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Necessities, Home Production, and Economic Impacts of Stay-at-Home Policies

Makoto Nirei* and Nao Sudo**

Abstract

Stay-at-home (SAH) policy is the most commonly used measure employed around the globe to contain the spreading of Covid-19. While its effectiveness is widely agreed, it comes with a cost of dampening market activities. In this paper, we study theoretically how a SAH policy in one market sector can affect other market sectors, shedding light on the roles of home production and the division of labor across members within a household. We develop a multi-sector general equilibrium model that incorporates multiple types of households consisting of two members, each of whom works differently in the market and at home. We show that the spillover effect arises from the interaction of which market goods are subject to the SAH policy, the degree of luxuriousness of the goods, and the working status of household members. First, spillover effects take place only when the SAH policy is imposed on necessities. Households that consider the good as a necessity allocate a large portion of time to home goods production, causing a reduction of their market labor inputs. Second, the spillover effects on workers are attenuated when their spouse is a homemaker or works for a sector producing goods that have a higher degree of luxuriousness. We also calibrate the model to Japan's data, identifying the size of subsistence points and spousal working status, to study the consequences of a hypothetical scenario in which a SAH policy is imposed on the education sector and discuss the roles of the degree of luxuriousness of goods and the spousal working status.

Keywords: Stay-at-home policies; Home production; Stone-Geary Preference; Sectoral spillover

JEL classification: E20, J11

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1 Introduction

Covid-19 spreads from person to person through economic activities that involve social contact. Infection takes place anywhere outside the home, including working places, communities, and schools. Stay-at-home (SAH hereafter) policies that request individuals to remain at home, isolating themselves from the rest of the economy, are therefore considered as one of the most effective tools to flatten the epidemic curve, reducing and delaying the peak of the infection, at least until proper medicines and vaccination for the disease are established.^{1,2} Indeed, governments in affected jurisdictions introduced some forms of SAH directives when the disease was spreading.³ There is also a consensus, however, that SAH policies come at the cost of economic downturns, along with other non-economic costs, such as mental health problems and disruptions to the education of children. For example, Baker et al. (2020) exploit transaction-level household financial data in the U.S. to study changes in individual spending after the outbreak of Covid-19. They document that the overall drop in spending is about twice as large in states that issued shelter-in-place orders. Similarly, Carvalho et al. (2020a, b) construct a daily expenditure measure of individual consumers in Spain from the transaction data collected by Banco Bilbao Vizcaya Argentaria, S.A (BBVA) and document that expenditure fell by 40% immediately after the nationwide lockdown in Spain.⁴

Because of this trade-off, the optimal level, length, and scope of SAH policies have been intensively explored from various perspectives. An early paper by Eichenbaum et al. (2020)

¹See, for example, the discussion in Doyle et al. (2020) regarding the benefits of SAH policies.

²Hale et al. (2020) collect various government responses to Covid-19 across the globe and classify containment and closure policies into seven categories, separating, for example, SAH policies and workplace closings. SAH policies in this paper are defined broadly so that they include the set of policies that forces workers to reduce and/or terminate working outside the home, which potentially leads to a decline in the market supply of goods produced by affected workers. These include SAH policies, workplace closings and school closings in Hale et al. (2020).

³In the U.S., for example, SAH directives started in California in mid-March and spread over the nation rapidly. By mid-April, 90% of the U.S. was under SAH orders..

⁴There are also some studies that argue that containment itself does not lead to a large economic downturn. Correia et al (2020) study the impacts of non-pharmaceutical interventions (NPIs) on economic activities of U.S. cities during the 1918 Flu Pandemic and find that economic disruptions were similar across cities with strict and lenient NPIs. Back et al. (2020) estimate the impact of SAH policies imposed to contain the spreading of Covid-19 in the U.S. on the unemployment rate and document that the impact is statistically significant but quantitatively minor.

combines a SIR model a la Kermack and McKendrick (1927) with the standard dynamic stochastic general equilibrium (DSGE) model and studies the implications of several classes of containment policies for social welfare. According to their simulation, when optimally pursued, the containment policy reduces aggregate consumption by 22%, limiting the death toll to 0.26% of the initial population, while aggregate consumption falls by 7%, and the death toll rises to 0.4% if such a policy is absent. Accemoglu et al. (2020) propose targeted lockdowns instead of uniform lockdowns, pointing out that elderlies face a higher rate of hospitalization and fatality associated with Covid-19. They argue that targeted lockdowns that incorporate age-specific risks can improve the trade-off between lives lost and economic loss.

In this paper, we focus on the costs of SAH policies, in particular the spillover effects of the policies, addressing the roles of luxuriousness of goods and home goods production. The typical household's reaction to SAH policies observed in the current pandemic is to stay at home and to produce goods and services at home for their own use. In particular, when SAH policies reduce the market supply of necessities, households fall into a difficult situation, cutting their working hours outside the home and producing necessities by themselves. One example is school closures.⁵ Once schools are closed, working parents have to reallocate their time to take care of their dependent(s) at home, a possible consequence of which is a fall in the labor force. In other words, disruptions to education (or childcare) services lead to disruptions to other market goods production through households' reallocation of labor inputs inside and outside the home.⁶ Indeed, partly due to the observation that the infection rate of the disease among small children is limited and the perceived knock-on effects on other sectors, national governments were divided regarding whether to close or

 $^{^{5}}$ According to Russel et al (2020), 107 countries had implemented national school closures as of mid-March to combat the virus.

⁶As the Financial Times (2020a) explains, "With schools shut, parents find it harder to work, with a knock-on effect on the economy."

reopen kindergartens and primary schools.^{7,8}

We construct a general equilibrium model and explore theoretically how spillover effects of SAH policies depend on the luxuriousness of goods and the structure of households. Our economy consists of multiple market goods, multiple home-produced goods, and multiple types of households. Goods differ from each other in terms of luxuriousness. Households consume necessities at least up to a certain point, referred to as the subsistence point that varies across goods and households. Households differ from each other in terms of how its members work inside and outside the home. The distribution of households that consist of members who work differently is characterized by what we refer to as the spousal matrix. Once a SAH policy is in place, households reallocate labor inputs across market and home production. We derive two theoretical predictions regarding the spillover effects of a SAH policy. First, the spillover effects exist and are pronounced only when SAH policies target the market production of necessities. In this situation, households produce necessities at home by increasing their home labor inputs. This reallocation reduces market labor inputs, causing a decline in other market goods. The spillover effects are attenuated or even absent, however, when SAH policies are imposed on goods that are not necessities. Second, the spillover effects of a worker depend on the working status of the worker's spouse. This is because the division of labor between the two members within a household affects how home goods for the specific household are produced. We show that, other things being equal, the spillover effects to a specific market sector are attenuated when a large number of workers in the sector belong to households that consist of a homemaker or households that consist of a member who works in a sector producing a market good that is not a necessity.

 $^{^{7}}$ School closures are not the only case where the interaction between necessities and SAH policies plays out. For example, the knock-on effects have manifested themselves when day care services for elderlies were suspended due to the outbreak in some areas in Japan. See, for example, the Japan Times (2020) for details.

⁸In practice, the authorities in affected areas have often carefully chosen the scope of SAH policies, taking into consideration residents' access to basic necessities and the safety of workplaces. For example, in the lockdown imposed in the State of California in March 2020, the government outlined a specific set of sectors considered necessary to maintain continuity of operations of critical federal infrastructure and ordered all individuals other than those working in those sectors to stay at home. These exemptions have the potential to moderate the knock-on effects.

In addition, we calibrate the model to Japan's data and study, as an example, how SAH policies in the education services sector affects the rest of the economy. We first show, using Japanese consumers' expenditure data, that both educational and medical service are generally considered as necessities by households. We also develop the spousal matrix in Japan and show that the medical sector may work as a "large column sum" sector, since male workers are more likely to have a spouse who works in the medical sector compared with other sectors in the economy. We then compute the spillover effects of a SAH policy, addressing the role played by this property of the matrix.

Our paper is built upon rapidly-growing studies about the impacts of containment policies. In particular, it is related to three strands of literature. The first strand of literature includes works that focus on macroeconomic costs and/or benefits of containment policies such as Acemoglu et al. (2020), Alvarez et al. (2020), Baker et al. (2020), Carvalho et al. (2020a, b), Eichenbaum et al. (2020), Jang and Yum (2020), Kikuchi et al. (2020). The second strand of literature includes studies that address the role of home production, such as Alon et al. (2020), Hupkau and Petrongolo (2020), Jessen et al. (2020), Leukhina and Yu (2020), and Sevilla and Smith (2020). Hupkau and Petrongolo (2020) document that housework has declined by 25% due to the pandemic in the U.K. Alon et al. (2020) point out that the pandemic has had outsized impacts on women's market work in the U.S. partly due to increasing childcare obligations. Though there are important similarities between these works and our work, ours differs in its focus on the role played by the spousal matrix and luxuriousness of goods. The third strand of literature includes studies that explore sectoral implications of Covid-19 or the contaminant policies, including Barrot et al. (2020), Glover et al. (2020), Guerrieri et al. (2020), and Krueger et al. (2020). Our study is particularly close to Barrot et al. (2020), which explores spillover effects from affected sectors to the rest of the economy. In addition, our model borrows ideas and methodologies developed in the literature on the subsistence point of goods consumption initiated by Geary (1950) and Stone (1954) and recently studied by Matsuyama (2002) and Morten et al. (2008).

The rest of the paper is organized as follows. The next section introduces our model.

Section 3 examines the qualitative implications of our model. Section 4 provides simulation results, including those that calibrate the model to the Japanese economy. Section 5 concludes.

2 Model

Our model highlights three aspects of households - home production, subsistence points, and differences regarding how each member of the household works. A summary of the model is provided below.

- 1. There are 3 market goods producing sectors sector 1, 2, and 3 each of which produces a different market good using labor inputs attached to the sector.
- 2. Some goods are necessities to some or all households. Households consume a certain amount of goods they consider necessities before consuming other goods.
- 3. There are $3 \times 3 + 6$ types of households, each of which has two family members who work in the market, at home, or both. Households differ in terms of how the two household members work. The distribution of households is characterized by what we refer to as the spousal matrix.
- 4. Household members produce home goods as well as market goods and consume a composite of market and home goods.

We borrow settings regarding home production from Benhabib et al. (1991). We borrow settings regarding subsistence points from Matsuyama (2002) and Morten et al. (2008).

2.1 Setting

Households

The economy consists of three sectors k = 1, 2, 3, each of which produces market good k using labor only. A household consists of two members x = a, b, and each member either works in a market goods producing sector $k_x = 1, 2, 3$ or does not supply labor in

the market, denoted by $k_x = 0$. Households are classified by the pattern of labor market participation $[k_a k_b]$. We assume that at least by one member of each household supplies labor to the market. Thus, there are 15 types of households in this economy: $j \in \mathcal{J} =$ $\{01, 02, 03, 10, 11, 12, 13, 20, 21, 22, 23, 30, 31, 32, 33\}$. Each type of household j is populated with measure ω_j , and the total measure of households is normalized to $\sum_{j \in \mathcal{J}} \omega_j = 1$. We summarize the distribution of households in the economy by what we refer to as the spousal matrix Γ_{ω} defined below.

$$\Gamma_{\omega} = \begin{bmatrix} N.A. & \omega_{01} & \omega_{02} & \omega_{03} \\ \omega_{10} & \omega_{11} & \omega_{12} & \omega_{13} \\ \omega_{20} & \omega_{21} & \omega_{22} & \omega_{23} \\ \omega_{30} & \omega_{31} & \omega_{32} & \omega_{33} \end{bmatrix}.$$
 (1)

Note that the row represents the sector that member a of a household works for and the column represents the sector that member b of a household works for. A pair (j, x) uniquely determines the sector k which the household member works for. For example, if j = 11, then both members of household j work for sector k = 1. This mapping of household members to sectors is denoted by k(j, x). If j = 30, then member a works for sector 3 and member b does not work in the market. For convenience in the analysis below, we refer to the set of households in which at least one member works for sector 1, 2, or 3, as group 1, 2, or 3 households, and denote them by G_1 , G_2 , and G_3 , respectively. Clearly, households that belong to each group overlap.

Each household receives utility from consuming three composites of market-produced and home-produced goods and leisure. Variables related to home production are denoted by a tilde. For example, consumption of home-produced good k is denoted as \tilde{C}_k .

The utility function of a type j household, denoted as U^{j} , is expressed as follows.

$$U^{j} = \sum_{k=1}^{3} \sigma_{k} \log \left(c_{k}^{j} - b_{k}^{j} \right) + \theta \sum_{x=a,b} \log \left(T - h^{j,x} - \sum_{k'=1}^{3} \tilde{h}_{k'}^{j,x} \right)$$
(2)

where

$$c_k^j = (C_k^j)^{\eta} (\tilde{C}_k^j)^{1-\eta}, \quad k = 1, 2, 3.$$
 (3)

In equation (2), the first term represents utility gains from consuming the three composite goods. Note that c_k^j refers to the composite goods that is made of market-produced goods

 C_k^j and home-produced goods \tilde{C}_k^j for k = 1, 2, 3 and $\eta \in (0, 1)$ is the share of the market good in the composite good. The parameter b_k^j is the subsistence point of consumption of composite good k, and the parameter σ_k is the utility weight on consumption of the composite good k, for k = 1, 2, 3. Both b_k^j and σ_k are exogenously given. As in Matsuyama (2002), we assume that the subsistence point of consumption may differ across goods. We also assume that the subsistence points can differ across households j.

The second term represents utility gains from the leisure of household members a and b. T is the time endowment for each member and $\theta > 0$ is the utility weight on leisure. Labor input for the market production of member x is denoted by $h^{j,x}$. For households of type j = 01, 02, 03, the market labor input of member a $h^{j,a}$ is zero and for those of type j = 10, 20, 30, the market labor input of member b $h^{j,b} = 0$. Finally, $\tilde{h}^{j}_{k'}$ denotes the hours worked to produce home good k' = 1, 2, 3. Note that both members of the household, regardless of type j, engage in home production of all of the three goods k' = 1, 2, 3.

Market goods production

We assume that the production function of market good Y_k for k = 1, 2, 3 is given by the following equations,

$$Y_k \le Ah_k^{\alpha}, \tag{4}$$
$$h_k = \prod_{j \in \mathcal{J}} \prod_{x=a,b} \left(\omega_j h^{j,x} \right)^{\gamma^{j,x}}, \qquad (4)$$

where $\sum_{j \in \mathcal{J}} \sum_{x=a,b} \gamma^{j,x} = 1$. A is a parameter that represents the technology level of market production, and α and $\gamma^{j,x}$ are the technology parameters that govern the share of labor inputs provided by member x of type j households that is set proportionately to the weight ω_j of each type of household.

Home goods production

Each home goods $\tilde{C}_{k'}^{j}$ for k' = 1, 2, 3 is produced by the two members of type j households and consumed by the household. For each type $j \in \mathcal{J}$, the following equations hold.

$$\tilde{C}_{k'}^{j} \leq \tilde{A} \left(\sum_{x=a,b} \delta^{j,x} (\tilde{h}^{j,x})^{\rho} \right)^{\frac{\rho}{\rho}},$$
(5)

where $\sum_{x=a,b} \delta^{j,x} = 1$ for any $j \in \mathcal{J}$. \tilde{A} represents the technology level of home goods production, $\beta \in (0,1)$ is the return to scale parameter of home goods production, ρ determines the degree of substitutability of the labor inputs provided by members a and bwithin a household, and $\delta^{j,x} \in (0,1)$ is the labor share of member x in home production.

2.2 Equilibrium

We consider the social planner's problem that allocates the set of labor inputs $\{h^{j,x}, \{\tilde{h}^{j,x}_{k'}\}_{k'=1,2,3}\}_{x=a,b}$ for $j \in \mathcal{J}$ so as to maximize social welfare U, defined as follows,

$$U = \sum_{j \in \mathcal{J}} \omega_j U^j, \tag{6}$$

subject to the set of resource constraints regarding home goods (5) k' = 1, 2, 3 and the resource constraint regarding market goods k = 1, 2, 3.

$$\sum_{j \in \mathcal{J}} \omega_j C_k^j \le Y_k = A h_k^{\alpha}, \quad \text{for } k = 1, 2, 3.$$
(7)

Equilibrium Conditions

Let λ_k be the Lagrange multiplier for (7), the resource constraint on market good k.

The first order conditions with respect to C_k^j , $h^{j,x}$, and $\tilde{h}_{k'}^{j,x}$ are given by

$$\lambda_k = \frac{\sigma_k}{1 - b_k^j / c_k^j} \frac{\eta}{C_k^j}, \quad \forall k, j$$
(8)

$$\lambda_k \gamma^{j,x} \alpha \frac{Y_k}{h^{j,x}} = \frac{\theta \omega_j}{T - h^{j,x} - \sum_{k'} \tilde{h}^{j,x}_{k'}}, \quad k = k(j,x), \forall j,x$$
(9)

$$\frac{\sigma_k}{\left(1 - b_k^j / c_k^j\right)} \frac{(1 - \eta)\beta \delta^{j,x} (h_k^{j,x})^{\rho - 1}}{\left(\sum_{x = a, b} \delta^{j,x} (\tilde{h}_k^{j,x})^{\rho}\right)} = \frac{\theta}{T - h^{j,x} - \sum_{k'} \tilde{h}_{k'}^{j,x}}. \quad \forall k, j, x$$
(10)

The first set of equations (8) implies that the marginal utility from consuming market good C_k^j is equalized across household types. The second set of equations (9) implies that the marginal gain from providing market labor inputs is equalized to the marginal disutility from the market labor inputs of workers who work in the market goods sector. The third set of equations (10) implies that the marginal gain from providing home labor inputs is equalized to the marginal disutility from the marginal disutility from the home labor inputs of the worker.

2.3 Stay-at-home policy

We study the effects of a SAH policy by comparing the allocation of labor inputs under two different economic environments, one without a SAH policy and one with a SAH policy. First, we compute the equilibrium set of labor inputs, $\{h^{j,x}\}_{x=a,b}$ and $\{\{\tilde{h}_{k'}^{j,x}\}_{x=a,b}\}_{k'=1,2,3}$ for $j \in \mathcal{J}$ under the environment described above. By definition, this allocation is socially optimal and considered to represent the economy without a SAH policy. We denote this allocation as follows.

$$\{h^{*j,x}\}_{x=a,b}$$
 and $\left\{\left\{\tilde{h}_{k}^{*j,x}\right\}_{x=a,b}\right\}_{k'=1,2,3}$

Second, we impose a cap on the amount of market labor inputs to a specific sector, say $k = \overline{k}$, and compute the equilibrium set of labor inputs again. More precisely, regarding the market labor input of member x of household type j with $k(j, x) = \overline{k}$, we impose the following constraints and refer them as the stay-at-home (SAH) constraints.

$$h^{j,x} \le \phi h^{*j,x} \text{ for } \phi \in [0,1].$$

$$(11)$$

The parameter ϕ represents the degree of the SAH constraint. Because the SAH constraint affects the labor inputs of all workers in sector \overline{k} , reducing their labor inputs equally by $(1-\phi) h^{*j,x}$ from the socially optimal level causes market goods production of $Y_{\overline{k}}$ to fall by $(1-\phi)^{\alpha} Y_{\overline{k}}^*$.

For the convenience of the analysis, we assume that the SAH policy is imposed only on the sector of goods 2, i.e., $\overline{k} = 2$, and refer to workers with k(j, x) = 2 as affected workers and workers with k(j, x) = 1 and 3 as unaffected workers, respectively.

3 Stay-at-home and the Spillover Effects

Before going to the quantitative analysis, we illustrate qualitatively how SAH policies on a specific goods sector is translated to changes in the labor input allocation of workers. We then discuss how the distribution of necessities of households, represented by the set of parameter $\left\{b_k^j\right\}_{k=1,2,3}$ and how the worker's spouse's working sector, represented by the matrix Γ_{ω} affect the spillover effects.

3.1 The socially optimal allocation

We start with describing labor input allocations in an economy where the SAH policy is absent. In this case, by arranging the first order equations (8)–(10), the labor input allocations of member x of household j are given by the following equations.

$$h^{*j,x} = \left(\sum_{j' \in \mathcal{J}} \frac{\omega_{j'}}{1 - b_k^{j'}/c_k^{*j'}}\right) \xi^{j,x} L^{*j,x}, k = k(j,x), \forall j,x$$
(12)

$$\tilde{h}_{k'}^{*j,x} = \frac{1}{1 - b_{k'}^j / c_{k'}^{*j}} \frac{(\tilde{h}^{*j,x})^{\rho} \, \tilde{\xi}^{j,x} L^{*j,x}}{\left(\sum_{x=a,b} \delta^{j,x} (\tilde{h}^{*j,x})^{\rho}\right) \theta}, \forall k', j, x$$
(13)

where

$$\begin{split} L^{*j,x} &\equiv T - h^{*j,x} - \sum_{k'} \tilde{h}_{k'}^{*j,x} \\ \xi^{j,x} &: = \frac{\gamma^{j,x} \alpha \eta \sigma_{k(j,x)}}{\theta \omega_j}, \\ \tilde{\xi}^{j,x} &: = \frac{\delta^{j,x} \beta (1 - \eta) \sigma_{k(j,x)}}{\theta}, \end{split}$$

where $L^{*j,x}$ denotes the amount of leisure a worker enjoys. Based on these equations, workers of type j households determine two market labor inputs $h^{*j,a}$ and $h^{*j,b}$ and six home labor inputs $\left\{\tilde{h}_{k'}^{*j,a}\right\}_{k'=1,2,3}$ and $\left\{\tilde{h}_{k'}^{*j,b}\right\}_{k'=1,2,3}$. The size of subsistence point b_k^j affects labor input allocations through the term $\left(1-b_k^j/c_k^{*j}\right)^{-1}$. Suppose first that the subsistence points are zero for all goods, i.e., $b_k^j = 0$ for all j and k, then this term vanishes and the labor input allocations of workers of type j households are pinned down by eight of the equations given in (12) and (13) independently from the labor input allocations of households of other types. In addition, when $\rho = 0$, there are no interactions of labor inputs across members within the same household. The labor input allocations of a worker are determined by four of the equations given in (12) and (13).

When the subsistence point of good b_k^j is above zero, as shown in equation (12), workers of sector k face an added incentive to allocate a larger portion of their time to producing market goods k. The incentive is reinforced as the consumption level of the corresponding composite good c_k^{*j} of the household to which the worker belongs becomes closer to the subsistence point b_k^j . Similarly, as shown in equation (13), workers of a household of type jface an added incentive to allocate a larger portion of their time to producing home goods C_k^{*j} again as the consumption level of the corresponding composite c_k^{*j} becomes closer to the subsistence point of the household b_k^j . Other things being equal, in both cases, utility gains that arise from increasing labor inputs $h^{*j,x}$ and $\tilde{h}^{*j,x}$ are high, since a rise in goods supply Y_k or C_k^{*j} widens the gap between the consumption level c_k^{*j} and the subsistence point b_k^j . It is important to note that when the subsistence points are positive, the labor input allocations of a worker are no longer independent from the labor input allocations of workers from other households. For example, once a decline in market labor inputs of workers with k(j, x) = 2 reduces the consumption level c_k^{*j} , home labor inputs of workers with $k(j, x) \neq 2$ should increase through equation (13).

3.2 The role of necessities and the spousal matrix

Labor input allocations under the SAH policy

How does the SAH policy affect labor input allocations? Because of constraint (11), equation (12) no longer holds for workers with k(j, x) = 2. Denoting by $\mu^{j,x}$ the Lagrange multiplier associated with the constraint, labor input allocations are determined by the following equations.

$$\left(\sum_{j'\in\mathcal{J}}\frac{\omega_{j'}}{1-b_k^{j'}/c_k^{j'}}\right)\frac{\sigma_k\eta\gamma^{j,x}\alpha}{h^{j,x}} = \frac{\theta\omega_j}{L^{j,x}} + \mu^{j,x}, \quad k = k(j,x), \forall j,x$$
(14)
$$\tilde{h}_{k'}^{j,x} = \frac{1}{1-b_{k'}^{j}/c_{k'}^{j}}\frac{(\tilde{h}^{j,x})^{\rho}\tilde{\xi}^{j,x}L^{j,x}}{\left(\sum_{x=a,b}\delta^{j,x}\left(\tilde{h}^{j,x}\right)^{\rho}\right)\theta}, \forall k', j,x$$
(15)

where

$$L^{j,x} \equiv T - h^{j,x} - \sum_{k'} \tilde{h}^{j,x}_{k'}.$$

Note that $\mu^{j,x} > 0$ for (j,x) : k(j,x) = 2 and 0 otherwise. First, consider the case in which the subsistence points are zero for all goods, i.e., $b_k^j = 0$ for all j and k. Regarding affected workers, the market labor input $h^{j,x}$ is determined by the SAH constraint (11) and home labor inputs $\tilde{h}_{k'}^{j,x}$ are determined by equation (15), which implies that the following equations hold.

The SAH policy therefore increases home labor inputs $\tilde{h}_{k'}^{j,x}$, while reducing market labor inputs $h^{j,x}$. These workers are forced to reduce their market labor inputs $h^{j,x}$, shifting their working time to home goods production. Regarding unaffected workers, equations (14) and (15) continue to hold, with the multiplier $\mu^{j,x}$ being zero, and the labor input allocations are constant. Consequently, labor input allocations under the SAH policy coincide with the socially optimal allocation, $h^{j,x} = h^{*j,x}$ and $\tilde{h}_{k'}^{*j,x} = \tilde{h}_{k'}^{j,x}$.

Now suppose that goods 2 is a necessity. For simplicity, we assume that $b_2^j = b > 0$ and $b_k^j = 0$ for $k \neq 2$ for all households. Because the SAH policy reduces the market production of good 2 Y_2 , through the equation (7), the consumption level of market goods 2 of households C_2^j changes, leading to labor input reallocations by both affected and unaffected workers, through the term $\left(1 - b_2^j/c_2^j\right)^{-1}$ in equations (14) and (15). So long as good 2 consumption C_2^j falls for household j, which indeed is the case under our parameterization shown below, the term $\left(1 - b_2^j/c_2^j\right)^{-1}$ in equation (15) rises, leading to a larger home labor inputs $\tilde{h}_{k'}^{j,x}$ by workers. A rise in home labor inputs $\tilde{h}_{k'}^{j,x}$ is then straightforwardly translated to a decline in leisure $L^{j,x}$, which in turn reduces the market labor inputs of unaffected workers $h^{j,x}$ as indicated by the following equation.

$$h^{j,x} = \xi^{j,x} L^{j,x} = \frac{\xi^{j,x} T}{\xi^{j,x} + \left(1 - b_2^j / c_2^j\right)^{-1} \tilde{\xi}^{j,x} + \tilde{\xi}^{j,x} + \tilde{\xi}^{j,x}}$$

$$\leq \frac{\xi^{j,x} T}{\xi^{j,x} + \left(1 - b_2^j / c_2^{*j}\right)^{-1} \tilde{\xi}^{j,x} + \tilde{\xi}^{j,x} + \tilde{\xi}^{j,x}} = h^{*j,x}.$$
(17)

The role of necessities

Spillover effects manifest themselves through an endogenous response by workers. As shown above, the level of the response depends on the subsistence point of goods consumption b_k^j that can potentially be different across goods k and across households j. For example, households with schoolchildren may consider education services as necessities, but may not consider elderly care as a necessity. Similarly, the demographic characteristics of households will affect how households view the luxuriousness of a particular good. This heterogeneity across households matters for the spillover effects. Suppose that goods 2 is a necessity for group 1 households, i.e., $b_2^j > 0$ for $j \in G_1$, but not for group 3 households i.e., $b_2^j = 0$ for $j \in G_3$. When the SAH policy is in place, workers who belong to group 1 households increase their home labor inputs $\tilde{h}_2^{j,x}$, reducing market labor input $h^{j,x}$ as indicated by the equation (17). By contrast, workers who belong to group 3 households do not reallocate labor inputs, since the term $(1 - b_2^j/c_2^j)$ is constant. Consequently, the spillover effects of good 3 production Y_3 are attenuated.

The role of the spousal matrix

Spillover effects also change depending on the characteristics of the spousal work allocation represented by the spousal matrix Γ_{ω} . Because the subsistence point b_k^j matters for household-level utility, the division of labor between members a and b within the household is optimally chosen so as to achieve the highest efficiency at the household level. As indicated by equation (15), the parameter ρ plays an important role in determining the division of labor within a household. When $\rho = 0$, the elasticity of substitution between member workers in home goods production is unity and how each member works is independent from how their spouse works. When $\rho \neq 0$, the optimal home labor inputs of member a $\tilde{h}_{k'}^{j,a}$ change depending on the home labor inputs of their spouse $\tilde{h}^{j,b}$, and vice versa. When home labor inputs of the two members are substitutes (complements), i.e., $\rho > 0$ ($\rho < 0$), an increase in a spouse's home labor inputs $\tilde{h}_{k'}^{j,b}$ reduces (increases) the worker's home labor inputs $\tilde{h}_{k'}^{j,a}$, because the marginal productivity of home goods production with respect to the worker's home labor inputs decreases (increases). For example, suppose that, as shown in equation (16), sector 2 workers increase their home labor inputs as a result of a SAH policy. If $\rho > 0$, then their spouse faces a weaker incentive to increase their own home labor inputs by themselves, which in turn can mitigate a fall in the market labor inputs of the worker, compared with the case where $\rho < 0$.

4 Quantitative Exercise

4.1 Calibration

We set the technology level of market goods production \tilde{A} to unity and the labor share α to 0.65, the standard value in the literature. We set the technology level of home goods production \tilde{A} to 0.8, implicitly assuming that home goods production is less efficient than market goods production, and set the labor share of home goods production β to 0.65. We set the utility weight on the three composite goods $\sigma_{\overline{k}}$ to unity, the weight on leisure θ to 1/3, and the time endowment T to 10. Regarding the share parameter of member a in home production $\delta^{j,x}$, we set $\delta^{j,x} = 0.5$ for worker a in a dual-income household and set $\delta^{j,x} = 0.1$ (0.9) for worker a if their spouse is a homemaker. We set the share parameter of market goods in producing the composite η to 0.9. We choose these values for η and $\delta^{j,x}$ so that the model-generated socially optimal labor allocations for market labor inputs relative to those for home labor inputs agree with the data.⁹

4.2 Simulation

4.2.1 Economy in which there are no necessities

We start with the case where there are no necessities in the economy, i.e., $b_k^j = 0$ for all j and k. Figure 1 shows market labor inputs $h^{j,x}$ of three groups of households G_1 , G_2 , and G_3 , in panel (1), output of the three market goods Y_1 , Y_2 , and Y_3 in panel (2), the welfare U in panel (3), and labor inputs to home production $\tilde{h}_{k'}^{j,x}$ of the three groups of households in panels (4), (5), and (6), respectively, when the SAH policy is imposed on market good 2 production.¹⁰ The horizontal axis represents the value of the parameter ϕ ,

⁹Based on the "Survey on Time Use and Leisure Activities," released by the Ministry of Internal Affairs and Communications in Japan, labor allocation for market labor inputs relative to that for home labor inputs within a household j, namely $\left(\sum_{x=a,b} h^{j,x}\right) \left(\sum_{x=a,b} \left(\sum_{k'} \tilde{h}^{j,x}_{k'}\right)\right)^{-1}$ in our notation, is about 2.5 for a dual-income household and about 1.0 for a single-income family. We compute the model counterpart of these variables using the parameter values described here and the spousal matrix Γ_{ω} that is calibrated to Japan's data, which is defined as Γ_{JP} below, and choose values for η and $\delta^{j,x}$ that bring the model-generated value close to the data.

¹⁰Note that each household group, G_1 , G_2 , and G_3 , consists of multiple types of households each with a different weight ω_j . In computing a variable shown in the figure, say home labor inputs $\tilde{h}_{k'}^{j,x}$ for k' = 1 of households that belong to group 1 G_1 , we compute the weighted average of a variable $\tilde{h}_{k'}^{j,x}$ of a corresponding

ranging from 0.9 to 1, and the vertical axis represents the deviation of a variable from its socially optimal level.

Because labor inputs to market good 2 are forced to fall, the production of market good 2 \hat{Y}_2 falls as well. Affected workers, all of them belonging to group 2 households G_2 , reallocate their labor inputs from market goods production $h^{j,x}$ to home goods production, as shown in panel (5). Such a shift in labor reallocations reduces welfare since home goods production is by assumption less efficient than market goods production. Unaffected workers do not change their labor input allocations in response to a decline in good 2 supply Y_2 , as seen in panels (1) and (2), due to the fact that the term $\left(1 - b_{k'}^j/c_{k'}^j\right)$ in equation (15) is unchanged when a SAH policy is introduced.

4.2.2 Economy in which there are necessities

We next consider a scenario where only goods 2 is a necessity, i.e., $b_2^j = 15 > 0$ and $b_k^j = 0$ for $k \neq 2$ for all households.

Figure 2 shows the impacts of the SAH policy on labor input allocations and the key variables under this scenario. The key difference from Figure 1 is that there are spillover effects. Market labor inputs of households of group 1 and 3 G_1 and G_3 fall and market goods production of goods 1 and 3 Y_1 and Y_3 also fall as the parameter ϕ is reduced from unity to 0.9. A fall of these market goods become quantitatively greater than a fall in good 2 itself when ϕ is sufficiently small. As discussed already, the central mechanism is the reallocation of labor inputs by unaffected workers. As the supply of market good 2 Y_2 becomes scarce, market goods consumption of good 2 $c_{j,2}$ falls, which implies that the marginal gains from producing home good \tilde{C}_2^j becomes higher, adding incentives for unaffected workers to increase their home labor inputs $\tilde{h}_{2}^{j,x}$. The rise in home labor inputs $\tilde{h}_{k'}^{j,x}$ is then translated into a decline in market labor inputs to sectors 1 and 3 through equation (14).

The size of subsistence points

household j that belongs to the group using the weight ω_j .

The spillover effects become larger as the size of subsistence points is elevated. Figure 3 shows changes in labor input allocations due to the introduction of a SAH policy on good 2 \hat{Y}_2 in three different economies, each of which sees a different value for the subsistence point of good 2, i.e., $b_2^j = b = 5$, 10,and 15, while those of other goods remain zero, i.e., $b_k^j = 0$ for $k \neq 2$. A higher subsistence point incurs a higher marginal gain from consuming the composite \tilde{C}_2^j , which gives an added incentive for all households to provide more home labor inputs $\tilde{h}_2^{j,x}$. Consequently, as shown in panels (4), (5), and (6), workers reduce their market labor inputs $h^{j,x}$ more.

The distribution of subsistence points

Reallocation of labor inputs takes place only among households who consider the target goods as necessities. In aggregate, therefore, the spillover effects depend on which households consider affected goods necessities, namely the distribution of subsistence points. Figure 4 considers three economies; an economy in which the composite good 2 $c_{j,2}$ is a necessity to all households except for the group 1 households G_1 (Case 1), an economy in which the composite good 2 $c_{j,2}$ is a necessity to all households (Case 2), and an economy in which the composite good 2 $c_{j,2}$ is a necessity to all households except for the group 2 households G_2 (Case 3). It is seen that in all cases production of market goods 2 and 3 Y_2 and Y_3 fall. This is because households G_3 households consider good 2 as a necessity in all cases. By contrast, production of market good 1 falls only in Cases 2 and 3. In Case 1, workers of group 1 households do not change their market labor input allocations, yielding no spillover effects. In Case 3 when good 2 is not a necessity for group 2 households, they demand market good 2 less than in Case 2. Thus, a larger portion of market good 2 is available to households of groups 1 and 3, leading to mitigated spillover effects of market goods 1 and 3 in Case 3 than in Case 2. The distribution of necessities across households therefore matters to the quantitative impacts of spillover effects.

When there are two necessities

Indirect spillover effects may arise in an economy where there are multiple necessity

goods. Figure 5 shows three cases. Case 1 studies an economy in which composite goods 1 and 2, $c_{j,1}$ and $c_{j,2}$, are considered as necessities to all households. Case 2 studies an economy in which only composite good 2 $c_{j,2}$ is considered as a necessity to all households. Case 3 studies an economy in which composite good 2 $c_{j,2}$ is considered as a necessity to all households except for the group 3 households G_3 and composite good 1 $c_{j,1}$ is considered as a necessity to all households.

The comparison between Cases 1 and 2 shows that spillover effects to necessities are attenuated compared with those to ordinary goods. When composite good 1 $c_{j,1}$ is a necessity, workers of market good 1 reduce their market labor inputs less compared with workers of market good 3. While a decline in production of market good 2 Y_2 adds an incentive for both of the workers to increase their home goods production \tilde{C}_2^j , workers of market good 1 also have one other incentive which prevents them from reducing their market labor inputs $h^{j,x}$ to the production of market good 1. Because such an incentive is absent among workers of sector 3, the spillover effects are more pronounced for market good 3.

Indirect spillover effects can be seen from Case 3. In this case, households of group 3 G_3 do not change their labor input allocations to a change in market production of good 2. This is because the composite good $c_{j,2}$ is not a necessity for them. By contrast, they increase their home labor inputs $\tilde{h}_1^{j,x}$, reducing market labor inputs $h^{j,x}$, when the supply of market good 1 falls. A decline in market labor inputs of households of group 2 due to the direct impact of SAH policies leads to a decline in market labor inputs of households of group 1, which in turn induces a fall in market good 3 Y_3 as well.

4.2.3 The role of spousal allocations

In our model, the key mechanism of the spillover effects is changes in home labor inputs and therefore the division of labor between member a and b within a household plays an important role. To see this, we study how each member of a household responds to SAH policies. Table 1 shows the deviation from the socially optimal level of market labor inputs of member a in each of 15 types of households $j h^{j,x}$ when the SAH policy with ϕ equal to 0.9 is imposed on market good 2 production. Similarly to the spousal matrix (1), the row represents the sector that member a of a household works for and the column represents the sector that their spouse, namely member b, of a household works for.

Table 1(1) shows changes in market labor inputs in an economy where only the composite good 2 $c_{j,2}$ is a necessity, i.e., $b_2^j = b > 0$ and $b_k^j = 0$ for $k \neq 2$ for all households.¹¹ Two observations are notable. First, a decline in market labor inputs is attenuated among workers whose spouse is a homemaker. Compared with workers of type ω_{11} and ω_{13} households that see a change in labor inputs of -10.95%, workers of type ω_{10} and ω_{30} see a change of about -0.75%. When the spouse is a homemaker, workers do not increase their home labor inputs $\tilde{h}_2^{j,x}$ much, since their spouse instead increases their home labor inputs $\tilde{h}_2^{j,x}$ and increases home goods $\tilde{C}_{j,2}$. Second, a decline in market labor inputs is attenuated for workers whose spouse is an affected worker, i.e., workers of sector 2. Member *a* of type ω_{12} and ω_{32} households sees a change only of -8.25%. As indicated by equation (16), affected workers allocate a large portion of their time to home labor inputs $\tilde{h}_{k'}^{j,b}$, which reduces the incentive for their spouse to increase their home goods production $\tilde{C}_{j,2}$, mitigating a decline in their market labor inputs $h^{j,a}$.

Table 1(2) shows changes in market labor inputs when composite goods 1 and 2 $c_{j,1}$ and $c_{j,2}$ are necessities i.e., $b_1^j = b_2^j = b > 0$ and $b_3^j = 0$ for all households. Again, there are two observations. First, because the composite good 1 is now a necessity, market labor inputs of workers of sector 1 do not fall much compared with those of workers of sector 3 regardless of how their spouse works. This is because, as indicated in Figure 5, the aggregate demand for good 1 is higher than that for good 3. Second, a decline in market labor inputs is attenuated for workers whose spouse works for sector 1 compared with workers whose spouse works for sector 3. For example, while workers of type ω_{31} households see a change in market labor inputs of -30.82%, those of type ω_{33} households see a change of -27.52%. Because composite good 1 $c_{j,1}$ is a necessity and good 3 $c_{j,3}$ is not, workers of sector 3 have less of a motive to maintain their market labor inputs $h^{j,a}$.

Not surprisingly, how market labor inputs of member a respond to the SAH policy

¹¹Throughout simulations in this subsection, we set the value of b equal to 15.

depends on the substitutability of home labor inputs across members. Table 1(3) and (4) show the results of the model simulations with a different value for the parameter ρ , $\rho = 0.1$. As the value of the parameter ρ falls from the baseline value of 0.5, home labor inputs provided by each member become less substitutable with those provided by their spouse. Consequently, home labor inputs of the two members become less dependent on each other and market labor inputs fall more in response to the implementation of SAH policies.

4.3 Necessities and spousal matrix in the data

4.3.1 Necessities in the data

Lastly, we discuss the model's quantitative implications by calibrating it to Japan's economy. First, we seek the subsistence point of good b that is consistent with the data. To do this, we borrow the idea of Bils and Klenow (1994) to estimate "Engel curves" -elasticities of expenditure with respect to households' total nondurable consumption, categorizing as necessities goods whose spending does not increase much when total nondurable consumption increases. We seek a value of the subsistence point b_k that generates an elasticity from the model that is quantitatively similar to the data.

Figure 6(a) shows the elasticity of consumption spending of goods k, i.e., $\Delta \ln c_k$, to a change in total consumption $\Delta \ln c$, estimated using data from the Family Income and Expenditure Survey released by the Ministry of Internal Affairs and Communications in Japan, for nine goods and services, "food (Good 1)," "housing (Good 2)," "fuel, light, and water charges (Good 3)," "furniture and household utensils (Good 4)," "clothing & footwear (Good 5)," "medical care (Good 6)," "transportation & communication (Good 7)," "education (Good 8)," and "elderly care (Good 9)."¹² The estimated results generally agree with the conventional view on the luxuriousness of goods. For example, the elasticities of "clothing & footwear" and "transportation & communication" exceed unity,

¹²In computing the elasticity from the data, we first construct the annual real growth rate of consumption spending of each of the nine categories from 2000 to 2019 using nominal consumption spending data reported in the Family Income and Expenditure Survey deflated by the Consumer Price Index (CPI) and regress the series of each category on that of total real consumption.

which implies that these goods are luxuries. By contrast, the estimates of the elasticities of "medical care," "education," and "elderly care" fall below unity, implying that these goods are necessities. The three bars at the right of the panel show the model-generated elasticity of good 2 $c_{j,2}$ to a change in consumption of good 1 that is computed separately under three different assumptions about the value of the subsistence point b, b = 1, 2, and $3.^{13}$ The model-generated elasticity of good 2 $c_{j,2}$ is about 0.95, 0.87, and 0.35, respectively, suggesting that subsistence points of b equal to a value between 2 and 3 are able to replicate the actual size of the elasticity of necessities. For example, the estimated elasticity of "medical care" is about 0.4, which is close to the model-generated value when b = 3.

Admittedly, the size of demand for some categories of goods and services, such as medical care, may differ across households depending on the characteristics of each household. Households that differ in terms of the age of the members should see different demand structures. Figure 6(2) shows the proportion of individuals that attend a hospital by age group, the proportion of individuals that provide elderly care within the same household by age group, and the proportion of households consisting of a member younger than 18 by the age of the household's head. We employ these variables as proxies for the difference in the size of households' demand for medical care, elderly care, and education services for children, respectively.¹⁴ The panel indicates that the size of demand is indeed age-specific. For example, a household consisting a member younger than 40 is more likely to have a higher demand for child care than a household consisting of a member older than 50, and the former household is less likely to have a higher demand for medical care or elderly care.

While demand for goods and services can differ across households depending on the

¹³In computing the elasticity of spending from the model, we consider the socially optimal equilibrium in which only good 2 is a necessity, i.e., $b_2^j = b > 0$ and $b_k^j = 0$ for $k \neq 2$ and for all households, and compute the consumption of good 2 $\Delta c_{j,2}^*$ and 1 $\Delta c_{j,1}^*$ by changing the level of productivity of the three market goods A in the resource constraint (7) marginally from unity. The elasticity is obtained by dividing the former by the latter.

¹⁴The proportion of individuals that attend a hospital by age group, the proportion of individuals that provide elderly care within the household by age, and the proportion of households consisting of a member younger than 18 by the age of the household's head are all constructed from the data reported in the Comprehensive Survey of Living Conditions released by the Ministry of Health, Labour and Welfare in Japan. The second series is defined as the number of individuals of a specific age who provide elderly care for other household members divided by the total number of individuals who provide elderly care for other household members.

ages of the members, as we see above, what matters for the spillover effects of the SAH policy in our model is how workers in each sector differ in terms of their demand. We therefore examine the age structure of workers in each sector to determine whether or not there is a significant degree of heterogeneity across workers in different sectors regarding what they consider as necessities. Figure 6(3) shows the average age of male and female workers in 18 sectors in Japan.¹⁵ While there are some differences across sectors in terms of the workers' average age, the difference is not substantial. Indeed, in 14 out of the 18 sectors the average age falls into the 40s for both male and female workers and three sectors see an average age in the 40s either for male or female workers. One notable exception is sector A, which includes agriculture, whose workers are aged above 60.

4.3.2 Spousal matrix in the data

Next, we construct the spousal matrix. Table 2 shows the distribution of married-couple households in Japan by sector based on the 2015 Population Census.¹⁶ In the table, the row represents the sector that the husband works for and the column represents the sector that the wife works for. Similar to matrix (1), element (l, m) of the matrix, for example, represents the proportion of spouse pairs whose husband works for sector l and wife works for sector $m.^{17,18}$

Three points are noteworthy. First, an important portion of households is single-income households in which only either the husband or wife supplies market labor inputs. The share of such households varies across sectors. Figure 7(1) shows the share of single-income households over the total number of married-couple households by sector. The share is the

¹⁵The numbers are constructed from the Census conducted in 2015. The sectoral classification of the panel is based on the Japanese industry classification (major group).

¹⁶The figures are constructed from Table 23, "Number of Couples, by Labour Force Status (3 Groups) and Industry (Major Groups) of Husband and Wife" in the census.

¹⁷The figures in the table are shown in terms of the share of each type of household over all marriedcouple households in which either a husband or wife works outside the home, which amounts to 22,093,661 households in 2015. NW includes individuals who are unemployed, out of the labor force, and whose labor force status is unknown.

¹⁸Similar to Figure 6(3), the sectoral classification of the table is based on the Japanese industry classification (major group). In this categorization, the medical and nursing industry and childcare service provided through nurseries belong to "Medical, health care and welfare (sector P)" and pre-school education provided through kindergartens belongs to "Education, learning support (sector O)."

highest in sectors F and G, both of which exceed 30%, and lowest in sectors A, M, and P, all of which are about 15%. Second, in most of the sectors the largest number of households belongs to the type in which both members work in the same sector. In other words, the diagonal elements in the spousal matrix Γ_{ω} are large relative to the off-diagonal elements. This can be seen in the shaded diagonal elements in the table. Figure 7(2) shows the share of workers whose partner works in the same sector by sector, namely the share of workers who belong to households located in the diagonal elements of the matrix. For the purpose of comparison, we also show the mean across sectors, which is the average share of workers who belong to households located in off-diagonal elements. Sectors A and T are the sector that see the highest shares for the diagonal element, of about 60%. The shares in other sectors are significantly lower, at about 20% on average. Third, there are two sectors that exhibit a "large column sum" property.¹⁹ Those are sectors I and P. As seen from the colored columns in Table 2, for a husband working in any sector, the probability that his wife works for either of the two sectors I or P is high relative to other sectors. Figure 7(3)shows for each sector the share of workers who have a spouse working for sector I or P. The share of workers whose spouse works for one of the two sectors exceeds the average share in almost all sectors and is twice as high as some sectors.

Combined with the discussion regarding Table 1, there are two takeaways. First, other things being equal the spillover effects on market goods production may be large for sectors A, M, and P. Because the share of single-income households is small, it is likely that workers in these sectors need to increase home labor inputs when supply of necessities is scarce. Second, due to the "large column sum" property, changes in market labor inputs to sectors I and P, including those arising from a SAH policy, are likely to affect market labor inputs in a broad set of sectors through changes in the market labor inputs of the spouses of the workers in these two sectors.

¹⁹In his seminal work on the role of input-output relationships on aggregate fluctuations, Dupor (1999) coined the term "large row sum," pointing out that the row sums measure the importance of each sector's output as an input material to all other sectors.

4.3.3 Simulation using the model calibrated to the actual economy

Using the model calibrated to Japan's economy, we simulate the spillover effects of a SAH policy. We study a hypothetical scenario in which the SAH policy is imposed on the education sector. We consider an economy that consists of three sectors: sector O, "Education, learning support," sector P, "Medical, health care and welfare," and sector "Others" that includes the rest of the sectors in the economy listed in Table 2. Regarding parameterizations, we assume that the subsistence point of the two composite goods produced from sectors O and P b_O and b_P are equal to 3 and that of the composite good produced from the product of "Others" b_{Others} is equal to 0. Other parameter values are the same as those described in Section 4.1. The spousal matrix of this economy shown below is given by rearranging the elements of Table 2.

$$\Gamma_{JP} = \begin{bmatrix} No \text{ market work} & \hat{k}_{jb} = P & \hat{k}_{jb} = O & \hat{k}_{jb} = \text{Others} \\ No \text{ market work} & N.A. & 0.012 & 0.003 & 0.039 \\ \hat{k}_{ja} = P & 0.016 & 0.021 & 0.002 & 0.011 \\ \hat{k}_{ja} = O & 0.013 & 0.004 & 0.010 & 0.008 \\ \hat{k}_{ja} = \text{Others} & 0.309 & 0.102 & 0.025 & 0.426 \end{bmatrix}$$
(18)

It can be seen from the matrix that the unique feature of sector P is preserved in this three-sector setting. A disproportionately large number of workers belong to a household that consists of a member b who works for sector P.

Figure 8 shows the spillover effects of the SAH policy on market good O to sectors P and Others under the baseline parameterization, shown in the black solid line, and two other alternative parameterizations, shown in the red dotted line. Under the baseline scenario, the output of market goods P and Others both fall and the former falls more than the latter through the mechanisms explained above. When the subsistence point of composite good $P c_P^j$ is zero, namely $b_P = 0$, households demand market good $P c_P^j$ less, leading to a larger decline in the market production of good Y_P , as shown in the left column. When the substitutability of home goods is low, namely $\rho = 0.1$, compared with the baseline

results, a fall in market goods production in the two unaffected sectors, P and Others, is mitigated. Because home goods produced by the spouse become less substitutable, each of the workers needs to devote more home labor input, which in turn makes the economy more susceptible to the spillover effects.

To see the spillover effects from a different angle, we show in Table 3 how member aand b in each type of households responds to the SAH policy imposed on market good Oproduction. Note that for simplicity we only consider the case when $\phi = 0.9$. In terms of market labor inputs to sector P, a worker whose spouse works for the same sector sees the largest decline, followed by a worker whose spouse works for sector O and for Others. A worker whose spouse is a homemaker even sees an increase in market labor input. As already discussed above, when their spouse works for sector O or Others, a worker faces a weaker incentive to reduce their market labor inputs because their spouse allocates a relatively large amount of labor input to home production $\tilde{C}_{i,O}$. Notice also that this quantitative relationship between market labor input to sector P and Others is linked to the "large column sum" property of sector P. In so far as good P is considered less luxurious than other goods by households in the economy, this property helps mitigate a fall in market good P production and amplify a fall in the production of other market goods following the introduction of SAH policies. Needless to say, a similar argument holds regarding how market labor inputs to Others are affected by this "large column sum" property. Indeed, other things being equal, a decline in market labor inputs to the sector is amplified by the fact that a large number of workers in the sector have a spouse working for sector P.

The lower two tables of Table 3 show the simulation results in an economy where $\rho = 0.1$ instead of 0.5. Indeed, as suggested in Figure 8, the division of labor inside the home is the key not only for the relative size of changes across workers but also for the absolute size of changes in market labor inputs. That is, while the ordering of the decline in market labor inputs across sectors is unaltered from the case of $\rho = 0.5$, the magnitude of the decline is larger except for households whose members both work for sector P.

5 Conclusion

The benefits of SAH policies have been widely accepted as our understanding of Covid-19 has deepened. These policies come, however, with economic costs. Among them, spillover effect or alternatively "knock-on effects" from the affected sectors to the rest of the economy are considered a key ingredient of the costs. In this paper, we focus on the spillover effect led by households' reallocation of working hours inside and outside the home. While our observations on the experience of SAH policies up to now is rather clear that such a channel exists, the nature of the channel, including why it exists or what determines its size, has not been explored theoretically yet. In this paper, we aim to fill this gap, shedding light on the interaction between the luxuriousness of goods, home production, and the spousal matrix.

We develop a simple general equilibrium model with multiple goods-producing sectors and multiple types of households whose members work inside or outside the home or both. Using the model, we derive some theoretical predictions. First, goods that a large number of households consider necessities are likely to cause large spillover effects, though the size is easily affected by characteristics of the spousal matrix. When the stay-at-home policy is imposed on necessities, households increase home production, aiming to mitigate the decline in the market supply of necessities at the expense of reducing their market labor inputs. Consequently, market goods production of unaffected sectors falls. Quantitative impacts of the spillover change, however, depending also on characteristics of the spousal matrix, i.e., how each member of a household works at home or outside the home. For example, a decline in market good production for a sector can be significant when the spouses of workers in the sector works for sectors that produce necessities.

Theoretical predictions drawn from our model have important policy implications regarding how SAH policies should be calibrated in practice. As shown in our quantitative example using Japan's data, households' preferences, spousal working status, and the distribution of households with different characteristics are all responsible in determining the spillover effects. These predictions, however, also indicate that the nature of the spillover effects are not unique across jurisdictions and depend on the age structure and preferences of households, as well as the distribution of household types.

There are three caveats to our analysis. First, while our analysis addresses costs associated with SAH policies from a view point that has not been much explored in existing studies, it is silent about the benefits of the policies. Indeed, similar to the costs, the benefits of SAH policies are considered as arising from multiple channels, depending on various characteristics of workers in affected sectors and their customers, including ages and the composition of the household members. Targeted SAH policies, if conducted, need to address heterogeneity of workers, consumers, and households in both aspects. Second, our analysis employs a static model in which households' intertemporal considerations are absent. This is because we are interested in short-run reallocations of households' labor inputs, in particular interactions between labor inputs outside and inside the home, keeping households' saving and investment decisions out of the scope of the analysis. Admittedly, however, dynamic aspects of SAH policies have attracted attention, in particular when the welfare implications of the policies are discussed.²⁰ Third, our analysis is silent about other potential channels that can enhance the spillover effects. For example, interactions across different production sectors through input-output linkages, along the lines of Dupor (1999), can importantly alter how the spillover effects are translated to the rest of the economy. Incorporating these three features is left for future research.

 $^{^{20}}$ Some existing studies argue that benefits of lockdown are highest at the early stage of the pandemic and these benefits decline over time. See, for example, Alvarez et al (2020), Demirguc-Kunt et al. (2020) and Ornelas (2020).

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Notes: 1. Households of group 1, 2, and 3 stand for the set of households that consists of a worker who works in sector 1, 2, and 3, respectively.

2. 1, 2, and 3 in panels (4), (5), and (6) stand for the amount of labor inputs allocated for home goods production of home goods 1, 2, and 3, among each household group, respectively.



a worker who works in sector 1, 2, and 3, respectively. 2. 1, 2, and 3 in panels (4), (5), and (6) stand for the amount of labor inputs allocated

for home goods production of home goods 1, 2, and 3, among each household group, respectively.





Notes: 1. The panels show the effects of SAH policies in economies with different values for the subsistence points regarding goods 2, b = 1, 3, and 5.

2. Households of group 1, 2, and 3 stand for the set of households that consists of a worker who works in sector 1, 2, and 3, respectively.





Notes: 1. The panels show the impacts of SAH policies in economies with three different distributions of the subsistence point regarding goods 2 and 3 across households. Case 1 considers an economy where good 2 is a necessity for all types of household groups other than group 1 households. Case 2 considers an economy where good 2 is a necessity for all households. Case 3 considers an economy where good 2 is a necessity for all households.

2. Households of group 1, 2, and 3 stand for the set of households that consists of a worker who works in sector 1, 2, and 3, respectively.





Notes: 1. The panels show the impacts of SAH policies in economies with three different distributions of the subsistence point regarding goods 2 and 3 across households. Case 1 considers an economy where goods 1 and 2 are necessities for all types of household groups. Case 2 considers an economy where good 2 is a necessity for all types of household groups. Case 3 considers an economy where good 1 is a necessity for all types of household groups while good 2 is a necessity only for households of groups 1 and 2.

2. Households of groups 1, 2, and 3 stand for the set of households that consist of a worker who works in sector 1, 2, and 3, respectively.



(1) Elasticity of goods consumption







(3) Average age of workers by sector

Notes: 1. Goods 1 to 9 correspond to "Food," "Housing," "Fuel, light and water charges," "Furniture and household utensils," "Clothing and footwear," "Medical care," "Transportation and communication," "Education," and "Elderly care," respectively.
2. "Attend a hospital" is computed as the proportion of individuals attending a hospital

due to injury or illness over the total individuals by age group. "Elderly care" is computed as the proportion of individuals in the age group who provide elderly care for a family member among all individuals providing elderly care to a family member. "Child care" is computed by dividing the number of households that consist of a family member below the age of 18 by the total number of households by age.

3. See footnote of Table 2 for the definitions of sectors A to T in panel (c).

Figure 7: Properties of married-couple household distribution



(1) Share of single-income households

(2) Share of workers from a household of diagonal elements



30 25 Share of sector P Mean Share of Sector I 20 15 10 5 0 D Е F G К 0 Ρ Н I J L Ν Q R S Т А Μ

(3) Share of workers from a household that consists of a worker of sector P and I

Notes: 1. Single-income households are the subset of married-couple households in which only either husband or wife has a market job and his/her partner is either unemployed, out of the labor force, or has a labor status that is unknown.

2. Panel (b) represents the number of husbands and wives from a household in which the couple works for the same sector divided by the total number of marriedcouple households.

- 3. Panel (b) represents the number of husbands and/or wives working in sector P or I divided by the total number of married-couple households.
- 4. "Mean" is computed from unity less the figure described in footnote 2 divided by the total number of households in each sector.



Figure 8: Effects of SAH on sector O

Notes: 1. Actual household matrix is constructed from 2015 population census for sectors O, P, and other sectors.

2. Panels on the left show the simulation results in an economy where the subsistence point of good P is zero and the baseline, respectively, and panels on the right show the simulation results in an economy where the degree of substitution is low and the baseline.

Table 1: Changes in market labor inputs of member a (1) Composite 2 is a necessity (rho = 0.5)

	No work	sector 1	sector 2	sector 3
No work	N.A.	N.A.	N.A.	N.A.
sector 1	-0.75	-10.95	-8.25	-10.95
sector 2	-10.00	-10.00	-10.00	-10.00
sector 3	-0.75	-10.95	-8.25	-10.95

(2) Composites 1 and 2 are necessities (rho = 0.5)

	No work	sector 1	sector 2	sector 3
No work	N.A.	N.A.	N.A.	N.A.
sector 1	0.30	-2.33	-0.58	-0.87
sector 2	-10.00	-10.00	-10.00	-10.00
sector 3	-9.15	-30.82	-22.40	-27.52

(3) Composite 2 is a necessity (rho = 0.1)

	No work	sector 1	sector 2	sector 3
No work	N.A.	N.A.	N.A.	N.A.
sector 1	-2.58	-11.00	-10.42	-11.00
sector 2	-10.00	-10.00	-10.00	-10.00
sector 3	-2.58	-11.00	-10.42	-11.00

(4) Composites 1 and 2 are necessities (rho = 0.1)

	No work	sector 1	sector 2	sector 3
No work	N.A.	N.A.	N.A.	N.A.
sector 1	-0.05	-2.33	-1.93	-1.98
sector 2	-10.00	-10.00	-10.00	-10.00
sector 3	-12.12	-29.14	-26.81	-28.00

				Female																
r		А	D	Е	F	G	Н	Ι	J	Κ	L	М	N	0	Р	Q	R	S	Т	NW
	А	2.44	0.01	0.16	0.00	0.00	0.02	0.18	0.02	0.01	0.01	0.09	0.05	0.05	0.30	0.02	0.05	0.03	0.02	1.11
	D	0.10	1.48	0.75	0.01	0.05	0.15	1.14	0.19	0.08	0.10	0.50	0.32	0.23	1.49	0.05	0.33	0.09	0.13	4.03
Male	Е	0.11	0.15	3.31	0.02	0.12	0.29	1.97	0.34	0.12	0.22	0.74	0.41	0.56	2.40	0.08	0.50	0.19	0.17	6.95
	F	0.00	0.01	0.03	0.03	0.01	0.01	0.07	0.02	0.01	0.01	0.03	0.01	0.04	0.12	0.00	0.02	0.03	0.01	0.32
	G	0.01	0.03	0.15	0.00	0.32	0.04	0.33	0.10	0.04	0.09	0.11	0.07	0.16	0.37	0.01	0.12	0.05	0.05	1.35
	Н	0.05	0.06	0.57	0.00	0.03	0.40	0.94	0.13	0.05	0.06	0.39	0.22	0.14	0.99	0.04	0.25	0.06	0.08	2.50
	Ι	0.06	0.09	0.62	0.01	0.08	0.16	3.60	0.22	0.10	0.12	0.48	0.28	0.31	1.39	0.05	0.32	0.10	0.13	4.06
	J	0.01	0.02	0.07	0.00	0.02	0.03	0.20	0.25	0.02	0.03	0.07	0.04	0.10	0.22	0.01	0.07	0.04	0.02	0.94
	K	0.01	0.02	0.08	0.00	0.01	0.02	0.20	0.04	0.41	0.03	0.08	0.05	0.06	0.21	0.01	0.06	0.02	0.03	0.98
	L	0.02	0.03	0.16	0.00	0.05	0.04	0.32	0.08	0.04	0.62	0.11	0.07	0.19	0.43	0.01	0.11	0.07	0.04	1.58
	М	0.01	0.01	0.10	0.00	0.01	0.03	0.27	0.04	0.02	0.02	1.08	0.07	0.05	0.25	0.01	0.06	0.02	0.03	0.68
	Ν	0.01	0.01	0.09	0.00	0.01	0.02	0.21	0.03	0.02	0.02	0.09	0.62	0.05	0.22	0.01	0.06	0.02	0.02	0.72
	0	0.01	0.02	0.09	0.00	0.02	0.02	0.21	0.05	0.02	0.05	0.07	0.05	1.02	0.41	0.01	0.07	0.08	0.02	1.29
	Р	0.02	0.02	0.14	0.00	0.02	0.03	0.32	0.05	0.03	0.04	0.11	0.07	0.16	2.07	0.01	0.09	0.07	0.04	1.64
	Q	0.02	0.01	0.06	0.00	0.00	0.01	0.09	0.02	0.01	0.01	0.03	0.02	0.04	0.16	0.08	0.02	0.02	0.01	0.26
	R	0.04	0.04	0.34	0.00	0.03	0.08	0.61	0.10	0.05	0.06	0.25	0.15	0.16	0.74	0.02	0.77	0.06	0.07	2.47
	S	0.03	0.03	0.15	0.01	0.02	0.04	0.35	0.10	0.03	0.06	0.14	0.08	0.31	0.79	0.03	0.12	0.48	0.03	1.73
	Т	0.00	0.00	0.04	0.00	0.00	0.01	0.06	0.01	0.01	0.01	0.03	0.02	0.01	0.07	0.00	0.03	0.00	1.95	1.21
	NW	0.14	0.07	0.67	0.01	0.03	0.11	1.00	0.13	0.09	0.07	0.48	0.35	0.31	1.22	0.04	0.44	0.11	0.21	NA

Table 2: Married-couple Household Composition

A: Agriculture and forestery+Fisheries+Mining and quarrying of stone and gravel, D: Construction, E: Manufacturing

F: Electricity, gas, head supply and water, G: Information and communication, H: Transport and postal activities, I: Wholesale and retail trade

J: Finance and insurance, K: Real estate and goods rental and leasing, L: Scientific research and professional and technical services

M: Accommodations, eating and drinking services, N: Living-related and personal services and amusement services, O: Education, learning support

P: Medical, health care and welfare, Q: Compound services, R: Services, N.E.C., S: Government except elsewhere classified, T; Industries unable to classify

	No work	sector P	sector O	Others
No work	N.A.	N.A.	N.A.	N.A.
sector P	0.52	-14.92	-13.10	-11.56
sector O	-10.00	-10.00	-10.00	-10.00
Others	-21.14	-52.16	-48.91	-51.47

Table 3: Changes in market labor inputs of members a and b (1) Market work of members a (upper) and b (lower), when rho = 0.5

	No work	sector P	sector O	Others
No work	N.A.	0.52	-10.00	-21.14
sector P	N.A.	-14.92	-10.00	-52.16
sector O	N.A.	-13.10	-10.00	-48.91
Others	N.A.	-11.56	-10.00	-51.47

(2) Market work of members a (upper) and b (lower), when rho = 0.1

	No work	sector P	sector O	Others
No work	N.A.	N.A.	N.A.	N.A.
sector P	-2.01	-14.81	-13.99	-13.98
sector O	-10.00	-10.00	-10.00	-10.00
Others	-26.61	-52.86	-49.53	-52.45

	No work	sector P	sector O	Others
No work	N.A.	-2.01	-10.00	-26.61
sector P	N.A.	-14.81	-10.00	-52.86
sector O	N.A.	-13.99	-10.00	-49.53
Others	N.A.	-13.98	-10.00	-52.45