IMES DISCUSSION PAPER SERIES

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Discussion Paper No. 2021-E-12

IMES

INSTITUTE FOR MONETARY AND ECONOMIC STUDIES

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JAPAN

IMES Discussion Paper Series 2021-E-12
December 2021

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Abstract

This paper measures the transparency of the Federal Reserve Board (FRB) regarding its target inflation rate before its adoption of inflation targeting using data on the disagreement in inflation expectations among U.S. consumers. We construct a model of inflation forecasters employing the frameworks of both an unobserved components model and a noisy information model. We estimate the model and extract the transparency of the FRB regarding the target as the standard deviation of the heterogeneous noise in the inflation trend signal, where the trend proxies the FRB's inflation target. The results show a great improvement in transparency after the mid-1990s as well as its significant contribution to the decline in the disagreement in long-horizon inflation expectations.

Keywords: Central bank transparency; forecast disagreement; inflation dynamics; imperfect information

JEL classification: E50, E37, D83

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The author would like to thank S. Basu, C. Baum, R. Chahrour, and P. Ireland, together with seminar participants at the 2019 BOK/ERI-BOJ/IMES Joint Research Workshop and staff of the Bank of Japan, for their useful comments. The views expressed in this paper are those of the author and do not necessarily reflect the official views of the Bank of Japan.

1 Introduction

One consensus view in recent central banking is that central banks around the world have become more transparent over the past few decades. In fact, the FRB has allowed the disclosure of information related to its monetary policy-making, such as moves in the target federal funds rate, past transcripts of policy meetings, longer economic forecasts, and inflation targets since the 1990s. A number of central banks in other economies have also increased the disclosure of their internal information related to monetary policy, such as economic projections and policy goals. How these initiatives have influenced the transparency of central banks and hence affected the expectation formation of private agents are interesting questions.

Nonetheless, studies that measure the degree of transparency are scarce. An independent central bank needs to be accountable and accountability requires the bank to be transparent. Therefore, the degree of transparency of a central bank is itself an important topic. Several studies have tried to measure the degree of transparency and they evaluate central bank transparency using discrete values, such as whether a bank has a numerical inflation target or not. Assessing a central bank's actions or communication mechanically in this manner is one natural way to measure transparency. However, even among central banks that have an explicit numerical target there are differences in the institutional setup or the wording of the target. Moreover, even for central banks that do not declare an explicit target there exist numerous ways to deliver information about the target, such as economic projections, speeches, transcripts, and monetary policymaking itself.[1] In this study we focus on transparency regarding the inflation target,[2] but we measure central bank transparency from the information that economic agents finally have rather than following each action of the central bank.

We focus on the relationship between the transparency of a central bank regarding the target and the disagreement in inflation expectations, which is defined as the standard deviation of inflation expectations across forecasters. Figure 1 shows the disagreement in inflation expectations from the Michigan Survey of Consumers. It shows that, while the disagreement in longer-run (LR) inflation expectations 5 to 10 years ahead was larger than that in shorter-run (SR) inflation expectations 1 year ahead in the 1990s, the reverse was true in the 2000s. Our conjecture is that this change in the relationship, where the disagreement in LR expectations has become smaller than the disagreement in SR expectations, can be attributed to the change in the FRB's transparency regarding the target. In fact, existing studies show empirically such a relationship between transparency

[1] While the Fed did not announce a numerical target for the inflation rate in the past, we find that they began to discuss a specific number in the transcripts of FOMC meetings in the mid-1990s.

[2] Geraats (2002) notes that there are several aspects of central bank transparency, such as transparency regarding economic information or that regarding operational procedure. Our focus, i.e., transparency regarding the inflation target, corresponds to "political transparency" in her terminology.

and disagreement.

To measure the degree of transparency, this study employs a model-based approach using data on disagreement in inflation expectations. Since the amount of disagreement would have been affected not only by the FRB's transparency but also by other factors related to inflation dynamics, including the level of inflation, we construct a model of inflation forecasters that captures these factors by combining two methodologies in the literature. One is a traditional method that decomposes inflation into a temporary gap component and a permanent component, where we assume the trend component proxies the FRB's inflation target. The second method uses dispersed information, where each forecaster knows the model structure and the parameters but does not know current economic variables correctly. Instead, each forecaster observes a heterogeneous noisy signal of current inflation and the inflation trend. We interpret the standard deviation of the distribution of the noise (hereafter, the size of the noise) in the inflation trend signal as the degree of the FRB's transparency regarding its inflation target. In addition, we generalize the model and consider the case where the actual size of the noise and the size of the noise as perceived by forecasters can be different. We estimate all the parameters employing both maximum likelihood and the method of moments, matching the theoretical moments from the model to the data. The data are the actual inflation rate in the U.S. and the inflation expectations of consumers from the Michigan Survey of Consumers. We split the sample into two parts, before and after 1993 (up until 2008), and compare the change in the parameters. In doing so, we quantify the change in the degree of the FRB's transparency regarding the target. Moreover, we decompose the change in the disagreement into the differences in the underlying parameters, including transparency.

Our findings are three-fold. First, we find a great improvement in our transparency measure after the mid-1990s. The actual size of the noise in the inflation trend, our index of central bank transparency, declines by more than half. This finding will be attributed to the increased disclosure by the FRB of information related to its inflation target. We also find a great decline in both the persistence of the inflation gap and the size of shocks to the inflation trend, where the latter will reflect the decline in the level of the inflation rate. Second, we find that the increase in the FRB's transparency played an important role in the change in the relationship between LR and SR disagreements. Our decomposition analysis shows that the decline in the LR disagreement relative to the SR disagreement can be attributed to several factors, including the decline in the size of trend shocks, but the increase in the FRB's transparency has contributed the most to the change in the relationship between the two disagreements. Third, we find a decline in the actual size of the noise in the inflation trend after the mid-1990s for almost all subgroups of consumers. However, there is heterogeneity and the decline was especially pronounced for the young, the educated, and those with high incomes. This suggests that whether information on the improvement in the FRB's transparency regarding the target is finally conveyed to an economic agent depends on the agent's characteristics.

Literature Review

This paper combines three strands of literature: inflation dynamics, imperfect information, and central bank transparency. The first strand of literature studies the change in inflation dynamics after the great inflation period. As is done in this study, these studies decompose inflation into a temporary component and a trend component and discuss the change in the dynamics of these components. Two stylized facts are related to this study. One is that the volatility of shocks to the permanent component declined from the great inflation period to the great moderation period (Stock and Watson (2007)).[3] The other is that the persistence of the inflation gap, which is defined as the deviation of inflation from its permanent component, declined during the same sample period (Cogley et al. (2010)). The decline in the fluctuation of trend inflation is consistent with Figure 1. Supposing forecasters have information with heterogeneous noise, the smaller is the trend fluctuation, the smaller will be the LR disagreement. This result comes from the decline in the signal-to-noise ratio of the trend, which means that the same noisy information becomes less informative for forecasters. Also, the decline in inflation gap persistence will also affect disagreements, though the direction of the effect is not obvious. Our contribution to this literature is that we extend this basic framework to incorporate dispersed information and provide novel empirical facts about informational frictions after the great inflation period while retaining the stylized facts about inflation dynamics.

The second strand of related literature studies imperfect information. The basic idea is that there exist informational frictions and economic agents cannot have full information about economic variables. One method to model imperfect information, proposed by Woodford (2001), is to assume that each agent observes heterogeneous noisy signals, or that agents have heterogeneous beliefs, about the current economic states. One of the reasons, then, behind the changing relationship between disagreements in our figure can be the change in the noisiness of the signals. Suppose that there exist informational frictions not only regarding current inflation, as is usually assumed in the noisy information literature, but also regarding its trend. Then, it is straightforward to conjecture that the noisiness of the trend signals or the heterogeneity in beliefs about the trend diminished over the sample. There are a few studies which use a dispersed information model with trend inflation. Patton and Timmermann (2010) employ a univariate model of inflation and assume that people have different priors about the long-run end point of the economy to study the source of aggregate disagreement in the Survey of Professional Forecasters. Their idea of heterogeneity in the long-run end point of the economy is similar to our

[3]Other papers that have emphasized the important role of time-varying inflation trends to explain inflation and other variables are Kozicki and Tinsley (2001, 2005, 2012), Gürkaynak et al. (2005), Cogley and Sbordone (2008), and Coibion and Gorodnichenko (2011).

heterogeneous noisy signal of the trend, but we are in particular interested in the time variation of the end point. Andrade et al. (2016) use an unobserved components VAR model with dispersed information, together with that with sticky information, to investigate the term structure of disagreements in the Blue Chip survey. They contribute to the literature by incorporating time-varying trends in the imperfect information framework and we extend their model by incorporating heterogeneous signals of the trend, similar to the idea of Patton and Timmermann (2010), in particular focusing on the inflation rate. Our contribution to the literature is that we discuss how people form their LR inflation expectations by explicitly incorporating the heterogeneous noisy information about the time-varying trend component, which is often associated with the FRB's policy goal.[4]

The third strand of literature studies central bank transparency. One group of papers, including Eijffinger and Geraats (2006), Crowe and Meade (2008), and Dincer and Eichengreen (2010), measures the degree of transparency. They evaluate transparency by observing the communication or actions of central banks, but we take a different approach for the reasons mentioned above. A second group of papers about central bank transparency evaluates the effect of central bank communication on economic variables. They discuss how differences in communication strategies, either over time or across central banks, influence economic outcomes. Among them, Capistrán and Ramos-Francia (2010), Ehrmann et al. (2012), and Siklos (2013) use regression analysis to argue that the transparency of central banks significantly affects the disagreement in inflation expectations. However, their goal is different from ours since they take transparency indices obtained from the first group of papers for instance, as given while we extract the transparency index from inflation expectations.

This paper is organized as follows. Section 2 presents our model. Section 3 describes our estimation strategies. Section 4 shows our results and analysis. Finally, Section 5 concludes.

2 Model

In this paper we employ two types of similar models of inflation forecasters: a standard model and a generalized model. We first propose the standard model, and then present the generalized model to better capture the actual data.

[4]Mertens and Nason (2020) is also close to our paper. They employ an unobserved components model of inflation with sticky information and time-varying parameters and estimate the model focusing on the change in the informational stickiness parameter, which is a parameter of informational rigidity similar to the size of the noise parameter in our model. Our paper is different from theirs in that we employ data about disagreements and focus on informational rigidity related to the inflation trend.

2.1 Standard Model

Our model of inflation is a standard univariate unobserved components model, and we employ the method of noisy information to describe inflation forecasters. Inflation x_t is composed of an unobserved trend component x_t^*, which is associated with the inflation target, and a gap component γ_t. The trend follows a random walk while the gap between inflation and the trend component follows an AR(1) process:

$$x_t \equiv \gamma_t + x_t^*, \tag{1}$$
$$x_t^* = x_{t-1}^* + \varepsilon_t^*, \tag{2}$$
$$\gamma_t = \phi\gamma_{t-1} + \varepsilon_t, \tag{3}$$

where exogenous shocks ε^* and ε are i.i.d. $N(0, \sigma_{\varepsilon*}^2)$ and i.i.d. $N(0, \sigma_\varepsilon^2)$ respectively, and ϕ shows the persistence of the gap. Each forecaster i can observe neither inflation x_t nor the trend x_t^* directly, but instead observes two heterogeneous noisy signals, s_{it} and s_{it}^*, of them:

$$s_{it} = x_t + w_{it}, \tag{4}$$
$$s_{it}^* = x_t^* + w_{it}^*, \tag{5}$$

where the heterogeneous noise components w_{it} and w_{it}^* are i.i.d. $N(0, \sigma_w^2)$ and i.i.d. $N(0, \sigma_{w*}^2)$ respectively. Here, we interpret the size of the noise in the signal of the inflation trend σ_{w*} as the index of the FRB's transparency regarding its target inflation rate. The specification is compactly described by

$$\boldsymbol{\xi}_t = F\boldsymbol{\xi}_{t-1} + \mathbf{e}_t, \tag{6}$$
$$\mathbf{s}_{it} = \boldsymbol{\xi}_t + \mathbf{w}_{it}, \tag{7}$$

where $\mathbf{e}_t$ is distributed $N(0, Q)$ and $\mathbf{w}_{it}$ is distributed $N(0, R)$. Furthermore,

$$\boldsymbol{\xi}_t = \begin{bmatrix} x_t \\ x_t^* \end{bmatrix}, F = \begin{bmatrix} \phi & 1-\phi \\ 0 & 1 \end{bmatrix}, \mathbf{e}_t = \begin{bmatrix} \varepsilon_t + \varepsilon_t^* \\ \varepsilon_t^* \end{bmatrix}, Q = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sigma_\varepsilon^2 & 0 \\ 0 & \sigma_{\varepsilon*}^2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}',$$
$$\mathbf{s}_{it} = \begin{bmatrix} s_{it} \\ s_{it}^* \end{bmatrix}, \mathbf{w}_{it} = \begin{bmatrix} w_{it} \\ w_{it}^* \end{bmatrix}, R = \begin{bmatrix} \sigma_w^2 & 0 \\ 0 & \sigma_{w*}^2 \end{bmatrix}.$$

As is common in the noisy information literature, we suppose that each agent i uses Kalman filtering to infer the unobserved processes $\boldsymbol{\xi}_t$ from his noisy signals $\mathbf{s}_{it}$. The updating equation for agent i is

$$\boldsymbol{\xi}_{t|it} \equiv E_{it}[\boldsymbol{\xi}_t] = \boldsymbol{\xi}_{t|it-1} + G\left(\mathbf{s}_{it} - \boldsymbol{\xi}_{t|it-1}\right) = (I-G)\,\boldsymbol{\xi}_{t|it-1} + G\mathbf{s}_{it}. \tag{8}$$

where we denote the Kalman gain matrix as G. Agent i uses this equation to "nowcast" the current state of the economy. G is defined as $G \equiv P_1 (P_1 + R)^{-1}$, and P_1 is the steady state mean squared error (hereafter, MSE) matrix of the 1-step ahead forecast. We assume that the MSE is at the steady state as is often assumed in this literature. P_1 is obtained from the Riccati equation:

$$P_1 = F \left[P_1 - P_1 (P_1 + R)^{-1} P_1 \right] F' + Q. \tag{9}$$

Aggregating the nowcasting equation for each agent to obtain the mean nowcast across forecasters leads to the following equation:

$$\bar{\boldsymbol{\xi}}_{t|it} \equiv \bar{E} \left[\boldsymbol{\xi}_{t|it} \right] = \bar{E} \left[(I - G) \boldsymbol{\xi}_{t|it-1} + G \mathbf{s}_{it} \right] = (I - G) F \bar{\boldsymbol{\xi}}_{t-1|it-1} + G \boldsymbol{\xi}_t, \tag{10}$$

where $\bar{E}$ denotes the mean across forecasters. As we can derive the h-step ahead forecast of forecaster i as $\boldsymbol{\xi}_{t+h|it} = F^h \boldsymbol{\xi}_{t|it}$, the h-step ahead mean forecast is

$$\bar{\boldsymbol{\xi}}_{t+h|it} = \bar{E} \left[\boldsymbol{\xi}_{t+h|it} \right] = \bar{E} \left[F^h \boldsymbol{\xi}_{t|it} \right] = F^h \bar{\boldsymbol{\xi}}_{t|it}. \tag{11}$$

Therefore, the h-step ahead mean forecast of the inflation rate is given by

$$\bar{x}_{t+h|it} = S_x \bar{\boldsymbol{\xi}}_{t+h|it} = S_x F^h \bar{\boldsymbol{\xi}}_{t|it}, \tag{12}$$

where $S_x \equiv [1, 0]$ is a selector matrix used to obtain inflation x_t from vector $\boldsymbol{\xi}_t$.

Next, we derive the theoretical disagreements, which we define as the standard deviation of forecasts across forecasters, $\sqrt{\bar{V}}$. The variance of nowcasts for the vector $\boldsymbol{\xi}_t$ is

$$\begin{aligned} \bar{V} \left[\boldsymbol{\xi}_{t|it} \right] &\equiv \bar{E} \left[\left(\boldsymbol{\xi}_{t|it} - \bar{\boldsymbol{\xi}}_{t|it} \right) \left(\boldsymbol{\xi}_{t|it} - \bar{\boldsymbol{\xi}}_{t|it} \right)' \right] \\ &= \bar{E} \left[\left(\left\{ (I - G) \boldsymbol{\xi}_{t|it-1} + G \mathbf{s}_{it} \right\} - \left\{ (I - G) \bar{\boldsymbol{\xi}}_{t|it-1} + G \boldsymbol{\xi}_t \right\} \right) (...)' \right] \\ &= \bar{E} \left[\left((I - G) F \left(\boldsymbol{\xi}_{t-1|it-1} - \bar{\boldsymbol{\xi}}_{t-1|it-1} \right) + G \mathbf{w}_{it} \right) (...)' \right] \\ &= (I - G) F \bar{V} \left[\boldsymbol{\xi}_{t-1|it-1} \right] F' (I - G)' + G R G'. \end{aligned} \tag{13}$$

Even though it looks like an AR(1) process, GRG' is not a stochastic shock, but rather a constant term. Hence, the variance does not fluctuate with exogenous shocks, but simply converges to its steady state. Solving equation (13) backwards, we can derive the steady state variance of nowcasts for the vector as follows:

$$\begin{aligned} \bar{V} \left[\boldsymbol{\xi}_0 \right] &= \sum_{T=0}^{\infty} \left[(I - G) F \right]^T G R G' \left[F' (I - G)' \right]^T \\ &\quad + \lim_{T \to \infty} \left\{ \left[(I - G) F \right]^T \bar{V} \left[\boldsymbol{\xi}_{t-T|it-T} \right] \left[F' (I - G)' \right]^T \right\}. \end{aligned} \tag{14}$$

If all the eigenvalues $(I-G)F$ are inside the unit circle, the second term disappears and the variance converges to a constant. Then, we can express the variance of the h-step ahead forecasts as a function of the variance of the nowcasts:

$$\begin{aligned}\bar{V}\left[\boldsymbol{\xi}_{t+h|it}\right] &= \bar{E}\left[\left(F^h\left(\boldsymbol{\xi}_{t|it}-\bar{\boldsymbol{\xi}}_{t|it}\right)\right)(...)'\right] = F^h\bar{E}\left[\left(\boldsymbol{\xi}_{t|it}-\bar{\boldsymbol{\xi}}_{t|it}\right)(...)'\right]\left[F'\right]^h \\ &= F^h\bar{V}\left[\boldsymbol{\xi}_{t|it}\right]\left[F'\right]^h. \end{aligned} \tag{15}$$

The variance of the h-step ahead inflation expectations is

$$\bar{V}\left[x_{t+h|it}\right] = S_x\bar{V}\left[\boldsymbol{\xi}_{t+h|it}\right]S_x'. \tag{16}$$

and the h-step ahead variance of inflation expectations at the steady state is

$$\bar{V}\left[x_h\right] = S_xF^h\bar{V}\left[\boldsymbol{\xi}_0\right]\left[F^h\right]'S_x'. \tag{17}$$

Finally, the steady state "disagreement" about inflation expectations is given by

$$\sqrt{\bar{V}\left[x_h\right]} = \sqrt{S_xF^h\bar{V}\left[\boldsymbol{\xi}_0\right]\left[F^h\right]'S_x'}. \tag{18}$$

2.2 Generalized Model

This subsection proposes a generalized model of inflation forecasters. In the standard model we assumed that forecasters know the size of noise components, σ_w and σ_{w*}, precisely, while in this generalized model we assume that forecasters do not know the true size of the noise components. This means that the size of the noise that forecasters perceive and the true size of the noise can be different.[5] For instance, forecasters might believe that their signals are precise even though the actual signal is very noisy. We describe the size of the noise components which forecasters perceive as the "subjective" size of the noise components, σ_w^{sbj} and σ_{w*}^{sbj}, and the true size of the noise components as the "objective" size of the noise components, σ_w^{obj} and σ_{w*}^{obj}. In addition, we describe the corresponding matrices of the size of the noise components R as R^{sbj} and R^{obj} respectively. We assume that all the forecasters have the same bias regarding the subjective size of the noise components.

Next, we show that we can express the generalized model by slightly modifying the standard model in Section 2.1. The inflation dynamics are the same as those in the standard model. In addition, forecasters can observe neither inflation x_t nor the trend

[5] Geraats (2007) theoretically investigates the communication strategy of the central bank under a similar setup. Moreover, this type of informational friction is seen in other fields, such as psychology, microeconomics, and finance. This type of friction is called overprecision (or underprecision). It assumes that people are overconfident about the precision of their knowledge and make forecasts based on their incorrect information about that precision.

x_t^* directly, but instead observe two heterogeneous signals, s_{it} and s_{it}^*, as in the standard model. Moreover, each agent i uses Kalman filtering again to infer the unobserved processes $\boldsymbol{\xi}_t$ $(= [x_t; x_t^*])$ from his noisy signals $\mathbf{s}_{it}$ $(= [s_{it}; s_{it}^*])$. Since each forecaster uses the noisy signals in his forecast, taking into account his perceived noisiness, the updating equation for forecaster i changes to

$$\boldsymbol{\xi}_{t|it} = \left(I - G^{sbj}\right) \boldsymbol{\xi}_{t|it-1} + G^{sbj} \mathbf{s}_{it}, \tag{19}$$

where the Kalman gain is $G^{sbj} \equiv P_1 \left(P_1 + R^{sbj}\right)^{-1}$, and the variance-covariance matrix for the one-period-ahead forecast error P_1 is similarly obtained from the Riccati equation:

$$P_1 = F \left[P_1 - P_1 \left(P_1 + R^{sbj}\right)^{-1} P_1\right] F' + Q. \tag{20}$$

Aggregating the nowcasting equation for each agent to obtain the mean nowcast equation results in:

$$\bar{\boldsymbol{\xi}}_{t|it} \equiv \bar{E}\left[\boldsymbol{\xi}_{t|it}\right] = \bar{E}\left[\left(I - G^{sbj}\right) \boldsymbol{\xi}_{t|it-1} + G^{sbj} \mathbf{s}_{it}\right] = \left(I - G^{sbj}\right) F \bar{\boldsymbol{\xi}}_{t-1|it-1} + G^{sbj} \boldsymbol{\xi}_t. \tag{21}$$

The actual noise in signals $\mathbf{s}_{it}$ disappear in the mean equation while the subjective parameters for the size of noise components R^{sbj} remain in G^{sbj}. The h-step ahead mean forecast is

$$\bar{\boldsymbol{\xi}}_{t+h|it} = F^h \bar{\boldsymbol{\xi}}_{t|it}. \tag{22}$$

and the variance of nowcasts across forecasters for the vector $\boldsymbol{\xi}_t$ is

$$\begin{aligned}
\bar{V}\left[\boldsymbol{\xi}_{t|it}\right] &\equiv \bar{E}\left[\left(\boldsymbol{\xi}_{t|it} - \bar{\boldsymbol{\xi}}_{t|it}\right)\left(\boldsymbol{\xi}_{t|it} - \bar{\boldsymbol{\xi}}_{t|it}\right)'\right] \\
&= \bar{E}\left[\left(\left(I - G^{sbj}\right) F \left(\boldsymbol{\xi}_{t-1|it-1} - \bar{\boldsymbol{\xi}}_{t-1|it-1}\right) + G^{sbj} \mathbf{w}_{it}\right)(\ldots)'\right] \\
&= \left(I - G^{sbj}\right) F \bar{V}\left[\boldsymbol{\xi}_{t-1|it-1}\right] F' \left(I - G^{sbj}\right)' + G^{sbj} R^{obj} G^{sbj\prime}.
\end{aligned} \tag{23}$$

The objective size of noise components R^{obj} shows up in this equation. This is because the people who calculate the variance of the noise components are not the biased forecasters but rather us, econometricians who know the true size of the noise components. Finally, the steady state "disagreement" about inflation expectations is given by

$$\sqrt{\bar{V}\left[x_h\right]} = \sqrt{S_x F^h \bar{V}\left[\boldsymbol{\xi}_0\right] \left[F^h\right]' S_x'}, \tag{24}$$

where

$$\bar{V}\left[\boldsymbol{\xi}_0\right] = \sum_{T=0}^{\infty} \left[\left(I - G^{sbj}\right) F\right]^T G^{sbj} R^{obj} G^{sbj\prime} \left[F' \left(I - G^{sbj}\right)'\right]^T.$$

In Section 2 we noted that we interpret σ_{w*} as the transparency of the FRB regarding the target, but with this generalized model we have two sizes of noise regarding the trend:

the subjective size σ_{w*}^{sbj} and the objective size σ_{w*}^{obj}. σ_{w*}^{sbj} shows the perceived size of noise in the information regarding the trend while σ_{w*}^{obj} shows the actual size of the noise. Since the objective size of the noise indicates the true accuracy of the information about the FRB's target which private agents actually have, we interpret this index as the FRB's transparency regarding the target. We will discuss how to identify these two indices in Section 3.2. It is important to note that, according to our model, the change in each index affects disagreement differently and this point will be further discussed in Section 4.

3 Estimation

This section explains our estimation procedure. First, we discuss the data, focusing on the MSC. Next, we explain our estimation strategy. Finally, we present our two methods of estimation: maximum likelihood estimation (hereafter, MLE) and the method of moments (hereafter, MoM).

3.1 Data

We estimate the model parameters using quarterly CPI inflation data from the U.S. Bureau of Labor Statistics and four time series from the MSC.[6] The four time series are the mean and standard deviation of consumers' short-run (SR) and long-run (LR) inflation expectations. To be precise, the SR measures correspond to Question 32 of the survey, which asks "By about what percent do you expect prices to go (up/down), on the average, during the next 12 months?" The LR measures correspond to Question 33, which asks "By about what percent per year do you expect prices to go (up/down) on the average, during the next 5 to 10 years?"

We use the two forecast mean series, depicted in Figure 2 along with the CPI inflation rate, for the maximum likelihood estimation below. Both the SR and LR mean expectations were very high around 1980, but they declined quickly and have been relatively stable since that period. The notable difference with the disagreements is the drastic decline during the Volcker disinflation period. In contrast, the decline of disagreements in Figure 1 was sluggish and they remained relatively high even after the disinflation period.

There are advantages and disadvantages to using the MSC. One big advantage is that it has a very long sample for LR inflation forecasts,[7] available from 1979Q1. This is crucial

[6] The MSC is a monthly survey of consumers and the most famous series is the index of consumer sentiment. The MSC surveys more than 500 consumers via telephone interview.

[7] Almost all other surveys have shorter samples for LR inflation forecasts. The Survey of Professional Forecasters is a quarterly survey which started LR forecasts of the consumer price index in 1991Q4. The Livingston Survey is a semi-annual survey, asking about 30 individuals in a variety of institutions, which

for our analysis because this includes the last part of the great inflation period. Another advantage is that consumers' inflation expectations are a good proxy for firms' inflation expectations. Although the price setters in the economy are not consumers but firms, the survey of firms' inflation expectations in the U.S. is limited. Coibion and Gorodnichenko (2015) discuss this point and argue that inflation expectations of consumers proxy firms' expectations better than those of professional forecasters. On the other hand, one of the disadvantages of the MSC is that the survey does not specify the inflation index. Thus, some consumers may answer based on the CPI or PCE deflator while others may answer based on their own consumption baskets. Another disadvantage of the MSC is the measurement error. There is potential misreporting because consumers answer the survey on the phone. Since these disadvantages can generate additional heterogeneity, the estimated size of the noise could be biased upward.

We split the sample into the periods: 1978Q1-1992Q4 and 1993Q1-2008Q3, estimate the model for both samples, and then compare the parameters. We exclude the sample after the Lehman shock period since the inclusion of the period greatly changes the statistical properties of the inflation process. As the motivation of this research is the comparison between the relatively high disagreement period and the relatively stable moderation period after that, the effect of the financial crisis on inflation disagreements is not our focus.

3.2 Estimation Strategy

In our estimation, we first estimate the standard model and the first stage of the generalized model using MLE and then conduct the second stage estimation of the generalized model using the MoM. The background of this strategy is as follows. The observable variables are the five time series explained in Section 3.1. One might think of simply putting all these time series into observation equations, setting the dynamics equations in Section 2 as state equations, and estimating all the parameters using MLE. However, we will not use the disagreements data for the MLE. The disagreement is theoretically not affected by idiosyncratic shocks, as shown in equation 13, and it does not fluctuate but rather simply converges to the steady state from the initial value.[8] Therefore, even if we put the disagreements into a state space model, it follows a deterministic path to the steady state perfectly depending on the initial value, which will not yield stable es-

started the 10-year ahead CPI inflation forecasts in 1990. Consensus Economics is a monthly survey of market economists and it provides not only forecasts of the U.S. but also those of several major countries. This survey started in 1989. Finally, some other papers, such as Andrade et al. (2016) and Erceg and Levin (2003), employ the LR forecasts from Blue Chip Economic Indicators, which is also a monthly survey of professional forecasters. The length of the LR forecasts of this survey, starting in 1979Q4, is comparable to the MSC.

[8] Coibion and Gorodonichenko (2012) empirically shows that disagreement is not affected by exogenous shocks.

timation results. For this reason we use the sample mean of the disagreement data and match it to the theoretical steady state values using the MoM so that the initial value does not affect the result.

Next, we specifically lay out how we estimate each parameters. We employ only MLE in estimating the standard model because all the parameters are included in the equations of actual inflation (6) and mean inflation expectations (10). Meanwhile, in the generalized model, the equation of mean inflation expectations (21) includes the subjective size of the noise components $\{\sigma_w^{sbj}, \sigma_{w*}^{sbj}\}$ but does not include the objective size of the noise components $\{\sigma_w^{obj}, \sigma_{w*}^{obj}\}$. These are included only in the equation of disagreement (23). Therefore, we first estimate all parameters other than the objective size of noise components in the MLE using the data on mean inflation expectations and, given the estimated parameters, estimate the remaining parameters for the objective size of noise components using the MoM. It can be argued that this two stage estimation procedure allows us to identify the objective size of noise components separately from the subjective ones.

3.3 Maximum Likelihood Estimation

We estimate the parameters of inflation dynamics and mean forecast models using three time series: the quarterly inflation rate x_t^{obs}, the 1-year ahead mean forecast $\bar{x}_{t+1y|it}^{obs}$, and the 5-to-10-years ahead mean forecast $\bar{x}_{t+5y|it}^{obs}$. The theoretical 1-year ahead and 5-to-10 years ahead mean forecasts are

$$\bar{x}_{t+1y|it} = \frac{1}{4}\sum_{h=1}^{4}\bar{x}_{t+h|it} = S_x\frac{1}{4}\sum_{h=1}^{4}F^h\bar{\boldsymbol{\xi}}_{t|it} = S_xF_{1y}(F)\bar{\boldsymbol{\xi}}_{t|it}, \tag{25}$$

$$\bar{x}_{t+5y|it} = \frac{1}{6}\sum_{j=5}^{10}\frac{1}{4}\sum_{h=4*(j-1)+1}^{4*(j-1)+4}\bar{x}_{t+h|it} = S_x\frac{1}{24}\sum_{h=17}^{40}F^h\bar{\boldsymbol{\xi}}_{t|it} = S_xF_{5y}(F)\bar{\boldsymbol{\xi}}_{t|it}, \tag{26}$$

where

$$F_{1y}(F) \equiv \frac{1}{4}\sum_{h=1}^{4}F^h \text{ and } F_{5y}(F) \equiv \frac{1}{24}\sum_{h=17}^{40}F^h.$$

The above equations suggest that both mean forecasts are perfectly correlated because both of them include the mean nowcast, $\bar{\boldsymbol{\xi}}_{t|it}$, which is obviously not the case in the data. We therefore assume that the observable mean forecasts include white noise measurement errors. Thus, we have

$$\bar{x}_{t+1y|it}^{obs} = S_xF_{1y}(F)\bar{\boldsymbol{\xi}}_{t|it} + v_{1y,t}, \tag{27}$$

$$\bar{x}_{t+5y|it}^{obs} = S_xF_{5y}(F)\bar{\boldsymbol{\xi}}_{t|it} + v_{5y,t}, \tag{28}$$

where the measurement errors $v_{1y,t}$ and $v_{5y,t}$ are i.i.d. $N\left(0,\sigma_{v1y}^2\right)$ and i.i.d. $N\left(0,\sigma_{v5y}^2\right)$, respectively. We can construct a state space model with these equations. To fit them to a state space model, the mean nowcasts equation (10) is rewritten as

$$\bar{\boldsymbol{\xi}}_{t|it} = (I-G)\,F\bar{\boldsymbol{\xi}}_{t-1|it-1} + GF\boldsymbol{\xi}_{t-1} + G\mathbf{e}_t, \tag{29}$$

where we assume the standard model. Then, the state equations are,

$$\begin{bmatrix}\boldsymbol{\xi}_t\\ \bar{\boldsymbol{\xi}}_{t|it}\end{bmatrix} = \begin{bmatrix}F & 0\\ GF & (I-G)\,F\end{bmatrix}\begin{bmatrix}\boldsymbol{\xi}_{t-1}\\ \bar{\boldsymbol{\xi}}_{t-1|it-1}\end{bmatrix} + \begin{bmatrix}I\\ G\end{bmatrix}\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}\varepsilon_t\\ \varepsilon_t^*\end{bmatrix}. \tag{30}$$

The innovation process is distributed $N\left(0,\Sigma^{\epsilon}\right)$, where

$$\Sigma^{\epsilon} = \begin{bmatrix}I\\ G\end{bmatrix}\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}\sigma_{\varepsilon}^2 & 0\\ 0 & \sigma_{\varepsilon*}^2\end{bmatrix}\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}'\begin{bmatrix}I\\ G\end{bmatrix}'. \tag{31}$$

The observation equations are

$$\begin{bmatrix}x_t^{obs}\\ \bar{x}_{t+1y|it}^{obs}\\ \bar{x}_{t+5y|it}^{obs}\end{bmatrix} = \begin{bmatrix}S_x & 0\\ 0 & S_xF_{1y}(F)\\ 0 & S_xF_{5y}(F)\end{bmatrix}\begin{bmatrix}\boldsymbol{\xi}_t\\ \bar{\boldsymbol{\xi}}_{t|it}\end{bmatrix} + \begin{bmatrix}0\\ v_{1y,t}\\ v_{5y,t}\end{bmatrix}. \tag{32}$$

The innovation process is distributed $N\left(0,\Sigma^{v}\right)$, where

$$\Sigma^{v} = \begin{bmatrix}0 & 0 & 0\\ 0 & \sigma_{v1y}^2 & 0\\ 0 & 0 & \sigma_{v5y}^2\end{bmatrix}. \tag{33}$$

The state space equations and the observation equations can be simplified as,

$$\boldsymbol{\Xi}_t = D\boldsymbol{\Xi}_{t-1} + \boldsymbol{\epsilon}_t, \tag{34}$$

$$\mathbf{X}_t = H'\boldsymbol{\Xi}_t + \boldsymbol{v}_t, \tag{35}$$

where $\boldsymbol{\epsilon}_t \sim N\left(0,\Sigma^{\epsilon}\right)$ and $\boldsymbol{v}_t \sim N\left(0,\Sigma^{v}\right)$. As can be observed in Figure 2, some of the mean forecasts of the 5-to-10-years ahead inflation expectations are missing. To deal with this problem, we employ the method of Harvey (1990), which is described in detail in Appendix A. For the estimation of the standard model, the parameters to be estimated are those for the actual process of inflation, $\{\phi,\sigma_{\varepsilon},\sigma_{\varepsilon*}\}$, those for the informational frictions, $\{\sigma_w,\sigma_{w*}\}$, and the measurement error parameters $\{\sigma_{v1y},\sigma_{v5y}\}$. We do not distinguish between the subjective and objective size of noise components in this model. On the other hand, in the first stage estimation of the generalized model, the parameters to be estimated are the same as those in the case of the standard model, but we replace the parameters of the size of the noise components $\{\sigma_w, \sigma_{w*}\}$ with those of the subjective size of the noise components $\{\sigma_w^{sbj}, \sigma_{w*}^{sbj}\}$.

3.4 Method of Moments Estimation

Next, we estimate the objective size of the noise components $\{\sigma_w^{obj}, \sigma_{w*}^{obj}\}$, our measures of transparency in the generalized model, by matching the theoretical steady state disagreements to the data given the estimated parameters in the MLE. We employ the method of moments estimation and use data on the average (across time) of the variance (across forecasters) of the 1-year ahead inflation expectations, $\bar{V}_{1y}^{obs}$, which is the square of the SR disagreement, and that of the 5-to-10-years ahead inflation expectations, $\bar{V}_{5y}^{obs}$, which is the square of the LR disagreement,

$$\bar{V}_{1y}^{obs} = \frac{1}{n_{1y}} \sum_{t=1}^{n_{1y}} \bar{V}_{t+1y|t}^{obs} \tag{36}$$

$$\bar{V}_{5y}^{obs} = \frac{1}{n_{5y}} \sum_{t=1}^{n_{5y}} \bar{V}_{t+5y|t}^{obs}, \tag{37}$$

where n_{1y} and n_{5y} are the number of observations for each moment. We match theoretical variances at the steady state to average variances in the sample. The theoretical variance of the 1-year ahead inflation expectations at the steady state is

$$\bar{V}\left[x_{1y}\right] = S_x F_{1y}(F) \bar{V}\left[\boldsymbol{\xi}_0\right] F_{1y}(F)' S_x'. \tag{38}$$

Similarly, the theoretical variance for the 5 to 10 years ahead expectations at the steady state is

$$\bar{V}\left[x_{5y}\right] = S_x F_{5y}(F) \bar{V}\left[\boldsymbol{\xi}_0\right] F_{5y}(F)' S_x'. \tag{39}$$

We choose parameters $\hat{\boldsymbol{\theta}}$ which satisfy the following equation:

$$\mathbf{m}^{\mathbf{obs}} - \mathbf{m}\left(\hat{\boldsymbol{\theta}}\right) = 0, \tag{40}$$

where

$$\hat{\boldsymbol{\theta}} = \begin{bmatrix} \hat{\sigma}_w^{obj} \\ \hat{\sigma}_{w*}^{obj} \end{bmatrix}, \ \mathbf{m}^{\mathbf{obs}} = \begin{bmatrix} \bar{V}_{1y}^{obs} \\ \bar{V}_{5y}^{obs} \end{bmatrix}, \ \mathbf{m}\left(\theta\right) = \begin{bmatrix} \bar{V}\left[x_{1y}\right] \\ \bar{V}\left[x_{5y}\right] \end{bmatrix}.$$

In this way we estimate the objective size of the noise components in the generalized model.

Table 1: MLE for the Standard Model

	ϕ	σ_{ε}	$\sigma_{\varepsilon *}$	σ_{w}	σ_{w*}	σ_{v1y}	σ_{v5y}
1978Q1 to 1992Q4	0.825	1.943	0.635	2.862	1.967	0.525	0.342
	(0.025)	(0.181)	(0.142)	(0.549)	(0.896)	(0.052)	(0.058)
1993Q1 to 2008Q3	0.564	1.482	0.174	1.143	0.301	0.516	0.245
	(0.074)	(0.133)	(0.049)	(0.570)	(0.147)	(0.050)	(0.026)

Note: Standard errors are in parentheses.

Table 2: Fitted Disagreements for the Standard Model

	Data			Estimated		
	S.run	L.run	Diff.	S.run	L.run	Diff.
1978Q1 to 1992Q4	7.305	7.678	-0.373	1.038	0.749	0.288
	(1.435)	(1.205)	(2.292)	(0.154)	(0.222)	(0.175)
1993Q1 to 2008Q3	4.330	3.714	0.616	0.259	0.157	0.102
	(1.137)	(1.327)	(0.605)	(0.091)	(0.059)	(0.091)

Note: "Diff." shows disagreement in SR inflation expectations minus disagreement in LR inflation expectations. Standard errors are in parentheses.

4 Results

4.1 Estimation Results for the Standard Model

We begin with the standard model. Table 1 shows the estimation results of the MLE for the standard model, but we will point out crucial problems with the model. With the estimated parameters in Table 1, we can calculate theoretical disagreements at the steady state, as in equations (38) and (39).

Table 2 shows the theoretical disagreements together with actual disagreements.[9] Two problems can be observed. First, the estimated disagreements are far smaller than the actual disagreements for both SR and LR disagreements. All the estimated disagreements are almost one-tenth of actual disagreements. Second, the estimated disagreements do not show the crossing of disagreements, which is a notable characteristic of the data. In the first row, which covers the period before the crossing, the LR disagreement in the data is larger than the SR disagreement while in the second row, the LR disagreement in the data is lower than the SR disagreement. However, the estimated LR disagreement is always lower than the estimated SR disagreement and the model fails to describe the dynamics of disagreements in the data. We will discuss the limitations of this model in the Appendix.

[9] The standard errors for estimated disagreements are obtained using the Delta method.

Table 3: MLE for the Generalized Model

	ϕ	σ_ε	$\sigma_{\varepsilon*}$	σ_w^{sbj}	σ_{w*}^{sbj}	σ_{v1y}	σ_{v5y}
1978Q1 to 1992Q4	0.825	1.943	0.635	2.862	1.967	0.525	0.342
	(0.025)	(0.181)	(0.142)	(0.549)	(0.896)	(0.052)	(0.058)
1993Q1 to 2008Q3	0.564	1.482	0.174	1.143	0.301	0.516	0.245
	(0.074)	(0.133)	(0.049)	(0.570)	(0.147)	(0.050)	(0.026)

Note: Standard errors are in parentheses.

4.2 Estimation Results for the Generalized Model

This section shows our estimation results for the generalized model. The first stage estimation to derive the dynamics of inflation $\{\phi, \sigma_\varepsilon, \sigma_{\varepsilon*}\}$ and the subjective size of the noise components $\{\sigma_w^{sbj}, \sigma_{w*}^{sbj}\}$ are exactly the same as those for the standard model in Section 4.1. We present the results again in Table 3 for reference. We find that the inflation gap is less persistent in the latter sample compared to that in the earlier sample. This implies that the FRB has become more aggressive in preventing inflation from deviating from the target (Cogley et al. (2010)). In addition, comparing the first and the second rows again, the size of the gap shocks σ_ε is 24% smaller in the latter sample while the size of trend shocks $\sigma_{\varepsilon*}$ is 73% smaller in the latter period. The results basically reflect the level effect: as the level of inflation has declined, the volatility of the shocks has also declined. Moreover, these results are consistent with the literature: the decline in the volatility of trend inflation contributed much to the stabilization of the inflation rate (Stock and Watson (2007)). Finally, the two subjective sizes of the noise components σ_w^{sbj} and σ_{w*}^{sbj} are smaller in the latter sample than in the earlier sample. In particular, the subjective size of the noise component in the inflation trend is 85% smaller. This implies that people have come to believe that the size of noise is smaller and put a lot more weight, or trust, on the signal of the inflation trend in making their forecast.

Table 4 shows the fitted disagreements for the generalized model using the estimated objective size of the noise components $\{\sigma_w^{obj}, \sigma_{w*}^{obj}\}$ from method of moments estimation.[10] In contrast to Table 2, the model tracks the disagreements in the MSC. In particular, the estimated model can produce the crossing of the SR and LR disagreements. Table 5 shows the estimated objective size of the noise components in the second stage estimation. The table shows that the objective size of the noise components is smaller in the latter sample than in the earlier sample. Specifically, the objective size of inflation noise is 12% lower while that of trend noise is 63% lower. The decline in the latter parameter, our measure of the transparency, is one of the key findings of this paper. This implies that people received more precise information about the inflation trend in the latter

[10] The standard errors for estimated disagreements are obtained using the Delta method by assuming there's no correlation between the parameters estimated in the MLE and those in the method of moments.

Table 4: Fitted Disagreements for the Generalized Model

	Data			Estimated		
	S.run	L.run	Diff.	S.run	L.run	Diff.
1978Q1 to 1992Q4	7.305	7.678	-0.373	7.305	7.678	-0.373
	(1.435)	(1.205)	(2.292)	(0.477)	(1.385)	(1.132)
1993Q1 to 2008Q3	4.330	3.714	0.616	4.330	3.714	0.616
	(1.137)	(1.327)	(0.605)	(0.474)	(0.525)	(0.471)

Note: "Diff." shows disagreement in SR inflation expectations minus disagreement in LR inflation expectations. Standard errors are in parentheses.

Table 5: Estimated Objective Size of Noise Components for the Generalized Model

	σ_w^{obj}	σ_{w*}^{obj}
1978Q1 to 1992Q4	18.057	20.382
	(0.515)	(0.560)
1993Q1 to 2008Q3	16.148	7.117
	(0.512)	(0.318)

Note: Standard errors are in parentheses.

sample and it suggests an improvement in the FRB's transparency. This finding can be attributed to the increased disclosure by the FRB regarding information related to its inflation target. In addition, the estimated objective size of the noise components is larger than the subjective size of the noise components, implying that the consumers answering the survey are overconfident about the precision of their knowledge. The results are consistent with the literature, which shows that people tend to be overconfident about their information.[11]

Finally, Table 6 shows the decomposition of the crossing of disagreements into the effects of parameter changes. As shown in the table, both the SR and LR disagreements are smaller in the latter sample, but the decline is relatively larger in the LR disagreement, by 0.989, which results in the crossing of the two disagreement as in Figure 1. To

[11] We briefly discuss the plausibility of the degree of overconfidence estimated in our study. We define the measure of overprecision as $\delta_w = \sigma_w^{sbj}/\sigma_w^{obj}$ and $\delta_{w*} = \sigma_{w*}^{sbj}/\sigma_{w*}^{obj}$ following Grubb and Osborne (2015). The overconfidence of each signal is 0.155 and 0.095 respectively in the earlier sample, and 0.070 and 0.040 respectively in the latter sample. Grubb and Osborne (2015) measure the overconfidence of U.S. consumers regarding the accuracy of their own forecasts by how much calling they do per month when choosing cell phone plans. Their estimate is 0.383, which is relatively higher than our estimates. This means that the consumers in their survey have more accurate understanding of the precision of their knowledge. One possible reason why their parameter is higher is that consumers may think more seriously in selecting cell phone plans since it affects their expenditure directly. Thus, the gap between their perceived precision and their actual precision could be smaller. On the other hand, consumers may have less incentive to be careful when they answer questions about the future inflation rate on a phone interview with the MSC because the answer will not affect their future expenditures. This could be the reason why our estimates of the parameter are smaller.

Table 6: Decomposition of the Difference in Disagreements

	S.run	L.run	Diff.
1978Q1 to 1992Q4	7.305	7.678	-0.373
1993Q1 to 2008Q3	4.330	3.714	0.616
Change in disagreements	-2.975	-3.964	0.989
Contribution of the change in			
ϕ	-2.992	0.235	-3.228
σ_{ε}	-0.137	0.215	-0.352
$\sigma_{\varepsilon *}$	-1.353	-2.026	0.673
σ_{w}^{sbj}	0.881	0.211	0.671
σ_{w*}^{sbj}	0.594	2.356	-1.762
σ_{w}^{obj}	-0.149	0.226	-0.374
σ_{w*}^{obj}	-4.374	-6.691	2.317
Cross terms	4.555	1.511	3.044

Note: "Diff." shows disagreement in SR inflation expectations minus disagreement in LR inflation expectations.

calculate the contribution of each parameter to the crossing, we conduct counterfactual simulations. Specifically, we calculate a hypothetical disagreement using parameter A from the first sample and the remaining parameters from the second sample. We then take the difference between the disagreement in the second sample and the hypothetical disagreement to be the contribution of the change in parameter A. We calculated the hypothetical disagreements for both the SR and the LR disagreements for each parameter.

Our results show that the decline in the objective size of noise components σ_{w*}^{obj}, the FRB's transparency, has contributed the most to the crossing of disagreements. The mechanism works as follows. As we can expect from equation 22, the longer the horizon of the inflation expectation is, the smaller the effect of the inflation gap becomes. Thus, when a consumer makes a forecast on LR inflation, he relies far more on the trend signal. As a result, the effect of the change in the size of the noise for the trend signal is larger for the LR disagreement than that for SR disagreement. In addition, the decline in the size of trend shocks $\sigma_{\varepsilon *}$ and that of the subjective size of inflation noise σ_{w}^{sbj} have also contributed to the crossing. The former suggests that the key parameter in the literature for the stabilization of the great inflation has also played a role in the crossing of the two disagreements. Finally, the decline in the persistence parameter ϕ and the subjective size of noise components σ_{w*}^{sbj} have affected the relationship between the two disagreements in opposite directions.

In conclusion, we find a large improvement in the FRB's transparency regarding its policy objective, which is extracted from the consumer survey data using a generalized noisy information framework. Moreover, this factor has contributed the most to the decline in the LR disagreement in inflation expectations relative to that in the SR dis-

Table 7: Estimated Objective Size of Noise Components of Trend Signal, Age Subgroups

	age 18-34	age 35-44	age 45-54	age 55-64	age 65-97
1978Q1 to 1992Q4	18.699	16.715	7.754	6.813	7.679
	(0.489)	(0.655)	(0.341)	(0.366)	(0.368)
1993Q1 to 2008Q3	7.604	7.062	6.106	6.309	5.753
	(0.372)	(0.328)	(0.302)	(0.299)	(0.234)

Note: Standard errors are in parentheses.

Table 8: Estimated Objective Size of Noise Components of Trend Signal, Income Subgroups

	Bottom 25%	Second 25%	Third 25%	Top 25%
1978Q1 to 1992Q4	9.761	17.352	22.510	12.613
	(0.274)	(0.545)	(0.949)	(0.631)
1993Q1 to 2008Q3	10.985	6.802	6.392	4.840
	(0.555)	(0.401)	(0.263)	(0.216)

Note: Standard errors are in parentheses.

agreement.

4.3 Estimation Using Subgroup Data

This section shows the estimation results of our generalized model using subgroup data from the MSC. The MSC collects data on several characteristics of the forecasters, and we estimate the model using data for age, income, and education subgroups. The estimation procedure is the same as that for the full sample of the MSC in Section 4.2. We estimate all parameters other than the objective size of the noise components using MLE and then estimate the objective size of the noise components using the method of moments given the estimated parameters from the MLE.

Tables 7 through 9 show the estimated objective size of the noise component in the signal of inflation trend, which is our measure of the FRB's transparency.[12] Overall, it declined after the mid-1990s for all the subgroups except for the bottom 25% income subgroup. The results are consistent with our main results in Section 4.2. There is also some heterogeneity. Regarding age subgroups, we found a large decline in the size of noise for young people, those aged 18 to 44, while the change is relatively modest for older people, those 45 and older. This suggests that the improvement in the FRB's transparency was most effective for young people. Regarding income subgroups the size of noise for the bottom 25% did not changed much, while for the education subgroups the

[12] We also estimate the parameters for gender and regional subgroups, but the results are similar to our main results in Section 4.2.

Table 9: Estimated Objective Size of Noise Components of Trend Signal, Education Subgroups

	Less than HS	HS deg.	Some Cllg	Cllg deg.	Grad
1978Q1 to 1992Q4	22.398	17.423	21.524	17.156	12.927
	(0.682)	(0.529)	(0.691)	(0.734)	(0.860)
1993Q1 to 2008Q3	10.318	7.767	7.768	5.109	4.784
	(0.542)	(0.440)	(0.344)	(0.241)	(0.216)

Note: Less than HS: Less than High School, HS deg.: High School degree, Some Cllg: Some college, Cllg deg.: College degree, Grad: Graduate studies. Standard errors are in parentheses.

decline in the size of noise for those with less than a high school education subgroup is smaller than that for the other education subgroups. This suggests that the improvement in the transparency of the FRB was relatively ineffective for the people in these subgroups. In summary, we found a decline in the actual size of noise in the inflation trend after the mid-1990s for almost all subgroups. However, there exists heterogeneity and the decline was especially great for the young, educated, and those in high income groups. This suggests that whether or not the improvement in the FRB's transparency regarding the target is conveyed to an economic agent depends on his/her characteristics to some extent.

5 Conclusion

This study measures the FRB's transparency regarding its target inflation rate using survey data of the disagreement in inflation expectations among U.S. consumers from the late 1970s to the early 2000s. We construct a model of inflation forecasters employing the frameworks of both an unobserved components model and a noisy information model. In particular, we explicitly model how people make use of heterogeneous noisy signals about the inflation trend, which proxies the FRB's inflation target. With this model in hand, we estimate the parameters including the size of noise in the trend signal, which we interpret as the degree of the FRB's transparency regarding the target inflation rate. We estimate all the parameters including the transparency employing both maximum likelihood and the methods of moments, matching the theoretical moments from the model to the data. The results show that the index of transparency largely improved after the mid-1990s. In addition, we show that the change in transparency regarding the target had a large impact on the term structure of disagreements in inflation expectations. Finally, we run the same estimation using subgroup data and show that the improvement in the FRB's transparency was more effective on the young, educated, and those in high income groups.

While we have focused on U.S. consumers, this approach can be applied to other economic agents. One of the reasons for using the MSC is its long sample for long horizon forecasts since our focus is on the great inflation period, a relatively old episode. However, if one focuses, for example, on the introduction of an inflation target in the U.S. or the global financial crisis, other rich survey data are also available. In addition, our model can be extended to focus on specific events. Since we simply split the sample into two parts and estimate the parameters for each sample, we do not argue what specific actions by the FRB have improved the transparency measure. One possible extension is to incorporate stochastic volatility in the size of the noise and evaluate the effect of specific actions by central banks on the estimated size of noise.

A Maximum Likelihood Estimation with Missing Observations

The state space equation and the observation equation are described in Section 3.3:

$$\Xi_t = D\Xi_{t-1} + \boldsymbol{\epsilon}_t, \tag{A.1}$$

$$\mathbf{X}_t = H'\Xi_t + \boldsymbol{\upsilon}_t, \tag{A.2}$$

where $\boldsymbol{\epsilon}_t \sim N(0, \Sigma^{\epsilon})$, $\boldsymbol{\upsilon}_t \sim N(0, \Sigma^{\upsilon})$, and $\mathbf{X}_t \equiv \left[x_t^{obs}; \bar{x}_{t+1y|it}^{obs}; \bar{x}_{t+5y|it}^{obs}\right]$. Unfortunately, some of the mean forecast of 5 to 10 years ahead inflation expectations $\bar{x}_{t+5y|it}^{obs}$ are missing as in Figure 2. To deal with this problem, this section applies the method proposed in Harvey (1990) to our estimation. The idea is simple: we update state variables without making use of unavailable observations. The parameters to be estimated are summarized as $\theta = \{\phi, \sigma_{\varepsilon}, \sigma_{\varepsilon *}, \sigma_w, \sigma_{w*}, \sigma_{v1y}, \sigma_{v5y}\}$.

Given the initial values $\Xi_{0|0}$ and $P_{1|0}$, we begin with a period when all three observations are available. The updating equation is

$$\Xi_{t|t} = D\Xi_{t-1|t-1} + K_t \left(\mathbf{X}_t - H'D\Xi_{t-1|t-1}\right), \tag{A.3}$$

where the Kalman gain, a 3×3 matrix, is given by

$$K_t = P_{t|t-1}H' \left(HP_{t|t-1}H' + \Sigma^{\upsilon}\right)^{-1}. \tag{A.4}$$

The MSE of the forecast of Ξ_t with information up until $t-1$, denoted by $P_{t|t-1}$, evolves according to

$$P_{t+1|t} = D\left[P_{t|t-1} - P_{t|t-1}H\left(H'P_{t|t-1}H + \Sigma^{\upsilon}\right)^{-1} H'P_{t|t-1}\right]D' + \Sigma^{\epsilon}. \tag{A.5}$$

Then, denoting the error by $\mathbf{u}_t = \mathbf{X}_t - \mathbf{X}_{t|t-1} = \mathbf{X}_t - H'D\Xi_{t-1|t-1}$, the MSE of the forecast of $\mathbf{X}_t$ is given by

$$\Omega_t \equiv E\left[\mathbf{u}_t\mathbf{u}_t'\right] = H'P_{t|t-1}H + \Sigma^{\upsilon}. \tag{A.6}$$

Finally, the log likelihood function for these observations is

$$\ln f\left(\mathbf{X}_t | \theta, \mathbf{X}_{t-1}, ..., \mathbf{X}_0\right) = -\frac{3}{2}\ln 2\pi - \frac{1}{2}\ln|\Omega_t| - \frac{1}{2}\mathbf{u}_t'\Omega_t^{-1}\mathbf{u}_t. \tag{A.7}$$

Next, we will describe the likelihood function in periods when we do not have the LR mean inflation expectations, specifically $\mathbf{X}_t = \left[x_t^{obs}; \bar{x}_{t+1y|it}^{obs}; N/A\right]$. We introduce a selector matrix to obtain available observations as below,

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}. \tag{A.8}$$

The updating equation is

$$\boldsymbol{\Xi}_{t|t} = D\boldsymbol{\Xi}_{t-1|t-1} + K_t \left(S\mathbf{X}_t - SH'D\boldsymbol{\Xi}_{t-1|t-1} \right), \tag{A.9}$$

where the Kalman gain, a 3×2 matrix, is given by

$$K_t = P_{t|t-1} (SH)' \left((SH) P'_{t|t-1} (SH)' + S\Sigma^v S' \right)^{-1}. \tag{A.10}$$

Thus, we simply skip the unavailable LR mean forecast. The MSE of the forecast of $\boldsymbol{\Xi}_t$ with information up until $t-1$, denoted by $P_{t|t-1}$, evolves according to

$$P_{t+1|t} = D \left[P_{t|t-1} - P_{t|t-1} (SH) \left((SH)' P_{t|t-1} (SH) + S\Sigma^v S' \right)^{-1} (SH)' P_{t|t-1} \right] D' + \Sigma^\epsilon. \tag{A.11}$$

Then, the MSE of the forecast of $\mathbf{X}_t$, denoted by $\mathbf{u}_t = S\mathbf{X}_t - S\mathbf{X}_{t|t-1} = S\mathbf{X}_t - SH'D\boldsymbol{\Xi}_{t-1|t-1}$, is given by

$$\Omega_t \equiv E\left[\mathbf{u}_t \mathbf{u}'_t\right] = (SH)' P_{t|t-1} SH + S\Sigma^v S'. \tag{A.12}$$

Finally, the log likelihood function for these observations is

$$\ln f\left(\mathbf{X}_t | \theta, \mathbf{X}_{t-1}, ..., \mathbf{X}_0\right) = -\ln 2\pi - \frac{1}{2} \ln |\Omega_t| - \frac{1}{2} \mathbf{u}'_t \Omega_t^{-1} \mathbf{u}_t. \tag{A.13}$$

B Properties of the Standard and the Generalized Model

In this section, we discuss why the standard model cannot describe disagreements in the survey data while the generalized model can.

We address two properties of the standard model, which is a basic noisy information model. First, both noise components need to be very large so that the model has a high level of disagreement according to our numerical exercise. Figure A-1 shows the level of theoretical disagreement for SR and LR inflation forecasts, respectively. We use the parameters of the inflation process $\{\phi, \sigma_{\varepsilon}, \sigma_{\varepsilon*}\}$ for the sample from 1978Q1 to 2008Q3. The horizontal axis shows the size of noise in the signal of the inflation rate while the vertical axis shows the size of noise in the signal of the inflation trend. The contours show the level of theoretical disagreement. Since both disagreements in the survey data are higher than 3.5, as in Table 2, we need a value of more than 60 for both noise components. This means that that an average forecaster gets a signal about the quarterly percentage growth of the inflation rate, that is different from the actual inflation rate by 60 percentage points. This is not realistic.

The mechanism explaining this point is the two opposing effects of noise on disagreement, which is also discussed in Andrade and Le Bihan (2013). The first effect is the positive "direct effect." If the size of the noise is larger, the information is more dispersed, which raises the disagreement in the variable. The other effect is the negative "weight effect." If the size of noise is larger, the forecaster puts less weight on the signal because the information is not reliable, which decreases the disagreement. The latter effect arises due to the optimal filtering assumption, that is, the Kalman filtering, where a forecaster puts optimal weights on her current noisy signals considering the usefulness of the signal. Since the forecaster optimally reduces the weight on the signal as the size of the noise increases, the pace of increase in the level of disagreement is slow.

Second, SR disagreement in the model is always larger than LR disagreement when we have large noise components in both signals. As a result, we cannot generate the crossing of the SR and LR disagreements, which is crucial for this study. Figure A-2 shows the difference in the level of theoretical disagreements, that is, SR disagreements minus LR disagreements. As the figure shows, almost the entire area is positive, which means that SR disagreements are higher than LR disagreements. In particular, the difference is always positive in the upper right region where both sizes of the noise components are large and the level of disagreement is high.

The mechanism explaining this point is again the weight effect. Given the size of noise, the weight on the signal increases as the MSE of the 1-period-ahead forecast of the variable increases. This is because the signal-to-noise ratio increases - in other words, the signal becomes relatively reliable. The MSE of the inflation rate is higher than that of the trend component because the inflation rate is the sum of the gap and the trend. As a result, suppose that a forecaster uses the same noisy signal to infer each variable and the level of disagreement in inflation tends to be higher than that of the trend because the weight becomes larger. Then, since the SR disagreement is the weighted average of the nowcast disagreement in inflation and the trend while the LR disagreement is almost entirely the nowcast disagreement in the trend, the SR disagreement tends to be higher than the LR disagreement, as shown in Figure A-2.

As seen above, the standard model, which is the basic noisy information model, is unable to capture the observed patterns of the term structure of disagreement in the survey data. This problem is related to the negative weight effect that comes from the optimal forecast. Thus, we use the generalized model explained in Section 2.2 and slightly relax the theoretical restriction on the weights. We continue to use Kalman filtering, but the generalized model has weights on signals which are different from the optimal weights and result from assuming that people have imperfect information about the distribution of the noise components: $R^{sbj} \neq R^{obj}$.[13]

Next, we discuss the properties of the generalized model and show why the model can produce the dynamics of the disagreements in the survey data. Figure A-3 describes numerical examples of SR and LR disagreements from the model when we change the objective/true the size of noise components. We employ the parameters of the true inflation process $\{\phi, \sigma_\varepsilon, \sigma_{\varepsilon*}\}$ and the subjective size of the noise components $\{\sigma_w^{sbj}, \sigma_{w*}^{sbj}\}$ from the estimation results of the sample from 1978Q1 to 2008Q3. We can observe two properties from the figures. First, the level of disagreement is much higher than that in the standard model, which is presented in Figure A-1. Since this model does not impose an optimal weight restriction, which gives rise to the negative weight effect seen in the standard model, but rather employs estimated fixed subjective sizes of the noise components, the negative weight effect disappears. As the figure shows, given the estimated weights, the model can express a high level of disagreement with smaller noise components. Second, the elasticity of SR disagreement with respect to the size of inflation noise is comparable to that with respect to the size of trend noise. Also, the elasticity of LR disagreement with respect to inflation noise is almost zero while that with respect to trend noise is very large. The different elasticities of the disagreements reflect different informativeness of signals in forecasting each variable. In fact, the weights on the signals, which depend on the estimated subjective size of the noise components, suggest that

[13] As noted in the footnote in Section 2.2, we use this setting following the literature. In addition, there can be other reasons why the weighting matrix can deviate from the optimal weighting matrix, the Kalman gain, which is used in pure noisy information models. One reason is that there may exist strategic interaction across forecasters, as discussed in Morris and Shin (2002) and Coibion and Gorodnichenko (2012). Alternatively, there may exist naive forecasters, who do not use optimal filtering to extract information from signals. In either case, the weighting matrix is different from the optimal weighting matrix.

forecasters rely far more on the inflation signal to nowcast inflation while they rely far more on the trend signal to nowcast the trend.

Finally, Figure A-4 describes the theoretical difference between the SR and the LR disagreements: SR minus LR. The important properties of the model are that the difference between disagreements can be large, and, more importantly, that the model can describe not only the case where SR disagreement is greater than LR disagreement but also the opposite case. The mechanism is the same as discussed above in relation to the previous figures: the disappearance of the negative weight effect. Thus, when the size of trend noise increases, the LR disagreement increases relative to the SR disagreement. Meanwhile, when the size of inflation noise increases, the SR disagreement increases relative to the LR disagreement. Overall, we can describe the dynamics of the term structures of disagreement in the data.

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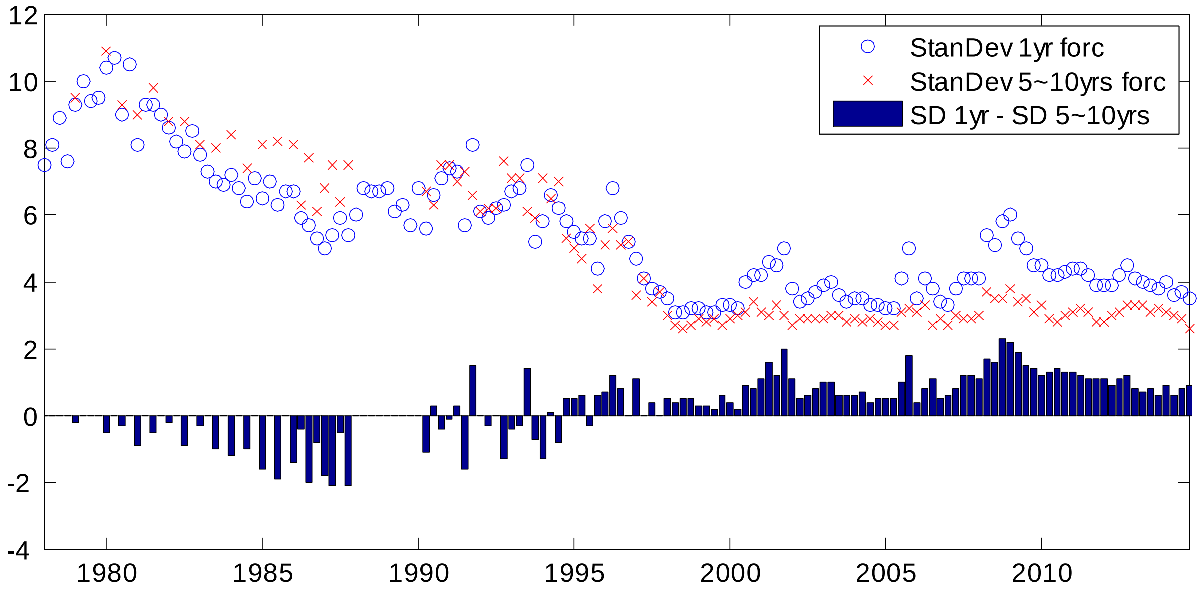
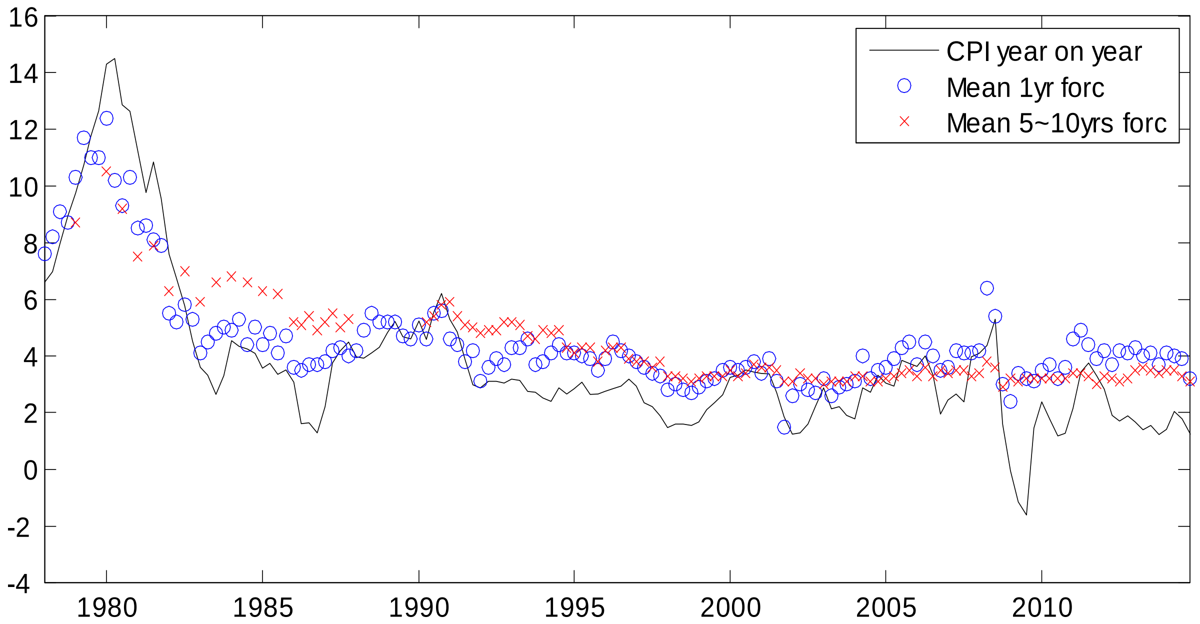
Figure 1: Disagreement in Inflation Forecasts



Source: Survey of Consumers, University of Michigan

Figure 2: Mean Inflation Forecasts



Source: Survey of Consumers, University of Michigan

Figure A-1: Theoretical Disagreement in Short-run (top) and Long-run (bottom) Inflation Forecasts: the Standard Model

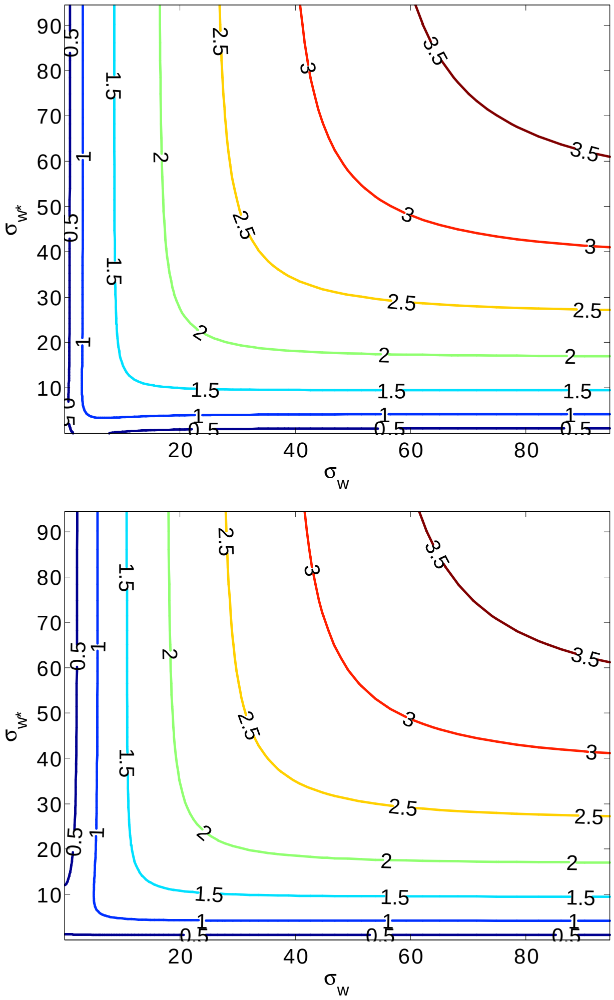


Figure A-2: Difference between Theoretical Disagreements: the Standard Model

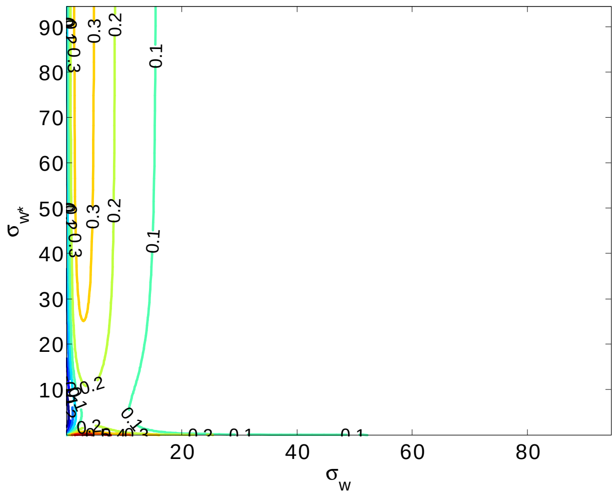


Figure A-3: Theoretical Disagreement in Short-run (top) and Long-run (bottom) Inflation Forecasts: the Generalized Model

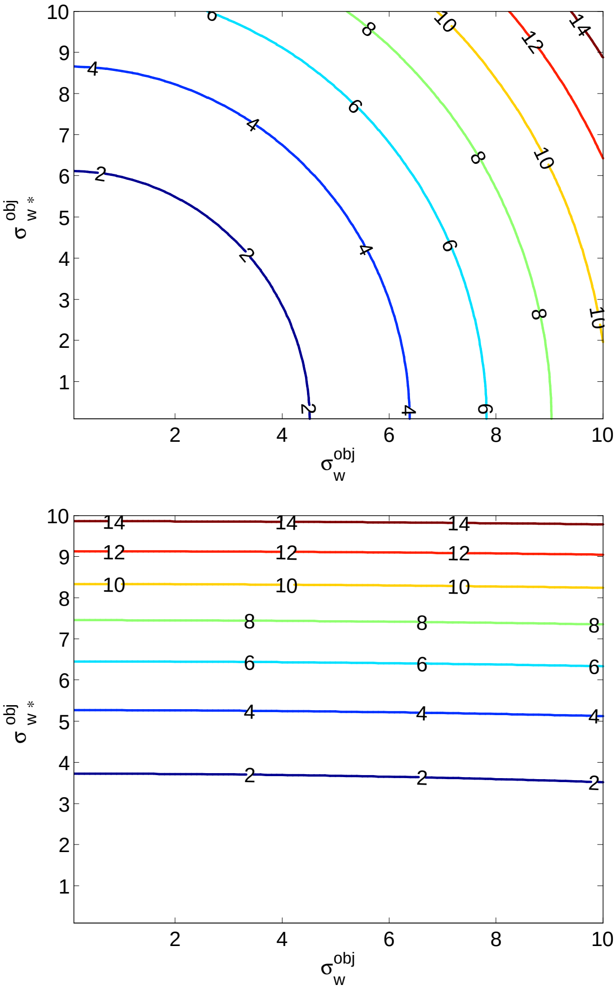


Figure A-4: Difference between Theoretical Disagreements: the Generalized Model