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Hidehiko Matsumoto*

Abstract

This paper studies the optimal monetary and macroprudential policies in a small open economy that borrows from abroad in foreign currency. The model features a novel mechanism in which sudden stops due to an occasionally binding borrowing constraint trigger a sharp currency depreciation through balance of payments adjustments, thereby increase the domestic-currency value of foreign debt and cause severe economic downturns. A policy analysis shows that a contractionary monetary policy mitigates depreciation during a crisis, but the anticipation of policy interventions during the crisis induces larger borrowings ex ante and destabilizes the economy. A combination of an ex ante macroprudential tax on foreign borrowing and ex post monetary policy interventions can stabilize the economy and improve social welfare.

Keywords: Exchange rate; Balance of payments; Sudden stops; Monetary policy; Macroprudential policy

JEL classification: F31, F32, F38, F41

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1 Introduction

Emerging economies have been accelerating their international bond issuance over the past decade. [Acharya et al. \(2015\)](#) report that financial corporations in 15 emerging economies have increased their international bond issuance from less than 400 billion dollars per year in 2010 to nearly 1 trillion dollars in 2014. They also report that roughly 80% of these foreign bonds are denominated in foreign currencies. [Figure 1](#) confirms this fact using more recent data. The figure shows the outstanding foreign debt of 14 emerging economies. The number in each bar indicates the percentage share of dollar-denominated debt in total debt. Along with active foreign debt accumulation since 2010, the dollar debt share has increased from 75% in 2010 to 81% in 2018.¹ Foreign-currency debt is known to deteriorate financial stability when sudden stops in capital inflows cause sharp currency depreciation, and thereby increase the domestic-currency value of foreign debt.

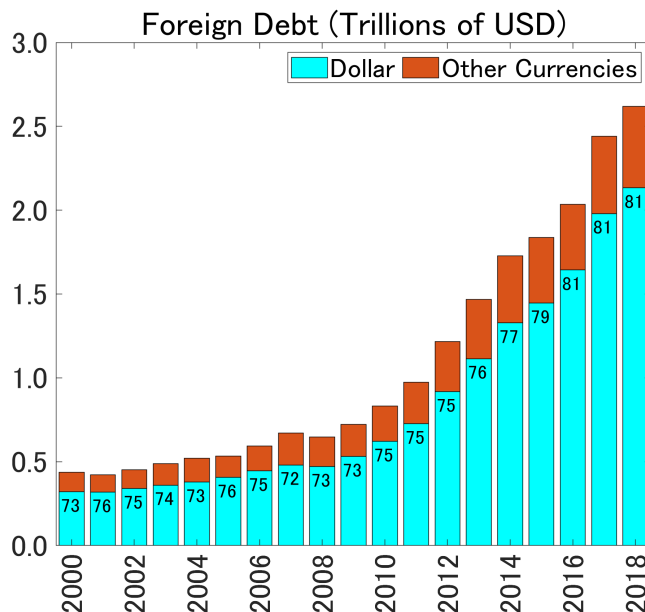
A recent study on sudden stops has shown substantial progress both in positive and normative analyses. In particular, studies have extensively used an occasionally binding borrowing constraint on foreign debt to examine theoretically the macroprudential policy effects against sudden stops.² However, existing studies do not model the fact that domestic currency depreciation increases the value of dollar-denominated foreign debt and thereby exacerbates economic downturns during sudden stops. Accordingly, some important questions have remained unanswered: Does macroprudential policy help to mitigate currency depreciation and economic downturns during sudden stops when a country holds dollar-denominated foreign debt? How should monetary policy be conducted when currency depreciation increases the value of foreign debt? How should monetary and macroprudential policies cooperate in this regard?

This paper addresses these questions by developing a model of a small open economy that borrows from abroad in foreign currency subject to the risk of sudden stops in capital inflows. The main contribution of this paper is twofold. First, I propose a novel mechanism in which sudden stops due to borrowing constraints trigger sharp currency depreciation through

¹[Bénétrix et al. \(2019\)](#) argue that the Bank for International Settlements (BIS) does not report the currency composition of domestically issued debt securities held by non-resident investors, and thus, may underestimate the share of domestic currency.

²[Bianchi and Mendoza \(2020\)](#) review the stylized facts of sudden stops with a survey on the literature.

Figure 1: Foreign debt by emerging economies



Note: This figure shows the outstanding foreign debt of 14 emerging economies: Argentina, Brazil, Chile, China, Colombia, Indonesia, India, Korea, Mexico, Malaysia, Philippines, Thailand, Turkey, and South Africa. Debt amounts are calculated by the author using data from the BIS debt securities statistics.

balance of payments adjustments, and thereby increase the domestic-currency value of foreign debt and cause a severe crisis. As private agents take the exchange rate as given, they socially overborrow in normal times and import too heavily during a crisis, both of which lead to inefficiently large depreciations and thus rationalize the need for policy interventions. Second, using the model, I provide policy implications for the optimal combination of monetary and macroprudential policies against sudden stops under dollar-denominated foreign debt.

The model is a small open economy as in [Bianchi and Mendoza \(2018\)](#), where households produce and consume tradable goods and borrow from abroad, subject to an occasionally binding borrowing constraint. To highlight the novel mechanism of the model, I abstract away the collateral asset price and an associated pecuniary externality from the model.³ I introduce three innovations into the model. First, I denominate foreign debt explicitly in foreign currency. Second, I assume that the tradable goods produced in this economy (home

³Section E in the appendix studies the model with pecuniary externality as in [Bianchi and Mendoza \(2018\)](#).

tradable goods) face a downward-sloping demand from foreign countries. Third, I introduce New Keynesian price stickiness to study the role of monetary policy.⁴

The key mechanism of the model is that real depreciation triggered by a binding borrowing constraint is amplified through the interaction between balance of payments adjustments and the borrowing constraint. When the borrowing constraint binds, the country will have to repay the outstanding foreign debt through limited new borrowings, leading to large net capital outflows. Large net capital outflows need to be matched with large net exports through balance of payments adjustments. This will require a real depreciation because exports face a downward-sloping demand from abroad. The real depreciation will in turn increase the domestic-currency value of foreign debt repayment, but new borrowings will be limited by the borrowing constraint.⁵ This implies further net capital outflows, leading to a second-round real depreciation. The loop continues and leads to large real depreciation during crises.

As households take the real exchange rate as given, the amplification loop of depreciation causes externalities and distorts the households decisions both before and during a crisis. When the borrowing constraint is not binding but could bind in the next period, reducing the foreign debt ex ante would reduce the debt repayment and mitigate the real depreciation if the constraint actually binds in the next period. Households do not internalize this effect and hence take socially excessive foreign debt in normal times. During a crisis when the borrowing constraint becomes binding, households do not realize that reducing imported inputs for production would improve the trade balance and mitigate the real depreciation. This externality would induce households to use socially excessive imported inputs during the crisis. Both these externalities would result in inefficiently large real depreciation during a crisis through the amplification loop. The social cost of large depreciation is twofold. First, imported inputs become inefficiently expensive and thus reduce the output. The model, therefore, explains the drop in output without working capital financing, which is commonly assumed in the literature. Second, an inefficiently large fraction of output is

⁴Devereux et al. (2018) and Coulibaly (2018) introduce New Keynesian price stickiness into a model with pecuniary externalities to study the optimal monetary and macroprudential policies.

⁵The key assumption is that the borrowing limit is denominated in domestic currency. Thus there will be a currency mismatch between foreign borrowings and the borrowing limit.

exported, resulting in inefficiently low domestic consumption.

Given this model economy, I examine the effects of monetary and macroprudential policies on crises mitigation.⁶ I first characterize the optimal discretionary monetary policy analytically without a macroprudential policy. When the borrowing constraint is not binding but may bind in the next period, the optimal discretionary monetary policy is contractionary through lowering inflation. This is because a contractionary monetary policy would result in real appreciation, thereby increases the effective interest rate and discourages foreign borrowing, and partially correct overborrowing. When the constraint is binding, the optimal discretionary monetary policy again becomes contractionary. This intervention discourages the use of imported inputs for production and thereby mitigates real depreciation, partially correcting ex post externality.

Next, I characterize the optimal combination of discretionary monetary policy and macroprudential tax on foreign borrowing. I show that in normal times, the monetary policy should focus only on minimizing the inflation cost, because overborrowing can be corrected through a macroprudential tax. However, in a crisis, the monetary policy should still intervene through lowering inflation. This is because an ex ante macroprudential tax cannot correct the ex post externality that induces too much imported inputs.

In quantitative analysis, I set the parameters to the standard values in the literature on sudden stops in emerging economies. I solve the model numerically using a global method to deal with an occasionally binding constraint. Non-linear crisis dynamics in the model are consistent with the empirical regularities of sudden stops in emerging economies, characterized by drops in output and consumption, sharp reversal of capital flows, and sharp real depreciation. I compare the crisis dynamics under four policy regimes: strict inflation targeting monetary policy, optimal discretionary monetary policy, and both of these policy regimes with the optimal macroprudential tax on foreign borrowings. I show that a discretionary monetary policy without taxes induces large foreign borrowings in normal times and destabilizes the economy. This is because the anticipation of monetary policy intervention during a crisis period reduces the effective interest rate ex ante and induces large borrow-

⁶I focus on time-consistent policies without commitment to future policies in order to avoid the time inconsistency due to firms' forward-looking price decisions.

ings in normal times. Macroprudential policy helps in stabilizing the economy by reducing foreign borrowings in normal times and thereby mitigating capital flow reversals and real depreciation during sudden stops.

Finally, I compare the welfare implications of the four policy regimes. Without macroprudential taxes, a discretionary monetary policy will result in slightly lower expected welfare than inflation targeting, by 0.02%, in terms of permanent consumption. In contrast, with an optimal macroprudential tax, a discretionary monetary policy will result in higher welfare than inflation targeting. A discretionary monetary policy with optimal tax will lead to 0.07% higher permanent consumption than inflation targeting without taxes, whereas the welfare gain from inflation targeting with the optimal tax is 0.03%. This result suggests that monetary policy intervention during a crisis will help stabilize the economy and improve welfare only if it comes with an ex ante macroprudential tax to correct overborrowing. Another finding is that the welfare gain under discretionary monetary policy with tax can be as high as 0.2% if the simulation starts when the borrowing constraint is binding, suggesting a sizable welfare gain through monetary policy interventions.

Related literature This paper contributes to the growing literature on policies managing sudden stops in capital inflows. The majority of the studies assume a borrowing limit based on the loan-to-value or debt-to-income ratio, both of which lead to pecuniary externality and overborrowing of private agents. Under this assumption, [Bianchi \(2011\)](#), [Bianchi and Mendoza \(2018\)](#), and [Jeanne and Korinek \(2019\)](#) study the optimal macroprudential tax on foreign debt to correct overborrowing. [Benigno et al. \(2013\)](#), [Benigno et al. \(2016\)](#), and [Jeanne and Korinek \(2020\)](#) study the optimal combination of ex ante macroprudential policy and ex post intervention.

[Fornaro \(2015\)](#) and [Ottonello \(2015\)](#) introduce nominal wage rigidities into this class of models and study the optimal exchange rate policy. They highlight the benefit of depreciation during crises and argue that exchange rate depreciation helps overcome sudden stops by boosting exports or reducing unemployment. The present paper, in contrast, emphasizes the negative effects of depreciation during a crisis by increasing the domestic-currency value of foreign-currency foreign debt. [Mendoza and Rojas \(2019\)](#) introduce a simple financial

intermediary for transforming foreign-currency foreign debt into domestic-currency domestic loans. Unlike the present paper, real depreciation in their model reduces the burden of debt repayment during a crisis, because real depreciation is a decline in the non-tradable goods price relative to the tradable goods price, and thus lowers the real interest rate denominated in consumption composites.

Studies in the literature most closely related to the present paper are [Devereux et al. \(2018\)](#) and [Coulibaly \(2018\)](#). They introduce New Keynesian price stickiness and study the optimal combination of monetary and macroprudential policies. [Devereux et al. \(2018\)](#) assume that the collateral value of an asset depends on the next-period expected asset price rather than realized asset price. They show that the ex ante policy to reduce foreign debt is not optimal, and that the government intervenes only when a crisis occurs. [Coulibaly \(2018\)](#) uses a model with tradable and non-tradable sectors and shows that inflation targeting dominates a discretionary monetary policy in welfare without macroprudential taxes, but the discretionary policy becomes preferable when combined with the optimal tax. Welfare analysis in the present paper gives similar results. The main contribution of the present paper relative to these preceding studies is that I examine the optimal policy in a situation where a depreciation increases the domestic-currency value of foreign debt and causes a severe crisis.

This study is also related to the literature examining the implications of foreign-currency debt in policy designs. [Aghion et al. \(2000\)](#), [Céspedes et al. \(2004\)](#), [Cook \(2004\)](#), [Choi and Cook \(2004\)](#), and [Devereux et al. \(2006\)](#) introduce currency mismatches in financial accelerator models to study the monetary and exchange rate policies. These studies typically do not consider a macroprudential policy. A growing literature studies the interaction between monetary and macroprudential policies both in closed and open economies; for example, [Unsal \(2013\)](#), [Angelini et al. \(2014\)](#), [Céspedes et al. \(2017\)](#), and [Davis and Presno \(2017\)](#). [Aoki et al. \(2018\)](#) introduce a currency mismatch in the balance sheet of financial intermediaries and study the interaction between monetary and financial policies.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 characterizes the externalities and key mechanism of the model. Section 4 analyzes the optimal policies. Section 5 conducts quantitative analyses. Section 6 concludes the paper.

2 Model

The model describes a small open economy similar to that in [Bianchi and Mendoza \(2018\)](#), where households produce goods and borrow from abroad subject to an occasionally binding borrowing constraint. I introduce three innovations into the model. First, foreign debt is denominated in foreign currency. Second, the export of tradable goods produced in this economy (home tradable goods) faces a downward-sloping demand from foreign countries. Third, I introduce New Keynesian price stickiness. Thus, the model includes intermediate firms that produce differentiated intermediate goods and the final good producer that assembles them.

2.1 Households

The economy is inhabited by a unit measure of identical households. The utility of a representative household is given as

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \log \left(c_t - \chi \frac{\ell_t^{1+\omega}}{1+\omega} \right) \right], \quad (1)$$

where c_t is the amount of goods consumed and ℓ_t is the labor supply. Households consume only home tradable goods. They produce wholesale goods and sell their output to intermediate firms. Wholesale goods are produced using the production function

$$y_t^w = A_t (k)^{\alpha_k} (\ell_t)^{\alpha_\ell} (m_t)^{\alpha_m}. \quad (2)$$

A_t is a stochastic aggregate productivity shock, k represents the productive assets that households own, and the amount of assets is fixed at 1. m_t is the imported inputs. The input share parameters satisfy $\alpha_k + \alpha_\ell + \alpha_m = 1$.

As most of the outstanding foreign bonds issued by emerging economies are denominated in foreign currency, I assume that households borrow from abroad in terms of foreign currency. Let S_t denote the nominal exchange rate, with the unit of foreign currency measured in domestic currency. Let P_t denote the nominal price of home tradable goods in the do-

mestic currency and P_t^* denote that of foreign tradable goods in foreign currency. The real exchange rate e_t , defined as the price of foreign tradable goods relative to home tradable goods, is given by $e_t = S_t P_t^* / P_t$. The price of foreign goods P_t^* is assumed to be constant and normalized to 1; this implies that $e_t = S_t / P_t$. A higher e_t corresponds to real depreciation.

The household's budget constraint in nominal terms is given as

$$P_t c_t + S_t \left(\frac{B_t^*}{R_t^*} - B_{t-1}^* \right) + \left(\frac{B_t}{R_t} - B_{t-1} \right) = P_t^w y_t^w - (1 + \tau_m) S_t m_t + T_t + \Pi_t, \quad (3)$$

where B_t^* is the foreign nominal bond holdings and R_t^* is the nominal interest rate on foreign bonds. As there is no inflation in foreign countries, B_t^* and R_t^* are also real foreign bond and real foreign interest rates. R_t^* is assumed to be stochastic and satisfy $\beta R_t^* < 1$. This condition implies that households always borrow from abroad and $B_t^* < 0$. B_t is the domestic nominal bond holdings, and R_t is the nominal interest rate set by the government. As households are homogeneous, $B_t = 0$ holds at equilibrium. P_t^w is the nominal wholesale goods price. The price of imported inputs is assumed to be 1 in foreign currency. This means that the real exchange rate e_t indicates the terms of trade as well. The tax τ_m on imported inputs is introduced to correct the terms-of-trade externality, and set to $\tau_m = 1/(\rho - 1)$ where ρ is the price elasticity of exports.⁷ T_t is the lump-sum transfer that rebates the tax on imported inputs and finances the subsidy on intermediate goods sales explained below. Π_t is the intermediate firms' profits paid to households.

Foreign borrowing of households is limited by fraction κ_t of the productive assets they own:

$$- S_t \frac{B_t^*}{R_t^*} \leq \kappa_t k, \quad (4)$$

where κ_t is a stochastic shock to the borrowing limit that takes two values following a first-order Markov process. [Bianchi \(2016\)](#) adopts the borrowing constraint in this form, and [Bianchi and Mendoza \(2018\)](#) show that the borrowing constraint in this form can be derived from an imperfect enforceability problem. As in [Bianchi \(2016\)](#), I assume that assets are evaluated at the book rather than market value. This assumption is made in order to shut

⁷Section A in the appendix proves that this constant tax rate corrects the externality associated with the terms of trade.

off the pecuniary externality associated with the asset price and focus on the externalities due to balance of payments adjustments. Section E in the appendix examines the model with asset price pecuniary externality.

The household's problem is to choose $\{c_t, \ell_t, m_t, B_t^*, B_t\}$ given $\{P_t, P_t^w, S_t, R_t, R_t^*\}$ to maximize their expected utility (1) subject to the production function (2), budget constraint (3), and borrowing constraint (4). Let λ_t be the Lagrange multiplier on the budget constraint, and μ_t be the multiplier on the borrowing constraint. To express the first-order conditions in real terms, let p_t^w denote the wholesale goods price relative to P_t , and $b_t = B_t/P_t$ and $b_t^* = B_t^*/P_t$ denote the real domestic and foreign bonds respectively. The first-order conditions by households are summarized as follows:

$$c_t : \lambda_t = \frac{1}{c_t - \chi \frac{\ell_t^{1+\omega}}{1+\omega}}, \quad (5)$$

$$\ell_t : p_t^w \alpha_\ell \frac{y_t^w}{\ell_t} = \chi \ell_t^\omega, \quad (6)$$

$$m_t : p_t^w \alpha_m \frac{y_t^w}{m_t} = e_t \frac{\rho}{\rho - 1}, \quad (7)$$

$$b_t : \lambda_t = \beta R_t E_t \left[\lambda_{t+1} \frac{1}{1 + \pi_{t+1}} \right], \quad (8)$$

$$b_t^* : \lambda_t - \mu_t = \beta R_t^* E_t \left[\lambda_{t+1} \frac{e_{t+1}}{e_t} \right], \quad (9)$$

$$\mu_t \left[-e_t \frac{b_t^*}{R_t^*} - \kappa_t k \right] = 0. \quad (10)$$

Equations (6) and (7) are the standard first-order conditions for production. Equations (8) and (9) are the Euler equations with respect to domestic bond and foreign bond, respectively. In (9), the expected real depreciation rate e_{t+1}/e_t affects the real interest rate on foreign bond, which plays the key role in policy analysis. $\pi_{t+1} = P_{t+1}/P_t - 1$ is the inflation rate from period t to $t + 1$. Equation (10) is the complementary slackness condition for the borrowing constraint.

2.2 Intermediate Firms

Intermediate firms are modeled as standard New Keynesian models with price adjustment costs. The unit measure of intermediate firms produces differentiated intermediate goods using wholesale goods purchased from households in the competitive market. Their production technology is to convert one unit of wholesale good to one unit of intermediate good. They sell their products to the final good producer given the demand equation

$$y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} Y_t, \quad (11)$$

where $y_t(i)$ is the amount of intermediate goods sold to the final good producer, and $p_t(i)$ is its price with $i \in [0, 1]$ indicating the type of intermediate goods. Y_t is the output of the final goods producer. This demand equation is the first-order condition with respect to intermediate inputs by the final good producer explained below.

The price setting of intermediate firms is subject to a price adjustment cost. A quadratic adjustment cost proposed in Rotemberg (1982) is assumed, with the target inflation rate equal to zero. In addition, a subsidy $\tau_y = 1/(\theta - 1)$ on intermediate goods sales is introduced to eliminate the distortion by market power of intermediate firms. The profit maximization problem of an intermediate firm i is then defined as

$$\max_{\{p_t(i)\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \Lambda_{0,t} \left\{ \left((1 + \tau_y) \frac{p_t(i)}{P_t} - \frac{P_t^w}{P_t} \right) y_t(i) - \frac{\psi}{2} \left(\frac{p_t(i)}{p_{t-1}(i)} - 1 \right)^2 Y_t \right\} \right],$$

subject to demand equation (11). $\Lambda_{0,t}$ is the households' stochastic discount factor, given as $\beta^t \lambda_t / \lambda_0$. Parameter ψ determines the price adjustment cost, which governs the extent of price stickiness. Rearranging the first-order condition with respect to $p_t(i)$ results in the following New Keynesian Phillips curve:

$$[-\theta + \theta p_t^w - \psi \pi_t (1 + \pi_t)] Y_t + \beta E_t [(\lambda_{t+1} / \lambda_t) \psi \pi_{t+1} (1 + \pi_{t+1}) Y_{t+1}] = 0. \quad (12)$$

As is standard in the New Keynesian models, the current inflation π_t is increasing in the current marginal cost p_t^w and expected future inflation.

2.3 Final Good Producer

A representative final goods producer assembles differentiated intermediate goods into home tradable goods. The production function is the standard aggregation with constant elasticity of substitution:

$$Y_t = \left(\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}},$$

where θ is the elasticity of substitution across different intermediate goods. The first-order condition for each type of intermediate goods $y_t(i)$ gives the demand equation (11).

The home tradable goods output is consumed by domestic households, exported to foreign countries, or used to settle the intermediate firms' price adjustment cost. The market clearing condition is therefore given as

$$Y_t = c_t + x_t + \frac{\psi}{2}(\pi_t)^2 Y_t, \quad (13)$$

where x_t is the foreign country exports. The foreign demand for home tradable goods is assumed downward sloping as follows:

$$x_t = \left(\frac{P_t}{S_t P_t^*} \right)^{-\rho} Y^*,$$

where $\rho > 1$ is the parameter for the price elasticity of demand for exports and Y^* determines the size of foreign demand.⁸ The foreign demand for exports in this form is commonly assumed in New Keynesian open economy models, such as [Aoki et al. \(2018\)](#) and [Devereux et al. \(2018\)](#). Using the definition of the real exchange rate e_t , exports can be written as a function of e_t as follows:

$$x_t = e_t^\rho Y^*. \quad (14)$$

2.4 Decentralized Equilibrium

The decentralized equilibrium of the model is defined as follows. Final, intermediate, and wholesale good outputs satisfy $Y_t = y_t(i) = y_t^w = A_t(k)^{\alpha_k} (\ell_t)^{\alpha_\ell} (m_t)^{\alpha_m}$. The domestic

⁸[Simonovska and Waugh \(2014\)](#) estimate the price elasticity of exports as between 2.79 and 4.46.

bond market clearing condition is $b_t = 0$. Transfers in real terms T_t/P_t involve financing a subsidy on intermediate goods sales $-\tau_y Y_t = -1/(\theta - 1)Y_t$ and tax rebates on imported inputs $\tau_m e_t m_t = 1/(\rho - 1)e_t m_t$. Intermediate profits paid to households are given as $\Pi_t/P_t = (1 + 1/(\theta - 1) - p_t^w)Y_t - (\psi/2)(\pi_t)^2 Y_t$. Substituting these equations, the household budget constraint (3) can be written in real terms as:

$$c_t + e_t \left(\frac{b_t^*}{R_t^*} - b_{t-1}^* \right) = Y_t - \frac{\psi}{2} (\pi_t)^2 Y_t - e_t m_t. \quad (15)$$

This equation is combined with the market clearing condition for final goods (13) to obtain the following balance of payments identity:

$$e_t^\rho Y^* - e_t m_t = e_t \left(\frac{b_t^*}{R_t^*} - b_{t-1}^* \right). \quad (16)$$

This balance of payments identity shows that the net exports on the left-hand side are equal to the net capital outflows on the right-hand side.

The decentralized equilibrium of the model is defined as allocations $\{Y_t, c_t, \ell_t, m_t, b_t^*, b_t\}_{t=0}^\infty$, prices $\{p_t^w, e_t, \pi_t\}_{t=0}^\infty$, and Lagrange multipliers $\{\lambda_t, \mu_t\}_{t=0}^\infty$ that satisfy (2), (5), (6), (7), (8), (9), (10), (12), (15), (16), $b_t = 0$, given the initial state b_{-1}^* and $b_{-1} = 0$, policy $\{R_t\}_{t=0}^\infty$ and exogenous shocks $\{A_t, R_t^*, \kappa_t\}_{t=0}^\infty$. This completes the exposition of the model economy.

3 Flexible Price Model

In this section, I examine the flexible-price version of the model and characterize the externalities by laying aside nominal rigidities. In the flexible price model, households directly produce home tradable goods, and there are no wholesale goods, intermediate goods, intermediate firms, or final good producers. As a monetary policy is irrelevant, the model has no domestic bonds. The decentralized equilibrium of this flexible-price economy is defined by allocations $\{Y_t, c_t, \ell_t, m_t, b_t^*\}_{t=0}^\infty$, prices $\{e_t\}_{t=0}^\infty$, and Lagrange multipliers $\{\lambda_t, \mu_t\}_{t=0}^\infty$ that satisfy (2), (5), (6) with $p_t^w = 1$, (7) with $p_t^w = 1$, (9), (10), (15), and (16), given the initial state b_{-1}^* and exogenous shocks $\{A_t, R_t^*, \kappa_t\}_{t=0}^\infty$.

3.1 Social Planner's Problem and Overborrowing

To characterize the externalities in the model, I set the social planner's problem and derive the first-order conditions. The planner's problem is defined in a recursive form as

$$V(b_{t-1}^*, s_t) = \max_{c_t, b_t^*, \ell_t, m_t, e_t} \log \left(c_t - \chi \frac{\ell_t^{1+\omega}}{1+\omega} \right) + \beta E_t V(b_t^*, s_{t+1}),$$

where $s_t = \{A_t, R_t^*, \kappa_t\}$ denotes stochastic shocks to the economy. The constraints to this problem are the production function (2), resource constraint (13) with $\pi_t = 0$ and the exports given by (14), balance of payments identity (16), and borrowing constraint in real terms:

$$-e_t \frac{b_t^*}{R_t^*} \leq \kappa_t k. \quad (17)$$

Section A in the appendix characterizes all the first-order conditions. Rearranging the first-order conditions leads to the following equations:

$$\gamma_t^{SP} = \frac{\rho}{\rho - 1} u_c(t) - \frac{b_t^*/R_t^*}{(\rho - 1)e_t^{\rho-1}Y^*} \mu_t^{SP}, \quad (18)$$

$$\gamma_t^{SP} - \mu_t^{SP} = \beta R_t^* E_t \left[\frac{e_{t+1}}{e_t} \gamma_{t+1}^{SP} \right], \quad (19)$$

where $u_c(t)$ is the marginal utility of consumption at period t , and μ_t^{SP} and γ_t^{SP} are the Lagrange multipliers on the borrowing constraint (17) and balance of payments identity (16), respectively.

The key variable is γ_t^{SP} , which is the social value of real appreciation through balance of payments adjustments. As shown in (18), γ_t^{SP} is always strictly positive given $\rho > 1$ and $-b_t^* < 0$. To understand the interpretation of γ_t^{SP} , assume that foreign debt repayment $-e_t b_{t-1}^* > 0$ decreases by one unit in terms of domestic currency. The effect of this reduction in repayment on the real exchange rate e_t can be obtained by applying the implicit function theorem to the balance of payments identity (16):

$$-\frac{\partial e_t}{\partial(-e_t b_{t-1}^*)} \frac{1}{e_t} = -\frac{1}{(\rho - 1)e_t^{\rho-1}Y^*}. \quad (20)$$

As this value is negative given $\rho > 1$, a reduction in repayment by one unit leads to a real appreciation of this size that has two consequences corresponding to the two terms of γ_t^{SP} in (18). First, a real appreciation reduces exports and enables households to consume more of their output. Applying the implicit function theorem to the resource constraint (13), a real appreciation of size (20) will increase the consumption as follows:

$$\frac{\partial c_t}{\partial e_t} \times \left(-\frac{1}{(\rho - 1)e_t^{\rho-1}Y^*} \right) = \frac{-\rho e_t^{\rho-1}Y^*}{-(\rho - 1)e_t^{\rho-1}Y^*} = \frac{\rho}{\rho - 1}.$$

The social value of this consumption is given by $u_c(t)[\rho/(\rho - 1)]$, which is the first term in (18). Second, when the borrowing constraint is binding, a real appreciation would relax the constraint by reducing the domestic-currency value of the foreign-currency debt. As the borrowing constraint (17) shows, a marginal real appreciation relaxes the borrowing constraint by b_t^*/R_t^* units. Then, by relaxing the binding borrowing constraint, the social value of a real appreciation of size (20) will be

$$-\frac{1}{(\rho - 1)e_t^{\rho-1}Y^*} \times \frac{b_t^*}{R_t^*} \mu_t^{SP},$$

which is the second term in (18). As μ_t^{SP} is the Lagrange multiplier on the borrowing constraint, this value will be positive only when the constraint is binding.

As the balance of payments identity (16) indicates, a one-unit reduction in payment for imported inputs $e_t m_t$ or one-unit increase in new borrowing $e_t b_t^*/R_t^*$ (one unit larger negative value) would result in real appreciation of the same size and the same amount of social value. Therefore, γ_t^{SP} captures the social value of real appreciation due to a one-unit reduction in payment for imported inputs or net capital outflows through balance of payments adjustments. As households take the real exchange rate e_t as given, γ_t^{SP} is the source of externalities, as shown below.

Substituting (18) into (19) gives the explicit expression for the Euler equation with respect to foreign bonds. If the constraint is presently not binding but may bind in the next period,

the Euler equation can be given as follows:

$$u_c(t) = \beta R_t^* E_t \left[\frac{e_{t+1}}{e_t} \left(u_c(t+1) - \underbrace{\frac{b_{t+1}^*/R_{t+1}^*}{\rho e_{t+1}^{\rho-1} Y^*} \mu_{t+1}^{SP}}_{\text{ex ante BOP externality}} \right) \right]. \quad (21)$$

Compared with the Euler equation in the decentralized equilibrium (9) where $\mu_t = 0$, the last term is added in the Euler equation by the social planner. This term comes from the second term of γ_t^{SP} in (18) and captures the social value of real appreciation when the constraint is binding. It indicates that the planner internalizes that reducing foreign debt at period t will reduce the net capital outflows at $t+1$ and lead to real appreciation at $t+1$. Real appreciation in the case of a binding borrowing constraint reduces the domestic-currency value of foreign-currency debt $-e_t b_t^*/R_t^*$ and relaxes the binding borrowing constraint. Because this additional term is positive, this externality induces households to socially overborrow when there is a possibility that the borrowing constraint binds in the next period. As this externality works through balance of payments (BOP) adjustments and distorts the household's decisions ex ante in normal times, I call this an "ex ante" BOP externality.

3.2 Amplification Loop of Depreciation

The key mechanism of the model is that when the borrowing constraint actually binds, the overborrowing induced by ex ante BOP externality leads to inefficiently large real depreciation through an amplification loop. This mechanism can be understood by substituting the binding borrowing constraint into the balance of payments identity (16) to obtain:

$$e_t^\rho Y^* - e_t m_t = -\kappa_t k - e_t b_{t-1}^*. \quad (22)$$

The effect of an additional unit of foreign borrowing at period $t-1$ on the real exchange rate at period t can be obtained by applying the implicit function theorem to this equation

as follows:

$$\begin{aligned} \frac{\partial e_t}{\partial(-b_{t-1}^*)} &= \frac{e_t}{(\rho - 1) e_t^{\rho-1} Y^* + b_t^*/R_t^*} \\ &= \frac{e_t}{(\rho - 1) e_t^{\rho-1} Y^*} \left\{ 1 + \left(\frac{-b_t^*/R_t^*}{(\rho - 1) e_t^{\rho-1} Y^*} \right) + \left(\frac{-b_t^*/R_t^*}{(\rho - 1) e_t^{\rho-1} Y^*} \right)^2 + \dots \right\} \end{aligned} \quad (23)$$

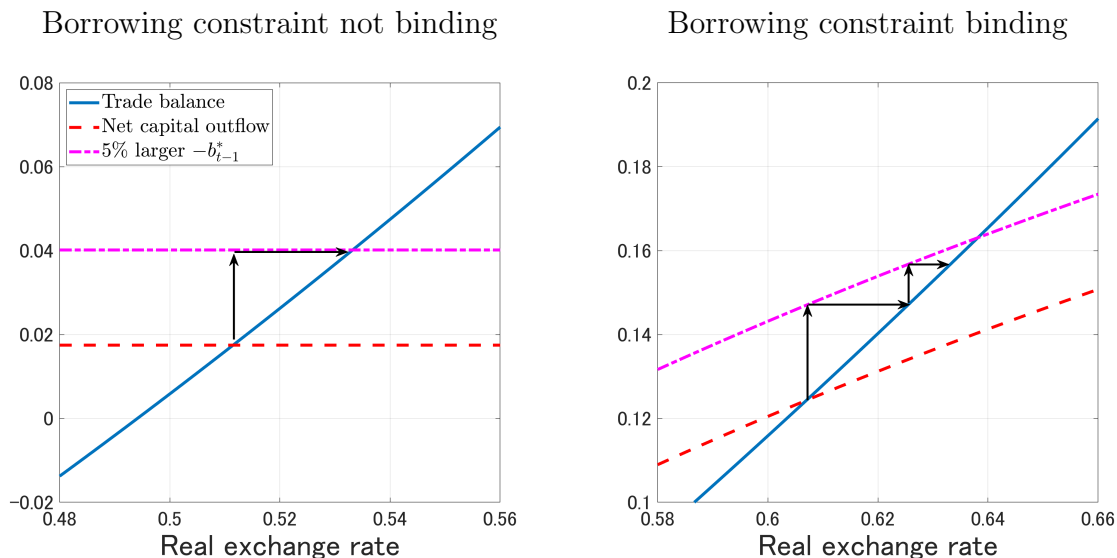
The denominator in the first line $(\rho - 1) e_t^{\rho-1} Y^* + b_t^*/R_t^*$ is always strictly positive under the parameters used in quantitative analysis. Thus, the term in parentheses in the second line $(-b_t^*/R_t^*) / [(\rho - 1) e_t^{\rho-1} Y^*]$ is strictly between 0 and 1.

This equation can be explained as follows. An additional unit of foreign borrowing (overborrowing) at period $t - 1$ increases the net capital outflows at period t on the right-hand side of (22). To meet the balance of payments identity, the real exchange rate will have to depreciate in order to enable the net exports on the left-hand side to increase by the same amount. As shown in (20), an additional unit of b_{t-1}^* leads to a real depreciation of $e_t / [(\rho - 1) e_t^{\rho-1} Y^*]$. This is a direct effect of overborrowing on the real exchange rate, the first term in (23). However, when the borrowing constraint is binding, new foreign borrowing $-e_t b_t^*/R_t^*$ is constrained by the borrowing limit $\kappa_t k$. Thus, the real depreciation (an increase in e_t) will force a cut in the borrowing amount $-b_t^*$ to keep the domestic-currency value of new foreign borrowing unchanged at $\kappa_t k$. The real depreciation of $e_t / [(\rho - 1) e_t^{\rho-1} Y^*]$ will force $-b_t^*$ to reduce by $b_t^* / [(\rho - 1) e_t^{\rho-1} Y^*]$ to keep the domestic-currency value of the foreign borrowing unchanged. This reduction in $-b_t^*$ will in turn lead to a real depreciation through a second-round balance of payments adjustment as follows:

$$\frac{\partial e_t}{\partial(-b_t^*)} \times \frac{b_t^*}{(\rho - 1) e_t^{\rho-1} Y^*} = \frac{e_t}{(\rho - 1) e_t^{\rho-1} Y^*} \times \left(\frac{-b_t^*/R_t^*}{(\rho - 1) e_t^{\rho-1} Y^*} \right),$$

where the partial derivative comes from the implicit function theorem applied to the balance of payments identity (16). This is the second term in (23). Thus, the interaction between balance of payments adjustments and the binding borrowing constraint results in an amplification loop of real depreciation. Note that the currency mismatch between foreign borrowing and the borrowing limit is crucial for this mechanism. If the borrowing limit is denominated in foreign currency, this amplification loop will not occur.

Figure 2: Amplification loop of depreciation



Note: The trade balance is given by $e_t^{\rho-1}Y^* - m_t$, and net capital outflows are $b_t^*/R_t^* - b_{t-1}^*$ in the left-hand side panel and $-\kappa_t k/e_t - b_{t-1}^*$ in the right-hand side panel. Parameters are set to the calibrated values in Section 5, and endogenous variables are set to the calibrated model steady state.

Figure 2 graphically illustrates the mechanism. Two panels plot the trade balance (blue solid line) and net capital outflows (red dashed line) on the vertical axis, given the real exchange rate on the horizontal axis. To emphasize the qualitative difference between not binding and binding, each balance of payments component is divided by the real exchange rate e_t to obtain the trade balance $e_t^{\rho-1}Y^* - m_t$, which is upward-sloping, and net capital outflows $b_t^*/R_t^* - b_{t-1}^*$, which is a horizontal line.⁹ To examine how foreign borrowings in the previous period b_{t-1}^* affect the real exchange rate, m_t and b_t^* are fixed at the steady-state values. The balance of payments identity implies that the real exchange rate is determined at the intersection between the trade balance and net capital outflows.

The left-hand side panel shows how larger foreign borrowings in the previous period would affect the present real exchange rate when the borrowing constraint is not binding today. The purple dashed-dotted line plots the net capital outflows when $-b_{t-1}^*$ is larger by 5% relative to the red dashed line. Larger net capital outflows correspond to an upward parallel shift from the dashed line to the dashed-dotted line. To meet the balance of payments identity, the

⁹Net capital outflows $e_t(b_t^*/R_t^* - b_{t-1}^*)$ would be slightly upward sloping if it is not divided by e_t , even when $b_t^* = b_{t-1}^*$.

net exports need to increase by the same amount, and this would require a real depreciation as depicted by the horizontal arrow. The length of the horizontal arrow indicates the real depreciation due to a 5% larger foreign borrowing. If households increase their borrowing today, the dashed-dotted line would shift downward and the real depreciation would be even smaller.

The right-hand side panel shows the case when the borrowing constraint is binding. New foreign borrowing is now limited by $-b_t^*/R_t^* = \kappa_t k/e_t$, and the net capital outflows are given by $-\kappa_t k/e_t - b_{t-1}^*$. This means that the net capital outflows are also increasing in the real exchange rate, in contrast to the case when the borrowing constraint is not binding. When the previous period's foreign borrowing is larger by 5%, the net capital outflows become larger for any real exchange rate, as plotted by the dashed-dotted line relative to the dashed line. To meet the balance of payments identity, the net exports need to increase through real depreciation, depicted by the first horizontal arrow. However, a real depreciation would cause further net capital outflows because the foreign debt repayment $e_t b_{t-1}^*$ increases, whereas new foreign borrowings are fixed at $\kappa_t k$. Thus, a second-round real depreciation becomes necessary to meet the balance of payments, as the second horizontal arrow depicts. The real depreciation loop continues until the solid and dashed-dotted lines intersect. Thus, overborrowing induced by ex ante BOP externality results in an inefficiently large real depreciation through the amplification loop.

The social cost of an inefficiently large real depreciation is twofold. First, imported inputs become inefficiently expensive and reduces the output. Second, an inefficiently large part of the output is exported, and domestic consumption becomes inefficiently low.

3.3 Ex Post Externality

The externality associated with balance of payments adjustments also distorts the household decision on imported inputs m_t when the borrowing constraint is binding. The first-order

condition with respect to m_t in the above planner's problem is given as follows:

$$\alpha_m \frac{Y_t}{m_t} = e_t \left[\frac{\rho}{\rho - 1} - \underbrace{\frac{b_t^*/R_t^*}{(\rho - 1)e_t^{\rho-1}Y^*} \mu_t^{SP}}_{\text{ex post BOP externality}} \right]. \quad (24)$$

The first term in bracket is the terms-of-trade externality. As this externality works even without a borrowing constraint and is not the focus of this study, it is corrected by the fixed rate tax τ_m . The second term indicates that when the borrowing constraint is binding and $\mu_t^{SP} > 0$, the social cost of buying imported inputs is higher than the cost in normal times. This second term is derived from the second term of γ_t^{SP} in (18), namely, the social value of real appreciation by relaxing the binding constraint. Therefore, this term captures the social cost of buying one additional unit of imported inputs through real depreciation and tightening the binding constraint. This externality implies that households buy socially excessive imported inputs when the borrowing constraint is binding, thereby deteriorating the trade balance and triggering an amplification loop of real depreciation as in the case of overborrowing.¹⁰ I call this an “ex post” BOP externality, because this externality works only when the borrowing constraint is binding.

3.4 Decentralization of Planner's Allocation

The social planner's optimal allocation can be decentralized by taxes on foreign bonds and imported inputs. The ex ante BOP externality associated with foreign bonds can be corrected by the following macroprudential tax on foreign bonds:

$$u_c(t) = \beta R_t^* (1 + \tau_t^b) E_t \left[\frac{e_{t+1}}{e_t} u_c(t+1) \right],$$

$$\tau_t^b = \frac{E_t \left[\frac{e_{t+1}}{e_t} \left(-\frac{b_{t+1}^*/R_{t+1}^*}{\rho e_{t+1}^{\rho-1} Y^*} \mu_{t+1}^{SP} \right) \right]}{E_t \left[\frac{e_{t+1}}{e_t} u_c(t+1) \right]},$$

¹⁰In Figure 2, larger imported inputs correspond to a downward shift in the trade balance curve.

which is strictly positive in the case of the positive probability that the borrowing constraint binds in the next period. Unlike the pecuniary externalities due to a drop in the collateral asset prices, the optimal macroprudential tax in this model is time consistent. The ex post BOP externality can be corrected by the following tax on imported inputs:

$$\alpha_m \frac{Y_t}{m_t} = (1 + \tau_t^m) e_t,$$

$$\tau_t^m = \frac{1}{\rho - 1} - \frac{b_t^*/R_t^*}{(\rho - 1)e_t^{\rho-1}Y^*} \frac{\mu_t^{SP}}{u_c(t)} = \tau_m - \frac{b_t^*/R_t^*}{(\rho - 1)e_t^{\rho-1}Y^*} \frac{\mu_t^{SP}}{u_c(t)},$$

where the second term corresponds to the tax to correct the ex post BOP externality. This additional tax is strictly positive only when the borrowing constraint is binding. In reality, however, introducing taxes on imported inputs only during crises may be difficult to implement appropriately. Therefore, the next section examines the optimal combination of monetary and macroprudential policies to address the externality.

4 Policy Analysis

In this section, I study the optimal monetary and macroprudential policies in the full model with price stickiness as described in Section 2. Although BOP externalities do not result in time inconsistency in an optimal policy, introducing New Keynesian price stickiness may lead to time inconsistency because firms' price decisions are forward-looking. Therefore, I follow [Bianchi and Mendoza \(2018\)](#) and assume that the government cannot commit to future policies and therefore focus on time-consistent optimal policies in a Markov perfect equilibrium. A Markov perfect equilibrium is characterized by the following two features: (1) the planner at each period chooses his/her policy rules optimally, taking the future planners' policy rules as given, but internalizing how his/her policies affect the policies of future planners; (2) the optimal policy rules coincide with the future planners' policy rules that are taken as given when choosing the current policy rules. The second feature implies that the planner's decision rules are time invariant, and that the planner has at no point in time an incentive to deviate from the decision rules expected by the past planners, ensuring that the planners' decision rules are time consistent.

4.1 Monetary Policy

I first consider the case where monetary policy is the only policy tool. If there were no borrowing constraint in the model, the optimal discretionary monetary policy would be strict inflation targeting, i.e. $\pi_t = 0 \forall t$.¹¹ Therefore, the main question here is whether and how the borrowing constraint and the associated BOP externalities cause the optimal discretionary monetary policy to deviate from strict inflation targeting. As $\pi_t = 0$ is set as the target inflation rate in the price adjustment cost, I call the monetary policy setting a positive inflation rate as “expansionary” monetary policy, and a negative inflation rate as “contractionary” monetary policy. The following proposition and corollary provide the main result of this subsection:

Proposition 1 *In the model described in Section 2, when the monetary policy is the only policy tool, strict inflation targeting is not the optimal discretionary monetary policy.*

Proof: See Section B.4 in the appendix.

The following corollary arises from the proof of Proposition 1:

Corollary 1 *Given $E_t(\pi_{t+1}) = 0$, the optimal discretionary monetary policy is contractionary both when the borrowing constraint is binding and when it is not binding.*

Proof: See Section B.5 in the appendix.

To prove the proposition, I set up a Ramsey planner’s problem, where the planner chooses an inflation rate and all the endogenous variables to maximize the household’s expected utility, subject to the decentralized equilibrium conditions derived in Section 2. Formally,

$$V(b_{t-1}^*, s_t) = \max_{c_t, b_t^*, \ell_t, m_t, e_t, p_t^w, \pi_t, \mu_t} \log \left(c_t - \chi \frac{\ell_t^{1+\omega}}{1+\omega} \right) + \beta E_t V(b_t^*, s_{t+1}),$$

subject to (2), (6), (7), (9), (10), (12), (15), (16), (17).¹² The full description of the first-

¹¹Section B.2 in the appendix provides the formal proof of this statement. Intuitively, given that the terms-of-trade externality and the distortion due to market power can be corrected by a tax and subsidy, the monetary policy should focus only on minimizing the price adjustment cost.

¹²The nominal interest rate R_t and Euler equation with respect to domestic bonds are not included in this setup. The nominal interest rate R_t can be obtained using the Euler equation after all the other endogenous variables are pinned down by the first-order conditions.

order conditions is provided in Section B in the appendix. Here, I show the key first-order conditions and a sketch of the proof and then discuss the intuition. The first-order conditions with respect to the inflation rate π_t and wholesale goods price p_t^w are given as follows:

$$\pi_t : \eta_t^{PC} \psi(2\pi_t + 1)Y_t = \lambda_t^{RP} \psi \pi_t Y_t, \quad (25)$$

$$p_t^w : \eta_t^{PC} \theta Y_t = \eta_t^\ell + \eta_t^m. \quad (26)$$

λ_t^{RP} is the Lagrange multiplier on the budget constraint (15), which captures the social value of the final goods. η_t^{PC} , η_t^ℓ , and η_t^m are the Lagrange multipliers on the New Keynesian Phillips curve (12) and the implementability constraints with respect to labor and imported inputs (6) and (7) in the decentralized equilibrium. In the first equation (25), the left-hand side shows the effect of a marginal increase in π_t on social welfare by increasing the wholesale goods price p_t^w through the New Keynesian Phillips curve. The right-hand side shows the marginal change in price adjustment cost evaluated by the social value of final goods. Given that $2\pi_t + 1$ is strictly positive, which always holds in the quantitative analysis below, the sign of the optimal inflation rate π_t is the same as that of the Lagrange multiplier η_t^{PC} , and $\pi_t = 0$ if and only if $\eta_t^{PC} = 0$. The second equation (26) provides the intuition for η_t^{PC} . It shows that the effect of a marginal increase in p_t^w on social welfare consists of two terms, η_t^ℓ and η_t^m . As these are the Lagrange multipliers on the implementability constraints with respect to labor and imported inputs, these multipliers take non-zero values if and only if the government has an incentive to distort production factor inputs relative to the decentralized equilibrium. Along with the first equation (25), these equations show that the optimal π_t deviates from the target 0 if and only if the government has an incentive to distort production factor inputs by manipulating p_t^w through monetary policy.

The proof of Proposition 1 follows the steps of setting $E_t(\pi_{t+1}) = 0$ and showing that $\pi_t = 0$ does not satisfy the first-order conditions of the planner's problem. By setting

$E_t(\pi_{t+1}) = 0$ and $\pi_t = 0$, η_t^ℓ and η_t^m satisfy the following equations:

$$\frac{\eta_t^\ell}{\lambda_t^{RP}} = \alpha_\ell \frac{Y_t}{\ell_t} - \chi \ell_t^\omega, \quad (27)$$

$$\frac{\eta_t^m}{\lambda_t^{RP}} = \alpha_m \frac{Y_t}{m_t} - e_t - \frac{\gamma_t^{RP}}{\lambda_t^{RP}} e_t. \quad (28)$$

The right-hand side of (27) clearly coincides with the first-order condition in the decentralized equilibrium (6), and so $\eta_t^\ell = 0$. This means that the government has no incentive to distort labor inputs through monetary policy. As regard to imported inputs, the right-hand side of (28) would coincide with the first-order condition in the decentralized equilibrium (7) if and only if $\gamma_t^{RP}/\lambda_t^{RP} = 1/(\rho - 1)$, where γ_t^{RP} is the Lagrange multiplier on the balance of payments identity.

The expression for $\gamma_t^{RP}/\lambda_t^{RP}$ differs depending on whether the borrowing constraint is binding or not:

$$\frac{\gamma_t^{RP}}{\lambda_t^{RP}} = \frac{1}{\rho - 1} + \frac{1}{(\rho - 1) e_t^{\rho-1} Y^* + \varepsilon_t^{pm}} \left(\frac{\eta_t^{EE}}{\lambda_t^{RP}} \frac{u_c(t)}{e_t} \right) \text{ when not binding,} \quad (29)$$

$$\frac{\gamma_t^{RP}}{\lambda_t^{RP}} = \frac{1}{\rho - 1} + \frac{1}{(\rho - 1) e_t^{\rho-1} Y^* + \varepsilon_t^{pm}} \left(-\frac{\mu_t^{RP}}{\lambda_t^{RP}} \frac{b_t^*}{R_t^*} \right) \text{ when binding.} \quad (30)$$

When not binding, η_t^{EE} in (29) is the Lagrange multiplier on the Euler equation with respect to foreign bonds in the decentralized equilibrium (9). The appendix shows that $\eta_t^{EE} > 0$ when the constraint is not binding, implying that $\gamma_t^{RP}/\lambda_t^{RP} > 1/(\rho - 1)$ and thus $\eta_t^m < 0$ in (28). $\eta_t^\ell = 0$ and $\eta_t^m < 0$ imply $\pi_t < 0$ from (26), leading to a contradiction with $\pi_t = 0$ and thus proving Proposition 1. This proof provides an intuition for the optimal monetary policy. When the borrowing constraint is not binding, households socially overborrow and the planner has an incentive to correct it. The planner's incentive to correct overborrowing is captured by $\eta_t^{EE} > 0$ in (29). As γ_t^{RP} is the social value of real appreciation, $\eta_t^{EE} > 0$ adds to the social value of real appreciation by correcting the overborrowing. As the Euler equation with respect to foreign bond (9) shows, a real appreciation today (lower e_t) increases the effective interest rate on foreign borrowing, thereby discouraging foreign borrowing by households. The planner thus conducts contractionary monetary policy ($\pi_t < 0$) to cause a

real appreciation and correct the overborrowing in normal times.

When the borrowing constraint is binding, the term in parentheses in (30) is again positive, proving that $\pi_t = 0$ is not optimal. The intuition is straightforward. When the borrowing constraint is binding, the planner wants to reduce the use of socially excessive imported inputs due to ex post BOP externality. A contractionary monetary policy is intended to discourage production and the use of imported inputs, and thereby correct the externality, at least partially.

Note that this discussion does not necessarily mean that the optimal discretionary monetary policy dominates a strict inflation targeting policy in terms of social welfare. This is because the optimal discretionary monetary policy is determined by period-by-period optimization without considering how expectations of future policies affect private agents' behavior. In fact, the quantitative analysis in Section 5 shows that expectations of monetary policy interventions during crises induce larger ex ante borrowings and exacerbate the crises, and thereby reduce the expected welfare relative to strict inflation targeting.

4.2 Monetary and Macroprudential Policies

In this subsection, I introduce macroprudential taxes on foreign borrowing and study interactions between monetary and macroprudential policies. In line with the previous subsection, I assume that the government cannot commit to future policies and focus on time-consistent policy in a Markov perfect equilibrium. I consider two policy regimes. In the first, monetary policy is strict inflation targeting, that is, $\pi_t = 0 \forall t$, and the government optimally chooses a macroprudential tax on foreign borrowing. In the second, the government optimally chooses both monetary policy and a macroprudential tax. For each policy regime, I set up the Ramsey planner's problem and characterize the optimal policies. The full description of the planner's problem is provided in Section C and D in the appendix. The key results and intuitions are as follows.

Under a strict inflation targeting monetary policy, the planner's problem is similar to that in the flexible price model described in Section 3, in that $\pi_t = 0 \forall t$. The key difference is that in the planner's problem discussed here, the first-order conditions with respect to labor ℓ_t and imported inputs m_t in the decentralized equilibrium, (6) and (7), are considered

to be implementability constraints, because tax on imported inputs is assumed to be not available. Although no externality directly distorts the labor decisions, the distortions on imported inputs due to ex post BOP externality affect the labor decisions indirectly through the marginal product of each other. The fact that the planner cannot intervene during crises through the use of either a monetary policy or taxes affects the design of the macroprudential tax on foreign borrowing ex ante. The Euler equation with respect to foreign borrowing in the planner's problem when the borrowing constraint is not binding now but may bind in the next period is given as

$$u_c(t) = \beta R_t^* E_t \left[\frac{e_{t+1}}{e_t} \left(u_c(t+1) - \underbrace{\frac{b_{t+1}^*/R_{t+1}^*}{\rho e_{t+1}^{\rho-1} Y^*} \mu_{t+1}}_{\text{ex ante BOP externality}} - \frac{1}{\rho e_{t+1}^{\rho-1} Y^*} \frac{\rho}{\rho-1} \eta_{t+1}^m \right) \right], \quad (31)$$

where η_{t+1}^m is the Lagrange multiplier on the first-order condition with respect to imported inputs in the decentralized equilibrium (7). The ex ante BOP externality indicates the need to impose a macroprudential tax on foreign borrowing in order to correct overborrowing. The last term comes from the ex post BOP externality and is negative when the borrowing constraint binds at $t+1$. The intuition is as follows. As shown in Section 3, the planner aims to reduce the use of imported inputs when the borrowing constraint binds. This intervention is intended to mitigate the real depreciation and relax the binding constraint. However, the planner here has no policy tool for intervention during crises. Therefore, the planner lowers the macroprudential tax rate slightly ex ante, so that when the borrowing constraint actually binds, the real exchange rate depreciates slightly more, discouraging the use of imported inputs and correcting the ex post BOP externality at least partially.

In the second policy regime, where both monetary and macroprudential policies are optimally chosen, the planner's problem is similar to that in the previous subsection with the optimal discretionary monetary policy. The difference is that the planner here is not subject to the Euler equation with respect to foreign bonds in the decentralized equilibrium (9) as an implementability constraint because the macroprudential tax on foreign borrowing is available. The question here is then whether and how the optimal discretionary monetary policy changes as the macroprudential taxes become available. The following proposition

and corollary answer this question.

Proposition 2 *In the model described in Section 2, when a time-consistent optimal macroprudential tax on foreign borrowings is available, strict inflation targeting is not an optimal discretionary monetary policy.*

Proof: See Section D.2 in the appendix.

The following corollary arises from the proof of Proposition 2:

Corollary 2 *Given $E_t(\pi_{t+1}) = 0$, the optimal discretionary monetary policy is (1) $\pi_t = 0$ when the borrowing constraint is not binding; and (2) $\pi_t < 0$ when the borrowing constraint is binding.*

Proof: See Section D.2 in the appendix.

The discussion on Proposition 1 in the previous subsection helps to explain the intuition of Proposition 2. When the borrowing constraint is not binding at present but may bind in the next period, the planner aims to discourage foreign borrowings and correct overborrowing. In the previous case where monetary policy is the only policy tool, the planner tries to achieve this goal through a contractionary monetary policy, which is captured by the positive η_t^{EE} in (29). However, in the present case with a macroprudential tax on foreign borrowing, the tax corrects the overborrowing, and $\eta_t^{EE} = 0$ in (29). Therefore, the planner has no incentive to use a monetary policy to correct overborrowing and thus sets $\pi_t = 0$ to minimize the price adjustment cost.

When the borrowing constraint is binding, the planner has an incentive to use a monetary policy. This can be understood from the expression for γ_t^{RP} in (30) when the constraint is binding. The last term is still positive and adds to the value of γ_t^{RP} , implying that a contractionary monetary policy is optimal. The intuition is that a macroprudential tax on foreign borrowing cannot mitigate the excessive imported inputs and the associated real depreciation when the constraint is binding, thus requiring a monetary policy intervention.

5 Quantitative Analysis

In this section, I analyze the model quantitatively. I solve the model numerically using a global method to deal with an occasionally binding constraint. The detailed algorithm is explained in Section G in the appendix. I first set the parameter values in the model.

5.1 Calibration

Each period in the model represents a year. The calibration strategy is to set the standard parameters to standard values in the literature, and choose the other parameters by targeting the average of the 14 countries in Figure 1. Table 1 presents the parameter values in the model. The discount factor β is set to 0.92 so that the mean foreign debt-to-GDP ratio in stochastic simulations under inflation targeting without macroprudential taxes becomes 40% to match the average external debt-to-GNI ratio across the 14 countries in 2019. The baseline interest rate on foreign borrowing 1.04 is standard for annual models. The labor disutility coefficient χ is set such that the labor supply at the steady state is 1. ω is set such that the Frisch elasticity of labor supply is 1, which is standard in the literature. The labor share in production α_ℓ is set to the conventional value of 0.66. The share of imported inputs α_m is set to target the import-to-GDP ratio at 22%, which is the average imported-input share of exported goods across the 14 countries. The asset share α_k is then set to $1 - \alpha_\ell - \alpha_m$. The elasticity of substitution across the differentiated intermediate goods θ and price adjustment cost parameter ψ are set to 8 and 50 respectively, which are standard values in the New Keynesian models. The price elasticity of demand for exports is set to 3, which is within the range of the empirical estimates in [Simonovska and Waugh \(2014\)](#). Foreign demand Y^* is normalized at 1. The tight borrowing limit $\kappa_t = 0.2$ is set such that the unconditional probability of crises is 7.2%, which is in line with the empirical finding in [Eichengreen and Gupta \(2016\)](#). The transition matrix for κ_t is set following [Bianchi and Mendoza \(2018\)](#). The stochastic process of aggregate productivity and interest rate is taken from [Mendoza \(2010\)](#), which calibrates the model based on Mexican data. Specifically, productivity takes two values, $A_t = \exp(\pm 0.0134)$, and the interest rate takes two values, $R_t^* = R^* \times \exp(\pm 0.0196)$, with the same autocorrelation 0.59 and negative correlation -0.67

Table 1: Parameter values

Variable	Value	Source/Target
β	Discount factor	Debt-to-GDP 40%
R^*	Baseline interest rate	Standard
χ	Labor disutility coefficient	Unit labor supply
ω	Frisch elasticity $1/\omega$	Standard
α_k	Asset share	$1 - \alpha_\ell - \alpha_m$
α_ℓ	Labor share	Standard
α_m	Imported-input share	Import-to-GDP 22%
θ	Elasticity of substitution	Standard in NK models
ψ	Price adjustment cost	Standard in NK models
ρ	Price-elasticity of exports	Simonovska and Waugh (2014)
Y^*	Foreign demand	Normalization
low κ_t	Tight borrowing limit	Crisis probability
P_{HL}, P_{LH}	Transition matrix for κ_t	Bianchi and Mendoza (2018)

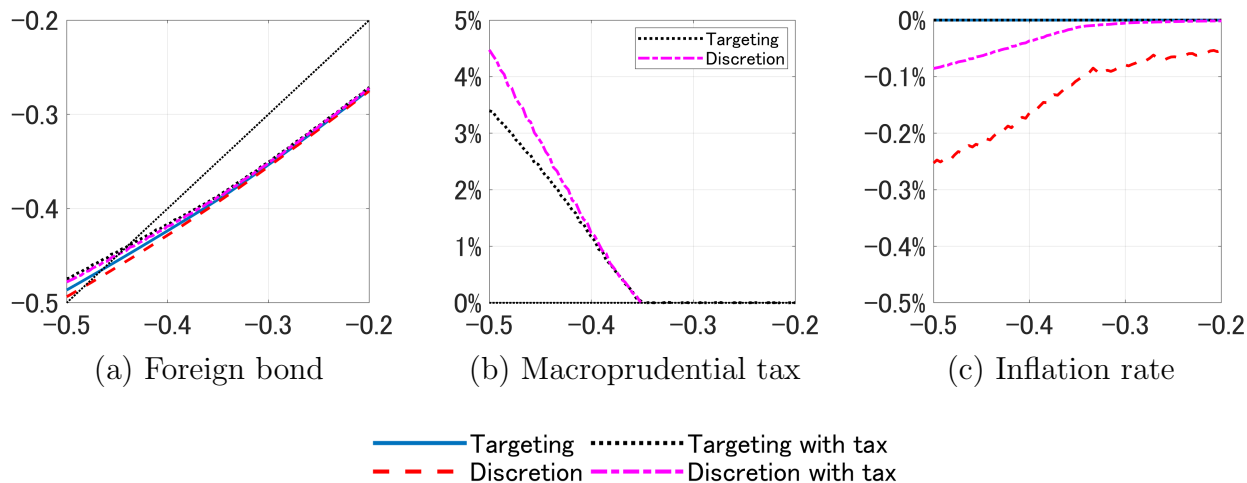
between productivity and the interest rate.

5.2 Decision Rules

In the following quantitative analysis, I compare four policy regimes: strict inflation targeting monetary policy, the optimal discretionary monetary policy discussed in Section 4.1, and both these monetary policies respectively with the corresponding optimal macroprudential tax on foreign borrowing discussed in Section 4.2. I start by plotting the decision rules for the key variables.

Figure 3 plots the decision rules under the four policy regimes when the borrowing limit shock is not hitting the economy. The horizontal axis in each panel is the amount of foreign bonds at the beginning of the period b_{t-1}^* , which is the state variable of the model. Business cycle shocks are set to high productivity and low interest rate. Panel (a) plots the decision rules for foreign bond b_t^* . It shows that foreign borrowing is smaller under macroprudential taxes. The intersection between the decision rules and the 45-degree line indicates the stochastic steady state. Under macroprudential taxes, foreign debt at the stochastic steady state is smaller by 6 to 7%. Another observation in Panel (a) is that foreign borrowing is larger under discretionary monetary policy than inflation targeting, both with and with-

Figure 3: Decision rules when the borrowing limit shock is not hitting



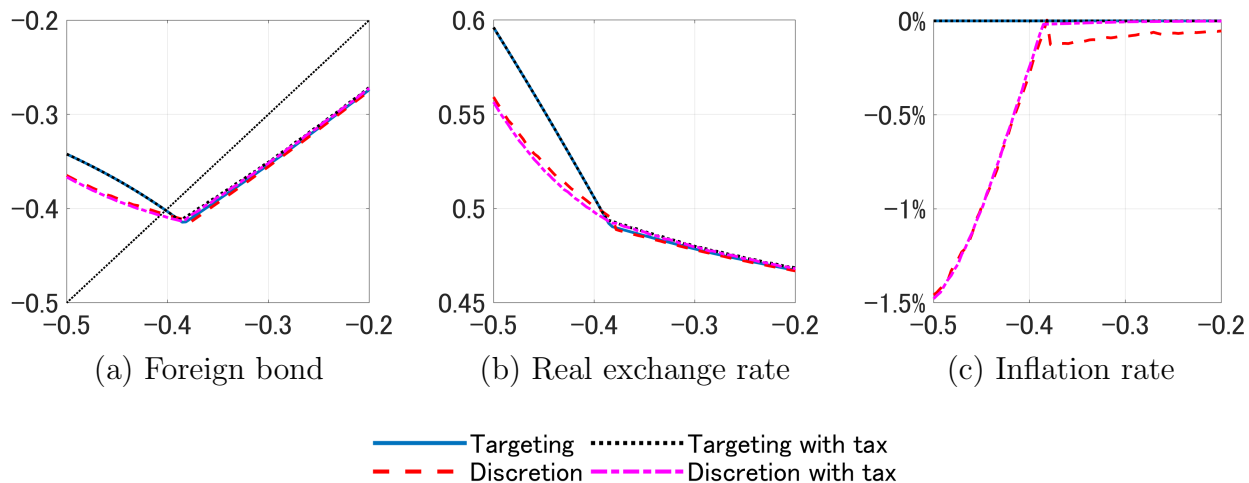
Note: These panels plot the decision rules under different policy regimes when the borrowing limit shock is not hitting. The horizontal axis gives the amount of foreign bonds at the beginning of the period.

out tax. This is because monetary policy intervention when the borrowing constraint binds would mitigate real depreciation and lower the expected e_{t+1} , thereby reducing the effective interest rate on foreign bond and inducing larger borrowings. Panel (b) shows that the macroprudential tax is imposed in the case of a positive probability that the borrowing constraint would bind in the next period. The tax rate becomes higher as the outstanding debt becomes larger, reflecting a larger ex ante BOP externality. A higher tax rate under a discretionary monetary policy is intended to reduce the overborrowing induced by expectations of monetary policy interventions, as discussed above. Panel (c) plots the inflation rate under a discretionary monetary policy. As discussed in Section 4, the optimal discretionary monetary policy without tax is contractionary in normal times to mitigate overborrowing, but it is almost zero when combined with a macroprudential tax.¹³

Figure 4 plots the decision rules when the borrowing limit shock hits the economy. Panel (a) shows that when the borrowing constraint binds, which is on the left-hand side of the kink in the decision rules, foreign borrowings shrink. This is because a large debt repayment with limited new borrowing triggers an amplification loop of real depreciation, forcing a cut

¹³A slightly negative inflation rate when combined with a tax reflects the possibility of the constraint binding and monetary policy intervening in the next period.

Figure 4: Decision rules when the borrowing limit shock hits



Note: These panels plot the decision rules under different policy regimes when a borrowing limit shock hits the economy. The horizontal axis is the amount of foreign bonds at the beginning of the period.

in new borrowings to meet the borrowing constraint. Panel (b) shows the real exchange rate depreciating when the borrowing constraint binds. For all the four policy regimes, $-e_t b_t^*/R_t^*$ is equal to the borrowing limit $\kappa_t k$ in the left-hand side of the kink, but each of e_t and b_t^* takes a different value depending on the policy. Panel (b) shows that a monetary policy intervention during crises mitigates the real depreciation under a discretionary monetary policy, both with and without tax. This mitigated real depreciation causes the foreign borrowings b_t^* to shrink less under discretionary monetary policy in Panel (a). Panel (c) shows that monetary policy intervention during a crisis is contractionary. As discussed in Section 4, a contractionary monetary policy discourages the production and use of imported inputs, thereby improves the trade balance, and mitigates the real depreciation.

5.3 Crisis Dynamics

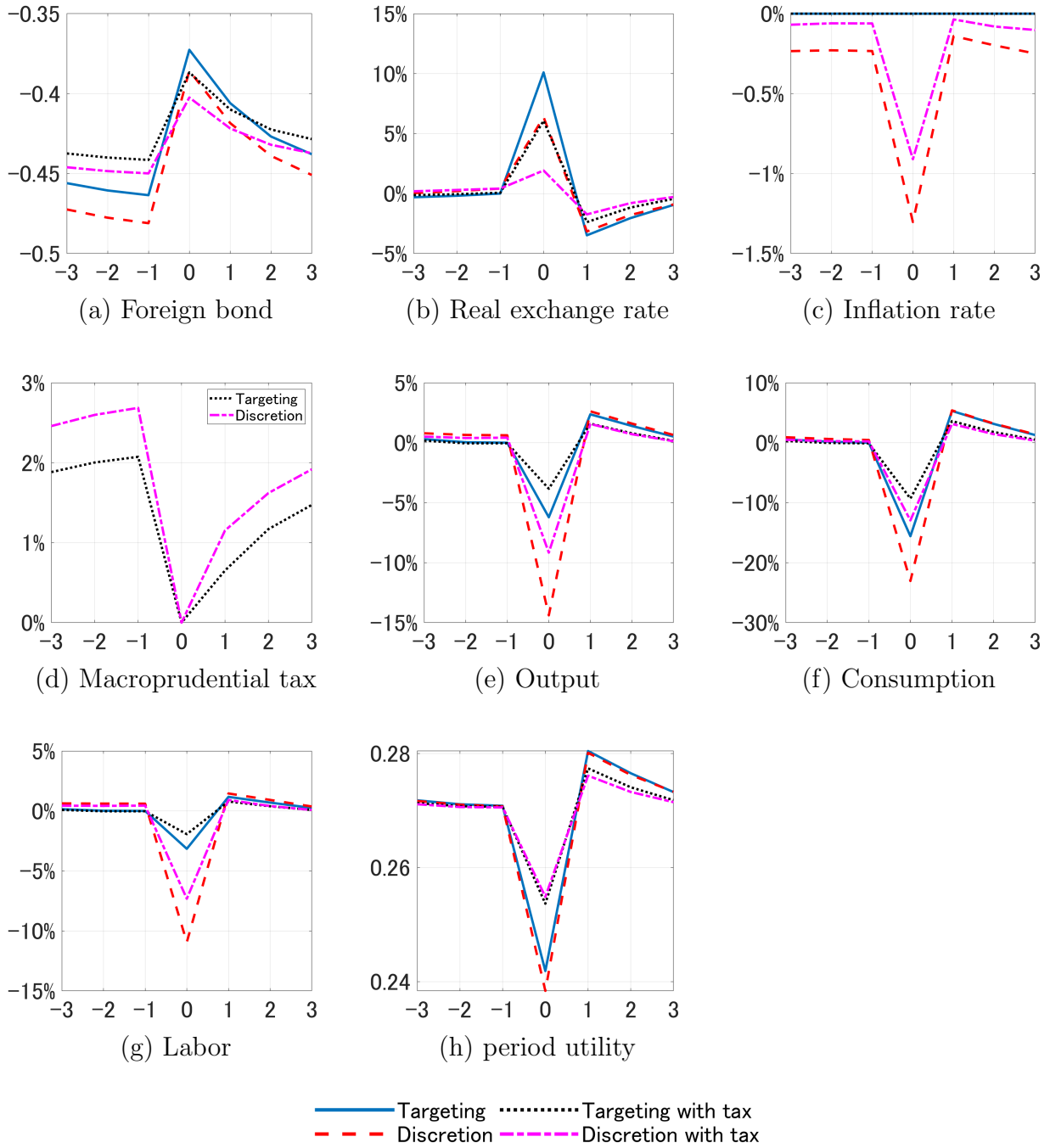
Next I compare the crisis dynamics under different policy regimes. I simulate the model for 10,000 periods with stochastic shocks, drop the first 1,000 periods, and then use the remaining 9,000 periods for this analysis. Following the literature, a crisis is defined as an event in which the current account is more than two standard deviations above its long-run

mean. I pick up all the crisis events under inflation targeting without taxes, and take the average dynamics of the variables around the crisis events under the four policy regimes. Figure 5 plots the crisis dynamics of the key variables in a seven-period window around the crisis events at period 0. The real exchange rate, output, consumption, and labor are expressed in terms of percentage gap from the values at period -1 under inflation targeting without taxes. These panels thus contain information on the relative levels across different policies as well.

Panel (a) shows that a crisis is associated with a sharp reversal in capital flows, but the size of the reversals varies widely across policies. First, macroprudential taxes reduce the pre-crisis foreign debt and thereby mitigate the capital flow reversals during the crisis. Second, the pre-crisis foreign debt is larger under discretionary monetary policy, both with and without a tax. As discussed above, a discretionary monetary policy induces larger foreign borrowings through the expectation of monetary policy interventions during crises. Panel (b) shows the dynamics of real exchange rate. Without macroprudential taxes, the real exchange rate depreciates by 10% under inflation targeting and 6% under discretionary monetary policy. Macroprudential taxes mitigate the real depreciation by reducing the capital outflows during crises. Along with macroprudential taxes, the real exchange rate depreciates by 6% under inflation targeting but only 2% under discretionary monetary policy. Monetary policy interventions under discretionary monetary policy are observed to mitigate depreciation. Panel (c) shows the size of monetary policy interventions, while Panel (d) gives the macroprudential taxes. The tax rate is higher under a discretionary monetary policy by about 0.6% before the crisis. This is to correct the overborrowing induced by expectations of monetary policy interventions.

As regards the real side of the economy, Panel (e) shows that the output drops during a crisis, because a real depreciation makes the imported inputs expensive. The model, therefore, explains the drop in output without working capital financing commonly assumed in the literature. Panel (e) also shows that monetary policy interventions have a negative impact on output. Without taxes, the output drops by 6.2% under inflation targeting and 14.4% under discretion. With taxes, the output drops by 3.8% under inflation targeting and 9.2% under discretion. Panel (f) shows a higher drop in consumption under discretionary

Figure 5: Crisis dynamics



Note: These figures plot the crisis dynamics under different policy regimes. The horizontal axis gives the time, and a crisis occurs at period 0. The real exchange rate, output, consumption, and labor are expressed in percentage gap from the level at period -1 under inflation targeting without a tax. The other variables are expressed in actual values.

Table 2: Standard deviations

	Without tax		With tax	
	Targeting	Discretion	Targeting	Discretion
Output	3.67	5.21 (142.1%)	3.33 (90.9%)	4.10 (111.7%)
Consumption	6.00	7.71 (128.5%)	4.66 (77.6%)	5.30 (88.3%)
Real exchange rate	3.13	2.28 (72.7%)	2.02 (64.5%)	1.19 (37.9%)
Current account-GDP	2.88	2.96 (102.9%)	2.44 (85.0%)	2.36 (82.0%)

Note: This table presents the standard deviations of selected variables under each policy regime. The values in parentheses are the percentages relative to the values under inflation targeting without taxes.

monetary policy.

These panels, however, do not necessarily imply that inflation targeting dominates a discretionary monetary policy in welfare. Panel (g) shows the labor supply much higher under inflation targeting than discretion. As a result, period utility in Panel (h), which is consumption minus labor disutility, becomes slightly higher under discretion than inflation targeting if combined with taxes. This result can be explained as follows. Monetary policy interventions during a crisis mitigate the inefficiently large depreciations due to ex post BOP externality. Mitigated depreciation implies smaller exports and thus a larger fraction of output consumed domestically. This means that consumption does not drop much although the output and associated labor disutility drop much from a contractionary monetary policy. Monetary policy interventions thus are intended to improve the balance between consumption and labor disutility distorted by externality.¹⁴ I conduct a formal welfare analysis in the next subsection.

Table 2 presents the standard deviations of selected variables. The standard deviations of output, consumption, and real exchange rate are divided by the simulation mean of each variable, so that all values can be interpreted as a percentage. The values in parentheses are the percentages relative to the values under inflation targeting without taxes. Macroprudential taxes are found to reduce the volatility of all variables except output under a discretionary monetary policy with taxes. In particular, the standard deviations of a real exchange rate

¹⁴Section F in the appendix examines the model with fixed labor supply and no labor disutility. In this case, monetary policy interventions under discretion actually lead to higher consumption during a crisis than under inflation targeting.

are substantially lower with taxes than without taxes. The current account-to-GDP ratio is also less volatile with taxes. The less volatile current account under taxes is partly due to less volatile exchange rates because the current account is given by $e_t(b_t^* - b_{t-1}^*)$ and thus is affected by changes in the real exchange rate. A comparison of different monetary policy regimes shows that a discretionary monetary policy leads to higher output and consumption volatility both with and without taxes. In contrast, the real exchange rate is less volatile under a discretionary monetary policy because monetary policy interventions mitigate real depreciation during a crisis.

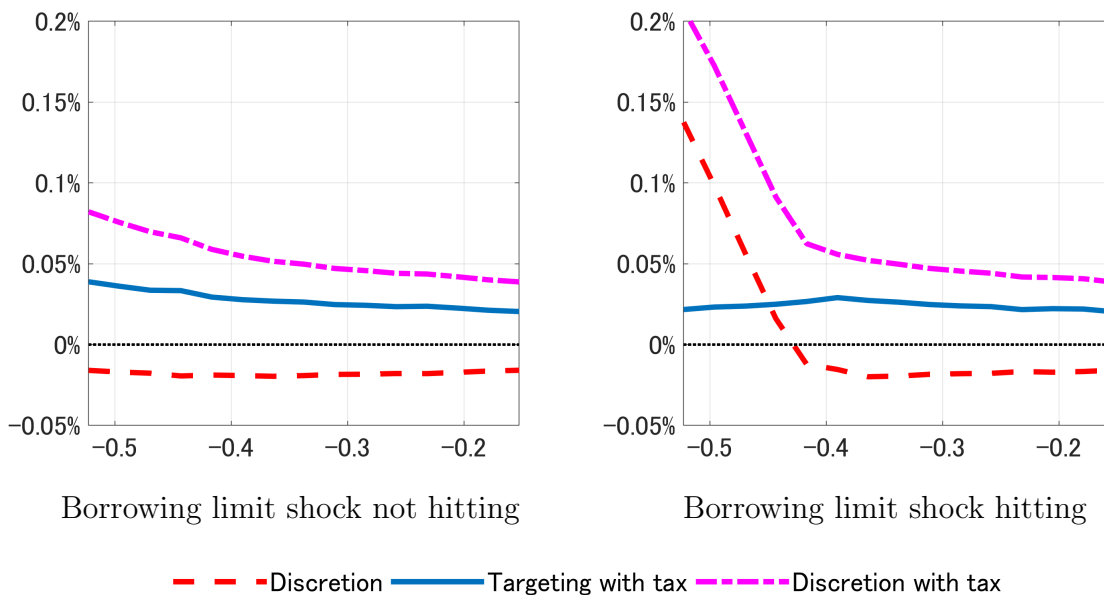
5.4 Welfare Analysis

Finally, I compare the welfare implications of different policy regimes. Using the expected utility under inflation targeting without macroprudential taxes as the benchmark welfare, I express the welfare gain/loss of the other policy regimes in terms of permanent consumption gain/loss relative to the benchmark welfare. Specifically, let $V^{IT}(b_{-1}^*, s_0)$ denote the expected utility under inflation targeting without taxes when the initial state is (b_{-1}^*, s_0) . Then the welfare gain/loss of an alternative policy regime is expressed as $\gamma(b_{-1}^*, s_0)$ satisfying the following equation:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \log \left([1 + \gamma(b_{-1}^*, s_0)] c_t - \chi \frac{\ell_t^{1+\omega}}{1+\omega} \right) \right] = V^{IT}(b_{-1}^*, s_0)$$

Figure 6 plots the results. The horizontal axis gives the initial foreign bond b_{-1}^* and the vertical axis gives the welfare gain/loss in percentage, which is $\gamma(b_{-1}^*, s_0) \times 100$. Business cycle shocks are set to low productivity and low interest rate, but these do not affect the results much. The left-hand side panel plots the welfare gain/loss when a borrowing limit shock does not hit the economy at the initial period. A discretionary monetary policy without taxes brings slightly lower welfare than inflation targeting by 0.02%. As shown in the previous subsection, a discretionary monetary policy without taxes induces larger borrowings ex ante and destabilizes the economy, thereby reducing welfare. In contrast, with the optimal macroprudential tax, a discretionary monetary policy brings higher welfare than inflation targeting. This result is consistent with the welfare analyses in [Coulibaly](#)

Figure 6: Welfare gain/loss under different policy regimes



Note: These panels plot the expected welfare gain/loss under different policy regimes in terms of permanent consumption relative to inflation targeting without taxes.

(2018). Evaluated at the simulation mean of foreign bond under inflation targeting, which is -0.45 , the welfare gain under discretionary monetary policy with taxes is 0.07% , whereas that under inflation targeting with taxes is 0.03% .

The right-hand side panel shows the welfare gain/loss when a borrowing limit shock hits the economy at the initial period. A welfare gain by monetary policy interventions under discretion can clearly be observed. Evaluated at the simulation mean of foreign bond under inflation targeting, the welfare gain under discretion without taxes is 0.03% and with taxes is 0.09% . The welfare gain can be higher if the initial foreign debt is larger. Conversely, if the initial debt is small and the borrowing constraint does not bind at the initial period, which is on the right-hand side of the kink in welfare curves, the welfare effects under discretion would be similar to that in the case of no borrowing limit shock on the left-hand side panel.

These results suggest that the optimal combination of monetary and macroprudential policies is an ex ante macroprudential tax to correct overborrowings and an ex post monetary policy intervention to mitigate real depreciations. The anticipation of monetary policy interventions without macroprudential taxes would induce even larger overborrowings in

normal times, thereby destabilize the economy and reduce welfare. Monetary policy interventions will stabilize the economy and improve welfare only if it is combined with an ex ante macroprudential tax to deal with overborrowing in normal times.

6 Conclusion

In this study, I developed a model of a small open economy that borrows from abroad in foreign currency subject to an occasionally binding borrowing constraint. The model features a novel mechanism in which the interaction between balance of payments adjustments and the borrowing constraint triggers an amplification loop of real depreciation, increasing the domestic-currency value of foreign debt and leading to severe crises. Private agents take the real exchange rate as given, and therefore overborrow in normal times and import excessively during crises, both of which result in inefficiently large real depreciation during crises.

Given this model economy, I examine the optimal monetary and macroprudential policies. When monetary policy is the only available policy tool, the optimal discretionary monetary policy becomes contractionary both in normal times and during a crisis to mitigate the depreciation. With the optimal macroprudential tax on foreign borrowing, the optimal discretionary monetary policy focuses only on minimizing the price adjustment cost in normal times, but still intervenes during crises to mitigate depreciation.

In the quantitative analysis, I show that a discretionary monetary policy induces larger borrowing in normal times and destabilizes the economy through an anticipation of policy interventions during crises. Macroprudential taxes help to stabilize the economy regardless of the monetary policy regime. The welfare analysis shows that a discretionary monetary policy brings lower welfare than inflation targeting without macroprudential taxes, but higher welfare when combined with the optimal taxes. This result suggests that the optimal policy mix is an ex ante macroprudential tax that corrects overborrowings and an ex post monetary policy intervention that mitigates real depreciation.

The tractable model structure provides a useful framework to study other policy questions. First, as in [Arce et al. \(2019\)](#), it would be interesting to examine how foreign reserve accumulation and reserve interventions can stabilize the economy through balance of pay-

ments adjustments. Second, introducing an endogenous choice of borrowing currency would be a useful extension. Third, introducing financial intermediaries and a currency mismatch as in [Aoki et al. \(2018\)](#) and [Mendoza and Rojas \(2019\)](#) would be an important research agenda.

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Appendix

A Flexible Price Model

This section gives the full description of the social planner's problem and the first-order conditions in the flexible price model in Section 3. The social planner's problem is set up as follows:

$$\begin{aligned}
 V(b_{t-1}^*, s_t) &= \max_{c_t, b_t^*, \ell_t, m_t, e_t} \log \left(c_t - \chi \frac{\ell_t^{1+\omega}}{1+\omega} \right) + \beta E_t V(b_t^*, s_{t+1}) \\
 &\quad - \lambda_t^{SP} [c_t + e_t^\rho Y^* - Y_t] \\
 &\quad - \mu_t^{SP} \left[-e_t \frac{b_t^*}{R_t^*} - \kappa_t k \right] \\
 &\quad + \gamma_t^{SP} \left[e_t^\rho Y^* - e_t m_t - e_t \left(\frac{b_t^*}{R_t^*} - b_{t-1}^* \right) \right].
 \end{aligned}$$

The first-order conditions are as follows:

$$c_t : u_c(t) - \lambda_t^{SP} = 0, \quad (\text{A1})$$

$$b_t^* : \mu_t^{SP} e_t \frac{1}{R_t^*} - \gamma_t^{SP} e_t \frac{1}{R_t^*} + \beta E_t V_b(b_t^*, s_{t+1}) = 0, \quad (\text{A2})$$

$$\ell_t : u_c(t)(-\chi \ell_t^\omega) + \lambda_t^{SP} \alpha_\ell \frac{Y_t}{\ell_t} = 0 \quad (\text{A3})$$

$$m_t : -\lambda_t^{SP} \left(-\alpha_m \frac{Y_t}{m_t} \right) - \gamma_t^{SP} e_t = 0, \quad (\text{A4})$$

$$e_t : -\lambda_t^{SP} \left(\frac{b_t^*}{R_t^*} - b_{t-1}^* + m_t \right) + \mu_t^{SP} \frac{b_t^*}{R_t^*} + \gamma_t^{SP} \left(\rho e_t^{\rho-1} Y^* - m_t - \frac{b_t^*}{R_t^*} + b_{t-1}^* \right) = 0. \quad (\text{A5})$$

Rearranging (A3),

$$u_c(t)(\chi \ell_t^\omega) = \lambda_t^{SP} \alpha_\ell \frac{Y_t}{\ell_t}.$$

Plugging (A1) into this equation,

$$\chi \ell_t^\omega = \alpha_\ell \frac{Y_t}{\ell_t},$$

which coincides with (6) with $p_t^w = 1$, the decentralized equilibrium condition with respect

to ℓ_t .

Rearranging (A5), the expression for γ_t^{SP} is:

$$\gamma_t^{SP} = \frac{\rho}{\rho - 1} \lambda_t^{SP} - \frac{b_t^*/R_t^*}{(\rho - 1)e_t^{\rho-1}Y^*} \mu_t^{SP}. \quad (\text{A6})$$

Plugging this equation into (A4),

$$\alpha_m \frac{Y_t}{m_t} = e_t \frac{\gamma_t^{SP}}{\lambda_t^{SP}} = e_t \left[\frac{\rho}{\rho - 1} - \frac{b_t^*/R_t^*}{(\rho - 1)e_t^{\rho-1}Y^*} \mu_t^{SP} \right],$$

which is (24) in the main text. This equation also shows that the terms-of-trade externality can be corrected by the fixed rate tax $\tau_m = 1/(\rho - 1)$.

Plugging λ_t^{SP} from (A1) and γ_t^{SP} from (A5) into the first-order condition with respect to b_t^* (A2),

$$\frac{\rho}{\rho - 1} u_c(t) - \mu_t^{SP} \frac{b_t^*/R_t^*}{(\rho - 1)e_t^{\rho-1}Y^*} - \mu_t^{SP} = \beta R_t^* E_t \left[\frac{e_{t+1}}{e_t} \left\{ \frac{\rho}{\rho - 1} u_c(t+1) - \mu_{t+1}^{SP} \frac{b_{t+1}^*/R_{t+1}^*}{(\rho - 1)e_{t+1}^{\rho-1}Y^*} \right\} \right].$$

When the borrowing constraint is not binding today but may bind in the next period, this equation reduces to the following equation:

$$u_c(t) = \beta R_t^* E_t \left[\frac{e_{t+1}}{e_t} \left\{ u_c(t+1) - \mu_{t+1}^{SP} \frac{b_{t+1}^*/R_{t+1}^*}{\rho e_{t+1}^{\rho-1}Y^*} \right\} \right],$$

which is (21) in the main text.

B Optimal Discretionary Monetary Policy

This section considers the case where monetary policy is the only policy tool and characterizes the optimal discretionary monetary policy.

B.1 Planner's Problem

The planner chooses inflation π_t and all the endogenous variables to maximize the household's expected utility subject to all the decentralized equilibrium conditions as the implementability constraints. With the Lagrange multipliers on the constraints, the Ramsey planner's

problem is defined as follows:

$$V(b_{t-1}^*, s_t) = \max_{\pi_t, c_t, b_t^*, \ell_t, m_t, e_t, p_t^w, \mu_t} \log \left(c_t - \chi \frac{\ell_t^{1+\omega}}{1+\omega} \right) + \beta E_t V(b_t^*, s_{t+1}) \quad (\text{A7})$$

$$-\lambda_t^{RP} \left[c_t + e_t \left(\frac{b_t^*}{R_t^*} - b_{t-1}^* \right) - \left\{ Y_t - \frac{\psi}{2} \pi_t^2 Y_t \right\} + e_t m_t \right] \quad (\text{A8})$$

$$-\mu_t^{RP} \left[-e_t \frac{b_t^*}{R_t^*} - \kappa_t q_t k \right] \quad (\text{A9})$$

$$+\gamma_t^{RP} \left[e_t^\rho Y^* - e_t m_t - e_t \left(\frac{b_t^*}{R_t^*} - b_{t-1}^* \right) \right] \quad (\text{A10})$$

$$-\eta_t^{PC} \left[\{-\theta + \theta p_t^w - \psi \pi_t (1 + \pi_t)\} Y_t + \beta E_t \left\{ \frac{u_c(t+1)}{u_c(t)} \psi \pi_{t+1} (1 + \pi_{t+1}) Y_{t+1} \right\} \right] \quad (\text{A11})$$

$$-\eta_t^{EE} \left[u_c(t) - \mu_t - \beta R_t^* E_t \left\{ \frac{e_{t+1}}{e_t} u_c(t+1) \right\} \right] \quad (\text{A12})$$

$$+\eta_t^\ell \left[p_t^w - \chi \ell_t^\omega \frac{\ell_t}{\alpha_\ell} \frac{1}{Y_t} \right] \quad (\text{A13})$$

$$+\eta_t^m \left[p_t^w - e_t \frac{\rho}{\rho-1} \frac{m_t}{\alpha_m} \frac{1}{Y_t} \right] \quad (\text{A14})$$

$$-\eta_t^\mu \left[\mu_t \left\{ -e_t \frac{b_t^*}{R_t^*} - \kappa_t k \right\} \right], \quad (\text{A15})$$

where Y_t should be taken as the Cobb-Douglas production defined in (2). The variables with a subscript $t+1$ should be taken as functions of b_t^* and s_{t+1} . The first-order conditions are given as follows:

$$\pi_t : \eta_t^{PC} \psi (2\pi_t + 1) Y_t = \lambda_t^{RP} \psi \pi_t Y_t, \quad (\text{A16})$$

$$c_t : u_c(t) - \lambda_t^{RP} + \eta_t^{PC} \left[\frac{u_{cc}(t)}{u_c(t)} \beta \psi E_t \{ u_c(t+1) \pi_{t+1} (1 + \pi_{t+1}) Y_{t+1} \} \right] - \eta_t^{EE} u_{cc}(t) = 0, \quad (\text{A17})$$

$$e_t : \gamma_t^{RP} = \frac{1}{\rho-1} \lambda_t^{RP} + \frac{1}{\rho-1} \frac{1}{e_t^{\rho-1} Y^*} \left[\eta_t^m \frac{\rho}{\rho-1} \frac{m_t}{\alpha_m} \frac{1}{Y_t} + \eta_t^{EE} \frac{1}{e_t} (u_c(t) - \mu_t) - (\mu_t^{RP} + \eta_t^\mu \mu_t) \frac{b_t^*}{R_t^*} \right], \quad (\text{A18})$$

$$\begin{aligned} \ell_t : \lambda_t^{RP} \left[\left\{ 1 - \frac{\psi}{2} \pi_t^2 \right\} \alpha_\ell \frac{Y_t}{\ell_t} - \chi \ell_t^\omega \right] - \eta_t^{PC} [-\theta + \theta p_t^w - \psi \pi_t (1 + \pi_t)] \alpha_\ell \frac{Y_t}{\ell_t} \\ - \eta_t^\ell \chi (\omega + 1) \ell_t^\omega \frac{1}{\alpha_\ell Y_t} + p_t^w \frac{\alpha_\ell}{\ell_t} (\eta_t^\ell + \eta_t^m) = 0, \end{aligned} \quad (\text{A19})$$

$$\begin{aligned} m_t : \lambda_t^{RP} \left[\left\{ 1 - \frac{\psi}{2} \pi_t^2 \right\} \alpha_m \frac{Y_t}{m_t} - e_t \right] - \gamma_t^{RP} e_t - \eta_t^{PC} [-\theta + \theta p_t^w - \psi \pi_t (1 + \pi_t)] \alpha_m \frac{Y_t}{m_t} \\ - \eta_t^m e_t \frac{\rho}{\rho-1} \frac{1}{\alpha_m Y_t} + p_t^w \frac{\alpha_m}{m_t} (\eta_t^\ell + \eta_t^m) = 0, \end{aligned} \quad (\text{A20})$$

$$b_t^* : e_t \frac{1}{R_t^*} \left[\lambda_t^{RP} + \gamma_t^{RP} - (\mu_t^{RP} + \eta_t^\mu \mu_t) \right] + \eta_t^{PC} \frac{\partial RHS_{t+1}^{PC}}{\partial b_t^*} - \eta_t^{EE} \frac{\partial RHS_{t+1}^{EE}}{\partial b_t^*} = \beta E_t V_b(b_t^*, s_{t+1}), \quad (\text{A21})$$

$$p_t^w : \eta_t^{PC} \theta Y_t = \eta_t^\ell + \eta_t^m, \quad (\text{A22})$$

$$\mu_t : \eta_t^{EE} - \eta_t^\mu \left(-e_t \frac{b_t^*}{R_t^*} - \kappa_t k \right) = 0, \quad (\text{A23})$$

where the two partial derivatives in (A21) are the terms that collect all the partial derivatives of the next planner's decision rules in (A11) and (A12) with respect to b_t^* . The envelope condition is given by:

$$V_b(b_{t-1}^*, s_t) = \lambda_t^{RP} e_t + \gamma_t^{RP} e_t.$$

Combining the envelope condition at $t+1$ and (A21) gives the following Euler equation with respect to foreign bond:

$$\lambda_t^{RP} + \gamma_t^{RP} - (\mu_t^{RP} + \eta_t^\mu \mu_t) + \frac{R_t^*}{e_t} \left[\eta_t^{PC} \frac{\partial RH S_{t+1}^{PC}}{\partial b_t^*} - \eta_t^{EE} \frac{\partial RH S_{t+1}^{EE}}{\partial b_t^*} \right] = \beta R_t^* E_t \left[\frac{e_{t+1}}{e_t} (\lambda_{t+1}^{RP} + \gamma_{t+1}^{RP}) \right]. \quad (\text{A24})$$

B.2 Without Borrowing Constraint

This subsection proves that without borrowing constraint, strict inflation targeting is the optimal discretionary monetary policy. Removing the borrowing constraint from the model implies $\mu_t^{RP} = \mu_t = \eta_t^\mu = 0$. The proof proceeds in the following steps: I assume that the next period expected inflation is zero, i.e. $E_t(\pi_{t+1}) = 0$. Then I set $\pi_t = 0$ and show that all the first-order conditions in the above planner's problem are satisfied.

First, $\pi_t = E_t(\pi_{t+1}) = 0$ implies $p_t^w = 1$ by the New Keynesian Phillips curve in (A11). $\pi_t = 0$ also implies $\eta_t^{PC} = 0$ by (A16), the first-order condition with respect to π_t . I further make a guess that $\eta_t^{EE} = 0$, which implies that the Euler equation with respect to foreign bond in the decentralized equilibrium holds in the planner's problem as well. Later I will verify that this guess is correct. Now (A17) implies:

$$u_c(t) = \lambda_t^{RP}.$$

Plugging this into (A19),

$$u_c(t) \left[\alpha_\ell \frac{Y_t}{\ell_t} - \chi \ell_t^\omega \right] - \eta_t^\ell \chi (\omega + 1) \ell_t^\omega \frac{1}{\alpha_\ell Y_t} + \frac{\alpha_\ell}{\ell_t} (\eta_t^\ell + \eta_t^m) = 0.$$

The first term is canceled out by (A13), thus:

$$\eta_t^\ell \chi(\omega + 1) \ell_t^\omega \frac{1}{\alpha_\ell Y_t} = \frac{\alpha_\ell}{\ell_t} (\eta_t^\ell + \eta_t^m).$$

(A22) with $\eta_t^{PC} = 0$ implies $\eta_t^\ell + \eta_t^m = 0$. Plugging this result into the above equation implies $\eta_t^\ell = 0$, and therefore $\eta_t^m = 0$.

Now, the first-order condition with respect to m_t (A20) is

$$u_c(t) \left[\alpha_m \frac{Y_t}{m_t} - e_t \right] = \gamma_t^{RP} e_t.$$

(A14) in the decentralized equilibrium condition with $p_t^w = 1$ implies:

$$\alpha_m \frac{Y_t}{m_t} = e_t \frac{\rho}{\rho - 1}.$$

Combining these two equations implies the following:

$$\gamma_t^{RP} = \frac{1}{\rho - 1} u_c(t).$$

Plugging this equation into the first-order condition with respect to e_t (A18) proves that (A18) is satisfied. Plugging the expression for γ_t^{RP} into (A24), the Euler equation with respect to foreign bond in the planner's problem is:

$$\frac{\rho}{\rho - 1} u_c(t) = \beta R_t^* E_t \left[\frac{e_{t+1}}{e_t} \frac{\rho}{\rho - 1} u_c(t + 1) \right],$$

where $\rho/(\rho - 1)$ is canceled out. Therefore, the Euler equation coincides in the decentralized equilibrium and in the planner's allocation, which verifies that the guess $\eta_t^{EE} = 0$ is correct. This completes the proof.

B.3 Interpretations of Lagrange Multipliers

This subsection provides interpretations and intuitions for Lagrange multipliers η_t^ℓ , η_t^m , and η_t^{PC} . By structure of the planner's problem, η_t^ℓ is the social value of a change in ℓ_t responding

to a marginal increase in p_t^w , and η_t^m is that of m_t .

Plugging the first-order condition with respect to π_t (A16) into the one with respect to ℓ_t (A19) and rearranging,

$$\begin{aligned}\eta_t^\ell &= \frac{\alpha_\ell Y_t}{\chi(\omega+1)\ell_t^\omega} \left[\lambda_t^{RP} \left\{ \left(1 - \frac{\psi}{2}\pi_t^2\right) \alpha_\ell \frac{Y_t}{\ell_t} - \chi \ell_t^\omega \right\} - \eta_t^{PC} \left\{ LHS_t^{PC} \alpha_\ell \frac{Y_t}{\ell_t} - \theta p_t^w \alpha_\ell \frac{Y_t}{\ell_t} \right\} \right] \\ &= \varepsilon_t^{p^\ell} \left[\lambda_t^{RP} \left\{ \left(1 - \frac{\psi}{2}\pi_t^2\right) \alpha_\ell \frac{Y_t}{\ell_t} - \chi \ell_t^\omega \right\} \right] - \eta_t^{PC} \frac{1}{\omega+1} \frac{\alpha_\ell Y_t}{p_t^w} [LHS_t^{PC} - \theta p_t^w], \quad (A25)\end{aligned}$$

where $LHS_t^{PC} = \{-\theta + \theta p_t^w - \psi \pi_t(1 + \pi_t)\}$. In the first term, $\varepsilon_t^{p^\ell} = \alpha_\ell Y_t / \chi(\omega+1)\ell_t^\omega$ indicates a change in ℓ_t responding to a marginal increase in p_t^w , because it is obtained by applying the implicit function theorem to $\chi \ell_t^\omega / (\alpha_\ell Y_t) - p_t^w = 0$ to derive $\partial \ell_t / \partial p_t^w$, which is strictly positive. The first term is then the social value of an increase in ℓ_t responding to a marginal increase in p_t^w by affecting production and labor disutility. In the second term, the coefficient comes from the following equation, using (A13):

$$\frac{\alpha_\ell Y_t}{\chi(\omega+1)\ell_t^\omega} \times \alpha_\ell \frac{Y_t}{\ell_t} = \frac{1}{\omega+1} \frac{\ell_t}{p_t^w} \times \alpha_\ell \frac{Y_t}{\ell_t} = \frac{1}{\omega+1} \frac{\alpha_\ell Y_t}{p_t^w}.$$

The second term is the social value of an increase in ℓ_t responding to a marginal increase in p_t^w by affecting p_t^w through the New Keynesian Phillips curve. Since η_t^{PC} contains η_t^ℓ in its expression as shown in (A22), there is a feedback loop in which a marginal change in p_t^w increases ℓ_t , which in turn affects p_t^w through the New Keynesian Phillips curve, and the loop continues.

Similar algebra with respect to m_t by plugging (A16) into (A20) leads to:

$$\begin{aligned}\eta_t^m &= \frac{\alpha_m Y_t}{e_t \rho / (\rho - 1)} \left[\lambda_t^{RP} \left\{ \left(1 - \frac{\psi}{2}\pi_t^2\right) \alpha_m \frac{Y_t}{m_t} - e_t \right\} - \gamma_t^{RP} e_t - \eta_t^{PC} \left\{ LHS_t^{PC} \alpha_m \frac{Y_t}{m_t} - \theta p_t^w \alpha_m \frac{Y_t}{m_t} \right\} \right] \\ &= \varepsilon_t^{p^m} \left[\lambda_t^{RP} \left\{ \left(1 - \frac{\psi}{2}\pi_t^2\right) \alpha_m \frac{Y_t}{m_t} - e_t \right\} - \gamma_t^{RP} e_t \right] - \eta_t^{PC} \frac{\alpha_m Y_t}{p_t^w} [LHS_t^{PC} - \theta p_t^w], \quad (A26)\end{aligned}$$

where $\varepsilon_t^{p^m} = \alpha_m Y_t / [e_t \rho / (\rho - 1)]$ is a change in m_t responding to a marginal increase in p_t^w , which is strictly positive given $\rho > 1$. The first term is the social value of an increase in m_t responding to a marginal increase in p_t^w by affecting production, the payment to imported inputs, and the effect on real exchange rate captured by γ_t^{RP} . The coefficient in the second

term comes from the following, using (A14):

$$\frac{\alpha_m Y_t}{e_t \rho / (\rho - 1)} \times \alpha_m \frac{Y_t}{m_t} = \frac{m}{p_t^w} \alpha_m \frac{Y_t}{m_t} = \frac{\alpha_m Y_t}{p_t^w}.$$

The second term is the social value of an increase in m_t responding to a marginal increase in p_t^w by affecting p_t^w through the New Keynesian Phillips curve. This term contains the same feedback loop as in the case of ℓ_t .

Plugging η_t^ℓ and η_t^m into the first-order condition with respect to p_t^w (A22) gives the following equation:

$$\begin{aligned} \theta Y_t \eta_t^{PC} &= \eta_t^\ell + \eta_t^m \\ &= \left(\varepsilon_t^{p\ell} \lambda_t^\ell + \varepsilon_t^{pm} \lambda_t^m \right) - \eta_t^{PC} [LHS_t^{PC} - \theta p_t^w] \left[\frac{1}{\omega + 1} \frac{\alpha_\ell Y_t}{p_t^w} + \frac{\alpha_m Y_t}{p_t^w} \right]. \end{aligned}$$

where λ_t^ℓ is the bracket term that starts with λ_t^{RP} in (A25), and λ_t^m is the bracket term that starts with λ_t^{RP} in (A26). Solving for η_t^{PC} ,

$$\begin{aligned} \eta_t^{PC} &= \frac{\varepsilon_t^{p\ell} \lambda_t^\ell + \varepsilon_t^{pm} \lambda_t^m}{\theta Y_t \left[1 + \frac{1}{\theta Y_t} \left\{ (LHS_t^{PC} - \theta p_t^w) \left(\frac{1}{\omega + 1} \frac{\alpha_\ell Y_t}{p_t^w} + \frac{\alpha_m Y_t}{p_t^w} \right) \right\} \right]} \\ &= \frac{\varepsilon_t^{p\ell} \lambda_t^\ell + \varepsilon_t^{pm} \lambda_t^m}{\theta Y_t \left[1 + \frac{1}{\theta} \left\{ (-\theta - \psi \pi_t (1 + \pi_t)) \left(\frac{1}{\omega + 1} \frac{\alpha_\ell}{p_t^w} + \frac{\alpha_m}{p_t^w} \right) \right\} \right]}. \end{aligned} \quad (\text{A27})$$

The sign of the denominator is ambiguous. However, in the special case of $E_t(\pi_{t+1}) = 0$, the New Keynesian Phillips curve implies $-\theta - \psi \pi_t (1 + \pi_t) = -\theta p_t^w$, and this equation reduces to the following:

$$\eta_t^{PC} = \frac{\varepsilon_t^{p\ell} \lambda_t^\ell + \varepsilon_t^{pm} \lambda_t^m}{\theta Y_t \left[1 - \left(\frac{1}{\omega + 1} \alpha_\ell + \alpha_m \right) \right]}. \quad (\text{A28})$$

The denominator is now strictly positive, and the bracket term is between 0 and 1. The interpretation is as follows. By structure of the implementability constraint (A11), η_t^{PC} can be interpreted as the effect of an increase in p_t^w by $1/(\theta Y_t)$ units on the social value. A direct effect of an increase in p_t^w is the effect on ℓ_t and m_t , captured by the numerator of (A28), which are the first terms in (A25) and (A26). There is also an indirect effect. These changes in ℓ_t and m_t affect p_t^w through the New Keynesian Phillips curve, captured by the last terms

in (A25) and (A26). This second-round change in p_t^w causes the second-round effects on ℓ_t and m_t , and also the third-round change in p_t^w . The bracket term in the denominator of (A28) captures the amplification effect through this feedback loop. Therefore, η_t^{PC} captures the total effect of an increase in p_t^w on the social welfare including the effect of the amplifying feedback loops. Although the clear expression in (A28) is obtained only under the special case of $E_t(\pi_{t+1}) = 0$, the interpretation of η_t^{PC} basically holds in the full model, where η_t^{PC} is given by (A27).

Given the interpretation of η_t^{PC} , the first-order condition with respect to π_t (A16) can be understood intuitively. Rewriting the equation just for convenience,

$$\eta_t^{PC} \psi(2\pi_t + 1) Y_t = \lambda_t^{RP} \psi \pi_t Y_t.$$

The right-hand side is understood as follows: When inflation π_t is marginally increased, it causes a loss (a gain if $\pi_t < 0$) of resources by $\psi \pi_t Y_t$ units through the price adjustment cost. The social value of this loss (or gain) is given by the right-hand side. The left-hand side is understood as follows: By increasing π_t marginally, the wholesale goods price p_t^w increases by $(1/\theta) \psi(2\pi_t + 1)$ through the New Keynesian Phillips curve. A marginal increase in p_t^w gives the social value of $\eta_t^{PC} \theta Y_t$. Therefore, a marginal increase in π_t gives the social value of $(1/\theta) \psi(2\pi_t + 1) \times \eta_t^{PC} \theta Y_t = \eta_t^{PC} \psi(2\pi_t + 1) Y_t$ by increasing p_t^w through the New Keynesian Phillips curve, which is the left-hand side. The first-order condition with respect to π_t (A16) states that the government chooses π_t so that the marginal social benefit (cost if $\pi_t < 0$) from increasing π_t and p_t^w in the left-hand side becomes equal to the marginal cost (benefit if $\pi_t < 0$) from a change in the price adjustment cost in the right-hand side.

B.4 Proof of Proposition 1

Proposition 1 *In the model described in Section 2, when the monetary policy is the only policy tool, strict inflation targeting is not the optimal discretionary monetary policy.*

I prove this proposition using proof by contradiction: I set the next period expected inflation to zero, i.e. $E_t(\pi_{t+1}) = 0$. Then, I assume $\pi_t = 0$ and show that the first-order conditions in the planner's problem are not satisfied.

First, $\pi_t = E_t(\pi_{t+1}) = 0$ implies $p_t^w = 1$ and $\pi_t = 0$ implies $\eta_t^{PC} = 0$. The previous subsection shows that η_t^{PC} given $E_t(\pi_{t+1}) = 0$ satisfies (A28). Since the denominator is strictly positive, the proof reduces to showing that the numerator in (A28) is not zero, which would imply $\eta_t^{PC} \neq 0$ and contradict with $\pi_t = 0$ by (A16).

Dividing the numerator in (A28) by λ_t^{RP} , which is strictly positive because it is the social value of an additional unit of home tradable goods, I obtain the following equation:

$$\varepsilon_t^{pl} \frac{\lambda_t^\ell}{\lambda_t^{RP}} + \varepsilon_t^{pm} \frac{\lambda_t^m}{\lambda_t^{RP}} = \varepsilon_t^{pl} \left[\alpha_\ell \frac{Y_t}{\ell_t} - \chi \ell_t^\omega \right] + \varepsilon_t^{pm} \left[\alpha_m \frac{Y_t}{m_t} - e_t - \frac{\gamma_t^{RP}}{\lambda_t^{RP}} e_t \right]. \quad (\text{A29})$$

Given $p_t^w = 1$ and $\pi_t = 0$, the first bracket coincides with the implementability constraint with respect to ℓ_t in (A13), and thus disappears. Given the implementability constraint with respect to m_t in (A14) with $p_t^w = 1$, the second term is zero if and only if:

$$\frac{\gamma_t^{RP}}{\lambda_t^{RP}} = \frac{1}{\rho - 1}.$$

Note that this equation holds in the model without borrowing constraint as shown in Section B.2. Now I show that this equation does not hold in the full model with the borrowing constraint.

The expression for γ_t^{RP} is given in (A18). Before examining this equation, it is useful to focus on the Lagrange multipliers included in (A18), η_t^{EE} , μ_t^{RP} , μ_t , and η_t^μ , in the case of the constraint not binding and binding respectively.

(1) When the borrowing constraint is not binding, $\mu_t^{RP} = \mu_t = 0$. As shown below, $\eta_t^{EE} \neq 0$ because private agents do not internalize the externalities and overborrow in normal times. η_t^μ has the opposite sign to η_t^{EE} because the inside of the parenthesis in (A23) is negative. $\eta_t^\mu \neq 0$ implies that the complementary slackness condition is binding with $\mu_t = 0$.

(2) When the borrowing constraint is binding, $\mu_t^{RP} > 0$, and $\eta_t^{EE} = 0$ because of (A23) and the binding constraint. μ_t is determined by the private Euler equation. The complementary slackness condition is satisfied with any μ_t because the constraint is binding. This implies that the complementary slackness condition is not binding, and $\eta_t^\mu = 0$.

Given these results, γ_t^{RP} in (A18) can be written separately in the case of not binding

and binding as follows. In the case of not binding,

$$\gamma_t^{RP} = \frac{1}{\rho - 1} \lambda_t^{RP} + \frac{1}{\rho - 1} \frac{1}{e_t^{\rho-1} Y^*} \left[\eta_t^m \frac{\rho}{\rho - 1} \frac{m_t}{\alpha_m Y_t} + \eta_t^{EE} \frac{u_c(t)}{e_t} \right]. \quad (\text{A30})$$

In the case of binding,

$$\gamma_t^{RP} = \frac{1}{\rho - 1} \lambda_t^{RP} + \frac{1}{\rho - 1} \frac{1}{e_t^{\rho-1} Y^*} \left[\eta_t^m \frac{\rho}{\rho - 1} \frac{m_t}{\alpha_m Y_t} - \mu_t^{RP} \frac{b_t^*}{R_t^*} \right]. \quad (\text{A31})$$

The expression for η_t^m is given in (A26). When $\pi_t = \eta_t^{PC} = 0$, this expression reduces to:

$$\frac{\eta_t^m}{\lambda_t^{RP}} = \varepsilon_t^{pm} \left[\alpha_m \frac{Y_t}{m_t} - e_t - \frac{\gamma_t^{RP}}{\lambda_t^{RP}} e_t \right]. \quad (\text{A32})$$

Because η_t^m includes γ_t^{RP} , I plug η_t^m in (A32) into (A30) and (A31) and derive the explicit expression for γ_t^{RP} . After some algebra, I obtain the following expressions for $\gamma_t^{RP}/\lambda_t^{RP}$. In the case of not binding,

$$\frac{\gamma_t^{RP}}{\lambda_t^{RP}} - \frac{1}{\rho - 1} = \frac{1}{(\rho - 1) e_t^{\rho-1} Y^* + \varepsilon_t^{pm}} \left(\frac{\eta_t^{EE} u_c(t)}{\lambda_t^{RP} e_t} \right). \quad (\text{A33})$$

In the case of binding,

$$\frac{\gamma_t^{RP}}{\lambda_t^{RP}} - \frac{1}{\rho - 1} = \frac{1}{(\rho - 1) e_t^{\rho-1} Y^* + \varepsilon_t^{pm}} \left(-\frac{\mu_t^{RP} b_t^*}{\lambda_t^{RP} R_t^*} \right). \quad (\text{A34})$$

The denominator of the fraction is strictly positive given $\rho > 1$. Therefore, whether $\gamma_t^{RP}/\lambda_t^{RP} = 1/(\rho - 1)$ holds or not depends on whether the bracket term is zero or not. In the case of binding, the bracket term is clearly strictly positive, because $\mu_t^{RP} > 0$ and $-b_t^*/R_t^* > 0$. This implies $\gamma_t^{RP}/\lambda_t^{RP} \neq 1/(\rho - 1)$ and thus $\eta_t^{PC} \neq 0$, which contradicts the initial conjecture that $\pi_t = 0$. This completes the proof of Proposition 1.

B.5 Proof of Corollary 1

The equations (A33) and (A34) suggest in which direction inflation should deviate from 0, expansionary ($\pi_t > 0$) or contractionary ($\pi_t < 0$).

When the borrowing constraint is not binding today but may bind next period, whether $\gamma_t^{RP}/\lambda_t^{RP}$ is greater, smaller, or equal to $1/(\rho - 1)$ depends on the sign of the Lagrange multiplier on the Euler equation η_t^{EE} in (A33). As shown in the social planner's problem in Section A, the ex ante BOP externality increases the value of the right-hand side in the Euler equation as shown in (21). This means that the implementability constraint (A12) can be interpreted as the binding inequality constraint $u_c(t) \leq \beta R_t^* E_t [(e_{t+1}/e_t)u_c(t+1)]$, and thus $\eta_t^{EE} > 0$. Positive η_t^{EE} implies $\gamma_t^{RP}/\lambda_t^{RP} > 1/(\rho - 1)$ in (A33), which lowers the value of η_t^{PC} and the optimal π_t . Therefore, the ex ante BOP externality pushes down the optimal inflation rate. The intuition is as follows: when monetary policy is the only policy tool, the planner lowers π_t and p_t^w , which discourages production and use of imported inputs. Smaller amount of imported inputs causes real appreciation, which is a decline in e_t . Lower e_t increases the effective real interest rate on foreign bond, and discourages foreign borrowing by households. This helps to correct overborrowing at least partially.

In the case of binding, the right-hand side of (A34) is strictly positive, which implies $\gamma_t^{RP}/\lambda_t^{RP} > 1/(\rho - 1)$. The last term in (A34) captures the social value of real appreciation when the constraint is binding. Comparing (A18) and (A6) in the flexible price model makes clear that this additional term is the ex post BOP externality that causes too much use of imported inputs during crises. Therefore, the optimal monetary policy deviates from $\pi_t = 0$ to deal with this externality. $\gamma_t^{RP}/\lambda_t^{RP} > 1/(\rho - 1)$ implies λ_t^m is negative in (A29), which pushes down the value of η_t^{PC} in (A27) and π_t in (A16). Therefore, the ex post BOP externality that causes too much imported inputs during crises pushes down the optimal inflation rate.

Based on this discussion, the following corollary can be formally proved:

Corollary 1 *Given $E_t(\pi_{t+1}) = 0$, the optimal discretionary monetary policy is contractionary both when the borrowing constraint is binding and when it is not binding.*

Given $E_t(\pi_{t+1}) = 0$, the denominator in (A28) is strictly positive. Thus the sign of η_t^{PC} and π_t is the same as the sign of the numerator:

$$\varepsilon_t^{p\ell} \left[\left(1 - \frac{\psi}{2} \pi_t^2 \right) \alpha_\ell \frac{Y_t}{\ell_t} - \chi \ell_t^\omega \right] + \varepsilon_t^{pm} \left[\left(1 - \frac{\psi}{2} \pi_t^2 \right) \alpha_m \frac{Y_t}{m_t} - e_t \left(1 + \frac{\gamma_t^{RP}}{\lambda_t^{RP}} \right) \right]. \quad (\text{A35})$$

Proposition 1 states that $\pi_t = 0$ does not satisfy the first-order conditions. To prove $\pi_t < 0$, it is sufficient to prove that $\pi_t > 0$ does not satisfy the first-order conditions. To the contrary, suppose $\pi_t > 0$. The New Keynesian Phillips curve with $\pi_t > 0$ and $E_t(\pi_{t+1}) = 0$ implies $p_t^w > 1$. From the implementability constraints (A13) and (A14), $p_t^w > 1$ implies the following:

$$\alpha_\ell \frac{Y_t}{\ell_t} < \chi \ell_t^\omega, \quad (\text{A36})$$

$$\alpha_m \frac{Y_t}{m_t} < e_t \frac{\rho}{\rho - 1}. \quad (\text{A37})$$

(A36) with $\pi_t > 0$ implies that the first bracket in (A35) is strictly negative. For the second bracket in (A35), the discussion above proves $\gamma_t^{RP}/\lambda_t^{RP} > 1/(\rho - 1)$ both when the constraint is not binding and when binding as shown in (A33) and (A34). Combined with (A37) and $\pi_t > 0$, the second bracket is also strictly negative. This means that (A35) is strictly negative, which implies that η_t^{PC} and π_t are also negative. This contradicts with the initial assumption of $\pi_t > 0$, thus $\pi_t > 0$ cannot satisfy the first-order conditions. The only possibility is $\pi_t < 0$ both when the constraint is not binding and when binding. This proves Corollary 1.

C Inflation Targeting and Optimal Tax

This section provides the planner's problem and the first-order conditions under the optimal macroprudential tax and inflation targeting monetary policy. The planner's problem is given as follows:

$$\begin{aligned}
V(b_{t-1}^*, s_t) &= \max_{c_t, b_t^*, \ell_t, m_t, e_t} \log \left(c_t - \chi \frac{\ell_t^{1+\omega}}{1+\omega} \right) + \beta E_t V(b_t^*, s_{t+1}) \\
&\quad - \lambda_t^{RP} \left[c_t + e_t \left(\frac{b_t^*}{R_t^*} - b_{t-1}^* \right) - Y_t + e_t m_t \right] \\
&\quad - \mu_t^{RP} \left[-e_t \frac{b_t^*}{R_t^*} - \kappa_t k \right] \\
&\quad + \gamma_t^{RP} \left[e_t^\rho Y^* - e_t m_t - e_t \left(\frac{b_t^*}{R_t^*} - b_{t-1}^* \right) \right] \\
&\quad - \eta_t^\ell \left[\alpha_\ell \frac{Y_t}{\ell_t} - \chi \ell_t^\omega \right] \\
&\quad - \eta_t^m \left[\alpha_m \frac{Y_t}{m_t} - \frac{\rho}{\rho-1} e_t \right].
\end{aligned}$$

This problem is similar to the case in the flexible-price version of the model in Section A. The difference is that the first-order conditions with respect to ℓ_t and m_t in the decentralized equilibrium are included in the implementability constraints. This is because the planner here cannot use taxes on imported inputs. The first-order conditions are given as follows:

$$c_t : u_c(t) - \lambda_t^{RP} = 0, \quad (\text{A38})$$

$$b_t^* : -\lambda_t^{RP} e_t \frac{1}{R_t^*} + \mu_t^{RP} e_t \frac{1}{R_t^*} - \gamma_t^{RP} e_t \frac{1}{R_t^*} + \beta E_t V_b(b_t^*, s_{t+1}) = 0, \quad (\text{A39})$$

$$\ell_t : u_c(t)(-\chi \ell_t^\omega) + \lambda_t^{RP} \alpha_\ell \frac{Y_t}{\ell_t} - \eta_t^\ell \left[\frac{\alpha_\ell Y_t}{\ell_t^2} (\alpha_\ell - 1) - \chi \omega \ell_t^{\omega-1} \right] - \eta_t^m \left[\alpha_\ell \alpha_m \frac{Y_t}{\ell_t m_t} \right] = 0 \quad (\text{A40})$$

$$m_t : \lambda_t^{RP} \left(\alpha_m \frac{Y_t}{m_t} - e_t \right) - \gamma_t e_t - \eta_t^\ell \left[\alpha_\ell \alpha_m \frac{Y_t}{\ell_t m_t} \right] - \eta_t^m \left[\frac{\alpha_m Y_t}{m_t^2} (\alpha_m - 1) \right] = 0, \quad (\text{A41})$$

$$e_t : -\lambda_t^{RP} \left(\frac{b_t^*}{R_t^*} - b_{t-1}^* + m_t \right) + \mu_t^{RP} \frac{b_t^*}{R_t^*} + \gamma_t^{RP} (\rho - 1) e_t^{\rho-1} Y^* + \eta_t^m \frac{\rho}{\rho-1} = 0, \quad (\text{A42})$$

Plugging (A38) and the implementability constraint with respect to ℓ_t into (A40),

$$-\eta_t^\ell \left[\frac{\alpha_\ell Y_t}{\ell_t^2} (\alpha_\ell - 1) - \chi \omega \ell_t^{\omega-1} \right] = \eta_t^m \left[\alpha_\ell \alpha_m \frac{Y_t}{\ell_t m_t} \right]. \quad (\text{A43})$$

The inside of the bracket in the left-hand side is strictly negative, which implies η_t^ℓ and η_t^m have the same sign, and $\eta_t^\ell = 0$ if and only if $\eta_t^m = 0$. The intuition is that the social value of distorting ℓ_t emerges as a side effect of distorting m_t . If the planner has an incentive to distort m_t , then the planner also adjusts ℓ_t accordingly, because ℓ_t and m_t are connected through the marginal product of each other.

Rearranging (A43) further using the implementability constraint with respect to ℓ_t ,

$$\eta_t^\ell = \frac{\alpha_m}{1 + \omega - \alpha_\ell} \frac{\ell_t}{m_t} \eta_t^m.$$

Plugging this equation into (A41),

$$\lambda_t^{RP} \left[\alpha_m \frac{Y_t}{m_t} \right] = \lambda_t^{RP} e_t + \gamma_t^{RP} e_t + \eta_t^m \frac{\alpha_m Y_t}{m_t^2} \left[\frac{\alpha_\ell \alpha_m}{1 + \omega - \alpha_\ell} + (\alpha_m - 1) \right].$$

It can be shown that the last bracket term is strictly negative. I denote this value as $\bar{\alpha} < 0$.

Solving the first-order condition with respect to e_t (A42) for γ_t^{RP} and plugging into this equation,

$$\begin{aligned} \lambda_t^{RP} \left[\alpha_m \frac{Y_t}{m_t} \right] &= \lambda_t^{RP} e_t + \eta_t^m \frac{\alpha_m Y_t}{m_t^2} \bar{\alpha} \\ &+ \left[\frac{1}{\rho - 1} \lambda_t^{RP} + \frac{-b_t^*/R_t^*}{(\rho - 1) e_t^{\rho-1} Y^*} \mu_t^{RP} - \frac{1}{(\rho - 1) e_t^{\rho-1} Y^*} \frac{\rho}{\rho - 1} \eta_t^m \right] e_t. \end{aligned}$$

Using the implementability constraint with respect to m_t , the left-hand side, the first term in the right-hand side, and the first term in brackets cancel out. Therefore,

$$\frac{-e_t b_t^*/R_t^*}{(\rho - 1) e_t^{\rho-1} Y^*} \mu_t^{RP} - \frac{e_t}{(\rho - 1) e_t^{\rho-1} Y^*} \frac{\rho}{\rho - 1} \eta_t^m + \eta_t^m \frac{\alpha_m Y_t}{m_t^2} \bar{\alpha} = 0.$$

The first two terms come from γ_t^{RP} , thus they are the social value of increasing m_t through real exchange rate. The first term is the effect of a change in e_t on the binding borrowing constraint. The second term is the effect of a change in e_t on the cost of buying m_t . The last term comes from the effect of increasing m_t on the production margins of ℓ_t and m_t .

Solving this equation for η_t^m ,

$$\eta_t^m = \frac{\frac{-e_t b_t^*/R_t^*}{(\rho-1)e_t^{\rho-1}Y^*}}{\frac{e_t}{(\rho-1)e_t^{\rho-1}Y^*} \frac{\rho}{\rho-1} - \frac{\alpha_m Y_t}{m_t^2} \frac{1}{\alpha}} \mu_t^{RP}.$$

The coefficient on μ_t^{RP} is strictly positive. Therefore, this equation implies that $\eta_t^m > 0$ if and only if $\mu_t^{RP} > 0$, i.e. the borrowing constraint is binding. Intuitively, the planner wants to distort m_t only when the constraint is binding. A positive η_t^m implies that the planner wants to increase the cost of m_t and discourages the use of imported inputs.

Finally, I derive the Euler equation when the constraint is not binding but may bind in the next period. The constraint presently not binding implies $\mu_t^{RP} = \eta_t^m = 0$. Plugging the expression for γ_t^{RP} from (A42) into (A39) and combined with the envelope condition,

$$u_c(t) = \beta R_t^* E_t \left[\frac{e_{t+1}}{e_t} \left(u_c(t+1) - \frac{b_{t+1}^*/R_{t+1}^*}{\rho e_{t+1}^{\rho-1} Y^*} \mu_{t+1}^{RP} - \frac{1}{\rho e_{t+1}^{\rho-1} Y^*} \frac{\rho}{\rho-1} \eta_{t+1}^m \right) \right],$$

which is (31) in the main text. As discussed in the main text, the last term captures the planner's incentive to correct the ex post BOP externality by slightly lowering the macroprudential tax rate ex ante and discouraging the use of imported inputs when the constraint binds in the next period.

When the borrowing constraint binds, the five implementability constraints in the planner's problem pin down the five endogenous variables, $c_t, \ell_t, m_t, b_t^*, e_t$. Thus the Euler equation is just to pin down the Lagrange multiplier μ_t^{RP} . This also implies that imposing a tax on foreign borrowing when the constraint is binding does not affect allocations. It would affect only the private Lagrange multiplier on the borrowing constraint μ_t . Without loss of generality, I assume that the planner sets the tax rate to zero when the constraint binds.

D Discretionary Monetary Policy and Optimal Tax

This section provides the planner's problem and the first-order conditions under the optimal macroprudential tax and optimal discretionary monetary policy.

D.1 Planner's Problem

The planner's problem is given as follows:

$$V(b_{t-1}^*, s_t) = \max_{\pi_t, c_t, b_t^*, \ell_t, m_t, e_t, p_t^w} \log \left(c_t - \chi \frac{\ell_t^{1+\omega}}{1+\omega} \right) + \beta E_t V(b_t^*, s_{t+1}) \quad (\text{A44})$$

$$- \lambda_t^{RP} \left[c_t + e_t \left(\frac{b_t^*}{R_t^*} - b_{t-1}^* \right) - \left\{ Y_t - \frac{\psi}{2} \pi_t^2 Y_t \right\} + e_t m_t \right] \quad (\text{A45})$$

$$- \mu_t^{RP} \left[-e_t \frac{b_t^*}{R_t^*} - \kappa_t k \right] \quad (\text{A46})$$

$$+ \gamma_t^{RP} \left[e_t^\rho Y^* - e_t m_t - e_t \left(\frac{b_t^*}{R_t^*} - b_{t-1}^* \right) \right] \quad (\text{A47})$$

$$- \eta_t^{PC} \left[\{-\theta + \theta p_t^w - \psi \pi_t (1 + \pi_t)\} Y_t + \beta E_t \left\{ \frac{u_c(t+1)}{u_c(t)} \psi \pi_{t+1} (1 + \pi_{t+1}) Y_{t+1} \right\} \right] \quad (\text{A48})$$

$$+ \eta_t^\ell \left[p_t^w - \chi \ell_t^\omega \frac{\ell_t}{\alpha_\ell} \frac{1}{Y_t} \right] \quad (\text{A49})$$

$$+ \eta_t^m \left[p_t^w - e_t \frac{\rho}{\rho - 1} \frac{m_t}{\alpha_m} \frac{1}{Y_t} \right]. \quad (\text{A50})$$

This problem is similar to the one in which monetary policy is the only policy tool in Section B. The difference is that the Euler equation with respect to foreign bond in the decentralized equilibrium is not included in the implementability constraints, because a macroprudential tax on foreign bond is available. I also omit the complementary slackness condition from the implementability constraint because private μ_t appears only in this equation, thus any μ_t is consistent with the first-order conditions here. The first-order conditions are the same as those in Section B with $\eta_t^{EE} = 0$.

D.2 Proof of Proposition 2 and Corollary 2

Proposition 2 *In the model described in Section 2, when a time-consistent optimal macroprudential tax on foreign borrowings is available, strict inflation targeting is not an optimal discretionary monetary policy.*

I follow the same strategy as the proof of Proposition 1 in Section B.4. Namely, I set $E_t(\pi_{t+1}) = 0$, and then assume $\pi_t = 0$ and show that the first-order conditions in the planner's problem are not satisfied. Most of the steps in Section B.4 carry over to this case. In particular, $\pi_t = 0$ if and only if $\eta_t^{PC} = 0$, which depends on whether $\gamma_t^{RP}/\lambda_t^{RP} = 1/(\rho - 1)$ or not. Given $\eta_t^{EE} = 0$, the expressions for $\gamma_t^{RP}/\lambda_t^{RP}$ when not binding and when binding

are given as follows. In the case of not binding,

$$\frac{\gamma_t^{RP}}{\lambda_t^{RP}} - \frac{1}{\rho - 1} = 0. \quad (\text{A51})$$

In the case of binding,

$$\frac{\gamma_t^{RP}}{\lambda_t^{RP}} - \frac{1}{\rho - 1} = \frac{1}{(\rho - 1) e_t^{\rho-1} Y^* + \varepsilon_t^{pm}} \left(-\frac{\mu_t^{RP}}{\lambda_t^{RP}} \frac{b_t^*}{R_t^*} \right). \quad (\text{A52})$$

The second equation proves that $\pi_t \neq 0$ when binding.

Following the same steps as in Section B.5, the following corollary is immediately proved:

Corollary 2 *Given $E_t(\pi_{t+1}) = 0$, the optimal discretionary monetary policy is (1) $\pi_t = 0$ when the borrowing constraint is not binding; and (2) $\pi_t < 0$ when the borrowing constraint is binding.*

Given $E_t(\pi_{t+1}) = 0$, (A51) implies that $\eta_t^{PC} = 0$, thus $\pi_t = 0$ is the optimal inflation rate when the constraint is not binding. When the constraint is binding, (A52) implies that $\pi_t < 0$ is optimal.

E Model with Asset Price Pecuniary Externality

This section introduces a collateral asset price into the borrowing limit, as in Bianchi and Mendoza (2018). It is assumed that productive assets can be traded among households, thus the household's budget constraint is modified as follows:

$$\begin{aligned} & P_t c_t + S_t \left(\frac{B_t^*}{R_t^*} - B_{t-1}^* \right) + \left(\frac{B_t}{R_t} - B_{t-1} \right) + Q_t (k_t - k_{t-1}) \\ &= P_t^w y_t^w - (1 + \tau_m) S_t m_t + T_t + \Pi_t, \end{aligned} \quad (\text{A53})$$

where Q_t is the asset price in nominal terms. Supply of assets remains unity as in the main model, thus $k_t = k_{t-1} = 1$ in the equilibrium. The borrowing limit is now modified as follows:

$$-S_t \frac{B_t^*}{R_t^*} \leq \kappa_t Q_t k_{t-1}. \quad (\text{A54})$$

As shown in [Bianchi and Mendoza \(2018\)](#), this borrowing constraint is derived from the imperfect enforceability problem in which lenders can seize and liquidate a κ_t fraction of assets when borrowers default on debt. The first-order condition with respect to k_t gives the following asset price equation in real terms q_t :

$$k_t : q_t u_c(t) = \beta E_t \left[u_c(t+1) \left(q_{t+1} + p_{t+1}^w \alpha_k \frac{y_{t+1}}{k_t} \right) + \mu_{t+1} \kappa_{t+1} q_{t+1} \right]. \quad (\text{A55})$$

Introducing the asset price into the borrowing limit introduces a pecuniary externality through an endogenous drop in the asset price when the borrowing constraint binds. The social planner's first-order condition with respect to foreign bond when the constraint is not binding but may bind in the next period is given as follows:

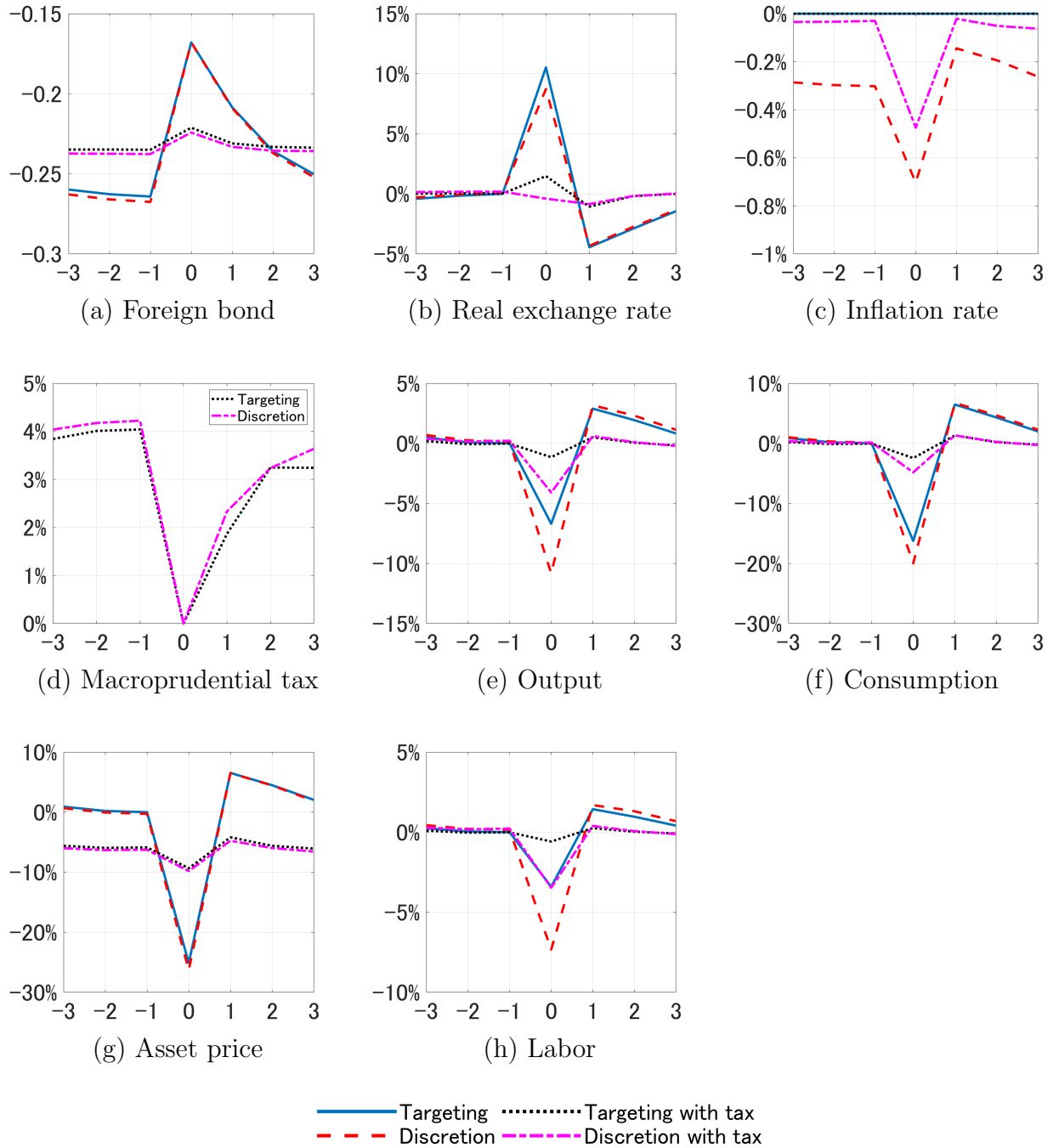
$$u_c(t) = \beta R_t^* E_t \left[\frac{e_{t+1}}{e_t} \left(u_c(t+1) \underbrace{-\xi_{t+1} q_{t+1} u_{cc}(t+1)}_{\text{pecuniary externality}} \underbrace{-\frac{b_{t+1}^*/R_{t+1}^* \mu_{t+1}^{RP}}{\rho e_{t+1}^{\rho-1} Y^*}}_{\text{ex ante BOP externality}} \right) \right]. \quad (\text{A56})$$

where ξ_{t+1} is the Lagrange multiplier on the asset price equation ([A55](#)). This equation shows that a pecuniary externality induces even larger overborrowing by households in addition to the ex ante BOP externality.

I consider the four policy regimes as in the main text: inflation targeting monetary policy, discretionary monetary policy, and each of these monetary policies with the optimal macroprudential taxes. The parameter values are set to be the same values as the baseline model in the main text. Under the same parameter values, the asset price when the borrowing constraint binds is about 0.5. Therefore, the amount of foreign debt becomes smaller relative to the baseline model even in normal times.

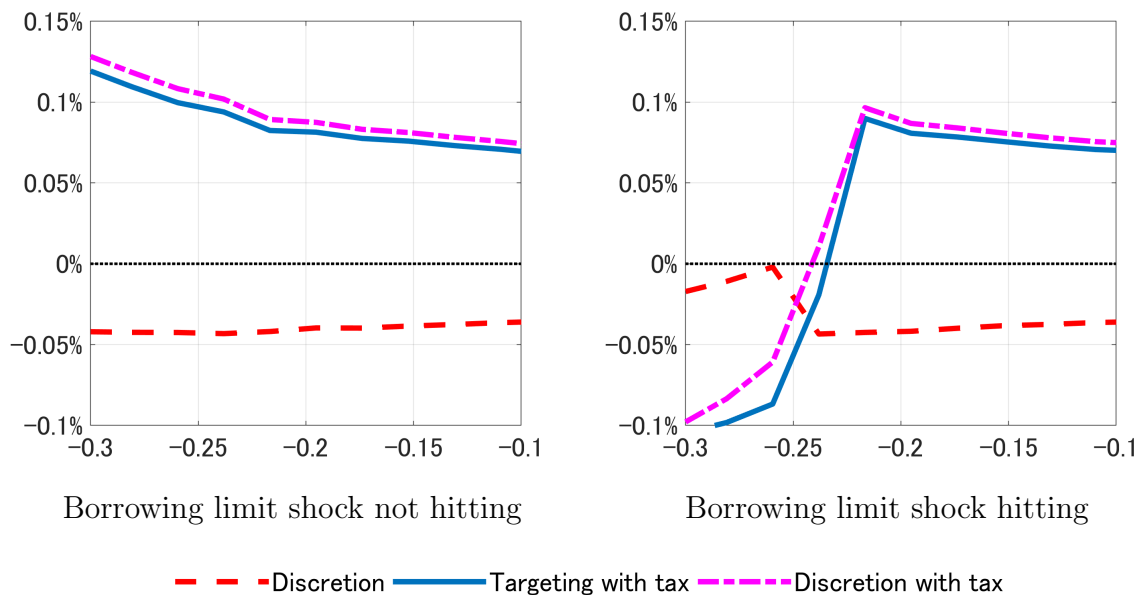
Figure [A1](#) plots the crisis dynamics under the four policy regimes. These dynamics are

Figure A1: Crisis dynamics in the model with collateral price



Note: These figures plot the crisis dynamics in the model with collateral asset price. The horizontal axis is the time, and a crisis occurs at period 0. Real exchange rate, output, consumption, asset price, and labor are expressed in percentage gaps from the level at period -1 under inflation targeting without a tax. The other variables are expressed in actual values.

Figure A2: Welfare gain/loss in the model with collateral price



Note: These panels plot the expected welfare gain/loss under different policy regimes, in terms permanent consumption relative to inflation targeting without taxes.

created in the same way as Figure 5 in the main text. Compared with the baseline model, the stabilization effect of macroprudential taxes is stronger. Foreign bond in (a), real exchange rate in (b), and consumption in (f) are substantially more stable under macroprudential taxes, regardless of monetary policy. The size of monetary policy intervention in (c) is smaller than that in the baseline model. This is because the amount of foreign borrowing substantially shrinks during crises, which makes the ex post BOP externality smaller as suggested in (24) in the main text. Output in (e) is still negatively affected by monetary policy interventions. The tax rate in (d) is about 1% higher than the baseline model due to the asset price pecuniary externality in addition to the ex ante BOP externality. A drop in asset price is substantially mitigated by the tax.

Figure A2 plots the welfare gain/loss by different policies relative to the expected welfare under inflation targeting without taxes. In the left panel when the borrowing limit is not hitting the economy, the order of the expected welfare is the same as that in the baseline model. The difference from the baseline model is that the welfare gain by macroprudential taxes is larger, and welfare gain by monetary policy intervention is smaller. In the right panel

when the borrowing limit shock is hitting the economy, macroprudential policies negatively affect welfare, unlike the baseline model. As shown in Panels (a) in Figure A1, foreign borrowing substantially shrinks under no taxes during crises due to a drop in the asset price. Then households increase foreign debt in the next period, which causes an overshoot of consumption in the period following the crisis, as shown in Panel (f). However, the macroprudential tax reduces this overshoot of consumption. Therefore, when a simulation starts with the borrowing constraint binding, the macroprudential tax reduces consumption in the next period, thereby reducing welfare. Although this effect exists in the baseline model as well, it is muted without an endogenous collateral asset price.

F Model with Fixed Labor Supply

Welfare analyses in the baseline model show that labor disutility plays the key role in determining the welfare effects of monetary policy. This raises a question of whether and how the results would be affected if labor supply is inelastic and there is no labor disutility. This section addresses this question. Removing labor disutility, household's utility function is given as follows:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \log(c_t) \right]. \quad (\text{A57})$$

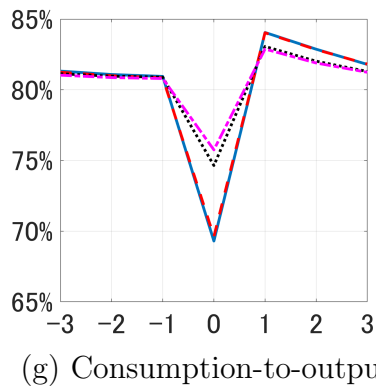
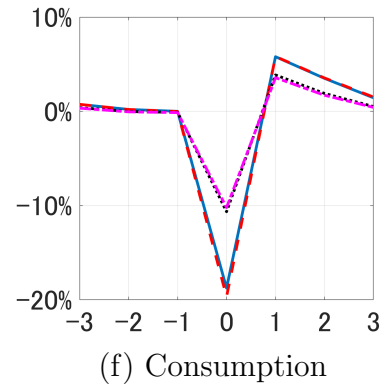
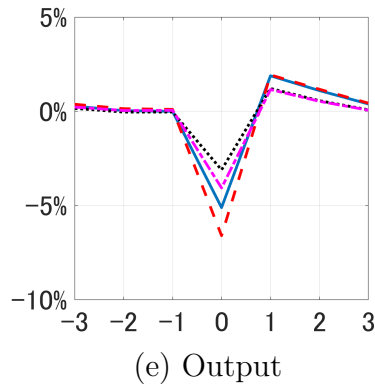
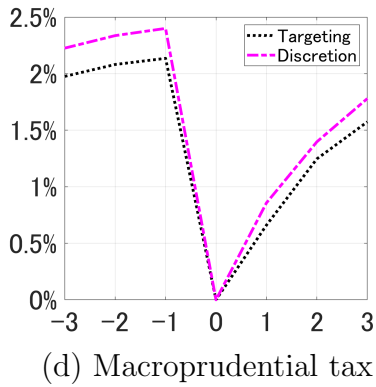
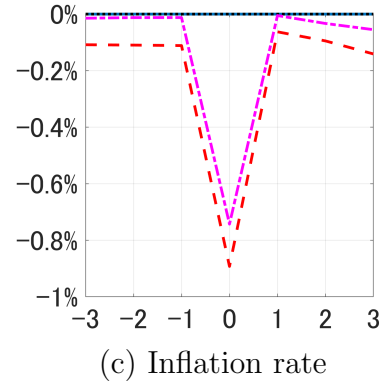
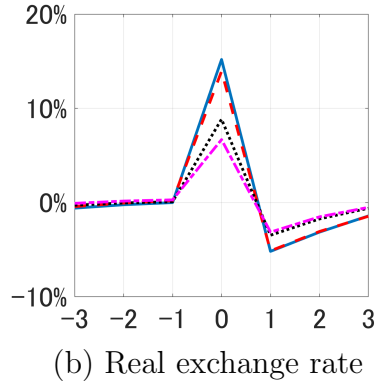
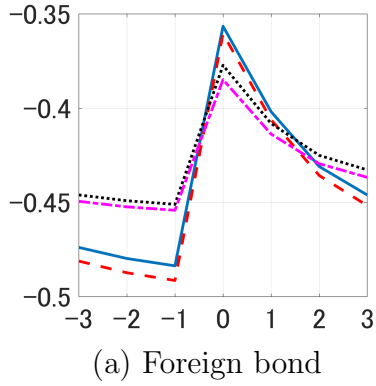
Households produce wholesale goods using assets, labor, and imported inputs as in the baseline model. But imported inputs are the only variable inputs:

$$y_t^w = A_t (k)^{\alpha_k} (\ell)^{\alpha_\ell} (m_t)^{\alpha_m}, \quad (\text{A58})$$

where ℓ is fixed at 1, which is the value in normal times in the baseline model. The other parts of the model remain unchanged, including the borrowing constraint and the balance of payments identity. The parameters are set to the same values in the baseline model.

Figure A3 plots the crisis dynamics under the four policy regimes. The dynamics are qualitatively similar to those in the baseline model. Macroprudential taxes stabilize foreign bond, real exchange rate, output and consumption. Monetary policy interventions during crises are contractionary, and output drops more under discretionary monetary policy. The

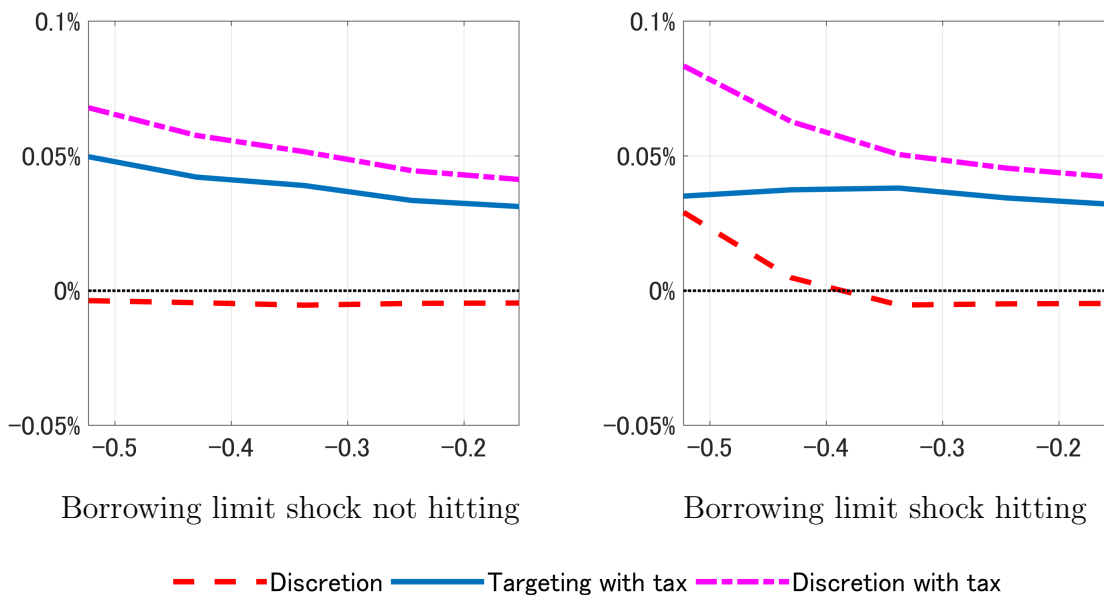
Figure A3: Crisis dynamics in the model with fixed labor



— Targeting Targeting with tax
 - - - Discretion - - - Discretion with tax

Note: These figures plot the crisis dynamics in the model with fixed labor supply. The horizontal axis is the time, and a crisis occurs at period 0. Real exchange rate, output, and consumption are expressed in percentage gaps from the level at period -1 under inflation targeting without a tax. The other variables are expressed in actual values.

Figure A4: Welfare gain/loss in the model with fixed labor



Note: These panels plot the expected welfare gain/loss under different policy regimes, in terms permanent consumption relative to inflation targeting without taxes.

important difference from the baseline model is that combined with taxes, consumption in Panel (f) drops less by 0.5% under discretionary monetary policy than in inflation targeting, which is the opposite to the result in the baseline model. This result is understood as follows. As in the baseline model, contractionary monetary policy mitigates real depreciation, thereby reducing exports and enabling households to consume a larger fraction of output. Panel (g) shows that the consumption-to-output ratio during crises is 75.8% under discretion and 74.6% under inflation targeting, both combined with taxes. But in the model with fixed labor, the optimal size of monetary policy intervention is designed such that consumption becomes actually higher under discretion, because there is no labor disutility and only consumption determines welfare. This explains the smaller size of monetary policy interventions in Panel (c) compared with the baseline model.

G Numerical Solution

G.1 Inflation Targeting without Taxes

The model under inflation targeting without taxes is solved using policy function iterations modified to deal with an occasionally binding constraint. I set 101 grid points for the debt space. There are four states for productivity and interest rate shocks, and two states for a borrowing limit shock κ_t . Thus, there are 808 grid points in total. The numerical solution is obtained by the following steps:

1. I set the initial guess for the decision rules of labor $\ell(b_{t-1}^*, s_t)$, real exchange rate $e(b_{t-1}^*, s_t)$, and the right-hand side of the Euler equation with respect to foreign debt $RHSEE(b_{t-1}^*, s_t)$.
2. For each grid point (b_{t-1}^*, s_t) , I solve the simultaneous equations of the equilibrium conditions using a non-linear solver and obtain the decision rules for $\ell(b_{t-1}^*, s_t)$, $e(b_{t-1}^*, s_t)$, $RHSEE(b_{t-1}^*, s_t)$. In this step, I first assume that the borrowing constraint is not binding and solve the equations. Then I check if the constraint is violated or not. If it is violated, I solve the equations with the binding constraint.
3. After deriving the decision rules at every grid point, I compare the initial guess and the obtained decision rules. If they are close enough, I stop. If not, I update the initial guess with the obtained decision rules and go back to step 2.

G.2 Discretionary Monetary Policy without Taxes

I use a combination of policy function iterations and value function iterations. This method consists of an outer loop of policy function iterations and an inner loop of value function iterations.

1. I set the initial guess for the decision rules of $\ell(b_{t-1}^*, s_t)$, $m(b_{t-1}^*, s_t)$, and $\pi(b_{t-1}^*, s_t)$. I use the same initial guess for the next period decision rules $\tilde{\ell}(b_{t-1}^*, s_t)$, $\tilde{m}(b_{t-1}^*, s_t)$, $\tilde{\pi}(b_{t-1}^*, s_t)$. I also set the initial guess for the value function $\tilde{V}(b_{t-1}^*, s_t)$, which is household's expected utility at each state.

2. Inner loop: for each grid point (b_{t-1}^*, s_t) , given the next period decision rules $\tilde{\ell}(b_{t-1}^*, s_t)$, $\tilde{m}(b_{t-1}^*, s_t)$, $\tilde{\pi}(b_{t-1}^*, s_t)$, and the guessed value function $\tilde{V}(b_{t-1}^*, s_t)$, I find the inflation rate $\pi(b_{t-1}^*, s_t)$ that maximizes the value function by using non-linear minimizer. I also obtain the corresponding value $V(b_{t-1}^*, s_t)$ at each grid point.
3. After obtaining $\pi(b_{t-1}^*, s_t)$ and $V(b_{t-1}^*, s_t)$ for every grid point, I check if $V(b_{t-1}^*, s_t)$ and $\tilde{V}(b_{t-1}^*, s_t)$ are close enough. If they are close enough, I proceed to step 4. If not, I update the guess for the value function $\tilde{V}(b_{t-1}^*, s_t)$ with the obtained value functions $V(b_{t-1}^*, s_t)$, and go back to step 2.
4. Outer loop: I compare the next period decision rules $\tilde{\ell}(b_{t-1}^*, s_t)$, $\tilde{m}(b_{t-1}^*, s_t)$, $\tilde{\pi}(b_{t-1}^*, s_t)$ and the obtained decision rules $\ell(b_{t-1}^*, s_t)$, $m(b_{t-1}^*, s_t)$, $\pi(b_{t-1}^*, s_t)$. If they are close enough, I stop. If not, I update the next period decision rules with the obtained decision rules and go back to step 2.

G.3 Inflation Targeting with Optimal Tax

I use a combination of policy function iterations and value function iterations.

1. I set the initial guess for the decision rules of $c(b_{t-1}^*, s_t)$ and $\ell(b_{t-1}^*, s_t)$. I use the same initial guess for the next period decision rules $\tilde{c}(b_{t-1}^*, s_t)$, $\tilde{\ell}(b_{t-1}^*, s_t)$. I also set the initial guess for the value function $\tilde{V}(b_{t-1}^*, s_t)$, which is households' expected utility at each state.
2. Inner loop: for each grid point (b_{t-1}^*, s_t) , I first assume that the borrowing constraint is not binding. Given the next period decision rules $\tilde{c}(b_{t-1}^*, s_t)$, $\tilde{\ell}(b_{t-1}^*, s_t)$ and the guessed value function $\tilde{V}(b_{t-1}^*, s_t)$, I find foreign debt $b^*(b_{t-1}^*, s_t)$ that maximizes the value function by using a non-linear minimizer. I also obtain the corresponding value $V(b_{t-1}^*, s_t)$ at each grid point. Then I check the constraint. If the constraint is not violated, I proceed to next grid point. If the constraint is violated, I solve the equilibrium conditions with the binding constraint using a non-linear solver to obtain $b^*(b_{t-1}^*, s_t)$ and $V(b_{t-1}^*, s_t)$.

3. After obtaining $b^*(b_{t-1}^*, s_t)$ and $V(b_{t-1}^*, s_t)$ for every grid point, I check if $V(b_{t-1}^*, s_t)$ and $\tilde{V}(b_{t-1}^*, s_t)$ are close enough. If they are close enough, I proceed to step 4. If not, I update the guess for the value function $\tilde{V}(b_{t-1}^*, s_t)$ with the obtained value functions $V(b_{t-1}^*, s_t)$, and go back to step 2.
4. Outer loop: I compare the next period decision rules $\tilde{c}(b_{t-1}^*, s_t)$, $\tilde{\ell}(b_{t-1}^*, s_t)$ and the obtained decision rules $c(b_{t-1}^*, s_t)$, $\ell(b_{t-1}^*, s_t)$. If they are close enough, I stop. If not, I update the next period decision rules with the obtained decision rules and go back to step 2.

G.4 Discretionary Monetary Policy with Optimal Tax

I use a combination of policy function iterations and value function iterations.

1. I set the initial guess for the decision rules $\ell(b_{t-1}^*, s_t)$, $m(b_{t-1}^*, s_t)$, and $\pi(b_{t-1}^*, s_t)$. I use the same initial guess for the next period decision rules $\tilde{\ell}(b_{t-1}^*, s_t)$, $\tilde{m}(b_{t-1}^*, s_t)$, $\tilde{\pi}(b_{t-1}^*, s_t)$. I also set the initial guess for the value function $\tilde{V}(b_{t-1}^*, s_t)$, which is household's expected utility at each state.
2. Inner loop: for each grid point (b_{t-1}^*, s_t) , I first assume that the borrowing constraint is not binding. Given the next period decision rules $\tilde{\ell}(b_{t-1}^*, s_t)$, $\tilde{m}(b_{t-1}^*, s_t)$, $\tilde{\pi}(b_{t-1}^*, s_t)$, and the guessed value function $\tilde{V}(b_{t-1}^*, s_t)$, I find a combination of foreign debt $b^*(b_{t-1}^*, s_t)$ and inflation $\pi(b_{t-1}^*, s_t)$ that maximizes the value function by using a non-linear minimizer. I also obtain the corresponding value $V(b_{t-1}^*, s_t)$ at each grid point. Then I check the constraint. If the constraint is not violated, I proceed to next grid point. If the constraint is violated, I find inflation $\pi(b_{t-1}^*, s_t)$ that maximizes the value function by using a non-linear minimizer, taking into account this time that the borrowing constraint is binding.
3. After obtaining $b^*(b_{t-1}^*, s_t)$, $\pi(b_{t-1}^*, s_t)$, and $V(b_{t-1}^*, s_t)$ for every grid point, I check if $V(b_{t-1}^*, s_t)$ and $\tilde{V}(b_{t-1}^*, s_t)$ are close enough. If they are close enough, I proceed to step 4. If not, I update the guess for the value function $\tilde{V}(b_{t-1}^*, s_t)$ by the obtained value functions $V(b_{t-1}^*, s_t)$, and go back to step 2.

4. Outer loop: I compare the next period decision rules $\tilde{\ell}(b_{t-1}^*, s_t)$, $\tilde{m}(b_{t-1}^*, s_t)$, $\tilde{\pi}(b_{t-1}^*, s_t)$, and the obtained decision rules $\ell(b_{t-1}^*, s_t)$, $m(b_{t-1}^*, s_t)$, $\pi(b_{t-1}^*, s_t)$. If they are close enough, I stop. If not, I update the next period decision rules with the obtained decision rules and go back to step 2.

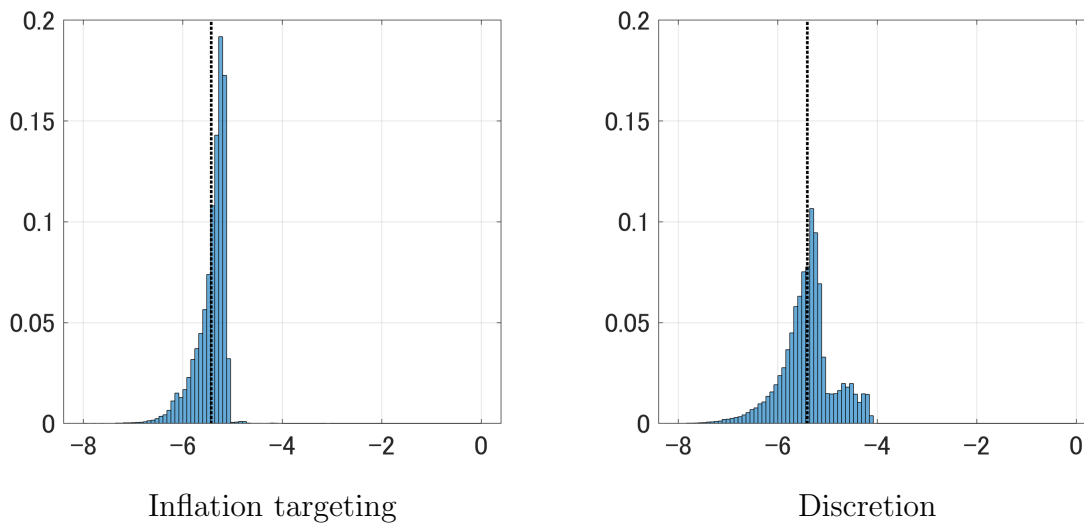
G.5 Accuracy of Solution

This subsection presents accuracy of the numerical solution obtained by the method described above. Following [Aruoba et al. \(2006\)](#), I compute the Euler equation errors as a measure of accuracy of the numerical solution. I use the Euler equation with respect to foreign bond, because it is subject to the occasionally binding borrowing constraint, and thus is likely to cause largest errors. Under each policy regime, I simulate the model with stochastic shocks for 100,000 periods, drop the first 1,000 periods, and compute the Euler equation errors for the 99,000 periods. For each period t of each simulation i , I compute the Euler equation error as follows:

$$error_{t,i} = \log_{10} \left(\left| 1 - \frac{c_{t,i}^{EE}}{c_{t,i}} \right| \right),$$

where $c_{t,i}$ is consumption at period t of simulation i directly derived from the decision rules, and $c_{t,i}^{EE}$ is consumption computed from the right-hand side of the Euler equation which is explicitly computed. [Figure A5](#) presents the distribution of the Euler equation errors under inflation targeting and discretionary monetary policy. Under both regimes, the mean of errors is smaller than -5 , and the maximum error is smaller than -3 .

Figure A5: Euler equation error



References in Appendix

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