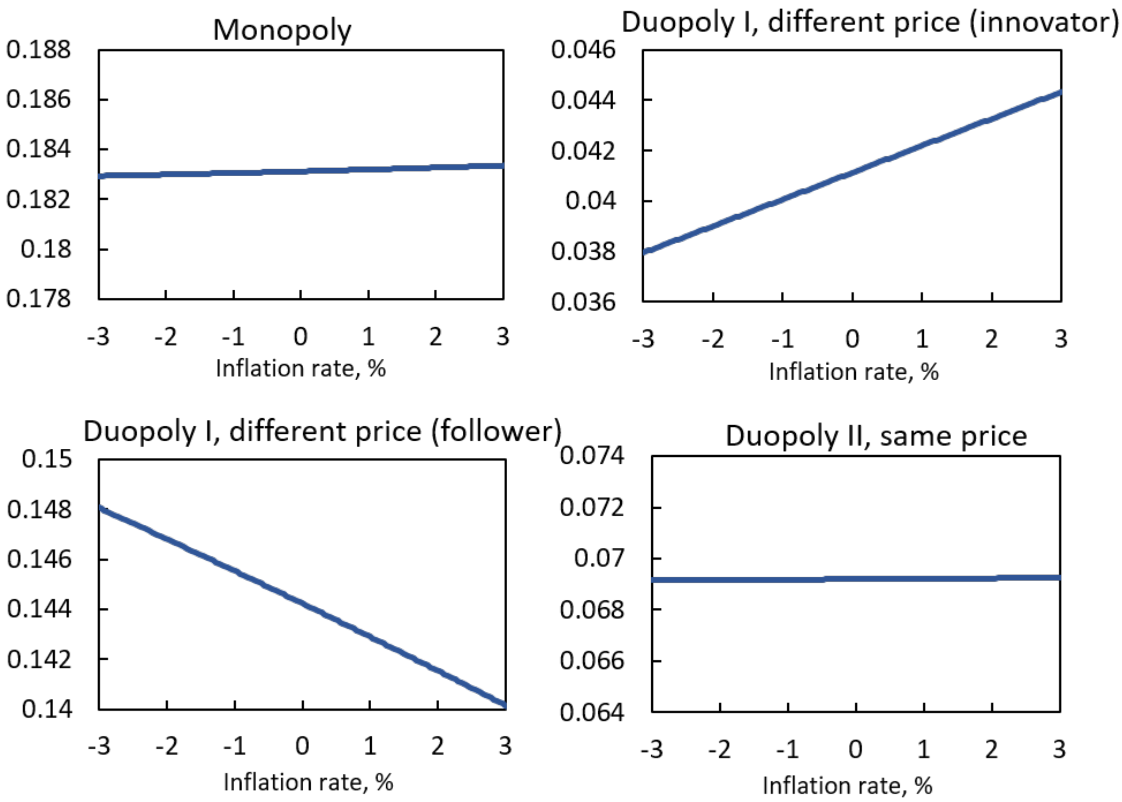

Figure 3: Effect of inflation on average profit of firms by competition status

Note: The relationship between the trend inflation rate and the firms' profits for four states: monopoly (top-left), duopoly where the firms set different prices for innovators (top-right) and for followers (bottom-left), and duopoly where the two firms set the same price (bottom-right).

its price. Clearly, when product prices converge, the profits of the producing firms also converge (the duopoly II state).

When innovators first enter the market with creative destruction, they act as a monopolist, setting a high price and earning the highest profits, as can be seen in a comparison of the top-left graphs in Figures 3 and 4 and the bottom two graphs. The innovators then migrate to the state shown in the top-right graphs after the entry of a follower until they can adjust their prices. The innovators continue on to a high price and earn smaller profits, as depicted at the top-right graphs, as the market changes from monopoly to duopoly. The profits and prices of followers at the time of entry and before innovators can adjust their prices is shown in the bottom-left graphs. Notice that the prices set by followers are lower than those set by the innovators regardless of the trend inflation rate. The followers attain higher profits than the innovators because they optimally set their product prices lower than the innovators under the

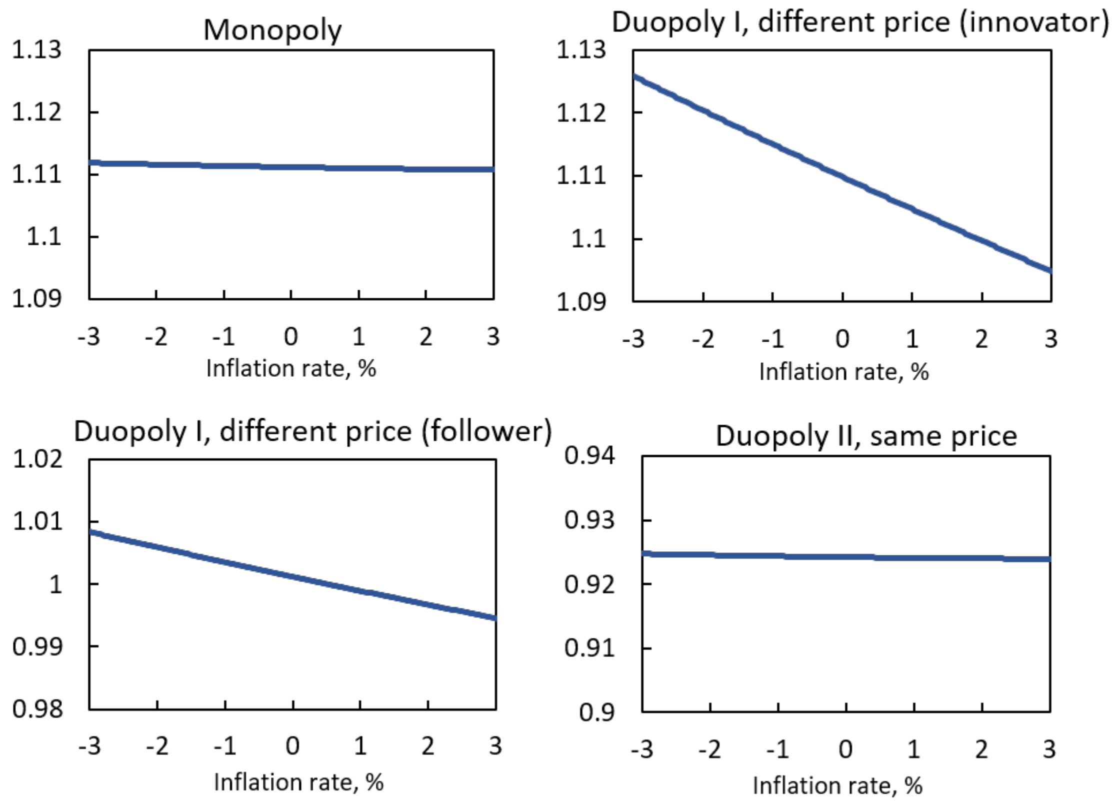

Figure 4: Effect of inflation on relative price by competition status

Note: The relationship between the trend inflation rate and the firms' relative price for four states: monopoly (top-left), duopoly where the two firms set different prices for innovators (top-right) and for followers (bottom-left), and duopoly where the two firms set the same price (bottom-right).

assumption that they can freely set the price at the time of entry. Both the innovators and followers then migrate to the state depicted in the bottom-right graph after Calvo shock occurs and the innovators renew their prices. There, the prices set by the firms and their profits converge.

As seen in these two figures, the trend inflation rate does not substantially affect firm profits and prices when a sector is a monopoly market (top-left graphs) or a duopoly market with the two firms setting the same price (bottom-right graphs). By contrast, as shown in the top-right and the bottom-left graphs, in the other two states, the trend inflation rate affects prices rather markedly, thus changing the profits of innovators and followers asymmetrically. In terms of profits, a higher trend inflation rate works favorably for innovators and unfavorably for followers.

The mechanism behind the asymmetric impact of the trend inflation rate on innovators and followers can be described as follows: When the trend inflation rate is positive, for example, since the absolute prices of the innovators' products are adjusted only sluggishly due to nominal rigidity, the relative prices of their products decline over time, which in turn mitigates a decline in innovator profits caused by the entry of followers. While the innovators' prices remain higher than the prices of followers, as shown in the top-right and bottom-left graphs of Figure 4, innovators do not experience a substantial loss in demand for their products, even if they are unable to reduce their absolute prices. This is because the price differential between the products remains relatively small. By contrast, such a decline in the relative prices does not occur when a zero or negative trend inflation rate prevails in the market. Indeed, as shown in Figure 4, when the trend inflation rate is negative, the relative price of an innovator's product in the duopoly I state becomes higher on average than its price in the monopoly state due to nominal rigidity. Under a negative inflation rate, although followers set higher prices compared to their prices under a positive trend inflation rate, the price differential between the products grows, which implies that the demand differential between the products becomes larger, resulting in a larger loss in profits from the innovator's perspective.

It is also important to note that the average profits of firms in the competition states depicted in the top-left and bottom-right graphs are not completely independent from the trend inflation rate, although the quantitative impacts are relatively minor. In the bottom-right graph (duopoly II) of Figure 3, there is a negative trend inflation rate value that produces the largest profits in that competition state. While real wages grow at a positive rate in the economy, a deviation from this value results in a nonzero growth rate in the nominal wage, generating distortions due to price stickiness, as discussed in Oikawa and Ueda (2018). In the state depicted in the top-left graph (monopoly) of Figure 3, it is increasing with respect to the inflation rate. This relationship reflects the precautionary motive of innovators to set a lower price than the ideal price in that state. If there were no risk of declining markups due to a follower's entry, monopolistic firms would set the price close to the ideal; thus, a negative inflation rate would be optimal with respect to innovator profits with nominal rigidity due to the same mechanism as in duopoly II. However, with a positive probability of declining markups, these firms are motivated to set their actual prices lower than they would otherwise in order to mitigate a decline in profits at the time a follower enters the market.[14]

4.3.2 Effects on Productivity Growth and Efficiency

Next, given the asymmetric effects of trend inflation on innovator and follower profits, we examine the effects of the trend inflation rate on the productivity growth rate $g_{\bar{A}_t} = \bar{A}_t / \bar{A}_{t-1}$. Figure 5 illustrates the effect of the trend inflation rate on the probability of market entry as either an innovator or a follower (left graph) and its impact on productivity growth (right graph). The left graph shows that the probability of firms entering the market as an innovator increases with the trend inflation rate, while the probability of entering the market as a follower decreases. As we show above, a positive trend inflation rate works favorably for innovators in the monopoly or duopoly I state and unfavorably for followers in the duopoly I state. This provides an added incentive for firms to enter as innovators rather than followers when they choose their type of R&D activities. Consequently, they will choose a larger value for σ_1 and a smaller value for σ_2 compared to the case in which the inflation rate is zero or negative. As illustrated in the right graph, a rise in the R&D activities for becoming an innovator due to a higher trend inflation rate promotes more creative destruction, which leads to a higher productivity growth rate for the economy as a whole, as is shown in equation (18).

4.3.3 Effects on Markups

Figure 6 depicts the effect of the trend inflation rate on variables related to the degree of competition in the economy. The top-left graph shows that the effect on the aggregate markup, computed as the sum of the markups set by firms in every sector of the economy. The aggregate markup is a monotonically increasing function of the trend inflation rate. Note that, as seen above, with a higher trend inflation rate, firms allocate more resources when conducting R&D activities to become innovators rather

[14] A positive inflation rate works in line with the motivation of such innovators. The relative prices of innovators approach the price they would set if they were able to adjust their prices flexibly in the duopoly II competition state, mitigating the potential decline in profits. This favorable effect of a positive trend inflation rate on innovator profits outweighs the adverse effects of price dispersions associated with a rise in the real wage.

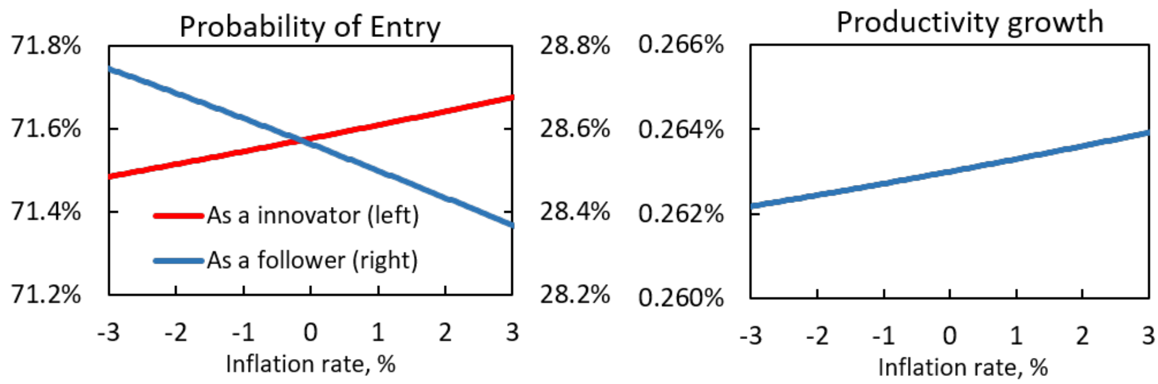

Figure 5: Effect of inflation on some statistics related to productivity growth

Note: The relationship between the trend inflation rate and rate on the probability of market entry as either an innovator or a follower (left graph) and its impact on productivity growth (right graph)

than followers, which in turn implies that a larger number of firms enter the economy as innovators. In the economy as whole, therefore, a larger portion of sectors become monopolies rather than duopolies, as shown at the top-right graph. Though, as shown in the bottom-left graph, neither the average markup of innovators nor that of followers increases monotonically with the trend inflation rate, the effect that arises from the share of monopolies quantitatively dominates, which leads to higher markups under higher inflation. It is noteworthy that with a larger portion of monopolistic sectors in which a higher markup prevails, other things being equal, the economy on average becomes less efficient. Therefore, as shown in the bottom-right graph, the level of aggregate labor productivity falls with the trend inflation rate.

There are substantial variations in firm markups in this economy. Figure 7 shows the probability density of markups when the trend inflation rate is the calibration baseline, 0%. While the average markup is around 20%, the level of markup significantly varies across firms. These variations stem from three factors. The first is the nominal price stickiness. As in the standard New Keynesian model, in every period a fixed share of firms are unable to adjust their product prices even if their current prices do not achieve desirable markups, leading to variations across the prices of firms depending on when the firms last adjusted their prices. The second factor is the difference in market condition. Obviously, monopolistic firms enjoy higher markups than firms in a duopoly market, and a change in the share of the monopoly market affects variations

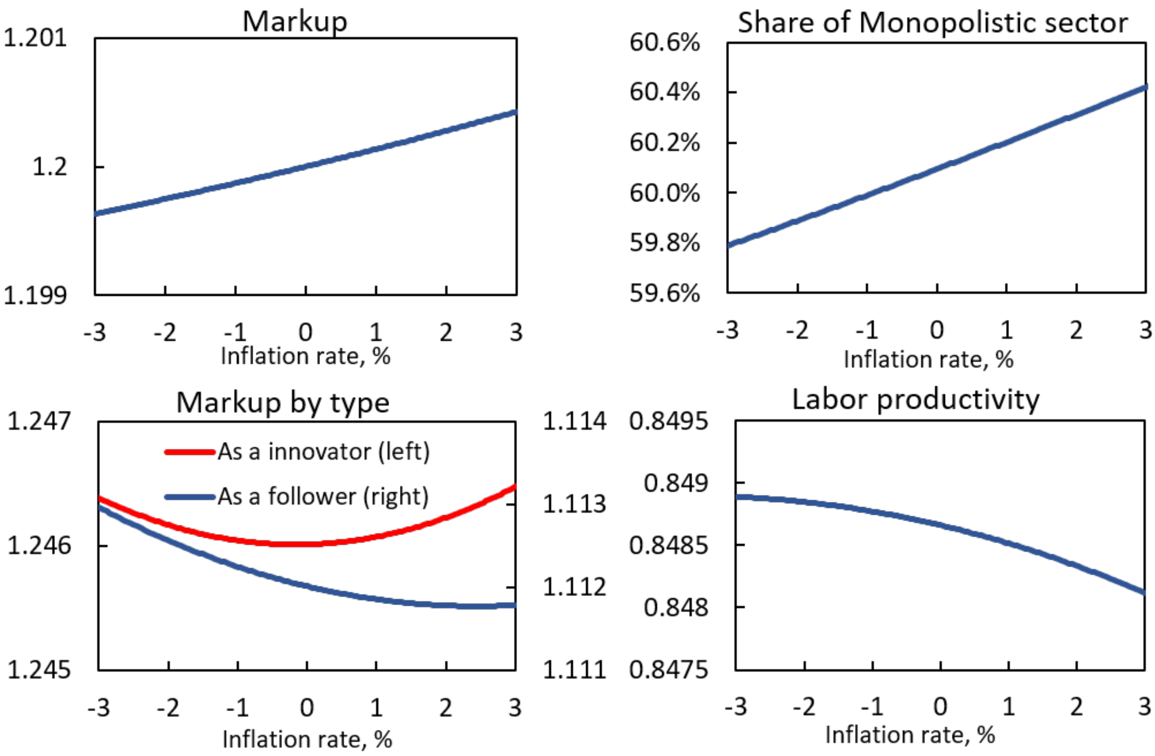

Figure 6: Effect of inflation on measures of markups

Note: The relationship between the trend inflation rate and the aggregate markup (top-left), the share of monopolistic sectors (top-right), the markup as either an innovator or a follower (bottom-left), and the level of aggregate labor productivity (bottom-right)

in the firms' markups. The third factor is the variation in productivity across sectors. Creative destruction occurs randomly across sectors and as implied by equation (18), which generates an invariant distribution of productivity as described in equation (19). The productivity level therefore differs across sectors depending on how frequently creative destruction occurs, which in turn acts as an additional source of variation in markups.[15]

4.4 Optimal Inflation Rate

The quantitative analysis detailed above indicates that there are four principal mechanisms through which a higher trend inflation rate affects social welfare in the economy.

[15]The two-humped shape shown in the figure, which derives from the second factor, stems from the simplification in which $n(S) \leq 2$. The fact that this shape is not consistent with the data does not affect the macroeconomic implications. The shape of the distribution will be smoothed in the more general case that includes $n(S) > 2$.

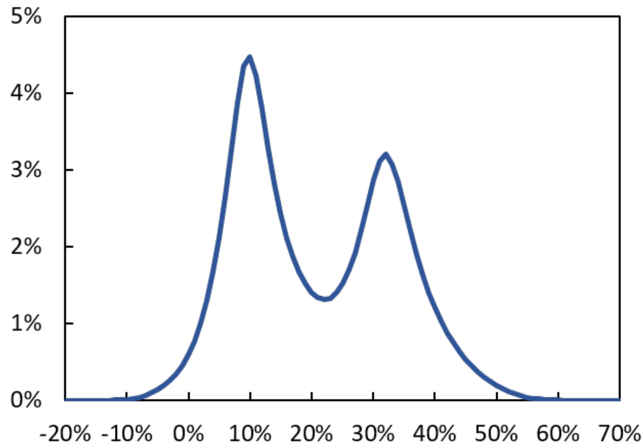

Figure 7: Model result of probability density of markups

- Channel 1: Declining markup of innovators
- Channel 2: Price dispersions associated with nominal wage growth
- Channel 3: Higher productivity growth due to creative destruction
- Channel 4: Inefficiency due to an increase in the share of monopolistic markets.

Channel 1: Declining Markup of Innovators

The trend inflation rate affects a change in the markups of innovators through the product life cycle, particularly in the duopoly I state, as seen in Figures 4 and 3. For positive trend inflation rates, the relative prices of innovators are lower and profits are higher than in the case of negative trend inflation rates, implying that the gap between the actual price and the optimal price, i.e., the price innovators would set if they are able to adjust their prices in the period, is lower compared to the case in which negative trend inflation prevails. In other words, other things being equal, the costs associated with the nominal rigidity are smaller under positive trend inflation rates.[16] Admittedly, as shown in the figures, positive trend inflation rates do not work favorably for followers, and the net gains from positive trend inflation rates through

[16]From a different point of view, if we consider an economy that consists only of firms with no risk of declining markups, the optimal trend inflation rate would be zero. Conversely, if we consider an economy that consists only of firms that see successive declines in ideal markups every period, determined exogenously, the optimal trend inflation rate would be the inverse of the change in ideal markups. One way of framing the argument here is that the optimal trend inflation rate in an economy like that in our model, which includes a mixture of the two types, is positive but not as large as that claimed in the second economy.

this channel (i.e., channel 1) depend on the relative size of the distortions facing both firms or, equivalently, the distortions facing households. Notice that the duopoly I state develops only if nominal rigidity is present. In other words, without nominal rigidity, both innovators and followers would set the same prices as in duopoly II, and distortions would arise from the fact that prices differ in the duopoly I state. As depicted in 4, while price differentials for the products produced by innovators and followers are present regardless of the trend inflation rate, they are more pronounced when the trend inflation rate falls below zero. Such differentials generate distortions in household spending for innovator products vs. follower products, resulting in a net loss from a household welfare perspective.

Adam and Weber (2019, 2023) argued that optimal trend inflation is positive based on similar reasoning. In their model, if the trend inflation rate is zero, then the prices that would be set would fall over time because of positive productivity growth, contrasting with our model, in which these drops in prices occur due to the entry of followers.[17]

Channel 2: Price Dispersions Associated with a Rise in Aggregate Labor Cost

A higher trend inflation rate also affects firms by changing the nominal growth rate of their production input prices, i.e., wages. In our economy, as productivity grows, the real wage grows at the same rate, which in turn implies that, other things being equal, nominal wages grow at a non-zero rate so that all the firms, except for a new entrant, must increase their prices over time. If, on the other hand, a negative inflation rate prevails around a rate close to the inverse of the real wage growth rate, firms are no longer forced to adjust their prices continuously over time because the optimal nominal wage growth will be close to zero. Therefore, so long as the real wage growth rate is positive, $g_{\bar{A}} > 1$, a negative trend inflation rate is favorable from the perspective of saving the costs of price adjustments. This mechanism is close to the mechanism discussed in Oikawa and Ueda (2018), although price rigidity appears in the form of

[17] As they describe, "the inflation rate can be used to ensure that relative prices between the set of price-adjusting firms and the set of non-adjusting firms are optimal."

menu costs in their model, which differs from our setting.[18] [19]

Channel 3: A Rise in Aggregate Labor Productivity Growth Due to Creative Destruction

One unique feature of our model is that there is an interplay between product life cycle and R&D activities, as shown in Figure 5. Because of the first channel, a positive inflation rate benefits innovator profits through the product life cycle. Therefore, with a higher trend inflation rate, gains from entering the market as an innovator rise and gains from entering the market as a follower fall. Because firms allocating their R&D resources for the two options take these gains into consideration, the share of entrants opting to become innovators becomes larger.[20] With more entrants choosing to become innovators, creative destruction occurs more frequently, and with more creative destruction, productivity grows faster, as shown in the right graph of Figure 5.

Channel 4: Inefficiency Due to Low Competitive Environment

When more of the firms choose to be innovators, the proportion of sectors that are monopolies increases at equilibrium. As shown in Figure 6, firms in monopoly sectors set higher markups compared to firms in duopoly sectors, which dampens the production efficiency of the sector. While such a tilt promotes a higher productivity growth rate, it reduces the aggregate productivity level. This channel therefore favors a more negative inflation rate.

Table 2 summarizes the characteristics of the four channels from the perspective of whether a positive or negative inflation rate is desirable and whether the choice is

[18] While there are several ways of modeling price stickiness in the literature, they all agree that price stickiness incurs some form of costs.

[19] Mineyama et al. (2022) introduced the assumption of a menu cost of price settings not only for incumbents but also for new entrants and showed that the optimal trend inflation rate becomes higher than the values advocated in Oikawa and Ueda (2018) due to the mechanism by which higher inflation increases the share of innovative firms in the economy.

[20] The discussion here is not specific to the case where the maximum number of products in a sector is 2; rather, it can be applied to more general cases in which innovators can face a greater number of subsequent markup declines due to the entry of a greater number of followers. In the general case allowing $n(S) > 2$ while followers also face the risk of subsequent markup declines, the markup declines are most pronounced when the number of firm transits from $n(S) = 1$ to $n(S) = 2$ to and becomes gradually less pronounced for entries beyond 2.

related to the product life cycle or the reallocation of entrant types. Indeed, since the sign of the effect of the four channels differs among the channels, whether the optimal trend inflation rate exceeds zero is not obvious. It is important to note, however, that the empirical regularity of product prices generally declining after introduction suggests that the combined effects of the first two channels, channels 1 and 2, support a positive inflation rate. This is because given a declining pattern of product relative prices over the life cycle, as in Figure 1, the distortions associated with nominal rigidity can be reduced by introducing a positive trend inflation rate so that the absolute prices of the products are flat throughout the product life cycle. In other words, given the declining trend for relative product prices over the life cycle, the optimal trend inflation is positive when the benefits from creative destruction (channel 3) outweigh the losses from a monopolistic environment (channel 4).

	Positive	Negative
Product life cycle	Channel 1	Channel 2
Reallocation of entrant types	Channel 3	Channel 4

Table 2: Summary of four channels

Note: The table indicates whether a channel is related to the product life cycle or the reallocation of entrant types and whether it favors a positive or negative inflation rate.

Using this model, we can calculate the optimal trend inflation rate that maximizes household welfare. Indeed, when the model is calibrated to the Japanese economy, the quantitative result suggests that the optimal trend inflation rate is a moderately positive value. Admittedly, some elements such as the downward nominal wage rigidities or the zero lower bound of the policy rate that have been considered in the existing studies as important determinants of the optimal trend inflation rate are not incorporated in this calculation.

As described above, there are four channels through which a positive trend inflation rate affects the economy: (1) declines in markups, (2) rises in price dispersions associated with a rise in aggregate labor cost, (3) a rise in aggregate labor productivity growth due to creative destruction, and (4) inefficiency due to a low competitive environment. To quantitatively assess the impact of each channel on the level of optimal trend inflation, we conducted counterfactual simulations under three alternative settings: (a) a setting in which the entry and exit of firms occur at an exogenously fixed rate, i.e., at the rate that would prevail at the zero trend inflation rate in the baseline economy, (b) a setting in which no productivity growth occurs in the economy, i.e., step size $\epsilon = 0$, and (c) a setting in which entry and exit rates are constant, as in setting (a), and productivity does not grow, as in setting (b). Notice that alternative setting (a) is absent for channels 3 and 4, since firm entries and exits are independent of the trend inflation rates, while alternative setting (b) is absent for channels 2 and 3, as a positive productivity growth rate causes price dispersions and output growth in the baseline economy. Clearly, alternative setting (c) is absent for channels 2, 3, and 4. Given that the optimal trend inflation rate is zero without the effects of the four channels, we can decompose the quantitative impacts of each channel by comparing the optimal trend inflation rate in these counterfactual settings and the baseline setting.[21]

Figure 8 shows the decomposition of the effect of the four channels in a waterfall chart. The figure implies that the optimal trend inflation rate increases by 0.88% due to channel 1 (markup), while it falls by 0.28% due to channel 2 (wage). The figure also shows that channel 3 (growth) has the largest impact, raising the level of optimal trend inflation by 2.05%, and that channel 4 (monopoly) lowers the level by 1.28%. Thus, it can be seen that the positive effects on social welfare of positive trend inflation rates through channels 1 and 3 dominate the negative effects through channels 2 and 4.

One might expect that the sum of the effects of channels 1 and 2 should be close to the inverse of the calibrated target of the product life cycle of prices, i.e., -3.09%, as in this case the absolute prices remain unchanged throughout the product life cycle. The reason that this is not the case is because the size of the product price decline is determined endogenously by the trend inflation rate and the interplay between innovators and followers in this model. Recall the follower price-setting problem. At the time of

[21]Channel 4 is present in both the baseline economy and setting (b), but precisely speaking, the size of the effect through channel 4 differs between the two cases, as the difference in the step size of innovation influences the choice between the two. In our model, the expected productivity level as an innovator is closely linked to the step size of innovation at entry, since, other things being equal, a higher step size implies a higher productivity level. For the purpose of measuring the difference of the effect through channel 4 between the baseline model and setting (b), we compared the optimal trend inflation rate in setting (b) and in one other alternative setting in which the innovation takes place with zero step size but the firms mistakenly perceive the step size as positive, as in the baseline. The two settings are therefore the same in terms of the effect on the aggregate productivity growth rate, but are different in terms of the incentive of becoming a follower or an innovator, which affects the share of monopolistic markets in the economy. We found that the quantitative difference in the optimal trend inflation rate between the two settings is less than 0.01%, which implies that the effect through channel 4 between setting (b) and the baseline is quantitatively minor with the current calibrated parameters.

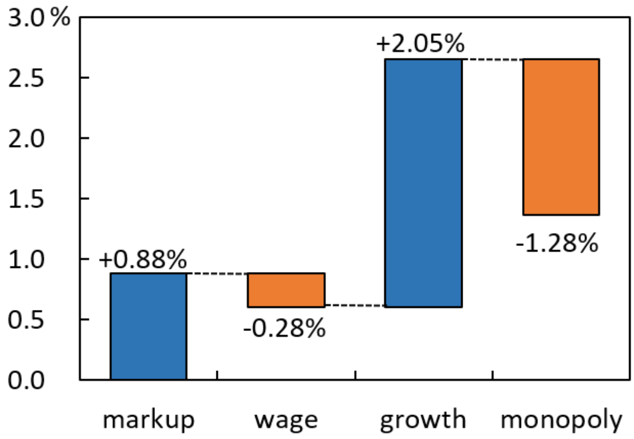

Figure 8: Decomposition of optimal trend inflation rate into the four channels

Note: Markup, wage, growth, and monopoly correspond to channels 1, 2, 3, and 4, respectively. The x-axis shows the optimal trend inflation rate. The sum of the values for the four channels is the quantitative result of the optimal trend inflation rate.

entry, namely in the state of duopoly I, followers initially set a high price but not one so high that it would cause the loss of a large portion of demand. How high a follower sets its product price depends on the price set by the innovator. When the actual price set by an innovator is higher than its optimal price due to nominal rigidity, a follower will set a higher price so as to increase its profits, which implies a larger price decline for both innovators and followers moving from the duopoly I state to duopoly II.[22]

4.5 Robustness Check

Admittedly, the value of the optimal trend inflation rate could vary when key parameter values are altered. Figure 9 illustrates the sensitivity of the optimal trend inflation rate to changes in the parameter values. Here, we computed the quantitative impacts of a 10% change in six key parameters: price elasticity of substitution within a sector γ, price elasticity of substitution across sectors θ, innovation step size ϵ, the cost parameter for being a follower c_2, entry cost c_E, and the Calvo parameter α. As shown in Figure 9, an increase in γ and ϵ raise the optimal trend inflation rate, while an increase in θ and c_E makes it lower, compared with the baseline setting. Changes in c_2 and α

[22]It is also important to note that we calibrated the pace of the product life cycle of product prices in the model using the data for the first year, which is generally greater than the pace seen in the second year and beyond.

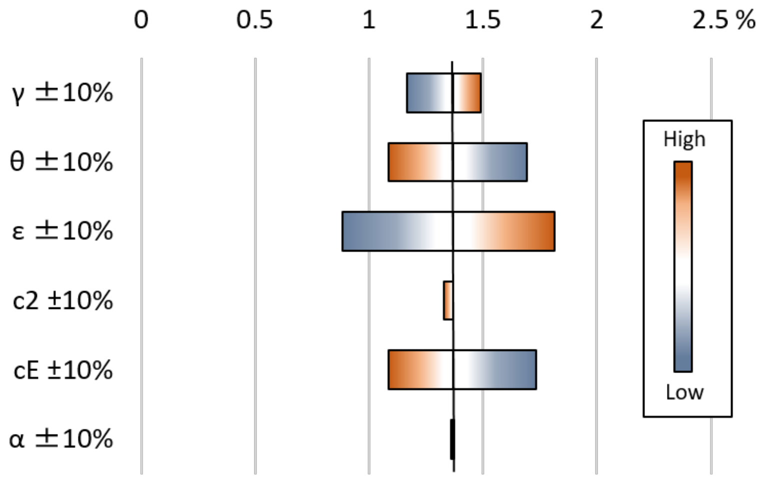

Figure 9: Optimal inflation rate and its sensitivity to some model parameters

Note: Quantitative impacts of a ±10% change in six key parameters: price elasticity of substitution within a sector γ, price elasticity of substitution across sectors θ, innovation step size ϵ, the cost parameter for being a follower c_2, entry cost c_E, and the Calvo parameter α.

have a relatively minor impact on the optimal trend inflation.[23]

Increase in γ Households become more price sensitive for products within a sector, which implies that the average markup is reduced and the difference between the markups that prevail in a monopoly and those in a duopoly widens. In other words, within a sector, the drop in relative prices due to the entry of a follower becomes greater, calling for a higher positive trend inflation rate to stabilize the absolute value of product prices over the life cycle through channel 1. With a positive trend inflation rate, a change in this parameter makes becoming an innovator more attractive, encouraging more creative destruction and promoting higher productivity growth through channel 3.

Increase in θ Households become more price sensitive between sectors; consequently, the average markup falls and the difference of markups between monopoly and duopoly sectors shrinks. Drops in relative prices due to the entry of followers become smaller,

[23]In the following sensitivity analyses, we explain the results based on dominant factors. Consequently, we only mention channels 1 and 3, but channels 2 and 4 also play a roll. For example, more creative destruction increases not only productivity growth (channel 3), but also the wage growth (channel 2) and the share of monopolistic firms (channel 4).

meaning that a lower trend inflation rate is necessary to stabilize the absolute value of product prices through channel 1. Thus, gains from stabilizing the product cycles of prices within a sector are reduced and those from containing price dispersions across sectors become higher, leading to a greater need for a low trend inflation rate. In addition, a change in this parameter makes the option of becoming an innovator less attractive, leading to a lower productivity growth through channel 3.

Increase in ϵ Productivity growth is promoted by increasing the step size per creative destruction, enhancing the benefits of creative destruction, which increases the optimal trend inflation rate through channel 3.

Increase in c_2 More innovator entrants are admitted at equilibrium, which, other things being equal, makes the product life cycle of relative prices more gradual, as fewer innovators face the entry of followers. This effect decreases the optimal trend inflation through channel 1. On the other hand, with a larger portion of firms becoming innovators, the frequency of creative destruction becomes higher, which increases the optimal trend inflation rate through channel 3. These opposing effects result in minimal changes to the optimal trend inflation rate for changes in c_2.

Increase in c_E An increase in costs reduces the number of entrants to the market, thus hampering productivity growth through channel 3. Consequently, the optimal trend inflation rate falls.

Increase in α A higher degree of price stickiness somewhat discourages innovation by increasing the risk of not being able to adjust prices for innovators when followers enter the market. This makes the optimal trend inflation rate modestly more positive, since a higher inflation rate can mitigate the innovators' profit losses, which in turn encourages more creative destruction and a higher productivity growth rate through channel 3.

5 Conclusion

Based on the notion of a product life cycle of prices and the role of markups in economic activities, this study investigated theoretically the optimal trend inflation rate by examining the R&D choices of firms seeking to enter a market, the product life cycle, and the interplay between the two. We constructed and analyzed a simple and tractable sticky price endogenous growth model in which firms choose the relative size of two possible types of R&D investment based on whether the firm intends to become an innovator or a follower once their investment is successful. Innovators innovate new products, bringing about creative destruction in the economy, and become a monopolist in their sector; followers develop products that are slightly different from those of the innovator, enter a sector, and compete with existing firms. Gains from being an innovator or a follower depend on the precise nature of the product life cycles of prices, output, and profits. We show that the level of trend inflation impacts the profits of firms over the product life cycle, which, in turn, alters the firm's decisions regarding what type of R&D activities to choose, resulting in a change in the productivity growth rate. This particular channel is novel and has not been addressed in prior studies on the optimal trend inflation rate.

Using data from Japan as a reference, we show that the trend inflation rate that maximizes household welfare could exceed zero. A positive inflation rate benefits households insofar as it tends to boost innovator profits and reduce the incentive for firms to become followers. As a result, under higher inflation rates, a larger volume of R&D activities required for firms to become innovators will be conducted and a larger amount of creative destruction will take place, which ultimately promotes a higher growth rate for the economy.

There are three caveats to be noted here. First, our model assumes that a decline in price over the product life cycle is attributable solely to declining product markups resulting from increased competition among incumbent firms and followers. In reality, however, other factors such as the learning-by-doing effect (Adam and Weber (2019, 2023)) and the fashion effect (Ueda et al. (2019)) may play a role in shaping the product life cycle. Second, our model does not comprehensively feature all the elements that are considered important in the literature on the optimal trend inflation rate. These include, for example, the gains from ensuring a positive inflation rate in the presence of an effective lower bound on the policy rate, as discussed in Coibion et al. (2012), or downward nominal wage rigidities The inclusion of these elements into the model could change the numerical implications of our model. Third, our quantitative exercise is calibrated to the Japanese economy. Parameter values such as the cost of R&D activities or the markup on a firm's products will clearly differ across regions and times. Augmenting the current model with these elements and further exploring the quantitative role of our model's parameters using data from other countries and periods will be an important part of our future research.

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Appendix A Proof of Proposition 1

From equation (18), the productivity of a sector in the next period A' given current productivity A is

$$A' = A\left(1+\epsilon\frac{\bar{A}}{A}\right) \tag{32}$$

if creative destruction occurs, and

$$A' = A \tag{33}$$

if creative destruction does not occur. Since the average productivity in the next period $\bar{A}'$ is the weighted average of sectors with and without creative destruction,

$$\begin{aligned}
\bar{A}' &= \delta\int_0^\infty A\left(1+\epsilon\frac{\bar{A}}{A}\right)d\Phi(A) + (1-\delta)\int_0^\infty Ad\Phi(A) \\
&= \int_0^\infty Ad\Phi(A) + \epsilon\delta\int_0^\infty \bar{A}d\Phi(A) \\
&= (1+\epsilon\delta)\bar{A},
\end{aligned} \tag{34}$$

so that $g_{\bar{A}} = 1+\epsilon\delta$. Next, since the optimal price $p_i^*(S)$ is proportional to $W/A(S)$, the following equations hold:

$$\Phi(Ap_i(S \in \mathcal{S}(A)) = \Phi(\bar{A}p_i(S \in \mathcal{S}(\bar{A})), \tag{35}$$

$$\Phi(Ap(S \in \mathcal{S}(A)) = \Phi(\bar{A}p(S \in \mathcal{S}(\bar{A})), \tag{36}$$

where $\Phi(\cdot)$ represents a cdf. Therefore,

$$\begin{aligned}
P &= \left(\int_0^1 p(S)^{1-\theta}dS\right)^{\frac{1}{1-\theta}} \\
&= \left(\int_0^1\int_0^\infty p(S \in \mathcal{S}(A))^{1-\theta}d\Phi(A)dS\right)^{\frac{1}{1-\theta}} \\
&= \left(\int_0^1 p(S \in \mathcal{S}(\bar{A}))^{1-\theta}dS\right)^{\frac{1}{1-\theta}}\left(\int_0^\infty\left(\frac{1}{A}\right)^{1-\theta}d\Phi(A)\right)^{\frac{1}{1-\theta}}\bar{A} \\
&= P(S \in \mathcal{S}(\bar{A}))\left(\int_0^\infty\left(\frac{1}{A}\right)^{1-\theta}d\Phi(A)\right)^{\frac{1}{1-\theta}}\bar{A},
\end{aligned} \tag{37}$$

where $P(S \in \mathcal{S}(\bar{A}))$ can be interpreted as the aggregate price when all sectors have the same productivity $\bar{A}$. From equations (12) and (37),

$$
\begin{aligned}
Y &= \frac{L}{\int_0^1 \frac{1}{A(S)} \sum_{i=1}^{n(S)} \left(\frac{p_i(S)}{p(S)}\right)^{-\gamma} \left(\frac{p(S)}{P}\right)^{-\theta} dS} \\
&= \frac{L}{\left(\int_0^\infty \left(\frac{1}{A}\right)^{1-\theta} d\Phi(A)\right)^{\frac{1}{1-\theta}} \left(\int_0^1 \sum_{i=1}^{n(S)} \left(\frac{p_i(S\in\mathcal{S}(\bar{A}))}{p(S\in\mathcal{S}(\bar{A}))}\right)^{-\gamma} \left(\frac{p(S\in\mathcal{S}(\bar{A}))}{P(S\in\mathcal{S}(\bar{A}))}\right)^{-\theta} dS\right)} \\
&= \frac{\left[\int_0^\infty \left(\frac{A}{\bar{A}}\right)^{\theta-1} d\Phi\left(\frac{A}{\bar{A}}\right)\right]^{\frac{1}{\theta-1}}}{\int_0^1 \sum_{i=1}^{n(S)} \left(\frac{p_i(S\in\mathcal{S}(\bar{A}))}{p(S\in\mathcal{S}(\bar{A}))}\right)^{-\gamma} \left(\frac{p(S\in\mathcal{S}(\bar{A}))}{P(S\in\mathcal{S}(\bar{A}))}\right)^{-\theta} dS} \bar{A}L.
\end{aligned}
\tag{38}
$$

Note that $\left(\frac{p_i(S\in\mathcal{S}(\bar{A}))}{p(S\in\mathcal{S}(\bar{A}))}\right)^{-\gamma} \left(\frac{p(S\in\mathcal{S}(\bar{A}))}{P(S\in\mathcal{S}(\bar{A}))}\right)^{-\theta}$ corresponds to the share of output of product i in an economy with identical productivity $\bar{A}$ across sectors (see equation (10)); the denominator of equation (38) equals 1. Thus,

$$
Y = \left[\int_0^\infty \left(\frac{A}{\bar{A}}\right)^{\theta-1} d\Phi\left(\frac{A}{\bar{A}}\right)\right]^{\frac{1}{\theta-1}} \bar{A}L, \tag{39}
$$

and $g_Y = g_{\bar{A}}$, as $d\Phi\left(\frac{A}{\bar{A}}\right)$ is an invariant distribution and the amount of labor L is constant.

Appendix B Data

Product life cycle. The product life cycle of prices is the aggregate of the product life cycle of durable goods, non-durable goods, and services. We used Figure 4 of Abe et al. (2016) for durable goods, Figure 2 of Dong et al. (2019) for non-durable goods, and Table 5 of Adam and Weber (2023) for services. We used the 2020-based consumer price index weights from the Statistics Bureau of Japan to aggregate components and the CPI by product to convert absolute prices into relative prices.

Exit rate. The exit rate data come from the top left graph of Chart 11 in Higo and Saita (2007). Assuming that the hazard rate in period 30 lasts from period 31 to period 120, we computed the average lifetime and took the reciprocal. We converted monthly data into quarterly values.

Price stickyness. The parameter for price stickiness α comes from Chart 7 of Higo and Saita (2007). We computed the average for all sample periods and converted results into quarterly values.

Markup and its distribution. Regarding the level of markup, we used the average from four main papers that computed the aggregate markup in Japan: Aoki et al. (2023), De Loecker and Eeckhout (2018), Diez et al. (2018), and Cabinet Office (2023). The variance of the markup comes from Figure 3-2-2 of Cabinet Office (2023). While this is the variance of the markup across firms and would differ from the variance across products, which we focus on, it is most reflective of the product-level variance among the available data.

Productivity growth. Productivity growth was computed using real GDP from the System of National Accounts published by the Cabinet Office; the number of employed persons was taken from the Labour Force Survey published by the Statistics Bureau. We used the 2001-2020 average.