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Unconventional Monetary Policy and the Bond Market in Japan: A New-Keynesian Perspective

Parantap Basu* and Kenji Wada**

Abstract

In this paper, we set up a medium scale new-Keynesian dynamic stochastic general equilibrium (DSGE) model to analyze the effects of various phases of unconventional monetary policy (UMP) on the Japanese bond market. Our model has two novel features: (i) a banking friction in the form of an aggregate bank run risk to motivate commercial banks' demand for excess reserve, and (ii) dynamic linkage between Central Bank resource constraint and the government budget constraint via a transfer payment by the Central Bank to the Treasury. We do three policy simulations to analyze the effects of various phases of UMP shocks on the bond market, namely: (i) effect of a quantitative easing (QE) shock; (ii) the effect of a negative shock to the overnight borrowing rate; and (iii) the effect of a negative shock to the interest rate on banks' excess reserve (IOER). In light of these results, we do an evaluation of the recent yield curve control policy of the Bank of Japan.

Keywords: QE; QQE; Excess Reserve; Overnight Borrowing Rate; IOER; Yield Curve Control

JEL classification: E43, E44, E58

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1 Introduction

During the last two decades, the Japanese economy experienced several episodes of monetary policy changes. Starting from an era of near zero interest rate, from the beginning of the millennium, Bank of Japan (BoJ) officially implemented Quantitative Easing (QE) policy to inject liquidity into the banking system. After the Great Recession, with a growing concern of a deflationary bubble, BoJ adopted a monetary policy with an explicit two percent inflation target which is the cornerstone of Qualitative and Quantitative Easing (QQE). QQE is a broader unconventional monetary policy (UMP) which combines three features in a chronological order: (i) quantitative easing with a two percent inflation target, (ii) negative interest rate on bank reserve and (iii) zero nominal long term bond yield target.

The aim of this research is to analyze the effects of these various episodes of UMP on the bond market behaviour in a dynamic stochastic general equilibrium (DSGE) framework. In the context of Japan, understanding the bond market implications is important because domestic bonds constitute about 27.67% of the Government Pension Investment Funds in Japan (December 2017; www.gpif.go.jp). In addition, about 84.7% of the total assets of BoJ is in government bonds (January 31, 2018; www.boj.or.jp). Also, the amount of public debt outstanding at the end of March 2018 is expected to be 865 trillion yen. If the price of government bond falls and the yield rises, the Japanese government might not be able to refinance the current debt. Fluctuations of bond prices and yields have major implications for commercial banks and the BoJ balance sheet given that about 39.5% of government bond is held by BoJ and 20.9% is held by commercial banks and securities houses at the end of March 2017.

Our model is a stylized medium scale new Keynesian model similar in spirit to extant models such as Smets and Wouters (2007), and Gerali et al. (2010). The advantage of using DSGE model is that it enables us to see the linkage between the real and financial sectors of the economy when a policy change occurs. We derive the pricing kernel which we use to price general fixed income securities, its yield to maturity and term premia. Our aim is to analyze the impulse responses of yield to maturity, holding period returns and term premia of long term government bonds to various monetary shocks pertaining to QQE.

As in any standard DSGE model, our model economy has the following decision units: (i) representative household, (ii) capital good producers, (iii) wholesale good producers, (iv) retail good producers, (v) banking sector, (vi) the Central Bank (CB hereafter), and (vii) the government. The model has standard frictions such as aggregate habit persistence, investment adjustment cost, monopolistic price formation, and nominal stickiness. A nonstandard feature of our DSGE model is the banking friction in the form of an aggregate bank run risk

as in Chang et al. (2014). Banks anticipate the risk of an excess withdrawal of deposits over its reserve which could force them to take recourse to emergency overnight borrowing from the CB at a penalty rate. Such an anticipation disciplines the banks to hold precautionary excess reserve and not push loans recklessly. This precautionary demand for bank reserve depends on the overnight borrowing rate and the interest rate on reserve making these two key policy rates of the CB.

A novel feature of our study is that we explicitly study the link between the government, CB and commercial banks via the long term government bonds and bank reserves which helps us study the effect of QQE policy on the bond market. We formulate the CB's resource constraint in line with the recent work of Hall and Reis (2015) which characterizes the supply function of bank reserve. CB creates reserve (which is the monetary base) keeping in mind that it has to pay interest and principal on existing bank reserves, and also cover the net QE purchase of government bonds from commercial banks. In addition, CB pays some transfer to the government after netting out the revenue that it receives from banks as a penalty from overnight loans as well as the seigniorage revenue. This CB transfer to the government helps finance the current budget deficit of the Treasury subject to the outstanding debt/GDP ratio. The dynamic linkage between the CB resource constraint, commercial bank's flow budget constraint and the government budget constraint is crucial for understanding the monetary transmission mechanism of UMP.

In the extant literature, the real effects of QE arise from limit to arbitrage. This limit to arbitrage can arise from lender's moral hazard as in Gertler and Karadi (2013) or some market segmentation due to preferred habitat (Vayanos and Villa, 2009) or transaction cost as in Chen et al. (2012). We model market segmentation in a simple way within the framework of frictionless financial markets. Households in our model do not trade in long term government bonds and hold short term bank deposits which are perfect substitutes of short term government bonds. Bank deposits provide direct convenience utility to households as in Hansen and Imrohorglu (2013) in addition to transaction utility of money. This assumption gives rise to a natural steady state borrowing-lending spread in our model. On the other hand, commercial banks specialize in dealing with long term government bonds and loans. The assumption of bond market segmentation in our model is motivated by the Japanese financial structure where commercial banks and the Central Bank are primary dealers of government bonds issued by the Treasury.

We formulate the basic QQE operation as a stochastic open market operation with a two percent long run inflation target and a target debt/GDP ratio. Such a policy entails a positive shock to monetary base with an offsetting increase in the share of CB holding of long term Japanese government bonds (JGB). Higher inflationary expectations triggered by

a positive monetary base shock entails an inflation tax on bank excess reserve. Commercial banks thus lower their reserve:deposit ratio and issue more loans. In equilibrium, the nominal loan rate also rises together with higher investment in response to inflationary expectations. On the other hand, the price of long term bond falls raising the nominal yield to maturity. The overall effect of QE is thus positive on the economy. We compare UMP with the conventional monetary policy (CMP) where CMP is modelled as a standard Taylor rule shock to the overnight borrowing rate in our model. Such a policy rate directly affects the precautionary demand for reserve by the commercial banks and through this channel it impacts the bond yields via the stochastic discount factor. We find that the real effect of a QQE shock is considerably stronger than a CMP shock. However, a QQE shock is unable to predict the observed decline in the nominal yield to maturity while a CMP predicts such a decline but rather weakly in terms of magnitude.

We also study the effects of UMP and CMP on the term premia. Recently a few papers examine the effects of large scale asset purchases (LSAP) of US Federal Reserve on term premia. Gagnon et al. (2011) use a reduced form term premium equation to assess the effect of LSAP on term premia. Chen et al. (2012) use a DSGE model to analyze the same effect but they do not model the Central Bank balance sheet and link it to the Central Bank purchase of government bond as we do. As in Gagnon et al. (2011) and Chen et al. (2012), we find that the Central Bank's bond purchase programme lowers the term premia of bonds of all maturities but shorter maturity bonds experience a sharper decline in term premium. We further investigate the effect of a negative shock to the interest rate on excess reserve (IOER) to depict the recent QQE experiment in Japan. We find that a negative IOER shock lowers the term premia and also stimulates the aggregate economy through bank lending channel because banks loan out their excess reserve to avoid penalty which stimulates investment. In contrast, an expansionary CMP shock has a much smaller effect on term premia. The macroeconomic effects of IOER shock through the banking channel is getting more highlighted in the recent literature (Bratsiotis, 2018).

Our DSGE model predicts that a negative IOER shock is the most effective way to stimulate the economy within the framework of the recent yield curve control experiment of BoJ with a positive inflation target. Nominal yield to maturity declines while inflation rises marginally in response to such negative IOER shock. However, such a policy might work only in the short run. We find that targeting a zero long term nominal yield is inconsistent with a two percent long run inflation target policy because it violates the fundamental Fisher's relationship.

The paper is organized as follows. In the next section, we briefly review the extant literature on DSGE modelling of the Japanese economy. In section 3, we give an overview

of the monetary policy history of Japan. In section 4, our basic DSGE model is laid out. Section 5 is devoted to present quantitative analysis of the model. Section 6 concludes.

2 Connections to Literature

There is a growing literature on DSGE modelling of the Japanese economy. The literature can be broadly classified in two strands: (i) with no explicit zero lower bound on the nominal interest rate and (ii) with zero lower bound on the interest rate. In the first group, Sugo and Ueda (2007) is one of the first articles that estimate a DSGE model of the Japanese economy. Although they model monetary policy rule and use call rate as a proxy for the short term nominal interest rate, they do not explicitly model the role of CB and abstract from any analysis of monetary or fiscal policy effects on bond market except that there is an interest rate shock through a discount bond. Iwata (2009) focuses on the fiscal policy under DSGE setting. Hirose (2014) estimates a DSGE with a deflationary steady state for Japan and considers whether several shocks to the economy have an inflationary effect. McNelis and Yoshino (2016) compare the performance of three policy rules on reducing the government debt using a DSGE model. However, they do not explicitly model the role of CB and there is no government bond in the model. Fueki et al. (2016) set up a DSGE model to analyze potential output and output gap for the Japanese economy.

In the second group of literature, Adjemian and Julliard (2010) estimate a DSGE model with zero lower bound for nominal interest rate. Michaelis and Watzka (2017) consider the change in the effectiveness of quantitative easing policy at the zero lower bound. Although there are liquidity shocks in their model, they do not have a DSGE model. Instead they estimate time varying parameter using VAR analysis and do not study the effect of monetary policy on bonds.

Our study contributes more to the first strand of literature. In contrast with extant studies, we explicitly model the role of CB and the nexus between the government budget constraint, the CB budget constraint and the commercial bank's flow budget constraint. We focus on the transmission channels of QQE policy of BoJ to the Japanese bond market via the dynamic linkage between CB and government budget constraints which is a new contribution in the literature.

While a plethora of literature exists on various applications of DSGE models, what is less understood is its bond yield implications. Rudebusch and Swanson (2012) show some innovative applications of DSGE model to understand bond pricing behaviour. However, they do not focus on the monetary policy effects on the bond market behaviour, which is the scope of our study. Chen et al. (2012) is one of the few studies that uses DSGE modelling

to assess the effects of UMP on long term bond yields in the US who find that the QE in the US has rather insignificant effects on long term bond yield. They, however, do not formulate the CB balance sheet and commercial banks' asset portfolio which we do. Moreover, their focus is on the QE operation in the US, while our focus is on QQE in Japan which involves additional monetary policy instruments including IOER. As in Chen et al. (2012), our model also predicts that a simple QE in the form of CB's open market purchase of long term government bonds has insignificant effect on the long term bond yield. However, we find that QE has nontrivial effects on real bond yield the term premia. In addition, we find that a negative shock to the interest rate on excess reserve has a stronger effect on the term premium as opposed to QE. We also find that a CMP in the form of an overnight borrowing rate shock has near zero effect on bond yield and term premia.

3 An Overview of BoJ Monetary Policy

3.1 Background

BoJ changed the policy rate from the official discount rate to short term market rate in March 1995, although they were not explicit about the exact short term rate as the operating target. From 1998, BoJ specified the uncollateralized overnight call rate as the policy rate and they also started to announce the target level for the call rate. On February 12, 1999, the BoJ decided to take the following two actions: (i) Provide ample liquidity and encourage the uncollateralized overnight call rate to move as low as possible, and (ii) Guide the call rate to move around 0.15%, and subsequently induce further decline in view of the market developments.¹ The well-known “zero interest rate” policy was announced, although the initial target call rate was at 0.15%. The call rate dropped from around 0.3% before the announcement to around 0.05% after the announcement. On August 11, 2000, BoJ decided to lift the zero interest policy by stating that the BoJ will encourage the uncollateralized overnight call rate to float on the average around 0.25%.² The call rate surged from around 0.05% before the announcement to around 0.3% after the announcement. Figure 1 shows a plot of the overnight call rate.³

¹This is based on the “Announcement of the Monetary Policy Meeting Decisions” released on February 12, 1999.

²This is based on the press release on August 11, 2000, titled “Change of the Guideline for Money Market Operations.”

³The appendix provides the details of all data sources.

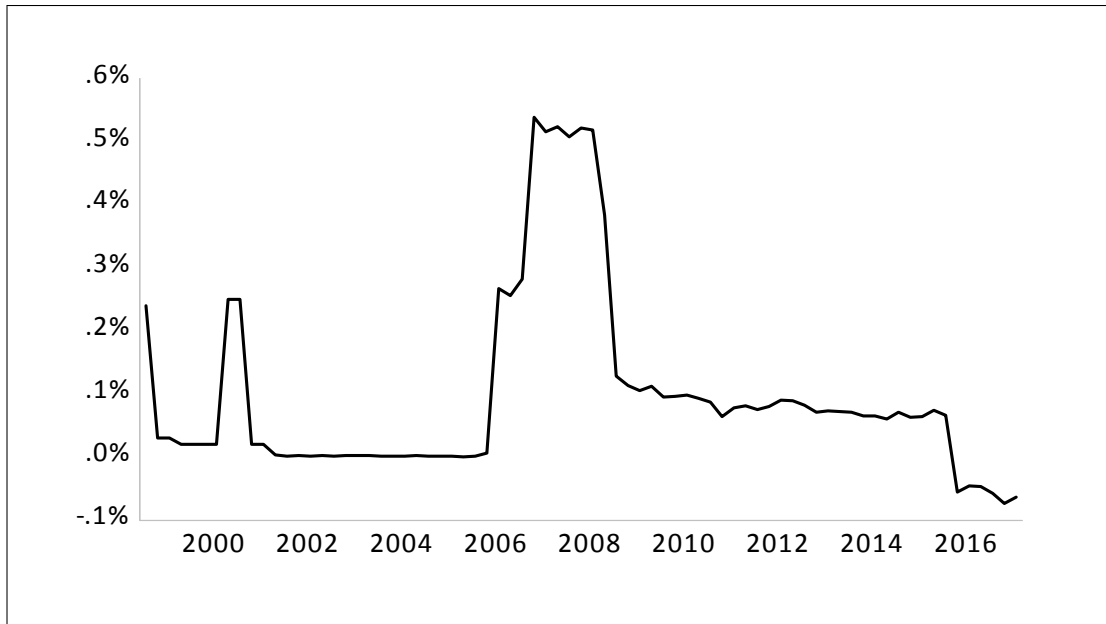


Figure 1: Overnight Call Rate

3.2 Quantitative Monetary Easing Policy (QME)

On March 19, 2001 the BoJ announced four major changes in monetary policy.⁴ (i) The main operating target for money market operations was changed from the current uncollateralized overnight call rate to the outstanding balance of the current accounts at BoJ;⁵ (ii) The new procedures for money market operations would continue until a zero percent CPI annual inflation target is achieved, (iii) The balance outstanding at BoJ’s current accounts would increase by 1 trillion yen from the average outstanding of 4 trillion yen in February 2001. (iv) The BoJ would increase the amount of its outright purchase of long-term government bonds from the current 400 billion yen per month to provide liquidity smoothly to the private sector.

With this announcement, the call rate spiked and then the level of call rate dropped to almost zero as seen in Figure 1. The zero coupon yield to maturity of 10 year JGB showed a mixed reaction to this announcement. It kept rising for more than four weeks after the announcement. After that the yield dropped gradually with occasional bumps but it rose sharply later after mid 2015 as seen in Figure 2.

⁴See the press release published on March 19, 2001, titled “New Procedures for Money Market Operations and Monetary Easing.”

⁵We use the phrase current account and reserve synonymously here. In practice there is a difference of the order of 2%.

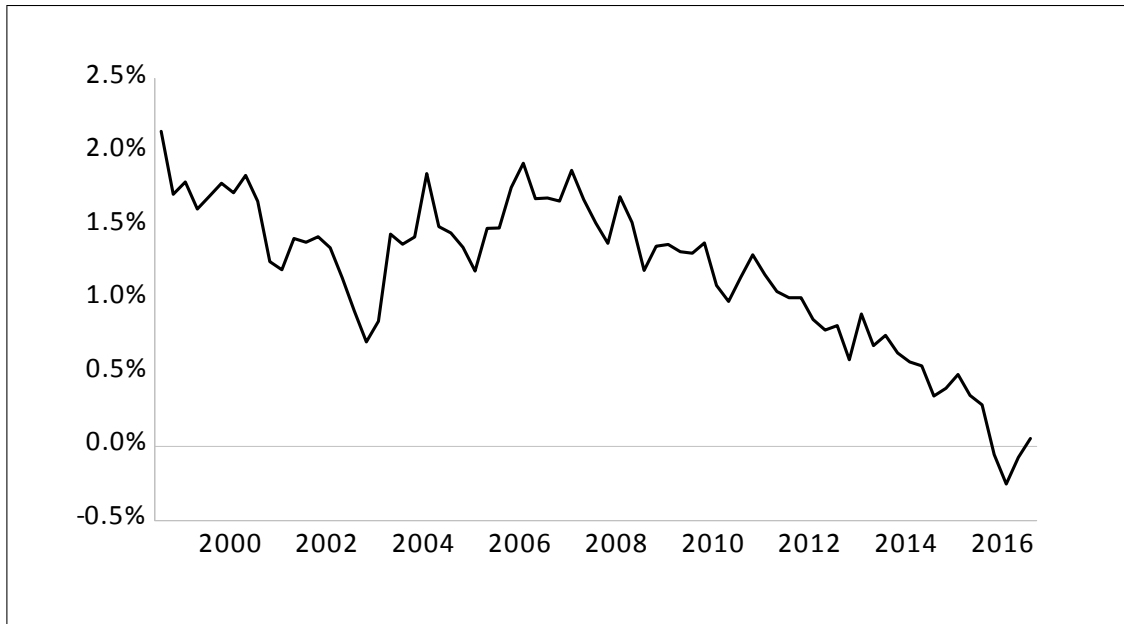


Figure 2: Yield to Maturity of Ten Year JGB

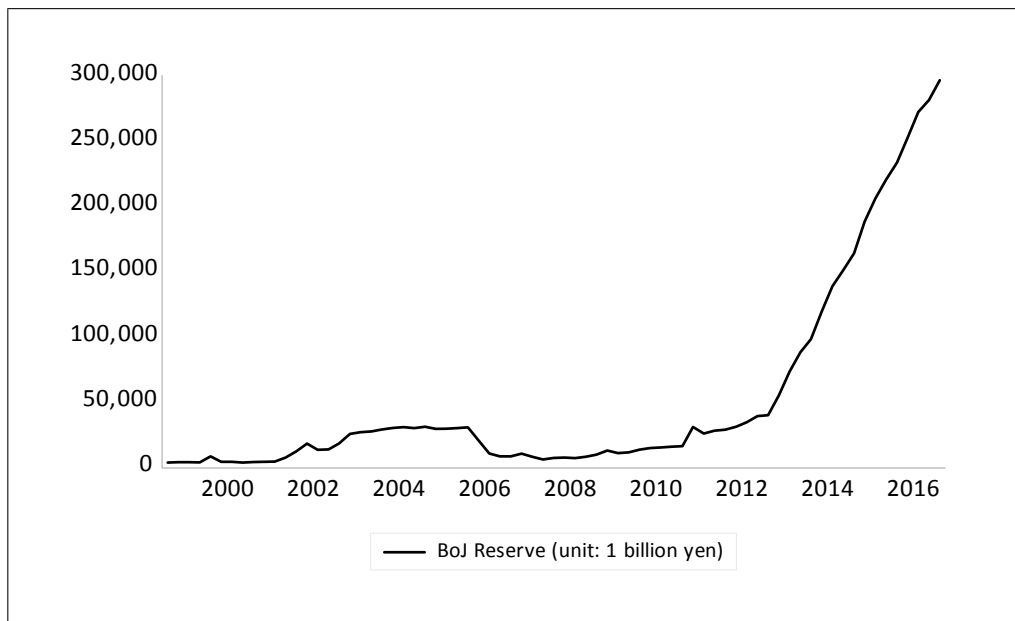


Figure 3: BoJ Reserve Balance

On March 9, 2006 BoJ changed the operating target from the reserve to uncollateralized overnight call rate.⁶ After the announcement, there was a gradual increase in the level of the overnight call rate. The reserve balance also declined initially reaching 7.6 trillion yen in May 2008 but it has increased afterwards reaching 58.1 trillion yen in March 2013. Figure 3 plots the time series of BoJ reserve balance.

⁶Based on a press release published on March 9, 2006, titled “Change in the Guideline for Money Market Operations.”

3.3 Qualitative and Quantitative Monetary Easing Policy

On April 4, 2013, the BoJ decided to introduce Qualitative and Quantitative Monetary Easing Policy (QQE). The main objective was to achieve an annual CPI inflation target of 2 percent. In order to achieve this objective BoJ decided to implement the “monetary base control” with an announcement to double the monetary base and the outstanding amounts of Japanese government bonds (JGB) in two years, and more than double the average remaining maturity of JGB purchases.⁷ The monetary base nearly doubled between April 2013 and April 2015 (Figure 4). Similar trend was reflected in the reserve balances held at the BoJ by banks as seen in Figure 3. From 2009 onward, long term bonds held by BoJ exponentially increased while the holding of the same by the banks decreased. This reflects a large scale purchase of long term government bonds by the BoJ which is the very essence of QQE (Figure 5).

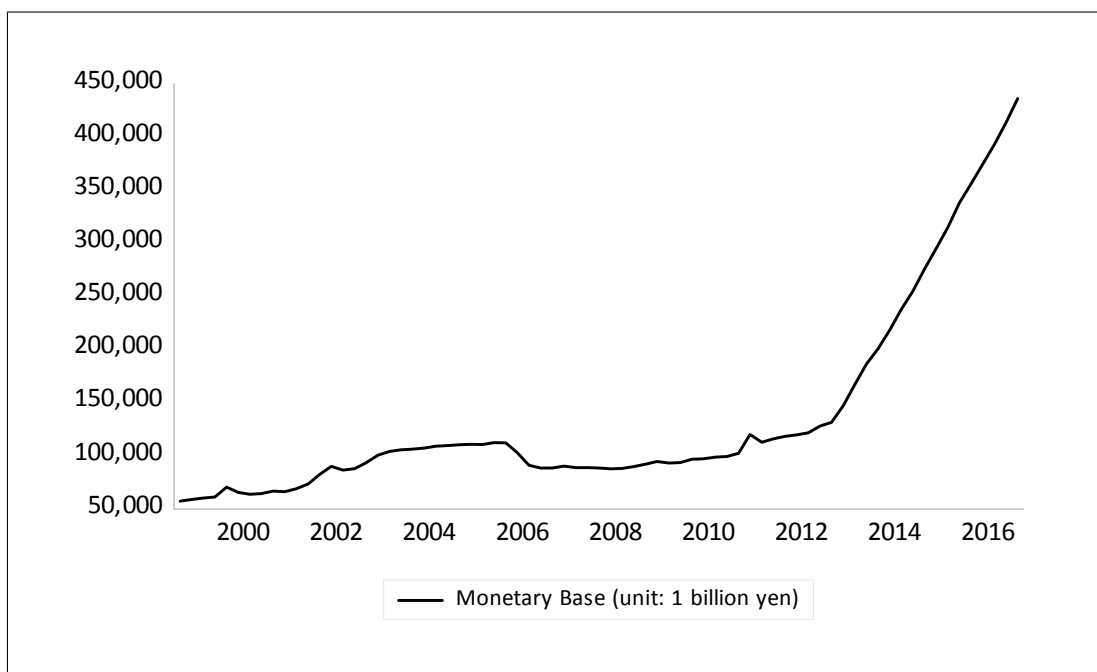


Figure 4: Monetary Base

⁷See the press release published on April 4, 2013, “Introduction of the Quantitative and Qualitative Monetary Easing.”

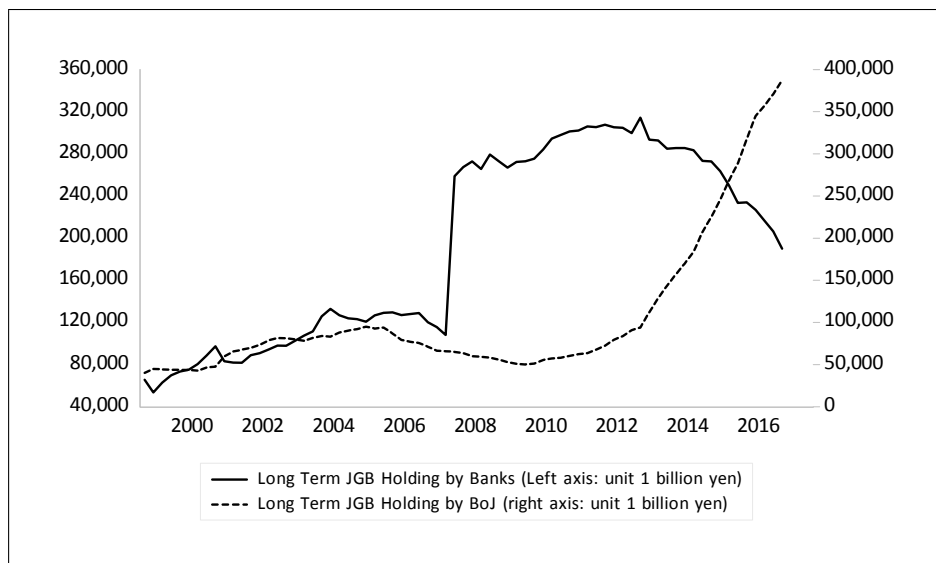


Figure 5: Long Term JGB Holding by Banks and BoJ

3.4 QQE with Negative Interest Rate

On January 29, 2016, BoJ decided to implement QQE with a negative Interest rate on excess reserve. In order to achieve the inflation target of two percent at the earliest possible time, the BoJ set the negative interest rate on current accounts at -0.1% .⁸ They also adopted a three-tier system in which the outstanding balance of each bank’s current account at the Bank will be divided into three tiers offering positive, zero and negative interest rates respectively.⁹

3.5 QQE with Yield Curve Control

On September 21, 2016, the BoJ introduced a new policy framework that consists of two major components: the first is “yield curve control” in which the Bank will control short-term and long-term interest rates; and the second is an “inflation-overshooting commitment” in which the Bank commits itself to expanding the monetary base until the year-to-year CPI inflation rate exceeds the inflation target of two percent and stays above the target in a stable manner.¹⁰

These various policy changes are summarized in Table 1.

The aim of our study is to primarily analyze the effect of QE and QQE on the bond

⁸See the press release published on January 29, 2016, titled ‘Introduction of “Quantitative and Qualitative Monetary Easing with a Negative Interest Rate”.’

⁹See the same report published on January 29, 2016.

¹⁰See the press release published on September 21, 2016, titled “New Framework for Strengthening Monetary Easing: Quantitative and Qualitative Monetary Easing with Yield Curve Control.”

Table 1: A History Table of Japanese Monetary Policy Regimes, 2000–2017

Date	Event
August 11, 2000	Abandonment of Zero Interest Rate Policy
March 19, 2001	Quantitative Monetary Easing Policy (from call rate to current account target)
March 9, 2006	Revival of Call Rate Target
April 4, 2013	QQE (monetary base control with a 2% inflation target)
January 29, 2016	QQE with a Negative Interest Rate (−0.1% interest rate on current account)
September 21, 2016	QQE with Yield Curve Control (0% yield on long term bond)

market and the aggregate economy. Thus we focus our attention on the period 2001 to the current. In the next section, we lay out a new Keynesian DSGE model to understand the effect of QE and other structural shocks on the aggregate economy and bond pricing aggregates.

4 Model

4.1 Story

We have seven players in the economy: the representative household, three types of firms, commercial banks, CB and the government. Household owns all productive units and thus profits are received as transfers to the household. Households save in the form of short term bank deposits (which are perfect substitutes for short-term government bonds). They supply labour to wholesale goods firms. Their income consists of labour, interest income from deposit and cash flows generated from the ownership of firms and the banks.

Three types of firms are: retail, wholesale and capital goods producers. Competitive risk neutral one period lived wholesale firms finance their capital spending from banks. Competitive capital goods producers buy used capital from wholesalers and refurbish it to new capital using investment goods bought from the retail producers. Retail producers convert wholesale goods costlessly to final goods and has some monopoly power of price fixing. Final goods can be used for household consumption, capital goods producers' investment and government use.

Banks collect household deposit and intermediate this to wholesale firms and also hold long term government bonds and excess reserve since they anticipate an aggregate bank run risk. If banks experience a shortfall of deposits, they borrow from the lender of last resort,

CB at a penalty rate. Excess reserve also earns an interest rate.

The government consumes some final goods which is financed by lump sum taxes on households and borrowing from the commercial banks and the CB via issuing long term government bonds. The CB finances its government bond holding by reserve creation, seigniorage and the revenue earned from banks resulting from penalty loans.

4.2 Households

Households solve the following maximization problem:

$$\max_{c_t, D_t, M_t^{TD}, H_t} E_t \sum_{t=0}^{\infty} \beta^t [U(c_t - \gamma_c C_{t-1}) + V(D_t/P_t) + W(M_t^{TD}/P_t) - \Phi(H_t)]$$

subject to the following budget constraint:

$$P_t c_t + D_t + M_t^{TD} \leq W_t H_t + (1 + i_t^D) D_{t-1} + M_{t-1}^{TD} + TR_t \quad (1)$$

where P_t is aggregate price index, c_t is the representative agent's consumption after adjusting for the previous period's aggregate consumption C_{t-1} up to a fraction γ_c which means a habit persistence relative to aggregate consumption. D_t is one period deposit in nominal terms which are perfect substitutes for short term government bonds (as in Gertler and Karadi, 2013), H_t is labour hours, W_t is nominal wage, i_t^D is the riskfree nominal interest rate on deposits, M_t^{TD} is transaction demand for cash, TR_t is lump sum transfer to the households which include cash flows from capital goods firms, retail goods firms, and banks as well as transfer from the government and cash injection from the CB. We assume that household receives direct utility from bank deposits and cash holding.¹¹ $U(\cdot), V(\cdot), W(\cdot)$ are instantaneous continuous, strictly concave utility functions from consumption, real deposit and real money balance with the usual regularity conditions and $\Phi(H_t)$ is the continuous disutility function from work.

The first order conditions are:

$$D_t : U_{ct} = V'(d_t) + \beta E_t U_{ct+1} (1 + i_{t+1}^D) (P_t/P_{t+1}) \quad (2)$$

¹¹We put both real cash balance and real deposits in the utility function motivated by the fact that both money and short term bank deposits provide different kinds of transaction conveniences to the household. Putting real cash balance in the utility function has a long tradition following Sidrauski (1967). The idea of real deposits in the utility function is borrowed from Hansen and Imrohroglu (2013) who put short term government bonds in the utility function. Since households value the liquidity service of short term bank deposits, they are willing to accept a lower rate on bank deposits than the loan rate the banks charge to the wholesale goods firms which are also owned by households. A natural borrowing-lending spread or limits to arbitrage thus arises in our model.

$$M_t^{TD} : U_{ct} = W'(m_t^{TD}) + \beta E_t U_{ct+1} (P_t/P_{t+1}) \quad (3)$$

$$H_t : \Phi'(H_t) = (W_t/P_t) U_{ct} \quad (4)$$

where $d_t = D_t/P_t$, $m_t^{TD} = M_t^{TD}/P_t$ and U_{ct} is the derivative of $U(c_t - \gamma_c C_{t-1})$ with respect to c_t . Equation (2) shows that marginal utility cost of holding a dollar of deposit balances the temporal marginal utility of liquidity service from deposits and the discounted utility benefits of the interest on deposit adjusted for inflation tax. Likewise equation (3) shows the marginal equivalence condition of cost and benefit of holding a dollar money balance. Equation (4) is the standard static efficiency condition for labour supply.

4.3 Capital goods producing firms

Capital goods producers buy last period's used capital from the wholesale firms/entrepreneurs $\{(1 - \delta_k) K_{t-1}\}$ at real prices Q_t . They produce new capital stock K_t by investing I_t of final goods using a linear investment technology:

$$K_t = (1 - \delta_k) K_{t-1} + Z_{xt} I_t \quad (5)$$

where Z_{xt} is an investment specific technology shock and δ_k is the physical rate of depreciation of capital. After investment this new capital is sold to the wholesalers at a relative price Q_t . For one unit investment, the capital goods producers purchases $(1 + S(\frac{I_t}{I_{t-1}}))$ of final goods where $S(\cdot)$ is a continuous flow investment adjustment cost function.¹² The capital goods producer then solves

$$\max_{I_{t+j}} E_t \sum_{j=0}^{\infty} \Omega_{t,t+j} \Pi_{t+j}^k$$

where, $S(1) = S'(1) = 0$ and $S''(1) = \kappa$ is the investment adjustment cost parameter and $\Omega_{t,t+s}$ is the inflation adjusted stochastic discount factor which is equal to $\frac{\beta^s U_{ct+s}}{U_{ct}} \cdot \frac{P_t}{P_{t+s}}$. Π_{t+j}^k is the cash flow of the capital goods producer given by:

$$\Pi_{t+j}^k = P_{t+j} \left[Q_{t+j} I_{t+j} - \left\{ 1 + S\left(\frac{I_{t+j}}{I_{t+j-1}}\right) \right\} I_{t+j} \right]$$

The first order condition gives the following Euler equation:

$$Q_t = 1 + S\left(\frac{I_t}{I_{t-1}}\right) + S'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}} - E_t \frac{\beta U_{ct+1}}{U_{ct}} \left[S'\left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2 \right] \quad (6)$$

¹²Note that this investment adjustment cost is incurred before investment is undertaken to install new capital K_t . That is why it does not appear in the linear investment technology (5).

This Q equation is similar to Gertler and Karadi (2013).

4.4 Wholesale goods producing firms

Wholesale firms are run by risk neutral entrepreneurs who produce intermediate goods (Y_t^W) for the final goods producing retailers in a perfectly competitive environment. The entrepreneurs hire labour force from the households and purchase new capital from the capital good producing firms. They borrow L_t amount of loan from the bank in order to meet the value of new capital ($Q_t K_t$). We assume that all capital spending is debt financed. Used capital at date t is sold at the resale market at the price Q_t .

Balance sheet condition of wholesale firms is:

$$Q_t K_t = \left(\frac{L_t}{P_t} \right) \quad (7)$$

The wholesale goods production function is specified as $Y_t^W = A_t K_{t-1}^\alpha (\Theta_t H_t)^{1-\alpha}$ with $0 < \alpha < 1$, A_t as the TFP shock, Θ_t as a labour augmenting technical progress component. We assume that Θ_t grows at a deterministic gross rate Λ which means the balanced growth rate of the economy is Λ . The equilibrium real wage, $W_t/P_t = (1 - \alpha) \frac{(P_t^W/P_t) Y_t^W}{H_t}$ where P_t^W is the nominal price of the wholesale good.

The gross rate of return from capital is given by,

$$\begin{aligned} 1 + r_{t+1}^k &= \frac{(P_{t+1}^W/P_{t+1}) Y_{t+1}^W - (W_{t+1}/P_{t+1}) H_{t+1} + (1 - \delta_k) K_t Q_{t+1}}{Q_t K_t} \\ &= \frac{(P_{t+1}^W/P_{t+1}) \left(\frac{Y_{t+1}^W}{K_t} \right) - (1 - \alpha) \frac{(P_{t+1}^W/P_{t+1}) Y_{t+1}^W}{H_{t+1}} \left(\frac{H_{t+1}}{K_t} \right) + (1 - \delta_k) Q_{t+1}}{Q_t} \\ &= \frac{(P_{t+1}^W/P_{t+1}) MPK_{t+1} + (1 - \delta_k) Q_{t+1}}{Q_t} \end{aligned}$$

where MPK_{t+1} denotes the the marginal product of capital at date $t + 1$. Defining i_t^L as the net nominal interest rate on loans, the optimality condition for firms demand for capital (or the arbitrage condition) can be written as,

$$1 + r_{t+1}^k = (1 + i_{t+1}^L) \frac{P_t}{P_{t+1}}$$

which yields,

$$(1 + i_{t+1}^L) = \frac{P_{t+1}^W MPK_{t+1} + (1 - \delta_k) P_{t+1} Q_{t+1}}{P_t Q_t}$$

In other words,

$$1 + i_{t+1}^L = \left[\left(\frac{P_{t+1}^W}{P_{t+1}} \right) \frac{MPK_{t+1}}{Q_{t+1}} + 1 - \delta_k \right] \left[\frac{P_{t+1}Q_{t+1}}{P_t Q_t} \right] \quad (8)$$

4.5 Final goods retail firms

Retailers buy intermediate goods at a price P_t^W , convert it one-to-one to final goods, and differentiate the goods at a zero cost. The i th retailer sells his/her unique variety of final product $y_t(i)$ applying a markup over the wholesale price after factoring in the market demand condition which is characterized by the price elasticity (ε^Y). Retailer's prices are sticky and indexed partly to last period's to steady state inflation. They also face a quadratic price adjustment cost which is parameterized by ϕ_P . Hence, the i th retailer chooses $\{P_{t+j}(i)\}_{j=0}^{\infty}$ to maximize present value of their expected cash flows ($\Pi_{t+j|t}^r(i)$) conditional on the information at date t .

$$\max_{\{P_t(i)\}} E_t \sum_{j=0}^{\infty} \Omega_{t,t+j} \{ \Pi_{t+j|t}^r(i) \}$$

subject to

$$y_{t+j|t}(i) = \left(\frac{P_{t+j}(i)}{P_{t+j}} \right)^{-\varepsilon^Y} y_{t+j}$$

where $y_{t+j|t}(i)$ is the demand for the i th retailer's good conditional on the aggregate demand y_{t+j} and

$$\begin{aligned} \Pi_{t+j|t}^r(i) &= P_{t+j}(i) y_{t+j}(i) - P_{t+j}^W(i) y_{t+j}^W(i) \\ &\quad - \frac{\phi_p}{2} \left[\left\{ \frac{P_{t+j}(i)}{P_{t+j-1}(i)} - (1 + \pi_{t+j-1})^{\theta_p} (1 + \bar{\pi})^{1-\theta_p} \right\}^2 P_{t+j} y_{t+j} \right] \end{aligned}$$

where $\phi_p > 0$ and $0 < \theta_p < 1$. Note that θ_p is an indexation parameter. This price adjustment cost specification is borrowed from Gerali et al. (2010). The aggregate demand at date t (y_t) is given by a standard CES aggregator:

$$y_t = \left[\int_0^1 y_t(i)^{\frac{\varepsilon^Y - 1}{\varepsilon^Y}} di \right]^{\frac{\varepsilon^Y}{\varepsilon^Y - 1}}; \quad \varepsilon^Y > 1$$

The first order condition after imposing symmetric equilibrium yields

$$1 - \varepsilon^Y + \varepsilon^Y \left(\frac{P_t}{P_t^W} \right)^{-1} - \phi_p \{ 1 + \pi_t - (1 + \pi_{t-1})^{\theta_p} (1 + \bar{\pi})^{1-\theta_p} \}$$

$$+\Omega_{t,t+1}\phi_p \left\{1 + \pi_{t+1} - (1 + \pi_t)^{\theta_p}(1 + \bar{\pi})^{1-\theta_p}\right\} (1 + \pi_{t+1})^2 \frac{y_{t+1}}{y_t} = 0 \quad (9)$$

In the steady state, when $\pi_{t+1} = \pi_t = \pi$, the above price equation reduces to a simple static markup equation:

$$\frac{P}{P^W} = \frac{\varepsilon^Y}{\varepsilon^Y - 1}. \quad (10)$$

4.6 The Banking sector

Denote L_{t-1} as outstanding loans issued at date $t-1$, and B_{t-1}^P as the corresponding outstanding net government bonds held by the commercial banks. Likewise, let M_{t-1}^{RD} be commercial banks' outstanding reserve holding at date $t-1$. Banks plan to hold excess reserve because they face the risk of a bank run at the end of period. As in Chang et al. (2014) at the end of each period, deposits can be withdrawn stochastically. If the withdrawal (say \widetilde{WX}_{t-1}) exceeds bank reserve (cash in vault), banks fall back on the CB for emergency loan at a penalty rate i_t^p mandated by CB. Banks pay back to the CB at the end of the period. We assume that this withdrawal risk is an aggregate risk which cannot be mitigated through any interbank market.¹³ This withdrawal uncertainty necessitates a demand for excess reserve by the banks even though the interest rate on excess reserve can become negative.

Define i_t^R as the interest rate on excess reserve (IOER) mandated by BoJ, \widetilde{WX}_{t-1} is the stochastic withdrawal realized at the end of period $t-1$ and S_t is the date t price of a nominal default free long term bond with geometrically declining coupon payments at rate ν as in Rudebusch and Swanson (2012). Bank's cash flow at date t can be rewritten as:

$$\begin{aligned} CF_t^b &= (1 + i_t^L)L_{t-1} + (1 + i_t^R)M_{t-1}^{RD} + (1 + \nu S_t)B_{t-1}^P - (1 + i_t^D)D_{t-1} \\ &\quad - (1 + i_t^p)\chi_t(\widetilde{WX}_{t-1} - M_{t-1}^{RD}) - (1 - \chi_t)\widetilde{WX}_{t-1} \\ &\quad - S_t B_t^P - L_t - M_t^{RD} + D_t \end{aligned} \quad (11)$$

where χ_t is an indicator function which is unity if $\widetilde{WX}_{t-1} - M_{t-1}^{RD} > 0$ and zero otherwise. Timing of decisions is crucial in our setting to make the bank's problem meaningful. At date t , banks make decisions about loans (L_t) bond holding (B_t^P) and reserve (M_t^{RD}) after observing the deposit (D_t). On the other hand, depositors could partially withdraw their deposit randomly before loans are repaid.¹⁴ This basically gives a motivation to banks to

¹³We do not have any interbank market for loans in this model because banks are all homogenous and subject to the same aggregate risk of bank run. However, since a higher penalty rate induces banks to hold more excess reserve (which we show later), this rate is a reasonable proxy for the overnight call rate.

¹⁴Here is an example that illustrates the stochastic withdrawal and contingent penalty. Suppose the withdrawal (\widetilde{WX}_{t-1}) is 30 dollars and bank reserve (M_{t-1}^{RD}) is 10 dollars. The bank falls short of reserve by 20 dollars and thus takes recourse to emergency loan from the CB. The bank incurs 10 percent penalty (i_t^p)

hold excess reserve as in Chang et al. (2014).¹⁵

Given the assets at date t , and deposit sequence $\{D_t\}$ determined by the household's problem, banks choose M_t^{RD}, B_t^P, L_t which solves the following dynamic optimization:

$$\max_{\{M_t^{RD}, B_t^P, L_t\}} E_t \sum_{s=0}^{\infty} \Omega_{t,t+s} C F_{t+s}^b$$

s.t. the statutory reserve requirement:

$$M_t^{RD} \geq \alpha_r D_t \quad (12)$$

where α_r is the statutory reserve ratio.

The Euler equation for M_{t+1}^{RD} is given by:

$$M_t^{RD} : 1 = E_t \Omega_{t,t+1} \left[(1 + i_t^R) + (1 + i_t^P) \text{Prob}(\widetilde{WX}_t / D_t \geq M_t^{RD} / D_t) \right] + \varkappa_t \quad (13)$$

The first term in the square bracket in (13) is the bank's interest income from reserve and the second term is the expected saving of penalty because of holding more reserve and \varkappa_t is the Lagrange multiplier associated with the reserve constraint (12).

The Kuhn–Tucker condition states that

$$\frac{M_t^{RD}}{D_t} = \alpha_r \text{ if } \varkappa_t > 0$$

Otherwise

$$E_t \Omega_{t,t+1} \left[(1 + i_t^R) + (1 + i_t^P) \text{Prob}(\widetilde{WX}_t / D_t \geq M_t^{RD} / D_t) \right] = 1 \quad (14)$$

Assuming a uniform distribution for \widetilde{WX}_t over $[0, D_t]$, (14) reduces to:¹⁶

$$M_t^{RD} : 1 = E_t \Omega_{t,t+1} \left[(1 + i_t^R) + (1 + i_t^P) \left(1 - \frac{M_t^{RD}}{D_t}\right) \right] \quad (15)$$

on its loan. Thus at the end of the day bank's payment to the CB with penalty is 22 dollars which includes the principal and interest. On the other hand, if the bank reserve is 40 dollars instead of 10 dollars, the bank does not need to approach the CB for an emergency loan but the bank's cash flow still falls by 20 dollars. Taking this into consideration, bank chooses the reserve holding optimally at the start of date t .

¹⁵We do not model here the withdraw decision of households and assume that the withdrawal, \widetilde{WX}_t is random i.i.d. process and it cannot exceed deposits. This basically rules out a sudden stop of the economy with a full bank run. This random withdrawal makes the cash flow of the bank risky. This cash flow is ploughed back as transfer (TR_t) to the household.

¹⁶We assume that the \widetilde{WX}_t cannot exceed the available deposit D_t which makes \widetilde{WX}_t / D_t bounded above by unity.

Solve $\frac{M_t^{RD}}{D_t}$ as follows:

$$\frac{M_t^{RD}}{D_t} = 1 - \frac{1 - (1 + i_t^R)E_t\Omega_{t,t+1}}{(1 + i_t^p)E_t\Omega_{t,t+1}} \quad (16)$$

Since $(1 + i_t^R)E_t\Omega_{t,t+1} < 1$, given the discount factor, $\Omega_{t,t+1}$, a higher i_t^R or i_t^p means a higher proportion of deposits held as reserve (M_t^{RD}/D_t) by the banks. It is straightforward to verify that at the steady state $\frac{\partial(M_t^{RD}/D_t)}{\partial i_t^R} = \frac{1}{1+i^p}$ and $\frac{\partial(M_t^{RD}/D_t)}{\partial i_t^p} = \frac{1}{(1+i^p)^2} \left[\frac{1+\bar{\pi}-\beta(1+i^R)}{\beta} \right]$. For the baseline calibrated values of the steady state parameters reported later in Table 2, $\frac{\partial(M_t^{RD}/D_t)}{\partial i_t^R} > \frac{\partial(M_t^{RD}/D_t)}{\partial i_t^p}$. In other words, a decline in the IOER has a larger quantitative effect than a drop in the overnight borrowing rate on depressing the banks' excess reserve.¹⁷ Thus banks respond by loaning out excess reserve more with respect to a negative shock to IOER than a negative shock to overnight borrowing rate. This result is crucial in understanding later why a the recent QQE experiment of negative IOER is a more effective tool for promoting the economy than a CMP of lowering the overnight borrowing rate.

Next the bank solves a recursive problem of choosing B_t^P and L_t given B_{t-1}^P and L_{t-1} which were chosen in the previous period. This is a dynamic allocation problem. We have the two Euler equations:

$$B_t^P : 1 = E_t\Omega_{t,t+1}(1 + \nu S_{t+1})/S_t \quad (17)$$

$$L_t : 1 = E_t\Omega_{t,t+1}(1 + i_{t+1}^L) \quad (18)$$

Since $\Omega_{t,t+1}(s)$ is nothing but a contingent claims price of a dollar for state s , above two equations basically mean a no arbitrage condition that the discounted value of the expected excess returns on bond and loan is zero. This suggests that loans and government bonds are perfect substitutes while the bank reserve is not because the banking friction in the form of a withdrawal risk drives a wedge between the interest rate on reserve and the loan or government bond rate.

4.7 CB budget constraints

We now characterize the supply of bank reserve (denoted M_t^R) and supply of currency (denoted M_t^T). As in Hall and Reis (2015), CB must create enough reserve to pay for the interest and principal on existing commercial bank reserves, cover the purchase of government

¹⁷For the steady state baseline parameter values, the effect of an IOER reduction on bank reserve is about 20 times larger than the effect of an equal sized reduction in the call rate.

bonds (B_t^{CB}) net of bond income held from previous bonds (which is $(1 + \nu S_t)B_{t-1}^{CB}$). The rest it pays to the government after netting out the seigniorage revenue from printing cash ($M_t^T - M_{t-1}^T$) and the penalty revenue received from banks with overnight loans. In other words, CB's budget constraint is given by:

$$M_t^R = (1 + i_t^R)M_{t-1}^R + S_t B_t^{CB} - (M_t^T - M_{t-1}^T) - (1 + \nu S_t)B_{t-1}^{CB} - (1 + i_t^p)\chi_t(\widetilde{WX}_{t-1} - M_{t-1}^{RD}) + Div_t \quad (19)$$

Note that the idea of dividend payment by the CB to the government is borrowed from Hall and Reis (2015). Literally, the CB does not pay such dividend but it should generate sufficient revenue to cover the deficits of the government. Thus Div_t is the link between CB and the government.

Dividing through the price level, the real dividend to the government (div_t) can be written as:

$$div_t = m_t^R - (1 + i^R)\frac{m_{t-1}^R}{1 + \pi_t} + m_t^T - \frac{m_{t-1}^T}{1 + \pi_t} - S_t b_t^{CB} + (1 + \nu S_t)\frac{b_{t-1}^{CB}}{1 + \pi_t} + (1 + i_t^p)\chi_t(\widetilde{WX}_{t-1} - M_{t-1}^{RD}) \quad (20)$$

where $b_t^{CB} = B_t^{CB}/P_t$, $m_t^R = M_t^R/P_t$, $m_t^T = M_t^T/P_t$.¹⁸

4.8 Government Budget Constraint

The government spends exogenous stream (G_t) of final goods. This spending is financed by lump sum taxes on households (T_t), the penalty dividends (Div_t) received from the CB. All government borrowing is in the form of long term government bonds (B_t^G). The government budget constraint in nominal form is given by:

$$P_t G_t + (1 + \nu S_t)B_{t-1}^G = P_t T_t + S_t B_t^G + Div_t \quad (21)$$

4.9 Equilibrium

In equilibrium, the following market clearing conditions hold:

1. Goods market clears which means that GDP equals the sum of consumption, private

¹⁸ $S_t b_t^{CB}$ (which is $S_t B_t^{CB}/P_t$) is the real holding of government bonds.

investment, government spending, and price adjustment costs.

$$c_t + \left\{ 1 + S \left(\frac{I_t}{I_{t-1}} \right) \right\} I_t + G_t + \frac{\phi_p}{2} \left[\left\{ (1 + \pi_t) - (1 + \pi_{t-1})^{\theta_p} (1 + \bar{\pi})^{1-\theta_p} \right\}^2 y_t \right] = y_t$$

2. The loan market clears in the sense that the balance sheet constraint (7) of the wholesaler binds:

$$L_t/P_t = Q_t K_t$$

3. Given that all public debt is nationally held, the bond market equilibrium requires that JGB held by banks and the CB sum to the treasury issued bonds

$$B_t^P + B_t^{CB} = B_t^G$$

4. Money market clears which means that the demand for bank reserve (M_t^{RD}) equals the supply bank reserve (M_t^R) and the transaction demand for money (M_t^{TD}) equals the corresponding supply (M_t^T):

$$M_t^{RD} = M_t^R$$

$$M_t^{TD} = M_t^T$$

4.10 Debt/GDP target

The value of the public debt is $S_t B_t^G$. We assume that the debt to GDP ratio is exogenously fixed at Γ . In other words,

$$S_t B_t^G = \Gamma P_t y_t$$

Given that S_t and y_t are stationary, the government has to issue nominal debts at the target rate of inflation ($\bar{\pi}$).

4.11 QE operation

In practical terms, a QQE is analogous to an open market operation where the CB is buying JGBs from the banks in the short run and place more reserve in the banking system. This alters the composition of assets (reserve/bond ratio) of the commercial banks in the short run. The commercial banks have a higher proportion of bank reserve and lower proportion of government bonds following the QQE operation. Since it is a short run policy exercise, such an operation should not permanently alter the asset composition of the CB. In addition, in the context of Japanese QQE operation, a two percent inflation target is also taken into consideration by the CB while undertaking the QQE operation.

Keeping these features in mind, we formulate the QE operation as a stochastic shock to monetary reserve and the ratio of JGBs held by CB (B_t^{CB}/B_t^G) as follows:¹⁹

$$\frac{M_t^R/M_{t-1}^R}{1 + \bar{\pi}} = \left(\frac{M_{t-1}^R/M_{t-2}^R}{1 + \bar{\pi}} \right)^{\rho_\mu} \exp(\xi_t^\mu) \quad (22)$$

$$(\lambda_t/\bar{\lambda}) = (\lambda_{t-1}/\bar{\lambda})^{\rho_\mu} \exp(\xi_t^\mu) \quad (23)$$

where $\lambda_t = B_t^{CB}/B_t^G$ and ξ_t^μ is an i.i.d. QE shock. Notice that the same shock, ξ_t^μ appear in both the monetary base and the asset composition equations (22) and (23). A positive QE shock (ξ_t^μ) boosts the monetary base and raises the ratio of bank reserve in the monetary base $M_t^R/(M_t^R + M_t^T)$. It also raises the share of CB holding of government bonds, λ_t in the short run. This feature reflects the basic tenets of QE operation that the CB purchases government bonds from commercial banks by injecting bank reserve which changes the asset composition of the CB and commercial banks. In the long run monetary base continues to grow at the target inflation rate and CB and commercial bank's asset composition reverts to the steady state ($\bar{\lambda}$) which is calibrated in the model. The rates of convergence of M_t^R/M_{t-1}^R and λ_t to the respective steady states are assumed to be the same which explains why the same smoothing coefficient, ρ_μ appears in both (22) and (23).

This law of motion (22) for the monetary base means that in a deterministic steady state the monetary base grows at the target rate of inflation $\bar{\pi}$. Such a money supply process imposes restriction on the short run growth rate of real reserve (m_t^R) and inflation as follows:

$$\frac{(1 + \pi_t)(m_t^R/m_{t-1}^R)}{1 + \bar{\pi}} = \left(\frac{(1 + \pi_{t-1})(m_{t-1}^R/m_{t-2}^R)}{1 + \bar{\pi}} \right)^{\rho_\mu} \exp(\xi_t^\mu) \quad (24)$$

where ξ_t^μ is an i.i.d. shock to the monetary base with zero mean.

What is the implication of such QQE for the balance sheets of BoJ and commercial banks? Note that when BoJ buys JGBs, it is buying debt of the government from the commercial banks. Thus the BoJ augments its asset by holding more government bonds and it simultaneously creates more liability by increasing monetary base. Thus the CB's balance sheet grows. Commercial Bank's assets just undergo a maturity transformation from long term bonds to equivalent short term bank reserves.

Since real reserve is proportional to deposit as shown in the bank's reserve demand

¹⁹Such an operation can also be thought of as a Repo operation by the CB. In other words, the CB is making a collateralized loans to the commercial banks. A QE shock with this feature makes the monetary base and the share of CB holding of government bonds to change in the same proportion relative to the steady state. Such a restriction keeps the CB budget constraint (19) binding by an offsetting adjustment of dividend paid to the government.

function, it also imposes restriction on the dynamics of deposits, interest rate on loans and consumption.

4.12 Policy Rates

There are two key policy rates: (i) the penalty rate i_t^p and (ii) the IOER, i_t^R . A reasonable proxy for i_t^p is the overnight borrowing rate. The official discount rate may be another proxy for i_t^p . However, it does not change much over time in response to macroeconomic condition. One can thus interpret i_t^p as a policy rate that would have prevailed if BoJ had used interest rate as a policy tool instead of quantitative easing. Viewed from this perspective, one can interpret i_t^p as a shadow interest rate in line with Iwasaki and Sudo (2017). We assume the following standard Taylor rule that characterizes the law of motion for i_t^p :

$$\frac{1 + i_t^p}{1 + \bar{i}^p} = \left(\frac{1 + i_{t-1}^p}{1 + \bar{i}^p} \right)^{\rho_{ip}} \left(\frac{1 + \pi_{t-1}}{1 + \bar{\pi}} \right)^{\phi_\pi(1-\rho_{ip})} \exp(\xi_t^{ip}) \quad (25)$$

where ρ_{ip} is the smoothing coefficient and ϕ_π is the inflation sensitivity of the interest rate, $\bar{\pi}$ is the target inflation rate and ξ_t^{ip} is an i.i.d. shock to the policy rate, i_t^p with zero mean.²⁰

The law of motion for IOER is posited as follows:

$$1 + i_t^R = (1 + i_{t-1}^R)^{\rho_{iR}} \exp(\xi_t^{iR}) \quad (26)$$

where ρ_{iR} is the smoothing coefficient and ξ_t^{iR} is an i.i.d. shock to the IOER with a zero mean. The underlying assumption is that IOER gravitates to a zero rate in the steady state.

4.13 Forcing Processes

We assume the following specifications for the three forcing processes, namely TFP (A_t), an investment specific technology (IST) shock (Z_{xt}), the fiscal spending shock (G_t):

$$A_t = \bar{A}^{1-\rho_A} A_{t-1}^{\rho_A} \xi_t^A \quad (27)$$

$$Z_{xt} = \bar{Z}_x^{1-\rho_z} Z_{xt-1}^{\rho_z} \xi_t^z \quad (28)$$

²⁰A few clarifications about the penalty rate i_t^p as a shadow policy rate are in order. If it is a reasonable shadow policy rate, it should pass the test of being a good price stabilizer. We follow a reverse engineering approach to evaluate the efficacy of penalty rate as a reasonable shadow policy rate. We back out the shock (ξ_t^{ip}) to this policy rate by executing the Bayesian estimation of our DSGE model (which is reported later) and correlate this backed out shock with the observed CPI inflation. We find that the correlation is -0.27 and statistically significant at 1% level. Thus the penalty rate (i_t^p) could be interpreted as a shadow policy rate because it is reasonable price stabilizer in our estimated DSGE model. Details of the calculations are available from the authors upon request.

$$G_t = \bar{G}^{1-\rho_G} G_{t-1}^{\rho_G} \xi_t^G \quad (29)$$

where ρ_A, ρ_z, ρ_G are the serial correlation coefficients, \bar{A}, \bar{Z}_x and \bar{G} are the steady states of the three respective processes. $\{\xi_t^A\}, \{\xi_t^z\}$ and $\{\xi_t^G\}$ are white noises.

The monetary policy block is summarized by three equations, namely (24), (25) and (26). Since monetary base and policy rates were used in different regimes quite randomly, we shock one of these three policy equations at a time. This justifies the assumption that $\{\xi_t^\mu\}, \{\xi_t^{ip}\}$ and $\{\xi_t^{iR}\}$ are i.i.d. processes which are contemporaneously uncorrelated among themselves.

4.14 Interest rates and bond yields

4.14.1 Borrowing-lending spread

A steady state borrowing-lending spread emerges in this model because deposit appears in the utility function and provides transaction convenience to the household. Thus the household is willing to accept a lower interest on its bank deposit than the loan rate. This means an endogenous credit rationing emerges in the model in the steady state because the borrowing-lending spread is positive. To see it combine (2) and (18) to get the following steady state borrowing-lending spread:

$$i^L - i^D = \frac{(1 + \pi) V'(d)}{\beta U'(c)} > 0 \quad (30)$$

4.14.2 Long term bond yield and term premium

Following Rudebusch and Swanson (2008, 2012), we focus on two important bond yield variables, (i) the yield to maturity and (ii) the term premium. Long term bond price (S_t) is modelled as perpetual consol type with geometrically declining coupon payments in a sequence $\{1, \nu, \nu^2, \dots\}$ at the end of each period where the duration of such a bond is given by $1/(1 - \beta\nu)$.²¹ Such a bond pricing equation obeys the Euler equation (17). The parameter ν pins down the duration of such a bond. We assume that these nominal long term bonds are default free and can be bought and sold every period.

A continuously compounded nominal yield to maturity (*nytm*) of such a long term bond

²¹See page 20 of Woodford (2001).

is given by the following formula as in Rudebusch and Swanson (2008).²²

$$nytm_t = \ln \left(\frac{1 + \nu S_t}{S_t} \right) \quad (31)$$

The real yield to maturity ($rytm_t$) is defined as $nytm_t$ minus the rate of inflation (π_t). Note that S_t fluctuates as the stochastic discount factor $\Omega_{t,t+1}$ shows fluctuations that reflect inflation and consumption uncertainty.

Following Rudebusch and Swanson (2008), we define the term premium on a long term bond as the difference between yield on the bond and the unobserved risk neutral yield on the same bond. The risk neutral counterpart of (17) can be written as:

$$S_t^R = \beta E_t \frac{[1 + \nu S_{t+1}^R]}{[1 + \pi_{t+1}]}$$

The term premium (tp) is thus defined as:

$$tp_t = \ln \left(\frac{1 + \nu S_t}{S_t} \right) - \ln \left(\frac{1 + \nu S_t^R}{S_t^R} \right) \quad (32)$$

Note that the term premium is a measure of risk of long term bonds. It is larger if the absolute value of the covariance between stochastic discount factor ($\Omega_{t,t+1}$) and the price of bond is higher.²³ Since the covariance depends on the inflation and the consumption growth, the term premium represents inflation and consumption risk associated with the long term bond.

The nominal one period holding period return of bond with maturity n for any investor sitting at date t as:²⁴

$$nhpr_{t,t+1} = \frac{1 + \nu S_{t+1}}{S_t} \quad (33)$$

The real holding period return ($rhpr_{t,t+1}^n$) is $nhpr_{t,t+1}$ minus the corresponding gross rate of inflation.

²²To get this formula, write (17) in continuous time as:

$$S_t = e^{-nytm_t} + e^{-nytm_t} \nu S_t$$

where $e^{-nytm_t} = \Omega_{t,t+1}$. This can be rewritten as (31). Our expression of $nytm$ differs from equation 15 in Rudebusch and Swanson (2008) because they assume that the coupon is given at the start of each period while we assume that the coupon is given at the end of each period.

²³See Rudebusch and Swanson (2008).

²⁴To see this, note that a perpetual bond bought at date t at the price S_t has a resale value νS_{t+1} at date $t+1$ which explains the numerator term. Thus the total proceeds next period including the coupon payment is $1 + \nu S_{t+1}$.

5 Estimation and Simulation

5.1 Baseline Calibration

In this section, we report the results of some policy experiments with our estimated DSGE model. For the baseline model, we fix the standard deep parameters β and α at the conventional levels of 0.99 and 0.36 respectively. The capital depreciation rate is fixed at a conventional level of 0.025. A long run 2% inflation target is set which means the steady state $\pi = .02$. The penalty interest rate $i^p = 0.001$ based on the quarterly average call rate. The steady state IOER, $i^R = 0$ and the required reserve ratio around 0.0067 (the average of all existing reserve ratios in Japan). For these baseline values, we find that the marginal benefit of holding an extra excess reserve exceeds the cost. Thus it is reasonable to assume that all banks start off with an excess reserve meaning that the Lagrange multiplier $\varkappa_t = 0$ in the steady state. The steady state proportion of government bonds held by BoJ ($\bar{\lambda}$) is fixed at 0.40.²⁵ The long run debt/GDP ratio (Γ) is fixed at 2.324.²⁶ The bond decay parameter ν is fixed at 0.985 to set the steady state duration of these perpetual bonds equal to 40 quarters. The interest rate on bank deposits is fixed at 4 basis points based on an average of short term ordinary deposits over our sample period 1999Q1–2017Q1. We specialize our simulation to a utility function: $\ln(c_t - \gamma_c C_{t-1}) + \eta_1 \ln d_t + \eta_2 \ln m_t^T - H_t$ and quadratic investment adjustment cost function: $S(I_t/I_{t-1}) = \frac{\kappa}{2}(I_t/I_{t-1} - \Lambda)^2$. The bank deposit preference parameter (η_1) is fixed at 0.0435 based on the steady state deposit/consumption ratio of 1.5 available for the year 2017Q1. The cash preference parameter η_2 is fixed at 0.0318 based on the cash/consumption ratio of 1.08 over our sample period. Table 2 presents the baseline calibrated parameter values.

Table 2: Baseline Parameter Values

β	η_1	η_2	α	δ_k	$\bar{\lambda}$	ν	$\bar{\pi}$	Γ	\bar{i}^r	α^r	\bar{i}_D	\bar{i}^P
0.99	0.0435	0.0318	0.36	0.025	0.40	0.985	0.02	2.324	0	0.0067	0.0004	0.001

²⁵According to the Ministry of Finance, 39.5% of government bond is held by BoJ at the end of March 2017 which is the last quarter of the sample period of this paper. Thus $\bar{\lambda}$ equal to 0.40 is a reasonable approximation. Change in this parameter has negligible effects on the results.

²⁶On May 31st 2017 the Japanese government set a new fiscal policy target and said that they would try to decrease the debt/GDP ratio. The debt/GDP ratio is 232.4% at the end of 2016. Unless GDP increases or debt decreases, it is hard to decrease the debt/GDP ratio. Thus we fix 232.4% as a reasonable target for the debt/GDP ratio.

5.2 Bayesian Estimation

We undertake a Bayesian estimation to compute the remaining parameters for which there is less conventional wisdom. Since there are six forcing processes, we choose six observable for our estimation. Four key macroeconomic variables are chosen as observable namely (i) the real per capita GDP, (ii) The real per capita private nonresidential investment, (iii) the real per capita government consumption and (iv) the annualized CPI inflation. All these four series are seasonally adjusted. The remaining two variables are: (v) the annualized overnight call rate as a proxy for i_p and (vi) the annualized nominal yield to maturity ($nytm$). Choice of observable is based on a trial and error process and these six observable give the best fit of the model. The sample period is 1999Q1–2017Q1. Details of the data sources are explained in the appendix.

Following Smets and Wouters (2007), we assume that all log level variables are trend stationary around a linear trend. In other words, a generic level variable (say v_t) in the model is written as V_t/Λ^t , where Λ is the balanced gross growth rate and V_t is the observed counterpart of the generic level variable. This means $\Delta \ln V_t = \ln \Lambda + \Delta \ln v_t$ which parallels the observation equation system in Smets and Wouters (2007). This means that the three key macro observable, GDP, investment and consumption are log differenced to be consistent with the model counterparts while the balanced growth rate Λ is estimated.

Our selection of the probability density functions for the priors are based on educated guesses and available estimates from extant studies. For prior, the Beta distribution is used for the fractions while the Inverse Gamma distribution is specified for the parameters with non-negativity constraints in line with Smets and Wouters (2007). In the absence of any knowledge about the priors of ϕ_p , ϕ_π , ε_Y and κ , their prior standard deviations are fixed at infinity.²⁷

The joint posterior distribution of the estimated parameters is obtained by standard procedure. First, the model equation system is loglinearized around the balanced growth path of the economy and written in a linear rational expectation recursive form.²⁸ Second, the system of equations is written in a Kalman filter observation equation form. Third, using this observation equation, the loglikelihood function of the relevant parameter vector is constructed. Fourth, the log posterior kernel is expressed using the prior density of the

²⁷The inverse gamma distribution of a random variable x with two parameters of v and ϱ has $E(x) = \varrho/(v-1)$ and $Var(x) = \varrho^2/((v-1)^2(v-2))$. Setting the prior standard deviation to infinity means $v = 2$ and $\varrho = E(x)$. The rationale for setting the prior standard deviation to infinity is that it imposes no bound on the relevant non-negative estimable parameter and let the data determine its value using Bayesian updating.

²⁸All nonstationary macroeconomic variables are deflated by the labour augmenting technical progress component, Θ_t to make the model stationary. The stationarized system is then log linearized around the steady state.

parameter. Fifth, the mode of this posterior kernel is computed using standard numerical optimization routines. Finally, a Gaussian approximation is constructed in the neighbourhood of this posterior mode using the Markov Chain Monte Carlo Metropolis–Hastings (MCMC-MH) algorithm. This algorithm simulates the smoothed histogram that approximates the posterior distributions of parameters of our interest. Five parallel chains are used in the MCMC-MH with a 30% acceptance rate from the draws. The univariate and multivariate diagnostic statistics are computed to check for MCMC convergence. All the computations are done by using dynare 4.5.4 version.

Table 3 reports the baseline parameter estimates from the Bayesian estimation routine. Priors are fixed at conventional levels. The price adjustment cost coefficient is taken from Keen and Wang (2005). The priors of the standard error of all forcing processes are kept at a very low level (0.001) to ensure that data provide enough information about the posterior counterparts. The only exception is the call rate ip . We fixed its prior at a higher level (0.01) to check the robustness of the low posterior estimate of its standard error.²⁹ The posterior estimate of the balanced growth rate 0.52% ($\Lambda = 1.0052$) is in line with the per capita GDP growth rate in our sample period. The posterior means of estimated parameters are reported with 90% highest posterior density (HPD) intervals. In most cases, data provide a lot of information about the parameters demonstrated by a significant difference between prior and posterior estimates. The only exceptions are for θ_p , and ϕ_p . The posterior estimate of the inflation targeting Taylor rule coefficient (ϕ_π) and the investment adjustment cost parameter κ are higher than the estimates of Fueki et al. (2016). All our estimable parameters are properly identified in the model following the criteria of asymptotic information matrix and collinearity patterns of the parameters as suggested in Iskrev (2010a, b) and Iskrev and Ratto (2010a, b).³⁰

5.2.1 Variance Decomposition

Table 4 shows the variance decompositions of fundamental shocks based on the average of all parameter draws for the whole sample period.³¹ Several points are in order. First, government spending shock and the overnight call rate shock have near zero contribution to any real and financial variables. Second, TFP shock picks up the half of the share of output variation in all periods. Next to TFP, monetary base and IOER account for a large variation

²⁹This was done because we find that the call rate has insignificant effects of the real and financial aggregates as seen by the irfs later. We want to ensure that this insignificance result is not due to a choice of a near zero prior σ_{ip} .

³⁰The details of the identification diagnostics are available from the authors upon request.

³¹The variance decomposition is based on 20,000 replications and five parallel chains for Metropolis–Hasting algorithm. The variance decomposition is based on the average of the posterior means over all these five Markov chains.

Table 3: Prior Densities and Posterior Estimates for Baseline Model

Parameters	Prior mean	Posterior Mean	90% HPD interval	Distribution	Prior sd
ρ_A	0.90	0.4659	(0.4270, 0.5245)	beta	0.05
ρ_{zx}	0.90	0.8662	(0.8224, 0.9075)	beta	0.05
ρ_G	0.90	0.9807	(0.9667, 0.9969)	beta	0.05
ρ_μ	0.90	0.9979	(0.9963, 0.9995)	beta	0.05
ρ_{ip}	0.90	0.9976	(0.9961, 0.9991)	beta	0.05
ρ_{iR}	0.90	0.9413	(0.9051, 0.9641)	beta	0.05
ϕ_π	3.50	3.6725	(3.587, 3.749)	invgamma	inf
ϕ_p	176.0	176.12	(176.05, 176.18)	invgamma	inf
θ_p	0.20	0.1957	(0.1820, 0.2101)	beta	0.01
γ_c	0.60	0.5128	(0.4632, 0.5677)	beta	0.10
ε_Y	4.00	3.9799	(3.9334, 4.0150)	invgamma	inf
κ	2.0	2.4038	(2.3721, 2.4445)	invgamma	inf
σ_A	0.001	0.7913	(0.7240, 0.8784)	invgamma	2
σ_{zx}	0.001	0.3466	(0.2702, 0.3958)	invgamma	2
σ_G	0.001	0.0110	(0.0096, 0.0124)	invgamma	2
σ_μ	0.001	0.0014	(0.0011, 0.0016)	invgamma	2
σ_{ip}	0.010	0.0013	(0.0012, 0.0014)	invgamma	2
σ_{iR}	0.001	0.0930	(0.0712, 0.1199)	invgamma	2
Λ	1.01	1.0052	(0.9943, 1.0134)	invgamma	2

of output. The sizable component of the variation of output contributed by IOER makes it a potent monetary policy instrument. Similar relative importance of shocks is observed for aggregate consumption. However, for investment, TFP and IST shock receive prominence. Third, on the bond market front, the nominal yield to maturity ($nytm$) and holding period return ($nhpr$) are influenced primarily by QE shock and IOER shock. Fourth, inflation is primarily accounted by TFP shock and QE shocks. Fifth, the real yield to maturity ($rytm$) and real holding period returns ($rhpr$) are driven significantly by TFP shock because TFP accounts mostly for the variance of inflation for the whole sample period.

5.3 Policy Simulations

We do policy experiments keeping in mind the six episodes of Japanese monetary policy since the inception of QE in 2001. First we look at the effect of a positive innovation ξ_t^μ to the monetary base equation (22) and the bond share equation (23). A positive ξ_t^μ means that the CB is injecting bank reserve to the private sector and the concomitant increase in λ_t means a larger holding of government bonds by the CB. This closely mimics a QE operation because such an operation entails CB purchase of JGB by open market operation. Since the growth rate of monetary base is a strictly mean reverting process (with a convergence to a

Table 4: Posterior Mean Variance Decomposition

	ε^A	ε^G	ε^μ	ε^{zx}	ε^{ip}	ε^{iR}
y	46.95	0.46	21.17	6.56	0.00	24.86
c	46.20	0.00	23.46	1.02	0.00	29.32
i	42.70	0.05	16.13	23.57	0.00	17.56
h	99.88	0.00	0.04	0.02	0.00	0.05
π	60.38	0.00	36.14	0.77	0.00	2.72
$nytm$	0.00	0.00	97.06	0.00	0.00	2.94
$rytm$	86.62	0.00	6.20	1.11	0.00	6.06
$nhpr$	0.00	0.00	91.11	0.00	0.00	8.89
$rhpr$	83.99	0.00	5.46	1.08	0.00	9.47

2% growth rate), it means that such a QE operation is temporary and it is phased out over time until the economy reverts to its steady state with a 2% target inflation. This policy experiment closely reflects the QE phase 1 and the onset of the QE tapering phase 2.

From phase 2, BoJ changed the operating target from the outstanding balance of current accounts at the Bank to the uncollateralized overnight call rate. In terms of our stylized model, such an operation may be formulated by giving up the open market purchase of government bonds and using the interest rate on overnight loans (i_t^p) as the control instrument. We, therefore, examine how a shock to i_t^p which is governed by a Taylor rule impacts the aggregate economy.

From phase 3, the focus is shifted more on the interest rate on bank reserve as the policy instrument. We, therefore, ask how a negative shock to i_t^R impacts the aggregate economy.

5.3.1 Phase 1: Effect of a positive QE shock

Impulse response functions (irf) in Figure 6 summarize the results of the phase 1 QE experiment.³² A positive one standard deviation shock to the monetary base growth (ξ_t^μ) immediately translates into a positive inflation shock via the money supply rule (22). Higher inflation raises the real marginal cost via the staggered price adjustment cost equation (9) as in any standard new-Keynesian model which means P_t^w/P_t rises. Higher real marginal cost makes the value of the marginal product of capital and labour shift out, which means wholesale firms buy more capital and hire more labour. This translates into a higher real price of capital Q_t . Nominal interest rate on loan rises due to two reasons: (i) higher inflationary expectation (Fisher effect) and (ii) greater demand for loan. This resembles a Tobin effect of inflation on investment. Retail output supply also rises along the standard new Keynesian channel as real marginal cost rises. Higher real wage encourages workers to supply more

³²All irfs are based on stochastic simulation given the posterior point estimate of the structural parameters.

labour. A wealth effect promotes consumption. Higher GDP boosts the transaction demand for currency by households. On the banking front, the ratio of reserve to deposit falls because a higher anticipated inflation imposes a tax on holding reserve. Banks thus advance more loans which are reflected by higher investment and higher Q . The CB dividend to the government also rises through this QE operation.

On the bond market front, several things happen. Because of a spark in inflationary expectations, the nominal bond price declines which reflects a decline in the stochastic discount factor. Consequently, nominal yield to maturity of 10 year bonds rises but quite insignificantly.³³ On the other hand, the real yield to maturity falls sharply which is due to higher inflation. Similar responses are also seen for nominal and real holding period returns. Overall effects of a positive QE shock are expansionary, output, investment, consumption and employment rise.

³³The effect on alternative maturity bonds is similar. Effectively a positive QE makes the yield curve shift outward.

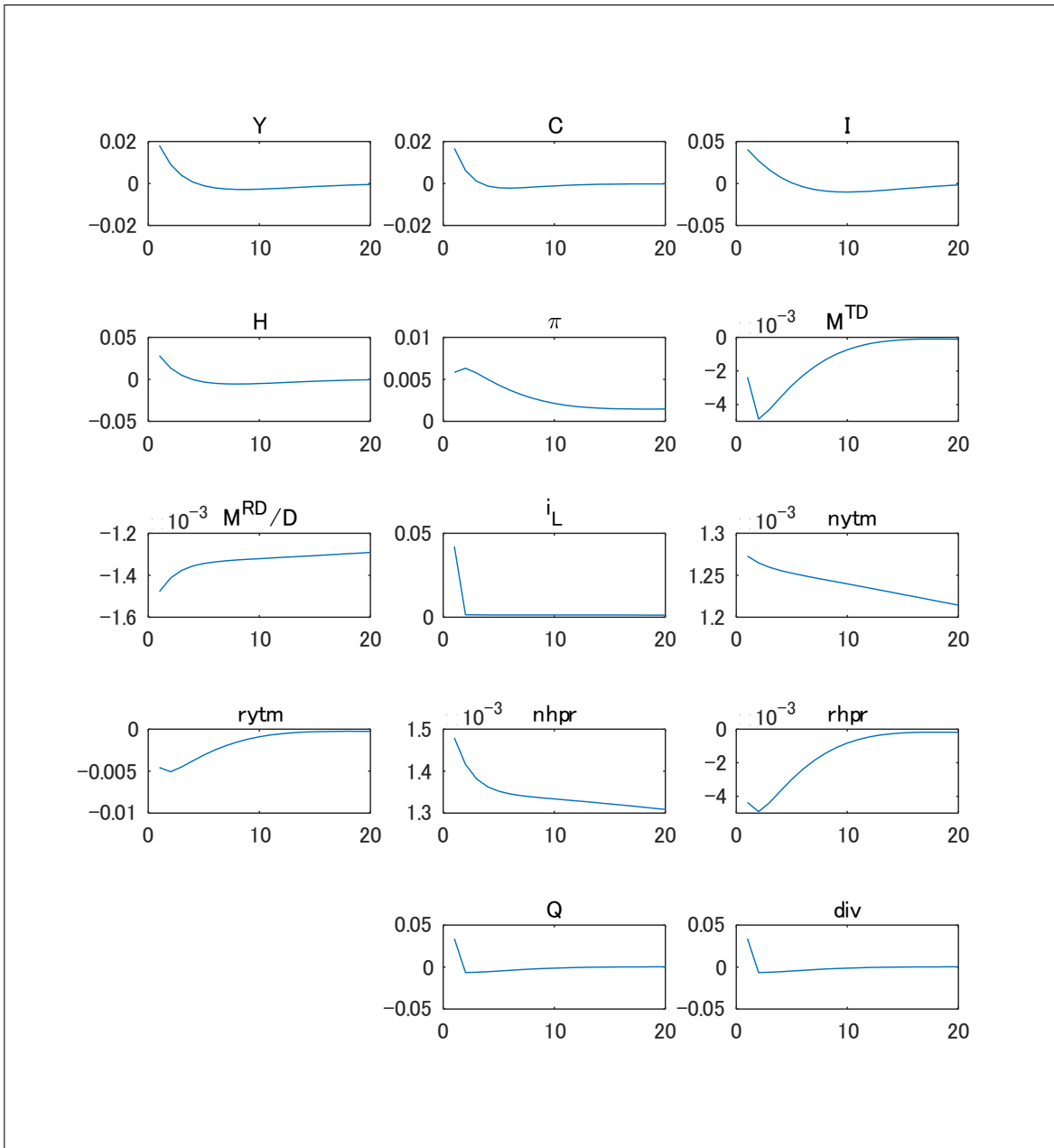


Figure 6: Macroeconomic effects of a positive QE (μ) shock

5.3.2 Phase 2: Effect of a negative shock to the overnight borrowing rate

We next report the phase 2 policy simulation where the CB abandons reserve balance as an operating target and switches to the overnight call rate as the control instrument. Such a policy experiment is akin to a conventional monetary policy based on a Taylor rule (CMP). Figure 7 plots the effects of a negative shock to the overnight call rate which is deemed to be an easy money policy by the CB. A one standard deviation negative shock to overnight call rate lowers excess reserves of banks reflected by a decline in reserve/deposit ratio. Since

more loan is released, investment rises which results in a higher Q . Higher Q also drives up the rental price of capital which via raising the real marginal cost of production makes the economy inflationary through the price adjustment equation (9). Consumption, investment and employment respond positively.

In contrast with a positive QE shock, on the bond market front both the nominal yield to maturity and nominal holding period returns declines marginally. However, all these effects are rather miniscule in nature. The overall effects on the macro economy are stimulative and similar to a positive QE shock. However, the effects are significantly weaker than a QE shock.

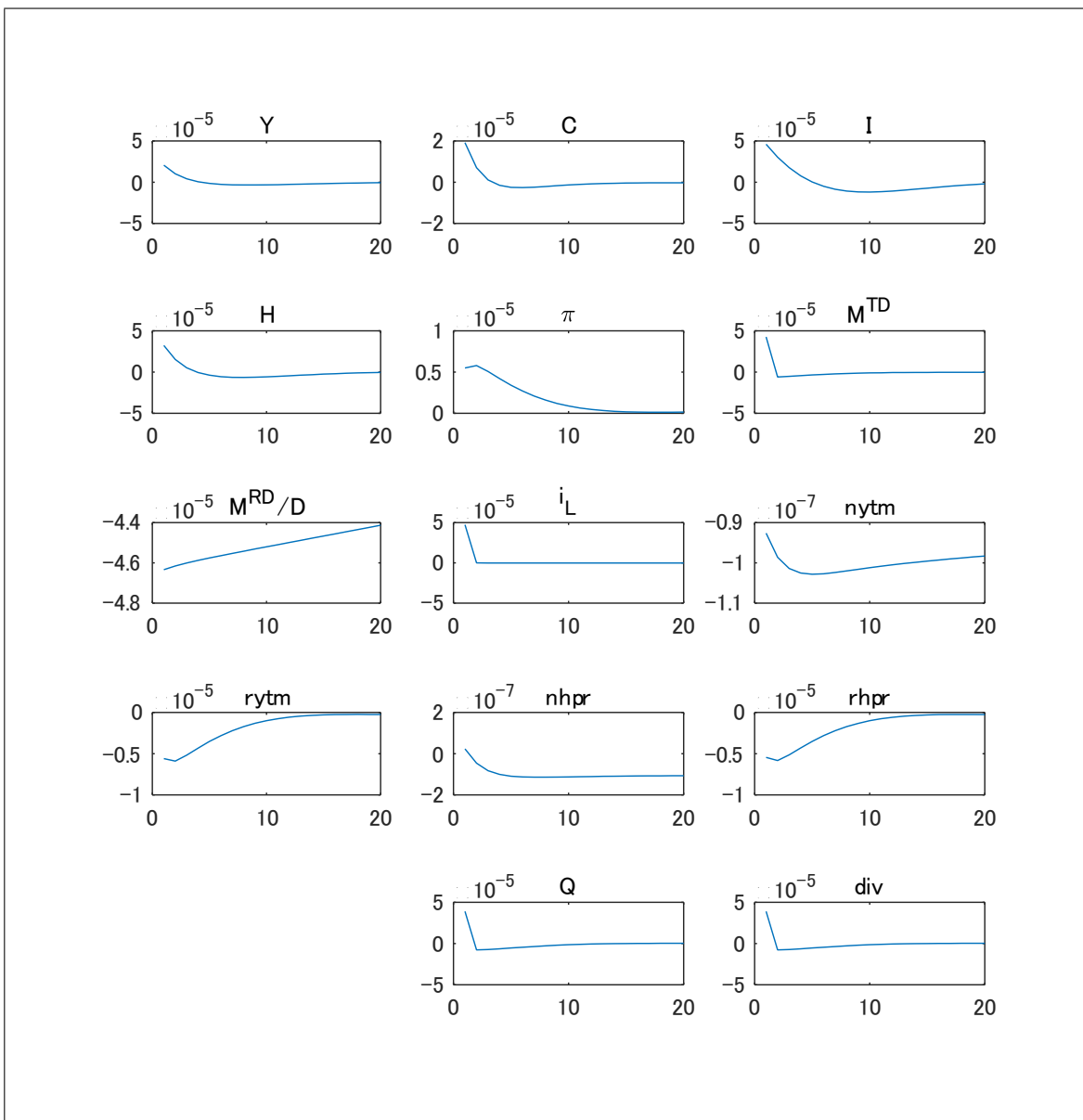


Figure 7: Macroeconomic effects of a positive QE (i^P) shock

5.3.3 Phase 3: Effect of a negative IOER shock

We now turn to the phase 3 policy simulation. What is the effect of a negative shock to IOER starting from a zero baseline rate? The effects and transmission mechanisms are quite similar to a drop in overnight borrowing rate shown in Figure 8. However, the effect of a drop in IOER is considerably stronger than the reduction in the overnight borrowing rate. The intuition for this stems from the comparative statics properties of the banks' excess reserve demand function noted in section 4.6. Banks respond more by loaning out excess reserve to a negative shock to IOER than an equivalent reduction in the overnight borrowing rate. In a recent paper, Bratsiotis (2018) demonstrates the effectiveness of IOER as a monetary policy tool through the deposit channel. In contrast, our transmission channel works through the lending channel.

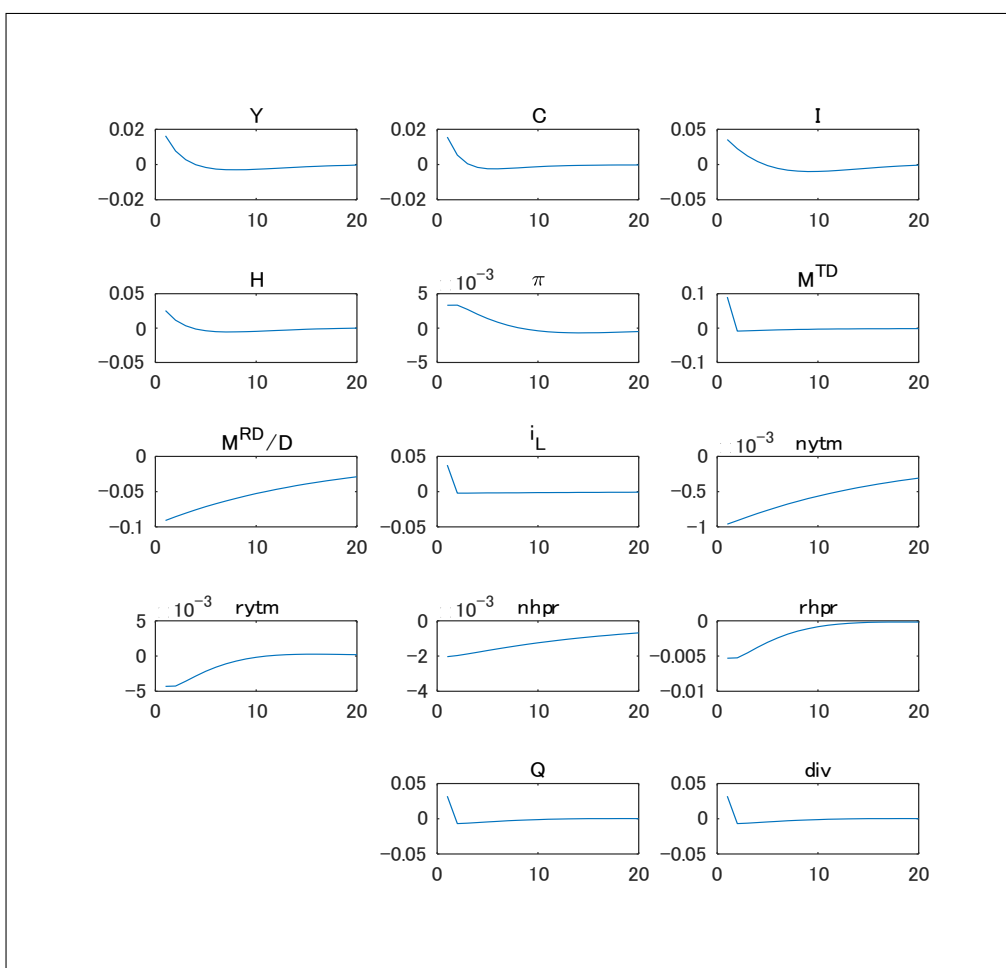


Figure 8: Macroeconomic effects of a negative IOER (i^R) shock

5.3.4 Term premia

Although QE and policy rate shocks have different effects on the nominal yield to maturity, its effects on the term premia are similar. We now take a closer look at term premia of bonds of various durations. Since the duration of these perpetual bonds (equal to $(1 - \beta\nu)^{-1}$) is positively related to the decay parameter ν , by varying ν we can conveniently compute the term premia (32) of bonds of different durations.³⁴ Since the term premium picks up the consumption risk and is related to the covariance between marginal utility of consumption and asset excess returns, a second order approximation of the model is used to compute the term premium formula (32). Figures 9 through 11 summarize the effects of three monetary policy shocks on term premia of bonds of four durations, namely 5, 10, 20 and 30 years. To do a fair comparison of the effects of three monetary policy shocks, we restrict the sizes of all three monetary policy shocks to a standard deviation of 0.01. A pruning procedure is used to smooth the shocks. Since the size of the shock is chosen to be small, each shock has quantitatively a small effect on term premia. Two observations are in order. First, the term premia uniformly decline for all these four bonds in response to each of these monetary policy shocks with the maximum effect on the shortest duration bonds (5 years). Second, QE shock has relatively the largest effect on term premia. IOER shock ranks next to QE shock in terms of its effect on the term premia.

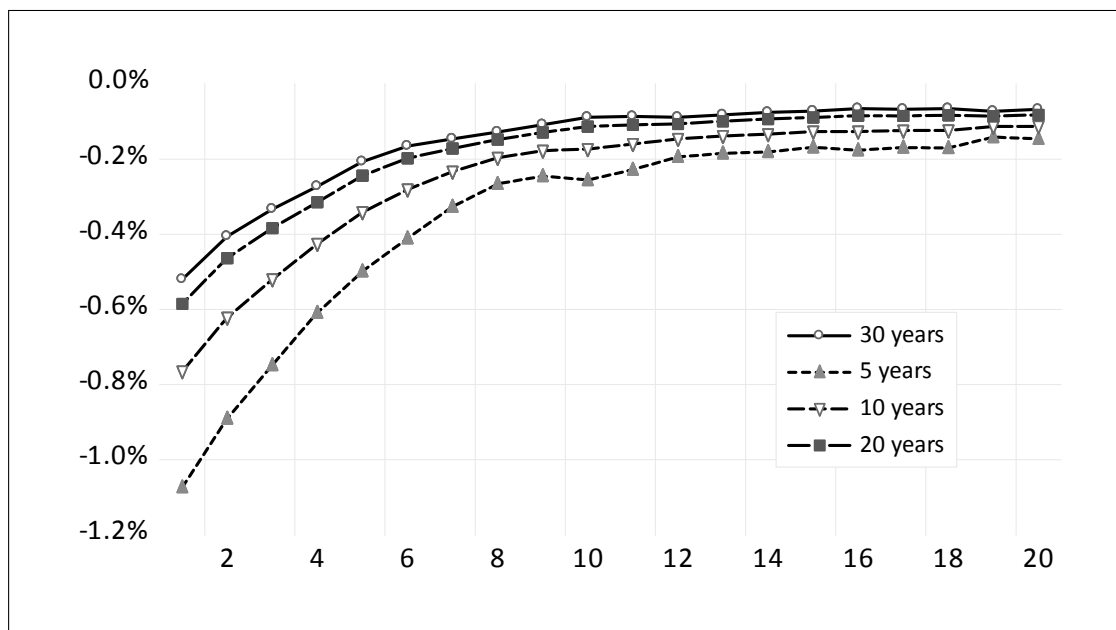


Figure 9: Effect of a QE (μ) shock on term premia of various durations

³⁴For zero coupon bonds, *duration* and *maturity* are the same. Since we use yields for zero coupon bonds from Ministry of Finance in our empirical analysis, we use these two terms interchangeably.

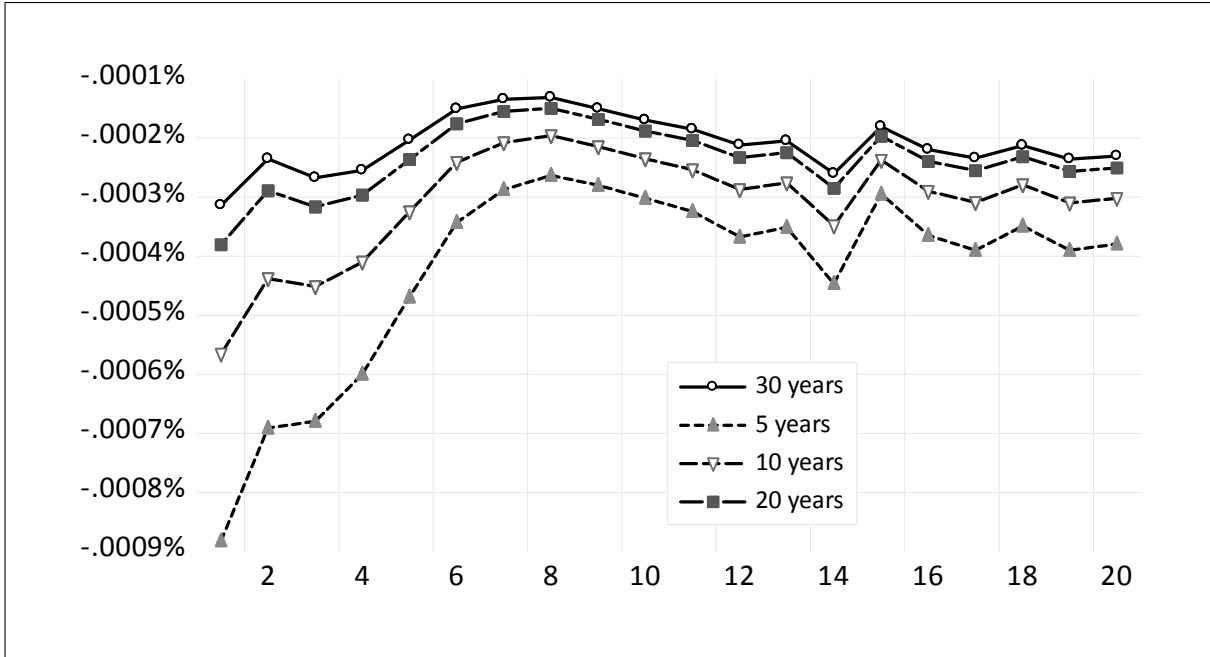


Figure 10: Effect of a call rate (i^P) shock on term premia of various durations

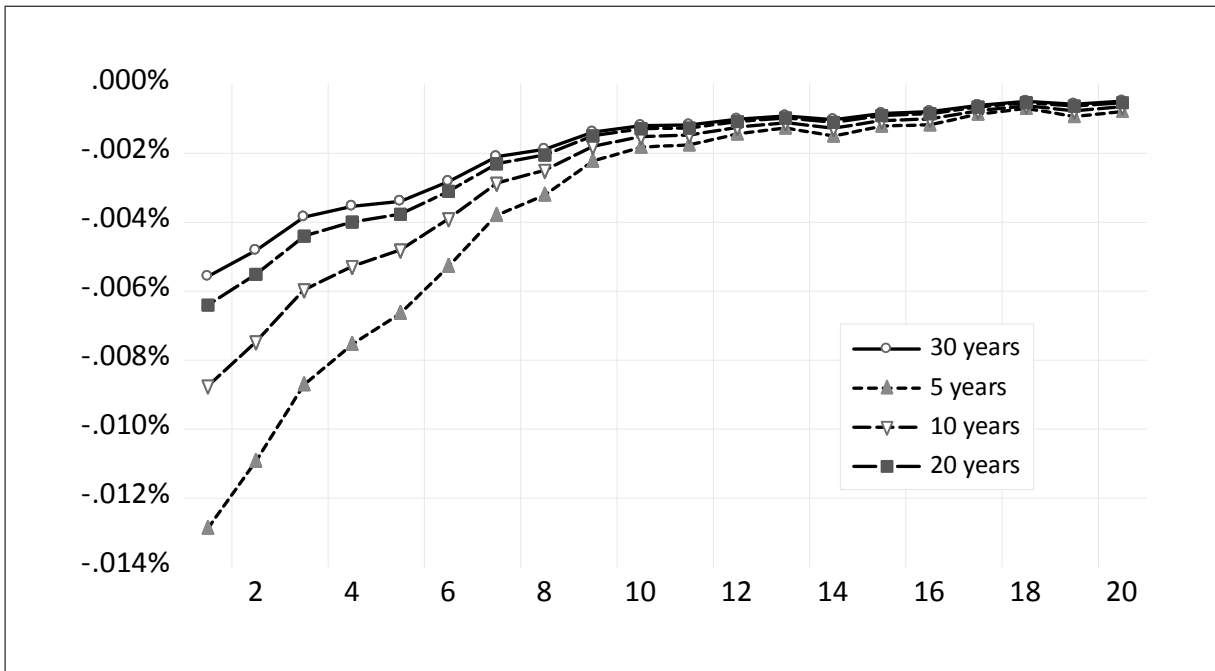


Figure 11: Effect of an IOER (i^R) shock on term premia of various durations

5.3.5 Call rate vs. IOER rates

A curious result from all our impulse response analysis is that in terms of impact effect, the IOER (i_R) always dominates the call rate (i_p). A change in i_p has minimal effects on the real and financial sectors of the economy as seen from all the irf in Figures 12 through 16. One may wonder whether this result is just an accident due to a larger estimate of the standard deviation of IOER shock (σ_{i_R}) as reported in Table 3. The irfs of term premia with respect to call rate and IOER shocks reported in Figures 9 through 11 involve the same size of each shock and it preserves the same relative ordering IOER and call rate shock in impacting the term premia of all maturities. In Figure 12, we report a similar counterfactual experiment for output responses to the call rate and IOER shocks. We set the standard deviation of both i_p and IOER shocks again at 0.01 while the rest of the shocks are fixed at the estimated standard errors. Notice that the impact effect of an IOER shock on GDP dominates the call rate shock.

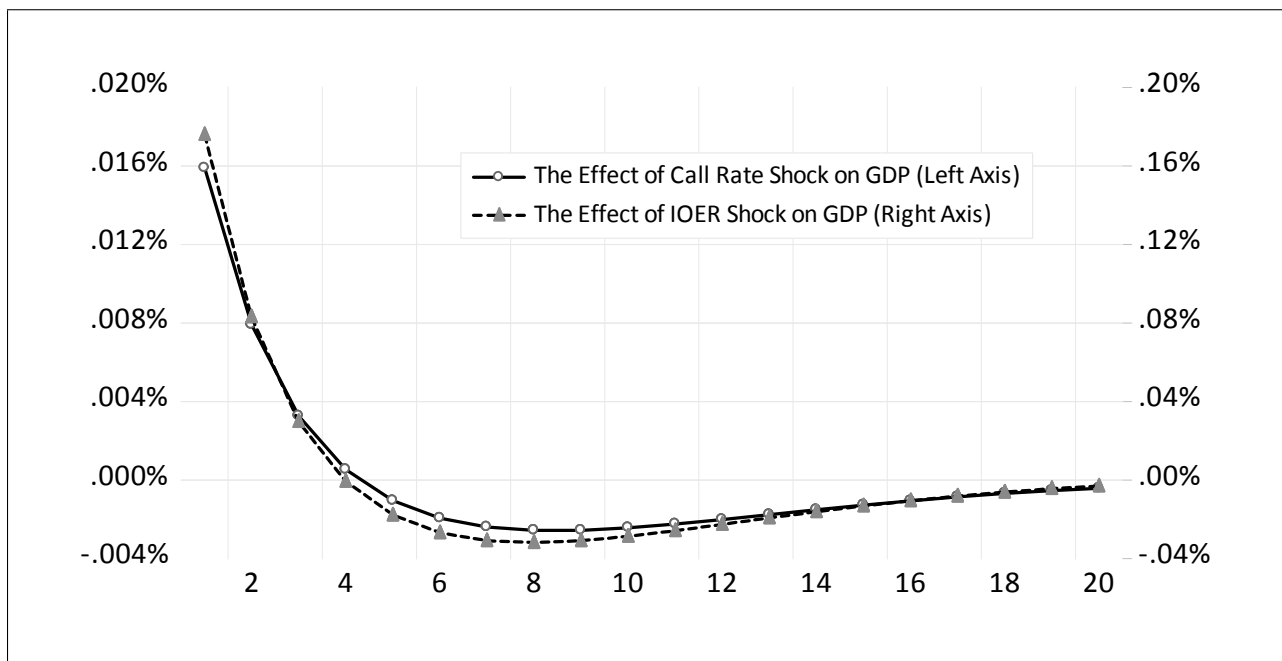


Figure 12: Effect of Call rate and IOER shocks on Output

The higher relative importance of IOER shock vis-a-vis the overnight call rate shock is, therefore, not just an accident due to a large estimate of σ_{i_R} . It happens primarily through the lending channel of monetary policy. Recall from the discussion in section 4.6 that the impact effect of a lower IOER shock on the banks' excess reserve decumulation is higher compared to a lower overnight borrowing rate shock. Banks respond more to a negative IOER shock by loaning out funds than a negative call rate shock. This stimulative effect

of a negative rate shock is also seen in the bond market in terms of a lower term premia. The underlying intuition is that in response to a negative rate shock consumption rises but the real yield to maturity falls. This means a negative covariance between consumption and real bond yield making bonds a good hedge which means a lower term premia. Since the responses of consumption and bond yields are more for a negative IOER shocks, the term premia decline more to IOER shocks compared to the call rate shock.

5.4 Lessons for Yield Curve Control

The basic tenets of yield curve control by BoJ since September 2016 consist of two elements: (i) keep the nominal long term and short term bond yield at a low level, and (ii) let the inflation stay at least above two percent. A short run analysis with our DSGE model predicts that these two goals may not be attainable using a standard quantitative easing policy. The irf plots as seen in Figure 7 show that a positive QE shock raises inflation and the nominal yield to maturity simultaneously. This happens due to the fact that a positive inflationary expectation resulting from a QE shock raises all nominal rates including the yield to maturity. The real yield to maturity, however, falls but it is not enough to outweigh the inflationary expectation ignited by QE. The recent QQE experiment of lowering the IOER is, however, encouraging in this context. As seen in the irf charts in Figure 9, a negative IOER shock lowers the nominal yield to maturity but it also raises inflation making both goals attainable at least in the short run.

The contradiction between two policy goals is sharpened in the long run. This can be easily seen by noting that in the long run the basic arbitrage condition dictates that the loan rate and the bond yield are equal and proportional to the inflation rate meaning (see equations (17) and (18))

$$1 + i_L = \frac{1 + \nu S}{S} = \frac{1 + \pi}{\beta} \quad (34)$$

Note that this is the basic Fisher's relationship between nominal interest rate and inflation which prevails in the long run. The immediate implication is that a yield curve control and inflation targeting are mutually contradictory long run goals. A simple back-of-the-envelope calculation based on the long run arbitrage condition (34) indicates that a long run inflation target of 2% means a long run yield of 3.03% (given $\beta = .99$) which is far away from near zero yield target.

The lesson for the policy of yield curve control is that BoJ may target a higher inflation and lower nominal yield in the short run by relying more on negative IOER shock but this should be phased out soon because in the long run these two targets are not mutually consistent.

6 Conclusion

Hardly any country has ever experienced so many monetary policy rules and regime switches within a short period of time as Japan. On the other hand, fiscal policy has been relatively stable. In this paper, we set up a monetary business cycle model of the Japanese economy with a particular focus on the bond market. Using this model, we study the effect of various phases of quantitative easing on the bond market fluctuations in a macro financial setting. Quantitative easing is modelled as a positive shock to monetary base with an offsetting purchase of long term government bonds by BoJ which causes maturity transformation of commercial bank assets. Our study spans the period 1999:Q1 to 2017:Q1 over which the Japanese monetary policy underwent several transitions which include switch between interest rate control and monetary base control. We put both these control instruments in a new Keynesian model. We find that the monetary base fluctuations significantly explain macro financial fluctuations. Among the two policy rates namely overnight borrowing rate and IOER, our estimated DSGE model predicts that the latter explains aggregate fluctuations next to the QE shock.

In terms of the effects of unconventional monetary policy on bond market yields, we find that the traditional QE raises the nominal yield to maturity and the nominal holding period returns of all maturity bonds. This happens particularly because a QE shock triggers inflationary expectations which raise all nominal yields. The term premia of all maturity bonds decline in response to a positive QE shock while shorter maturity bonds experience larger drops. About policy rate changes, we find that a negative shock to IOER is a more effective tool in stimulating the economy than lowering the overnight call rate. A negative IOER also lowers the nominal yield to maturity and raises inflation thus making positive inflation target with a zero yield achievable in the short run. The lesson for yield curve control experiment of BoJ is that a long term zero yield target is not consistent with a long run 2% inflation target. Thus it might be more effective to pursue the yield curve control only for the short run.

7 Appendix

7.1 Data Sources

The data for uncollateralized overnight call rate came from BoJ. Regarding bond yield series, yield observations are based on the estimated yield to maturity of zero coupon bonds for various maturities published by the Japanese Ministry of Finance. The amount of total bank reserve (required reserve and excess reserve) and the amount of monetary base came

from BoJ sources. GDP data came from Economic and Social Research Institute, Cabinet Office. The series for CPI (all items) are from Statistics Bureau, Ministry of Internal Affairs and Communications. Annualized real yields to maturity are computed as follows. Denote nominal yield to maturity of 10 year bond each quarter by $y_{t,10}^n$ (annual rate), real yield to maturity of 10 year bond each quarter by $y_{t,10}^r$, and the quarterly CPI price level by p_t^{CPI} . The quarterly inflation between t and $t+1$ is then calculated as $\pi_t = \frac{p_{t+1}^{CPI}}{p_t^{CPI}} - 1$. The annualized net CPI inflation for each quarter is calculated by $(1 + \pi_t)^4 - 1$. Then annualized real yield to maturity is calculated from annual nominal yield to maturity and annualized inflation by $y_{t,10}^r = \frac{1+y_{t,10}^n}{(1+\pi_t)^4} - 1$.

References

- Adjemian, S., and M. Juillard (2010), “Dealing with ZLB in DSGE models An application to the Japanese Economy,” *ESRI Discussion Paper Series*, No.258.
- Bratsiotis, G.J. (2018), “Credit Risk, Excess Reserves and Monetary Policy: The Deposits Channel,” CGBCR Discussion Paper. <http://hummedia.manchester.ac.uk/schools/soass/cgbcrcr/discussionpapers/dpcgbcrcr236.pdf>.
- Chang, S., S. Contessi, and J. Francis (2014), “Understanding the Accumulation of Bank and Thrift Reserves during the U.S. Financial Crisis,” *Federal Reserve Bank of St. Louis Working Paper Series*, 2013-029.
- Chen, H., V. Curdia, and A. Ferrero (2012), “The Macroeconomic Effects of Large-Scale Asset Purchase Programs,” *The Economic Journal*, 122(November), F289–F315.
- Fueki, T., I. Fukunaga, H. Ichiue, and T. Shirota (2016), “Measuring Potential Growth with an Estimated DSGE Model of Japan’s Economy,” *International Journal of Central Banking*, 12(1), 1–32.
- Gagnon, J, M. Rasin, J. Ramache, and B. Sack (2011) “Large-scale Asset Purchases by the Federal Reserve: Did they work?” *Federal Reserve Bank of New York Staff Reports* No 441.
- Gali, J. (2015), *Monetary Policy, Inflation and the Business Cycle*, 2nd Edition, Princeton Universtiy Press.
- Gerali, A., S. Neri, L. Sessa, and F. Signoretti (2010), “Credit and Banking in a DSGE Model of the Euro Area,” *Journal of Money, Credit and Banking*, 42(6), 107–141.

- Gertler, M., and P. Karadi (2013), “QE 1 vs. 2 vs...3: A Framework for Analyzing Large-Scale Asset Purchases as a Monetary Policy Tool,” *International Journal of Central Banking*, 9(S1), 5–53.
- Hall, R., and R. Reis (2015), “Maintaining Central-Bank Financial Stability under New-Style Central Banking,” *National Bureau of Economic Research Working Paper*, No. 21173.
- Hansen, G., and S. Imrohroglu (2013), “Fiscal Reform and Government Debt in Japan: A Neoclassical Perspective,” *National Bureau of Economic Research Working Paper*, No.19431.
- Hirose, Y. (2014), “An Estimated DSGE Model with a Deflation Steady State,” *JSPS Grants-in-Aid for Scientific Research (S) Understanding Persistent Deflation in Japan Working Paper Series*, No.43.
- Iskrev, N. (2010a), “Local Identification in DSGE models,” *Journal of Monetary Economics*, 57(2), 189–202.
- Iskrev, N. (2010b), “Parameter Identification in Dynamic Economic models,” *Economic Bulletin and Financial Stability Report Articles*, Banco de Portugal, Economics and Research Department.
- Iskrev, N., and M. Ratto (2010a), “Computational advances in analyzing identification of DSGE models,” In *Proceedings of the 6th DYNARE Conference*, June 3–4, Gustavelund, Tuusula, Finland: Bank of Finland, DSGE-net and Dynare Project at CEPREMAP.
- Iskrev, N., and M. Ratto (2010b), “Identification toolbox for DYNARE,” In *Proceedings of the 1st MONFISPOL Conference*, London, 4–5 November. The London Metropolitan University and the European Research project (FP7-SSH) MONFISPOL.
- Iwasaki, Y., and N. Sudo (2017), “Myths and Observations on Unconventional Monetary Policy –Takeaways from Post-Bubble Japan–,” *Bank of Japan Working Paper Series*, No.17-E-11.
- Iwata, Y. (2009), “Fiscal Policy in an Estimated DSGE Model of the Japanese Economy: Do Non-Richardian Households Explain All?” *ESRI Working Paper Series*, No.216.
- Keen, B., and Y. Wang (2005), “What is a realistic value for price adjustment cost in New Keynesian models?” *Applied Economics Letters*, 14(11), 789–793.

- McNelis, P., and N. Yoshino (2016), “Finding stability in a time of prolonged crisis: Unconventional policy rules for Japan,” *Journal of Financial Stability*, 27, 122–136.
- Michaelis, H., and S. Watzka (2017), “Are there differences in the effectiveness of quantitative easing at the zero-lower bound in Japan over time?” *Journal of International Money and Finance*, 70, 204–233.
- Rudebusch, G., and E. Swanson (2008), “Examining the Bond Premium Puzzle with a DSGE Model,” *Journal of Monetary Economics*, 55(S), S111–S126.
- Rudebusch, G., and E. Swanson (2012), “The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks,” *American Economic Journal: Macroeconomics*, 4(1), 105–143.
- Sidrauski, M. (1967), “Rational Choice and Patterns of Growth in a Monetary Economy,” *American Economic Review*, 57(2), 534–544.
- Smets, F., and R. Wouters (2007), “Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach,” *American Economic Review*, 97(3), 586–606.
- Sugo, T., and K. Ueda (2007), “Estimating a DSGE Model for Japan: Evaluating and Modifying a CEE/SW/LOWW Model,” *Bank of Japan Working Paper Series*, No.07-E-2.
- Vayanos, D., and J. Vila (2009), “A Preferred-Habitat Model of the Term Structure of Interest Rates,” *Working Paper*, London School of Economics.
- Woodford, M. (2001), “Fiscal Requirements for Price Stability,” *National Bureau of Economic Research Working Paper*, No.8072.