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Abstract

This paper quantitatively evaluates the long-run effects of changes in inflation on the real economy, with a focus on deflation and population aging in Japan. It develops an overlapping generations model that incorporates household demand for safe assets. The model features two channels through which a decline in inflation affects the real economy in the long run, that is, the Mundell-Tobin effect and the redistribution effect. Calibrated to the Japanese economy, the model shows that a decline in inflation does more damage to young households and impairs capital accumulation, thus reducing output and social welfare, and moreover, that the damage can be magnified by population aging. This result could provide a certain rationale for central banks to pursue and maintain a positive rate of inflation in an aging economy.

Keywords: Demographics; life cycle; deflation; Mundell-Tobin effect; redistribution; overlapping generations model
JEL classification: E21, E31, J11

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1 Introduction

Over the past few decades, the Japanese economy experienced a prolonged period of low economic growth and long-lasting mild deflation together with rapid population aging. Faced with this economic adversity, the Bank of Japan (BOJ) throughout this period continuously implemented an accommodative monetary policy, including unconventional measures, to stimulate economic activity and escape from deflation. Nonetheless, it turned out to be extremely hard for the Japanese economy to overcome the stagnant situation.[1] Given the unique phenomenon of long-lasting mild deflation in Japan, what impact this had on the Japanese economy during the period remains open to question.

While there has been a relatively large body of literature examining the long-run effects of inflation on aggregate output and social welfare,[2] to the best of my knowledge, there have not been many studies that measure the effects of deflation quantitatively using a general equilibrium monetary model.[3] The reason may be that, in the postwar era, no major countries except for Japan experienced a deflationary situation for such a long period. Another reason could be that it appears difficult to explain the cost of long-lasting deflation in commonly used models with an infinitely lived representative agent.[4]

On the other hand, there is also a view that mild deflation might be a sign of price stability and rather desirable, given the little cost involved in light of the Friedman (1969) rule, which is the most famous doctrine in monetary theory.[5] Some have stated that a

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[1] For instance, Uchida (2024) documents the compound causes and background of long-lasting mild deflation in Japan.

[2] As for literature based on a general equilibrium monetary model, Cooley and Hansen (1989), Imrohoroglu (1992), Dotsey and Ireland (1996), and others measure the steady-state welfare costs of inflation for the U.S. However, these works restrict their attention to an inflationary situation.

[3] The literature on New Keynesian economics has emphasized the role of the zero lower bound (ZLB) on nominal interest rates in discussing the optimal rate of inflation. For example, Coibion, Gorodnichenko, and Wieland (2012) argue that the cost of deflation is significant and hence the optimal inflation rate should be positive or even above 2%. In contrast, Takahashi and Takayama (2024) argue that the cost of deflation under the ZLB has been elusive for the Japanese economy.

[4] At the steady state of the representative agent model that is commonly used for macroeconomic analysis, when the nominal interest rate is zero, the real interest rate is pinned down by the subjective discount factor and the inflation rate is uniquely determined by the Fisher equation. Meanwhile, the real money balance is not uniquely determined regardless of the inflation (deflation) rate, and the economy falls into a liquidity trap, as described by Krugman (1998) and Svensson (1999). In fact, the Friedman rule remains optimal in their models, which is associated with mild deflation as long as the real return on capital is positive.

[5] As a recent example, see Cochrane (2024).

grayer economy tends to prefer deflation to inflation because elderly people, who make up the majority of voters, are basically retirees and live on asset income including nominal financial claims.[6]

Regarding the effectiveness of monetary policy, there is a view that monetary policy becomes less effective in a grayer economy. Specifically, such a view argues that, against the background of population aging and prolonged deflation in Japan, monetary policy might not be able to stimulate consumption and investment enough to overcome deflation. Behind this argument lies the conjecture that elderly people tend to prefer safe assets to risky assets, and that deflation increases the real return of safe nominal assets such as cash and deposits to a greater extent, making households and firms more willing to retain safe assets and less willing to hold risky assets.

Against this backdrop, this paper quantitatively evaluates the long-run effects of deflation on the real economy using a general equilibrium monetary model with overlapping generations (OG). Specifically, it aims at making a quantitative assessment of the impact of past deflation on aggregate output in Japan over four decades (1980-2021). In addition, given the fact that Japan has the most advanced aging population in the world, this paper provides some insights into how and how much the effects of deflation (and inflation) vary depending on population aging.[7] It is believed that Japan can serve as an interesting reference point in this regard. To this end, this paper extends a standard OG model with production technology by incorporating household demand for safe assets into the utility function and calibrates the extended model to the Japanese data.[8]

The model features two possible channels through which deflation affects aggregate output adversely in the long run for both steady states and transition dynamics: the

[6] Bullard, Garriga, and Waller (2012) develop an original argument, which they call "political economy hypothesis." It was later tested by Juselius and Takáts (2021).

[7] To reiterate, the focus of this paper is on the impact of deflation on real aggregates in an aging economy, and the inflation rate itself is taken as given exogenously in this paper. However, it is of great interest for sure how population aging can cause deflation as well as stagnant growth. Braun and Ikeda (2022), for example, study the causality theoretically by using a life-cycle framework that allows for an endogenous mechanism determining the price level.

[8] To enhance its tractability for the computation of transition dynamics, the model abstracts away the ZLB. For the role of the ZLB, see Oda (2016), who conducts steady-state analyses in a similar OG model with the ZLB. For reasonable changes in the inflation rate ranging from minus 3 to 3 percent, the results of the steady-state analyses on the cost of inflation/deflation are similar between Oda (2016) and the current paper.

Mundell-Tobin effect (Mundell [1963] and Tobin [1965]) and the redistribution effect. The Mundell-Tobin effect operates through a return differential between money and capital. The basic idea behind this effect is that lower inflation increases the real return on money holdings, thus shifting agents' demand for assets from capital to money and putting downward pressure on output. The redistribution effect focuses on the fact that a change in the inflation rate acts as a kind of taxation or subsidy on money holdings by changing the real value of money. For example, a decline in the inflation rate can transfer wealth (resources) from cash-poor agents (say, young borrowers) to other cash-rich agents (older lenders). Agents can respond differently to these wealth transfers, depending on the characteristics of their life-cycle -- for example, a marginal propensity to consume or save, whether they are of working age or are retirees, etc. -- which in turn implies that the effect of changes in the inflation rate on the economy depends on the age-specific profiles of households' money holdings. Since the 2000s, both of the above effects have received growing attention in the literature studying the sub-optimality of the Friedman rule and the distributional consequences of inflation or monetary policy for inequality.

The calibrated model of this paper can reproduce actual developments in many of the key macroeconomic variables well enough to assess the impact of deflation quantitatively by counterfactual experiments. The quantitative analysis finds that a decline in inflation likely does more damage to the young and impairs capital accumulation, thus reducing output and social welfare. It also suggests that population aging can enlarge the adverse effects of deflation, partly due to the increase in the proportion of the population of the old who have a stronger preference for money holdings, while also imposing additional burdens on the young. This result provides a certain rationale for central banks to pursue and maintain a positive rate of inflation in an aging economy. It also suggests that (unconventional) monetary policy might become less effective for encouraging households and firms to consume and invest in a grayer economy.

Related Literature

This paper is closely related to a few strands in the literature. One is associated with the optimal inflation rate and the long-run costs of inflation. The Friedman (1969) rule has been recognized as surprisingly robust in various monetary models (Chari, Christiano, and Kehoe [1996]). Since the 2000s, however, there have been growing moves to rethink the Friedman rule as most advanced countries have introduced a price stability target of around 2 percent. Several studies point to the sub-optimality of the Friedman rule, focusing on the Mundell-Tobin effect and the redistribution effect. For example, Smith (2002) and Altermatt and Wipf (2022) emphasize the role of the Mundell-Tobin effect and argue that inflation can be welfare-improving by encouraging capital accumulation and output. Using life-cycle models, Bhattacharya, Haslag, and Russell (2005), Ireland (2005), Palivos (2005), and Oda (2016) stress the importance of the redistribution effect.[9] They demonstrate the possibility that inflation makes the whole economy better off in terms of output and/or social welfare, while deflation severely damages young generations with little asset holdings. They also show that a small but positive rate of inflation would be desirable even if the Mundell-Tobin effect were absent.

There are also studies that focus on the distributional effects of unanticipated inflation along transition paths, assessing quantitatively the effects of a temporary change in inflation on the real values of nominal assets (e.g., Doepke and Schneider [2006], Doepke, Schneider, and Selezneva [2015] for the U.S. and Meh, Ríos-Rull, and Terajima [2010] for Canada). This paper differs from these others in that it investigates the effects of a permanent change in inflation both at steady states and along transition paths. In addition, the model features households' endogenous asset portfolio choices, which give rise to endogenous redistribution effects as well as the Mundell-Tobin effect. Moreover, this paper studies how the effects of inflation/deflation vary depending on population aging.

Another strand in the literature addresses possible effects of population aging on the effectiveness of monetary policy. First, the channel receiving the most focus is the downward pressure exerted by population aging on the natural rate of interest, which seems to have surfaced in the context of the secular stagnation hypothesis originally proposed by Hansen (1939) and resurrected by Summers (2013). Several studies (e.g., Carvalho, Ferrero, and Nechio [2016] for advanced countries, Eggertsson, Mehrotra, and Robbins [2019] for the U.S., and Sudo and Takizuka [2020] for Japan), which use OG

[9] Ireland (2005) introduces money into the Blanchard-Yaari model that can take account of differences in the birth date of households and the population growth rate. He argues that with population growth, the Ricardian equivalence does not hold, supplied money becomes net wealth, and thus the liquidity trap is removed. As a result, his model can identify the welfare costs of different rates of deflation even when the nominal interest rate is zero. Bhattacharya, Haslag, and Russell (2005) demonstrate the role of inter-generational transfers in breaking the Friedman rule by incorporating money into an OG model that can deal with finite lifespans as well. Palivos (2005) introduces money demand in the form of money-in-the-utility and investigates the role of the distributional effects of inflation by allowing for a difference in the preference for altruism. Oda (2016) finds that the Friedman rule does not hold and a positive rate of inflation can be desirable even if the model incorporates elastic labor supply and money through the cash-in-advance constraint into an OG model.

models, report that population aging had pushed down the natural rate of interest considerably.[10] A decrease in the natural rate and the resulting binding ZLB constrain the capacity of central banks to mitigate deflationary pressure by cutting the interest rates, as documented by Summers (2014), Brzoza-Brzezina, Kolasa, and Bielecki (2019), and others.

Second, a small but growing body of literature tackles more directly the question whether monetary policy becomes more or less effective in a grayer economy.[11] Fujiwara and Teranishi (2008), Sterk and Tenreyro (2018), and Leahy and Thapar (2022) show that monetary policy shocks have asymmetric effects on heterogeneous agents at different life-cycle stages and hence the demographic structure can alter the size of macroeconomic responses to the shocks. Yoshino and Miyamoto (2017) and Imam (2015) demonstrate that the effectiveness of fiscal and monetary policies becomes weaker with a larger proportion of retirees. Miles (2002) points out, however, that the impact of population aging may tend to be ambiguous, because there are various channels and factors through which population aging can influence monetary policy transmission (e.g., credit constraints, the pension system, etc.) and the impact is likely to vary depending on the relative importance of the channels and factors.

Finally, this paper is related to the literature on the costs of deflation, such as Baig (2003), Fuhrer and Tootell (2003), Federal Reserve Bank of San Francisco (2006), and Ueda (2003), etc. Among these, Fuhrer and Tootell (2003) summarize the costs as follows. Regarding the costs during a transition, (1) deflation may reflect weak demand, which may lead to a decrease in spending and aggregate output; (2) in the presence of wage rigidity, firms may counteract a reduction in their revenues by curbing employment; (3) as many financial contracts are written in nominal terms under inflation expectation, deflation may induce redistributions of wealth between borrowers and lenders, which

[10] As a population ages, households accumulate more savings for their increased longevity, while the labor force population shrinks. This leads to more capital stock relative to the labor force, thus exerting downward pressure on the natural rate. On the other hand, the proportion of the retirement-age population who dissave increases, exerting downward pressure on savings and upward pressure on the natural rate of interest. These studies, however, agree that the effect of workers saving more for increased longevity quantitatively dominates.

[11] In contrast, most of the literature (to give a few representative ones: Coibion et al [2017], Kaplan, Moll, and Violante [2018], and Auclert [2019]) analyzing the distributional effects of monetary policy on macroeconomic variables such as consumption, basically aims to analyze the effects of monetary policy on inequality rather than the effects of demographic structure or population aging on the effectiveness of policy.

entail increased burdens of debt and decreased values of collaterals in real terms, thus likely leading to a further rise in delinquency and default rates. Regarding the costs at a steady state, (4) all price changes may produce some distortions due to imperfectly indexed contracts, tax codes, or menu costs; (5) as tax rates are not fully indexed to prices, under deflation, the government may lose part of the revenues that it raised from income taxes as well as its seigniorage or inflation tax revenues, and may then raise other tax rates to recover the losses; and (6) in the presence of a zero lower bound (ZLB) on nominal interest rates, the central bank may not be able to lower real interest rates enough to alleviate deflationary pressure. Although a single paper could not cover all six of these effects, this paper aims at constructing a general equilibrium monetary model that takes account of effects (1), (3), and (5) to evaluate the impact of deflation on the real economy.

Outline

The rest of this paper is organized as follows. Section 2 shows some stylized facts relevant to the motivation of this paper. Section 3 presents the set-up of the model and some theoretical background of the channels in the model through which changes in inflation affect household behavior and hence real aggregates. Section 4 describes data sources and calibration methodology. Section 5 provides the simulation results of steady-state analysis and dynamic analysis. Section 6 offers a conclusion.

2 Stylized Facts

In terms of the motivation for the model to be introduced in the next section, this section presents three stylized facts about the Japanese economy, regarding population aging, household asset holdings, the return on money, and private savings-investment.

First, the Japanese economy is facing rapid population aging due to a sharp decline in the fertility rate and a rapid increase in longevity. Figure 1 gives an international comparison of demographic trends. It shows that, in Japan, the old-age dependency ratio has increased rapidly since the 1990s: from roughly 10 percent in 1990 to approximately 30 percent in 2021. The pace of this increase is also found to have been eminent in Japan compared to other G7 countries.

Second, elderly households in Japan tend to own more safe assets, in terms of not only amount but also proportion, than young or middle-aged households, as shown in Figure 2. In addition, according to a survey, older households have a higher preference for safety and liquidity rather than profitability in their asset choices, as shown in Figure 3.[12] These facts suggest that, in Japan, as the population ages, aggregate money equivalents (cash and deposits) would grow at a faster pace than risky assets, with the average (or aggregate) preference (demand) for safe assets being higher. In other words, it is possible that the economy as a whole has come to hold more money equivalents as a result of population aging.

Third, the real return on money holdings increased and often stayed positive from the late 1990s to the 2010s, as the inflation rate fell and turned negative. The former Governor of BOJ, Kuroda (2014) noted, "Under such circumstances, accumulating retained earnings mainly by cost reduction and hoarding them in cash and deposits became relatively more advantageous for firms as a form of investment than taking risks and making fixed investment." In fact, Figure 4 shows that cash and deposits held by households and firms increased substantially, while the accumulation of tangible fixed assets remained depressed.[13] Given the above second fact, it is also suggested that population aging as well as deflation may have played a role in these developments in private savings-investment.

3 Model

This section presents the model and provides theoretical overviews of how changes in the inflation rate affect household behavior and of how the effects can vary depending on population aging. The basic structure of the model is a standard overlapping generations (OG) model with production and flexible prices, which consists of the three sectors: a representative firm, the government, and households. The model features household preferences for money and bond holdings, and so three different types of assets exist:

[12] Some theoretical studies -- Cocco (2005) and Cocco, Gomes, and Maenhout (2005), for example -- argue that workers whose main income source is wages prefer risky assets while retirees whose main income source is asset returns prefer safe assets. They also argue that working-age households do not hold as many risky assets as they may desire when home-ownership and borrowing constraints are considered. Using data on U.S. household asset holdings, Poterba and Samwick (2001) and Ameriks and Zeldes (2004) show that the age-specific profile of risky asset holdings has an inverted V-shape.

[13] To the best of my knowledge, there have been few studies focusing on the relationship between inflation and asset portfolio choice. Of the few studies, Aoki et al (2024), pointing out a negative correlation between inflation and bond returns, show that higher inflation induces household demand shifts away from money and bonds to stocks which are regarded as inflation-proof assets.

money, government bonds, and capital. One period of the model corresponds to one year of data.

3.1 Firm

There is a representative firm that produces final consumption goods with a Cobb-Douglas constant returns to scale technology:

$$Y_t = Z_t K_{t-1}^{\alpha} L_t^{1-\alpha} \tag{1}$$

where Y_t and L_t are the aggregate output and aggregate inputs of labor in period t, respectively, K_{t-1} is the aggregate stock of capital that was chosen in the previous period $t-1$ and is available at the beginning of the current period t, and $0 < \alpha < 1$ is a time-invariant parameter corresponding to the income share of capital. Z_t is deterministic total factor productivity (TFP) in period t, which grows by $(1+z_t)^{1-\alpha}$ in every period.

The firm rents capital at the rental rate R_t and hires labor at the wage rate W_t from households in competitive markets. The firm's profit maximization yields the first-order conditions that determine the factor prices:

$$\max_{L_t, K_{t-1}} \; Y_t - W_t L_t - R_{t-1} K_{t-1} \; ; \tag{2}$$

$$W_t = \frac{\partial Y_t}{\partial L_t} = (1-\alpha) Z_t \left(\frac{K_{t-1}}{L_t}\right)^{\alpha} \quad \text{and} \quad R_{t-1} = \frac{\partial Y_t}{\partial K_{t-1}} = \alpha Z_t \left(\frac{K_{t-1}}{L_t}\right)^{\alpha-1} . \tag{3}$$

The real rate of return on capital is the marginal product of capital net of the depreciation of capital: $R_{t-1} - \delta_t$, where δ_t is the depreciation rate of capital. Here the after-tax real rate of return on capital r_{t-1} is defined as $r_{t-1} = \left(1 - \tau_t^k\right)(R_{t-1} - \delta_t)$.

3.2 Government

The government raises revenues by injecting non-interest-bearing money M_t into the economy according to the rule at the growth rate σ_t in every period t:

$$M_t = (1 + \sigma_t) M_{t-1} \, . \tag{4}$$

The government sets the growth rate of the money supply so as to control the inflation rate, $\pi_t = P_t / P_{t-1} - 1$, where P_t is the price level in period t. To see this point, this money supply rule (4) can be transformed in real terms by dividing both sides of the equation by the price level:

$$(1+\pi_t)\frac{M_t}{P_t} = (1+\sigma_t)\frac{M_{t-1}}{P_{t-1}} \,. \tag{5}$$

In the model simulations in this paper, the inflation rate π_t is taken as exogenously given, while the growth rate of money σ_t and the real balance of money M_t/P_t are endogenously determined.

The government also raises revenues by issuing one-period nominal bonds B_t and by levying taxes on households' labor income, capital income, and consumption at the flat rates τ_t^l, τ_t^k, and τ_t^c, to finance its expenditures, which are the sum of government purchases and social transfers. The flat rates of these taxes are taken as exogenously given. Then, the budget constraint is expressed in nominal terms as follows:

$$\begin{aligned} &P_tG_t + M_{t-1} + (1+i_{t-1})B_{t-1} + P_tSS_t \\ &= M_t + B_t + P_t\tau_t^cC_t + P_t\tau_t^lw_tL_t + P_t\tau_t^k(R_{t-1}-\delta_t)K_{t-1} + P_tT_t \,, \end{aligned} \tag{6}$$

where G_t, C_t, and T_t are government purchases, aggregate consumption of households, and aggregate lump-sum tax/transfer to all the households living in period t, respectively. G_t and B_t are basically determined by rules that set the ratios of corresponding variables to aggregate output Y_t at exogenous values derived from data. On the other hand, the lump-sum tax/transfer T_t is adjusted to satisfy the budget constraint. SS_t is the total amount of social security transfers to retirees. The real and nominal returns (x_t and i_t) on government bonds are related by the Fischer equation:

$$1 + i_t = (1+\pi_{t+1})(1+x_t) \,. \tag{7}$$

3.3 Demographics and Households

Households consist of 80 cohorts. In every period t, a new generation of age-21 households comes into the economy and the other existing generations of households become older by one year, while the oldest generations of age 100 exit from the economy in the subsequent period. Each household can be economically active for a maximum of 80 periods from age 21 to age 100, but is exposed to mortality risk in every period. Namely, households of age $j-1$ in period $t-1$ can survive to age j in period t with the conditional probability $\psi_{j,t}$, where $\psi_{101,t} = 0$ by assumption. Then the population of age-j households in period t, $n_{j,t}$, is given by the law of motion:

$$n_{j,t} = \psi_{j,t} n_{j-1,t-1} \quad \text{for} \quad j = 22, 23, \ldots, 100, \tag{8}$$

while the population of age-21 households is determined by the growth rate f_t in period t: $n_{21,t} = (1 + f_t) n_{21,t-1}$. The total population of households N_t, the population share of age-j households $\mu_{j,t}$, and the population growth rate ρ_t in period t are given by

$$N_t = \sum_{j=21}^{100} n_{j,t} \ , \quad \mu_{j,t} = \frac{n_{j,t}}{N_t} \ , \quad \text{and} \quad 1 + \rho_t = \frac{N_t}{N_{t-1}} \ , \tag{9}$$

where, by definition,

$$\mu_{j,t} = \frac{\psi_{j,t} n_{j-1,t-1}}{(1 + \rho_t) N_{t-1}} = \frac{\psi_{j,t}}{1 + \rho_t} \mu_{j-1,t-1} \quad \text{and} \quad \sum_{j=21}^{100} \mu_{j,t} = 1 \, . \tag{10}$$

Until mandatory retirement, households of age j in period t supply labor $h_{j,t}$ (hours worked), and earn labor income $\varepsilon_j W_t h_{j,t}$, where W_t is the wage rate and ε_j is the age-specific labor efficiency. The labor income is subject to the tax rate of τ_t^l. Households are assumed to retire at age 71 and thereafter receive the social security benefit ss_t from the government, which is assumed to cover a certain proportion (the replacement ratio θ) of the average labor income in the contemporaneous period: $ss_t = \theta W_t L_t$. Then, the total amount of the social security benefits (included in Equation (6)) is expressed as

$$SS_t = \sum_{j=71}^{100} n_{j,t} ss_t = \theta_t W_t L_t \sum_{j=71}^{100} n_{j,t} \ . \tag{11}$$

Each household also equally pays/receives the lump-sum tax/transfer τ_t to/from the government:

$$T_t = \sum_{j=21}^{100} n_{j,t} \tau_t = N_t \tau_t \, . \tag{12}$$

In every period, age-j households consume final goods $c_{j,t}$ in exchange for part of their income by paying consumption tax on the rate τ_t^c. They also purchase capital $k_{j,t}$, rent it to the firm at the rental rate R_t, and pay capital income tax at the rate τ_t^k. They buy government bonds $b_{j,t}$ as well, and receive interest payments at the rate i_t. Furthermore, they hoard part of their income in the form of non-interest-bearing assets, namely money $m_{j,t}$.

Although newly emerging households of age 21 enter the economy with no initial assets: $k_{21,t} = m_{21,t} = b_{21,t} = 0$, they obtain accidental (unintended) bequests as an additional source of income in every period t.[14] The government is assumed to collect all accidental bequests from households that died in period $t-1$ and redistribute them equally among all living households in period t. The total amount of accidental bequests redistributed in period t, Ξ_t, is given by

$$P_t \Xi_t = \sum_{j=22}^{101} (1-\psi_{j,t})(P_t k_{j-1,t-1} + b_{j-1,t-1} + m_{j-1,t-1}) n_{j-1,t-1} \ . \tag{13}$$

Subject to the budget constraints described below, households born in period s, whose age is j in the subsequent period $t = s - 21 + j$ for $j = 21, 22, \ldots, 100$, choose sequences of consumption $c_{j,t}$, labor inputs $h_{j,t}$, capital, government bonds, and money holdings, $\{k_{j,t}, b_{j,t}, m_{j,t}\}$ to maximize their expected lifetime utility discounted by the subjective discount factor β in every period:

$$\mathrm{U}_s = E\left[\sum_{j=21}^{100} \beta^{j-21} \Psi_{j,s+j-21} u\left(c_{j,s+j-21}, h_{j,s+j-21}, \frac{b_{j,s+j-21}}{P_t}, \frac{m_{j,s+j-21}}{P_t}\right)\right] , \tag{14}$$

where $\Psi_{j,s+j-21}$ is the unconditional probability of surviving from birth to age j:

[14] Household motives for intended bequests to descendants are not explicitly incorporated in this model, especially regarding capital stock: $k_{100,t} = 0$, which means that households that have lived up to 100 years old use up their capital stock before they die. Regarding money and bond holdings, as described later on, it is assumed that households have age-specific preferences for holding these instruments from the perspective of liquidity and safety throughout their lives, so that $b_{100,t} \neq 0$ and $m_{100,t} \neq 0$. This does not necessarily deny the possibility that some bequest motives are included, at least in the form of money and bond holdings, as long as those age-specific parameters are calibrated to the data.

$$\Psi_{j,s+j-21} = \prod_{i=21}^{j} \psi_{i,s-j+i} \ . \tag{15}$$

$u(\cdot)$ is a period utility function having consumption, hours worked, real balance of money, and bond holdings as arguments. Then, the budget constraints in nominal terms for $j = 21, 22, \dots, 100$ are as follow:

$$\begin{aligned} P_t(1+\tau_t^c)c_{j,t} + P_t k_{j,t} + b_{j,t} + m_{j,t} \ \le \ P_t \omega_t + P_t \frac{\Xi_t}{N_t} - P_t \tau_t \\ + P_t(1+r_{t-1})k_{j-1,t-1} + (1+i_{t-1})b_{j-1,t-1} + m_{j-1,t-1} \, , \end{aligned} \tag{16}$$

$$\text{where } \omega_t = \begin{cases} (1-\tau_t^l) W_t \varepsilon_j h_{j,t} & \text{for } j \le 70 \text{ (Workers)} \\ ss_t = \theta_t W_t L_t & \text{for } j \ge 71 \text{ (Retirees)} \end{cases} . \tag{17}$$

Following the convention originally proposed by Sidrauski (1967), the model introduces the real balance of money into the utility function. The utility function also includes the real balance of bonds, as is the case with Poterba and Rotemberg (1987), Krishnamurthy and Vissing-Jorgensen (2012), and Hansen and İmrohoroğlu (2016). The assumption that households obtain utility directly from money and bond holdings can be justified by the idea that they have a particular preference for liquidity and safety for which money and government bonds provide holders. In addition, this "money/bond-in-the-utility" specification possibly captures the functioning of transaction services. The degree of preference over real money and bond holdings is assumed to differ with age to capture the age-specific profiles of these holdings as shown in Figures 2 and 3.

The functional form of a household's utility is assumed to be logarithmic and additively separable[15]:

$$u\left(c_{j,t}, h_{j,t}, \frac{b_{j,t}}{P_t}, \frac{m_{j,t}}{P_t}\right) = \chi_j \log\left(\frac{c_{j,t}}{\chi_j}\right) + \gamma_{j,t} \log\left(\frac{b_{j,t}}{P_t}\right) + \eta_{j,t} \log\left(\frac{m_{j,t}}{P_t}\right) - \zeta_{j,t} \frac{h_{j,t}^{1+\nu}}{1+\nu}$$

$$\text{for } j \le 70 \text{ (Workers)} \, , \tag{18}$$

[15] This type of additively separable preference proposed by King, Plosser, and Rebelo (1998) is known to be compatible with a balanced growth path.

$$u\left(c_{j,t}, 0, \frac{b_{j,t}}{P_t}, \frac{m_{j,t}}{P_t}\right) = \chi_j \log\left(\frac{c_{j,t}}{\chi_j}\right) + \gamma_{j,t} \log\left(\frac{b_{j,t}}{P_t}\right) + \eta_{j,t} \log\left(\frac{m_{j,t}}{P_t}\right)$$

$$\text{for } j \geq 71 \text{ (Retirees)}, \quad (19)$$

where ν is the reciprocal of the Frisch elasticity of hours worked by households. To capture the actual life-cycle pattern of consumption, the utility function allows for a family scale factor, denoted by χ_j, which depends on the number of dependent children at the household head's age j. $\zeta_{j,t}$ is a time-variant and age-specific disutility parameter for labor, which represents the degree of age-j household preferences for leisure in period t. Higher $\zeta_{j,t}$ implies that households put a higher value on leisure. In addition, this parameter ($\zeta_{j,t}$) is divided into two components: a time-varying one and an age-specific one: $\zeta_{j,t} = \zeta_t \zeta_j$. The former ($\zeta_t$) is common for all the ages in period t, which is useful for capturing the influence of the institutional reductions in hours worked implemented in Japan from 1988 to 1993.[16] The latter (ζ_j) is time-invariant and useful for capturing the relative preference for leisure of a particular age, which depends on the actual life-cycle features such as schooling, parenting, and caring for family.

The parameters representing age-j household preferences for bond and money holdings, $\gamma_{j,t}$ and $\eta_{j,t}$, are also time-varying and age-specific. Higher $\gamma_{j,t}$ or $\eta_{j,t}$ implies that households place a higher value on bond or money holdings. Similarly, they are divided into two components: $\gamma_{j,t} = \gamma_t \gamma_j$ and $\eta_{j,t} = \eta_t \eta_j$. The time-varying components (γ_t and η_t) are introduced to capture various changes in the economic environment and financial conditions, including macroeconomic policy changes. The age-specific components (γ_j and η_j) may reflect the relative preference of a particular age that cannot be captured by the life-cycle features of a standard OG model with no uncertainty. For example, a difference in preference among ages may come from a difference in information technology skill or financial literacy; older agents may become poorer at financial transactions due to less skill in information technology or lower financial literacy because they are unable to catch up with progress in these areas.

In any case, the set-up of these three parameters $\{\zeta_{j,t}, \gamma_{j,t}, \eta_{j,t}\}$ including time-varying and age-specific components is a useful shortcut for adequately replicating the actual movements and life-cycle profiles in relevant variables, as shown later. Even at the

[16] For details, for example, see Hayashi and Prescott (2002).

expense of a theoretically strict foundation, this matters for making a quantitative assessment of the impact of deflation, which is the main objective of this paper.

3.4 Competitive Equilibrium

Taking the initial distribution of asset holdings (capital, bonds, and money) $\{k_{j,0}, b_{j,0}, m_{j,0}\}$ as given, a competitive equilibrium consists of sequences of prices $\{W_t, R_t, r_t, i_t, x_t, \pi_t, P_t\}$, household decisions $\{c_{j,t}, h_{j,t}, k_{j,t}, b_{j,t}, m_{j,t}\}$, government policy $\{G_t, B_t, M_t, \sigma_t, \tau_t, \tau_t^c, \tau_t^l, \tau_t^k, T_t\}$, and aggregate factor inputs $\{K_t, L_t\}$ and other aggregate quantities $\{Y_t, C_t\}$, such that

- the household decisions solve the households' utility maximization problem subject to their budget constraints,
- the aggregate factor inputs solve the firm's profit maximization problem,
- the government budget constraint and money supply rule are satisfied,
- the market clearing conditions hold:

$$L_t = \sum_{j=21}^{70} n_{j,t}\varepsilon_j h_{j,t} \quad \text{for the labor market;} \tag{20}$$

$$K_t = \sum_{j=21}^{100} n_{j,t}k_{j,t} \quad \text{for the capital market;} \tag{21}$$

$$B_t = \sum_{j=21}^{100} n_{j,t}b_{j,t} \quad \text{for the bond market;} \tag{22}$$

$$M_t = \sum_{j=21}^{100} n_{j,t}m_{j,t} \quad \text{for the money market;} \tag{23}$$

$$Y_t = C_t + K_{t+1} - (1-\delta_t)K_t + G_t \quad \text{for the final goods market,} \tag{24}$$

$$\text{where } C_t = \sum_{j=21}^{100} n_{j,t}c_{j,t} \text{ is aggregate consumption.} \tag{25}$$

3.5 Theoretical Background

In the model economy described above, changes in the inflation rate can affect real aggregates through at least two possible channels: the Mundell-Tobin effect and the redistribution effect. This subsection provides some theoretical background of how these effects can work in the model and how they can change with population aging, with a focus on the relative importance of the two effects. Specifically, deflation or a decline in the inflation rate can have adverse effects on capital stock and labor supply, and hence on output through the Mundell-Tobin effect. On the other hand, deflation or a decline in the inflation rate can put downward pressure on capital accumulation but upward pressure on labor supply through the redistribution effect. Consequently, at a first glance, it is not trivial whether deflation affects output positively or negatively in the model. Thus, it is important to use a general equilibrium model to assess quantitatively the impact on output.

Mundell-Tobin Effect

This effect operates through a return differential between money and capital. The basic idea behind the effect is simple: higher inflation reduces the real return on money holdings, thus shifting agents' demand for assets from money to capital.[17] As derived in Appendix A, the optimality conditions of the household problem yields

$$1 + r_t = \frac{1}{1 + \pi_t} \cdot \left[1 - (1 + \tau_t^c) \frac{u_m^j}{u_c^j} \right]^{-1} . \tag{26}$$

A higher rate of inflation (increasing π_t) reduces the real return on money holdings, which is captured by $1/(1 + \pi_t)$. Assuming that the marginal rate of substitution (MRS) between the real money balance and consumption (u_m^j / u_c^j) is constant, this implies that the real return on capital r_t should decrease, which is possible only when aggregate capital stock increases. Therefore, a rise in the inflation rate puts upward pressure on capital accumulation and hence output. In the case of a fall in the inflation rate, this effect works in the opposite direction.

It is worth mentioning a general equilibrium effect of a change in the inflation rate. In response to a rise in the inflation rate, the real return on capital decreases and the real

[17] There is another type of asset in this model: government bonds, whose nominal return is indexed by the inflation rate, as long as the inflation rate is fully anticipated. The same shift occurs between bonds and money.

wage rate increases. The labor supply curve derived from the household problem increases in the wage rate, so that, all other things being equal, the labor supply would increase. In the case of a fall in the inflation rate, the real return on capital rises and this general equilibrium effect acts to reduce labor supply. These labor supply responses are consistent with the complementarity between labor and capital inputs in the Cobb-Douglas production function.

From Equation (26), we notice that the effect of a change in the inflation rate possibly depends on the MRS in the term $[1-(1+\tau_t^c)MRS]^{-1}$, which corresponds to sensitivity. With the specification of the household's utility function described above, this MRS is given by

$$MRS_{m/c}^{j} \equiv \frac{u_m^j}{u_c^j} = \frac{\eta_{j,t}}{\chi_j}\frac{\tilde{c}_{j,t}}{\widetilde{m}_{j,t}} \,. \tag{27}$$

It is worth noting that this MRS hinges on age and therefore the average propensity to hold capital/money at a macro-level (across living households) varies according to the population distribution in the model. Under the assumption that older households have a higher preference for money holdings ($\eta_{j,t}$) and a smaller family scale (χ_j), it is possible that population aging tends to exert upward pressure on the average MRS across all ages, thus leading to an increase in the sensitivity of capital accumulation to changes in the inflation rate. In other words, this suggests that a decline in the inflation rate can bring about a larger decrease in capital stock in a grayer economy.

Wealth Redistributions

The redistribution effect that is caused by changes in the inflation rate is well studied by Ireland (2005) and Doepke and Schneider (2006). The center of the redistribution effect that affects output is the so-called wealth effect on labor and capital inputs. On top of this, a key factor determining the impact on aggregates is the differences in marginal propensities among cohorts to consume/save and to work, more specifically, between "winners" and "losers" -- here, the young vs. the old.

Ireland (2005) deals with the distributional effects on the economy of money supply (monetary transfers/taxes) implemented by the government. He demonstrates why money becomes net wealth for households breaking the Ricardian equivalence, a famous proposition advocated by Barro (1974). Then, he explores the impact of deflation as well as inflation at the steady state while taking account of the lower bound on the nominal interest rate. Doepke and Schneider (2006) deal with the wealth effects of zero-sum redistributions from old lenders to young borrowers along a transition path, which can occur under the assumption that nominal interest rates on financial claims do not adjust immediately against inflation. They analyze asymmetric responses to changes in households' wealth according to the differences in life-cycle features (specifically, marginal propensities to save and labor status) between the young and the old. These types of distributional effects operate in the monetary OG model used here, which takes explicit account of changes in population and TFP and a finite lifespan of households.

First, consider the way that inflation or deflation gives rise to wealth redistributions through the government lump-sum transfers/taxes from a perspective of an intra-temporal horizon. In a monetary model, inflation (deflation) acts as a kind of tax (subsidy) on money holdings, through a decrease (increase) in its real value. The amount of the tax (subsidy) for a household of age j in period t, denoted by $\tau_{j,t}^{m}$, depends on the amount of its money holdings carried over from the previous period:

$$\tau_{j,t}^{m} = \frac{1}{1+\pi} m_{j-1,t-1} - m_{j-1,t-1} = -\frac{\pi}{1+\pi} m_{j-1,t-1} \,. \tag{28}$$

When the government supplying the money receives the inflation tax revenues T_t^m from households:

$$T_t^m = -\sum_{j=22}^{101} \mu_{j,t} \tau_{j,t}^{m} = \frac{\pi}{1+\pi} \sum_{j=22}^{101} \mu_{j,t} m_{j-1,t-1} = \frac{\pi}{1+\pi} M_{t-1} \,, \tag{29}$$

redistributions occur between households and the government. Besides, when the government rebates the inflation tax revenues to households in some way, redistributions among households can also emerge. Here, suppose that the government does this in a lump-sum manner. In this case, all the living households obtain an equal amount of transfers from the government after having paid different amounts of inflation taxes to the government. The net gain or loss to an age-j household caused by these redistributions is given by

$$T_t^m + \tau_{j,t}^m = \frac{\pi}{1+\pi}\left(M_{t-1} - m_{j,t-1}\right). \quad (30)$$

As seen in this equation, when inflation (tax) occurs in the economy ($\pi > 0$), there are gains to households that have less money holdings than the average (quantity per capita) of money (if $M_{t-1} - m_{j-1,t-1} > 0$), while there are losses to households that have more money holdings than the average (if $M_{t-1} - m_{j-1,t-1} < 0$). The reverse holds when deflation (subsidy) occurs in the economy ($\pi < 0$). In other words, inflation induces resource transfers from households with more money holdings to households with less money holdings; deflation induces resource transfers from households with less money holdings to households with more money holdings. As observed in the data, on average, the young own relatively less money, while the old own relatively more money, thus implying that in the case of inflation (deflation), the young (old) are winners; the old (young) are losers. This mechanism of redistribution is schematically depicted in Figure 5.

Rewriting the net resource gains or losses to households in Equation (30) using detrended per-capita counterparts in real terms,

$$\tilde{T}_t^m + \tilde{\tau}_{j,t}^m = \frac{1}{(1+\rho_t)(1+z_t)}\frac{\pi}{1+\pi}\left(\widetilde{M}_{t-1} - \widetilde{m}_{j-1,t-1}\right). \quad (31)$$

It is worth noting that, as population ages and decreases, namely, with higher $\widetilde{M}_{t-1}$ and lower ρ_t, the magnitude of gains or losses likely becomes larger, especially for the youngest households. Therefore, in the case of deflation ($\pi < 0$), the young suffer from much more losses and are much worse-off in a grayer economy, given that the marginal utility of wealth tends to be greater in general because they have few assets just after birth.

Next, to see how the above redistribution (monetary transfers/taxes) becomes net wealth to households, we check how the real money balance per household changes from a perspective of an inter-temporal horizon.[18] Suppose that the government expands the aggregate money supply at a constant rate σ by making lump-sum transfers υ_t equally among all the households living in each period: $\upsilon_t = \sigma\widetilde{M}_{t-1}$. Here, define aggregate monetary transfers per household made in period $u \geq t$ to households that were alive in

[18] This argument relies on Ireland (2005).

period t by $v_u^t = \sigma \widetilde{M}_{u-1}$. As a simplification to calculate the monetary wealth of households analytically, now abstract from the opportunity cost of carrying money instead of bonds or capital over all future periods by assuming that the nominal interest rate is zero: $i = 0$. Also, suppose that each household can live for sufficiently many periods (say, infinitely). Then, the monetary wealth of households Υ_t is composed of the value of money supplied in the current period and the discounted present value of all future transfers/taxes that households alive in the current period will receive/pay:

$$\Upsilon_t = \widetilde{M}_{t-1} + \sum_{u=t}^{\infty}\left(\prod_{v=t}^{u-1}\frac{1}{1+x_v}\right)v_u^t = \left[1+\sigma\sum_{u=t}^{\infty}\left(\prod_{v=t}^{u-1}\frac{1}{1+x_v}\right)\right]\widetilde{M}_{t-1}. \tag{32}$$

At a steady state equilibrium with all the detrended variables constant: $a_t = a$ for all period t,

$$\Upsilon = \left\{1+\sigma\left(\frac{1+x}{x}\right)\right\}\widetilde{m}. \tag{33}$$

Combining with $1 + x = 1/(1 + \pi)$ derived from the Fisher equation and $(1 + z)(1 + \rho)(1 + \pi) = 1 + \sigma$ derived from the money supply rule (5), Equation (33) is expressed as

$$\Upsilon = \left(1-\frac{\sigma}{\pi}\right)\widetilde{m} = \left\{1-\frac{\sigma(1+\rho)(1+z)}{1+\sigma-(1+\rho)(1+z)}\right\}\widetilde{m}. \tag{34}$$

In the special case without population growth and TFP growth: $\rho = 0$ and $z = 0$, this equation implies that $\Upsilon = 0$. The households receiving monetary transfers in the current period are exactly the same households that pay all of the taxes required to implement the money supply rule (for achieving $i = 0$ in this case) in the future periods. In this case, the Ricardian equivalence holds and so the real money balance will not be net wealth for households. On the other hand, with either population growth or TFP growth: $\rho \neq 0$ or $z \neq 0$, then $\Upsilon \neq 0$. This implies that households alive in the current period pay only a part of the future taxes or receive only a part of the future transfers which are associated with future changes in money supply; other households born in subsequent periods incur the remaining taxes or transfers. Consequently, monetary transfers/taxes become the net wealth of households even at the steady state. In short, the intuition here is that, in a world with population growth rate of 1 percent, 100 households receive 100 units of monetary transfers today, but each of them has to pay only 100/101 tomorrow because newly born households will pay the rest of 1/101.

Wealth Effects of Redistributions

Finally, we turn to changes in household behavior, such as consumption-savings choices and labor-leisure choices, in response to the additional wealth gains/losses described above. As outlined in Table 1, wealth effects create asymmetric responses of the young and the old according to consumption smoothing out over their lifetimes -- more specifically, depending on their marginal propensities to consume/save and their labor status.[19] This asymmetry results in persistent effects on economic aggregates, even if the wealth redistributions are zero-sum as a whole, because the remaining lifespans also differ among the young and the old.

To smooth out consumption over their life cycle, while the young can adjust labor supply and have the higher marginal propensity to save for longer life expectancy, the old cannot adjust labor supply after mandatory retirement and have the higher marginal propensity to consume during their shorter life expectancy. In the case of deflation, where wealth transfers occur from the young to the old, the old winners consume much more and dissave less due to additional gains. On the other hand, the young losers are forced to save less and to work more even while cutting their consumption little by little. As for labor supply, only the young's response drives the aggregate upward. As for consumption, the old's response outweighs the young's response for a while, due to their higher propensity to consume and shorter life expectancy, thus pushing up aggregate consumption. In return, as for savings (in terms of capital accumulation), the reduction in savings by the young is not fully offset by the slowdown of dissaving by the old, thus leading to a decrease in aggregate capital stock.

Importantly, the impact on output depends on which of the wealth effects on the two factor inputs is dominant -- on labor supply or on capital stock. Considering that capital stock is fixed in the initial period and its movements are likely delayed as a state variable,[20] however, output tends to move in the same direction as labor supply and consumption, at least during the initial several periods of the transition path. Consequently,

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[19] The argument presented here follows Doepke and Schneider (2006), who provide a detailed discussion based on households' optimization within a much simpler OG framework.

[20] Labor input is a jump variable and can respond immediately and strongly to a shock at any given point in time.

a move into deflation likely induces increases in labor supply and consumption, at least in the short run, through redistribution effects. On top of this, these effects on aggregates can to some extent be persistent, because they do not fade out until households influenced by a redistribution shock start to die. If redistributions are not zero-sum and last permanently (i.e., the real money balance becomes net wealth), then these effects can persist even at a steady state. In the long run (at a steady state), however, the impact on output is ambiguous at the moment.

Qualitative Postulation

To summarize the argument, potential effects of deflation or a decline in the inflation rate can be organized in Table 2. Capital stock would likely fall in the model economy presented by this paper, while the qualitative effects on other aggregates are ambiguous at the moment. Those effects may depend on a set of parameters and exogenous variables fed into the model. In any case, it is important to evaluate the effects quantitatively within a general equilibrium framework.

4 Data and Calibration

We calibrate the model presented in the previous section to the Japanese economy during the period from 1980 to 2021. We first describe the calibration of the model's structural parameters and then outline the definition and construction of the exogenous inputs used in both in-sample and out-of-sample simulation.

4.1 Constant Parameters and Age-Specific Parameters

Many of the key real aggregates are defined and calculated from the Japanese National Account (JNA),[21] following closely the data construction methodology proposed by Hayashi and Prescott (2002) and İmrohoroğlu and Sudo (2011). Data on monetary aggregates held by the private sector are obtained from the Currency in Circulation report published by the BOJ. Money is defined as M0 (monetary base) in the baseline model, but M2 (money supply including bank deposits) is used as an alternative in a part of the

[21] Data on the national account is on the basis of the 2008 System of National Accounts (08SNA). Data prior to 1994 are connected and traced backward using the year-on-year rates of change in each series obtained from 93SNA.

analysis, as seen in the next section.[22] More importantly, the BOJ has purchased a huge amount of Japanese government bonds (JGBs) under its quantitative and qualitative monetary easing (QQE) policy since 2013 -- it has become the largest holder of JGBs, holding about half of the amount outstanding. As Hansen and İmrohoroğlu (2023) and others point out, while monetary aggregates have increased sharply, the rise in the ratio of net government debt to GNP has slowed since 2013. In light of this, we define government bonds in the model as net government debt by subtracting the JGBs held by the BOJ. This treatment with respect to this data is also consistent with the assumption of a consolidated budget constraint (6) between the government and the central bank, including the aggregate money supply in the model.

The calibrated constant parameters are listed in Table 3.[23] The four parameters α, β, ν, and θ are basically constant throughout the simulation periods. We use the sample average for the income share of capital in GNP α for the sample period. We choose the value of the subjective discount factor β so that our benchmark model should closely replicate the historical average of the capital-output ratio during the sample period. We set the value of the reciprocal of the Frisch elasticity ν at 2.0 so that our benchmark model can capture an average life-cycle pattern of labor inputs during the sample period. The replacement ratio θ for public pension is set to a long-run average value with reference to an estimate by Oshio and Yashiro (1997), similarly to Chen, İmrohoroğlu, and İmrohoroğlu (2007).

We also set the five age-specific parameters that characterize the life-cycle features of households as constant over time: labor efficiency ε_j, family scale factor χ_j, preference for money holdings η_j, preference for bond holdings γ_j, and disutility weight on labor ζ_j. These values are basically either taken or estimated from the Basic Survey and Wage Structure (BSWS) by the Ministry of Health, Labour and Welfare and the National Survey

[22] Specifically, M0 is defined by "banknotes in circulation + coins in circulation + current account balances (current account deposits in the Bank of Japan)," and M2 is defines by "currency in circulation + deposits deposited at domestically licensed banks, etc."

[23] For a dynamic simulation, we set as a sample period the overall period from 1980 to 2021, termed the "full sample period." For the steady-state analysis, we chose the latter half of the overall period as a sample period, termed the "sub-sample period." This sub-sample period includes the years when stagnating growth, the resulting deflation, and population aging grew more serious in Japan and when there were a few important changes (e.g., the introduction of QE/QQE) with respect to policy regimes. The reason for this latter choice is that we are focusing on the more recent situation in Japan and thereby make the results of the steady-state analysis more vivid and more plausible from the perspective of implications for the future.

of Family Income and Expenditure (NSFIE) by the Ministry of Internal Affairs and Communications, so that they can closely replicate the related age-specific profiles of consumption, labor supply (employment rate times hours worked), and asset holdings.[24, 25]

4.2 Time Series of Exogenous Variables

In our dynamic simulation under the assumption of perfect foresight, we compute the equilibrium transition paths of allocations and prices from the initial period to the terminal steady state. We take the year 1980 as the starting period of the dynamic simulation.[26] The year 2021 is the last period for which we have available data on the national account series at present. The model also employs the observed values as its exogenous inputs for about four decades (the 1980-2021 period) and the assumed values of these inputs from 2022 and onward.

In order to conduct perfect foresight simulation from 1980 to 2021, we need to specify the time series of exogenous variables throughout the simulation period. These series consist of the growth rate of the age-21 population f_t, the age-specific conditional survival rate $\psi_{j,t}$, the inflation rate π_t, the time-varying component of disutility for labor ζ_t, time-varying components of preferences for money and bond holdings, η_t and γ_t, the depreciation rate of capital δ_t, the TFP growth rate z_t, and a set of fiscal variables $\{G/Y_t, B/Y_t, M/Y_t, \tau_t^c, \tau_t^l, \tau_t^k\}$.[27] This subsection provides a brief description of the construction of these variables in order.

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[24] Basically, the data on the age-specific profiles of households are provided only for some age groups by five-year or ten-year increment. Therefore, we needed to interpolate each series by a single age for use in the model.

[25] The detailed definition and construction methodology for these exogenous variables basically follow Muto, Oda, and Sudo (2012), which is a working paper version of Muto, Oda, and Sudo (2016).

[26] We do not have a detailed data set on asset distribution at the beginning of 1980, which is the initial period of the model's simulation. To set the initial distribution of assets $\{k_{j,0}, b_{j,0}, m_{j,0}\}$, we first compute the life-cycle profiles of assets from a steady state that is calibrated to the key macroeconomic variables for 1980. In computing the transition path, we multiply the simulated distribution of capital stock ($\bar{k}_j$ for all the ages) by a scalar q_k so that the aggregate capital-output ratio computed from the model should coincide with its data equivalent K_{1980}/Y_{1980}; $k_{j,0} = q_k \cdot \bar{k}_j$. We follow the same treatment for $\bar{b}_j$ and $\bar{m}_j$ so that the data B_{1980}/Y_{1980} and M_{1980}/Y_{1980} should be reproduced by the model; $b_{j,0} = q_b \cdot \bar{b}_j$ and $m_{j,0} = q_m \cdot \bar{m}_j$.

[27] Because there is no detailed data available for the life-cycle profile of government bond holdings $b_{j,t}$, we assume that it is the same as the life-cycle profile of cash and deposits, as is the case with that of money holdings $m_{j,t}$. However, the amounts of money and bond holdings themselves are assumed

Here, it is worthwhile to note that the main objective of this paper is not to make a plausible forecast of future periods, but to quantify the impact of deflation during the actual periods with counterfactual experiments. In this sense, the baseline forecast for future periods is less important than the baseline projection for the actual period. Assuming the future paths of these exogenous variables, especially the fiscal rules, is not an easy task, but it matters less for our purpose.[28] Therefore, we make only simple assumptions about the out-of-sample paths of these exogenous variables, at least as long as the assumptions allow the transition paths to converge computationally while satisfying the transversality conditions for household and government budget constraints.

It is also notable that the age distribution of the population at the initial state of the model $\mu_{j,0}$ is exogenously set as equal to the actual distribution in 1980. For the actual period, the demographic variables (f_t and $\psi_{j,t}$) are computed from the Population Estimates published by the Ministry of Internal Affairs and Communications. For the future horizons, these variables are obtained from the medium variant projections in the Population Projection for Japan published by the National Institute of Population and Social Security Research. Together with this initial age distribution of the population, taking these demographic variables as given enables us to reproduce the actual age distribution of the population thereafter almost perfectly.

The disutility weight on labor ζ_t is assumed to rise from 1988 to 1993 in a quadratic fashion, so as to capture the effects of institutional changes in labor inputs (hours worked by households) in the model. As discussed by Hayashi and Prescott (2002), this reduction in working hours was legally established in the late 1980s, and the workweek length dropped from 44 hours in 1988 to 40 hours in 1993.[29] In the period after 1993, including the forecast horizon, this parameter (ζ_t) is kept constant over time in the model at the value for 1993.

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to be different, reflecting the ratios of those aggregates to the aggregate of total assets or to GNP. In other words, the life-cycle profiles of $b_{j,t}$ and $m_{j,t}$ are assumed to be similar, but the former is larger than the latter by the ratio of B_t to M_t.

[28] In this regard, for example, Hansen and İmrohoroğlu (2016) and İmrohoroğlu, Kitao, and Yamada (2011) investigate in detail the feasibility of fiscal balance and the sustainability of government debt in Japan under various fiscal regimes.

[29] Based on a simple growth model, Hayashi and Prescott (2002) point out that the fall in workweek length as well as the slowdown in TFP growth is important for the Japanese economic stagnation during the 1990s. Note that they treat working hours as being exogenous before 1993 (and endogenous thereafter).

The inflation rate is given exogenously in all the simulation experiments. In the in-sample period corresponding to the actual period (1980-2021), the rate is set at the actual values for the growth rate of the GNP deflator obtained from the JNA. In the out-of-sample period corresponding to the projection period (2022 and beyond), the value is kept constant over time at the in-sample average (0.4 percent), which is equal to the steady-state value.

The in-sample sequence of utility weights on government bond holdings γ_t is set so that the model-generated government bond yield should match the corresponding data series. On the other hand, the in-sample sequence of utility weights on money holdings η_t is set so that the model-generated ratio of monetary aggregates to GNP should match the corresponding data series. The out-of-sample sequences of these two weights are set constant at the steady-state values presented below, which are chosen as the in-sample averages.

TFP (Z_t) is calculated as the Solow residual series from equation (1) and the corresponding data in line with the methodology used in Hayashi and Prescott (2002) and İmrohoroğlu and Sudo (2011). For the period after 2021, the TFP is assumed to grow by $1.012^{1-\alpha}$ constantly, which is based on the historical average of the growth rate of the Solow residual from 1980 to 2021, implying that per-capita GNP grows at a rate of 1.2 percent along the balanced growth path. The depreciation rate δ_t is set at its actual values until 2021 and constantly at 0.072 beyond 2021, which is the historical average from 1980 to 2021.

Finally, during the in-sample period, the fiscal variables $\{G/Y_t, M/Y_t, \tau_t^c, \tau_t^l, \tau_t^k\}$ are calculated from the data and exogenously given in the model, while the variable B_t (hence B/Y_t) is endogenously computed in the model given the preference parameter $\gamma_{t,j}$. For the out-of-sample period, $\{G/Y_t, B/Y_t, \tau_t^c, \tau_t^l, \tau_t^k\}$ are also assumed to change linearly from the values of 2021 to the in-sample average value for ten years (2022-2031), and to be kept constant at the in-sample average value ever thereafter.[30] On the other hand, the variable M_t (hence M/Y_t) is endogenously computed in the model, given the preference parameter $\eta_{t,j}$. With the lump-sum taxes/transfers $\tilde{\tau}_t$ adjusted so as to satisfy the government budget constraint in every period under these assumptions, the transition paths of allocations and prices converge to the terminal steady state in the computation.

[30] τ_t^c is assumed to be constant at 10 percent throughout the forecast horizon.

The calibrated values of key exogenous variables at the steady state are summarized in Table 4.[31] Figures 6 and 7 show some of the main age-specific profiles that characterize the life cycle of households and the time series of key exogenous variables fed into the baseline simulation of the model, respectively. As demonstrated in related studies on other countries, the conditional surviving probability decreases with age and labor efficiency exhibits a hump-shape, peaking around the early 50s. The disutility weight on labor is higher among youth, likely reflecting schooling, parenting, caring for family, and other factors. The utility weights on money and bond holdings, which are regarded as proxies of household preferences for liquidity and safety, increase with age until around the 60s.[32] The growth rate of the age-21 population f_t was above zero throughout most of the 1980s, but was negative (around -2 percent) in many years throughout the 2000s and 2010s. Movement in the time-varying component of the disutility weight on labor reflects the legislative reduction in the workweek length described above. The time-varying component of household preferences for money holdings η_t is found to have risen and stayed at high levels since 2013, albeit with some fluctuations, while the BOJ was conducting super-accommodative monetary easing.

5 Quantitative Results

This section reports the results of three types of simulations: steady-state analysis, impulse responses to a deflationary shock, and transition dynamics. In each analysis, we evaluate the impact of deflation on aggregate variables by altering the inflation rate exogenously. For steady-state and impulse response analyses, we also examine how the impact can vary between an aging economy and a non-aging economy. For transition dynamics, we quantitatively assess the impact of past deflation in Japan by comparing the results of the baseline simulation and those of a counterfactual experiment.

5.1 Steady-State Analysis

This subsection studies steady states with different rates of inflation. A comparison between steady states represents the impact of anticipated changes in the inflation rate under the assumption of perfect foresight. This type of analysis can provide us with some

[31] As for the full and sub-samples, see Footnote 23.

[32] Data on cash and deposit holdings by age group are available in ten-year increments until the 70s. Due to this data restriction, the values for households above the 70s are assumed to be the same as those for the 70s.

useful insights about how the model economy responds to changes in the inflation rate in the long run.

Before comparing steady states, let us start by studying the model's performance in the dimension of life-cycle profiles. Figure 8 depicts household life-cycle profiles of consumption ($c_{j,ss}$), labor supply ($h_{j,ss}$), total asset holdings ($k_{j,ss} + m_{j,ss} + b_{j,ss}$), and safe asset holdings ($m_{j,ss} + b_{j,ss}$). In each of the panels, the dotted lines correspond to the actual profiles (the averages of the relevant data) and the solid lines correspond to the profiles simulated by the model. As seen in this figure, the model can reasonably mimic general patterns in these life-cycle profiles.

The consumption profile has a hump-shape, with a peak at the 50s. Labor supply is relatively small among the youngest group (20s) and declines significantly on the 60s and 70s. Safe asset holdings and total asset holdings continue to increase until the 50s or 60s. The young (20s and 30s) hold a relatively small amount of assets, while the old (the 60s and the 70s) hold a larger amount of assets. Therefore, deflation or a decline in the inflation rate likely induces wealth redistributions from the 20s and 30s to the 60s and 70s, when the government returns inflation tax revenues equally to all households in a lump-sum manner. However, households consume more and dissave more after the retirement age (71 years old) in the model than they do in reality observed from the data. This difference may arise partly from the fact that the model does not incorporate housing and the intended bequests of households in terms of capital ($k_{j,t}$) for their descendants.[33] Instead, housing is assumed to be indifferent to other capital in this model, which means that the amount (value) of housing is included in that of capital stock, not only for the household's age-specific profile but also for the aggregate variable based on JNA data.[34] This is why the oldest group of households in the model is more reluctant to retain capital stock than they do in reality.

[33] This point is discussed in Chen, İmrohoroğlu, and İmrohoroğlu (2007) as well. As we saw in Section 3, the "money/bond-in-the-utility" specification in the model does not necessarily exclude the possibility of households' intended bequests in terms of safe assets ($m_{j,t}$ and $b_{j,t}$), of which the ratio to total assets is usually less than one third.

[34] In this regard, for example, Ogawa and Yoshida (2024) point out that the proportion of housing to asset holdings increases with age and remains high over old ages after retirement in Japan as well as in the U.S., because they likely regard housing as buy-and-hold safe assets partly due to bequest motives. Using an OG model that explicitly allows for housing, they find that population aging in Japan had an adverse impact on economic growth and real interest rates.

Now we are in a position to examine the effects of changes in the inflation rate. Figure 9 shows a set of variations in key real aggregates for different rates of inflation ranging from -3 to 3 percent on the horizontal axis. These variations are expressed as percentage deviations from the level for zero percent inflation. To check the difference in the impact according to the degree of aging as well, suppose two simulation cases: (a) The "aging case," where the growth rate of the age-21 population is assumed to be low (-1.4 percent) and (b) the "non-aging case," where the growth rate is assumed to be high (1.0 percent).[35] In the panels of Figure 9, the aging case and the non-aging case are denoted by the black and gray lines, respectively. The results suggest that the Mundell-Tobin effect and redistribution effects are likely at work, as discussed in Section 3 and documented below.[36] The results are qualitatively analogous but quantitatively different between both cases.

First of all, the super-neutrality of money does not hold even at the steady state, in the sense that real aggregates vary for different rates of inflation; different growth rates of money supply affect real aggregates.[37] In addition, the Friedman rule does not hold either, in the sense that social welfare is not maximized when the nominal interest rate is zero in the territory of deflation,[38] although the ZLB is not imposed on the nominal interest rate in this model. As pointed out by Bhattacharya, Haslag, and Russell (2005) and Ireland (2005), and also discussed in Section 3, these results derive from the fact that the real money balance becomes net wealth in the OG model with population growth and TFP growth as well as finite lifespans, collapsing the Ricardian equivalence even at the steady state.

For both cases, the panels (b) to (e) of Figure 9 indicate that, as the inflation rate declines (going to the right along the horizontal axis), labor supply increases and capital stock decreases, while consumption falls moderately and the real money balance rises. Following the qualitative discussion in Section 3, these results suggest that the Mundell-

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[35] At the steady state, the growth rate of total population ρ is equated to the growth rate of the age-21 population f.

[36] Unfortunately, in this large-scale computational model, it is too difficult to identify the two effects and decompose the overall impact into their contributions.

[37] In a representative agent model, if the MRS between consumption and labor is independent of the real money balance as shown by Equation (A1-12), then the super-neutrality holds at the steady state; from Equation (A1-15), the real return on capital r is pinned down by exogenous parameters, such as the subjective discount factor: $1 + z = \beta(1 + r)$.

[38] Social welfare is calculated as household expected lifetime utility U_s that is the sum of the discounted present values of the period utility over its lifetime as shown in Equation (14).

Tobin effect should work in a deflationary direction, causing decreases in capital stock and increases in the real money balance. Figure 10 confirms this substitution between money and capital in terms of the share of total assets. On the other hand, the redistribution effect also operates with respect to increases in labor supply. Furthermore, with the current parametrization, relying on the Japanese data, this increase in labor supply is dominated by a decrease in capital stock, which is induced by the redistribution effect as well as by the Mundell-Tobin effect. As a result, it leads to a decline in output as seen in the panel (f). A decline in consumption seems relatively small compared to the decline in output, because the redistribution effect on consumption, specifically an increasing wealth effect on the consumption of old winners, partly counteracts a decrease in the consumption of young and middle-aged losers associated with a decrease in output as well as the direct effect of redistribution.

The panel (g) shows that welfare declines in line with declining output. Besides, as the inflation rate falls into negative territory, wealth redistributions from the young to the old occur and expand, thus making the young with fewer asset holdings even poorer. This forces them not only to work more while incurring greater disutility for labor, but also to cut down their consumption. This is why deflation results in such a great impairment of welfare.

Two points are worth mentioning here. One is the non-linearity or asymmetry of the impact between inflation and deflation. The negative impact of deflation is larger than the positive impact of inflation. There are a few reasons for this. First, the real return on money holdings $1/(1+\pi)$ decreases non-linearly and is convex in the inflation rate π. Money is a non-interest-bearing asset and its real return rises rapidly as deflation occurs and deepens, so that households are more willing to hold a great deal of money instead of bonds and capital, thus crowding out capital stock and further pushing down output. Second, the direction of redistributions changes between inflation and deflation: from the old to the young for inflation and from the young to the old for deflation. The marginal welfare losses of a young household to an additional unit of wealth losses are larger than the marginal welfare gains of an old household to an additional unit of wealth gains. In other words, because deflation makes the youngest households with little wealth much poorer, deeper deflation does a great deal more damage to them. This brings about a considerable decline in welfare in a deflationary situation.

The other point is that the impact of deflation is larger in the aging case than in the non-aging case.[39] There are a few reasons for this. As discussed in Section 3, the sensitivity of capital depletion (the substitution of money for capital in household asset choices) to a reduction in the inflation rate likely becomes greater in a grayer economy with a higher proportion of older population, under the assumption that older households have a higher preference for money holdings. Also, the negative redistributions (wealth losses) to young households (especially, the youngest ones) possibly get larger in a grayer economy. Moreover, for the same reason as above, the ratio of capital stock to total assets gets relatively smaller and the ratio of the real money balance to total assets gets relatively larger in a grayer economy. Consequently, a further decrease in capital stock likely has a greater adverse impact on output and hence social welfare.

5.2 Impulse Responses

This subsection reports the results of impulse response simulations with a one-time deflationary shock that permanently reduces the inflation rate by 1 percentage point. This experiment includes the impact of an unanticipated change in the inflation rate in the initial period. The shock is a surprise only in the initial period when it hits the model economy, in the sense that it was unanticipated by all the living households (not incorporated into their decision-making) at the initial steady state. At the same time, the Fisher equation (7) does not hold in the initial period because the nominal interest rate was pre-determined according to the inflation rate of the initial steady state. As a result, the real balance of government bonds also increases with the deflationary shock in the initial period only.

In the subsequent periods, however, the paths of the inflation rate and other variables are fully anticipated by all the living households and incorporated into their decision-making under the assumption of perfect foresight. This type of analysis can provide us with some useful insights about how the model behaves dynamically in response to changes in the inflation rate, as the model economy moves from the initial state to the

[39] Additionally, it may be worth mentioning the difference in the impact on output and welfare between the two cases. The difference in the impact on output may seem small compared with that in the impact on welfare. On the one hand, regarding the impact on output, the decrease in capital stock is partly counteracted by the increase in labor supply. On the other hand, regarding the impact on welfare, as discussed above, the redistribution effects on young losers -- especially, the resulting increase in labor supply -- aggravate their utility directly. Therefore, the negative impact on welfare tends to become larger in the aging case.

terminal state. This is also instructive for checking whether both the Mundell-Tobin effect and redistribution effects indeed operate or not, and to what extent.

Let the inflation rates at the initial and terminal steady states be 0 and -1 percent, respectively. Similar to the steady-state analysis reported in the previous subsection, suppose two cases: the aging case and the non-aging case, with the same assumption about the growth rate of the age-21 population. Figure 11 shows the impulse responses of key aggregate variables, in terms of percentage deviations from the value of the initial steady state on the vertical axis. In each of the panels, the aging case and the non-aging case are denoted by the solid and dotted lines, respectively.

In the panels of (c) capital stock, (d) the real balance of money, and (e) the real balance of bonds, a demand shift is observed away from bonds and capital to money, with the Mundell-Tobin and redistribution effects operating in a deflationary direction. In the experiment here, the Mundell-Tobin effect, shifting asset demands toward money (in terms of the proportion to total assets), occurs only once because the change (decline) in the inflation rate occurs only once in the initial period and because consumption and money holdings are additively separable in the utility function. Even so, capital stock (in terms of the level) continues to decrease for a while as the redistribution effects persist. The panel of (a) labor supply also reveals that the redistribution effect dominates the complementarity between capital and labor through the Mundell-Tobin effect. Labor supply increases in response to the deflationary shock, because the young and middle-aged losers work more while the old winners are already retired and hence cannot adjust their labor inputs.

Except in the initial period, where the beginning-of-period capital stock is fixed, this deflationary shock impacts output negatively, because the decrease in capital stock is dominant over the increase in labor supply. In the panel (b), consumption increases in the short run through the wealth redistributions, which enable the old winners to consume much more while the young losers consume less. However, this increase in consumption fades away as the old -- who are living in the initial period and benefit from the wealth redistributions caused by deflation -- die, and eventually consumption decreases slightly because output declines and young workers cannot avoid reducing their consumption in the long run. These effects of deflation are larger in the aging case than in the non-aging case, for the same reason as mentioned in the previous subsection.

5.3 Transition Dynamics

In this subsection, we assess quantitatively the impact of past deflation in Japan by conducting a dynamic simulation under two scenarios and then comparing the two results: (1) a baseline scenario where the inflation rate is set at the actual value, and (2) a counterfactual scenario where the inflation rate is assumed to be constant at 2 percent since 1993. Notice that the baseline simulation entails the actual scenario where Japan experienced deflation, while the counterfactual simulation is under an alternative scenario without deflation. Before showing the assessment, it is interesting to note briefly the transition path of the baseline simulation, as a result of the calibration presented in the previous section, from the viewpoint of the model's performance in the dimension of developments in key macroeconomic variables over the in-sample period of 1980-2021.

Figure 12 shows the model-generated series of per-capita real GNP (output), the capital-GNP ratio, the ratio of capital to total assets, the net government indebtedness-GNP ratio,[40] the real interest rate (real return on government bonds), and the after-tax real return on capital, together with the data counterparts. The black and gray lines correspond to the simulated and actual series, respectively. The general fitness of the model seems reasonably good. This is important as a premise for making a quantitative assessment of the impact of a macroeconomic shock by counterfactual experiments.

The stagnant per-capita real GNP since the late 1990s is reproduced in the model. As demonstrated by Sudo and Takizuka (2020) and others, partly due to population aging, the real interest rate has exhibited a declining trend in Japan over the past several decades. The model also closely replicates this movement in the real interest rate. In addition, the model captures the increasing trend in the net government indebtedness-GNP ratio, as also demonstrated by Hansen and İmrohoroğlu (2023). The fitness of these two series reveals that the kind of reverse-engineering method of calibration described in Section 3 has functioned successfully.

To examine the impact of past deflation, Figure 13 shows movements in the key aggregates of the baseline simulation in terms of either the deviation rate (percentage) or the deviation width (percentage points) from those of the counterfactual simulation. The qualitative observations are basically as would be expected from the discussion in the above two subsections but are a little tricky to interpret, because a difference in the

[40] The net government indebtedness is defined as the sum of net government debt (B) and money balance (M).

inflation rate between the baseline and counterfactual scenarios emerges from the year 1993, although both simulations start from the year 1980. That is to say, although the direct influence of the Mundell-Tobin effect and redistributions themselves begins to materialize from 1993, households that were alive before 1993 (and can live after 1993) will change their behavior as soon as they update their expectations about the future paths of the inflation rate and other variables from the starting year of the simulations.[41]

The movements in consumption and labor supply are mixed in this experiment, because wealth effects operate as a kind of anticipation effect prior to the year 1993 when the deflationary situation is assumed to begin. Once the counterfactual simulation starts in 1980, retirees who can be alive after a few decades cannot adjust their labor supply but can consume much more, because they anticipate that subsequent deflation will make them richer due to wealth redistributions (wealth gains induced by the deflationary shock from 1993). Older middle-aged and younger old workers (the late 50s and 60s in the model) will also consume more goods and pursue more leisure, because of their anticipated wealth gains after 1993, around the time when they retire.[42] Conversely, young workers work more and save less, anticipating wealth losses induced by future deflation. With the wealth effects on retirees and older workers slightly dominant, the impact on labor supply becomes slightly negative until the mid-2000s, while the impact on consumption becomes slightly positive. After that, however, such wealth effects decay as retirees die and older middle-aged workers retire, both of which enjoyed the wealth gains. In the meantime, the redistribution effects for younger workers (causing them to work more) become dominant, thus resulting in increases in the labor supply, as discussed in the previous subsections.

Regarding household asset choices, the ratio of government bonds decreases and the ratio of capital increases temporarily prior to 1993. In other words, substitutions from bonds to capital occur, as households expect the real return on bonds to decline under

[41] As an alternative, of course, the model could start the simulation from the year 1993. However, shortening the simulation period (postponing the starting period) may cause a stronger influence of the initial distribution of household assets described in Footnote 26 to remain in the 2000s and afterwards. As another alternative, the model could set an inflation rate of 2 percent from the year 1980 (the same period that the simulation starts). However, the result would still be mixed in this case because the inflation rate was often above 2 percent during the period before 1993.

[42] As seen in Figure 6, a peak is observed around the early 50s in the age-specific profile of labor efficiency. Consequently, the contribution of the reduction in hours worked by households around the early 50s to a decrease in aggregate labor supply is considered relatively large.

deflation from 1993 onward. Here, it is notable that the aggregate level of capital stock does not necessarily increase, but rather decreases, because retirees dissave more and older middle-aged workers save less due to the anticipated wealth effects discussed above. During the deflationary period from 1993, due to the Mundell-Tobin effects as well as the redistribution effects, the ratio of the real money balance exhibits an up-trend and the ratio of capital shows a down-trend, albeit with some fluctuations, which reflect the difference in the inflation rate between the baseline and counterfactual scenarios. Over the past few decades, deflation is found to have caused the substitution from capital to money in the aggregate asset portfolio, which is suggested by the Mundell-Tobin effect and redistribution effects.

On an annual basis, the overall impact on output may seem to be small but is continuously negative throughout the simulation period. Per-capita real GNP declines slowly but steadily, with labor supply declining in the initial few decades and with capital stock declining under deflation. In addition, while the inflation target of 2 percent is not achieved, the negative impact continues to expand incrementally for about three decades, up to more than -0.1% of annual GNP in 2021. The cumulative impact is sizable, amounting to -2.1% of annual GNP during the period of 1993-2021 and -2.3% of annual GNP during the period 1980-2021.

The impact on output possibly varies to a quantitatively significant degree depending on the definition of money in the model. The scale of wealth redistributions induced by inflation or deflation depends on how inflation-proof or how deflation-proof nominal financial assets are, in other words, on how quickly agents adjust their expectations to changes in the inflation rate. In the model, money is defined as a non-interest-bearing asset and so is not inflation-proof at all but rather its real value (real return) is appreciated by deflation.[43] In the baseline setting used so far, money is regarded as M0 (monetary base). In reality, for example, nominal interest rates on bank deposits held by the private sector do not necessarily adjust smoothly to changes in the inflation rate so as to maintain their real returns (their real values).[44] On the other hand, it is not so straightforward to assume or take a position on how well interest-rate adjustments in financial assets and

[43] In contrast, government bonds are defined as interest-bearing assets without the lower bound of zero in the model, so they are not only inflation-proof but also deflation-proof in the sense that its real value (real return) is not appreciated by deflation, as long as a change in the inflation rate is fully anticipated.

[44] In this regard, nominal interest rates on bank deposits remained stuck to almost zero from the mid-1990s in Japan, while the BoJ continuously implemented an accommodative monetary policy against long-lasting deflation.

updates of household expectations work in response to inflation or deflation. Instead, it is thought to be easier and more useful to make an additional assessment of the impact by conducting the same types of simulations with an alternative definition of money (M2: money supply including bank deposits).

Naturally, because the quantity of M2 is much larger than that of M0, it can be guessed that the impact under this alternative setting likely becomes larger than under the baseline setting. Altering the definition of the monetary aggregate in the model from M0 to M2, as shown in Table 4, the household preference parameter for money holdings, η, becomes five times as large. From the perspective of the Mundell-Tobin effect described by Equation (26) and (27), a higher value of η possibly likely leads to the higher sensitivity of capital depletion to a reduction in the inflation rate. From the perspective of the redistribution effect, given Equation (30) or (31), a larger quantity of money, M, possibly brings about larger losses per younger household. As a result, the quantitative result under the alternative setting (M2) is expected to provide an upper bound for the magnitude of the negative impact of deflation on output.

Figure 14 shows the impact of past deflation on per-capita real GNP under this alternative setting (the dotted line), which looks qualitatively similar to that under the baseline simulation. As expected, however, the magnitude is much larger in the alternative setting. It is about -0.47% of annual per-capita real GNP in 2021, which is more than four times larger than the magnitude under the baseline setting. The cumulative impact amounts to -14.2% of annual GNP during the period of 1980-2021. Consequently, regarding past deflation in Japan, the assessment here suggests that the negative impact on GNP can lie in the range between -0.1% and -0.5% of annual per-capita real GNP in 2021.

6 Conclusion

This paper has quantified the impact of deflation in Japan, a country that also has experienced rapid population aging over the past few decades. We made quantitative assessments of the long-run effects of changes in the inflation rate on the real economy by conducting counterfactual experiments based on a neoclassical monetary model with overlapping generations of households. To this end, the OG model is extended by incorporating age-specific household preferences for safe assets and calibrated to the Japanese economy during the most recent four decades. This calibrated model succeeded in reproducing actual developments in many of the key macroeconomic variables.

Using the model, we considered two possible channels through which a decline in the inflation rate has real effects on economic aggregates in the long run: the Mundell-Tobin effect and the redistribution effect. Through the Mundell-Tobin effect, falling into deflation can affect adversely capital stock and labor supply, and hence output. Through the redistribution effect, falling into deflation can exert downward pressure on capital accumulation but upward pressure on labor supply, because it gives rise to redistributions of wealth from the young, who have fewer asset holdings, to the old, who have more asset holdings. Consequently, it has been considered important to use a general equilibrium framework to assess quantitatively whether deflation or a decline in the inflation rate impacts output and social welfare, positively or negatively.

The quantitative analysis found that a decline in the inflation rate likely does more damage to the young and impairs capital accumulation, thus reducing output and social welfare. The experiments also suggested that population aging can aggravate the adverse impact of deflation, partly due to the increase in the proportion of the population of the old who have a stronger preference for money holdings, while also imposing additional burdens on the young. This result provides a certain rationale for pursuing and maintaining a positive rate of inflation in an aging economy. The result also suggests the possibility that (unconventional) monetary policy might have become less effective in encouraging households and firms to invest rather than save in Japan.

As a caveat, this paper does not take account of heterogeneity within households of the same age. Naturally, there are significant differences in the size and composition of wealth within a cohort, so inflation/deflation induces redistribution within each cohort as well as among all cohorts. In this setting, a household that receives (pays) resource transfers under inflation (deflation) is not necessarily a young worker who has a higher propensity to save. There could be additional redistribution effects worth investigating.

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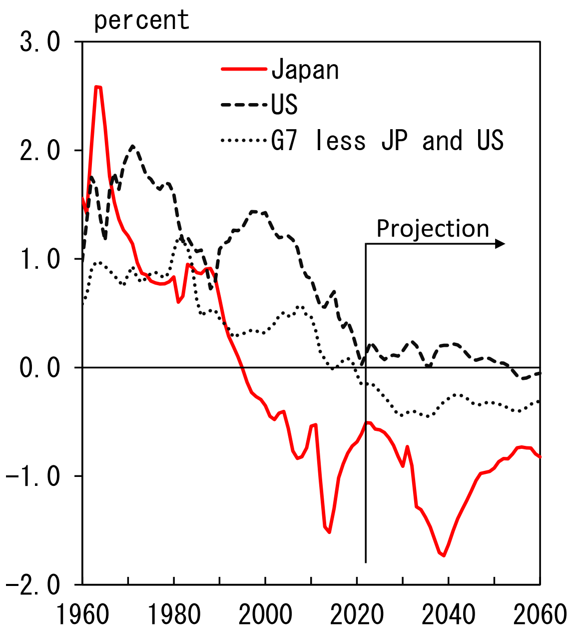
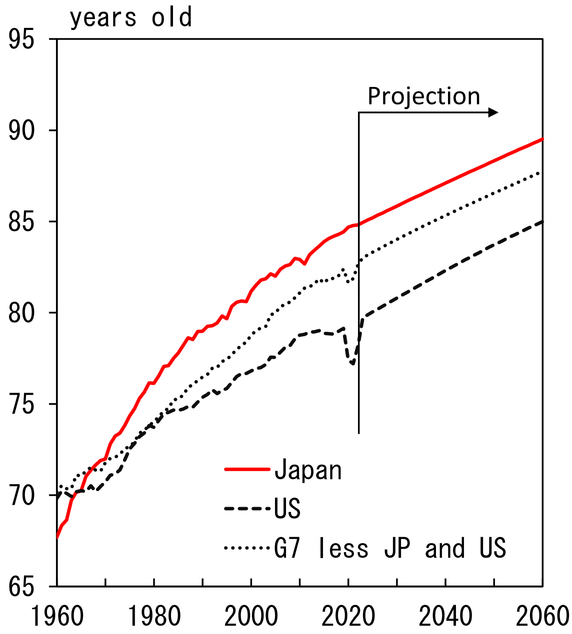
Figure 1: International Comparison of Demographic Trends

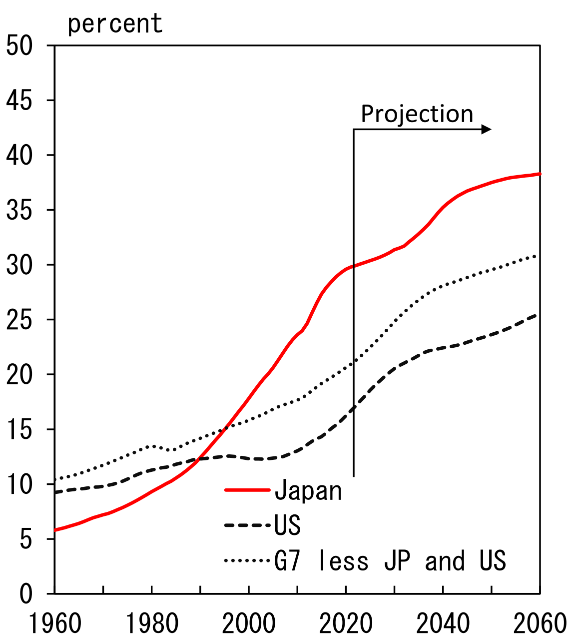
Working-Age Population Growth



Average Life Expectancy

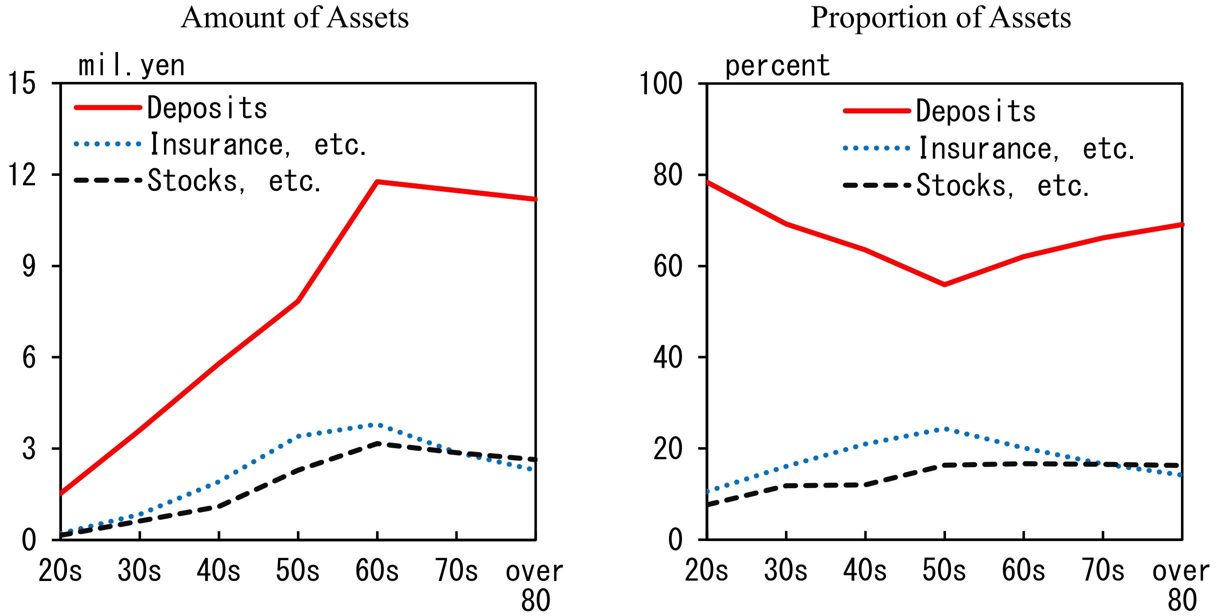


Old-Age Dependency Ratio



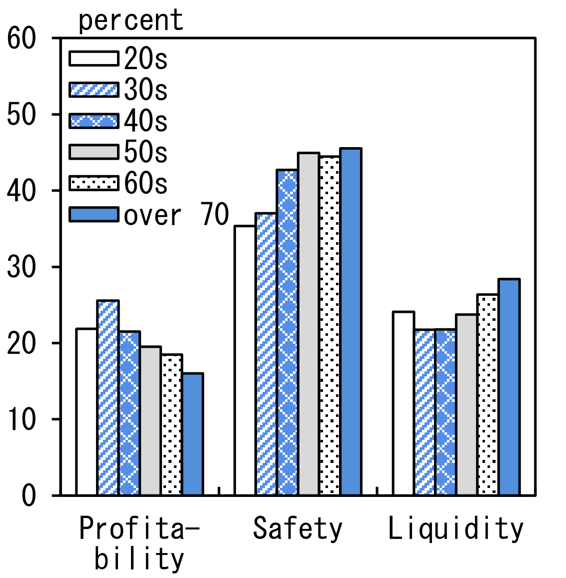
Note: Working-age population is the population aged 15 to 64. Life expectancy is the average of the life expectancies of men and women. The old-age dependency ratio is the ratio of the population aged 65 and over to the total population.
Source: United Nations, "World Population Prospects."

Figure 2: Age-Specific Profile of Household Asset Holdings



Note: Values are per household by age group of the head of household.
Source: Ministry of Internal Affairs and Communications, "2019 National Survey of Family Income, Consumption and Wealth."

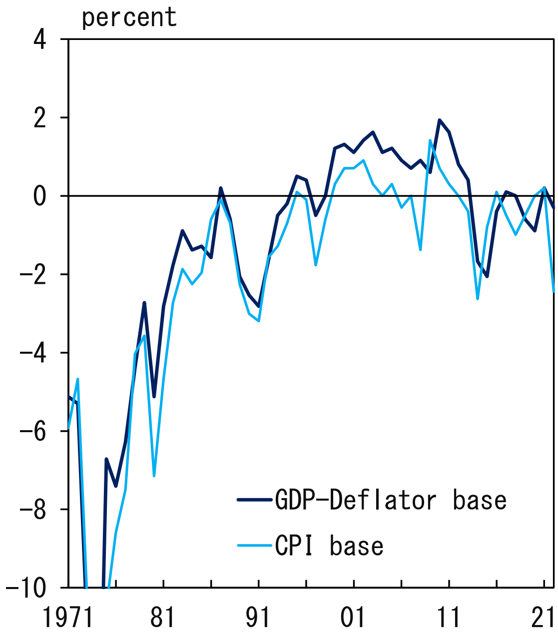
Figure 3: Household Motives for Holding Financial Assets



Note: Values are simple averages for CY 2007-2022.
Source: Central Council for Financial Services Information, "Survey of Household Finances (Japanese only)."

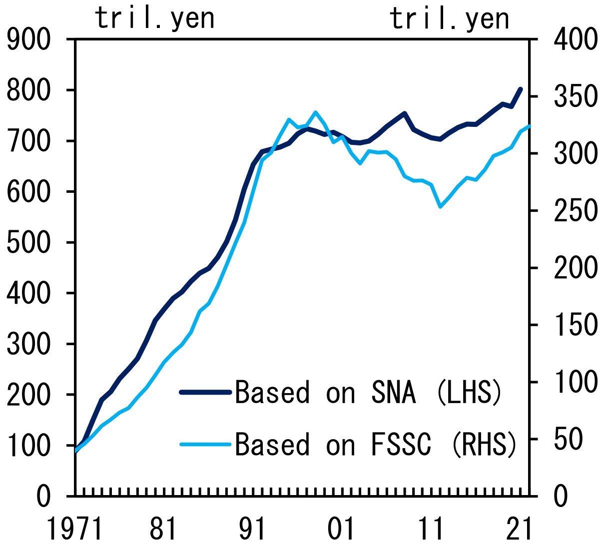
Figure 4: Movements in Real Return on Money and Private Savings-Investment

Real Return on Money

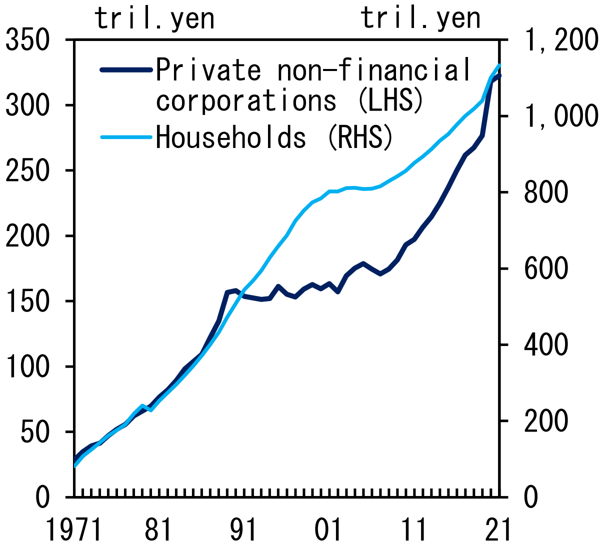


Note: Real return on money is defined as the inverse of $(1+\pi)$, where π is the inflation rate.
Sources: Cabinet Office, "System of National Accounts (SNA)"; Ministry of Internal Affairs and Communications, "Consumer Price Index."

Tangible Fixed Assets



Cash and Deposits



Note: In the left panel, values based on SNA are CY values excluding general government; values based on FSSC are FY values for all industries and all sizes excluding "Finance and insurance."
Sources: Cabinet Office, "System of National Accounts (SNA)"; Ministry of Finance, "Financial Statements Statistics of Corporations by Industry (FSSC)."

Figure 5: Redistributions among Households via Government Transfers/Taxes

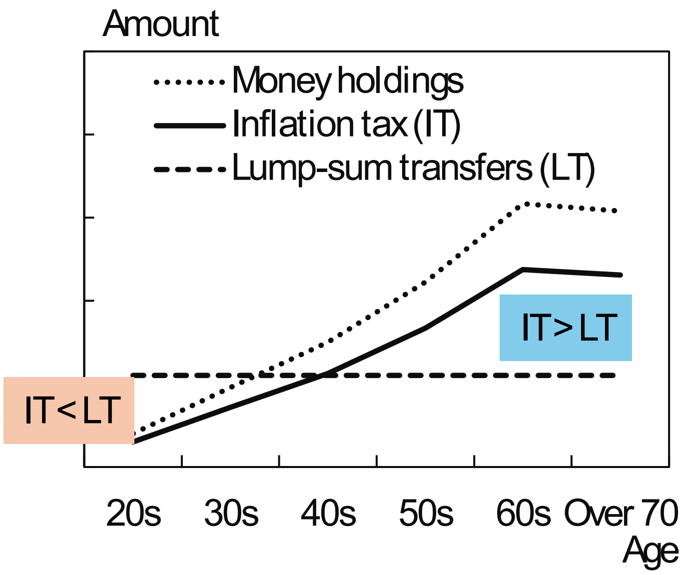


Table 1: Asymmetric Responses between the Young and the Old

Case of deflation	Young workers/borrowers	Old retirees/lenders	Aggregates
Redistribution	Losses	Gains	Zero-Sum
Remaining lifespan	Long	Short	——
Propensity to consume	Low	High	C: Upward ↑
Propensity to save	High	Low	K: Downward ↓
Propensity to work	Some	None	L: Upward ↑

Note: This summary follows the analogy to the discussion offered by Doepke and Schneider (2006), who deal with a case of inflation.

Table 2: Potential Long-Run Effects of Deflation or Disinflation

Aggregates	Mundell-Tobin effect & others	Redistribution (wealth) effects	Overall
Capital stock	Downward	Downward	Downward
Labor supply	Downward	Upward	Ambiguous
Consumption	Downward	Upward	Ambiguous
Output	Downward	Ambiguous	Ambiguous

Note: "Others" includes complementarity between capital and labor.

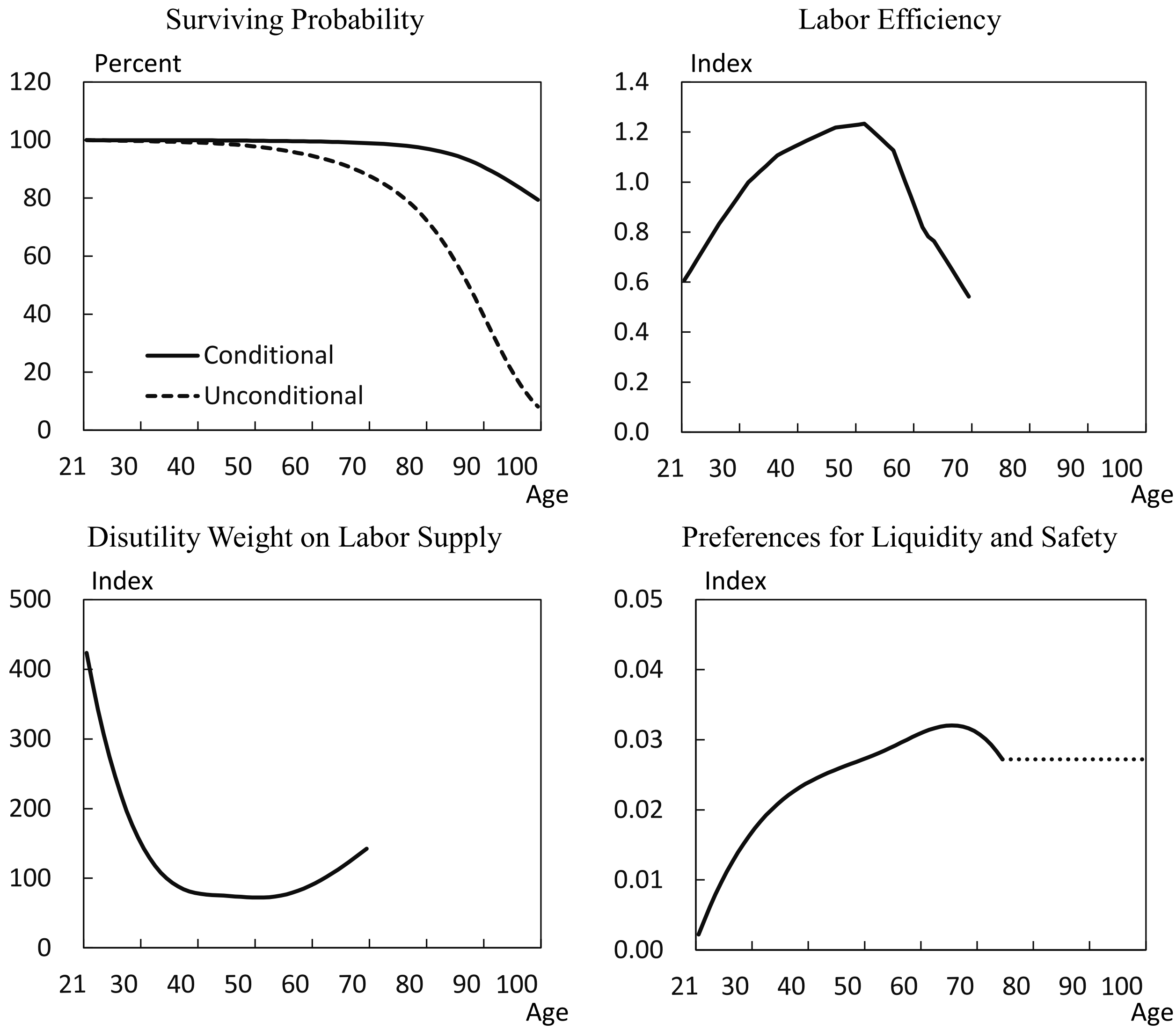
Table 3: Constant Parameters

Symbol	Description	Full sample: 1980-2021	Sub-sample: 2000-2021
α	Capital share	0.412	0.440
β	Subjective discount factor	0.999	0.989
$1/\nu$	Frisch elasticity of labor supply	0.5	0.5
θ	Replacement ratio of public pension	0.40	0.40

Table 4. Steady-State Values of Exogenous Variables

Symbol	Description	Full sample: 1980-2021	Sub-sample: 2000-2021
f	Growth rate of age-21 population	-0.004	-0.014
π	Inflation rate	0.004	-0.002
ζ	Disutility weight on labor	280	300
η	Preference for money (M0) holdings	0.032	0.040
	Preference for money (M2) holdings	0.160	Not used
γ	Preference for bond holdings	0.0075	0.037
$(1+z)^{1-\alpha}$	TFP growth rate	$1.012^{1-\alpha}$	$1.001^{1-\alpha}$
δ	Depreciation rate	0.072	0.066
τ^c	Consumption tax rate	0.1	0.1
τ^l	Labor income tax rate	0.368	0.401
τ^k	Capital income tax rate	0.392	0.342
G/Y	Government expenditure-GNP ratio	0.19	0.20
B/Y	Net government debt-GNP ratio	0.35	0.53
M/Y	Money (M0)-GNP ratio	0.25	0.41
	Money (M2)-GNP ratio	1.25	1.51

Figure 6: Exogenous Age-Specific Parameters



Note: The values for the surviving probability are the ones for the terminal steady state. The values for household preference for liquidity and safety above 75 years old are assumed to be the same as those for 75 years old. Also see Footnote 32 in this regard.

Figure 7: Exogenous Time Series of Macroeconomic Variables

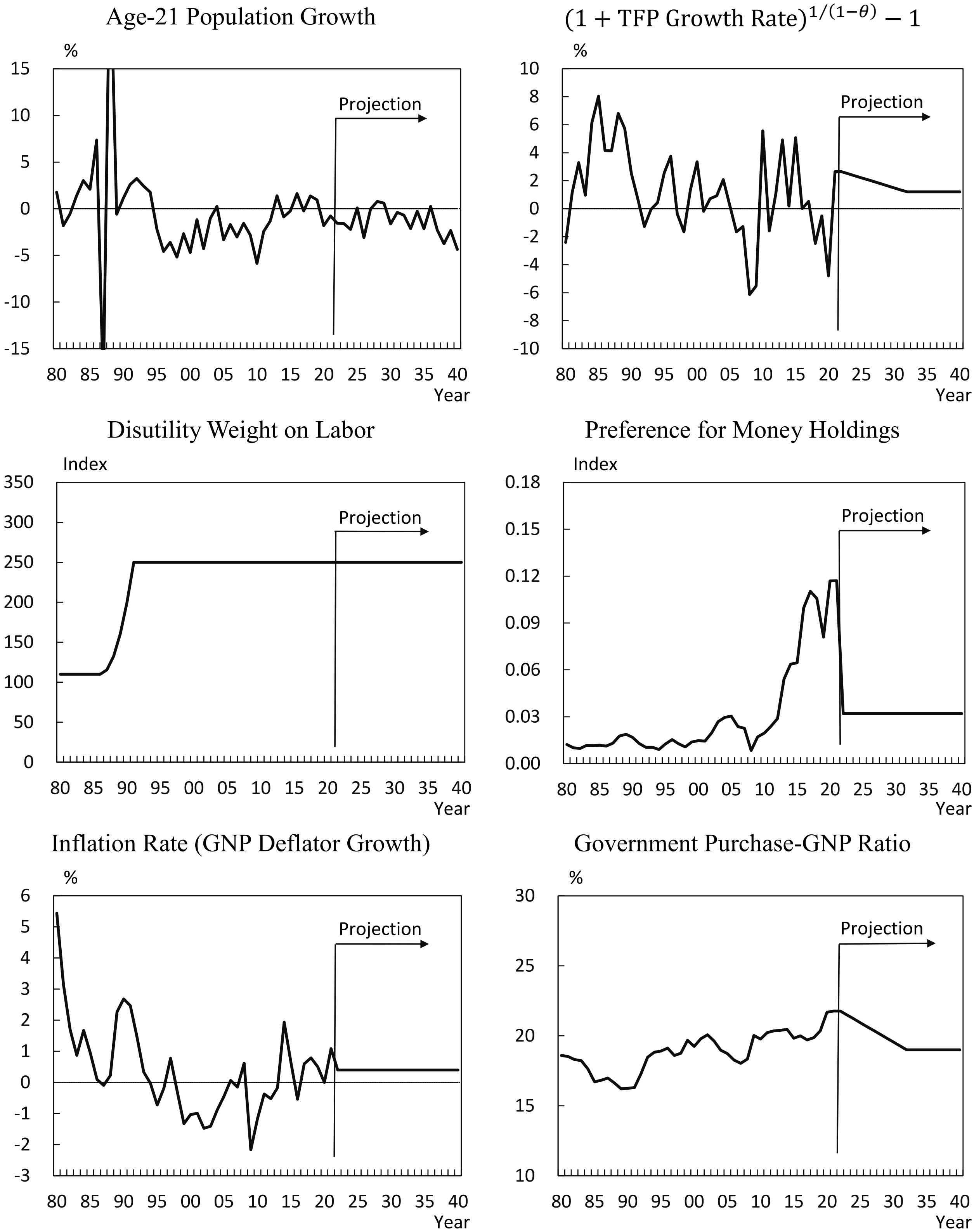
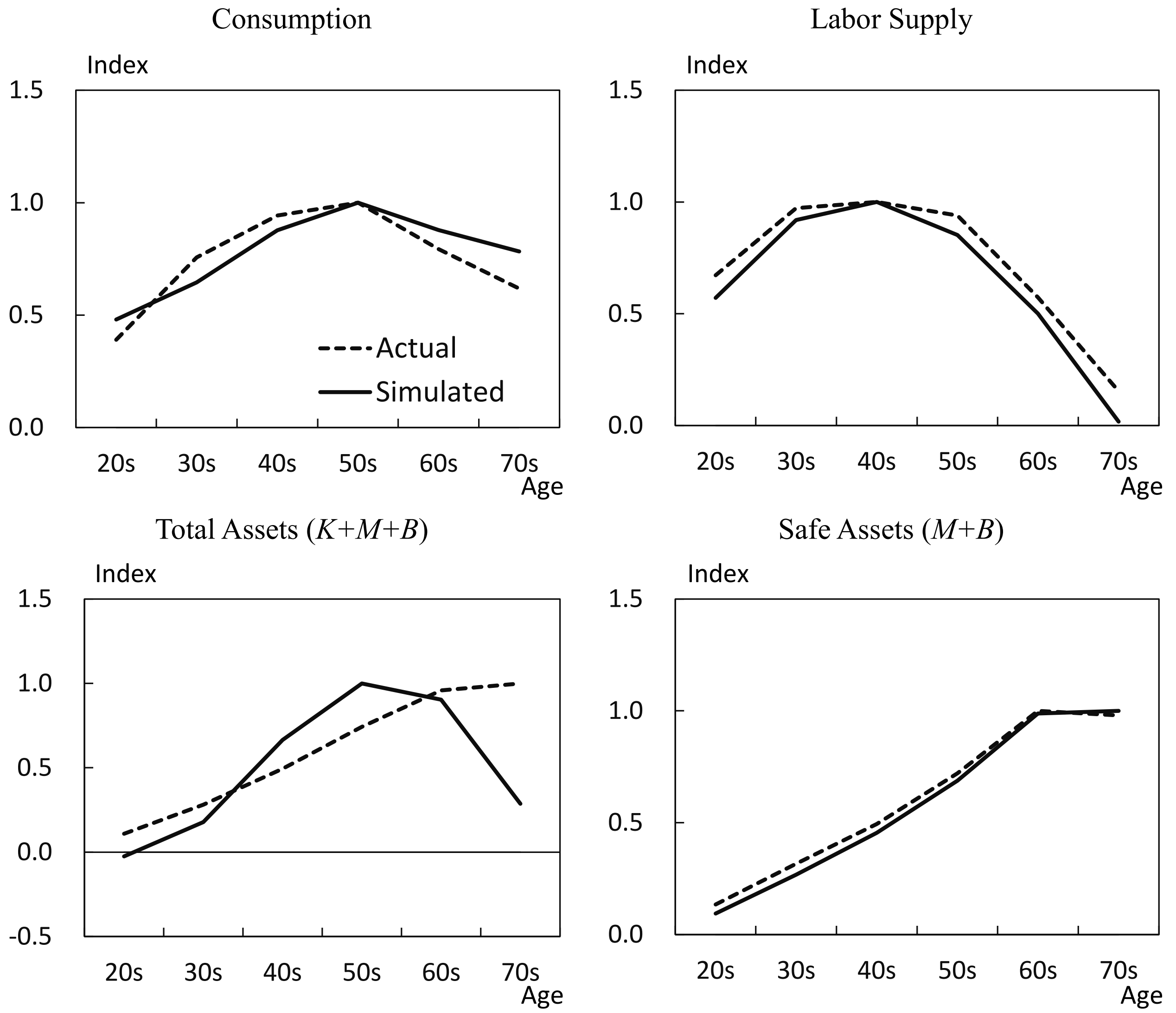


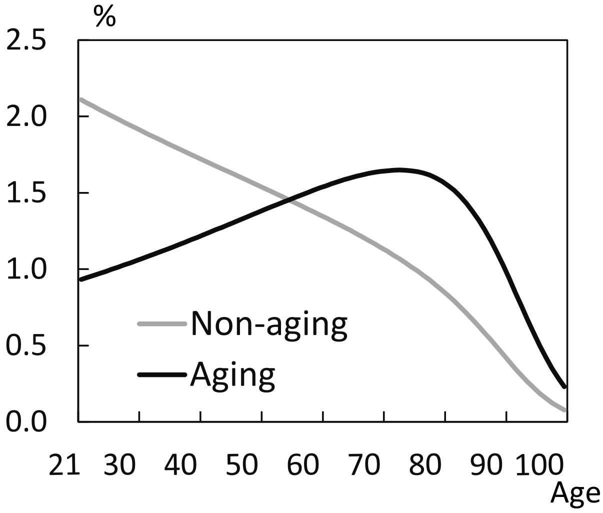
Figure 8: Model Performance: Age-Specific Profiles



Note: In each of the panels, figures are normalized by a maximum value that is equal to one. Labor supply is defined as the employment rate times hours worked.

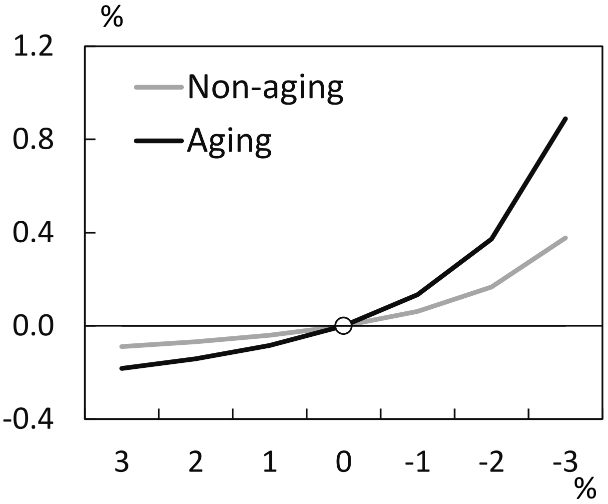
Figure 9: Steady-State Analysis

(a) Population Distribution

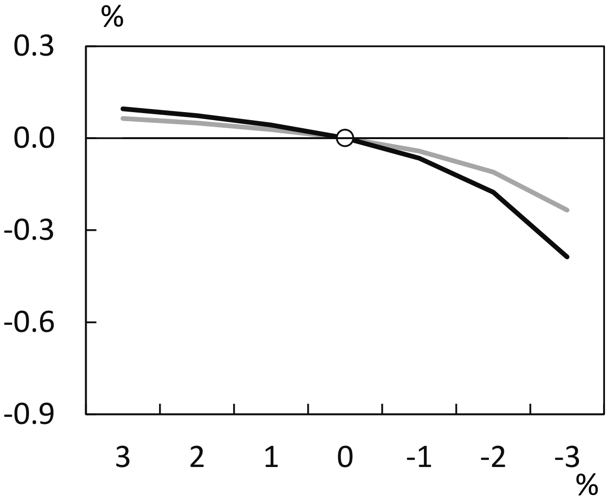


Note: The inflation rate is shown on the horizontal axis; the percentage deviation from the value for zero percent inflation is shown on the vertical axis. Welfare is calculated as the sum of the discounted present values of period utility for households from age 21 at birth to age 100 at death in the model.

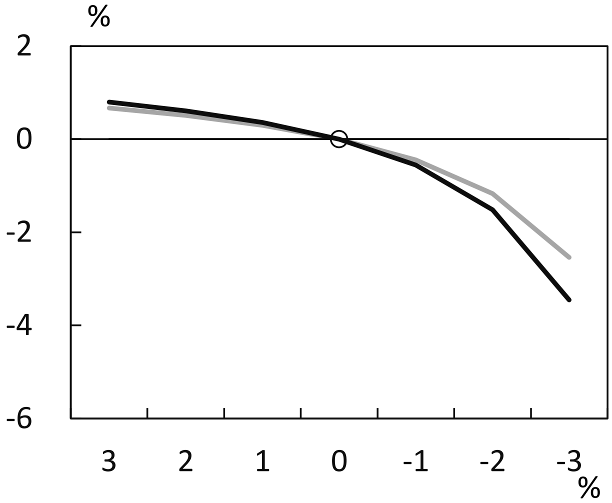
(b) Labor Supply



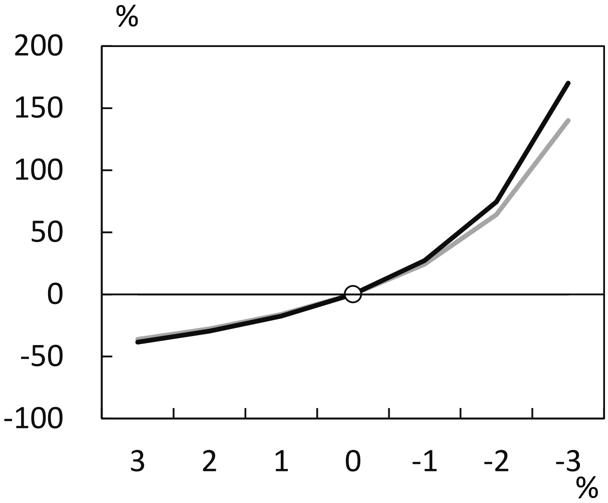
(c) Consumption



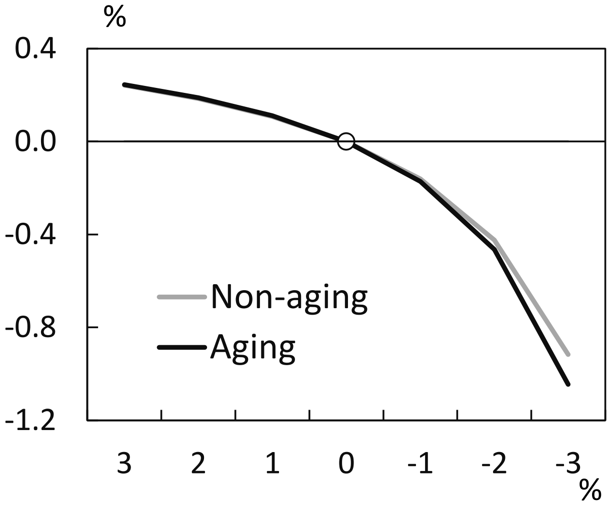
(d) Capital Stock



(e) Real Money Balance



(f) Output



(g) Welfare (Lifetime Utility)

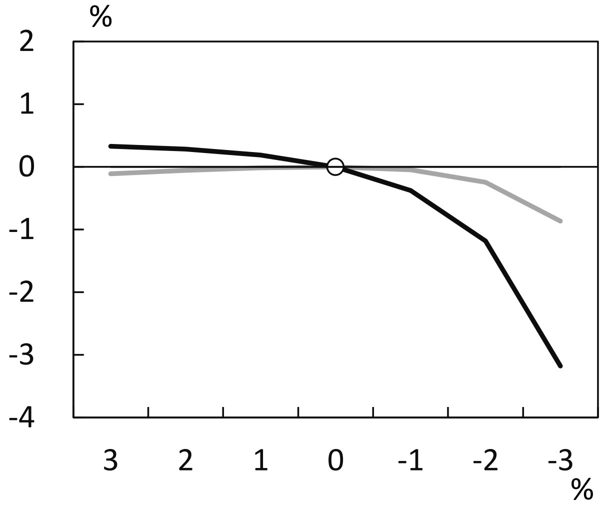


Figure 10: Proportions of Assets at Steady States for the "Aging Case"

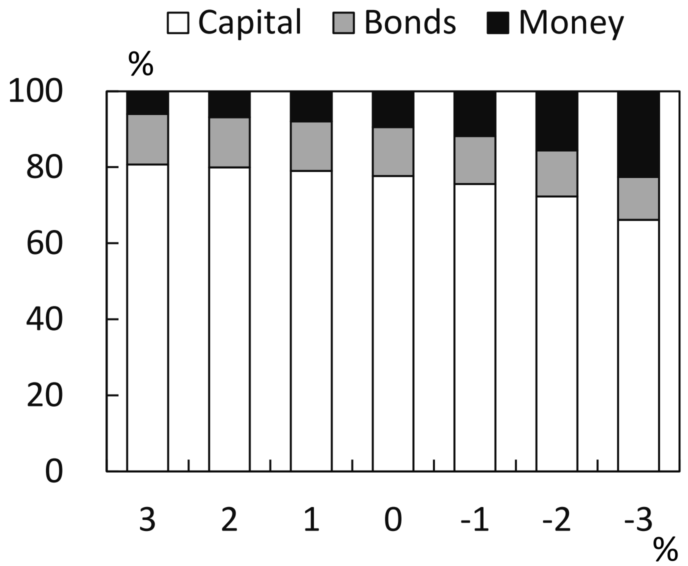
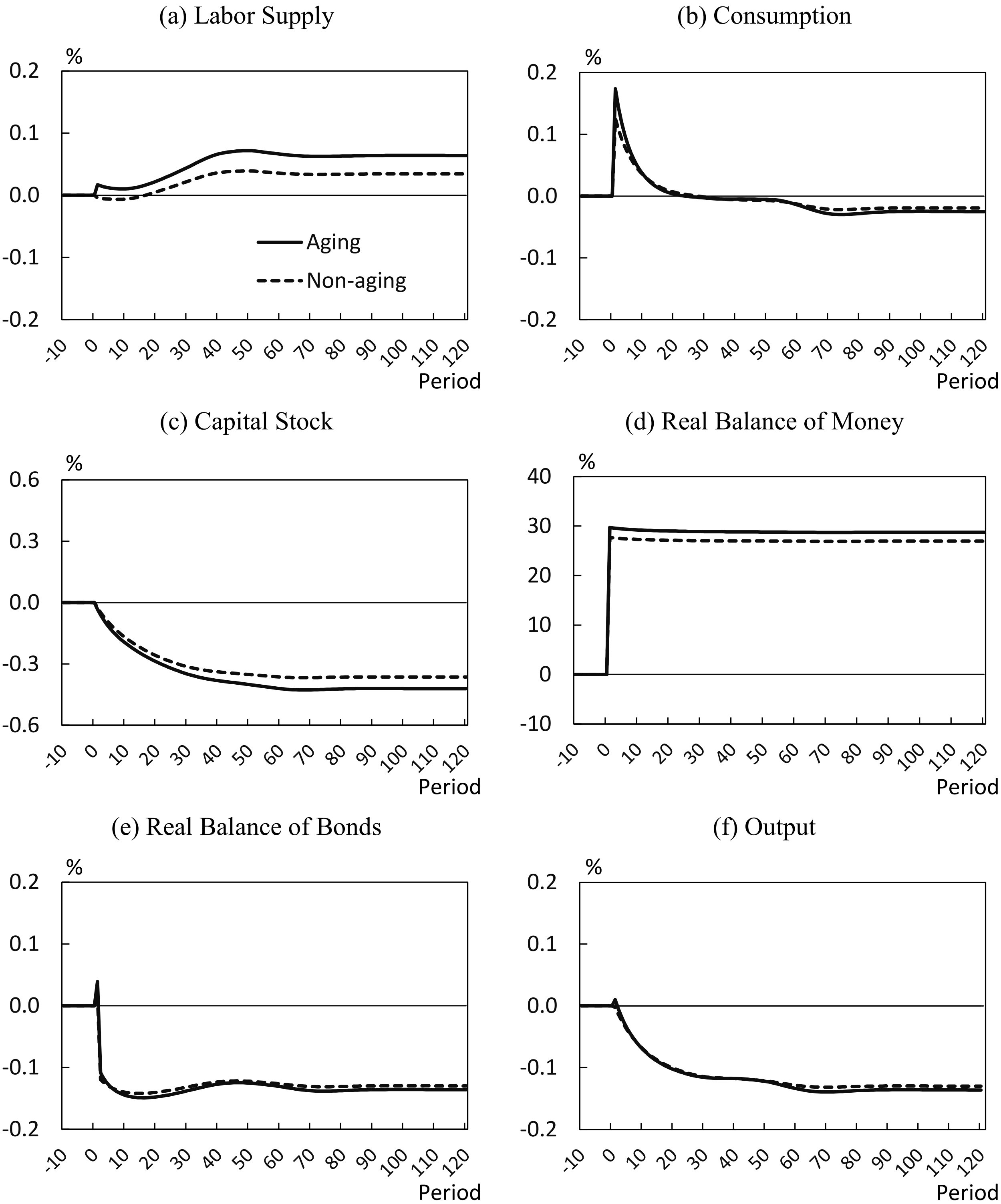


Figure 11: Impulse Responses to a Deflationary Shock:
"Aging Case" vs. "Non-Aging Case"



Note: This simulation assumes the one-time surprise shock that permanently reduces the inflation rate from zero to minus 1 percent by 1 percentage point. The percentage deviation from the initial steady state with the inflation rate of zero percent is shown on the vertical axis.

Figure 12: Model Performance: Time Series of Macroeconomic Variables

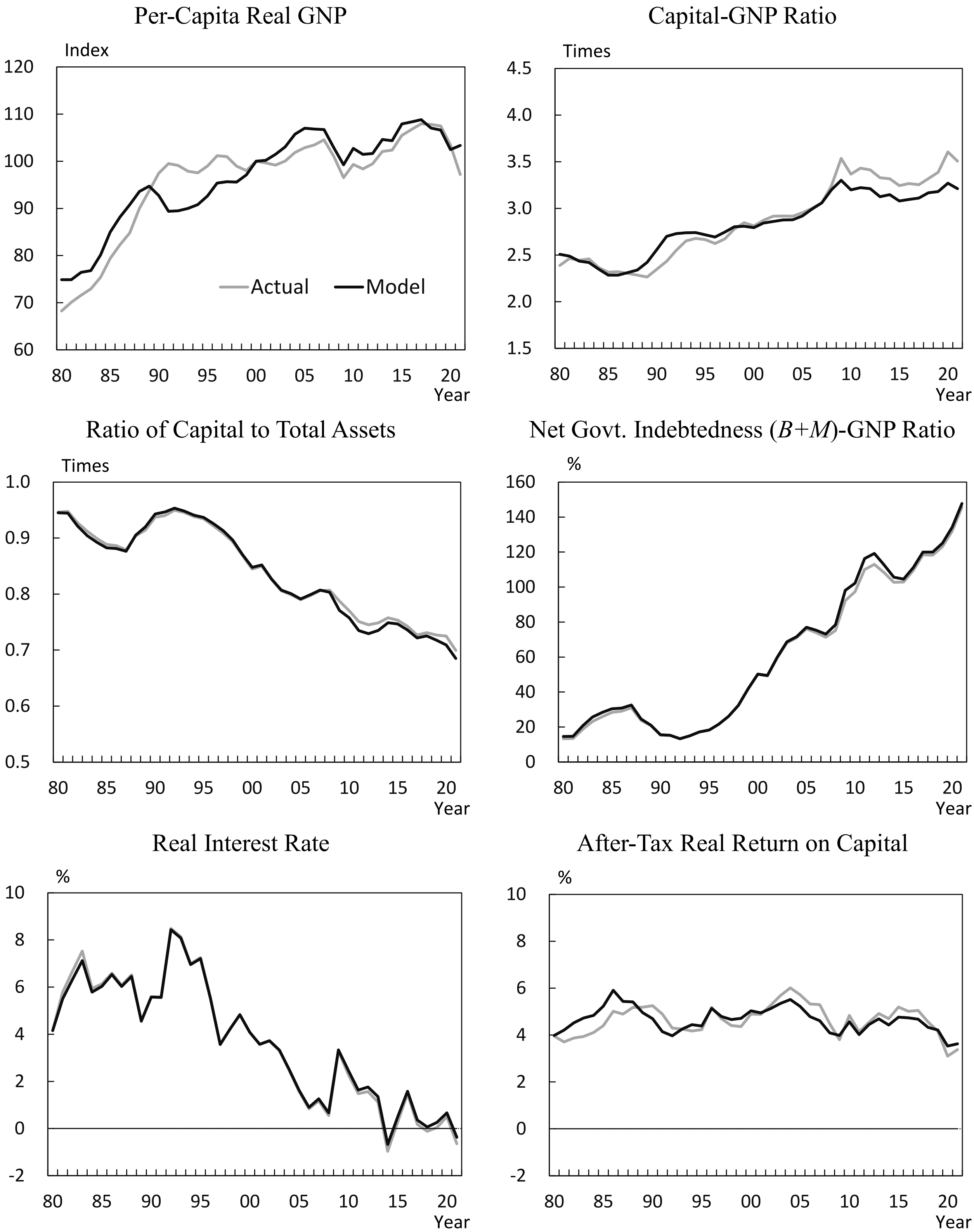
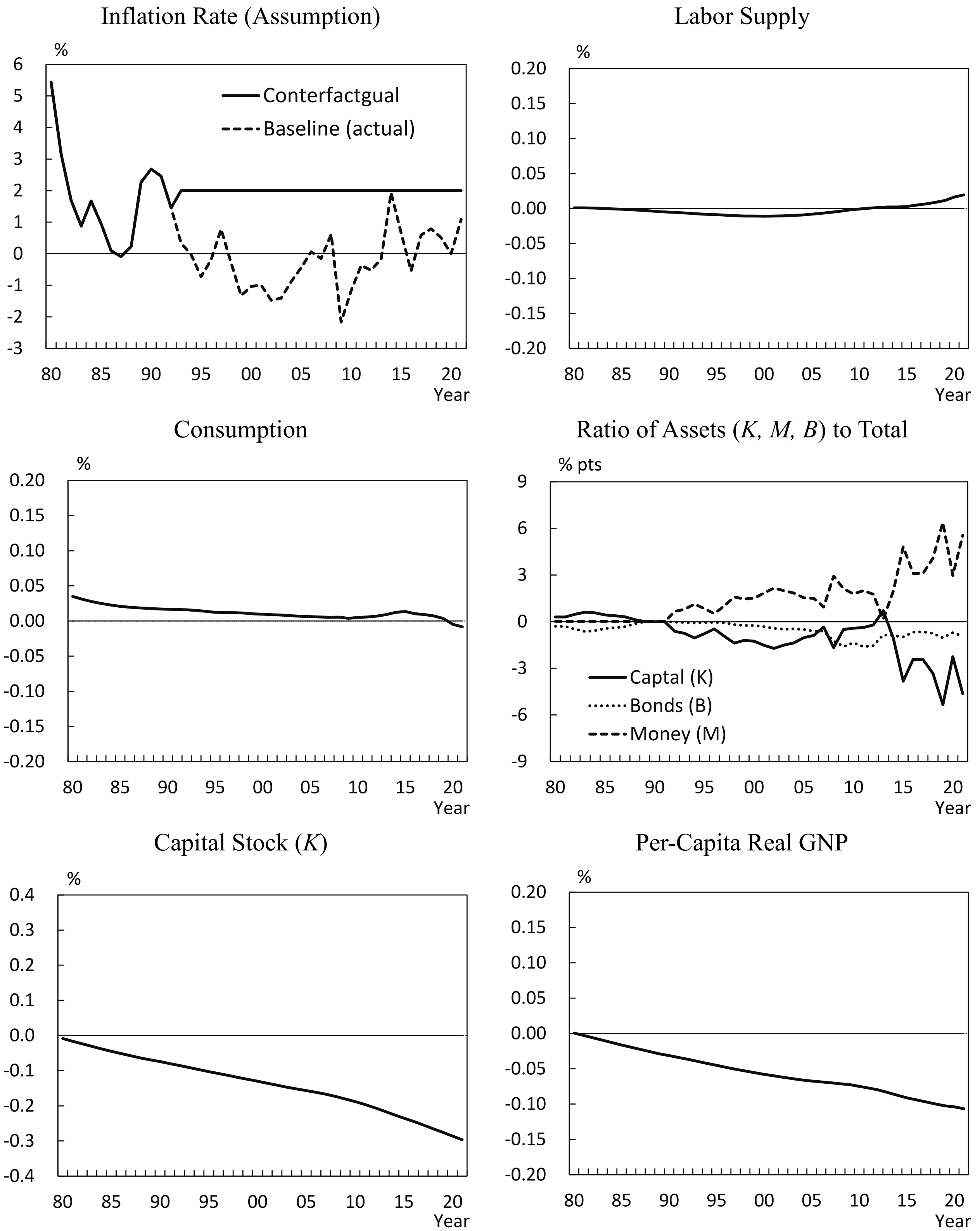
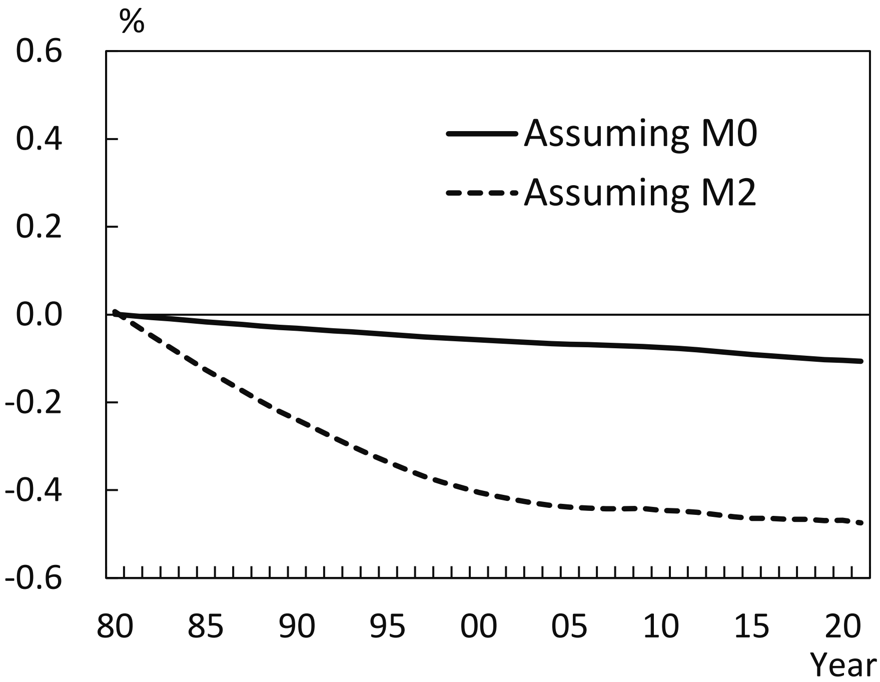


Figure 13: The Impact of Deflation in Japan



Note: Except in the first panel, the percentage deviation or percentage point deviation of the counterfactual simulation from the baseline simulation are shown on the vertical axis.

Figure 14: Impact of Deflation on Per-Capita Real GNP



Note: The percentage deviation of the counterfactual simulation from the baseline simulation is shown on the vertical axis.

Appendix

A1. Detrended and Deflated Version of the Model

This appendix derives the detrended equilibrium conditions in real terms that are used to solve the model numerically. First of all, a real aggregate variable A_t and a real per-capita variable a_t are transformed to detrended per-capita variables $\tilde{A}_t$ and $\tilde{a}_t$, respectively:

$$\tilde{A}_t = \frac{A_t}{N_t Z_t^{1/(1-\alpha)}} \quad \text{and} \quad \tilde{a}_t = \frac{a_t}{Z_t^{1/(1-\alpha)}} \,. \tag{A1-1}$$

Regarding nominal variables $A_t \in \{B_t, M_t\}$ and $a_t \in \{b_{j,t}, m_{j,t}\}$, the detrended per-capita variables are also deflated by the price level P_t:

$$\tilde{A}_t = \frac{A_t}{P_t N_t Z_t^{1/(1-\alpha)}} \quad \text{and} \quad \tilde{a}_t = \frac{a_t}{P_{t_t} Z_t^{1/(1-\alpha)}} \,. \tag{A1-2}$$

By this transformation of variables, government and household budget constraints (6) and (16) are rewritten as follows:

$$\begin{aligned} &\tilde{G}_t + \frac{\widetilde{M}_{t-1}}{(1+\pi_t)(1+z_t)(1+\rho_t)} + \frac{(1+x_{t-1})\tilde{B}_{t-1}}{(1+z_t)(1+\rho_t)} + \widetilde{SS}_t \\ &= \widetilde{M}_t + \tilde{B}_t + \tau_t^c \tilde{C}_t + \tau_t^l \widetilde{w}_t \tilde{L}_t + \frac{\tau_t^k (R_{t-1} - \delta_t) \widetilde{K}_{t-1}}{(1+z_t)(1+\rho_t)} + \tilde{\tau}_t \end{aligned} \tag{A1-3}$$

and

$$\begin{aligned} &(1+\tau_t^c)\tilde{c}_{t,j} + \tilde{k}_{t,j} + \tilde{b}_{t,j} + \widetilde{m}_{t,j} \le \tilde{\Omega}_t + \tilde{\xi}_t - \tilde{\tau}_t \\ &+ \frac{(1+r_{t-1})\tilde{k}_{t-1,j-1}}{1+z_t} + \frac{(1+x_{t-1})\tilde{b}_{t-1,j-1}}{1+z_t} + \frac{\widetilde{m}_{t-1,j-1}}{(1+\pi_t)(1+z_t)} \,. \end{aligned} \tag{A1-4}$$

together with the factor prices expressed by

$$\widetilde{w}_t = (1-\alpha)\left[\frac{1}{(1+z_t)(1+\rho_t)} \frac{\widetilde{K}_{t-1}}{\tilde{L}_t}\right]^{\alpha} \quad \text{and} \tag{A1-5}$$

$$R_{t-1} = \alpha \left[\frac{1}{(1+z_t)(1+\rho_t)} \frac{\widetilde{K}_{t-1}}{\tilde{L}_t} \right]^{\alpha-1} . \tag{A1-6}$$

and with the money supply rule given by

$$(1+z_t)(1+\rho_t)(1+\pi_t)\widetilde{M}_t = (1+\sigma_t)\widetilde{M}_{t-1} . \tag{A1-7}$$

The clearing conditions for the labor market and the asset markets are

$$\tilde{L}_t = \sum_{j=21}^{70} \mu_{j,t}\varepsilon_j h_{j,t} \; ; \tag{A1-8}$$

$$\widetilde{K}_t = \sum_{j=21}^{100} \mu_{j,t}\tilde{k}_{j,t} \; ; \; \widetilde{B}_t = \sum_{j=21}^{100} \mu_{j,t}\tilde{b}_{j,t} \; ; \; \widetilde{M}_t = \sum_{j=21}^{100} \mu_{j,t}\widetilde{m}_{j,t} \; . \tag{A1-9}$$

The aggregate resource constraint is also transformed to

$$\tilde{Y}_t = \left[\frac{\widetilde{K}_{t-1}}{(1+z_t)(1+\rho_t)} \right]^{\alpha} \tilde{L}_t^{1-\alpha} = \tilde{C}_t + \widetilde{K}_t - \frac{(1-\delta_t)\widetilde{K}_{t-1}}{(1+z_t)(1+\rho_t)} + \tilde{G}_t \, , \tag{A1-10}$$

where $\tilde{C}_t = \sum_{j=21}^{100} \mu_{j,t}\tilde{c}_{j,t}$ is aggregate consumption. (A1-11)

The set of the first order conditions from the household's utility maximization for $j = 22, 23, \ldots, 100$, is given in the form of the intra-temporal and inter-temporal marginal rates of substitution (MRS) with consumption:

$$(1+\tau_t^c)\frac{-u_h^j}{u_c^j} = \widetilde{w}_t \varepsilon_j \left(1-\tau_t^l\right) ; \tag{A1-12}$$

$$(1+\tau_t^c)\frac{u_b^j}{u_c^j} = 1 - \frac{1+x_t}{1+r_t} \; ; \tag{A1-13}$$

$$(1+\tau_t^c)\frac{u_m^j}{u_c^j} = 1 - \frac{1}{(1+r_t)(1+\pi_t)} \ ; \tag{A1-14}$$

$$\frac{1+\tau_t^c}{1+\tau_{t-1}^c} \cdot \frac{u_c^{j-1}}{u_c^j} = \beta\psi_{j,t}\frac{1+r_{t-1}}{1+z_t} \ , \tag{A1-15}$$

where let u_a^j denote the marginal utility of a variable $a \in \{c, h, b, m\}$ chosen by age-j households in period t. Then, there are 10 kinds of equations (except for the aggregation formula (A1-8), (A1-9), and (A1-11)) with respect to 10 kinds of unknowns $\{\tilde{c}_{j,t}, \tilde{h}_{j,t}, \tilde{k}_{j,t}, \tilde{b}_{j,t}, \widetilde{m}_{j,t}, \widetilde{w}_t, r_t(R_t), \sigma_t(\pi_t), x_t(i_t), \tau_t\}$ in each period t.