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Abstract

A tighter monetary policy is generally associated with higher real interest rates on deposits and loans, weaker performance of equities and real estate, and slower growth in employment and wages. How does a household's exposure to monetary policy vary with its age? The size and composition of both household income and asset portfolios exhibit large variation over the lifecycle in Japanese data. We formulate an overlapping generations model that reproduces these observations and use it to analyze how household responses to monetary policy shocks vary over the lifecycle. Both the signs and the magnitudes of the responses of a household's net worth, disposable income and consumption depend on its age.

Keywords: Monetary policy; Lifecycle; Portfolio choice; Nominal government debt
JEL classification: E52, E62, G51, D15

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1 Introduction

Monetary policy is generally thought to operate through financial markets. A higher policy interest rate increases the real interest rate on liquid assets like deposits and bonds and increases borrowing costs. These changes in turn have a negative effect on the performance of the stock and real estate markets and eventually depress aggregate consumption, investment, output and employment. Formal empirical evidence supporting this perspective is provided by the structural vector autoregression (SVAR) literature on monetary policy shocks (see e.g. Christiano et al. (2005)).

Given that the size and composition of household disposable income and wealth differs across households, it is likely that they have heterogeneous exposures to monetary policy shocks. Auclert (2019) raises this possibility and decomposes individual exposures to monetary policy into income, unexpected inflation, real interest rate exposure, and intertemporal substitution components. Doepke and Schneider (2006) provide evidence that households have heterogenous and in some cases large exposures to the unexpected inflation component. Cloyne et al. (2020) and Holm et al. (2020) provide empirical evidence that the real interest rate exposure component is important. They document large cross-sectional variation in the size and composition of individual disposable income and wealth and provide evidence that the sign and magnitude of households' consumption response to an identified monetary policy shock depends on the size and composition of their financial and real asset portfolios.

Considerable progress has been made in building quantitative structural monetary models that reproduce key cross-sectional observations about earnings and wealth inequality (see e.g. Kaplan et al. (2018) and Bayer et al. (2019)). The main source of heterogeneity in quantitative heterogeneous agent New Keynesian (HANK) models is idiosyncratic earnings risk. Individuals are ex ante identical and they save to self-insure against the risk of a long sequence of low earnings.

Uninsured earnings risk is one of the main sources of earnings inequality. However, it may be less important for understanding the distributional effects of changes in monetary policy. Recent empirical evidence by Kehoe et al. (2020) suggests that fixed characteristics such as age, educational attainment, and gender may be more important than uninsured earnings risk in understanding the differential exposure of individuals to monetary policy shocks. Age is a fixed characteristic and we find that household portfolio and income exposures to monetary policy exhibit large variation over the lifecycle. According to our results a surprise tightening in monetary policy is bad news for a 31 year old working-aged household but good news for a 71 year old household.

We start by providing empirical evidence that both the size and composition of household portfolios of financial and real assets exhibit large variation over the lifecycle. In Japanese household level data, young working-age households have low net worth and hold leveraged long positions in illiquid assets such as homes and durable goods whereas older-working aged households hold large and positive amounts of both liquid and illiquid assets.

The sources of household income also vary systematically with age in Japan. Age-wage profiles are hump-shaped and most employees in both the private and public sectors have a mandatory retirement age. Retirees have no direct exposure to labor market risk and their income consists of asset income from their savings and public pensions.

Next we propose an overlapping generations (OG) model that captures how wealth and income vary over the lifecycle. Households can borrow or lend liquid and illiquid assets but illiquid assets are costly to adjust. Young working-age households have low initial resources and face a hump-shaped labor efficiency profile and face mandatory retirement at the age of 68. Retirees receive public pensions and have a maximum lifespan. Finally, individuals face idiosyncratic mortality risk that is increasing in age.

Government debt is a nominal liability that is held constant in all periods and the monetary authority sets the nominal interest rate on government debt. Finally, the model includes nominal price rigidities as in Rotemberg (1996). These elements help the model to produce empirically relevant responses for the key variables that govern a household's overall exposure to monetary policy. A surprise increase in the nominal interest rate lowers wages, increases the real return on savings and real borrowing costs, lowers stock prices and increases the real value of outstanding government debt.

We find that the sign and magnitude of disposable income and consumption responses depend on the age of the household. In response to a tighter monetary policy, retirees aged 85+ experience a decline in disposable income and reduce their consumption. Households in this group have low wealth and short planning horizons so their marginal propensity to consume (MPC) is large. However, disposable income and consumption increase for households aged between 57 and 81. They have high wealth and the return on their diversified portfolio of assets increases. They prefer to save most of the income bonus and their MPCs are small in the impact period. Younger working-age households hold leveraged long positions in illiquid assets. The shock results in lower asset income from their portfolio and lower labor income. So consumption also falls for members of this group.

A surprise tightening in monetary policy also influences how much wealth a household accumulates over its remaining lifetime. For example, households aged 71 adjust their asset portfolio in a way that allows them enjoy more consumption at older ages and their consumption response is hump-shaped. A household that experiences the same shock at age 31, in contrast, reduces its consumption most on impact but then experiences persistently low consumption for the remainder of its life.

The pattern of exposures to monetary policy in our model implies that a tighter monetary policy increases both wealth and consumption inequality. High wealth households close to retirement see their wealth increase. Young working-aged households and older retirees are less affluent and see their wealth fall. Given that the highest wealth households have the lowest MPCs, it follows that consumption inequality increases by more than wealth inequality.

Our paper has ties to the large but primarily theoretical literature on monetary policy in OG models as in Wallace (1980). Several recent papers, including Hu et al. (2019) and Sterk and Tenreyro (2018), have analyzed monetary policy in tractable OG models. We propose a quantitative OG model that allows us to analyze the empirical relevance of mechanisms such as real assets and asset substitution that have been analyzed in this previous literature.

Previous research by Wong (2019) finds that a shock to monetary policy has different effects on the size, but not the sign, of consumption of different age groups in the U.S. In our model, not only the size of the consumption response but also its sign varies over the lifecycle. Our result is consistent with empirical evidence in Cloyne et al. (2020) for the U.S. and the U.K. and Holm et al. (2020) for Norway. The peak response of nondurable consumption expenditures of outright homeowners is smaller and has a different sign as compared to homeowners with mortgages in Cloyne et al. (2020). Holm et al. (2020) find that households who are net borrowers reduce their consumption in response to a tighter monetary policy but that households who are creditors increase their consumption.

Hagedorn (2018) has emphasized that monetary and fiscal policy jointly determine the price level in a class of economies where Ricardian equivalence fails. Our OG model falls into this class of models. Monetary policy changes the price level which in turn alters the real value of government debt and net wealth. Our results suggest that the sign and magnitude of the net wealth effect depends on the age of the household.

Finally, our results also have implications for the response of macroeconomic variables to monetary policy in HANK frameworks. Broer et al. (2020) and Kaplan et al. (2018)

find that countercyclical profits induce countercyclical responses in investment and hours in their HANK models and propose particular remedies. In our model wages are flexible and there are no restrictions on how households allocate dividends from profits between liquid and illiquid assets. Moreover, profits increase when monetary policy is tightened but investment and hours fall.

2 Motivation

One reason we focus our analysis on Japan is because nationally representative survey data is available on household portfolios of real and financial assets by age group. The Japanese National Survey of Family Income and Expenditure (NSFIE), which is conducted every five years, provides detailed information on household holdings of real assets including real estate and durable goods as well as a range of financial asset categories by 10-year age group.

Table 1 reports our imputations of household net worth and holdings of liquid and illiquid assets by age group using data from the 2014 NSFIE. Age refers to the age of the household head and asset holdings are relative to peak pretax income over the lifecycle, which occurs in the group aged 50–59. Liquid assets include liquid securities like deposits and illiquid assets consist of less liquid real and financial assets like residential real estate and equity. Our strategy for classifying assets into these two categories follows Kaplan et al. (2018) and Appendix C provides specific details on how we implement this strategy on the NSFIE. We do want to point out that all loans are assigned to liquid assets in Table 1. Liquid asset holdings are net of all borrowing and illiquid asset holdings are gross. This way of organizing the data facilitates comparison with results from our model.[1]

The age-asset profile in the NSFIE illustrates that there is large variation in both the size and composition of net worth over the lifecycle. There are four key properties of the data. First, net worth is hump-shaped. It increases steadily during working ages, peaking at over 6 times peak income. Full public pensions become available at age 65 and the 60–69 age group has the highest net worth. Net worth then falls during retirement as older households draw down their savings. Second, households aged 49 or below have negative net holdings of liquid assets but positive holdings of illiquid assets. In other words, they are taking leveraged long positions in illiquid assets. Third, older working-age individuals

[1] In Japan most mortgages have adjustable interest rates and the costs of prepaying a mortgage in Japan are small. So it is not unreasonable to treat a mortgage as a liquid liability.

Table 1: Household net worth, liquid and illiquid asset holdings by age in Japan

Age	Net Worth	Liquid assets	Illiquid assets
Under 30	0.65	-0.08	0.73
30–39	1.60	-0.58	2.18
40–49	2.58	-0.31	2.90
50–59	4.52	0.76	3.76
60–69	6.29	1.70	4.60
70+	6.01	1.77	4.25

Note: Liquid assets are net of total borrowing and illiquid assets are gross. Net worth and assets are divided by peak income of the 50–59 year old age group. The main data source is the 2014 NSFIE. More details on the construction of the data can be found in Appendix C.

and retirees have positive holdings of both liquid and illiquid assets. Fourth, individuals maintain their asset holdings until late in life.

Another reason why households risk exposure to monetary policy varies with age is because the composition of their income depends on the age of the household. Unfortunately, the publicly available data on real and financial assets by age-group in the 2014 NSFIE doesn't provide a detailed breakdown of sources of income. However, wages also exhibit significant variation over the lifecycle (see Braun and Joines (2015)). In addition, Japanese law allows firms to impose mandatory retirement ages and most employees are subject to mandatory retirement at either 55 or 60 years of age. Companies are required to offer to re-employ workers after mandatory retirement until they can qualify for public pension benefits at the age of 65. But, salary and fringe benefits of these fixed-term re-employment contracts are often less attractive. A recent supreme court ruling in 2019 largely reaffirmed this practice.[2]

A tighter monetary policy is conventionally thought to increase the real return on liquid securities, increase real borrowing costs, and to have a negative impact on the performance of equity and real estate. If this conventional wisdom is correct, then the results reported in Table 1 suggest that the impact of a tighter monetary policy could vary systematically over the lifecycle. Our next step is to construct a model that we can use to analyze how shocks to monetary policy impact the situation of households at different stages of their lifecycle.

[2] An English language summary of the mandatory retirement conventions in Japan and the 2019 Supreme Court ruling can be found in Puckett (2019).

3 Model

The model we propose strikes a compromise in terms of its microeconomic detail. On the one hand, it captures the main sources of income and savings over the lifecycle. On the other hand, it abstracts from mechanisms like uninsured earnings risk that produce cross-sectional heterogeneity within an age group. This simplification allows us to readily discern how monetary policy affects households at different stages of the lifecycle.

We consider an OG economy with representative cohorts along the lines of Braun et al. (2009) and Braun and Joines (2015) and extend these papers in two ways. First, households can save and/or borrow two assets that differ in terms of their liquidity services. Illiquid assets such as homes and equity offer a high return but are costly to acquire and sell. Liquid assets offer a lower return but are costless to adjust. Depending on where households are in their lifecycle, they choose to borrow liquid assets to purchase illiquid assets or hold positive amounts of both assets. Second, the model includes nominal price rigidities and a central bank that pursues a nominal interest rate rule.

3.1 Demographic structure

The economy has an OG structure that evolves in discrete time with a period length of one year. Let j denote the age of the individual as $j = 1, ..., J$. We start keeping track of individuals at age 21 and individuals survive until at most age 120. Thus, model age of $j = 1$ corresponds to age 21, model age $J = 100$ corresponds to age 120 and $J = 100$ cohorts are active in a given year. We consider an economy with a stationary population distribution and no population growth. Let $N_{j,t}$ be the number of individuals of age j in period t, then $N_{j,t} = N_j$ for all t. Then the aggregate population is $N = \sum_{j=1}^{J} N_j$. The number of individuals of each age is defined recursively from conditional survival probabilities as $N_{j+1} = \psi_j N_j$, where ψ_j is the conditional probability that an individual of age j survives to the next year.[3]

3.2 Households

Individuals are organized into households. Each household consists of one individual (adult) and children. The number of children varies with the age of the adult and the age of the

[3] Note also that the unconditional probability of surviving from birth to age $j = 2, ..., J$ is given by $\Psi_j = \psi_{j-1}\Psi_{j-1}$ where $\Psi_1 = 1$ and that the population share of each cohort is given by $\mu_j = \frac{\Psi_j}{\sum_{j=1}^{J} \Psi_j}$.

household is indexed by the age of the adult. Adults face mortality risk and have no bequest motives.[4] At the beginning of each period the adult learns whether she will die at the end of the current period as in Braun et al. (2016). Let $z^i_{j,t} \in \{0, 1\}$ index the survival state for the i^{th} household where a value of zero denotes the death state.[5] Under our assumption, households consume all of their resources in their final year of life and there are no accidental bequests. This, in turn, reduces the costs of death and helps the model to reproduce the empirical observation that households retain wealth until late in life. Death is the only source of idiosyncratic risk faced by households and there are only two types of households in any cohort: surviving households ($z^i_{j,t} = 1$) and non-surviving households ($z^i_{j,t} = 0$).

Households work, consume, and save for retirement. A household of age j in period t earns an after-tax wage rate of $(1 - \tau^w)w_t\epsilon_j$, where τ^w denotes a labor-income tax rate and ϵ_j is the efficiency of labor of an age-j household.[6] All cohorts face the same age-efficiency profile and the efficiency index ϵ_j is assumed to drop to zero for all $j \geq J_r$, where J_r is the mandatory retirement age.

Households provision for retirement by acquiring liquid and illiquid assets. They may save or borrow using either asset and the liquid asset is nominally denominated because monetary policy directly controls the nominal interest rate on liquid assets in this economy. The liquid asset earns the nominal interest rate R_{t-1} between period $t-1$ to t and its after-taxed real return is given by $\tilde{R}_{t-1}/\pi_t$, where $\tilde{R}_{t-1} = 1 + (1 - \tau^a)(R_{t-1} - 1)$. The real return on illiquid assets in period t is R^a_t and its after-taxed return is $\tilde{R}^a_t = 1 + (1 - \tau^a)(R^a_t - 1)$.

From the perspective of the household the only distinction between liquid and illiquid assets is that households face costs of adjusting their holdings of illiquid assets as in Aiyagari and Gertler (1991) and Kaplan and Violante (2014). When we parameterize our model, we follow Kaplan et al. (2018) and include real assets such as homes and durable goods and illiquid financial assets such as equities in our measure of illiquid assets. So the adjustment costs can be interpreted as representing service flows to the financial service sector when, for instance, a household purchases or sells a home. Following Kaplan et al. (2018), we

[4] Horioka (2021) in a survey article concludes that the assumptions of the selfish lifecycle model work particularly well in Japan but less well in other countries such as the U.S. One reason for this is bequest taxes. Bequest taxes are higher and exemptions are much lower in Japan than in the U.S.

[5] Children only affect consumption demand as we explain in more detail below. Thus, we do not track their membership or reassign them if the household is dissolved due to a death event.

[6] Given that there is only one type of heterogeneity in a cohort, to conserve on notation we do not explicitly index the identity of each household of age j in period t in the ensuing discussion unless it is required to avoid confusion.

also abstract from the service flow of utility services provided by real assets. Thus, the benefit from holding illiquid assets is entirely pecuniary in our model. Adjustment costs on holdings illiquid assets are given by

$$\chi(a_{j,t}, a_{j-1,t-1}, z) = \begin{cases} \frac{\gamma_a(z)}{2}(a_{j,t} - a_{j-1,t-1})^2, & a_{j-1,t-1} > 0 \\ \frac{\gamma_a(z)}{2}a_{j,t}^2, & a_{j-1,t-1} = 0 \end{cases} \tag{1}$$

where $a_{j,t}$ denotes the holdings of illiquid assets in the end of period t and $\gamma_a(z) \geq 0$ is a parameter that governs the size of the adjustment costs for $z = z^i_{j,t} \in \{0, 1\}$. These costs have two main features. First, they vary with the level of the change in assets. Second, they depend on whether the household experiences the death event in the current period. Our specification of adjustment costs has several attractive features. It creates a wedge between the after-tax return on liquid and illiquid assets even though there is no aggregate uncertainty in the model. In addition, the incidence of the adjustment costs varies systematically with age in a way that is consistent with how one might to expect them to vary over the lifecycle. For instance, newly formed households face relatively high costs of accumulating illiquid assets and older working-age households who experience the death event face a relatively high cost of liquidating their relatively large holdings of illiquid assets. Finally, the costs are convex, so they don't create kinks in the household's budget set. Essentially, these costs can be interpreted as those pertaining to a representative individual of a given age and we parameterize these costs to fit the age profile of average asset holdings reported in Table 1.

Given these definitions, the decisions of a surviving household of age-j in period t (i.e., a household with $z^i_{j,t} = 1$) are constrained by:

$$\begin{aligned}(1 + \tau^c)c_{j,t} &+ a_{j,t} + \chi(a_{j,t}, a_{j-1,t-1}, 1) + d_{j,t} \\ &\leq \tilde{R}^a_t a_{j-1,t-1} + \frac{\tilde{R}_{t-1}}{\pi_t} d_{j-1,t-1} + (1 - \tau^w)w_t \epsilon_j h_{j,t} + b_{j,t} + \xi_t,\end{aligned} \tag{2}$$

where $c_{j,t}$ is total household consumption for a household of age j in period t, τ^c is a consumption tax rate, $d_{j,t}$ denotes holdings of the liquid asset, expressed in terms of the final good, at the end of period t, $h_{j,t}$ denotes hours worked, $b_{j,t}$ denotes public pension (social security) benefits, ξ_t is a lump-sum government transfer, and $\chi(\cdot)$ is the transaction cost of adjusting individual holdings of the illiquid asset.[7] We wish to emphasize that there

[7]We are omitting here the dependence of individual choices on the survival event to save on notation.

are no ad hoc restrictions on borrowing of surviving households. They are free to borrow against their future earnings and they are also free to take leveraged long positions on illiquid assets, which have a higher return in equilibrium. The only constraint on borrowing of surviving households is the natural borrowing constraint.

If instead the household is in its final period of life ($z^i_{j,t} = 0$), the event is publicly observed by lenders and borrowing is not possible. Thus, the optimal strategy for the household is to consume all of its income and wealth during the current period

$$(1+\tau^c)c_{j,t} = \tilde{R}^a_t a_{j-1,t-1} + \frac{\tilde{R}_{t-1}}{\pi_t} d_{j-1,t-1} + (1-\tau^w) w_t \epsilon_j h_{j,t} + b_{j,t} + \xi_t - \chi(0, a_{j-1,t-1}, 0). \quad (3)$$

The period utility function for a household of age j in period t is given by

$$u(c_{j,t}, h_{j,t}; \eta_j) = \frac{\eta_j \left(c_{j,t}/\eta_j\right)^{1-\sigma}}{1-\sigma} - \frac{\upsilon}{1+1/\nu} h_{j,t}^{1+1/\nu}, \quad (4)$$

where $\sigma > 0$ is the inverse of the elasticity of inter-temporal substitution, $\nu > 0$ governs the Frisch elasticity of labor supply, $\upsilon > 0$ is a labor dis-utility parameter, and η_j is a family scale, which we assume is time-invariant. In the model, children are essentially age-specific deterministic demand shocks to household consumption.

We assume that working-age households belong to a labor union. The union respects their marginal utilities and wages are flexible. We analyze the symmetric equilibrium. Thus, hours worked are identical for all workers in period t, $h_{j,t} = \bar{h}_t$ for all $j < J_r$ with $\bar{h}_t$ given by

$$(1-\tau^w)\bar{\epsilon}_t w_t = \upsilon \bar{\lambda}_t^{-1} \bar{h}_t^{\frac{1}{\nu}} \quad (5)$$

where $\bar{\lambda}_t$ is the weighted average of the marginal utilities of working households and $\bar{\epsilon}$ is the weighted average of the efficiency of labor. For the modeling of the labor supply decision and the derivation of equation (5), see Appendix A.3. This specification implies that workers who experience a shock are unable to self-insure by adjusting their hours worked differently from the average worker. It is worth pointing out that earnings vary by age because the efficiency of a worker's labor depends on the worker's age.

Formally, we have for $z^i_{j,t} \in \{0,1\}$: $c_{j,t}(z^i_{j,t})$, $a_{j,t}(z^i_{j,t})$, $d_{j,t}(z^i_{j,t})$, and $h_{j,t}(z^i_{j,t})$. In what follows this dependence is only made explicit when required.

Under these assumptions the household's optimal choices are given by the solution to

$$U_j(a_{j-1,t-1},d_{j-1,t-1},z_{j,t}) = \max_{\{c_{j,t},a_{j,t},d_{j,t}\}} \Big\{ u\left(c_{j,t},\bar{h}_t;\eta_j\right)$$
$$+\beta z_{j,t}\left[(1-\psi_{j+1})U_{j+1}(a_{j,t},d_{j,t},0)+\psi_{j+1}U_{j+1}(a_{j,t},d_{j,t},1)\right]\Big\}, \tag{6}$$

subject to equations (2) and (3) for $z_{j,t} \in \{0,1\}$ and for $j = 1,..J-1$, and $z_{J,t} = 0$, where $\beta > 0$ is the preference discount factor and ψ_{j+1} is the conditional probability that a household of age $j+1$ survives to the next period.[8] Note that we have imposed no restrictions on the sign or magnitude of asset holdings beyond the natural borrowing constraint. It is thus conceivable, for instance, that households might want to borrow both types of assets. However, in equilibrium, the return on illiquid assets exceeds the return on liquid assets and all private borrowing will be in the form of liquid assets. Appendix A.1 reports the optimality conditions for the household problem. Appendix A.2 reports results about the sign of the liquidity premium and a characterization of the optimal portfolios in a simpler version of the household problem.

3.3 Production of goods and services

The production of goods and services is organized into four sectors.

Final good sector. Firms in this sector are perfectly competitive and combine a continuum of intermediate goods, $\{Y_t(i)\}_{i\in(0,1)}$, to produce a homogeneous final good Y_t, using the production technology: $Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{\theta}} di\right]^{\theta}$ with $\theta > 1$. Let $P_t(i)$ denote the price of intermediate good i, and P_t denote the price of the final good. Final good firms are price takers in input markets and it follows that demand for intermediate good i is:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\theta}{\theta-1}} Y_t. \tag{7}$$

The final good is either consumed by households or used as an input in the capital good sector.

[8] There is a theoretical possibility that adjustment costs on illiquid assets could exceed the size of beginning of period illiquid assets. Our strategy for parameterizing the adjustment costs on illiquid assets rules this possibility out.

Intermediate goods sector. Firms in this sector are monopolistically competitive and each firm produces a unique good indexed by $i \in (0, 1)$. Intermediate goods firm i produces $Y_t(i)$ by combining capital $K_t(i)$ and effective labor $H_t(i)$ with a Cobb-Douglas production function:

$$Y_t(i) = K_t(i)^\alpha H_t(i)^{1-\alpha}, \quad 0 < \alpha < 1. \tag{8}$$

Intermediate goods firm i faces demand curve (7), and sets its price $P_t(i)$ to maximize profits subject to a quadratic price adjustment cost function. The optimality condition of this problem is derived in Appendix A.4. In a symmetric equilibrium, the condition can be expressed as

$$(\pi_t - 1)\pi_t = \frac{1}{\gamma}\frac{\theta}{\theta - 1}(mc_t - 1) + \Lambda_{t,t+1}\frac{Y_{t+1}}{Y_t}(\pi_{t+1} - 1)\pi_{t+1}, \tag{9}$$

where $\pi_t = P_t/P_{t-1}$ is the gross inflation rate. Equation (9) is the New Keynesian Phillips curve that relates the current inflation rate π_t to the real marginal cost mc_t and the future inflation rate π_{t+1}. In a symmetric equilibrium the aggregate output is

$$Y_t = K_t^\alpha H_t^{1-\alpha}, \tag{10}$$

where K_t denotes the aggregate capital and H_t denotes the aggregate effective labor. The aggregate total profits of intermediate goods firms in period t, $\Omega_t \equiv \int_{i\in(0,1)} \Omega_t(i)di$, are given by

$$\Omega_t = \left[\theta - mc_t - \frac{\gamma}{2}\left(\pi_t - 1\right)^2\right] Y_t. \tag{11}$$

Capital good sector. Capital good firms are perfectly competitive and use a linearly homogeneous production technology to produce capital. The representative firm purchases $(1 - \delta)K_t$ units of old (depreciated) capital from the mutual fund and I_t units of the final good from the final good firms, and uses the two inputs to produce K_{t+1} units of new capital that is sold back to the mutual fund. Then, the conventional investment identity obtains:

$$K_{t+1} = (1 - \delta)K_t + I_t. \tag{12}$$

Mutual fund sector. Our economy has two types of illiquid assets – capital and shares in intermediate goods firms – and there is no aggregate uncertainty in the model after time-zero. Thus, a no arbitrage argument implies that the return on the two illiquid assets is the same in all periods except possibly time-zero when their returns will differ if an aggregate time-zero shock occurs. We allocate ownership and the potential time-zero capital gains and losses among households by assuming that households invest in a mutual fund produced by perfectly competitive financial service firms. Each firm holds the market portfolio of the two illiquid assets and pays households the market return on illiquid assets.

To derive the market return on illiquid assets note that the return on capital in period t is given by

$$R_t^k = r_t^k + 1 - \delta. \tag{13}$$

The one period return from investing one unit of the period $t-1$ final good into shares is

$$R_t^v = \frac{\Omega_t + V_t}{V_{t-1}}, \tag{14}$$

where V_t is the share price. We assume that the return on capital and equity is subject to a corporate tax as well as an asset income tax paid by households. Liquid assets, in contrast, will consist primarily of government debt in equilibrium and are taxed once at the household level. To reduce the notational burden, we assume that corporate taxes are paid by the mutual fund. Let τ^k denote the corporate tax rate. Then, perfect competition leads to the arbitrage conditions:

$$R_t^a - 1 = (1-\tau^k)(R_t^k - 1) = (1-\tau^k)(R_t^v - 1). \tag{15}$$

for all $t > 0$. From this no-arbitrage restriction the share price is given by

$$V_t = \sum_{i=1}^{\infty} \left(\prod_{j=1}^{i} \frac{1}{R_{t+j}^k} \right) \Omega_{t+i}. \tag{16}$$

Hence, the discount factor $\Lambda_{t,t+1}$ in equation (9) is given by $\Lambda_{t,t+1} = 1/R_{t+1}^k$.

We will analyze the dynamic responses to shocks to monetary policy by assuming that the economy is in a steady-state in all periods prior to $t = 0$ and that an unexpected shock hits the economy in period $t = 0$. Equation (15) does not obtain in period $t = 0$. A time-zero shock creates a wedge between the ex ante and ex post return of each real asset and thereby produces an unexpected capital gain or loss. Both the sign and size of these time-zero revaluation effects will generally be different for the two real assets.

3.4 Government

The government consists of a central bank and a fiscal authority.

Central bank. The central bank sets the nominal interest rate R_t following a simple rule that depends on the current inflation rate and the past nominal interest rate:

$$\log\left(\frac{R_t}{R}\right) = \rho_r \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_r)\phi_\pi \log(\pi_t) + \epsilon_t, \tag{17}$$

where R is a constant and ϵ_t is an i.i.d. monetary policy shock. The parameter ρ_r governs the inertia of the nominal interest rate, and the parameter $\phi_\pi > 1$ captures the central bank's stance on inflation. A high ϕ_π implies a strong anti-inflation stance and vice versa.

Fiscal authority. The fiscal authority raises revenue by taxing consumption, labor income, capital income, and mutual funds. Total tax revenue is

$$T_t = \sum_{j=1}^{J}\left[\tau^c \bar{c}_{j,t} + \tau^{ka}(R_t^k - 1)a_{j-1,t-1} + \tau^a \frac{(R_{t-1}-1)}{\pi_t} d_{j-1,t-1} + \tau^w w_t \epsilon_j \bar{h}_t\right] N_{j,t}, \tag{18}$$

where $\bar{c}_{j,t} = \psi_{j,t} c_{j,t}(1) + (1-\psi_{j,t})c_{j,t}(0)$ is the average consumption by surviving and non-surviving households and $\tau^{ka} = \tau^a + \tau^k - \tau^a\tau^k$ is the total tax rate on illiquid assets.

Let D_t^n denote the face value of nominal government debt issued in period t. Then aggregate government expenditures consist of government purchases G_t, nominal interest payments on its debt, net of new issuance, $(R_{t-1}D_{t-1}^n - D_t^n)/P_t$, subsidies to intermediate goods firms, $\tau^f Y_t = (\theta - 1)Y_t$, public pension benefits $B_t \equiv \sum_{j=J_r}^{J} b_{j,t}N_{j,t}$, and lump-sum transfers to households, $\Xi_t \equiv \sum_{j=1}^{J} \xi_t N_{j,t}$. It follows that the government flow budget constraint is given by

$$G_t + \frac{R_{t-1}D_{t-1}^n - D_t^n}{P_t} + \tau^f Y_t + B_t + \Xi_t = T_t \tag{19}$$

and the government bond market clearing condition is given by

$$\frac{D_t^n}{P_t} = D_t \equiv \sum_{j=1}^{J} \bar{d}_{j,t} N_{j,t}, \tag{20}$$

where $\bar{d}_{j,t} = \psi_j d_{j,t}(1) + (1-\psi_j)d_{j,t}(0)$ is the average government bond holdings by surviving and non-surviving households.[9]

We assume that the fiscal authority is passive in that it does not adjust the size of its nominal debt when monetary policy is changed. In other words, nominal government debt is constant: $D^n_{t-1} = D^n_t$, for $t = 0, 1, ...$ Consequently, a change in monetary policy affects both tax revenues and the real value of government debt. The size of the lump-sum transfer, ξ_t, is adjusted in each period to close the government budget constraint, equation (19). This assumption matters because in our economy changing the size of the lump-sum transfer induces redistribution. This same assumption is maintained in other models where monetary policy has distributional effects such as Sterk and Tenreyro (2018), Hagedorn et al. (2019) and Hu et al. (2019).

Benefits from the pubic pension program are modeled in the same way as Braun et al. (2009). A household starts receiving a public pension benefit at the mandatory retirement age of J_r. The real size of the benefit during the household's retirement is constant at a level that is proportional to its average real wage income before retirement:

$$b_{j,s+j-1} = \begin{cases} 0 & \text{for } j = 1, ..., J_r - 1 \\ \lambda \left(\frac{1}{J_r - 1} \sum_{j=1}^{J_r - 1} w_{s+j-1} \epsilon_j \bar{h}_{s+t-j} \right) & \text{for } j = J_r, ..., J, \end{cases} \tag{21}$$

where λ is the replacement ratio of the pension benefit and s is the household's birth year. Thus, the public pension system implicitly assumes perfect inflation indexation of pension benefits.

3.5 Competitive equilibrium

In the impulse response analysis that follows, we will assume that the shock arrives at the beginning of time zero and that households have perfect foresight about the subsequent evolution of prices and government policy.[10] Consequently, perfect foresight is assumed in the following definition of a competitive equilibrium.

Given prices, all firms maximize their profits, all households maximize their utility, and all markets clear. Appendix B provides specific details on the definition and algorithms

[9] Because $d_{j,t}(0) = 0$, the aggregate bond can be arranged as

$$D_t \equiv \sum_{j=1}^{J} [\psi_j d_{j,t}(1) + (1 - \psi_j) d_{j,t}(0)] N_{j,t} = \sum_{j=1}^{J} \psi_j d_{j,t}(1) N_{j,t} = \sum_{j=1}^{J} d_{j,t}(1) N_{j+1,t+1}.$$

[10] Boppart et al. (2017) provide a justification for using this approach in heterogeneous agent economies.

used to compute steady-state and dynamic equilibria for this economy. Here we simply state the two market clearing conditions that have not yet been reported.

First, the aggregate household illiquid assets, denoted by $A_t \equiv \sum_{j=1}^{J} \bar{a}_{j,t} N_{j,t}$ with $\bar{a}_{j,t} = \psi_j a_{j,t}(1) + (1-\psi_j) a_{j,t}(0)$, are equal to the sum of capital and the value of all ownership shares of intermediate goods firms:

$$A_t = K_{t+1} + V_t. \tag{22}$$

Second, as shown in Appendix B.1, Walras' Law implies the market clearing condition for the final good

$$C_t + I_t + G_t = Y_t - \frac{\gamma}{2} (\pi_t - 1)^2 Y_t - X_t, \tag{23}$$

where $X_t = \sum_{j=1}^{J} \bar{\chi}_{j,t} N_{j,t}$, with $\bar{\chi}_{j,t} = \psi_j \chi_{j,t}(1) + (1-\psi_j)\chi_{j,t}(0)$, is the aggregate cost of adjusting illiquid assets. Observe that the aggregate costs of price adjustments and illiquid asset adjustments are modeled as explicit resource costs and consequently subtracted from the aggregate output.

4 Model parameterization and assessment

4.1 Capital, saving and debt

Our general equilibrium model has implications for household-level holdings of liquid and illiquid assets by age and also the aggregate net stocks of liquid and illiquid assets. We parameterize the model to reproduce the aggregate magnitudes of net liquid and illiquid assets in Japanese macro data. We then assess the model by comparing its implications for household-level holdings of liquid and illiquid assets by age with Japanese micro survey data.

Before we calibrate the model, we first need to classify household assets and liabilities as either liquid or illiquid. Our classification scheme is similar in spirit to that used by Kaplan et al. (2018). Table 2 provides an overview of the main components of the two categories.[11]

A comparison of the results in Table 2 with the similar results for U.S. data for the year 2004 reported in Kaplan et al. (2018) (See Table 2 of their paper) reveals some important distinctions between the U.S. and Japan. The biggest difference between the two countries

[11] See Appendix C for complete details.

Table 2: Aggregate stocks of liquid and illiquid assets relative to GDP in Japan

Liquid assets and liabilities	
Currency and domestic deposits	1.74
Bonds (total public and private)	0.052
Consumer credit	-0.069
Total net liquid assets as defined in Kaplan et al. (2018)	1.73
Total net liquid assets in our model	1.23
Illiquid assets and liabilities	
Household physical assets	2.02
Equity and options	0.49
Insurance and private pensions	0.99
Mortgages and installment credit	-0.37
Other non-housing loans	-0.12
Total net illiquid assets as defined in Kaplan et al. (2018)	3.01
Total net illiquid assets in our model	3.50
Net worth	4.73

Note: The financial data are taken from the Flow of Funds Accounts (FFA) for the fiscal year 2014. The stock of household physical assets is the 2014 end of calendar year value from the Japanese National Income and Product Accounts (NIPA). All variables are expressed as a multiple of GDP for the fiscal year 2014.

is that Japanese households hold a lot more liquid assets compared to Americans. The net stock of liquid assets in Japan is 1.73 but only 0.26 in the U.S., where all variables are expressed as a multiple of GDP. Japanese hold more deposits and cash than Americans. Deposits (plus cash) are 1.86 in Japan but only 0.23 in the U.S. In addition, over 90% of Japanese government debt is held domestically. So this difference between the two countries reflects the fact that Japanese households are indirectly holding a large amount of government debt.

The net stock of illiquid assets is about the same in Japan and the U.S. Illiquid assets are 3.0 in Japan and 2.9 in the U.S. However, Japanese households have smaller direct holdings of equity than Americans (0.49 in Japan versus 1.61 in the U.S.) and Japanese hold more real assets. Real assets are 1.53 in Japan versus 1.32 in the U.S.[12]

Given that Japanese households have much higher levels of liquid wealth than Americans while illiquid wealth is about the same in the two countries, it follows then that Japanese

[12]The remaining difference in holdings of illiquid assets is insurance assets which are not reported for the U.S. in Kaplan et al. (2018).

households have higher net worth (relative to GDP).[13] Aggregate household net worth in Japan is 4.73 times GDP and only 3.18 times GDP in the U.S.

An important distinction between our model and Kaplan et al. (2018) is that individuals save and borrow for different reasons. In our model, individuals save to smooth consumption over the lifecycle and only face life-expectancy risk. Individuals in our model borrow for two reasons. First, young individuals borrow against their future higher income. Second, some individuals in our model borrow liquid assets and use them to purchase illiquid assets because this strategy enhances the overall return on their portfolio of liquid and illiquid assets. In Kaplan et al. (2018), in contrast, individuals borrow if they experience negative earnings shocks. This difference in the savings motive affects how we organize the data. In our model, all borrowing by individuals is made in the form of liquid assets and we rearrange the data to reflect this property of the model. When calibrating the model's net aggregate stock of liquid and illiquid assets, we assign all household borrowing to the liquid asset category.[14] This adjustment results in a lower total stock of aggregate household (net) liquid assets of 1.23 and higher aggregate stock of (gross) illiquid assets of 3.5. We reproduce these two targets in the model by varying the stock of government debt and the preference discount factor. The resulting net debt-GDP ratio in the model is of 1.23 and the resulting value of β is 0.996 as reported in Table 3.

4.2 Costs of adjusting illiquid assets

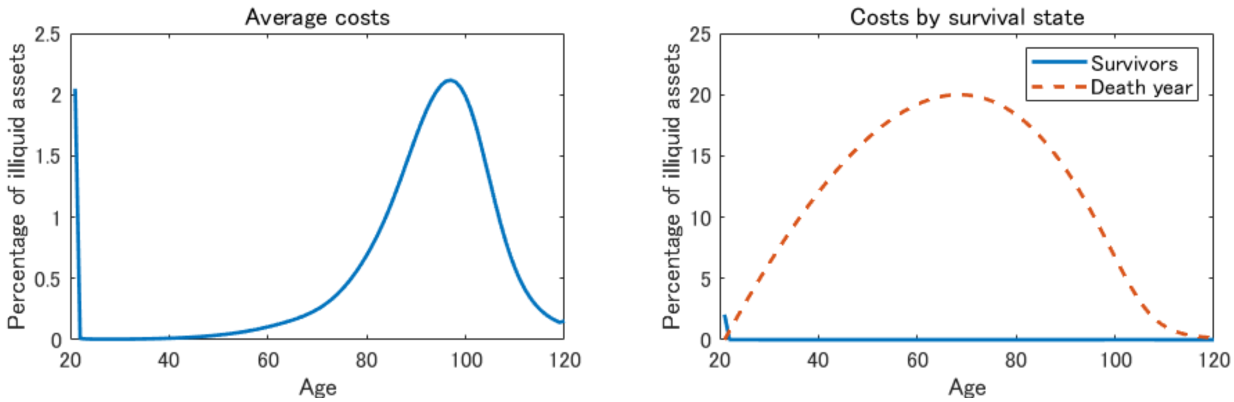
Household-level adjustment costs on illiquid assets produce a small liquidity premium and make it possible for the model's steady-state to reproduce the main features of the empirical age-profiles of net worth and its components reported in Table 1. The model's implications for net worth and its components are discussed in Section 4.4.2. Here we report the size of the baseline steady-state age-profile of adjustment costs, provide some intuition for how they work and explain how we selected the parameters that govern them.

Figure 1 reports the age-profile of adjustment costs on illiquid assets relative to total assets using the baseline parameterization of the model which assumes that the maximum cost at any age is two percent of assets conditional on surviving and 20 percent of assets conditional on it being the death year. The right panel of the figure reports the adjustment

[13] We define net worth to be the sum of illiquid and liquid assets and abstract from human wealth throughout the paper.

[14] This property of the model captures the main features of Japanese data. The biggest component of illiquid liabilities using the Kaplan et al. (2018) scheme is mortgage debt. As discussed above most Japanese mortgages have adjustable interest rates and the costs of prepaying a mortgage in Japan are small.

Figure 1: Adjustment costs on illiquid assets relative to total asset holdings by age



costs by survival state. It has two main properties. Newly formed households of age 21 have the highest adjustment costs conditional on survival. This is because they enter the economy with no assets. Newly retired households (age 68) have the highest adjustment costs if they experience a death event shock at this age because they have the largest holdings of illiquid assets. Put differently, a 68 year old household who discovers that this is the last year of her life and quickly sells off her large stock of illiquid assets pays a fee of 20 percent of total assets.

The left panel of Figure 1, which shows the age-profile of the average costs of adjusting illiquid assets, has two modes. The first mode occurs at age 21 and is a cost of acquiring assets. It captures in a simple way the idea that young households may not be very sophisticated purchasers of a home or car and must allocate more resources to acquiring them. The adjustment costs then fall sharply and are less than 0.5 percent of total assets until age 76. The second mode in average adjustment costs occurs at age 98 and is 2.3 percent of total assets. Households older than age 76 face an interesting trade off. On the one hand, they are attracted to the higher return offered by illiquid assets. On the other hand, mortality risk is increasing and it is costly for them to have to rapidly liquidate their entire holdings of illiquid assets if they discover that this is their death year.

We selected our baseline values of the two parameters that govern the age-profile of adjustment costs on illiquid assets after experimenting with a range of values for each parameter. We varied $\gamma_a(1)$ so that the initial cost of acquiring illiquid assets for 21 year old households ranged from 2% to 5% of their initial purchase of assets. The parameter $\gamma_a(0)$ was chosen by varying the maximum costs for a household in their death year of liquidating all illiquid assets from 10% to 20% of total assets. The baseline targets of 2 percent of assets for survivors and 20 percent of assets for non-survivors help the model

Table 3: Parameterization of the model

Parameter	Description	Value
Demographics		
J_r	Retirement age	48 (Age 68)
J	Maximum lifespan	100 (Age 120)
$\{\psi_j\}_{j=1}^{J}$	Survival probabilities	Braun and Joines (2015)
Technology		
θ	Gross markup	1.05
γ	Price adjustment cost	41.2
α	Capital share parameter	0.30
δ	Depreciation rate	0.102
$\gamma_a(0)$	Cost of adjusting illiquid assets in death year	0.0723
$\gamma_a(1)$	Cost of adjusting illiquid assets in non-death year	0.203
Preferences		
σ	Inverse elasticity of intertemporal substitution	3
ν	Frisch labor supply elasticity	2
υ	Preference weight on leisure	6.9
β	Preference discount factor	0.996
Monetary Policy		
ρ_r	Interest rule persistence	0.35
ϕ_π	Interest rule inflation elasticity	2
Fiscal Policy		
τ^c	Consumption tax rate	0.05
τ^k	Corporate tax rate	0.35
τ^a	Tax rate on asset income	0.2
τ^w	Tax rate on labor income	0.232
τ^f	Subsidy to intermediate goods firms	$\theta - 1$
G/Y	Government share of output	0.16
λ	Public pension replacement ratio	0.094
D/Y	Net government debt output ratio	1.23

to account for the long number of years that working-age households borrow and induce retirees to hold positive amounts of liquid assets until relatively late in life.

4.3 Other model parameters

Table 3 reports the entire parameterization of the model. The remainder of our calibration strategy follows Braun and Joines (2015). In particular, we adjust Japanese NIPA account data to recognize some differences between our model and the data. For instance, our model has no external sector and there is no government investment in the model. The specific adjustments follow the strategy of Hayashi and Prescott (2002).

Starting with demographics, we assume that new households are formed at age 21 and the size of the household is parameterized in the same way as Braun et al. (2009). In the model individuals face mandatory retirement at age 68 ($J_r = 48$). This is two years older than the age where one can qualify for full public pension benefits in Japan and is chosen to be consistent with the effective labor-market exit age in 2014 for Japan estimated by the OECD.[15] Finally, the maximum lifespan is set to 120 years ($J = 100$).

We use the same depreciation rate as Braun and Joines (2015). But we use a smaller value of the capital share parameter $\alpha = 0.3$. This value in conjunction with the rest of our parameterization results in an after-tax return on illiquid assets of 1.80% per annum. The parameter θ governs the elasticity of substitution of intermediate goods. We set this parameter to produce a gross markup of 1.05 in steady-state.

Many real business cycle and NK models assume that preferences are additively separable in consumption and leisure and posit log-preferences over consumption. Lifecycle analyses though often set the relative-risk aversion coefficient on consumption at a higher level of about 3 (see e.g. Brown and Finkelstein (2008)). We set σ to this value. Hours worked in our model are determined in a way that is close to the representative agent framework and there is no distinction between extensive and intensive labor supply decisions. It follows that the Frisch labor supply elasticity in our model, given by ν, reflects the combined effects of adjustments along both margins. We set this parameter to 2.[16]

The extent of nominal price rigidities in the model is determined by γ, the cost of price adjustment for intermediate goods firms. We set $\gamma = 41.2$, which implies that intermediate goods prices adjust on average every 2 years.[17]

The monetary policy and fiscal policy parameters are set in the following way. It is common to allow for persistence in the central bank's interest rate rule. We set the serial correlation to $\rho_r = 0.35 = 0.77^4$. The inflation elasticity is set to $\phi_\pi = 2$. Both of these choices are common choices used elsewhere in the literature.

In terms of fiscal policy variables we set the net Japanese government debt ratio to 1.23, which is the size of net liquid assets held by households in the 2014 FFA data discussed above. Intermediate goods firms receive a subsidy that is chosen so that the steady-state markup is zero. Capital income is taxed twice in Japan. The overall tax rate on capital in-

[15]See Pensions at a Glance OECD, 2015.

[16]We assign zero weight to individuals in their death year when computing $\bar{\lambda}$ in equation (5). If we include them and set $\nu = 1$, the model results are essentially indistinguishable from what is reported here.

[17]Using a log-linearized version of the model we can map back and forth between γ and the corresponding Calvo parameter and derive the average duration of price changes.

come faced by households in the model is 48%, which is a combination of a corporate profits tax rate (τ^k) of 35% and a 20% personal tax rate on asset income (τ^a). The consumption tax rate is set to 5% and the labor income tax rate is set to 23.2%. The personal tax rate on labor income, government purchases relative to output and the replacement rate of the public pension are calibrated using the same targets as Braun and Joines (2015).

4.4 Assessment of the model parameterization

4.4.1 Aggregate moments

The steady-state of our model reproduces some of the main features of the Japanese economy including the composition of aggregate output, the size of aggregate net worth and the main revenue sources and expenditures of the public sector. Lump-sum transfers are used to close the government budget constraint and amount to only -0.2% of output. More information is provided in Table 11 of Appendix F. Here we discuss the main gaps between the model and Japanese data.

The pre-tax premium on illiquid assets is 1.47 percent in the model. It consists of a liquidity premium of 0.13 percent and a tax treatment premium of 1.34 that arises because illiquid assets are subject to both the corporate profits tax and the household tax on asset income. For purposes of comparison, Damodaran (2020) estimates that the overall equity premium in Japan is 5.4 percent. Given that there is no aggregate uncertainty in the model and no individual-specific earnings risk, or limited participation effects, it is to be expected that the excess return on illiquid assets in the model is smaller than the excess return on equity in Japanese data.

The model's steady-state also understates the aggregate amount of borrowing in Japanese FFA data. In the model, household borrowing is 35 percent of GDP whereas in our FFA data it is 56 percent. This result is surprising because we have not imposed any constraints on individual borrowing other than the requirement that they have enough resources to repay their loans. Still, we haven't modeled all of the reasons for why individuals borrow and introducing other borrowing motives such as uninsured borrowing risk might enhance the model's performance along this dimension.

Finally, the steady-state level of the real interest rate on government debt may be too high in the model. The nominal yield on 10-year Japanese government bonds is currently close to zero and Japan's inflation rate is also close to zero. The model, in contrast, produces a pre-tax real return on government debt of 2.25 percent. We are not too concerned about

Table 4: Household net worth, liquid and illiquid asset holdings by age: model and data

	Net Worth		Liquid assets		Illiquid assets	
Age	Model	Data	Model	Data	Model	Data
Under 30	0.06	0.65	-0.59	-0.08	0.65	0.73
30–39	0.97	1.60	-0.78	-0.58	1.76	2.18
40–49	2.87	2.58	0.18	-0.31	2.69	2.90
50–59	5.39	4.52	2.05	0.76	3.34	3.76
60–69	7.13	6.29	3.47	1.70	3.66	4.60
70+	4.28	6.01	1.11	1.77	3.28	4.25

Note: Liquid assets are net of total borrowing and illiquid assets are gross. Net worth and assets are divided by peak income of the 50–59 year old age group. The main source for the data is the 2014 NSFIE. More details on the construction of the data can be found in Appendix C.

this gap between the model and Japanese data because this interest rate is also the interest rate on private loans in the model.

4.4.2 Net worth and asset portfolios by age

There are two key features we want the model to have. The first one is that it reproduce the main properties of the age-profile of household net worth, net liquid asset holdings, and gross illiquid asset holdings, reported in Table 1. Table 4 reproduces the results shown in Table 1 and reports the analogue of the simulated data from the model. Both the model and data results are reported as relative to peak (pre-tax) income of the 50–59 year old age group.[18]

We explained in Section 2 that our data has four main properties. First, the age-profile of net worth is hump-shaped. Second, households under age 50 hold leveraged long positions in illiquid assets. Third, older working-age households and retirees have positive holdings of both liquid and illiquid assets. Fourth, individuals maintain their asset holdings until late in life.

Our model reproduces each of these properties of our data. The hump-shaped pattern of saving over the lifecycle produced by the model reflects primarily the hump-shaped age-earning profile and the fact that Japan's public pension insurance program provides incomplete coverage. The reason younger households are taking leveraged long positions in illiquid assets in the model is because their mortality risk is relatively low and thus investing

[18]The lifetime peak in income occurs in the 50–59 year old age group in both the data and our model.

in illiquid assets is a good deal for them. On balance, the benefits of a higher expected return on illiquid assets exceed the cost of having to suddenly liquidate their holdings if they experience a death event. Older working-age households and retirees choose to hold a diversified portfolio of liquid and illiquid assets. Working-age individuals experience a sudden income loss at retirement and they plan for retirement by acquiring liquid assets. Once they retire, they immediately start to draw down their holdings of liquid assets. Interestingly, retired households continue to increase their holdings of illiquid assets well into retirement. Holdings of illiquid assets are increasing in the model up until age 68. Finally, net worth declines monotonically during retirement but the pace of decline is gradual and net worth remains high until late in life. To see this final observation, note that the average age of the 70+ age group is 76 in our 2014 NSFIE sample. In our model households of this age have net worth that is more than five times their peak earnings. The main reason individuals hold large levels of assets until late in life in the model is because they know their death year has arrived at the start of their final year of life and have the opportunity to consume their wealth before they pass away. In practice, there are a variety of reasons for why households choose to hold wealth until late in life. For instance, Kopecky and Koreshkova (2014) highlight the role of nursing home risk and Lockwood (2018) argues that bequests are a luxury good. We have omitted these motives from the model. But they may be less important in Japan. Japan has a public long-term care insurance program and imposes large taxes on bequests.

There are also several differences between the age-asset profiles in the model and the NSFIE survey. The biggest gap between the model and the data is that households between the ages of 40–69 have higher holdings of liquid assets and higher net worth in the model as compared to the NSFIE survey. These gaps may reflect under-reporting biases in the NSFIE as we explain in Appendix C. We have calibrated the model to reproduce aggregate net worth in the FFA. So the model results in Table 4 can be interpreted as an estimate of the size of missing assets in the NSFIE survey.

Finally, observe that households in the 70+ age group have higher net worth in the data as compared to the model. Three factors may be contributing to this difference. The first factor is selection. Zaninotto et al. (2020) find that individuals with high wealth have a higher life expectancy. In the model, all households of a given age have identical wealth and this selection effect is absent. Second, the NSFIE may be under-sampling the oldest individuals because they are in nursing homes, hospitals or other institutions. This conjecture is supported by the fact that the average age of the 70+ group is 76 in the NSFIE and 80 in our model. Third, the oldest retirees borrow liquid securities in the model but it may be difficult for them to borrow in practice. In the model 86+ year old individuals take leveraged long positions in illiquid assets. Our NSFIE data doesn't break out asset positions of 86+ year old retirees. However, the aggregate size of reverse mortgages is about 0.03% of GDP in Japan.[19] For purposes of comparison total borrowing by retirees is 4.4% of output in the model. Reverse mortgages are not the only way for older retirees to borrow in Japan. Some older retirees may still have existing mortgage debt and older retirees can also secure new bank loans if a child or other person serves as a guarantor.

4.5 Partial equilibrium marginal propensities to consume

We conclude this section by reporting age-profiles of MPCs for a 50,000 yen or approximately a $500 increase in disposable income. Figure 2 reports the marginal propensity to consume for households with ages between 21–100 for two scenarios. The first scenario increases the lump-sum transfer by 50,000 yen for one year and the second scenario increases it by the same amount for two years. These are partial equilibrium marginal propensities to consume that hold prices and government policy variables fixed at their steady-state values. However, the individual responses are dynamic. Households optimally choose how to divide up the bonus to their income among consumption in all future periods of their life. The marginal propensities to consume are calculated as expected values over the two survival states.

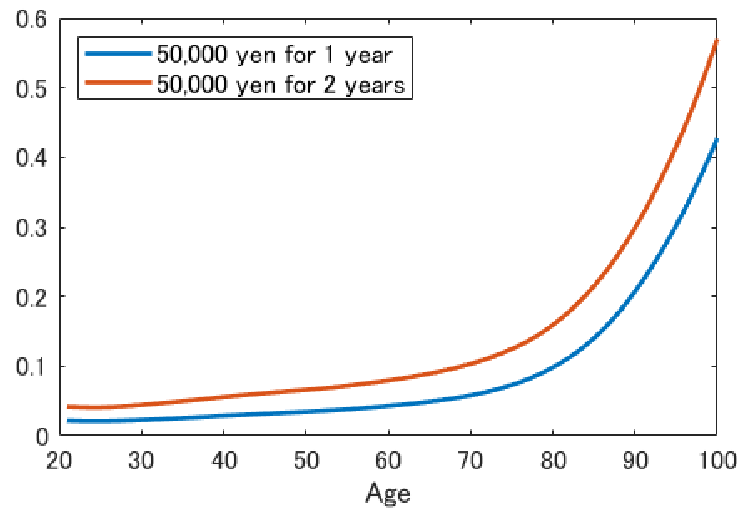
Figure 2 shows that the pattern of the marginal propensity to consume increases monotonically with age. The main reason a household's MPC increases with age is because its planning horizon becomes shorter.[20] They face relatively high mortality risk and if this is their death year, they consume all of their wealth. Average life expectancy in the model is 83 years. So the fraction of households with high MPCs is small. It follows that the cross-sectional average value of the MPC is also relatively small. It is 0.05 for a one year 50,000 yen increase in the lump-sum transfer and rises to 0.09 if the transfer is increased by this amount for two years.[21] Average MPCs of this magnitude are somewhat smaller than recent estimates using Japanese data. Koga and Matsumura (2020) estimate that the average MPC out of transitory income shocks is 0.15 using data from the Japanese

[19] The Nikkei reported on May 19, 2021 that the size of the reverse mortgage market was about 160 billion yen in Japan.

[20] We don't report MPCs for households over age 100 in Figure 2 because their share of the population is very small.

[21] The size of the MPCs is virtually identical if the size of the increase in transfers is 5,000 yen instead.

Figure 2: Partial equilibrium marginal propensities to consume by age



Household Panel Survey.

5 The aggregate effects of monetary policy

We now analyze the aggregate effects of a surprise tightening in monetary policy in our model.[22] Table 5 reports responses in the goods and labor markets and Table 6 reports responses of financial and fiscal variables to a 1 percentage point surprise increase in the nominal interest rate. The tables report results for two models: our OG heterogeneous agent New Keynesian (OG HANK) model and a representative agent New Keynesian (RANK) model with the same parameterization. The main distinctions between the two models are that the RANK model has no demographic structure, and makes no distinction between liquid and illiquid assets.

5.1 Comparison of OG HANK with RANK

The results in Tables 5–6 indicate that the OG HANK model responses are in good accord with our understanding about how a tighter monetary impacts the situation of households. On impact, a higher nominal interest rate increases the real interest rate on liquid assets, r^d. In our model this represents the real interest rate on liquid savings like deposits and also the real interest on private loans. A higher nominal interest rate also reduces the value

[22] Recall from Section 3.4 that we are assuming that the size of nominal debt remains fixed when monetary policy is changed and that lump-sum transfers are adjusted in each period to close the government budget constraint.

Table 5: Goods and labor market responses to a 1% shock to monetary policy

OG HANK								
Year	Y	C	I	K	H	w	r^k	π
1	-0.651	-0.080	-2.49	0.000	-0.929	-0.788	-0.238	-0.561
2	-0.102	-0.003	-0.349	-0.254	-0.037	-0.137	0.011	-0.015
3	0.061	0.012	-0.241	-0.264	0.026	-0.092	0.027	0.023
4	0.049	0.020	-0.218	-0.261	0.042	-0.089	0.030	0.026
5	0.040	0.026	-0.203	-0.257	0.053	-0.090	0.031	0.026
RANK								
1	-1.121	-0.065	-3.50	0.000	-1.60	-0.995	-0.308	-0.731
2	-0.043	-0.058	-0.007	-0.357	0.092	-0.130	0.038	0.022
3	-0.039	-0.053	-0.007	-0.321	0.083	-0.117	0.031	0.020
4	-0.035	-0.047	-0.005	-0.289	0.074	-0.065	0.028	0.019
5	-0.031	-0.043	-0.005	-0.260	0.067	-0.059	0.025	0.016

r^k–rental rate on capital

Note: Interest rates are percentage point changes from steady-state. All other variables are percentage point deviations from steady-state.

of stocks and the overall real return on illiquid assets, r^a, in the impact period. Finally, the model predicts that this impulse has a negative impact on the labor market. The real wage and hours both fall.

A comparison of the responses in our OG HANK model with the RANK model indicates that the nominal interest rate responds more in the OG HANK model but produces smaller responses in most other real aggregate variables than the RANK model. The impact response of the nominal interest rate is about 5 times larger in the OG HANK model, 0.28 percentage points as compared to 0.05 percentage points in the RANK model. However, the inflation rate falls by more in the RANK model. The response of the real interest rate on liquid assets is about the same in the two models (0.85 OG HANK and 0.78 RANK).[23]

Most aggregate variables in the goods and labor market exhibit larger responses in the RANK model (see Table 5). Investment falls by 3.50 percent and hours fall by 1.60 percent in the RANK model while investment and hours only fall by 2.50 percent and 0.93 percent

[23] In the RANK model all assets are liquid so we compare the real return on liquid assets from the OG HANK model with the real interest rate response in the RANK model.

Table 6: Financial and government sectors' responses to a 1% shock to monetary policy

OG HANK								
Year	R	r^d	r^a	Spread	V	Ω	ξ/y	d
0	0.277	0.855	-0.034	-0.889	-0.423	19.24	-0.403	0.564
1	0.078	0.093	0.011	-0.081	-0.477	1.33	-0.483	0.578
2	0.057	0.034	0.027	0.007	-0.469	0.045	-0.250	0.555
3	0.055	0.028	0.030	0.002	-0.454	-0.085	-0.224	0.529
4	0.055	0.027	0.031	0.003	-0.436	-0.097	-0.219	0.502
RANK								
0	0.052	0.783	-0.073	-0.855	-0.345	28.39	NA	NA
1	0.047	0.025	0.038	0.000	-0.310	-0.141	NA	NA
2	0.042	0.023	0.034	0.000	-0.279	-0.127	NA	NA
3	0.038	0.020	0.031	0.000	-0.251	-0.114	NA	NA
4	0.034	0.018	0.028	0.000	-0.226	-0.103	NA	NA

R – nom. rate, ξ/y–LS transfers over output, d– real gov. debt,
r^d – real rate of bonds, r^a – real rate of assets, V – share price , Ω– profits.

Note: Interest rates are percentage point changes from steady-state and transfers are expressed as a percentage of output. All other variables are percentage point deviations from steady-state.

in the OG HANK model. One reason for the bigger responses of investment and hours in the RANK model is because profits, Ω, increase by more (the marginal cost falls by more). The relatively large decline in the marginal cost acts to depress investment and hours by more. The response of aggregate consumption, in contrast, is larger in the OG HANK model. On net, the larger declines in hours and investment translate into a larger decline in output on impact in the RANK model. Neither of the two models produce a significant amount of propagation and the responses of most aggregate variables are small in subsequent years.[24]

However, a tighter monetary policy has a more persistent effect on asset prices and returns in our OG HANK model compared to the RANK model. In the RANK model the spread on government debt and real assets only differs from its steady-state level in

[24]In our OG HANK model a tightening in monetary policy has real effects even when prices are flexible. We don't report results for that version of the model because it is inconsistent with evidence from SVARs. Both hours and output increase on impact and the magnitude of the aggregate non-neutralities is much smaller than what we report here.

year 0. In subsequent periods the after-tax return on the two assets is identical. However, households face costs of adjusting their holdings of illiquid assets in the OG HANK model and this creates more persistence in profits, equity prices and the liquidity spread. The difference in persistence between the two models is most prominent in the price of equity, which declines for an additional two years in the OG HANK model.

In the OG HANK model, a tighter monetary policy also has distributional effects that operate through a decline in lump-sum transfers. We have already seen that the real stock of government debt increases when the nominal interest rate is increased. This is due entirely to the decline in the price level as the nominal debt is held fixed. Table 6 shows that lump-sum transfers fall. Most of the decline is due to higher real payments on government debt. This can be seen comparing the pattern of the responses of the two variables. However, other aspects of the government budget constraint are also changing. Labor tax revenue, for instance, declines.

5.2 Asset returns and co-movement

A tighter monetary policy has rich implications for the response of real returns on liquid and illiquid assets on impact and in subsequent years (see Table 6). In the impact period a higher nominal interest rate lowers the price level and increases the real return on liquid assets. It also induces a pattern of capital gains and asset price adjustments that differs across the two types of illiquid assets. The return on equity in intermediate goods firms increases on impact. On the one hand, the price of equity falls. At the same time though profits increase due to a higher markup. The second effect is larger and this is why the return on equity increases. The return on capital, in contrast, falls on impact. The stock of capital is predetermined and output falls on impact, implying a fall in the rental rate on capital. The portfolio share of capital in total illiquid assets is 0.62 in the OG HANK model and it follows that the overall return on illiquid assets falls on impact.[25] To summarize, a surprise tightening in monetary policy lowers equity prices and lowers the excess return of illiquid assets relative to liquid assets in the impact period.

[25] The return on equity also increases on impact in the RANK model. It is worth noting though that the steady-state values of interest rates, and the shares of equity and capital differ considerably in the two models. In the OG HANK model, illiquid assets offer a steady-state premium over liquid assets. There are also more borrowers (government and households) in the OG HANK model. These differences translate into higher steady-state interest rates in the OG HANK model and a lower level of real assets (capital and equity) as compared to the RANK model. In Japanese data the ratio of equity, held by domestic private agents, to GDP was about 1.65 in 2014. The steady-state equity output ratio is more consistent with this figure in the OG HANK model. It is 1.34 in the OG HANK model and 3 in the RANK model.

The fact that the overall return on illiquid assets falls on impact in the OG HANK model is noteworthy. Broer et al. (2020) argue that the large positive response of profits along the lines reported in Table 6 has counterfactual implications for the response of hours and output to monetary policy shocks in HANK models. The effect of higher profits on individual decisions is particularly stark in their model which abstracts from physical capital. Higher profits largely or even fully offset the decline in wages, and hours and output don't respond at all. They find that introducing nominal wage rigidities attenuates the profit response and induces larger responses in employment and output. When physical capital is modeled, higher profits induce countercyclical movements in aggregate investment in some standard HANK models. For instance, Kaplan et al. (2018) find that households prefer to invest most of the higher profits from a tighter monetary policy in illiquid assets and investment increases in their model if households are free to do so. Kaplan et al. (2018) produce a procyclical response of aggregate investment in their model by restricting the share of profits that can be reinvested in illiquid assets.

Our OG HANK model has different dynamic properties. Profits are large and positive in period 0 and wages are flexible. Yet, hours worked and output fall in period 0. In our model the return on illiquid assets exceeds its steady-state level in period 1 and households are free to invest all profits they receive in period 0 into illiquid assets but aggregate investment falls in period 0. We will defer a discussion of these properties of the model to Section 6.3. Here we just point out that households of all ages prefer to reduce their allocations to illiquid assets in period 0.

5.3 Evidence from SVARs

A second reference point for our OG HANK model is empirical evidence from SVARs. A formal comparison of our OG HANK model with SVAR evidence is hampered by two factors. First, Japan has experienced a protracted period of very low interest rates and the impulse responses reported here assume the initial value of the nominal interest rate is positive and don't impose a lower bound on the nominal interest rate.[26] Second, the OG HANK model operates at annual frequencies whereas SVAR analyses are typically conducted at monthly or quarterly frequencies.

Still the responses of our OG HANK model are qualitatively similar to results reported

[26]We report results that provide information about how the effects of a tightening in monetary policy might differ at the effective lower bound in Section 7.

in e.g. Ikeda et al. (2020) who estimate impulse responses to monetary policy shocks in Japan using SVARs with the effective lower bound. According to their analysis, the nominal interest rate response peaks on impact with about a 0.6% increase and drops monotonically to zero in six quarters. Table 6 shows that the OG HANK model predicts a peak response of the nominal interest rate of 0.28% on impact and then a positive but small response in years 1–4. Given that our model period is one year, a model nominal interest rate response of 0.3% is about right. Output and the inflation rate fall by a peak of 0.8% and 0.2%, respectively, in Ikeda et al. (2020) and by 0.65% and 0.56%, respectively, in our model. Both of the model's values are within the error bands of the SVARs reported in Ikeda et al. (2020).

6 Monetary policy over the lifecycle

We have seen that our OG HANK model reproduces the empirical age-profile of household assets and also produces plausible responses of a range of aggregate variables to a shock to monetary policy. Thus, we are now in a position to use the model to analyze how a tighter monetary policy affects the situation of households at each stage of the lifecycle.

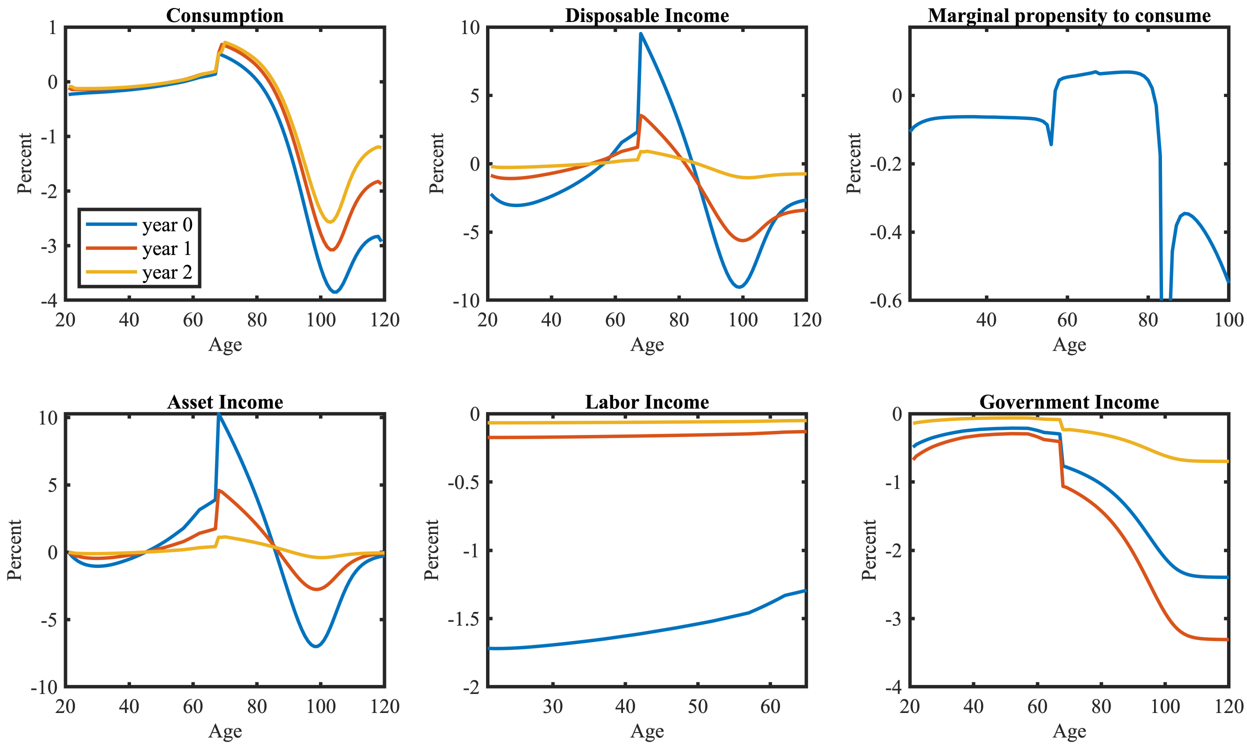
6.1 Responses of disposable income and consumption by age

Figure 3 reports household MPCs by age. To facilitate their interpretation, the figure also reports the components of the MPCs including the deviations of consumption, the deviations of disposable income and the deviations of the three main sub-components of disposable income. The deviations are expressed as a percentage of steady-state disposable income of a household of the given age.

Responses of disposable income. The upper left two panels of Figure 3 illustrate one of the main properties of our model. Both the sign and the magnitude of the responses of consumption and disposable income to a tightening in monetary policy depend on the age of the household. In the impact year, disposable income falls for households younger than age 57 and older than age 84. However, households close to the mandatory retirement age of 68 see their disposable income rise.

In most, but not all cases, households with lower disposable income choose to reduce their consumption and households with higher disposable income increase their consump-

Figure 3: Impulse responses by age to a 1% tightening in monetary policy



Note: All of the panels except the upper right one are deviations from steady-state as a percentage of steady-state disposable income. The upper right panel is the ratio of the consumption deviation and the absolute value of the disposable income deviation.

tion in the impact year. Consumption and disposable income both fall for working age households younger than age 57 and for retired households older than age 84. Households with ages 58–81 experience increases in disposable income and choose to increase their consumption.

To help understand why the sign and the magnitude of the response of disposable income vary with age, consider its three main components – asset income, labor income, and government income. Previous empirical work by Cloyne et al. (2020) and Holm et al. (2020) has found that the response of labor income to a tighter monetary policy is relatively homogeneous at different wealth levels. Our model has the same property. Wealth increases with age and the age profile of the labor income response is relatively flat between ages 21 and 67. Workers must retire at age 68 and the steady-state level of their disposable income falls at that point. This is why the labor income response is only reported until age 67 and it is also the reason why the magnitudes of the asset income and government income responses are larger for the 68+ age group.

The biggest asset income deviations occur right after retirement. Households in this group have no labor earnings but have the largest and most diversified portfolio of liquid and illiquid assets (see Table 4). The fact that their asset income increases on impact indicates that the higher return on their liquid assets is more than offsetting the decline in the return on their illiquid assets. Variations in asset income are also important for the 86+ year old group. From the steady-state analysis we know that households in this age group borrow liquid assets and take long positions in illiquid assets. The combination of a higher interest rate on their loans in conjunction with a lower return on their illiquid assets results in a large decline in their asset income.

Consider finally retirees older than age 99. Households who survive this long have lived well beyond the model life expectancy of 83 years.[27] As a result they have low wealth and the decline in lump-sum transfers associated with a tighter monetary policy is an important contributing factor to the decline in their disposable income.

Marginal propensities to consume. The upper right panel of Figure 3 reports the magnitude of the MPC, which is measured as the ratio of the consumption deviation to the absolute value of the disposable-income deviation for households with ages 21–100 in the impact year.[28] We report the MPCs using the absolute value of the disposable income deviation because the signs of the consumption and disposable income deviations may individually or jointly be positive or negative and we are interested in understanding how the sign of the individual consumption responses are related to the response of aggregate consumption that falls.

The average value of the MPC is -0.068. However, it exhibits considerable variation in sign and magnitude over the lifecycle. For working-age households younger than age 55, it lies between -0.14 and -0.062. However, the sign of the response of disposable income deviation becomes positive at age 57. The MPC is positive between ages 57 and 80 with a typical value of about 0.06. The sign of the disposable income response changes again at age 85 and the MPC is large in the neighborhood of this age. Beyond age 89, the MPC falls monotonically from -0.35 to minus one.

The age profile of MPCs is very different in Figures 2 and 3. The absolute magnitude of

[27] The size of this group is less than 1 percent of the population.

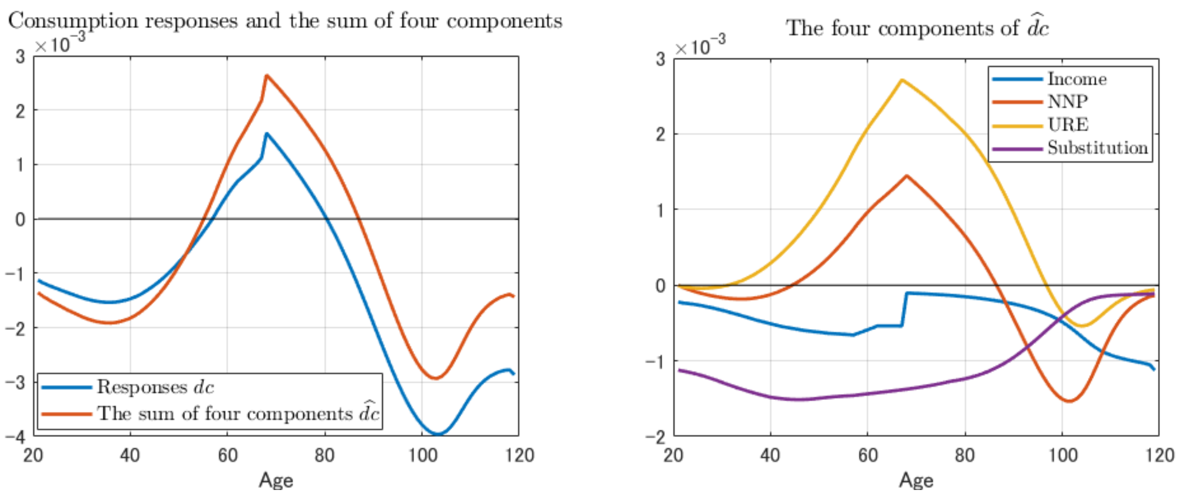
[28] The panel omits households over age 100 because they are a very small faction of the population (less than 0.15 percent) and including their MPCs makes it hard to read the figure. We only report MPCs in the impact year because wealth effects are large in some age groups and the consumption deviation for them in years 2 and 3 is large relative to the disposable income deviation.

the monetary policy MPCs is larger than the partial equilibrium MPCs in most age groups and the monetary policy MPCs are not monotonic in age. It is clear from this comparison that there are large differences in household exposures to monetary policy.

Our finding that the absolute magnitude of the decrease in consumption tends to decline with age up until about age 57 (as shown in the top-left panel of Figure 3) is in line with empirical evidence for the U.S. in Wong (2019). She finds that consumption of younger working-age individuals with ages 25–34 is more responsive to changes in monetary policy as compared to older working-age individuals with ages 35–64 and also the oldest individuals in her sample whose ages range between 65–75. Wong (2019) also finds that the sign of the consumption response is identical in all three age groups. However, we find that the response of consumption to a tighter monetary policy differs across age groups. Neither Cloyne et al. (2020) nor Holm et al. (2020) decompose households into age groups but their results suggest that the sign of the consumption response to a monetary policy shock depends on the household's portfolio of assets. Cloyne et al. (2020) report impulse responses of nondurable consumption expenditures to an *easing* in monetary policy for households who own their house outright and for households who have a mortgage in Figure 3 of their paper. The pattern of the responses in that figure is similar to what we are finding here. Consumption of outright owners falls but the size of the decline is small and not statistically significant. Households who have a mortgage though increase their consumption and the size of the increase is large and significant. Holm et al. (2020) find that households who are creditors increase their consumption when monetary policy is tightened, but that households who are borrowers reduce their consumption.

Auclert decomposition of consumption responses. Doepke and Schneider (2006) have found that household exposures to inflation exhibit large variation over the lifecycle. How important is an unexpected change in the inflation rate in our model? To investigate this question we apply a decomposition formula proposed by Auclert (2019) to the impulse responses of consumption on impact by age to a tightening in monetary policy in our model. The formula decomposes the difference in consumption from steady-state, dc, into four components: i) *income*, ii) *net nominal position* (NNP), iii) *unhedged interest rate exposure* (URE), and iv) *substitution*. The income component captures the impact of changes in earned and unearned income. The NNP component captures the effect of changes in the real value of nominal assets due to a surprise change in the inflation rate, and the URE component reflects the effect of a change in the real interest rate on net wealth – the

Figure 4: The decomposition of consumption responses on impact by age



Note: dc denotes the level of the consumption deviation from steady-state by age and $\widehat{dc}$ is the sum of the four components: income, NNP, URE, and substitution.

discounted present value of assets net of liabilities. Finally, the substitution component captures intertemporal substitution between consumption and saving arising from a change in the real interest rate. Appendix D reports the formula for this decomposition and explains why this decomposition is not exact for our model.

Let $\widehat{dc}$ denote the sum of the four components. Under the specific assumptions considered by Auclert (2019), the response of dc coincides with $\widehat{dc}$. The left panel of Figure 4 reports the impact responses of dc and $\widehat{dc}$ by age in our model.[29] Inspection of this panel indicates that this decomposition is not exact in our model. Still, we believe that it is a useful way to identify the relative contributions of these four factors.

The right panel of Figure 4 plots the four components of $\widehat{dc}$. We know from Figure 3 that changes in asset income play a central role in generating the pattern of the age-consumption-response profiles in our model. Using this decomposition we can ascertain the relative contributions of the surprise inflation components and the net wealth components. This decomposition suggests that the net wealth component (URE) is even larger than the inflation component (NNP) analyzed by Doepke and Schneider (2006). In addition,the intertemporal substitution component is particularly important for younger working-age households using this decomposition. This component is also influential at other stages of the lifecycle. For instance, it is about as large in absolute magnitude as the NNP component

[29]Please keep in mind that dc is the level of the consumption deviation from steady-state at a given age. The upper left hand panel of Figure 3, in contrast, reports the deviation of consumption from steady-state as a percentage of steady-state disposable income – dc divided by steady-state disposable income.

at age 95.

6.2 Responses of net worth and portfolios by age

Given the important role played by changes in the URE and NNP components in determining the age-profile of consumption responses in our model, it makes sense to analyze how households adjust their portfolios and how their net worth changes.

In the discussion that follows, it will be helpful to recall from our discussion in Section 4.4.2 that the household-level steady-state asset portfolios have the following three properties. First, surviving households of all ages hold positive levels of illiquid assets. Second, younger working-age households and older retirees borrow liquid assets. That is, they hold leveraged long positions in illiquid assets. Third, households with ages 21–29 have low net worth.[30]

Table 7 shows how households adjust their portfolios in response in the impact period to a tighter monetary policy. The responses for liquid and illiquid assets and net worth reported in this table are percentage deviations from the steady-state at the *end* of the impact year for the given age group. We have adjusted the sign of the responses of liquid assets and net worth so that a positive number corresponds to higher holdings for savers and less borrowing for borrowers. Table 7 also reports the leverage ratio which is defined as the percentage change in the ratio of liquid to illiquid assets for households who are borrowers.

Perhaps the most noteworthy feature of Table 7 is that households of all ages reduce their holdings of illiquid assets. One reason for this can be seen by inspecting the responses of r^a and r^d in Table 6. Both interest rates are temporarily high in period 1, but the deviation of r^d is larger. However, there is considerable variation in how households adjust their holdings of liquid assets and net holdings of assets (net worth). Households in the 21–25 age group have low or even negative net worth when the shock arrives and they respond by reducing their holdings of both assets. Net worth falls by 35.09 percent but the leverage ratio on their portfolio increases by 2.75 percent. Households with ages 31–45 reduce their holdings of illiquid assets and increase their borrowing. These households have experienced declines in their labor income and also capital losses on their leveraged long positions on illiquid assets and they leave the impact year with lower net wealth.

Households between the ages 56 and 85 have positive holdings of both liquid and illiquid

[30] In fact, households with ages 21–24 have negative steady-state net worth.

Table 7: Portfolio adjustments by age to a tighter monetary policy

Age group	Liquid assets	Illiquid assets	Leverage	Net worth
21–25	0.39	-3.04	2.75	-35.09
26–30	-0.47	-0.88	1.37	-8.43
31–35	-0.70	-0.55	1.26	-2.15
36–40	-1.08	-0.41	1.27	-2.25
41–45	-3.63	-0.33	2.51	-0.53
46–50	-0.21	-0.29	0.00	-0.26
51–55	0.24	-0.26	0.00	-0.09
56–60	0.39	-0.24	0.00	0.04
61–65	0.50	-0.22	0.00	0.13
66–70	0.66	-0.22	0.00	0.21
71–75	0.83	-0.22	0.00	0.20
76–80	1.06	-0.23	0.00	0.13
81–85	3.74	-0.25	0.00	0.04
86–90	2.59	-0.28	-2.31	-0.12
91–95	0.34	-0.36	0.02	-0.40
96+	5.02	-5.06	0.01	0.74

Note: These are asset holdings at the end of the impact year. Liquid, illiquid assets and net worth are expressed as percentage deviations from the steady-state for the given age group. The sign of the liquid asset response has been adjusted so that a positive number implies that the household has increased holdings (or reduced borrowing for borrowers). Leverage is expressed as a percentage change from the steady-state leverage ratio.

assets when the monetary policy shock arrives and exit the impact period with higher net worth. They experience a capital gain on impact because the higher real return on their holdings of liquid assets exceeds the decline in the return on their holdings of illiquid assets. From their perspective, liquid assets are also a more attractive investment. Thus, they reduce their allocation to illiquid assets and increase their allocation to liquid assets.

Households over age 85 are also borrowers. Even in this age group one can discern heterogeneous responses to a tighter monetary policy. All households in the 86+ age group reduce the level of their borrowing and their holdings of illiquid assets. Leverage ratios also fall for households with ages 86–90. However, 91+ year old households increase their leverage ratios. They find that the expected benefit of a higher return on their leveraged portfolio exceeds the cost of experiencing a death event and having to liquidate their position in illiquid assets.

6.3 Linking the household level responses to the aggregate responses

Our discussion up to this point has focused on the heterogeneous impact of a tighter monetary policy on household decisions. It is not immediately clear from this discussion though what the contribution of the various household level responses are to the aggregate responses. Household MPCs and net worth vary by age and the fraction of households of a given age in the total population also varies due to mortality risk.

Table 8 provides information on the contribution of the main household level responses to the declines in average consumption, average net worth and holdings of illiquid assets in the impact period of the shock.[31] Results are reported for three age groups of households. We chose the specific age ranges so that the sign of the consumption response is the same for all households in each age group. Inspection of Table 8 reveals that the increase in consumption and net worth of households with ages 57–79 partially offsets the negative impacts of a tighter monetary policy on younger working-age households and older retirees. The effect of the 57–79 age group is particularly pronounced on average net worth which declines by -0.215 percentage points. Net worth falls by -0.436 percent for working-age households but increases by 0.147 percent for the 57–79 age group. This same group also mitigates the decline in average consumption. The oldest age group experiences the largest consumption declines but their population share is relatively small. As a result this group contributes only -0.031 percentage points to the total -0.098 percentage point decline in average consumption.

Aggregate investment falls in our model in the impact period and this property of our model is different from other HANK models as we discussed in Section 5. We are now in a position to discuss the result in more detail. Table 8 shows that on net working-age households with ages 21–56 reduce their illiquid asset holdings most. This reflects several considerations. First, they have experienced losses on their portfolios. The ex post return on illiquid assets in period 0 is negative and they are borrowers so their ex post borrowing costs were high. Moreover, in period 1 real asset returns are temporarily high but real borrowing costs are even higher. Since their preferred investment strategy is now less profitable, they choose to partially unwind their position. Finally, members of this group are also facing lower labor and government income.

Members in the group with ages 81–120 are retired but their situation is similar to that

[31] The responses reported here are population weighted averages of individual responses

Table 8: Population shares and impact responses of average consumption and net worth responses to a tighter monetary policy by age group

Age	Population Share	Consumption	Net worth	Illiquid assets
21–56	0.563	-0.194	-0.436	-0.812
57–80	0.328	0.130	0.147	-0.224
81–120	0.108	-0.283	-0.162	-0.303
21–120	1.000	-0.098	-0.215	-0.564

Note: Population shares are percentages of the total population. Consumption and net worth are averages of percentage deviations from the aggregate steady-state value by age in the impact year.

of the 21–56 year old age group. Their preferred asset allocation strategy is no longer so lucrative and their net government income is down.

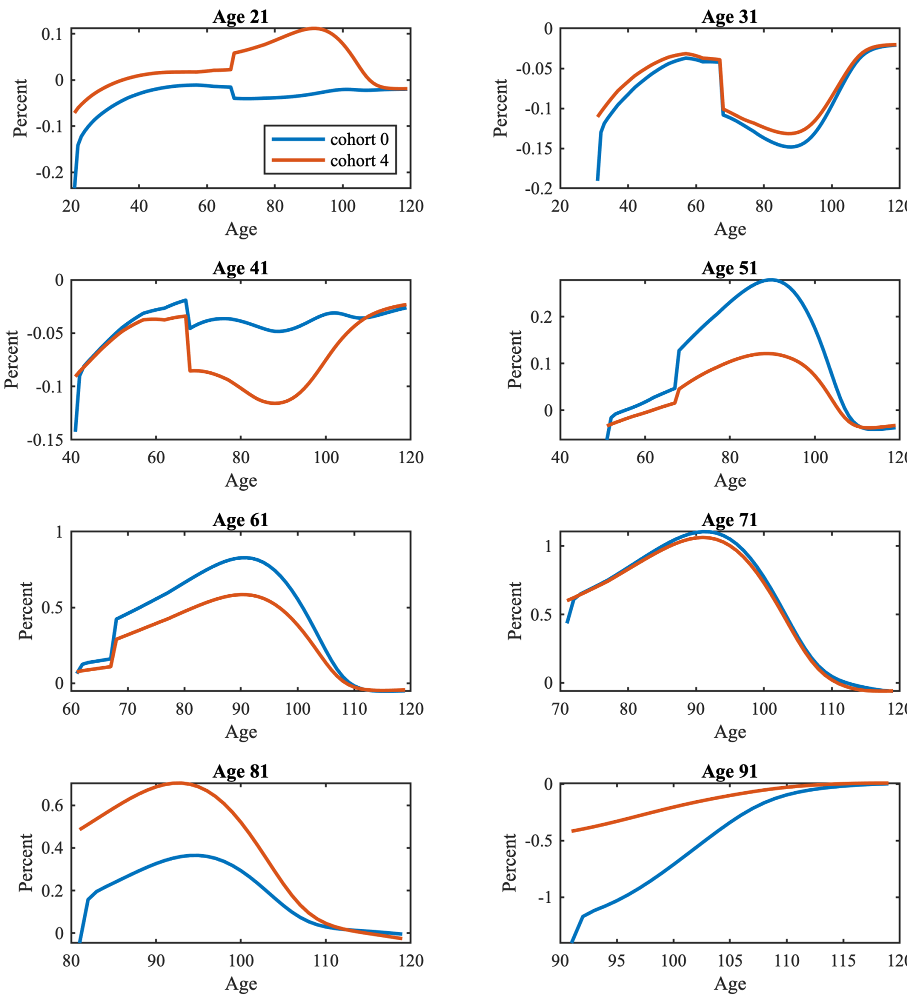
Perhaps the most interesting age group is households with ages 57–80. Their disposable income and net worth both increase in period 0 and they respond by consuming more but also choose to tilt their portfolios towards liquid assets. Moreover, as we will show next, this group finds their preferred investment strategy so attractive that they choose to save most of the increase in wealth that they experience in period 0. We now turn to analyze in more detail how the consumption plans of households change over the remainder of their lives.

6.4 Cohort effects and inequality

We have seen that monetary policy shocks have heterogeneous effects on households of different ages in the impact period. We now show that shocks to monetary policy also have large and persistent effects on consumption in subsequent periods for members of particular cohorts. To illustrate this point consider first members of the cohort that has age 71 when the monetary policy shock arrives. We choose this cohort because the decomposition of consumption we performed above suggests that wealth effects are particularly pronounced for households around this age. These households have just retired and have high and positive holdings of both liquid and illiquid assets.

Figure 5 displays age-consumption profiles of households in a cohort starting from the age specified in the title of the panel. The line labeled cohort 0 shows the age-consumption profile for the cohort with that age (e.g. 21 years old in the upper left panel) in the year that the shock arrives. The consumption-age profiles are reported as the deviation

Figure 5: Consumption-age profiles for different cohorts



Note: The figure reports consumption deviations from steady-state as a percentage of steady-state disposable income for an household of the age listed on the horizontal axis. Cohort 0 has the age listed in the title of the panel when the shock arrives in time zero. Cohort 4 reaches the age specified in the title of the panel five years later.

of consumption at the age listed on the horizontal axis from its steady-state level relative to steady-state disposable income at the same age. The panel titled "Age 71 of Figure 5 shows that a tighter monetary policy has a positive and hump-shaped effect on the consumption of households who are 71 years old in the impact period. Surprisingly, the peak deviation of their consumption occurs at age 90 or nearly 20 years after the shock arrives. To see why their consumption responds in this way it is helpful to decompose the response of the age profile of consumption of a 71 year old household into three components using the household's flow budget constraint: changes in cash flows from illiquid assets, changes in cash flow from liquid assets and changes in net cash flows from the government.

Appendix E performs this decomposition and shows that cash flows from liquid assets exhibit particularly large, positive and persistent deviations from the steady-state profile. Returns on liquid and illiquid assets are slightly above their steady-state levels for many periods but the deviation of liquid asset holdings is larger (see Figure 10 in Appendix E). Households in this cohort respond by persistently tilting their portfolio towards liquid assets.

Why does the peak consumption deviation occur at age 90? This is because the households' preferred asset allocation strategy changes if they survive to age 91. Up until this point they preferred to hold positive amounts of both assets. But, after age 90 they prefer to leverage up on illiquid assets and this investment strategy is not as lucrative because borrowing costs are now higher. Still, the households have accumulated more wealth at this juncture of the lifecycle and they are able to enjoy higher consumption in each remaining period of their life, if they pass away prior to age 108.[32]

Next we turn to describe how consumption-age profiles respond to a tighter monetary policy for households in other age groups. Notice that the age 51 and 61 cohort 0 households also experience large and hump-shaped increases in their consumption-age profile deviations. However, the cohort that is of age 91 at the time that monetary policy is tightened experiences a large loss in wealth and it responds by shifting its entire age-consumption profile down. Consumption declines by over 2 percent on impact and is down by more than 0.5 percent in all subsequent years of life (conditional on surviving). The shape of the consumption deviations is monotonic in this cohort. This reflects the fact that mortality risk is high and increasing rapidly with age.

The cohort whose age is 21 when the nominal interest rate rises also experiences a relatively large and persistent decline in its subsequent age-consumption profile. Households are born with zero wealth so the MPC of this cohort is large in the impact period. However, the shock to monetary policy is affecting not only this cohort's current income but also the return on its preferred investment strategy.

An alternative way to measure the size and persistence of cohort effects is to compare two cohorts who attain the same age in different years. The second line in each panel, labeled cohort 4, shows the consumption-age profile of the cohort that reaches the age specified in the title of the panel five years later. In most cases members of cohort 4 have smaller consumption-age profile deviations (in absolute value) from steady-state. This is not true in all instances. For example, at age 41, the deviation in the consumption-age

[32]See Appendix E for more details.

Table 9: Wealth and consumption inequality

Year	0	1	2	3	4
Wealth Inequality	0.16	0.24	0.24	0.18	0.15
Consumption Inequality	0.44	0.54	0.46	0.41	0.38

Note: Inequality is measured as the difference between the 90th and 10th percentiles of wealth (consumption) by age. We do not adjust for differences in the population share of different age groups when computing percentiles and the numbers reported in the table are percentage deviations from steady-state.

profile of cohort 4 is larger than the deviation of cohort 0. The reason why this occurs is because cohort 4 households experienced the shock at age 36. At that stage of their lifecycle their preferred investment strategy was to borrow liquid assets and invest them in illiquid assets. A tighter monetary policy lowered the return from that investment strategy and they arrive at age 41 with lower wealth compared to cohort 0. The cohort 4 consumption-age profile deviation is also larger at age 81. At this stage of the lifecycle cohort 0 is worse off than cohort 4. Members of cohort 0 were too old to enjoy the big positive effects that a tighter monetary had on the asset portfolio of the younger cohort.

Monetary policy also induces persistent increases in wealth and consumption inequality as reported in Table 9. We measure inequality as the difference between the 90th and 10th percentiles of the cross-sectional age-distribution in a given year. Inequality in net worth increases by 0.16 percent on impact and increases further to 0.24 percent in year one and then gradually falls back to its steady-state level.[33] Wealth inequality increases because the highest wealth households, who are close to age 65, have positive returns on their portfolios while younger and older households have negative returns. Consumption inequality increases by 0.44 percent on impact, rises to 0.54 percent in year 1 and then falls back toward its steady-state level thereafter. Consumption inequality increases because some age groups are reducing their consumption while other age groups are increasing their consumption. The reason consumption inequality increases more than wealth inequality is because households with low wealth have higher MPCs as compared to households with high wealth.

[33] The numbers are not adjusted for differences in the population share of different age groups.

7 Robustness

Taxing the old. In our baseline analysis we close the government budget constraint by adjusting the lump-sum transfer in each period. We have seen that the absolute magnitude of the MPC is particularly large for older households. They have relatively short planning horizons. Moreover, a lower lump-sum transfer associated with the tighter monetary policy has a large impact on their disposable income. This raises the question of how robust our conclusions are to this strategy for balancing the government budget constraint. We investigate this question by performing a counterfactual where the lump-sum transfer to all households over age 75 is held fixed at its steady-state value. Changing the government financing scheme in this manner doesn't impact our main results. The sign of the MPCs of older retirees is still negative and the absolute magnitude of their MPCs becomes smaller but is still quite high. The tax base for the lump-sum transfer (tax) is smaller in this simulation. Thus, the lump-sum transfer has to fall by more to close the government budget constraint, but disposable income of households close to age 68 still increases and their MPCs continue to be positive. This suggests that our finding about households' heterogeneous exposures to monetary policy shocks is reasonably robust to the specific details of how the group that is most sensitive to the size and timing of taxes is treated.

Table 13 in Appendix F reports results on inequality and Table 12 in the Appendix reports responses of macro aggregates in the impact period. Using this alternative tax scheme attenuates the increases in wealth and consumption inequality. The responses of the macro aggregates though are very close to the baseline responses.

Effective lower bound on the nominal interest rate. We have not analyzed unconventional monetary policies at the effective lower bound. But, these types of policies can be considered in the framework we have developed. One of the features of our model is that the price level is determinant even when the coefficient on the inflation rate in the central bank's interest rate targeting rule is set to zero. We have investigated how the dynamics of the model differ under this assumption. When we tighten monetary policy by the same amount, the inflation rate and profits fall by more and this induces larger responses in wages, returns and also the fiscal variables that impact household exposures to monetary policy (see Table 12 in Appendix F). The age-profiles of disposable income and MPCs have the similar signs and shapes as the baseline. Consumption inequality is exacerbated by this shock, the average MPC falls from -0.068 to -0.35 and aggregate consumption now falls by more. The responses of other macro aggregates are larger too but have the same qualitative features as our baseline specification.

Easier monetary policy. Given that there is large variation of households' exposures to a monetary policy shock, it is interesting to ascertain whether there are asymmetries in the impacts of tighter as compared to looser monetary policy shocks. An easier monetary policy lowers borrowing costs and increases the spread on illiquid assets, lump-sum transfers and wages. These changes increase disposable income of younger workers and older retirees but lower disposable income of households who are close to age 68. It follows that wealth inequality and consumption inequality both fall. An easier monetary policy shock of the same magnitude also produces small but meaningful differences in the absolute magnitude of the aggregate responses. The absolute magnitudes of the responses of output, hours, and the inflation rate are larger, but those of consumption and investment are smaller here as compared to the baseline.

Higher government debt. Japan and many other advanced economies have seen their debt/GDP levels increase in recent years. A higher debt-GDP ratio changes the aggregate composition of liquid and illiquid assets in our model and ultimately the shares of liquid and illiquid assets held by households. Given the important role that asset income plays in determining a household's overall exposures to monetary policy shocks, it is interesting to understand how the results change when we posit a higher debt-output level in the model. To explore this question we increased the debt-output level from its baseline level of 1.23 to 1.5 while holding fixed the other structural parameters. An increase of this magnitude has large impacts on the model's steady-state allocations. The ratio of private illiquid assets in output falls from 3.5 to 3.3 and private borrowing is also crowded out by higher government borrowing. These declines are associated with higher returns on both liquid and illiquid assets.

We find that households close to the age of 68 are better hedged against a tighter monetary policy and that young workers and older retirees are more exposed to this shock when there is more government debt in the economy. Net worth and consumption of the first group increase by more and net worth and consumption of the other two groups fall by more in the impact period relative to the baseline. The main reason for these differences is that the lump-sum transfers now fall by more. Recall that a tighter monetary policy lowers the price level and this increases the real stock of outstanding government debt.

This economy has a higher (fixed) nominal debt level and thus its real liabilities increase by more. Table 13 in Appendix F shows that a tighter monetary policy induces larger increases in wealth inequality and consumption inequality in this scenario. However, these differences in the household level responses largely average out and the response of the macro aggregates are close to the baseline.

8 Conclusion

We have found that both the sign and magnitude of disposable income and consumption responses to a monetary policy shock differ over the lifecycle. Our results also suggest that monetary policy has particularly large and persistent impacts on certain age groups. In light of these findings, in our future work we are interested in understanding how an optimal monetary policy strikes a balance between macro-economic stabilization and equity objectives.

References

Aiyagari, S. R. and M. Gertler (1991). Asset returns with transactions costs and uninsured individual risk. *Journal of Monetary Economics 27*, 311–331.

Auclert, A. (2019, June). Monetary policy and the redistribution channel. *American Economic Review 109*(6), 2333–67.

Bayer, C., R. Luetticke, L. Pham-Dao, and V. Tjaden (2019). Precautionary savings, illiquid assets, and the aggregate consequences of shocks to household income risk. *Econometrica 87*(1), 255–290.

Boppart, T., P. Krusell, and K. Mitman (2017). Exploiting MIT shocks in heterogeneous-agent economies: The impulse response as a numerical derivative. NBER Working Paper 24138.

Braun, R. A., D. Ikeda, and D. J. Joines (2009). The saving rate in Japan: Why it has fallen and why it will remain low. *International Economic Review 50*(1), 291–321.

Braun, R. A. and D. J. Joines (2015). The implications of a graying Japan for government policy. *Journal of Economic Dynamics and Control 57*, 1–23.

Braun, R. A., K. A. Kopecky, and T. Koreshkova (2016, 03). Old, sick, alone, and poor: A welfare analysis of old-age social insurance programmes. *The Review of Economic Studies 84*(2), 580–612.

Broer, T., P. Krusell, and E. Öberg (2020). The New Keynesian transmission mechanism: A heterogeneous-agent perspective. *Review of Economic Studies 87*, 77–101.

Brown, J. and A. Finkelstein (2008). The interaction of public and private insurance: Medicaid and the long-term care insurance market. *American Economic Review 98*, 1083–1102.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005, February). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy 113*(1), 1–45.

Cloyne, J., C. Ferreira, and P. Surico (2020). Monetary policy when households have debt: New evidence on the transmission mechanism. *Review of Economic Studies 87*(1), 102–129.

Damodaran, A. (2020). Equity risk premiums: Determinants, estimation and implications–the 2020 edition. mimeo.

Doepke, M. and M. Schneider (2006). Inflation and the redistribution of nominal wealth. *Journal of Political Economy 114*(6), 1069–1097.

Hagedorn, M. (2018). Prices and inflation when government bonds are net wealth. mimeo.

Hagedorn, M., I. Manovski, and K. Mitman (2019, February). The fiscal multiplier. mimeo.

Hayashi, F. and E. Prescott (2002). The 1990s in Japan: A Lost Decade. *Review of Economic Dynamics 5*(1), 206–235.

Holm, M., P. Paul, and A. Tischbirek (2020, January). The transmission of monetary policy under the microscope. Working Paper Series 2020-03, Federal Reserve Bank of San Francisco.

Horioka, C. Y. (2021). Is the selfish life-cycle model more applicable in Japan and, if so, why? A literature survey. *Review of Economics of the Household 19*, 157–187.

Hu, G., G. Ma, W. Qiao, and N. Wallace (2019). Conventional monetary policy in OLG models: Revisiting the asset substitution channel. mimeo.

Ikeda, D., S. Li, S. Mavroeidis, and F. Zanetti (2020). Testing the effectiveness of unconventional monetary policy in Japan and the United States. Institute for Monetary and Economic Studies Discussion Paper No. 2020-E-10, Bank of Japan.

Kaplan, G., B. Moll, and G. L. Violante (2018, March). Monetary policy according to HANK. *American Economic Review 108*(3), 697–743.

Kaplan, G. and G. L. Violante (2014). A model of the consumption response to fiscal simulus payments. *Econometrica 82*(4), 1199,1239.

Kehoe, P., V. Midrigan, E. Pastorino, and S. Salgado (2020). On the dynamic effects of monetary policy with heterogenous agents. mimeo.

Koga, M. and K. Matsumura (2020). Marginal propensity to consume and the housing choice. Bank of Japan Working Paper Series No. 20-E-3.

Kopecky, K. A. and T. Koreshkova (2014, July). The impact of medical and nursing home expenses on savings. *American Economic Journal: Macroeconomics 6*(3), 29–72.

Lockwood, L. M. (2018, September). Incidental bequests and the choice to self-insure late-life risks. *American Economic Review 108*(9), 2513–50.

Puckett, B. (2019, June). Japan: Supreme court rules on mandatory retirement. International newsletter, Olgetree Deakins.

Rotemberg, J. (1996). Prices, output and hours: An empirical analysis based on a sticky price model,. *Journal of Monetary Economics 37*, 503–533.

Sterk, V. and S. Tenreyro (2018). The transmission of monetary policy through redistributions and durable purchases. *Journal of Monetary Economics 99*(C), 124–137.

Takayama, N. and Y. Kitamura (1994, January). Household saving behavior in Japan. In *International Comparisons of Household Saving*, NBER Chapters, pp. 125–168. University of Chicago Press.

Wallace, N. (1980). The overlapping generations model of fiat money. In *Models of Monetary Economies*, pp. 49–82. Federal Reserve Bank of Minneapolis.

Wong, A. (2019). Refinancing and the transmission of monetary policy to consumption. manuscript.

Zaninotto, P., G. D. Batty, S. Stenholm, I. Kawachi, M. Hyde, M. Goldberg, H. Westerlund, J. Vahtera, and J. Head (2020, 01). Socioeconomic Inequalities in Disability-free Life Expectancy in Older People from England and the United States: A Cross-national Population-Based Study. *The Journals of Gerontology: Series A 75*(5), 906–913.

Appendix

A Model

A.1 Household problem

Consider the household problem (6). Let $\lambda_{j,t}(z)$ denote a Lagrange multiplier on the budget constraint (2) for $z = 1$ or (3) for $z = 0$. The first-order conditions with respect to $c_{j,t}$, $a_{j,t}$, and $d_{j,t}$ are given respectively by:

$$\lambda_{j,t}(z) = \frac{1}{1+\tau_t^c}\left(\frac{c_{j,t}(z)}{\eta_j}\right)^{-\sigma}, \tag{24}$$

$$\begin{aligned}&\lambda_{j,t}(1)\left(1+\gamma_a(1)\Delta a_{j,t}\right)\\ &\qquad = \beta\left[(1-\psi_{j+1,t+1})\frac{\partial U_{j+1}(a_{j,t},d_{j,t},0)}{\partial a_{j,t}} + \psi_{j+1,t+1}\frac{\partial U_{j+1}(a_{j,t},d_{j,t},1)}{\partial a_{j,t}}\right],\end{aligned} \tag{25}$$

$$\lambda_{j,t}(1) = \beta\left[(1-\psi_{j+1,t+1})\frac{\partial U_{j+1}(a_{j,t},d_{j,t},0)}{\partial d_{j,t}} + \psi_{j+1,t+1}\frac{\partial U_{j+1}(a_{j,t},d_{j,t},1)}{\partial d_{j,t}}\right], \tag{26}$$

where $\Delta a_{j,t} = a_{j,t}(1) - a_{j-1,t-1}$. For $z = 0$, conditions (25) and (26) are replaced by $a_{j,t} = 0$ and $d_{j,t} = 0$. The envelope conditions imply

$$\frac{\partial U_{j+1}(a_{j,t},d_{j,t},1)}{\partial a_{j,t}} = \lambda_{j+1,t+1}(1)\left(\tilde{R}^a_{t+1} + \gamma_a(1)\Delta a_{j+1,t+1}\right), \tag{27}$$

$$\frac{\partial U_{j+1}(a_{j,t},d_{j,t},0)}{\partial a_{j,t}} = \lambda_{j+1,t+1}(0)\left(\tilde{R}^a_{t+1} - \gamma_a(0)a_{j,t}\right), \tag{28}$$

$$\frac{\partial U_{j+1}(a_{j,t},d_{j,t},z_{j+1})}{\partial a_{j,t}} = \lambda_{j+1,t+1}(z_{j+1})\frac{\tilde{R}_t}{\pi_{t+1}}, \quad z_{j+1}\in\{0,1\} \tag{29}$$

Arranging conditions (24)-(29) yields

$$\begin{aligned}(1+\gamma_a(1)\Delta a_{j,t})\left(\frac{c_{j,t}(1)}{\eta_{j,t}}\right)^{-\sigma} &= \beta\Bigg[(1-\psi_{j+1,t+1})\left(\frac{c_{j+1,t+1}(0)}{\eta_{j+1}}\right)^{-\sigma}\left(\tilde{R}^a_{t+1} - \gamma_a(0)a_{j,t}\right)\\ &\quad + \psi_{j+1,t+1}\left(\frac{c_{j+1,t+1}(1)}{\eta_{j+1}}\right)^{-\sigma}\left(\tilde{R}^a_{t+1} + \gamma_a(1)\Delta a_{j+1,t+1}\right)\Bigg],\end{aligned} \tag{30}$$

$$\left(\frac{c_{j,t}(1)}{\eta_{j,t}}\right)^{-\sigma} = \beta\left[(1-\psi_{j+1,t+1})\left(\frac{c_{j+1,t+1}(0)}{\eta_{j+1}}\right)^{-\sigma} + \psi_{j+1,t+1}\left(\frac{c_{j+1,t+1}(1)}{\eta_{j+1}}\right)^{-\sigma}\right]\frac{\tilde{R}_t}{\pi_{t+1}}. \tag{31}$$

In the state $z = 0$, the household is in the final period of life and consumes all of its wealth:

$$c_{j,t}(0) = \frac{\tilde{R}^a_t a_{j-1,t-1} - \chi(0, a_{j-1,t-1}, 0) + \frac{\tilde{R}_{t-1}}{\pi_t}d_{j-1,t-1} + (1-\tau^w)w_t\epsilon_j h_{j,t} + b_{j,t} + \xi_t}{1+\tau_t^c}. \tag{32}$$

A.2 Liquidity premium and asset portfolios

Suppose that adjustment costs on illiquid assets are the same for surviving and non-surviving households: $\gamma_a(1) = \gamma_a(0) \equiv \gamma_a$. In addition, consider a stationary equilibrium so that time subscripts are dropped. Then, combining equations (30) and (31) yields

$$\Delta a_{j+1} = \frac{\tilde{R}}{\pi \kappa_{j+1}} \Delta a_j - \frac{1}{\gamma_a \kappa_{j+1}} \left(\tilde{R}^a - \frac{\tilde{R}}{\pi} \right) + \frac{1 - \kappa_{j+1}}{\kappa_{j+1}} a_j, \tag{33}$$

where $\Delta a_j \equiv a_j(1) - a_{j-1}$ and $0 < \kappa_{j+1} < 1$, given by

$$\kappa_{j+1} = \frac{\psi_{j+1} \left(c_{j+1}(1)/\eta_{j+1} \right)^{-\sigma}}{(1 - \psi_{j+1}) \left(c_{j+1}(0)/\eta_{j+1} \right)^{-\sigma} + \psi_{j+1} \left(c_{j+1}(1)/\eta_{j+1} \right)^{-\sigma}}.$$

From equation (33) we can establish two results. In doing so, we guess and verify that the initial real asset holding is positive $a_1 > 0$. Since $a_0 = 0$, an increase in the real asset in the initial age is also positive: $\Delta a_1 > 0$. The first result is that the interest rate spread between the return on real assets and the real government bond yield has to be positive:

$$\tilde{R}^a - \frac{\tilde{R}}{\pi} > 0.$$

To show this, suppose contrarily that the interest rate spread is non-positive: $\tilde{R}^a - \tilde{R}/\pi \leq 0$. Then, because $\Delta a_1 > 0$, equation (33) implies $\Delta a_j > 0$ for all $j = 2, ..., J$, so that $a_J > 0$, violating the terminal condition. Hence, the interest rate spread has to be positive in equilibrium. Second, because of the positive interest rate spread, individuals will prefer to borrow at the same interest rate as the liquid assets. The second result implies $a_j \geq 0$ for all $j = 1, ..., J$. Hence, the household may leverage real assets by borrowing liquid assets: $a_j > 0$ and $d_j < 0$. Since the problem of the real asset choice is smooth in the initial age, the real asset holding is positive in the initial age $a_1 > 0$, verifying the guess assumed in this discussion.

A.3 Labor supply decision

Working households belong to a labor union and work for an identical amount of hours $h_{j,t} = \bar{h}_t$ for all $j = 1, ..., J_r - 1$. The labor union consists of a continuum of union groups $l \in (0, 1)$ and distributes total hours worked $\sum_{j=1}^{J_r-1} \bar{h}_t N_{j,t}$ among union groups. Each union group l has a one-to-one linear technology that transforms hours per worker into specific labor supply per worker $\tilde{h}_t(l)$. An employment agency combines a continuum of specific labor and produces homogeneous labor $\tilde{H}_t = \tilde{h}_t \sum_{j=1}^{J_r-1} N_{j,t}$ following the technology: $\tilde{h}_t =$

$\left[\int_0^1 \tilde{h}_t(l)^{\frac{1}{\theta_w}} dl\right]^{\theta_w}$, where $\theta_w > 1$ is a wage markup. The employment agency is competitive and it follows that demand for specific labor l is given by

$$\tilde{h}_t(l) = \left(\frac{W_t(l)}{W_t}\right)^{-\frac{\theta_w}{\theta_w - 1}} \tilde{h}_t, \tag{34}$$

where $W_t(l)$ is nominal wage for specific labor supply $\tilde{h}_t(l)$.

Each union group chooses $W_t(l)$ to maximize the benefits of the members of the labor union, i.e. working households. Then, the problem of the union group is given by:

$$\max_{\{W_t(l)\}} \ (1-\tau^w) \sum_{j=1}^{J_r - 1} \frac{W_t(l)}{P_t} \epsilon_j \tilde{h}_t(l) \mu^w_{j,t} - \sum_{j=1}^{J_r - 1} \left(\frac{\psi_{j,t}}{\lambda_{j,t}(1)} + \frac{1-\psi_{j,t}}{\lambda_{j,t}(0)}\right) \frac{\upsilon}{1+\frac{1}{\nu}} \tilde{h}_t(l)^{1+\frac{1}{\nu}} \mu^w_{j,t},$$

subject to the labor demand curve (34), where $\lambda_{j,t}(z)$ with $z \in \{0, 1\}$ is a Lagrange multiplier, given by equation (24), and $\mu^w_{j,t} = N_{j,t} / \sum_{j=1}^{J_r-1} N_{j,t}$ is the ratio of population with age j to the working population. The second term of the problem is the weighted average of disutility of supplying labor, which is transformed into the units of the final good, over working households. This statement of the problem weights the disutilities of both surviving households and also households who experience a death shock. Our baseline specification, however, assumes that only surviving households receive weight and $\psi_{j,t}$ is set to 1 for all j in the problem. The first-order condition of this problem is equation (5) with $\bar{\lambda}_t$ and $\bar{\epsilon}_t$ are given, respectively, by[34]

$$\bar{\lambda}_t = \left[\sum_{j=1}^{J_r - 1} \left(\frac{\psi_{j,t}}{\lambda_{j,t}(1)} + \frac{1-\psi_{j,t}}{\lambda_{j,t}(0)}\right) \mu^w_{j,t}\right]^{-1}$$

$$\bar{\epsilon}_t = \sum_{j=1}^{J_r - 1} \epsilon_j \mu^w_{j,t}.$$

A.4 Intermediate goods firms

The i-th intermediate goods firm's problem can be solved in two steps. First, it minimizes the real costs of production, $w_t H_t(i) + r^k_t K_t(i)$, subject to equation (8), where w_t is the real wage per effective unit of labor and r^k_t is the rental rate of capital. Let $K_t \equiv \int_{i\in(0,1)} K_t(i) di$ and $H_t = \int_{i\in(0,1)} H_t(i) di$ denote aggregate capital and aggregate labor in effective units,

[34] Equation (5) abstracts from a markup because it is not identified under our additive preference structure.

respectively. Then the cost minimizing input demands can be expressed as:

$$w_t = mc_t(1-\alpha)\left(\frac{K_t(i)}{H_t(i)}\right)^{\alpha} = mc_t(1-\alpha)\left(\frac{K_t}{H_t}\right)^{\alpha}, \tag{35}$$

$$r_t^k = mc_t\alpha\left(\frac{K_t(i)}{H_t(i)}\right)^{\alpha-1} = mc_t\alpha\left(\frac{K_t}{H_t}\right)^{\alpha-1}, \tag{36}$$

where mc_t is real marginal cost and the second equality follows from the linear-homogeneity of the production function.

Second, the firm chooses $P_t(i)$ to maximize the present value of profits, $\Pi_t(i)+\Lambda_{t,t+1}\Pi_{t+1}(i)+ ...$, subject to equation (7), where the discount factor, $\Lambda_{t,t+1}$, is derived from preferences in the next subsection, and where the period-t profits, $\Omega_t(i)$, are given by

$$\Omega_t(i) = (1+\tau^f)\frac{P_t(i)}{P_t}Y_t(i) - mc_tY_t(i) - \frac{\gamma}{2}\left(\frac{P_t(i)}{P_{t-1}(i)}-1\right)^2 Y_t, \quad \gamma > 0. \tag{37}$$

In equation (37) τ^f is a subsidy and the quadratic term is the price adjustment cost. We assume that the subsidy is set at $\tau^f = \theta - 1$, so that marginal cost is one in steady-state. The optimality condition for the firm's price setting problem is:

$$\gamma\left(\frac{P_t(i)}{P_{t-1}(i)}-1\right)\frac{Y_t}{P_{t-1}(i)} = \frac{\theta}{\theta-1}\frac{mc_tY_t}{P_t} - \frac{1+\tau^f}{\theta-1}\frac{Y_t}{P_t} + \Lambda_{t,t+1}\gamma\left(\frac{P_{t+1}(i)}{P_t(i)}-1\right)\frac{P_{t+1}(i)}{P_t(i)^2}Y_{t+1}.$$

In a symmetric equilibrium the previous equation simplifies to equation (9), which is a nonlinear version of the NK Phillips curve.

B Competitive equilibrium

B.1 Derivation of the final goods market equilibrium condition

To derive the final goods market clearing condition, observe that the household budget constraints given by equations (2)–(3) hold with equality in equilibrium and can be summed over j, to obtain

$$\begin{aligned}
&(1+\tau_t^c)C_t + A_t + X_t + D_t \\
=&\tilde{R}_t^a\sum_{j=1}^{J}a_{j-1,t-1}(1)N_{j,t} + \frac{\tilde{R}_{t-1}}{\pi_t}\sum_{j=1}^{J}d_{j-1,t-1}(1)N_{j,t} + (1-\tau^w)w_tH_t + B_t + \Xi_t, \\
=&\tilde{R}_t^aA_{t-1} + \frac{\tilde{R}_{t-1}}{\pi_t}D_{t-1} + (1-\tau^w)w_tH_t + B_t + \Xi_t,
\end{aligned}$$

where $H_t = \sum_{j=1}^{J} \epsilon_j \bar{h}_t N_{j,t}$ and

$$X_t = \sum_{j=1}^{J} \left[\psi_{j,t}\chi\left(a_{j,t}(1), a_{j-1,t-1}(1), 1\right) + (1-\psi_{j,t})\chi\left(0, a_{j-1,t-1}(1), 0\right)\right] N_{j,t}$$

with $a_{0,t-1} = 0$. Substituting the government budget constraint, equation (19), into the previous equation and using the formulas for the after-tax interest rates we obtain

$$\begin{aligned}&(1+\tau_t^c)C_t + A_t + X_t \\ =&[1+(1-\tau^{ka})(R_t^k - 1)]A_{t-1} - \tau^a \frac{R_t}{\pi_t} D_{t-1} + (1-\tau^w)w_t H_t + T_t - G_t - \tau^f Y_t.\end{aligned}$$

Further substituting the tax equation (18) for T_t into this condition yields

$$C_t + A_t + X_t = R_t^k A_{t-1} + w_t H_t - G_t - \tau^f Y_t.$$

Recall that equation (15) implies $R_t^k = (\Omega_t + V_t)/V_{t-1}$. Thus, income from real assets can be expressed as

$$\begin{aligned}R_t^k A_{t-1} =& R_t^k (K_t + V_{t-1}), \\ =& [r_t^k + 1 - \delta]K_t + \Omega_t + V_t \\ =& r_t^k K_t + K_{t+1} - I_t + \Omega_t + V_t.\end{aligned}$$

Substituting (35), (36), (11), and (22) for w_t, r_t^k, Ω_t, and A_t, respectively, into the budget constraint yields[35]

$$C_t + I_t + G_t = Y_t - \frac{\gamma}{2}\left(\pi_t - 1\right)^2 Y_t - X_t, \tag{38}$$

B.2 Stationary equilibrium

In the economy with non-zero population growth, the aggregate variables such as Y_t grow at the rate of the population growth of $n_t = N_t/N_{t-1}$ in a stationary equilibrium. We scale aggregate variables in capital letters by using population and denote corresponding

[35] Substituting equation (22) into the aggregate household budget constraint yields

$$C_t + I_t + G_t = r_t^k K_t + w_t H_t + \Omega_t - \tau^f Y_t - X_t.$$

Substituting (11) into this equation:

$$C_t + I_t + G_t = r_t^k K_t + w_t H_t + \left[\theta - mc_t - \frac{\gamma}{2}\left(\pi_t - 1\right)^2\right] Y_t - \tau^f Y_t - X_t.$$

Note that equations (36) and (35) imply $r_t^k K_t + w_t H_t = mc_t Y_t$. And note that $\tau^f = \theta - 1$. Substituting these into the above equation yields equation (38).

per-capita variables as their small letters, e.g., $y_t = Y_t/N_t$.

In a stationary equilibrium the inflation rate is at its target value of $\pi = 1$. This implies that the marginal cost is unity, $mc = 1$, from equation (9), and the price adjustment cost is zero. Instead of pinning down per-capita hours worked $h = H/N$, we normalize it to be unity in a stationary equilibrium and set the coefficient of disutility of labor, υ, to satisfy $h = 1$. Note that hours in efficiency units per total population and hours per working population are related as follows: $h = \sum_{j=1}^{J_r-1} \epsilon_j \bar{h} \mu_j$. Under the normalization of $h = 1$, hours per working population is given by

$$\bar{h} = \frac{1}{\sum_{j=1}^{J_r-1} \epsilon_j \mu_j}.$$

Only when $\sum_{j=1}^{J_r-1} \epsilon_j \mu_j = 1$ e.g. for normalization, the two variables coincide: $h = \bar{h} = 1$.

Fix r^k, $\tilde{R}$, and ξ.[36] The return on capital is $R^k = r^k + 1 - \delta$. After the deduction of the capital income tax, the return on illiquid assets is given by $R^a = 1 + (1 - \tau^k)(R^k - 1)$. The interest rate income tax τ^a is also imposed. So, the after-taxed return is $\tilde{R}^a = 1 + (1 - \tau^a)(R^a - 1)$. The capital-labor ratio is given by $K/H = (r^k/\alpha)^{-1/(1-\alpha)}$ from equation (36). The output is given by $y = (K/H)^\alpha h$ with $h = 1$ by normalization. The real wage is given by $w = (1 - \alpha)(K/H)^\alpha$ from equation (35). The pension benefit b is given by

$$b = \lambda \frac{1}{j_r - 1} \sum_{j=1}^{j_r-1} w \epsilon_j h_j,$$

where $h_j = \bar{h}$. The liquid asset holding is $d = (D/Y)y$, where D/Y is a targeted net government debt output ratio. The government spending is $g = (G/Y)y$, where G/Y is a targeted government expenditure output ratio.

Consider a backward shooting algorithm for $c_J(0)$ and $a_{J-1}(1)$. Fix $c_J(0) > 0$ and $a_{J-1}(1) \geq 0$. Consumption $c_{J-1}(1)$ is given by equation (31) with $\psi_J = 0$ as

$$c_{J-1}(1) = \left(\frac{\beta \tilde{R}}{\pi} \right)^{-\frac{1}{\sigma}} \frac{\eta_{J-1}}{\eta_J} c_J(0).$$

From equation (30) with $\psi_J = 0$, we obtain $a_{J-2}(1)$ as

$$a_{J-2}(1) = a_{J-1}(1) - \frac{1}{\gamma_a(1)} \left[\beta \left(\frac{c_J(0)}{c_{J-1}(1)} \frac{\eta_{J-1}}{\eta_J} \right)^{-\sigma} \left(\tilde{R}^a - \gamma_a(0) a_{J-1}(1) \right) - 1 \right].$$

[36]Practically the initial values for r^k and ξ are set at the corresponding steady-state values with no cost of adjusting illiquid assets.

The liquid asset holding $d_{J-1}(1)$ is given by equation (32) as

$$d_{j-1}(1) = \frac{1}{\tilde{R}}\left\{c_j(0) + \chi(0, a_{j-1}(1), 0) - \tilde{R}^a a_{j-1}(1) - \left[(1-\tau^w)w\epsilon_j\bar{h} + b + \xi\right]\right\},$$

for $j = J$. From the budget constraint (2), the liquid asset holding $d_{J-2}(1)$ is given as

$$\begin{aligned} d_{j-1}(1) = \frac{1}{\tilde{R}}\Big\{ c_j(1) &+ a_j(1) + \chi(0, a_j(1), a_{j-1}(1)) + d_j(1) \\ &- \tilde{R}^a a_{j-1}(1) - \left[(1-\tau^w)w\epsilon_j\bar{h} + b + \xi\right] \Big\}, \end{aligned} \tag{39}$$

for $j = J-1$. Then, $c_{J-1}(0)$ is given by equation (32). With $c_{J-1}(0)$ and $c_{J-1}(1)$ on hand, $c_{J-2}(1)$ is given by equation (31) as

$$c_j(1) = \left(\frac{1}{\beta\tilde{R}}\right)^{\frac{1}{\sigma}} \left(\frac{\eta_j}{\eta_{j+1}}\right) \left[(1-\psi_{j+1})c_{j+1}(0)^{-\sigma} + \psi_{j+1}c_{j+1}(1)^{-\sigma}\right]^{-\frac{1}{\sigma}}, \tag{40}$$

for $j = J-2$. Then, Δa_j is given by equation (30) and the real asset holding in the previous period is given by $a_{j-1}(1) = a_j(1) - \Delta a_j$ for $j = J-2$. Then, the liquid asset holding in the previous period is given by (39), the consumption $c_{j-1}(0)$ is given by equation (32), and the consumption $c_{j-1}(1)$ is given by equation (40). Continuing this process for $j = J-3, ..., 1$ yields the initial asset holdings of $a_{-1}(1)$ and $d_{-1}(1)$. Consumption $c_J(0)$ and the real asset holding $a_{J-1}(1)$ are adjusted so as to satisfy $d_{-1}(1) = 0$ and $a_{-1}(1) = 0$.

The loop for r^k, $\tilde{R}$, and ξ is closed as follows. Let τ_t denote per capita tax revenue: $\tau_t = T_t/N_t$. From equation (19), the updated value of the lump-sum transfer ξ' is given by

$$\xi' = \tau - g - \left(\frac{R}{\pi n} - 1\right)d - \tau^f y - b\sum_{j=j_r}^{J}\mu_j.$$

where $R = 1 + (\tilde{R}-1)/(1-\tau^a)$, n is the gross growth rate of population, and τ is given by equation (18) as

$$\tau = \tau^c\sum_{j=1}^{J}\bar{c}_j\mu_j + \tau^{ka}(r^k-\delta)\sum_{j=1}^{J}a_{j-1}(1)\mu_j + \tau^a\frac{R-1}{\pi}\sum_{j=1}^{J}d_{j-1}(1)\mu_j + \tau^w w\sum_{j=1}^{J}\epsilon_j\bar{h}\mu_j, \tag{41}$$

where $\bar{c}_j = \psi_j c_j(1) + (1-\psi_j)c_j(0)$. From equation (20),

$$d_t = \sum_{j=1}^{J} d_{j,t}(1)\mu_{j+1,t+1}n_{t+1},$$

with $d_{J,t}(1) = 0$, so that in evaluating τ, $\sum_{j=1}^{J} d_{j-1}(1)\mu_j$ in equation (41) should be set at

$$\sum_{j=1}^{J} d_{j-1}(1)\mu_j = d/n.$$

The capital stock is given by $k = (a - v)/n$ from equation (22), where the illiquid asset holding is given by $a \equiv \sum_{j=1}^{J} a_{j-1}(1)\mu_j/n$ and the value of firms v is given by combining equation (16) and (11) as

$$v = \frac{(\theta - 1)y}{R^k/n - 1}.$$

The updated value of r^k is given by

$$r^{k\prime} = \alpha k^{\alpha-1} h^{1-\alpha},$$

with $h = 1$. The value of the liquid asset holding, implied from the household optimization problem is $d' = \sum_{j=1}^{J} d_{j-1}(1)\mu_j/n$. The corresponding value that is consistent with the target value of debt output ratio is d. Since households are willing to hold liquid assets more as the interest rate increases, adjust the updated value of $\tilde{R}'$ as follows: increase (decrease) $\tilde{R}'$ if $d' < d$ ($d' > d$). In doing so, make sure that there is a positive spread between $\tilde{R}^a$ and $\tilde{R}/\pi$: $\tilde{R}^a - \tilde{R}/\pi > 0$.

Guessed values for r^k, $\tilde{R}$, and ξ are adjusted until $|r^k - r^{k\prime}| + |d - d'| + |\xi - \xi'|$ becomes close enough to zero. This completes the description of the computation of the stationary equilibrium.

B.3 Dynamic equilibrium

Consider the computation of a transition path from period t_s to period t_e. Without loss of generality we assume that the economy reaches a stationary equilibrium in period t_e. In what follows we will assume that the initial condition is also a steady-state, that is, that the economy is in steady-state in period $t_s - 1$. In order to induce a transition we assume that in period t_s an MIT shock arrives. The MIT shock could consist of a perturbation to the central bank's nominal interest rate targeting rule, and/or some element of fiscal policy.

In the main text we have assumed that the population distribution is stationary with zero population growth in the dynamic equilibrium. However, here we relax that assumption and allow for the possibility of time-varying population during the transition. In particular, the size of the population in period t follows $N_t = n_t N_{t-1}$ where n_t is the population growth rate and survival probabilities ψ_t can depend on t during the transition. In what follows our use of the term steady-state when referring to the initial or terminal condition consists of a steady-state population distribution with a constant population as well as the steady-state price system and allocations that we defined in the previous section.

The aggregate state variables in period $t = t_s$ consist of the per-capita capital stock k_t, per-capita value of equity in intermediate goods firms v_{t-1}, per capita government debt d_{t-1}, and the nominal interest rate R_{t-1}. The initial conditions for age-specific variables are liquid asset holdings $\{d_{j,t-1}\}_{j=1}^{J}$ and illiquid asset holdings $\{a_{j,t-1}\}_{j=1}^{J}$. Recall that illiquid assets consist of the capital stock and equity in intermediate goods firms and that each age-group holds the market portfolio of the two underlying assets. The market portfolio shares of capital stock and equity are k_{t_s}/a_{t_s-1} and v_{t_s-1}/a_{t_s-1}, respectively, where $a_{t_s-1} = k_{t_s}+v_{t_s-1}$.

We solve this two point boundary problem by guessing and verifying a path for $\{r_t^k, \pi_t, \xi_t, \gamma_{y,t+1}, \gamma_{k,t+1}\}_{t=t_s}^{t_e}$ and $R_{t_s}^a$, where $\gamma_{y,t+1} = y_{t+1}/y_t$, $\gamma_{k,t+1} = k_{t+1}/k_t$, and $R_{t_s}^a$ is the ex post return on the market portfolio of illiquid assets in the initial period.

Since the initial condition is a steady-state, a sequence of capital is given by $k_{t+1} = \gamma_{k,t+1}k_t$ with $k_{t_s} = k$. From the law of motion for capital (12), the newly produced investment good in per capita is given by

$$i_t = n_{t+1}k_{t+1} - (1-\delta)k_t.$$

where $i_t \equiv I_t/N_t$. The return on real assets is given by the arbitrage condition (15) as $R_{t+1}^a = 1 + (1-\tau^k)(R_{t+1}^k - 1)$, where $R_{t+1}^k = r_{t+1}^k + 1 - \delta$. Its after-taxed return is given as $\tilde{R}_s^a = 1 + (1-\tau^a)(R_s^a - 1)$ for $s = t_s+1, t_s+2, ...$ For $s = t_s$, the guessed value of $R_{t_s}^a$ is used to compute its after-taxed return. Compute the nominal interest rate R_t using the monetary policy rule (17) for $t = t_s, ..., t_e$. Compute the marginal cost mc_t using the Phillips curve (9) as

$$mc_t = 1 + \frac{\gamma(\theta-1)}{\theta}\left[(\pi_t - 1)\pi_t - \frac{\gamma_{Y,t+1}n_{t+1}}{R_{t+1}^k}(\pi_{t+1}-1)\pi_{t+1}\right].$$

for $t = t_s, ..., t_e$. Compute k_t/h_t from equation (36) and compute w_t from equation (35). Since k_t is known, hours per capita in efficiency units h_t can be computed from the ratio k_t/h_t. Hours per working population is given by $\bar{h}_t = h_t/\sum_{j=1}^{J_r-1}\epsilon_j\mu_{j,t}$.

Given these prices and aggregate variables, solve the household problem for those with age $j = 2, ..., J$ in period $t = t_s$ and those who are born in period $t = t_s, ..., t_e$. The solution yields $\{a_{j,t}(z), d_{j,t}(z), c_{j,t}(z)\}_{j=1}^{J}$ for $t = t_s, ..., t_e$ and for $z \in \{0, 1\}$. From these individual decisions, the illiquid asset holding, the liquid asset holding, and consumption in per capita terms are given, respectively, by $a_t = \sum_{j=1}^{J}[\psi_{j,t}a_{j,t}(1) + (1-\psi_{j,t})a_{j,t}(0)]\mu_{j,t}$, $d_t = \sum_{j=1}^{J}[\psi_{j,t}d_{j,t}(1) + (1-\psi_{j,t})d_{j,t}(0)]\mu_{j,t}$, and $c_t = \sum_{j=1}^{J}[\psi_{j,t}c_{j,t}(1) + (1-\psi_{j,t})c_{j,t}(0)]\mu_{j,t}$, where $\mu_{j,t} = N_{j,t}/N_t$ is the share of population with age j.

Now we are in a position to aggregate the economy and derive conditions to confirm whether the initially guessed values for $R^a_{t_s}$ and $\{r^k_t, \pi_t, \xi_t, \gamma_{y,t+1}, \gamma_{k,t+1}\}_{t=t_s}^{t_e}$ are in an equilibrium. By using the endogenously computed d_t and the exogenously given $d^n_t = D^n_t/N_t$, the price level P_t can be computed from the liquid asset market clearing condition (20) for all t. This yields the updated sequence of the inflation rate, $\pi'_t = P_t/P_{t-1}$ for all t, where P_{t_s-1} is in the initial steady-state. From equation (20), using $d_{0,t-1}(1) = d_{J,t-1}(1) = 0$, the liquid asset market clearing condition is written as

$$\frac{d^n_{t-1}}{P_{t-1}} = \sum_{j=1}^{J} d_{j-1,t-1}(1)\mu_{j,t}n_t.$$

Then, the liquid asset term in the tax equation (18) can be written as

$$\begin{aligned}\tau^a \frac{R_{t-1}-1}{\pi_t}\sum_{j=1}^{J} d_{j-1,t-1}(1)N_{j,t} =& \tau^a \frac{R_{t-1}-1}{P_t/P_{t-1}}\sum_{j=1}^{J} d_{j-1,t-1}(1)\mu_{j,t}N_t \\ =& \tau^a \frac{R_{t-1}-1}{P_t}\frac{d^n_{t-1}}{n_t}N_t.\end{aligned}$$

From equations (18) and (19), the updated value of the lump-sum transfer ξ'_t is given by: for $t = t_s+1, ..., t_e$

$$\begin{aligned}\xi'_t =& \tau^c_t \sum_{j=1}^{J} \bar{c}_{j,t}\mu_{j,t} + \tau^{ka}(R^k_t - 1)\sum_{j=1}^{J} a_{j-1,t-1}(1)\mu_{j,t} + \tau^a \frac{(R_{t-1}-1)d^n_{t-1}/n_t}{P_t} \\ &+ \tau^w w_t \sum_{j=1}^{J} \epsilon_j \bar{h}_t \mu_{j,t} - g_t - \frac{R_{t-1}d^n_{t-1}/n_t - d^n_t}{P_t} - \tau^f y_t - \sum_{j=j_r}^{J} b_{j,t}\mu_{j,t}.\end{aligned} \tag{42}$$

where $g_t = G_t/N_t$ and $\tau^{ka} = \tau^k + \tau^a - \tau^k\tau^a$. In period $t = t_s$ when the monetary policy shock hits, the returns earned by investing in capital and ownership shares can be different so that the lump-sum transfer is given by:

$$\xi'_{t_s} = \sum_{j=1}^{J} \left[\tau_t^c \bar{c}_{j,t} + \frac{\tau^{ka}}{1-\tau^k}(R^a_{t_s} - 1)a_{j-1,t_s-1} + \tau^w w_{t_s}\epsilon_j \bar{h}_{t_s} \right] \mu_{j,t_s} + \tau^a \frac{(R_{t_s-1}-1)d^n_{t_s-1}/n_{t_s}}{P_{t_s}}$$
$$- g_{t_s} - \frac{R_{t_s-1}d^n_{t_s-1}/n_{t_s} - d^n_{t_s}}{P_{t_s}} - \tau^f y_{t_s} - \sum_{j=j_r}^{J} b_{j,t_s}\mu_{j,t_s}.$$

From equation (22), the updated value of the capital stock is given by

$$k'_{t+1} = (a_t - v_t)/n_{t+1},$$

where $v_t \equiv V_t/N_t$ is the per-capita value of the sum of the intermediate goods firms and investment good firms. Its aggregate value V_t is given by (16) with Ω_t given by (11). Thus, v_t can be computed forward using

$$v_t = \frac{\left[\theta - mc_{t+1} - \frac{\gamma}{2}(\pi_{t+1}-1)^2\right] y_{t+1}}{R^k_{t+1}/n_{t+1}} + \frac{v_{t+1}}{R^k_{t+1}/n_{t+1}},$$

With a sequence of k'_{t+1} in hand, the updated gross growth rate of capital is simply computed as $\gamma'_{k,t+1} = k'_{t+1}/k'_t$. From the wage Phillips curve (5), the updated value of the hours worked $\bar{h}'_t$ can be computed. The updated value for the net return on capital $r^{k\prime}_t$ is given by equation (36). The updated output is given by

$$y'_t = (k'_t)^\alpha \left(\sum_{j=1}^{J} \epsilon_j \bar{h}'_t \mu_{j,t} \right)^{1-\alpha},$$

for all $t = t_s, ..., t_e$. Then, the updated value of $\gamma_{y,t+1}$ is given by $\gamma'_{y,t+1} = y'_{t+1}/y'_t$. The updated value of $R^a_{t_s}$ is the weighted average of the returns on capital and equity

$$R^{a\prime}_{t_s} = 1 + (1-\tau^k)\Bigg\{ \frac{k_{t_s}n_{t_s}}{k_{t_s}n_{t_s} + v_{t_s-1}}(r^k_{t_s} + 1 - \delta)$$
$$+ \frac{v_{t_s-1}}{k_{t_s}n_{t_s} + v_{t_s-1}} \frac{\left(v_{t_s} + \left[\theta - mc_{t_s} - \frac{\gamma}{2}(\pi_{t_s}-1)^2\right] y_{t_s}\right) n_{t_s}}{v_{t_s-1}} - 1 \Bigg\},$$

where $k_{t_s} = k$ and $v_{t_s-1} = v$. Finally, the updated value of pension benefits is computed as:

$$b_{j,t} = \lambda \left(\frac{1}{j_r - 1} \sum_{i=1}^{j_r-1} w_{t-j+i}\epsilon_i \bar{h}_{t-j+i} \right).$$

for $j = j_r, ..., J$ and $t = t_s, t_s+1, ...$

The values of $\{r^k_t, \pi_t, \xi_t, \gamma_{Y,t+1}\}_{t=t_s}^{t_e}$ and $R^a_{t_s}$ are adjusted until $\max_{t\in\{t_s,...,t_e\}}(|r^k_t - r^{k\prime}_t| +$

$|\pi_t - \pi_t'| + |\xi_t - \xi_t'| + |\gamma_{Y,t+1} - \gamma_{Y,t+1}'| + |\gamma_{k,t+1} - \gamma_{k,t}'|) + |R_{t_s}^a - R_{t_s}^{a\prime}|$ becomes close enough to zero. This completes the computation of the transition.

C Data measures of liquid and illiquid assets

We calibrate two model parameters, β, the preference discount factor and d/y the public debt output ratio, to data on aggregate stocks of (net) liquid and (gross) illiquid assets. Here we provide more detail on how we construct the aggregate stocks of liquid and illiquid assets. We have two alternative data sources. First, we can construct them using aggregate data from the Japanese FFA for financial variables and data from the Japanese NIPA accounts for aggregate holdings of real assets. Secondly, we derive aggregate stocks from the NSFIE, which is nationally representative and conducted once every five years. We describe each of these strategies in turn. Table 10 reports assets and liabilities as a fraction of GDP. The GDP shares in the column with the heading FFA/NIPA are constructed using 2014 fiscal year FFA data for the household sector from the Bank of Japan and using 2014 NIPA accounts for calendar year GDP and holdings stocks of physical assets held by households. We discuss construction of the data in this column first. In Japanese FFA data the household sector consists of households and private unincorporated enterprises. Aggregate liquid assets consist of household holdings of cash, domestic deposits, and public and private debt securities. This amounts to 1.79 times GDP. Liquid liabilities, which consist of consumer credit, are 0.069 of GDP. Net liquid assets relative to GDP are then 1.73 using the classification scheme of Kaplan et al. (2018). As explained in the main text it is more suitable in our model to treat all household borrowing as liquid borrowing. Under this assumption net liquid assets are 1.23.

Illiquid assets have two components: physical and financial assets. Physical assets are the end-of-calendar-year stock of household (and unincorporated private business) non-financial assets taken from the 2014 NIPA and amount to 2.02 times GDP. Financial assets include household holdings of: non-life insurance reserves, life insurance reserves, annuity entitlements, private pensions (defined benefit and defined contribution), and equity and options from the FFA. The resulting magnitude of illiquid financial assets is 3.50 times GDP. Illiquid liabilities consist of net non-financial sector loans, installment credit and non-financial sector loans, mortgages, and other loans by financial institutions plus non-housing loans by public financial institutions. Total illiquid liabilities constructed in this way amount to 0.49 of GDP. Net illiquid assets are then 3.01 using the classification scheme

Table 10: Liquid and illiquid assets relative to GDP in FFA/NIPA and NSFIE

	FFA/NIPA	NSFIE
A. Liquid assets and liabilities		
Liquid assets		
Currency and domestic deposits	1.74	0.99
Debt securities (total public and private)	0.052	0.047
Total liquid assets	1.79	1.04
Liquid liabilities		
Consumer credit	-0.069	-0.025
Total liquid liabilities	-0.069	-0.025
Net liquid assets: Kaplan et al. (2018)	1.73	1.01
Net liquid assets: Our model	1.23	0.85
B. Illiquid assets and liabilities		
Illiquid assets		
Physical assets	2.02	2.17
Equity and options	0.49	0.19
Insurance	0.99	0.30
Non-life insurance reserves	0.09	NA
Life insurance	0.88	0.30
Life insurance reserves	0.39	NA
Annuity entitlements	0.18	NA
Private pensions	0.31	NA
Total illiquid assets	3.50	2.66
Illiquid liabilities		
Installment credit and net non-financial sector loans	-0.00	-0.01
Mortgages	-0.37	-0.34
Other loans fin. inst. and non-home loans public inst.	-0.11	NA
Total illiquid liabilities	-0.49	-0.35
Net illiquid assets: Kaplan et al. (2018)	3.01	2.31
Net illiquid assets: Our model	3.50	2.66
C. Net worth	4.73	3.32

Note: Data are expressed as a multiple of GDP. Results under the heading FFA/NIPA are based on Flow of Funds and NIPA aggregate data. Data under the heading NSFIE are constructed from the National Family Income and Expenditure Survey and the Family Income and Expenditure Survey.

of Kaplan et al. (2018) and 3.50 using our classification scheme. Summing together net liquid and illiquid assets implies that total household net-worth is 4.73 times GDP.

An alternative way to construct these aggregates is to aggregate up micro survey data. Our primary source for the results reported in the final column of Table 10 is the 2014 NSFIE. An attractive feature of this survey is that it reports family income, financial and physical assets, and liabilities by 10 year age group of the household head. This survey is large and nationally representative but is only conducted at 5 year intervals. The most recent survey for which data is publicly available as of 2020 is from the year 2014. This is why we have chosen to use the year 2014 as our reference point.

A much smaller household survey called the Family Income and Expenditure Survey (FIES) is conducted on a quarterly basis. This survey provides more detail on the composition of financial assets and liabilities than the NSFIE but doesn't report results for holdings of physical assets. We use data from the FIES 2019 end of year survey to impute shares of financial liquid assets and liabilities by 10-year age group when these categories are not available in the NSFIE. Namely, we impute liquid and illiquid shares of financial securities and equity using their shares in the 2019 FIES for each 10 year age group. The 2014 NSFIE reports mortgages but does not provide detail on other liabilities. We thus impute non-mortgage illiquid liabilities and liquid liabilities by 10-year age group using their shares in the 2019 FSIE. A final issue is that holdings of cash are not reported in the NSFIE. We impute cash holdings assuming that they are 9% of deposits, which is the ratio of cash to deposits in the aggregate FFA data.

With household level data on liquid and illiquid assets in hand, we then construct aggregate NSFIE assets by multiplying average per-household levels of each variable times the total number of households in Japan in 2014 and then dividing by calendar year nominal GDP for the year 2014. A comparison of the two columns of data in Table 10 reveals that the size of net liquid and illiquid assets is smaller in the NSFIE than in the FFA/NIPA. For instance, liquid assets are 1.79 times GDP in the FFA/NIPA and only 1.04 times GDP in the NFSIE. Deposits in the FFA, in particular, are much higher than in the NSFIE.

Illiquid assets are also smaller in the NSFIE as compared to the FFA. Physical assets are about the same in the two datasets, but, holdings of life insurance and equity are much smaller in the NSFIE. A final and smaller difference is that illiquid liabilities are also smaller in the NSFIE. This second difference appears to be primarily due to non–mortgage lending by financial institutions and non-housing loans by public financial institutions, categories that are not broken out in the NSFIE. These differences translate into smaller average net worth and a larger share of illiquid assets in total (net) assets. Net worth is 3.32 in the NSFIE while it is 4.73 in the FFA data. Using definitions suitable for our model, the share of illiquid assets in total assets is 0.76 for the NSFIE and 0.74 for the FFA.

Takayama and Kitamura (1994) report that the size of average household financial assets is substantially smaller in the NSFIE survey than the FFA data using the NSFIE in earlier years. They suggest three reasons for the gap. First, a disproportionately large number of high wealth households may be refusing to participate in the NSFIE. Second, some self-employed respondents may be confused about what to report and are not reporting assets and liabilities of their businesses. Third, there may be measurement error because the household sector is treated as a residual in the SNA/FFA commodity flow method.

The results in Table 10 are consistent with the claim that the NSFIE may not be adequately capturing assets and *liabilities* of proprietorships. Not only are deposits small in the NSFIE, but the NSFIE doesn't appear to be capturing other loans of financial institutions and non-home loans of public institutions. We conjecture that most of these loans are to proprietorships. Finally, our results also suggest that respondents in the NSFIE are also under-stating the value of their private pensions and annuities.

D The decomposition of consumption responses

Applicability of decomposition. The decomposition formula developed by Auclert (2019) holds up to first-order approximation under specific assumptions about a model and a monetary policy shock. The assumptions hold for a one-time transitory monetary policy shock in a standard RANK model without persistence. But, our model does not satisfy the assumptions strictly as the model has some degree of persistence and has positive nominal government debt, and it is solved nonlinearly. Yet, the model may have features that are not so away from the assumptions.

Figure 6: Impulse responses of aggregate variables

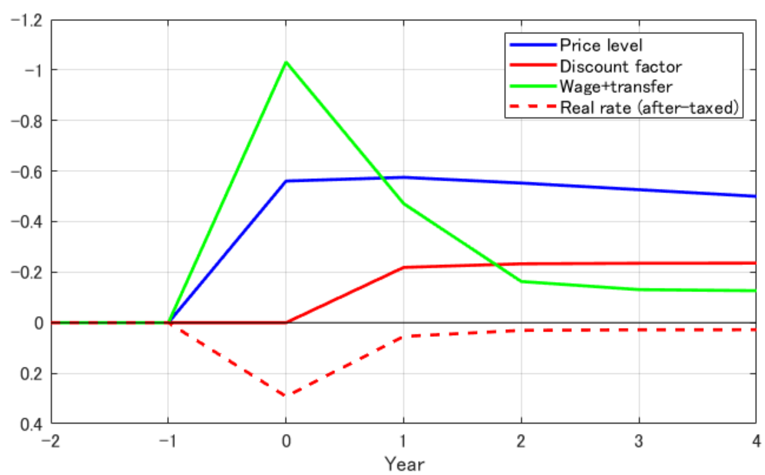


Figure 6 plots the impulse responses of the price level P_t, the ex-post real interest rate R_t/π_{t+1}, the real discount factor R_0^k/R_t^k, and real wage plus real unearned income $w_t + \xi_t$ to a tightening in monetary policy in $t = 0$ in our model. The vertical axis is flipped for ease of comparison with Figure 1 of Auclert (2019). Although the response of the wage plus transfers is somewhat persistent, the responses of the variables are broadly in line with those assumed in Auclert (2019): one time permanent changes in the price level and the discount factor; one time transitory changes in the real rate and the wage plus transfers.

Decomposition formula. Consider the impact response of consumption by an age-j household to a tightening in monetary policy in $t = 0$. The decomposition formula developed by Auclert (2019) can be written as

$$dc_{j,0} = \underbrace{\widehat{MPC}_j \left(d\mathcal{Y}_{j,0} - NNP_{j,0}\frac{dP_0}{P} + URE_{j,0}\frac{d\tilde{R}_1^r}{\tilde{R}^r} \right) - \frac{c_j}{\sigma}\left(1 - \widehat{MPC}_j\right)\frac{d\tilde{R}_1^r}{\tilde{R}^r}}_{\widehat{dc}_{j,0}} + \text{disc}_j, \quad (43)$$

where $dc_{j,0} \equiv c_{j,0} - c_j$ is a difference in consumption by an age-j household from steady-state, dP_0/P is a deviation of the price level from steady-state, $d\tilde{R}_1^r/\tilde{R}^r$ is a deviation of the ex-post real interest rate $\tilde{R}_1^r = \tilde{R}_0/\pi_1$, and disc_j is a discrepancy between $dc_{j,0}$ and $\widehat{dc}_{j,0}$. In formula (43), there are four key statistics: the modified marginal propensity to consume, $\widehat{MPC}_j$, a change in earned and unearned income, $d\mathcal{Y}_{j,0}$, net nominal position, $NNP_{j,0}$, and unhedged interest rate exposure, $URE_{j,0}$, which are defined, respectively, as

$$\widehat{MPC}_j \equiv \frac{\partial c_j/\partial \xi}{1 + (1-\tau^w)w\epsilon_j(\partial h/\partial \xi)},$$

$$d\mathcal{Y}_j \equiv d\xi_0 + db_{j,0} + (1-\tau^w)h\epsilon_j dw_0 + (1-\tau^w)w\epsilon_j dh,$$

$$NNP_j \equiv \frac{\tilde{R}_{-1}}{\pi_0}d_{j-1,-1},$$

$$URE_{j,0} \equiv (1-\tau^w)w_0\epsilon_j h_0 + b_{j,0} + \xi_0 - (1+\tau^c)c_{j,0} + \frac{\tilde{R}_{-1}}{\pi_0}d_{j-1,-1} + a_{j,0}^m,$$

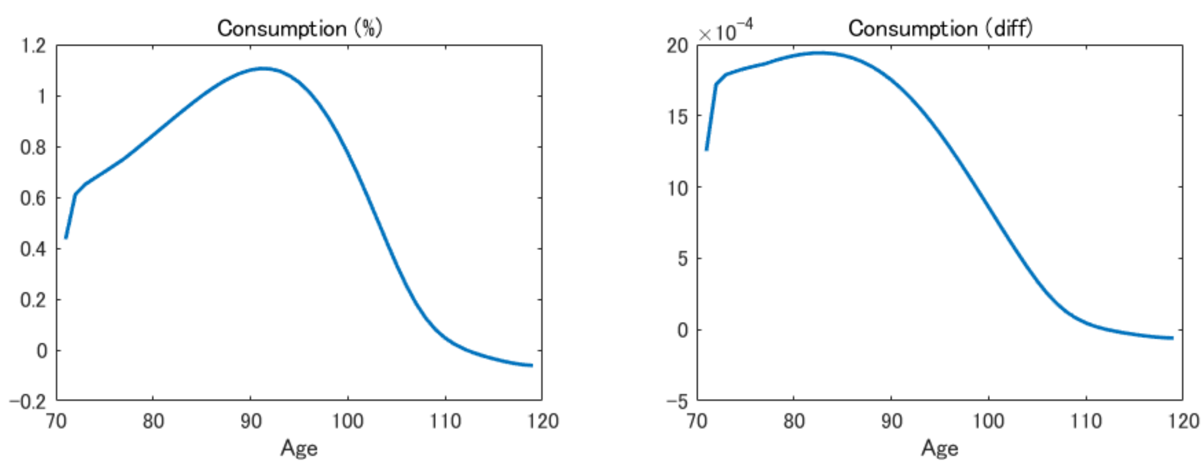
where $\partial c_j/\partial \xi$ denotes the partial equilibrium marginal propensity to consume for an age-j household, $\partial h/\partial \xi$ denotes the partial equilibrium marginal propensity to supply labor, and $a_{j,0}^m$ is the illiquid assets maturing in period 0, held by an age-j household. The partial equilibrium marginal propensity to consume is already calculated in Figure 2 for all ages. Since the hours worked are determined by the labor union to which all working households belong, an increase in ξ for a single household does not affect hours worked, and thus $\partial h/\partial \xi = 0$. The illiquid assets maturing in period 0 is given as $a_{j,0}^m = \mu_{j,0}^a A_0^m$, where $\mu_{j,0}^a$ is a ratio of the illiquid assets held by an age-j household to the total illiquid assets, given by $\mu_{j,0}^a \equiv a_{j-1,-1}/A_{-1}$, and A_0^m is the aggregate illiquid assets maturing in period 0, given by $A_0^m \equiv \left[1 + (1-\tau^a)(1-\tau^k)(r_0^k - \delta)\right] K_0 + (1-\tau^a)(1-\tau^k)\Omega_0$. The four components shown in Figure 4 are then given by

$$
\begin{aligned}
\mathit{Income} &= \widehat{MPC}_j \times d\mathcal{Y}_{j,0}, \\
\mathit{NNP} &= -\widehat{MPC}_j \times NNP_{j,0} \times \frac{dP_0}{P}, \\
\mathit{URE} &= \widehat{MPC}_j \times URE_{j,0} \times \frac{d\tilde{R}^r_1}{\tilde{R}^r}, \\
\mathit{Substitution} &= -\frac{c_j}{\sigma}\left(1 - \widehat{MPC}_j\right) \times \frac{d\tilde{R}^r_1}{\tilde{R}^r}.
\end{aligned}
$$

E The effects of a tighter monetary policy on age-consumption profile of a 71 year old household.

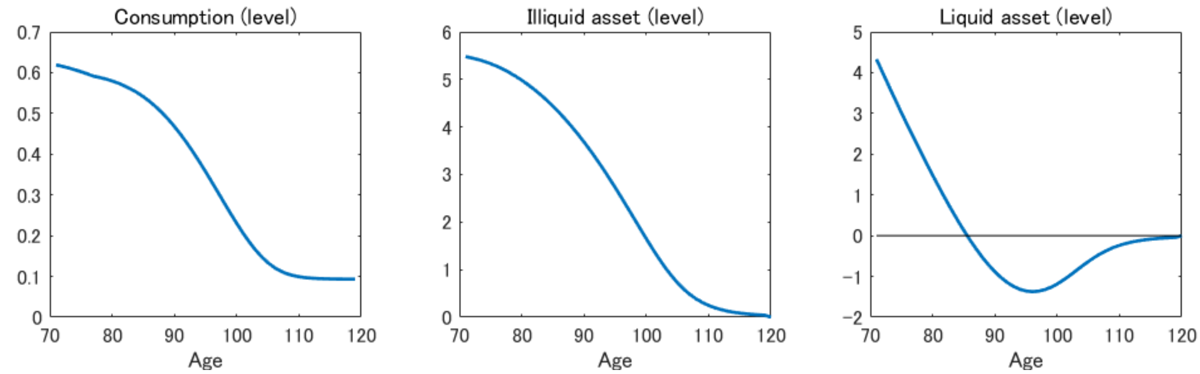
This appendix provides more details about why a tightening in monetary policy produces hump-shaped consumption a response for households who are close to retirement age by analyzing the response of the cohort that is aged 71 when monetary policy is tightened. The left panel of Figure 7 plots the response of the household's consumption in terms of a difference from steady-state for each age consumption, $c_{j,t} - c_j$, divided by the steady-state disposable income in each age, $(\tilde{R}^a - 1)a_{j-1} + (\tilde{R} - 1)d_{j-1} + b_j + \xi$. The response is hump-shaped and persistent with peak arrived more than 20 years later after the shock hits. The response of consumption in terms of a difference from steady-state, $c_{j,t} - c_j$, is also hump-shaped as shown in the right panel of Figure 7.

Figure 7: Impulse responses by a household aged 71



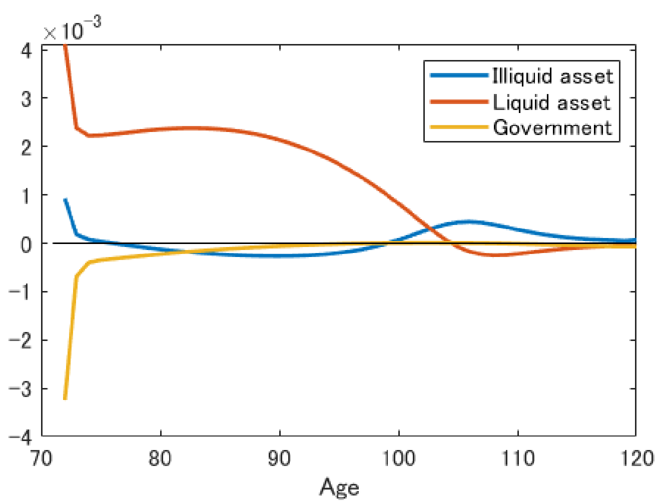
To understand why the response of consumption is hump-shaped, let's start considering the steady-state age profiles of consumption $c_{j,t}$, illiquid asset holdings $a_{j,t}$, and liquid asset

Figure 8: Age profiles of consumption and asset holdings



holdings $d_{j,t}$ (Figure 8). Since the mortality risk is relatively high after age 71, in spite of consumption smoothing motive, the age consumption profile is decreasing in age. The illiquid asset holding is also decreasing in age as the household draws down the illiquid assets for consumption. The household draws down the liquid assets more quickly than the illiquid assets. After age 85 the household borrows to keep the illiquid asset position. It is important to keep in mind that it is relative to this age-profile of consumption (the left panel of Figure 8) that the response of consumption is hump-shaped.

Figure 9: Decomposition of the consumption response (diff)

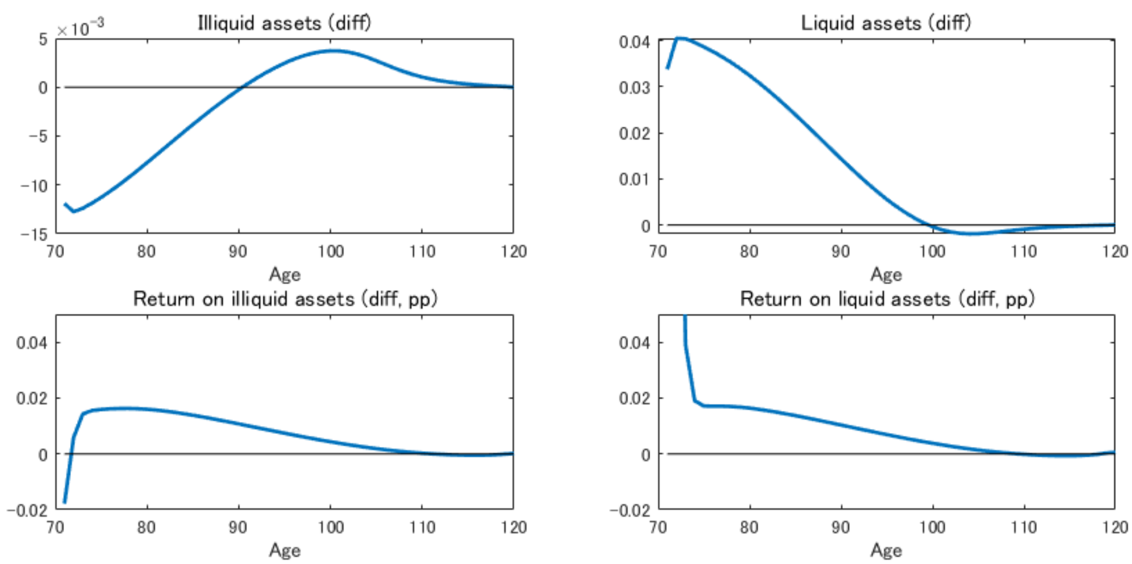


Consumption of a household with age $j \geq 71$ in period t is given by

$$c_{j,t} = \underbrace{(\tilde{R}^a_t - 1)a_{j-1,t-1} - \triangle a_{j,t}}_{\text{Illiquid asset}} + \underbrace{(\tilde{R}_t - 1)d_{j-1,t-1} - \triangle d_{j,t}}_{\text{Liquid asset}} + \underbrace{b_{j,t} + \xi_t}_{\text{Government}}, \quad (44)$$

where $\triangle a_{j,t} = a_{j,t} - a_{j-1,t-1}$ and $\triangle d_{j,t} = d_{j,t} - d_{j-1,t-1}$. Figure 9 plots a decomposition of the consumption response into an illiquid asset cash flow component, $(\tilde{R}^a_t - 1)a_{j-1,t-1} - \triangle a_{j,t}$, a liquid asset cashflow component, $(\tilde{R}_t - 1)d_{j-1,t-1} - \triangle d_{j,t}$, and the government transfer component, $b_{j,t} + \xi_t$, in terms of a differences from steady-state, from $t = 1$ (age 72) onward, where the impact period of $t = 0$ (age 71) is omitted because the responses in period $t = 0$ are much greater than those in the remaining periods. Figure 9 indicates that the liquid asset cash flow component is driving the hump-shaped response of consumption deviations from steady-state.

Figure 10: Illiquid and liquid asset holdings (diff)



In response to a tightening in monetary policy, the household reduces its holdings of illiquid assets and increases its holdings of liquid assets (Figure 10) in a persistent fashion. Its preferred asset allocation strategy is to hold positive amounts of both assets up until age 85 and tilting its portfolio towards liquid assets is attractive to it given the relatively high deviation of liquid assets from their steady-state level. The consumption difference peaks at age 83 and then begins to fall. This decline reflects the fact that beyond age 85 the household's preferred asset allocation strategy is to take a leveraged long position in illiquid securities and this strategy is not so attractive because it now is facing relatively high borrowing costs as shown in the bottom right panel of Figure 10.

Then, why does the real return on liquid assets remain relatively high? Since the nominal debt is fixed and the price level is the same in the initial and terminal steadystates, a monetary policy tightening causes deflation initially but it is followed by inflation later. The central bank in the model responds to this small increase in the inflation rate by raising the nominal interest rate.

F Additional results

Table 11: Model aggregate steady-state moments

Variable	description	steady-state value
$R^k - 1$	return on capital	3.72%
$R/\pi - 1$	return on government debt	2.25%
$\tilde{R}^a - 1$	After–tax return on illiquid assets	1.93%
$\tilde{R}/\pi - 1$	After–tax return on liquid assets	1.80%
D^G/Y	Gross liquid assets relative to output	1.58
$(D^G - D)/Y$	Private borrowing relative to output	0.35
$(A + D - D^G)/Y$	Net stock of illiquid assets relative to output	3.15
$(A + D)/Y$	Net worth	4.73
K/Y	Capital-output ratio	2.16
V/Y	Value of shares relative to output	1.34
Ξ/Y	Lump–sum transfers relative to output	-0.002
B/Y	Social Security outlays relative to output	0.091
C/Y	Consumption share of output	0.67
I/Y	Investment share of output	0.24
$\bar{\gamma}_a/y$	Financial services share of output	0.011

Table 12: Impact responses of aggregate variables to monetary policy innovation under alternative scenarios.

A. Response of goods and labor market variables

Scenario	Y	C	I	H	w	r^k	π
Baseline	-0.651	-0.080	-2.49	-0.929	-0.788	-0.238	-0.561
Lump-sum	-0.633	-0.063	-2.46	-0.903	-0.790	-0.235	-0.557
$\phi_\pi = 0$	-0.882	-0.164	-3.04	-1.26	-1.16	-0.334	-0.499
Negative	0.725	0.062	2.21	1.037	0.811	0.258	0.581
d/y=1.5	-0.659	-0.076	-2.60	-0.940	-0.790	-0.246	-0.560

B. Response of financial and fiscal variables

Scenario	R	r^d	r^a	Spread	V	Ω	ξ	d
Baseline	0.278	0.855	-0.034	-0.889	-0.423	19.24	-0.403	0.564
Lump-sum	0.282	0.856	-0.040	-0.897	-0.440	19.15	-0.457	0.560
$\phi_\pi = 0$	1.028	1.55	-0.145	-1.69	-0.902	28.51	-0.658	0.501
Negative	-0.253	-0.842	-0.018	0.825	0.406	-23.31	-0.053	-0.578
d/y=1.5	0.278	0.857	-0.032	-0.888	-0.454	19.33	-0.884	0.563

Note: "Baseline" is the baseline scenario. "Lump-sum" assumes that lump-sum taxes are fixed at their steady-state level for households aged 76+ and $\phi_\pi = 0$ assumes that the coefficient on the inflation rate in the interest rate targeting rule is 0. "Negative" assumes that the monetary policy innovation is -0.01 instead and "d/y=1.5" assumes that the debt output ratio is 1.5 instead of its baseline value of 1.23.

Table 13: Wealth and consumption inequality under alternative scenarios

Wealth Inequality					
Year	0	1	2	3	4
Baseline	0.16	0.24	0.24	0.18	0.15
Lump-sum	0.16	0.21	0.22	0.16	0.14
$\phi_\pi = 0$	0.036	0.24	0.26	0.17	0.15
Negative	-0.18	-0.25	-0.25	-0.18	-0.16
d/y=1.5	0.18	0.26	0.26	0.20	0.17
Consumption Inequality					
Year	0	1	2	3	4
Baseline	0.44	0.54	0.46	0.41	0.38
Lump-sum	0.21	0.32	0.34	0.34	0.34
$\phi_\pi = 0$	0.84	1.18	0.91	0.65	0.48
Negative	-0.38	-0.50	-0.45	-0.41	-0.39
d/y=1.5	0.47	0.60	0.52	0.47	0.45

Note: Inequality is measured as the difference between the 90th and 10th percentiles of wealth (consumption) by age. We do not adjust for differences in the population share of different age groups when computing percentiles and the numbers reported in the table are percentage deviations from steady-state.