IMES DISCUSSION PAPER SERIES

Monetary Policy, Labor Force Participation, and Wage Rigidity

Hiroyuki Kubota, Ichiro Muto, and Mototsugu Shintani

Discussion Paper No. 2022-E-17

IMES

INSTITUTE FOR MONETARY AND ECONOMIC STUDIES

BANK OF JAPAN

2-1-1 NIHONBASHI-HONGOKUCHO
CHUO-KU, TOKYO 103-8660
JAPAN

IMES Discussion Paper Series 2022-E-17
November 2022

Monetary Policy, Labor Force Participation, and Wage Rigidity

Hiroyuki Kubota*, Ichiro Muto, and Mototsugu Shintani*****

Abstract

To understand the role of monetary policy in determining the labor force participation rate, we present empirical evidence for Japan and the US. The data suggests that labor force participation declines in Japan but increases in the US in response to a monetary tightening. To inspect the mechanism, we develop and estimate a New Keynesian model of endogenous labor force participation decisions incorporating wage rigidity. We find that the opposite response of labor force participation can be attributed to a difference in the degree of wage rigidity. Counterfactual analysis based on the estimated models shows that the large-scale monetary easing in recent years helped boost the labor force participation rate in Japan, while its effect was almost neutral in the US.

Keywords: Labor force participation; Monetary policy; Unemployment; Wage rigidity

JEL classification: E24, E32, E52, E58

* University of California, Los Angeles (E-mail: hkubota@ucla.edu)
** Associate Director-General, Institute for Monetary and Economic Studies (currently, General Manager, Aomori Branch), Bank of Japan (E-mail: ichirou.mutou@boj.or.jp)
*** The University of Tokyo (E-mail: shintani@e.u-tokyo.ac.jp)

The authors would like to thank Kosuke Aoki, Kenta Fukuda, Masao Fukui, Yasuo Hirose, Takatoshi Ito, Yuto Iwasaki, Munechika Katayama, Takuji Kondo, Takushi Kurozumi, Toshihiko Mukoyama, Taisuke Nakata, Nobuyuki Oda, Ryohei Oishi, Kazuhiro Teramoto, Kozo Ueda, Toshiaki Watanabe, Tsutomu Watanabe, and the seminar and conference participants at the Bank of Japan, Osaka University, and the 2022 Summer Workshop on Economic Theory in Otaru for their helpful comments and discussions. Kubota gratefully acknowledges the financial support of the Grants-in-Aid for Scientific Research (22J12457). Shintani gratefully acknowledges the financial support of the Grants-in-Aid for Scientific Research (20H01482). The views expressed in this paper are those of the authors and do not necessarily reflect the official views of the Bank of Japan.

1 Introduction

In recent years, monetary policy authorities have been paying more attention to the development of the labor force participation rate (LFPR) as one of the key variables in understanding labor market conditions. For example, the Fed was closely monitoring the LFPR during the time of recovery from the global financial crisis (see Yellen, 2014). Figure 1 shows the historical evolution of the LFPR in Japan and the US. The LFPR in Japan continued to decline in the 1990s and the 2000s, but began to rise around 2013, which corresponds to the timing when the Bank of Japan commenced quantitative and qualitative monetary easing (QQE). On the other hand, the LFPR in the US was relatively stable in the 1990s and the 2000s, but sharply declined after the global financial crisis in 2008. Despite the large-scale monetary easing taken by the Fed in response, the LFPR did not revert to the level before the Great Recession. More recently, the LFPR suddenly fell in 2020 due to the Great Resignation triggered by the outbreak of COVID-19, and the Fed has emphasized promoting maximum employment in response (see Hobijn and Sahin, 2021). Against this backdrop, monetary policy discussions these days have increasingly focused on the development in the LFPR.[1] While macroeconomists have investigated the cyclical properties of the LFPR, the accumulation of theoretical and empirical studies on the relationship between the LFPR and monetary policy has so far been limited.

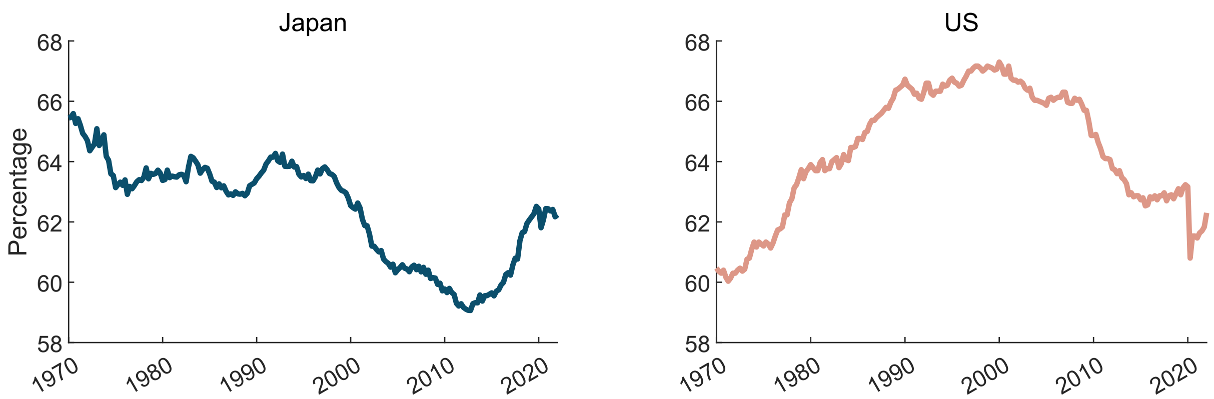

Figure 1. Labor Force Participation Rate in Japan and the US

In this paper, we investigate the effects of monetary policy on labor force participa-

[1]Indeed, a recent estimate by Elsby et al. (2015) implies that the labor force participation margins account for around one-third of unemployment fluctuations in the US.

tion. First, we provide empirical evidence on whether labor force participation reacts to monetary policy in Japan and the US. Second, we theoretically investigate the mechanism in which the LFPR is responsive to monetary policy.

In the first half of the analysis, we present empirical evidence from vector autoregressive (VAR) models for Japan and the US. In the analysis, monetary policy shocks are identified by using monetary policy surprises in futures rates as external instruments, as in Gertler and Karadi (2015). We find that labor force participation reacts to monetary policy in opposite directions across Japan and the US: the LFPR falls in Japan in response to a monetary tightening, while rising temporarily in the US. Further analysis based on disaggregated measures suggests the presence of both discouraged workers and added workers as the result of a monetary tightening in the two countries.

In the second half of the analysis, we introduce and estimate a simple New Keynesian model of labor force participation. Through this model, we provide a structural interpretation of the different responses of the LFPR in Japan and the US found in the VAR analysis. By introducing wage rigidity into a model of labor force participation originally developed by Erceg and Levin (2014), we show that the degree of wage rigidity changes the relative importance of the discouraged worker and the added worker effects. As the degree of wage rigidity increases, the LFPR response to a monetary tightening changes from negative to positive because the added worker effect becomes larger than the discouraged worker effect. By estimating our model using a standard Bayesian technique as in Smets and Wouters (2007) and others, we find that the degree of wage rigidity is higher in the US than in Japan, and the estimated model successfully reproduces the opposite responses of the LFPR to a monetary policy shock in the two countries. Finally, using our estimated model, we also conduct counterfactual analysis to quantify the effects of the recent monetary easing on the LFPR in the two countries. The results imply that the monetary easing partly increased labor force participation in Japan, while the decline in labor force participation due to the monetary easing was weak in the US.

Our paper is related to two strands of literature. First, our paper is related to studies on the macroeconomic models on the interaction between monetary policy and the labor market. Galí (2011) and Galí et al. (2012) model unemployment to derive the New Keynesian wage Phillips curve, which is grounded on the introduction of wage rigidity into a New Keynesian model by Erceg et al. (2000). This simple framework of describing unemployment has been extended in various directions, including Iwasaki et al. (2021), who evaluate the role of monetary policy in the presence of downward nominal wage rigidity. In contrast to the topic of unemployment, relatively fewer studies focus on the role of monetary policy in the determination of the LFPR. Among a few exceptions, Erceg and Levin (2014) incorporate endogenous labor force participation decisions in a standard New Keynesian model for the purpose of analyzing the monetary policy shock and LFPR. On the other hand, Christiano et al. (2015) and Campolmi and Gnocchi (2016) employ New Keynesian models with search and matching frictions and investigate the effects of monetary policy on the LFPR. However, these three papers focus on a flexible wage economy. In this sense, the model in our analysis integrates sticky wage models of unemployment in the first group and models of labor force participation in the second group.

Second, our paper is also related to the literature on the cyclicality of labor force participation. As pointed out by Shimer (2013), in theory, the LFPR can be either procyclical or countercyclical. For example, in a model with search and matching frictions considered by Nucci and Riggi (2018) and Cairó et al. (2022), the LFPR declines in response to a positive productivity shock when the degree of wage rigidity is sufficiently high, while it rises when the degree of wage rigidity is low.[2] Similarly, in our New Keynesian model, the degree of wage rigidity plays an important role in determining the cyclical property of the LFPR. Empirically, the LFPR has often been considered procyclical in the US (e.g., Shimer, 2013; Erceg and Levin, 2014; Nucci and Riggi, 2018), but the evidence regarding the effects of structural shocks on the LFPR has been limited. Several papers, including Cairó et al. (2022), Tüzemen and Van Zandweghe (2018), and Van Zandweghe (2017), investigate the response of the LFPR to productivity shocks identified in the VAR framework. However, no consensus has been reached regarding

[2] Tüzemen (2017) also shows that the LFPR can be countercyclical in an on-the-job search model without the wage stickiness.

the sign of its response. Unlike these studies that investigate the impulse responses of the LFPR to a productivity shock, our paper focuses on evaluating the response to a monetary policy shock.

The remaining part of the paper proceeds as follows. In Section 2, we provide empirical evidence of the relationship between the LFPR and monetary policy by estimating a VAR model. We develop a New Keynesian model in Section 3, and estimate the structural parameters of the model in Section 4. In Section 5, we conduct counterfactual analysis to assess the effects of the recent monetary easing in the two countries. Concluding remarks are presented in Section 6.

2 Empirical Evidence on the Effect of a Monetary Policy Shock on Labor Force Participation

In this section, we use a monthly VAR model and present empirical evidence on whether the LFPR reacts to monetary policy in Japan and the US. We follow Gertler and Karadi (2015) and identify monetary policy shocks by using monetary policy surprises in futures rates as the external instrument. We evaluate the effects of monetary policy shocks on the LFPR based on impulse responses and variance decomposition. In an additional analysis, we further investigate the responses of gross labor force inflows and outflows and their effects on labor force participation.

2.1 Data and Identification

Our VAR model includes six aggregate variables: log industrial production, the log consumer price index, the one-year government bond rate, a stock price, a credit spread, and the LFPR. To avoid the nonlinearity arising from the zero lower bound (ZLB), the one-year government bond rate is employed as the policy indicator for both countries. As a credit spread, we use the difference between the medium-term bond index and five-year government bond rate for Japan. For the US, we use the series provided by Favara et al. (2016), which extends the excess bond premium series originally estimated by Gilchrist and Zakrajšek (2012). A detailed description of the data is presented in Table A1 in Appendix A. The sample period for Japan is from January 1990 to January 2020 and that for the US is from July 1979 to January 2020. The lag order is set to twelve following Gertler and Karadi (2015).

In identifying monetary policy shocks, we make use of monetary policy surprises as the external instruments, which measure changes in futures interest rates within a tight window around monetary policy announcements. Let Z_t denote a monetary policy surprise, ε_t^p denote a monetary policy shock, and ε_t^q denote a vector of other structural shocks. For the instruments to be relevant and exogenous, they must be correlated with the monetary policy shock ε_t^p and uncorrelated with the other structural shocks ε_t^q:

$$\mathrm{E}\left[Z_t \varepsilon_t^p\right] = \phi, \quad \mathrm{E}\left[Z_t \varepsilon_t^q\right] = 0. \tag{1}$$

By measuring changes in futures rates within a sufficiently tight window around policy announcements, monetary policy surprises Z_t reflect only unanticipated components in the policy decisions. If we assume that those changes reflect only exogenous shocks to monetary policy, monetary policy surprises are valid as the external instruments because they satisfy both requirements in equation (1).

For the external instruments for Japan, we use the target factor constructed in Kubota and Shintani (2022). Since the target factor summarizes the policy surprises in the three-, six-, nine-, and twelve-month ahead Euroyen futures rates, our instrument reflects expectations of the interest rate up to a one-year horizon. In particular, we calculate the target factor from the extended surprise series by combining the following three: the high-frequency surprises within a thirty-minute window around every Monetary Policy Meeting (MPM) after October 1999, the daily surprises around every MPM from January 1998 to September 1999, and the daily surprises based on the policy change dates specified in Honda and Kuroki (2006) before 1997, which is when the MPMs had not been held. We convert the target factor into the monthly frequency, using the method explained in Gertler and Karadi (2015, footnote 11).[3] For the US, we make use of the high-frequency

[3]The path factor is also available in Kubota and Shintani (2022), but we focus on the target factor

policy surprises used in Gertler and Karadi (2015), which captures the three-month ahead expectation of the short-term interest rate. The policy surprises are measured within a thirty-minute window around every FOMC and in the three-month ahead monthly federal funds futures (FF4). Due to the availability of instruments, the sample periods for the first-stage regression is from July 1992 to January 2020 for Japan, and from January 1991 to June 2012 for the US, respectively.[4]

2.2 Estimated Impulse Responses: Main Result

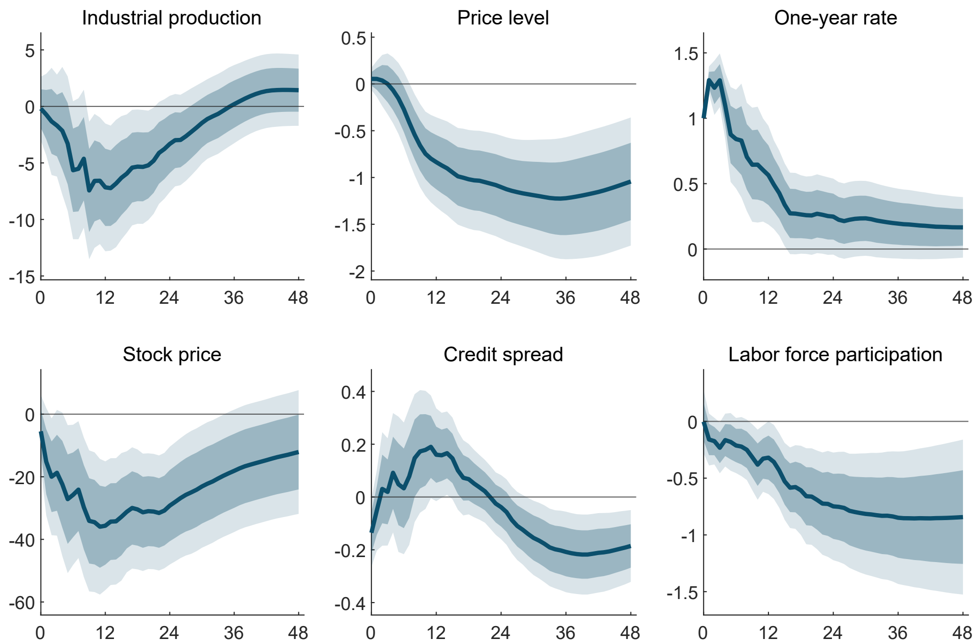

Figure 2. Impulse Responses to a Tightening Monetary Policy Shock: Japan

Note: The darker and lighter shaded areas indicate the 68 percent and 90 percent confidence bands, respectively. The unit of the horizontal axes is a month.

Estimated impulse responses to a tightening monetary policy shock for Japan are presented in Figure 2. The policy shock is normalized to induce a 1 percentage point increase in the 1-year rate on impact. For each variable, 68 percent and 90 percent confidence bands are indicated with the darker and lighter shaded areas, respectively.[5]

in this paper because the path factor turns out to be an invalid external instrument in the first-stage regression.

[4]For Japan, the F-statistic and the robust F-statistic in the first-stage regression are 43.00 and 13.08, respectively. For the US, the F-statistic and the robust F-statistic are 12.35 and 7.97, respectively.

[5]We compute the confidence bands using the asymptotic formula obtained by Montiel Olea et al. (2021) along with a HAC covariance matrix estimator.

As presented in the figure, the labor force decreases in response to a monetary tightening in Japan, regardless of the definition of the LFPR. After the shock, the LFPR gradually declines by 0.3 percent points within a year. The shock has a very persistent effect on the LFPR and the negative response becomes more than 0.8 percent points after two years.[6] We also note that the rest of the variables also respond in consistent directions with conventional theory: in terms of the real side of the economy, outputs and prices decline significantly after the shock hits the economy. The shock also affects financial variables: the stock price drops instantaneously and the credit spread increases, the latter of which is consistent with the credit channel.

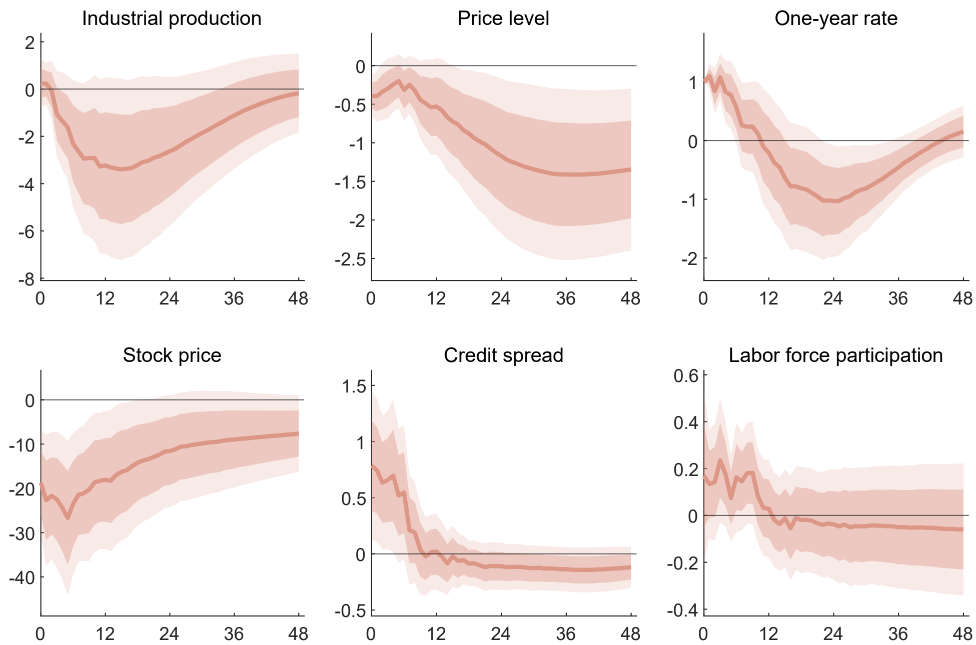

Figure 3. Impulse Responses to a Tightening Monetary Policy Shock: US

Note: The darker and lighter shaded areas indicate the 68 percent and 90 percent confidence bands, respectively. The unit of the horizontal axes is a month.

Similarly, Figure 3 shows estimated impulse responses to a tightening monetary policy shock for the US. In contrast to the estimation result for Japan, the LFPR temporarily increases after a monetary tightening in the US. A monetary policy shock that induces a 1 percentage point increase in the 1-year rate on impact raises the LFPR by 0.1 percent points for a year, while the statistical evidence is not as strong as the case of Japan.[7]

[6] Our main estimation result remains unchanged even if we limit our sample to the period before the QQE was introduced in 2013.

[7] While our sample period for the US contains the great inflation period of the 1980s, we also obtain

For the remaining variables, the estimated responses are in line with Gertler and Karadi (2015).

	Japan	US
Impact	1.14	2.15
One year	7.04	3.96
Two years	13.84	2.19
Three years	15.43	1.70
Four years	15.16	1.55

Table 1. Contribution of Monetary Policy Shocks to the LFPR

To understand the contribution of monetary policy shocks to the variations in the LFPR, the results of the forecast error variance decomposition for the two countries are reported in Table 1. The results reflect the fact that the effect of a monetary policy shock on the LFPR is highly persistent in Japan, but lasts for only about a year in the US as we see in Figures 2 and 3. In Japan, the contribution of monetary policy shocks on the variations in labor force participation is small on impact but increases to above 15 percent after three years. In the US, while the contribution of monetary policy is relatively large in the short run but becomes smaller in the long run.

2.3 Estimated Impulse Responses: Further Analysis

The cyclicality of labor force participation has long been studied in the context of the relative importance of the discouraged worker effect and the added worker effect since the seminal works by Mincer (1966), Cain (1967), and Bowen and Finegan (1969). In an economic downturn, the discouraged workers leave the labor force due to reduced wages or more costly job search, while labor force participation can increase because of the added workers who wish to compensate for income loss due to the layoff of the primary earner in a family (e.g., Lundberg, 1985).[8] With these arguments in the literature of labor

the positive response of the LFPR to a monetary tightening in subsample analysis after 1990. Our estimation results contrast to those by Christiano et al. (2015), who find the positive response of the LFPR to an expansionary monetary policy shock. This difference may reflect the choice of the sample period (1951 to 2008) and the identification strategy.

[8]Furthermore, Finegan et al. (2008) discuss that outflows from the labor force during a recession do not include only discouraged workers but also market timers who have attractive nonmarket uses of their time, and countercyclical enrollees who wish to continue or return to school. They also discuss that, in

economics, it will be useful to further investigate the effect of a monetary tightening on the elements comprising labor force participation. To this end, we disaggregate labor force participation in three ways, namely, gross flows, genders, and demographics. We then reestimate six-variable VAR models which replace the LFPR with a variable of interest.

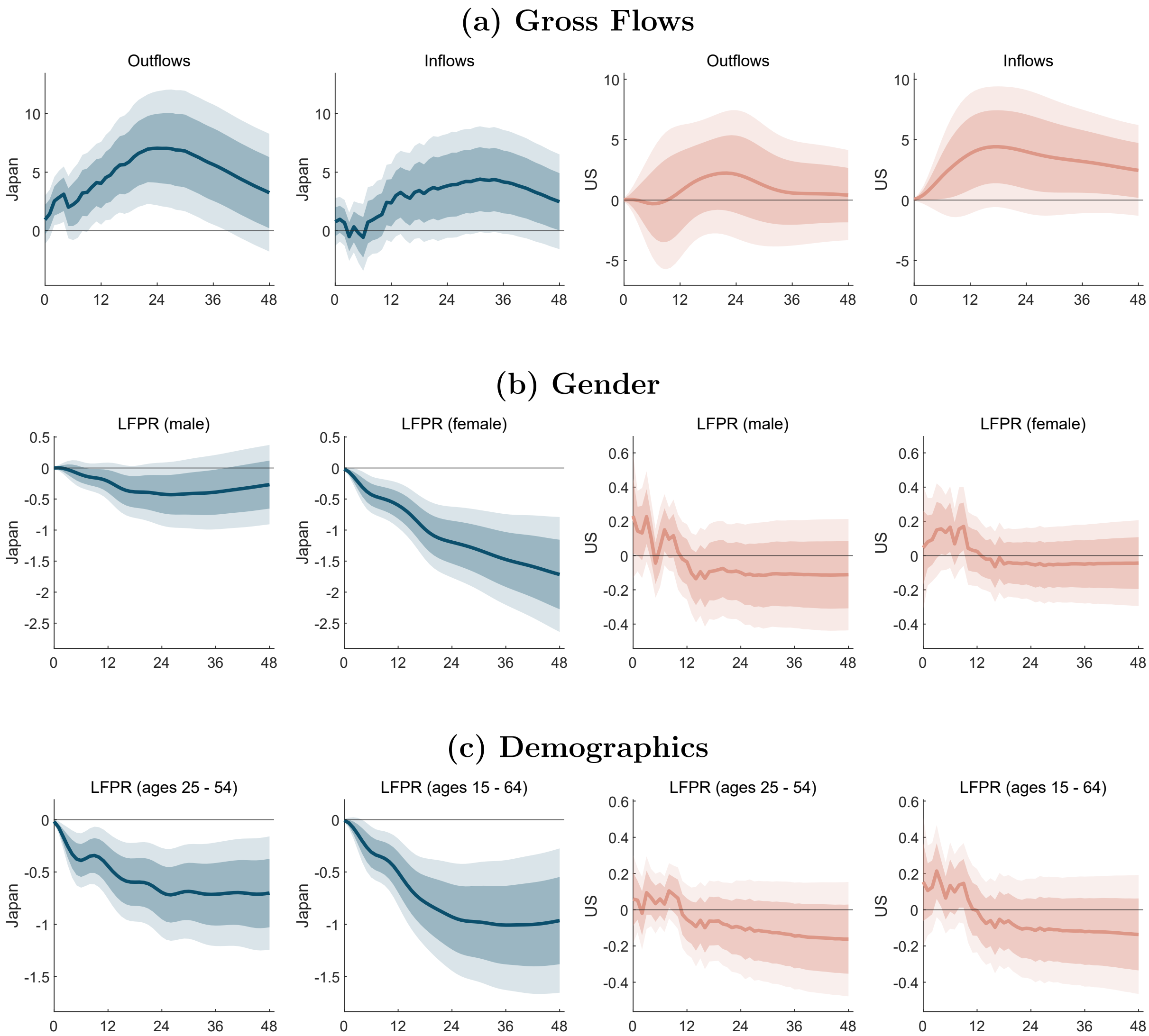

Figure 4. Impulse Responses to a Tightening Monetary Policy Shock: Disaggregated Measures

Note: The darker and lighter shaded areas indicate the 68 percent and 90 percent confidence bands, respectively. The unit of the horizontal axes is a month.

First, we look at the responses of gross labor flows between the labor force and not-in-the-labor force. Panel (a) of Figure 4 shows the estimated impulse responses of labor force

addition to the added workers, the extended workers, who stay in the labor market by postponing their retirement, can increase labor force participation in an economic downturn.

outflows and inflows in Japan and the US. The vertical axes represent how much the gross labor flows raise the LFPR in terms of a percentage point. For the two countries, both labor force outflows and inflows increase in response to a monetary tightening. These positive responses suggest the presence of the discouraged workers and added workers in an economic downturn. At the same time, we observe that the difference in the relative size of the increases in outflows and inflows is consistent with the opposite response of the LFPR across the two countries. If we simply interpret the increase in outflows as the discouraged workers and the increase in inflows as the added workers, the former dominates in Japan, while the latter dominates in the US.

Second, we separately investigate the responses of labor force participation of male and female workers. Given that the husband is typically the primary earner in couples, labor force participation behaviors are likely to differ between male and female workers.[9] We show the responses of labor force participation of male and female workers in Japan and the US in panel (b) of Figure 4. The sign of the responses are consistent with the result obtained from the aggregate LFPR. While the responses of male and female workers are similar in the US, the response of female workers is much larger than that of male workers in Japan. This result is in line with the view that female workers in Japan are heavily discouraged in a economic downturn, and thus the female LFPR is highly procyclical (e.g., Tachibanaki and Sakurai, 1991).

Third, we investigate the responses of the labor force participation of specific age groups. If young or elderly workers behave differently in a recession, the estimated LFPR response will be affected by the difference in the age profile across the two countries. While we use the LFPR for ages 15 and over in the main result, in this analysis, we consider the two alternative definitions of the LFPR, which are based on workers of ages 25–54 (prime age) and 15–64. We report the impulse responses of the LFPR based on ages 25–54 and 15–64 in panel (c) of Figure 4, respectively. For both countries, the main

[9]In the family type in which the primary earner is the husband, the wife could easily leave the labor force in response to the reduced wage, as she is the secondary earner. Thus, we expect the discouraged worker effect for married female workers to be larger than that for married male workers. At the same time, in such a family, female labor supply functions as an important consumption insurance device against income shocks faced by the husband, as estimated by Blundell et al. (2016) using panel data. Thus, the added worker effect is expected to be large for married female workers as well.

conclusion remains almost unchanged: In response to a tightening monetary policy, the LFPR declines in Japan and rises in the US.

3 A Macroeconomic Model of Labor Force Participation

In this section, we introduce a simple New Keynesian model, which can explain the empirical evidence in the previous section, namely, the difference in the LFPR responses to monetary policy shocks between the two countries. To this end, we extend a model originally developed by Erceg and Levin (2014), in which households endogenously determine their labor force participation. In particular, we introduce wage rigidity to their model along the lines of Erceg et al. (2000) and Galí (2011). With such an extension, we can look at how the difference in the degree of wage rigidity can generate different responses of labor force participation to a monetary policy shock. In particular, the sign of the LFPR response is determined by the degree of wage rigidity, which affects the relative importance of the discouraged worker effect and the added worker effect of monetary tightening.

3.1 Household

3.1.1 Household Structure

We first present a description of the household side of the economy in Erceg and Levin (2014), in which the household explicitly chooses labor force participation. In their model, a large representative household with a continuum of members on the unit square are indexed by a pair $(l, k) \in [0, 1] \times [0, 1]$: the first dimension l is homogeneous, and the second dimension k determines disutility from work. The representative household allocates members $l \in [0, L_t]$ to the labor market and $l' \in (L_t, 1]$ to home production sectors. The L_t fraction of members allocated to the labor market decides to work or not, according to their heterogeneous disutility of work represented by the index k as in Galí

(2011). Following Erceg and Levin (2014), we assume that the labor disutility k of each member is revealed only after the labor force participation decision has been made.[10] All the members of type $k \in [0, e_t]$ in the labor force are employed and the remaining members of type $k' \in (e_t, 1]$ are unemployed. Since any labor adjustment takes place at the extensive margin under the assumption of indivisible labor, L_t corresponds to the LFPR, $E_t \equiv L_t e_t$ corresponds to the employment-population ratio, $U_t \equiv L_t(1 - e_t)$ corresponds to the number of the unemployed members, and $u_t \equiv U_t/L_t = 1 - e_t$ corresponds to the unemployment rate.

In a model of Erceg and Levin (2014) under a flexible wage assumption, the period utility of the representative household is formulated as

$$\begin{aligned} U\left(C_t, e_t, L_t\right) &\equiv \frac{\widetilde{C}_t^{1-\tau}}{1-\tau} - \chi_t L_t \int_0^{e_t} k^{\frac{1}{\nu}} dk + \kappa \frac{\left(1 - L_t\right)^{1-\xi}}{1-\xi} \\ &= \frac{\widetilde{C}_t^{1-\tau}}{1-\tau} - \chi_t L_t \frac{e_t^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} + \kappa \frac{\left(1 - L_t\right)^{1-\xi}}{1-\xi}, \end{aligned} \tag{2}$$

where $\widetilde{C}_t \equiv C_t/A_t - h\left(C_{t-1}/A_{t-1}\right)\left(A_{t-1}/A_t\right)$, A_t denotes the productivity level, h denotes the internal habit formation parameter, τ denotes the inverse of the intertemporal elasticity of substitution, χ_t denotes an exogenous preference shifter with mean χ, ν denotes the Frisch elasticity, and κ and ξ are a scaling parameter and a curvature parameter for the home production term, respectively. The preference shifter χ_t follows the AR(1) process in log as $\ln \chi_t = (1 - \rho_\chi) \ln \chi + \rho_\chi \ln \chi_{t-1} + \varepsilon_{\chi,t}$, where χ is the steady state of χ_t and $\varepsilon_{\chi,t}$ denotes a labor supply shock.

To incorporate wage rigidity into the model, we now introduce labor differentiation following Erceg et al. (2000) and Galí (2011). We assume that a large representative household consists of a continuum of the families on a unit interval $j \in [0, 1]$, which represents the labor service in which family members are specialized. Firms regard each family's labor services j as an imperfect substitute for those of other families. As in (2), members of each family j are indexed by their labor force status and disutility of

[10] Erceg and Levin (2014) emphasize the importance of this assumption. Without this assumption, unemployment disappears as the representative household has an incentive to allocate members with a higher disutility of work to home production. Unemployed members are not beneficial to the household as they neither gain utility nor earn any income.

work within a unit square given by $(l, k) \in [0,1] \times [0,1]$. This structure of households is visualized in Figure 5.

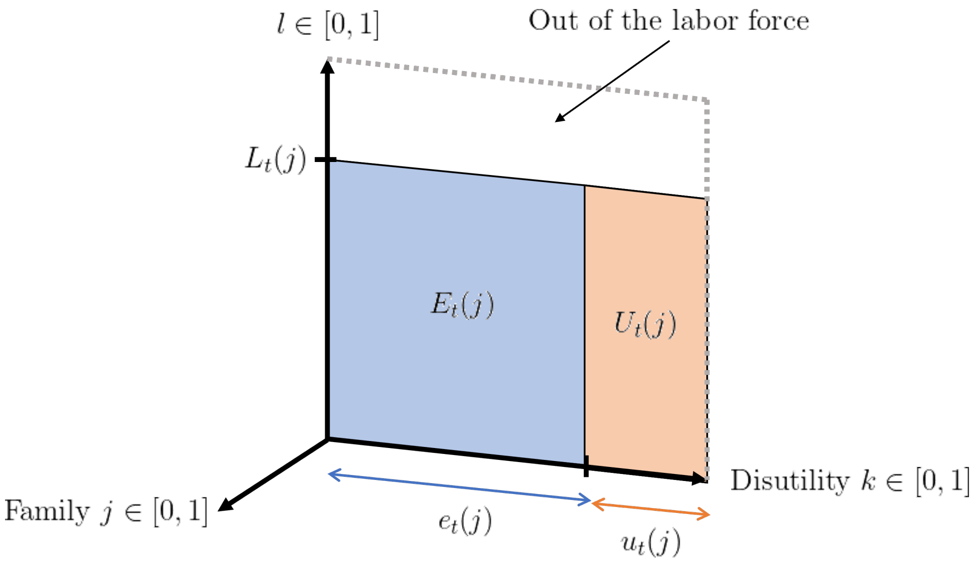

Figure 5. Household Structure

In our model, the representative household allocates members $l \in [0, L_t(j)]$ to the labor force, and in each family j, members of type $k \in [0, e_t(j)]$ are employed and earn $W_t(j)$. The representative household has the following form of a period utility:

$$\begin{aligned} U\left(C_t, \{e_t(j)\}, \{L_t(j)\}\right) &\equiv \frac{\widetilde{C}_t^{1-\tau}}{1-\tau} - \chi_t \int_0^1 L_t(j) \int_0^{e_t(j)} k^{\frac{1}{\nu}} dk dj + \kappa \int_0^1 \frac{(1-L_t(j))^{1-\xi}}{1-\xi} dj \\ &= \frac{\widetilde{C}_t^{1-\tau}}{1-\tau} - \chi_t \int_0^1 L_t(j) \frac{e_t(j)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} dj + \kappa \int_0^1 \frac{(1-L_t(j))^{1-\xi}}{1-\xi} dj. \end{aligned} \tag{3}$$

Because we assume full risk sharing as in Merz (1995), the level of consumption is common to all individuals regardless of their employment status. In this specification, $L_t(j)$ corresponds to the LFPR within family j, $E_t(j) \equiv L_t(j)e_t(j)$ corresponds to the employment-population ratio within family j, $U_t(j) \equiv L_t(j)(1 - e_t(j))$ corresponds to the number of the unemployed members in family j, and $u_t(j) \equiv U_t(j)/L_t(j) = 1 - e_t(j)$ corresponds to the unemployment rate within family j.

As in Erceg et al. (2000) and Galí (2011), workers supply a labor service as monopolistic competitors. However, since individuals typically cannot affect their wage setting until they enter the labor force in practice, we assume that household decisions of labor force participation and labor supply are made in a sequential manner in the model. First, the representative household chooses $L_t(j)$ as a wage taker before the type k of each member is revealed. Second, individual families determine $W_t(j)$, the nominal wage for labor service supplied by family j, given the labor demand by firms as monopolistic competitors.

3.1.2 Budget Constraint

The budget constraint for the representative household is given by

$$P_t C_t + B_t = \int_0^1 W_t(j) L_t(j) e_t(j) (1 - \Phi_w(\pi_{w,t}(j))) \, dj + R_{t-1} B_{t-1} + \Pi_t. \tag{4}$$

where P_t is the price of final goods, B_t is purchases of government bonds, $\Phi_w(\pi_{w,t}(j))$ is the wage adjustment cost, $\pi_{w,t}(j) \equiv W_t(j) / W_{t-1}(j)$ is the (gross) wage inflation for worker j, R_t is the risk-free nominal interest rate on government bonds, and Π_t is total dividend payments from firms. The wage adjustment is subject to a Rotemberg-style quadratic cost function with partial indexation. Along the line of Galí (2011), we assume that the wage adjustment cost is zero when workers set their wages following the indexation rule given by:

$$\overline{\pi}_{w,t} = \gamma (\pi_{p,t-1})^{\iota_w} (\pi_p)^{1-\iota_w},$$

where $\pi_{p,t} \equiv P_t / P_{t-1}$ denotes the (gross) price inflation, $\iota_w \in [0, 1]$ is the degree of wage indexation to past price inflation, π_p denotes the steady-state price inflation, and γ denotes the steady-state growth rate of productivity. Accordingly, the wage adjustment cost function takes the following form:

$$\Phi_w(\pi_{w,t}(j)) = \frac{\phi_w}{2} \left(\frac{\pi_{w,t}(j)}{\overline{\pi}_{w,t}} - 1 \right)^2,$$

where ϕ_w (> 0) measures the degree of nominal wage rigidity.

The families supply their differentiated labor to the labor packers under monopolistic competition. The labor packers aggregate the imperfectly substitutable labor services of household members to produce the homogeneous labor services E_t, and supply them to the intermediate goods producers under perfectly competitive wages W_t. The production function for the labor packers is given by:

$$E_t = \left(\int_0^1 E_t(j)^{1-\lambda_{w,t}} dj \right)^{\frac{1}{1-\lambda_{w,t}}},$$

where $\lambda_{w,t}$ denotes the time-varying inverse demand elasticity for each labor service. The inverse demand elasticity follows the AR(1) process in log as $\ln \lambda_{w,t} = (1-\rho_w)\ln \lambda_w + \rho_w \ln \lambda_{w,t-1} + \varepsilon_{w,t}$, where λ_w is the steady state of $\lambda_{w,t}$ and $\varepsilon_{w,t}$ denotes a wage markup shock. Then the demand for the labor service j is given by

$$E_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\frac{1}{\lambda_{w,t}}} E_t, \tag{5}$$

and the labor cost W_t that the firms face is related to wages paid to workers $W_t(j)$ through $W_t = \left(\int_0^1 W_t(j)^{(\lambda_{w,t}-1)/\lambda_{w,t}} dj \right)^{\lambda_{w,t}/(\lambda_{w,t}-1)}$.

3.1.3 First Order Conditions

Finally, the household's lifetime utility is

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s d_{t+s} U\left(C_{t+s}, \{e_{t+s}(j)\}, \{L_{t+s}(j)\} \right) \right], \tag{6}$$

where β is the constant component of the discount factor and d_t is a time-varying component of the discount factor. The time-varying component follows an AR(1) process in log as $\ln d_t = \rho_d \ln d_{t-1} + \varepsilon_{d,t}$, where $\varepsilon_{d,t}$ denotes a discount factor shock. The household maximizes their lifetime utility (6) by choosing $\{C_t, B_t, \{L_t(j)\}, \{W_t(j)\}\}_{t=0}^{\infty}$ in a sequential manner. The household first chooses $\{L_t(j)\}$ subject to the budget constraint (4) without information of the demand schedule (5), and then chooses $\{W_t(j)\}$ subject to

(4) and (5) along with C_t and B_t. The consumption Euler equation is derived as

$$1 = \beta \mathbb{E}_t \left(\frac{d_{t+1}}{d_t} Q_{t+1|t} \frac{R_t}{\pi_{p,t+1}} \right), \tag{7}$$

where $Q_{t+s|t} \equiv U_{C,t+s}/U_{C,t}$ denotes the time t value of a unit of the consumption good in period $t+s$, and $U_{C,t}$ denotes the marginal utility with respect to consumption at period t.

The optimality condition for labor force participation in a symmetric equilibrium, where $e_t(j) = E_t(j)/L_t(j) = E_t/L_t$, $W_t(j) = W_t$, and $\pi_{w,t} = \pi_{w,t}(j)$ hold for all j, is given by

$$\chi_t \frac{(E_t/L_t)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} + \kappa \left(1 - L_t\right)^{-\xi} = U_{C,t} \frac{W_t}{P_t A_t} \frac{E_t}{L_t} \left(1 - \Phi_w\left(\pi_{w,t}\right)\right), \tag{8}$$

where the first term on the left side corresponds to a marginal increase in disutility from work, the second term on the left side corresponds to a marginal decrease in utility from labor force participation, and the right side corresponds to utility gain from a marginal increase in labor income evaluated in terms of consumption. Based on a trade-off between consumption, work, and home production, the optimal labor force participation is determined so that utility losses from a marginal increase in labor force participation equate to utility gains.

Given the labor demand schedule (5) as well as the budget constraint (4), each family chooses $W_t(j)$ to maximize the lifetime utility (6). In a symmetric equilibrium, the wage inflation dynamics can be described by the New Keynesian wage Phillips curve (NKWPC):

$$\begin{aligned} 0 = {} & \frac{1}{\lambda_{w,t}} \frac{1}{\mu_{w,t}} + \left(1 - \frac{1}{\lambda_{w,t}}\right) \left(1 - \Phi_w\left(\pi_{w,t}\right)\right) - \Phi_w'\left(\pi_{w,t}\right) \pi_{w,t} \\ & + \beta \mathbb{E}_t \left(\frac{d_{t+1}}{d_t} Q_{t+1|t} \frac{E_{t+1}}{E_t} \frac{1}{\pi_{t+1}} \Phi_w'\left(\pi_{w,t+1}\right) \pi_{w,t+1}^2 \right). \end{aligned} \tag{9}$$

Here, $\mu_{w,t}$ is the wage markup defined as the ratio of the real wage to the marginal rate of substitution of labor to consumption, or

$$\mu_{w,t} \equiv -\frac{W_t}{P_t}\left(\frac{U_{E,t}}{U_{C,t}}\right)^{-1} = \frac{W_t}{P_t}\left(\frac{\chi_t E_t^{\frac{1}{\nu}} L_t^{-\frac{1}{\nu}}}{U_{C,t}}\right)^{-1}, \tag{10}$$

where $U_{E,t}$ denotes the symmetric equilibrium level of $U_{E,t}(j) \equiv \partial U\left(C_t, \{e_t(j)\}, \{L_t(j)\}\right)/\partial E_t(j)$.[11]

3.2 Firm

3.2.1 Final Goods Producers

We assume perfectly competitive final goods producing firms that combine a continuum of intermediate goods indexed by $i \in [0, 1]$, according to the CES technology:

$$Y_t = \left(\int_0^1 Y_t\left(i\right)^{1-\lambda_{p,t}} di\right)^{\frac{1}{1-\lambda_{p,t}}}, \tag{11}$$

where $\lambda_{p,t}$ denotes the time-varying inverse demand elasticity for each intermediate good. The inverse demand elasticity follows the AR(1) process in log as $\ln\lambda_{p,t} = (1-\rho_p)\ln\lambda_p + \rho_p \ln\lambda_{p,t-1} + \varepsilon_{p,t}$, where λ_p is the steady state of $\lambda_{p,t}$ and $\varepsilon_{p,t}$ denotes a price markup shock. Under the assumption of a perfectly competitive market, profit maximization and free entry imply that the demand for intermediate goods is

$$Y_t\left(i\right) = \left(\frac{P_t\left(i\right)}{P_t}\right)^{-\frac{1}{\lambda_{p,t}}} Y_t, \tag{12}$$

where $P_t\left(i\right)$ is the price of intermediate goods. The relationship between $P_t\left(i\right)$ and P_t is given by $P_t = \left(\int_0^1 P_t(i)^{(\lambda_{p,t}-1)/\lambda_{p,t}} di\right)^{\lambda_{p,t}/(\lambda_{p,t}-1)}$.

[11] In defining the marginal disutility of labor $U_{E,t}(j)$, we measure a marginal increase in labor in terms of $E_t(j)$, rather than $e_t(j)$. Accordingly, we know that the marginal disutility of labor $U_{E,t}(j)$ is given by

$$U_{E,t}(j) = \frac{\partial U\left(C_t, \{e_t(j)\}, \{L_t(j)\}\right)}{\partial E_t(j)} = \frac{\partial U\left(C_t, \{e_t(j)\}, \{L_t(j)\}\right)}{\partial e_t(j)} \frac{\partial e_t(j)}{\partial E_t(j)} = -\chi_t E_t(j)^{\frac{1}{\nu}} L_t(j)^{-\frac{1}{\nu}}.$$

3.2.2 Intermediate Goods Producers

Intermediate good i is produced by a monopolistically competitive firm, according to the following production technology:

$$Y_t(i) = A_t E_t(i). \tag{13}$$

where $E_t(i)$ denotes the labor input for the intermediate goods producing firm i. Intermediate goods producers purchase labor services at a nominal wage of W_t. The price adjustment is subject to a Rotemberg-style quadratic cost function with partial indexation. We assume that the price adjustment cost is zero when firms set their prices following the indexation rule given by:

$$\overline{\pi}_{p,t} = (\pi_{p,t-1})^{\iota_p} (\pi_p)^{1-\iota_p} ,$$

where $\iota_p \in [0, 1]$ is the degree of price indexation to past price inflation. Accordingly, the price adjustment cost function takes the following form:

$$\Phi_p (\pi_{p,t}(i)) = \frac{\phi_p}{2} \left(\frac{\pi_{p,t}(i)}{\overline{\pi}_{p,t}} - 1 \right)^2 ,$$

where ϕ_p (> 0) measures the degree of price rigidity.

Given the demand for intermediate goods (12), each firm maximizes its profit by choosing its output price and labor input. Assuming a symmetric equilibrium where all intermediate goods producers choose the same prices and labor inputs ($P_t(i) = P_t$, $E_t(i) = E_t$, and $\pi_{p,t}(i) = \pi_{p,t}$), the aggregate price inflation dynamics is described by the New Keynesian Phillips curve (NKPC):

$$0 = \frac{1}{\lambda_{p,t}} \frac{W_t}{P_t A_t} + \left(1 - \frac{1}{\lambda_{p,t}} \right) (1 - \Phi_p(\pi_{p,t})) - \Phi_p'(\pi_{p,t})\pi_{p,t} + \beta \mathbb{E}_t \left(\frac{d_{t+1}}{d_t} Q_{t+1|t} \frac{Y_{t+1}}{Y_t} \Phi_p'(\pi_{p,t+1})\pi_{p,t+1} \right) . \tag{14}$$

3.3 Government

Monetary policy is governed by an interest rate feedback rule:

$$R_t = R_t^{*1-\rho_r} R_{t-1}^{\rho_r} e^{\varepsilon_{r,t}}, \tag{15}$$

where $\varepsilon_{r,t}$ is a monetary policy shock, R_t^* is the notional rate of the nominal interest rate, and ρ_r is a degree of interest smoothing. The notional rate R_t^* is given by

$$R_t^* = R \left(\frac{\pi_{p,t}}{\pi_p} \right)^{\psi_\pi} \left(\frac{Y_t}{\gamma Y_{t-1}} \right)^{\psi_y}, \tag{16}$$

where $R \ (= \pi_p \gamma / \beta)$ is the steady-state nominal interest rate, ψ_π is the sensitivity parameter on inflation, and ψ_y is the sensitivity parameter on output growth.

The fiscal authority issues a risk-free bond B_t to consume a fraction $\zeta_t = 1 - 1/g_t$ of aggregate output Y_t and finance the interest payment $R_{t-1} B_{t-1}$. Accordingly, the government's budget constraint is given by:

$$B_t = R_{t-1} B_{t-1} + \zeta_t P_t Y_t \tag{17}$$

The government spending variable g_t follows the AR(1) process in log as $\ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \varepsilon_{g,t}$, where $g = 1/(1 - \zeta)$, ζ is the steady state of ζ_t, and $\varepsilon_{g,t}$ denotes a government spending shock.

3.4 Remaining Part of the Model

Intermediate goods producers' total dividend payments to the household are given by

$$\Pi_t = (1 - \Phi_p(\pi_{p,t}))\, P_t Y_t - W_t E_t, \tag{18}$$

Combining the household budget constraint (4) and the government budget constraint (17), we obtain the following aggregate resource constraint:

$$P_t C_t + \zeta_t P_t Y_t = (1 - \Phi_p(\pi_{p,t}))\, P_t Y_t - \Phi_w(\pi_{w,t}) W_t E_t. \tag{19}$$

The unemployment rate is defined as follows.

$$u_t \equiv 1 - \frac{E_t}{L_t} \tag{20}$$

The productivity level A_t follows a stochastic trend in log:

$$\Delta \ln A_t = (1 - \rho_a) \ln \gamma + \rho_a \Delta \ln A_{t-1} + \varepsilon_{a,t}, \tag{21}$$

where $\varepsilon_{a,t}$ denotes a productivity shock.

There are seven exogenous shocks in this economy: a labor supply shock $\varepsilon_{\chi,t}$, a productivity shock $\varepsilon_{a,t}$, a discount factor shock $\varepsilon_{d,t}$, a price markup shock $\varepsilon_{p,t}$, a wage markup shock $\varepsilon_{w,t}$, a monetary policy shock $\varepsilon_{r,t}$, and a government spending shock $\varepsilon_{g,t}$. For all $i \in \{\chi, a, d, w, p, r, g\}$, $\varepsilon_{i,t}$ is an iid normal random variable with mean zero and variance σ_i^2. The dynamics of the eleven endogenous variables in this model (Y_t, C_t, E_t, L_t, u_t, $\pi_{p,t}$, $\pi_{w,t}$, W_t/P_t, R_t, R_t^*, and $\mu_{w,t}$) are governed by the eleven equations ((7), (8), (10), (9), (13), (14), (15), (16), (19), (20), and $\pi_{w,t} = W_t/W_{t-1}$) and the seven exogenous shocks.

3.5 Discouraged Worker Effect and Added Worker Effect of a Monetary Tightening

Let us emphasize that, in this model, the degree of wage rigidity plays an important role in determining the sign of the response of labor force participation to monetary policy. To see this mechanism, it is useful to decompose changes in labor force participation due to monetary policy shocks into the procyclical and countercyclical components. This decomposition is analogous to the well-known regression analysis in the labor literature regarding the relative importance of the discouraged worker and added worker effects in determining the cyclicality of labor force participation.[12]

Under the assumption of no habit formation ($h = 0$) and the absence of both government spending shocks and labor supply shocks, the log-linearized version of the optimal labor force participation condition (7) can be written as:

$$\widehat{l}_t = \psi_{\mathrm{D}} \widehat{w}_t + \psi_{\mathrm{A}} \widehat{y}_t, \tag{22}$$

where $\widehat{l}_t$, $\widehat{w}_t$, and $\widehat{y}_t$ denote the log deviations of L_t, $W_t/(A_t P_t)$, and Y_t/A_t, respectively. Under typical choices of the parameter values, we expect that $\psi_{\mathrm{D}} > 0$ and $\psi_{\mathrm{A}} < 0$ hold.[13]

We note that equation (22) can be interpreted analogously to the regression of the LFPR of married women on the wife's wage and the husband's income, conditional on permanent components, considered by Mincer (1966) and Cain (1967). They interpret the positive coefficient on the wife's wage as representing the discouraged worker effect, while the negative coefficient on the husband's income represents the added worker effect. With some abuse of terminology, we similarly refer to the first term regarding the real wage $\widehat{w}_t$ in (22) as the discouraged worker effect of a monetary tightening, and to the second term regarding the output $\widehat{y}_t$ as the added worker effect of a monetary tightening. For the first term, a monetary tightening decreases the real wage $\widehat{w}_t$ due to the reduced labor demand, which discourages individuals from work, and thus has a negative effect on labor force participation. For the second term, when we interpret the output $\widehat{y}_t$ as the household income, the income declines in response to a monetary tightening, which increases the marginal net benefit of work for individuals, and thus has a positive effect

[12] Our decomposition differs from the dynamic version of Hicksian decomposition to the substitution effect (the wage effect and the interest rate effect) and the wealth effect popularly used in the literature of real business cycle models (e.g., King and Rebelo, 1999). The reason is that the decomposition to the discouraged worker and added worker effects is simple and intuitive for the purpose of understanding the role of wage rigidity on the LFPR.

[13] Let L and E denote the steady-state values of L_t and E_t, respectively. The coefficients in (22) are given by $\psi_{\mathrm{D}} = 1/\psi_l$ and $\psi_{\mathrm{A}} = (\psi_e - \tau)/\psi_l$, where

$$\psi_e = \left(\frac{\nu}{1+\nu} \chi \left(\frac{E}{L} \right)^{1+\frac{1}{\nu}} + \kappa (1-L)^{-\xi} \right)^{-1} \chi \left(\frac{E}{L} \right)^{1+\frac{1}{\nu}} - 1; \text{ and}$$

$$\psi_l = \left(\frac{\nu}{1+\nu} \chi \left(\frac{E}{L} \right)^{1+\frac{1}{\nu}} + \kappa (1-L)^{-\xi} \right)^{-1} \left(\kappa \xi (1-L)^{-1-\xi} L - \chi \left(\frac{E}{L} \right)^{1+\frac{1}{\nu}} \right) + 1.$$

on labor force participation. If the discouraged worker effect, represented by the first term, dominates the added worker effect, represented by the second term, then labor force participation declines in response to a monetary tightening. Likewise, if the added worker effect dominates the discouraged worker effect, then labor force participation increases in response to a monetary tightening.

Let us now consider how wage rigidity affects the relative importance of the discouraged worker and added worker effects in our model. In a sticky wage economy, a tightening monetary policy induces a smaller decline in the real wage $\widehat{w}_t$ than in a flexible wage economy, which implies that the discouraged worker effect becomes smaller. At the same time, in a sticky wage economy, a decline in the output $\widehat{y}_t$ in response to a tightening monetary policy becomes larger than in a flexible wage economy, which implies that the added worker effect becomes larger. Therefore, as the degree of wage rigidity increases, the added worker effect becomes more likely to dominate the discouraged worker effect, and thus labor force participation becomes more likely to increase.[14]

	Wage rigidity ϕ_w				
	0.1	1	5	10	20
Discouraged worker effect	−0.667	−0.615	−0.471	−0.370	−0.275
Added worker effect	0.468	0.479	0.515	0.545	0.582
LFPR response	−0.243	−0.182	−0.005	0.118	0.252

Table 2. LFPR Response on Impact to a Monetary Tightening: Numerical Examples

To see how the degree of wage rigidity affects the relative importance of the two effects, it is helpful to numerically evaluate the decomposition (22).[15] In this exercise, we use the sample averages from 1990 to 2019 in Japan to set some parameters. First, we set the annual trend inflation rate π_p to the -0.32 percent, which is the average GDP deflator inflation rate. Second, we calibrate χ and κ so that the steady-state value of the

[14]Similarly, Nucci and Riggi (2018) and Cairó et al. (2022) show that the degree of wage rigidity affects the sign of the response of the LFPR to a productivity shock. The decomposition between the two effects through the optimal labor force participation condition (22) also holds for productivity shocks, as well as price markup shocks, wage markup shocks, and discount factor shocks.

[15]We can also evaluate the role of wage rigidity in the model in terms of the NKWPC. The discussion regarding the log-linearized version of the NKWPC is provided in detail in Appendix B.

employment-population ratio E matches 59.16 percent, and that of the LFPR L matches 61.47 percent. The remaining parameter values used in this exercise correspond to the prior means that we later use in the Bayesian estimation in Section 4 (except for $h = 0$). Under this choice of the parameter values, we have $\psi_{\mathrm{D}} = 0.429$ and $\psi_{\mathrm{A}} = -0.900$.

We conduct this exercise for different degrees of wage rigidity by setting ϕ_w at 0.1, 1, 5, 10, and 20. In Table 2, we report the impact responses of the discouraged worker effect, the added worker effect, and the LFPR for each degree of wage rigidity. Since the coefficients ψ_{D} and ψ_{A} do not depend on ϕ_w, the relative importance of the two effects is determined by how ϕ_w changes the size of the decline in the real wage and the output. When the wage is relatively flexible at $\phi_w = 0.1$ or 1, the discouraged worker effect is stronger than the added worker effect, and thus labor force participation declines. It should also be noted that the negative response of labor force participation in the flexible wage case is consistent with the result obtained by Erceg and Levin (2014). When the degree of wage stickiness increases to $\phi_w = 5$, a monetary tightening has a near zero effect on labor force participation because the two effects almost cancel out each other. When the wage is relatively sticky at $\phi_w = 10$ or 20, the discouraged worker effect is dominated by the added worker effect, and thus labor force participation increases.

4 Bayesian Estimation

In this section, we estimate our New Keynesian model for Japan and the US using a Bayesian approach. Along with the posterior estimates, we present estimated impulse responses and variance decomposition of the model. We take a further look at impulse responses by changing wage rigidity for the two countries and investigate if the degree of wage rigidity affects the direction of the response of labor force participation to monetary policy, the implication obtained from the previous section.

4.1 Data and Prior Distribution

We use seven quarterly data series in the Bayesian estimation: log per-capita real GDP growth, log per-capita real consumption growth, GDP deflator inflation, wage inflation, the shadow interest rate, the employment-population ratio, and the unemployment rate. As proxies to the policy rates, we use the shadow interest rates estimated in Krippner (2015) to take into account the unconventional monetary policies without explicitly incorporating the nonlinear ZLB constraint. All the per-capita variables are divided by the working age population (15 years old and over) to match the definition of the LFPR. While the LFPR data is not included as an observable variable in estimation, L_t can be recovered from the employment-population ratio E_t and the unemployment rate u_t through the relationship $u_t = 1 - E_t/L_t$. A detailed description of the data is presented in Table A2 in Appendix A.

The sample periods for the Bayesian estimation are set similarly to those used in VAR estimation. Specifically, the sample period is from 1990Q1 to 2019Q3 for Japan, and from 1979Q3 to 2019Q4 for the US, respectively. The end of the sample period for Japan slightly differs from VAR estimation due to availability of the shadow rate series. We log-linearize the model to obtain the linear state space representation, and evaluate the likelihood function using the Kalman filter. The estimation procedure follows that of Smets and Wouters (2007) and so forth.

Several parameters are set as follows. The discount factor β is set to 0.99, and the steady-state level of the government spending fraction ζ is set to 15 percent, following Iwasaki et al. (2021). The annual trend inflation rate π_p is set to the sample averages: -0.32 percent for Japan and 2.71 percent for the US. The parameters χ and κ are calculated so that the steady-state values of the employment-population ratio E and the LFPR L match the averages in the data.[16]

Table 3 describes the prior distribution for the parameters to be estimated. We use the common prior distributions for the two countries. As a whole, we closely follow the prior distributions used in Smets and Wouters (2007), Galí et al. (2012), and Iwasaki et al.

[16]The sample average of the employment-population ratio is 59.16 percent in Japan and 61.20 percent in the US. The sample average of the LFPR is 61.47 percent in Japan and 65.23 percent in the US.

(2021). The prior means of the inverse of the intertemporal elasticity of substitution parameter τ and the Frisch labor supply elasticity ν are set at 2.0 and 0.5, respectively, as in Galí et al. (2012). Regarding price and wage adjustment cost parameters ϕ_p and ϕ_w, we follow Iwasaki et al. (2021) to set prior means of 30 and 20, and standard deviations of 15 and 20, respectively. The prior means for the steady-state values of the inverse demand elasticity for intermediate goods and labor services, λ_p and λ_w are set at 0.1. The prior for the annual steady-state growth rate γ is centered around 2 percent.

The parameters regarding the monetary policy rule are also centered around standard values. The prior mean is set to 0.5 for the interest rate smoothing parameter ρ_r, 1.5 for the sensitivity parameter of inflation ψ_π, and 0.2 for the sensitivity parameter of the output growth ψ_y, respectively. The prior mean of the curvature parameter of the home production term in utility, ξ, is set to 2, following the prior mean for the inverse of the intertemporal substitution elasticity.[17] Following Smets and Wouters (2007) and Galí et al. (2012), consumption habit parameter h is set at 0.7 with a standard deviation of 0.1, and the prior means for the price and wage indexation parameters ι_p and ι_w are set at 0.5 with a standard deviation of 0.25. As priors for the autoregressive parameters of the exogenous processes, we use a Beta distribution with a mean of 0.5. For the standard deviations of the exogenous shocks, we use an Inverse Gamma distribution with degrees of freedom $(0.5, 2.0)$, implying the prior mean of approximately 0.88 percent.

4.2 Impulse Responses and Variance Decompositions Implied by the Estimated Model

In Table 3, we report the means and the 90 percent credible intervals of the posterior distributions along with the prior distribution. The posterior result for Japan is summarized in the fifth to sixth columns, and that for the US is summarized in the seventh to eighth columns. The posterior distribution is obtained using the Markov chain Monte Carlo method.

[17] Erceg and Levin (2014) set ξ at 2.82 in their calibration. This value is within one standard deviation from the mean in our prior distribution.

	Prior			Posterior (Japan)		Posterior (US)	
	Type	Para1	Para2	Mean	90% CI	Mean	90% CI
Structural Parameters							
τ	G	2.0	1.0	0.60	[0.46, 0.75]	0.15	[0.09, 0.21]
ν	G	0.5	0.25	0.85	[0.37, 1.29]	0.75	[0.49, 1.00]
ϕ_p	G	30	15	63.30	[33.07, 91.89]	49.05	[25.68, 71.66]
ϕ_w	G	20	10	22.87	[9.73, 36.08]	118.12	[84.94, 149.81]
λ_p	B	0.1	0.05	0.23	[0.13, 0.32]	0.18	[0.10, 0.25]
λ_w	B	0.1	0.05	0.06	[0.02, 0.11]	0.17	[0.12, 0.21]
γ	G	2.0	1.0	0.88	[0.31, 1.43]	0.91	[0.53, 1.28]
ρ_r	B	0.5	0.1	0.86	[0.81, 0.91]	0.81	[0.79, 0.84]
ψ_π	G	1.5	0.2	2.54	[2.19, 2.88]	2.57	[2.27, 2.87]
ψ_y	G	0.2	0.1	1.17	[0.64, 1.70]	0.42	[0.20, 0.63]
ξ	G	2.0	1.0	1.83	[1.38, 2.25]	3.82	[3.09, 4.53]
h	B	0.7	0.1	0.24	[0.12, 0.35]	0.85	[0.80, 0.91]
ι_p	B	0.5	0.25	0.05	[0.00, 0.09]	0.06	[0.00, 0.12]
ι_w	B	0.5	0.25	0.19	[0.00, 0.38]	0.24	[0.03, 0.43]
AR(1) Parameters of Exogenous Variables							
ρ_g	B	0.5	0.1	0.95	[0.93, 0.97]	0.96	[0.95, 0.98]
ρ_p	B	0.5	0.1	0.78	[0.70, 0.86]	0.98	[0.97, 0.99]
ρ_w	B	0.5	0.1	0.87	[0.81, 0.93]	0.97	[0.96, 0.98]
ρ_d	B	0.5	0.1	0.97	[0.96, 0.98]	0.91	[0.88, 0.94]
ρ_χ	B	0.5	0.1	0.71	[0.64, 0.79]	0.98	[0.97, 0.99]
ρ_a	B	0.5	0.1	0.18	[0.12, 0.24]	0.18	[0.13, 0.24]
Standard Deviation of Shocks							
$100\sigma_r$	IG	0.5	2.0	0.31	[0.21, 0.42]	0.29	[0.26, 0.32]
$100\sigma_g$	IG	0.5	2.0	0.79	[0.70, 0.88]	0.57	[0.52, 0.63]
$100\sigma_p$	IG	0.5	2.0	0.19	[0.16, 0.23]	0.13	[0.10, 0.15]
$100\sigma_w$	IG	0.5	2.0	1.28	[0.97, 1.58]	0.19	[0.16, 0.23]
$100\sigma_d$	IG	0.5	2.0	3.44	[2.38, 4.51]	2.54	[1.72, 3.33]
$100\sigma_\chi$	IG	0.5	2.0	3.36	[2.12, 4.74]	4.30	[3.18, 5.40]
$100\sigma_a$	IG	0.5	2.0	0.80	[0.71, 0.88]	0.59	[0.53, 0.65]

Table 3. Prior and Posterior Distribution: Japan and US

Notes: Para1 and Para2 denote the means and the standard deviations for Beta (B) and Gamma (G) distributions; and s and v for the Inverse Gamma (IG) distribution $P_{IG}(\sigma|v,s) \propto \sigma^{-v-1}e^{-vs^2/2\sigma^2}$, respectively. CI stands for the credible interval.

Our main interest is in the posterior estimates of the wage rigidity parameter ϕ_w. The posterior mean for ϕ_w is 22.87 in Japan, and 118.12 in the US. Our estimation result of higher wage rigidity in the US than in Japan is consistent with the evidence at the micro level.[18] For example, the estimates based on administrative payroll data from 2008 to 2016 by Grigsby et al. (2021) imply that approximately 35 percent of workers receive no base wage change in a given year. As for Japan, using the panel data from 1993 to 1998, Kuroda and Yamamoto (2003) report that approximately 20 percent of workers have a nominal base wage change rate around zero. There are several possible reasons why wage rigidity is expected to be lower in Japan than in the US. Taylor (1989) states that the low wage rigidity in Japan reflects the synchronized wage determination process in the annual spring wage negotiations called *Shunto*.[19] Freeman and Weitzman (1987) stress the role of adjustment through the bonus payment in making wages relatively flexible in Japan. Furthermore, the low mobility of the labor market due to the strict employment protection in Japan may also contribute to wage flexibility: firms respond to business cycles by wage adjustment rather than quantity adjustment. On the other hand, Barattieri et al. (2014) show that wage changes realize only in the timings of the job-to-job transitions, which is one of the reasons for high wage rigidity in the US.

We also report the estimated impulse responses implied by our New Keynesian model under the posterior distribution. In Figure 6, we show the estimated impulse responses to a tightening monetary policy shock for Japan. The shaded areas indicate 90 percent credible intervals. The policy shock is normalized to induce a 1 percentage point increase in the 1-year rate on impact. Consistent with our VAR finding in Section 2, labor force participation declines in response to a monetary tightening in Japan. The LFPR instantly drops by approximately 2.5 percent within a half year after the shock, and gradually reverts to the original level within three years. Obviously, this result is also consistent with our view: the lower degree of wage rigidity in Japan strengthens the discouraged

[18]Similar evidence at the aggregate level is obtained by Muto and Shintani (2020) and Iwasaki et al. (2021), who compare parameter estimates of the New Keynesian wage Phillips curve between the two countries.

[19]In Japan, the annual wage negotiations between the enterprise unions and the employers take place simultaneously in many firms from the beginning of March.

Figure 6. Impulse Responses to a Tightening Monetary Policy Shock: Japan

Note: The shaded areas indicate the 90 percent credible bands. The unit of the horizontal axes is a quarter.

worker effect and weakens the added worker effect, which results in declining labor force participation.

Figure 7. Impulse Responses to a Tightening Monetary Policy Shock: US

Note: The shaded areas indicate the 90 percent credible bands. The unit of the horizontal axes is a quarter.

The estimated impulse response functions for the US are reported in Figure 7. In contrast with the case of Japan, the LFPR in the US increases on impact in response to a monetary tightening, which is consistent with our VAR estimation result. The LFPR rises by 0.6 percent on impact after the shock, and then declines to the original level after three years. In line with our interpretation, the higher degree of wage rigidity in the US sets up the added worker effect to dominate the discouraged worker effect, leading to an increase in labor force participation.[20]

	Japan		US	
	Model	Data	Model	Data
Impact	0.00	1.14	21.66	2.15
One year	8.81	7.04	5.84	3.96
Two years	7.47	13.84	2.98	2.19
Three years	7.09	15.43	1.98	1.70
Four years	6.88	15.16	1.52	1.55

Table 4. Contribution of Monetary Policy Shocks to the LFPR: Model Prediction

Note: CI stands for the confidence interval.

To make a comparison with VAR evidence in terms of the contribution of monetary policy, the results of the forecast error variance decompositions at the posterior means for the two countries are reported in the second and fourth columns in Table 4. The variance decomposition results from the VAR models in Table 1 are also relisted in the third and fifth columns. The predictions from the estimated models in the two countries are consistent with what we find in the VAR analysis. For Japan, the estimated model implies that the contribution of monetary policy shocks on the LFPR variations is close to zero on impact, while it becomes larger over time and reaches 6.88 percent after four years. For the US, on the other hand, the contribution of monetary policy is large on impact, and it becomes smaller over time and close to zero after four years.

[20]While our posterior estimates imply the countercyclicality of the LFPR conditional on monetary shocks in the US, the predicted correlation implies that the LFPR is procyclical in the US, which is consistent with previous results reported by Erceg and Levin (2014) and Nucci and Riggi (2018), among others.

4.3 Evaluating the Role of Wage Rigidity

To investigate the role of wage rigidity in determining the sign of the response of labor force participation, we further calculate impulse responses under alternative parameter values for wage rigidity. To this end, we consider a hypothetical case where wage rigidity is sufficiently high in Japan, and make a comparison to the estimated impulse responses. Specifically, the posterior mean of the wage rigidity parameter ϕ_w for Japan (22.87) is replaced with that for the US (118.12), while all the rest of the parameters are unchanged from the posterior means for Japan.

Figure 8. Evaluating the Role of Wage Rigidity: Japan

Note: The unit of the horizontal axes is a quarter.

In Figure 8, we plot the impulse response functions to a tightening monetary policy shock if wage rigidity is hypothetically high in Japan, as well as the those under the posterior means. The solid lines represent the impulse responses under the posterior means for Japan, and the dashed lines represent the impulse responses in the hypothetical case. In the hypothetical economy, in contrast with the actual case, labor force participation initially increases in response to a monetary tightening. At the same time, we also observe that, in the hypothetical case, a decline of the real wage becomes smaller and a decline of the output becomes larger. This observation is in line with our view on how wage rigidity affects the LFPR response: Under higher wage rigidity, the discouraged worker effect becomes weaker due to a smaller decline in the real wage, and the added worker effect becomes larger due to a larger decline in the output. In the hypothetical economy, LFPR responds positively because the latter effect dominates the former.

Importantly, this simulation result implies that the labor force participation response to monetary policy depends on wage rigidity to a nontrivial extent. The degree of wage rigidity affects the sizes of the declines in the real wage and the output, which changes the relative importance of the discouraged worker and added worker effects. In Japan, the low degree of wage rigidity contributes to the stronger discouraged worker effect and the weaker added worker effect, leading to the negative response of the LFPR. Because the wage rigidity parameter solely changes the sign of the LFPR response, the role of wage rigidity is quantitatively important.

5 Counterfactual Analysis

In this section, we quantify the effects of the recent large-scale monetary easing policies in Japan and the US on labor force participation by counterfactual exercises. In our counterfactual scenario, we assume that the central banks did not conduct monetary easing by setting all expansionary monetary policy shocks during the periods of the recent quantitative easing to zero. While our model does not explicitly incorporate the ZLB, negative monetary policy shocks on the shadow interest rate when the ZLB binds can be interpreted as the quantitative easing shocks.

For Japan, we consider a counterfactual scenario where the Bank of Japan gave no expansionary monetary policy shocks after 2013Q2, when the Bank of Japan commenced the QQE involving large-scale bond and other asset purchases. In each panel in Figure 9, we plot the realized paths and the counterfactual paths by the solid lines and the dashed lines, respectively. The units of the vertical axes are a percentage. In the top left panel, we present the realized and counterfactual paths of monetary policy shocks used in the simulation. The realized path of the shock is estimated with Kalman smoothing. As the

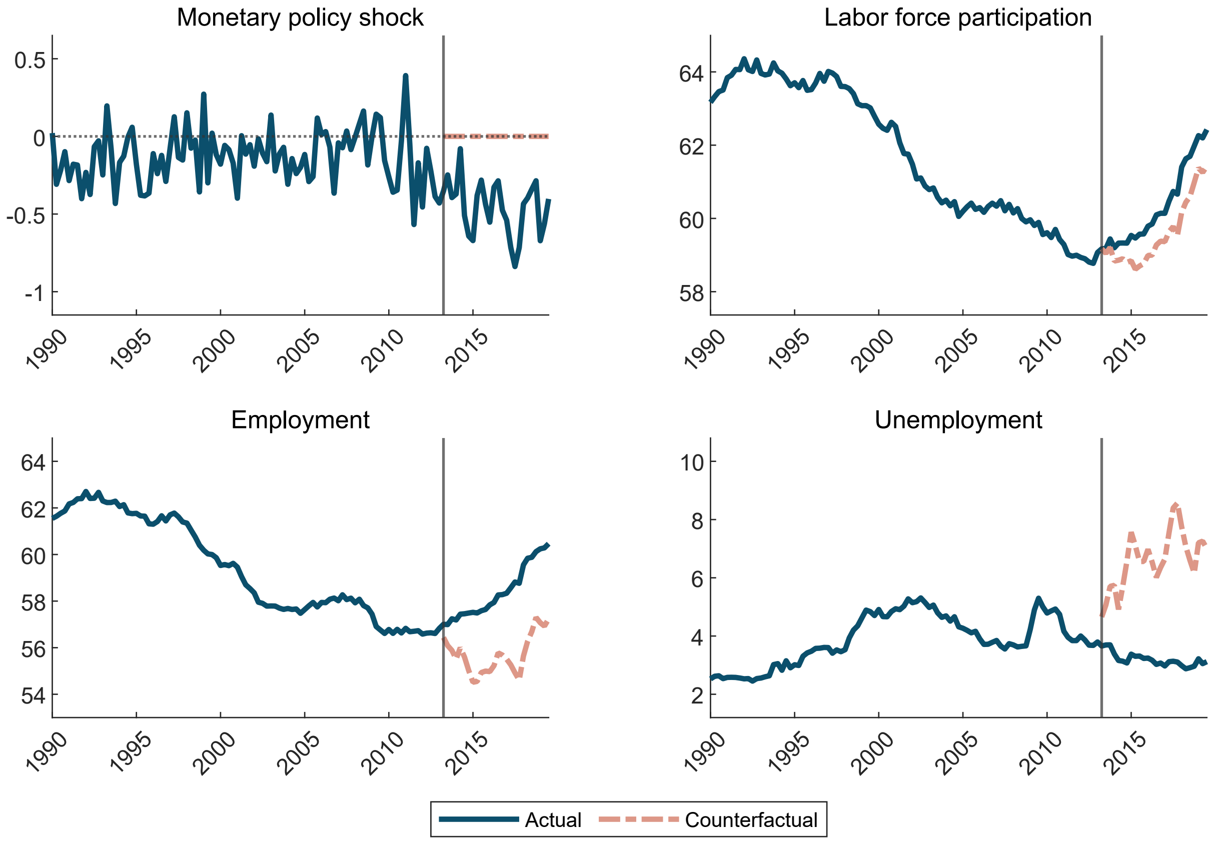

Figure 9. Counterfactual Simulation: Japan

Note: The unit of the vertical axis is a percentage.

panel shows, all the monetary policy shocks after 2013Q2 until the end of the sample are negative.

The remaining three panels in Figure 9 report the actual and counterfactual evolution of the LFPR, the employment-population ratio, and the unemployment rate in Japan. Two observations stand out in the evolution of the LFPR. First, in the counterfactual scenario, as the monetary policy stance is more tightening than the actual, the path of the LFPR shifts downward, as predicted in the estimated impulse response in Figure 6. Second, the growing trend of the LFPR after 2013 is still observed even under the assumption of no monetary easing. This observation implies that labor force participation started to increase not solely due to the QQE but also to other factors. For example, measures taken by the government and firms to support childcare contributed to promoting female workers to participate in the labor force.

On the other hand, employment continues to decrease in the counterfactual scenario. This result implies that the recovery in employment after 2013 is largely attributed to the large-scale monetary expansion. While both paths of labor force participation and employment shift downward in the counterfactual scenario, the change of the path in employment is larger than that in labor force participation. For this reason, the unemployment rate would have increased rather than decreased in the 2010s.

We apply the same simulation for the US, assuming a counterfactual scenario where the Fed gave no easing monetary policy shocks after 2008Q4, when the Fed introduced its first quantitative easing ("QE1") to deal with the global financial crisis. The counterfactual analysis for the US is reported in Figure 10. In contrast to Japan's case, the model implies that monetary policy affects the evolution of the LFPR only to a small extent, while we can observe that the quantitative easing induces a slight decrease in the LFPR in the short run. Under the posterior estimates for the US, as observed in Figure 7, the LFPR initially responds in a positive direction and then falls below the original level in response to a monetary tightening. We conjecture that, as the positive effect in the short run and the negative effect in the long run offset each other, the historical path of the LFPR has been barely affected by monetary policy. These results are also consistent

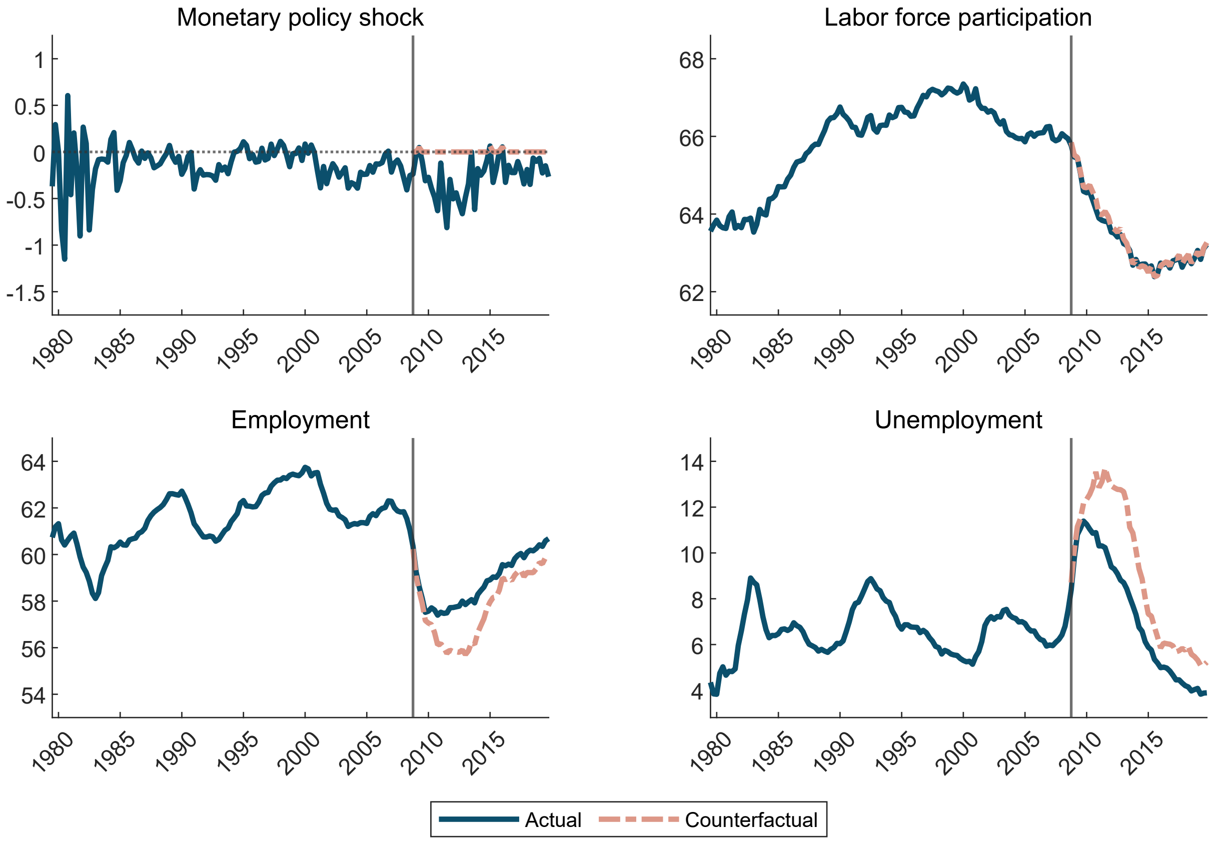

Figure 10. Counterfactual Simulation: US

Note: The unit of the vertical axis is a percentage.

with Table 4, which implies that the contribution of monetary policy to variations in the LFPR becomes trivial in the long run.

6 Conclusion

How does the LFPR respond to monetary policy? Our paper begins with tackling this question by presenting empirical evidence from VAR models. By using monetary policy surprises as the external instruments in identifying shocks, our VAR estimation results imply the following two facts. First, in response to a monetary tightening, labor force participation persistently declines in Japan. Second, in the US, labor force participation temporarily increases. These findings imply a stronger discouraged worker effect or a weaker added worker effect in Japan than in the US.

To provide a structural interpretation for the differences between the two countries, we develop and estimate a New Keynesian model of endogenous labor force participation decisions. Our model extends the model of Erceg and Levin (2014) in allowing for wage rigidity. We find that wage rigidity is one of the determinants of the sign of the response of labor force participation to a tightening monetary policy shock. For a higher degree of wage rigidity, a decline in the real wage becomes smaller, which implies the weaker discouraged worker effect. At the same time, a decline in the output become larger, which leads to the larger added worker effect. Thus, the wage rigidity affects the sign of the LFPR response by changing the relative importance of the two effects. The estimated New Keynesian model implies higher wage rigidity in the US than in Japan, and successfully reproduces the differences with regard to the direction of the response of labor force participation. Furthermore, counterfactual analysis based on the estimated models shows that the large-scale monetary easing in recent years helped in boosting the labor force participation rate in Japan, while its effect was almost neutral in the US.

There are several directions in which this study could be extended. First, it is possible to introduce search and matching frictions in a model for the further investigation of the discouraged worker effect. Second, since labor force participation decisions are likely to differ across their income level and between male and female workers, it is also insightful to extend the model to incorporate various types of heterogeneity. These extensions remain for future work.

References

Barattieri, A., Basu, S., and Gottschalk, P. (2014). Some Evidence on the Importance of Sticky Wages. *American Economic Journal: Macroeconomics*, 6(1):70–101.

Blundell, R., Pistaferri, L., and Saporta-Eksten, I. (2016). Consumption Inequality and Family Labor Supply. *American Economic Review*, 106(2):387–435.

Bowen, W. G. and Finegan, T. A. (1969). *The Economics of Labor Force Participation*, volume 2479. Princeton University Press.

Cain, G. G. (1967). Unemployment and the Labor-Force Participation of Secondary Workers. *Industrial and Labor Relations Review*, 20(2):275–297.

Cairó, I., Fujita, S., and Morales-Jiménez, C. (2022). The Cyclicality of Labor Force Participation Flows: The Role of Labor Supply Elasticities and Wage Rigidity. *Review of Economic Dynamics*, 43:197–216.

Campolmi, A. and Gnocchi, S. (2016). Labor Market Participation, Unemployment and Monetary Policy. *Journal of Monetary Economics*, 79:17–29.

Christiano, L. J., Eichenbaum, M. S., and Trabandt, M. (2015). Understanding the Great Recession. *American Economic Journal: Macroeconomics*, 7(1):110–167.

Elsby, M. W., Hobijn, B., and Şahin, A. (2015). On the Importance of the Participation Margin for Labor Market Fluctuations. *Journal of Monetary Economics*, 72:64–82.

Erceg, C. J., Henderson, D. W., and Levin, A. T. (2000). Optimal Monetary Policy with Staggered Wage and Price Contracts. *Journal of Monetary Economics*, 46(2):281–313.

Erceg, C. J. and Levin, A. T. (2014). Labor Force Participation and Monetary Policy in the Wake of the Great Recession. *Journal of Money, Credit and Banking*, 46(S2):3–49.

Favara, G., Gilchrist, S., Lewis, K. F., and Zakrajšek, E. (2016). Recession Risk and the Excess Bond Premium. *FEDS Notes*, pages 168–194.

Finegan, T. A., Penaloza, R. V., and Shintani, M. (2008). Reassessing Cyclical Changes in Workers' Labor Market Status: Gross Flows and the Types of Workers Who Determine Them. *ILR Review*, 61(2):244–257.

Freeman, R. B. and Weitzman, M. L. (1987). Bonuses and Employment in Japan. *Journal of the Japanese and International Economies*, 1(2):168–194.

Galí, J. (2011). The Return of the Wage Phillips Curve. *Journal of the European Economic Association*, 9(3):436–461.

Galí, J., Smets, F., and Wouters, R. (2012). Unemployment in an Estimated New Keynesian Model. *NBER Macroeconomics Annual*, 26(1):329–360.

Gertler, M. and Karadi, P. (2015). Monetary Policy Surprises, Credit Costs, and Economic Activity. *American Economic Journal: Macroeconomics*, 7(1):44–76.

Gilchrist, S. and Zakrajšek, E. (2012). Credit Spreads and Business Cycle Fluctuations. *American Economic Review*, 102(4):1692–1720.

Grigsby, J., Hurst, E., and Yildirmaz, A. (2021). Aggregate Nominal Wage Adjustments: New Evidence from Administrative Payroll Data. *American Economic Review*, 111(2):428–471.

Hobijn, B. and Sahin, A. (2021). Maximum Employment and the Participation Cycle. *NBER Working Paper*.

Honda, Y. and Kuroki, Y. (2006). Financial and Capital Markets' Responses to Changes in the Central Bank's Target Interest Rate: The Case of Japan. *The Economic Journal*, 116(513):812–842.

Iwasaki, Y., Muto, I., and Shintani, M. (2021). Missing Wage Inflation? Estimating the Natural Rate of Unemployment in a Nonlinear DSGE Model. *European Economic Review*, 132:103626.

King, R. G. and Rebelo, S. T. (1999). Resuscitating Real Business Cycles. In Taylor, J. B. and Woodford, M., editors, *Handbook of Macroeconomics*, volume 1, Part B, chapter 14, pages 927–1007. Elsevier, 1 edition.

Krippner, L. (2015). *Zero Lower Bound Term Structure Modeling: A Practitioner's Guide*. Springer.

Kubota, H. and Shintani, M. (2022). High-Frequency Identification of Monetary Policy Shocks in Japan. *The Japanese Economic Review*, 73(3):483–513.

Kuroda, S. and Yamamoto, I. (2003). Are Japanese Nominal Wages Downwardly Rigid?(Part I): Examinations of Nominal Wage Change Distributions. *Monetary and Economic Studies*, 21(2):1–29.

Lundberg, S. (1985). The Added Worker Effect. *Journal of Labor Economics*, 3(1, Part 1):11–37.

Merz, M. (1995). Search in the Labor Market and the Real Business Cycle. *Journal of Monetary Economics*, 36(2):269–300.

Mincer, J. (1966). Labor Force Participation and Unemployment: A Review of Recent Evidence. *Prosperity and Unemployment*, 73:112.

Montiel Olea, J. L., Stock, J. H., and Watson, M. W. (2021). Inference in Structural Vector Autoregressions Identified with an External Instrument. *Journal of Econometrics*, 225(1):74–87.

Muto, I. and Shintani, K. (2020). An Empirical Study on the New Keynesian Wage Phillips Curve: Japan and the US. *The BE Journal of Macroeconomics*, 20(1):20160168.

Nucci, F. and Riggi, M. (2018). Labor Force Participation, Wage Rigidities, and Inflation. *Journal of Macroeconomics*, 55:274–292.

Shimer, R. (2013). *Job Search, Labor-Force Participation, and Wage Rigidities*, volume 50. Cambridge University Press.

Smets, F. and Wouters, R. (2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review*, 97(3):586–606.

Sugo, T. and Ueda, K. (2008). Estimating a Dynamic Stochastic General Equilibrium Model for Japan. *Journal of the Japanese and International Economies*, 22(4):476–502.

Tachibanaki, T. and Sakurai, K. (1991). Labour Supply and Unemployment in Japan. *European Economic Review*, 35(8):1575–1587.

Taylor, J. B. (1989). Differences in Economic Fluctuations in Japan and the United States: The Role of Nominal Rigidities. *Journal of the Japanese and International Economies*, 3(2):127–144.

Tüzemen, D. (2017). Labor Market Dynamics with Endogenous Labor Force Participation and On-the-Job Search. *Journal of Economic Dynamics and Control*, 75:28–51.

Tüzemen, D. and Van Zandweghe, W. (2018). The Cyclical Behavior of Labor Force Participation. *Federal Reserve Bank of Kansas City Working Paper*, 18-08.

Van Zandweghe, W. (2017). The Changing Cyclicality of Labor Force Participation. *Federal Reserve Bank of Kansas City Economic Review*, Q3:5–34.

Yellen, J. L. (2014). Labor Market Dynamics and Monetary Policy. In *Proceedings of the Remarks at the Federal Reserve Bank of Kansas City Economic Symposium*, Jackson Hole, Wyoming.

Appendix A Data Description

Series	Japan	US
Industrial production	Ministry of Economy, Trade and Industry, Indices of industrial production	Board of Governors of the Federal Reserve System, Industrial production
Price level	Ministry of Internal Affairs and Communications, Consumer price index (tax adjusted)	BLS, Consumer price index for all urban consumers: all items less food and energy in U.S. city average
Interest rate	Ministry of Finance, One-year government bond yield	Board of Governors of the Federal Reserve System, Effective federal funds rate
Stock price	Nikkei 225 stock price index	S&P 500 index
Credit spread	Calculated by differencing five-year government bond yields (Bank of Japan) from Nikkei bond index (Nikkei Inc.)	Favara et al. (2016)
Labor force participation rate	Monthly Labor Survey, Labor force participation rate	BLS, Labor force participation rate
Gross labor flows	Labor Force Survey, Labor force flows	BLS, Labor force flows

Table A1. Data Description: VAR Estimation

Series	Japan	US
Real GDP	Cabinet Office, SNA, Real GDP	BEA, Gross domestic product, Real GDP
Consumption	Cabinet Office, SNA, Consumption of households	BEA, Personal income and outlays, Real personal consumption expenditure
Inflation	Cabinet Office, SNA, GDP deflator	BEA, Gross Domestic Product, Implicit price deflator
Wage inflation	Labor Force Survey, Total cash earning, divided by hours worked[21]	Average weekly earnings of production and nonsupervisory employees, total private
Interest rate	Bank of Japan, Overnight call rate	Board of Governors of the Federal Reserve System, Effective federal funds rate
Shadow rate	Krippner (2015)	Krippner (2015)
Employment-population ratio	Labor Force Survey, Employment-population ratio	OECD, Employment-population ratio: Aged 15 and over: All persons for the United States
Unemployment rate	Labor Force Survey, Unemployment rate	BLS, Civilian unemployment rate
Population	Labor Force Survey, Working age population	OECD, Working age population: aged 15 and over: all persons for the United States

Table A2. Data Description: Bayesian Estimation

[21]We adjust institutional factors that bias hours worked data following Sugo and Ueda (2008). Total hours worked series is retrieved from Monthly Labor Survey, Japan.

Appendix B New Keynesian Wage Phillips Curve

The log-linearized version of the NKWPC (9) is given by:

$$\widehat{\pi}_{w,t} - \iota_w \gamma \widehat{\pi}_{p,t-1} = \beta \left(\mathrm{E}_t \widehat{\pi}_{w,t+1} - \iota_w \gamma \widehat{\pi}_{p,t} \right) - \varphi \widehat{\mu}_{w,t} + \widehat{\lambda}_{w,t}, \tag{A1}$$

where $\varphi = \pi_w(1-\lambda_w)/(\lambda_w \phi_w)$, $\widehat{\pi}_{w,t}$ and $\widehat{\pi}_{p,t}$ denote the log deviations of $\pi_{w,t}$ and $\pi_{p,t}$, respectively, $\widehat{\lambda}_{w,t}$ denotes the rescaled log deviations of $\lambda_{w,t}$, and

$$\widehat{\mu}_{w,t} = \widehat{w}_t - \widehat{mu}_t - \frac{1}{\nu}\widehat{e}_t + \frac{1}{\nu}\widehat{l}_t - \widehat{\chi}_t \tag{A2}$$

is the log deviations of $\mu_{w,t}$ derived from (10). Because $\widehat{u}_t = \widehat{l}_t - \widehat{e}_t$ where $\widehat{u}_t$ denotes the deviation of the unemployment rate u_t from its steady state, we obtain a linear NKWPC given by

$$\widehat{\pi}_{w,t} - \iota_w \gamma \widehat{\pi}_{p,t-1} = \beta \left(\mathrm{E}_t \widehat{\pi}_{w,t+1} - \iota_w \gamma \widehat{\pi}_{p,t} \right) - \varphi \left(\widehat{w}_t - \widehat{mu}_t - \widehat{\chi}_t \right) - \frac{\varphi}{\nu}\widehat{u}_t + \widehat{\lambda}_{w,t}. \tag{A3}$$

Thus, the unemployment gap $\widehat{u}_t$ appears in our NKWPC.

Unlike Erceg and Levin (2014)'s formulation of labor force participation decisions, Galí (2011) introduces labor force participation in his model so that the labor force corresponds to the level of employment where the real wage is equal to the marginal rate of substitution under the sticky wage assumption. By imposing $L_t = 1$ in (3), Galí (2011)'s definition of labor force participation in the log deviation from the steady state is given by

$$\widetilde{l}_t = \nu(\widehat{w}_t - \widehat{mu}_t - \widehat{\chi}_t). \tag{A4}$$

Since the wage markup reduces to

$$\widehat{\mu}_{w,t} = \widehat{w}_t - \widehat{mu}_t - \frac{1}{\nu}\widehat{e}_t - \widehat{\chi}_t,$$

substituting $\widehat{\mu}_{w,t}$ into (A1) yields Galí (2011)'s NKWPC given by:

$$\widehat{\pi}_{w,t} - \iota_w \gamma \widehat{\pi}_{p,t-1} = \beta \left(\mathrm{E}_t \widehat{\pi}_{w,t+1} - \iota_w \gamma \widehat{\pi}_{p,t} \right) - \frac{\varphi}{\nu} \widetilde{u}_t + \widehat{\lambda}_{w,t}, \tag{A5}$$

where $\widetilde{u}_t = \widetilde{l}_t - \widehat{e}_t$ is Galí (2011)'s definition of the unemployment gap. For comparison, our NKWPC (A3) can also be written as:

$$\widehat{\pi}_{w,t} - \iota_w \gamma \widehat{\pi}_{p,t-1} = \beta \left(\mathrm{E}_t \widehat{\pi}_{w,t+1} - \iota_w \gamma \widehat{\pi}_{p,t} \right) - \frac{\varphi}{\nu} \widehat{l}_t - \frac{\varphi}{\nu} \widetilde{u}_t + \widehat{\lambda}_{w,t}.$$

Our formulation adds our definition of the labor force participation $\widehat{l}_t$ to Galí (2011)'s NKWPC (A5). This result is also related to the fact that the LFPR enters the NKPC under the formulation of Erceg and Levin (2014).