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Abstract

What are the main drivers of the recent increase in wealth concentration in the U.S.? This paper quantifies the role played by digitalization using a tractable model with heterogeneous agents with risk aversion. The model combines (1) digital capital that substitutes for labor in the production process and (2) households' investments in risky digital assets to replicate the asset growth of the wealthy since the 1990s. In the equilibrium, a small number of prosperous households with low risk aversion, i.e., digital entrepreneurs, hold most of the risky digital capital, whereas a large number of risk-averse households rely mainly on labor income. Hence, when digitalization advances, these risk-tolerant households enjoy higher returns from digital capital, further accumulating digital capital disproportionately. Based on the model calibrated to the U.S. economy, we show that digitalization (an increase in digital productivity by 21-43 percent) has contributed to more than about 50 percent of the increase in the share of wealth of the top 1 percent of households and more than about 80 percent of that of the top 0.1 percent of households observed over the last 30 years. Moreover, it explains about 20-40 percent increase in the annual savings of the top 1 percent of households. Finally, the comparative statics on the macroeconomic variables show that while advances in digitalization decrease the labor share by 3-5 percentage points, which is in line with the empirical literature, it also increases wages, meaning that risk-averse households, who rely mainly on labor earnings, also gain some benefits from digitalization.

Keywords: Digitalization; Entrepreneurship; Wealth inequality; Savings inequality
JEL classification: E21, E22, E24, E25

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1 INTRODUCTION

Wealth inequality is a salient feature of the modern U.S. economy, and elucidating its mechanisms has become one of the most critical research agendas in macroeconomics. Saez and Zucman (2016, 2020) show that wealth concentration in the U.S. economy has increased substantially in recent decades, with most of this increase being driven by the wealthiest classes of the population. In other words, wealth inequality has arisen as a thicker tail of the asset distribution. Gomez (2023) decomposes the growth of the top percentile of the wealth distribution and finds that more than half of the rise in top wealth inequality is explained by the rise in the fortunes of individuals who have newly appeared in the top percentile.

Evidence suggests that many of the recent wealthiest are entrepreneurs who have actively invested in the development of digital technologies. According to the Federal Reserve Board's Distributional Financial Accounts presented in Table 1, as of 2022, private business accounts for 31.8% of the assets of the top 0.1% of households in the wealth distribution, second only to corporate equities and mutual fund shares at 41.6%. This means that the wealthiest part of the population includes not only investors who earn large returns on their financial assets but also entrepreneurs who run their own businesses. Consistent with this finding, using U.S. administrative tax data, Smith et al. (2019) report that a large portion of the income of the top 1% in the U.S. as of 2014 is attributable to the human capital of entrepreneurs. The Forbes list of the richest people in the U.S. in Table 2 shows more directly that most of the wealthiest are entrepreneurs who have actively developed and used digital technologies. Digital entrepreneurs now make up the majority of the top tier of U.S. billionaires, and the expansion of wealth inequality in the U.S. has been driven by the rise of these digital entrepreneurs.

Table 1: Asset composition by wealth percentile group in 2022:Q4

Income Source	Bottom 50 %	10-50 %	1-10 %	0.1-1.0%	Top 0.1%
Real estate	56.9	38.1	24.1	16.6	8.2
Consumer durable goods	18.9	6.7	2.9	1.9	2.4
Corporate equities and mutual fund shares	2.1	7.5	21.6	40.3	41.6
Defined benefit pension entitlements	3.2	18.0	15.9	1.7	0.3
Defined contribution pension entitlements	5.5	9.6	9.8	3.1	0.8
Private businesses	1.7	4.2	8.8	16.8	31.8
Other assets	11.7	15.9	16.8	19.7	14.8

Source: Federal Reserve Board Distributional Financial Accounts (DFA).

Table 2: Forbes, The Ten Richest Americans in 2023

Rank	Name	Net Worth (dollar, billion)	Source of Wealth
1	Elon Musk	251	Tesla, SpaceX
2	Jeff Bezos	161	Amazon
3	Larry Ellison	158	Oracle
4	Warren Buffett	121	Berkshire Hathaway
5	Larry Page	114	Google
6	Bill Gates	111	Microsoft
7	Sergey Brin	110	Google
8	Mark Zuckerberg	106	Facebook
9	Steve Ballmer	101	Microsoft
10	Michael Bloomberg	96.3	Bloomberg LP

Source: Forbes.

Against this backdrop, this paper investigates how the development of digital technology has had a significant impact on wealth inequality. To this end, we construct a tractable dynamic stochastic general equilibrium model with heterogeneous agents. In an otherwise standard neoclassical growth model, our model includes three key ingredients. First, we assume that firms use three types of inputs for production: labor, traditional capital, and digital capital. Following recent studies on labor replacement with new technology, such as Alonso et al. (2022) and Eden and Gaggl (2018), we assume that digital capital is a substitute for labor, while labor is a complement of traditional capital. In addition, we assume that digital capital is complementary to traditional capital, as in Berg et al. (2018).[1] Second, we assume that investment in digital capital is risky: Autor et al. (2020) argue that high-tech industries have a "winner take most" nature with network effects and scale-biased innovations that provide a very high return on investment for winners but little to nothing for losers. Brynjolfsson et al. (2020) also point out that the value of digital capital is more volatile than that of traditional capital because it is intangible, less traded on secondary markets, and its value may be more closely tied to a particular firm. This high-risk/high-return characteristic of digital technology firms can be empirically confirmed in U.S. data (Rajgopal et al., 2022). Third, to introduce digital entrepreneurs, we assume that households are heterogeneous in preference parameters (Gârleanu and Panageas, 2015), and decide how much to invest in assets with different risks depending on their risk aversion

[1] Berg et al. (2018) refer to what we call digital capital as "robot capital," which is "the combination of computers, artificial intelligence, big data and the digitalization of information, networks, sensors and servos that are emphasized in the literature on the new machine age (Brynjolfsson and McAfee (2014))."

and time preference. A small fraction of households who are sufficiently risk-tolerant to invest in risky digital assets are interpreted as entrepreneurs in our model. Their investment in risky digital assets builds up digital capital, an input to the production side of the economy.

The contribution of this study is three-fold. First, we develop a parsimonious heterogeneous agent model replicating the U.S. wealth distribution well. In our heterogeneous settings, the households with low risk aversion actively invest in risky digital assets because their return is higher, whereas households with high risk aversion refrain from investing in such assets due to their uncertain returns. Thus, in the equilibrium, successful entrepreneurs willing to take risks accumulate high-return digital wealth, while risk averse households mainly rely on labor earnings and accumulate a moderate amount of wealth, creating wealth inequality. Comparing the wealth distribution between the data and the model, we show that the stationary equilibrium of the benchmark economy calibrated to the U.S. economy replicates the wealth distribution reported by Saez and Zucman (2020) and the Survey of Consumer Finances (SCF). This demonstrates that our parsimonious setting of the model, which incorporates heterogeneity in household preferences on risk aversion and risky digital assets in the production function, are capable of capturing the wealth inequality in the U.S.

Second, we evaluate the importance of advances in digital technology in explaining the rise in wealth inequality in the past 30 years. To capture the effects of digitalization, we conduct comparative statics on the impact of increased productivity of digital capital. We obtain the analytical result that, under certain conditions, an increase in the productivity of digital capital widens wealth inequality and lowers the labor share. Our model also replicates the wealth concentration in the U.S. economy from 1989 to 2019 reported by Saez and Zucman (2020) and the SCF well. In particular, quantitative analysis based on our model confirm that digitalization has significantly increased the share of households with low risk aversion in the top 0.1% and 1% of the wealth distribution. This result indicates that the concentration of risky digital capital in risk-tolerant households due to advances in digital technology can explain the magnitude of the rise in U.S. wealth inequality in the last three decades.

Third, we validate our heterogeneous agent model by cross-checking the testable implications of the model along several dimensions of macroeconomic data. Savings and investing decisions of households play a key role in our analysis. Mian et al. (2021) elucidate the savings behavior of households over the past 40 years based on an empirical study, pointing out a significant rise in savings by the top 1% of the wealth distribution. Fagereng et al. (2020) use Norwegian administrative panel data and conclude that capital gains are the crucial factor in explaining the differences in savings rates between the rich and the poor. Compared to these studies, we use a model with risky digital capital to assess the importance of changes in household behavior and changes in the wealth distribution in explaining the increase in the savings of the rich and investment in digital assets. In particular, we decompose the change in savings into two components —changes in the savings policy function and changes in the distribution of wealth —to quantify the driving forces behind the rise in savings inequality. Our decomposition suggests that the change in the distribution of wealth is the predominant factor in the change in savings, meaning that the wealthy group accelerated savings mainly because their wealth position grew. We also explore other distributional implications of digitalization. Advancement in digitalization increases output through improvements in productivity, but the increase in output is disproportionately distributed to risk-tolerant households. In particular, we find a rise in the return on digital capital and a decline in the labor share. Thus, entrepreneurs who own most of the digital assets benefit from the advances in digital technology, whereas the risk-averse households who rely mainly on labor earnings are mostly excluded from the benefits, leading to a rise in wealth inequality. Nevertheless, the wage slightly increases, implying that risk-averse households also benefit moderately from digitalization.

RELATED LITERATURE

This article is related to a recent body of work studying the impacts of automation. Acemoglu and Restrepo (2018, 2020) show that automation would reduce employment and wages, incorporating a task-based model in the spirit of Zeira (1998) into a representative agent framework. Extending the task-based model to a heterogeneous agent framework, Moll et al. (2022) and Hémous and Olsen (2022) show that advances in automation could be a key driver of the observed trends in income and wealth inequality. Specifically, Moll et al. (2022) analyze the impact of automation on increasing capital income inequality, while Hémous and Olsen (2022) investigate the impact of automation on increasing labor income inequality through an expanding skill premium, using an endogenous growth model.

Our model differs from theirs because we aim to study the impact of digitalization, building on Berg et al. (2018) rather than a task-based model by incorporating digital capital as a complement to traditional capital while being a substitute for labor in the production function. Therefore, our model potentially captures the positive impact of digitalization on traditional capital accumulation. In addition, our model introduces the high-risk, high-return characteristic of digital capital that distinguishes it to from traditional capital. In this respect, our model departs from Berg et al. (2018) by embedding their approach into a standard heterogeneous-agent framework, as in Aoki and Nirei (2017). Based on previous studies (Cagetti (2003), Gomes and Michaelides (2008), Calvet et al. (2021)), we introduce heterogeneities in time preference and risk aversion, enabling households to decide how much to invest in assets with different risks depending on their risk aversion and time preference. This model framework illustrates in a parsimonious way the channel through which an advancement in digitalization leads to the concentration of wealth at the top of the distribution.

This paper is also related to the extensive literature studying the forces behind rising wealth inequality. Similar to our study, two relatively early studies on wealth inequality in the United States, Quadrini (1999) and Cagetti and De Nardi (2006) focus on the presence of entrepreneurs in the top tier of the wealth distribution. However, they consider the possibility that the presence of borrowing constraints on entrepreneurs increases their savings rate and affects wealth inequality, which differs in perspective from our analysis, which focuses on digital capital and its risk characteristics. De Nardi and Fella (2017) list several possible factors behind the rise in U.S. wealth inequality and examine the relevance of each, concluding that the presence of entrepreneurs is one important determinant. İmrohoroğlu and Zhao (2022) demonstrate that, in a period of lower interest rates, entrepreneurs benefit from lower borrowing costs and workers suffer lower returns, thus increasing wealth inequality. However, these studies do not pay special attention to digital capital or digital entrepreneurs. Gomes et al. (2024) investigate how automation affects the accumulation of household wealth. They show that households reduce their exposure to the stock market in response to the rise in human capital risks due to increased automation. These studies differ from ours in that they focus on the portfolio choices of ordinary households rather than entrepreneurs. Lee (2021, 2023) and Straub (2019) examine the implications of increased wealth or income inequality on macroeconomic variables such as aggregate demand and real interest rates. However, these studies do not analyze the relationship between wealth inequality and digitalization.

The rest of the paper is organized as follows. In section 2 we develop the model to study the quantitative predictions of digitalization, and present a basic analytical result on inequality. Section 3 describes the calibration strategy by explaining how we match the model to data in the U.S. Section 4 discusses the quantitative results on how digitalization affects inequality, savings behavior, and macroeconomic variables. Section 5 concludes.

2 MODEL

In this section, we develop a parsimonious and tractable model to quantify the impact of digitalization on the concentration of wealth and savings in the top portion of the wealth distribution. We use a continuous time setting so that we explicitly obtain a double-power law stationary distribution of wealth using the result of Toda (2014).

There are two key features of the model. First, households are heterogeneous in relative risk aversion γ_i and time preference ρ_i. With this setup, we allow households to selectively choose entrepreneurial investments depending on their preferences, thereby generalizing the model of household wealth distribution by Aoki and Nirei (2017). In this way, digitalization affects not only production technology but also the differential savings choices of households.

Second, we introduce risky labor-substituting capital in addition to traditional capital, following the formulation of Berg et al. (2018). We interpret the labor-substituting capital as digital capital. Digital capital is risky, meaning that the volatility of the return is higher than that of safe assets. Risk-averse households have the incentive to invest in risky digital capital when its mean return is higher than that of the safe asset.

The underlying uncertainty that drives the ex-post dispersion of household wealth stems from distributional shocks to the return to digital capital. We posit that an individual investor of digital assets is exposed to an uninsurable idiosyncratic risk to the rate of return, whereas the mean return to digital capital is not affected by the idiosyncratic risk. For example, owners of digital capital are subject to stochastic depreciation of the capital, which makes their asset holdings volatile. These distributional shocks aid our model analysis by simplifying the transmission mechanism.

2.1 HOUSEHOLDS

There is a continuum of households of measure 1, indexed by $i \in [0,1]$. They differ in risk aversion γ_i as well as time preference ρ_i. We use a perpetual youth model where each household dies at Poisson hazard rate ζ and is replaced by a newborn household with the same preferences. Households participate in a public pension program that seizes all their assets upon death while living households receive ζ times their financial wealth. Thus, if a household dies, all of its assets except for human capital are distributed to other living households.

Households have the following Epstein-Zin recursive utility, where we can separate relative risk aversion from the elasticity of intertemporal substitution.

$$J_t^i = \max \ \mathbb{E}_t \left[\int_t^{\infty} f^i(C_s, J_s^i) ds \right].$$

f^i is a normalized aggregator of current consumption C and continuation utility J^i as follows,

$$f^i(C, J^i) = (\rho_i + \zeta)(1-\gamma_i)J^i \left[\log(C) - \frac{1}{1-\gamma_i} \log\Big((1-\gamma_i)J^i\Big) \right],$$

where the stochastic death rate ζ is added to the time preference discount rate ρ. In our setting, the elasticity of intertemporal substitution is assumed to be one.

Households inelastically supply a unit of labor and receive labor income ω. The portfolio of households consists of risk-free assets and risky digital assets. Risk-free assets consist of traditional capital K and human wealth H. Household i chooses its portfolio depending on its level of risk aversion and time preference. Let θ_i be the fraction of total wealth W_i invested in digital capital and $1-\theta_i$ be the fraction invested in risk-free capital. The net rate of return to traditional capital is equal to the risk-free rate r, and the depreciation rate of traditional capital is denoted by δ. Human wealth H_t is defined by

$$H_t = \int_t^{\infty} \omega_s e^{-\int_t^s (\zeta + r_u) du} ds.$$

The steady-state value of human wealth is $H = \omega / (\zeta + r)$.

In addition to the risk-free asset, households can invest in a risky asset, that is, digital assets A_t. A unit of output can be converted to a unit of digital assets, which is used by firms as digital capital to produce goods. A unit of digital assets generates a mean rate of return r_A. Moreover, we assume that digital assets are subject to a household-specific rate-of-return risk σdZ_t, where Z_t follows the Wiener process. That is, the aggregate return of digital capital is deterministic and given by r_A, whereas digital assets held by each household generate stochastic returns with diffusion parameter σ, making this investment risky since the realized return to individuals can be volatile. Therefore, the instantaneous rate of return to digital assets is written as

$$r_A dt + \sigma dZ_t.$$

Our motivation for introducing the term σdZ_t is to capture the highly volatile nature of investment in digital assets. As described in previous studies (Autor et al. (2020), Brynjolfsson et al. (2020), and Rajgopal et al. (2022)), the high-tech industry tends to show high-risk, high-return properties: successful investment yields significant profits, whereas households lose a disproportionate amount of wealth if a negative shock is realized. To tractably incorporate the rate of return risk into our model, we consider the following setup. The digital asset A_t in aggregate generates deterministic return $r_{A,t}$, which is equal to the deterministic marginal product of A_t minus its depreciation. However, an individual household is exposed to household-specific distributional shocks at each instant, meaning that the distribution of its yield is different across investors.[2] In this setup, each household faces an idiosyncratic rate of return risk when it holds digital assets. In our economy with heterogeneous risk aversion, risk-tolerant households invest disproportional amounts of their wealth in digital capital. Thus, in the stationary equilibrium, successful risk-tolerant households dominate the wealthy class, as they accumulate their wealth through prosperous investment projects. Since the successful risk-tolerant households generate their income mostly from their digital assets rather than risk-free assets, including labor earnings, they represent entrepreneurs in our model.

Households maximize the Epstein-Zin recursive utility by optimally choosing sequences of

[2] Thus, the aggregated excess return on digital assets, $\theta_i W \sigma dZ_t$, is always equal to zero.

consumption, C_t, and an asset portfolio, θ_i, under the following budget constraint.

$$dW_i = [(\theta_i r_A + (1-\theta_i)r + \zeta)W_i - C]\,dt + \theta_i W_i \sigma\,dZ. \tag{1}$$

Here, W_i denotes the "total wealth" of households, which is the sum of traditional capital, digital assets, and human wealth.[3] Assuming that return on assets r, r_A and labor earnings ω are constant over time, the Hamilton-Jacobi-Bellman equation is given by

$$0 = \max_{\theta_i, C}\left\{ f^i(C, J^i) + J^i_W((\theta_i r_A + (1-\theta_i)r + \zeta)W_i - C) + \frac{1}{2}J^i_{WW}\theta_i^2 W_i^2 \sigma^2 \right\}. \tag{2}$$

2.2 FIRMS

The production side of the model follows Berg et al. (2018). Firms use three types of inputs for production: labor L_t, traditional capital K_t, and digital capital A_t. Following recent studies on labor replacement with new technology, such as Alonso et al. (2022) and Eden and Gaggl (2018), we assume that digital capital is a substitute for labor, while labor is a complement of traditional capital. In addition, we assume that digital capital is complementary to traditional capital, as in Berg et al. (2018) and Alonso et al. (2022).

More specifically, the firm produces a final good Y_t according to a CES production function

$$Y_t = \left[\lambda_k K_t^{\alpha} + \lambda_V V_t^{\alpha}\right]^{\frac{1}{\alpha}}. \tag{3}$$

V_t denotes the composite of labor L_t and digital capital A_t,

$$V_t = \left[\lambda_A (bA_t)^{\phi} + \lambda_L L_t^{\phi}\right]^{\frac{1}{\phi}}, \tag{4}$$

where b represents digital capital-augmenting productivity. λ_k, λ_V, λ_A, and λ_L denote CES weight parameters for traditional capital K_t, composite V_t, digital capital A_t, and labor L_t, respectively. The elasticity of substitution between K_t and V_t and between A_t and L_t are denoted by α and ϕ, respectively. We assume $\alpha < 0$ to capture the idea that K and V are complements, while $0 < \phi < 1$ so that digital capital A is a substitute of labor L.

[3]Given that human wealth H is the same across all households, we exclude this term when showing the results in later sections. In other words, "wealth" in the results section represents the sum of traditional capital and digital assets.

From (3) and (4), the competitive factor prices of K_t, A_t, and L_t are derived as,

$$r = \lambda_K K^{\alpha-1} Y^{1-\alpha} - \delta, \tag{5}$$

$$r_A = Y^{1-\alpha} \lambda_V V^{\alpha-\phi} \lambda_A b^{\phi} A^{\phi-1} - \delta_A, \tag{6}$$

$$\omega = Y^{1-\alpha} \lambda_V V^{\alpha-\phi} \lambda_L L^{\phi-1}, \tag{7}$$

where δ and δ_A denote the rate of depreciation of traditional capital and digital capital, respectively. We will discuss the relationship between δ and δ_A in the calibration section in more detail.

2.3 STATIONARY EQUILIBRIUM

Here, we define the equilibrium of the model. To simplify the analysis, we exclusively focus on the steady-state solutions and omit the time subscript t henceforth. A competitive equilibrium can be defined as a set of household variables, $\{C_i, \theta_i\}$, price variables $\{r, r_A, \omega\}$, and aggregate variables $\{K, A, L, Y\}$ such that:

- Households maximize their utility subject to budget constraints and taking prices as given.
- Firms maximize their profits. That is, the competitive factor prices satisfy equations (5), (6), and (7).
- Goods, traditional capital, digital capital, and labor markets clear.

First, we provide the solution for the households' problem. Household i maximizes its utility by choosing its consumption C_i and asset portfolio θ_i subject to the budget constraint given the wage and interest rates. Household behavior is characterized by the following equations:

$$\begin{aligned}
\frac{C_i}{W_i} &= \rho_i + \zeta, \\
\theta_i &= \frac{r_A - r}{\sigma^2 \gamma_i}, \\
dW_i &= ((\theta_i r_A + (1-\theta_i) r + \zeta) W_i - C_i)\, dt + \theta_i \sigma W_i dB_{it}, \\
H &= \frac{\omega}{r + \zeta}.
\end{aligned}$$

The first two equations are the linear consumption and portfolio choices familiar from the Merton problem. The first equation shows that the consumption of each household depends only on its time preference and the size of its wealth. The second equation shows that the asset portfolio (share of risky digital assets) is proportional to the risk premium (return on digital asset r_A

minus risk-free rate r), whereas it is inversely proportional to the household's risk aversion and volatility of the rate of return in digital assets σ^2. The third equation shows the stochastic law of motion for wealth. The fourth equation is the definition of human wealth.

Market clearing conditions for digital capital, traditional capital, labor, and goods markets yield the following set of equations:

$$\begin{aligned} A &= \int_0^1 \theta_i W_i di, \\ K &= \int_0^1 (1-\theta_i) W_i di - H, \\ L &= 1, \\ F(b, K, A, L) &= C + \delta K + \delta_A A, \end{aligned}$$

where in the last equation, C denotes the aggregated consumption demand,

$$C = \int_0^1 C_i di.$$

Note that in the steady state, the investment demand for traditional capital is equal to the depreciated amount δK, and the same applies to A as well because we assume that a good is converted to one unit of digital capital. Also note that, in the labor market, we assume that one unit of labor is supplied inelastically by a unit continuum of households.

2.4 STATIONARY DISTRIBUTION: ANALYTICAL SOLUTIONS

In this model, the stationary distribution of wealth can be derived analytically for each type of household. In the following, we show that the wealth distribution follows a double Pareto distribution, as discussed in Toda (2014). First, from the budget constraint, household i's wealth, conditional on its survival, evolves according to

$$d \log W_{it} = \left(\mu_{wi} - \frac{\sigma_{wi}^2}{2} \right) dt + \sigma_{wi} dB_{it}$$

where

$$\mu_{wi} = \theta_i r_A + (1-\theta_i) r - \rho_i, \tag{8}$$

$$\sigma_{wi} = \theta_i \sigma. \tag{9}$$

Household i dies at Poisson rate ζ, in which event W_{it} is reset to the initial wealth H. We can obtain the wealth distribution of households by integrating households' wealth over time.[4] Specifically, the stationary distribution of the logarithm of W_{it} is described by

$$f(\log W_i) = \frac{\kappa_i \zeta}{\chi_i} \left(\frac{W_i}{H}\right)^{-\psi_{1,i}}, \quad \text{if } W_i \geq H$$
$$f(\log W_i) = \frac{\kappa_i \zeta}{\chi_i} \left(\frac{W_i}{H}\right)^{\psi_{2,i}}, \quad \text{if } W_i < H$$

where H is the initial wealth of households, κ_i is the normalization parameter defined shortly, and $\chi_i, \eta_i, \psi_{1,i}$, and $\psi_{2,i}$ are parameters determined as follows:

$$\psi_{1,i} = \frac{\chi_i - \eta_i}{\sigma_{wi}^2}, \tag{10}$$
$$\psi_{2,i} = \frac{\chi_i + \eta_i}{\sigma_{wi}^2}, \tag{11}$$
$$\chi_i = \sqrt{2\zeta\sigma_{wi}^2 + \eta_i^2}, \tag{12}$$
$$\eta_i = \mu_{wi} - \frac{\sigma_{wi}^2}{2}. \tag{13}$$

Thus, by a change of variables, we can obtain the stationary distribution of wealth as

$$f(W_i) = \frac{\kappa_i \zeta}{\chi_i W_i} \left(\frac{W_i}{H}\right)^{-\psi_{1,i}}, \quad \text{if } W_i \geq H \tag{14}$$
$$f(W_i) = \frac{\kappa_i \zeta}{\chi_i W_i} \left(\frac{W_i}{H}\right)^{\psi_{2,i}}, \quad \text{if } W_i < H \tag{15}$$

The normalization parameter κ_i is determined from the following relationship:

$$\int_0^1 \int_0^\infty f(W_i) dW_i di = \int_0^1 \frac{\kappa_i \zeta}{\chi_i} \left\{ \frac{1}{\psi_{1,i}} + \frac{1}{\psi_{2,i}} \right\} di = \int_0^1 \kappa_i di = 1$$

Therefore, for each κ_i, we can compute the fraction of each type of household. Thus, the derived wealth distribution follows a double-Pareto distribution whose Pareto exponent at the upper tail is ψ_{1_i}. We will describe the comparative statics of this Pareto exponent as a function of digital productivity b in the next section. Specifically, we analytically show that an increase in b will lead to a rise in inequality under certain conditions:

[4] See Aoki and Nirei (2017) for a more detailed explanation of this derivation.

The average level of wealth is derived as follows:

$$W_i^{average} = \int_0^\infty W_i f(W_i) dW_i.$$

Using the derived stationary distribution with respect to wealth above, the average level of wealth can be obtained when $\psi_{1,i} > 1$, from the following explicit-form expression:

$$W_i^{average} = \frac{\kappa_i \zeta H}{\chi_i} \left\{ \frac{1}{\psi_{2,i}+1} + \frac{1}{\psi_{1,i}-1} \right\}.$$

We can also compute the savings rate for each wealth bin, defined as the ratio of savings to income in the bin. To this end, we first define τ_v as the top v percentile of the wealth distribution and determine τ_v implicitly by the following equation:[5]

$$\sum_i \int_{\tau_v}^\infty f(W_i) dW_i = \sum_i \frac{\kappa_i \zeta H^{\psi_{1,i}}}{\chi_i \psi_{1,i}} (\tau_v)^{-\psi_{1,i}} = v. \tag{16}$$

Using Ito's lemma, the savings of the top v%, ΔSR_v, can be calculated from the following equations:

$$\Delta SR_v = \sum_i \int_{\tau_v}^\infty \Delta W_i f(W_i) dW_i = \sum_i (e^{\mu_W} - 1) \frac{\kappa_i \zeta H^{\psi_{1,i}}}{\chi_i (\psi_{1,i} - 1)} (\tau_v)^{-\psi_{1,i}+1}, \tag{17}$$

where ΔW_i is the savings (change in wealth), which can be calculated from the budget constraint and the formula for geometric Brownian motion. Also, μ_w is defined in the following equation:

$$\mu_w = \theta_i r_A + (1-\theta_i) r - \rho_i. \tag{18}$$

Similarly, income ΔI_v can be characterized from the following equation:

$$\Delta I_v = \sum_i (e^{\mu_I} - 1) \frac{\kappa_i \zeta H^{\psi_{1,i}}}{\chi_i (\psi_{1,i} - 1)} (\tau_v)^{-\psi_{1,i}+1}, \tag{19}$$

where

$$\mu_I = \theta_i r_A + (1-\theta_i) r + \zeta. \tag{20}$$

Using these relationships, the savings rate of the top v% group is computed as $\frac{\Delta SR_v}{\Delta I_v}$.

[5] This formula holds when v is small enough so that the wealth level is greater than the human capital ($W_i \geq H$). A slightly modified version of the equation applies to the case when households' wealth is less than human capital.

2.5 COMPARATIVE STATICS ON INEQUALITY

The stationary distribution (14, 15) shows that the wealth distribution follows a double Pareto distribution with upper-tail index $\psi_{1,i}$. Moreover, from (8), (9), and (10), the upper-tail index of $\psi_{1,i}$ depends on the productivity of digital capital, b. That is, changes in the productivity of digital capital have a distributional impact. Our parsimonious model allows for analytical results on comparative statics of wealth inequality, the labor share, and capital returns when digital productivity b changes. In this section, we derive the comparative statics in the case of homogeneous agents $\kappa = 1$, where all households have common preferences (ρ, γ).

As we show in Appendix A, the stationary equilibrium conditions boil down to the following two equations that simultaneously determine (r, θ).

$$\rho + \zeta = \left(\left(\frac{r+\delta}{\lambda_K} \right)^{\frac{-1}{\alpha-1}} - \delta \right) \left(\frac{r - \rho + (\sigma\theta)^2 \gamma}{\zeta} - \theta \right) - \delta_A \theta \tag{21}$$

$$1 = \lambda_K \left(\frac{r+\delta}{\lambda_K} \right)^{\frac{\alpha}{\alpha-1}} + \lambda_V \left[\frac{r + \delta_A + \sigma^2 \gamma \theta}{\lambda_A \lambda_V b^{\phi}} \left\{ \frac{\theta}{\rho + \zeta + \delta_A \theta} \left(1 - \left(\frac{r+\delta}{\lambda_K} \right)^{\frac{1}{\alpha-1}} \delta \right) \right\}^{1-\phi} \right]^{\frac{\alpha}{\alpha-\phi}} \tag{22}$$

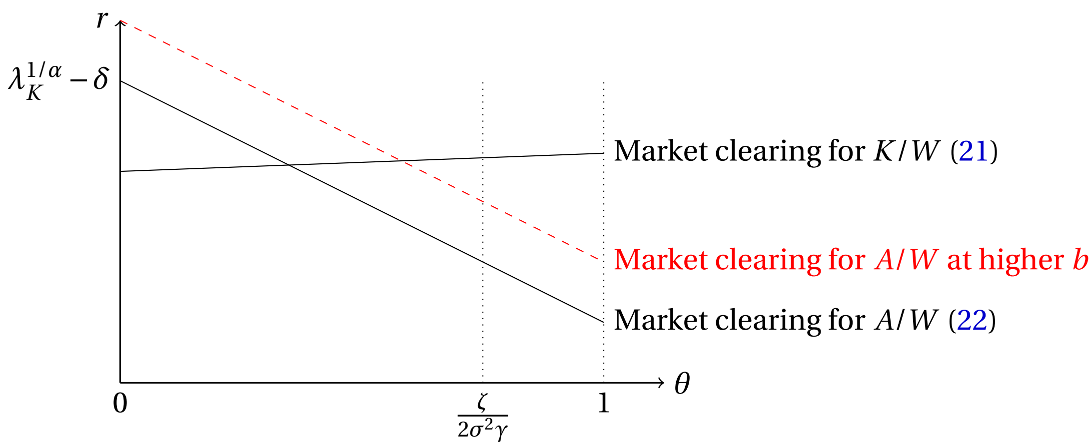
Given the optimal investment strategy $\theta = (r_A - r)/(\sigma^2\gamma)$, the two equations can be equivalently seen as determining the pair (r, r_A). The first equation (21) shows a relationship between r and θ which must hold to clear the market for traditional capital K, whereas the second equation (22) indicates the market-clearing condition for digital capital A.

As depicted in Figure 1, the stationary equilibrium (r, θ) is determined by the intersection of curves corresponding to (21, 22). The market-clearing schedule for digital capital (22) is downward sloping. When r increases, the firm's demand for K decreases, as in (5), which negatively affects the demand for digital capital per output, A/Y. Meanwhile, the stationary output-to-wealth ratio Y/W is negatively related to r as we show in Appendix A. Hence, demand for A/W must decrease even further. To meet the decreased demand for A/W, $r_A - r$ decreases so that supply θ decreases.

The curve for A/W (22) shifts to the right when digital capital productivity b increases, because the productivity gain raises firms' demand for digital capital for any fixed (r, r_A). Since b does not affect the market clearing condition for traditional capital (21), we conclude that an increase in b unambiguously increases the stationary digital capital share θ of wealth and risk premium $r_A - r$ as long as the slope of (21) exceeds that of (22). A sufficient condition for this is obtained when (21) is upward sloping, which holds if the digital asset share is sufficiently small, in particular if $\theta < \zeta/(2\sigma^2\gamma)$. This condition is satisfied at the equilibrium value in our benchmark calibration.

We state this result on θ as well as other comparative statics in the following proposition.

Figure 1: Market clearing conditions for traditional capital K/W and digital capital A/W.



Note: When the productivity of digital capital b increases, the A/W schedule shifts to the right. The market clearing condition for K/W is sloping upward in region $0 < \theta < \zeta/(2\sigma^2\gamma)$. Since θ is proportional to the risk premium, the horizontal axis can be also considered a linear function of $r_A - r$.

Proposition 1 (Comparative Statics of b)**.** *Suppose that a stationary equilibrium $r > 0$, $r_A > r$, ω, Y, $\psi_1 > 1$, and $\theta \in (0,1)$ exists for a given b. Then, a stationary equilibrium exists for digital capital productivity in the neighborhood of b and has the following properties.*

(a) If $\theta < \zeta/(2\sigma^2\gamma)$ holds, then $d\theta/db > 0$, $d(r_A - r)/db > 0$, and $dr/db > 0$.

(b) If $\theta < \zeta/(2\sigma^2\gamma)$ and $\psi_1 < 2$ hold, then $d\psi_1/db < 0$.

(c) If $|dr/db|$ is sufficiently small, then $d(\omega L/Y)/db < 0$.

Proof: See Appendix A.

Item (a) states that both θ and r are increasing in b if the digital capital to total wealth ratio θ is sufficiently small in equilibrium. The condition $\theta < \zeta/(2\sigma^2\gamma)$ is equivalent to saying that household supply of K/W is increasing in r_A for any fixed r. That is, an increase in the return to digital capital induces a greater increase in the supply of digital and traditional capital combined than the increase in digital capital alone. This condition is satisfied under our benchmark calibration where the equilibrium digital wealth share is sufficiently small.

The slope of (21) tends to be close to zero under our calibrated parameters. This is interpreted as follows. An increase in the risk premium $r_A - r$ raises the stationary investment of digital capital and the output-wealth ratio, leading to a greater demand for K per unit of wealth by firms (see (36) and (39) in Appendix A). At the same time, households respond to a higher $r_A - r$ by raising θ as well as traditional capital under the condition $\theta < \zeta/(2\sigma^2\gamma)$, while shrinking the share of human wealth. This leads to an increased supply of traditional capital. Hence, an increase in $r_A - r$ raises both demand and supply of K/W, leaving r little affected in order to keep the capital market balanced.

The proposition presumes that a stationary equilibrium exists uniquely. Establishing the existence of a stationary equilibrium is generally difficult. However, we note that at $\theta = 0$, equation (22) is solved by $\bar{r} := \lambda_K^{1/\alpha} - \delta$. At $\theta = 0$ and $r = \bar{r}$, the right-hand side of (21) equals $\bar{r}(\bar{r} - \rho)/\zeta$, which exceeds the left-hand side value $\rho + \zeta$ for a wide range of parameter values. In that case the market-clearing r for K/W at $\theta = 0$ is below $\bar{r}$, which is necessary for a solution $\theta > 0$ to exist. Furthermore, if a stationary equilibrium exists satisfying $0 < \theta < \zeta/(2\sigma^2\gamma)$, it exists uniquely because (21) is upward sloping.

Item (b) states that the stationary Pareto exponent ψ_1 decreases when b increases if the conditions of item (a) and $\psi_1 < 2$ are satisfied. That is, digitalization increases inequality in the upper tail of the household wealth distribution. Empirically estimated Pareto exponents are consistently below 2 for U.S. households after the mid-1980s (see Aoki and Nirei (2017)), satisfying the condition $\psi_1 < 2$. The driving force of a small ψ_1 is the heavy investment by risk-tolerant households in risky digital capital when the digital capital productivity increases.

Finally, item (c) states that an increase in b lowers the labor share $\omega L/Y$ when the impact of b on the stationary risk-free rate is negligible. In our calibration scenarios, the change in the risk-free rate when b changes is smaller than 10 basis points, which justifies ignoring the effect through r. In that case, the labor share is mainly driven by a change in θ. Since digital capital is a substitute for labor (i.e., $\phi > 0$), an increase in b shifts factor demand away from labor to digital capital, leading to a depressed wage income share in the labor market equilibrium.

Thus, our model captures two forces of digital capital advancement that drive inequality: first, digitalization enriches households who are willing and able to invest in digital capital, and second, digital capital substitutes firms' factor demand away from labor and lowers the labor share of income.

3 CALIBRATION

This section describes how we map our framework to the data. We calibrate our model by matching the household wealth distribution, their savings behavior, and various properties of digital capital in the U.S.

3.1 CALIBRATION IN STEADY STATE

Table 3 shows our parameter values. Most of the parameters are based on statistical data or the literature, while other parameters are jointly calibrated by solving the model to match wealth shares in the U.S.

Table 3: Parameter Values

Parameter	Value	Target/Source	Data Source
ζ	1/40		
σ	0.5	Business income share in 1989	Kuhn and Rios-Rull (2020)
δ	0.06	Literature	
δ_A	0.18	Depreciation rate of digital capital	BEA
α	-0.72	Capital stock to output ratio in 1989	Feenstra et al. (2015)
ϕ	0.5	Literature	
λ_K	0.5	Aggregate saving in 1989	Saez and Zucman (2016)
λ_L	0.83	Labor share in 1989	Grossman and Oberfield (2022)
ρ^H	0.032	Internally calibrated	
ρ^L	0.024	Internally calibrated	
γ^H	6.5	Internally calibrated	
γ^L	0.16	Internally calibrated	

First, ζ is set to be 1/40, implying that households accumulate their wealth for 40 years on average after they start working. We use the depreciation rate of traditional capital $\delta = 0.06$, which is the standard value. For the depreciation rate of digital capital, we use $\delta_A = 0.18$, which is higher than the traditional capital depreciation rate δ.[6] To calibrate δ_A, we first specify digital capital in our model as the "information processing equipment" and "nonresidential intellectual property products" in private fixed assets from the statistics of the Bureau of Economic Analysis (BEA), following Nordhaus (2021). Then we set δ_A to be the weighted average of depreciation rates based on the stock of capital in 1989.[7]

Our choice for the value of ϕ, the elasticity of substitution between labor and digital capital, is set based on the following literature. Alonso et al. (2022) indicate that ϕ may lie somewhere between 0.33 and 0.67. Eden and Gaggl (2018) conclude that the elasticity of substitution between labor and ICT-related capital has risen sharply since the late 90s, and thus, ϕ rises from 0.6 to 0.7. Berg et al. (2018), who calibrate the elasticity of substitution of a production function similar to ours, find that ϕ is 0.52 based on the elasticity of substitution between ICT capital and unskilled labor observed in the U.S. data. We set ϕ to 0.5 which is in line with these previous estimates.

[6] Li and Hall (2020) also discusses the estimation of capital depreciation, stating that the depreciation rate for digital capital is generally higher than traditional capital.

[7] Eden and Gaggl (2018) also estimate the depreciation rates of digital capital. Although their definition of digital capital is not exactly the same as ours, the depreciation rates lie between 0.15 and 0.20, which is in line with the value used in this paper.

As for σ, α, λ_K, and λ_L, we match the following statistics consistent with the U.S. data in 1989, which we refer to as the benchmark economy. First, we calibrate σ so that the stationary business income share in our model is 0.11. This value is taken from Kuhn and Rios-Rull (2020), which uses the SCF for the estimation. α is set to -0.72 so that the capital stock to output ratio is 3.8. We set λ_K to 0.5 so that the aggregate savings rate becomes 0.11, consistent with Saez and Zucman (2016). They calculated the savings rate based on the tax returns micro-data, which are from NBER micro-files and from internal IRS files.[8] In addition, we calibrate λ_L so that the labor share is 0.62, which is in line with Grossman and Oberfield (2022), as we discuss later.

For computational simplicity, we assume that there are two types of households in terms of risk aversion: households with high risk aversion (γ^H) and those with low risk aversion (γ^L). In addition, households of each level of risk aversion are divided equally into two types of time preference: impatient households (ρ^H) and patient households (ρ^L). In total, we have four types of households: (γ^L, ρ^L), (γ^L, ρ^H), (γ^H, ρ^L), and (γ^H, ρ^H). Following Quadrini (1999), we assume that 10% of the population has low risk aversion. That is, the percentage of each household (ρ^L, γ^L), (ρ^H, γ^L), (ρ^L, γ^H), (ρ^H, γ^H) is (0.05, 0.05, 0.45, 0.45) respectively. To calibrate ρ^H, ρ^L, γ^H, and γ^L, we use the simulated method of moments to choose the remaining four parameters, assuming that the U.S. economy achieved a stationary equilibrium in 1989. Specifically, our targets are the wealth shares of the distribution at the top 0.1, top 1 percent, top 5 percent, and top 10 percent.[9] The resulting values of the preference parameters are $\rho^H = 0.032$, $\rho^L = 0.027$, $\gamma^H = 6.5$, and $\gamma^L = 0.16$.

Table 4: Calibration targets: Model vs Data

Calibration Targets	Data in 1989	Model
Labor share	0.62	0.62
Capital stock to output ratio	3.8	3.3
Business income share	0.11	0.13
Aggregate saving rate	0.11	0.10

Note: The data for the labor share are from Grossman and Oberfield (2022) and the capital stock to output ratio is calculated based on the data in Feenstra et al. (2015). For the business income share, we refer to Kuhn and Rios-Rull (2020). In the model, we define $\frac{A+K}{Y}$ and $\frac{(r_A+\delta_A)*A}{Y}$ as the capital stock to output ratio and business income share, respectively. As for the aggregate savings, the data come from Saez and Zucman (2016).

[8] They shed light on wealth inequality by focusing on capitalized income tax data. For example, if the stock of claims of households is equal to 50 times the flow of interest income in tax data, they estimate the size of the claims by multiplying 50 by the corresponding income amount.

[9] The data of the wealth distribution is provided by Moritz Kuhn. These data underlie the analysis by Kuhn and Rios-Rull (2020).

Table 5: Wealth Distribution: Data vs Model

	SCF 1989 (%)	SZ 1989 (%)	Benchmark (%)
Bottom 90 %	32.9	35.3	28.9
Top 10 %	67.1	64.7	71.1
Top 5 %	54.2	50.6	54.0
Top 1 %	29.9	28.6	27.2
Top 0.1 %	10.5	12.1	10.8

Note: The SCF data come from Kuhn and Rios-Rull (2020). SZ shows the data from Saez and Zucman (2020).

Table 4 presents a comparison of key statistics of the stationary equilibrium in the benchmark economy and their empirical counterparts. This table indicates that the model's outputs match their empirical counterparts very well. The labor share in the model is 0.62, which is in line with the data. The capital stock to output ratio is 3.3 in the model, which is close to the data (3.8). The business income share and aggregate savings rate are 0.13 and 0.11 in the model, respectively, which are also consistent with the data.

The distribution of wealth is shown in Table 5. The second column shows the distribution observed in the 1989 Survey of Consumer Finances (SCF). The third column shows the data from Saez and Zucman (2016). The fourth column reports the corresponding wealth share estimated in the model for the benchmark calibration. The table reveals that the model generates a wealth distribution that is similar to that obtained from the data, since the degree of wealth concentration in the benchmark economy is remarkably consistent with the wealth shares in the data in 1989. This demonstrates that our parsimonious model with four types of household preferences (ρ, γ) reasonably captures the observed patterns of U.S. wealth concentration in 1989.

3.2 Properties of the Benchmark Economy

Table 6 summarizes the business income share for households of various wealth groups. As we explained earlier, business income, defined as $\frac{(r_A+\delta_A)*A}{Y}$ originates from the risky investment and is closely related to the return on digital assets. The first column refers to percentiles of the wealth distribution. In the second column, data from Kuhn and Rios-Rull (2020) are presented, and the corresponding statistics calculated from the benchmark calibration are reported in the third column. This table makes it clear that households with greater wealth tend to have higher shares of business income, and overall the model matches the data well, especially for the top income households. This result indicates that our simple model of risky capital investment can quantitatively replicate the salient patterns of the business income distribution that drives wealth inequality, as we argue in the following sections.

Table 6: Business income share by percentile of the wealth distribution: Model vs Data

	Data (%)	Benchmark (%)
Top 10 %	17.7	20.8
Top 5 %	19.1	24.3
Top 1 %	34.4	33.6

Note: The data for business income share are based on Kuhn and Rios-Rull (2020), using the 1989 data.

3.3 DIGITALIZATION SCENARIOS

We capture recent advances in digital technology by an exogenous increase in b, the productivity of digital capital. In this paper, we prepare two scenarios given that it is not straightforward to assess the advance in digitalization. In scenario A, following Nordhaus (2021), we use stock-based statistics: we match the changes in the ratio $\frac{A}{K}$ consistent with the BEA data. In this approach, digital capital is defined as the “information processing equipment” and “nonresidential intellectual property products” in private fixed assets from the statistics of the BEA. This definition is consistent with our calibration of δ_A. According to this dataset, the ratio of digital assets to traditional capital $\frac{A}{K}$ increases by 19 percent, which corresponds to an increase of b from 1 to 1.21 in our model specification.

In scenario B, we use flow-based statistics. Specifically, we match the digital capital productivity so that the digital investment to GDP ratio is consistent with that in Corrado and Hulten (2010).[10] Based on the database, the digital investment to GDP ratio increased by four percentage points from 1989 to 2017. Here, digital capital is defined as software, research and development, organization capital, and computer and network design consulting.[11] This gives us $b = 1.43$. Thus, scenario A suggests moderate advances in digitalization, while scenario B implies greater advances. These two scenarios provide us with a relevant range of magnitudes on the productivity impacts of digitalization, which allows for various scopes and methods for

[10]Given the digital capital depreciation rate δ_A and stock of the digital capital at the steady state, we can calculate the annual digital investment. We increase b to match the four percentage points rise in the digital investment to GDP ratio.

[11]Since digital investment can also be related to the broader category of investment, we also check the robustness of our results by including other types of investment. For example, even if we include brand and design (industrial), this also gives a four percentage point increase. Moreover, when we include all types of intangible investment, the increase is less than 5 percentage points. Thus, the increase in digital investment does not change even if we use alternative definitions.

measurement.

4 QUANTITATIVE RESULTS

In this section, we numerically solve for a stationary equilibrium and investigate the quantitative impacts of digitalization. We conduct comparative statics, showing that the change in digital capital technology explains the concentration of wealth at the top of the distribution by raising returns on digital capital because rich entrepreneurs disproportionately benefit from its higher return. In addition, we quantify the macroeconomic consequences of digitalization on output, the labor share, and the interest rate.

4.1 WEALTH DISTRIBUTION

In this section, we evaluate how digitalization alters the savings distribution and wealth inequality in the model economy by numerically conducting the comparative statics of the stationary equilibrium when digital productivity b increases.

Table 7 and Table 8 compare the wealth distributions implied by the model and those from the data. The first column in Table 7 presents percentiles of the wealth distribution. The second and the third columns show the data from the 1989 and 2019 Survey of Consumer Finances (SCF).[12] The fifth and sixth columns show the data from Saez and Zucman (2020). The "dif" column shows the difference between the data in 2019 and 1989.

Table 8 shows the wealth distribution of the stationary equilibrium of the model. Baseline, Case A, and Case B show the computed results in the benchmark economy and in the digitalization scenarios A and B described in Section 3.3, respectively. Column "dif" shows the difference between the results in the digitalization scenario and those in the benchmark economy.

[12] Bricker et al. (2021) augment the SCF data with the wealth of the Forbes 400 families. Based on their calculations, the wealth share of the top 1% of households was 25.1% in 1989 and 31.7% in 2019, representing an increase of 6.6 percentage points during this period.

Table 7: Wealth Distribution (Data)

	Data SCF			Data SZ		
	1989 (%)	2019	dif	1989	2019	dif
Bottom 90 %	32.9	23.6	-9.3	35.3	28.6	-6.7
Top 10 %	67.1	76.4	+9.3	64.7	71.4	+6.7
Top 5 %	54.2	64.9	+10.7	50.6	57.9	+7.3
Top 1 %	29.9	37.2	+7.3	28.6	34.9	+6.3
Top 0.1 %	10.5	14.1	+3.6	12.1	17.6	+5.5

Note: The second and the third columns show the data from the Survey of Consumer Finances based on Kuhn and Rios-Rull (2020). SZ shows the data from Saez and Zucman (2020). The “dif" column shows the difference between the data in 2019 and 1989.

Table 8: Wealth Distribution (Model)

	Baseline	Case A	dif (Case A)	Case B	dif (Case B)
Bottom 90 %	28.9	27.2	-1.7	25.5	-3.4
Top 10 %	71.1	72.8	+1.7	74.5	+3.4
Top 5 %	54.0	56.5	+2.5	59.0	+5.0
Top 1 %	27.2	30.6	+3.4	34.1	+6.9
Top 0.1 %	10.8	13.9	+3.1	17.5	+6.7

Note: Baseline, Case A, and Case B show the model results in the benchmark economy and in the digitalization scenarios A and B described in Section 3.3, respectively. The “dif" column shows the difference between the results of the “advances in digitalization" scenario and those in the benchmark economy.

Comparing Table 7 and Table 8 highlights that digitalization could be one of the main drivers of wealth concentration from 1989 to 2019. Our model is capable of reproducing the wealth concentration in the U.S. economy from 1989 to 2019. Saez and Zucman (2020) report that the increase in the share of wealth of the top 0.1% and 1% of households over this period is about 5.5 and 6.3 percentage points, respectively. The SCF data indicates that the increase in the share of the wealth of the top 0.1% and 1% of households is 3.6 and 7.3 percentage points. Our model predicts that the increase in the share of the wealth of the top 0.1% and 1% is 3.1 and 3.4 percentage points in Case A, and 6.7 and 6.9 percentage points in Case B, respectively. These results indicate that over the past 30 years, digitalization has contributed to more than about 50 percent of the increase in the share of wealth of the top 1 percent of households and more than about 80 percent of that of the top 0.1 percent of households.

The intuition behind this concentration in top-wealth households is as follows. In equilibrium, risk-tolerant households behave like entrepreneurs and hold more risky digital capital. They enjoy higher returns from risky investment in digital capital as digitalization becomes more advanced and the realized return on digital capital higher. Thanks to this additional return, they can accumulate more wealth, implying wealthy households disproportionately end up holding a larger share of wealth. On the other hand, risk-averse households become workers. Since they hold little in risky assets, they do not benefit from digital capital investments. Moreover, the risk-averse households rely on labor income relatively more. Labor is substituted by digital capital as an input of production as digitalization advances. Even though the wage rate increases slightly due to the accumulation of traditional and digital capital, as we describe in our discussion of the macroeconomic consequences of digitalization in section 4.3, the effect of an increasing wage is dominated by the substitution effect, leading to a decline in the labor share. Thus, the bottom 90% of households do not benefit from the increased capital income from digitalization and suffer from the declining labor share caused by the labor-substitution effects of digitalization. Hence, digitalization widens the wealth disparity in our model by treating successful risk-taking households favorably.

4.2 SAVINGS BEHAVIOR

Table 9 shows the increase in annual savings of the top 1% of the wealth distribution over the past 30 years, indicating that the wealthiest 1% contributed significantly to the increase in the aggregate saving-output ratio. The second and the third columns show the stationary saving-output ratio in the benchmark scenario and in the digitalization scenario. The fourth column shows the difference between the two: savings of the top 1% of households increase by 0.3% of output through digitalization in Case A and 0.6% in Case B. The third row shows the estimate reported by Mian et al. (2021) for the increase in the savings-to-GDP ratio that accrued to the top 1% of households from 1983–1997 to 2008–2016.[13] As can be seen from Table 9, the increase in savings of the top 1% of households in the model accounts for 21%-43% of the estimated increase in savings in the U.S. This indicates that our model captures an important portion of the savings increase of the rich, but leaves room for other factors. For example, the effect of capital gains caused by the productivity change, which may have contributed significantly to the savings of the rich as suggested by Fagereng et al. (2021), is not incorporated in our model.

[13] We cite an estimate using the income-less-consumption approach of the CBO, but other estimates reported by Mian et al. (2021) give similar results.

Table 9: Savings in the top 1% increase compared to output: Model vs Data

	Benchmark (%)	Advance in digitalization (%)	Saving increase (% point)
Case A	0.9	1.2	0.3
Case B	0.9	1.5	0.6
Data			1.4

Note: This table shows the annual savings of the top 1% of households scaled by output. The second and third columns show the savings to output ratio of the top 1% of households in the benchmark model and the digitalization scenarios, respectively. The fourth column shows the difference between the two rows, describing the increase in savings due to digitalization. The third row cites the data from Mian et al. (2021).

The aggregate savings rate is an integral of household savings rates weighted by the wealth distribution. Thus, the increase in the savings-to-output ratio can be decomposed into the contribution of the shift in savings rates and the contribution of the increase in wealth amounts. Specifically, for each top v percent group, the increase in savings of the wealth group is decomposed as:

$$\frac{\Delta SR_v}{Y} = \frac{\Delta SR_v}{W_v}\frac{W_v}{Y}, \tag{23}$$

where Y denotes aggregate output, ΔSR_v the increase in the savings rate of group v defined in equation (17), and W_v the wealth holdings of households in group v. The first component captures the shift in the savings policy function, which represents the propensity to save out of wealth. The second component captures the effect of wealth concentration that changes the composition of the set of policy functions across households.

Table 10 shows the savings decomposition for each wealth percentile group. In the table, the overall change shows the change in savings in each percentile of wealth divided by output, corresponding to the left hand side of the equation. The contribution of the savings policy function and the contribution of wealth holdings correspond to the change in the first term and the second term, respectively. The correlated term is the residual coming from the second-order contribution. The numbers in parentheses show the share of contribution to overall change. There are two findings from this exercise. First, richer households tend to increase overall savings as digitalization advances. For example, in Case A, the overall change in savings is 9 percent for the top 10 percent of households while it is more than 45 percent for the top 0.1 percent of households. The difference becomes striking in Case B: the change in savings is more than 100 percent for the top 0.1 percent of households, which is almost 6 times the savings of the top 10 percent of households. This shows that richer households tend to save large amounts of wealth because of their higher propensity to save and the sheer amount of wealth they possess, even without capital gains taken into consideration.

Second, the driver of savings is different among the wealth percentiles. For instance, in Case B, the contribution of the savings function is around 30 percent while the contribution from the wealth distribution is 63 percent for the top 10 percent of households. On the other hand, the savings policy function explains less than 20 percent and the wealth distribution accounts for around 70 percent of the savings for the top 0.1 percent of households. Thus, the contribution of the change in the savings behavior of the rich is modest. Rather, the increase in the wealth level of the high-wealth group significantly contributes to the overall increase in savings.

In sum, the productivity gain in digital capital in our model can account for about 20%-40% of the increase in aggregate savings. Aggregate savings are affected by both the change in savings behavior and wealth concentration, and we find wealth concentration to be the major factor, accounting for about 70% of the aggregate savings change.

Table 10: Savings Decomposition (percent)

	Overall change	Savings policy	Wealth dist.	Correlated term
Case A				
Top 10 %	8.6	2.6	5.8	0.2
		(30.6)	(67.6)	(1.8)
Top 5 %	12.4	4.0	8.1	0.3
		(32.0)	(65.4)	(2.6)
Top 1 %	24.3	7.1	16.0	1.2
		(29.3)	(66.0)	(4.7)
Top 0.1 %	45.6	9.7	32.8	3.2
		(21.2)	(71.9)	(6.9)
Case B				
	Overall change	Savings policy	Wealth dist.	Correlated term
Top 10 %	18.8	6.3	11.8	0.7
		(33.2)	(62.9)	(3.9)
Top 5 %	27.4	9.2	16.7	1.5
		(33.6)	(60.8)	(5.6)
Top 1 %	55.1	15.9	33.8	5.4
		(28.8)	(61.4)	(9.8)
Top 0.1 %	107.7	20.5	72.4	14.8
		(19.1)	(67.2)	(13.8)

Note: Overall change shows the change in savings in each percentile of wealth divided by output. “Savings policy" and “Wealth dist." show contributions of the savings policy function (savings divided by wealth), and contribution of wealth (wealth divided by output), respectively. Numbers in parentheses show the share of contribution to overall change. The calculation for the decomposition is described in the main text.

Table 11 shows the share of each type of household in various wealth percentiles. In our model, we have four types of preferences depending on parameters of risk aversion and the time-discount rate, namely $(\gamma_L, \rho_L), (\gamma_L, \rho_H), (\gamma_H, \rho_L)$, and (γ_H, ρ_H). The table in the baseline section shows the share in percent, while those in Case A and Case B describe the increase/decrease compared to the baseline results. There are three findings from this table.

First, the table highlights that in the baseline economy, risk-averse and patient households (γ_H, ρ_L) constitute 82.5 percent of mildly rich households (top 10 percent of the wealth distribution). Households of type (γ_H, ρ_H) are concentrated around the median level of wealth. In our model, 90 percent of households are risk-averse, and they have little chance of achieving the 10th percentile of the wealth distribution if they are also impatient. However, risk-averse but patient households accumulate some wealth, a portion of which is invested into risky assets, and successful ones amount to more than 80 percent of the top 10 percent group.

Second, households with low risk aversion, and especially those with a low time-discount rate, (γ_L, ρ_L), account for the majority of households in the top 0.1 percent of the wealth distribution. In our model, the savings policy function depends on their preferences. Households with lower time-discount rates save more, and those with lower risk aversion tend to invest in risky digital capital. Thus, low risk aversion and low time-discount rate households in particular have a large incentive to invest in risky digital assets. Some of those households draw high returns and grow their wealth remarkably, which we interpret as successful entrepreneurs in our model.

Third, as digitalization advances, the share of high risk aversion households decreases, while that of low risk aversion households increases. Especially, among the top 0.1 percent of households in Case B, households with γ_L increase by almost 10 percent compared to the baseline scenario, while households with γ_H decrease by the same amount. This is also intuitive because these risk-tolerant households benefit from advances in digitalization, which leads to a higher return of risky digital capital, enabling them to accumulate more wealth. In particular, the share of (γ_L, ρ_H) increases remarkably compared to (γ_L, ρ_L) in the top wealth percentile (top 0.1 percent). Households with (γ_L, ρ_H) are characterized by low risk aversion and a high time-discount rate, implying that their propensity to save is low compared to ρ_L households, ceteris paribus. Thus, in the baseline economy, households with (γ_L, ρ_L) comprise the majority of the rich since they have a higher propensity to save, and more incentives to invest in risky digital capital. On the other hand, in Case B, the benefits of holding risky digital capital increase also for ρ_H households as the risk premium increases. Moreover, as we can see in Table 10, the contribution from wealth holdings plays an important role, indicating that they can accumulate their wealth even if their marginal propensity to save is low. Thus, the increase in the share of (γ_L, ρ_L) is relatively subdued, leading to a change in the composition of wealthy households in the digitalization scenario.

Table 11: Share of each preference type (percent)

	(γ_L, ρ_L)	(γ_L, ρ_H)	(γ_H, ρ_L)	(γ_H, ρ_H)
Baseline				
Top 10 %	11.0	6.6	82.5	0.0
Top 5 %	14.9	8.0	77.1	0.0
Top 1 %	28.8	11.7	59.4	0.0
Top 0.1 %	58.7	15.2	26.1	0.0
Case A (Change from the baseline)				
Top 10 %	-0.2	+0.4	-0.3	0.0
Top 5 %	0.0	+0.8	-0.8	0.0
Top 1 %	+1.0	+2.4	-3.2	0.0
Top 0.1 %	+1.7	+4.2	-5.9	0.0
Case B (Change from the baseline)				
Top 10 %	-0.4	+0.6	-0.3	0.0
Top 5 %	-0.1	+1.3	-1.3	0.0
Top 1 %	+1.5	+4.1	-5.4	0.0
Top 0.1 %	+2.2	+7.4	-9.6	0.0

Note: These tables show the share of each type of household in each wealth percentile. While the table of the baseline shows the level of the share, those of Case A and Case B describe the change compared to the baseline results.

4.3 MACROECONOMIC CONSEQUENCES OF DIGITALIZATION

Table 12 summarizes the steady-state impacts of increases in b on key macroeconomic variables. Four findings emerge. First, digitalization increases output, as shown in the second row in Table 12. As the productivity of digital capital increases, output increases from 1.78 to 1.93 in Case A and 2.12 in Case B, which implies that the digitalization considered here contributes to aggregate economic growth by 8 to 19 percentage points.

Second, the return on digital capital increases as digitalization advances, which is also discussed in the previous section. The fourth row in Table 12 presents the return on digital capital, r_A. In the benchmark economy, the return on digital capital is 0.044. As the productivity of digital capital increases, the return on digital capital increases to 0.046 in Case A and to 0.048 in Case B. This result suggests that digitalization permanently increases returns to wealth for households who hold digital capital. On the other hand, the return on traditional capital is almost constant, as seen in the table. Thus, the return on digital capital increases, while that on traditional capital remains the same, indicating that the risk premium rises due to digitalization. This result confirms that the gains from the high productivity of digital capital are distributed to risk-tolerant households, whereas risk-averse households gain little in their capital income. Also, since the returns to digital capital are volatile, the stationary distribution of wealth among risk-tolerant households becomes more dispersed as their portfolio shifts to digital assets. The wealth disparity within risk-tolerant households is manifested by the low Pareto exponent ψ_1 of this type of household.[14] Also, the lack of response in r, observed in this table, validates the premise of Proposition 1(c): if $|dr/db|$ is sufficiently small, then $d(\omega L/Y)/db < 0$. We find that the response of the risk-free rate is sufficiently small when the productivity of digital capital increases, which confirms the assumption of this proposition in our quantitative exercise. This indicates that the labor share, which is the main income source for ordinary households, declines as b increases. We describe the implications of the decline in the labor share in more detail next.

Table 12: Macroeconomic Consequences

	Benchmark	Case A	Case B
Output	1.78	1.93	2.12
Wage	1.11	1.16	1.21
Return on digital (risky) capital	0.044	0.046	0.048
Return on risk-free capital	0.032	0.032	0.033
Labor share	0.62	0.60	0.57
Digital capital/Output	0.58	0.69	0.80

Third, as the sixth row of the table shows, the labor share $\omega L/Y$ decreases due to digitalization. The labor share is 62 percent in the benchmark economy, and it falls to 60 percent in Case A and to 57 percent in Case B. We note that the magnitude of decrease in Case B is comparable to the decline in the labor share of the U.S. nonfarm business sector shown by Grossman and Oberfield (2022) (from 0.62 in 1989 to 0.57 in 2019). Although digitalization increases the aggregate pie of the economy (output), the labor share decreases as the return to digital capital increases. This benefits households with low risk aversion disproportionately because these

[14]Indeed, ψ_1 of (γ_L, ρ_L) households decreases from 1.48 to 1.36 in Case A and to 1.28 in Case B. That of (γ_L, ρ_H) decreases from 1.90 to 1.69 in Case A and to 1.55 in Case B. Note that the Pareto tail index of overall wealth distribution coincides with the lowest ψ_1 among household types.

households hold considerable amounts of digital capital, whereas most of the risk averse households rely mainly on labor earnings, which is consistent with the literature.[15]

Finally, wages increase in the long-run in response to digitalization. In our quantitative exercise, the wage increases by five percent in Case A and nine percent in Case B. As we mentioned in the model section, the composite of labor and digital capital V and traditional capital K are complements, and thus K (scaled by output) increases by 9 percent in Case A and 19 percent in Case B. Even though labor and digital capital are substitutes, this increase in traditional (risk-free) capital eventually raises labor demand and the real wage, which is consistent with Berg et al. (2018). Reflecting the increase in the real wage, Table 13 indicates that digitalization increases consumption for every type of household in both Case A and Case B. Specifically, while low risk-averse households increase their spending significantly (by 20–30 percent) in case B compared to the baseline, risk averse households also increase their consumption by around 10 percent. Although this is still less than the increase in aggregate output, reflecting the rise in inequality, these results demonstrate that working households with little digital capital also benefit from wage increases due to digitalization.

Table 13: Consumption increase for each preference type (percent)

	(γ_L, ρ_L)	(γ_L, ρ_H)	(γ_H, ρ_L)	(γ_H, ρ_H)	Aggregate output
Case A	15.1	10.6	5.1	4.7	8.8
Case B	34.8	23.3	10.8	10.0	19.2

Note: This table shows the increase in consumption compared to the baseline for each type of household. As a reference, it also shows the increase in aggregate output, Y.

5 CONCLUSION

This study investigates the role played by digitalization in the increased wealth concentration in the U.S. over the last three decades using a tractable model with agents that are heterogeneous in risk aversion. The key feature is introducing risky digital capital that can substitute for labor. A small number of prosperous households with low risk aversion, i.e., digital entrepreneurs, actively invest in risky digital assets because their return is higher, whereas households with high risk aversion refrain from investing in such assets due to their uncertain return. In the equilibrium, successful entrepreneurs willing to take risks accumulate high-return digital wealth,

[15] Berg et al. (2018), Acemoglu and Restrepo (2018) and Moll et al. (2022) analyze the impact of digitalization on inequality. Although the model settings are different across the literature (Berg et al. (2018) uses a similar production function to ours, while Acemoglu and Restrepo (2018) and Moll et al. (2022) employ a task-based approach), they all point out the decrease in the labor share that accompanies digitalization.

while risk averse households mainly rely on labor earnings with a moderate amount of wealth, creating wealth inequality. When digitalization advances, these risk-tolerant households enjoy higher returns from digital capital, further accumulating digital capital disproportionately.

Using comparative statics, we analytically show that the increase in the productivity of digital capital widens wealth inequality and lowers the labor share. Based on a numerical exercise using a model calibrated to match the U.S. economy, we demonstrate that our model replicates the wealth concentration in the U.S. economy from 1989 to 2019 reported by Saez and Zucman (2016, 2020) and the SCF well. In particular, quantitative analysis based on our model confirm that digitalization has significantly increased the share of households with low risk aversion in the top 0.1% and 1% of the wealth distribution. This result indicates that the concentration of risky digital capital among risk-tolerant households due to advances in digital technology can explain the magnitude of the rise in U.S. wealth inequality in last three decades. We also find that the large increase in the savings of the rich in the U.S. reported by Mian et al. (2021) can also be considered a result of digitalization through the concentration of wealth, while other factors such as the effect of capital gains also contribute. Finally, comparative statics describe that while digitalization increases output and decreases the labor share, which is in line with the empirical literature, it also increases wages, implying that risk-averse households who mainly rely on labor earnings also gain some benefits from digitalization.

REFERENCES

Acemoglu, Daron and Pascual Restrepo, "The Race Between Man and Machine: Implications of Technology for Growth, Factor Shares and Employment," *American Economic Review*, June 2018, *108* (6), 1488–1542.

— **and** — , "Robots and Jobs: Evidence from US Labor Markets," *Journal of Political Economy*, June 2020, *128* (6), 2188–2244.

Alonso, Cristian, Andrew Berg, Siddharth Kothari, Chris Papageorgiou, and Sidra Rehman, "Will the AI Revolution Cause a Great Divergence?," *Journal of Monetary Economics*, April 2022, *127*, 18–37.

Aoki, Shuhei and Makoto Nirei, "Zipf's Law, Pareto's Law, and the Evolution of Top Incomes in the United States," *American Economic Journal: Macroeconomics*, May 2017, *9* (3), 36–71.

Autor, David, David Dorn, Lawrence F. Katz, Christina Patterson, and John Van Reenen, "The Fall of the Labor Share and the Rise of Superstar Firms," *Quarterly Journal of Economics*, May 2020, *135* (2), 645–709.

Berg, Andrew, Edward F. Buffie, and Luis-Felipe Zanna, "Should We Fear the Robot Revolution? (The Correct Answer is Yes)," *Journal of Monetary Economics*, August 2018, *97*, 117–148.

Bricker, Jesse, Sarena Goodman, Kevin B. Moore, and Alice Henriques Volz, "A Wealth of Information: Augmenting the Survey of Consumer Finances to Characterize the Full U.S. Wealth Distribution," August 2021. Federal Reserve Board Finance and Economics Discussion Series, 2021-053.

Brynjolfsson, Erik and Andrew McAfee, *The Second Machine Age: Work, Progress, and Prosperity in a Time of Brilliant Technologies*, WW Norton Company, 2014.

— **, Lorin Hitt, Daniel Rock, and Prasanna Tambe**, "Digital Capital and Superstar Firms," December 2020. NBER Working Paper, 28285.

Cagetti, Marco, "Wealth Accumulation over the Life Cycle and Precautionary Savings," *Journal of Business and Economic Statistics*, 2003, *21* (3), 339–353.

— **and Mariacristina De Nardi**, "Entrepreneurship, Frictions, and Wealth," *Journal of Political Economy*, October 2006, *114* (5), 835–870.

Calvet, Laurent E., John Y. Campbell, Francisco Gomes, and Paolo Sodini, "The Cross-Section of Household Preferences," May 2021. NBER Working Paper, 28788.

Corrado, Carol A. and Charles R. Hulten, "How Do You Measure a "Technological Revolution"?," *American Economic Review*, 2010, *100* (2), 99–104.

De Nardi, Mariacristina and Giulio Fella, "Saving and wealth inequality," *Review of Economic Dynamics*, October 2017, *26*, 280–300.

Eden, Maya and Paul Gaggl, "On the Welfare Implications of Automation," *Review of Economic Dynamics*, July 2018, *29*, 15–43.

Fagereng, Andreas, Luigi Guiso amd Davide Malacrino, and Luigi Pistaferri, "Heterogeneity and Persistence in Returns to Wealth," *Econometrica*, January 2020, *88* (1), 115–170.

_, Martin Blomhoff Holm, Benjamin Moll, and Gisle Natvik, "Saving Behavior Across the Wealth Distribution: The Importance of Capital Gains," 2021. mimeo.

Feenstra, Robert C., Robert Inklaar, and Marcel P. Timmer, "The Next Generation of the Penn World Table," *American Economic Review*, October 2015, *105* (10), 3150–3182.

Gârleanu, Nicolae B. and Stavros Panageas, "Young, Old, Conservative, and Bold: The Implications of Heterogeneity and Finite Lives for Asset Pricing," *Journal of Political Economy*, May 2015, *123* (3), 670–685.

Gomes, Francisco and Alex Michaelides, "Asset Pricing with Limited Risk Sharing and Heterogeneous Agents," *The Review of Financial Studies*, January 2008, *21* (1), 415–449.

_, Thomas Jansson, and Yigitcan Karabulut, "Do Robots Increase Wealth Dispersion?," *The Review of Financial Studies*, 2024, *37* (1), 119–160.

Gomez, Matthieu, "Decomposing the Growth of Top Wealth Shares," *Econometrica*, 2023, *91* (3), 979–1024.

Grossman, Gene M. and Ezra Oberfield, "The Elusive Explanation for the Declining Labor Share," *Annual Review of Economics*, April 2022, *14* (1), 93–124.

Hémous, David and Morten Olsen, "The Rise of the Machines: Automation, Horizontal Innovation, and Income Inequality," *American Economic Journal: Macroeconomics*, January 2022, *14* (1), 179–223.

Kuhn, Moritz and Jose-Victor Rios-Rull, "2019 Update on the U.S. Earnings, Income, and Wealth Distributional Facts: A View from Macroeconomics," October 2020. Manuscript.

Lee, Byoungchan, "Wealth Inequality, Aggregate Consumption, and Macroeconomic Trends under Incomplete Markets," October 2021. mimeo.

__ , "Wealth Inequality and Endogenous Growth," *Journal of Monetary Economics*, 2023, *133*, 132–148.

Li, Wendy CY and Bronwyn H Hall, "Depreciation of Business R&D Capital," *Review of Income and Wealth*, 2020, *66* (1), 161–180.

Mian, Atif, Ludwig Straub, and Amir Sufi, "The Saving Glut of the Rich," February 2021. Manuscript.

Moll, Benjamin, Lukasz Rachel, and Pascual Restrepo, "Uneven Growth: Automation's Impact on Income and Wealth Inequality," *Econometrica*, November 2022, *90* (6), 2645–2683.

Nordhaus, William D, "Are We Approaching an Economic Singularity? Information Technology and the Future of Economic Growth," *American Economic Journal: Macroeconomics*, January 2021, *13* (1), 299–332.

Quadrini, Vincenzo, "The Importance of Entrepreneurship for Wealth Concentration and Mobility," *Review of Income and Wealth*, March 1999, *45.*

Rajgopal, Shivaram, Anup Srivastava, and Rong Zhao, "Do Digital Technology Firms Earn Excess Profits? Alternative Perspectives," *The Accounting Review*, September 2022, pp. 1–25.

Saez, Emmanuel and Gabriel Zucman, "Wealth Inequality in the United States since 1913: Evidence from Capitalized Income Tax Data," *Quarterly Journal of Economics*, 2016, *131* (2), 519–578.

__ **and** __ , "Trends in US Income and Wealth Inequality: Revising After the Revisionists," October 2020. NBER Working Paper, 27921.

Smith, Matthew, Danny Yagan, Owen M. Zidar, and Eric Zwick, "Capitalists in the Twenty-First Century," *Quarterly Journal of Economics*, 2019, *134* (4), 1675–1745.

Straub, Ludwig, "Consumption, Savings, and the Distribution of Permanent Income," June 2019. mimeo.

Toda, Alexis Akira, "Incomplete Market Dynamics and Cross-sectional Distributions," *Journal of Economic Theory*, 2014, *154*, 310–348.

Zeira, Joseph, "Workers, Machines, and Economic Growth," *Quarterly Journal of Economics*, 1998, *113* (4), 1091–1117.

İmrohoroğlu, Ayse and Kai Zhao, "Rising Wealth Inequality: Intergenerational Links, Entrepreneurship, and the Decline in Interest Rate," *Journal of Monetary Economics*, April 2022, *127*, 86–104.

A DERIVATION AND PROOF FOR SECTION 2.5

Stationary equilibrium conditions Here we assume homogeneous households: $\kappa = 1$ and common ρ and γ. We define the risk premium as $r_p := r_A - r$. Labor supply is fixed at $L = 1$. The steady state $(r, r_p, \omega, Y, K, A, W, C, H, \theta)$ satisfies the following conditions:

$$r + \delta = \lambda_K \left(\frac{K}{Y}\right)^{\alpha-1} \tag{24}$$

$$r + r_p + \delta_A = \lambda_A \lambda_V b^{\phi} \left(\frac{V}{Y}\right)^{\alpha-\phi} \left(\frac{A}{Y}\right)^{\phi-1} \tag{25}$$

$$\omega = \lambda_L \lambda_V \left(\frac{V}{Y}\right)^{\alpha-\phi} \left(\frac{L}{Y}\right)^{\phi-1} \tag{26}$$

$$Y = \omega L + (r+\delta)K + (r + r_p + \delta_A)A \tag{27}$$

$$Y = C + \delta K + \delta_A A \tag{28}$$

$$\theta = \frac{r_p}{\sigma^2 \gamma} \tag{29}$$

$$\frac{C}{W} = \rho + \zeta \tag{30}$$

$$\frac{A}{W} = \theta \tag{31}$$

$$\frac{K+H}{W} = 1 - \theta \tag{32}$$

$$H = \frac{\omega}{r+\zeta} L \tag{33}$$

Equilibrium human wealth We obtain the following equilibrium relationship:

$$\frac{\zeta H}{W} = \zeta - \mu_w. \tag{34}$$

This is derived using (27, 28, 30, 33) as follows:

$$\rho + \zeta = \frac{C}{W} = \frac{(r+\zeta)H + rK + r_A A}{W} = \frac{(r+\zeta)H}{W} + r\left(1 - \frac{H}{W}\right) + r_p\theta = \frac{\zeta H}{W} + r + r_p\theta.$$

This leads to (34), using $\mu_w = \theta r_A + (1-\theta) r - \rho = r + \theta r_p - \rho$.

Households' supply of digital capital θ Equations (32, 34) imply the following supply function of K/W:

$$\frac{K}{W} = \frac{r - \rho + (\sigma\theta)^2 \gamma}{\zeta} - \theta. \tag{35}$$

The supply of A/W is simply $\theta = r_p/(\sigma^2\gamma)$. Thus, the pair (r, r_p) determines the households' supply of K/W and A/W.

Firms' demand for digital capital From (24), firms' demand for capital per total wealth, given Y/W, is

$$\frac{K}{W} = \left(\frac{r+\delta}{\lambda_K}\right)^{\frac{1}{\alpha-1}} \frac{Y}{W} \tag{36}$$

which is decreasing in r. Also, our production function requires V/Y to satisfy $1 = \lambda_K(K/Y)^\alpha + \lambda_V(V/Y)^\alpha$. These relationships imply that V/Y is increasing in r. Substituting out V/Y from the demand function for A/Y in (25) and using (36), we obtain a demand function for A/W given Y/W in an implicit form as

$$1 = \lambda_K\left(\frac{r+\delta}{\lambda_K}\right)^{\frac{\alpha}{\alpha-1}} + \lambda_V\left(\frac{r+\delta_A+\theta\sigma^2\gamma}{\lambda_A\lambda_V b^\phi}\left(\frac{A}{W}\frac{W}{Y}\right)^{1-\phi}\right)^{\frac{\alpha}{\alpha-\phi}}. \tag{37}$$

Derivation of (21) and (22) that determine stationary equilibrium prices (r, r_p) We have $Y = C + \delta K + \delta_A A$ from (28). Dividing both sides by W we obtain

$$\frac{Y}{W} = \rho + \zeta + \delta\frac{K}{W} + \delta_A\theta. \tag{38}$$

Combining with (36), we have

$$\left(1 - \left(\frac{r+\delta}{\lambda_K}\right)^{\frac{1}{\alpha-1}}\delta\right)\frac{Y}{W} = \rho + \zeta + \delta_A\theta. \tag{39}$$

Hence, we have a set of equations (31, 35, 36, 37, 39) that determines r and θ (or r_p) along with K/W, A/W, and Y/W.

Equating the supply and demand for K/W (35, 36) and substituting out Y/W using (39), we obtain an equilibrium relationship between r and θ in the market for traditional capital K:

$$\rho + \zeta = \left(\left(\frac{r+\delta}{\lambda_K}\right)^{\frac{-1}{\alpha-1}} - \delta\right)\left(\frac{r-\rho+(\sigma\theta)^2\gamma}{\zeta} - \theta\right) - \delta_A\theta. \tag{40}$$

Similarly, equating the supply and demand for A/W (31, 37) using (39), we obtain a relationship between r and θ in the market for digital capital A:

$$1 = \lambda_K\left(\frac{r+\delta}{\lambda_K}\right)^{\frac{\alpha}{\alpha-1}} + \lambda_V\left[\frac{r+\delta_A+\sigma^2\gamma\theta}{\lambda_A\lambda_V b^\phi}\left\{\frac{\theta}{\rho+\zeta+\delta_A\theta}\left(1-\left(\frac{r+\delta}{\lambda_K}\right)^{\frac{1}{\alpha-1}}\delta\right)\right\}^{1-\phi}\right]^{\frac{\alpha}{\alpha-\phi}}. \tag{41}$$

Thus we have derived (21) and (22).

Proof of Proposition 1 (a) Now we inspect the properties of the stationary equilibrium. Here we assume that the stationary equilibrium exists uniquely. In particular, all equilibrium conditions are satisfied with non-negativity conditions for A, K, W, Y, H.

In (40), the expression in the last pair of brackets on the right-hand side is equal to K/W (35) and thus positive. Hence the expression in the first pair of brackets must also be positive. Then, we observe that the right-hand side is increasing in r.

In (41), the right-hand side is strictly increasing in r since $0 < \phi < 1$ and since we have established that $((r+\delta)/\lambda_K)^{-1/(\alpha-1)} - \delta \geq 0$. Also, the right-hand side is strictly increasing in θ. Hence, $dr/d\theta < 0$ holds for (41).

Moreover, an increase in b lowers the right-hand side of (41) since $\phi > 0$. At any fixed r, an increase in b must raise θ in (41) to clear the market for A/W. Hence, an increase in b shifts the curve (41) to the right in Figure 1.

In (35), we have $d(K/W)/d\theta \leq 0$ if $\theta \leq \zeta/(2\sigma^2\gamma)$. The supply function for K/W is decreasing in r_p given r for this region of θ. When this holds, the right-hand side of (40) is decreasing in θ. Thus, $dr/d\theta \geq 0$ must hold for (40).

In summary, the stationary equilibrium pair (r, θ) is determined as shown in Figure 1. If the stationary equilibrium exists in the region $\theta < \zeta/(2\sigma^2\gamma)$, equation (40) is upward sloping at θ. Moreover, an increase in b does not affect (40) while it shifts (41) to the right. Therefore, an infinitesimal increase in b increases both r_p and r. This completes the proof for Item (a).

Item (b): Impact of b on Pareto exponent ψ_1 We know from the stationary distribution of wealth that

$$W = \frac{\zeta H}{\chi}\left(\frac{1}{\psi_2+1} + \frac{1}{\psi_1-1}\right) \tag{42}$$

where $\psi_1 = (\chi-\eta)/\sigma_w^2$, $\psi_2 = (\chi+\eta)/\sigma_w^2$, $\chi = \sqrt{2\zeta\sigma_w^2 + \eta^2}$, $\eta = \mu_w - \sigma_w^2/2$, $\mu_w = \theta r_A + (1-\theta)r - \rho = r + \theta r_p - \rho$, and $\sigma_w = \theta\sigma$. Note that $(\psi_1, -\psi_2)$ are the two roots of the quadratic equation $0 = \sigma_w^2\psi^2 + 2\eta\psi - 2\zeta$. By Vieta's formulas, $\psi_1 - \psi_2 = -2\eta/\sigma_w^2$ and $-\psi_1\psi_2 = -2\zeta/\sigma_w^2$. Thus, ψ_1 satisfies

$$0 = f(\psi_1, \theta, r) := \psi_1 - \frac{2\zeta}{\psi_1(\sigma\theta)^2} + \frac{r-\rho}{(\sigma\theta)^2} - 1 + 2\gamma.$$

Note that $\partial f/\partial\psi_1 > 0$ and $\partial f/\partial r > 0$. Also, we assume an environment where the equilibrium satisfies $\theta < \zeta/(2\sigma^2\gamma)$ and therefore we have $dr/db \geq 0$ and $d\theta/db \geq 0$ from Item (a). Hence, we obtain $d\psi_1/db < 0$ if $\partial f/\partial\theta > 0$. Condition $\partial f/\partial\theta > 0$ is equivalent to

$$r - \rho - 2\zeta/\psi_1 < 0. \tag{43}$$

Note that $\psi_1 > 1$ must hold in equilibrium because the household wealth distribution has a finite mean. Since $\psi_1 = (\chi - \eta)/\sigma_w^2 = \left(\sqrt{2\zeta\sigma_w^2 + \eta^2} - \eta\right)/\sigma_w^2$, the condition $\psi_1 > 1$ is equivalent to $2(\zeta - \eta) > \sigma_w^2$. Using $\eta = r - \rho + \sigma_w^2(\gamma - 1/2)$, the condition is equivalently transformed to $\zeta - \gamma\sigma_w^2 > r - \rho$. Hence, (43) holds if $2\zeta/\psi_1 > \zeta - \gamma\sigma_w^2$. This implies that a sufficient condition for (43) is $\psi_1 < 2$.

In summary, if the stationary equilibrium satisfies $\psi_1 < 2$, we obtain $d\psi_1/db < 0$.

Item (c): Impact of b on labor share $\omega L/Y$ We use

$$\frac{\omega}{Y} = \frac{(r+\zeta)H}{W}\frac{W}{Y} = \frac{\zeta + r}{\zeta}\frac{\zeta + \rho - r - (\sigma\theta)^2\gamma}{\zeta + \rho + \delta_A\theta}\left\{1 - \left(\frac{r+\delta}{\lambda_K}\right)^{\frac{1}{\alpha-1}}\delta\right\}.$$

The derivative of the right-hand side with respect to r is at most finite. Thus we investigate the impact of θ on ω/Y, given r is unchanged.

$$\begin{aligned}\frac{\partial \log(\omega/Y)}{\partial\theta} &= -\frac{2\sigma^2\theta\gamma}{\zeta + \rho - r - (\sigma\theta)^2\gamma} - \frac{\delta_A}{\zeta + \rho + \delta_A\theta} = -\frac{2r_p}{\zeta H/W} - \frac{\delta_A}{(C + \delta_A A)/W} \\ &< 0\end{aligned}$$

Hence, we obtain the desired result.

B ALTERNATIVE SCENARIO

In this appendix, we study an alternative scenario of digitalization. In the main text, we only increase b, the productivity of digital capital, to capture the advances in digitalization. In this alternative scenario, we capture recent advances in digital technology by an exogenous increase in ϕ, the elasticity of substitution between digital capital and labor, in addition to b. Specifically, we change ϕ from 0.5 to 0.6, which is in line with the analysis shown in Eden and Gaggl (2018). Given this increase, we match the digital capital productivity so that the digital investment to GDP ratio rises by 4 percentage points, corresponding to Case B. Table 14 present the wealth distribution for alternative scenarios. The table highlights that even if we change both b and ϕ, the change in wealth distribution is very similar to the results in Case B. This result indicates that even if we also change the substitutability between digital capital and labor in addition to the productivity of digital capital, the results are very similar as long as we use the same calibration target.

Table 14: Wealth Distribution in alternative scenario

	Data SCF		Data SZ		Model	
	1989 (%)	2019	1989	2019	Bench	AD (II)
Bottom 90 %	32.9	23.6	35.3	28.6	28.9	25.5
Top 10 %	67.1	76.4	64.7	71.4	71.1	74.5
Top 5 %	54.2	64.9	50.6	57.9	54.0	59.0
Top 1 %	29.9	37.2	28.6	34.9	27.2	34.1
Top 0.1 %	10.5	14.1	12.1	17.6	10.8	17.4

Second and third columns show the data from the Survey of Consumer Finances based on Kuhn and Rios-Rull (2020). SZ shows the data coming from Saez and Zucman (2020). Bench, AD(II) show the results based on the quantitative analysis in benchmark economy, scenario where both b and ϕ increase, respectively.