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Abstract

Uncertainty is a fact of life for central banks, and the effective lower bound (ELB) of short-term nominal interest rates has become one source of uncertainty for many of them. This paper analyzes the effects of uncertainty about monetary policy transmission on inflation in a canonical New Keynesian model with optimal discretionary monetary policy under the ELB. The main finding is that a greater degree of uncertainty enlarges the "deflationary bias" of the economy. In the model, the central bank reacts to the uncertainty by attenuating the response of the nominal interest rate to exogenous shocks. Such inactive policy response leaves the fall in inflation caused by the ELB risk partially untreated, which lowers the inflation expectations of private agents and results in undershooting of the inflation target.

Keywords: Model Uncertainty; Effective Lower Bound; Deflationary Bias; Risky Steady State
JEL classification: D81, E32, E52

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1 Introduction

Policy decision making under uncertainty is a classic issue in economics. A widely acknowledged view is that policymakers facing uncertainty should take a cautious policy stance. In fact, there is an influential argument proposed by Brainard (1967), often referred to as the "Brainard attenuation (conservatism) principle," which states that when policymakers are unsure about the effect of their policy, they should change their policy instrument by less in response to exogenous shocks than in the absence of uncertainty. The intuition is simple and clear-cut: the movement in the policy instrument may enlarge uncertainty about future outcomes when its effect is uncertain. While the policy attenuation leads policymakers to partially refrain from pursuing the first best outcome that would be achieved in the absence of uncertainty, it enables them to make their policymaking "robust" by reducing the probability that the economy deviates significantly from the objective target. Therefore, if the costs of attenuating the policy response are not that substantial, Brainard's recommendation should be reasonable overall. Indeed, several studies (Levin et al. (2005), Sala et al. (2008), Edge et al. (2010), and Williams (2013)) point out that the policy changes needed for central banks to obtain such robustness are quantitatively small and thus the welfare cost relative to the first best outcome is also modest. Against this background, the Brainard attenuation principle has been viewed by many central banks as a practical strategy to date.[1]

This paper argues that there is at least one important caveat that should be noted, though it has not been considered in previous studies, in applying such an argument to the current economic environment. It is related to the existence of the effective lower bound (ELB) constraint on the nominal interest rate. Since the ELB was not a serious constraint for almost all central banks prior to the global financial crisis, it is not surprising that past studies have abstracted from it when studying policy decisions under uncertainty. Nevertheless, given the much smaller room for conventional interest rate cuts while uncertainty surrounding central banks remains pervasive, the relevant question is "is there any concern or pitfall in applying the Brainard attenuation principle to the current economic environment?"

This paper addresses the question by considering discretionary monetary policy by a central bank that faces parameter uncertainty about the effect of monetary policy. The analysis is based on a canonical New Keynesian model with the occasionally binding ELB constraint on the nominal interest rate. In the model, the central bank faces uncertainty about the intertemporal elasticity of substitution, which is one of the key structural parameters that affect the transmission of monetary policy. Due to the presence of such parameter uncertainty, the central bank is uncertain about the slopes of the IS curve and the Phillips curve. As in Brainard's original set-up, the central bank takes

[1] As recent examples, Powell (2018) and Praet (2018) explictly referred to Brainard's result. In explaining the ECB's policy intentions, Draghi (2019) states that "*You just do what you think is right and you temper with a consideration that there is uncertainty. In other words, in a dark room you move with tiny steps.*" The argument here is in line with the Brainard attenuation principle.

a Bayesian approach in the sense that it has a unique prior distribution of the uncertain parameter and minimizes the expected loss based on that distribution. Under a standard calibration, such settings lead the central bank to conduct less aggressive policy than under certainty, in line with the Brainard attenuation principle.[2] In addition to parameter uncertainty, the model includes the ELB constraint on the nominal interest rate to consider the current environment surrounding central banks. In the model, the natural real rate occasionally fluctuates with exogenous shocks. When the natural real rate falls sharply due to a large negative shock, the central bank is forced to lower the nominal interest rate to the ELB. The real interest rate stays higher than the level of the natural real rate, which leads to a sizable decline in output and inflation. Households and firms in the model rationally anticipate that the ELB constraint may bind in the future, and such a possibility reduces their inflation expectations even when the policy rate is above the ELB. Since the reduced inflation expectations work as a negative cost-push shock to the economy, the discretionary central bank cuts the policy rate while facing a trade-off between inflation and output stabilization. As a result, the economy experiences lower inflation along with higher output than the central bank's targets in the states where the ELB is not binding, a phenomenon called "deflationary bias". Using such a framework, this paper asks how the presence of parameter uncertainty affects stabilization policy and social welfare under the ELB constraint.

The main finding of this paper is that the attenuated policy stance of the central bank in the face of parameter uncertainty, which is in line with the Brainard attenuation principle, can make the inflation rate in the economy permanently lower in the presence of the ELB constraint. In particular, such policy attenuation increases the size of the deflationary bias induced by discretionary policymaking with the ELB constraint, which makes it more difficult for the central bank to achieve the inflation target compared to the case without parameter uncertainty. In the numerical exercise with the standard calibration, inflation in the steady state is about 14 basis points lower than in the absence of parameter uncertainty. As a result, the undershooting of the steady state inflation rate from the target is 36 basis points, which is sizable given the small standard deviation of inflation. In the baseline case, the presence of parameter uncertainty leads to a 0.10% welfare loss in terms of steady state consumption. Overall, the message of this paper is that ignoring the presence of the ELB constraint on the nominal interest rate can lead to a significant underestimation of the cost of uncertainty for the central bank.

The key mechanism behind the result is the response of inflation expectations to the attenuated

[2] In general, whether parameter uncertainty leads to less or more aggressive policy responses depends on the assumptions of the model. For example, Kimura and Kurozumi (2007) consider uncertainty about the parameters of inflation persistence and conclude that such uncertainty leads the central bank to pursue a more, rather than less, aggressive policy. Using a model similar to the one in this paper, Ferrero et al. (2019) show that uncertainty about the slope of the Phillips curve can lead the central bank to react more aggressively in response to supply-side shocks, if shocks are persistent enough. Since the objective of this paper is to draw implications of the Brainard attenuation principle in the current circumstances, it focuses on the case where the central bank implements less aggressive policy in response to uncertainty.

stance of the central bank. In line with the Brainard attenuation principle, a central bank that is uncertain about its own policy effects reacts less to supply-side shocks. Households and firms interpret the attenuated response of the central bank as if it puts a smaller weight on inflation stabilization compared to the social optimum. They then expect that even if there are negative shocks to the natural real rate and their inflation expectations fall with the anticipation of the ELB's likely binding in the near future, the central bank's reaction to mitigate it will be milder. This further strengthens the downward pressure on their inflation expectations. Since the central bank takes the fallen inflation expectations as given when it acts under discretion, inflation further undershoots the central bank's inflation target and a larger deflationary bias arises in equilibrium.

One caveat should be noted here: the results of this paper do not imply that it is always optimal for central banks to avoid a cautious policy stance in the face of uncertainty. Indeed, the Brainard attenuation principle has been supported by a number of previous studies and still provides a good starting point for the debate on policymaking with uncertainty. On the other hand, it is also well-known that there are several cases where the Brainard attenuation principle is overturned and uncertainty calls for a more aggressive response. The two most typical cases are those in which the central bank faces uncertainty about the persistence of the inflation rate (Moessner (2005) and Kimura and Kurozumi (2007)) and those in which the central bank adopts the minimax principle to deal with uncertainty (Giannoni (2002, 2007)). Given that none of these studies accounts for the ELB on nominal interest rates, this paper contributes to the literature by showing that there are additional policy costs of the attenuated response, which matter especially in the current economic environment.

This paper is related to the literature that considers the implications of model uncertainty for the conduct of monetary policy. So far, a great number of studies, such as Svensson (1999), Clarida, Gali, and Gertler (1999), Estrella and Mishkin (1999), Martin and Salmon (1999), Kimura and Kurozumi (2007), Kurozumi (2010), Williams (2013), and Ferrero et al. (2019), re-examine the Brainard attenuation principle by modeling parameter uncertainty using the Bayesian approach, as in the original work of Brainard (1967) but employing various models. Among these studies, this paper is closest to Kurozumi (2010), which considers the effect of uncertainty about the effect of policy on discretionary policymaking using the canonical New Keynesian model.

This paper is also related to papers that consider the implications of the deflationary bias, a systematic undershooting of the inflation target induced by the ELB constraint on the policy rate. Adam and Billi (2007) and Nakov (2008) first point out that the deflationary bias arises when the nominal interest rate is occasionally constrained by the ELB. Nakata and Schmidt (2019a) analytically shows that the deflationary bias can be reduced by assigning monetary policy to an inflation-conservative central banker. In this context, the result of this paper implies that uncertainty about the effects of policy can lead central banks to take an "anti-inflation-conservative" stance and results in a larger deflationary bias in the economy.

The paper proceeds as follows: Section 2 describes the baseline setup. Section 3 presents the main results of this paper. Section 4 presents additional results. Finally, Section 5 concludes.

2 The Setup

2.1 The model

The analysis is based on the canonical New Keynesian model developed in Woodford (2003) and Gali (2008). The economy consists of a continuum of identical households, a continuum of monopolistically competitive firms who face restrictions on the frequency of price adjustments à la Calvo (1983), and a central bank. The utility maximization of the households leads to the following IS equation

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n), \tag{1}$$

where x_t denotes the output gap, E_t is the expectation operator conditional on private agents' period t information set, and $\sigma > 0$ denotes the intertemporal elasticity of substitution of consumption. The variable i_t is the nominal interest rate, π_t is the inflation rate, and r_t^n is the natural real rate of interest. The natural real rate r_t^n follows the AR(1) process described below:

$$r_t^n = (1 - \rho_r) r^n + \rho_r r_{t-1}^n + \varepsilon_t^r, \tag{2}$$

where $\rho_r \in [0, 1)$ and $\varepsilon_t^r \sim N(0, \sigma_r^2)$. The steady state value of the natural real rate r^n is set to $\frac{1}{\beta} - 1$, where $\beta \in (0, 1)$ is the subjective discount factor of the households.

The profit-maximizing price setting behavior of firms leads to the following New Keynesian Phillips curve

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t, \tag{3}$$

where the slope parameter $\kappa > 0$ is given by

$$\kappa = \kappa_u(\omega + \sigma^{-1}), \quad \kappa_u = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\omega\theta)},$$

with $\alpha \in (0, 1)$ denoting the share of firms keeping prices unchanged in a given period, $\omega > 0$ the inverse of the labor-supply elasticity, and $\theta > 1$ the price elasticity of demand for differentiated goods.

As pointed out in Woodford (2003), we can derive the central bank's objective function as a second-order approximation to the households' expected lifetime utility

$$L_0 = E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda x_t^2), \tag{4}$$

where the relative weight on the output gap is defined as $\lambda = \frac{\kappa}{\theta}$.[3] In each period t, the central bank acts discretionarily by solving

$$\min_{\pi_t, x_t, i_t} \; L_t = E_t \sum_{j=0}^{\infty} \beta^{t+j} (\pi_{t+j}^2 + \lambda x_{t+j}^2), \tag{5}$$

subject to the equations (1), (2), (3), and the effective lower bound (ELB) constraint

$$i_t \geq 0, \tag{6}$$

given the current level of the natural real rate r_t^n and the level of the variables in subsequent periods $\{\pi_{t+j}, x_{t+j}, i_{t+j} \geq 0\}$ for $j \geq 1$.

2.2 Parameter uncertainty and the Bayesian approach

To focus solely on the implication of uncertainty faced by the central bank, private agents are assumed to know the model structure and the values of all model parameters perfectly. Therefore, the behavior of private agents is described by the IS equation (1) and the New Keynesian Phillips curve (3). They can observe the current state of the economy, x_t, π_t, i_t, and r_t^n. Every period t, they form rational expectations, $E_t\pi_{t+1}$ and $E_t x_{t+1}$, in the sense that their subjective probability distributions about the realization of the future variables coincide with the objective probability distributions according to the model.

The central bank has the same information as private agents about the current state of the economy. In particular, the central bank's information set in period t contains x_t, π_t, i_t, and r_t^n. The central bank also knows that the expectations of private agents are formed according to the equations (1) to (3).

The difference between the information set of private agents and that of the central bank arises from parameter uncertainty. In particular, the central bank does not know the true value of the structural parameter σ. As in Brainard (1967), the central bank takes a Bayesian approach in the sense that it has a unique prior distribution of the uncertain parameter σ and minimizes the expected loss based on this parameter distribution. In other words, the central bank assigns probabilities to every possible value of the unknown parameter σ and seeks the policy that results in the best outcome on average. The mean and the variance of the prior distribution over the true

[3] To consider the efficient deterministic steady state, I assume that distortions associated with imperfect competition in goods and labor markets are eliminated by fiscal subsidies.

value of σ are denoted by

$$E_t^{CB}[\sigma] = \bar{\sigma}, \quad V_t^{CB}[\sigma] = v_\sigma, \tag{7}$$

respectively. In what follows, E_t^{CB} and V_t^{CB} denote the expectation and variance operators conditional on the central bank's period t information set. The mean $\bar{\sigma}$ is the value which the central bank considers most likely and the variance v_σ indicates the magnitude of the central bank's subjective uncertainty about the true value of the parameter.[4]

This paper focuses on the parameter σ for the following two reasons. First, the parameter plays a key role in the transmission mechanism of monetary policy. The parameter σ determines the strength of the household's desire to smooth consumption, which is the key transmission mechanism of monetary policy in the New Keynesian model. Second, in reality there exists great uncertainty about the true value of σ. While the parameter has been estimated in hundreds of previous studies, a wide range of values has been reported depending on sample selection (periods, micro or aggregate data, countries) or estimation techniques.[5] Reflecting this fact, a variety of calibration patterns are employed, ranging from $\sigma = 6.25$ in Woodford (1997, 2003) to $\sigma = 0.2$ in Nakov (2008), even for the same canonical New Keynesian model. For now, there is still no consensus about the true value of σ and considering uncertainty about it is reasonable in this sense.

It should be noted that parameter uncertainty about σ is propagated through the microfoundations of the model not only to σ itself but also to the slope of the New Keynesian Phillips curve κ and to the weight on the output gap in the loss function λ. Due to such parameter uncertainty, the central bank has less information than private agents. Of particular importance is that the central bank does not know their expected inflation $E_t\pi_{t+1}$ and output gap $E_t x_{t+1}$. For example, note that the New Keynesian Phillips curve (3) implies that

$$E_t\pi_{t+1} = \frac{1}{\beta}(\pi_t - \kappa x_t). \tag{8}$$

Since the value of the parameter κ is now uncertain, the central bank cannot infer $E_t\pi_{t+1}$ even if it knows the structure of the New Keynesian Phillips curve (3) and observes π_t and x_t correctly.

Therefore, the central bank has to form expectations based on its own information set to conduct policy. The expectations of the central bank are formed according to the IS equation (1) and the New Keynesian Phillips curve (3), but conditional on its own information set, which is smaller than that of private agents. By taking the expectations of (1) and (3) conditional on the prior

[4]For simplicity, the prior distribution of the central bank is assumed to be constant overtime. If the central bank can learn about the model of the economy through conducting policy, v_σ may decrease and $\bar{\sigma}$ may approach the true value of σ over time. It should be noted, however, that even if such learning is allowed, v_σ would remain nearly constant if the structure of the economy changes more quickly than the learning of the central bank.

[5]Regarding the range of estimated values of σ, see Havránek (2015) for example.

distribution of the central bank, we obtain

$$x_t = E_t^{CB} x_{t+1} - \bar{\sigma}(i_t - E_t^{CB}\pi_{t+1} - r_t^n) + \bar{\sigma}\beta^{-1}\left\{\bar{\kappa} - \kappa_u(\omega + \sigma^{-1})\right\} x_t, \tag{9}$$

$$\pi_t = \beta E_t^{CB}\pi_{t+1} + \bar{\kappa} x_t, \tag{10}$$

where $\bar{\kappa}$ denotes the central bank's expectation of κ calculated based on the prior distribution of σ. Note that the central bank's subjective expectations, $E_t^{CB} x_{t+1}$ and $E_t^{CB}\pi_{t+1}$, do not coincide with the rational expectations, $E_t x_{t+1}$ and $E_t \pi_{t+1}$ in general.

In a similar fashion, the central bank minimizes the loss function L_t based on its own expectations. The objective function of the central bank in period t can be written as

$$E_t^{CB}[L_t] = \pi_t^2 + \bar{\lambda} x_t^2 + \beta(E_t^{CB}[\pi_{t+1}^2] + \bar{\lambda} E_t^{CB}[x_{t+1}^2]) + \cdots, \tag{11}$$

where $\bar{\lambda}$ denotes the central bank's expectation of λ.[6]

In considering (11), it is helpful to first consider the loss function in the absence of parameter uncertainty, which is given by

$$L_t = \pi_t^2 + \lambda x_t^2 + \beta(E_t[\pi_{t+1}^2] + \lambda E_t[x_{t+1}^2]) + \cdots.$$

In the above standard loss function, the expected squared deviations of future inflation and the output gap from the target, $E_t[\pi_{t+1}^2]$ and $E_t[x_{t+1}^2]$, depend only on the variability of exogenous shocks. Therefore, when the central bank does not face parameter uncertainty (i.e., when it minimizes the objective expected loss function as in standard rational expectation models), these terms do not affect its policymaking.

In contrast, when the central bank faces parameter uncertainty, the terms $E_t^{CB}[\pi_{t+1}^2]$ and $E_t^{CB}[x_{t+1}^2]$ do, in fact, affect its policymaking. To make the point clear, it is convenient to rewrite them as

$$E_t^{CB}[\pi_{t+1}^2] = (E_t^{CB}[\pi_{t+1}])^2 + V_t^{CB}[\pi_{t+1}],$$
$$E_t^{CB}[x_{t+1}^2] = (E_t^{CB}[x_{t+1}])^2 + V_t^{CB}[x_{t+1}].$$

The subjective variance of future inflation and the output gap are given by

$$V_t^{CB}[\pi_{t+1}] = v_\kappa \beta^{-2} x_t^2 + t.i.p., \tag{12}$$

$$V_t^{CB}[x_{t+1}] = v_\sigma \beta\left\{i_t - \beta^{-1}(\pi_t - \kappa_u \omega x_t) - r_t^n\right\}^2 + t.i.p., \tag{13}$$

where v_κ is the subjective variance of κ based on the central bank's prior distribution and "*t.i.p.*"

[6] See the appendix for the detailed derivations of (9), (10) and (11).

denotes terms which are independent of the policy.[7] When the central bank faces parameter uncertainty (i.e., $v_\sigma, v_\kappa > 0$), $V_t^{CB}[\pi_{t+1}]$ and $V_t^{CB}[x_{t+1}]$ depend on the central bank's current choices $\{\pi_t, x_t, i_t\}$. In other words, the central bank can affect its own subjective uncertainty about the future state by controlling its policy in the current period. For example, (12) implies that the conditional variance of the future inflation rate $V_t^{CB}[\pi_{t+1}]$ depends on the choice of the output gap in the current period x_t, so the current policy choice can affect $V_t^{CB}[\pi_{t+1}]$ through changing x_t. To minimize the objective function (11), the central bank now has to consider the effects of the current policy choice on $V_t^{CB}[\pi_{t+1}]$ and $V_t^{CB}[x_{t+1}]$.

In summary, due to the presence of parameter uncertainty, the central bank cares not only about the current level of inflation and the output gap, but also about the distribution of the likely values of these variables in the future. As a result, it starts to avoid policy choices which increase uncertainty about the future, which is mathematically represented by the variance of the subjective distribution of future inflation and output, $V_t^{CB}[\pi_{t+1}]$ and $V_t^{CB}[x_{t+1}]$. Such newly added motivation generates an additional trade-off between inflation and output stabilization, which changes the central bank's policymaking in a non-trivial way.

In each period t, the central bank chooses π_t, x_t, and i_t to minimize (11) subject to (9), (10), (12), (13) and the ELB constraint.[8] The optimality conditions of the central bank can be written as

$$\pi_t + \phi_{2t} - v_\sigma \bar{\lambda} \left\{ i_t - \beta^{-1}(\pi_t - \kappa_u \omega x_t) - r_t^n \right\} = 0, \tag{14}$$

$$\begin{aligned} &\bar{\lambda} x_t + \left[1 - \bar{\sigma}\beta^{-1} \left\{ \bar{\kappa} - \kappa_u(\omega + \bar{\sigma}^{-1}) \right\} \right] \phi_{1t} - \bar{\kappa}\phi_{2t} \\ &\qquad + v_\kappa \beta^{-1} x_t + v_\sigma \bar{\lambda} \kappa_u \omega \left\{ i_t - \beta^{-1}(\pi_t - \kappa_u \omega x_t) - r_t^n \right\} = 0, \end{aligned} \tag{15}$$

$$\bar{\sigma}\phi_{1t} - \phi_{3t} + v_\sigma \beta \bar{\lambda} \left\{ i_t - \beta^{-1}(\pi_t - \kappa_u \omega x_t) - r_t^n \right\} = 0, \tag{16}$$

$$i_t \phi_{3t} = 0, \ i_t \geq 0, \ \phi_{3t} \geq 0, \tag{17}$$

where ϕ_{1t} is the Lagrange multiplier associated with (9), ϕ_{2t} with (10), and ϕ_{3t} with the ELB constraint (6). These conditions and the equations (1) to (3) lead to a Markov-Perfect equilibrium of the model.

Note that if $\bar{\sigma} = \sigma$ and $v_\sigma = 0$, (14) to (17) are reduced to the following well-known targeting rule of the discretionary central bank

$$i_t(-\lambda x_t + \kappa \pi_t) = 0, \ i_t \geq 0, \ -\lambda x_t + \kappa \pi_t \geq 0. \tag{18}$$

Structural parameters		
Discount factor	β	0.99
Intertemporal elasticity of substitution	σ	1
Inverse of labor supply elasticity	ω	0.47
Price elasticity of demand	θ	10
Share of firms keeping prices unchanged	α	0.8
Shock process		
Steady state value of natural real rate	r^n	$1/\beta$
AR-coefficient of natural real rate shocks	ρ_r	0.8
S.d. of natural real rate shocks (annual)	σ_r	$\frac{0.6}{100}$
Central bank's prior distribution of σ		
mean	$\bar{\sigma}$	1
variance	v_σ	[0, 0.25]

Table 1: Baseline calibration

3 Main Results

Table 1 summarizes the parameter values used for the numerical exercises. The setting here basically follows Nakata and Schmidt (2019b), which calibrates a similar model using U.S. data for the period 1984 Q1 to 2016 Q4.

It is well known that if one employs a traditional calibration, where the target is usually pre-2007 data, the probability of the nominal interest rate hitting the ELB results in less than 5% in almost all existing DSGE models. Several studies argue that the figure is too small given that the major economies have had many recent episodes of hitting the ELB.[9] In the baseline case, the persistence and standard deviation of the natural real rate shock are set to $\rho_r = 0.8$ and $\sigma_r = \frac{0.6}{100}$, respectively. The implied unconditional probability of the ELB constraint binding in the model is 20%, which is still lower than the 30% estimated by Hills et al. (2019) using U.S. data for the period 1996 Q1 to 2019 Q2.[10] Given the difficulty of determining a reasonable figure due to the scarcity of ELB episodes, the results for different settings are checked in the sensitivity analysis section.

The true value of the uncertain parameter σ is set to 1, the figure most commonly used in the

[7]See the appendix for the detailed derivations of (12) and (13).

[8]See the appendix for further details.

[9]Regarding the risk of underestimating the probability of hitting the ELB , see Reifschneider and Williams (2000) and Chung et al. (2012) for examples.

[10]The probability is computed by simulating the model 2000 times under the calibration in Table 1. Each simulation consists of 1,100 periods with the first 100 periods discarded as burn-in periods. Since the value of v_σ has little effect on the probability, it is fixed at 0 throughout the simulation.

macroeconomics literature. Throughout the paper, the central bank's subjective distribution of σ is set to a normal distribution with mean $\bar{\sigma} = 1$ and variance $v_\sigma \in [0, 0.25]$. When $v_\sigma = 0.25$, the central bank in the model believes that there is a 95% chance that the true value of σ falls within the interval $[1 - 1.96 \cdot \sqrt{0.25}, 1 + 1.96 \cdot \sqrt{0.25}] = [0.02, 1.98]$. Since the focus of this paper is not on the error of the parameter value but rather the uncertainty about it, the analysis focuses on the case where $\sigma = \bar{\sigma}$, which implies that the expectation of the central bank about σ is correct on average.

3.1 Parameter uncertainty and the deflationary bias

The main result of this paper is related to the "risky steady state" of the economy. Both the risky steady state and the deterministic steady state are defined as the position at which the economy will settle if it has not been hit by exogenous shocks for a sufficiently long time. However, the risky steady state differs from the deterministic steady state in the sense that the agents in the economy internalize the risk of future realizations of shocks in their actions.[11] While these two concepts of the steady state coincide in linear models, the difference matters in non-linear models such as the one in this paper.

In the present model, the wedge between the risky steady state and the deterministic steady state arises due to the occasionally binding ELB constraint. The deterministic steady state of the model is efficient in the sense that the inflation rate and the output gap coincide with the targets of the central bank. On the other hand, the inflation rate (the output gap) becomes inefficiently low (high) in the risky steady state. Private agents in the model correctly anticipate that even if the interest rate is currently above the ELB, when a large negative shock arises in the future, the ELB constraint will bind and the economy will run into a severe recession. The possibility of hitting the ELB in the future reduces their inflation expectations today. Since these reduced inflation expectations are isomorphic to a negative cost-push shock, the central bank cuts the nominal rate while facing a trade-off between inflation and output stabilization, which leads to lower inflation and higher output than the targets in the risky steady state. Nakov (2008) terms this phenomenon the "deflationary bias".

Figure 1 shows how the degree of the central bank's parameter uncertainty v_σ affects the size of the deflationary bias in the economy. The blue solid line indicates the annualized inflation rate (the left panel) and the output gap (the right panel) in the risky steady state of the model, respectively. The risky steady state is calculated by simulating the model for 5000 periods while setting the realization of exogenous shocks to zero. As explained, both the inflation rate and the output gap in the deterministic steady state coincide with the central bank's target levels, both of which are 0. Therefore, the reported inflation and output in Figure 1 represent the wedge between the deterministic and risky steady states caused by the risk of hitting the ELB. In other words, the

[11] For the formal definition of the risky steady state, see Coeurdacier et al. (2011).

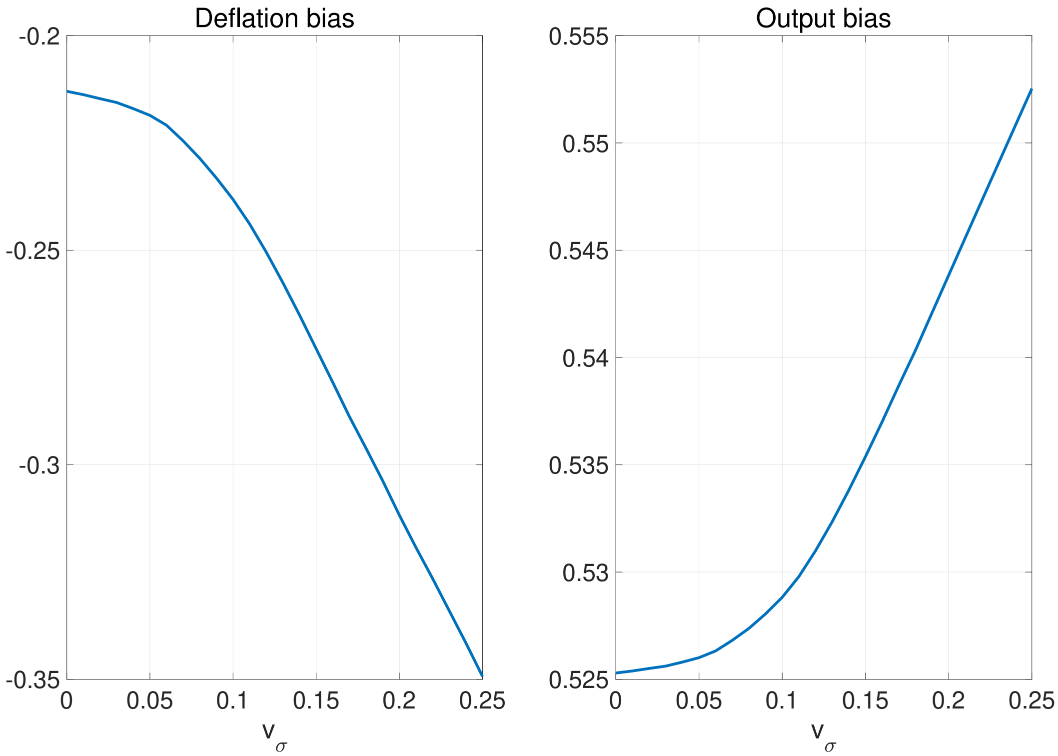

Figure 1: Parameter uncertainty and deflationary bias

figure shows the size of the deflationary bias caused by parameter uncertainty.

The figure gives the main result of this paper: the size of the deflationary bias becomes larger as the degree of the central bank's parameter uncertainty becomes larger. When the central bank faces parameter uncertainty, lower inflation and higher output gap are realized even without exogenous shocks. In other words, parameter uncertainty makes the inflation rate in the economy permanently lower in the presence of the ELB constraint. Without parameter uncertainty ($v_\sigma = 0$), inflation is -0.21% and the output gap is 0.53% in the risky steady state. With parameter uncertainty ($v_\sigma = 0.25$), inflation is -0.35% and the output gap is 0.55%. The presence of parameter uncertainty results in a 14 basis point fall in inflation, which is sizable given its small standard deviation.

The larger deflationary bias caused by parameter uncertainty also brings changes to the short-run responses of equilibrium inflation and output to exogenous shocks, as shown in Figure 2. The blue solid and black dashed lines indicate the equilibrium responses of inflation and output to the natural real rate with and without parameter uncertainty, respectively. The dashed vertical line indicates the steady state value of the natural real rate.

With parameter uncertainty, the inflation rate falls more severely when the natural real rate falls due to negative exogenous shocks (the left panel of Figure 2). To understand the mechanism, it is useful to consider the situation where the natural real rate is low in the current period, but

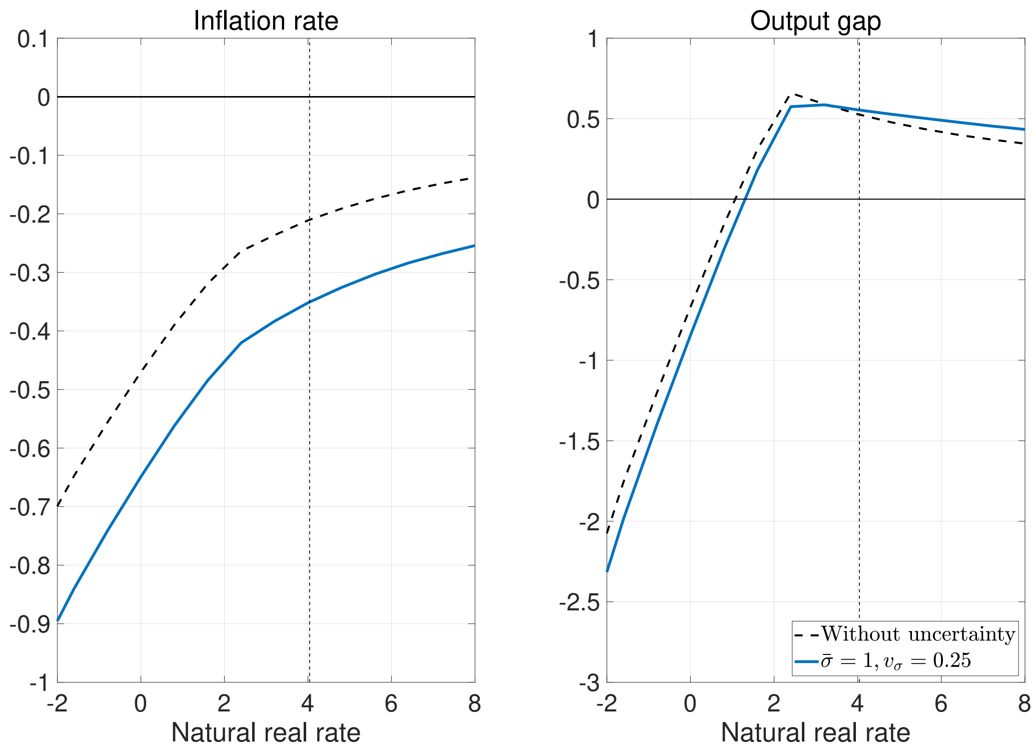

Figure 2: Equilibrium response to the natural rate with and without parameter uncertainty

expected to recover towards the steady state value in the near future. Such anticipation increases the inflation expectations of the private agents since, if these expectations are realized, inflation will actually increase. However, due to the lower inflation in the risky steady state with parameter uncertainty, the increase in inflation expectations becomes smaller than in the absence of parameter uncertainty. At the same time, such lower inflation expectations lead to a higher real interest rate and so to a lower output gap today (see the right panel of Figure 2). Both the lower output gap and inflation expectations today result in lower inflation today according to the New Keynesian Phillips curve (3).

Following the existing literature, I express the social welfare cost of the economy in terms of the perpetual consumption loss as a share of its steady state value. The welfare measure is calculated as

$$W = (1-\beta)\frac{\varepsilon}{\kappa_u}E[L], \tag{19}$$

where the unconditional expectation E here is taken with respect to the unconditional distribution of r_t^n.[12] For the calculation, I conduct 2000 simulations, each of which considers 1100 periods with

[12]For a detailed derivation of the welfare measure (19), see Nakata and Schmidt (2019a).

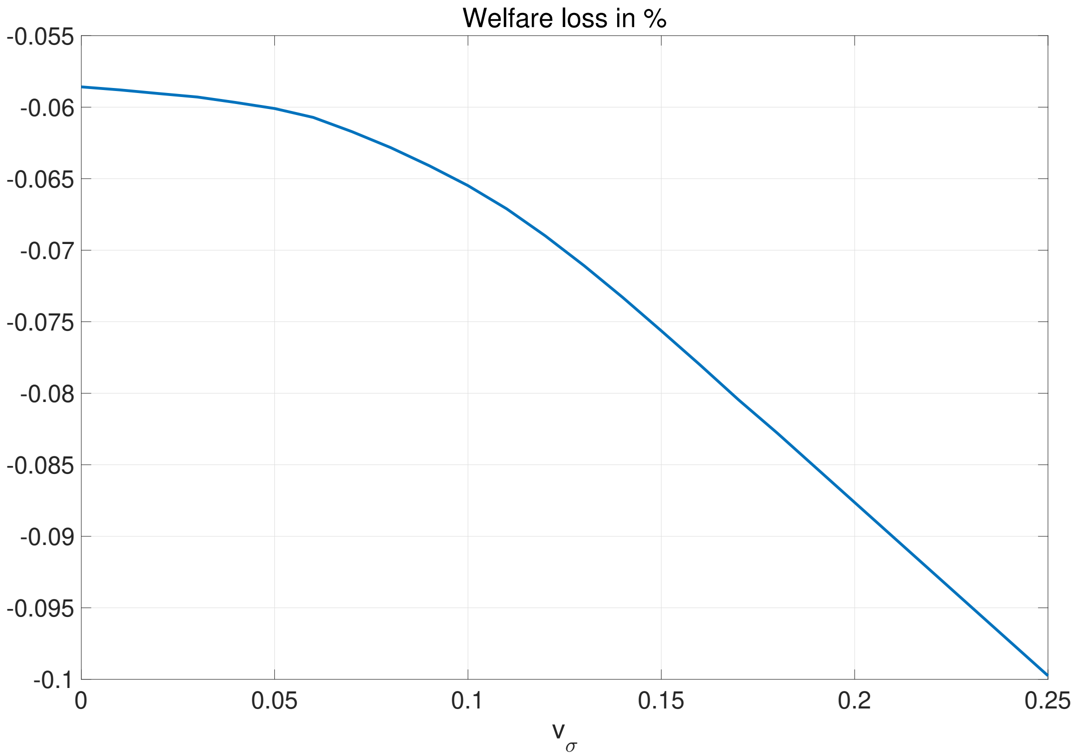

Figure 3: Welfare effects of parameter uncertainty

the first 100 periods discarded as burn-in periods.

The solid line in Figure 3 plots the welfare loss for alternative values of v_σ over $v_\sigma \in [0, 0.25]$. An increase in the central bank's uncertainty makes the economy fluctuate around the risky steady state with lower inflation and a higher output gap. On average, lower inflation and a higher output gap are realized and such changes lead to a larger welfare loss. Figure 4, which shows the ergodic distributions of the inflation rate π_t and the output gap x_t in the model, supports this.

3.2 Discussion

Brainard attenuation principle revisited

With parameter uncertainty about σ, the central bank has to be concerned about the subjective variances of the future inflation rate and the future output gap, $V_t^{CB}[\pi_{t+1}]$ and $V_t^{CB}[x_{t+1}]$. These two additional terms change the trade-off which the central bank faces. As explained below, the change makes the central bank conduct less aggressive policy in response to exogenous shocks, in particular to supply-side shocks.

First, through the micro-foundations of the model, uncertainty about σ translates into the slope of the New Keynesian Phillips curve. The equation (12) captures the effect of uncertainty about

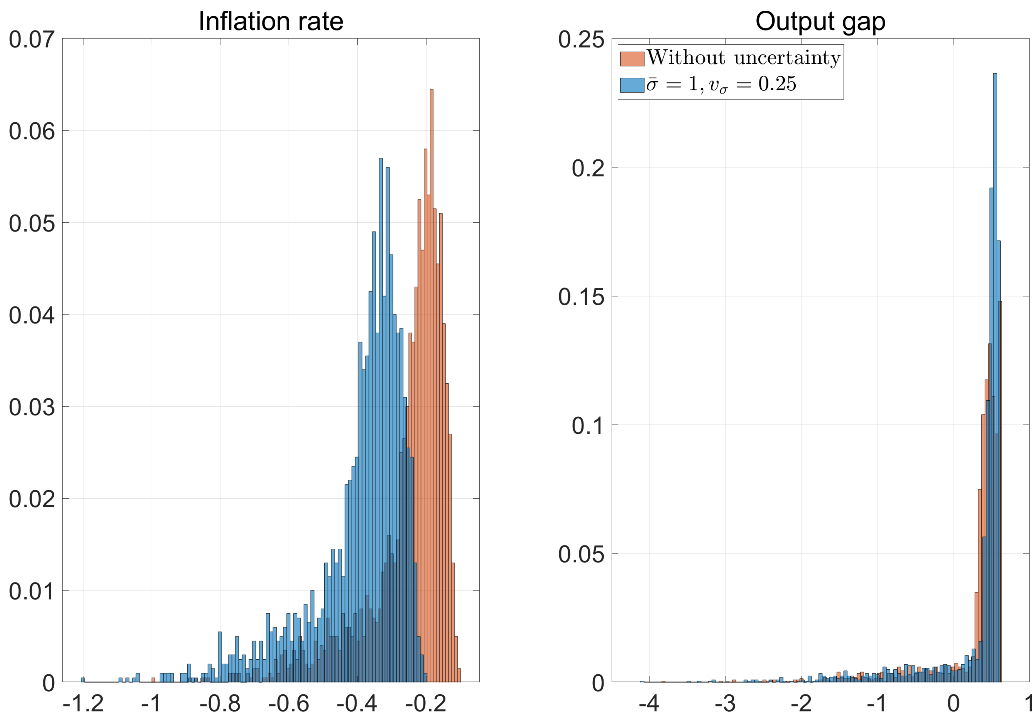

Figure 4: Ergodic distribution with and without parameter uncertainty

the slope of the New Keynesian Phillips curve on the stance of policymaking by the central bank. To illustrate this, suppose that a negative cost-push shock hits the economy. In response to the shock, the central bank cuts the nominal interest rate and allows the output gap to be larger than the target in order to bring the lowered inflation rate back to its target. When the slope of the New Keynesian Phillips curve is uncertain, however, the current output gap far from the target (larger x_t in (12)) makes the prediction of future inflation more difficult (see (8)) and increases the subjective variance of future inflation ($V_t^{CB}[\pi_{t+1}]$ in (12)). This makes the central bank more hesitant to allow output to deviate from the target than in the absence of parameter uncertainty. As a result, the central bank attenuates its policy response in the sense that it cuts the nominal interest rate by less and tolerates smaller booms in response to the negative cost-push shock. The same mechanism also holds for any type of supply-side shock, which generates a trade-off between inflation and output stabilization.

On the other hand, the mechanism through (13) is a bit more difficult to interpret. To get a rough intuition, it is convenient to consider the independent effect of σ on the slope of the IS curve by temporarily ignoring uncertainty about κ and λ. When only the slope of the IS curve is uncertain, we can ignore the covariance of σ and $E_t\pi_{t+1}$. Then the equation (9) is simplified to

$$x_t = E_t^{CB} x_{t+1} - \bar{\sigma}(i_t - E_t^{CB}\pi_{t+1} - r_t^n). \tag{20}$$

At the same time, (13) is simplified to

$$V_t^{CB}[x_{t+1}] = v_\sigma \beta \left(i_t - E_t\pi_{t+1} - r_t^n\right)^2 + t.i.p. \tag{21}$$

The above equation implies that the central bank's subjective uncertainty about the future output gap $V_t^{CB}[x_{t+1}]$ increases with the gap between the real interest rate and the natural real rate, $i_t - E_t\pi_{t+1} - r_t^n$. Since it is this gap that households respond to when substituting between their current and future consumption, widening the gap makes the prediction of their future consumption, which is equal to the future aggregate demand in equilibrium, more difficult. Fearful of such increasing uncertainty about future output, the central bank avoids widening the gap in response to exogenous shocks. Given that only supply-side shocks require the central bank to widen the gap between the real rate and the natural real rate, this means that a central bank facing uncertainty about σ attenuates the response to such supply-side shocks.[13]

In summary, because of the increased uncertainty represented by $V_t^{CB}[\pi_{t+1}]$ and $V_t^{CB}[x_{t+1}]$, the central bank conducts a more attenuated policy than in the case without parameter uncertainty in response to supply-side exogenous shocks, which generate a short-run trade-off between inflation and output stabilization.

Attenuated stance towards inflation stabilization and larger deflationary bias

However, private agents see such an attenuated stance from the central bank as analogous to a policy which is conducted based on a modified loss function such as

$$L_t = E_t \sum_{j=0}^{\infty} \beta^j (\pi_{t+j}^2 + \tilde{\lambda} x_{t+j}^2), \quad \tilde{\lambda} > \lambda. \tag{22}$$

In other words, from the perspective of private agents, a central bank that cares about parameter uncertainty about σ looks like a banker who puts a smaller weight on inflation stabilization relative to output stabilization compared to the social optimum. If they observe the central bank's weaker stance towards inflation stabilization, they will expect that even if negative natural real rate shocks occur and their inflation expectations start to fall as they anticipate a higher possibility of the ELB constraint binding in the near future, the central bank's reaction to mitigate this will be milder than in the case without parameter uncertainty. This anticipation strengthens the downward pressure on

[13] On the other hand, the existence of (21) does not change the response of the central bank's policy to demand-driven shocks. Regardless of whether the central bank cares about (21) or not, equating the real rate to the natural real rate is always optimal in dealing with this type of shock.

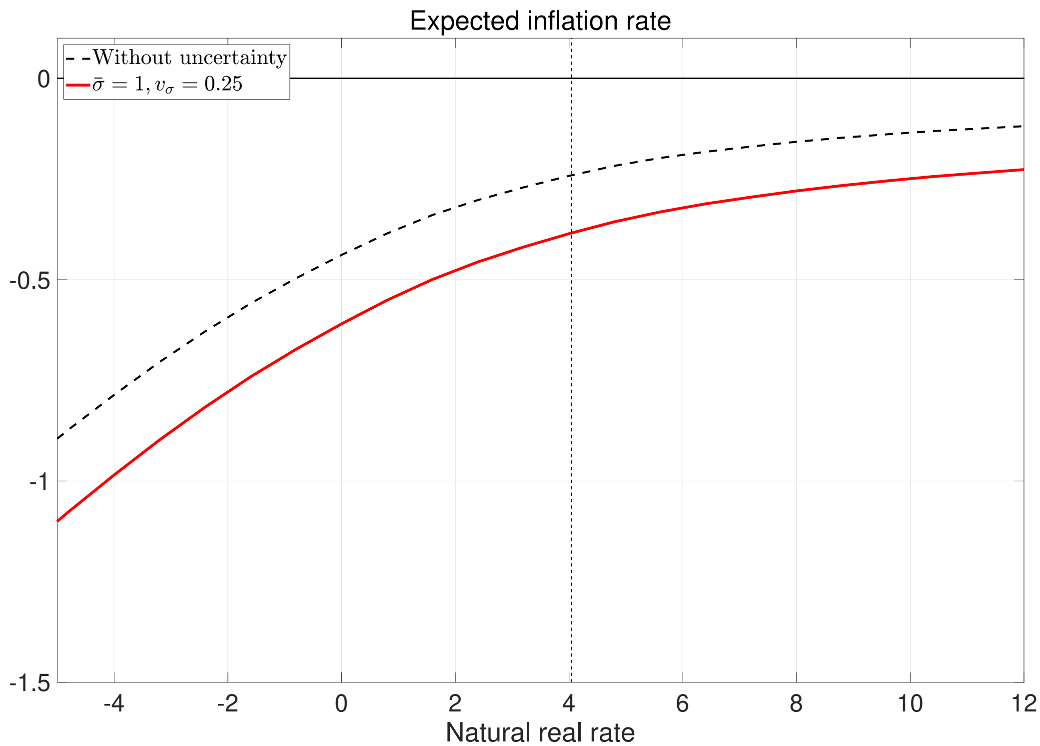

Figure 5: Inflation expectations of private agents

their inflation expectations, which leads to lower inflation. Figure 5 shows the equilibrium responses of the private agents' expectations to the natural real rate without parameter uncertainty (black dashed line) and with parameter uncertainty (red solid line). The dashed vertical line indicates the steady state of the natural real rate. As the figure clearly shows, inflation expectations are always lower when the central bank faces parameter uncertainty.

Through such reduced inflation expectations induced by the attenuated response of the central bank, the presence of parameter uncertainty results in a larger deflationary bias in the economy. The intuition is graphically illustrated in Figure 6. The figure illustrates the condition of the model economy when there are no exogenous shocks and the ELB constraint does not bind. In the figure, the blue line represents the New Keynesian Phillips curve (3). On the other hand, the red line represents the targeting rule of the central bank. The solid and dashed lines represent the case with and without parameter uncertainty, respectively.

Private agents rationally anticipate that when the natural real rate falls sharply due to a large negative shock, the central bank will be unable to avoid the economy running into a severe recession. The possibility of hitting the ELB in the future reduces their inflation expectations even if the nominal interest rate is currently above the ELB. The reduced inflation expectations result in an upward shift of the blue line, the New Keynesian Phillips curve, in Figure 6. Since the

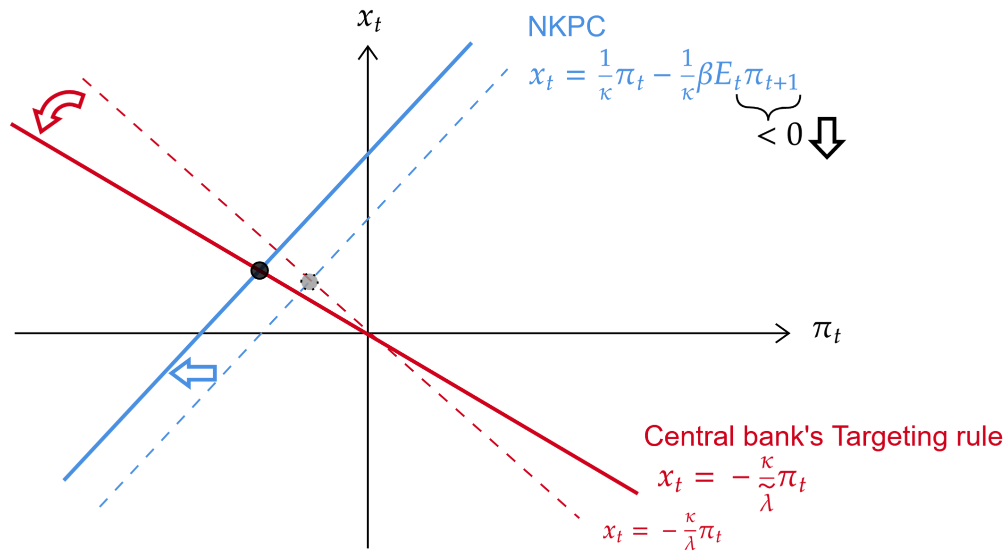

Figure 6: Deflationary bias with parameter uncertainty

discretionary central bank takes such reduced inflation expectations as given, the economy settles at the intersection of the blue line and the red line. The intersection corresponds to the risky steady state of the economy.

As pointed out above, the attenuated response of the central bank to supply-side shocks is analogous to putting a larger weight on the output gap in the loss function, which is shown as the flatter targeting rule (red solid line) in the figure. By correctly anticipating the central bank's attenuated response, the fall in inflation expectations of the private agents becomes more severe than would be in the case without parameter uncertainty. As a result, the New Keynesian Phillips curve shifts upward further (blue solid line) and the intersection moves towards the upper left of the figure, which leads to lower inflation and higher output in the risky steady state.

Nakata and Schmidt (2019a) analytically show that the deflationary bias can be reduced by assigning monetary policy to an inflation-conservative central banker. According to their result, a central bank that puts comparatively more weight on inflation stabilization mitigates the deflationary bias away from the ELB at the cost of a potentially higher output gap. In this context, the result of this paper implies that uncertainty about the effects of policy can lead central banks to take an "anti-inflation-conservative" stance and results in a larger deflationary bias in the economy.

Both during and after periods of decline in the natural real rate, a central bank facing parameter uncertainty moves inflation expectations more aggressively, albeit with the difference of raising or lowering them.

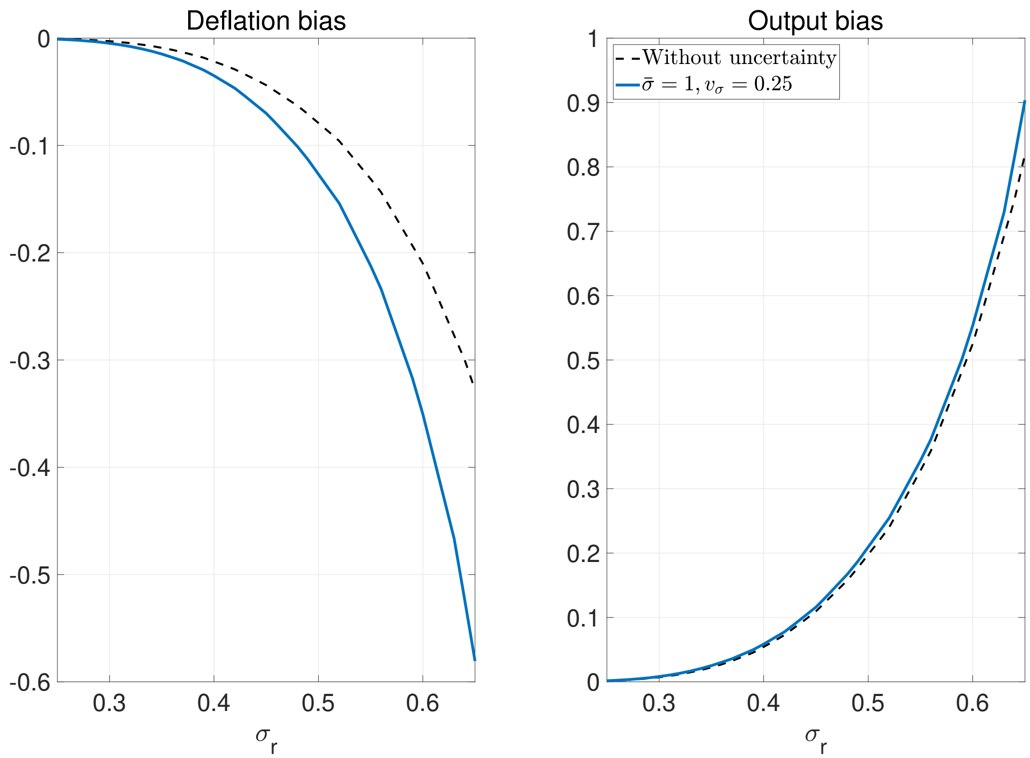

Figure 7: The risky steady state inflation and output with changing σ_r

4 Additional Results

4.1 Sensitivity analysis

Several studies, such as Bianchi et al. (2019) and Seneca (2019), point out that the size of the deflationary bias depends on the probability of the economy being at the ELB, which in turn is affected by the calibration of the shock process in the baseline model. Since almost all major economies have only one ELB episode in their history, it is generally difficult to choose a reasonable value for the probability of hitting the ELB. Therefore, this section reports the results of robustness exercises regarding the standard deviation of exogenous shocks σ_r and the persistence of the natural real rate process ρ_r.

Figure 7 shows how the standard deviation σ_r, affects the risky steady state of the economy in the model. In varying σ_r, the persistence ρ_r is fixed so that the unconditional standard deviation of the level of the natural rate changes along with σ_r. When the standard deviation is smaller than 0.25, the deflationary bias vanishes due to nearly zero probability of hitting the ELB. As the blue and dashed lines in the figure indicate, the deflationary bias is larger in the presence of parameter uncertainty as long as the standard deviation is larger than 0.25. In other words, the effect of parameter uncertainty on the size of the deflationary bias is qualitatively robust against

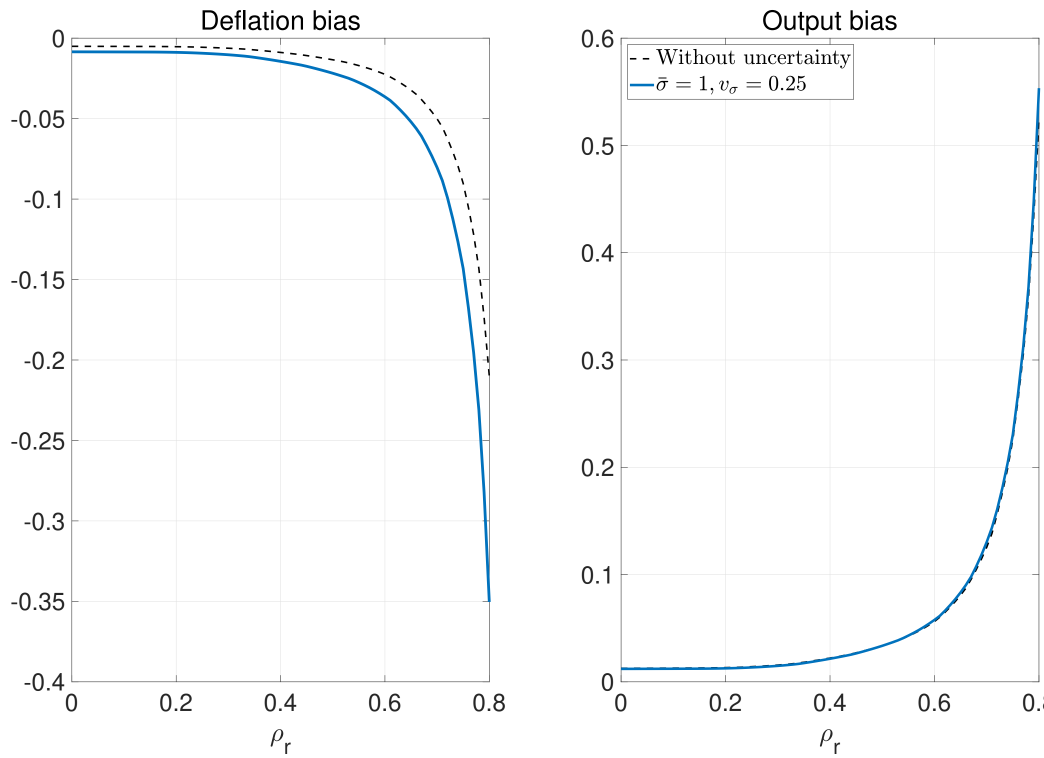

Figure 8: The risky steady state inflation and output with changing ρ_r

different values of the parameters. Quantitatively, as the standard deviation σ_r increases, raising the probability of hitting the ELB, the effect of parameter uncertainty also becomes larger.

The same thing can be said for the persistence parameter of the natural real rate process ρ_r. Figure 8 shows how ρ_r affects the risky steady state of the economy in the model. As the persistence parameter ρ_r increases, the deepening of the deflationary bias induced by parameter uncertainty becomes more severe.

4.2 Two shocks

In the baseline case, the only exogenous disturbance in the model is a shock to the natural real rate. This section reports the results when a cost-push shock is added to the New Keynesian Phillips curve as

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t. \tag{23}$$

The cost-push shock u_t follows an AR(1) process:

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u, \tag{24}$$

	Inflation	Output gap	Welfare loss
Deterministic steady state	0	0	0
Risky steady state			
w/o parameter uncertainty, $v_\sigma = 0$	-0.46	1.16	0.30
(the baseline case)	(-0.21)	(0.53)	(0.06)
with parameter uncertainty, $v_\sigma = 0.25$	-0.56	0.87	0.34
(the baseline case)	(-0.35)	(0.55)	(0.10)

Table 2: The risky steady state and welfare in the cost-push-shocks-augmented model

where $\rho_u \in [0, 1)$ and $\varepsilon_t^u \sim N(0, \sigma_u^2)$. Following the result of Ireland (2011), the persistence and the standard deviation of the cost-push shock are set to $\rho_u = 0$ and $\sigma_u = \frac{0.17}{100}$, respectively.

Table 2 reports the deterministic and the risky steady state values of inflation and the output gap and the resulting welfare loss with and without parameter uncertainty. As in the baseline case, the inflation rate in the risky steady state becomes lower with parameter uncertainty. On the other hand, the output gap in the risky steady state also becomes lower with parameter uncertainty, which is not observed in the baseline case. As a result, the increase in welfare loss due to parameter uncertainty is milder than in the baseline version of the model.

In interpreting the results, it is important to note that the existence of negative, not positive, cost push shocks in the economy is truly related to the level of the output gap in the risky steady state. The wedge between the risky steady state and the deterministic steady state in the model is generated by the possibility of the nominal interest rate hitting the ELB. Since the central bank can raise the nominal interest rate without restriction in response to positive cost-push shocks, the existence of such positive shocks is not directly related to the properties of the risky steady state. In contrast, the likelihood of negative cost push shocks in the economy increases the probability of the ELB constraint binding and results in a larger wedge between the two steady states.

As pointed out in the baseline analysis, in the face of parameter uncertainty the central bank takes an attenuated stance towards inflation stabilization, which means that the central bank hesitates in lowering the nominal interest rate and generating an output boom in response to negative supply-side shocks. Since the baseline version of the model does not include any disturbances other than the shocks to the natural real rate, the only relevant supply-side shock in the model is a fall in inflation expectations due to the possibility of future interest rates reaching the ELB. On the other hand, the model here also includes direct cost-push shocks to the New Keynesian Phillips curve. This additional possibility of negative cost-push shocks results in a larger effect on the attenuated stance induced by parameter uncertainty, which leads to a smaller output gap in the risky steady state.

5 Conclusion

This paper revisits the policymaking of a central bank that faces parameter uncertainty by explicitly considering the existence of the effective lower bound (ELB). The analysis is based on a canonical New Keynesian model with an occasionally binding ELB constraint on the nominal interest rate. The central bank in the model takes a Bayesian approach in the sense that it has a unique prior distribution of the uncertain parameter and minimizes the expected loss based on this parameter distribution.

The main finding is that the attenuated policy stance in the face of parameter uncertainty, which is in line with the Brainard attenuation principle, can make the inflation rate in the economy permanently lower in the presence of the ELB constraint. In particular, such policy attenuation increases the size of the "deflationary bias" in the economy and makes it more difficult for the central bank to achieve its inflation target compared to the case without parameter uncertainty. The message of this paper is that ignoring the existence of the ELB constraint on the nominal interest rate can lead to a significant underestimation of the cost of uncertainty for the central bank.

Appendix

Derivation of (9) and (10)

Note that the realized values of the inflation rate π_t and output gap x_t are included in the information set of the central bank in period t. We then obtain (10) by taking the conditional expectation on both sides of the Phillips curve (3). Similarly, by taking the conditional expectation on both sides of (1), we obtain

$$E_t^{CB} x_t = E_t^{CB} x_{t+1} - \bar{\sigma}(i_t - E_t^{CB}\pi_{t+1} - r_t^n) + Cov_t^{CB}(\sigma, E_t\pi_{t+1}). \tag{25}$$

We can calculate the third term on the right-hand side of the above equation as follows. First, note that

$$\begin{aligned} Cov_t^{CB}(\sigma,\kappa) &= E_t^{CB}(\sigma\kappa) - E_t^{CB}(\sigma)E_t^{CB}(\kappa) \\ &= E_t^{CB}(\sigma\kappa_u(\omega+\sigma^{-1})) - \bar{\sigma}\bar{\kappa} \\ &= \bar{\sigma}\left\{\bar{\kappa} - \kappa_u(\omega+\sigma^{-1})\right\}. \end{aligned} \tag{26}$$

By combining (3) and (26), we get

$$\begin{aligned} Cov_t^{CB}(\sigma, E_t\pi_{t+1}) &= Cov_t^{CB}(\sigma, \beta^{-1}(\pi_t - \kappa x_t)) \\ &= -\beta^{-1} Cov_t^{CB}(\sigma, \kappa) x_t \\ &= \bar{\sigma}\beta^{-1}\left\{\bar{\kappa} - \kappa_u(\omega + \sigma^{-1})\right\} x_t. \end{aligned} \tag{27}$$

Then, by substituting (27) into (25), we obtain (9).

Derivation of (11)

The central bank's objective function in the initial period $t = 0$ can be written as

$$\begin{aligned} E_0^{CB}[L_0] &= E_0^{CB}\left[E_0 \sum_{t=0}^{\infty} \beta^t(\pi_t^2 + \lambda x_t^2)\right] \\ &= E_0^{CB}\left[E_0^{CB}(\pi_0^2 + \lambda x_0^2) + \sum_{t=1}^{\infty} \beta^t E_{t-1}^{CB}(\pi_t^2 + \lambda x_t^2)\right] \\ &= E_0^{CB}\left[\pi_0^2 + \bar{\lambda} x_0^2 + \sum_{t=1}^{\infty} \beta^t \left[(E_{t-1}^{CB}[\pi_t])^2 + V_{t-1}^{CB}[\pi_t] + \bar{\lambda}\left\{(E_{t-1}^{CB}[x_t])^2 + V_{t-1}^{CB}[x_t]\right\}\right]\right] \\ &= \pi_0^2 + \bar{\lambda} x_0^2 + \beta(V_0^{CB}[\pi_1] + \bar{\lambda} V_0^{CB}[x_1]) \\ &\quad + E_0^{CB} \sum_{t=1}^{\infty} \beta^t \left[(E_{t-1}^{CB}[\pi_t])^2 + \bar{\lambda}(E_{t-1}^{CB}[x_t])^2 + \beta\left\{V_t^{CB}[\pi_{t+1}] + \bar{\lambda} V_t^{CB}[x_{t+1}]\right\}\right]. \end{aligned}$$

We can easily derive the objective function in the discretionary policy problem as (11).

Derivation of (12) and (13)

We can rewrite (1) as

$$x_{t+1} = x_t + \sigma(i_t - \pi_{t+1} - r_t^n) + \sigma\epsilon_{t+1}^{\pi} + \epsilon_{t+1}^{x}, \tag{28}$$

where $\epsilon_{t+1}^{\pi} \equiv \pi_t - E_t\pi_{t+1}$ and $\epsilon_{t+1}^{x} \equiv x_t - E_t x_{t+1}$, respectively. Similarly, we can rewrite (3) as

$$\pi_{t+1} = \beta^{-1}(\pi_t - \kappa x_t) + \epsilon_{t+1}^{\pi}. \tag{29}$$

By combining (25), (28) and (29), we obtain

$$x_{t+1} - E_t^{CB} x_{t+1} = (\sigma - \bar{\sigma})\left\{i_t - \beta^{-1}(\pi_t - \kappa_u \omega x_t) - r_t^n\right\} + \epsilon_{t+1}^{x}.$$

Then, we can obtain (13) as

$$\begin{aligned} V_t^{CB}[x_{t+1}] &= E_t^{CB}(x_{t+1} - E_t^{CB} x_{t+1})^2 \\ &= v_\sigma \beta \lambda \left\{ i_t - \beta^{-1}(\pi_t - \kappa_u \omega x_t) - r_t^n \right\}^2 + t.i.p. \end{aligned}$$

Similarly, subtracting (10) from (29) leads to

$$\pi_{t+1} - E_t^{CB} \pi_{t+1} = -\beta^{-1}(\kappa - \bar{\kappa}) x_t + \epsilon_{t+1}^{\pi}$$

and from this, we obtain (12) as

$$\begin{aligned} V_t^{CB}[\pi_{t+1}] &= E_t^{CB}(\pi_{t+1} - E_t^{CB} \pi_{t+1})^2 \\ &= v_\kappa \beta^{-2} x_t^2 + t.i.p. \end{aligned}$$

Equilibrium conditions under discretionary policy

The period t Lagrangian for the loss minimization problem is given by

$$\begin{aligned} \mathcal{L}_t^D = -\frac{1}{2} & \left\{ \pi_t^2 + \bar{\lambda} x_t^2 + \beta (V_t^{CB}[\pi_{t+1}] + \bar{\lambda} V_t^{CB}[x_{t+1}]) \right\} \\ & - \phi_{1t}[x_t - E_t^{CB} x_{t+1} + \bar{\sigma}(i_t - \pi_{t+1} - r_t^n) - \bar{\sigma}\beta^{-1} \left\{ \bar{\kappa} - \kappa_u(\omega + \sigma^{-1}) \right\} x_t] \\ & - \phi_{2t}(\pi_t - \beta E_t^{CB} \pi_{t+1} - \bar{\kappa} x_t) \\ & + \phi_{3t} i_t \end{aligned}$$

where ϕ_{1t} is the Lagrange multiplier associated with (9), ϕ_{2t} with (10), and ϕ_{3t} with the ELB constraint (6).

Substituting (12) and (13), we can see that the Kuhn-Tucker conditions are

$$\frac{\partial \mathcal{L}_t}{\partial \pi_t} = -\pi_t - \phi_{2t} + v_\sigma \bar{\lambda} \left\{ i_t - \beta^{-1}(\pi_t - \kappa_u \omega x_t) - r_t^n \right\} = 0, \quad (30)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial x_t} &= -(\bar{\lambda} + v_\kappa \beta^{-1}) x_t - \left[1 - \bar{\sigma}\beta^{-1} \left\{ \bar{\kappa} - \kappa_u(\omega + \sigma^{-1}) \right\} \right] \phi_{1t} + \bar{\kappa} \phi_{2t} \\ & \quad - v_\sigma \bar{\lambda} \kappa_u \omega \left\{ i_t - \beta^{-1}(\pi_t - \kappa_u \omega x_t) - r_t^n \right\} = 0, \end{aligned} \quad (31)$$

$$\frac{\partial \mathcal{L}_t}{\partial i_t} = -\bar{\sigma} \phi_{1t} + \phi_{3t} - v_\sigma \beta \bar{\lambda} \left\{ i_t - \beta^{-1}(\pi_t - \kappa_u \omega x_t) - r_t^n \right\} = 0, \quad (32)$$

$$i_t \phi_{3t} = 0, \; i_t \geq 0, \; \phi_{3t} \geq 0. \quad (33)$$

As a result, the equilibrium conditions can be written as (14), (15), (16), and (17).

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