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Abstract

We establish important empirical regularities on firms’ expectations about aggregate and idiosyncratic components of sectoral demand using industry-level survey data for the universe of Japanese firms. Expectations about the idiosyncratic component of demand differ across sectors, and they positively co-move with those about aggregate component. To study the implications for firms’ price setting, we develop a theoretical framework that captures systematic features in firms’ expectation formation based on inference of different shocks from a common signal—a chief modelling approach to imperfect information. We show that the sensitivity of inflation to changes in demand decreases with the volatility of idiosyncratic component of demand that proxies the degree of shock heterogeneity. We use principal component analysis on Japanese sectoral-level data to estimate the degree of shock heterogeneity, and we establish that the observed increase in shock heterogeneity plays a significant role for the reduced sensitivity of inflation to movements in real activity since the late 1990s.

Keywords: Imperfect information; Shock heterogeneity; Inflation dynamics

JEL classification: E31, D82, C72

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1 Introduction

We establish important empirical regularities on firms’ expectations about aggregate and idiosyncratic components of sectoral demand using industry-level survey data for the universe of Japanese firms. Expectations about the idiosyncratic component of demand differ across sectors, and they positively co-move with those about aggregate component.\footnote{As we discuss in section 2, the dataset is the Annual Survey of Corporate Behavior compiled by the Cabinet Office of Japan for 25 sectors for the period 2003-2017.}

Motivated by these findings, we develop a theoretical model of imperfect information that provides a structural interpretation behind these findings and its implications on firms’ price settings. Our theoretical framework embeds nominal price rigidities. In the spirit of Lucas (1972), it allows sectoral demand to compound exogenous aggregate and idiosyncratic components, requiring the profit-maximizing firm to infer the effect of each different component from the commonly observed signal of sectoral demand. The model provides a parsimonious framework that links imperfect information on different components of sectoral demand to the systematic decision of optimal price changes, which determine the sensitivity of prices to changes in demand. If a change in demand comes from the aggregate component that equally applies to all firms in the economy, each firm optimally adjusts the price and complement the optimal change in the price by other firms. Thus, optimal price setting entails strategic complementarity that makes the firm’s price more responsive to changes in aggregate demand compared to when the idiosyncratic, sector-specific component generates the change in demand.\footnote{Koga et al. (2019) show that strategic complementarity in price setting is important to describe the price setting behavior of Japanese firms. Cornand and Heinemann (2018) show that with monetary policy at the zero lower bound, pricing decisions are strategic complement. More generally, Angeletos and Pavan (2007) provide a discussion on the role of strategic complementarity for the response of agents’ actions to changes in fundamentals under dispersed information.} Therefore, distinguishing the shock that changes demand is critical to the firm price-setting decision and the dynamics of inflation.

However, since the firm cannot disentangle changes in aggregate and idiosyncratic demand, it conjectures that changes in sectoral demand is partially caused by changes in idiosyncratic demand that have no effect on the price-setting decisions of other firms in the economy. This misperception induces firms to decrease the response to aggregate shocks. In addition, shock heterogeneity in demand exacerbates the misperception. The larger the degree of shock heterogeneity—represented by the ratio between the volatility of the idiosyncratic- and aggregate-component of demand—the lower the response of inflation to
changes in current demand. This is because if the relative volatility of the idiosyncratic component of demand is large, the firm conjectures that a large portion of the sectoral demand shock occurs due to idiosyncratic shock and that aggregate demand does not change. Consequently, the firm expects that the average price in the period is almost the same as that in the previous period and adjusts its prices less strongly to changes in demand. A central prediction of the theoretical framework is the negative relationship between shock heterogeneity—encapsulated by a rise in the volatility of idiosyncratic shocks—and the sensitivity of inflation to changes in economic activity.

We empirically test the prediction of the theoretical model. First, we construct the measure that captures shock heterogeneity in demand by conducting a principal component analysis to disentangle the volatility of exogenous movements in idiosyncratic demand relative to the volatility of exogenous movements in aggregate demand.\(^3\) The estimates show that shock heterogeneity—proxied by the relative size of volatility of idiosyncratic shocks to that of the aggregate shocks—has steadily increased over the sample period, with the ratio of variance of idiosyncratic demand relative to the variance of aggregate demand doubling from the mid-1970s to the late-2000s. Second, we estimate standard Phillips curve regressions that include our estimated measure of shock heterogeneity to find that the data robustly support a significant inverse relationship between shock heterogeneity and the sensitivity of prices to movements in aggregate demands. The empirical results show that the sensitivity of inflation to aggregate demand has halved since the late 1990s, coinciding with a period of substantial increase in shock heterogeneity.

The analysis is tightly linked with the literature on the role of imperfect information on firms’ price settings. It relates to studies that focus on imperfect information in models with flexible prices (Woodford, 2003; Hellwig and Venkateswaran, 2009; Mackowiak et al., 2009; Crucini et al., 2015; and Kato and Okuda, 2017) and nominal price rigidities (Fukunaga, 2007; Nimark, 2008; Angeletos and La’O, 2009a; Melosi, 2017; and L’Huillier, 2019). It also relates to studies that allow for coexistence of idiosyncratic and aggregate shocks in the presence of costly information acquisition (Veldkamp and Wolfers, 2007; and Acharya, 2017). More specifically, the analysis relates to the literature that investigates the effect of imperfect information on the Phillips curve. Mankiw and Reis (2002) and Dupor et al. (2010) develop sticky-information models to investigate the effect of informational frictions on the empirical

\(^3\)We use sector-level data for the universe of Japanese firms across 29 sectors for the period 1975-2017.
performance of the Phillips curve. Coibion and Gorodnichenko (2015) establish that information frictions are critical in reconciling an empirically-consistent formation of expectations that explain the missing disinflation between 2009 and 2011. Mackowiak and Wiederholt (2009) investigate the effect of rational inattention on the Phillips curve, and they establish a negative relationship between the relative variance of idiosyncratic shocks to aggregate shocks and the sensitivity of inflation to real activity. Different from those studies, we empirically assess the relevance of imperfect information on demand and study the interplay between shock heterogeneity and the sensitivity of inflation to changes in demand.

The remainder of the paper is organized as follows. Section 2 provides evidence on the formation of expectations from survey data. Section 3 presents the model with imperfect information about sectoral demand and lays out the formation of expectations. Section 4 introduces nominal price rigidities and investigates the relationship between shock heterogeneity and the sensitivity of inflation dynamics to real activity, and it uses sector-level data to test theoretical predictions. Section 5 concludes.

2 Evidence from Survey Data

We study the formation of expectations on aggregated and sector-specific demand using the Annual Survey of Corporate Behavior produced by the Cabinet Office of Japan for 25 sectors over the period 2003-2017. The data provide aggregate responses from surveys of

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4 While we focus on the role of imperfect information, changes in the relationship between inflation and real activity are studied in the literature from more broader perspective, as generated by the anchoring effect of inflation targets (Roberts, 2004 and L’Huillier and Zame, 2014), increasing competition in the goods market (Sbordone, 2008; Zanetti, 2009; and IMF, 2016), downward wage rigidities (Akerlof et al., 1996), structural reforms (Thomas and Zanetti, 2009, Zanetti, 2011; Cacciatore and Fiori, 2016), and changes in trend inflation (Ball and Mazumder, 2011). Mavroeidis et al. (2014) reviews the empirical literature on U.S. data. Mourougane and Ibaragi (2004), Veirman (2007), Nishizaki et al. (2014), Kaihatsu et al. (2017) and Kaihatsu and Nakajima (2015) provide evidence on the reduced sensitivity of inflation to real activity in Japan.

5 The industries included in the sample are Foods, Textiles and Apparels, Pulp and Paper, Chemicals, Pharmaceutical, Rubber Products, Glass and Ceramics Products, Iron and Steel, Nonferrous Metals, Metal Products, Machinery, Electric Appliances, Transportation Equipment, Precision Instruments, Other Products, Construction, Wholesale Trade, Retail Trade, Real Estate, Land Transportation, Warehousing and Harbor Transportation Services, Information and Communication, Electric Power and Gas, Services, and Banks. The Economic and Social Research Institute in the Cabinet Office of Japan directly surveys approximately 1,000 public-listed Japanese firms on nominal and real growth rates of the Japanese economy as well as nominal and real growth rates of demand in their respective sectors. Sectoral averages are publicly available at: http://www.esri.cao.go.jp/en/stat/ank/ank-e.html. The survey is conducted each January, and questionnaires are available at: http://www.esri.cao.go.jp/en/stat/ank/lh28ank/lh28ank_questionnaire.pdf. We proxy expectations on aggregate demand with survey data on expectations on one-year-ahead GDP growth, and we proxy expectations on sectoral demand with survey data on expectations on one-year-ahead growth
the universe of Japanese firms on expectations about one-year-ahead growth rate of sectoral and aggregate demand. Since sectoral demand compounds aggregate and idiosyncratic (i.e., sector-specific) components, we infer measures for expectations on the changes in aggregate and idiosyncratic demand as the difference between the expectations of the changes in sectoral and aggregate demands that we use to characterize systematic patterns in the formation of expectations.

Table 1 provides summary statistics from survey data. Columns (1) and (2) shows historical averages of the changes in the expectations of one-year-ahead growth rate of aggregate and idiosyncratic demand, respectively. Entries reveal large differences in the changes of expectations between the two different components of sectoral demand. Changes in the expectations of aggregate demand are broadly similar across sectors while changes in the expectations of idiosyncratic demand differ widely across sectors. Columns (3) and (4) show that both components of sectoral demand have sizeable variation over the sample period and that the average volatility of the two components is similar. Finally, columns (5) and (6) show that both components have high serial correlation and that the average persistence of aggregate component is twice as large as the persistence of the idiosyncratic component.

Figures 1 provides an illustrative example on firms’ expectations about sectoral demand partitioned between aggregate and idiosyncratic components for the chemical sector (panel (a)) and the retail sector (panel (b)). The figures show that expectations of the growth of sectoral demand (black line) widely vary over the period, becoming negative during 2008 at the time of the global financial crisis. The contribution of aggregate demand (white bar) and idiosyncratic demand (gray bar) also change over time. Movements in sectoral expectations are different across sectors, slow-moving in the retail sector and fast-moving in the chemical sector. On average, changes in the expectations of idiosyncratic demand are of similar magnitude to those of aggregate demand, consistent with the evidence in Table 1.

Next, we focus on co-movements between expectations of the different aggregate and idiosyncratic components. Table 2 in column (1) shows results from regressing the measure of firm’s expectations of the growth rate of aggregate demand on the firm’s expectations about the growth rate of idiosyncratic demand. The regression coefficient is significant, indicating

\[ \text{Table 1 about here.} \]

\[ \text{Figure 1 about here.} \]

\[ \text{Table 2 about here.} \]
a positive correlation between changes in the expectations of aggregate and idiosyncratic demand. Since historical averages of expected growth rates of idiosyncratic demand vary across sectors while those of aggregate demand are similar across sectors, panel (b) reports regression estimates that include dummy variables to control for sector-level fixed effects, and shows that results continue to hold.

[Table 2 about here.]

To summarize, the analysis establishes the following empirical regularities: (i) expectations about the idiosyncratic component of demand largely vary across sectors, (ii) expectations about both components are largely and equally volatile, and (iii) they positively co-move.

In the next section, we develop a model that replicates and interprets these findings.

3 Theoretical Framework

The model is based on Angeletos and La’O (2009b) and Mackowiak et al. (2009) and comprises monopolistic competitive firms that face a sectoral demand that additively compounds aggregate and idiosyncratic components. Firms observe sectoral demand and past realizations of aggregate and idiosyncratic components. However, they cannot observe current realizations of each different component of demand, and therefore need to infer each component to maximize revenues. The economy is populated by a representative household and a continuum of monopolistic competitive firms that produce differentiated goods indexed by $j \in [0, 1]$ in a continuum of sectors indexed by $i \in [0, 1]$. Each representative household consumes the whole income, and there is no saving in equilibrium. Time is discrete and indexed by $t$.

**Households.** Preferences of the representative household over consumption, $C_t$, and labor, $N_t$, are described by the utility function:

$$\sum_{t=0}^{\infty} \beta^t (\log C_t - N_t),$$

where $\beta \in (0, 1)$ is the discount rate. The household’s aggregate consumption, $C_t$, and consumption of goods in sector $i$, $C_t(i)$, are defined by the CES consumption aggregators:

$$C_t \equiv \left[ \int_0^1 (C_t(i) \Theta_t(i))^{\frac{n-1}{n}} \, di \right]^{\frac{n}{n-1}},$$

and

$$C_t(i) \equiv \left[ \int_0^1 (C_t(i, j))^{\frac{n-1}{n}} \, dj \right]^{\frac{n}{n-1}}.$$
where \( \eta > 1 \) is the elasticity of the substitution across sectors, \( \tilde{\eta} > 1 \) is the elasticity of the substitution across goods in the same sector, \( C_t(i, j) \) is consumption of good \( j \) in sector \( i \), and \( \Theta_t(i) \) is the sector-specific preference shocks.

**Firms.** Each firm \( j \) in sector \( i \) (we refer to is as \((i, j)\)) faces the following demand:

\[
C_t(i, j) = \Theta_t^{-1}(i) \left( \frac{P_t(i, j)}{P_t(i)} \right)^{-\tilde{\eta}} \left( \frac{P_t(i)}{P_t} \right)^{-\eta} C_t,
\]

where \( P_t(i) \equiv \left[ \int_0^1 P_t^{1-\tilde{\eta}}(i, j) dj \right] \frac{1}{1-\eta} \) is the sector \( i \) price index, \( P_t \equiv \left[ \int_0^1 P_t^{1-\eta}(i) \Theta_t^{\eta-1}(i) di \right] \frac{1}{1-\eta} \) is the aggregate price index, and the idiosyncratic preference shock, \( \Theta_t(i) \), acts as an exogenous demand shifter.\(^7\)

Each firm \((i, j)\) manufactures a single good \( Y(i, j) \), according to the production technology:

\[
Y_t(i, j) = AL_t(i, j),
\]

where \( A \) is aggregate productivity and \( \epsilon \in (0, 1) \) determines the degree of diminishing marginal returns in production.

**Market Clearing.** In a symmetric equilibrium, market clearing implies \( Y_t(i, j) = C_t(i, j) \) for each firm \((i, j)\), and \( Y_t = C_t \) for the economy. Aggregate nominal demand, \( Q_t \), is given by the following cash-in-advance constraint:

\[
Q_t = P_tC_t.
\]

In the rest of the analysis, we use lower-case variables to indicate logarithms of the corresponding upper-case variables (i.e., \( x_t \equiv \log X_t \)).

**Optimal Price Setting.** We first derive the optimal price setting rule with flexible prices, assuming perfect information about current nominal shocks. During each period \( t \), the firm \((i, j)\) sets the optimal price as a mark-up over the marginal cost:

\[
p_t(i, j) = \mu + mc_t(i, j),
\]

where \( \mu \equiv \tilde{\eta}/(\eta - 1) > 0 \) is the mark-up and \( mc_t(i, j) \) is the nominal marginal cost faced by firm \((i, j)\). The nominal marginal cost is the difference between the nominal wage, \( w_t \), and

\(^7\)See Appendix A for the derivation of the demand function for each firm \((i, j)\) and price indexes.
the marginal product of labor:

\[ mc_t(i, j) = w_t + (1 - \epsilon) l_t(i, j) - a - \log(\epsilon). \] (4)

Using the production technology in equation (2), we express labor input as: 

\[ l_t(i, j) = \frac{y_t(i, j) - a}{\epsilon}, \]

which we use in equation (4) to rewrite the nominal marginal cost as:

\[ mc_t(i, j) = w_t + \frac{1 - \epsilon}{\epsilon} y_t(i, j) - \frac{1}{\epsilon} a - \log(\epsilon). \]

The optimal labor supply condition for the representative household is:

\[ w_t - p_t = c_t, \] (5)

and the linearized-version of consumer demand in equation (1) is:

\[ c_t(i, j) = -\tilde{\eta}(p_t(i, j) - p_t(i)) - \eta(p_t(i) - p_t) + c_t + (\eta - 1) \theta_t(i), \] (6)

where the idiosyncratic preference shock, \( \theta_t(i) \), follows the AR(1) process:

\[ \theta_t(i) = \rho_v \theta_{t-1}(i) + \tilde{\epsilon}_t(i), \] (7)

and \( \tilde{\epsilon}_t(i) \sim \mathcal{N}(0, (1 - \epsilon)^{-2} (\eta - 1)^{-2} \tau^2) \).

We derive the optimal price-setting rule for firm \((i, j)\) by using equations (5), (6), the equilibrium conditions, \( y_t(i, j) = c_t(i, j) \) and \( y_t = c_t \), and the cash-in-advance constraint, \( y_t = q_t - p_t \), which yields:

\[ p_t(i, j) = r_1 p_t(i) + r_2 p_t + (1 - r_1 - r_2) x_t(i) + \xi, \] (8)

where

\[ x_t(i) = q_t + v_t(i), \] (9)

\[ v_t(i) = (1 - \epsilon)(\eta - 1) \theta_t(i), \] (10)

\[ \xi = \frac{\epsilon}{\epsilon + \tilde{\eta}(1 - \epsilon)}(\mu - \frac{1}{\epsilon} a - \log(\epsilon)), \] (11)

\[ r_1 = \frac{(\tilde{\eta} - \eta)(1 - \epsilon)}{\epsilon + \tilde{\eta}(1 - \epsilon)}, \] (12)

\[ r_2 = \frac{(\eta - 1)(1 - \epsilon)}{\epsilon + \tilde{\eta}(1 - \epsilon)}, \] (13)

\[ ^8 \text{Appendix B shows the derivation of the price setting rule.} \]
and \( p_t = \int_0^1 p_t(i) \, di \). Equation (8) shows that the optimal pricing rule for firm \((i, j)\) is a weighted average of sectoral prices \((p_t(i))\), aggregate prices \((p_t)\), and sectoral demand \((x_t(i))\). The weight between these terms is determined by parameters \( r_1 \) and \( r_2 \) that reflect the degree of strategic complementarity among firms in the same sector and across sectors, respectively.

Equation (9) shows that sectoral demand \((x_t(i))\) additively combines the aggregate \((q_t)\) and idiosyncratic components \((v_t(i))\), and equation (10) shows that idiosyncratic demand depends on the idiosyncratic preference shock \( \theta_t(i) \). The parameter \( \xi \), defined in equation (11), is a linear transformation of the level of aggregate productivity, \( a \), and without loss of generality, we normalize aggregate productivity such that \( \xi = 0 \).

Since firms in the same sector face same marginal costs and access the same information, \( p_t(i) = p_t(i, j) = p_t(i, j') \) for \( j \neq j' \). Thus, equation (8) reduces to:

\[
    p_t(i) = rp_t + (1 - r) x_t(i),
\]

where

\[
    r \equiv \frac{r_2}{1 - r_1} = \frac{(\eta - 1)(1 - \epsilon)}{\epsilon + \eta(1 - \epsilon)}.
\]

Equation (14) shows that the optimal pricing rule for firm \((i, j)\) is a weighted average of aggregate prices \((p_t)\) and sectoral demand \((x_t(i))\). The weights for average prices and sectoral demand are determined by the parameter \( r \), which similarly to equation (8) reflects the degree of strategic complementarity between firms in different sectors.\(^{10}\)

**Information Structure and Shocks.** Next, we describe the change in the environment under imperfect information. Each firm in sector \( i \) observes sectoral demand that changes in response to aggregate demand and idiosyncratic demand, according to \( x_t(i) = q_t + v_t(i) \), as described by equation (9), without observing the different aggregate and idiosyncratic components. Aggregate demand follows stochastic process:

\[
    q_t = q_{t-1} + u_t,
\]

\(^9\)Appendix C shows the derivation of the index of aggregate prices.

\(^{10}\)Equation (14) shows that if production technology converges to constant returns (i.e., \( \epsilon \to 1 \)), average prices become less important in the determination of the price for firm \( i \) (i.e., \( r \to 0 \)) since the marginal cost converges to the aggregate nominal wage across firms (i.e., \( mc_t(i) \to w_t \)) and heterogeneity in the firms’ prices decreases. The magnitude of the idiosyncratic shock decreases (i.e., \( v_t(i) \to 0 \)) as the production technology converges to constant returns (i.e., \( \epsilon \to 1 \)). As a result, in the limiting case of a linear production technology (i.e., \( \epsilon = 1 \)), the optimal pricing rule is \( p_t(i) = q_t + \xi \).
and $u_t$ follows the AR(1) process:

$$u_t = \rho_u u_{t-1} + \epsilon_t,$$

where $0 \leq \rho_u < 1$, and $\epsilon_t \sim \mathcal{N}(0, \sigma^2)$. The idiosyncratic component of demand ($v_t(i)$) depends on the idiosyncratic preference shock ($\theta_t(i)$) described in equation (7). By normalizing $v_t(i) = (1 - \epsilon)(\eta - 1)\theta_t(i)$, the idiosyncratic demand follows the AR(1) process:

$$v_t(i) = \rho_v v_{t-1}(i) + \epsilon_t(i),$$

where $0 \leq \rho_v < 1$, and $\epsilon_t(i) \sim \mathcal{N}(0, \tau^2)$.

The information set of each firm in sector $i$ in period $t$ comprises knowledge on present realization of sectoral demand and observed past aggregate and idiosyncratic components of demands (i.e., $\mathcal{H}_t(i) \equiv \{\{x_s(i)\}_{s=0}^t, \{q_s, u_s, v_s(i), \theta_s(i), e_s, \epsilon_s, \theta_s(i), \eta_s(i)\}_{s=0}^{t-1}\}$). To simplify the notation, we denote $\mathbb{E}_t \equiv \mathbb{E}[\mathbf{H}_t(i)]$. The information structure requires inference on the different and unobserved components of aggregate ($q_t$) and idiosyncratic demand ($v_t(i)$) by using information from the common signal of sectoral demand (i.e., $x_t(i) = q_t + v_t(i)$) and knowledge about past realizations of aggregate and idiosyncratic components of demand, i.e., $q_t \sim \mathcal{N}(q_{t-1} + \rho_u u_{t-1}, \sigma^2)$ and $v_t(i) \sim \mathcal{N}(\rho_v v_{t-1}(i), \tau^2)$, respectively.

**Mapping the model in the data.** The model characterizes the expectations on the level of sectoral demand whereas the data refer to the expectations on the changes of sectoral demand. To map the model into empirical measurements, we derive changes in sectoral demand by taking the first difference of $\Delta x_t(i) = \Delta q_t + \Delta v_t(i)$. To simplify the notation, we label $\Delta x_t(i) = \Delta x_t(i)$, $\Delta v_t(i) = \Delta v_t(i)$, and from equation (15) $u_t = \Delta q_t$. Using equations (16)-(17), the change in sectoral demand, $\Delta x_t(i)$, depends on the change in aggregate demand, $u_t$, and the change in idiosyncratic demand, $\Delta v_t(i)$:

$$\Delta x_t(i) = u_t + \Delta v_t(i).$$

**The formation of expectations.** We use the model to study the link between imperfect information on the different components of sectoral demand and co-movements in the expectations about these components.

Using equation (18), we derive expectations at time $t$ about sectoral demand in $k$-period ahead:

$$\mathbb{E}_t \left[ \sum_{h=1}^{k} \Delta x_{t+h}(i) \right] = \mathbb{E}_t \left[ \sum_{h=1}^{k} u_{t+h} \right] + \mathbb{E}_t \left[ \sum_{h=1}^{k} \Delta v_{t+h}(i) \right].$$
We use equation (19) to investigate co-movements in the expectations of future changes in aggregate and idiosyncratic demand. If firms observe different realizations of aggregate and idiosyncratic components of sectoral demand, such that \( \mathbb{E}_t[u_t] = u_t \) and \( \mathbb{E}_t[\tilde{v}_t] = \tilde{v}_t \), the expectations on the different components of sectoral demand are independent from each other and therefore uncorrelated. We exploit this property to link imperfect information on the current realizations of the different components of sectoral demand with co-movements in the expectations of the different components, as outlined in the next proposition. To simplify notation, we denote the unconditional covariance operator with \( \mathbb{C} \).

**Proposition 1** If sectoral demand compounds singularly unobservable aggregate and idiosyncratic components (i.e., \( \bar{x}_t(i) = u_t + \tilde{v}_t(i) \)), the following relationship holds:

\[
\mathbb{C}(\mathbb{E}_t[u_t], \mathbb{E}_t[\tilde{v}_t]) > 0 \Rightarrow \mathbb{C}\left(\mathbb{E}_t\left[\sum_{h=1}^{k} u_{t+h}\right], \mathbb{E}_t\left[\sum_{h=1}^{k} \tilde{v}_{t+h}(i)\right]\right) > 0
\]

**Proof**: See Appendix E.1. \( \square \)

Proposition 1 shows that co-movements in the expectations on current changes in aggregate and idiosyncratic components of sectoral demands are critical for the correlation between expectations on future changes of these components. The specific information structure in our model implies a positive correlation between current realizations of aggregate and idiosyncratic components, as a result of imperfect information about the realization of each single component. Since the firm observes a compounded signal of the two components, they optimally attribute movements in the signal to both components, and thus expectations about each different component of demand positively co-move.

The findings in Proposition 1 enable us to interpret the positive empirical co-movements in expectations on aggregate and idiosyncratic demand from survey data outlined in section 2. If we use the model to estimate the regression equation in Table 2, it yields:

\[
\mathbb{E}_t\left[\sum_{h=1}^{k} u_{t+h}\right] = \beta_0 + \beta_1 \mathbb{E}_t\left[\sum_{h=1}^{k} \tilde{v}_{t+h}(i)\right], \quad (20)
\]

where \( \beta_0 \) is the constant term and \( \beta_1 \) is the coefficient that establishes the correlation between changes in aggregate and idiosyncratic demand. The value for \( \beta_1 \) is equal to:

\[
\beta_1 = \frac{\mathbb{C}\left(\mathbb{E}_t\left[\sum_{h=1}^{k} u_{t+h}\right], \mathbb{E}_t\left[\sum_{h=1}^{k} \tilde{v}_{t+h}(i)\right]\right)}{\mathbb{V}\left(\mathbb{E}_t\left[\sum_{h=1}^{k} \tilde{v}_{t+h}(i)\right]\right)}, \quad (21)
\]
Equation (21) shows that the value for the correlation coefficient $\beta_1$ depends on the correlation of expectations about future realizations of aggregate and idiosyncratic demand, which in turn is determined by the expectations on these components in the *current* period. Proposition 1 shows that imperfect information implies positive correlation in the expectations on aggregate and idiosyncratic demand at period $t$, which generates a positive correlation in the expectations on future values of these components. The regression coefficient $\beta_1$ is therefore positive, consistent with the evidence in survey data.

### 4 Shock Heterogeneity and Inflation Dynamics

This section investigates role of shock heterogeneity, represented by the relative volatility of the idiosyncratic component relative to aggregate component, for the sensitivity of inflation to sectoral demand.

To link demand to prices, we enrich the model with nominal price rigidities. Under nominal rigidities, the optimal price-setting rule in equation (14) continues to hold, but with expectations formed under imperfect information such that:

$$p_t(i) = rE_t[p_t] + (1-r)E_t[q_t + v_t(i)] = rE_t[p_t] + (1-r)E_t[x_t(i)],$$

where $r \equiv (\eta - 1)(1-\epsilon)/ (\epsilon + \eta (1-\epsilon))$. Equation (22) shows that imperfect information plays a critical role for the formation of expectations and therefore is important for optimal price setting in each sector $i$.

We embed nominal price rigidities, as in Calvo (1983), and we assume that a firm retains the same price with exogenous probability $\theta \in (0,1)$ and otherwise changes prices optimally. The optimal price for firms in sector $i$, denoted as $p_t^*(i)$, depends on expectations formed at time $t$ on present and future prices, as described by the pricing rule:

$$p_t^*(i) = (1 - \beta \theta) \sum_{j=0}^{\infty} (\beta \theta)^j E_t[p_{t+j}(i)]$$

$$= (1 - \beta \theta) \sum_{j=0}^{\infty} (\beta \theta)^j [rE_t[p_{t+j}] + (1-r)E_t[x_{t+j}(i)]] ,$$

where the second equation is derived by substituting the optimal pricing rule in equation (22). Equation (23) shows that each firm in sector $i$ sets prices as a weighted average of the firm’s expectations about current and expected future prices whose expectations depend on the information set at time $t$. 

11
The Equilibrium Average Price. Equation (23) provides the equilibrium average price once we derive expectations for prices and sectoral demand. The next proposition characterizes the equilibrium average price.

Proposition 2 (Analytical solution to the equilibrium average price).

The equilibrium average price is given by

\[ p_t^* = [\theta + (1 - \theta)a_1] p_{t-1} + (1 - \theta) a_2 q_t + (1 - \theta) a_3 q_{t-1} + (1 - \theta) a_4 u_{t-1}, \tag{24} \]

where \((a_1, a_2, a_3, a_4, a_5)\) are determined from the following conditions.

\[
\begin{align*}
    a_1 &= \left[(1 - \beta \theta) r + \beta \theta a_1\right] \left[\theta + (1 - \theta) a_1\right], \\
    a_2 &= (1 - \beta \theta) (1 - r) \\
    &\quad + \left[(1 - \beta \theta) r + \beta \theta a_1\right] (1 - \theta) + \beta \theta \left[\rho_u \frac{\sigma^2}{\sigma^2 + \tau^2} + \rho_v \frac{\tau^2}{\sigma^2 + \tau^2}\right] a_2 \\
    &\quad + \beta \theta \frac{\sigma^2}{\sigma^2 + \tau^2} a_3 + \beta \theta \frac{\sigma^2}{\sigma^2 + \tau^2} a_4 + \beta \theta \frac{\tau^2}{\sigma^2 + \tau^2} a_5, \\
    a_3 &= \left[(1 - \beta \theta) r + \beta \theta a_1\right] (1 - \theta) + \beta \theta \left[\rho_u \frac{\sigma^2}{\sigma^2 + \tau^2} + \rho_v \frac{\tau^2}{\sigma^2 + \tau^2}\right] a_2 \\
    &\quad + \beta \theta \frac{\tau^2}{\sigma^2 + \tau^2} - \left[(1 - \beta \theta) r + \beta \theta a_1\right] (1 - \theta) a_3 \\
    &\quad - \frac{\sigma^2}{\sigma^2 + \tau^2} \beta \theta a_4 - \frac{\tau^2}{\sigma^2 + \tau^2} \beta \theta a_5, \\
    a_4 &= \left[(1 - \beta \theta) r + \beta \theta a_1\right] (1 - \theta) + \beta \theta + \beta \theta \rho_u - \beta \theta \rho_v \left[\rho_u \frac{\tau^2}{\sigma^2 + \tau^2} a_2 + \beta \theta \frac{\tau^2}{\sigma^2 + \tau^2} \rho_u a_3 \right] \\
    &\quad + \left[(1 - \beta \theta) r + \beta \theta a_1\right] (1 - \theta) + \beta \theta \left[\rho_u \frac{\sigma^2}{\sigma^2 + \tau^2} + \rho_v \frac{\tau^2}{\sigma^2 + \tau^2}\right] a_4 - \beta \theta \frac{\tau^2}{\sigma^2 + \tau^2} \rho_u a_5, \\
    a_5 &= -\left[(1 - \beta \theta) r + \beta \theta a_1\right] (1 - \theta) + \beta \theta + \beta \theta \rho_u - \beta \theta \rho_v \left[\rho_u \frac{\sigma^2}{\sigma^2 + \tau^2} a_2 + \beta \theta \frac{\sigma^2}{\sigma^2 + \tau^2} \rho_u a_3 \right] \\
    &\quad - \beta \theta \frac{\sigma^2}{\sigma^2 + \tau^2} \rho_u a_3 - \beta \theta \frac{\sigma^2}{\sigma^2 + \tau^2} \rho_v a_4 + \beta \theta \frac{\sigma^2}{\sigma^2 + \tau^2} \rho_v a_5.
\end{align*}
\]

Proof: See Appendix E.2. □

Equation (24) shows that the equilibrium average price depends on the equilibrium price in the past period \((p_{t-1})\) and the sequence of present and past aggregate demand.
Important to our subsequent analysis, the proposition indicates that the degree of shock heterogeneity, encapsulated by the relative volatility of idiosyncratic shocks and described by the idiosyncratic-to-aggregate-shock ratio \((\tau/\sigma)\), plays an important role for the sensitivity of aggregate price to present and past aggregate demands. In the next subsection, we discuss and numerically assess the relationship between inflation and shock heterogeneity.

Using the average price defined in equation (24), we derive the gross inflation rate \((\pi_t \equiv p_t - p_{t-1})\) that describes the change in the average price from period \(t\) to period \(t-1\):

\[
\pi_t = \left[ \theta + (1-\theta)a_1 \right] \pi_{t-1} + (1-\theta) a_2 u_t + (1-\theta) (a_3 + a_4) u_{t-1} - (1-\theta) a_4 u_{t-2}
\]

(25)

where \(a_1 \equiv \theta + (1-\theta)a_1\), \(a_2 \equiv (1-\theta)a_2\), \(a_3 \equiv (1-\theta)(a_3 + a_4)\), and \(a_4 \equiv -(1-\theta)a_4\). Equation (25) is the closed-form solution for inflation under imperfect information. This formulation is similar to Angeletos and La'O (2009a) since the inflation rate \((\pi_t)\) depends on past inflation \((\pi_{t-1})\) and reacts to current and past changes in demand \((u_t, u_{t-1})\), respectively since demand in the past period \(t-1\) is fully revealed in the current period \(t\).

Note that \(a_1 = a_4 = 0\) holds if \(\theta = 0\) since nominal price rigidity \(\theta > 0\) generates persistence (i.e., dependence on \(\pi_{t-1}\)). Note also that \(a_3 = a_3 = a_4 = 0\) holds if aggregate price is perfectly known by firms as \(\tau^2 = 0\) and there exists no persistence in aggregate demand fluctuations \((\rho_u = 0)\). Under perfect information and flexible prices, the current inflation rate depends on current changes in aggregate demand.\(^{11}\)

### 4.1 Quantitative Assessment

Using equation (25), we investigate the effect of shock heterogeneity represented by the ratio of the volatility of the idiosyncratic to aggregate shock \((\tau/\sigma)\) and the degree of nominal rigidities represented by the parameter \(\theta\) on the coefficients \(a_1\) and \(a_2\), which determine the response of inflation.

**Sensitivity of Inflation to Changes in Demand.** To study the properties of the system, we calibrate the model using standard parameter values. We set \(\eta = 2\), \(\epsilon = 2/3\), \(r =\)

\(^{11}\)These findings resemble those in Angeletos and La'O (2009a), but they differ across two important dimensions. First, the coefficients \((a_2, a_3, a_4)\) depend on the volatility of idiosyncratic shocks \((\tau^2)\), and second, inflation depends on the changes in demand two period before \(u_{t-2}\) since aggregate shocks have positive persistence \((\rho_u > 0)\).
For the sake of completeness, let us consider the expression:

\[
\frac{(\eta - 1)(1 - \epsilon)}{[\epsilon + \eta(1 - \epsilon)]} = 0.5, \quad \text{and} \quad \beta = 0.99.
\]

While we estimate the degree of shock heterogeneity \((\tau/\sigma)\) in the next section to investigate the properties of the model, we allow the ratio \(\tau/\sigma\) to cover a wide range of values \([0, 2]\). Similarly, we allow the degree of nominal price rigidity \((\theta)\) to cover the whole range of admissible values \([0, 1]\). The parameters for the persistence of aggregate and idiosyncratic shocks are set equal to \(\rho_u = 0.35\) and \(\rho_v = 0.15\), respectively, to replicate the average persistence of expectations in aggregate and idiosyncratic components of demand in survey data.

Panel (a) in Figure 2 shows the sensitivity of parameters \(\alpha_1\) and \(\alpha_2\) in the closed-form solution for inflation in equation (25) to the degree of nominal price rigidity \((\theta)\). The increase in nominal price rigidities generates a rise in the coefficient \(\alpha_1\) since a low frequency of price changes increases the importance of past inflation in the determination of current inflation. The increase in the degree of nominal price rigidity also generates a decrease in the coefficient \(\alpha_2\) since the sensitivity of individual prices to the current aggregate shock is lowered by less sensitive average prices, and the sensitivity of average prices to the same shock is directly dampened by the increase in nominal price rigidity \((\theta)\).

Panel (b) in the figure shows that the coefficient \(\alpha_2\) depends on the relative volatility of idiosyncratic shocks (i.e., \(\tau/\sigma\)). Individual prices become less sensitive to the current aggregate shock \((\alpha_2\) decreases). Consequently, the average price becomes less sensitive to aggregate shocks. Strategic complementarity in the optimal price setting (encapsulated by \(r > 0\) in equation (14)) induces the firm to largely adjust prices if it perceives the change in sectoral demand is from the aggregate component, which is common across firms in the economy. A widening in the volatility of the idiosyncratic component of demand relative to that of the aggregate component of demand, decreases the response of prices to changes in sectoral demand. This mechanism explains the negative relationship between \(\tau/\sigma\) and \(\alpha_2\).

Response of Inflation to Shocks. How does the degree of shock heterogeneity influence the sensitivity of inflation to changes in demand? To address this important question, we simulate the model and determine the response of inflation to a one-period, positive aggregate demand shock. Figure 3 shows that the larger the degree of shock heterogeneity, as represented by the ratio \(\tau/\sigma\), the lower the response of inflation to changes in current demand. Since the firm cannot disentangle changes in aggregate and idiosyncratic demand, it
conjectures that changes in sectoral demand are partially caused by changes in idiosyncratic demand that have no effect on the price-setting decisions of other firms in the economy. This misperception induces firms to decrease the response to aggregate shocks. If the ratio of $\tau/\sigma$ is large, the firm conjectures that a large portion of the sectoral demand shock occurs due to idiosyncratic shock and that aggregate demand does not change. Consequently, the firm expects that the average price in the period is almost the same as that in the previous period and adjusts its prices less strongly to changes in demand.

4.2 Empirical Assessment

This section first applies principal component analysis to estimate the degree of shock heterogeneity. It then applies standard regression analysis to test the relevance of shock heterogeneity to the sensitivity of inflation to changes in aggregate demand.

Monte Carlo Experiment. To ensure regression analysis is powerful in detecting the effect of shock heterogeneity on the sensitivity of inflation to real activity, we conduct a Monte Carlo experiment. We use the theoretical model as the data-generating process and feed the system with aggregate shocks, $u_t$, to generate data series for inflation, $\pi_t$, for 1,000,000 periods. We allow for different degrees of information heterogeneity, as represented by the ratio, $\tau/\sigma$, within the range of values $[0, 2]$ and for degrees of nominal price rigidities, represented by the parameter $\theta$ in the wide range of values $\{0.2, 0.4, 0.6, 0.8\}$. To make results consistent with widely used specifications of the Phillips curve, we estimate the slope coefficient that captures the sensitivity of prices to real activity for two representative versions of the Phillips curve. First, a New Keynesian Phillips curve that features forward-looking expectations on inflation and second, a hybrid Phillips curve with backward- and forward-looking expectations on inflation:

$$\pi_t = \beta E[\pi_{t+1}|H_t(i)] + \kappa \bar{y}_t,$$

$$\pi_t = (1 - \gamma) E[\pi_{t+1}|H_t(i)] + \kappa \bar{y}_t + \gamma \pi_{t-1},$$

where the proxy of output gap $\bar{y}_t$ is defined as cumulative changes in output from three period before $\bar{y}_t \equiv y_t - y_{t-3}$. In our model, $Y_t = Q_t/P_t$, and thus $\bar{y}_t = (y_t - y_{t-1}) + (y_{t-1} - y_{t-2}) + (y_{t-2} - y_{t-3}) = (q_t - q_{t-1}) + (q_{t-1} - q_{t-2}) + (q_{t-2} - q_{t-3}) - (p_t - p_{t-1}) - (p_{t-1} - p_{t-2}) - (p_{t-2} -$
\[ p_{t-3} = \Sigma_{j=0}^{2} (u_{t-j} - \pi_{t-j}) \] Panels (a) and (b) in Figure 4 show estimates for the coefficient \( \kappa \) in the New Keynesian and hybrid Phillips curve, respectively, for values of \( \tau/\sigma \) within the range [0, 1] (on the x-axes) and different degrees of nominal price rigidity (\( \theta \), different lines). For both specifications, the slope coefficient \( \kappa \) is monotonically decreasing in \( \theta \) and \( \tau/\sigma \), indicating that the empirical estimation correctly attributes the increase in information heterogeneity to a reduction in the sensitivity of inflation to real activity, irrespective of the degree of nominal price rigidity, as predicted by the theoretical model.

[Figure 4 about here.]

**Estimation of Shock Heterogeneity.** We use the Financial Statements Statistics of Corporations by Industry, compiled by the Ministry of Finance of Japan, which provides quarterly data on sector-level sales of Japanese firms.\(^{13}\) The data cover the period 1975:Q3-2018:Q3 for 29 major sectors in the economy. We proxy aggregate shocks by the principal component of movements in sales growth across sectors. We estimate changes in aggregate sales, \( u_t \), as the principal (first) component of \( \tilde{x}_t(i) \) across sectors, \( i \in \{1, 2, ..., 29\} \), by calculating it as \( u_t = \Sigma_{i=1}^{29} \Lambda_i \tilde{x}_t(i) \), where \( \Lambda_i \) is the loading factor of \( \tilde{x}_t(i) \).\(^{14}\) We proxy changes in idiosyncratic demand, \( \tilde{v}_t(i) \), by subtracting the estimated principal component from changes in sectoral demand:\(^{15}\) \( \tilde{x}_t(i) - (\Sigma_{i=1}^{29} \Lambda_i)^{-1} u_t = \tilde{x}_t(i) - (\Sigma_{i=1}^{29} \Lambda_i)^{-1} \Sigma_{i=1}^{29} \Lambda_i \tilde{x}_t(i) \), where the term \((\Sigma_{i=1}^{29} \Lambda_i)^{-1}\) normalizes \( u_t \).\(^{16}\)

We proxy the variance of aggregate fluctuations, \( \frac{1}{1-\rho_{\pi}^2} \sigma_t^2 \), with the average of the square

---

\(^{12}\)In the estimation, we set \( \beta = 0.99 \) and estimate parameters \( \gamma \) and \( \kappa \) using GMM with lagged inflation. Although not the main focus of this study, \( \gamma \) changes only slightly along with \( \tau/\sigma \) and \( \theta \).


\(^{14}\)The proportion of the variance of the first component is around 19%, which is considerably larger than the variance of the second component (7%), suggesting that the second principal component plays a limited role in aggregate shocks.

\(^{15}\)To ensure results are robust to alternative normalizations, we implement alternative specifications. First, we define \( u_t = \Sigma_{i=1}^{29} \Lambda_i \tilde{x}_t(i) \) and \( \tilde{x}_t(i) - u_t \), and second, we define \( u_t = (\Sigma_{i=1}^{29} \Lambda_i)^{-1} \Sigma_{i=1}^{29} \Lambda_i \tilde{x}_t(i) \) and \( \tilde{x}_t(i) - u_t \). Results remain unchanged across different normalization assumptions.

\(^{16}\)Since the proxy for aggregate shock is \( u_t = \Sigma_{i=1}^{29} \Lambda_i \tilde{x}_t(i) \) and the sectoral shock is \( \tilde{x}_t(i) \), the scale of aggregate shocks \( \Sigma_{i=1}^{29} \Lambda_i \) may differ from the scale of sectoral shocks. Estimation results reveal that \( \Sigma_{i=1}^{29} \Lambda_i \approx 4.7 \), which we use to normalize \( u_t \).
of the extracted principle component for alternative moving windows of size $2k + 1$:

$$
\frac{1}{1 - \rho_u^2} \sigma_t^2 = \frac{1}{2k + 1} \sum_{s=-k}^{k} \left( \Sigma_{i=1}^{29} \Lambda_i \bar{x}_{t+s}(i) \right)^2.
$$

(26)

Similarly, we proxy the variance of the idiosyncratic fluctuations, $\frac{1}{1 - \rho_v^2} \tau_t^2$, with the average of the square of the proxy of idiosyncratic demand for alternative moving windows of size $2k + 1$:

$$
\frac{1}{1 - \rho_v^2} \tau_t^2 = \frac{1}{2k + 1} \sum_{s=-k}^{k} \sum_{i=1}^{29} \left[ \bar{x}_{t+s}(i) - \left( \Sigma_{i=1}^{29} \Lambda_i \right)^{-1} \Sigma_{i=1}^{29} \Lambda_i \bar{x}_{t+s}(i) \right]^2.
$$

(27)

To ensure robustness of results, we compute the variance of each of the shocks in equations (26) and (27), using four alternative time windows: two-years ($k = 4$), three-years ($k = 6$), five-years ($k = 10$), and ten-years ($k = 20$). We exclude the upper and lower 10% of the samples as outliers. Because the dataset shows that persistence of the aggregate and idiosyncratic fluctuations in each sector does not have time trends, we assume constant values for the parameters $\rho_u$ and $\rho_v$. The variance of the aggregate component of demand, $\frac{1}{1 - \rho_u^2} \sigma_t^2$, and the variance of the idiosyncratic component of demand, $\frac{1}{1 - \rho_v^2} \tau_t^2$, are monotonic with respect to the aggregate shocks $\sigma_t$ and $\tau_t$. By setting to zero the constant terms in the variances (i.e., $1/(1 - \rho_u^2)$ and $1/(1 - \rho_v^2)$), we measure shock heterogeneity as the ratio of the square root of the estimate of the variance of idiosyncratic shocks ($\tau_t^2$) to that of aggregate shocks for each period ($\sigma_t^2$). In our specification, the level of $\tau_t / \sigma_t$ does not indicate the absolute level of shock heterogeneity, but it provides a good proxy for the relative variation in shock heterogeneity.

Panel (a) in Figure 5 shows the estimated series for shock heterogeneity, defined as the ratio of the variance of idiosyncratic shocks to the variance of aggregate shocks ($\tau_t / \sigma_t$), for the alternative time windows. Entries show that the degree of information heterogeneity has steadily increased throughout the sample period, with the ratio $\tau_t / \sigma_t$ rising from a value of 2 in the early 1980s to 4 in the mid-2000s, subsequently reaching a value of approximately 3 after 2010 in the 10-year window. Shorter time windows show similar dynamics, despite increasing volatility. Overall, the analysis detects a robust increase in shock heterogeneity in the post-2008 period.\(^{17}\)

\[^{17}\]Movements in $\tau_t / \sigma_t$ are primarily driven by changes in $\tau_t$ since the value for $\sigma_t$ remains broadly stable across the sample period, except during the period of the global financial crisis (2007:4Q to 2010:1Q). Appendix D shows that the aggregate shock series extracted from industry-level data are consistent with measures of aggregate shocks as proxied by the output gap.
Estimation of the Phillips Curve. In this section, we use the proxy for information heterogeneity to assess the empirical importance of shock heterogeneity for the reduced sensitivity of inflation to changes in demand over time.

To implement the estimation of the Phillips curve, we use insights from the theoretical model in equation (25) that links information frictions to inflation. We regress current inflation on past inflation ($\pi_{t-1}$), changes in current aggregate demand ($u_t$), and an interaction term between changes in current aggregate demand and the degree of shock heterogeneity ($u_t \times \tau_t/\sigma_t$) that captures the differential effect of shock heterogeneity for the effect of aggregate demand on current inflation. Following the insights from the theoretical model, we include aggregate demand with two lags.\(^{18}\) Table 3 shows the estimates for the Phillips curve with measures of shock heterogeneity based on time windows of two years (column (1)), three years (columns (2)), five years (column (3)) and ten years (column (4)), respectively. All entries show that current inflation is positively correlated with past inflation and current demand. This finding is in line with the fundamental prediction of the Phillips curve. The interaction term is negative, implying that a rise in shock heterogeneity reduces the positive correlation between inflation and aggregate demand, in accordance with the results of the analysis, and shows that the raise in shock heterogeneity plays an empirically significant part in the reduced sensitivity of inflation to real activity.

The theoretical analysis in section 4.1 shows that the degree of nominal price rigidity is positively related to the flattening of the Phillips curve. To ensure empirical results are not driven by the reduced degree of nominal price rigidity over the sample period, we control the estimation for the degree of nominal price rigidity by using a dummy variable equal to 1 for the period 2000-2018 when nominal price rigidities decreased (see evidence in Sudo et al. (2014) and Kurachi et al. (2016)). We also enrich the estimation of the Phillips curve with two additional interaction terms. The first term interacts the dummy variable for nominal price rigidities with past inflation ($\pi_{t-1} \times dummy$) to capture the interplay between the degree of nominal price rigidity and the effect of past inflation on current inflation. The second term interacts the dummy variable for nominal price rigidities with current aggregate demand ($u_t \times dummy$) to capture the interplay between nominal price rigidities and current aggregate demand. Table 4 reports the results. Columns (1) to (4) show that the coefficient for the interaction term of past inflation with the dummy variable ($\pi_{t-1} \times dummy$) is

\(^{18}\)We also add cross-terms between shock heterogeneity and first-to-second lags of aggregate shocks.
negative, indicating that the positive correlation between current inflation and past inflation decreases with a decline in nominal price rigidities, in line with the predictions of our model outlined in section 4.1. The estimates for the interaction term of changes in demand with the dummy variable \((u_t \times \text{dummy})\) are either close to zero and insignificant (columns (1)-(3)) or positive (columns (2)-(4)), showing that the relationship between inflation and changes in aggregate demand remains broadly unchanged across periods with different degrees of nominal price rigidity. This finding corroborates the empirical evidence in Sudo et al. (2014) and Kurachi et al. (2016), which shows a consistent rise in the frequency of price adjustment across Japanese firms since the early 2000s. Important for our analysis, the interaction term between aggregate demand and the degree of nominal price rigidity \((u_t \times \tau_t/\sigma_t)\) remains negative and retains the same magnitude of the benchmark estimates in Table 2, showing that the effect of the degree of shock heterogeneity is broadly similar across periods with different degrees of nominal price rigidity.\(^{19}\)

[Table 3 and 4 about here.]

5 Conclusion

Sector-level surveys for the universe of Japanese firms reveal important empirical regularities about expectations on aggregate and idiosyncratic components of demand. Expectations about the idiosyncratic component of demand largely vary across sectors, and they positively co-move.

We find that a model with nominal price rigidities and imperfect information based on inference of different unobservable components from a common signal—a canonical approach to incorporate imperfect information—realistically encapsulates firms’ expectations about demand. A central prediction of the model is the negative relationship between the degree of shock heterogeneity—proxied by the ratio of the volatility of idiosyncratic shocks to the volatility of aggregate shocks—and the sensitivity of inflation to real activity. We apply principal component analysis to extract the degree of shock heterogeneity from sector-level sales data for Japanese firms across 29 sectors, and use it to test the theoretical prediction of the model. We find that the observed increase in shock heterogeneity plays a significant role for the reduced sensitivity of inflation to movements in real activity since the late 1990s.

\(^{19}\)Results continue to hold if we use changes in real aggregate demand. An appendix that details results is available from the authors on request.
The analysis can be extended across several dimensions. Within the realm of models with information frictions, an interesting extension would be to endogenize the acquisition of information, which is likely to interact with the degree of shock heterogeneity in determining the reaction of aggregate variables to exogenous disturbances, thus playing a potentially important role for the sensitivity of inflation to aggregate demand. Two promising approaches to endogenize the information structure are the rational inattention approach (Mackowiak and Wiederholt, 2009; Mackowiak et al., 2009; and Matejka et al., 2017) and the choice of information acquisition (Hellwig and Veldkamp, 2009). Another interesting extension would be to use the framework to investigate whether optimal monetary policy is different from alternative models of imperfect information (Adam, 2007; Lorenzoni, 2010; Angeletos et al., 2016; and Tamura, 2016). Extending the analysis across these interesting dimensions remains open to future research.
References


A Derivation of Demand Functions and Price Indexes

A.1 Demand Functions

A representative household first determines the allocation of consumption across sectors and then determines that to goods in each sector taking the expenditure level to each sector as given.

Define the expenditure level by

\[ Z_t = \int_0^1 P_t(i)C_t(i)di. \]

The Lagrangean is:

\[ L = \left[ \int_0^1 (C_t(i)\Theta_t(i))^{\frac{\eta - 1}{\eta}} di \right]^{\frac{\eta}{\eta - 1}} - \lambda_t \left( \int_0^1 P_t(i)C_t(i)di - Z_t \right), \]

and the first-order conditions are:

\[ C_t(i)^{\frac{1}{\eta}} C_t^\frac{1}{\eta} (\Theta_t(i))^{\frac{\eta - 1}{\eta}} = \lambda_t P_t(i). \]  

Thus, for any two sectors, the following equation holds:

\[ C_t(i) = C_t(j) \left( \frac{P_t(i)}{P_t(j)} \right)^{-\eta} \left( \frac{\Theta_t(i)}{\Theta_t(j)} \right)^{\eta - 1}. \]

By substituting the equations into the equation for consumption expenditures \( Z_t = \int_0^1 P_t(i)C_t(i)di \), we have

\[ \int_0^1 P_t(i) \left[ C_t(j) \left( \frac{P_t(i)}{P_t(j)} \right)^{-\eta} \left( \frac{\Theta_t(i)}{\Theta_t(j)} \right)^{\eta - 1} \right] di = Z_t \]

\[ \Leftrightarrow C_t(j) = P_t^{-\eta}(j)\Theta_t^{\eta - 1}(j)Z_t \frac{1}{\int_0^1 P_t^{1-\eta}(i)\Theta_t^{\eta - 1}(i)di}. \]  

By substituting the equation:

\[ \int_0^1 P_t(i)C_t(i)di = Z_t = P_tC_t, \]

into equation (29), it yields:

\[ C_t(i) = \Theta_t^{\eta - 1}(i) \left( \frac{P_t(i)}{P_t} \right)^{-\eta} C_t \frac{P_t^{1-\eta}}{\int_0^1 P_t^{1-\eta}(i)\Theta_t^{\eta - 1}(i)di}. \]  

By defining \( P_t = \left[ \int_0^1 P_t^{1-\eta}(i)\Theta_t^{\eta - 1}(i)di \right]^{\frac{1}{1-\eta}} \), we can re-write equation (30) as:

\[ C_t(i) = \Theta_t^{\eta - 1}(i) \left( \frac{P_t(i)}{P_t} \right)^{-\eta} C_t. \]
We show the derivation of aggregate price index \( P_A \). Price Indexes

Making the same calculation for \( C_2 \), we can derive sectoral price index \( P_{i,j} \).

By combining these demand functions, we obtain the demand for good \((i, j)\) as follows.

\[
C_t(i, j) = \Theta_t^{\eta-1}(i) \left( \frac{P_t(i, j)}{P_t(i)} \right)^{-\bar{\eta}} \left( \frac{P_t(i)}{P_t} \right)^{-\eta} C_t.
\]

**A.2 Price Indexes**

We show the derivation of aggregate price index \( P_t \equiv \left[ \int_0^1 P_t^{1-\eta}(i) \Theta_t^{\eta-1}(i) \, di \right]^{1/\eta} \) because we can derive sectoral price index \( P_t(i) \equiv \left[ \int_0^1 P_t^{1-\bar{\eta}}(i, j) \, dj \right]^{1/1-\bar{\eta}} \) by exactly the same method.

Recall that \( \lambda_t^{-1} \) indicates the price of one unit of utility. From the first-order condition (28),

\[
C_t(i)^{-\bar{\eta}} C_t^{\frac{1}{\eta}} (\Theta_t(i))^{\frac{\eta-1}{\eta}} = \lambda_t P_t(i)
\]

\[
\Leftrightarrow C_t(i)^{-\bar{\eta}} C_t^{\frac{1}{\eta}} (\Theta_t(i))^{\frac{\eta-1}{\eta}} = \lambda_t C_t(i) P_t(i)
\]

\[
\Leftrightarrow \int_0^1 C_t(i)^{-\bar{\eta}} (\Theta_t(i))^{\frac{\eta-1}{\eta}} \, di C_t^{\frac{1}{\eta}} = \lambda_t \int_0^1 C_t(i) P_t(i) \, di
\]

\[
\Leftrightarrow C_t \lambda_t^{-1} = Z.
\]

We next derive the aggregate price index. From the first-order condition (28),

\[
C_t(i)^{-\bar{\eta}} C_t^{\frac{1}{\eta}} (\Theta_t(i))^{\frac{\eta-1}{\eta}} = \lambda_t P_t(i)
\]

\[
\Leftrightarrow (C_t(i) \Theta_t(i))^{-\bar{\eta}} C_t^{\frac{1}{\eta}} \Theta_t(i) = \lambda_t P_t(i)
\]

\[
\Leftrightarrow (C_t(i) \Theta_t(i))^{\frac{1}{\eta}} = C_t^{\frac{1}{\eta}} \Theta_t(i) \lambda_t^{-1} P_t^{-1}(i)
\]

\[
\Leftrightarrow (C_t(i) \Theta_t(i))^{\frac{\eta-1}{\eta}} = C_t^{\frac{\eta-1}{\eta}} \Theta_t^{\eta-1}(i) \lambda_t^{1-\eta} P_t^{1-\eta}(i)
\]

\[
\Leftrightarrow \int_0^1 (C_t(i) \Theta_t(i))^{\frac{\eta-1}{\eta}} \, di = C_t^{\frac{\eta-1}{\eta}} \lambda_t^{1-\eta} \int_0^1 (P_t^{1-\eta}(i) \Theta_t^{\eta-1}(i)) \, di
\]

\[
\Leftrightarrow 1 = \lambda_t^{1-\eta} \int_0^1 (P_t^{1-\eta}(i) \Theta_t^{\eta-1}(i)) \, di
\]

\[
\Leftrightarrow \lambda_t^{-1} = \left[ \int_0^1 (P_t^{1-\eta}(i) \Theta_t^{\eta-1}(i)) \, di \right]^{1/\eta}.
\]
B Derivation of the Price Setting Rule

From
\[ p_t(i, j) = \mu + mc_t(i, j), \]
\[ c_t(i, j) = -\tilde{\eta}(p_t(i, j) - p_t(i)) - \eta(p_t(i) - p_t) + c_t + (\eta - 1)\theta_t(i), \]
and
\[ mc_t(i, j) = w_t + \frac{1 - \epsilon}{\epsilon} y_t(i, j) - \frac{1}{\epsilon}a - \log(\epsilon), \]
p_t(i, j) is given by,
\[ p_t(i,j) = \mu + mc_t(i, j) = \mu + y_t + p_t - \frac{1}{\epsilon}a - \log(\epsilon) + \frac{1 - \epsilon}{\epsilon} [-\tilde{\eta}(p_t(i, j) - p_t(i)) - \eta(p_t(i) - p_t) + c_t + (\eta - 1)\theta_t(i)] \]
\[ = -\frac{1 - \epsilon}{\epsilon} \tilde{\eta}p_t(i, j) + \frac{1 - \epsilon}{\epsilon} (\tilde{\eta} - \eta) p_t(i) + \left(1 + \frac{1 - \epsilon}{\epsilon}\eta \right) p_t \]
\[ + (\mu - \frac{1}{\epsilon}a - \log(\epsilon)) + \left(1 + \frac{1 - \epsilon}{\epsilon}\right)y_t + \frac{1 - \epsilon}{\epsilon}(\eta - 1)\theta_t(i) \]
\[ = -\frac{1 - \epsilon}{\epsilon} \tilde{\eta}p_t(i, j) + \frac{1 - \epsilon}{\epsilon} (\tilde{\eta} - \eta) p_t(i) + (\mu - \frac{1}{\epsilon}a - \log(\epsilon)) + \left(1 + \frac{1 - \epsilon}{\epsilon}\right)q_t + \frac{1 - \epsilon}{\epsilon}(\eta - 1) p_t + \frac{1 - \epsilon}{\epsilon}(\eta - 1)\theta_t(i) \]
\[ = \frac{1 - \epsilon}{\epsilon} (\tilde{\eta} - \eta) p_t(i) + \frac{1}{1 + \frac{1 - \epsilon}{\epsilon}\tilde{\eta}} (\mu - \frac{1}{\epsilon}a - \log(\epsilon)) \]
\[ + \frac{1 + \frac{1 - \epsilon}{\epsilon}}{1 + \frac{1 - \epsilon}{\epsilon}\tilde{\eta}}q_t + \frac{\frac{1 - \epsilon}{\tilde{\eta}}(\eta - 1)}{1 + \frac{1 - \epsilon}{\epsilon}\tilde{\eta}} p_t + \frac{\frac{1 - \epsilon}{\epsilon}(\eta - 1)}{1 + \frac{1 - \epsilon}{\epsilon}\tilde{\eta}}\theta_t(i) \]
\[ = \frac{(\tilde{\eta} - \eta)(1 - \epsilon)}{\epsilon + \tilde{\eta}(1 - \epsilon)} p_t(i) + \frac{\epsilon}{\epsilon + \tilde{\eta}(1 - \epsilon)} (\mu - \frac{1}{\epsilon}a - \log(\epsilon)) \]
\[ + \frac{1}{\epsilon + \tilde{\eta}(1 - \epsilon)}q_t + \frac{(1 - \epsilon)(\eta - 1)}{\epsilon + \tilde{\eta}(1 - \epsilon)} p_t + \frac{(1 - \epsilon)(\eta - 1)}{\epsilon + \tilde{\eta}(1 - \epsilon)}\theta_t(i). \]

C Derivation of the Index of Aggregate Prices

Recall that: \( P_t \equiv \left[ \int_0^1 p_t^{1-\eta}(i)\Theta_{\tilde{\eta}}^{\eta-1}(i)di \right]^{\frac{1}{1-\eta}} \) can be expressed as, \( P_t = \left[ \int_0^1 \left( \frac{P_t(i)}{\Theta_{\tilde{\eta}}(i)} \right)^{1-\eta}di \right]^{\frac{1}{1-\eta}} \)
\[ \left[ \int_0^1 \left( \tilde{P}_t(i) \right)^{1-\eta}di \right]^{\frac{1}{1-\eta}}, \text{where } \tilde{P}_t(i) \equiv \frac{P_t(i)}{\Theta_{\tilde{\eta}}(i)}. \text{ We then define } p_t \equiv \int_0^1 \tilde{P}_t(i)di, \text{ such that:} \]
\[ p_t \equiv \int_0^1 \tilde{P}_t(i)di = \int_0^1 p_t(i)di - \int_0^1 \theta_t(i)di = \int_0^1 p_t(i)di, \]
since \( \theta_t(i) \sim \mathcal{N}(0, (1 - \epsilon)^{-2}(\eta - 1)^{-2}\tau^2) \) and \( \int_0^1 \theta_t(i)di = 0. \)
D Aggregate Shocks and the Output Gap

To evaluate whether the extracted shock \( u_t = \sum_{i=1}^{29} \Lambda_i \tilde{x}_t (i) \) is a plausible measure of aggregate disturbances that is consistent with alternative measures, we compare the eight-quarters backward moving averages of the aggregate shocks, \( \frac{1}{8} \sum_{s=0}^{7} u_{t-s} \), with the output gap published by the Bank of Japan.\(^{21}\)

Figure A examines the relationship between the dynamics of our estimates for aggregate shocks and the output gaps. It shows that the two series are highly correlated, with a correlation coefficient equal to 0.72, suggesting that our identified measure for the aggregate shock is consistent with alternative measures of aggregate shocks.\(^{22}\)

[Figure A about here.]

\(^{20}\)Our measure of the aggregate shock is a flow rather than stock concept. By comparing moving averages of the aggregate shocks (i.e., the averages of flow data) with the output gap (i.e. stock data), we ensure that our measure is consistent with conventional measures.

\(^{21}\)The series is available here. https://www.boj.or.jp/en/research/research_data/gap/index.htm/


\(^{22}\)We conduct the same exercise with the eight-quarters backward moving averages of the normalized real shocks, \( \frac{1}{8} \sum_{s=0}^{7} \left( \sum_{i=1}^{29} \Lambda_i \right)^{-1} u_{t-s} - \pi_{t-s} \), and obtain the broadly same results (a correlation coefficient equal to 0.65).
E  Proofs

E.1 Proof of Proposition 1

We divide the proof of Proposition 1 in two parts: part (i) and part (ii).

**Part (i)** The terms $\mathbb{E}_t \left[ \sum_{h=1}^k u_{t+h} \right]$ and $\mathbb{E}_t \left[ \sum_{h=1}^k \tilde{v}_{t+h}(i) \right]$ are equal to:

$$
\mathbb{E}_t \left[ \sum_{h=1}^k u_{t+h} \right] = \frac{1 - \rho_u^{k+1}}{1 - \rho_u} \mathbb{E}_t [u_t],
$$

$$
\mathbb{E}_t \left[ \sum_{h=1}^k \tilde{v}_{t+h}(i) \right] = \frac{1 - \rho_v^{k+1}}{1 - \rho_v} \mathbb{E}_t [\tilde{v}_t],
$$

respectively. It follows that:

$$
\mathbb{C} \left( \mathbb{E}_t \left[ \sum_{h=1}^k u_{t+h} \right], \mathbb{E}_t \left[ \sum_{h=1}^k \tilde{v}_{t+h}(i) \right] \right) = \frac{1 - \rho_u^{k+1}}{1 - \rho_u} \frac{1 - \rho_v^{k+1}}{1 - \rho_v} \mathbb{C} (\mathbb{E}_t [u_t], \mathbb{E}_t [\tilde{v}_t]). \tag{31}
$$

From equation (31), co-movements in expectations of $k$-period ahead aggregate and idiosyncratic demand are determined by co-movements in the expectations about current aggregate and idiosyncratic demand, according to:

$$
\mathbb{C} \left( \mathbb{E}_t \left[ \sum_{h=1}^k u_{t+h} \right], \mathbb{E}_t \left[ \sum_{h=1}^k \tilde{v}_{t+h}(i) \right] \right) = 0 \text{ if } \mathbb{C} (\mathbb{E}_t [u_t], \mathbb{E}_t [\tilde{v}_t]) = 0
$$

$$
\mathbb{C} \left( \mathbb{E}_t \left[ \sum_{h=1}^k u_{t+h} \right], \mathbb{E}_t \left[ \sum_{h=1}^k \tilde{v}_{t+h}(i) \right] \right) > 0 \text{ if } \mathbb{C} (\mathbb{E}_t [u_t], \mathbb{E}_t [\tilde{v}_t]) > 0
$$

$$
\mathbb{C} \left( \mathbb{E}_t \left[ \sum_{h=1}^k u_{t+h} \right], \mathbb{E}_t \left[ \sum_{h=1}^k \tilde{v}_{t+h}(i) \right] \right) < 0 \text{ if } \mathbb{C} (\mathbb{E}_t [u_t], \mathbb{E}_t [\tilde{v}_t]) < 0.
$$

Part (i) of the proof outlines the general relationship between the co-movements in the expectations on distinct components of demand at the present period $t$ and implied co-movements in the expectations on realizations of these components $k$-period ahead. If expectations of current aggregate and idiosyncratic demand are mutually independent (i.e., $\mathbb{C} (\mathbb{E}_t [u_t], \mathbb{E}_t [\tilde{v}_t]) = 0$), the correlation between expectations of each component $k$-period ahead also is independent. However, if their correlation at time $t$ is positive (negative), it generates a positive (negative) correlation in the expectations of these components.
Part (ii) The terms $E_t [u_t]$ and $E_t [v_t(i)]$ are equal to:

\[
E_t [u_t] = \frac{\tau^2}{\sigma^2 + \tau^2} (q_{t-1} + \rho_u u_{t-1}) + \frac{\sigma^2}{\sigma^2 + \tau^2} [x_t(i) - \rho_v v_{t-1}(i)]
\]
\[
= \rho_u u_{t-1} + \frac{\sigma^2}{\sigma^2 + \tau^2} [x_t(i) - q_{t-1} - \rho_u u_{t-1} - \rho_v v_{t-1}(i)]
\]
\[
= \rho_u u_{t-1} + \frac{\sigma^2}{\sigma^2 + \tau^2} [e_t + \epsilon_t(i)]
\]

\[
E_t [v_t(i)] = \frac{\sigma^2}{\sigma^2 + \tau^2} \rho_v v_{t-1}(i) + \frac{\tau^2}{\sigma^2 + \tau^2} [x_t(i) - q_{t-1} - \rho_u u_{t-1}]
\]
\[
= \rho_v v_{t-1}(i) + \frac{\tau^2}{\sigma^2 + \tau^2} [x_t(i) - q_{t-1} - \rho_u u_{t-1} - \rho_v v_{t-1}(i)]
\]
\[
= \rho_v v_{t-1}(i) + \frac{\tau^2}{\sigma^2 + \tau^2} [e_t + \epsilon_t(i)].
\]

Thus, $E_t [\tilde{v}_t]$ is given by,

\[
E_t [\tilde{v}_t(i)] = \rho_v v_{t-1}(i) + \frac{\tau^2}{\sigma^2 + \tau^2} [e_t + \epsilon_t(i)] - v_{t-1}(i)
\]
\[
= (\rho_v - 1)v_{t-1}(i) + \frac{\tau^2}{\sigma^2 + \tau^2} [e_t + \epsilon_t(i)].
\]

It follows that:

\[
\mathbb{C}(E_t [u_t], E_t [\tilde{v}_t]) = \frac{\sigma^2}{\sigma^2 + \tau^2} \frac{\tau^2}{\sigma^2 + \tau^2} \mathbb{V}[e_t + \epsilon_t(i)] = \frac{\sigma^2 \tau^2}{\sigma^2 + \tau^2} > 0.
\]

Since equation (32) establishes $\mathbb{C}(E_t [u_t], E_t [\tilde{v}_t]) > 0$, part (i) proves Proposition 1. □
E.2 Proof of Proposition 2

First, we guess that $p^*_t(i)$ takes the following form:

$$p^*_t(i) = a_1p_{t-1} + a_2x_t(i) + a_3q_{t-1} + a_4u_{t-1} + a_5v_{t-1}(i).$$

Given the guess, and since only a randomly selected fraction $1 - \theta$ of firms adjusts prices in any given period, we infer that the aggregate price level must satisfy:

$$p_t = \theta p_{t-1} + (1 - \theta) \int_0^1 p^*_t(i)di$$

Therefore, $p^*_t(i)$ is obtained as:

$$p^*_t(i) = (1 - \beta\theta)[(1 - r)x_t(i) + rE_t[p_t]] + \beta\theta E_t[p^*_{t+1}(i)]$$

$$ = (1 - \beta\theta)(1 - r)x_t(i) + (1 - \beta\theta)rE_t[p_t] + \beta\theta E_t[p^*_{t+1}(i)]$$

$$ = (1 - \beta\theta)(1 - r)x_t(i) + (1 - \beta\theta)rE_t[p_t] + \beta\theta [a_1p_t + a_2x_{t+1}(i) + a_3q_t + a_4u_t + a_5v_t(i)]$$

$$ = (1 - \beta\theta)(1 - r)x_t(i) + [(1 - \beta\theta)r + \beta\theta a_1]E_t[p_t] + \beta\theta a_2E_t[x_{t+1}(i)] + \beta\theta a_3E_t[q_t] + \beta\theta a_4E_t[u_t] + \beta\theta a_5E_t[v_t(i)]$$

$$ = (1 - \beta\theta)(1 - r)x_t(i) + [(1 - \beta\theta)r + \beta\theta a_1]E_t[p_t] + \beta\theta a_2E_t[q_t + u_{t+1} + v_{t+1}(i)] + \beta\theta a_3E_t[q_t] + \beta\theta a_4E_t[u_t] + \beta\theta a_5E_t[v_t(i)]$$

$$ = (1 - \beta\theta)(1 - r)x_t(i) + [(1 - \beta\theta)r + \beta\theta a_1]E_t[p_t] + \beta\theta (a_2 + a_3)E_t[q_t] + \beta\theta (a_2\rho_t + a_4)E_t[u_t] + \beta\theta (a_2\rho_t + a_5)E_t[v_t(i)].$$

The term $E_t[p_t]$ is given by:

$$E_t[p_t] = [\theta + (1 - \theta) a_1]p_{t-1} + (1 - \theta) a_2E_t[q_t] + (1 - \theta) a_3q_{t-1} + (1 - \theta) a_4u_{t-1},$$
which yields:

\[
p_t^*(i) = (1 - \beta \theta)(1 - r)x_t(i) + [(1 - \beta \theta)r + \beta \theta a_1][\theta + (1 - \theta) a_1]p_{t-1} \\
+ [((1 - \beta \theta)r + \beta \theta a_1)(1 - \theta)a_2 + \beta \theta (a_2 + a_3)]E_t[q_t] \\
+ [(1 - \beta \theta)r + \beta \theta a_1](1 - \theta)a_3q_{t-1} \\
+ [(1 - \beta \theta)r + \beta \theta a_1](1 - \theta)a_4u_{t-1} \\
+ \beta \theta (a_2 \rho_u + a_4)E_t[u_t] + \beta \theta (a_2 \rho_v + a_3)E_t[v_t(i)] \\
= (1 - \beta \theta)(1 - r)x_t(i) + [(1 - \beta \theta)r + \beta \theta a_1][\theta + (1 - \theta) a_1]p_{t-1} \\
+ [((1 - \beta \theta)r + \beta \theta a_1)(1 - \theta)a_2 + \beta \theta (a_2 + a_3)]E_t[q_t] \\
+ [(1 - \beta \theta)r + \beta \theta a_1](1 - \theta)a_3q_{t-1} \\
+ [(1 - \beta \theta)r + \beta \theta a_1](1 - \theta)a_4u_{t-1} \\
= (1 - \beta \theta)(1 - r)x_t(i) + [(1 - \beta \theta)r + \beta \theta a_1][\theta + (1 - \theta) a_1]p_{t-1} \\
+b_1q_{t-1} + b_2E_t[u_t] + b_3E_t[v_t(i)] + b_4u_{t-1}.
\]

Since

\[
x_t(i) = q_{t-1} + \rho_u u_{t-1} + e_t + \rho_v v_{t-1}(i) + \epsilon_t(i) \\
\Leftrightarrow e_t = x_t(i) - q_{t-1} - \rho_u u_{t-1} - \rho_v v_{t-1}(i) - \epsilon_t(i), \\
\Leftrightarrow \epsilon_t(i) = x_t(i) - q_{t-1} - \rho_u u_{t-1} - \rho_v v_{t-1}(i) - e_t,
\]

The terms \(E_t[u_t]\) and \(E_t[v_t(i)]\) are equal to:

\[
E_t[u_t] = \rho_u u_{t-1} + E_t[e_t] \\
= \rho_u u_{t-1} + \frac{\sigma^2}{\sigma^2 + \tau^2}[x_t(i) - q_{t-1} - \rho_u u_{t-1} - \rho_v v_{t-1}(i)]
\]

\[
E_t[v_t(i)] = \rho_v v_{t-1}(i) + \frac{\tau^2}{\sigma^2 + \tau^2}[x_t(i) - q_{t-1} - \rho_u u_{t-1} - \rho_v v_{t-1}(i)].
\]
It follows that:

\[ p_t^*(i) = (1 - \beta \theta)(1 - r)x_i(i) + [(1 - \beta \theta)r + \beta \theta a_1][\theta + (1 - \theta) a_1]p_{t-1} + b_1q_{t-1} + b_2E_t[u_i] + b_3E_t[v_i] + b_4u_{t-1} \]

\[ = (1 - \beta \theta)(1 - r)x_i(i) + [(1 - \beta \theta)r + \beta \theta a_1][\theta + (1 - \theta) a_1]p_{t-1} + b_2\rho_u u_{t-1} + b_2\frac{\sigma^2}{\sigma^2 + \tau^2}[x_i(i) - q_{t-1} - \rho_u u_{t-1} - \rho_v v_{t-1}(i)] + b_3\rho_v v_{t-1}(i) + b_4u_{t-1} + b_1q_{t-1} \]

\[ = [(1 - \beta \theta)r + \beta \theta a_1][\theta + (1 - \theta) a_1]p_{t-1} + \left[ (1 - \beta \theta)(1 - r) + b_2\frac{\sigma^2}{\sigma^2 + \tau^2} + b_3\frac{\tau^2}{\sigma^2 + \tau^2} \right]x_i(i) + b_1\frac{\sigma^2}{\sigma^2 + \tau^2} - b_3\frac{\tau^2}{\sigma^2 + \tau^2} q_{t-1} \]

\[ + \left[ b_4 + (b_2 - b_3)\frac{\tau^2}{\sigma^2 + \tau^2}\rho_u \right] u_{t-1} + [b_3 - b_2]\frac{\sigma^2}{\sigma^2 + \tau^2}\rho_v v_{t-1}(i) \]

and thus the equilibrium conditions are:

\[ a_1 = [(1 - \beta \theta)r + \beta \theta a_1][\theta + (1 - \theta) a_1], \]

\[ a_2 = (1 - \beta \theta)(1 - r) + b_2\frac{\sigma^2}{\sigma^2 + \tau^2} + b_3\frac{\tau^2}{\sigma^2 + \tau^2}, \]

\[ a_3 = b_1 - b_2\frac{\sigma^2}{\sigma^2 + \tau^2} - b_3\frac{\tau^2}{\sigma^2 + \tau^2}, \]

\[ a_4 = b_4 + (b_2 - b_3)\frac{\tau^2}{\sigma^2 + \tau^2}\rho_u, \]

\[ a_5 = [b_3 - b_2]\frac{\sigma^2}{\sigma^2 + \tau^2}\rho_v, \]

where

\[ b_1 = \left[ [(1 - \beta \theta)r + \beta \theta a_1](1 - \theta)a_2 + \beta \theta (a_2 + a_3) + [(1 - \beta \theta)r + \beta \theta a_1](1 - \theta)a_3 \right], \]

\[ b_2 = \left[ [(1 - \beta \theta)r + \beta \theta a_1](1 - \theta)a_2 + \beta \theta (a_2 + a_3) + \beta \theta (a_2\rho_u + a_4) \right], \]

\[ b_3 = \beta \theta (a_2\rho_u + a_5), \]

\[ b_4 = [(1 - \beta \theta)r + \beta \theta a_1](1 - \theta)a_4, \]

By simplifying the conditions, we obtain:

\[ a_1 = [(1 - \beta \theta)r + \beta \theta a_1][\theta + (1 - \theta) a_1], \]
\[ a_2 = (1 - \beta \theta)(1 - r) \]
\[ + \left[ (1 - \beta \theta) r + \beta \theta a_1 \right] (1 - \theta) a_2 + \beta \theta (a_2 + a_3) \frac{\sigma^2}{\sigma^2 + \tau^2} \]
\[ + \beta \theta (a_2 \rho_u + a_4) \frac{\sigma^2}{\sigma^2 + \tau^2} + \beta \theta (a_2 \rho_v + a_5) \frac{\tau^2}{\sigma^2 + \tau^2} \]
\[ = (1 - \beta \theta)(1 - r) \]
\[ + \left[ (1 - \beta \theta) r + \beta \theta a_1 \right] (1 - \theta) a_2 + \beta \theta (a_2 + a_3) \frac{\sigma^2}{\sigma^2 + \tau^2} \]
\[ + \beta \theta (a_2 \rho_u + a_4) \frac{\sigma^2}{\sigma^2 + \tau^2} + \beta \theta (a_2 \rho_v + a_5) \frac{\tau^2}{\sigma^2 + \tau^2} \]
\[ a_3 = \left[ (1 - \beta \theta) r + \beta \theta a_1 \right] (1 - \theta) a_2 + \beta \theta (a_2 + a_3) \frac{\sigma^2}{\sigma^2 + \tau^2} \]
\[ + \left[ (1 - \beta \theta) r + \beta \theta a_1 \right] (1 - \theta) a_3 - \frac{\sigma^2}{\sigma^2 + \tau^2} \beta \theta (a_2 \rho_u + a_4) \]
\[ - \frac{\tau^2}{\sigma^2 + \tau^2} \beta \theta (a_2 \rho_v + a_5) \]
\[ = \left[ (1 - \beta \theta) r + \beta \theta a_1 \right] (1 - \theta) a_2 + \beta \theta (a_2 + a_3) \frac{\sigma^2}{\sigma^2 + \tau^2} \]
\[ + \left[ (1 - \beta \theta) r + \beta \theta a_1 \right] (1 - \theta) \frac{\tau^2}{\sigma^2 + \tau^2} + \beta \theta \frac{\tau^2}{\sigma^2 + \tau^2} \]
\[ a_4 = \left[ (1 - \beta \theta) r + \beta \theta a_1 \right] (1 - \theta) a_4 \]
\[ + \left[ (1 - \beta \theta) r + \beta \theta a_1 \right] (1 - \theta) a_2 + \beta \theta (a_2 + a_3) \frac{\tau^2}{\sigma^2 + \tau^2} \rho_u \]
\[ + \beta \theta (a_2 \rho_u + a_4) - \beta \theta (a_2 \rho_v + a_5) \]
\[ = \left[ (1 - \beta \theta) r + \beta \theta a_1 \right] (1 - \theta) a_4 + \beta \theta \frac{\tau^2}{\sigma^2 + \tau^2} \rho_a \]
\[ a_5 = \left[ (1 - \beta \theta) r + \beta \theta a_1 \right] (1 - \theta) a_2 + \beta \theta (a_2 + a_3) \frac{\tau^2}{\sigma^2 + \tau^2} \rho_v \]
\[ - \beta \theta (a_2 \rho_u + a_4) - \beta \theta (a_2 \rho_v + a_5) \]
\[ - \beta \theta \frac{\tau^2}{\sigma^2 + \tau^2} \rho_v a_3 - \beta \theta \frac{\tau^2}{\sigma^2 + \tau^2} \rho_v a_4 + \beta \theta \frac{\tau^2}{\sigma^2 + \tau^2} \rho_v a_5, \]
Table 1: Descriptive statistics about survey data

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</tr>
<tr>
<td>Warehousing &amp; Harbor Transportation Services</td>
</tr>
<tr>
<td>Information &amp; Communication</td>
</tr>
<tr>
<td>Electric Power &amp; Gas</td>
</tr>
<tr>
<td>Services</td>
</tr>
<tr>
<td>Banks</td>
</tr>
<tr>
<td>Averages across sectors</td>
</tr>
</tbody>
</table>
Table 2: Firms’ expectations on aggregate and idiosyncratic demands

Dataset: Survey of corporate behavior; 25 sectors; 2003y-2017y

Dependent Variable: firms’ expectations on the growth rate of the aggregate demand

<table>
<thead>
<tr>
<th></th>
<th>One year ahead expectations</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pooled OLS model</td>
<td>fixed effect model</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>1.05***</td>
<td>1.07***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.02)</td>
<td></td>
</tr>
<tr>
<td>Firms’ expectations on the</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>growth rate of the</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>idiosyncratic demand</td>
<td>0.18***</td>
<td>0.24**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.08)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>375</td>
<td>375</td>
<td></td>
</tr>
<tr>
<td>Adjusted-R²</td>
<td>0.04</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>SE</td>
<td>0.88</td>
<td>0.90</td>
<td></td>
</tr>
</tbody>
</table>

Note: Estimated by ordinary-least-squares. The standard errors are HAC estimators. 
*** Significant at the 1 percent level. ** Significant at the 5 percent level. * Significant at the 10 percent level.
Table 3: Estimation of the Phillips curve (part 1)

Dataset: Financial statement statistics of corporations by industry, consumer price index; 29 sectors; 1976/1Q-2018/3Q

Dependent Variable: Inflation rate ($\pi$, core consumer price index, seasonally adjusted, QoQ)

<table>
<thead>
<tr>
<th></th>
<th>2 years trimmed mean</th>
<th>3 years trimmed mean</th>
<th>5 years trimmed mean</th>
<th>10 years trimmed mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.083**</td>
<td>0.079**</td>
<td>0.078**</td>
<td>0.075***</td>
</tr>
<tr>
<td></td>
<td>(0.032)</td>
<td>(0.032)</td>
<td>(0.033)</td>
<td>(0.027)</td>
</tr>
<tr>
<td>Lag of inflation ($\pi_{t-1}$)</td>
<td>0.613***</td>
<td>0.610***</td>
<td>0.518***</td>
<td>0.443***</td>
</tr>
<tr>
<td></td>
<td>(0.103)</td>
<td>(0.109)</td>
<td>(0.120)</td>
<td>(0.110)</td>
</tr>
<tr>
<td>Aggregate shocks ($u_t$)</td>
<td>0.019**</td>
<td>0.025**</td>
<td>0.045***</td>
<td>0.065***</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.011)</td>
<td>(0.017)</td>
<td>(0.020)</td>
</tr>
<tr>
<td>Aggregate shocks×shock heterogeneity ($u_t \times \tau / \sigma_t$)</td>
<td>-0.006**</td>
<td>-0.007**</td>
<td>-0.013**</td>
<td>-0.018***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.005)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>Observations</td>
<td>171</td>
<td>171</td>
<td>171</td>
<td>171</td>
</tr>
<tr>
<td>Adjusted-$R^2$</td>
<td>0.58</td>
<td>0.58</td>
<td>0.61</td>
<td>0.63</td>
</tr>
<tr>
<td>SE</td>
<td>0.39</td>
<td>0.40</td>
<td>0.39</td>
<td>0.37</td>
</tr>
</tbody>
</table>

Note: Estimated by ordinary-least-squares. The standard errors are HAC estimators. Aggregate shocks are calculated as the developments in the principal component of 29 sectors. First and second lags of aggregate shocks are included in estimation as control variables. Data extrapolation using the values in the closest periods is applied for the missing values in the estimates of shock heterogeneity. The series for the core consumer price index is “all items, less fresh food (impact of consumption taxes are adjusted)”.

*** Significant at the 1 percent level. ** Significant at the 5 percent level. * Significant at the 10 percent level.
Table 4: Estimation of the Phillips curve (part 2)

<table>
<thead>
<tr>
<th>Dataset: Financial statement statistics of corporations by industry, consumer price index; 29 sectors; 1976/1Q-2018/3Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent Variable: Inflation rate ((\pi_t), core consumer price index, seasonally adjusted, QoQ)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>2 years trimmed mean</th>
<th>3 years trimmed mean</th>
<th>5 years trimmed mean</th>
<th>10 years trimmed mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.037</td>
<td>0.030</td>
<td>0.032</td>
<td>0.042</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.036)</td>
<td>(0.036)</td>
<td>(0.033)</td>
</tr>
<tr>
<td>Lag of inflation ((\pi_{t-1}))</td>
<td>0.689***</td>
<td>0.695***</td>
<td>0.669***</td>
<td>0.622***</td>
</tr>
<tr>
<td></td>
<td>(0.086)</td>
<td>(0.088)</td>
<td>(0.095)</td>
<td>(0.083)</td>
</tr>
<tr>
<td>Lag of inflation×time dummy (2000-2017)</td>
<td>-0.755**</td>
<td>-0.738***</td>
<td>-0.719***</td>
<td>-0.740**</td>
</tr>
<tr>
<td>((\pi_{t-1} \times 1_{(2000-2017)}))</td>
<td>(0.168)</td>
<td>(0.178)</td>
<td>(0.172)</td>
<td>(0.158)</td>
</tr>
<tr>
<td>Aggregate shocks ((u_t))</td>
<td>0.026**</td>
<td>0.026**</td>
<td>0.040**</td>
<td>0.093***</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.012)</td>
<td>(0.018)</td>
<td>(0.023)</td>
</tr>
<tr>
<td>Aggregate shocks × time dummy (2000-2017)</td>
<td>-0.006</td>
<td>-0.004</td>
<td>0.002</td>
<td>0.019**</td>
</tr>
<tr>
<td>((u_t \times 1_{(2000-2017)}))</td>
<td>(0.008)</td>
<td>(0.008)</td>
<td>(0.008)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Aggregate shocks×shock heterogeneity ((u_t \times \tau_t/\sigma_t))</td>
<td>-0.006**</td>
<td>-0.007**</td>
<td>-0.011**</td>
<td>-0.029***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.006)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>Observations</td>
<td>171</td>
<td>171</td>
<td>171</td>
<td>171</td>
</tr>
<tr>
<td>Adjusted-R²</td>
<td>0.66</td>
<td>0.65</td>
<td>0.65</td>
<td>0.68</td>
</tr>
<tr>
<td>SE</td>
<td>0.36</td>
<td>0.36</td>
<td>0.36</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Note: Estimated by ordinary-least-squares. The standard errors are HAC estimators. Aggregate shocks are calculated as the developments in the principal component of 29 sectors. First and second lags of aggregate shocks are included in estimation as control variables. Data extrapolation using the values in the closest periods is applied for the missing values in the estimates of shock heterogeneity. The series for the core consumer price index is “all items, less fresh food (impact of consumption taxes are adjusted)”.

*** Significant at the 1 percent level. ** Significant at the 5 percent level. * Significant at the 10 percent level.
Figure 1: Firms’ expectations about their sectoral demand, aggregate demand, and idiosyncratic demand

(a) Chemical

(b) Retail

- growth of idiosyncratic demand
- growth of aggregate demand
- growth of sectoral demand
Figure 2: The slope of the Phillips curve

(a) The degree of nominal price rigidity ($\theta$)

(b) The degree of shock heterogeneity ($\tau/\sigma$)

Notes: Parameters are $\tau/\sigma = 1$, $r = 0.5$, $\beta = 0.99$, $\rho_u = 0.35$, $\rho_v = 0.15$ for (a), and $\theta = 0.2$, $r = 0.5$, $\beta = 0.99$, $\rho_u = 0.35$, $\rho_v = 0.15$ for (b).
Figure 3: Responses of aggregate inflation to aggregate shocks (Simulation)

Notes: Parameters are $\theta = 0.5$, $r = 0.5$, $\beta = 0.99$, $\rho_u = 0.35$, $\rho_v = 0.15$. 
Figure 4: Estimates for the slope coefficient in the New Keynesian and the hybrid Phillip Curve

(a) Estimates for the slope coefficient ($\kappa$) in the New Keynesian Phillips Curve

(b) Estimates for the slope coefficient ($\kappa$) in the hybrid Phillips Curve

Notes: Parameters are $r = 0.5$, $\beta = 0.99$, $\rho_u = 0.35$, $\rho_v = 0.15$. 
Figure 5: Estimates of shock heterogeneity ($\tau_t/\sigma_t$)

Notes: Upper and lower 10% of the samples are excluded in estimation.
Source: Ministry of Finance “Financial statements statistics of corporations by industry”.
Figure A: Aggregate shocks and output gap

Sources: Ministry of Finance “Financial statements statistics of corporations by industry”,
       Bank of Japan “Output Gap and Potential Growth Rate”.