Macroeconomic Effects of Quantitative and Qualitative Monetary Easing Measures

Junko Koeda

Discussion Paper No. 2018-E-16
NOTE: IMES Discussion Paper Series is circulated in order to stimulate discussion and comments. Views expressed in Discussion Paper Series are those of authors and do not necessarily reflect those of the Bank of Japan or the Institute for Monetary and Economic Studies.
Macroeconomic Effects of Quantitative and Qualitative Monetary Easing Measures

Junko Koeda*

Abstract
We estimate a structural vector autoregressive model with an effective lower bound of nominal interest rates (ELB) using Japanese macroeconomic and financial data from the mid-1990s to the end of 2016. The estimated results show that the Bank of Japan’s quantitative and qualitative easing (QQE) policy increased output via “pure” quantitative easing when the first-year’s QQE level effect was controlled, complemented by qualitative easing. Our nonlinear counter-factual analyses show that raising the ELB or lowering an inflation threshold in forward guidance is not necessarily contractionary.

Keywords: effective lower bound of nominal interest rates; quantitative and qualitative monetary easing policy; forward guidance; structural vector autoregression; maximum likelihood

JEL classification: E58, E52, C32

* School of Political Science and Economics, Waseda University (E-mail: jkoeda@waseda.jp)

This paper was prepared in part while the author was a visiting scholar at the Institute for Monetary and Economic Studies, Bank of Japan. The author is grateful for the research opportunities at the Institute for Monetary and Economic Studies at the Bank of Japan. The author is grateful to the reviewers for most valuable comments. The author thanks Shin-ichi Fukuda, Yutaka Harada, Fumio Hayashi, Ryozo Miyao, Taisuke Nakata, Mototsugu Shintani, Etsuro Shioji, Shigenori Shiratsuka, Tomohiro Tsuruga, Kazuo Ueda, Kozo Ueda, and Yoichi Ueno for their helpful comments. The paper benefited from presentation at the Bank of Japan, Hitotsubashi University, Institute of Statistical Research, JSME, RCAST Macroeconomic Analysis Workshop, and Waseda University (Center for Positive Political Economy). The views expressed in this paper are those of the author and do not necessarily reflect the official views of the Bank of Japan.
1 Introduction

The Bank of Japan (BOJ) implemented its quantitative and qualitative monetary easing (QQE) policy in April 2013 under the (then) new Governor Haruhiko Kuroda. Since then, the BOJ has boosted the monetary base to another higher level (Figure 1a) to achieve its 2% inflation goal while aggressively purchasing unconventional assets, largely in the form of long-term Japanese government bonds. ¹ Additional measures followed. Besides additional easing, the BOJ lowered an effective lower bound of nominal interest rates (ELB) by introducing a negative interest rate policy in January 2016; and it has also been directly controlling 10-year term bond yield around zero under its yield curve control since September 2016. However, the BOJ’s 2% inflation goal has not yet been met. What are the macroeconomic effects of measures beyond initial QQE by raising bank reserves, compressing term spread, and deepening the negative interest rate policy? Should the BOJ postpone a liftoff until its 2% inflation goal is overshot?

[Figure 1]

To examine these questions, we estimate a regime-switching structural vector autoregressive (SVAR) model with an ELB and threshold-based forward guidance. More specifically, we extend Hayashi and Koeda’s (2018) SVAR framework in several dimensions. First, we include the recent QQE period in the sample period. A change in the central bank’s governor is often interpreted as a monetary policy regime change. If a policy changes, so should the vector autoregression (VAR) coefficients (Stock and Watson, 2001). Thus, we allow reduced-form model

---

¹ Prior to QQE, the size of JGB holdings by the BOJ was limited to that of banknotes in circulation by so called the “banknote principle.”
coefficients to change between the pre- and during QQE periods. These changes in estimated coefficients may capture the role of expectations, which is emphasized by the BOJ under the QQE. We find under the QQE that output increases with inflation (the coefficient of the lagged inflation in the output equation becomes statistically significantly positive under the QQE). Further, output tends to increase with term spread reduction (the coefficient of the term spread in the output equation becomes more negative under QQE than in the pre-QQE period).

Second, to capture the effects of qualitative as well as quantitative easing, we use term spread as a qualitative easing measure, and excess reserve rate (log of the actual-to-required reserve ratio) as a “pure” quantitative easing\(^2\) measure. The existing empirical literature that analyzes the macroeconomic effects of unconventional monetary policy uses various policy measures. One strand of this literature uses quantity-based measures. Quantity-based measures include liability-based measures, such as reserve targets (Honda, Kuroki, and Tachibana, 2013), monetary base (Sadahiro, 2005; Miyao, 2016; Miyao and Okimoto, 2017; and Nakashima, Shibamoto, and Takahashi, 2017), bank reserves held at the BOJ (Schenkelberg and Watzka, 2013; Kimura and Nakajima, 2016), and excess-reserve variables (Shioji, 2016; Hayashi and Koeda, 2018). Further, asset-based measures include the announced amount of asset-purchases (Weale and Wieladek, 2016), central bank assets (Gambacorta, Hofmann, and Peersman, 2014) and the share of unconventional assets to the total assets held by the BOJ (Nakashima et al., 2017). Another strand of the literature uses priced-based measures. For instance, Kapetanios, Mumtaz, Stevens, and Theodoridis (2012) and Baumeister and Benati (2013) include yield spreads in their VAR

\(^2\) For discussion on “pure” quantitative easing policy in Japan, see for example, Shiratsuka (2010), Ueda (2012), and Ito (2013).
models. The unconventional monetary policy measure in Wu and Xia (2014) is the U.S. shadow policy rate estimated via a term structure model. Similarly, Iwasaki and Sudo (2017) use a Japanese shadow rate.\(^3\) We use both quantity- and price-based measures in our analysis.

Third, we add financial variables to the SVAR model to capture possible monetary policy transmission channels via movements in stock, bond, and foreign exchange markets as well as in bank lending activity. We find that these financial variables (particularly, stock prices) are affected by changes in unconventional monetary measures. However, we did not find strong monetary policy transmission to macroeconomic variables via these financial channels. Finally, to control the possible short-run nature of QQE (e.g., Hayakawa, 2016), we add the first-year QQE dummy variable to the reduced-form output and inflation equations. Output gap tended to be higher in the first year of QQE (the dummy coefficient is statistically significantly positive in the output equation). Further, the coefficient for the lagged excess reserve rate becomes more positive than the one under the pre-QQE period.

We then compute nonlinear impulse responses to analyze the macroeconomic effects of QQE. These impulse responses are consistent with the existing findings (see Hayashi and Koeda, 2018, for a review) which find that “pure” quantitative easing increases inflation and output under pre-QQE periods. We find that this result also holds under QQE if the first-year QQE level effect is controlled by a dummy in the macroeconomic dynamic equations. However, without such a dummy, an excess reserve shock is no longer expansionary. On the other hand, qualitative easing

\(^3\) See Ueno (2017) for the corresponding estimation methodology.
that involves term spread compression\textsuperscript{4} can stimulate economic output.

Further, we conduct nonlinear counter-factual analyses to quantify the macroeconomic effects of

(i) Raising the ELB

(ii) Lowering the inflation threshold in forward guidance.\textsuperscript{5}

More specifically, we increase the ELB from -0.1\% to 0\% and lower the inflation threshold from 2\% to 1\% in the base period of September 2016 (the month in which the BOJ announced a new policy framework, including yield curve control and an over-shooting commitment,\textsuperscript{6} along with its comprehensive assessment of QQE).

Regarding (i), we find that an increase in ELB can be expansionary. It is accompanied by a drop in real interest rate, where inflation rises at an earlier and faster pace than nominal policy rate. This finding that an increase in ELB can be expansionary is not without theoretical possibilities, such as the effect of reverse interest rate (Brunnermeier and Koby, 2017), that of the central bank’s information (Nakamura and Steinsson, 2013), and that from a neo-Fisherian environment (e.g., Uribe, 2018). Regarding (ii), we find that weaker forward guidance does not

\textsuperscript{4} Term spread can be viewed as a monetary policy variable even before the implementation of yield-curve control, because term spread compression can be achieved without directly affecting the size of excess reserves, for example by maturity swaps. Fukunaga, Kato, and Koeda (2015) and Koeda (2017) estimate the effect of maturity swaps on reducing term spread and bond risk premium using a term structure model with preferred habitat investors and arbitrageurs.

\textsuperscript{5} In July 2018, the Bank of Japan officially introduced forward guidance which is not threshold based. Forward guidance in this paper refers to the liftoff conditions that have been officially announced during the sample period. For more discussion, see Hayashi and Koeda (2018).

\textsuperscript{6} In September 21, 2016, the BOJ announced an “inflation-overshooting commitment” to continue monetary expansion “until the year-on-year rate of increase in the observed CPI exceeds the price stability target of 2 percent and stays above the target in a stable manner.” https://www.boj.or.jp/en/announcements/release_2016/k160921a.pdf.
negatively affect inflation or output if trend growth is sufficiently strong. This finding that weaker forward guidance is not necessarily contractionary may deserve some explanation. In addition to the fact that the effectiveness of forward guidance depends on how much central bankers can commit in their policies, several other possible explanations exist. First, a New Keynesian DSGE model with an ELB implies a negative relation between the natural rate of interest and the optimal inflation target level (Andrade, Gali, Le Bihan, and Matheron, 2018). Since such a rate is commonly tied to trend growth in macro models, a higher inflation target may not be desirable if trend growth is strong. Second, the effects of longer-duration monetary easing at the ELB may depend on how much room is left for it. After the introduction of a negative interest rate policy as an additional QQE measure, even the term spread fell into a negative range, leaving limited room for additional stimulus. Third, as argued by Hayashi and Koeda (2018), a “liftoff” (which occurs when the net policy rate becomes positive) can be expansionary if it triggers the economy to move to a better state. Further, Hayashi (2018) presents theoretical examples in which an inflation condition for a liftoff leads an economy to a liquidity trap at equilibrium. Lastly, the higher the inflation condition for a liftoff, the more contractionary after the liftoff would be, if the BOJ were to follow a Taylor rule that responds more to inflation than to output.

---

7 The effectiveness of forward guidance is known as “policy duration effect” in Japan (Fujiki, Okina, and Shiratsuka, 2001; Ueda, 2002). See Ugai’s (2007) survey for the pre-QQE period and Ueda (2005) for policy discussion.

8 The standard New Keynesian dynamic stochastic general equilibrium (DSGE) models with ELB established the desirability of keeping the policy rate at the ELB for an extended period (Eggertsson and Woodford, 2003; Jung, Teranishi, and Watanabe, 2005), while some researchers point out the implausibly large effects of forward guidance, known as the forward guidance puzzle. Forward guidance becomes less powerful in an incomplete market setting (McKay, Nakamura, and Steinsson, 2016).

9 Related policy discussions are available in several books written in Japanese. For example, Iwata, Samikawa, and JCER (2016) and Shirai (2016).

10 Existing new Keynesian models with the ELB have equilibrium multiplicity (e.g., Benhabib, Schmitt-Grohe, and Uribe, 2001; Aruoba, Cuba-Borda, and Schorfheide, 2016). Hirose (2014) addresses this multiplicity issue in estimating a new Keynesian DSGE model with an ELB for Japan.

11 This does not necessarily mean that a central banker should be less conservative by responding less to inflation than to output. Nakata and Schmidt (2016) find that a central banker who implements discretionary monetary policy
hikes would have to be more aggressive after the liftoff, compared to a situation where a liftoff occurs with a combination of lower inflation and higher output levels.

The rest of the paper proceeds as follows. Section 2 describes the SVAR model. Section 3 explains the estimation strategy and discusses estimated results. Section 4 provides nonlinear impulse response and counter-factual analyses. Section 5 provides robustness checks. Finally, Section 6 concludes.

2 SVAR Model

This section provides an overview of the SVAR model. A full description of the model is provided in Appendix B. The baseline SVAR model contains five variables as follows:

- Two macroeconomic variables: monthly inflation rate ($p$) and output gap ($x$),
- Two monetary policy variables: policy rate ($r$) and excess reserve rate ($m$, log of the actual-to-required reserve ratio)
- One additional financial variable ($z$)

The model consists of the following three types of equations:

(i) Reduced-form macroeconomic equations, which consist of the two-variable VAR of inflation ($p$) and output ($x$) that depend on their lags, the trend variable (12-month growth rate in percent of potential output), the lagged monetary policy variables, and the lagged financial variable. We allow the reduced-form dynamics to differ across regimes.

---

can enhance welfare by being conservative, when an economy is away from the ELB but still faces the ELB risk. Their finding appears to hold when a central banker implements a rule-based monetary policy.
(ii) Monetary policy equations, which consist of

- Taylor rule equation with an ELB and forward guidance, and
- Excess reserve equation; and

(iii) A financial variable equation.

For the additional financial variable (z), we consider each of the following: (i) change in stock prices, (ii) 10-year term spread, (iii) change in yen/$ exchange rate, and (iv) change in bank loans. The change variables are the log differences of the corresponding level variables multiplied by 100. The term spread is the difference between the zero-coupon 10-year yield and the policy rate in annualized rate in percent. We regress each financial variable on the constant, current, and lagged values of p, x, r, and m, and the lagged dependent variable.

3 Estimation

3.1 Data

Figures 1 to 3 show the data used in the estimation. We extend the Hayashi and Koeda (2018) database except for data on Consumer Price Index (CPI) and the additional financial variables. Appendix A describes the data. For CPI, we use the consumption-tax adjusted “core core” CPI (CPI excluding food and energy) data series obtained from the Ministry of Internal Affairs and Communications of the Japanese government. This series starts from January 1995. Our sample period thus starts from January 1995 and ends in December 2016.¹²

¹² Miyao’s (2002) VAR analysis finds a break in 1995. Fujiwara (2006) and Inoue and Okimoto (2008) estimate regime-switching VAR and find that the probability of one of the regimes becomes very high after the late 1990s.
We assume that the regime is observable and identify the following two ELB regime periods following Hayashi and Koeda (2018):


We refer the combined ELB period to the quantitative easing (QE) period in which reserves are supply determined. The remaining sample period corresponds to the positive net policy regime (P).

Since the ELB ($\bar{r}_t$) is exogenous to the model, it is set at 0% until October 2008, and 0.1% from November 2008 (the month in which the interest rate on excess reserves was increased to 0.1%) until December 2015. From January 2016, it is set at −0.1%, that is, the lowest interest rate paid on excess reserves since the introduction of the negative interest rate policy.

### 3.2 Estimation strategy

We conduct maximum likelihood estimation exploiting the block-recursive SVAR structure. As in the standard block-recursive SVAR (see Christiano, Eichenbaum, and Evans, 1999), the identifying assumption for our model is that inflation and output are predetermined. This structure enables us to estimate each type of equation separately. Appendix C provides the technical details regarding the maximum likelihood estimation.

### 3.3 Estimated results

**Reduced-form macroeconomic dynamics**
Table 2 reports the lagged-financial-variable coefficients in the reduced-form macroeconomic equations. They are estimated on a) the lagged subsample QQE, b) lagged subsample pre-QQE, and c) lagged subsample P, because the inflation-output dynamics depends on the previous period’s regime. None of these coefficients are statistically significant suggesting that monetary policy transmissions to inflation and output via these financial variables are not strong. Given the weak significance levels of the lagged financial variable coefficients, the benchmark specification only includes the lagged term spread as an additional financial variable. Appendix Table 1 reports all the estimates for the macroeconomic equations.

[Table 2]

Table 3 reports the benchmark reduced-form macroeconomic dynamic estimates. Some may emphasize the short-run nature of QQE and view QQE as being more effective at the beginning of implementation (e.g., Hayakawa, 2016). To address this view, we also estimate the reduced-form macro dynamics by including a dummy variable that takes the value 1 for the first QQE year (from April 2013 to March 2014) and 0 otherwise (Table 3d).

Several observations can be made on how the reduced-form coefficients change between pre-QQE and QQE periods, which possibly reflect changes in expectations under QQE. First, output increases with excess reserve accumulation under QE as the coefficient for the lagged excess reserve rate is positive in the output equation (Table 3b). However, this result does not robustly hold under QQE. On one hand, as shown in Table 3d, the first-year QQE dummy coefficient is statistically significantly positive in the output-gap equation, and furthermore, the coefficient for the lagged excess reserve rate \( m \) becomes more positive than that under the pre-
QQE period. On the other hand, the lagged $m$ coefficient is no longer positive unless the first QQE year effect is controlled (Table 3c). A more careful analysis is needed to examine what exactly this dummy variable captures,\textsuperscript{13} but we leave such an analysis to future research owning to data limitation.

Second, output increases with term spread reduction under QQE as the lagged term spread coefficient in the output equation turns negative under QQE (Table 3c/3d). This coefficient was weakly positive under the pre-QQE period. Third, output increases with inflation under QQE as the coefficient of the lagged inflation in the output equation turns to statistically significantly positive under QQE (Tables 3c and 3d). This result contrasts with that obtained on lagged subsample pre-QQE, in which the corresponding coefficient is statistically insignificant.

[Table 3]

These estimates capture some differences (a) across regimes and (b) across different ELB periods. To check (a), we conduct the standard likelihood ratio test\textsuperscript{14} on whether the reduced-form dynamics differ across the positive-net-policy-rate ($P$) and ELB regimes. Specifically, we consider the null hypothesis that, in the reduced-form macroeconomic dynamics, the following coefficients are the same across the $P$ regime and QQE periods: (i) the coefficients for the lagged inflation and output gap, (ii) the coefficient for the lagged financial variable, and (iii) the constant term. With eight restrictions under the null hypothesis, we reject the null at the 5\% significance level. To

\textsuperscript{13} Some others may be concerned about the possible macroeconomic effects of consumption tax hikes (effective in April 2014). We address this concern to a certain degree by using consumption-tax-adjusted "core-core" (excluding food and energy) CPI as our inflation measure.

\textsuperscript{14} The corresponding likelihood ratio can be calculated by $-2(\ln L_0 - \ln L_1)$, where $\ln L_j$ is the log likelihood function evaluated with restriction ($j=0$) and without restriction ($j=1$).
check (b), we carry out a structural break test within the ELB regime to examine whether there is a break across the pre-QQE and QQE periods. We find a break somewhat earlier than the April 2013 announcement of QQE. The Andrews’ (1993) sup-F test finds a structural break for the output equation occurring in March 2011 ¹⁵ (the month in which the Great East Japan Earthquake occurred; this precedes QQE implementation) but follows the implementation of the Comprehensive Monetary Easing policy. The Chow breakpoint test finds no break occurring in April 2013, ¹⁶ though this outcome may be affected by observation trimming involved in the standard structural test (e.g., 15% in Andrew sup F test). In sum, statistical tests suggest that the macroeconomic dynamics clearly change across the regimes, but less clearly in terms of timing across different ELB periods.

**Equations for monetary policy variables**

Table 4 reports the Taylor rule estimates. As in Okina and Shiratsuka (2002) and Braun and Waki (2006), we include the trend variable in the desired Taylor rate to control for movements in the equilibrium real interest rate. ¹⁷ To address the possible change in the target inflation rate since QQE, we estimate the rate for the period only up to March 2013 (estimated to be 0.34% per year), then set it exogenously at 2% from April 2013 and onward. The estimated speed of adjustment per month is about 18 percent. The Taylor principle is violated because the inflation coefficient is less

---

¹⁵ We regress the output variable on a constant, the trend variable, the two lagged macroeconomic variables, and the two lagged monetary policy variables. The null hypothesis assumes no breaks on the coefficients of constant, the lagged macroeconomic variables, and the lagged monetary policy variables. We did not find a break for the inflation equation.

¹⁶ The existing empirical studies do not provide clear evidence for a break at the introduction of QQE in April 2013. Fukuda (2015) finds a structural change in financial markets driven by aggressive foreign investors from December 2012 only until May 2013. Fujiwara, Nakazono, and Ueda (2015), using a survey-based data, did not find any notable difference in perceptions before and after the introduction of “Abenomics.”

¹⁷ The equilibrium real interest rate (or the natural rate of interest) is commonly tied to the potential growth rate in macro models.
than one.\footnote{See Hayashi and Koeda (2018) for more discussion on this violation.}

Table 4

Table 5 reports estimates for the excess reserve equation. We select one $m$-lag per the BIC criterion for the pre-QQE period during which the BOJ announced the targeted level of reserves under its “quantitative easing.” For the QQE period, we include two $m$-lags to address the flow effect given that the BOJ announced the pace of asset purchases under QQE. Under QQE, the constant is estimated to be higher than in the pre-QQE period. Further, the inflation coefficient becomes statistically significantly negative, whereas the output coefficient is no longer statistically significantly negative. In other words, the supply of excess reserves responds to inflation rather than output gap under QQE, while it responded to output gap under the pre-QQE period.

Table 5

Financial-variable equation

Table 6 reports the estimated coefficients for monetary policy variables in each financial variable equation. Table 6a shows the corresponding estimates on the subsample QE (i.e., the subsamples pre-QQE and QQE combined). We combine these subsamples because we do not find a structural break in the financial variable equations over the combined period. We allow the financial-variable equations to differ across regimes as there are structural breaks based on the likelihood ratio test in the manner described previously.\footnote{Specifically, we consider the null hypothesis that, for each financial equation, (i) the coefficients for current and lagged inflation and output gap, (ii) the coefficient for the lagged additional financial variable, and (iii) the constant term, are the same across regimes. With six restrictions under the null, we reject the null hypothesis at the 5% significance level for all four financial variable equations.} Table 6b shows the corresponding estimates on the...
Tables 6a and 6b provide a few observations. First, at the ELB where the policy-rate volatility is small (less than 5 basis point on average under QE), the policy-rate cut effect on stock prices is much larger. Second, bank loans increase with excess reserves under QE. This result is consistent with bank-level panel regression results (Shioji, 2016). Third, the 10-year term spread is negatively associated with the policy rate under \( P \). The spread and the yen/dollar rate only weakly respond to the excess reserves under QE.

[Table 6]

4 Nonlinear Impulse Response and Counter-factual Analyses

We compute impulse responses (IR) of inflation (\( p \)), output gap (\( x \)), policy rate (\( r \)), excess reserve rate (\( m \)), additional financial variable (\( z \)), and regime variable (\( s \)) to an excess reserve shock and a term spread shock in subsections 4.1 and 4.2. We interpret the former shock as a pure quantitative easing shock and the latter shock as a quantitative easing policy shock that does not directly increase reserves. Since our nonlinear IRs depend on history, we set the base period at October 2015, the month in which the BOJ decided not to implement an additional monetary easing. We then conduct counter-factual analyses in subsections 4.3 and 4.4 to examine the effects of an increase in ELB and a lowering of the inflation threshold in forward guidance. We set the base period at September 2016, the month in which the BOJ published a comprehensive assessment of

\[20\] The estimated magnitude under \( P \), though it is not statistically significant, is comparable with Honda and Kuroki (2006) whose event study shows that a surprise decrease in the policy rate target leads to a 3% increase in stock prices for the period from August 1989 to March 2001; a similar magnitude to that is found for the US (Bernanke and Kuttner, 2003).
its monetary policy. We assume that the model’s exogenous variables (trend growth and the ELB) remain at the base period’s level over the simulated period.

For each baseline or alternative scenario, we compute the median of 10,000 simulated paths of \( p, x, r, m, z, \) and \( s \), given the history and the parameter estimates reported in the previous section. The median path of \( s \) captures the “liftoff” probability (the probability that the net policy rate becomes positive). Regarding the parameter of the inflation threshold in forward guidance (\( \bar{\pi} \)), we set \( \bar{\pi} = 2 \) over the simulation period in line with the BOJ’s 2% inflation goal under QQE, because the estimated value (0.34%) corresponds to the pre-QQE value (see Appendix C).

### 4.1 Responses to an excess reserve shock

Following Hayashi and Koeda (2018), the IR is defined to an excess reserve shock in terms of model variables (Gallant, Rossi, and Tauchen, 1993) as follows.

\[
E(y_{t+k}|s_t = Z, (p_t, x_t, \bar{r}_t, m_t + \delta_m, z_t), ...) - E(y_{t+k}|s_t = Z, (p_t, x_t, \bar{r}_t, m_t, z_t), ...),
\]

where we set \( \delta_m = 0.1 \) which corresponds to a reserve supply shock that increases excess reserve rate \( (m) \) by about 10 percent in the base period. Figures 4 shows the corresponding IRs with different base periods. For the pre-QQE period, we report the IRs with the base period of February 2004, simulated with the estimated macro-dynamics for the pre-QQE period. The figure indicates that an excess reserve rate shock increases output under pre-QQE, consistent with Hayashi and Koeda’s (2018) findings.

For the QQE period, we report the IRs with the base period of October 2015. We simulate these IRs with the estimated macro-dynamics without and with the QQE dummy (Figures 4b and
4c respectively). We observe that the output responses are sensitive to model specifications. In Figure 4b, the output gap responds negatively and bottoms out in four months, despite the fact that the term spread’s response is initially slightly negative because of the direct negative effect of the lagged excess reserves on output in the output equation. Further, the inflation response to the shock is not positive. Thus, the macroeconomic effects of “pure” quantitative easing have not been empirically observed in the QQE period contrary to the pre-QQE period. This result seems to be consistent with Nakashima et al. (2017), who find that an unanticipated monetary base shock is contractionary, except for the quantitative easing period of 2001–2006.21 However, when IRs are simulated with the estimates with the dummy, the excess reserve shock becomes expansionary (Figure 4c). It increases the output to a greater degree in comparison to pre-QQE (the top right figures in Tables 4a and 4c). This result appears to be consistent with Miyao (2016) who finds larger responses of real GDP and inflation to a positive monetary-base shock under QQE, using data up to March 2015.22

In either specification (that uses estimates with or without the dummy), term spread narrows only slightly with excess reserve shock as it was already compressed to approximately 25 basis points in the base period of October 2015. This effect of excess reserve shock should be distinguished from the effect of maturity swaps (between short-term and long-term bonds) that

21 In addition, Nakashima et al. (2017) find that an “anticipated” quantitative easing shock identified by the maximum forecast error variance approach is contractionary.
22 More specifically, Miyao (2016) estimates a VAR of the following five variables: monthly real GDP, 12-month CPI inflation excluding food and energy, monetary base, long-term government bond yield, and stock prices. Similarly, Miyao and Okimoto (2017) estimate a smooth-transition VAR of these five variables for Jan. 2001—Dec. 2015; they find that macroeconomic responses to a monetary base shock become larger during BOJ’s “aggressive easing” periods, which includes the first few years of QQE based on their estimation.
do not impact the size of the BoJ’s balance sheet. The next subsection discusses a qualitative easing policy that reduces term spread while maintaining the size of excess reserves.

4.2 Responses to a term spread shock

We define the IR to a term spread shock as follows:

\[
E(y_{t+k}|s_t = Z, (p_t, x_t, \tilde{r}_t, m_t, z_t + \delta_z), ...) - E(y_{t+k}|s_t = Z, (p_t, x_t, \tilde{r}_t, m_t, z_t), ...),
\]

where we set \(\delta_z = -0.1\), which corresponds to a negative term spread shock in the base period that lowers term spread between 10-year zero coupon Japanese government bond yield and the uncollateralized call rate by 10 basis points. Figure 5 shows the corresponding IRs. In response to this shock, output modestly increases peaking in about four months, and the increase slowly dies out. Thus, a qualitative easing policy that reduces term spread while maintaining the size of excess reserves, for example, a maturity swap, can have some positive macroeconomic effects.

[Figure 5]

4.3 Effects of an increase in ELB

Suppose the BOJ had moved back the ELB from −0.1% to 0% in the base period of September 2016, which is under the Z regime. The effect of this ELB can be captured by

\[
E(y_{t+k}|s_t = Z, (p_t, x_t, \tilde{r}_a, m_t, z_t), ...) - E(y_{t+k}|s_t = Z, (p_t, x_t, \tilde{r}_b, m_t, z_t), ...),
\]

where \(\tilde{r}_a = 0\) and \(\tilde{r}_b = -0.1\). Figure 6 shows the simulated differences of the above conditional expectations assuming that the ELB is equal to −0.1% (baseline scenario) and to 0% (alternative scenario) from the base period onward. The figure indicates that this increase in ELB is
accompanied by a drop in (ex-post) real interest rate and higher inflation and output. Thus, considering the increase in ELB, inflation rises faster than the speed of nominal policy rate hike. However, the increase in ELB does not instantly raise the likelihood of “liftoff” (bottom right, Figure 6).

[Figure 6]

4.4 Effects of lowering the inflation threshold in forward guidance

Suppose the inflation threshold (\(\bar{\pi}\)) had been lowered from 2% to 1% from the base period of September 2016 onward. Suppose also that this change does not induce a regime shift in the base period. The effect of this weakening of forward guidance can be expressed in terms of shocks:

\[
E \left( y_{t+k} \middle| \varepsilon_t \right) = \begin{bmatrix} p_t - \hat{\pi}_t \\ x_t - \hat{x}_t \end{bmatrix}, (v_{rt} \leq \bar{r}_t - r^e_t \text{ or } v_{\bar{\pi}t} + \bar{\pi}_a > \pi_t), v_{mt} = m_t - m^e_t, v_{zt} = z_t - z^e_t, \ldots
\]

\[
-E \left( y_{t+k} \middle| \varepsilon_t \right) = \begin{bmatrix} p_t - \hat{\pi}_b \\ x_t - \hat{x}_t \end{bmatrix}, (v_{rt} \leq \bar{r}_t - r^e_t \text{ or } v_{\bar{\pi}t} + \bar{\pi}_b > \pi_t), v_{mt} = m_t - m^e_t, v_{zt} = z_t - z^e_t, \ldots
\]

where \(\bar{\pi}_a = 1\) and \(\bar{\pi}_b = 2\); this is the only difference between the baseline and alternative scenarios. Alternatively, this difference can be expressed as having consistently lower \(\bar{\pi}\) shocks under the alternative scenario. Figure 7 shows the simulated differences of the above conditional expectations. Lowering the inflation threshold in the base period does not have a statistically significant impact on neither inflation nor output.

[Figure 7a]

Figure 7a assumes that the trend growth remains at the base period’s level (0.78%) over
the simulation period. If the trend growth were zero over the simulation period, then the simulated differences of the above conditional expectations look very different (Figure 7b), indicating that the weakening of forward guidance can be contractionary with weak potential growth. However, with a zero trend growth, the probability of satisfying the liftoff conditions is almost zero over the simulated period, thus the economy cannot get out of a liquidity trap. This trend-growth assumption is not common in the existing optimal monetary policy literature. As discussed by Nakajima, Shiratsuka, and Teranishi (2010), additional forces, such as structural reforms, may be required for an economy to get out of the liquidity trap in a weak trend growth environment.

[Figure 7b]

These contrasting results with different trend growth paths seem to be consistent with the existing theoretical implications. Using a New Keynesian DSGE model with an ELB, Andrade, Gali, Le Bihan, and Matheron (2018) find that the relation between the natural rate of interest (which is commonly driven by trend growth in macro models\(^\text{23}\)) and the optimal inflation target is “downward sloping” for US and the euro area. Therefore, a higher inflation target is not necessarily desirable if the trend growth is strong.

5 Robustness checks

BOJ’s output gap quarterly estimates

\(^{23}\) There is growing research on estimating the natural rate of interest for Japan. For example, a supplementary material to BOJ’s 2016 comprehensive assessment of QQE (Fujiwara, Iwasaki, Muto, Nishizaki, and Sudo, 2016) applies different estimation methods for the equilibrium real interest rate in Japan.
BOJ provides quarterly output-gap estimates\textsuperscript{24} by applying a different method from the Cabinet office of Japan (Figure 8). The correlation between the two official estimates over the sample period from 1995Q1 to 2016Q4 is high (0.92); however, under the QQE period, the Cabinet office’s output gap increases faster in 2013 and declines more sharply in 2014 than the BOJ’s. Given that quarterly data gives only 15 observations for the QQE period examined in this study (2013Q2–2016Q4), we apply the same interpolation method to convert the original quarterly output-gap series into a monthly series. We then re-estimate the model by replacing the Cabinet office’s output gap series with BOJ’s. In the output gap equation, the coefficients for the lagged excess reserve ratio and the lagged term spread now become weakly positive. As a result, the impulse response of output gap to the positive excess reserve rate or that to the negative term spread shock becomes less conclusive. Other key results remain broadly unchanged.

**Excess reserve rate vs. monetary base**

The correlation between excess reserve rate and monetary base over the sample period is over 0.9. When we re-estimate the model simply replacing the excess reserve rate with monetary base, the corresponding impulse response and counter factual results are broadly unchanged.

6 Conclusion

Using macroeconomic and financial data until the end of 2016, our empirical evidence shows that since the implementation of QQE,

- Boosting the size of the BOJ’s balance sheet alone does not \textit{robustly} increase inflation and

\textsuperscript{24} See Kawamoto, Ozaki, Kato, and Maehashi (2017) for methodology. This BOJ output gap series is available from 1983Q1 and is downloadable from https://www.boj.or.jp/research/research_data/gap/index.htm/
output. Qualitative easing is a complement, but at the cost of unwinding QE;

- Lowering the inflation threshold in the forward guidance is not necessarily contractionary if trend growth is sufficiently strong; and

- Raising the ELB can be expansionary. However, our estimates do not explicitly model the risk of Japanese government bond holdings in a rising policy rate environment.
References


Iwata, Kazumasa, Samikawa, Ikuko, and Japan Center for Economic Research. 2016. Negative Interest Rate Policy (In Japanese), Nikkei Publishing Inc.


24
Figure 1. Monetary Policy Variables

a) Excess reserve rate

b) Policy rates

Figure 1a is the excess reserve rate \( m \) on the left-hand side scale and the log of monetary base (in 100 million yen) on the right-hand side scale. Figure 1b shows the uncollateralized overnight call rate \( r \) and the lowest interest rate applied on excess reserves \( \text{rbar} \) in annualized rate in percent.
Figure 2. Macroeconomic Variables.

a) Inflation and trend growth (12-month change, %)

b) Monthly GDP gap, %

Figure 2a plots the 12-month core CPI inflation rate and the trend growth rate (the 12-month growth rate of potential output) in percent. Figure 2b plots estimated monthly GDP gap.
Figure 3. Other Financial Variables.

a) Stock price index

Figure 3a plots the Tokyo Stock Price Index. Figure 3b plots the difference between the zero-coupon 10-year yield and the uncollateralized overnight call rate in annualized rate in percent.

b) 10-year term spread
Figure 3. ctn.

(c) Yen/Dollar rate

(d) Bank loans

Figure 3c plots the Yen/Dollar spot rate and Figure 3d plots loans and bills discounted in the banking account of domestically licensed banks (in 100 million yen).
Figure 4. Responses to an Excess Reserve Shock

(a) Base period = Feb. 2004

The figure shows impulse responses to a reserve supply shock that increases $m$ by 0.1 in the base period. The 68% probability bands in dashed lines. The financial variable ($z$) is term spread.
Figure 4. ctn.
(b) Base period = Oct. 2015 (simulated with estimates without the QQE dummy)

The figure shows impulse responses to a reserve supply shock that increases $m$ by 0.1 in the base period. The 68% probability bands in dashed lines. The financial variable ($z$) is term spread.
The figure shows the differences in the median of simulated paths under the baseline and alternative scenarios. The 68% probability bands in dashed lines. The financial variable ($z$) is term spread. The dummy variable that takes the value of 1 from April 2013 to March 2014 and 0 otherwise is included in the reduced-form macro equations. The dummy is assumed to be 0 over the simulated period.
The figure shows impulse responses to a term spread shock that lowers term spread by 10 basis points in the base period. The 68% probability bands in dashed lines. The financial variable (z) is term spread.
The figure shows the differences in the median of simulated paths under the baseline and alternative scenarios. The 68% probability bands in dashed lines. The financial variable \((z)\) is term spread.
Figure 7. Lowering the Inflation Threshold in the Forward Guidance (base period = Sept 2016)
a) Trend growth over the simulated period = trend growth in the base period

These figures show the differences in the median of simulated paths under the baseline and alternative scenarios. The 68% probability bands in dashed lines. The financial variable ($z$) is term spread.
Figure 7. ctn.

b) Trend growth over the simulated period is zero

These figures show the differences in the median of simulated paths under the baseline and alternative scenarios. The 68% probability bands in dashed lines. The financial variable (z) is term spread.
Figure 8. Output-gap Estimates from Cabinet Office of Japan and BOJ

a) Official quarterly estimates

b) Estimated monthly estimates

Figure a) plots official output-gap estimates from the Cabinet office of Japan (CAO, solid) and BOJ (dashed). Figure b) shows our interpolated monthly output gap estimates.
Table 1. Summary Statistics.

<table>
<thead>
<tr>
<th>Subsample</th>
<th>p (monthly inflation rate, %)</th>
<th>(\pi) (output gap, %)</th>
<th>(r) (policy rate, % per year)</th>
<th>m (excess reserve rate)</th>
</tr>
</thead>
<tbody>
<tr>
<td>subsample QQE (2013.4-2016.12)</td>
<td>0.49</td>
<td>-0.18</td>
<td>0.04</td>
<td>2.92</td>
</tr>
<tr>
<td>mean</td>
<td>0.49</td>
<td>-0.18</td>
<td>0.04</td>
<td>2.92</td>
</tr>
<tr>
<td>std. dev.</td>
<td>0.85</td>
<td>0.66</td>
<td>0.05</td>
<td>0.42</td>
</tr>
<tr>
<td>subsample pre-QQE (2001.3-2006.6, 2008.12-2013.3)</td>
<td>-0.70</td>
<td>-1.95</td>
<td>0.04</td>
<td>1.20</td>
</tr>
<tr>
<td>mean</td>
<td>-0.70</td>
<td>-1.95</td>
<td>0.04</td>
<td>1.20</td>
</tr>
<tr>
<td>std. dev.</td>
<td>1.31</td>
<td>1.66</td>
<td>0.04</td>
<td>0.54</td>
</tr>
<tr>
<td>subsample P (1995.2-1999.2, 2000.8-2001.2, 2006.7-2008.11)</td>
<td>0.10</td>
<td>-0.21</td>
<td>0.49</td>
<td>0.03</td>
</tr>
<tr>
<td>mean</td>
<td>0.10</td>
<td>-0.21</td>
<td>0.49</td>
<td>0.03</td>
</tr>
<tr>
<td>std. dev.</td>
<td>1.38</td>
<td>1.41</td>
<td>0.31</td>
<td>0.04</td>
</tr>
</tbody>
</table>
Table 2. Reduced Form Estimates with Additional Financial Variables.

a) On the lagged subsample QQE

<table>
<thead>
<tr>
<th>t-1 is in</th>
<th>coefficient of</th>
<th>dependent variable</th>
<th>lagged term spread %</th>
<th>lagged % change in stock price index</th>
<th>lagged % change in yen/$ rate</th>
<th>lagged % change in bank loans</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>QQE</td>
<td>inflation</td>
<td>-0.46</td>
<td>0.03</td>
<td>0.01</td>
<td>0.23</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>[-0.31]</td>
<td>[0.69]</td>
<td>[0.08]</td>
<td>[0.60]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>output</td>
<td>-1.02</td>
<td>-0.01</td>
<td>-0.02</td>
<td>0.15</td>
<td>0.58</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>[-1.20]</td>
<td>[-0.23]</td>
<td>[-0.42]</td>
<td>[0.70]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) On the lagged subsample pre-QQE

<table>
<thead>
<tr>
<th>t-1 is in</th>
<th>coefficient of</th>
<th>dependent variable</th>
<th>lagged term spread %</th>
<th>lagged % change in stock price index</th>
<th>lagged % change in yen/$ rate</th>
<th>lagged % change in bank loans</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>pre-QQE</td>
<td>inflation</td>
<td>-0.65</td>
<td>0.05</td>
<td>0.02</td>
<td>0.07</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>[-1.06]</td>
<td>[1.55]</td>
<td>[0.42]</td>
<td>[0.29]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>output</td>
<td>-0.06</td>
<td>0.00</td>
<td>0.01</td>
<td>-0.11</td>
<td>0.73</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>[-0.13]</td>
<td>[0.14]</td>
<td>[0.16]</td>
<td>[-0.72]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c) On the lagged subsample P

<table>
<thead>
<tr>
<th>t-1 is in</th>
<th>coefficient of</th>
<th>dependent variable</th>
<th>lagged term spread %</th>
<th>lagged % change in stock price index</th>
<th>lagged % change in yen/$ rate</th>
<th>lagged % change in bank loans</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>inflation</td>
<td>-0.14</td>
<td>-0.03</td>
<td>0.17</td>
<td>0.21</td>
<td>0.31</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>[-0.09]</td>
<td>[-0.48]</td>
<td>[1.35]</td>
<td>[0.46]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>output</td>
<td>-0.86</td>
<td>0.05</td>
<td>-0.03</td>
<td>0.10</td>
<td>0.76</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>[-0.87]</td>
<td>[1.26]</td>
<td>[-0.37]</td>
<td>[0.35]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The numbers in brackets are t-values. The lagged subsample P (Table 2c) starts from April 1999 as the lagged change in bank loan is only available from that month.
Table 3. Reduced Form Estimates with the Lagged Term Spread

a) On the lagged subsample P

<table>
<thead>
<tr>
<th>t-1 is in</th>
<th>dependent variable</th>
<th>coefficient of</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>const.</td>
<td>gₜ</td>
<td>pₜ₋₁</td>
</tr>
<tr>
<td>P</td>
<td>inflation</td>
<td>-0.79</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>output</td>
<td>-0.23</td>
<td>-0.14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[-0.82]</td>
<td>[-0.38]</td>
</tr>
</tbody>
</table>

(85 obs.)

b) On the lagged subsample QE (the pre-QQE and QQE period combined)

<table>
<thead>
<tr>
<th>t-1 is in</th>
<th>dependent variable</th>
<th>coefficient of</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>const.</td>
<td>gₜ</td>
<td>pₜ₋₁</td>
</tr>
<tr>
<td>QE</td>
<td>inflation</td>
<td>-1.26</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>output</td>
<td>-1.21</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[-2.36]</td>
<td>[0.55]</td>
</tr>
</tbody>
</table>

(160 obs.)

c) On the lagged subsample QQE

<table>
<thead>
<tr>
<th>t-1 is in</th>
<th>dependent variable</th>
<th>coefficient of</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>const.</td>
<td>gₜ</td>
<td>pₜ₋₁</td>
</tr>
<tr>
<td>QQE</td>
<td>inflation</td>
<td>4.66</td>
<td>-4.19</td>
</tr>
<tr>
<td></td>
<td>output</td>
<td>3.85</td>
<td>-2.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[1.89]</td>
<td>[-1.00]</td>
</tr>
</tbody>
</table>

(45 obs.)

d) On the lagged subsample QQE, with the first year QQE dummy

<table>
<thead>
<tr>
<th>t-1 is in</th>
<th>dependent variable</th>
<th>coefficient of</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>const.</td>
<td>gₜ</td>
<td>pₜ₋₁</td>
</tr>
<tr>
<td>QQE</td>
<td>inflation</td>
<td>5.20</td>
<td>-3.71</td>
</tr>
<tr>
<td></td>
<td>output</td>
<td>0.65</td>
<td>-5.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.43]</td>
<td>[-3.13]</td>
</tr>
</tbody>
</table>

(45 obs.)

The numbers in brackets are t-values.

<table>
<thead>
<tr>
<th>trend growth rate</th>
<th>inflation</th>
<th>output gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.14</td>
<td>0.20</td>
<td>0.12</td>
</tr>
<tr>
<td>[0.01]</td>
<td>[2.16]</td>
<td>[2.82]</td>
</tr>
</tbody>
</table>

The numbers in brackets are t-values.

### Table 5. Excess Reserve Equation.

<table>
<thead>
<tr>
<th>t is in</th>
<th>coefficients of</th>
<th>$R^2$</th>
<th>$\sigma_s$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>QQE</td>
<td>const $\pi_t$ $x_t$ $m_{t-1}$ $m_{t-2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(45 obs.)</td>
<td>0.25 -0.05 -0.01 0.68 0.26 0.993 0.04</td>
<td>[5.23] [-2.28] [-0.87] [4.56] [1.87]</td>
<td></td>
</tr>
</tbody>
</table>

The numbers in brackets are t-values.
Table 6. Financial Variable Equations

a) On the subsamples of QE

<table>
<thead>
<tr>
<th>t is in dependent variable</th>
<th>coefficient of</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>QE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(161 obs.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>term spread</td>
<td>-0.74</td>
<td>-0.10</td>
</tr>
<tr>
<td></td>
<td>[-1.73]</td>
<td>[-1.52]</td>
</tr>
<tr>
<td>change in stock price index</td>
<td>-35.60</td>
<td>2.73</td>
</tr>
<tr>
<td></td>
<td>[-1.77]</td>
<td>[0.89]</td>
</tr>
<tr>
<td>change in yen/$ rate</td>
<td>6.17</td>
<td>2.30</td>
</tr>
<tr>
<td></td>
<td>[0.58]</td>
<td>[1.42]</td>
</tr>
<tr>
<td>change in bank loans</td>
<td>-1.46</td>
<td>1.21</td>
</tr>
<tr>
<td></td>
<td>[-0.62]</td>
<td>[3.27]</td>
</tr>
</tbody>
</table>

b) One the subsample of P

<table>
<thead>
<tr>
<th>t is in dependent variable</th>
<th>coefficient of</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(86 obs.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>term spread</td>
<td>-0.93</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>[-2.82]</td>
<td>[2.78]</td>
</tr>
<tr>
<td>change in stock price index</td>
<td>-5.22</td>
<td>4.18</td>
</tr>
<tr>
<td></td>
<td>[-0.59]</td>
<td>[0.59]</td>
</tr>
<tr>
<td>change in yen/$ rate</td>
<td>0.58</td>
<td>-0.92</td>
</tr>
<tr>
<td></td>
<td>[0.10]</td>
<td>[-0.20]</td>
</tr>
<tr>
<td>change in bank loans</td>
<td>-1.60</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>[-0.86]</td>
<td>[0.69]</td>
</tr>
</tbody>
</table>

The numbers in brackets are t-values.
Appendix A: Data Description

We extend the database in Hayashi and Koeda (2018), except for data on Consumer Price Index (CPI) and the additional financial variables. This appendix describes data construction on the CPI and financial variables.

Consumption-tax adjusted CPI (Consumer Price Index)

We construct (i) the monthly series on the monthly inflation rate ($p$) and (ii) the 12-month inflation rate ($\pi$) from the consumption tax adjusted “core-core” CPI (CPI excluding food and energy) data provided by the Ministry of Internal Affairs and Communications of the Japanese government. The Ministry has started providing this CPI series since May 2017. The series starts from January 1995 and ends in December 2014 with the base year of 2015. The series matches with the current core-core CPI series (without adjustment for consumption tax and with the base year of 2015) from January 2015. Thus, it is extended with the current core-core CPI series until December 2016.

We apply the U.S. Census X12-ARIMA method to the seasonally unadjusted (but consumption tax-adjusted) “core core” CPI from January 1995 through December 2016. Following the Ministry, we take the log for data transformation, use the ARIMA option of (0 1 1)(1 0 1), apply X-11 default for the seasonal filter, and set a level shift outlier in April 2014.

The 10-year interest rate

We construct the monthly 10-year interest rate series using daily data on the zero-coupon ten-year yield obtained from Bloomberg (tickname: I01810Y). These daily data are available from April 3, 1989. The rate for month $t$ is the average of daily values over the reserve maintenance period of the 16th of month $t$ to the 15th of month $t+1$. The unit is annual rate in percent.

The yen/$ exchange rate

We construct yen/$ exchange rate using daily data on the yen/$ closing spot rate obtained from WM/Reuters. This daily data series is available from December 31, 1993. The rate for month $t$ is the average of daily values over the reserve maintenance period of the 16th of month $t$ to the 15th
of month t+1.

**Stock price index**

We construct the monthly stock price index using daily data on the closing value of Tokyo Stock Price Index (TOPIX) obtained from Bloomberg. This daily data series is available from May 16, 1949.

**Bank loans**

We obtained monthly data on bank loans from the Bank of Japan. Specifically, we use “loans and bills discounted” in the banking account of domestically licensed banks (domestic branches only). The data series code is BS02'FAABK_FAAB2DBHA37. The value is the average of the calendar month. The unit is 100 million yen. This monthly data series is available from Apr. 1999.
Appendix B: The SVAR Model

The baseline SVAR model contains the following five variables:

- Two macroeconomic variables: monthly inflation rate ($p$) and output gap ($x$),
- Two monetary policy variables: policy rate ($r$) and excess reserve rate ($m$, log of the actual-to-required reserve ratio)
- One additional financial variable (see Section B.3)

The model consists of the following four types of equations: (i) reduced-form macroeconomic equations, (ii) a Taylor rule equation with an ELB and forward guidance, (iii) an excess reserve equation, and (iv) a financial variable equation.

There are two regimes: the positive net policy rate ($P$) and ELB regimes ($Z$). As defined previously, the net policy rate is the difference between the nominal policy rate and the ELB. The ELB regime has a “zero”$^{25}$ net policy rate. Regime is endogenous with ELB and forward guidance.$^{26}$ The corresponding transition probabilities depend on the state variables, as explained in Section 2.2.

B.1 Reduced-form macroeconomic dynamics

The reduced-form macroeconomic dynamics is modeled by the two-variable VAR of inflation ($p$) and output gap ($x$) that depend on their lags, the trend variable (12-month growth rate in percent

---

$^{25}$ During the first three months after the introduction of negative interest rate policy in January 2016, the net policy rate was positive (0.16, 0.1, and 0.09 percent, respectively). We treat these months as the ELB regime. For the remaining period, we allow small transaction costs (up to 0.06 percent annual rate) to allow the net policy rate to slightly deviate from zero under the ELB regime.

$^{26}$ Koeda (2013) introduces state-dependent policy regime shifts to analyze bond yield dynamics.
of potential output), the lagged monetary policy variables, and the lagged financial variable. We allow the reduced-form dynamics to differ across regimes.\textsuperscript{27}

### B.2 Equations for monetary policy variables

The policy rate \((r)\) follows a Taylor rule with an exogenous ELB \((\bar{r}_t)\).\textsuperscript{28} We define the Taylor rate as

\[
    r_t = (1 - \gamma_r)r_t^* + \gamma_r r_{t-1} + v_{rt}, \quad v_{rt} \sim N(0, \sigma_r^2),
\]

where the rule-based component of Taylor rate \((r_t^*)\) is assumed to be a linear function of output gap \((\chi)\) and the inflation rate over the past 12 months \((\pi_t)\); \(1 - \gamma_r\) is the speed of adjustment. Once the economy is at the ELB, a “lift-off” (the net policy rate becomes positive) will not occur unless inflation exceeds \(\bar{\pi}\) and the Taylor rate lies above the ELB. If the economy is in the positive net policy regime, the economy shifts to an ELB regime when the Taylor rate hits the ELB.

\[
\begin{align*}
    & \text{If } s_{t-1} = P, \quad s_t = \begin{cases} P & \text{if } (1 - \gamma_r)r_t^* + \gamma_r r_{t-1} + v_{rt} > \bar{r}_t, \\ Z & \text{otherwise.} \end{cases} \\
    & \text{If } s_{t-1} = Z, \quad s_t = \begin{cases} P & \text{if } (1 - \gamma_r)r_t^* + \gamma_r r_{t-1} + v_{rt} > \bar{r}_t \text{ and } \pi_t \geq \bar{\pi} + v_{\bar{\pi}t}, \\ Z & \text{otherwise.} \end{cases}
\end{align*}
\]

\textsuperscript{27} The possible parameter changes across regimes can be addressed by the standard regime-switching models with hidden states (Sims and Zha, 2006; for applications for Japan, see Fujiwara, 2006 and Inoue and Okimoto, 2008) and time-varying parameter models (e.g., Nakajima, Kasuya, and Watanabe, 2011). However, regime switching in our model can be policy induced.

\textsuperscript{28} Iwata and Wu (2006) estimate their SVAR while treating the policy rate as a censored variable. They assume that inflation and output dynamics under positive policy rates is the same as that under the ELB regime.
where \( r_t^* \equiv \alpha_r^* + \beta_r^* (\pi_t) [ x_t ] \), \( v_{rt} \sim N(0, \sigma_r^2) \)

The excess reserve rate \((m)\) depends on its lags and the current macroeconomic variables. Following Hayashi and Koeda (2018), the excess reserve rate is assumed to be zero under \( P \) and is supply-determined by the central bank under ELB.

### B.3 Financial variable equation

For the additional financial variable \((z)\), we consider each of the following variables: (i) change in stock prices, (ii) 10-year term spread (annual rate), (iii) change in yen/$ exchange rate, and (iv) change in bank loans. These variables are expressed in percent. The change variables are the log differences of the corresponding level variables multiplied by 100. We regress each financial variable on the constant, current, and lagged values of \( p, x, r, \) and \( m \), and the lagged dependent variable.

### B.4 Model mapping

The model mapping is as follows:

\[
(s_t, p_t, x_t, r_t, m_t, z_t) = f_t \left( s_{t-1}, p_{t-1}, x_{t-1}, r_{t-1}, m_{t-1}, z_{t-1} \ldots, \left( \begin{array}{c} \varepsilon_t \ \nu_{rt} \ \nu_{\pi t} \ \nu_{mt} \ \nu_{z t} \end{array} \right) ; \theta_A, \theta_B, \theta_C, \theta_D \right)
\]

1) \((p_t, x_t)\) determined: The reduced-form shocks for the macroeconomic dynamics \( \varepsilon_t \) are drawn.

2) \( s_t = P \) or \( Z \) determined: The central bank draws the shocks on policy rate and the inflation
threshold in forward guidance \(v_{rt}, v_{\bar{rt}}\).

3) \((r_t, m_t)\) determined: The central bank draws the reserve supply shock \(v_{mt}\).

4) \(z_t\) determined: The financial-variable shock \(v_{zt}\) is drawn.

All shocks \(\varepsilon_t, v_{rt}, v_{\bar{rt}}, v_{mt}, v_{zt}\) are assumed to be i.i.d. normal. The model parameters are \((\theta_A, \theta_B, \theta_C, \theta_D)\), where \(\theta_A\) are the reduced-form parameters for inflation and output, \(\theta_B\) are the parameters of the Taylor rule with regime evolution, \(\theta_C\) are the parameters of the excess reserve supply equation, and \(\theta_D\) are the parameters of the financial-variable equation.

**Appendix C: Maximum Likelihood Estimation**

Define

\[
y_t = \begin{pmatrix} y_{1t} \\ r_t \\ m_t \\ z_t \end{pmatrix}, \quad y_{1t} = \begin{pmatrix} p_t \\ x_t \end{pmatrix}
\]

The likelihood of the data is

\[
L = p(s_1, \ldots, s_T, y_1, \ldots, y_T | x, I_0)
\]

where \(x \equiv (x_T, x_{T-1}, \ldots)\) is a vector of exogenous variables and \(I_0 \equiv (s_1, \ldots, s_T, y_1, \ldots, y_T)\). The usual sequential factorization yields

\[
L = \prod_{t=1}^{T} p(s_t, y_t | x, I_{t-1})
\]

The likelihood for date \(t\) can be written as

\[
p(s_t, y_t | x, I_{t-1}) = p(z_t | m_t, r_t, s_t, y_{1t}, x, I_{t-1})
\]
\[ \times p(m_t | r_t, s_t, y_{1t}, x, I_{t-1}) \]
\[ \times p(r_t | s_t, y_{1t}, x, I_{t-1}) \]
\[ \times \text{Prob}(s_t | y_{1t}, x, I_{t-1}) \]
\[ \times p(y_{1t} | x, I_{t-1}). \]

Recursive identification enables each term on the RHS of this equation to be estimated separately. We set \( \bar{\pi} = 2\% \) for April 2013 and December 2016 in estimating \( \text{Prob}(s_t | y_{1t}, x, I_{t-1}) \). Thus the estimated value of \( \bar{\pi} \) corresponds to the pre-QQE value.
### Appendix Table 1. Reduced Form Estimates with One Additional Financial Variable

a) With the lagged term spread (This table is the same as Table 3)

<table>
<thead>
<tr>
<th>dependent variable</th>
<th>coefficient of</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>inflation</td>
<td>const.</td>
<td>( g_t )</td>
</tr>
<tr>
<td>inflation</td>
<td>(85 obs.)</td>
<td>-0.79</td>
</tr>
<tr>
<td>output</td>
<td></td>
<td>-1.64</td>
</tr>
<tr>
<td>output</td>
<td></td>
<td>-0.23</td>
</tr>
<tr>
<td>output</td>
<td></td>
<td>-0.82</td>
</tr>
</tbody>
</table>

b) With the lagged change in stock price index

<table>
<thead>
<tr>
<th>dependent variable</th>
<th>coefficient of</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>inflation</td>
<td>const.</td>
<td>( g_t )</td>
</tr>
<tr>
<td>inflation</td>
<td>(45 obs.)</td>
<td>4.66</td>
</tr>
<tr>
<td>output</td>
<td></td>
<td>1.30</td>
</tr>
<tr>
<td>output</td>
<td></td>
<td>3.85</td>
</tr>
<tr>
<td>output</td>
<td></td>
<td>1.89</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>dependent variable</th>
<th>coefficient of</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>inflation</td>
<td>const.</td>
<td>( g_t )</td>
</tr>
<tr>
<td>inflation</td>
<td>(160 obs.)</td>
<td>-1.26</td>
</tr>
<tr>
<td>output</td>
<td></td>
<td>-1.63</td>
</tr>
<tr>
<td>output</td>
<td></td>
<td>-1.21</td>
</tr>
<tr>
<td>output</td>
<td></td>
<td>-2.16</td>
</tr>
</tbody>
</table>

The numbers in italics are \( t \)-values.
Appendix Table 1. ctn.
c) With the lagged change in yen/$ rate

<table>
<thead>
<tr>
<th>t-1 is in</th>
<th>dependent variable</th>
<th>coefficient of</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>lagged subsample P</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>inflation</td>
<td>const.</td>
<td>$g_{t-1}$</td>
</tr>
<tr>
<td></td>
<td>(85 obs.)</td>
<td>-0.64</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>output</td>
<td>-1.80</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.11</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.51</td>
<td>0.10</td>
</tr>
</tbody>
</table>

d) With the lagged change in bank loans

<table>
<thead>
<tr>
<th>t-1 is in</th>
<th>dependent variable</th>
<th>coefficient of</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>lagged subsample Z</td>
<td></td>
<td></td>
</tr>
<tr>
<td>QQE</td>
<td>inflation</td>
<td>const.</td>
<td>$g_{t-1}$</td>
</tr>
<tr>
<td></td>
<td>(45 obs.)</td>
<td>3.02</td>
<td>-3.22</td>
</tr>
<tr>
<td></td>
<td>output</td>
<td>0.97</td>
<td>-0.80</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.60</td>
<td>-2.57</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.44</td>
<td>-1.10</td>
</tr>
</tbody>
</table>

The numbers in italics are t-values.