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Abstract

The Greek sovereign default episode in 2012 was characterized by its high debt-to-GDP ratio and the severe economic contraction following the default. Conventional strategic default models designed to analyze a government's incentive to default often fail to replicate these characteristics. To address this issue, we provide a dynamic stochastic general equilibrium (DSGE) model where a sovereign default is triggered by the government's inability to repay its debt. We show that the inability-to-repay model replicates the empirical features observed in Greece, while the conventional strategic default model calibrated to the Greek economy does not.

Keywords: Sovereign Default; Dynamic Stochastic General Equilibrium; Inability to Repay Debt; Strategic Decision to Default; Fiscal Limit; Laffer Curve
JEL classification: E32, E44, F34, H63

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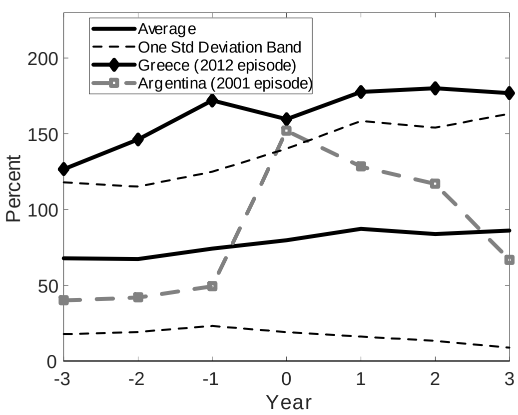
1 Introduction

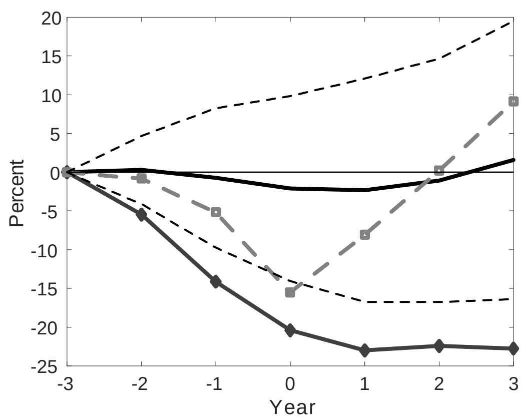
The Greek sovereign default episode in 2012 was extraordinary in two respects.[1] First, the size of the public debt was the highest in the history of default, reaching 199.2 billion euros at the outset of its default. The debt-to-GDP ratio was over 150 percent when it defaulted, approximately twice as high as the average of sovereign defaults in other countries that took place from 1982 to 2013, as shown in Figure 1 (a). Second, the Greek default had a huge impact on its domestic economy in terms of severity and length of the economic contraction after the default. GDP dropped by more than 20 percent, as shown in Figure 1 (b). In addition, the economic contraction in Greece continued for more than 3 years. This is in contrast with what most defaulting countries witnessed in previous episodes. As shown in Figure 1, on average, GDP in defaulting countries declined by about 3 percent when the government defaulted and recovered to the pre-default level at a faster pace, typically within 3 years.

Figure 1: Development of Debt-to-GDP ratio and GDP around Default Period

(a) Debt-to-GDP Ratio



(b) GDP



Notes 1: The horizontal axis represents the number of years after the onset of default.
2: The vertical axis in panel (b) represents the cumulative changes in GDP from 3 years before default.
3: The numbers of default episodes used for computing debt-to-GDP ratio and GDP in the figure are 86 and 104, respectively. These episodes are taken from Schmitt-Grohe and Uribe (2017).
Source: Historical Public Debt Database (IMF), and World Development Indicators (World Bank).

In the current paper, we argue that these empirical characteristics of the Greek default are not captured by conventional strategic default models in which the default is

[1] In this paper, following the *Standard and Poor's* definition, sovereign default is defined as the failure of a government to meet a principal or interest payment on the due date. The duration of default is defined as the period that the country is not able to access bond markets after the default.

triggered by the government's strategic decision. While some studies in the literature apply conventional strategic default models to explain the European debt crisis including Greece (e.g., D'Erasmo and Mendoza 2016, Salomao 2017, and Bocola and Dovis 2016), they do not replicate the high debt-to-GDP ratio and the severe contraction observed in the data.[2] As seminal strategic model studies such as Aguiar and Gopinath (2006), Arellano (2008), Yue (2010) and Mendoza and Yue (2012) state clearly that their models are developed to analyze emerging countries, strategic default models are originally designed to describe defaults in emerging countries in which the debt-to-GDP ratio tends to be low and the economic contraction is less severe. Regarding the debt-to-GDP ratio, as Bi (2012) and Paluszynski (2017) point out, the strategic default models cannot replicate the high debt outstanding observed in the data. For example, the average debt-to-GDP ratio in Aguiar and Gopinath (2006), Yue (2010), and Mendoza and Yue (2012) ranges from 10 percent to 23 percent, substantially lower than the level observed in Argentina (50 percent) or Greece (150 percent). To address this issue, Hatchondo and Martinez (2009), Chatterjee and Eyigungor (2012) and Hatchondo, Martinez and Sosa-Padilla (2016) introduce long-term bonds into sovereign default analysis to explain debt-to-GDP ratio.[3] However, these studies are focused on explaining defaults in emerging economies and not applied to Greece where debt-to-GDP ratio is substantially higher.

As for the severe and lengthy economic contraction, strategic default models cannot replicate the data observed in Greece. This is partly because the models assume the period of exclusion from financial markets after default is shorter than the data. For example, Arellano (2008) and Yue (2010) set the average exclusion period to be less than 1 year so that the economy in the model recovers to the normal state at a rapid pace. In the data, as reported in Dias and Richmond (2009), the exclusion period for defaulting emerging countries ranges from 5.7 to 8.4 years. To address this issue, some recent studies such as Benjamin and Wright (2009), Bi (2008) and Asonuma and Joo (2019a) analyze the lengthy default duration as a result of prolonged debt restructuring negotiations. Alternatively, Gordon and Gerron-Quintana (2018), Park (2017) and Asonuma and Joo (2019b) emphasize the role of capital with which economic recovery can be delayed by the weak investment after the default. However, none of these studies analyzed both high debt-to-GDP and the severe economic contraction in Greece in a unified modelling framework. We argue that when the debt-to-GDP ratio is high and the adverse effects

[2] Paluszynski (2017) and Bocola, Bornstein and Dovis (2019) point out that those models do not properly describe the crisis.

[3] D'Erasmo (2011) emphasizes the role of the government's reputation in explaining high debt-to-GDP ratios.

of default are considerable as in the case of Greece, the government in strategic default models is not willing to default because it is too costly.[4,5]

To address these issues, we propose a dynamic stochastic general equilibrium (DSGE) model where the sovereign default is triggered by the government's inability to repay its debt, rather than the government's strategic decision (Bi 2012, Bi and Leeper 2013, Juessen, Linnermann and Schabert 2016). As Bi (2012) points out, this class of models can describe high debt-to-GDP ratios observed in advanced countries including Greece. Following Bi (2012), our inability-to-repay default model assumes that the government defaults when the debt outstanding exceeds the sum of discounted maximum future fiscal surpluses determined by the shape of the Laffer curve. The government imposes a tax on final goods, and issues new bonds in order to finance its expenditure and repayment. When debt-to-GDP ratio is low, the tax rate follows a linear rule. When debt-to-GDP ratio is higher, the government tries to satisfy the intertemporal government budget constraint as much as possible by raising the rate nonlinearly. However, the government cannot raise tax revenue beyond the fiscal limit level. Once defaults, the government introduces the austerity measure, namely an increase in the tax rate, to maximize tax revenue. The high tax rate during this period leads to a severe economic contraction. Similarly to Mendoza and Yue's (2012) model, the economy suffers from the effects of financial autarky after the default as well as the adverse effects arising from the high tax rate, because domestic firms cannot raise working capital necessary to import intermediate goods from abroad.

We compare this model with Mendoza and Yue (2012)'s strategic default model, both of which are calibrated to the Greek economy. We conduct a quantitative simulation that demonstrates the inability-to-repay default model closely captures the empirical characteristics of the Greek episode, i.e., the high debt-to-GDP ratio at the time of default and the severe contraction in GDP and its components following the default. In contrast, the strategic default model cannot generate the default itself.

[4] In addition to these two issues, the discount factor in many strategic default models is set to a fairly low value in order to replicate the default probability in the data. For example, the discount factor used in Aguiar and Gopinath (2006), Yue (2010) and Mendoza and Yue (2012) ranges from 0.74 to 0.88, which is substantially lower than the value widely used in existing studies. For example, Smets and Wouters (2003) use 0.99 as the discount factor.

[5] In fact, there are several anecdotes that support the view that Greek government tried hard to avoid the default. For example, the government increased both value added and income tax rates, implemented cuts in salary for public employees and reduced public pension payments. Consequently, the tax revenue-to-GDP ratio surged from 38.9 percent in 2009 to 44.0 percent in 2011. Besides, the government had been under strong external pressure from EU nations and the IMF to avoid the disorderly default when receiving emergency funding. These anecdotes are consistent with the empirical results of Levy-Yeyati and Panizza (2011) that policymakers tend to defer default until the government's insolvency become apparent.

This paper contributes to the literature in two aspects. First, to the best of our knowledge, this is the first paper that develops an inability-to-repay type of sovereign default model that is calibrated in detail to the Greek economy.[6] Second, we quantitatively investigate which of the two default mechanisms can describe both the high debt-to-GDP ratio and the severe economic contraction observed in Greek episode in a unified framework. We replicate both features with our calibrated inability-to-repay model. We also show that the strategic default model such as Mendoza and Yue (2012) is not able to explain Greek episode.[7]

This paper is organized as follows. In Section 2, we present both an inability-to-repay default model and a strategic default model. In Section 3, we explain the calibration strategy and solution methods. In Section 4, we conduct a quantitative analysis using models calibrated to Greece and compare the implications of these two models. Finally, we present concluding remarks in Section 5.

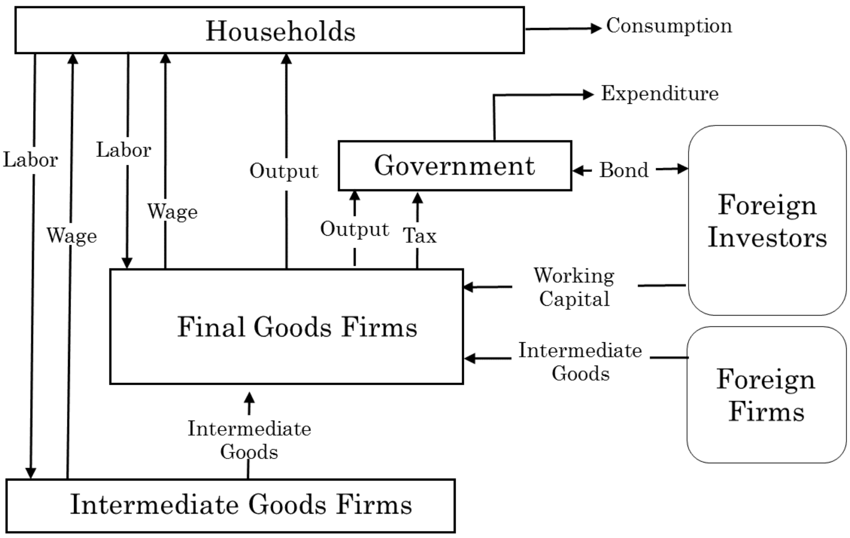
2 Model

In this paper, we construct two sovereign default models, each with its own distinct mechanism for triggering default. The common feature of the models is that both economies are small open economies, consisting of four domestic agents: households, final goods firms, intermediate goods firms and the government, and two foreign agents: foreign investors and foreign firms. An overview of the model is given in Figure 2. The setting of the private sector is borrowed from Mendoza and Yue (2012). Households consume final goods and supply their labor to both final and intermediate goods firms. Final goods firms produce final goods from labor and Armington-aggregated domestic and imported intermediate goods with time varying TFP that follows the AR(1) process as: $A_t = \rho_A A_{t-1} + \varepsilon_{A,t}$, where $\varepsilon_A \sim N(0, \sigma_A^2)$. Imported intermediate goods consist of a continuum of differentiated intermediate goods that are aggregated using a Dixit-Stiglitz aggregator. When importing the intermediate goods from abroad, final goods firms need to borrow working capital. This implies that when the government defaults, final goods

[6] In the literature, a range of research analyzes the interaction between sovereign default and nominal devaluation (e.g., Na et al. 2018). In this paper, however, following the existing studies on the Greek default episode in 2012 (e.g., D'Erasmo and Mendoza 2016), we abstract such effect on default. This is because Greece adopts euro as its currency, and its monetary policy is conducted by ECB which is literally independent from the Greek government.

[7] Acharya and Rajan (2013) develops a model in which a sovereign default can be triggered by either the inability to repay or by a strategic decision, and investigates the interaction between fiscal stability and financial stability.

Figure 2: Model Overview



firms lose access to the financial market and cannot import these intermediate goods. The absence of imported intermediate goods leads to a loss in production efficiency, and results in a decline in final goods production. Intermediate goods firms use only labor in their production processes. See Appendix A for details of the settings regarding private agents and the definition of the competitive equilibria under the two proposed models.

2.1 Government and Foreign Investors

At time t, the government repays its debt B_t issued in the previous period, spends government expenditure G_t, collects tax revenue T_t, and issues new government bonds B_{t+1} with price q_t. Government bonds are of one-period maturity, zero coupon and non-contingent, taking both negative and positive values. A negative value for issuance of government bonds indicates that the government's net assets are positive and the government receives interest income from foreign investors with the world risk-free interest rate r^f. The government imposes a tax on production of final goods Y_t. Thus, the total tax revenue T_t is given by:

$$T_t = \tau_t Y_t, \tag{1}$$

where $\tau_t \in [0,1]$ is the tax rate. In normal times, the government issues new government bonds B_{t+1} to meet the budget constraint:

$$q_t B_{t+1} \geq B_t + G_t - T_t. \tag{2}$$

If the government defaults, it does not repay its debt to foreign investors. Instead, international organizations (e.g. the IMF) bail out all of the debts and provide funding Φ_t^F to the government to make up its budget deficit. Note that at this state, the government cannot issue new bonds by itself due to exclusion from the financial market. Exclusion from the financial market continues for at least χ periods. After χ periods, the government returns to the financial markets with the exogenous probability ϑ, and with the exogenously given debt outstanding B^r.

Foreign investors are risk-neutral and behave as perfectly competitive agents, investing in both government bonds and working capital, taking all prices as given. If the government defaults, these investors receive only a reduced amount of government bond $(1-\delta)B_t$ from the international organizations, where δ is the haircut rate. From the expected zero profit condition of investment in government bonds, the bond is priced by:

$$q_t = \frac{1}{1+r^f}\{(1-P_t^e) + P_t^e(1-\delta)\}, \tag{3}$$

where P_t^e is the probability of default in the next period.

2.2 Default Scheme

In this section, we describe separately the two distinct default mechanisms, inability-to-repay debt and strategic decision to default.

2.2.1 Inability-to-Repay Default

Tax Rate Rule Under the inability-to-repay default model, the tax rate rule consists of three parts.

First, when the debt outstanding is sufficiently low, similar to Davig, Leeper and Walker (2010) and Bi (2012), the government increases its tax rate in proportion to the debt outstanding:

$$\tau_t^{lr} = \overline{\tau} + \kappa(B_t - \overline{B}), \tag{4}$$

where $\overline{\tau}$ and $\overline{B}$ are the steady-state tax rate and debt outstanding that are exogenously given, and $\kappa(\geq 0)$ is a tax rate adjustment parameter.

Second, when the debt outstanding reaches a threshold value above which the government cannot meet the intertemporal budget constraint, it adjusts the tax rate to satisfy the constraint.[8] The renewed intertemporal budget constraint is derived as:

$$B_t \leq \prod_{j=0}^{n} q_{t+j} B_{t+n+1} + \sum_{i=0}^{n} \prod_{j=0}^{i} q_{t+j-1}(T(\tau_{t+i}, A_{t+i}) - G(g_{t+i})), \tag{5}$$

where $q_{t-1} = 1$. Following Gali, López-Salido and Vallés (2007), g_t is the government expenditure expressed as deviations from the steady state, standardized by GDP, and following the AR(1) process, i.e., $g_t = \rho_g g_{t-1} + \varepsilon_{g,t}$, where $\varepsilon_{g,t} \sim N(0, \sigma_g^2)$. The transversality condition to ensure that the debt outstanding is not accumulated faster than the discount rate is:

$$\lim_{n\to\infty} \prod_{j=0}^{n} q_{t+j} B_{t+n+1} = 0. \tag{6}$$

The intertemporal budget constraint with the transversality condition is derived as:

$$B_t \leq \sum_{i=0}^{\infty} \prod_{j=0}^{i} q_{t+j-1}(T(\tau_{t+i}, A_{t+i}) - G(g_{t+i})). \tag{7}$$

This constraint indicates that the current debt obligation has to be smaller than the sum of the primary surplus over the entire future periods discounted by the price of government bonds. The government tries to satisfy this constraint by changing the tax rate τ_t that affects the tax revenue T_t while the debt outstanding B_t is assumed to be predetermined and government expenditure G_t is assumed to be exogenous. If the government maintains the current tax rate τ_t in the future, the sum of the expected fiscal surplus evaluated by the price of government bonds S_t (hereafter, expected surplus) is:

$$S_t = S(\tau_t, A_t, g_t) = \mathrm{E}_t \sum_{i=0}^{\infty} \prod_{j=0}^{i} q_{t+j-1}(T(\tau_t, A_{t+i}) - G(g_{t+i})). \tag{8}$$

If the tax rate determined by the linear tax rate rule (4) does not satisfy the condition $B_t \leq S(\tau_t, A_t, g_t)$, i.e., the amount of debt outstanding B_t must be less than or equal to

[8] More precisely, the government sets the sequence of a specific tax rate that takes the same value from the current period to the infinite future, such that it satisfies the intertemporal government budget constraint.

the expected surplus, the government needs to deviate from the linear rule and adjust the tax rate to satisfy the condition, $B_t = S(\tau_t, A_t, g_t)$. Thus, the tax rate in this case, τ_t^{es}, is:

$$\tau_t^{es} = S^{-1}(B_t, A_t, g_t). \tag{9}$$

The threshold of the debt outstanding B_t at which the debt outstanding exceeds the expected surplus under the linear rule is denoted by $Z^{int}(A_t, g_t)$.

Third, we assume that the tax rate is bounded from above. The government is not able to set a tax rate above some point at which the direct increase in tax revenue from the tax rate increase is fully offset by the decrease in tax revenue that arises from the drop in final goods due to the increase in the tax rate. In other words, in this case, the economy is at the top of the Laffer curve. Moreover, following Bi, Shen and Yang (2014), we assume that the government faces political disturbance $\varrho \in [0, 1]$ that prevents the government from setting the tax rate at the top of the curve. This political disturbance is interpreted as policymakers' lacking the power or willingness to impose a severe austerity policy because such policy is often accompanied by a heavy burden for taxpayers. Thus, the maximum tax rate that the government can set, τ_t^{ma}, is:

$$\tau^{ma}(A_t) = (1 - \varrho) \cdot \left(\arg\max_{\tau_t} T(\tau_t, A_t) \right). \tag{10}$$

The sum of expected maximum fiscal surplus discounted by the price of government bonds, denoted by Z_t^{ma}, is:

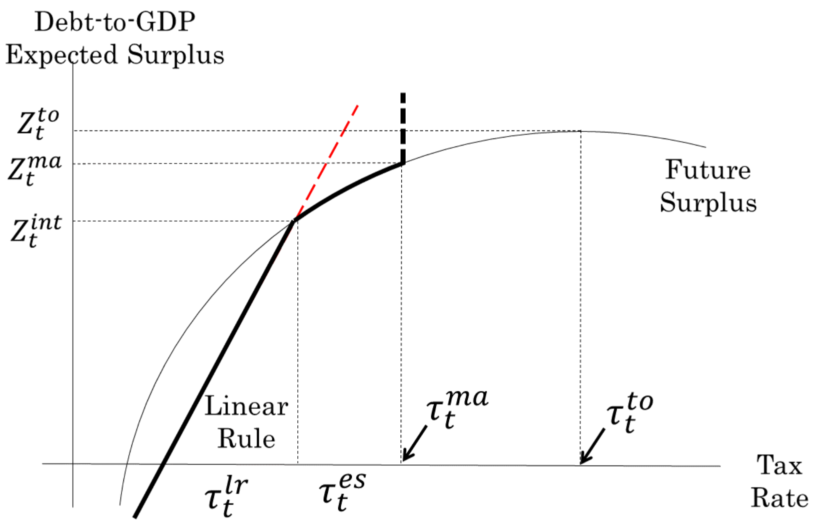
$$Z^{ma}(A_t, g_t) = \mathrm{E}_t \sum_{i=0}^{\infty} \prod_{j=0}^{i} q_{t+j-1}(T(\tau_{t+i}^{ma}, A_{t+i}) - G(g_{t+i})). \tag{11}$$

If the debt outstanding exceeds the expected maximum surplus $Z^{ma}(A_t, g_t)$, the government does not increase the tax rate further, because of political disturbance, and declares a default. The government's tax rate rule is summarized as:

$$\tau_t = \begin{cases} \tau_t^{lr} & \text{if } B_t \leq Z^{int}(A_t, g_t), \\ \tau_t^{es} & \text{if } Z^{int}(A_t, g_t) < B_t \leq Z^{ma}(A_t, g_t), \\ \tau_t^{ma} & \text{if } B_t > Z^{ma}(A_t, g_t) \end{cases} . \tag{12}$$

Figure 3 illustrates this tax rate rule.[9,10]

Figure 3: Tax Rate Rule under Inability-to-Repay Default Model



Finally, in the default state, the government is forced by international organizations to introduce the austerity measure and to set its tax rate so that it can collect the maximum tax revenue without political disturbance, namely the rate that corresponds to the top of the Laffer curve (τ_t^{to}).

Default Region In the inability-to-repay default model, the government defaults when the debt outstanding exceeds the fiscal limit. The fiscal limit b_t^* is drawn from the distribution of the sum of discounted maximum fiscal surpluses denoted by B^*:[11]

$$B^*(A_t, g_t) = \sum_{i=0}^{\infty} \prod_{j=0}^{i} q_{t+j-1}(T(\tau_{t+i}^{ma}, A_{t+i}) - G(g_{t+i})). \tag{13}$$

Whether the economy is in the default state or not at the beginning of period t is denoted by $\xi_t \in \{\xi_{n,t}, \xi_{d,t}\}$, where $\xi_{n,t}$ and $\xi_{d,t}$ represent non-default state and default

[9] If $Z^{int}(A_t, g_t) > Z^{ma}(A_t, g_t)$, the tax rate rule only consists of two parts: τ_t^{lr} and τ_t^{ma}.

[10] As shown in Figure 3, the linear tax rule alone does not correspond to the default at expected maximum surplus, failing to explain the high debt to GDP upon default as observed in the data.

[11] As Ghosh et al. (2013) points out, a small shock to the current primary balance can lead to a substantial difference in the fiscal limit. For example, if the interest rate is 1 percent, a 1 percentage point increase in the current fiscal surplus relative to GDP pushes up the fiscal limit relative to GDP by 100 percentage points.

state respectively.[12] The default region $\Gamma^I(B_t, \xi_{n,t}, b_t^*)$ that shapes the set of TFP and government expenditure in which the debt outstanding exceeds the fiscal limit in the next period is defined as:

$$\Gamma^I(B_t, \xi_{n,t}, b_t^*) = \{A_t \in \mathbb{A},\, g_t \in \mathbb{G} : B_t > b_t^*,\, \xi_t = \xi_{n,t}\}. \tag{14}$$

The probability of inability-to-repay default in the next period $P_t^{I,e}$ is the probability that TFP and government expenditure shocks fall into the default region:

$$P^{I,e}(A_t, g_t, B_{t+1}, \xi_{n,t+1}, b_{t+1}^*) = \int\int_{\Gamma^I(B_{t+1}, \xi_{n,t+1}, b_{t+1}^*)} f^A(A_{t+1}, A_t) f^g(g_{t+1}, g_t) dA_{t+1} dg_{t+1}, \tag{15}$$

where f^A and f^g are the transition probability function of TFP and government expenditure respectively. Note also that, foreign investors are assumed to price the government bonds depending on the probability of inability-to-repay default, i.e., the probability of default is set at $P_t^e = P_t^{I,e}$.

2.2.2 Strategic Default

In this subsection, we explain the alternative default mechanism; strategic decision to default that is widely studied in the existing literature. In this alternative model, the government chooses to default if and only if defaulting is the better option for the government.[13]

Tax Rate Rule Similarly to existing studies, we assume that the government sets its tax rate following the linear rule (4).

Default Region The government makes the decision regarding its default so as to maximize households' lifetime utility. Thus, the government's optimization problem is

[12] Obviously, the transition to the default state can take place only from the non-default state (i.e. $\xi_t = \xi_{n,t}$).

[13] The implicit assumption commonly made in strategic default models is that there is an upper bound regarding the debt-to-GDP ratio in the economy, and the government decides to default or not under this upper bound. This assumption rules out Ponzi schemes. For example, Mendoza and Yue (2012) assume the largest debt amount the government can repay with full commitment as output over risk-free interest rate. One key difference between this upper bound and the fiscal limit considered in the inability-to-repay model is that the former considers the use of lump-sum tax whereas the latter does not. Following the convention of the strategic default model, we assume in the analysis below that the government exploits the lump-sum tax to meet the intertemporal government budget constraint when studying the strategic default model. Note also that when studying the inability-to-default model, we assume that the government is unable to use a lump-sum tax.

formulated as the maximization of its value function defined as:

$$V(A_t, g_t, B_t, \xi_{n,t}) = \max_{d_t \in \{0,1\}} \{(1-d_t)V_n(A_t, g_t, B_t, \xi_{n,t}) + d_t V_d(A_t, g_t, \xi_{n,t})\}, \tag{16}$$

where d_t represents the government's default decision, taking 1 for default and 0 for non-default, and V_n and V_d represent the government's value functions corresponding to the non-default state and default state respectively. They are defined as:

$$V_n(A_t, g_t, B_t, \xi_{n,t}) = u(C_t, L_t) + \beta \mathrm{E}_t[V(A_{t+1}, g_{t+1}, B_{t+1}, \xi_{n,t+1})|(A_t, g_t, B_t, \xi_{n,t})], \tag{17}$$

and

$$\begin{aligned} V_d(A_t, g_t, \xi_{n,t}) &= u(C_t, L_t) + \beta\{(1-\vartheta)\mathrm{E}_t[V_d(A_{t+1}, g_{t+1}, \xi_{d,t+1})|(A_t, g_t, \xi_{n,t})] \\ &\quad + \vartheta \mathrm{E}_t[V_n(A_{t+1}, g_{t+1}, B^r, \xi_{n,t+1})|(A_t, g_t, \xi_{n,t})]\}, \end{aligned} \tag{18}$$

where $\beta \in (0,1)$ is the discount factor and ϑ is an exogenously given recovery probability with which the defaulted government reverts back to the non-default state.[14] The default region Γ^S and the probability of default $P^{S,e}$ are defined as:

$$\Gamma^S(B_t, \xi_{n,t}) = \{A_t \in \mathbb{A},\ g_t \in \mathbb{G} : V_d(A_t, g_t, \xi_{n,t}) > V_n(A_t, g_t, B_t, \xi_{n,t}),\ \xi_t = \xi_{n,t}\}, \tag{19}$$

and

$$P^{S,e}(A_t, g_t, B_{t+1}, \xi_{n,t+1}) = \int\int_{\Gamma^S(B_{t+1}, \xi_{n,t+1})} f^A(A_{t+1}, A_t) f^g(g_{t+1}, g_t) dA_{t+1} dg_{t+1}. \tag{20}$$

The price of the government bond depends on the probability of strategic default as $P_t^e = P_t^{S,e}$.

3 Calibration and Solution Method

For the purpose of the quantitative exercise focusing on the Greek default episode in 2012, we calibrate the two models to the Greek economy. In this section, we explain the calibration strategy and solution method.

[14] In this model, following Mendoza and Yue (2012), we assume $\chi = 0$.

3.1 Calibration Strategy

Most parameters are calibrated to the data from 1999Q1 to 2016Q4 except some deep parameters that are set to be consistent with existing studies.

Government Regarding the parameters related to the government's bond price, the discount factor β is set at 0.99 following Smets and Wouters (2003), which implies the risk-free interest rate r^f is 0.01. We set the haircut rate δ to 0.05 so as to be consistent with the actual bond spread, i.e., the quarterly long-term interest rate differential of Greek government bonds relative to the German bonds in 2012.[15] The government debt outstanding at the steady state $\overline{B}$ is set so that the debt-to-GDP ratio at steady state is 104 percent, which matches with the historical average in Greece from 1999 to 2008.[16,17]

As for the parameters related to fiscal surplus/deficit, we set the steady state government expenditure-to-GDP $\overline{G}$ to 0.385, to match the data from 1999 to 2008.[18] To close the fiscal surplus/deficit at the steady state, the steady-state tax rate $\overline{\tau}$ is set so that the tax revenues are equalized to $\overline{G}$.

Private Agents The households' utility function is of the Greenwood-Hercowitz-Huffman (1988) preference[19] as $u(c_t, L_t) = \frac{\left(C_t - \frac{L_t^{1+\eta}}{1+\eta}\right)^{1-\sigma} - 1}{1-\sigma}$, where σ is the degree of risk aversion, which we set to unity so that the function is the log-utility, and η is the inverse Frisch elasticity of labor supply, which we set at 0.455, the standard value used in existing RBC models.

As for firms, the intermediate input share in final goods production α_M is set to 0.39 to match the total intermediate consumption over gross final goods in Greece in the early 2000s, taken from the OECD's STAN Input-Output data. The labor input share in final goods production α_L is set at 0.42 so that the proportion of labor input to total input minus intermediate input share (i.e., $\alpha_L/(1-\alpha_M)$) is equivalent to 0.7,

[15] More precisely, the haircut rate is set by matching the model produced bond spread with the observed spread. The ex-post haircut rate of Greek debt restructuring was about 60 percent (Zettelmeyer, Trebesch and Gulati 2013). However, the amount of defaulted debt was limited to the equivalent of about 100 percent of its GDP. GDP-linked warrants were also granted to investors. Thus, the effective haircut rate could be much lower than the ex-post haircut rate observed in the data.

[16] The reason we drop the data in 2008 is that this is 1 year ahead of the year when underreporting of the government's debt outstanding and deficit is revealed.

[17] The government debt outstanding is not measured on a net basis but on a gross basis because of data limitations. However, as the gap between gross and net debt was less than 10 percent of GDP in Greece, it does not have any significant quantitative effect on the result.

[18] Government expenditure is measured as its total expenses, including social benefits, and excluding interest payments.

[19] Under the Greenwood-Hercowitz-Huffman preference, labor supply depends only on the wage rate.

which is often assumed in most literature. The Armington weight of domestic inputs λ is set at 0.63, to match the share of domestic intermediate goods-to-GDP, taken from the OECD's STAN Input-Output data. The substitution elasticity between domestic and foreign intermediate goods ψ is set at 0.9, following Evers (2015). Dixit-Stiglitz curvature parameter ν is set at 3, following Feenstra et al. (2018). The upper bound of imported inputs with working capital θ is set at 0.26 according to the World Bank data in 2005, and time invariant TFP for intermediate goods production A^I is set at 0.31, following Mendoza and Yue (2012). The labor share in the production of intermediate goods γ is set at 0.7.

Other Parameters Parameters associated with shocks are set using HP-filtered corresponding time series. These parameters include the auto-regressive parameter and the standard deviation of TFP shock, 0.600 and 0.015, and those of government expenditure shock, 0.650 and 0.040 respectively. We set the period of exclusion from the financial market after default χ to 26 quarters,[20] and the probability of recovery to the non-default state ϑ to 0.044 following Dias and Richmond (2009).

Tax Rate Rule under Inability-to-Repay Default Model To calibrate the parameters regarding the fiscal rule we observe developments in the tax rate and the debt-to-GDP ratio in Greece. Figure 4 (a) shows the historical relation between debt-to-GDP ratios and tax rates.

The data can be roughly categorized into four sub-periods. First, before the debt-to-GDP ratio hit around 100 percent, namely from 1984 to 1992, these two variables were positively correlated. Second, from 1993 to 2008, the tax rate ranged between 30 percent and 40 percent while the debt-to-GDP ratio remained at around 100 percent. Third, after the onset of the global financial crisis in 2008, the debt-to-GDP ratio rose at quite a rapid pace, while the pace of tax increase was moderate. Finally, the government defaulted in 2012, and both the debt-to-GDP ratio and the tax rate have been elevated since then.

In the model, the parameter of the linear tax rate rule is calibrated to the data from 1984 to 1992 using OLS estimation. The political disturbance ϱ is set at 0.08, so that the tax rate is consistent with the observed rate. In 2011, a year before the default, the tax rate was about 8 percentage points lower than the post-default average during 2012-16, which implies that the government could have increased the tax rate by about 8 percentage points to increase tax revenue, if it had not been facing political concerns.

[20]This is because it took about 6.5 years for the Greek government to complete the final bailout program funded by the European Stability Mechanism (ESM).

Table 1: Calibration to the Greek Economy

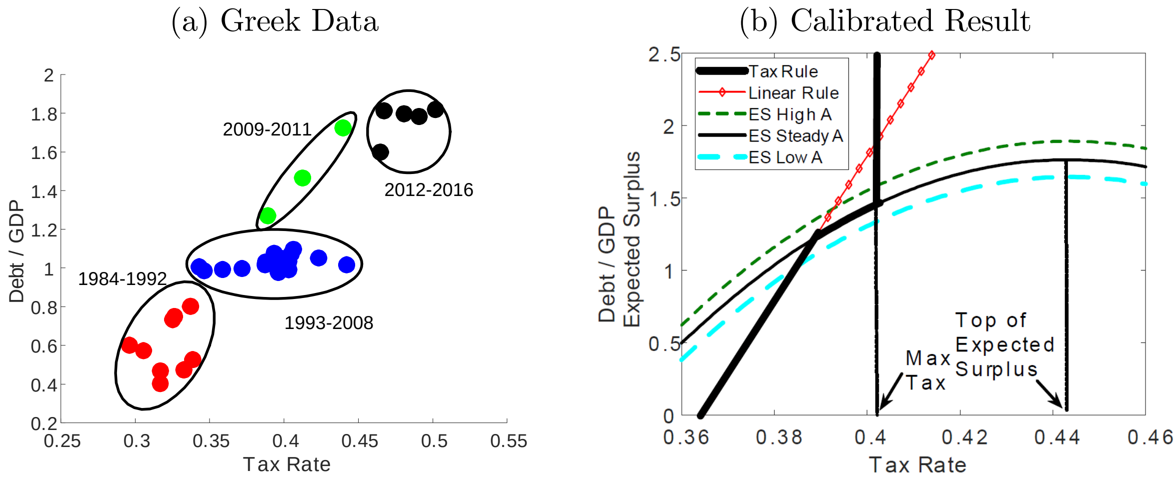
Parameter	Value	Target/Source
σ: Risk aversion	1	Log-utility
η: Inverse Frisch elasticity of labor supply	0.455	Standard value
β: Households' discount factor	0.99	Smets and Wouters (2003)
α_M: Intermediate input share in final goods production	0.39	OECD
α_L: Labor input share in final goods production	0.42	Standard value
λ: Armington weight of domestic inputs	0.63	OECD
ψ: Substitution elasticity across intermediate goods	0.9	Evers(2015)
ν: Substitution elasticity within intermediate goods	3	Feenstra et al. (2018)
θ: Imported goods with working capital	0.26	World Bank
A^I: TFP for intermediate goods production	0.31	Mendoza and Yue (2012)
r^f: World risk-free interest rate	0.01	Smets and Wouters (2003)
γ: Labor input share in intermediate goods production	0.7	Standard value
δ: Haircut rate	0.05	Bond premium in 2012
ρ_g: Persistence of gov't expenditure shock	0.65	IMF
σ_g: Standard deviation of gov't expenditure shock	0.04	IMF
ρ_A: Persistence of TFP shock	0.6	OECD
σ_A: Standard deviation of TFP shock	0.015	OECD
$\overline{G}$: Steady-state government expenditure	0.0163	IMF ($\overline{G}/GDP = 0.385$)
$\overline{B}$: Steady-state government debt	0.0440	IMF ($\overline{B}/GDP = 1.04$)
$\overline{\tau}$: Steady-state tax rate	0.385	Steady-state gov't expenditure
κ: Elasticity of tax rate	0.02	OLS
ϱ: Political disturbance	0.08	IMF
χ: Period of exclusion	26	Duration of Bailout Program
ϑ: Probability of recovery	0.044	Dias and Richmond (2009)

Figure 4 (b) shows the calibrated tax rate rule under the inability-to-repay default model. In the area where the debt-to-GDP ratio is below 100 percent and the tax rate is below 39 percent, the government sets its tax rate according to the linear rule (line marked with circles). In the area where the debt-to-GDP is higher than 100 percent and the tax rate is above 39 percent, the line exceeds the expected surplus (solid curve) derived from the equation (8). In this area, to sustain the high debt-to-GDP ratio, the government deviates from the linear rule and sets the higher tax rate on the curve. Once the tax rate reaches the maximum tax rate that is estimated to be 40.5 percent, the government cannot contain any further increase in the debt-to-GDP ratio by raising the tax rate, because of political pressures, and declares the default.

Table 1 summarizes all calibrated parameters for the Greek economy.[21]

[21] The explanation of calibrated parameter values for the Argentinean economy is provided in Appendix E.

Figure 4: Tax Rate Rule



Notes 1: The level of GDP in the right panel (b) is the steady-state value.

2: ES represents the expected surplus.

3: "High A" and "Low A" represent 5 percent higher and lower than the steady-state TFP respectively.

Source: OECD

3.2 Solution Method

There are two steps to solving the inability-to-repay model. In the first step, we compute the mapping from a given productivity level A_t and expenditure level g_t to the expected surplus and fiscal limit,[22] since the fiscal limit is a function of realizations of future shocks to these variables that are drawn by Monte-Carlo sampling. In the second step, based on the mapping, we solve the model on the discrete state space (DSS). To obtain the DSS, following Coleman (1991) and Davig (2004), we find a fixed point of the decision rule for government bonds by using the monotone map method for each of three tax rate cases, τ_t^{lr}, τ_t^{es} and τ_t^{ma} respectively. Then, we select the sequences of the tax rate and corresponding endogenous variables, following the tax rate rule in (12).

As for the solution of the strategic default model, we apply a two-loop algorithm. The outside and inside loops iterate the price of government bonds and government value functions in both non-default and default states respectively. A detailed explanation of the computational procedure is given in Appendix B.

[22] Following Bi (2012), to compute the fiscal limit, we assume that the future government bond is priced under the assumption of a zero default probability, i.e., the price of government bonds is only discounted by the risk-free interest rate.

4 Quantitative Analysis

In this section, we quantitatively investigate how each of the two proposed models fits with the Greek experience. As we explained in the previous section, the government defaults when the current TFP and government expenditure fall into the default region.

In what follows, we study the default regions under the two proposed models and investigate which of the two models can provide more plausible explanation of the Greek default episode in 2012. Then we examine the simulated path of the economy under the two models and see whether they can capture the severe and long-lasting economic contraction around the default.

4.1 Trigger of Default

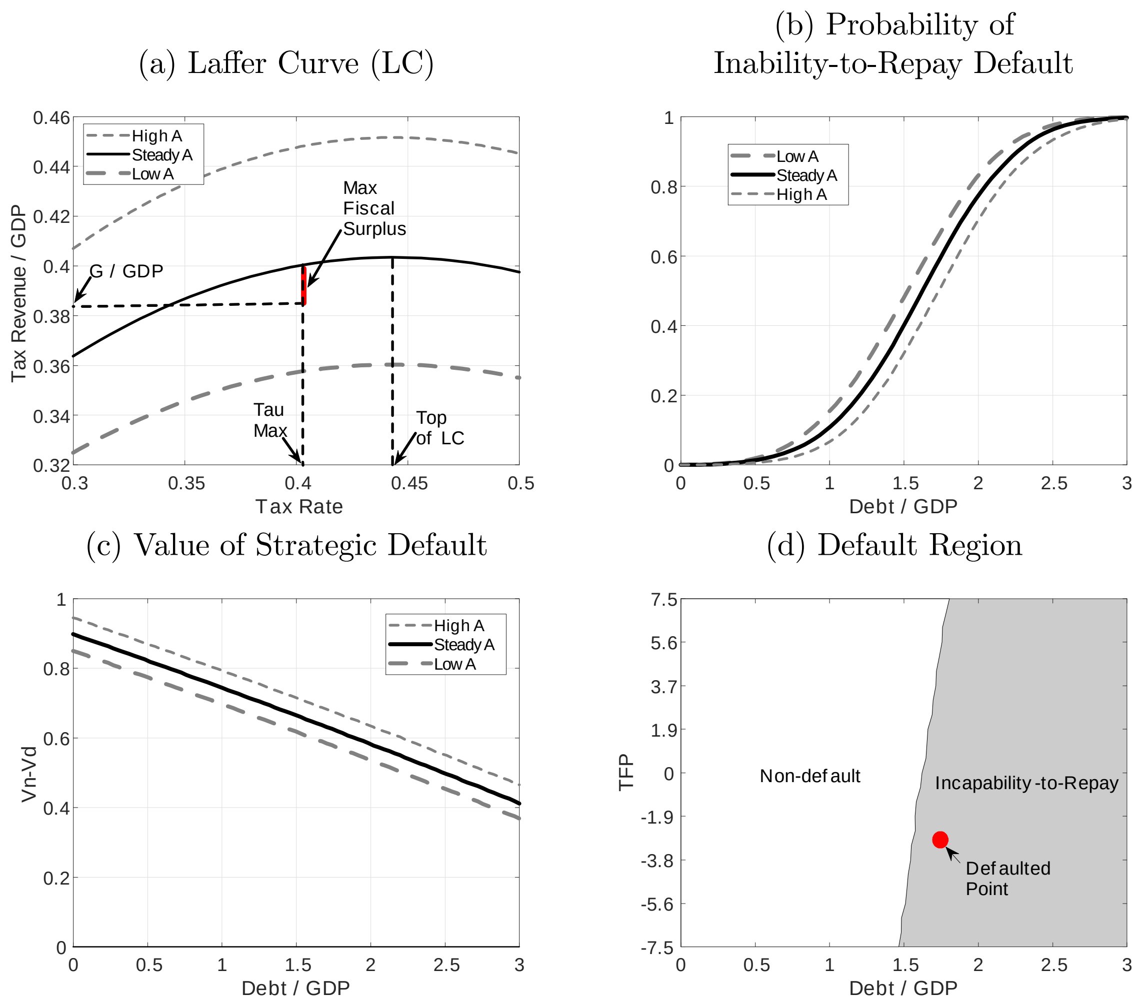
To investigate the trigger of the default, we compute the default regions constructed by the two models.

First, we compute the Laffer curve of the economy in the inability-to-repay default model. Panel (a) in Figure 5 shows the Laffer curve under different values of TFP, taking the tax rate on the horizontal axis and the tax revenue over steady-state GDP on the vertical axis.[23] The tax rate at the top of the Laffer curve is about 44.5 percent, and the maximum tax rate is about 40.5 percent. The maximum tax revenue over the steady-state GDP is about 40 percent. This means that the maximum fiscal surplus is about 1.5 percent, i.e., the government can potentially accumulate about 1.5 percent of fiscal surplus over GDP, provided that the steady-state government expenditure-to-GDP is maintained at 38.5 percent. When TFP is 5 percentage points higher (lower), the tax revenue relative to GDP is about 5 percentage points higher (lower) than the steady state. Panel (b) illustrates the probability of inability-to-repay default under different values of current TFP, where we suppose that government expenditure is fixed at the steady state. If the current TFP is at the steady state (bold line) and the debt-to-GDP ratio is around 100 percent, then the probability of default is around 10 percent. The probability increases with debt-to-GDP ratio and reaches 50 percent when the debt-to-GDP ratio reaches about 160 percent.

Next, we show the value functions for the strategic default model. Panel (c) shows the difference between the two value functions, i.e., the value function if the government chooses not to default and if it chooses to default, under different values of the current

[23] The steady-state GDP is computed, by feeding into the model the steady-state values of TFP and government expenditure.

Figure 5: Comparison of Two Sovereign Default Models (Greece)



Notes 1: Debt-to-GDP is measured relative to the steady-state GDP.
2: TFP in panel (d) shows the deviation from the steady state.
3: Government expenditure is fixed at the steady state.

TFP. The difference is positive over the plausible range of debt-to-GDP ratio below 300 percent. This suggests that the government has no incentive to choose to default.

For comparison, panel (d) illustrates the default regions with various values of TFP given that government expenditure is fixed at the steady state. The default region of the inability-to-repay default is located in the area where the debt-to-GDP ratio is higher than 150 percent, while the region of strategic default does not appear in this panel as implied in panel (c). In other words, over the range considered in panel (d), the government does not choose to default. The dotted point in the panel represents the pair of observations of debt-to-GDP ratio and TFP that was observed at the onset of the default in Greece in 2012. Overall, these pictures suggest that the Greek default was likely attributable to the government's inability to repay its debts rather than the government's strategic decision.[24]

4.2 Simulation Analysis

In this section we examine the quantitative performance of the inability-to-repay default model in explaining the severe economic contraction after the default in Greece. We set the initial debt-to-GDP ratio to 80 percent, and the initial values of both TFP and government expenditure at the steady-state values. Then we generate stochastic TFP and government expenditure shocks 5,000 times over 500 quarters, and compute the equilibrium time path of endogenous variables.

Table 2: Summary Statistics

		Average Before Default 2009Q1-2011Q4		Average After Default 2012Q1-2015Q1	
	Unit	Data	Model	Data	Model
Debt-to-GDP	Percent	141.1	148.1	167.6	170.9
Probability of Default	Percent	34.5	42.1	–	–
Bond Spread	% Points	7.2	7.0	–	–
Tax Rate	Percent	36.4	39.8	42.6	44.3

Table 3: Cumulative Changes around the Default

	Economic Impact by Default		
	Unit	Data	Model
GDP	Percent	-21.0	-13.8
Consumption	Percent	-20.1	-21.6
Imports	Percent	-13.2	-31.0

Note: Cumulative changes represent the percent change of the variables from 3 years before the default to 3 years after the default.

Table 2 reports the summary statistics of the key variables in the data and simulation before and after the default. This suggests that the inability-to-repay default model successfully replicates these variables.[25]

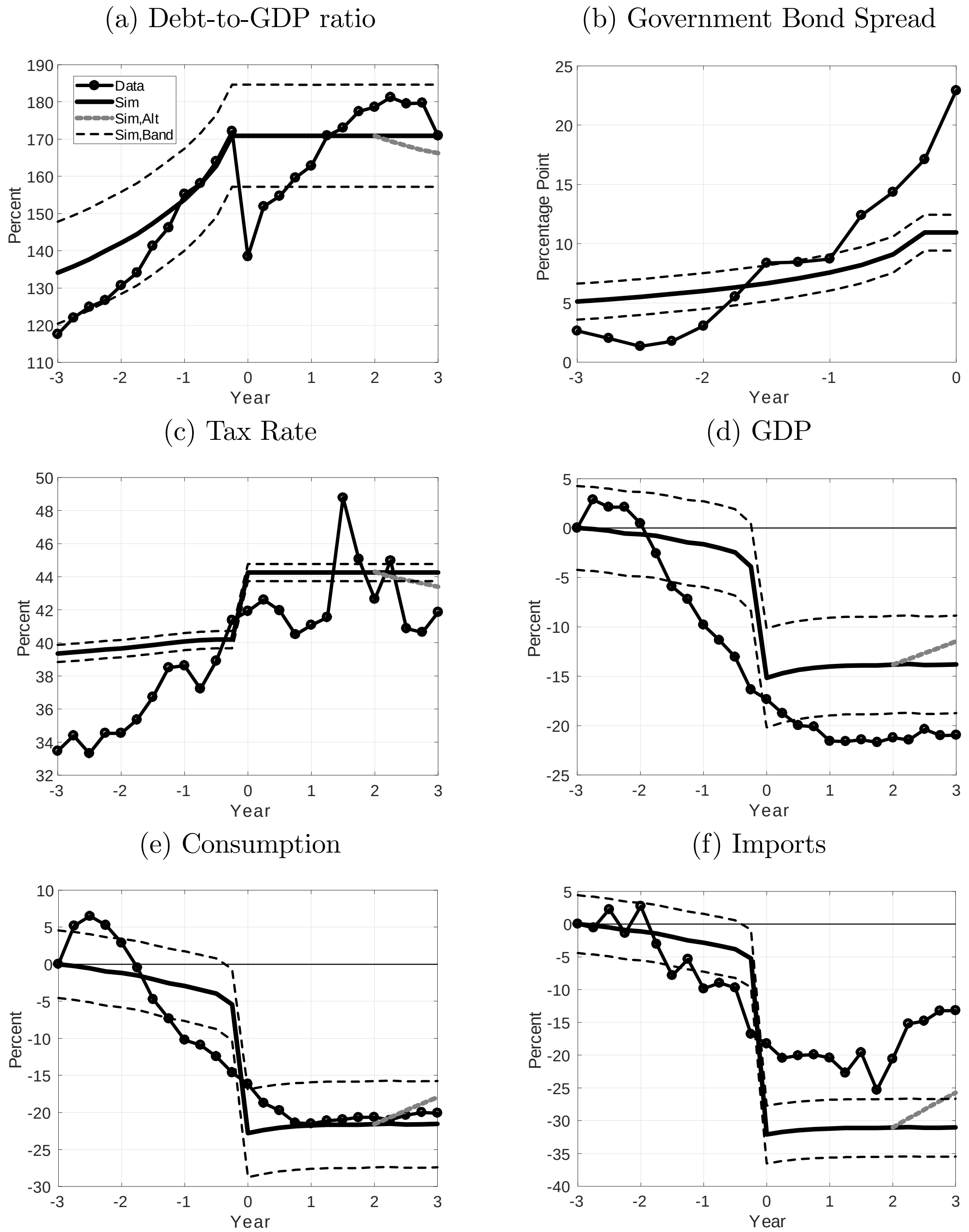
Regarding other endogenous economic variables, Table 3 shows the cumulative changes in GDP and its components from 3 years before the default until 3 years after the default.

[24] In Appendix C, we conduct robustness analysis for different values of government expenditure and confirm that the results are not substantially altered.

[25] The bond spread in the data is the 10-year Greek government bond yield against the 10-year German government bond yield on a quarterly basis.

This suggests that the model can replicate the significant drop in GDP, consumption, and imports around the default.

Figure 6: Key Endogenous Variables (Simulation vs Data)



Notes 1: Year 0 is set at 2012Q1 for the data.

2: 2Y-Alt simulation represents simulation results with 2 years of exclusion from the financial market.

Figure 6 illustrates the developments in the key variables in both data and simulation before and after the default. Panel (a) shows the mean of the baseline simulation path of debt-to-GDP ratio for defaulted samples. The simulated path (bold line) closely tracks the realized path of the debt-to-GDP ratio within the one standard error band. Panel (b) shows the mean of the simulation path of the spread. It illustrates that the nonlinear increase in spread is somewhat captured by the simulation path, although some periods of deviation from the error band are observed. Given the market turbulence in this period, the deviation could be explained by other factors such as the effect of overall uncertainty regarding the euro area sovereign debt problem. Panels (d), (e), and (f) describe the cumulative changes in GDP, consumption, and imports from 3 years before the default. The model closely replicates the data as the deviations fall mostly in the one standard error band. [26,27]

Overall, the model generally accommodates features of the highly nonlinear dynamics of key economic variables around the default.

4.3 Discussion: Argentinean Default in 2001

We have so far focused on the case of the Greek default throughout this paper. Because a good number of recent studies of the sovereign default analyze the Argentinean default in 2001, however, it would be worthwhile to apply both the inability-to-repay default model and the strategic default model to the Argentinean economy and highlight the differences between the two models.[28]

Essentially, the Argentinean default is attributable to the government's strategic decision rather than its inability to repay its debt. The Argentinean default is not considered as having been triggered by the government's inability to repay its debt because, given that economic contraction following the default was moderate, the government was considered to have been less reluctant to choose default. Panel (a) in Figure 7 shows the Laffer curve in Argentina and the maximum fiscal surplus. The tax revenue over GDP that the government can achieve when the tax rate is set at the top of the Laffer curve is about 32 percent, which is about 10 percentage points larger than the steady-state government expenditure. Following Bi, Shen and Yang (2014) we set ϱ to 0.33.[29] As a

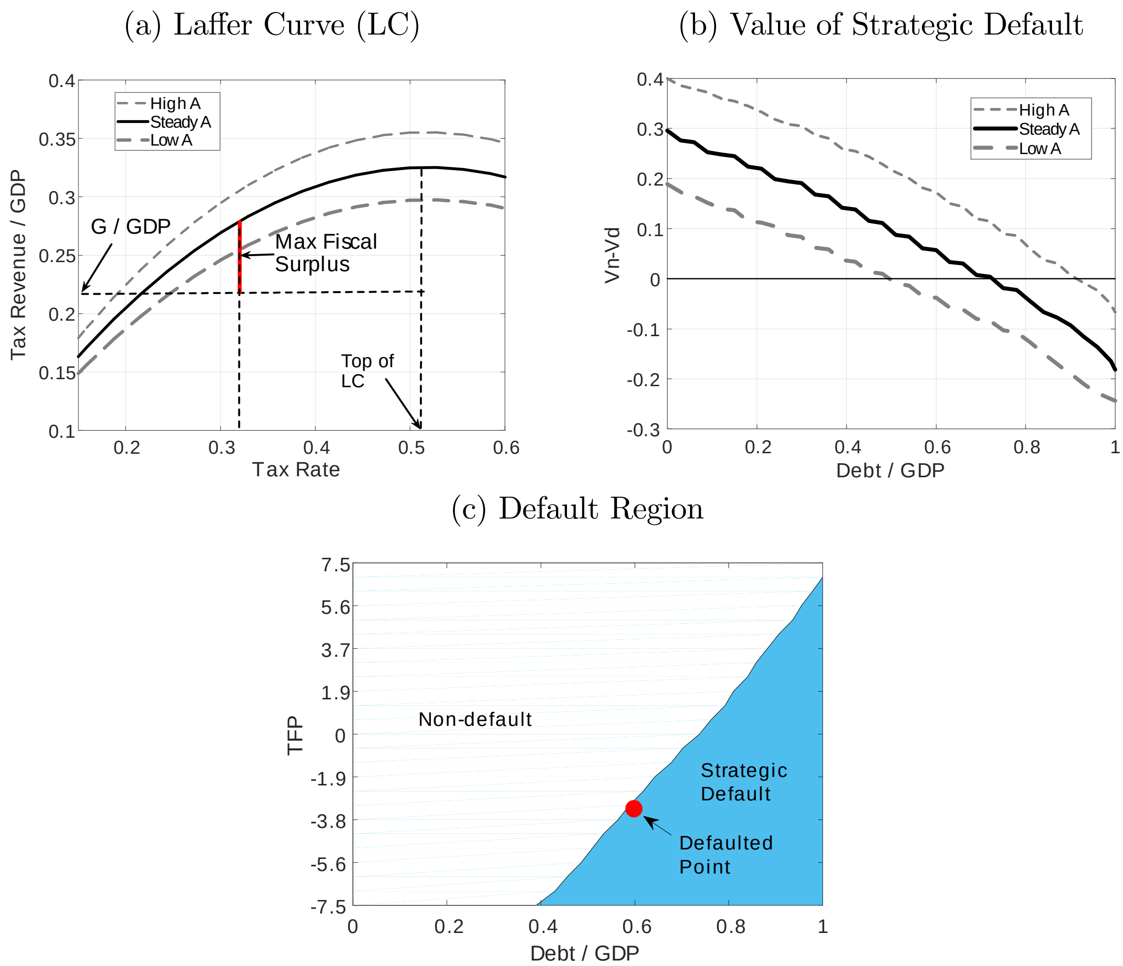
[26] In this analysis, we assumed that the period of exclusion from financial markets is at least 26 quarters. This is relatively long when compared with existing studies.

[27] To verify the robustness of the results, we also present the mean of simulated path in which the default state continues for only 8 quarters, the standard assumption used in the literature. We find that the overall picture is little altered by this change in the setting.

[28] We explain the calibration strategy and results of the Argentinean economy in Appendix E.

[29] Bi, Shen and Yang (2014) applies an inability-to-repay type model to Argentinean economy and subtract 33 percent of the tax rate in all future periods by referring to the Argentinean political risk

Figure 7: Comparison of Two Sovereign Default Models (Argentina)



Notes 1: Debt-to-GDP is measured relative to the steady-state GDP.
2: TFP in panel (c) shows the deviation from the steady state.
3: Government expenditure is fixed at the steady state.

result, the fiscal surplus would be about 5 percent at the steady state and the potential fiscal limit is about 250 percent of GDP, which is substantially higher than the actual debt-to-GDP ratio.

Regarding the strategic default model, we assume that the tax rate in the default state is 5 percentage points lower than that in the non-default steady state, based on the fact that the historical average of the tax rate in Argentina during 2002-03 was 5 percentage points lower than that of 1998-2002 in Argentina. Panel (b) shows the difference of value functions constructed by the strategic default model, i.e., non-default value function minus the default value function. Given the steady-state value of TFP, the value function at the default state exceeds the value function at the non-default state

factor taken from the International Country Risk Guide's (ICRG's) index in order to replicate the low fiscal limit in emerging countries such as Argentina.

for debt-to-GDP ratio by more than 70 percent. In other words, this value is considered as the threshold at which strategic default is triggered.

Panel (c) illustrates the default region for various values of TFP and debt-to-GDP ratio. Again, we assume that government expenditure is at the steady state. The inability-to-repay default region does not appear in this picture where debt-to-GDP ratio ranging from 0 to 100 percent is considered. The actual TFP and debt-to-GDP ratio at the time the government defaulted in 2001 falls into the strategic default region in the panel.

The key difference from the Greek case is that Argentina had more room to accumulate future fiscal surplus than Greece, which in turn made inability-to-repay default less likely. Besides, choosing to default was considered less attractive to the Greek government than the Argentinean government. This is because, given the low tax rate after default in Argentinean data at the time of its default, economic contraction was relatively mild, and the government could improve households' welfare by choosing to default and relieve itself of having to continuously make interest payments.

5 Conclusion

We propose a DSGE model in which a sovereign default is triggered by a government's inability to repay its debt because the government has limited capacity to increase its fiscal surplus. We compare this model with the strategic default model in which the government chooses to default to avoid the burden of debt. We conduct a quantitative analysis that shows the Greek default in 2012 was attributable to the government's inability rather than to its strategic decision, this being consistent with the view that, fearing default would lead to a large economic contraction, the Greek government had no incentive to choose to default. In addition, our simulation study shows that the model can replicate the high debt-to-GDP ratio and the substantial drop in GDP and its components.

Admittedly, the economic environments considered in the current paper are quite simple, and there is potential room to add various elements in the economy not covered here that have an important bearing upon a government's decision to default. For example, as Borensztein and Panizza (2009) mentioned, to bring the model closer to the data, it may be beneficial to consider the banking sector and the government's strategic decisions that depend on which sector is holding bonds. Further research along these lines is left as our future research agenda.

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Appendix A. Private Agents and Competitive Equilibrium

A.1 Households

A representative household derives utility from consumption C_t and disutility from labor L_t. The household provides its labor L_t to both final and intermediate goods firms, and receives labor income $w_t L_t$ from these two types of firms and profits π_t^f and π_t^m from each type of firm respectively, where w_t is the wage rate. The household solves the following utility maximization problem:

$$\max_{C_t, L_t} \mathrm{E}_t \left[\sum_{i=0}^{\infty} \beta^i u(C_{t+i}, L_{t+i}) \right], \tag{21}$$

subject to its budget constraint $C_t \leq w_t L_t + \pi_t^f + \pi_t^m$, where $\beta \in (0, 1)$ is the subjective discount factor, and the utility function $u : \mathbb{R}_+^2 \to \mathbb{R}$ is continuous, twice differential and satisfies $\frac{\partial u}{\partial C} > 0$, $\frac{\partial^2 u}{\partial C^2} < 0$, $\frac{\partial u}{\partial L} < 0$, $\frac{\partial^2 u}{\partial L^2} < 0$ and $\frac{\partial^2 u}{\partial C^2} \frac{\partial^2 u}{\partial L^2} - \left(\frac{\partial^2 u}{\partial C \partial L} \right)^2 > 0$.

A.2 Final Goods Firms

There is a continuum of final goods firms, producing final goods from intermediate goods M_t and labor inputs L_t^f.[30] The production function follows Cobb-Douglas:

$$Y_t = e^{A_t} \left(M \left(m_t^d, m_t^* \right) \right)^{\alpha_M} (L_t^f)^{\alpha_L}, \tag{22}$$

where $M\left(m_t^d, m_t^*\right)$ is intermediate goods which are composed of domestic intermediate inputs m_t^d and imported intermediate inputs m_t^*. Parameters $0 < \alpha_M, \alpha_L < 1$ are the share of intermediate goods and labor in final goods respectively, satisfying $\alpha_M + \alpha_L < 1$. The technology of combining these two types of intermediate goods follows the CES Armington aggregator:

$$M\left(m_t^d, m_t^*\right) = \left[\lambda \left(m_t^d\right)^{\frac{\psi-1}{\psi}} + (1-\lambda) \left(m_t^*\right)^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}}, \tag{23}$$

where $\lambda \in [0, 1]$ is the weight of domestic intermediate goods in total intermediate goods composition and $\psi (> 0)$ is the elasticity of substitution across domestic and imported

[30] Following Mendoza and Yue (2012), we abstract capital input for simplicity.

intermediate goods. The profit maximization problem of final goods firms is:

$$\max_{m_t^*, m_t^d, L_t^f} \pi_t^f = Y_t(1-\tau_t) - p_t^* m_t^* - p_t^m m_t^d - w_t L_t^f, \tag{24}$$

subject to the production function (22) and the combining technology of domestic and imported intermediate goods (23). As explained above, the government imposes a tax on their final goods production.

Then, the first-order conditions are:

$$e^{A_t} \alpha_M \left(M\left(m_t^d, m_t^*\right)\right)^{\alpha_M - \frac{\psi-1}{\psi}} (1-\lambda)(m_t^*)^{\frac{-1}{\psi}} \left(L_t^f\right)^{\alpha_L} (1-\tau_t) = p_t^*, \tag{25}$$

$$e^{A_t} \alpha_M \left(M\left(m_t^d, m_t^*\right)\right)^{\alpha_M - \frac{\psi-1}{\psi}} \lambda \left(m_t^d\right)^{\frac{-1}{\psi}} \left(L_t^f\right)^{\alpha_L} (1-\tau_t) = p_t^m, \tag{26}$$

$$e^{A_t} \alpha_L \left(M\left(m_t^d, m_t^*\right)\right)^{\alpha_M} \left(L_t^f\right)^{\alpha_{L-1}} (1-\tau_t) = w_t. \tag{27}$$

The technology of combining the continuum of differentiated imported intermediate goods $m_{j,t}^*$ is the Dixit-Stiglitz aggregator:

$$m_t^* \equiv \left[\int_0^1 \left(m_{j,t}^*\right)^{\frac{\nu-1}{\nu}} dj\right]^{\frac{\nu}{\nu-1}}, \tag{28}$$

where $\nu(>0)$ is the substitution elasticity among imported intermediate goods. If the government repays its debt, final goods firms can access the financial market to borrow working capital at the risk-free interest rate r^f from foreign investors in order to import a fraction $\theta \in [0,1]$ of intermediate goods from foreign firms. The foreign investors have enough resources to lend the working capital, and final goods firms repay it within the period t. Thus, the cost minimization problem with respect to the imported intermediate goods is:

$$\min_{m_{jt}^*} \int_\theta^1 p_{j,t}^* m_{j,t}^* dj + (1+r^f) \int_0^\theta p_{j,t}^* m_{j,t}^* dj. \tag{29}$$

Then, the first-order conditions with respect to $m_{j,t}^*$ are as follows:

$$m_{j,t}^* = \left(\frac{(1+r^f) p_{j,t}^*}{p_t^*}\right)^{-v} m_t^*, \quad \text{for } j \in [0,\theta], \tag{30}$$

$$m_{j,t}^* = \left(\frac{p_{j,t}^*}{p_t^*}\right)^{-v} m_t^*, \quad \text{for } j \in [\theta,1]. \tag{31}$$

The aggregate imported intermediate goods is provided by the CES index:

$$p_t^* = \left(\int_\theta^1 \left(p_{j,t}^*\right)^{1-v} dj + \int_0^\theta \left((1+r^f)p_{j,t}^*\right)^{1-v} dj \right)^{\frac{1}{1-v}}. \tag{32}$$

When the country is in a default state, final goods firms cannot raise the working capital necessary to import intermediate goods. Thus, the price of aggregated imported intermediate goods in this state is:

$$p_t^* = \left(\int_\theta^1 \left(p_{j,t}^*\right)^{1-v} dj \right)^{\frac{1}{1-v}}. \tag{33}$$

A.3 Domestic Intermediate Goods Firms

A representative domestic intermediate goods firm produces intermediate goods from labor L_t^m and sells them to final goods firms. The technology of domestic intermediate goods production is:

$$m_t^d = A^I \left(L_t^m\right)^\gamma, \tag{34}$$

where both parameters of A^I and $\gamma \in [0, 1]$ are the time-invariant state of TFP and the labor share respectively in intermediate goods production. The profit maximization problem of intermediate goods firms is:

$$\max_{L_t^m} \pi_t^m = p_t^m A^I \left(L_t^m\right)^\gamma - w_t L_t^m. \tag{35}$$

Then, the first-order condition with respect to L_t^m is:

$$w_t = \gamma p_t^m A^I \left(L_t^m\right)^{\gamma-1}. \tag{36}$$

A.4 Competitive Equilibrium

We define the competitive equilibria of both the inability-to-repay default model and the strategic default model. For convenience, we summarize state vectors as $s^I \in \{A,\ g,\ B,\ \xi,\ b^*\}$ for the former model and $s^S \in \{A,\ g,\ B,\ \xi\}$ for the latter model. The prime symbol represents variables at the next period (i.e., B' represents the debt outstanding at the next period.).

Definition 1

A recursive equilibrium of the inability-to-repay default model is defined as, given the vector of state variables s^I, *the fiscal limit in the next period* $b^{*\prime}$ *and tax rate* $\tau(s^I)$, *a set of government policies* $\{B'(s^I), G(g), T(s^I)\}$, *private allocation* $\{C(s^I), Y(s^I), L(s^I), M(s^I), m^d(s^I), m^*(s^I), L^f(s^I), L^m(s^I)\}$, *factor prices* $\{w(s^I), p^*(s^I), p^m(s^I)\}$, *and the price of government bonds* $q(A, g, B', \xi_n, b^{*\prime})$ *such that:*

(a) Government policies satisfy the rules of government expenditure and taxation (1), and for $\xi = \xi_n$, *given* $q(A, g, B', \xi_n, b^{*\prime})$, *the amount of newly issued government bonds follows the budget constraint (2); (b) For* $\xi = \xi_n$, *given the default set* $\Gamma^I(B, \xi_n, b^*)$ *and the probability of default* $P^e(A, g, B', \xi_n, b^{*\prime})$, *the price of government bonds* $q(A, g, B', \xi_n, b^{*\prime})$ *satisfies the foreign investors' zero profit condition (3); (c) Households, final goods firms, and intermediate goods firms solve their optimization problems respectively; (d) The market for domestic intermediate goods clears; (e) The market for labor clears,* $L_t = L_t^f + L_t^m$; *(f) The market for final goods clears,* $Y_t - T_t - p_t^* m_t^* = C_t$.

Definition 2

A recursive equilibrium of the strategic default model is defined as, given the vector of state variables s^S, *tax rate* $\tau(s^S)$ *and value functions* $V(s^S)$, $V_n(s^S)$ *and* $V_d(s^S)$, *a set of government policies* $\{d(s^S), B'(s^S), G(g), T(s^S)\}$, *private allocation* $\{C(s^S), Y(s^S), L(s^S), M(s^S), m^d(s^S), m^*(s^S), L^f(s^S), L^m(s^S)\}$, *factor prices* $\{w(s^S), p^*(s^S), p^m(s^S)\}$, *and the price of government bond* $q(A, g, B', \xi_n)$ *such that:*

(a) Government policies satisfy the rules of government expenditure and taxation (1), for $\xi = \xi_n$, *given* $q(A, g, B', \xi_n)$, *the amount of newly issued government bonds follows the budget constraint (2) and the government solves its optimization problem (16); (b) For* $\xi = \xi_n$, *given the default set* $\Gamma^S(B, \xi_n)$ *and the probability of default* $P^e(A, g, B', \xi_n)$, *the price of government bonds* $q(A, g, B', \xi_n)$ *satisfies the foreign investors' zero-profit condition (3); (c) Households, final goods firms, and intermediate goods firms solve their optimization problems respectively; (d) The markets for domestic intermediate goods, labor and final goods clear.*

In terms of the market clearing condition of final goods, the aggregate constraints are different in the normal and default states as $Y_t - p_t^* m_t^* - (B_t - q_t B_{t+1}) = C_t + G_t$ and $Y_t - p_t^* m_t^* + \Phi_t^F = C_t + G_t$ respectively. However, by substituting the government's budget constraints in the normal state (2) and that in the default state, $T_t + \Phi_t^F = G_t$, for each state of aggregate constraint, the same market clearing condition is derived regardless of the government state.

Appendix B. Computational Algorithm

B.1 Expected Surplus and Fiscal Limit

1. Discretize the state space of tax rate (τ), and TFP (A) and government expenditure normalized by GDP (g) in terms of deviation from the respective steady-state value. Take 101 grid points for the tax rate uniformly in the interval between 0.15 and 0.60.[31] Tauchen's method (1986) is applied to obtain the state space of TFP and government expenditure, taking 25 grid points uniformly by setting the center points to zero.

2. Calculate endogenous variables $(C_t, Y_t, L_t, m_t^d, m_t^*, L_t^f, L_t^m, T_t, G_t, w_t, p_t^m, p_t^*)$ for all grid points.

3. Calculate the expected surplus and fiscal limit separately.

   (a) Expected Surplus: Aggregate the expected future fiscal surplus discounted by the risk-free interest rate $E_t \sum_{i=0}^{200} \left(\frac{1}{1+r^f}\right)^{t+i} (T_{t+i} - G_{t+i})$ given the corresponding tax rate on each grid point.

   (b) Fiscal Limit: Find the tax rate that maximizes the tax revenue in each state variable (A_t, g_t), and subtract the political disturbance. Then, draw the future shocks for (A_{t+i}, g_{t+i}) given the initial state (A_t, g_t) and aggregate the discounted future maximum fiscal surplus over a 200-quarter horizon. Repeat this procedure 5,000 times for all initial states of (A_t, g_t). Finally, calculate the cumulative distribution function of default probability.

B.2 Discrete State Space of Inability-to-Repay Default

1. Discretize the state space of TFP (A_t), government expenditure (g_t) and government debt (B_t), taking the same number of grid points for TFP and government expenditure described in the previous section and 101 grid points for the government debt B_t in the interval between -0.5 and 3.0 times the steady-state level of GDP. Then, make an initial guess for the issuance of government bonds f_0^b.

[31] We only study the tax rates within this range. This is because if the tax rate is too high, the first-order conditions of final goods firms will not be obtained due to negative profits as firms produce at least the amount of government expenditure. Also, if the tax rate is too low, the fiscal limit will be negative because of low tax revenues.

2. Given the tax rate rules, τ_t^{lr}, τ_t^{es} and τ_t^{ma}, evaluate the probability of default $P^{I,e}$ with new government bond f_0^b using a piecewise linear interpolation, and compute other endogenous variables.

3. Update the guess for government bond issuance f_1^b from the government intratemporal budget constraint (2) and the pricing equation of government bond (3).

4. Accept the decision rule f_1^b if the difference between the updated and old decision rules f_0^b is small enough (i.e. $\sup \left\| f_1^b - f_0^b \right\| < \epsilon$).[32] Otherwise, go back to step 2 with the updated decision rule $f_0^b = f_1^b$.

5. Finally, select the tax rule τ_t from the state spaces for (τ_t^{lr}, τ_t^{es} and τ_t^{ma}) that satisfies the rule (12) and the corresponding decision rule for government bond issuance f^b and endogenous variables.

B.3 Discrete State Space of Strategic Default

1. Discretize the state space of TFP (A_t), government expenditure (g_t) and tax rate (τ_t). Set the tax rate (τ_t) from its linear rule (4), corresponding to the debt outstanding that ranges from -0.5 to 3 times the steady-state level of GDP. Take the same number of grid points for TFP, government expenditure and tax rate as the prior sections.

2. Make initial guesses for the non-default and default value functions $V_{n,0}$ and $V_{d,0}$ respectively, and initialize the government bond price q_0 to be $1/(1+r^f)$.

3. The government's value function iteration.

   (a) Given q_0, calculate the private allocations and utilities for each state space in non-default and default. Then, derive the tax rate in the next period and the corresponding expected value functions using a piecewise linear interpolation.

   (b) Calculate the updated value functions $V_{n,1}$ and $V_{d,1}$ from the expected value functions and utilities, following equations (17) and (18) respectively.

   (c) If the differences between current and updated government value functions are small enough (i.e. $\sup\{|V_{n,1} - V_{n,0}|,\ |V_{d,1} - V_{d,0}|\} < \varepsilon\}$), stop the iteration. Otherwise, go back to step (a) with the updated government value functions.

[32] We checked the local uniqueness of the solution by perturbing the policy function f^b, and confirmed its convergence.

4. Calculate the default set Γ^S and the probability of default $P_t^{S,e}$, using a piecewise linear interpolation on the tax rate grid points. Then, derive the price of government bond q_1.

5. If the difference between the current and updated government bond price is small enough (i.e. $\sup||q_1 - q_0|| < \varepsilon\}$), stop the iteration. Otherwise, go back to step 3 with the updated price of the government bond.

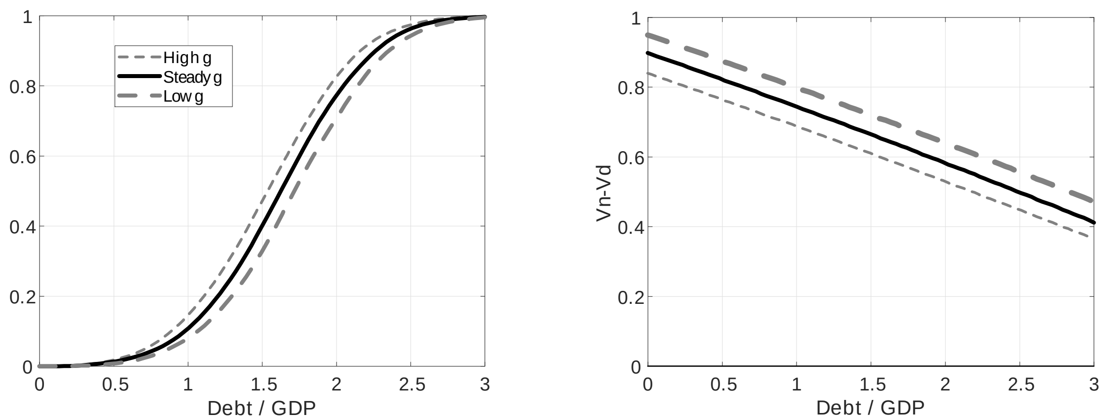
Appendix C. Quantitative Analysis under Different Values of Government Expenditure

In section 4, we explained how the current level of TFP affects the probability of default under both the inability-to-repay default model and the strategic default model. In this Appendix, we examine how the other state variable, i.e., government expenditure, influences the results. Figure C shows these two cases under different values of government expenditure, keeping the value of TFP at the steady-state value.

Figure C: Comparison of the Two Sovereign Default Models (Greece)

(a) Probability of Inability-to-Repay Default

(b) Value Function of Strategic Default



Notes 1: Debt-to-GDP is measured relative to the steady-state GDP.

2: "High g" and "Low g" represent government expenditure at a level 5 percent higher and lower than the steady state respectively.

3: TFP is fixed at the steady state.

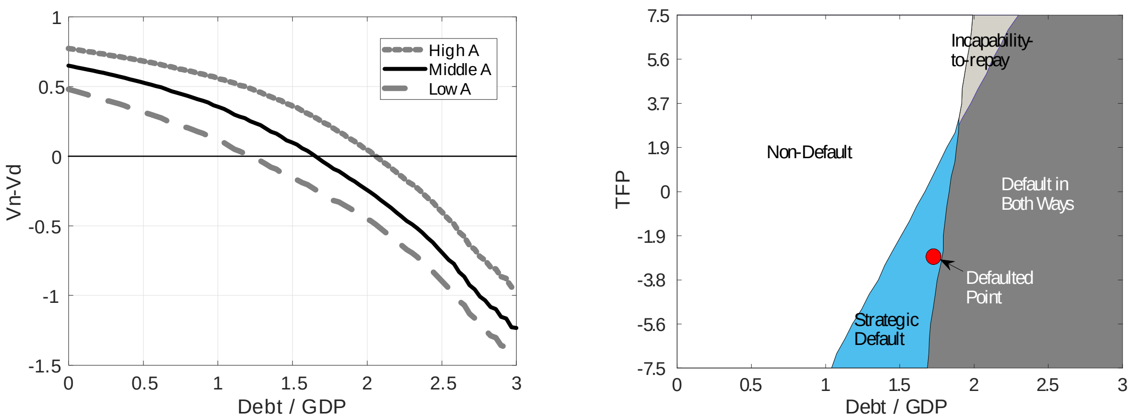
Panel (a) shows that current high (low) government expenditure increases (decreases) the probability of inability-to-repay default. However, similar to the TFP, variations in the probability are small because government expenditure is assumed to return to the steady-state level. Panel (b) shows that the non-default value (V_n) is larger than the default value (V_d) in the plausible region of debt-to-GDP ratio, indicating that without changes in TFP level, changes in government expenditure on their own would not have triggered the Greek default.

Appendix D. Counterfactual Analysis

In the current paper we argue that the Greek default is likely attributable to the government's inability to repay its debt as suggested by our simulation analysis. However, why does the strategic default model fail to explain the Greek default? To see this more clearly, in this appendix, we explore an extreme counterfactual scenario under which strategic default can emerge as a plausible scenario. To be specific, we examine the case where the tax rate under the default state is reduced by 8 percent from the baseline simulation, equivalent to political disturbance, and government expenditure is 5 percent lower than the baseline during the default state. Clearly, under these premises, the economic contraction after the default would become more moderate, and therefore the government has a higher incentive to choose to default.

Figure D: Trigger of Default

(a) Value Function of Strategic Default (b) Default Region



Notes 1: Debt-to-GDP is measured relative to the steady-state GDP.
2: TFP in panel (b) represents the deviation from the steady state.
3: The tax rate and government expenditure during default are assumed to be lower for 8% points and 5% points than the baseline respectively.

Panel (a) in Figure D shows the difference between the value function for non-default and that for default. The default value exceeds the non-default value at the point where the debt-to-GDP ratio is around 160 percent when TFP is at the steady-state level. The threshold is lower when TFP is lower. Panel (b) shows the default regions with respect to the TFP for different values of debt-to-GDP ratio. Again, we fixed the government expenditure at 5 percent lower than the steady-state value. In this case, the strategic default region emerges when the debt-to-GDP ratio is relatively high. For example, provided that TFP is at the steady-state value, the government chooses to default as a result of the strategic decision when the debt-to-GDP ratio exceeds 160 percent. When the ratio exceeds 180 percent, the government can default either by its inability to repay its debt or by strategic decision. These two panels show that strategic default can occur only under the assumptions of a moderate economic downturn during the default state, which is, however, at odds with observations in the case of the Greek default.

Appendix E. Calibration

In this appendix, we explain calibrated parameter values for the Argentinean economy. Most parameters are set following Mendoza and Yue (2012). The risk-free interest rate r^f is set at 2 percent, based on the average money market rate between 1993Q4 and 2001Q3. The households' subjective discount factor β is set at 0.98. The ratio of government expenditure to GDP is set at 21.9 percent, based on the historical average between 1993Q4 and 2001Q3. The steady-state tax rate $\overline{\tau}$ and the government expenditure-to-GDP $\overline{G}$ are set as equal. The government debt-to-GDP $\overline{B}$ is set at 34 percent, based on the historical average of the corresponding series between 1993Q4 and 2001Q3. The political disturbance ϱ is set at 0.33, in accordance with the International Country Risk Guide's (ICRG's) index. We summarize the calibrated parameters for the Argentinean economy in Table E. Other parameters not listed in this table are set at the same values used in the Greek case.

Table E: Calibration to the Argentinean Economy

Parameter	Value	Target/Source
β: Households' discount factor	0.98	$1-r^f$
α_M: Intermediate input share in final goods production	0.43	Mendoza and Yue (2012)
α_L: Labor input share in final goods production	0.40	Mendoza and Yue (2012)
ψ: Substitution elasticity across intermediate goods	2.86	Mendoza and Yue (2012)
ν: Substitution elasticity within intermediate goods	2.44	Mendoza and Yue (2012)
λ: Armington weight of domestic inputs	0.62	Mendoza and Yue (2012)
r^f: Risk-free interest rate	0.02	Money Market Rate
$\overline{G}$: Steady-state government expenditure	0.0106	IADB ($\overline{G}/GDP = 0.219$)
$\overline{B}$: Steady-state government debt	0.0163	IMF ($\overline{B}/GDP = 0.335$)
$\overline{\tau}$: Steady-state tax rate	0.219	Steady-state gov't expenditure
ϱ: Political disturbance	0.33	Bi, Shen and Yang (2014)

Appendix F. Data

Table F: Data

Variables	Term	Frequency	Sources
<Greece>			
Public debt-to-GDP	1999-2016	quarterly	OECD
Government expenditure-to-GDP	1999-2008	quarterly (SA)	IMF
Government tax revenue-to-GDP	1999-2016	quarterly (SA)	IMF
GDP	1999-2016	quarterly (SA)	OECD
Consumption	1999-2016	quarterly (SA)	OECD
Imported intermediate goods	early 2000s	-	OECD*
Imports of goods and services	1999-2016	quarterly (SA)	OECD
Domestic intermediate goods	1999-2016	annual	ECB
Labor	1999-2016	quarterly (SA)	OECD
Yields on Greek and German gov't bonds	1999-2016	quarterly	OECD
TFP	1999-2016	quarterly (SA)	OECD
<Argentina>			
Public debt-to-GDP	1993-2001	quarterly	IMF
Government expenditure-to-GDP	1993-2001	quarterly (SA)	IADB
Money market rate	1993-2001	quarterly	IMF
<World>			
Default Episodes	1982-2013	annual	SU
GDP	1979-2016	annual	WB

Notes 1: SA indicates that data is seasonally adjusted.
2: * represents the OECD's STAN Input-Output Data.
3: IADB represents the Inter-American Development Bank.
4: SU represents Schmitt-Grohe and Uribe (2017).

Most of the Greek data we use is that of from 1999Q1 to 2016Q4, except for government expenditure-to-GDP and imported intermediate goods, where the data for government expenditure-to-GDP is that from 1999Q1 to 2008Q4, since the steady state and exogenous shock associated with government expenditure-to-GDP needs to be calibrated with pre-crisis data, and the data for imported intermediate goods is that from the early 2000s due to data limitation. Most Argentinean data we use is from 1993Q4 to 2001Q3 which is 1 quarter prior to the crisis.

Note also that government expenditure is the government's total expenditure, including social benefits minus interest payments; tax revenue includes social contributions; and labor is total employment for those aged 15 and over. Yields on both Greek and

German government bonds are measured from 10-year bond yields. TFP is defined as GDP per person employed. The world default episodes are obtained from Schmitt-Grohe and Uribe (2017). The GDP data for defaulting countries is from World Development Indicators, published by the World Bank (WB).