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Abstract

This paper proposes a framework to jointly study productivity and trade dynamics during financial crises. The persistent output loss caused by crises is driven by lower productivity growth, which is determined by changes in product entry and exit margins in domestic and export markets. We calibrate and validate the model using unique data on firms' product portfolios, finding it closely matches the behavior of various margins during Chile's 1998 sudden stop. We decompose the sources of the welfare cost of sudden stops, finding a third of the welfare cost is due to a decline in productivity growth. Lower productivity growth, in turn, is due mostly to slower firm and product entry into the domestic market, while a decrease in production costs induces surviving firms to tilt their product portfolios towards export markets, boosting the productivity recovery in the aftermath of the crisis.

Keywords: Endogenous growth; Firm dynamics; Trade dynamics; Sudden Stops
JEL classification: F10, F41, F43, F44, O33

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1 Introduction

Sudden stops - sharp contractions in capital inflows - often cause severe economic downturns in emerging economies. The persistent effect on output of this type of financial crises is indicative of a decline in productivity. Unbundling the slowdown in aggregate productivity, recent work points to distortions affecting firm creation and the expansion of incumbent firms during financial crises. The literature, however, has ignored the role played by exporters in productivity dynamics during sudden stops. Because exporters are central to the trade surpluses observed during crises and are a key driver of productivity growth in open economies, this is potentially an important omission. In fact, exporting firms typically account for the bulk of productivity growth [Bernard and Jensen, 2004], and during sudden stop episodes, the ensuing exchange rate adjustment boosts the demand for exports relative to domestic sales. This mechanism gives rise to differential firm and product entry rates into export and domestic markets.

We propose a unified framework to study sudden stops in which the evolution of trade and productivity dynamics shape the aggregate response of the economy. Our model bridges the endogenous sudden stop literature [Mendoza, 2010] and theories of firm dynamics with endogenous growth [Klette and Kortum, 2004], adding to this framework product-level export dynamics. We use unique data on Chilean manufacturing firms' product portfolios to discipline our quantitative model, and show it matches macro and micro dynamics during sudden stops. With the calibrated model, we calculate that the slowdown in domestic product entry rates are the key driver of the productivity loss on impact, while export product entry shapes the productivity recovery in the aftermath.

We model a small open economy consisting of final and intermediate good producers. Firms in the intermediate-goods sector innovate to introduce domestic and export product lines. Final-good producers demand intermediate inputs to produce. These firms own a productive asset used as collateral for borrowing to finance working capital. Series of favorable productivity and interest rate shocks cause increases in leverage. Unfavorable shocks arriving at times of high leverage make the collateral constraint binding, raising the effective cost of borrowing during a financial crisis and decreasing the demand of tradable final good producers for intermediate inputs.

Sudden stops have a starkly different impact on exporters and non-exporters. Intermediate goods sold domestically face a lower demand, and therefore lower profits, due to the local crisis. The lower value of domestic product lines reduces product entry into the domestic market by both new and incumbent firms. In contrast, exported products do not rely entirely on local demand, and exports benefit from lower wages while facing a stable foreign demand. This is a classical exchange rate effect favoring exporters during crises. Therefore, the value of export product lines increases relative to domestic ones, generating incentives for incumbent firms to incur in innovation to introduce exported products. Consistent with the empirical evidence in Alessandria et al. [2014], the extensive margin of exports adjusts gradually and drives a sluggish recovery in productivity.

We discipline the model using novel data on firms' product portfolios. We use unique firm-level data from the Chilean manufacturing census that lists each firm's entire set of products with revenue split between local and foreign sales. The model is calibrated to match the firm-level product portfolio distribution. To validate the model, we document a series of facts. We show that firms typically introduce products first into the domestic market and later into export markets. Similarly, when firms drop products from the export market, they typically keep selling them in the domestic market; products are rarely drawn from the domestic and export markets simultaneously. In addition, we document how firms' existing product portfolios determine the probability of observing each of these transitions. For instance, the probability that a firm introduces a product in the domestic market is increasing in the firms' number of existing products, and the probability that a firm starts exporting a product previously sold only domestically is increasing in the number of domestic products. Although these moments were not used in the calibration, the model is able to replicate the frequency of these transitions along its balanced growth path.

We then explore the response of the model to a sudden stop, and contrast it to the behavior of Chilean firms' product portfolios during the 1998 sudden stop. This event was large, unanticipated and exogenous to the Chilean economy. We observe in the data that domestic product entry falls strongly during the crisis, while the probability of adding export products is much more stable. The model closely replicates these dynamics, which were not targeted in the calibration. The main mechanism in the model is the local contraction in demand due to the financial friction, and the larger this friction, the larger the asymmetry between export and domestic product dynamics. We take this prediction to the data, showing that domestic sales fall relative to export sales during the crisis, and this is especially the case in more financially-dependent sectors. This is indicative of a financial channel driving product dynamics during the 1998 sudden stop, in line with the mechanism in the model.

With the calibrated model, we find that following a sudden stop GDP and consumption fall by 7% and 10% below trend respectively, while the asset price falls by 12%. More importantly, firm and trade dynamics in the simulated model are consistent with the empirical evidence in the literature and with our empirical findings. First, both entry rates and incumbent firms' innovation rates decline substantially during sudden stops as in Ates and Saffie [2016]. This slows down aggregate productivity growth and in turn leads to a persistent negative impact on GDP, consistent with the empirical findings in Cerra and Saxena [2008] and Blanchard et al. [2015]. Second, while imports of intermediate goods decline, exports of intermediates remain stable, roughly in line with empirical facts documented in Alessandria et al. [2014]. Moreover, export profits rise by 23% relative to the trend and remain above-trend several years following a sudden stop, leading to a gradual expansion in the extensive margin of exports.

Finally, we use our model to study to what extent productivity and trade dynamics account for the welfare loss from a sudden stop. Consumers would be willing to forgo 4.6% of their consumption the period before the crisis to avoid an average sudden stop. About 30% of this welfare loss is due to the endogenous slowdown in productivity growth. Lower productivity growth on impact is explained entirely by lower entry rates for domestic products, while higher entry rates for exported products drive the recovery after the crisis.

The rest of the paper is organized as follows. Section 1.1 reviews the related literature. Section 2 introduces the model. Section 3 includes the calibration and validation of the model and the quantitative analysis. Section 4 concludes.

1.1 Contribution to the Existing Literature

Our paper contributes to a literature studying the response of the economy to sudden stops. Recent work [Mendoza, 2010, Jeanne and Korinek, 2019, Bianchi, 2011, Bianchi and Mendoza, 2018] models sudden stops as endogenous events using occasionally binding collateral constraints. This approach produces the amplification and asymmetry that these events epitomize, preserving long-run business cycle properties of standard models. Our contribution to this literature is to incorporate productivity and trade dynamics in a heterogeneous-firms framework. This is essential because the slow recovery following these episodes is characterized by slow TFP growth [Meza and Quintin, 2007, Pratap and Urrutia, 2012] and a key role played by the extensive margin of exports [Alessandria et al., 2014]. Recent developments in this literature [Seoane and Yurdagul, 2019, Akıncı and Chahrour, 2018] introduce growth- rate trend shocks and news shocks to improve the quantitative performance of these models. Endogenous technological change generates fluctuations in growth rates with similar properties to news and trend shocks, so by explicitly modeling endogenous trade and productivity dynamics we provide a measurable micro-foundation for these channels.

Our paper is also part of a nascent literature that blends endogenous technological change and international finance with the goal of studying the medium and long-run consequences of large but temporary external shocks. Comin and Gertler [2006] develop a model in which short-run shocks to the economy cause medium-term business cycles using a product-variety-expansion type of endogenous growth framework. A similar framework is used by Queralto [2020] to study Korea's 1997 financial crisis, by Guerron-Quintana and Jinnai [2019] to measure the cost of the U.S. Great Recession, by Gornemann [2014] to explain long-term costs of sovereign crises, and by Ma [2020] to study macroprudential policies. Closer to our paper Ates and Saffie [2016] bridge a version of the Schumpeterian growth model of Klette and Kortum [2004] and the business cycle model of Neumeyer and Perri [2005] and Uribe and Yue [2006] to show that sudden stops have a persistent effect on growth through the composition of entering firms. Matsumoto [2019] extends Ates and Saffie [2016] allowing for an occasionally binding constraint to study the interplay of FDI and reserve accumulation in emerging countries. A key contribution of our model to this literature is incorporating trade dynamics, which are essential to the understanding of sudden stops in emerging markets. In addition, we contribute to this literature by contrasting the model with micro-data on firms' domestic and export product portfolios.

These trade dynamics are important, as the literature studying the adjustment of exporters to crises or large devaluations has shown. In this regard, Alessandria et al. [2014] find that the sluggish response of exports to large devaluations is driven by the extensive margin, which adjusts slowly given its forward looking nature. Alfaro et al. [2018] document that exporting firms' productivity and innovation rise in response to depreciation. Blaum [2017] shows that the response of imports to devaluations is determined in part by the fact that large exporters – which expand during a devaluation – import inputs to produce.

We also relate to the literature on endogenous technical change. Recent work has studied the interaction between trade and productivity dynamics (e.g. [Perla et al., 2015, Buera and Oberfield, 2016, Sampson, 2015, Bloom et al., 2013]). Closest to our paper, Akcigit et al. [2018] also model competition between intermediate good producers across countries. The link between trade and productivity in our model is more stylized, allowing us to go beyond transitional dynamics and studying aggregate risk with occasionally binding financial constraints. On the empirical front we also make a key contribution to the endogenous technical change literature. In fact, the quantitative literature that builds on Klette and Kortum [2004] has relied on patent and plant level data to estimate the parameters governing the expansions and contractions of products [Akcigit and Kerr, 2018, Acemoglu et al., 2018, Lentz and Mortensen, 2008, Cao et al., 2018]. In contrast, we observe the portfolio of domestic and exported products at the plant level. Thus, this is the first paper in the Klette and Kortum [2004] framework that uses product-level data for calibration and validation.

2 Model

The model consists of an infinite-horizon small open economy (Home). A representative firm produces a tradable final good. This firm borrows working capital within each period and faces an endogenous collateral constraint. Shocks to aggregate productivity and the real interest rate can occasionally make this constraint binding and generate sudden stops.

A set of firms produce differentiated intermediate varieties used to assemble the final good in the domestic market or abroad. This intermediate sector is modeled as a version of the Schumpeterian growth model developed by Ates and Saffie [2016], which is a discrete time version of Klette and Kortum [2004] incorporating aggregate risk. These intermediate good producers innovate to introduce new product lines, competing among them and with foreign firms to become the lowest cost producer under Bertrand competition. These firms also innovate to be able to export these product lines. This setting in the intermediate sector generates endogenous productivity dynamics. In addition it gives rise to trade dynamics at the intensive and extensive margins. An overview of this environment is presented in Figure 1.

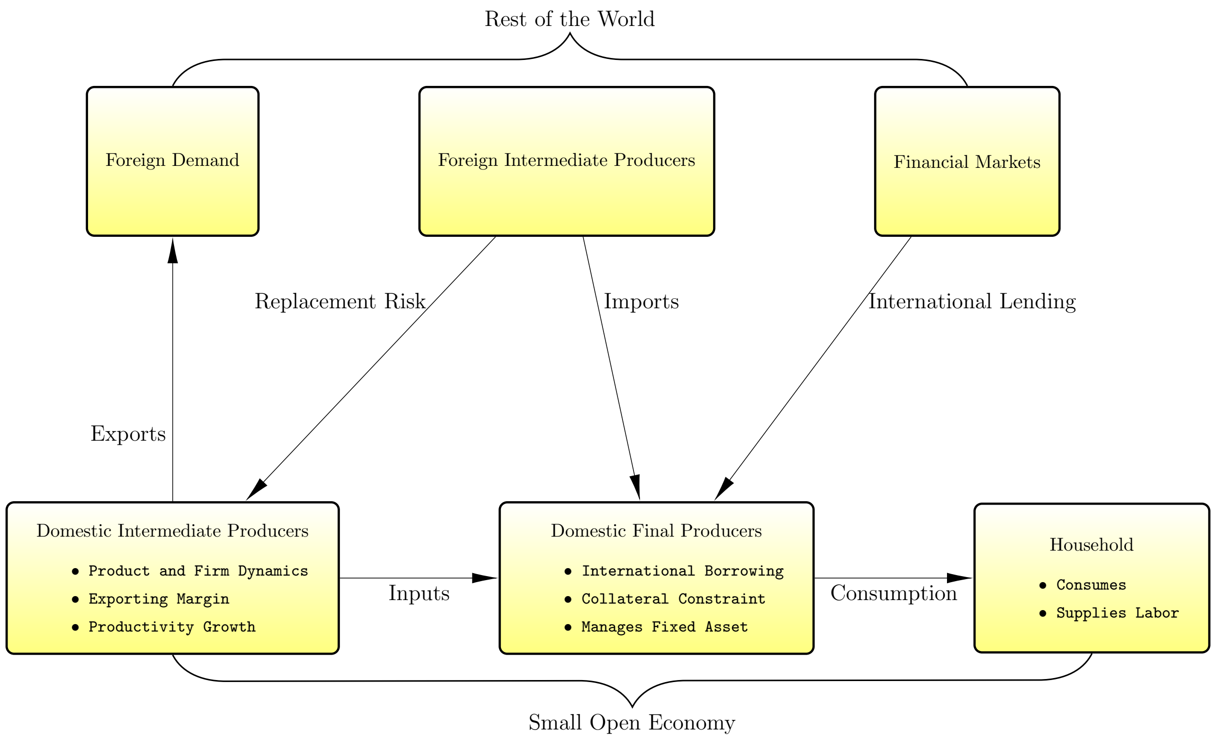


Figure 1: The Model Economy

2.1 Tradable Final Good

A representative firm produces a tradable final good using a set of differentiated intermediate goods $\{y_t(i)\}_{i=0}^{1}$ according to the production function:

$$Y_t = \exp(\varepsilon_t^A) \exp\left[\int_0^1 \ln y_t(i) di\right] , \tag{1}$$

in which ε_t^A is a stochastic productivity shock.

We assume the firm must pay in advance a fixed fraction ϕ of the cost of intermediate inputs. This working capital payment is financed by within-period borrowing from abroad without any interest. In addition, the firm borrows from abroad using a one-period non-contingent bond. The firm is subject to the following collateral constraint, which states that total borrowing must not be larger than a fixed fraction κ of the value of a productive asset owned by the firm and used as collateral.

$$-B_t + \phi\left[\int_0^1 p_t(i) y_t(i) di\right] \leq \kappa Q_t L_{t-1} \tag{2}$$

In this expression Q_t denotes the price of the asset and L_{t-1} denotes the amount owned by the firm. The firm rents this productive asset at a rate R_t^L to firms in the intermediate good sector, which require it to produce. This asset exists in fixed supply. Each period, the firm chooses amounts of each intermediate good $\{y_t(i)\}_{i=0}^1$, the amount of the productive asset L_t to hold, and foreign bond holdings B_t to maximize the discounted value of current and future profits:

$$\max_{\{\{y_t(i)\}_{i=0}^1, L_t, B_t\}_{t=0}^\infty} E_o \sum\nolimits_{t=0}^\infty \beta^t \lambda_t \Pi_t$$

subject to the collateral constraint (2). Future profits are discounted with the same discount rate used by the representative household.[1] Firm profits are:

$$\Pi_t = Y_t - \int_0^1 p_t(i) y_t(i) di - B_t + \exp(\varepsilon_{t-1}^R) R B_{t-1} - Q_t L_t + (Q_t + R_t^L) L_{t-1} \tag{3}$$

where λ_t is the marginal utility of tradable goods consumption by households, $p_t(i)$ is the price of intermediate good i, and $\exp(\varepsilon_t^R)R$ is a stochastic gross interest rate on the foreign bond. Each period, the final tradable producer chooses intermediate goods demand $\{y_t(i)\}_{i=0}^1$, productive asset holdings L_t, and foreign bond holdings B_t to maximize the expected profit discounted by household's discount rate adjusted by the marginal utility λ_t. The demand for each intermediate good is:

$$p_t(i) \left(1 + \phi \frac{\mu_t}{\lambda_t}\right) = \frac{Y_t}{y_t(i)} \tag{4}$$

In this expression, μ_t stands for the Lagrange multiplier on the borrowing constraint (2). When the borrowing constraint is slack, $\mu_t = 0$ and the demand function for intermediate goods (4) is standard, equating price and marginal product. When the borrowing constraint binds, a strictly positive μ_t appears as the external financing premium on working capital payments, which increases the effective cost of inputs.

2.2 Intermediate Goods

There is a continuum of differentiated tradable intermediate goods indexed by $i \epsilon [0, 1]$ used to assemble the final good. We refer to these as product lines. These intermediate goods can be

[1] The assumption that the firm can own an asset and can borrow from abroad instead of households makes the problem more tractable. A model in which households own the asset and borrow from abroad would be equivalent but less tractable.

produced by Home or Foreign firms. Each product line is produced by a single firm - the lowest cost producer - in a context of Bertrand competition.

These intermediate good producers have heterogeneous productivity levels $a_t(i)$. They produce using the productive asset $\ell_t(i)$ and labor $h_t(i)$ according to the following production function:[2]

$$y_t(i) = a_t(i)\,(\ell_t(i))^{\alpha}\,(h_t(i))^{1-\alpha} \tag{5}$$

Firms innovate to introduce new product lines by becoming the lowest cost producers. When a firm carries on a successful domestic innovation, it obtains a productivity lead equal to $(1+\sigma^D)$ times the previous leading technology, which becomes available to all firms. Firms also innovate to export existing domestic product lines. When a firm carries out a successful export innovation it obtains a larger productivity lead equal to $(1+\sigma^X) > (1+\sigma^D)$ times the previous existing technology. For each product line, only one successful innovation occurs at a time. The probability of successful innovations and the investment firms need to incur in to innovate are discussed in the next subsection.

Product lines can be classified into domestic lines (D) (in which the lowest cost producer is a domestic firm), export lines (X) (in which a domestic firm has innovated to be able to sell the product both domestically and abroad) and import lines (M) (in which the lowest cost producer is a foreign firm and the final tradable good producer imports the product). We describe the profits for each of these three types below.

Under Bertrand competition, the firm with the leading technology sets a price equal to that of the marginal cost of its competitors which have the second-best (i.e. the previous leading) technology. Firms' marginal cost depends on factor prices and trade costs (both of which differ between Home and Foreign firms), and their productivity.[3]

Foreign firms trying to sell in the Home market face an iceberg trade cost such that, shipping $1+\xi$ units is required to sell 1 unit. Home firms trying to sell abroad face the same iceberg trade cost.

■ **(D) Domestic lines** In this case the second lowest marginal cost belongs to domestic firms. Because all domestic firms face the same factor prices, differences in cost between the leading firm

[2] Both factors of production - the productive asset and labor - are internationally immobile

[3] Factor prices are a rental rate R_t^L for the productive asset and wage W_t. These are denoted R_t^{L*} and W_t^* in the case of foreign firms.

and its competitors are due only to differences in productivity. Let $a_t(i)$ denote the productivity level of the lowest cost producer (i.e. the leader) for line i. The price set is equal to the second lowest marginal cost:

$$p_t^D(i) = \widetilde{MC}_t^D(i) = \frac{1}{a_t(i)/(1+\sigma^D)} \overline{\alpha} \left(R_t^L\right)^{\alpha} (W_t)^{1-\alpha} \ , \tag{6}$$

where $\overline{\alpha} = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}$. Profits obtained from this line are:

$$\pi_t^D(i) = p_t(i) y_t(i) - R_t^L \ell_t(i) - W_t h_t(i)$$

Replacing in this expression the demand for intermediate goods by the final tradable good producer (4), profits can be written as:

$$\pi_t^D = Y_t \frac{1}{1+\phi\mu_t/\lambda_t} \frac{\sigma^D}{1+\sigma^D} \tag{7}$$

The following points are worth mentioning. First, profits are independent of the productivity level $a_t(i)$ of the lowest cost producer.[4] Second, profits are a decreasing function of the Lagrange multiplier on the borrowing constraint μ_t.[5] Third, profits are independent of factor prices. Factor prices impact both the cost and the price (which is equal to the cost of the second-best firm), canceling out.

■**(X) Export lines** Export lines are owned by domestic firms and sold both domestically and abroad. In the domestic market, prices and profits are identical to those discussed above for domestic lines, with the only difference that the productivity lead is $(1+\sigma^X)$:

$$p_t^X(i) = \widetilde{MC}_t^X(i) = \frac{1}{a_t(i)/(1+\sigma^X)} \overline{\alpha} \left(R_t^L\right)^{\alpha} (W_t)^{1-\alpha} \tag{8}$$

$$\pi_t^X = Y_t \frac{1}{1+\phi\mu_t/\lambda_t} \frac{\sigma^X}{1+\sigma^X} \tag{9}$$

Due to the larger productivity lead, profits from domestic sales are larger than those of domestic

[4]In Appendix 6.1 we show that asset and labor inputs for each product line are also independent of productivity. This property enables us to study the aggregate dynamics of the economy without keeping track of heterogeneous productivity levels across product lines.

[5]When the borrowing constraint binds, μ_t is strictly positive and profits decline. The reason for this is that when the borrowing constraint binds, the final tradable good producer lowers its demand for intermediate goods. This translates into lower profits for intermediate good producers.

lines. It is worth noting at this point the role of the multiplier μ_t: when the constraint binds, profits from domestic sales for exporters and non-exporters fall due to a decline in domestic demand.

In the Foreign market, a representative final tradable good producer demands intermediate goods according to the following production function:

$$Y_t^* = \exp\left[\int_0^1 \ln y_t^*(i)di\right]$$

Foreign production of the final tradable good is not subject to shocks, and Y_t^* grows at a constant rate. The demand of this foreign final good producer for each intermediate good is:

$$p_t^*(i) = \frac{Y_t^*}{y_t^*(i)} \tag{10}$$

In the case of export lines, the second lowest marginal cost belongs to foreign intermediate good producers. The price set for exports in the foreign market is equal to this second lowest marginal cost:

$$p_t^*(i) = \widetilde{MC}_t^*(i) = \frac{1}{a_t(i)/(1+\sigma^X)}\overline{\alpha}\left(R_t^{L*}\right)^{\alpha}(W_t^*)^{1-\alpha} \tag{11}$$

Using the expression for the foreign demand for intermediate goods (in equation (10)), profits from export lines' sales abroad are:

$$\pi_t^* = Y_t^*\left(1 - \frac{1+\xi}{1+\sigma^X}\frac{\left(R_t^L\right)^{\alpha}(W_t)^{1-\alpha}}{\left(R_t^{L*}\right)^{\alpha}(W_t^*)^{1-\alpha}}\right) \tag{12}$$

where ξ is the iceberg cost of exporting. Profits from export sales differ from profits from domestic sales of export lines in that they do depend (negatively) on factor prices. Lower domestic factor prices make domestic production cheaper, while the export price is determined by foreign factor prices.

■ **(M) Import lines** In this case the lowest cost producer is a Foreign firm and the domestic final good producer imports the intermediate good, As the demand for intermediate goods by this producer (in equation (4)) has a unit elasticity, the total payment to foreign firms is independent of the price charged:

$$p_t(i)y_t(i) = \frac{Y_t}{1+\phi\mu_t/\lambda_t} \tag{13}$$

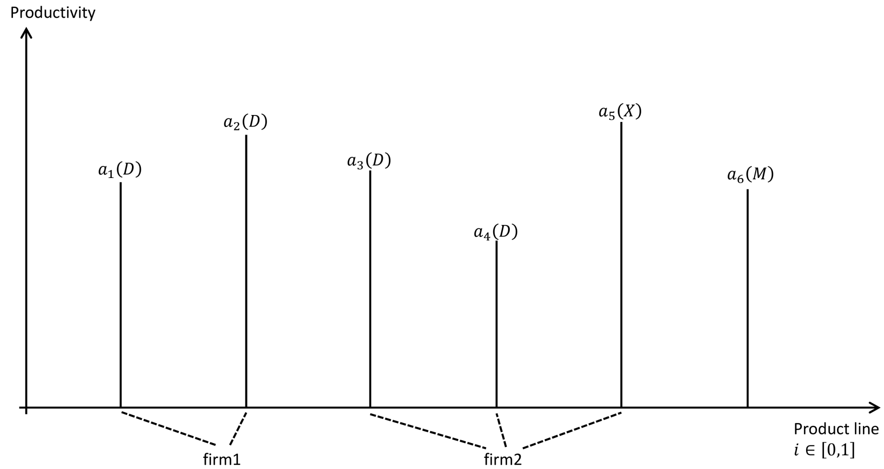
The price is equal to the second lowest marginal cost, which in this case belongs to a Home firm. The productivity lead by foreign firms is the same as that of Home exporting firms, $1 + \sigma^X$. Consequently the price is the same as that of export lines in equation (8). Note that in equation (13), the right hand side is independent of a product line i, so output $y_t(i)$ is also the same as for export lines.

2.3 Innovation and Firm Dynamics

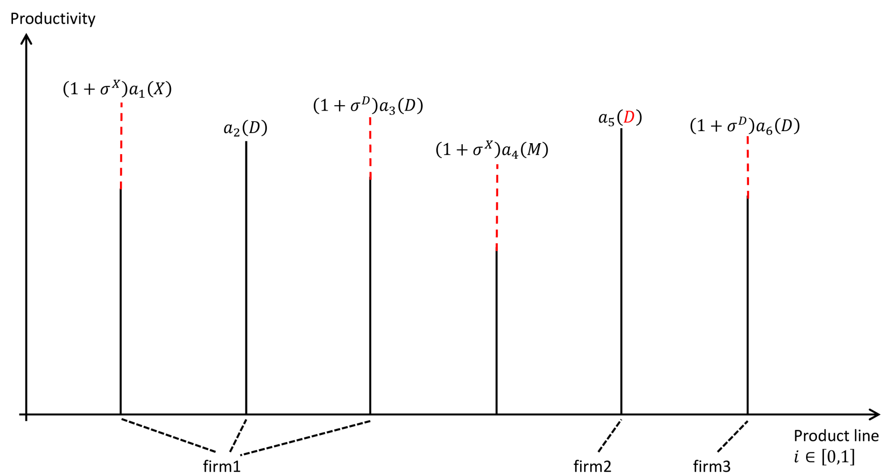
Firm dynamics are shaped by firm entry, innovation by incumbent Home firms, and innovation by Foreign firms. The productivity of each product line evolves with each technological improvement generated by successful innovations. A successful domestic innovation increases the existing productivity of a product line by an exogenous factor $1+\sigma^D$ and leads an entering or incumbent Home firm to acquire the product line. A successful export innovation increases the existing productivity of a product line by an exogenous factor $1 + \sigma^X$ and leads to adding a product sold domestically to the export market.

Due to entry and innovation, aggregate productivity in the intermediate sector increases over time. Firm dynamics change the status of each product line over time and endogenously determine the extensive margins of imports and exports. Below we explain in detail firms' innovation decisions.

■ **A Graphic Example** Figure 2 illustrates an example of the evolution of firms' product lines from a period t to $t + 1$. In period t (top panel) Home firm 1 produces two domestic product lines, denoted by (D). Home firm 2 produces two domestic and one export line (denoted by (X)). There is also one Foreign product line, denoted by (M). In period $t+1$ (bottom panel) Home firm 1 succeeds in an export innovation for product line 1, which becomes an export line. Home firm 1 also succeeds in a domestic innovation and acquires domestic product line 3. Foreign innovation occurs in product line 4 and 5 owned by Home firm 2. Home firm 2 loses product line 4 and this line becomes an import line. For product line 5, Home firm 2 exits from the Foreign market and this product line becomes a domestic line with no productivity change. Finally, firm entry in Home occurs on product line 6.



a) Period t



b) Period $t+1$

Figure 2: Firm Dynamics

Note: This figure provides an example of firms' product dynamics between an initial period t (top panel) and a subsequent period $t+1$ (bottom panel).

2.3.1 Innovation by Incumbent Domestic Firms

■ **Domestic innovation** A firm owning n^D domestic lines and n^X export lines has $n^D + n^X$ domestic innovation opportunities.[6] For each innovation opportunity, a firm chooses to invest an amount Z_t^D.[7] The probability of success of a domestic innovation is proportional to the amount invested:

$$i_t^D = \eta^D \left(\frac{Z_t^D}{A_t} \right)^{1/\rho} \tag{14}$$

[6] The underlying assumption is that a domestic innovation is a spin-off from existing technologies.
[7] This investment is measured in units of the tradable final good.

This probability is inversely proportional to the average productivity of intermediate firms A_t (including foreign firms). The functional form is consistent with the empirical patterns shown in Akcigit and Kerr [2018]. We assume that domestic innovation can take place only on domestic lines and import lines, and does not happen on export lines.

■ **Export innovation** A firm owning n^D domestic product lines has n^D export innovation opportunities. For each innovation opportunity, a firm chooses to invest an amount Z_t^X. The probability of success of an export innovation is:

$$i_t^X = \eta^X \left(\frac{Z_t^X}{A_t} \right)^{1/\rho} \tag{15}$$

When a firm's export innovation is successful, a product sold domestically can also be exported and the domestic line becomes an export line.

■ **Foreign innovation** Finally, there are two types of innovation by Foreign firms, both of which occur with exogenous probabilities. The first type is domestic innovation by Foreign firms (so "domestic" here means the domestic country for Foreign firms). In this case, an export product line owned by a Home firm is forced to exit from the Foreign market, and goes back to being a domestic line. This happens with an exogenous probability i^{FX} for each export product line. The second type of innovation by Foreign firms is an export innovation. In this case, a domestic product line is forced to exit from the Home market, and this product line becomes an import line. This happens with an exogenous probability i^{FD}.

■ **Incumbents' Innovation Decisions** As is common in Schumpeterian growth models, innovation is undirected in the sense that innovation is equally likely to apply to any product line. This feature is preserved in this model because operating profits are independent of a firm's productivity level, so firms with a given productivity are indifferent among any product lines that can be introduced. Undirected innovations carried on by a continuum of firms imply that each product line faces the same replacement probability d_t. The probability of i successes in n trials for a binomial process with success probability p is:

$$P(i, n, p) = \binom{n}{i} p^i (1-p)^{1-i}$$

The value of a Home firm with n^D domestic lines and n^X export lines can be written in a recursive form as follows:

$$
\begin{aligned}
V_t(n^D, n^X) = \max_{Z_t^D, Z_t^X} & \{n^D \pi_t^D + n^X(\pi_t^X + \pi_t^*) - (n^D + n^X) Z_t^D - n^D Z_t^X \\
& + \sum_{i=0}^{n^D+n^X} P(i, n^D + n^X, i_t^D) \sum_{j=0}^{n^D} P(j, n^D, d_t) \sum_{k=0}^{n^D-j} P(k, n^D - j, (1-d_t) i_t^X) \sum_{m=0}^{n^X} P(m, n^X, i^{FD}) \\
& E_t \left[\Lambda_{t,t+1} V_{t+1}(n^D + i - j - k + m, n^X + k - m)\right]\}
\end{aligned}
$$

The first line represents operating profits minus innovation investment costs. The second and third line add up the expected value of a firm across all the possible combinations of innovations and replacement on n^D domestic lines and n^X export lines in the next period. The first summation adds up across all the possibilities for domestic innovations from 0 to $n^D + n^X$ successes. The second summation adds up over the number of domestic lines being replaced from 0 to n^D. The third summation adds up over the number of successful export innovations. It is worth noting at this point that given that there is a continuum of product lines and innovation decisions are simultaneous, the probability that two or more innovations occur at the same time for a same product is zero. Thus the effective success probability is given by $(1-d_t)i_t^X$. The last summation adds up over the number of export lines being replaced from 0 to n^X. $\Lambda_{t,t+1}$ in the last line is the stochastic discount factor by households.

We use a guess-and-verify method to show that the value of a firm with n^D domestic lines and n^X export lines is equal to the sum of n^D times the value of a single domestic line and n^X times the value of a single export line:

$$V_t(n^D, n^X) = n^D V_t(1,0) + n^X V_t(0,1)$$

The proof is shown in Appendix 6.3. This linear relation enables us to aggregate firm dynamics in a tractable way and study how firm dynamics affect endogenous growth and the extensive margins of imports and exports. It enables us to do so without having to keep track of the firm size distribution. The value of a single domestic line is given by:

$$
\begin{aligned}
V_t(1,0) \quad = \quad & \max_{Z_t^D, Z_t^X} \left\{\pi_t^D - Z_t^D - Z_t^X \right. \\
& \left. + \left(i_t^D + (1-d_t)(1-i_t^X)\right) E_t\left[\Lambda_{t,t+1} V_{t+1}(1,0)\right] + (1-d_t) i_t^X E_t\left[\Lambda_{t,t+1} V_{t+1}(0,1)\right]\right\}
\end{aligned} \tag{16}
$$

and the value of a single export product line is:

$$V_t(0,1) = \max_{Z_t^D} \left\{ \pi_t^X + \pi_t^* - Z_t^D + (i_t^D + i^{FX}) E_t \left[\Lambda_{t,t+1} V_{t+1}(1,0) \right] + (1 - i^{FX}) E_t \left[\Lambda_{t,t+1} V_{t+1}(0,1) \right] \right\} \tag{17}$$

The first-order condition with respect to Z_t^D pins down the optimal investment for domestic innovation opportunities:

$$\eta^D \frac{1}{\rho} \left(\frac{Z_t^D}{A_t} \right)^{1/\rho - 1} \frac{1}{A_t} E_t \left[\Lambda_{t,t+1} V_{t+1}(1,0) \right] = 1 \tag{18}$$

The first-order condition with respect to Z_t^X pins down the optimal investment for export innovation opportunities:

$$(1 - d_t) \eta^X \frac{1}{\rho} \left(\frac{Z_t^X}{A_t} \right)^{1/\rho - 1} \frac{1}{A_t} \left(E_t \left[\Lambda_{t,t+1} V_{t+1}(0,1) \right] - E_t \left[\Lambda_{t,t+1} V_{t+1}(1,0) \right] \right) = 1 \tag{19}$$

Note that investment is forward-looking in the sense that as the expected value of a product line increases, firms increase their investment.

2.3.2 Domestic Entry

Firm entry results from innovation by households. Households invest in two different types of innovation to start firms with domestic or export product lines. In both cases, new firms poach a product line from incumbent firms and start with a single product line. Households invest an amount Z_t^{ED} to create new firms with a domestic line, and Z_t^{EX} to create new firms with an export line.[8] The number of firms created from Z_t^{ED} and Z_t^{EX} units of investment is:

$$e_t^D = \eta^{ED} \left(\frac{Z_t^{ED}}{A_t} \right)^{1/\rho} \tag{20}$$

and

$$e_t^X = \eta^{EX} \left(\frac{Z_t^{EX}}{A_t} \right)^{1/\rho} \tag{21}$$

[8] This investment is measured in units of the tradable final good.

respectively. In both cases, the optimal investment (Z_t^{ED} or Z_t^{EX}) is such that the marginal benefit and marginal cost of investment are equal:

$$\eta^{ED}\frac{1}{\rho}\left(\frac{Z_t^{ED}}{A_t}\right)^{1/\rho-1}\frac{1}{A_t}E_t\left[\Lambda_{t,t+1}V_{t+1}(1,0)\right]=1 \tag{22}$$

$$\eta^{EX}\frac{1}{\rho}\left(\frac{Z_t^{EX}}{A_t}\right)^{1/\rho-1}\frac{1}{A_t}E_t\left[\Lambda_{t,t+1}V_{t+1}(0,1)\right]=1\,. \tag{23}$$

2.3.3 Productivity Growth and The Extensive Margins of Trade

We can now characterize how firm dynamics translate into aggregate productivity growth and into the extensive margins of exports and imports. We denote the share of domestic lines by θ_t^D, and the share of export lines by θ_t^X. The share of imported product lines is then $1-\theta_t^D-\theta_t^X$. The rate at which domestic product lines are replaced (d_t) is the sum of the probability that a product line is replaced due to domestic entry, domestic innovation, or foreign innovation:

$$d_t=(e_t^D+e_t^X+(\theta_{t-1}^D+\theta_{t-1}^X)i_t^D)\frac{1}{1-\theta_{t-1}^X}+i^{FD} \tag{24}$$

In this expression, the firm entry rate and the domestic innovation rate are divided by the share of domestic and import lines. This is because these innovations affect only domestic and import lines, and thus the probability that each domestic product line is replaced due to these innovations is the number of product lines facing these innovations divided by the total number of product lines that could potentially receive these innovations. Note also that the domestic innovation rate is equal to the probability that a domestic innovation by an incumbent firm is successful (i_t^D) times the share of domestically-owned product lines (which is the sum of domestic and export lines). The law of motion for the share of domestic lines is:

$$\begin{aligned}\theta_t^D=\theta_{t-1}^D+(e_t^D+(\theta_{t-1}^D+\theta_{t-1}^X)i_t^D)\frac{1-\theta_{t-1}^D-\theta_{t-1}^X}{1-\theta_{t-1}^X}\\+\theta_{t-1}^X i^{FX}-\theta_{t-1}^D(1-d_t)i_t^X-\theta_{t-1}^D i^{FD}-e_t^X\frac{\theta_{t-1}^D}{1-\theta_{t-1}^X}\,.\end{aligned} \tag{25}$$

This share increases due to (1) entry of domestic lines and domestic innovation by incumbent firms that occur on import lines, and (2) innovation by Foreign firms that pushes export lines back to being domestic lines. It decreases due to (1) export innovation, (2) foreign innovation that forces domestic lines to exit, and (3) entry of export lines that occurs on domestic lines.

The law of motion for the share of export lines θ_t^X is:

$$\theta_t^X = \theta_{t-1}^X + \theta_{t-1}^D (1 - d_t) i_t^X + e_t^X - \theta_{t-1}^X i^{FX} \,. \tag{26}$$

This share increases due to export innovations by incumbent firms and entry of export lines, and decreases due to foreign innovations that turn export lines back into domestic lines. The share of import lines is consequently $1 - \theta_t^D - \theta_t^X$. Note that the extensive margin of imports is determined by endogenous changes in the share of import product lines. The extensive margin of exports is determined by endogenous changes in the share of export lines.

Finally, aggregate production of the tradable final good is:

$$Y_t = \exp\left[\int_0^1 \ln y_t(i) di\right] = \\ A_t \left[\left(\ell_t^D\right)^\alpha \left(h_t^D\right)^{1-\alpha}\right]^{\theta_{t-1}^D} \left[\left(\ell_t^X\right)^\alpha \left(h_t^X\right)^{1-\alpha}\right]^{\theta_{t-1}^X} \left[\frac{1}{1+\xi} \left(\ell_t^M\right)^\alpha \left(h_t^M\right)^{1-\alpha}\right]^{1-\theta_{t-1}^D - \theta_{t-1}^X} , \tag{27}$$

where ℓ_t^D,ℓ_t^X and ℓ_t^M are the amounts of the productive asset used by each product line, and h_t^D, h_t^X and h_t^M are the amounts of labor hired by each product line. Note that ℓ_t^M and h_t^M are factors employed abroad.[9] The average productivity of intermediate firms (A_t) is:

$$A_t = \exp\left[\int_0^1 \ln a_t(i) di\right] . \tag{28}$$

A_t grows as the productivity of each product line $a_t(i)$ improves through domestic firm entry, innovation by incumbent Home firms, and foreign innovation.[10] The growth rate of A_t is:

$$\frac{A_{t+1}}{A_t} = 1 + g_t = (1 + \sigma^D)^{e_t^D + (\theta_{t-1}^D + \theta_{t-1}^X) i_t^D} (1 + \sigma^X)^{e_t^X + \theta_{t-1}^D (1-d_t) i_t^X} (1 + \sigma^X)^{i^{FD}} \tag{29}$$

Note that the three terms in the right-hand side correspond to the sum of domestic firm entry

[9] Expressions for these variables are shown in the Appendix.

[10] Note that A_t is not necessarily the productivity level of this economy, because A_t includes productivity of Foreign firms. But the long-run growth rate of this economy is determined by growth in A_t.

and domestic innovations, exporting firm entry and export innovations, and foreign innovations respectively.

2.4 Households

The representative household consumes final tradable goods and supplies labor elastically. In addition it invests Z_t^{ED} and Z_t^{EX} units of the tradable good in domestic and export entry. It receives income from the wage $W_t H_t$, from profits from the tradable good producers, and from profits from domestic intermediate good producers. The representative household's optimization problem is then to maximize:

$$\max_{\{C_t, H_t, Z_t^{ED}, Z_t^{EX}\}_{t=0}^{\infty}} E_0 \sum\nolimits_{t=0}^{\infty} \beta^t \left[\ln \left(C_t - A_t \frac{(H_t)^{\omega}}{\omega} \right) \right]$$

subject to the budget constraint:

$$C_t + Z_t^{ED} + Z_t^{EX} = W_t H_t + \Pi_t + \theta_{t-1}^D \left(\pi_t^D - Z_t^D - Z_t^X \right) + \theta_{t-1}^X \left(\pi_t^X + \pi_t^* - Z_t^D \right) . \tag{30}$$

Optimal investment in domestic entry Z_t^{ED} and Z_t^{EX} are determined by equation (22) and (23) .

Finally, the trade balance is:

$$\begin{aligned} TB_t \;=\; & \underbrace{Y_t - C_t - Z_t^{ED} - Z_t^{EX} - \theta_{t-1}^D \left(Z_t^D + Z_t^X \right) - \theta_{t-1}^X Z_t^D}_{\text{final tradable output - absorption}} \\ & + \underbrace{\theta_{t-1}^X Y_t^*}_{\text{exports of intermediate goods}} - \underbrace{\left(1 - \theta_{t-1}^D - \theta_{t-1}^X\right) \frac{Y_t}{1 + \phi \mu_t / \lambda_t}}_{\text{imports of intermediate goods}} \end{aligned}$$

In Appendix 6.1 we define the equilibrium of the economy and describe the stationarized equilibrium conditions that we use to solve the model numerically.

3 Quantitative Analysis

In this section we show that a calibrated version of the model can replicate key micro and macroeconomic stylized facts of sudden stop episodes. After validating the model, we quantify the implications of sudden stops for productivity dynamics and welfare.

3.1 Data: Firms' Domestic and Export Product Portfolios

We use unique data on firms' product portfolios of Chilean manufacturing firms. Our firm-product level data comes from the Chilean Annual Survey of Manufactures, which contains data on the universe of manufacturing plants with 10+ employees. While the standard information on plant-level outcomes of this Census has been used extensively in the literature, we have access to an additional form that records each product produced by each firm. This data reports separately domestic and export sales of each of these products. We use annual firm-level product data for 1996-1999. We are able to aggregate the plant level data to the firm level. In addition, we construct firms' ages using the standard plant-level data of this annual Census of Manufactures starting in 1980. This data has been used by Navarro [2012] and Garcia-Marin and Voigtländer [2019]. To the best of our knowledge, the Chilean Census of Manufactures is the only one reporting firm-product level data and distinguishing between domestic and export markets.

3.2 Calibration

The model is calibrated at an annual frequency. There are 17 parameters to be determined in the model. We take conventional values from the literature, and calibrate others to target the Chilean economy.[11] In addition we use the firm-product level data described above. Table 1 shows the values of 9 externally-determined parameter values. The discount factor $\beta = 0.96$ and the interest rate on foreign bonds $R = 1.05$ are standard values for annual models. The parameter for the labor supply elasticity $\omega = 1.455$ is set following Mendoza [1991]. Regarding the production parameters, the asset's share in tradable production $\alpha = 0.08$ is set to target a capital to output ratio equal to 2, consistent with the Chilean economy. The iceberg trade cost $\xi = 0.21$ follows the estimation by Anderson and Van Wincoop [2004]. The fraction of the input cost subject to the working capital requirement ϕ varies widely depending on how it is estimated. We set its value to 0.2 so that the total credit-to-GDP ratio on the balanced growth path matches the data.[12] The coefficient on the borrowing constraint κ is set to 0.2 based on Mendoza [2010]. The amount of productive asset $k = 0.6$ is set to target the frequency of sudden stops of 7.7% [Eichengreen et al., 2008, Jeanne and Rancière, 2011]. The concavity parameter governing productivity-enhancing

[11] We calibrate the model to target the Chilean economy (a representative small open economy) because later in this section we use our Chilean data on firms' product portfolios to study a sudden stop episode.

[12] According to the External Wealth of Nations Database by Lane and Milesi-Ferretti [2007], the average net foreign asset position in Chile in 1990-2005 is 38.2%. We target this value and set $\phi = 0.2$ accordingly.

investment ρ is set to 1.5, which is within the range reported in the literature (including Comin and Gertler [2006], Akcigit and Kerr [2018], and their literature review).

Table 1: Externally-Determined Parameters

Variable		Value	Source
β	Discount factor	0.96	Standard
R	Foreign bond interest rate	1.05	Standard
ω	Frisch elasticity $1/(\omega - 1)$	1.455	Mendoza (1991)
α	Asset share in production	0.08	Targets Capital to Output ratio (Chile)
ξ	Iceberg trade cost	0.21	Anderson & van Wincoop (2004)
ϕ	Fraction of input s.t. working cap. req.	0.2	Targets Total credit to GDP ratio (Chile)
κ	Coefficient on borrowing constraint	0.2	Mendoza (2010)
L	Amount of productive asset	0.6	Targets Frequency of Sudden Stops
ρ	Concavity of innovation investment	1.5	Median value from literature

Seven parameters related to firm dynamics and growth (σ^D,σ^X, η^D, η^{ED}, η^{EX}, η^X, and Y^*) are jointly determined to match seven moments at the balanced growth path of the model with the Chilean data on firms' product portfolios described earlier. The seven targeted moments are (1) the aggregate growth rate, (2) the relative profit of non-exporting firms to exporting firms, (3) the share of single-product non-exporting firms, (4) the share of exporting firms among single-product firms, (5) the average number of products owned by non-exporting firms, (6) the average number of exported products owned by exporting firms, and (7) the share of exports in total revenue for exporting firms. In choosing these parameter values, the exogenous foreign innovation rate on domestic lines and export lines are also determined. Because foreign innovation on export lines corresponds to domestic innovation by foreign firms, its rate is set equal to the domestic innovation rate by domestic firms. Similarly, foreign innovation on domestic lines corresponds to export innovation by foreign firms, thus its rate is set equal to the export innovation rate by domestic firms. The values of these parameter values and the corresponding targeted moments are listed in Table 2.

The aggregate shocks to the economy determine the productivity of the final tradable sector ε_t^A and the interest rate on the foreign bond ε_t^R. We take the stochastic process for these shocks from Mendoza [2010], in which ε_t^A and ε_t^R follow a joint discrete Markov process with two realizations for each variable. In particular, ε_t^A takes ± 0.0134 and ε_t^R takes ± 0.0196 with the same autocorrelation 0.59 and the negative correlation -0.67 between ε_t^A and ε_t^R. Finally, the foreign factor prices R_t^{L*}

Table 2: Jointly-Determined Parameters

Variable		Value	Target	Model	Data
σ^D	Domestic innovation step size	0.06	Aggregate growth rate	2.5%	2.5%
σ^X	Export innovation step size	0.30	Relative profit of non-exporters to exporters	27.8%	26.2%
η^{ED}	Non-exporter entry coefficient	1.46	Share of single-product non-exporters	37.1%	38.3%
η^{EX}	Exporter entry coefficient	0.31	Share of exporters in single-product firms	20.8%	21%
η^D	Domestic innovation coefficient	2.97	Avg. number of non-exporters' products	2.24	2.56
η^X	Export innovation coefficient	0.52	Avg. number of exporters' exported products	1.05	1.7
y^*	Foreign demand	0.74	Export revenue share for exporters	30.5%	35.9%
i^{FX}	Foreign innovation rate on X lines	0.23	Domestic innovation rate by domestic firms		
i^{FD}	Foreign innovation rate on D lines	0.01	Export innovation rate by domestic firms		

and W_t^* are set equal to the domestic values R_t^L and W_t at the balanced growth path, under the assumption that the domestic foreign economies have a similar productivity level.

3.3 Model Validation

Our next step is to show that the assumptions in the model regarding product entry and exit are closely aligned to the data, and that quantitatively, the calibrated model matches non-targeted moments. We first contrast the microeconomic assumptions in the model to the data pre-sudden stop. We then compare the response of model and data to Chile's 1998 sudden stop episode.

■ **Product transitions** Our first step to contrast data and model is documenting how a firm's existing product portfolio shapes the addition or removal of products from the domestic and export markets. Note that these are moments that are not targeted in the calibration.

We study these transitions based on a balanced panel for 1996-1997 (i.e. prior to the sudden stop) with 3503 firms out of which 825 (23.5%) are exporters in 1996 and 870 (24.8%) are exporters in 1997. Appendix Table 8 documents the raw frequency of different type of firms' product portfolios' transitions. Here we restrict our analysis to the transitions that were established to occur more frequently: adding new products to the domestic market, adding domestic products to export markets, dropping products from the domestic market, and dropping export products turning them to solely domestic products.

To show how existing product baskets shape product transitions, equation 31 estimates the impact of the existing number of products on the probability that each of the transitions described in Table 8 takes place. In this equation, Y_f is a dummy variable taking a value of one if a transition takes place and zero otherwise. X_f represents measures of the existing number of products. Our regression includes firm age as an additional control. It also includes industry (s) fixed effects (at the four-digit level of disaggregation) in order to compare across firms within a given industry. Given the inclusion of fixed effects, we estimate this equation using a linear probability model. Similar results are obtained estimating a probit model.

$$Y_f = \beta_1 \cdot X_f + \beta_2 \cdot \text{Age}_f + \phi_s + \epsilon_f \tag{31}$$

The results are shown in Table 3. For each of the different transitions in Columns 1 through 4, the regressor X_f varies, as we are interested in choosing the regressor that speaks directly to the model in Section 2. Column 1 indicates that one additional product produced by a firm in period t is associated to a 1.6 percentage point increase in the probability that a firm introduces one or more new domestic products between periods t and $t+1$. This coefficient is relatively large, as the unconditional probability of adding new domestic product (shown in the first row of Table 8) is 0.15. Column 2 indicates that one additional domestic product sold by a firm in period t leads to a 0.6 percentage point increase in the probability that a firm introduces a new export product between periods t and $t+1$. Column 3 shows that one additional domestic product sold by a firm in period t is associated to a 6.6 percentage point increase in the probability that a firm drops a domestic product between periods t and $t+1$. Finally Column 4, which is restricted to firms that export in t, shows that one additional exported product sold by a firm in period t is associated to a 4.9 percentage point increase in the probability that a firm drops an exported product (subsequently sold only domestically) between periods t and $t+1$. In most cases, firm age seems to be uncorrelated with the probability of adding or dropping products once we control for the existing number of products.

We contrast these empirical results with the model. We simulate 5000 firms and estimate a regression equivalent to 31 on the simulated data for the 4498 firms present in two consecutive periods. This equation excludes of course industry fixed effects, as the model corresponds to a single industry.

$$Y_f = \beta_0 + \beta_1 \cdot X_f + \epsilon_f \tag{32}$$

The results are shown in panel B in Table 3. There is a close relationship between the behavior of the data, described earlier, and the model. While the elasticities are larger in the simulated data, the model effectively replicates the key patterns found.

Table 3: Transitions and Firm Characteristics

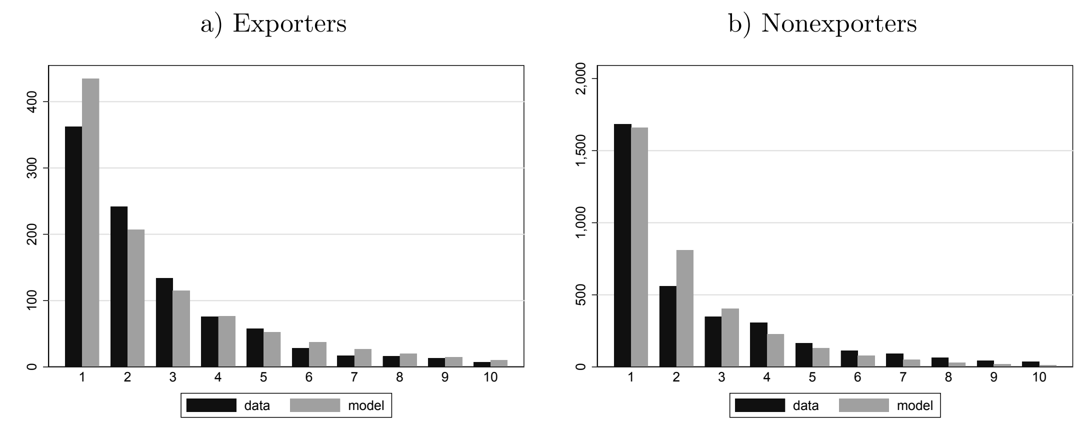
	(1) Not Produced to Domestic	(2) Domestic to Exported	(3) Domestic to Not Produced	(4) Exported to Domestic
Panel A: Data				
Number of Products	0.016***			
	(0.002)			
Number of Domestic Products		0.006***	0.066***	
		0.002	0.002	
Number of Exported Products				0.049***
				(0.012)
Age (log)	0.000	0.010**	0.005	-0.016
	(0.007)	(0.005)	(0.007)	(0.019)
Observations	3503	3503	3503	809
Panel B: Model				
Number of Products	0.080***			
	(0.003)			
Number of Domestic Products		0.010***	0.121***	
		0.001	0.003	
Number of Exported Products				0.103**
				(0.051)
Observations	4498	4498	4498	1118

NOTE: Panel A reports the results of the estimation of equation 31. Panel B reports the results of the estimation of equation 32 using simulated data. The dependent variable in each column is (1) the probability that a firm introduces to the domestic market a product not sold previously, (2) the probability that a firm introduces to the export market a product previously sold domestically, (3) the probability that a firm withdraws a product from the domestic market, subsequently not selling it, and (4) the probability that a firm withdraws a product from the export market, subsequently selling it domestically.

■ **The Firm Size Distribution** A next step in validating the model comes from contrasting the firm product distribution in the data and the simulated model. In the model, each firm is characterized by the number of domestic and export lines it owns, (n^D, n^X). Let $\delta_t(n^D, n^X)$ denote the measure of firms that own n^D domestic lines and n^X export lines at period t. We derive the firm-size distribution $\delta_t(n^D, n^X)$ at the balanced growth path of the model using the innovation rates at the balanced growth path. The detailed steps are explained in the Appendix. Note that while the shares of exporters and non-exporters selling a single product was targeted in the model calibration, the rest of the distribution is non-targeted.

First, panel a) in Figure 3 illustrates the firm-size distribution for exporting firms. The figure shows the number of firms selling a single product, two products, etc. Real and simulated data are shown next to each other, showing a close fit. Panel b) reports the same firm-size distribution among exporting firms, with data and model also aligned.

Figure 3: Number of Exporters and Nonexporters by Total Number of Products Sold



NOTE: This figure shows the distribution of the total number of products sold by exporting firms (panel A) and non-exporting firms (panel B) in the data and in the simulated model.

3.3.1 Sudden Stop Dynamics

We now turn to discussing the sudden stop dynamics in the model.[13] Following Bianchi and Mendoza [2018], sudden stops are identified as events in which the current account adjusted for its trend is at least two standard deviations above its mean. Under this definition, the unconditional probability of sudden stops in the model is 7.7%, which is in line with empirical estimations in Eichengreen et al. [2008] and Jeanne and Rancière [2011]. Figure 4 plots the average dynamics of key macroeconomic variables before and after sudden stops events. Panels 4a, 4b and 4c show the path of real GDP, consumption, and the asset price in log deviations from their linear trends.[14]

[13]Details on the simulation of the model are included in the Appendix.

[14]A linear trend is constructed by taking the log of 20-period series around each sudden stop (10 periods before and after sudden stop respectively) and taking a linear trend of this log series.

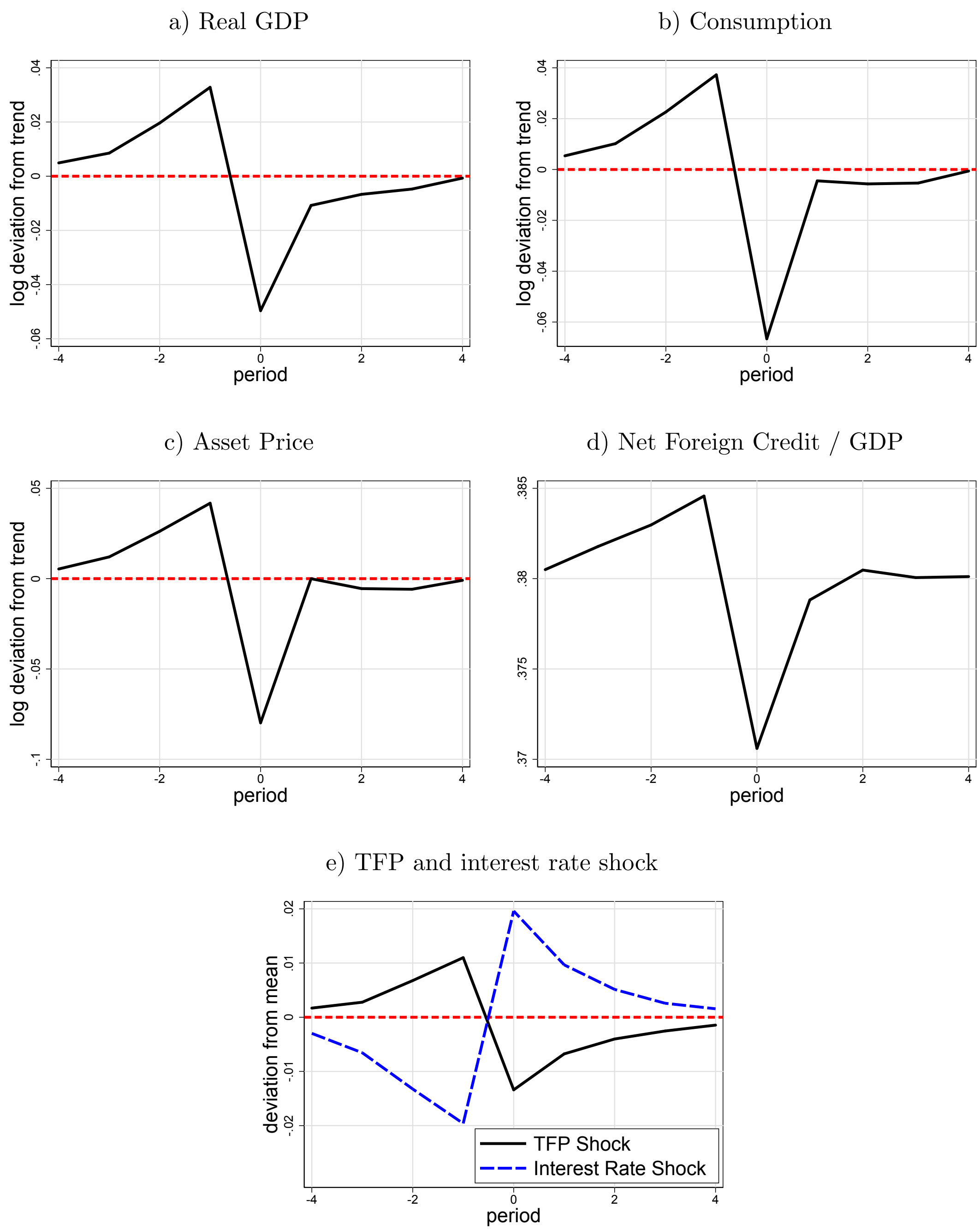
On average, GDP declines by 7%, consumption falls by 10%, and the asset price drops by 12% on impact following a sudden stop. The sharp fall in the asset price indicates that the borrowing limit substantially tightens during sudden stops, and an amplification effect sets in motion as in Mendoza [2010] and Bianchi and Mendoza [2018]. The net foreign credit-to-GDP ratio in panel 4d shrinks abruptly, indicating a sudden reversal of capital inflows.

Panel 4e illustrates the average path of productivity shocks and interest rate shocks, which cause the sudden stop. Before a sudden stop occurs, productivity is high and the interest rate is low, implying that the country is facing favorable shocks. When these favorable shocks reverse to bad shocks of low productivity and a high interest rate, the asset price declines and forces the borrowing constraint to bind. Households are then forced to cut consumption, which reduces the asset price further, and the amplification mechanism is set in motion. These developments of exogenous shocks and the subsequent endogenous dynamics are all consistent with Mendoza [2010].

We now examine trade and growth dynamics, which are the novel features of our model. Figure 5 plots the average dynamics of the key variables around sudden stop episodes. Panel 5c shows the dynamics of imports and exports of intermediate goods. We see that imports fall substantially, while exports are almost unaffected. A decline in imports occurs because the final tradable producer in this country is constrained by the borrowing limit and is forced to reduce its demand for intermediate goods. In contrast, foreign demand is not affected by the sudden stop in the domestic economy, so exports are not directly impacted. Panel 5d shows that the trade balance-to-GDP ratio improves during sudden stops, which is also in line with the empirical facts reported by [Mendoza, 2010]. Panel 5a shows firms' innovation rates. In a sudden stop, the rate of domestic innovation falls by 20% and is persistently lower, while the rate of export innovation diminishes only by 2%. As a result, the growth of the productivity index A_t declines sharply (panel 5b). This has a persistent effect on the economy. This long-lasting negative impact is consistent with empirical findings by Cerra and Saxena [2008] and Blanchard et al. [2015].

We analyze in Figure 6 how the extensive margins of trade react to sudden stops. Panel 6a shows a a 3% decline in the domestic wage. This results in a decline in the relative marginal cost of production for domestic firms compared to foreign firms. Panel 6b shows a decline in the relative marginal cost of the domestic economy compared to the foreign economy. Due to the decline in the relative marginal cost, export profits increase by 23% above trend as seen in Panel 6c. In

Figure 4: Sudden Stop Dynamics: Macro Variables

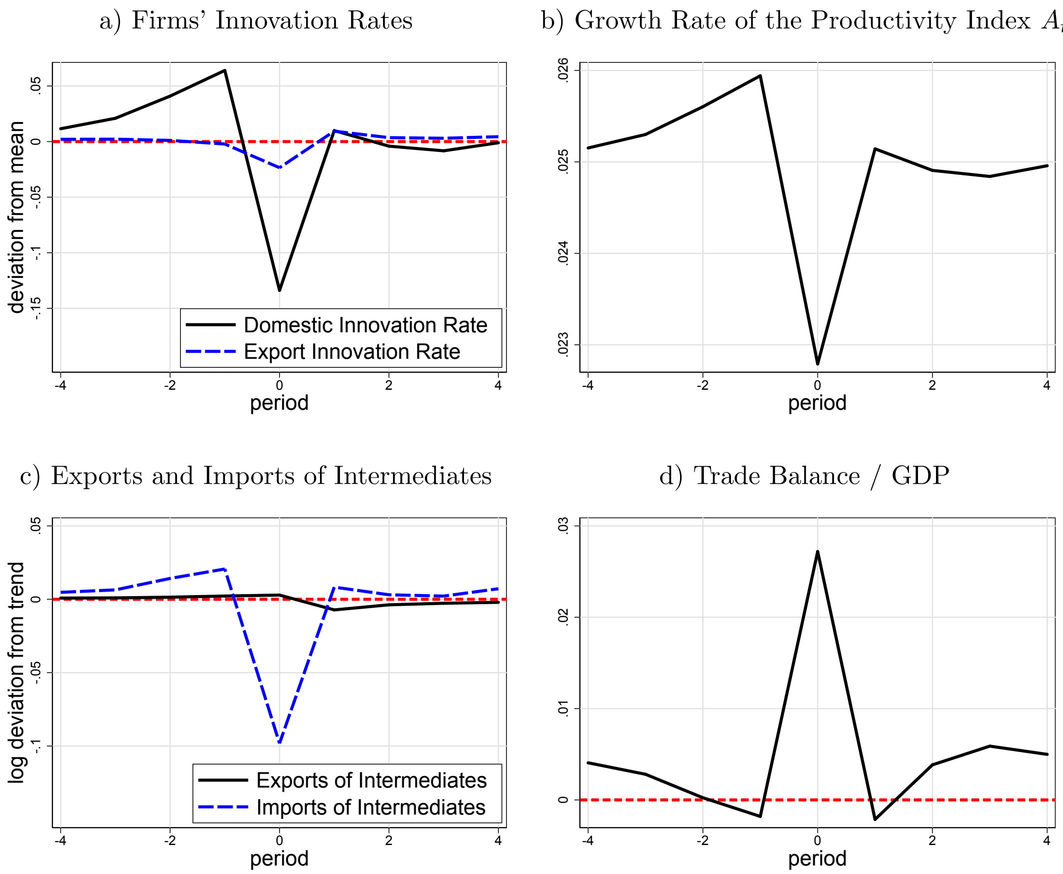


Note: This figure shows the path of real GDP (in log deviation from trend), aggregate consumption (in log deviation from trend), the asset price (in log deviation from trend), the ratio of net foreign assets to GDP (in levels), the productivity shock (in deviation from the mean), and the interest rate shock around sudden stop episodes (in deviation from the mean).

contrast, domestic profits fall by 7%.[15]

[15]Further, in Appendix 7.2 we discuss the behavior of the extensive margin.

Figure 5: Sudden Stops Dynamics: Trade and Growth



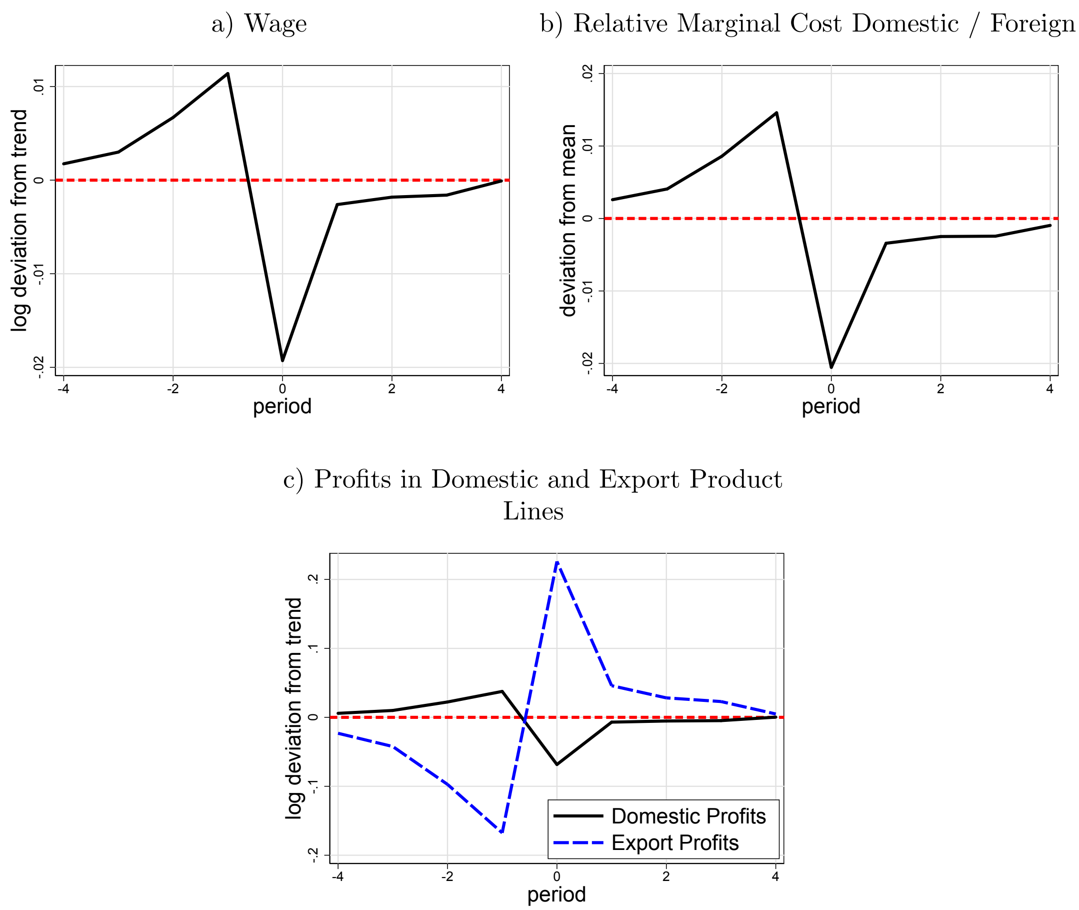
Note: This figure shows the path of the domestic and export innovation rates (in deviation from the mean), the growth rate of the productivity index (A_t) (in levels), exports and imports of intermediate goods (in log deviation from trend), and the trade balance to GDP ratio (in levels) around sudden stop episodes.

3.3.2 The Dynamics of Firm Product Portfolios during Chile's 1998 Sudden Stop

In 1998 Chile faced a severe sudden stop common to several other emerging markets as a consequence of the Russian default that year and the Asian financial crisis that had started in mid 1997. This event was both unanticipated and exogenous to the Chilean economy, which had few direct ties with the countries in which this crisis originated. The sudden stop sharply decreased capital inflows and GDP growth. Calvo and Talvi [2005] discuss this episode in detail.

Our model has clear predictions about the dynamics of firms' product portfolios during sudden stop events. Our goal in this section is to contrast these predictions to the behavior of Chilean manufacturing firms. First, at the extensive margin, the model predicts that the rate at which

Figure 6: Sudden Stop Dynamics: Trade Margins



NOTE: This figure shows the wage (in log deviation from trend), the ratio of domestic to foreign marginal cost (in deviation from the mean), and domestic and export profits (in log deviation from trend).

firms introduce products into the domestic market falls relative to the rate of product entry into the export market. Second, the model predicts that the profits and revenue of non-exporting firms falls relative to exporters. A third corollary of the model is that the relative decline of domestic to export sales is magnified in industries that rely more heavily on external finance.

Our empirical analysis is based on the dataset of firms' product portfolios described earlier in Section 3.1. We focus on a panel of firms between 1996 and 1999, with the last two years being considered as crisis years. We observe the revenue of each product sold by each firm, and can distinguish between revenue from domestic or export sales for each individual product. In addition, we see total firm revenue and profits.

We will compare the results obtained from the data on Chilean firms with data from a simulated panel of firms based on the calibrated model. We simulate two pre-crisis and two crisis periods, to be consistent with the horizon of the Chilean data. We start with a panel of 5000 firms along the balanced growth path. We assume these firms experience the innovation rates shown in panel a) in Figure 5 and domestic and export revenue profits as shown in panel c) in Figure 6 (for profits), allowing for entry and exit as a result of this.

■ **Product Entry** We documented earlier the frequencies with which firms introduce new products to the domestic market and export products previously sold domestically. Here we examine how the probability that each of these events take place evolves over time during the sudden stop episode. To this end, we estimate the following regression in which Y_{ft} is a dummy variable taking a value of one if a given transition takes place between years $t-1$ and t, and zero otherwise. This regression includes firm fixed effects and a dummy variable indicating the sudden stop period.

$$Y_{ft} = \beta_1 \times \mathbf{1}[\text{Sudden Stop}_t] + \phi_f + \epsilon_{ft} \tag{33}$$

The results are shown in the first two columns in Table 4. We find that during the sudden stop, the probability that firms introduce domestic products falls relative to the previous. In contrast, the probability of exporting products previously sold only domestically, falls to a much lesser extent, which is consistent with the predictions of the model. Specifically, column 1 indicates that the probability that a firm introduces one (or more) new domestic products during the sudden stop is 8.3 percentage points lower (0.22 standard deviations lower) than in the previous years. Column 2 shows that the probability that a firm introduces one or more new exported products previously sold domestically is 1.8 percentage points lower (0.10 standard deviations lower) during the sudden stop than before. The table also shows that with the simulated panel, in columns 3 and 4, we find quite similar patterns of product entry.

■ **Firm Revenue and Profits** Next, we study firm revenue and profits. We estimate firm-level regressions with these outcomes as functions of the interaction of a dummy variable indicating the sudden stop period and an exporter status dummy variable. We include firm and year fixed effects.

$$\log\left(\mathrm{Y}_{ft}\right) = \beta_1 \times \mathbf{1}[\text{Sudden Stop}_t] \times \mathbf{1}[\text{Exporter}_{ft}] + \phi_f + \delta_t + \epsilon_{ft} \tag{34}$$

Table 4: Probability of Adding Products during the Sudden Stop

	(1)	(2)	(3)	(4)
	Data		Model	
	Not Sold to Domestic	Domestic to Exported	Not Sold to Domestic	Domestic to Exported
$\mathbf{1}$[Sudden Stop$_t$]	-0.083***	-0.018***	-0.075***	-0.003
	(0.006)	(0.003)	(0.008)	(0.003)
Firm Fixed Effects	Yes	Yes	Yes	Yes
Observations	15523	15523	12785	12785

Note: This table reports the results of the estimation of equation 33. The transitions in each column are (1) a firm introduces to the domestic market a product not sold previously, (2) a firm introduces to the export market a product previously sold domestically. ***, **, and * denote statistical significance at a 1, 5 and 10 percent confidence level.

These results are shown in the first two columns in Table 5. We find an increase in revenue (column 1) and profits (column 2) for exporters relative to non-exporters during the sudden stop period compared to the previous years. Specifically, exporters obtain 7% higher revenue and 11% higher profits relative to non-exporters during the sudden stop. In contrast, columns 3 and 4 report the results using the simulated firm panel. The estimated coefficients are remarkably similar, being somewhat higher relative to the previous results in the case of revenue and extremely close in the case of profits.

Table 5: Revenue and Profits of Exporters vs. Non-exporters

	(1)	(2)	(3)	(4)
	Data		Model	
	Revenue	Profits	Revenue	Profits
$\mathbf{1}[\text{Sudden Stop}_t] \times \mathbf{1}[\text{Exporter}_{ft}]$	0.066***	0.107***	0.095***	0.118***
	(0.010)	(0.017)	(0.011)	(0.011)
Year Fixed Effects	Yes	Yes	Yes	Yes
Firm Fixed Effects	Yes	Yes	Yes	Yes
Observations	21213	20797	19022	19022

Note: This table reports the results of the estimation of equation 34. In column 1 the dependent variable is log revenue. In column 2 the dependent variable is log profits. ***, **, and * denote statistical significance at a 1, 5 and 10 percent confidence level.

■ **Domestic Sales, Export Sales and Financial Dependence** Finally, we examine the prediction that revenue from domestic sales falls during the sudden stop relative to revenue from export sales. Our model also predicts that the gap between export and domestic revenue observed in a sudden stop is magnified in industries that rely more heavily on external finance. Recall that the representative final good producer pays in advance a fraction ϕ of the cost of intermediate inputs (2). In normal times (when the lagrangian multiplier μ_t is zero), profits from domestic sales (equation 9 for exporters and equation 7 for non-exporters) and profits from export sales (equation 12) do not depend on ϕ. During a sudden stop (when the multiplier μ_t is strictly positive) profits from domestic sales depend negatively on the extent of financial dependence (ϕ). Thus, a sudden stop generates a larger decline in domestic relative to export sales in industries with a larger degree of financial dependence.

We use our data at the most disaggregate level, with each observation corresponding to a firm $\times$ product $\times$ market (domestic or export) $\times$ period combination. We compile data on industries' external finance dependence from Rajan and Zingales [1996].[16] We estimate the following equation of revenue as a function of the interaction between a dummy for exported products and a dummy for the sudden stop years. In addition we add the triple interaction between year dummies, the

[16] While these measures of industries' financial dependence were created by Rajan and Zingales [1996] using U.S. data, we believe that assigning these to the Chilean manufactures is reasonable, especially because we care about the relative ordering rather than precise values.

exported product dummy, and the financial dependence measures. We include firm, product, market and year fixed effects.

$$\log(\text{Revenue}_{fpmt}) = \beta_1 \times \mathbf{1}[\text{Sudden Stop}_t] \times \mathbf{1}[\text{Exported}_m] + \beta_2 \times \mathbf{1}[\text{Sudden Stop}_t] \times \text{Fin. Dep.}_p + \beta_3 \times \mathbf{1}[\text{Exported}_m] \times \text{Fin. Dep.}_p + \beta_4 \times \mathbf{1}[\text{Sudden Stop}_t] \times \mathbf{1}[\text{Exported}_m] \times \text{Fin. Dep.}_p + \phi_f + \rho_p + \delta_t + \nu_m + \epsilon_{fpmt} \tag{35}$$

The results are reported in Table 6. Column 1 includes firm and product fixed effects while in column 2 these are replaced by firm × product fixed effects. Both columns yield very similar results. From the coefficients on the triple interactions, it appears clearly that during the sudden stop, the additional revenue obtained from export relative to domestic products is magnified in industries that rely more on external finance. Consider the total elasticity of revenue to the interaction between the sudden stop dummy and the exported dummy (i.e. the additional revenue obtained from export sales relative to domestic sales during the sudden stop relative to the previous years). We can evaluate this elasticity for industries with different degrees of financial dependence. Based on column 1 in Table 6, this elasticity is -0.141 at the 10th percentile of financial dependence and 0.072 at the 90th percentile. This gap illustrates clearly the role of financial dependence, in line with the prediction of the model. For this final result we do not provide results of an equivalent regression with simulated data because we have a single-industry model.

Summing up all the previous results, we have described in detail for the first time in the literature the differential dynamics of revenue and product entry rates between export and domestic markets during a sudden stop episode. These findings are consistent with the model's predictions and further validate the calibrated model which we will use in the next section for counterfactuals.

3.4 Sudden Stops, Welfare and Productivity

Finally, we simulate counterfactuals to determine to what extent the growth and trade dynamics account for the welfare loss caused by sudden stops. We conduct the following simulations: we set the initial state of the economy as the average state when sudden stops take place in the previous simulation. Then we create two economies facing different shocks: the first economy receives a good shock of high TFP and a low interest rate in period 1. The second economy receives a bad

Table 6: Sudden Stops and External Finance Dependence

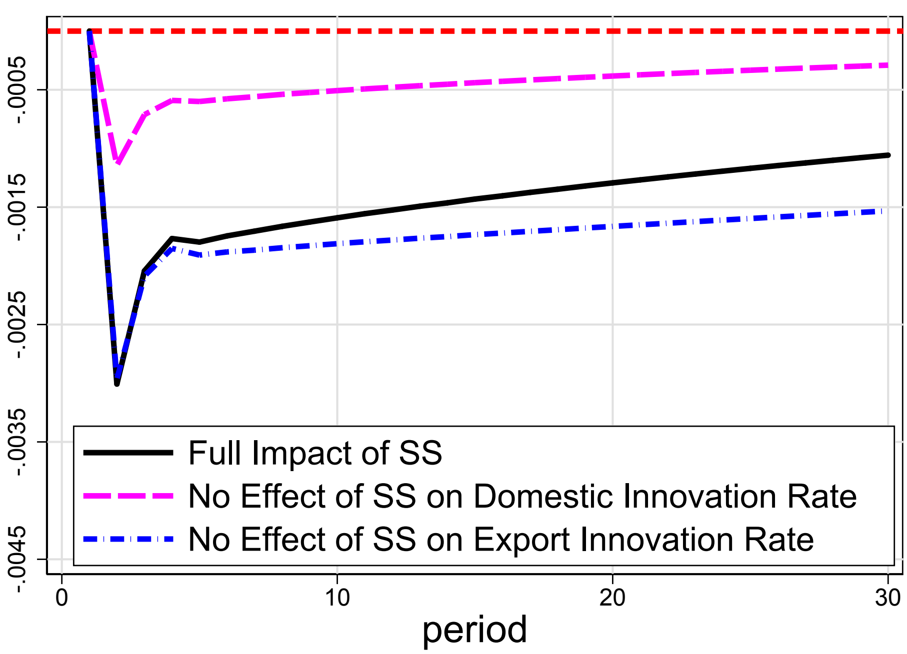
	(1)	(2)
$\mathbf{1}[\text{Sudden Stop}_t] \times \mathbf{1}[\text{Exported}_m]$	-0.156***	-0.177***
	(0.059)	(0.051)
$\mathbf{1}[\text{Sudden Stop}_t] \times \text{Fin. Dep.}_p$	0.016	-0.000
	(0.057)	(0.050)
$\mathbf{1}[\text{Exported}_m] \times \text{Fin. Dep.}_p$	-1.409***	-1.621***
	(0.106)	(0.092)
$\mathbf{1}[\text{Sudden Stop}_t] \times \mathbf{1}[\text{Exported}_m] \times \text{Fin. Dep.}_p$	0.507***	0.539***
	(0.152)	(0.130)
Year Fixed Effects	Yes	Yes
Market Fixed Effects	Yes	Yes
Firm Fixed Effects	Yes	No
Product Fixed Effects	Yes	No
Firm-Product Fixed Effects	No	Yes
Observations	50048	46981

Note: This table reports the results of the estimation of equation 35. Column 1 corresponds to the case with firm and product fixed effects. Column 2 corresponds to the case with firm × product fixed effects. ***, **, and * denote statistical significance at a 1, 5 and 10 percent confidence level.

shock of low TFP and a high interest rate in period 1. Only this second economy faces a sudden stop, triggered by the bad shock. For the subsequent periods, both economies face the same random shocks. Henceforth, we refer to the first economy as the *no-SS economy*, and the second economy as the *SS economy*. We simulate these two economies a thousand times with stochastic shocks from period 2 onwards, and compare the average productivity paths and expected welfare. This analysis allows us to compute the average productivity and welfare losses due to a sudden stop.

This exercise also allows us to disentangle the effects of the product and trade dynamics on productivity and welfare. In a first counterfactual we take the path of the productivity growth rate g_t from the no-SS economy and feed it into the SS economy. In this economy, a sudden stop happens in period 1, but it does not affect productivity at all. In a second counterfactual exercise, we take the domestic innovation rate i_t^D from the no-SS economy and feed it into the SS economy to assess how much the domestic innovation rate accounts for the productivity loss and the welfare loss. Finally, in the third counterfactual we take the exporting innovation rate i_t^x from the no-SS economy and feed it into the SS economy.

Figure 7: Productivity Loss from Sudden Stops



NOTE: This figure shows the productivty loss from sudden stops.

Figure 7 plots the productivity loss in percentage in each counterfactual simulation compared to the case of no sudden stop. The horizontal axis is the time period after a sudden stop happens. The red solid line shows the productivity loss by a sudden stop. It shows that the productivity level drops by 0.3% on impact following a sudden stop compared to the no-SS economy, and its recovery is very slow. The black line with dots is the productivity loss when the path of domestic innovation rate is replaced with the path in the no-SS case. In this case, the productivity loss is about one third of the full effect. This means that about two-thirds of the productivity loss by a sudden stop come from the lower domestic product entry rate. The blue dashed line is the productivity loss when the path of exporting innovation rate is replaced. In this case, the productivity drop on impact is not different from the baseline case, but the recovery is slower. This means that boosted exporting product entry after a sudden stop helps recovery in productivity.

Table 7: Welfare Loss from Sudden Stops

Economy	Welfare loss (%, relative to baseline)
Total Cost of Sudden Stop	4.62%
No Effect of SS on Productivity Growth g_t	3.24% (70.1%)
No Effect of SS on Domestic Innovation Rate i_t^D	3.49% (75.5%)
No Effect of SS on Export Innovation Rate i_t^X	5.04% (109.1%)

Table 7 shows the welfare loss in each counterfactual simulation compared to the case of no sudden stop. Following Durdu and Mendoza [2006], we translate the welfare loss into the compensating variation in period-1 consumption that equates the expected lifetime utility of each counterfactual to the no-SS case. Specifically, let V_1^{noss} denote the expected lifetime utility in no-SS economy, and $E_1[V_2^{cf}]$ denote the expected utility in a counterfactual economy from period 2 and onwards. The compensating variation v^{cf} satisfies the following equation:

$$V_1^{noss} = \ln\left((1+v^{cf})c_1 - A_1\frac{H_1^{\omega}}{\omega}\right) + E_1[V_2^{cf}] \ , \tag{36}$$

The result in Table 7 shows that the welfare loss by a sudden stop corresponds to 4.62% of consumption at period 1.[17] If the path of productivity growth is replaced with the path in the

[17] 4.62% may look small compared to the size of a drop in consumption when a sudden stop happens, which

case of no sudden stop, the welfare loss would reduce to 3.24%, which is 70.1% of the total welfare loss by a sudden stop. This implies that about 30% of the welfare loss by a sudden stop comes from the productivity slowdown due to lower product entries. Next, if the path of domestic innovation rate is replaced with the path in the case of no sudden stop, the welfare loss would reduce to 3.49%, which is close to the case where the productivity growth rate is replaced. This is in line with the above result that a large fraction of productivity loss comes from the lower domestic product entry rates. In contrast, if the path of exporting innovation rate is replaced with the path in the case of no sudden stop, the welfare loss would increase to 5.04%, which is 9.1% higher than the total welfare loss by a sudden stop. This implies that the boosted exporting product entry after the sudden stop helps productivity recovery and reduces the welfare loss by the sudden stop.

4 Conclusions

In this paper we introduce a new model of endogenous trade and productivity to study the impact of sudden stops on firm dynamics and economic growth. This theory stems from two key empirical facts. First, financial crises have persistent output effects [Cerra and Saxena, 2008] that can be explained by distortions on firm dynamics [Ates and Saffie, 2016]. Second, exports are not only an essential adjustment margin during sudden stops episodes [Alessandria et al., 2014], but also key contributor to productivity growth Bernard and Jensen [2004]. Our model captures these two facts by extending Ates and Saffie [2016] to include exporting dynamics and allowing for a leverage-driven financial crisis as in Bianchi and Mendoza [2018] and Jeanne and Korinek [2019].

We discipline the model using unique data on firms' products portfolios in domestic and export markets for Chile around the sudden stop episode of 1998. The balanced growth path of the model captures key product-level dynamics from the data showing for the first time that the Klette and Kortum [2004] model is successful when contrasted to this type of micro-data. Moreover, the model also replicates the product-level dynamics observed in the data in response to the 1998 sudden stop.

The calibrated model shows that exporters have a central role in the recovery from the crisis. Lower internal demand and lower domestic wages provide incentives for firms to enter export

is roughly 10%. This is because labor disutility also drops when a sudden stop happens, which reduces the compensating variation in terms of only consumption. If we compute the compensating variation in terms of consumption minus labor disutility, then it would be 12.24% instead of 4.62%. The decomposition analysis would not be affected by this alternative measure.

markets and fuel the productivity recovery. Therefore, a fruitful direction for future research is studying the role of macro-prudential policies [Benigno et al., 2013] and in particular firm-level subsidies that discriminate between exporters and non-exporters.

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Online Appendix

6 Theory Appendix

6.1 Equilibrium and Stationarized Equilibrium

This section defines the equilibrium of the economy and the stationarized equilibrium.

6.1.1 Factor Allocation

Before defining the equilibrium, we derive the expressions for asset and labor allocations. First we show that the total cost for production $R_t^L \ell_t(i) + W_t h_t(i)$ is equal to production $y_t(i)$ times the marginal cost. The latter can be written as:

$$y_t(i) \times MC_t(i) = a_t(i)(\ell_t(i))^{\alpha}(h_t(i))^{1-\alpha} \times \frac{1}{a_t(i)} \overline{\alpha}(R_t^L)^{\alpha}(W_t)^{1-\alpha} = \overline{\alpha}(R_t^L \ell_t(i))^{\alpha}(W_t h_t(i))^{1-\alpha}$$

Using the cost minimization condition $R_t^L \ell_t(i)/W_t h_t(i) = \alpha/(1-\alpha)$,

$$y_t(i) \times MC_t(i) = \frac{1}{\alpha} R_t^L \ell_t(i) = \frac{1}{1-\alpha} W_t h_t(i) = R_t^L \ell_t(i) + W_t h_t(i) \tag{37}$$

This shows that production times the marginal cost is equal to the total cost.

Next, profit for a product line can be written as follows:

$$\pi_t(i) = p_t(i) y_t(i) - (R_t^L \ell_t(i) + W_t h_t(i)) = (p_t(i) - MC_t(i)) y_t(i)$$

Recall that the optimal price is equal to the marginal cost for the second-best rival firm. Whether the rival firm is a domestic firm or a foreign firm depends on whether the product is sold domestically or exported. We first consider the case of domestic sales by domestic and exporting lines, for which the second-best rival is a domestic firm. The case for exports and imports is examined next.

For domestic sales, given that the rival firm is also a domestic firm, the rival's marginal cost is $1+\sigma^s$ times the marginal cost for the leader, where $s = D, X$ depending on the type of the product line. Therefore,

$$\pi_t(i) = \sigma^s MC_t(i) y_t(i)$$

Using (37),

$$\pi_t(i) = \sigma^s \frac{1}{\alpha} R_t^L \ell_t(i) = \sigma^s \frac{1}{1-\alpha} W_t h_t(i)$$

In the main text, we derived another expression for a profit in equation (7). Thus we have:

$$\sigma^s \frac{1}{\alpha} R_t^L \ell_t(i) = \sigma^s \frac{1}{1-\alpha} W_t h_t(i) = Y_t^T \frac{1}{1+\phi\mu_t/\lambda_t} \frac{\sigma^s}{1+\sigma^s} \tag{38}$$

This equation shows that the asset and labor input $\ell_t(i)$ and $h_t(i)$ are independent of productivity level $a_t(i)$, and depends only on the type of product lines, $s = D, X$. Combining this equation with $s = D$ and $s = X$, we obtain the relative factor input between domestic lines and exporting lines:

$$\frac{\ell_t^D}{\ell_t^X} = \frac{h_t^D}{h_t^X} = \frac{1+\sigma^X}{1+\sigma^D}$$

Next, for exports by exporting lines, demand is exogenously given by Y_t^*. Using the demand equation for each type of intermediate goods,

$$\begin{aligned} Y_t^* &= p_t(i) y_t(i) = MC_t^*(i) \times \frac{1}{1+\xi} a_t(i) \ell_t(i)^\alpha h_t(i)^{1-\alpha} \\ &= \frac{1}{a_t^*(i)} \overline{\alpha} (R_t^{L*})^\alpha (W_t^*)^{1-\alpha} \times \frac{1}{1+\xi} a_t(i) \ell_t(i)^\alpha h_t(i)^{1-\alpha} \end{aligned}$$

Note that the amount that exporting firms can actually sell to the foreign demand is their output minus the loss due to an iceberg cost. Using $a_t(i) = (1+\sigma^X) a_t^*(i)$,

$$Y_t^* = \overline{\alpha} (R_t^{L*})^\alpha (W_t^*)^{1-\alpha} \frac{1+\sigma^X}{1+\xi} \ell_t(i)^\alpha h_t(i)^{1-\alpha}$$

Combined with the cost minimization between $\ell_t(i)$ and $h_t(i)$, this equation tells us that the factor input for exporting lines is also independent of the productivity level. It follows that the relation between aggregate factor inputs L_t^*, H_t^* and individual factor inputs ℓ_t^*, h_t^* satisfy $\ell_t^* = L_t^*/\theta_{t-1}^X$ and

$h_t^* = H_t^*/\theta_{t-1}^X$. Plugging these equations,

$$Y_t^* = \overline{\alpha}(R_t^{L*})^{\alpha}(W_t^*)^{1-\alpha}\frac{1+\sigma^X}{1+\xi}\frac{1}{\theta_{t-1}^X}(L_t^*)^{\alpha}(H_t^*)^{1-\alpha}$$

We also utilize the fact that Y_t^*, R_t^{L*}, W_t^* all grow at the same exogenous rate $\overline{g}$. This impliles that $(R_t^{L*})^{\alpha}(W_t^*)^{1-\alpha}/Y_t^*$ is a constant. Let ω^* denote this constant. Then this equation can be written as follows:

$$1 = \overline{\alpha}\omega^*\frac{1+\sigma^X}{1+\xi}\frac{1}{\theta_{t-1}^X}(L_t^*)^{\alpha}(H_t^*)^{1-\alpha}$$

This equation pins down the factor inputs for exports.

Finally, we show that factor inputs by foreign firms for importing lines are linear in the factor inputs by exporting firms for domestic sales. Demand for imported intermediate goods from final producer is given by the same equation as other product lines:

$$p_t(i)y_t(i) = Y_t\frac{1}{1+\phi\mu_t/\lambda_t}$$

The optimal price is the marginal cost for the closest rival firm, and in this case it is a domestic firm. Therefore,

$$\frac{1+\sigma^X}{a_t(i)}\overline{\alpha}(R_t^L)^{\alpha}(W_t)^{1-\alpha} \times \frac{1}{1+\xi}a_t(i)(\ell_t^F)^{\alpha}(h_t^F)^{1-\alpha} = Y_t\frac{1}{1+\phi\mu_t/\lambda_t}$$

$$\overline{\alpha}(R_t^L)^{\alpha}(W_t)^{1-\alpha}\frac{1+\sigma^X}{1+\xi}(\ell_t^F)^{\alpha}(h_t^F)^{1-\alpha} = Y_t\frac{1}{1+\phi\mu_t/\lambda_t}$$

Note that foreign exporters are subject to an iceberg cost, and thus they can sell only a fraction $1/(1+\xi)$ of their output. The same equation for exporting lines' domestic sales is given as follows:

$$\overline{\alpha}(R_t^L)^{\alpha}(W_t)^{1-\alpha}(1+\sigma^X)(\ell_t^X)^{\alpha}(h_t^X)^{1-\alpha} = Y_t\frac{1}{1+\phi\mu_t/\lambda_t}$$

Comparing these two equation, foreign exporters' factor input is given by the following equation:

$$\frac{1}{1+\xi}(\ell_t^F)^{\alpha}(h_t^F)^{1-\alpha} = (\ell_t^X)^{\alpha}(h_t^X)^{1-\alpha}$$

This means that foreign exporters use more inputs than domestic exporting firms, but the effective inputs that contribute to production is the same due to an iceberg cost.

Using these result, final goods production can be written as follows:

$$\begin{aligned}
Y_t &= \exp\left[\int_0^1 \ln y_t(i)di\right] = \exp\left[\int_0^1 \ln a_t(i)di\right] \times \exp\left[\int_0^1 \ln\left(\ell_t(i)^\alpha h_t(i)^{1-\alpha}\right)\right] \\
&= A_t \exp\left[\theta_{t-1}^D \ln\left\{\left(\frac{L_t^D}{\theta_{t-1}^D}\right)^\alpha \left(\frac{H_t^D}{\theta_{t-1}^D}\right)^{1-\alpha}\right\} \times (1-\theta_{t-1}^D)\ln\left\{\left(\frac{L_t^X}{\theta_{t-1}^X}\right)^\alpha \left(\frac{H_t^X}{\theta_{t-1}^X}\right)^{1-\alpha}\right\}\right] \\
&= A_t \left\{\left(\frac{L_t^D}{\theta_{t-1}^D}\right)^\alpha \left(\frac{H_t^D}{\theta_{t-1}^D}\right)^{1-\alpha}\right\}^{\theta_{t-1}^D} \left\{\left(\frac{L_t^X}{\theta_{t-1}^X}\right)^\alpha \left(\frac{H_t^X}{\theta_{t-1}^X}\right)^{1-\alpha}\right\}^{1-\theta_{t-1}^D} \\
&= A_t \frac{\left(L_t^D\right)^\alpha \left(H_t^D\right)^{1-\alpha}}{\theta_{t-1}^D} \left(\frac{1+\sigma^D}{1+\sigma^X}\right)^{1-\theta_{t-1}^D}
\end{aligned}$$

6.1.2 Stationarized Equilibrium

To stationarize the model, we divide the equilibrium conditions by aggregate productivity A_t. We denote stationarized variables by the lower-case letters, and use g_t to denote the productivity growth rate $A_{t+1}/A_t - 1$. We also make some arrangements and reduce the number of equations. The following is the complete list of equations to characterize the stationarized equilibrium of the model:

■ **Final goods producer**

$$y_t = \exp(\varepsilon_t^A)\frac{(L_t^D)^\alpha (H_t^D)^{1-\alpha}}{\theta_{t-1}^D}\left(\frac{1+\sigma^D}{1+\sigma^X}\right)^{1-\theta_{t-1}^D}$$

$$\lambda_t - \mu_t = \beta R \exp(\varepsilon_t^R) E_t(\lambda_{t+1})$$

$$\lambda_t q_t = \beta E_t\left[\lambda_{t+1}(q_{t+1} + r_{t+1}^L) + \mu_{t+1}\kappa q_{t+1}\right]$$

$$\mu_t\left[-b_t + \phi y_t^T \frac{1}{1+\phi\mu_t/\lambda_t} - \kappa q_t L\right] = 0$$

■ **Intermediate goods producing firms**

$$r_t^L = \frac{1}{1+\sigma^D}\alpha y_t \frac{\theta_{t-1}^D}{L_t^D}\frac{1}{1+\phi\mu_t/\lambda_t}$$

$$w_t = \frac{1}{1+\sigma^D}(1-\alpha)y_t^T \frac{\theta_{t-1}^D}{H_t^D}\frac{1}{1+\phi\mu_t/\lambda_t}$$

$$L_t^X = L_t^D \frac{\theta_{t-1}^X}{\theta_{t-1}^D}\frac{1+\sigma^D}{1+\sigma^X}$$

$$H_t^X = H_t^D \frac{\theta_{t-1}^X}{\theta_{t-1}^D}\frac{1+\sigma^D}{1+\sigma^X}$$

$$\pi_t^D = \frac{\sigma_t^D}{1+\sigma_t^D} y_t \frac{1}{1+\phi\mu_t/\lambda_t}$$

$$\pi_t^X = \frac{\sigma_t^X}{1+\sigma_t^X} y_t \frac{1}{1+\phi\mu_t/\lambda_t}$$

$$\pi_t^* = y_t^* - \frac{1+\xi}{1+\sigma^X}\frac{1}{\omega^*}(r_t^L)^\alpha (w_t)^{1-\alpha}$$

$$1 = \frac{1+\sigma^X}{1+\xi}\overline{\alpha}\omega^* \frac{1}{\theta_{t-1}^X}(L_t^*)^\alpha (H_t^*)^{1-\alpha}$$

$$v_t(1,0) = \pi_t^D - z_t^D - z_t^X + \left[i_t^D + (1-d_t)(1-i_t^X)\right] E_t(\Lambda_{t,t+1}v_{t+1}(1,0)) + (1-d_t)i_t^X E_t(\Lambda_{t,t+1}v_{t+1}(0,1))$$

$$v_t(0,1) = \pi_t^X + \pi_t^* - z_t^D + (i_t^D + i^{FX})E_t(\Lambda_{t,t+1}v_{t+1}(1,0)) + (1-i^{FX})E_t(\Lambda_{t,t+1}v_{t+1}(0,1))$$

$$i_t^D = \eta^D (z_t^D)^{1/\rho}$$

$$\eta^D \frac{1}{\rho}(z_t^D)^{1/\rho-1} E_t(\Lambda_{t,t+1}v_{t+1}(1,0)) = 1$$

$$i_t^X = \eta^X (z_t^X)^{1/\rho}$$

$$(1-d_t)\eta^X \frac{1}{\rho}(z_t^X)^{1/\rho-1}\left(E_t(\Lambda_{t,t+1}v_{t+1}(0,1)) - E_t(\Lambda_{t,t+1}v_{t+1}(1,0))\right) = 1$$

■Aggregate variables

$$d_t = i^{FD} + (e_t^D + e_t^X + (\theta_{t-1}^D + \theta_{t-1}^X)i_t^D)\frac{1}{1-\theta_{t-1}^X}$$

$$\theta_t^D = \theta_{t-1}^D + (e_t^D + (\theta_{t-1}^D + \theta_{t-1}^X)i_t^D)\frac{1-\theta_{t-1}^D-\theta_{t-1}^X}{1-\theta_{t-1}^X} + \theta_{t-1}^X i^{FX} - \theta_{t-1}^D(1-d_t)i_t^X - \theta_{t-1}^D i^{FD} - e_t^X\frac{\theta_{t-1}^D}{1-\theta_{t-1}^X}$$

$$\theta_t^X = \theta_{t-1}^X + \theta_{t-1}^D(1-d_t)i_t^X + e_t^X - \theta_{t-1}^X i^{FX}$$

$$\frac{A_{t+1}}{A_t} = 1 + g_t = (1+\sigma^D)^{e_t^D + (\theta_{t-1}^D + \theta_{t-1}^X)i_t^D}(1+\sigma^X)^{e_t^X + \theta_{t-1}^D(1-d_t)i_t^X}(1+\sigma^X)^{i^{FD}}$$

$$a_t^* = \frac{1+\overline{g}}{1+g_t}a_{t-1}^*$$

■Households

$$c_t + z_t^{ED} + z_t^{EX} = y_t - \theta_{t-1}^D(z_t^D + z_t^X) - \theta_{t-1}^X z_t^D + \theta_{t-1}^X a_t^* y^* - (1-\theta_{t-1}^D - \theta_{t-1}^X)\frac{y_t}{1+\phi\mu_t/\lambda_t} - b_t + R\exp(\varepsilon_{t-1}^R)\frac{b_{t-1}}{1+g_{t-1}}$$

$$H_t^{\omega-1} = w_t$$

$$\lambda_t = \frac{1}{c_t - w_t^{\omega}/\omega}$$

$$e_t^D = \eta^{ED}(z_t^{ED})^{1/\rho}$$

$$\eta^{ED}(1/\rho)(z_t^{ED})^{1/\rho-1}E_t(\Lambda_{t,t+1}v_{t+1}(1,0)) = 1$$

$$e_t^X = \eta^{EX}(z_t^{EX})^{1/\rho}$$

$$\eta^{EX}(1/\rho)(z_t^{EX})^{1/\rho-1}E_t(\Lambda_{t,t+1}v_{t+1}(0,1)) = 1$$

■**Market clearing**

$$H_t = H_t^D + H_t^X + H_t^*$$

$$1 = L_t^D + L_t^X + L_t^*$$

6.2 Numerical Solution

In this section we sketch the numerical solution method. The solution method is a version of the policy function iteration, modified to deal with the occasionally binding constraint. Below is the procedure to obtain the numerical solution.

1. We set the equally-spaced grid points for the endogenous state variables, foreign debt $R\exp(\varepsilon_t^R)b_{t-1}/(1+g_{t-1})$, share of domestic product lines θ_{t-1}^D, share of exporting product lines θ_{t-1}^X, relative productivity of foreign countries over the domestic country $a_t^* = A_t^*/A_t$. There are also 2 states for stochastic shocks ε_t^A and ε_t^R respectively.D

2. For each grid point, we set the initial guess for five variables: $E_t(\Lambda_{t,t+1}v_{t+1}(1,0)$, $E_t(\Lambda_{t,t+1}v_{t+1}(0.1)$, L_t^D, b_t, and q_t.

3. For each grid point, we do the following:

   - We leave the five variables we have made guess for as unknown variables. Express all the other endogenous variables as functions of the state variables and the five unknowns. In this process, we first assume that the borrowing constraint is not binding and proceed. Later we check if the constraint is satisfied. If it is not satisfied, we recalculate all the variables using the binding borrowing constraint. The other endogenous variables, which include next-period state variables, are now functions of the five variables.

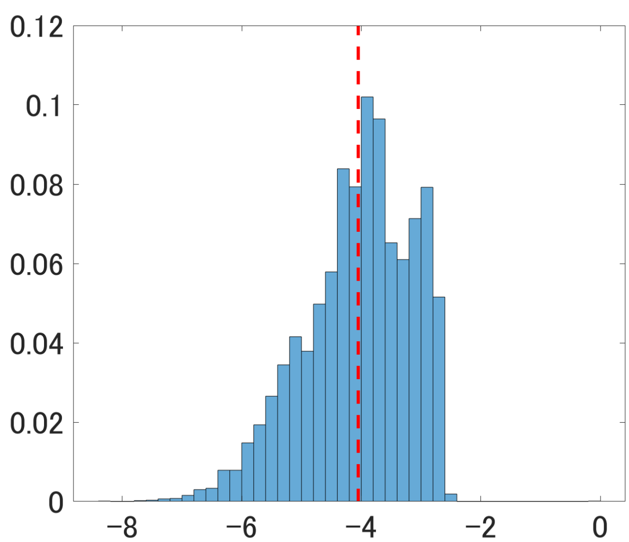
   - Using four-dimensional linear interpolation over the next-period state variables and the guess for the five variables ($E_t(\Lambda_{t,t+1}v_{t+1}(1,0)$, $E_t(\Lambda_{t,t+1}v_{t+1}(0.1)$, L_t^D, b_t, and q_t), we compute all the endogenous variables next period. We then calculate all the forward-looking expectation terms, such as the right-hand side of the Euler equations and the value functions.

- All the equilibrium conditions are now the functions of the initial five unknowns. There are five equations we did not use in step (a), thus five equations in total. We solve for the five unknowns using non-linear solver.

4. We check the gap between the guess and the newly-obtained values for the five variables. If they are close enough, we stop. If not, we update the guess by the newly-obtained values, and go back to step 3. Repeat this process until the gap becomes sufficiently small.

We check the accuracy of the numerical solution using the Euler equation error. We simulate the model for 100,000 periods with stochastic shocks and compute the Euler equation error for each period. Figure 8 plots the distribution of the Euler equation errors obtained in this way. The average error is -4 and the maximum error is -2.5, which is reasonably small compared to the literature.

Figure 8: Distribution of Euler Equation Errors



6.3 Proof of Linear Relations in Value Functions

This section shows the detailed procedure of the guess-and-verify method to prove the linear relation in value functions for intermediate producing firms. We guess the linear relation $V_t(n^D, n^X) = n^D V_t(1,0) + n^X V_t(0,1)$ and prove it. We first work on the value of a firm with a single domestic product line:

$$\begin{aligned} V_t(1,0) &= \max_{Z_t^D, Z_t^X} \left\{ \pi_t^D - Z_t^D - Z_t^X \right. \\ &\quad \left. + \left[\sum_{i=0}^{1} P(i,1,i_t^D) \left\{ \sum_{j=0}^{1} P(j,1,d_t) \left(\sum_{k=0}^{1-j} P\left(k,1-j,i_t^X\right) E_t\left[\Lambda_{t,t+1} V_{t+1}(1+i-j-k,k)\right] \right) \right\} \right] \right\} \end{aligned}$$

Using the linear relation, the summations in the second line can be written as follows:

$$\begin{aligned} & \sum_{i=0}^{1} P(i,1,i_t^D) \left\{ \sum_{j=0}^{1} P(j,1,d_t) \left(\sum_{k=0}^{1-j} P\left(k,1-j,i_t^X\right) E_t\left[\Lambda_{t,t+1} V_{t+1}(1+i-j-k,k)\right] \right) \right\} \\ = \; & \sum_{i=0}^{1} P(i,1,i_t^D) \left\{ \sum_{j=0}^{1} P(j,1,d_t) \left(\sum_{k=0}^{1-j} P\left(k,1-j,i_t^X\right) E_t\left[\Lambda_{t,t+1}[(1+i-j-k)V_{t+1}(1,0) + kV_{t+1}(0,1)]\right] \right) \right\} \\ = \; & E_t\left[\Lambda_{t,t+1} V_{t+1}(1,0)\right] \sum_{i=0}^{1} P(i,1,i_t^D) \sum_{j=0}^{1} P(j,1,d_t) \sum_{k=0}^{1-j} P\left(k,1-j,i_t^X\right)(1+i-j-k) \\ & + E_t\left[\Lambda_{t,t+1} V_{t+1}(0,1)\right] \sum_{i=0}^{1} P(i,1,i_t^D) \sum_{j=0}^{1} P(j,1,d_t) \sum_{k=0}^{1-j} P\left(k,1-j,i_t^X\right)(k) \\ = \; & (i_t^D + (1-d_t)(1-i_t^X)) E_t\left[\Lambda_{t,t+1} V_{t+1}(1,0)\right] + (1-d_t) i_t^X E_t\left[\Lambda_{t,t+1} V_{t+1}(0,1)\right] \end{aligned}$$

Therefore we have:

$$\begin{aligned} V_t(1,0) &= \max_{Z_t^D, Z_t^X} \left\{ \pi_t^D - Z_t^D - Z_t^X \right. \\ &\quad \left. + \left(i_t^D + (1-d_t)(1-i_t^X) \right) E_t\left[\Lambda_{t,t+1} V_{t+1}(1,0)\right] + (1-d_t) i_t^X E_t\left[\Lambda_{t,t+1} V_{t+1}(0,1)\right] \right\} \end{aligned}$$

Similarly, we can show that the value of a firm with a single exporting line is given as follows:

$$V_t(0,1) = \max_{Z_t^D} \left\{ \pi_t^X + \pi_t^* - Z_t^D + (i_t^D + i^{FX}) E_t\left[\Lambda_{t,t+1} V_{t+1}(1,0)\right] + (1 - i^{FX}) E_t\left[\Lambda_{t,t+1} V_{t+1}(0,1)\right] \right\}$$

which is equations (16) and (17) in the main text. Next I work on the value of a firm with general n^D domestic lines and n^X exporting lines:

$$
\begin{aligned}
V_t(n^D, n^X) = \max_{Z_t^D, Z_t^X} \{ & n^D \pi_t^D + n^X(\pi_t^X + \pi_t^*) - (n^D + n^X) Z_t^D - n^D Z_t^X \\
& + \sum_{i=0}^{n^D+n^X} P(i, n^D + n^X, i_t^D) \sum_{j=0}^{n^D} P(j, n^D, d_t) \sum_{k=0}^{n^D-j} P(k, n^D - j, i_t^X) \sum_{m=0}^{n^X} P(m, n^X, i^{FX}) \\
& E_t \left[\Lambda_{t,t+1} V_{t+1}(n^D + i - j - k, n^X + k - m) \right] \}
\end{aligned}
$$

Using the linear relation in the value function,

$$
\begin{aligned}
V_t(n^D, n^X) = \max_{Z_t^D, Z_t^X} \{ & n^D \pi_t^D + n^X(\pi_t^X + \pi_t^*) - (n^D + n^X) Z_t^D - n^D Z_t^X \\
& + E_t[\Lambda_{t,t+1} V_{t+1}(1,0)] \sum_{i=0}^{n^D+n^X} P(i, n^D + n^X, i_t^D) \sum_{j=0}^{n^D} P(j, n^D, d_t) \sum_{k=0}^{n^D-j} P(k, n^D - j, i_t^X) \sum_{m=0}^{n^X} P(m, n^X, i^{FX})(n^D + i - j - k + m) \\
& + E_t[\Lambda_{t,t+1} V_{t+1}(0,1)] \sum_{i=0}^{n^D+n^X} P(i, n^D + n^X, i_t^D) \sum_{j=0}^{n^D} P(j, n^D, d_t) \sum_{k=0}^{n^D-j} P(k, n^D - j, i_t^X) \sum_{m=0}^{n^X} P(m, n^X, i^{FX})(n^X + k - m)
\end{aligned}
$$

The second line can be written as follows:

$$
E_t[\Lambda_{t,t+1} V_{t+1}(1,0)](n^D + (n^D + n^X) i_t^D - n^D d_t - n^D(1 - d_t) i_t^X + n^X * i^{FX}
$$

The third line can be written as follows:

$$
E_t[\Lambda_{t,t+1} V_{t+1}(0,1)](n^X + n^D(1 - d_t) i_t^X - n^X i^{FX}
$$

Therefore $V_t(n^D, n^X)$ can be written as follows::

$$
\begin{aligned}
V_t(n^D, n^X) &= \max_{Z_t^D, Z_t^X} \{ n^D \pi_t^D + n^X(\pi_t^X + \pi_t^*) - (n^D + n^X) Z_t^D - n^D Z_t^X \\
&\quad + E_t[\Lambda_{t,t+1} V_{t+1}(1,0)](n^D + (n^D + n^X) i_t^D - n^D d_t - n^D(1 - d_t) i_t^X + n^X i^{FX} \\
&\quad + E_t[\Lambda_{t,t+1} V_{t+1}(0,1)](n^X + n^D(1 - d_t) i_t^X - n^X i^{FX}) \} \\
&= \max_{Z_t^D, Z_t^X} \{ n^D \pi_t^D + n^X(\pi_t^X + \pi_t^*) - (n^D + n^X) Z_t^D - n^D Z_t^X \\
&\quad + n^D \left\{ (i_t^D + (1 - d_t)(1 - i_t^X)) E_t[\Lambda_{t,t+1} V_{t+1}(1,0)] + (1 - d_t) i_t^X E_t[\Lambda_{t,t+1} V_{t+1}(0,1)] \right\} \\
&\quad + n^X \left\{ (i_t^D + i^{FX}) E_t[\Lambda_{t,t+1} V_{t+1}(1,0)] + (1 - i^{FX}) \right\} E_t[\Lambda_{t,t+1} V_{t+1}(0,1)] \} \\
&= n^D V_t(1,0) + n^X V_t(0,1)
\end{aligned}
$$

This verifies that the initial guess $V_t(n^D, n^X) = n^D V_t(1,0) + n^X V_t(0,1)$ is correct.

6.4 Firm Size Distribution

This section shows the law of motion for the share of each firm size and how to derive the firm size distribution. Each firm is characterized by the number of domestic and exporting lines it owns, (n^D, n^X). The law of motion for the firm size (n^D, n^X) is the formula that gives us the measure (number) of firms that own (n^D, n^X) given the firm size distribution in the previous period. Let $\delta_t(n^D, n^X)$ denote the measure of firms that own n^D domestic lines and n^X exporting lines at period t. Because the total measure of intermediate goods is one and each firm owns at least one product line, the measure of firms is between 0 and 1, i.e. $\delta_t(n^D, n^X) \in [0, 1]$ $\forall t, n^D, n^X$. In order for a firm to become a firm with (n^D, n^X) in the next period, there are some conditions to be satisfied. For example, a firm with (i, j) at period $t-1$ can own at most $2i + j$ domestic lines, because this is the case in which all domestic innovations $(i + j)$ are successful, all exporting innovations fail, and no replacement on domestic lines happens. So, if a firm owns (i, j) that satisfies $2i + j < n^D$, this firm cannot become a firm with (n^D, n^X) in the next period for any n^X. Let (i, j) denote the number of domestic and exporting lines that a firm owns at period $t - 1$. Let (k, ℓ) denote the number of successes in domestic innovation and exporting innovation respectively. Let q denote the number of exporting lines that are hit by foreign innovation and turn back to domestic lines. Let r^D denote the number of replacements that happen on domestic lines. Then consider a case in which this firm becomes a firm with (n^D, n^X). The table below lists up all the notations:

symbol	description
n^D	domestic lines next period
n^X	exporting lines next period
i	domestic lines this period
j	exporting lines this period
k	successful domestic innovation
ℓ	successful exporting innovation
q	exporting lines to become domestic
r^D	replacements on domestic lines

These variables satisfy the following equations and inequality:

- $n^D = i + k - \ell + q - r^D$

- $n^X = j + \ell - q$
- $n^D + n^X = i + j + k - r^D$
- $\ell + r^D \leq i$ (ℓ and r^D do not happen on the same domestic line)

Given these restrictions, the conditions that each variable needs to satisfy are given as follows:

i:

- $i \geq 0$

j:

- $j \geq 0$
- $n^D \leq i + k + j \rightarrow j \geq I_+ \left\{ n^D/2 - i \right\}$
- $n^X \leq i + j \rightarrow j \geq n^X - i$
- $n^D + n^X \leq 2i + 2j \rightarrow j \leq I_+ \left\{ (n^D + n^X)/2 - i \right\}$

k:

- $k \geq 0$
- $k \leq i + j$
- $n^D \leq i + k + j \rightarrow k \geq n^D - i - j$
- $n^D + n^X \leq i + j + k \rightarrow k \geq n^D + n^X - i - j$

ℓ:

- $\ell \geq 0$
- $\ell \leq i$
- $n^D \leq i + k - \ell + j \rightarrow \ell \leq i + k + j - n^D$
- $n^X \leq j + \ell \rightarrow \ell \geq n^X - j$

q:

- $\ell + r^D = i + k + q - n^D \leq i \rightarrow q = j + \ell - n^X \leq n^D - k \rightarrow \ell \leq n^D + n^X - j - k$

- $q = j + \ell - n^X \leq j \rightarrow \ell \leq n^X$

r^D:

- $\ell + r^D = i + k + q - n^D \leq i$

where $I_+\{x\}$ is the smallest integer that is equal to or greater than x. Summarizing these conditions, i, j, k, ℓ are subject to the following restrictions:

i:

- $i \geq 0$

j:

- $\max\{0, n^X - i\} \leq j \leq I_+\{(n^D + n^X)/2 - i\}$

k:

- $\max\{0, n^D + n^X - i - j\} \leq k \leq i + j$

ℓ:

- $\max\{0, n^X - j\} \leq \ell \leq \min\{i, i + k + j - n^D, n^D + n^X - j - k, n^X\}$

Using these conditions, the law of motion for $\delta_t(n^D, n^X)$ can be written as follows:

$$\delta_t(n^D, n^X) = \sum_i \sum_j \delta_{t-1}(i,j) \sum_k \sum_\ell P(k, i+j, i^D) P(\ell, i - r^D, i^X) P(q, j, i^{FX}) P(r^D, i, d)$$

subject to the above constraints on i, j, k, ℓ. There are two special cases, for which new firm entry is added: domestic firm entry e^{ED} is added to the case of $(n^D, n^X) = (1, 0)$, and exporting firm entry e^{EX} is added to the case of $(n^D, n^X) = (0, 1)$.

To derive the firm-size distribution at the balanced growth path, we use the values at the balanced growth path for new entry, innovation and replacement rate, $e^{ED}, e^{EX}, i^D, i^X, d$, and iterate this law of motion for large enough (n^D, n^X) until the distribution converges for every firm size.

7 Empirical Appendix

7.1 Product Transitions

Here we document the frequency with which firms add or drop products from the domestic and export markets. We document these transitions based on a balanced panel for 1996-1997 (i.e. prior to the sudden stop) with 3512 firms out of which 825 (23.5%) are exporters in 1996 and 870 (24.8%) are exporters in 1997.

We count the number of firms adding or dropping products and group them according to their initial and final status (not sold, sold exclusively domestically, or exported). We define six transitions of interest. The frequency of these transitions is shown in Column 1 in Table 8. We find that 15.4% of firms add one or more domestic product not produced the previous year. 2.6% of firms add one or more new products simultaneously to the domestic and export markets. We also find that 5.3% of firms add one or more products to the export market sold exclusively in the domestic market the previous year.

We also find that 16.0% of firms drop one or more domestic products. 2.7% of firms drop one or more product both sold domestically and exported. Finally, 4.5% of firms drop one or more products from the export market that transitions to be sold only domestically the next period.

Table 8 also documents the number of products added or dropped in each transition. In each case, there is a larger probability of adding or dropping a single product, and the probability of each event is decreasing in the number of products added or dropped. In all cases, the decrease in the probability of adding or dropping a single product to adding or dropping more than one is quite steep. For instance, while 10% of firms introduce a new product to the domestic market, only 3% introduce 2 products and 3% introduce three or more; and while 5% introduce a single product previously sold domestically to the export market, only 0.4% introduce two, and 0.4% introduce three or more.

Table 8: Number of Firms Per Transition

	Number of Products Added or Dropped			
	Any	1	2	3+
Not Produced to Domestic	0.15	0.10	0.03	0.03
Domestic to Exported	0.05	0.05	0.004	0.004
Domestic to Not Produced	0.16	0.10	0.03	0.03
Exported to Domestic	0.05	0.04	0.005	0.003
Not Produced to Domestic + Exported	0.03	0.02	0.003	0.002
Domestic + Exported to Not Produced	0.03	0.02	0.004	0.001

Note: This table reports the frequency of firms' product transitions. The transitions in each row are (1) a firm introduces to the domestic market a product not sold previously, (2) a firm introduces to the export market a product previously sold domestically, (3) a firm withdraws a product from the domestic market, subsequently not selling it, (4) a firm withdraws a product from the export market, subsequently selling it domestically, (5) a firm introduces simultaneously to the domestic and export market a product not sold previously , and (6) a firm simultaneously withdraws a product from the domestic and export markets, subsequently not selling it.

7.2 Extensive Margin: Domestic, Export and Import Lines

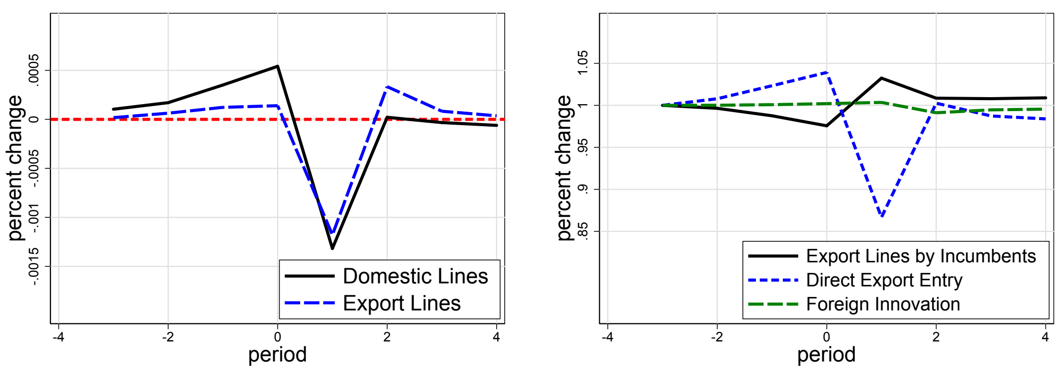
In the main text, Figure 6c documents the evolution of profits accrued from domestic and export lines during a sudden stop. Profit streams determine firm values and consequently investment decisions, with then shape the response of the extensive margin. In this appendix we discuss the response of the extensive margin to sudden stops. Figure 9a shows the change in percentage points in the shares of domestic and export lines. The fall in the share of domestic lines is about a third larger than that for export lines, which is consistent with the decline in profits from domestic lines (compared to an increase in profits from export lines). Even though the decline in the share of export lines is smaller, it is interesting to understand why this share falls at all. To do so we refer to the law of motion of the share of export lines in equation (26), which allows us to decompose changes in this share into three terms: the addition of export lines by incumbent firms, the addition of export lines by new firms that start exporting immediately, and foreign innovation.

Figure 9b plots the evolution of each of these components. The results indicate that the decline in the share of export lines is due to a large decline in direct export entry (i.e. entry into exporting by new firms). The reason for this difference between new export lines by incumbents versus direct export entry becomes clear from comparing equations 19 and 23. The first one, that determines the introduction of new export lines by incumbents, depends on the difference in value between export and domestic lines, which is more stable and increases during a sudden stop, while the latter depends only on the value of export lines, which faces a sharp drop in a sudden stop.

We contrast this with the data, and document in Table 9 that in fact we see a larger decline in export entry during the sudden stop among entrants (firms not active in the previous year) than among incumbent firms.

Figure 9: Sudden Stop Dynamics: Extensive Margin

a) Change in Domestic and Export Lines　　b) Components of Change in Export Lines



NOTE: Panel a) plots changes in percentage points in the shares of domestic and export lines. Panel b) plots the components of the share of export lines.

Table 9: Export Entry by Incumbent and New Firms

	(1) Share of Exported Products by New Firms	(2) Share of Exported Products by Incumbent Firms
1997	0.091	0.087
1998	0.051	0.068
1999	0.061	0.063

Note: This table reports, for each year, the share of new products introduced by firms to the export market. In column 1 we count new export products sold by entrants (firms not present in the previous period). In column 2 we count new export products sold by incumbent firms.