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Abstract

A macroprudential perspective posits a link between bank fundamentals and the likelihood of banking crises. We articulate this link by developing a dynamic general equilibrium model that features bank runs in a global game framework. The model endogenizes the probability of bank runs as a function of bank fundamentals, leverage in particular. The model generates procyclical leverage and shows that credit growth tends to precede banking crises, replicating the empirical finding of Schularick and Taylor (2012). Countercyclical leverage restrictions can improve social welfare by reducing the crisis probability despite dampening economic activities in normal times.

Keywords: Banking crises; global games; macroprudential policy
JEL classification: E32, E44, G21, G28

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1 Introduction

The global financial crisis (GFC) of 2007-08 crystalized the potential for calamity inherent in the financial system. Policymakers around the world responded by reforming the regulatory framework and established Basel III. The objective of Basel III reforms is to improve the banking sector's ability to absorb shocks, thereby reducing the risk of spillovers from the financial sector to the real economy (Basel Committee on Banking Supervision, 2010b), where spillovers from the financial system culminate in financial crises. The banking sector's shock-absorbing capacity has to do with bank fundamentals. Hence, simply put, the goal of Basel III is to reduce the probability of a financial crisis by enhancing bank fundamentals.

A key challenge facing policymakers and economists is to understand the relationship between the probability of a financial crisis and bank fundamentals. The empirical literature has found that credit growth is a significant precursor of banking crises and severe downturns (Schularick and Taylor, 2012; Aikman et al., 2019). It has also found that most banking crises feature bank runs (Reinhart and Rogoff, 2009; Gorton, 2012) including the GFC (Bernanke, 2018). The theoretical literature has developed various dynamic models of financial crises and, in light of the empirical observations, Gertler and Kiyotaki (2015) and Gertler et al. (2020b) have made substantial progress by developing dynamic bank run models. In these models, however, a bank run is sunspot driven and thus bank-run-led crisis probability is indeterminate, leaving some questions still open. Specifically, how do bank fundamentals such as bank leverage affect the probability of a crisis? What could be the potential mechanism underlying the empirical evidence that credit growth predicts financial crises?

This paper tackles these questions by developing a dynamic general equilibrium model that features bank runs in a global game framework. The contribution of the paper is three-fold. First, the model that we have developed explains a close link between crisis probability and bank fundamentals in an infinite horizon economy. Incorporating a global game allows the model to determine bank-run-led crisis probability endogenously. The analytical solution leads to crisis probability as a function of bank leverage – key fundamentals of the banking sector. The model thereby articulates the idea underlying Basel III reforms: enhancing bank fundamentals, and bank leverage in particular, can reduce crisis probability. Second, the model provides a unified framework for bridging two empirical facts: procyclical leverage and credit growth as a precursor of financial crises. The model generates procyclical leverage – a crucial feature of banking that exacerbated the GFC as pointed out by Adrian and Shin (2010). Combined with endogenous crisis probability as a function of leverage, the model replicates the seminal empirical finding by Schularick and Taylor (2012) that credit growth tends to precede financial crises. Third, the paper sheds light on the role of countercyclical macroprudential policy in reducing crisis probability and improving social welfare.

The basic framework of the model is a standard real business cycle (RBC) model, modified to incorporate banks that channel funds from households to firms. Each household consists of a large number of family members, each of whom is either a banker or a depositor. Bankers and depositors manage banks and deposits, respectively. Into this model, a bank run global game is incorporated. Depositors receive private information about bank asset returns and make a binary choice whether to run on banks (withdraw deposits early) or stay. This problem is formulated as a global game among depositors, as studied by Rochet and Vives (2004), with the game working as an equilibrium selection device. In this environment, the probability of bank default is endogenously determined as a function of bank fundamentals such as leverage.

Banks take this feedback effect of leverage on the default probability into account and choose leverage to maximize the expected profits. And so the banks balance risk and return: the higher probability of default generated by increased leverage balanced with the higher return generated by leveraging lending. Thus, the risk of bank runs disciplines bank behavior to some extent and helps determine the leverage without any binding constraints. The endogenous leverage in turn affects the probability of bank runs and resulting bank default, and thus the leverage drives the resilience of the banking system.

Leverage may be excessive and the crisis probability may be too high in a laissez-faire economy. In the model, the crisis probability is reflected in the deposit interest rate, but it is not priced on margin. The model assumes that neither bank leverage nor risk is observable to creditors as in Acharya (2009) and Mendicino et al. (2020). Bank leverage is not contractible and thereby banks maximize the expected profits given the deposit interest rate in the competitive market. Banks ignore the effect of their choice of risk – leverage – on the interest rate. This leads to bank risk shifting similar in spirit to that described by Jensen and Meckling (1976) and possibly to excessive leverage.

After the leverage is chosen today, should, at the beginning of the next period, a negative shock to the bank asset return be greater than the resilience level, a bank run occurs and bank capital is wiped out. The banking sector receives capital injection from the government and restarts its operation. Should, on the other hand, the negative shock be small so that a bank run does not occur, banks distribute some profits as dividends, retain the remaining profits, and operate banking in the next period. This process, along with household and firm decisions as in the standard RBC model, is repeated every period.

The model is calibrated to the United States economy and solved globally. The calibrated model generates procyclical leverage during normal and boom periods, consistent with the empirical evidence reported by Adrian and Shin (2010). Specifically, a change in bank assets is mainly driven by a change in bank debt or leverage. What drives the procyclical leverage is the assumption of no equity issuance and low contribution of retained earnings to bank capital, which makes bank capital sticky. A parameter that governs the contribution of retained earnings plays a critical role in generating procyclical leverage.

Just as bank run probability increases as leverage increases, so procyclical leverage implies that the probability of banking crises increases as credit expands. Indeed, the model simulation shows that banking crises tend to occur during booms with high levels of leverage and bank credit. In such credit booms, reflecting macro-financial linkages embedded in the model, output, investment, and consumption are also booming. But, at the same time, the banking system becomes vulnerable to negative shocks. If the banking system is hit by a negative shock that exceeds the resilience level of the banking system, a bank-run-led crisis ensues. The banking system collapses and becomes dysfunctional as bank capital plummets. Because rebuilding bank capital takes time, the real economy goes through a severe recession and the recovery is slow.

The dynamic nature of the model allows it to be tested against the seminal empirical finding by Schularick and Taylor (2012): credit growth tends to precede banking crises. The model is simulated and the simulated data are taken to banking crisis regressions, which are exactly the same specifications as in Schularick and Taylor (2012). The regression result shows that credit growth in the few years run-up to a crisis is a significant predictor of that crisis, consistent with their empirical finding. In addition, the regression result entails economic significance as well as statistical significance. The result implies that a one standard deviation increase in credit growth in the past three years raises the probability of a crisis occuring in the next year by 1.8 percentage points (pp). Since the crisis probability in the stochastic steady state is 5 percent, the increase of 1.8pp is substantial, albeit lower than the 2.8pp reported in Schularick and Taylor (2012). Procyclical leverage and endogenous crisis probability are underlying drivers of these results. However, if leverage were countercyclical instead, the results would break down: the significance of credit growth as a precursor of crises would disappear.

The tendency for vulnerability in the banking system to rise during credit booms may warrant introducing macroprudential policy. The model simulation shows that imposing countercyclical leverage restrictions, which suppress leverage during normal and boom periods but allow for high leverage during crises and downturns, can lower crisis probability substantially. Although this would decrease output and consumption during normal times, the countercyclicality of such policy is critical for managing crises. But, if the restriction imposed is flat rather than countercyclical, output and investment drop even more during crises when bank capital is scarce i.e., when leverage is needed to expand credit to the real economy. Thus, releasing leverage by loosening the restrictions during crises supports the real economy and makes the crises less severe.

To focus on endogenous crisis probability in a dynamic framework, the model abstracts away some important features of banking. Specifically, the model assumes a single type of bank for simplicity. Thus, banks in the model correspond to any financial institutions, including shadow banks, that are subject to bank runs. The model also abstracts away deposit insurance. But the model's main results hold as long as deposit insurance is imperfect, that is deposits are not 100 percent protected as is indeed the case in practice. Ikeda (2018), for example, uses a version of the two-period model with bank runs, and finds that imperfect deposit insurance makes excessive leverage even more excessive due to other risk shifting similar to Kareken and Wallace (1978).

Related literature This paper contributes to the literature on bank runs in a macroeconomic framework. The literature has been recently advanced by Gertler and Kiyotaki (2015), who characterize runs as self-fulfilling rollover crises, following the sovereign debt crisis models of Calvo (1988) and Cole and Kehoe (2000).[1] Gertler et al. (2020b) extend the Gertler and Kiyotaki model by embedding it into a New Keynesian framework with investment. Using the extended model, Gertler et al. (2020a) study the nature of crises and the role of macroprudential policy. Many extensions have emerged, including Gertler et al. (2016) and Poeschl (2020) who incorporate shadow banking, Aoki et al. (2019) who focus on the low interest rate environment, Paul (2020) who incorporates long-term loans, Faria-e-Castro (2020) who studies the role of countercyclical capital buffers, and Mikkelsen

[1] Other types of bank runs in a dynamic model include Ennis and Keister (2003) and Martin et al. (2014), both of whom build on Diamond and Dybvig (1983), and Angeloni and Faia (2013), who extend Diamond and Rajan (2000).

and Poeschl (2020) who focus on macroeconomic uncertainty.

This paper critically differs from these papers in that bank run probability is not indeterminate but endogenously determined by introducing a global game, as in Rochet and Vives (2004). The global game, introduced as an equilibrium selection device, provides a link between agent actions and economic fundamentals. The game has been adopted to study binary decision problems such as currency attacks (Morris and Shin, 1998) and bank runs (Rochet and Vives, 2004; Goldstein and Pauzner, 2005; Vives, 2014). Our contribution to this literature is to incorporate a static global game bank run problem into a dynamic general equilibrium model.

Broadly, this paper is positioned in the literature on macroeconomic models with financial crises that occur nonlinearly. In addition to the bank run papers in a macroeconomic framework, the literature explores occasionally binding collateral constraints (Mendoza, 2010; Brunnermeier and Sannikov, 2014; Akinci and Queralto, 2017; He and Krishnamurthy, 2019), focusing on the role of collateral in borrowing; and adverse selection and market shutdown (Kurlat, 2013; Bigio, 2015; Boissay et al., 2016), focusing on financial markets. Our paper is complementary to these strands of literature as it focuses on financial institutions vulnerable to runs.

Finally, this paper is related to the literature on time-varying tools of macroprudential policy in a dynamic general equilibrium model, including, in addition to those already mentioned, Bianchi (2011), Karmakar (2016), Bianchi and Mendoza (2018), Elenev et al. (2018), and Davydiuk (2019).

2 The Model

The model is based on a standard RBC model, extended to incorporate banking and bank runs in a global game framework. Time is discrete and its horizon is infinite. There is a single type of good, which can be used as either consumption or investment. The economy consists of households, firms, banks, and a government. Banks channel funds from households to firms, subject to bank run risk. The mechanism of bank runs is based on the global game of Rochet and Vives (2004), modified to endogenize lending and borrowing interest rates, and bank behavior, especially a choice of leverage. Section 2.1 presents the standard part of the model. Sections 2.2, 2.3, and 2.4 present the novel parts of the model, namely, run decisions, bank runs in a global game, and bank behavior, respectively. Section 2.5 defines an equilibrium.

2.1 Standard Part of the Model

Household There is a representative household, which consists of a continuum of family members with measure unity. Each family member, indexed by $j \in [0, 1]$, is either a depositor or a banker. Depositors manage deposits and make run decisions, while bankers manage banks and lend to firms. As family members, both depositors and bankers work at firms, earn wage income, and bring that income back to the household. The population of depositors and bankers is fixed at $0 < \zeta < 1$ and $1-\zeta$, respectively. Depositors and bankers switch their occupations with an exogenous probability in a way that each population stays constant over time.

Each family member has GHH preferences (Greenwood et al., 1988) over consumption $c_{j,t}$ and hours worked $h_{j,t}$ in period $t = 0, 1, 2, ...$, given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \log \left(c_{j,t} - \psi \frac{h_{j,t}^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \right), \tag{1}$$

where $0 < \beta < 1$ is a preference discount factor, $\nu > 0$ is the Frisch labor supply elasticity, $\psi > 0$ is a coefficient on the disutility of labor, and E_0 is an expectation operator conditional on information in period $t = 0$. GHH preferences are employed to help incorporate a global game bank run problem into the dynamic model as will be discussed in Section 2.3.

The representative household has a leader who collects all resources at hand, allocates resources as consumption to its family members, and decides the amount of saving. The only means of saving or investment is assumed to be bank deposits d_t. In addition, the leader decides how much labor each family member should supply.[2] Accordingly, the household leader chooses $\{c_{j,t}\}_{j=0}^1$, $\{h_{j,t}\}_{j=0}^1$, and d_t to maximize the expected sum of household member utility (1) over $j \in [0, 1]$,

$$E_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 \log \left(c_{j,t} - \psi \frac{h_{j,t}^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \right) dj, \tag{2}$$

[2]The result would be equivalent to another setting where each family member chooses labor supply. In this case, each member chooses $c_{j,t}$ and $h_{j,t}$ to maximize the periodic GHH utility subject to the budget constraint $c_{j,t} = w_t h_{j,t} + \Theta_{j,t}$, where $\Theta_{j,t}$ is a lump-sum transfer from the household leader to achieve the consumption allocation explained below.

subject to the flow budget constraint, given by

$$c_t + d_t \leq R_t d_{t-1} + w_t h_t + \Theta_t, \tag{3}$$

where $c_t \equiv \int_0^1 c_{j,t} dj$ is the household's total consumption, R_t is the deposit interest rate, w_t is the wage, $h_t \equiv \int_0^1 h_{j,t} dj$ is the household's total hours worked, and Θ_t is the sum of lump-sum taxes imposed by the government and the net lump-sum transfer from banks, which includes the value of the assets liquidated and sold by banks to households during bank runs as will be explained in Section 2.3. The deposit interest rate is state-contingent, given by

$$R_t = \begin{cases} \bar{R}_{t-1} & \text{if no bank default} \\ v_t \bar{R}_{t-1} & \text{if bank default} \end{cases}$$

In the case of no bank default, the bank pays the non-contingent interest rate $\bar{R}_{t-1}$ to depositors. In the case of bank default, the bank is able to pay only a fraction $0 \leq v_t < 1$ of the promised interest rate.

Consumption can be heterogeneous across family members of the household as will be explained in Section 2.2, but we focus on a special case in which such heterogeneity vanishes asymptotically as will be shown in Section 2.5. Then, the household leader's choice of the aggregate variables $\{c_t, h_t, d_t\}$ is obtained by maximizing utility (1), with index j omitted, subject to the budget constraint (3). The first-order conditions of the problem yield the consumption Euler equation and the labor supply curve, given respectively by

$$1 = E_t \beta \left(\frac{c_t - \psi \frac{h_t^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}}{c_{t+1} - \psi \frac{h_{t+1}^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}} \right) v_{t+1} \bar{R}_t, \tag{4}$$

$$w_t = \psi h_t^{\frac{1}{\nu}}. \tag{5}$$

Since GHH preferences abstract from income effects, the labor supply curve (5) does not depend on consumption, and thereby $h_{j,t} = h_t$ for all $j \in [0, 1]$. The allocation of individual consumption $\{c_{j,t}\}_{j=0}^1$ will be discussed in the following subsections.

Firm There is a representative firm, which produces output y_t by combining physical capital k_t and labor h_t using a Cobb-Douglas production technology, given by

$$y_t = A_t k_t^{\alpha} h_t^{1-\alpha}, \quad 0 < \alpha < 1, \tag{6}$$

where A_t is the total factor productivity (TFP) and $a_t \equiv \log(A_t)$ follows an AR(1) process

$$a_t = \rho_a a_{t-1} + \epsilon_{a,t}, \quad 0 \leq \rho_a < 1, \tag{7}$$

with $\epsilon_{a,t} \sim N(0, \sigma_a^2)$. The productivity shock $\epsilon_{a,t}$ is an only aggregate shock in the economy. The firm does not own physical capital, so that it has to borrow funds from banks to rent physical capital. Perfect competition leads to factor prices for physical capital and labor, r_t^k and w_t, given respectively by

$$r_t^k = \alpha A_t \left(\frac{k_t}{h_t}\right)^{\alpha-1}, \tag{8}$$

$$w_t = (1-\alpha) A_t \left(\frac{k_t}{h_t}\right)^{\alpha}. \tag{9}$$

Because no friction is assumed between the firm and the banking sector, the marginal return r_t^k becomes equal to the bank asset return – what banks receive in return for lending to the firm.

Market clearing Let i_t denote investment – the amount of goods used as new physical capital. For simplicity, the model assumes full capital depreciation. Then, the newly installed physical capital in period t is given by

$$k_{t+1} = i_t. \tag{10}$$

With full capital depreciation, the gross return on investment is reduced to r_t^k, making the return more volatile than would otherwise be the case. This feature is useful for incorporating bank runs as will be discussed in Section 2.3. Finally, the good market clearing condition is

$$y_t = c_t + i_t. \tag{11}$$

2.2 Run Decisions

Timing of events The role of depositors is to make a binary decision – run or stay. At the end of period $t-1$, the household leader makes deposits into a bank and delegates the management of deposits to the depositors. Each depositor oversees an identical amount of deposits d_{t-1}/ζ, where ζ is the population of depositors. Deposits can be withdrawn early at the beginning of each period.

As described in Figure 1, events unfold as follows. At the very beginning of period t, an aggregate shock $\epsilon_{a,t}$ is realized. But, at this point, no one knows the realized value. At the same time, before $\epsilon_{a,t}$ becomes common knowledge, each depositor j receives a private signal, $s_{j,t}$, about the log of the bank asset return, $\hat{r}_t^k \equiv \log(r_t^k)$, given by

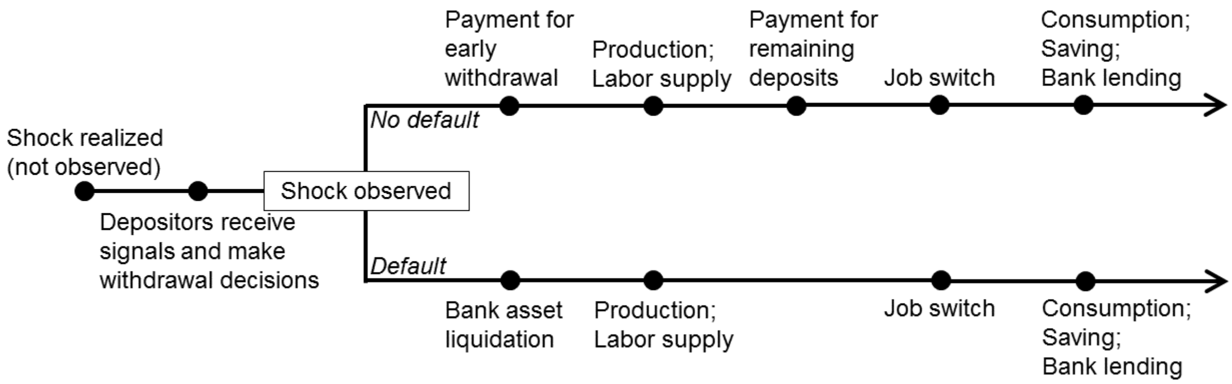
$$s_{j,t} = \hat{r}_t^k + \epsilon_{j,t}, \tag{12}$$

where $\epsilon_{j,t}$ is a noise that follows the normal distribution: $\epsilon_{j,t} \sim N(0, \sigma_\epsilon^2)$. Using the private information, each depositor decides whether to run or stay. After run decisions are made, the aggregate shock $\epsilon_{a,t}$ and thereby the realized bank asset return r_t^k become common knowledge. But the run decisions cannot be overturned. Depending on the size of a run – the amount of deposits withdrawn early, the bank may default.

In the rest of period t, events unfold differently depending on whether the bank defaults or doesn't default. In the case of bank survival, the bank pays the promised interest rate $\bar{R}_{t-1}$ per unit of deposits withdrawn early. Production takes place, where both depositors and bankers supply labor. The bank receives the asset return r_t^k per unit of lending. The bank then pays the promised interest rate $\bar{R}_{t-1}$ per unit of deposits withheld for a whole period. After that, job switches between depositors and bankers occur, and the bank capital in period t is determined. Finally, decisions on consumption, saving, and bank lending are made.

In the case of bank default, the bank liquidates assets and distributes resources at hand to depositors. Specifically, the amount paid back to an individual depositor differs depending on the withdrawal decision made: those who withdrew early receive more, and those who stayed receive less. For example, the following repayment rule generates such an outcome. The bank liquidates assets as much as it can to pay the promised interest rate $\bar{R}_{t-1}$ to depositors who withdrew early; whatever remains at the bank after the payment is distributed equally among the remaining depositors. After that, events unfold as in

Figure 1: Timing of events in each period



the case of bank survival: production takes place, job switches occur, and decisions on consumption, saving, and bank lending are made.

Costly withdrawals We assume that it is costly for depositors to withdraw deposits early. The costs reflect transaction costs in switching between banks (Klemperer, 1987) or changing portfolios (Constantinides, 1986; Duffie and Sun, 1990). The model captures these costs by assuming that depositors incur utility cost κ if they withdraw deposits early. Specifically, when depositor j runs on a bank, the periodic utility is given by

$$\ln\left(c_{j,t} - \psi\frac{h_{j,t}^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}\right) - \kappa, \tag{13}$$

for given $\{c_{j,t}, h_{j,t}\}$.

Imperfect insurance The utility function (2) implies that the household leader would seek to distribute consumption equally among the household members. We assume that nothing prevents such perfect insurance when the bank survives, i.e. in a normal state of the economy. Hence, in this case, $c_{j,t} = c_t$ holds for all $j \in [0, 1]$. However, we also assume that there is a limit to perfect insurance when the bank defaults, i.e. during a crisis. Let $c_{s,t}$, $c_{w,t}$, and $c_{b,t}$ denote consumption for depositors who stayed, those who withdrew, and bankers, respectively. It is assumed that, in the case of bank default, bankers and depositors who withdrew early are perfectly insured, $c_{b,t} = c_{w,t}$, but depositors who stayed are not.

The consumption of these depositors, $c_{s,t}$, is given by

$$c_{s,t} - \psi \frac{h_t^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} = \theta \left(c_{w,t} - \psi \frac{h_t^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \right), \tag{14}$$

where $0 < \theta < 1$ governs the degree of imperfect insurance. Depositors who stayed are paid less by the defaulted bank than those who withdrew early, as explained above. This income difference is not perfectly insured, giving rise to lower consumption in those who stayed as in equation (14).

Imperfect insurance in the form of (14) aims to capture the fact that depositors who did not withdraw their deposits lose a part of their deposits when the bank defaults. Jacewitz and Pogach (2018) show that depositors recovered only 55% of their uninsured deposits on average from defaulting banks in 2007–2014 in the United States. Egan et al. (2017) develop a structural empirical model of the US banking sector and argue that uninsured deposits are frequently impaired in the case of bank default and these deposits are therefore potentially prone to runs.

Run decision rules Given this environment, at the beginning of each period, each depositor j decides whether to run or stay by using a private signal about the bank asset return. Let $F_{j,t}$ denote a distribution function about the log of bank asset return $\hat{r}_t^k$, perceived by depositor j with private information $s_{j,t}$. As derived in the next subsection, there is a unique threshold, $\hat{r}_t^{k*}$, below which the bank defaults. Then, the expected periodic utility if depositor j withdraws is given by

$$u_{j,t}(\text{run}) \equiv \int_{-\infty}^{\hat{r}_t^{k*}} \left[\ln \left(c_{w,t} - \psi \frac{h_t^{1+1/\nu}}{1+1/\nu} \right) - \kappa \right] dF_{j,t} + \int_{\hat{r}_t^{k*}}^{\infty} \left[\ln \left(c_t - \psi \frac{h_t^{1+1/\nu}}{1+1/\nu} \right) - \kappa \right] dF_{j,t},$$

where $c_{w,t}, c_t$ and h_t depend on the realization of the bank asset return. The expected periodic utility when depositor j stays is given by

$$u_{j,t}(\text{stay}) \equiv \int_{-\infty}^{\hat{r}_t^{k*}} \left[\ln \theta + \ln \left(c_{w,t} - \psi \frac{h_t^{1+1/\nu}}{1+1/\nu} \right) \right] dF_{j,t} + \int_{\hat{r}_t^{k*}}^{\infty} \left[\ln \left(c_t - \psi \frac{h_t^{1+1/\nu}}{1+1/\nu} \right) \right] dF_{j,t}.$$

The periodic utilities from period $t+1$ onward are independent of a run choice in period t. Hence, depositor j decides to run if and only if $u_{j,t}(\text{run}) > u_{j,t}(\text{stay})$, giving rise to the simple run decision rule:

$$\text{run} \Longleftrightarrow P_{j,t} > \frac{-\kappa}{\ln \theta} \equiv \gamma, \tag{15}$$

where $P_{j,t} \equiv F_{j,t}(\hat{r}_t^{k*})$ is the probability of bank default, perceived by depositor j. We assume that the costs of early withdrawal, captured by κ, are small enough to satisfy $0 < \gamma < 1$.

2.3 Bank Runs

Banks intermediate funds from the household, which has funds but lacks an investment opportunity, to the firm, which has an investment (production) opportunity but needs funds for investment. Without loss of generality, a representative bank is considered. The bank is managed by bankers. In this subsection, we describe how bank runs cause bank default. By doing so, we derive bank-run led default (crisis) probability as a function of bank fundamentals.

Bank balance sheet and asset return In period t after family members switch occupations, the bank equity (or bank capital) n_t is determined. Given the equity, the bank offers a deposit contract with the promised interest rate $\bar{R}_t$, and takes in deposits d_t from the household. The bank combines the equity and deposits $n_t + d_t$ and lends to the firm, which uses the resources as physical capital. Thus, the bank balance sheet is written as

$$k_{t+1} = n_t + d_t = L_t n_t, \tag{16}$$

where $L_t = (n_t + d_t)/n_t$ is the bank leverage. The left-hand side of (16), k_{t+1}, is the bank asset and the right-hand side, $n_t + d_t$, is the bank liability. The log return of bank lending $\hat{r}_{t+1}^k$ follows a normal distribution, thanks to GHH preferences of the household (1) and the aggregate shock process (7), as shown in the following lemma.

Lemma 1 (Bank asset return) *The log of bank asset return follows the normal distribution, $\hat{r}_{t+1}^k \sim N(\mu_{t,k}, \sigma_k^2)$, where*

$$\mu_{t,k} = \frac{(1-\alpha)\nu}{\alpha\nu + 1} \log\left(\alpha\left(\frac{1-\alpha}{\psi}\right)\right) + \frac{1+\nu}{\alpha\nu+1}\rho_a a_t - \frac{1-\alpha}{\alpha\nu+1}\log(k_{t+1}),$$
$$\sigma_k = \frac{1-\alpha}{\alpha\nu+1}\sigma_a.$$

Proof. Appendix A.1. ■

The normality of the log bank asset return implies the normality of the private signal $s_{j,t}$ received by each depositor j from equation (12). This feature will be useful for deriving the probability of bank default perceived by depositor j, $P_{j,t}$, which is critical for bank runs, as indicated by the run decision rule (15).

Costly liquidation and bank default probability In the beginning of period $t+1$, some depositors may withdraw funds from the bank. In response, the bank has to liquidate its assets by terminating some lending to the firm. But such an early liquidation is costly. The assets are liquidated by selling to the household at a discounted value of only a fraction $1/(1+\lambda)$ of the bank asset return r^k_{t+1}, where $\lambda \geq 0$ governs the degree of the discount. This costly liquidation captures the idea of fire sales and the illiquidity of bank assets.

Let $x_{t+1} \in [0,1]$ denote the size of a bank run – a fraction of deposits withdrawn. Then, the bank defaults if and only if it cannot pay the promised interest rate $\bar{R}_t$ to the depositors:

$$r^k_{t+1}(n_t + d_t) - (1+\lambda)x_{t+1}\bar{R}_t d_t < (1 - x_{t+1})\bar{R}_t d_t. \tag{17}$$

The left-hand side of the inequality (17) is what the bank has after liquidating some assets early and the right-hand side is the liability to the remaining depositors who stayed. The default condition (17) can be rewritten in terms of the bank asset return as

$$r^k_{t+1} < \bar{R}_t \left(1 - \frac{1}{L_t}\right)(1 + \lambda x_{t+1}). \tag{18}$$

Given the interest rate $\bar{R}_t$, the leverage L_t, and the run size x_{t+1}, the bank defaults when its realized return r^k_{t+1} is low enough to satisfy condition (18). Even in the case of no run with $x_{t+1} = 0$, the bank can still fail if $r^k_{t+1} < \bar{R}_t(1 - 1/L_t) \equiv \underline{r}^k_{t+1}$. This case corresponds to fundamental default or insolvency. Bank failure in the case of $r^k_{t+1} > \underline{r}^k_{t+1}$ corresponds to illiquidity default.

The bank default condition (18) in terms of the bank asset return, which follows the log normal distribution from Lemma 1, suggests that the probability of default perceived by depositor j can be derived as

$$P_{j,t+1} = Pr\left(\hat{r}^k_{t+1} < \log\left[\bar{R}_t\left(1 - \frac{1}{L_t}\right)(1 + \lambda x_{t+1})\right] \middle| s_{j,t+1}\right), \tag{19}$$

where $Pr(z|s)$ is the probability of z conditional on information s, $\hat{r}^k_{t+1} \sim N(\mu_{k,t}, \sigma_k^2)$ from Lemma 1, and $s_{j,t+1} = \hat{r}^k_{t+1} + \epsilon_{j,t+1}$ with $\epsilon_{i,t+1} \sim N(0, \sigma_\epsilon^2)$ from equation (12).

Threshold strategy In this environment with the depositors' run decision rule (15), as shown by Rochet and Vives (2004), there is a unique equilibrium in which depositor j runs if and only if

$$s_{j,t+1} < s^*_{t+1} \tag{20}$$

for some common threshold s^*_{t+1} among depositors, if the standard deviation of the signal, σ_e, is small relative to the standard deviation of the log bank asset return, σ_k. Given such a threshold strategy, by using the signal equation (12), the run strategy (20) can be written as $\epsilon_{j,t+1} < s^*_{t+1} - \hat{r}^k_{t+1}$. Since the signal is normally distributed, the run size x_{t+1} is given by

$$x_{t+1} = x(\hat{r}^k_{t+1}, s^*_{t+1}) = \Phi\left(\frac{s^*_{t+1} - \hat{r}^k_{t+1}}{\sigma_\epsilon}\right), \tag{21}$$

where $\Phi(\cdot)$ is the standard normal distribution function. Then, the probability of default perceived by depositor j, (19), can be written as

$$P_{j,t+1} = Pr\left(\hat{r}^k_{t+1} < \hat{r}^{k*}_{t+1} | s_{i,t+1}\right), \tag{22}$$

where the threshold of the log bank asset return $\hat{r}^{k*}_{t+1}$, below which the bank fails with the log return $\hat{r}^k_{t+1}$, is a solution to

$$\hat{r}^{k*}_{t+1} = \log\left[\bar{R}_t\left(1 - \frac{1}{L_t}\right)\left(1 + \lambda\Phi\left(\frac{s^*_{t+1} - \hat{r}^{k*}_{t+1}}{\sigma_\epsilon}\right)\right)\right]. \tag{23}$$

To solve for a pair of thresholds, s^*_{t+1} and $\hat{r}^{k*}_{t+1}$, consider a marginal depositor j^* whose signal coincides with the threshold: $s_{j^*,t+1} = s^*_{t+1}$. Such a depositor has to be indifferent between withdrawing and staying. Then, from the depositor's decision rule (15), the perceived bank default probability has to be just equal to the threshold probability:

$$Pr(s^*_{t+1}) = Pr(\hat{r}^k_{t+1} < \hat{r}^{k*}_{t+1} | s^*_{t+1}) = \gamma. \tag{24}$$

Equations (23) and (24) can be jointly solved for the thresholds s^*_{t+1} and $\hat{r}^{k*}_{t+1}$ under the uniqueness assumption about the threshold strategy. The detail of the assumption can be found in Appendix A.2.

Crisis probability Since the log bank asset return $\hat{r}^k_{t+1}$ follows the normal distribution as shown in Lemma 1, and the bank defaults – a banking crisis erupts – when the return falls below $\hat{r}^{k*}_{t+1}$, the probability of a crisis can be derived as

$$P_t = \Phi\left(\frac{\hat{r}^{k*}_{t+1} - \mu_{t,k}}{\sigma_k}\right). \tag{25}$$

As implied by equation (23), the threshold $\hat{r}^{k*}_{t+1}$ is affected especially by leverage L_t, and so is the crisis probability through the threshold. The following lemma clarifies the impact of leverage on the thresholds including s^*_{t+1}.

Lemma 2 (Thresholds and leverage) *Let $s^*_{t+1} = s^*(L_t)$ and $\hat{r}^{k*}_{t+1} = \hat{r}^{k*}(L_t)$ denote a solution to equations (23) and (24) as a function of leverage L_t. Then, the thresholds, $s^*(L_t)$ and $\hat{r}^{k*}(L_t)$, are both increasing in leverage.*

Proof. Appendix A.2. ■

The corollary of Lemma 2 is that the crisis probability P_t is also increasing in leverage. Another corollary is that the run size, given by equation (21), is also increasing in leverage. These results highlight endogenous crisis probability: it is neither exogenous nor indeterminate but determined as a function of bank fundamentals, bank leverage in particular.

2.4 Bank Behavior

So far bank leverage and capital have been taken as given. This subsection describes how they are determined by considering bank behavior.

Bank problem The objective of bankers is to maximize the bank's expected profits in each period.[3] In period t, the bank chooses leverage L_t to maximize the expected profits taking into account the effects of its chosen leverage on the run size and accompanying costs of early liquidation:

$$\max_{\{L_t\}} \int_{\hat{r}^{k*}(L_t)}^{\infty} \left\{ e^{\hat{r}^k_{t+1}} L_t - \bar{R}_t \left[1 + \lambda x\left(\hat{r}^k_{t+1}, s^*(L_t)\right)\right] (L_t - 1) \right\} n_t dF_t(\hat{r}^k_{t+1}),$$

[3] The assumption that banks, or borrowers in general, maximize their expected profits is also adopted by Bernanke et al. (1999), Christiano et al. (2014), and Mendicino et al. (2020).

subject to the technical constraint $L_t \leq L_{\max}$, where $x(\cdot)$ denotes the run size, given by equation (21), and F_t denotes a normal distribution function with mean $\mu_{t,k}$ and standard deviation σ_k, given by Lemma 1. The first term in the integral is the gross return on bank assets. The second term captures the interest rate cost of deposits and the costs associated with early liquidation. In equilibrium, the technical constraint is non-binding as $L_{\max}$ is set high enough, although it does help exclude an 'uninteresting' solution, as will be explained in the next subsection.

The equilibrium level of leverage is characterized by the first-order condition, given by

$$\int_{\hat{r}^{k*}_{t+1}}^{\infty} e^{\hat{r}^k_{t+1}} dF_t(\hat{r}^k_{t+1}) = \bar{R}_t(1-P_t) + \lambda \bar{R}_t \int_{\hat{r}^{k*}_{t+1}}^{\infty} \left[x_{t+1} + \frac{\partial x_{t+1}}{\partial s^*_{t+1}} \frac{\partial s^*_{t+1}}{\partial L_t}(L_t - 1) \right] dF_t(\hat{r}^k_{t+1}), \quad (26)$$

where P_t is the probability of bank default, given by equation (25). The optimality condition (26) equates the expected bank asset return – the left-hand-side of (26) – with the expected deposit interest rate payment plus the expected marginal cost of early liquidation with respect to leverage – the right-hand-side of (26). Given a unique solution to (26), the solution also satisfies the second-order condition, as summarized in the following lemma.

Lemma 3 (Optimality) *Assume that the first-order condition (26) has a unique interior solution, $L_t < L_{max}$. Then, the solution also satisfies the second-order condition if the mean bank asset return is greater than the deposit interest rate:*

$$e^{\mu_{t,k} + \frac{\sigma_k^2}{2}} > \bar{R}_t. \quad (27)$$

Proof. Appendix A.2. ■

Assumption (27) implies that there is a positive spread between the mean bank asset return and the deposit interest rate. Under this assumption, the marginal profit with respect to leverage is positive at no borrowing of $L_t = 1$. The bank can increase the expected profits by increasing leverage until the leverage satisfies the first-order condition (26). Beyond that point, bank leverage becomes so high that the cost of early liquidation outweighs the benefits of borrowing. In this problem, bank leverage is determined endogenously without any binding constraint. The effects of leverage on the run size and the associated costly liquidation, summarized in Lemma 2, refrain banks from choosing leverage that is too high.

Bank capital The bank is assumed to distribute profits to the household following a certain rule. Specifically, it remits a fraction $1-\chi_0$ of net profits, if any, to the household. The net profits are given by

$$\pi_t^b = r_t^k L_{t-1} n_{t-1} - \bar{R}_{t-1}(1+\lambda x_t)(L_{t-1}-1)n_{t-1} - n_{t-1}, \tag{28}$$

where the run size x_t is given by equation (21). If the net profits are negative, the bank does not pay any dividend. Parameter χ_0, which governs this rule of thumb, plays a critical role in generating procyclical leverage as will be discussed in Section 3.1.

After the remittance, a fraction $1-\chi_1$ of bankers become depositors and take home their portion of bank capital to the household. The same number of depositors become bankers and receive an exogenous equity $n_0 > 0$ from the household in aggregate. Then, the bank capital evolves according to

$$n_t = \begin{cases} \chi_1 \left[\chi_0 \pi_t^b + (1-\chi_0)\pi_t^b \mathbf{1}_{\{\pi_t^b < 0\}} + n_{t-1} \right] + n_0 & \text{if no default, } \hat{r}_t^k \geq \hat{r}_t^{k*} \\ \bar{n} & \text{if default, } \hat{r}_t^k < \hat{r}_t^{k*} \end{cases} \tag{29}$$

where $\mathbf{1}_{\{\pi_t^b<0\}}$ is an indicator function taking unity if $\pi_t^b < 0$ and zero otherwise. In the case of bank runs and resulting bank failure, the bank capital is wiped out and the banking sector needs new capital to resume its operations. It is assumed that, in the case of bank failure, new bank capital $\bar{n} > 0$ is injected into the bank from the government, which finances the capital injection by lump-sum taxes on the household.

2.5 Equilibrium

Limit equilibrium To make the depositors' run decision and the bank problem analytically tractable, as in the standard global game literature, we focus on a limiting case in which the accuracy of the private information becomes infinite, $1/\sigma_\epsilon \to \infty$, or $\sigma_\epsilon \to 0$. Then, the thresholds s_{t+1}^* and $\hat{r}_{t+1}^{k*}$, characterized by equations (23) and (24), have an analytical solution, and the optimality condition of the bank problem (26) is simplified as summarized in the following proposition.

Proposition 1 (Limit equilibrium) *Consider the limiting case of $\sigma_\epsilon \to 0$. Then, a* *solution to equations (23) and (24) is given by*

$$s_{t+1}^* = \hat{r}_{t+1}^{k*} = \log\left[\bar{R}_t\left(1-\frac{1}{L_t}\right)(1+\lambda(1-\gamma))\right]. \tag{30}$$

In addition, the optimality condition of the bank problem (26) is reduced to

$$\int_{\hat{r}_{t+1}^{k*}}^{\infty} e^{\hat{r}_{t+1}^k} dF_t(\hat{r}_{t+1}^k) = \bar{R}_t(1-P_t) - \lambda(1-\gamma)\frac{\bar{R}_t}{L_t}\frac{1}{\sqrt{2\pi}\sigma_k}e^{-\frac{1}{2}\left(\frac{\hat{r}_{t+1}^{k*}-\mu_{t,k}}{\sigma_k}\right)^2}. \tag{31}$$

Proof. Appendix A.2 ■

In the limiting case, all depositors run on the bank if the realized log bank asset return is low enough to satisfy $\hat{r}_{t+1}^k < \hat{r}_{t+1}^{k*}$, and no depositors run on the bank otherwise. Because of a full-scale run in which all depositors withdraw funds, the bank defaults, giving rise to a bank-run-led financial crisis in the case of $\hat{r}_{t+1}^k < \hat{r}_{t+1}^{k*}$. This binary feature – run (default) or no run – helps the model to keep a representative framework, as explained shortly, and facilitates numerical analyses in Section 3.

The threshold solution (30) and the simplified optimality condition (31) shine light on the importance of the non-binding technical constraint, $L_t \leq L_{\max}$, in the bank problem. The threshold solution (30) implies that $\hat{r}_{t+1}^{k*} < \infty$ even in the case of $L_t \to \infty$. Then, the optimality condition (31) suggests that the bank can make more profits by increasing leverage indefinitely $L_t \to \infty$. The technical constraint, with an unrealistically high upper bound, e.g. $L_{\max} = 100$, can exclude such an uninteresting solution. The technical constraint can therefore be interpreted as the upper limit on leverage, imposed by bank business models.

Representative agent framework In general, the model has heterogeneity in depositors' consumption, $c_{w,t} > c_{s,t}$, as implied by equation (14), due to imperfect insurance in the case of bank default, but such heterogeneity vanishes in the limiting case of $\sigma_\epsilon \to 0$. To see this, the total consumption c_t can be written, by using $c_{w,t} = c_{b,t}$ and equation (14), as

$$\begin{aligned} c_t \equiv \int_0^1 c_{j,t} dj =& \zeta(1-x_t)c_{s,t} + \zeta x_t c_{w,t} + (1-\zeta)c_{b,t}, \\ =& \left[1-\zeta(1-x_t)(1-\theta_t)\right]c_{w,t} + \zeta(1-x_t)(1-\theta_t)\frac{h_t^{1+1/\nu}}{1+1/\nu}, \end{aligned} \tag{32}$$

where x_t is the run size, given by equation (21), and $\theta_t = \theta < 1$ in the case of bank default and $\theta_t = 1$ in the case of bank survival. In the case of bank default, as the noise of the private information vanishes, i.e. as $\sigma_\epsilon \to 0$, equation (32) implies $c_{w,t} \to c_t$ since $(1 - x_t)(1 - \theta_t) \to 0$. In the limit equilibrium, $c_{j,t} = c_t$ holds for all j, as all depositors make a correct decision by running on the bank only when the bank defaults later. Hence, combined with the assumption of perfect insurance in the case of no default, heterogeneity in consumption vanishes. This is another benefit of considering the limiting case from the perspective of model simplicity.

In the limiting case of $\sigma_\epsilon \to 0$, the household problem features a representative agent framework, which was assumed in deriving the household optimality conditions (4) and (5). The objective function for the household leader, (2), can be written, by using equations (13) and (14), as

$$\begin{aligned} & E_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 \log \left(c_{j,t} - \psi \frac{h_{j,t}^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \right) dj \\ =& E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln \left(c_t - \psi \frac{h_t^{1+1/\nu}}{1+1/\nu} \right) + \zeta \left[(1 - x_t) \ln \theta_t - x_t \kappa \right] \right\}, \end{aligned} \tag{33}$$

where $c_{w,t} \to c_t$ was imposed in the second equality. Since the terms in the square brackets are outside the control of the household leader, the leader maximizes the expected discounted sum of the log utility subject to the budget constraint (3) as in a standard representative agent model.

Competitive equilibrium The model is closed by deriving the recovery rate v_{t+1} that appears in the Euler equation (4). In response to a full-scale run, the bank liquidates all assets and defaults. Then, the recovery rate is given by

$$v_{t+1} = \begin{cases} 1 & \text{if no default, } \hat{r}^k_{t+1} \geq \hat{r}^{k*}_{t+1} \\ r^k_{t+1} \bar{R}_t^{-1} \left(1 - \frac{1}{L_t} \right)^{-1} - \lambda & \text{if default, } \hat{r}^k_{t+1} < \hat{r}^{k*}_{t+1}. \end{cases} \tag{34}$$

Under the assumption of the limiting case of $\sigma_\epsilon \to 0$, a competitive equilibrium for this economy is characterized by the following system of equations: the household optimality conditions (4) and (5); the production technology (6) and the shock process (7); the firm optimality conditions (8) and (9); the market clearing conditions (10) and (11); the bank balance sheet (16) and the bank optimality conditions (30) and (31); and the law of motion for bank capital (29) and the recovery rate equation (34), with twelve endogenous variables $\{c_t, h_t, \bar{R}_t, v_t, r_t^k, w_t, k_{t+1}, y_t, i_t, L_t, \hat{r}_t^{k*}, n_t\}$ and one exogenous shock a_t.

3 Numerical Analyses

This section presents the main results of the paper by analyzing the model numerically. In Section 3.1 we set the parameter values of the model and then solve the model globally using a parameterized expectation algorithm, with the details of the solution method in Appendix B. Section 3.2 studies two key features of the model, namely, procyclical leverage and endogenous bank run probability. Section 3.3 analyzes the dynamics of the model during a time of crisis. Section 3.4 examines the model by running crisis regressions à la Schularick and Taylor (2012) using the data simulated from the model. Finally, Section 3.5 studies the role of macroprudential policy in addressing bank-run-led crises.

3.1 Parameterization

Each period in the model represents a quarter. The model parameters are divided into two sets: parameters that are standard in the RBC literature and banking sector parameters that are specific to the model. The model is calibrated to broadly represent the US economy.

Standard parameters The preference discount factor β is set to $\beta = 1.03^{-1/4}$, implying the annual risk-free interest rate of 3 percent in steady state where there is no TFP shock. The coefficient for labor disutility ψ is set so that the labor supply in steady state is normalized to unity. The labor supply elasticity ν is set to 2, which is consistent with estimates in the literature reviewed in Keane and Rogerson (2012). The capital share in production α, TFP shock persistence ρ_a, and its standard deviation σ_a are also set in line with the RBC literature, such as King and Rebelo (1999).

Banking sector parameters The liquidation cost parameter λ is set to 0.1765, so that the liquidated value of one unit of bank asset is 15 percent lower than it would be were the asset held to maturity. This discount rate is identical to that used by Gertler and Kiyotaki (2015). The amount of capital injection after a bank run, $\bar{n}$, is set to 30 percent of

Table 1: Parameter values

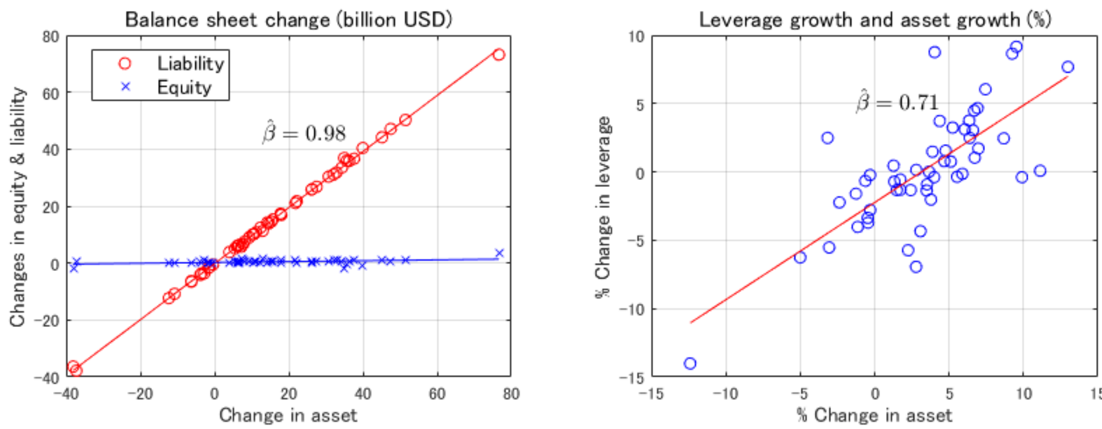
	Parameter description	Value	Target/Source
	Standard parameters		
β	Preference discount factor	0.9926	Steady state risk-free rate of 3%
ψ	Labor disutility coefficient	0.3830	Steady state labor supply of unity
ν	Labor supply elasticity	2	Keane and Rogerson (2012)
α	Capital share in production	0.33	Standard RBC literature
ρ_a	TFP shock persistence	0.95	Standard RBC literature
σ_a	TFP shock standard deviation	0.01	Standard RBC literature
	Banking sector parameters		
λ	Liquidation cost	0.1765	Liquidation discount by 15%
$\overline{n}$	Capital injection during a crisis	0.0055	30% of steady state bank capital
γ	Threshold default probability	0.5349	Crisis probability of annual 5%
n_0	New banker endowment	0.00085	Leverage of 10
χ_1	Law of motion for bank capital	0.95	Gertler and Kiyotaki (2015)
χ_0	Degree of retained earning	0.025	Leverage procyclicality

bank capital in the stochastic steady state.[4] This amount is consistent with Gertler et al. (2020b), who use a similar value in light of the evidence that bank equity measured by the XLF index dropped by 70 percent from 2007Q3 to 2008Q4. The threshold bank default probability γ and new banker endowment n_0 are jointly set for the model to hit bank-run-led crisis probability of 5 percent annually and leverage of 10, both in the stochastic steady state. The target value of the crisis probability is consistent with historical experience that in any given country, banking crises occur on average once every 20 to 25 years, i.e. the average annual crisis probability of 4 to 5 percent (Basel Committee on Banking Supervision, 2010a). The leverage of 10 is chosen following Gertler and Kiyotaki (2015).[5] Also, again following Gertler and Kiyotaki (2015), the survival probability of bankers χ_1 in the law of motion for bank capital is set to 0.95. Finally, parameter χ_0, which governs the fraction of net profits that the bank keeps at hand and adds to its own net worth, is set to $\chi_0 = 0.025$ in order to capture the procyclical bank leverage observed in the data, which

[4]The stochastic steady state is the state of the economy that is reached when agent behavior takes stochastic TFP shocks into account, but where the shocks never materialize and stay at zero.

[5]Laux and Rauter (2017) report that the average leverage over 934 US banks is around 10 during the period from 1990Q3 to 2013Q1.

Figure 2: Procyclical leverage: US investment banks before the GFC



Note: Each data point is the weighted average of the following five major investment banks in the US before the GFC: Bear Stearns, Goldman Sachs, Lehman Brothers, Merrill Lynch, and Morgan Stanley. The sample period is 1994Q2-2007Q3, where the starting period is chosen so that data for at least three of the five investment banks are available, and the end period corresponds to one period ahead of the Great Recession in the US. The recession periods of 2001Q1-2001Q4 are excluded to focus on 'normal' periods. $\hat{\beta}$ indicates a linear regression coefficient in each panel. The data source is Bloomberg.

will be further explored in Section 3.2. Parameter values are summarized in Table 1.

Procyclical leverage in the data Leverage has been procyclical for investment banks and commercial banks in the US, as pointed out by Adrian and Shin (2010, 2011) and later confirmed by Laux and Rauter (2017) in a larger sample. They argue that US banks managed balance sheet size by actively adjusting debt and leverage but keeping bank equity almost unchanged. Such bank behavior led to procyclical leverage. Figure 2 reproduces their results for major US investment banks during the periods before the GFC. The left panel shows that a change in assets on the bank balance sheets is driven mainly by a change in liabilities. It also shows little change in bank equity, indicating sticky bank equity (Adrian and Shin, 2011). The right panel shows that the asset growth is positively correlated with the leverage growth. These results indicate that the leverage was procyclical in the run-up to the GFC. Procyclical leverage especially during booms brings an important implication for our model as high leverage makes the banking system vulnerable to negative shocks that may trigger a bank run.

3.2 Key Features of the Model

Procyclical leverage We set parameter χ_0 in the law of motion for bank capital (29) in a way that the model replicates the facts about procyclical leverage, shown in Figure

2, during 'normal' periods, including boom periods. Here, a 'normal' period is defined as that where no bank run has occurred in the past 12 periods and where bank net profits are non-negative.[6] We focus on these periods in the model for two reasons. First, procyclical leverage during booms is critical in explaining credit booms, increases in bank vulnerability, and resulting increases in crisis probability, as highlighted by Schularick and Taylor (2012). Second, in this type of model, including Gertler and Kiyotaki (2015), during bank-run-led crises or fundamental hardships where bank net profits become negative, the bank capital drops sharply so that the bank leverage inevitably increases and becomes countercyclical.

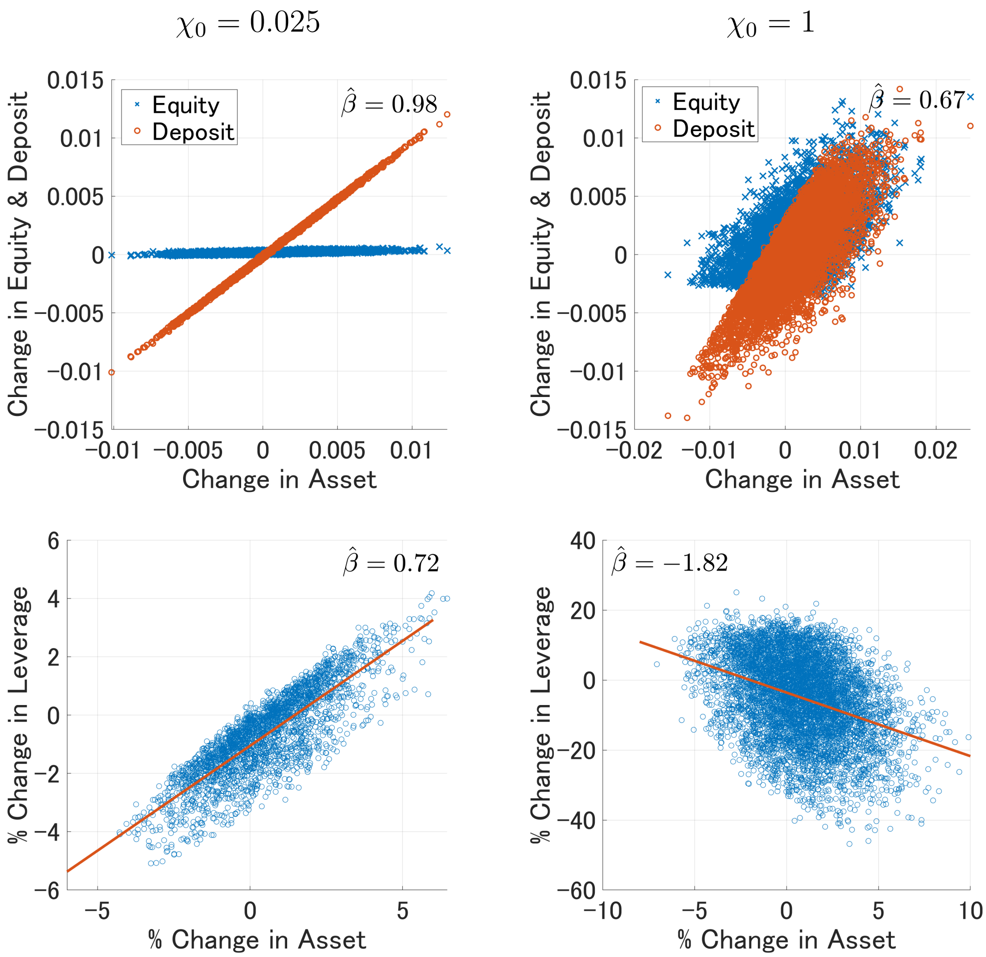
We simulate the model and plot the same variables as in Figure 2. The two left panels in Figure 3 present the results in the model with our parameter choice of $\chi_0 = 0.025$. The upper left panel shows that a change in the bank assets is mostly driven by a change in the bank borrowing (red plots) while the bank equity is almost flat (blue plots), consistent with the empirical evidence presented in Figure 2 and Adrian and Shin (2010, 2011). The lower left panel plots the same fact from a different perspective by plotting a percentage change in leverage against a percentage change in bank assets as in Figure 2 and Laux and Rauter (2017). The regression coefficient is 0.72, which implies a high degree of procyclicality in leverage, consistent with the empirical estimate of 0.71 reported in Figure 2 for US investment banks and the estimate of 0.69 reported by Laux and Rauter (2017) for US banks. These results indicate that our baseline model with $\chi_0 = 0.025$ captures the dynamics of bank balance sheets observed in the data well.

The value of $\chi_0 = 0.025$ implies a 97.5 percent dividend payout ratio in 'normal' periods. In light of empirical evidence that the dividend payout ratio was 40 percent or above during the periods from the late 1990s through 2006 for US banks (Floyd et al., 2015), is $\chi_0 = 0.025$ too low? What drives this discrepancy lies in the asset growth trend observed in the data (Figure 2) but missed in the model (Figure 3). In the data, the average growth rate in assets is about 12 percent annually and the average return on equity (ROE) is just below 20 percent. This implies that, with a 40 percent dividend payout ratio, the bank capital would increase by 12 ($= 20 \times (1 - 0.4)$) percent, keeping the leverage unchanged, as the assets also grow at the same rate. In our model, this situation corresponds to almost no retained capital – the value of χ_0 close to zero. Our model abstracts away such an asset growth trend, but captures banks' stable propensity to pay dividends (Floyd et al., 2015), making bank capital sticky relative to bank assets.

The right two panels in Figure 3 point to a critical role of χ_0 in generating procyclical

[6] Consistent with this notion, the sample data used in Figure 2 exclude periods of recession.

Figure 3: Balance sheet dynamics

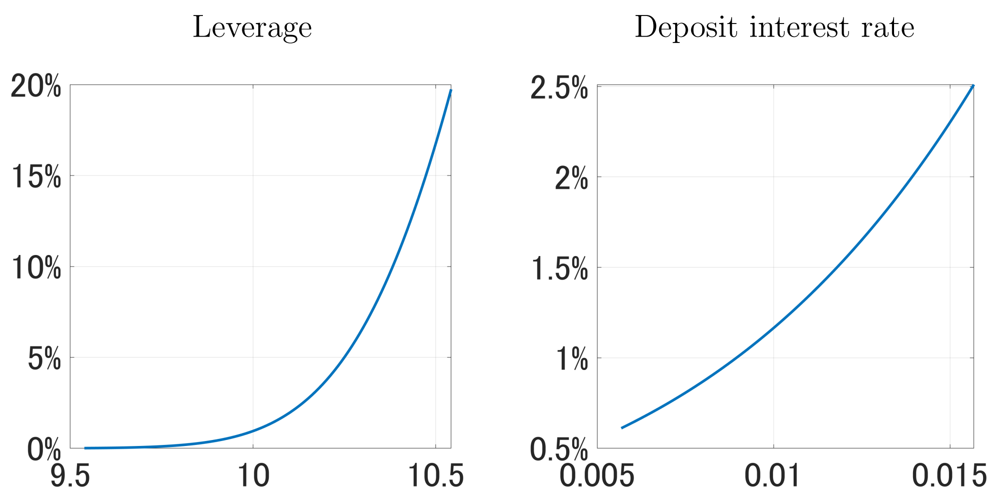


Note: The left panels correspond to the model with our parameter choice of $\chi_0 = 0.025$. The right panels correspond to the model with $\chi_0 = 1$. In the top panels, $\hat{\beta}$ is the regression coefficient of changes in deposit against changes in bank assets.

leverage by plotting the simulated data with $\chi_0 = 1$ instead. The case of $\chi_0 = 1$ is commonly used in the recent dynamic bank-run models such as Gertler and Kiyotaki (2015) and financial friction models such as Bernanke et al. (1999) and Christiano et al. (2014). In this case, bank capital would become much more volatile as shown in the upper panel. In addition, the lower panel shows that the regression coefficient of percentage changes in bank assets on percentage changes in leverage would turn to be negative, which makes a stark contrast to the empirical evidence of the corresponding coefficient of 0.71 mentioned earlier. With $\chi_0 = 1$, high profits during booms increase bank capital and in turn lower leverage, making leverage countercyclical.[7] The analyses here show that a low value of χ_0

[7]The case of $\chi_0 = 1$ is similar to what Adrian and Shin (2010) call "passive" financial intermediaries,

Figure 4: Bank run probability as a function of bank fundamentals



Note: The horizontal axis is leverage in the left panel and the deposit interest rate in the right panel. The vertical axis is the run probability in both panels.

is important in quantitatively capturing the observed dynamics of banks' balance sheets and procyclical leverage.

Endogenous crisis probability The salient feature of our model is endogenous banking crisis probability. In the model, all endogenous variables can be written as a function of state variables. As detailed in Appendix B, the state variables consist of physical capital k_t, bank capital n_{t-1}, and the deposit interest rate $\bar{R}_{t-1}$ as well as the productivity shock. The leverage is given by k_t/n_{t-1}. Hence, the crisis probability can be written as a function of these variables.

Figure 4 plots the crisis probability as a function of bank fundamentals, namely leverage and the deposit interest rate. The left panel shows that the crisis probability is monotonically increasing in leverage in a convex way, where a change in leverage is driven by a change in bank assets k_t with other state variables fixed at their stochastic steady state values. The right panel shows that the crisis probability is increasing in the deposit interest rate. As analytically shown in equation (30), leverage and the interest rate both increase the run threshold, which in turn raises the crisis probability. This analytical feature also holds for the numerically solved policy function for the crisis probability. As leverage and the interest rate increase, the banks become more vulnerable to negative shocks, and thereby

which do not adjust leverage actively, and letting their leverage decline during a boom through an increase in bank capital. However, it appears that this does not apply to US banks, where leverage has been procyclical as argued by Adrian and Shin (2010).

the crisis probability increases.

3.3 Crisis Dynamics

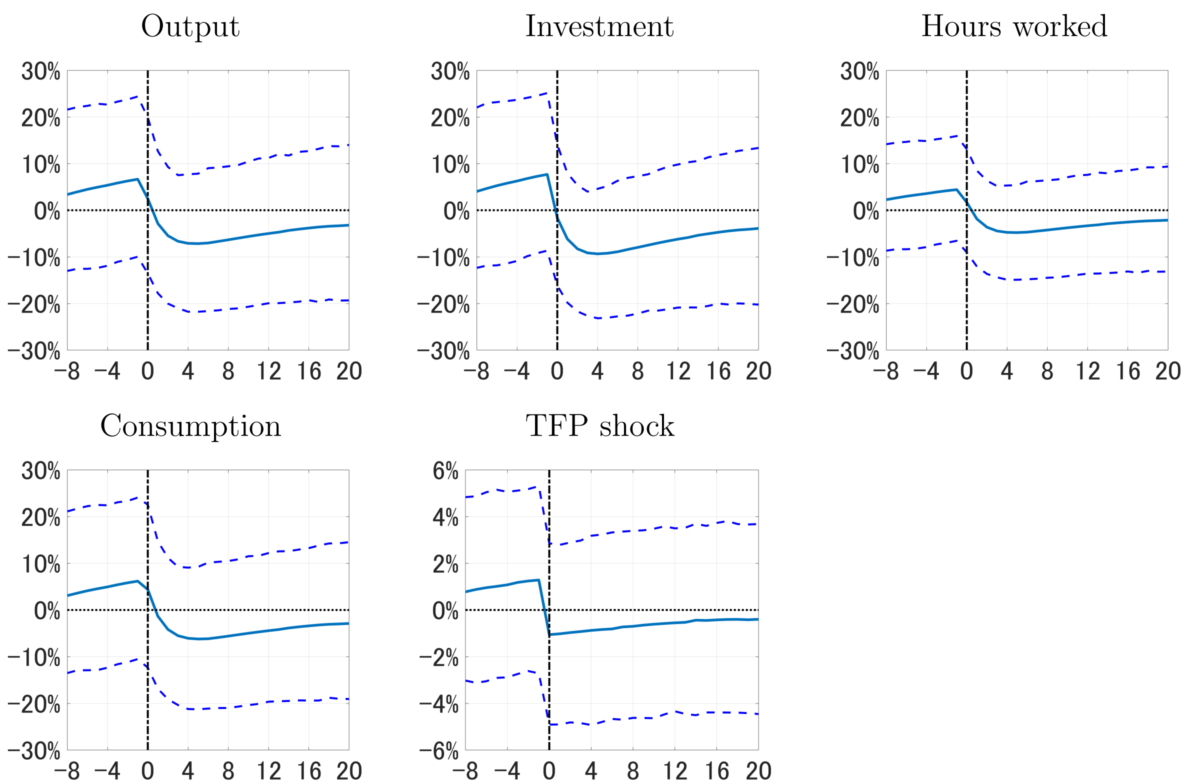
We next turn to the crisis dynamics in the model. To this end we simulate the calibrated model for 100,000 periods with stochastic shocks and drop the first 1,000 periods. In this simulated sample, we identify a banking crisis, which is defined as a bank run such that 3 years (12 periods) or more have passed since the previous bank run, if any. This definition is similar in spirit to Laeven and Valencia (2013), whose empirical study defines the end of a banking crisis as the point at which two consecutive years of output and credit growth has occurred. In addition, the empirical studies such as those of Schularick and Taylor (2012) and Aikman et al. (2019) suggest a build-up of vulnerabilities in the financial system in the three to five years leading up to a financial crisis. Our definition of a banking crisis aims to capture such crises in practice.

We pick up all banking crises thus defined from the 99,000-period simulation. We now analyze the dynamics of the real economy and the banking sector around banking crises, and consider the effects that bank runs exert beyond exogenous negative shocks.

Real economy The simulation shows that a typical banking crisis occurs when the real economy is booming, and causes a severe recession afterwards. Figure 5 plots the average dynamics of the real economy (thick lines) around banking crises that occur in period 0 in the horizontal axis with the 10th and 90th percentiles of the dynamics (dashed lines). All the variables are expressed in percentage deviations from the sample mean. The average dynamics show a typical boom-bust cycle: before a crisis, output, investment, consumption, and hours worked are all booming, well above the sample mean. When a banking crisis erupts, all the variables drop sharply and a persistent recession ensues. The lower middle panel shows the average path of the TFP that triggers this boom-bust cycle. In the run-up to a typical banking crisis the TFP increases and it becomes more than 1% higher than its sample average just before the crisis. The negative TFP shock at period 0, which draws down the TFP level by 2%, triggers a bank run, resulting in a severe recession.

Banking sector The simulation reveals a build-up of banking sector vulnerabilities in the run up to the crises. Figure 6 plots the average dynamics of the banking sector around the crises in the simulation. During the two years preceding a typical crisis, the bank

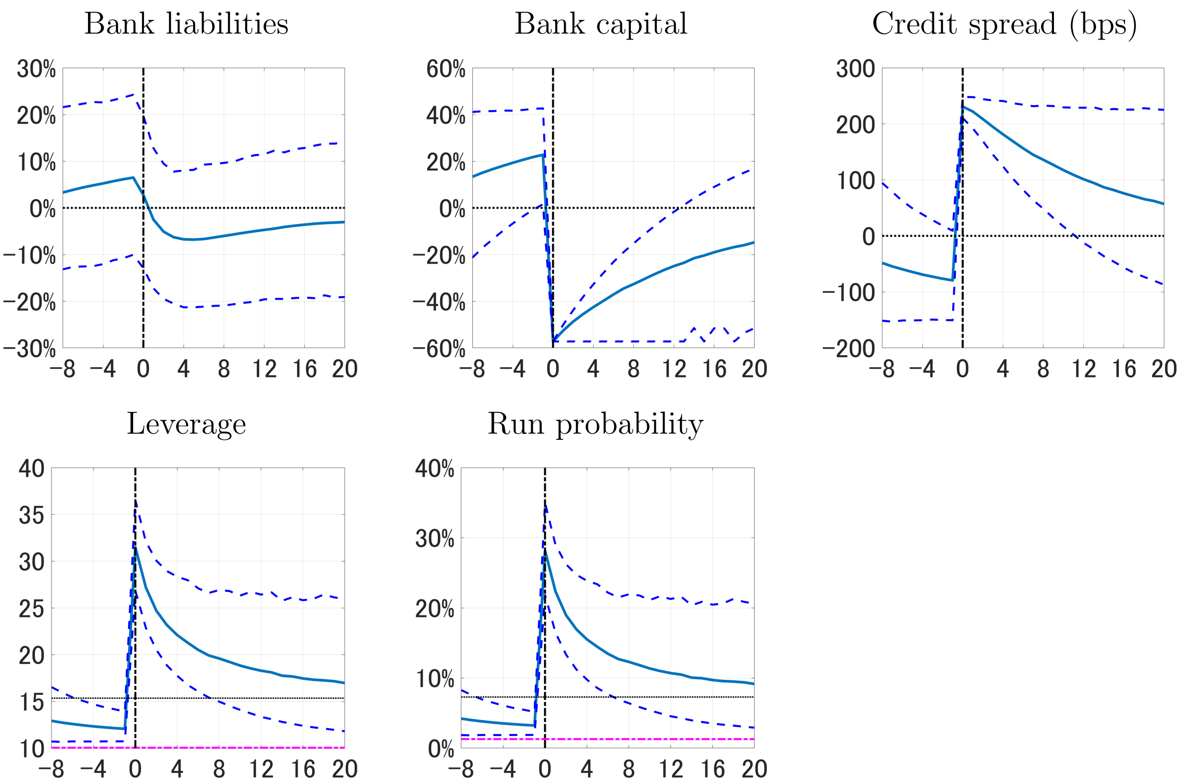
Figure 5: Crisis dynamics of the real economy



Note: The solid lines are the average path of each variable around banking crises. The dashed lines are the 10th and 90th percentiles of the dynamics. All variables are expressed in percentage deviations from the sample mean.

liabilities – borrowing from the household sector – expand and attain a much higher level than its sample mean (upper left panel). The bank capital also expands (upper middle panel), reflecting strong bank profits in the boom periods. The increases in bank liabilities and capital feed into an increase in the supply of credit to the real economy and boost investment as shown in Figure 5. During this credit boom, the credit spread – the difference between the expected return on bank assets and the deposit interest rate $(E_t r^k_{t+1} - \bar{R}_t)$ – is suppressed relative to its stochastic mean (upper right panel) as the marginal return on physical capital remains low due to an increase in physical capital. Furthermore, the leverage is higher than its stochastic steady state level (dot dashed line) by more than 2 (lower left panel). As a result, the average run probability climbs to about 13 percent annually, which is much higher than its stochastic steady state level of 5 percent annually (lower right panel). Together with the result on the real economy shown in Figure 5, Figure

Figure 6: Crisis dynamics of the banking sector

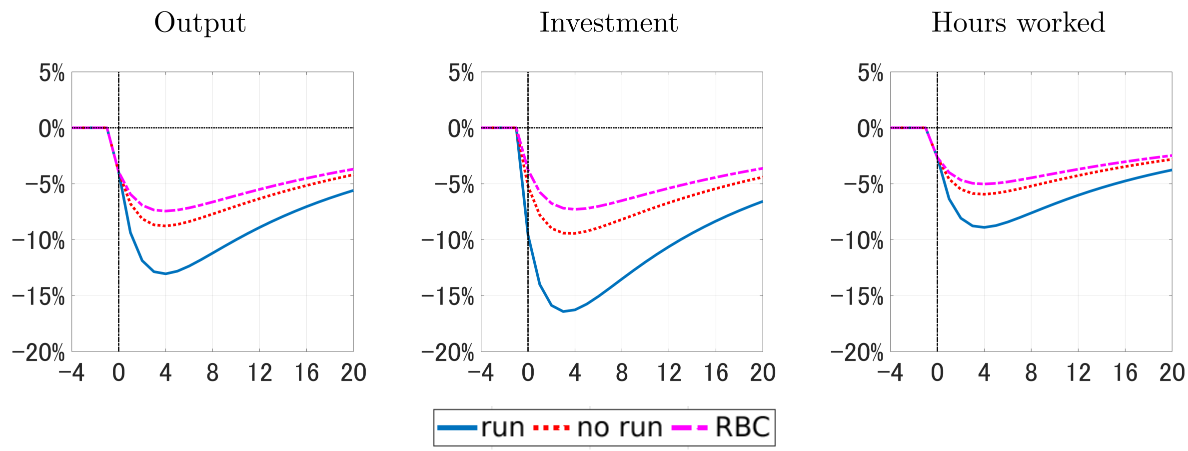


Note: The solid lines are the average path of each variable around banking crises. The dashed lines are the 10th and 90th percentiles of the dynamics. Bank liabilities and bank capital are expressed in percentage deviations from the stochastic mean. Credit spread is expressed in a difference from the stochastic mean in basis points (bps). Leverage and run probability are expressed in actual values.

6 indicates that a boom in the real economy preceding a crisis is fueled by credit expansion and high bank leverage. This is consistent with the empirical literature, which documents that credit expansion is a robust precursor of banking crises. We will discuss this point in more detail in the next subsection.

A banking crisis makes the banking system even more vulnerable to negative shocks in the aftermath of the crisis. Once a crisis erupts, bank leverage shoots up as bank capital plummets. The bank has to restart its operations with the small amount of bank capital $\bar{n}$ injected by the government. Because of such a low level of bank capital, the bank leverages its lending substantially, but the amount of lending stays subdued and only recovers slowly. The crisis probability shoots up in accordance with the increase in leverage and stays high for a while from the start of the crisis. The persistently high leverage and run probability reflect the effects of many possible bank runs repeatedly happening after the initial run in

Figure 7: The effects of bank runs on the real economy



Note: All lines are expressed in percentage deviations from the stochastic steady state values.

period 0 in Figure 6. The path of bank capital shows this point more clearly. The flat line of the bottom 10th percentile in bank capital indicates that bank runs can happen repeatedly, keeping bank capital at $\bar{n}$ potentially for three years or even longer. These repeated bank runs slow down the rebuilding of bank capital, which in turn slows down the economic recovery following a banking crisis.

Effects of bank runs In the model, a bank run is triggered by a negative shock. How large, beyond the effect of a negative shock, is the effects of a bank run on the severity of a following recession? To address this question, we compare the crisis dynamics in the following three cases. The first case corresponds to run dynamics where the negative TFP shock, which is just large enough to trigger a bank run, hits the economy at the stochastic steady state. The second case corresponds to no run where a negative shock that is marginally smaller than the initial shock hits the economy at the stochastic steady state. In this case, a run is not triggered although the size of the shock is essentially the same. The third case corresponds to the dynamics of the RBC model, which abstracts away the banking sector from the model presented in Section 2, in response to the same negative shock.

Comparison of the three cases reveals that a bank run and financial friction have substantial negative effects beyond the negative shock itself. Comparing the RBC case and the no-run case shines light on the role of the banking sector as an amplifier. Figure 7 shows that the banking sector in the model amplifies the effects of a negative shock on the economy and prolongs the effects, but to a limited extent. By contrast, a bank run *substantially* amplifies the effects of the shock. In the run case, output and hours decrease twice as much and investment drops more than double the amount that occurs in the RBC case. This comparison shows that a bank run plays a crucial role in shaping the dynamics of our model.

3.4 Credit Growth and Crisis Probability

The empirical literature on banking crises has found that credit expansion is a significant precursor of banking crises. In their influential paper, Schularick and Taylor (2012), using extensive data covering 14 countries over more than 100 years, show that a credit boom is a significant and robust indicator of a high risk of banking crises.[8]

To test the empirical performance of our model, we run regressions similar to those in Schularick and Taylor (2012) using artificial data generated from the model simulation. Specifically, we use observations in the 99,000-period stochastic simulation of our model and run two types of regressions. The first specification is a simple OLS regression, given by

$$p_t = \beta_0 + \beta_1 \frac{d_{t-1} - d_{t-5}}{d_{t-5}} + \beta_2 \frac{d_{t-5} - d_{t-9}}{d_{t-9}} + \beta_3 \frac{d_{t-9} - d_{t-13}}{d_{t-13}} + \varepsilon_t,$$

where ε_t is an error term that is i.i.d. with mean zero. The dependent variable p_t takes 1 if a banking crisis happens within a year (four quarters), and 0 otherwise, given by

$$p_t = \begin{cases} 1 & \text{if a banking crisis occurs in the period from } t \text{ to } t+3 \\ 0 & \text{if no crisis} \end{cases}$$

where a banking crisis is identified following the definition given in Section 3.3. The three explanatory variables are the annual growth rate of deposits in the past three years. To be consistent with the empirical analyses in Schularick and Taylor (2012), who use annual data, we transform our quarterly model observations into annual data by using time-t observations with t being a multiple of four.

[8]Gourinchas and Obstfeld (2012) also show empirically that a credit boom precedes financial crises both in advanced and emerging economies.

Table 2: Schularick and Taylor (2012) banking crisis regressions

	OLS	Logit	$\partial p_t / \partial x_t$
1 year ago	$0.120^{***}(0.044)$	$1.255^{***}(0.457)$	0.119
2 years ago	$0.160^{***}(0.044)$	$1.672^{***}(0.460)$	0.159
3 years ago	$0.187^{***}(0.043)$	$1.952^{***}(0.450)$	0.185

Note: *** denotes a 1% significance. Parentheses show standard deviations. $\partial p_t / \partial x_t$ denotes the marginal effects of the logit specification.

The second specification is a logit regression, given by

$$p_t = \frac{1}{1 + \exp\left(-\left(\beta_0 + \beta_1 \frac{d_{t-1} - d_{t-5}}{d_{t-5}} + \beta_2 \frac{d_{t-5} - d_{t-9}}{d_{t-9}} + \beta_3 \frac{d_{t-9} - d_{t-13}}{d_{t-13}}\right)\right)} + \varepsilon_t,$$

where the variables are defined similarly as in the OLS specification. The logit specification guarantees that the right-hand side takes at value between 0 and 1. Using these two regressions, we test whether a credit expansion in the past three years can predict a banking crisis in the next year.

Table 2 presents our results. In both specifications, all the explanatory variables are significant at 1% with a positive sign, implying that a credit expansion indicates a significantly high probability of a banking crisis in the next three years. The last column in the table evaluates the marginal effects of the credit growth in the past three years in the logit regression by using the unconditional probability of banking crises. These numbers suggest that the marginal effects are very similar in both specifications. In addition, the sum of the coefficients is about 0.47, which is very close to the 0.40 in Schularick and Taylor (2012).

We further follow Schularick and Taylor (2012) and study the impacts of credit growth on the probability of a banking crisis. In our model simulation, the standard deviation of the average annual credit growth over three years is 4 percent.[9] This implies that a one standard deviation increase in credit growth in the past three years raises the probability of a crisis in the next year by 1.8 percentage points (pp). Since the crisis probability in the stochastic steady state is 5 percent, the increase of 1.8pp is substantial, albeit lower than the 2.8pp reported in Schularick and Taylor (2012). Overall, our model successfully replicates the stylized facts about credit expansion and crisis probability documented by

[9] To derive this number, we first compute the gross deposit growth rate over three years and take its cubic root for each observation, which is the average annual deposit growth rate over three years. We compute the standard deviation of this variable through the entire sample.

Schularick and Taylor (2012).

3.5 Macroprudential Policy

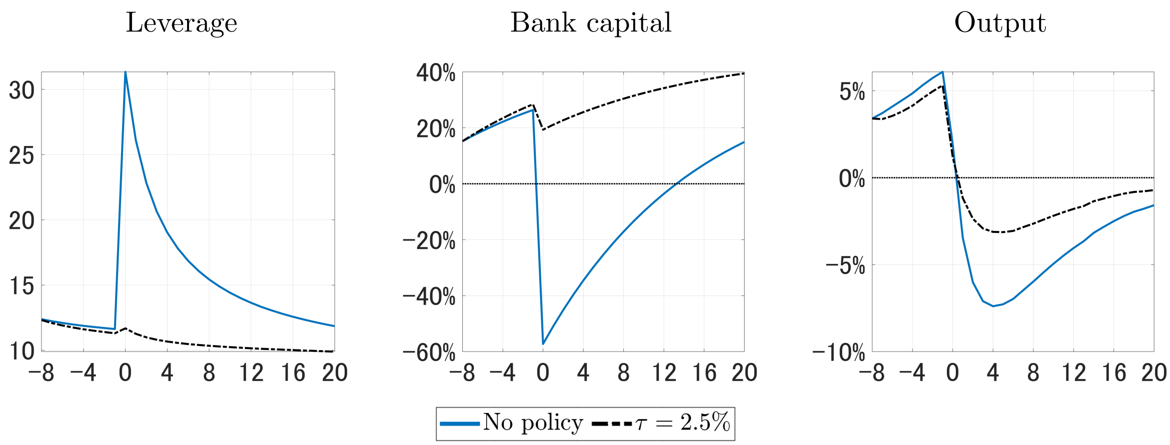
Thus far we have shown that the model generates two key features of banking and crises: (i) procyclical leverage, consistent with Adrian and Shin (2010), and (ii) banking crises that tend to follow credit booms, consistent with Schularick and Taylor (2012). Using the model, we now consider the role of macroprudential policy in addressing banking crises. We first discuss the sources of inefficiency in our model. We then introduce leverage restrictions into the model and present some quantitative results and welfare implications.

Sources of inefficiency The main source of inefficiency in our model is an externality associated with the choice of leverage by banks. For the purpose of exposition, consider a continuum of banks instead of a representative bank. When banks choose leverage, they take as given the deposit interest rate determined in the competitive market. However, if all banks were to reduce leverage, the bank run probability would be lower and the expected recovery rate of deposit would be higher. In this case, households would be willing to accept a lower deposit interest rate, and the interest rate would actually be lower. A lower deposit interest rate would in turn decrease liquidation costs, making bank runs less likely. It would also increase bank profits, and banks would be able to accumulate a larger amount of bank capital. As a result, the banking sector would be more resilient against negative shocks. But, because banks do not internalize the fact that lowering leverage would lower the deposit interest rate and make the banking sector more resilient, they choose instead socially excessive leverage.[10] This over-leverage by banks may justify policy interventions in our model.

Leverage restrictions We consider two types of leverage restrictions in the following analysis. The first type limits bank leverage by a fixed percentage relative to the leverage that would be chosen in the economy without any restrictions. Specifically, let L_t^* denote leverage that would be chosen by banks if there were no policy. Then, the leverage

[10] Ikeda (2018) shows that the source of inefficiency in the version of a two-period model of bank runs is two-fold: bank risk shifting and a pecuniary externality that works through the interest rate. The bank risk shifting arises because the bank takes the deposit interest rate as given in choosing leverage as in Acharya (2009) and Mendicino et al. (2020). The pecuniary externality, which is similar in spirit to that studied by Christiano and Ikeda (2016), arises because the interest rate constitutes the cost of early liquidation. These two types of inefficiency can be put simply as an externality associated with bank leverage choice.

Figure 8: Leverage restrictions by a fixed percentage



Note: Leverage is expressed in actual values. Bank capital and output are expressed in percentage deviations from the stochastic mean.

restriction is written as $L_t \leq (1-\tau)L_t^*$, where τ governs the tightness of the restriction. By construction, the restriction is always binding and the leverage becomes lower by $(100 \times \tau)\,\%$ at any time.

From a social welfare perspective, the tightness τ should balance the benefits of reducing crisis probability against the cost of restraining credit and thus economic activity. Indeed, the social welfare measured by the household's expected life-time utility is non-monotonic with respect to the tightness τ. The social welfare is increasing in τ when τ is small, reaches its maximum at $\tau = 0.025$, and is decreasing in τ when $\tau > 0.025$.

The restriction with $\tau = 0.025$ can make the banking sector more resilient and prevent a run from happening. To illustrate this, we simulate and compare two model economies, with and without the restriction, by using the average dynamics of banking crises presented in Section 3.3. Specifically, in both economies, we set the initial state of the simulation to the state variables at $t-8$ from the average dynamics. Then we feed the average path of TFP shocks from $t-8$ to $t+20$ to these economies and compare the dynamics. The result is plotted in Figure 8. Before the negative TFP shock hits the economy at period 0, the leverage restriction lowers leverage compared to the economy without restrictions. Lower leverage implies lower credit to the real economy, thus it reduces investment and output as shown in the right panel. This is the costs of leverage restrictions. When the negative TFP shock hits in period 0, a bank run erupts in the economy without restrictions. By contrast, a run does not occur in the economy with the restriction. The leverage restriction

Table 3: Stochastic simulation means of selected variables

Variable	Stochastic steady state No policy	Stochastic steady state $\tau = 2.5\%$	Stochastic simulation mean No policy	Stochastic simulation mean $\tau = 2.5\%$
Output	0.573	0.569 (-0.75%)	0.560	0.570 ($+1.80\%$)
Consumption	0.389	0.387 (-0.51%)	0.384	0.389 ($+1.24\%$)
Leverage	10.041	9.545 (-4.94%)	15.343	10.566 (-31.14%)
Run probability	1.3%	0.1% (-1.16%)	7.3%	0.4% (-6.91%)
Crisis probability	–	–	1.7%	0.2% (-1.46%)

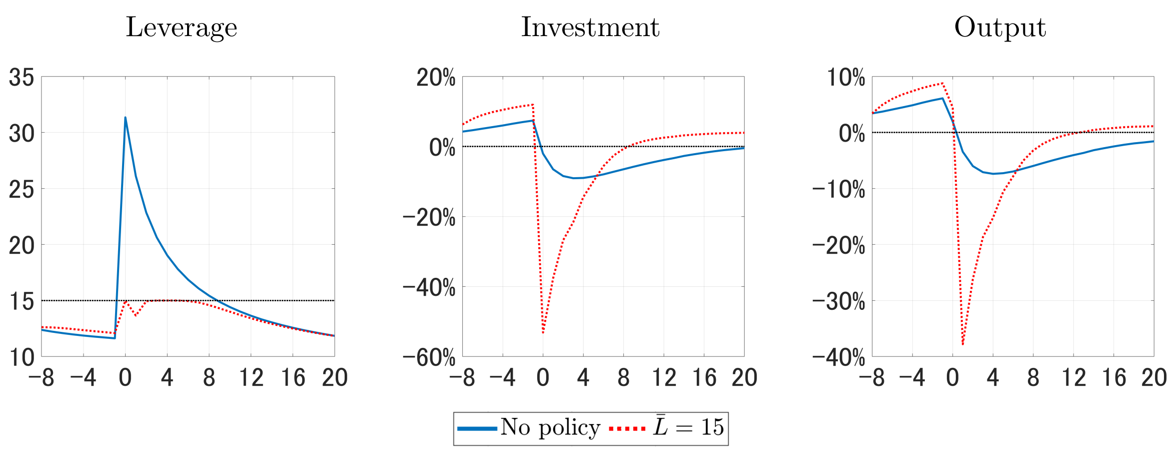
successfully prevents a run from happening. Bank capital remains positive and a decrease in output is substantially limited compared to the case without the restriction, illustrating the benefits of leverage restrictions.

Table 3 compares the key moments of the economy with and without the restriction. The first two columns of Table 3 present the values at the stochastic steady state. Numbers in parentheses are percentage gaps from the values under no policy. The leverage restriction lowers leverage, thereby lowering a run probability substantially from 1.3% without policy to 0.1% with policy. The cost of this lower leverage is that leverage restriction reduces output and consumption as well.

The latter two columns of Table 3 present the means of the entire sample of the 99,000-period stochastic simulation including bank runs. With no policy, the sample means of output and consumption are lower than those in the stochastic steady state in the first column. These lower values reflect the effects of bank runs, which are accompanied by persistently low output and consumption. For the same reason, leverage and run probability are substantially higher than those in the stochastic steady state. By contrast, under the leverage restriction, the sample means in the fourth column are essentially the same as those in the stochastic steady state in the second column. This is because the leverage restriction reduces the number of bank runs substantially in the stochastic simulation.

Comparing the sample means with and without the restriction in the third and fourth columns, both consumption and output become higher under the restriction by more than 1 percent, in contrast to the case of the stochastic steady state. The sample means of leverage and run probability are substantially lower under the restriction, implying that the economy becomes more stable and resilient under the restriction. The bottom row is the unconditional probability of banking crises, which is computed as the number of banking

Figure 9: Fixed leverage restrictions



Note: Leverage is expressed in actual values. Investment and output are expressed in percentage deviations from the stochastic mean.

crises divided by the total number of periods. The restriction reduces the probability of banking crises substantially.

Our results suggest that there is a trade-off associated with macroprudential policy, in line with the literature on this much discussed trade-off. Macroprudential policy reduces leverage and thereby discourages economic activity in normal times. But it also reduces the probability of a banking crisis and makes the economy and the financial system more resilient. But, what our model adds to the literature is the explicit modeling of endogenous crisis probability. In addition, our model shows that the welfare gained by reducing the probability of a banking crisis quantitatively dominates the cost of discouraging economic activity in normal times.[11]

Role of countercyclical restrictions The first type of leverage restrictions, namely $L_t \leq (1-\tau)L_t^*$, is countercyclical in that it allows banks to have high leverage especially when bank capital is damaged and the banking sector needs high leverage to supply credit to the real economy. To illustrate the role of this countercylicality, we consider the second type of leverage restrictions: a fixed upper limit on bank leverage. As an illustration, we

[11]We compute the expected welfare gain measured by a permanent consumption gap that equalizes households' expected utility with and without policy, which is standard in the literature. In the case of asymptotically zero disutility of early withdrawals, $\kappa \to 0$, the expected welfare gain is 0.05% of permanent consumption, which is substantially lower than the percentage gap in the sample mean of consumption in Table 3. This is because higher labor disutility offsets the welfare gain by higher consumption. Because the welfare gain is computed under the assumption of $\kappa \to 0$, it should be regarded as the lower bound.

consider $L_t \leq \bar{L} = 15$. We then conduct the same simulation as above, namely we start the simulation from the initial state at $t-8$ of the average crisis dynamics and feed the average TFP path from $t-8$ to $t+20$ into the two economies, with and without the restriction.

The result is plotted in Figure 9. The fixed leverage restriction does not prevent a bank run, because $L_t \leq 15$ is a relatively loose restriction. But our focus lies in the effects of this policy on the economy during and after a banking crisis. When a bank run occurs in period 0, the fixed leverage restriction becomes binding and leverage is restrained at $L_t = 15$. Given that bank capital is wiped out and only $\bar{n}$ units are injected, low leverage implies a sharp contraction in investment as shown in the middle panel. Output also drops sharply because capital shrinks substantially. As a result, this fixed leverage restriction exacerbates the crisis, thereby reducing welfare compared to the laissez-faire economy with no restriction. This experiment indicates the importance of the countercyclicality of leverage restrictions, as is often emphasized in the literature. When a banking crisis actually happens, macroprudential policy needs to be loosened so that the banking sector does not deleverage substantially and can continue to supply credit to the real economy.

4 Conclusion

This paper has developed a dynamic general equilibrium model of bank runs. By incorporating the global game bank run model of Rochet and Vives (2004) into the otherwise standard RBC model with a banking sector, the model features endogenous bank run probability, which is derived as a function of bank fundamentals, bank leverage in particular. Thus, the model articulates a link between bank fundamentals and banking crisis probability – the core idea of Basel III – in the infinite horizon economy.

The model generates procyclical leverage – a salient feature of the banking sector pointed out by Adrian and Shin (2010). In addition, by combining the procyclical leverage with the endogenous crisis probability, the model replicates the seminal empirical finding of Schularick and Taylor (2012) that credit growth tends to precede banking crises. The model simulation shows that countercyclical leverage restrictions, which allow banks to have high leverage when they actually need it, can improve social welfare by balancing the benefits of reducing the crisis probability against the costs of suppressed economic activity in normal times.

We would like to conclude with some caveats and limitations of the paper. First, the model generates procyclical leverage by introducing a parameter that makes bank capital sticky. While this paper already has made the model rich by introducing a global game, enriching the background of procyclical leverage would be important. For example, Nuño and Thomas (2017) explain leverage cycles by extending a value-at-risk bank model studied by Adrian and Shin (2014). Second, while the model exclusively focuses on leverage as a bank choice, banks in practice choose other important variables that could affect financial stability. Specifically, bank liquidity is an essential safeguard against run risk. Ikeda (2018) studies the interaction between leverage and liquidity choices and restrictions using a two-period model with bank runs in a similar global game framework. Third, the model abstracts away from monetary policy. The fact that the crisis probability depends on the deposit interest rate in the model suggests that monetary policy could affect the crisis probability. Incorporating monetary policy in the model is on our agenda for research next.

Despite all these caveats, we hope that this paper will be useful for understanding and promoting financial stability.

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Appendix

A Analytical Results

A.1 Proof of Lemma 1

Combining the labor supply and demand curves – equations (5) and (9) – yields

$$h_t = \left(\frac{1-\alpha}{\psi} e^{a_t} k_t^{\alpha}\right)^{\frac{\nu}{\alpha\nu+1}}.$$

Substituting this equation into the marginal return on capital (8) yields

$$r_t^k = \alpha e^{a_t} k_t^{\alpha-1} \left(\frac{1-\alpha}{\psi} e^{a_t} k_t^{\alpha}\right)^{\frac{\nu(1-\alpha)}{\alpha\nu+1}} = \alpha \left(\frac{1-\alpha}{\psi}\right)^{\frac{\nu(1-\alpha)}{\alpha\nu+1}} e^{\frac{1+\nu}{\alpha\nu+1}a_t} k_t^{-\frac{1-\alpha}{\alpha\nu+1}}.$$

Then, the log of bank asset return in period $t+1$ can be written as

$$\hat{r}_{t+1}^k = \log(r_{t+1}^k) = \frac{(1-\alpha)\nu}{\alpha\nu+1} \log\left(\alpha\left(\frac{1-\alpha}{\psi}\right)\right) + \frac{1+\nu}{\alpha\nu+1} a_{t+1} - \frac{1-\alpha}{\alpha\nu+1}\log(k_{t+1}).$$

Because the shock follows the stochastic process (7) and k_{t+1} is predetermined, the log bank asset return follows the normal distribution with mean and standard deviation, given by

$$\begin{aligned}
\mu_{t,k} &= \frac{(1-\alpha)\nu}{\alpha\nu+1} \log\left(\alpha\left(\frac{1-\alpha}{\psi}\right)\right) + \frac{1+\nu}{\alpha\nu+1}\rho_a a_t - \frac{1-\alpha}{\alpha\nu+1}\log(k_{t+1}), \\
\sigma_k &= \frac{1-\alpha}{\alpha\nu+1}\sigma_a.
\end{aligned}$$

A.2 The bank run global game

Uniqueness condition for equations (23) and (24) Equation (24) can be written explicitly as

$$\Phi\left(\sqrt{\frac{1}{\sigma_k^2} + \frac{1}{\sigma_\epsilon^2}}\hat{r}_{t+1}^{k*} - \frac{\frac{1}{\sigma_k^2}\mu_{t,k} + \frac{1}{\sigma_\epsilon^2}s_{t+1}^*}{\sqrt{\frac{1}{\sigma_k^2} + \frac{1}{\sigma_\epsilon^2}}}\right) = \gamma. \tag{A.1}$$

Let $\hat{r}^{k*}(s)$ denote a solution to equation (23) for $\hat{r}_{t+1}^{k*}$ as a function of $s_{t+1}^* = s$. This function has the following properties: $\lim_{s\to-\infty} \hat{r}^{k*}(s) = \log(\underline{r}_{t+1}^k)$ and $\lim_{s\to\infty} \hat{r}^{k*}(s) = \log[\underline{r}_{t+1}^k(1+\lambda)]$, where $\underline{r}_{t+1}^k = \bar{R}_t(1-1/L_t)$. The slope of $\hat{r}^{k*}(s)$ is derived by totally differentiating equation (23) with respect to $\hat{r}_{t+1}^{k*}$ and $s_{t+1}^* = s$ as

$$\frac{d\hat{r}^{k*}}{ds} = \left[1 + \frac{1}{\lambda}\frac{r_{t+1}^{k*}}{\underline{r}_{t+1}^k}\frac{\sigma_\epsilon}{\phi((s-\hat{r}_{t+1}^{k*})/\sigma_\epsilon)}\right]^{-1} \leq \left(1 + \frac{\sigma_\epsilon}{\lambda}\sqrt{2\pi}\right)^{-1},$$

where $\phi(\cdot) \leq 1/\sqrt{2\pi}$ and $r_{t+1}^{k*}/\underline{r}_{t+1}^k = 1 + \lambda\Phi(\cdot) \geq 1$ are used to derive the inequality.

Let the left-hand-side of equation (A.1) be written as

$$P(s) = \Phi\left(\sqrt{\frac{1}{\sigma_k^2}+\frac{1}{\sigma_\epsilon^2}}\hat{r}^{k*}(s) - \frac{\frac{1}{\sigma_k^2}\mu_{t,k}+\frac{1}{\sigma_\epsilon^2}s}{\sqrt{\frac{1}{\sigma_k^2}+\frac{1}{\sigma_\epsilon^2}}}\right).$$

Function $P(s)$ satisfies $\lim_{s\to-\infty} P(s) = 1$ and $\lim_{s\to\infty} P(s) = 0$. Hence, a sufficient condition for a unique solution for $P(s) = \gamma$ is that $P(s)$ is decreasing in s. Function $P(s)$ is decreasing in s if

$$P'(s) \propto \left(\frac{1}{\sigma_k^2}+\frac{1}{\sigma_\epsilon^2}\right)\frac{\partial \hat{r}^{k*}}{\partial s} - \frac{1}{\sigma_\epsilon^2} \leq \left(\frac{1}{\sigma_k^2}+\frac{1}{\sigma_\epsilon^2}\right)\left(1+\frac{\sigma_\epsilon}{\lambda}\sqrt{2\pi}\right)^{-1} - \frac{1}{\sigma_\epsilon^2} < 0,$$

or

$$\sigma_\epsilon < \frac{\sigma_k^2\sqrt{2\pi}}{\lambda}. \tag{A.2}$$

This condition ensures a unique pair of the thresholds, $\hat{r}_{k*}$ and s^*, and it is imposed on the model in the main text.

Proof of Lemma 2 Totally differentiating equations (23) and (A.1) with respect to $\hat{r}^k_{t+1}$, s^*_{t+1}, and L_t yields

$$\begin{aligned} r^{k*}_{t+1} d\hat{r}^{k*}_{t+1} =& \frac{\bar{R}_t}{L_t^2}\left(1+\lambda\Phi\left(\frac{s^*_{t+1}-\hat{r}^{k*}_{t+1}}{\sigma_\epsilon}\right)\right)dL + \bar{R}_t\left(1-\frac{1}{L_t}\right)\lambda\phi\left(\frac{s^*_{t+1}-\hat{r}^{k*}_{t+1}}{\sigma_\epsilon}\right)\frac{ds^*_{t+1}-d\hat{r}^{k*}_{t+1}}{\sigma_\epsilon}, \\ d\hat{r}^{k*}_{t+1} =& \frac{\sigma_k^2}{\sigma_\epsilon^2+\sigma_k^2}ds^*_{t+1}. \end{aligned} \tag{A.3}$$

Arranging these equations leads to

$$\frac{\partial s^*_{t+1}}{\partial L_t} = \frac{\left(1+\frac{\sigma_\epsilon^2}{\sigma_k^2}\right)\frac{\bar{R}_t}{L_t^2}\left(1+\lambda\Phi(\cdot)\right)}{r^{k*}_{t+1} - \frac{\sigma_\epsilon}{\sigma_k^2}\bar{R}_t\left(1-\frac{1}{L_t}\right)\lambda\phi(\cdot)}. \tag{A.4}$$

The denominator is positive under the uniqueness assumption (A.2) and the numerator is also positive. Hence, $\partial s^*_{t+1}/\partial L_t > 0$. From this result and equation (A.3), it follows that $\partial\hat{r}^{k*}_{t+1}/\partial L_t > 0$.

Proof of Lemma 3 Let π_{t+1} denote the bank gross profits in period $t+1$. Then the first-order condition (26) can be written as $\partial\mathbb{E}(\pi_{t+1})/\partial L_t = 0$. Under the assumption that $\partial\mathbb{E}(\pi_{t+1})/\partial L_t$ has a single crossing at zero for $L_t < L_{\max}$, the solution satisfies the second-order condition, $\partial^2\mathbb{E}(\pi_{t+1})/\partial L_t^2 < 0$ if $\lim_{L_t\to 0}\partial\mathbb{E}(\pi_{t+1})/\partial L_t > 0$. From equation (23), $\lim_{L_t\to 1}\hat{r}^{k*}_{t+1} = -\infty$ and thereby $\lim_{L_t\to 1} s^*_{t+1} = -\infty$. This implies $\lim_{L_t\to 1} x(\hat{r}^k_{t+1}, s^*_{t+1}) = 0$ for $\hat{r}^k_{t+1} > \hat{r}^{k*}_{t+1}$. Therefore, equation (26) implies $\partial\mathbb{E}(\pi_{t+1})/\partial L_t = \mathbb{E}(e^{\hat{r}^k_{t+1}}) - \bar{R}_t$ in the limit of $L_t \to 1$. Condition (27) is then equivalent to $\lim_{L_t\to 1}\partial\mathbb{E}(\pi_{t+1})/\partial L_t > 0$.

Derivation of equation (30) in Proposition 1 Consider equations (23) and (A.1) in the limiting case of $\sigma_\epsilon \to 0$. Equation (A.1) implies

$$\lim_{\sigma_\epsilon\to 0}\Phi\left(\frac{s^*_{t+1}-\hat{r}^{k*}_{t+1}}{\sigma_\epsilon}\right) = 1-\gamma. \tag{A.5}$$

This result and equation (A.1) imply

$$\lim_{\sigma_\epsilon \to 0} e^{\hat{r}^{k*}_{t+1}} = \bar{R}_t \left(1 - \frac{1}{L_t}\right) [1 + \lambda(1-\gamma)] \, .$$

Equation (A.5) also implies $\lim_{\sigma_\epsilon \to 0}(s^*_{t+1} - \hat{r}^{k*}_{t+1}) = 0$. This completes the derivation of equation (30).

Derivation of equation (31) in Proposition 1 Consider the first-order condition (26) in the limiting case of $\sigma_\epsilon \to 0$. Consider the second term of the RHS of the condition:

$$\lambda \bar{R}_t \int_{\hat{r}^{k*}_{t+1}}^{\infty} \left[x_{t+1} + \frac{\partial x_{t+1}}{\partial s^*_{t+1}} \frac{\partial s^*_{t+1}}{\partial L_t} (L_t - 1) \right] dF_t(\hat{r}^k_{t+1}).$$

Since $x_{t+1} = \Phi((s^*_{t+1} - \hat{r}^k_{t+1})/\sigma_\epsilon)$ and $s^*_{t+1} \to \hat{r}^{k*}_{t+1}$ as $\sigma_\epsilon \to 0$, it follows that $x_{t+1} = 0$ for $\hat{r}^k_{t+1} > \hat{r}^{k*}_{t+1}$. Then, the term shown above in the limiting case is reduced to

$$\lambda \bar{R}_t (L_t - 1) \lim_{\sigma_e \to 0} \left[\frac{\partial s^*_{t+1}}{\partial L_t} \int_{\hat{r}^{k*}_{t+1}}^{\infty} \frac{\partial x_{t+1}}{\partial s^*_{t+1}} dF_t(\hat{r}^k_{t+1}) \right] .$$

From equation (A.4):

$$\lim_{\sigma_\epsilon \to 0} \frac{\partial s^*_{t+1}}{\partial L_t} = \frac{\frac{\bar{R}_t}{L_t^2} \left(1 + \lambda(1-\gamma)\right)}{r^{k*}_{t+1}} = \frac{1}{L_t(L_t - 1)}.$$

Hence, the term above is reduced to

$$\frac{\lambda \bar{R}_t}{L_t} \lim_{\sigma_\epsilon \to 0} \int_{\hat{r}^{k*}_{t+1}}^{\infty} \phi\left(\frac{s^*_{t+1} - \hat{r}^k_{t+1}}{\sigma_\epsilon}\right) \frac{1}{\sigma_\epsilon} dF_t(\hat{r}^k_{t+1}),$$

where ϕ is the standard normal pdf. Consider the integral term, which is explicitly written as:

$$\int_{\hat{r}^{k*}_{t+1}}^{\infty} \phi\left(\frac{s^*_{t+1} - \hat{r}^k_{t+1}}{\sigma_\epsilon}\right) \frac{1}{\sigma_\epsilon} dF_t(\hat{r}^k_{t+1}) = \int_{\hat{r}^{k*}_{t+1}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{s^*_{t+1} - \hat{r}^k_{t+1}}{\sigma_\epsilon}\right)^2} \frac{1}{\sigma_\epsilon} \frac{1}{\sigma_k \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\hat{r}^k_{t+1} - \mu_{t,k}}{\sigma_k}\right)^2} d\hat{r}^k_{t+1}.$$

The terms in the power of e can be arranged as:

$$
\begin{aligned}
&-\frac{1}{2}\left(\frac{s^*_{t+1}-\hat{r}^k_{t+1}}{\sigma_\epsilon}\right)^2-\frac{1}{2}\left(\frac{\hat{r}^k_{t+1}-\mu_{t,k}}{\sigma_k}\right)^2\\
=&-\frac{1}{2}\left[\frac{(s^*_{t+1})^2-2s^*_{t+1}\hat{r}^k_{t+1}+(\hat{r}^k_{t+1})^2}{\sigma_\epsilon^2}+\frac{(\hat{r}^k_{t+1})^2-2\mu_{t,k}\hat{r}^k_{t+1}+\mu_{t,k}^2}{\sigma_k^2}\right]\\
=&-\frac{1}{2}\left[\left(\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}\right)(\hat{r}^k_{t+1})^2-2\left(\frac{s^*_{t+1}}{\sigma_\epsilon^2}+\frac{\mu_{t,k}}{\sigma_k^2}\right)\hat{r}^k_{t+1}+\frac{(s^*_{t+1})^2}{\sigma_\epsilon^2}+\frac{\mu_{t,k}^2}{\sigma_k^2}\right]\\
=&-\frac{1}{2}\left(\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}\right)\left[(\hat{r}^k_{t+1})^2-2\frac{\frac{s^*_{t+1}}{\sigma_\epsilon^2}+\frac{\mu_{t,k}}{\sigma_k^2}}{\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}}\hat{r}^k_{t+1}+\frac{\frac{(s^*_{t+1})^2}{\sigma_\epsilon^2}+\frac{\mu_{t,k}^2}{\sigma_k^2}}{\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}}\right]\\
=&-\frac{1}{2}\left(\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}\right)\left[\left(\hat{r}^k_{t+1}-\frac{\frac{s^*_{t+1}}{\sigma_\epsilon^2}+\frac{\mu_{t,k}}{\sigma_k^2}}{\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}}\right)^2-\left(\frac{\frac{s^*_{t+1}}{\sigma_\epsilon^2}+\frac{\mu_{t,k}}{\sigma_k^2}}{\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}}\right)^2+\frac{\frac{(s^*_{t+1})^2}{\sigma_\epsilon^2}+\frac{\mu_{t,k}^2}{\sigma_k^2}}{\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}}\right]\\
=&-\frac{1}{2}\left(\frac{\hat{r}^k_{t+1}-\frac{\frac{s^*_{t+1}}{\sigma_\epsilon^2}+\frac{\mu_{t,k}}{\sigma_k^2}}{\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}}}{\sqrt{\frac{\sigma_\epsilon^2\sigma_k^2}{\sigma_\epsilon^2+\sigma_k^2}}}\right)^2+\frac{1}{2}\left[\frac{\left(\frac{s^*_{t+1}}{\sigma_\epsilon^2}+\frac{\mu_{t,k}}{\sigma_k^2}\right)^2}{\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}}-\frac{(s^*_{t+1})^2}{\sigma_\epsilon^2}-\frac{\mu_{t,k}^2}{\sigma_k^2}\right].
\end{aligned}
$$

Then, $\int_{\hat{r}^{k*}_{t+1}}^\infty \phi(\cdot)/\sigma_\epsilon dF(\hat{r}^k_{t+1})$ is written as:

$$
\begin{aligned}
&\int_{\hat{r}^{k*}_{t+1}}^\infty \phi\left(\frac{s^*_{t+1}-\hat{r}^k_{t+1}}{\sigma_\epsilon}\right)\frac{1}{\sigma_\epsilon}dF_t(\hat{r}^k_{t+1})\\
=&\int_{\hat{r}^{k*}_{t+1}}^\infty \frac{1}{\sigma_\epsilon\sigma_k\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\hat{r}^k_{t+1}-\tilde{\mu}_{t,k}}{\tilde{\sigma}_k}\right)^2\right]d\hat{r}^k_{t+1}\times\frac{1}{\sqrt{2\pi}}\exp\left\{\frac{1}{2}\left[\frac{\left(\frac{s^*_{t+1}}{\sigma_\epsilon^2}+\frac{\mu_{t,k}}{\sigma_k^2}\right)^2}{\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}}-\frac{(s^*_{t+1})^2}{\sigma_\epsilon^2}-\frac{\mu_{t,k}^2}{\sigma_k^2}\right]\right\},\\
=&\int_{\hat{r}^{k*}_{t+1}}^\infty \frac{1}{\tilde{\sigma}_k\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\hat{r}^k_{t+1}-\tilde{\mu}_{t,k}}{\tilde{\sigma}_k}\right)^2\right]d\hat{r}^k_{t+1}\times\frac{1}{\sqrt{2\pi(\sigma_\epsilon^2+\sigma_k^2)}}\exp\left\{\frac{1}{2}\left[\frac{\left(\frac{s^*_{t+1}}{\sigma_\epsilon^2}+\frac{\mu_{t,k}}{\sigma_k^2}\right)^2}{\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}}-\frac{(s^*_{t+1})^2}{\sigma_\epsilon^2}-\frac{\mu_{t,k}^2}{\sigma_k^2}\right]\right\},
\end{aligned}
$$

where

$$
\tilde{\mu}_{t,k}=\frac{\frac{s^*_{t+1}}{\sigma_\epsilon^2}+\frac{\mu_{t,k}}{\sigma_k^2}}{\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}}\quad\text{and}\quad\tilde{\sigma}_k=\frac{\sigma_\epsilon\sigma_k}{\sqrt{\sigma_\epsilon^2+\sigma_k^2}}.
$$

Define $z=(\hat{r}^k_{t+1}-\tilde{\mu}_{t,k})/\tilde{\sigma}_k$. Note that $\lim_{\sigma_\epsilon\to 0}(\hat{r}^{k*}_{t+1}-\tilde{\mu}_{t,k})/\tilde{\sigma}_k=\Phi^{-1}(\gamma)$. Then, it follows that

$$
\begin{aligned}
&\lim_{\sigma_\epsilon\to 0}\int_{\hat{r}^{k*}_{t+1}}^\infty \frac{1}{\tilde{\sigma}_k\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\hat{r}^k_{t+1}-\tilde{\mu}_{t,k}}{\tilde{\sigma}_k}\right)^2\right]d\hat{r}^k_{t+1}=\int_{\Phi^{-1}(\gamma)}^\infty\phi(z)dz=1-\gamma,\\
&\lim_{\sigma_\epsilon\to 0}\frac{1}{\sqrt{2\pi(\sigma_e^2+\sigma_k^2)}}\exp\left\{\frac{1}{2}\left[\frac{\left(\frac{s^*_{t+1}}{\sigma_\epsilon^2}+\frac{\mu_{t,k}}{\sigma_k^2}\right)^2}{\frac{1}{\sigma_\epsilon^2}+\frac{1}{\sigma_k^2}}-\frac{(s^*_{t+1})^2}{\sigma_\epsilon^2}-\frac{\mu_{t,k}^2}{\sigma_k^2}\right]\right\}=f_t(s^*_{t+1})=f_t(\hat{r}^{k*}_{t+1}),
\end{aligned}
$$

Therefore, in the limiting case of $\sigma_\epsilon\to 0$, the second term of the RHS of the condition (26) is reduced to $\lambda(1-\gamma)f_t(\hat{r}^{k*}_{t+1})\bar{R}_t/L_t$.

B Numerical Solution

B.1 Solution Algorithm

We solve the model globally using a parameterized expectation algorithm. Specifically, we approximate the right-hand side of the Euler equation as a function of the state variables. The state variables in period t consist of $\left\{k_t, \bar{R}_{t-1}, n_{t-1}, a_t\right\}$. After $k_t, \bar{R}_{t-1}, n_{t-1}$ are chosen in period $t-1$, a stochastic shock determines the realized TFP a_t at the beginning of period t, which in turn determines whether a bank run happens or not. We define the following variable that indicates whether a bank run happens or not:

$$\xi_t = \frac{\exp\left(\hat{r}_t^k\right)}{\exp\left(\hat{r}_t^{k*}\right)}.$$

By construction, $\xi_t \geq 1$ implies that the realized bank return is above the threshold for runs, and a run does not happen. $\xi_t < 1$ implies a run happens. We use this variable as a state variable instead of a_t for approximation.

We create three approximation functions for the right-hand side of the Euler equation, depending on the state of the economy: the first approximation function is for 'normal times' – all cases except the second and third cases, the second one is when a run happens, and the third one is when a run does not happen but a bank profit is negative. The reason for these three functions is that the policy function for the right-hand side of the Euler equation has a kink and a jump across these three states of the economy. We find that having three approximation functions substantially reduces computational errors and improves accuracy of our numerical solution.

Accordingly, we approximate the right-hand side of the Euler equation in the following form:

$$\beta E_t\left[\frac{1}{c_{t+1} - \psi h_{t+1}^{1+\frac{1}{\nu}} / \left(1+\frac{1}{\nu}\right)} v_{t+1}\bar{R}_t\right] \approx \exp\left[P_n\left(\ln\left(k_t\right), \ln\left(\bar{R}_{t-1}\right), \ln\left(n_{t-1}\right), \ln\left(\xi_t\right); \eta_n^i\right)\right],$$

where P_n is the n-th order polynomial function with coefficient vector η_n^i, and $i = 1, 2, 3$ indicates the three states of the economy explained above. We solve the model using third-order polynomials according to the following steps:

(1) Make an initial guess for the polynomial coefficients $\eta_3^1(0), \eta_3^2(0), \eta_3^3(0)$. Generate a series of shocks $\{\epsilon_{a,t}\}_{t=0}^T$ that follows $N(0, \sigma_a^2)$ for $T = 10,000$. Set the initial state at the stochastic steady state.

(2) Simulate the model.

   (a) At each period, given the state variables $k_t, \bar{R}_{t-1}, n_{t-1}, a_t$, we compute the endogenous variables $h_t, y_t, \hat{r}_t^k, \hat{r}_t^{k*}, \xi_t, \pi_t^b$ using the equilibrium conditions. Depending on the values of ξ_t and π_t^b, we use a different polynomial function for the right-hand side of the Euler equation, denoted by g_t:

$$g_t = \begin{cases} \exp\left[P_3\left(\ln(k_t), \ln\left(\bar{R}_{t-1}\right), \ln(n_{t-1}), \ln(\xi_t); \eta_3^1(0)\right)\right] & \text{if } \xi_t \geq 1 \text{ and } \pi_t^b \geq 0 \\ \exp\left[P_3\left(\ln(k_t), \ln\left(\bar{R}_{t-1}\right), \ln(n_{t-1}), \ln(\xi_t); \eta_3^2(0)\right)\right] & \text{if } \xi_t < 1 \\ \exp\left[P_3\left(\ln(k_t), \ln\left(\bar{R}_{t-1}\right), \ln(n_{t-1}), \ln(\xi_t); \eta_3^3(0)\right)\right] & \text{if } \xi_t \geq 1 \text{ and } \pi_t^b < 0 \end{cases}$$

Using g_t, we compute c_t in the left-hand side of the Euler equation. We then compute all the other endogenous variables, including the next period state variables $k_{t+1}, \bar{R}_t, n_t$. a_{t+1} is computed using a_t, $\epsilon_{a,t+1}$, and the law of motion for a_t.

(b) Given the next period state variables, compute the right-hand side of the Euler equation explicitly. We create 301 grid points from -0.06 to 0.06 for realizations of $\epsilon_{a,t+1}$. For each grid of $\epsilon_{a,t+1}$, we compute the corresponding c_{t+1} and h_{t+1} in the same way as in (a). We then integrate over all grids of $\epsilon_{a,t+1}$ using trapezoidal integration to compute the right-hand side of the Euler equation, denoted by $\tilde{g}_t$.

(3) Obtain a new set of coefficients by regression. We regress log of the computed right-hand side of the Euler equation $\ln(\tilde{g}_t)$ on log of the state variables $\ln(k_t), \ln\left(\bar{R}_{t-1}\right), \ln(n_{t-1}), \ln(\xi_t)$ for observations $t = 500 \sim 10,000$. In this step, we conduct three regressions separately depending on whether a run happens or not and whether a bank profit is positive or not. We obtain a new set of the polynomial coefficients $\eta_3^1(1), \eta_3^2(1), \eta_3^3(1)$.

(4) Check convergence. If all the coefficients $\eta_3^1(0), \eta_3^2(0), \eta_3^3(0)$ and $\eta_3^1(1), \eta_3^2(1), \eta_3^3(1)$ are close enough, we stop. Otherwise, we update the coefficients in the following way:

$$\eta_3^i(0) = \lambda \times \eta_3^i(0) + (1-\lambda) \times \eta_3^i(1), i = 1, 2, 3.$$

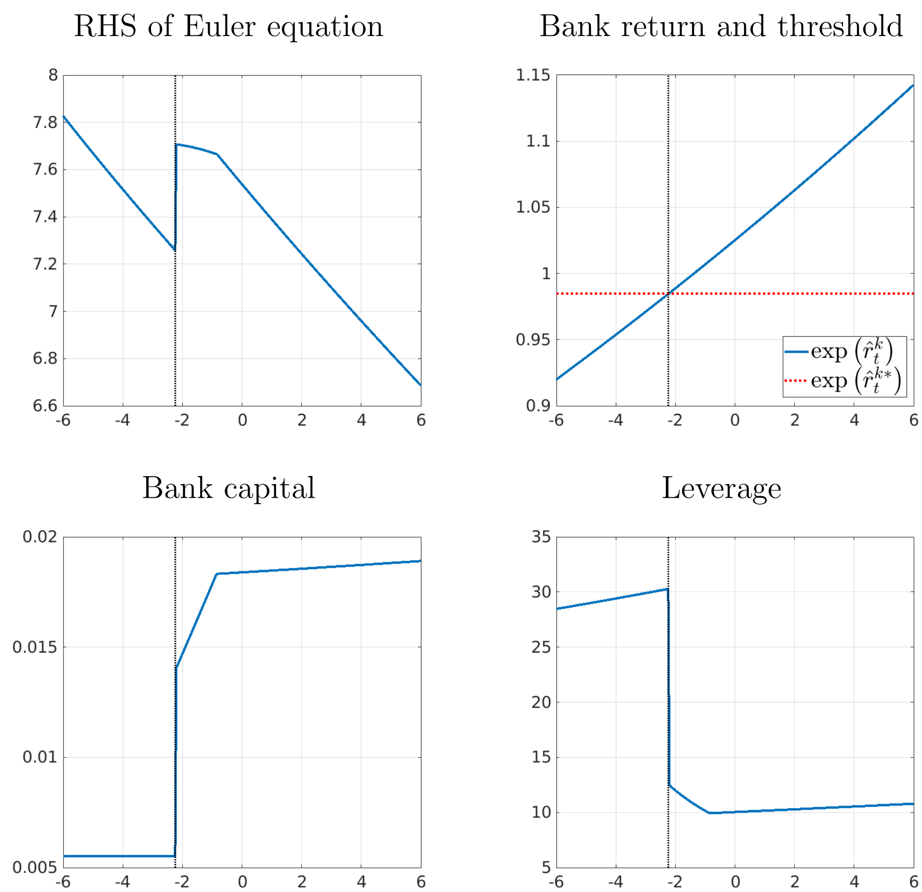
We find that $\lambda = 0.3$ works well. Go back to Step (2) and repeat the steps until the coefficients converge.

B.2 Policy Functions and Accuracy

Figure 10 plots the policy functions for the selected variables obtained by the solution method explained above. The state variables $k_t, \bar{R}_{t-1}, n_{t-1}$ are set at the stochastic steady state values, and the panels show how each variable changes as the realized TFP on the horizontal axis (expressed in percentage deviation from 1) changes. The policy function for the right-hand side of the Euler equation has a jump and a kink. A jump at around -2 is the threshold between a bank run and no run. This is where the realized bank return $\hat{r}_t^k$ and the threshold for a run $\hat{r}_t^{k*}$ coincide, as shown in the upper-right panel. If the realized TFP is lower than this value, indicated by the vertical dotted line, a bank run happens. When a run happens, the bank capital drops to the injected amount $\bar{n}$ and leverage jumps up, as shown in the bottom two panels.

A kink at around -1 is the threshold between a positive and negative bank profit. When a relatively low TFP shock but not low enough to trigger a run hits the economy, the bank profit becomes negative. The negative profit draws down the bank capital to some extent, which in turn increases leverage as shown

Figure 10: Policy functions over TFP shocks



Note: These panels plot the policy functions over TFP shocks, expressed in percentage deviation from 1 on the horizontal axis. The other state variables are set at the stochastic steady state values.

in the bottom two panels. The three separate approximation functions explained above are intended to capture these jump and kink accurately.

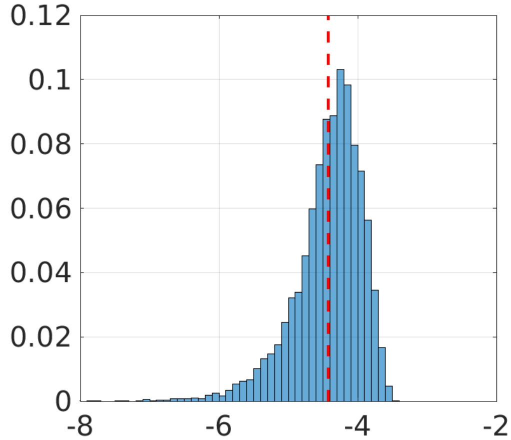
Figure 11 plots the histogram of the Euler equation errors over the 9,500-period stochastic simulation. At each period in the simulation, the Euler equation error is computed as follows:

$$error_t = \log_{10}\left(\frac{c_t - c_t^{EE}}{c_t}\right),$$

where c_t^{EE} is derived by first computing the right-hand side of the Euler equation $\tilde{g}_t$ explicitly as in Step (2)(b) of the numerical solution, and then computing consumption using the Euler equation as follows:

$$c_t^{EE} = (\tilde{g}_t)^{-1} + \psi\frac{h_t^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}$$

Figure 11: Euler equation errors



Note: The horizontal axis is log10 of the Euler equation errors. The vertical axis is expressed in terms of a ratio to total observations.

Errors are expressed in log10 as is standard in the literature. Figure 11 shows that the average error is -4.42 and the maximum error is -3.43, which is small enough compared to the literature. R^2 for the three regressions are all higher than 0.9999.