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Missing Wage Inflation? Estimating the Natural Rate of Unemployment in a Nonlinear DSGE Model

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Abstract

During the recovery from the global financial crisis, most advanced economies have experienced a surprisingly weak response of wage inflation to the decline in unemployment. In this study, we investigate whether downward wage rigidity (DWR) is the source of the flattening wage Phillips curve and the lack of wage inflation in the four advanced economies: Japan, the euro area, the UK, and the US. Specifically, we apply Markov chain Monte Carlo methods with a particle filter to estimate a nonlinear New Keynesian dynamic stochastic general equilibrium model incorporating asymmetric wage adjustment costs. This enables us to jointly estimate the degree of DWR as well as the natural rate of unemployment, that is, the rate of unemployment expected in the absence of (downward) wage rigidity. Our results indicate that wage adjustment costs are highly asymmetric in Japan, the euro area, and the UK, but not in the US. Especially, an L-shaped wage Phillips curve between wage inflation and the unemployment gap clearly emerges in Japan, due to the presence of DWR. As for the US, wage adjustment costs are large but symmetric, which means that wages are inherently quite sticky both in an upward and downward direction. Our results suggest that missing wage inflation in Japan, the euro area, and the UK is attributable largely to DWR, but not in the US.

Keywords: downward wage rigidity; natural rate of unemployment; Phillips curve; particle filter
JEL classification: E24, E31, E32

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1 Introduction

During the recovery from the global financial crisis and European debt crisis, advanced economies have experienced a surprisingly weak response of wage inflation to the decline in unemployment. As shown in Figure 1, the unemployment rate in advanced economies such as Japan, the euro area, the UK, and the US has fallen substantially from post-crisis peak levels, but wage inflation in these economies has not picked up markedly despite the clear improvement in labor market condition.[1] Against this background, central bankers have expressed their concern over "missing wage inflation," since this creates a weak inflationary environment in which the inflation rate persistently remains below the central bank's target.[2]

There have been a wide range of discussions regarding the possible causes of this missing wage inflation. Potential causes that have been mentioned include macroeconomic factors such as the weakness of trend productivity growth and inflation expectations, as well as recent structural changes in the labor market, such as changes in labor force participation or the increase of part-time workers in total employment (see, for example, IMF (2017)). However, there are still few analyses trying to quantify the relative importance of the possible causes and the discussion on this issue seems to be far from having reached a conclusion.

In this study, we investigate whether downward wage rigidity (DWR) is the source of the flattening wage Phillips curve and the lack of wage inflation in the four ad-

[1] Figure 1 shows wage inflation in terms of increases in labor compensation per worker. However, the picture with regard to subdued wage inflation remains the same when increases in hourly wages are used (Appendix Figure 1).

[2] For example, in her remarks on the weak developments in wage growth in the post-recovery phase, Yellen (2014) highlighted that "since wage movements have historically been sensitive to tightness in the labor market, the recent behavior of both nominal and real wages point to weaker labor market conditions than would be indicated by the current unemployment rate." Similarly, Constâncio (2017) observed that "[t]he fact that wages are not increasing more is an important puzzle in advanced economies. Higher wage increases were to be expected due to the strengthening of the economic recovery and are necessary in order to normalise inflation." Meanwhile, Haldane (2017) noted that "[a]s unemployment has fallen recently, this Phillips curve relationship would have led us to expect wage growth to pick up. That, plainly, has not happened. Over recent years, the Phillips curve relationship has been anything but strong and stable. And that same flatness in the Phillips curve has been found in a number of other countries." Finally, Cunliffe (2017) also expressed his concerns regarding the flattening or disappearance of the wage Phillips curve.

vanced economies. The possibility that DWR was the reason for the subdued wage inflation in recent years has been highlighted by Kuroda (2017), who observed that "*wage reductions of full-time employees had been conducted only marginally in an economic downturn, and wages barely rise even if the economy recovers and labor market conditions tighten. It also can be said that past downward rigidity in wages has led to the present upward rigidity.*"[3] DWR is an issue that has been widely discussed by Keynes (1936) and Tobin (1972), who identified it as a prominent feature of macroeconomic dynamics in a low-inflation environment.[4] As more recent studies have shown, the wage Phillips curve becomes highly nonlinear and quite flat at low rates of wage inflation in the presence of DWR (see e.g., Akerlof, Dickens and Perry (1996), Benigno and Ricci (2011), and Daly and Hobijn (2014)). The flat shape at lower wage inflation rates means that wage inflation is irresponsive not only to downside shocks but also to *upside* shocks in the labor market. The mechanism behind the latter is clearly explained by Elsby (2009), who argues that wage increases are irreversible to a certain degree. First, forward-looking firms temper wage increases as a precaution against future costly wage cuts. Second, even in the absence of forward-looking behavior, DWR raises the level of wages that firms inherit from the past and makes it less necessary for firms to raise wages as much to obtain their

[3]Similarly, Yellen (2014) observed that "the sluggish pace of nominal and real wage growth in recent years may reflect the phenomenon of 'pent-up wage deflation.' The evidence suggests that many firms faced significant constraints in lowering compensation during the recession and the earlier part of the recovery because of 'downward nominal wage rigidity' – namely, an inability or unwillingness on the part of firms to cut nominal wages. To the extent that firms faced limits in reducing real and nominal wages when the labor market was exceptionally weak, they may find that now they do not need to raise wages to attract qualified workers. As a result, wages might rise relatively slowly as the labor market strengthens. If pent-up wage deflation is holding down wage growth, the current very moderate wage growth could be a misleading signal of the degree of remaining slack. Further, wages could begin to rise at a relatively more rapid pace once pent-up wage deflation has been absorbed."

[4]Paul Krugman has also repeatedly stressed the importance of DWR in understanding inflation dynamics, especially after the global financial crisis. For example, he stated that "[w]hat employers learned during the long slump is that you can't cut wages even when people are desperate for jobs; they also learned that extended periods in which you would cut wages if you could are a lot more likely than they used to believe. This makes them reluctant to grant wage increases even in good times, because they know they'll be stuck with those wages if the economy turns bad again," (Krugman (2018)).

desired wage level.[5]

The highly nonlinear nature of the wage Phillips curve especially at low levels of wage inflation is easily discernible in scatter plots of wage inflation and the unemployment rate for the four advanced economies, as shown in Figure 2. The pattern is particularly notable in Japan, where the nonlinear relationship between wage inflation and the unemployment rate is highly stable and the wage Phillips curve is nearly vertical for high levels of wage inflation but becomes almost monotonically flatter for lower levels of wage inflation. The patterns for the euro area and the UK are quite similar, suggesting that in both economies the wage Phillips curve is nonlinear with a flat part for lower rates of wage inflation, especially since the 2000s. In contrast, for the US, no clear relationship between wage inflation and unemployment can be observed for the observation period as a whole. However, if we focus on data since the 1990s, there seems to be a downward-sloping wage Phillips curve with a flatter slope at low rates of wage inflation.[6]

These observations suggest the presence of DWR in all four advanced economies. However, based on this evidence only, it would be rash to conclude that DWR is an entrenched feature of these economies. This is particularly so since the unemployment rate is a relatively crude measure that does not necessarily provide an accurate measure of labor market slack once the possibility that the natural rate of unemployment is time-varying is taken into account. That is, as suggested by the natural rate hypothesis, what matters for wage inflation is the deviation of the actual unemployment rate from the natural rate. This means that we need to correctly identify developments in the natural rate of unemployment to assess the true pressure of labor market tightness on wage inflation. This means that the importance of

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[5] Our theoretical model emphasizes the former mechanism in explaining the irresponsiveness of wage inflation to upside shocks to the labor market.

[6] The emergence of an inverse relationship between wage growth and the unemployment rate in the US economy since the mid-1980s has been highlighted as the "return of wage Phillips curve" by Galí (2011a), who derives and estimates a linear forward-looking structural relationship between wage growth and the unemployment rate, the so-called "New Keynesian wage Phillips curve (NKWPC)." Muto and Shintani (2018) compare wage inflation dynamics in the US and Japan through the lens of the NKWPC taking into account time variations in the parameters of the NKWPC.

DWR can be correctly assessed only by using an appropriate measure of the natural rate of unemployment. This should be done through an empirical analysis using a coherent framework that explicitly incorporates both DWR and the natural rate of unemployment.

Against this background, the aim of this study is to examine the importance of DWR in the recent weak wage inflation observed in the four advanced economies. To this end, we apply Markov chain Monte Carlo (MCMC) methods with the particle filter developed by Fernández-Villaverde and Rubio-Ramírez (2007) to estimate a nonlinear New Keynesian dynamic stochastic general equilibrium (DSGE) model incorporating DWR for the four economies. Our model essentially follows the models developed by Kim and Ruge-Murcia (2009, 2011) and Aruoba, Bocola, and Schorfheide (2017), which introduce DWR through an asymmetric wage adjustment cost function to produce downwardly rigid wage inflation dynamics.[7] Using an asymmetric wage adjustment cost function provides a reasonable setup, because it does not preclude wage cuts, which are rare but observed in practice in micro- and macro-level data.[8] However, since their models do not incorporate unemployment and assume that wage inflation is determined by a wage markup (the ratio of real wages to the marginal rate of substitution between consumption and labor), we extend their models by introducing the link between the wage markup and the unemployment rate derived by Galí (2011a, b) based on the assumption of indivisible labor input à la Hansen (1985).

In order to obtain an accurate measure of labor market slack, we calculate the natural rate of unemployment fully taking DWR into account. In the New Keynesian framework, the natural rate of unemployment is defined as the hypothetical rate that should be realized in the absence of wage rigidity. There exist some previous studies which have attempted to estimate the natural rate of unemployment based

[7] As explained below, we use an asymmetric wage adjustment cost function that is close to the one employed by Fahr and Smets (2010) rather than the linex adjustment cost function used by Kim and Ruge-Murcia (2009, 2011) and Aruoba, Bocola, and Schorfeide (2017).

[8] An alternative setup to incorporate DWR would be to introduce an inequality constraint that strictly requires the wage inflation rate not to be below zero. However, this kind of setup would contradict the fact that wage cuts are observed in practice in the data.

on this concept. For example, Galí, Smets and Wouters (2012) develop a medium-scale sticky wage model with unemployment and present an estimate of the natural rate of unemployment through a counterfactual exercise in which wage rigidity is absent.[9] Other studies presenting estimates of the natural rate of unemployment are those by Gertler, Sala and Trigari (2008) and Sala, Söderström, and Trigari (2008), which employ search and matching models with staggered wage contracts. However, wage rigidity in these studies is assumed to be symmetric in an upward and downward direction. To the best of our knowledge, no previous studies calculate the natural rate of unemployment in a New Keynesian DSGE model with downward wage rigidity, despite its potential importance in understanding wage inflation dynamics in a low inflation environment. By employing the recently developed technique of using an MCMC algorithm with particle filter, our study contributes to the literature by jointly estimating (i) the degree of downward wage rigidity as well as (ii) the natural rate of unemployment within the coherent framework of a New Keynesian DSGE model for four advanced economies.

The main findings of our analysis can be summarized as follows. First, wage adjustment costs are highly asymmetric in Japan, the euro area, and the UK but not in the US. This result indicates that, except in the case of the US, it is extremely important to take DWR into account to accurately measure the natural rate of unemployment in the economies examined here. Second, in the case of Japan, an L-shaped wage Phillips curve between wage inflation and the unemployment gap emerges, once DWR is properly taken into account. This indicates that wage inflation in Japan is not responsive to labor market slack as long as the unemployment rate is above the natural rate but then accelerates once the unemployment rate falls well below the natural rate. Therefore, our results imply that the missing wage inflation observed in Japan is not a permanent phenomenon and wage inflation is likely to reappear with further improvement in the labor market. Third, in the case of the US, wage adjustment costs are large but symmetric, which means that wages are highly

[9] They also use the setup proposed by Galí (2011a, b) to introduce unemployment into the New Keynesian sticky-wage framework.

sticky both in an upward and downward direction. This suggests that wage inflation has responded only sluggishly to the decline in unemployment because the wage Phillips curve is inherently quite flat in the case of the US. These results suggest that missing wage inflation in Japan, the euro area, and the UK can be largely explained by DWR, but this is not the case for the US.

The rest of this study is organized as follows. Section 2 presents the model used for our analysis. Section 3 explains the data and empirical methodologies. Section 4 presents the empirical results of the model and the natural rate of unemployment. Section 5 compares the estimated wage adjustment cost functions for the four economies and examines whether missing wage inflation can be explained by DWR. Section 6 discusses some caveats and possible extensions. Finally, Section 7 concludes.

2 Model

Our model essentially follows the New Keynesian sticky wage models developed by Kim and Ruge-Murcia (2009, 2011) and Aruoba, Bocola, and Schorfheide (2017), which introduce asymmetric wage adjustment costs to produce downwardly rigid wage inflation dynamics. However, we modify and extend their models in the following respects. First, since their models do not incorporate unemployment, we extend their models to introduce the unemployment rate, by adopting the setup of Galí (2011a, b) and Galí, Smets and Wouters (2012). This means that our model focuses on variations in labor input in the form of the extensive margin (employment) rather than the intensive margin (hours).[10] Second, we add several types of exogenous shocks - namely, wage markup shocks, discount factor shocks, and labor supply shocks - to the model. Incorporating wage markup shocks is important to capture time-variations in the natural rate of unemployment, while incorporating discount factor shocks plays a key role in creating fluctuations in aggregate demand. Incorporating labor supply shocks is useful to prevent overestimating the importance of

[10] To be consistent, wage is defined as the wage per worker rather than the wage per hour.

wage markup shocks.[11] Third, in order to facilitate the identification of asymmetry in wage adjustment costs, we slightly change the specification of the wage adjustment cost function following Fahr and Smets (2010) and Abbritti and Fahr (2013).

2.1 Setup

2.1.1 Households

The economy is populated by a large number of identical households. Each household consists of a continuum of family members indexed by a pair $(j,k) \in [0,1] \times [0,1]$. Households provide differentiated types of labor services $N_t(j)$ and the index j represents the type of labor service in which a household member is specialized. We assume that labor input is indivisible in the sense that an individual works a fixed number of hours or does not work at all. This means that all endogenous variations in labor input take place at the extensive margin.

A household's period utility is specified as:

$$\begin{aligned} U\left(C_t, \{N_t(j)\}\right) &\equiv \frac{\left(C_t/A_t\right)^{1-\tau}-1}{1-\tau} - \chi_t \int_0^1 \int_0^{N_t(j)} k^{\frac{1}{\nu}} dk dj \\ &\equiv \frac{\left(C_t/A_t\right)^{1-\tau}-1}{1-\tau} - \chi_t \int_0^1 \frac{N_t(j)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} dj, \end{aligned} \tag{1}$$

where $1/\tau$ is the intertemporal elasticity of substitution. The index k determines the disutility from working, labor disutility is $\chi_t k^{\frac{1}{\nu}}$ if the member is employed and zero otherwise, χ_t is an exogenous preference shifter and ν is the Frisch labor supply elasticity. The exogenous preference shifter χ_t, which is referred to as a "labor supply shock," follows a first order autoregressive process in log:

$$\ln \chi_t = \rho_\chi \ln \chi_{t-1} + \varepsilon_{\chi,t}, \tag{2}$$

[11] As highlighted by Galí, Smets and Wouters (2012), introducing the unemployment rate makes it possible to distinguish between wage markup and labor supply shocks, which prevents the overestimation of the importance of wage markup shocks in the fluctuations of macroeconomic variables.

where $\varepsilon_{\chi,t}$ is an iid white noise with variance σ_{χ}^{2}.[12]

Household j obtains utility from consumption relative to a habit stock which is given by the level of technology A_t.This assumption is introduced in order to ensure that the economy fluctuates along a balanced growth path (Aruoba, Bocola, and Schorfheide (2017)). The utility function is additively separable and we assume full risk sharing within a household. This implies the same level of consumption for all household members, which is represented by C_t.

The household seeks to maximize intertemporal utility:

$$E_t\left[\sum_{s=0}^{\infty}\beta^s d_{t+s}U\left(C_{t+s},\{N_{t+s}\left(j\right)\}\right)\right], \tag{3}$$

where β is the constant component of the discount factor and d_t is a discount factor shock with law of motion:

$$\ln d_t = \rho_d \ln d_{t-1} + \varepsilon_{d,t}, \tag{4}$$

where $\varepsilon_{d,t}$ is an iid normal random variable with mean zero and variance σ_d^2. The budget constraint is given by

$$\begin{gathered} P_tC_t + B_t\left(j\right) + T_t \\ = W_t\left(j\right)N_t\left(j\right)\left(1-\Phi_w(\pi_{w,t}(j))\right) + R_{t-1}B_{t-1}\left(j\right) + P_tD_t\left(j\right) + P_tSC_t\left(j\right). \end{gathered} \tag{5}$$

where P_t is the price of final goods, $B_t\left(j\right)$ is purchases of government bonds, T_t is lump-sum taxes, $W_t\left(j\right)$ is the nominal wage for type i labor, $\Phi_w(\pi_{w,t}(j))$ is the wage adjustment cost (where $\pi_{w,t}(j) \equiv W_t\left(j\right)/W_{t-1}\left(j\right)$), R_t is the risk-free nominal interest rate on government bonds, $D_t\left(j\right)$ is residual real profits, and $SC_t\left(j\right)$ is the net cash inflow from a set of state-contingent securities.

Household j maximizes lifetime utility (3) with respect to consumption C_t, bond holdings B_t, and nominal wage $W_t\left(j\right)$. Let $\lambda_t(j)$ be the Lagrange multiplier on budget constraint (5) in the utility maximization problem for a household providing type j labor services. The first order condition with respect to consumption is as

[12] Following Galí, Smets and Wouters (2012), ρ_{χ} is fixed to 0.999.

follows:

$$U_{C,t} = \lambda_t(j)P_t, \tag{6}$$

where $U_{C,t}$ denotes the partial derivative of period utility with respect to consumption ($U_{C,t} = \frac{\partial U(C_t,N_t(j))}{\partial C_t} = \frac{1}{A_t}\left(\frac{C_t}{A_t}\right)^{-\tau}$), which is the same for all households.

Let $Q_{t+s|t}$ be the time t value of a unit of the consumption good in period $t+s$ ($Q_{t+s|t} = \frac{U_{C,t+s}}{U_{C,t}} = \left(\frac{A_{t+s}}{A_t}\right)^{\tau-1}\left(\frac{C_{t+s}}{C_t}\right)^{-\tau}$). The first order condition with respect to bond holdings can then be expressed as follows:

$$1 = \beta E_t\left(\frac{d_{t+1}}{d_t}Q_{t+1|t}\frac{R_t}{\pi_{t+1}}\right). \tag{7}$$

Labor services provided by households are aggregated in the following constant elasticity of substitution form:

$$N_t = \left(\int_0^1 N_t\left(j\right)^{1-\lambda_{w,t}}dj\right)^{\frac{1}{1-\lambda_{w,t}}}, \tag{8}$$

where the inverse demand elasticity $\lambda_{w,t}$ is time-varying and evolves according to a first order autoregressive process in log:

$$\ln\lambda_{w,t} = (1-\rho_w)\ln\overline{\lambda}_w + \rho_w\ln\lambda_{w,t-1} + \varepsilon_{w,t}, \tag{9}$$

where $\overline{\lambda}_w$ is the steady state value of $\lambda_{w,t}$ and $\varepsilon_{w,t}$ is an iid normal random variable with mean zero and variance σ_w^2. The value of $\lambda_{w,t}$ determines workers' market power, and, as is explained later, it is the sole determinant of the wage markup in this model. Therefore, we refer to $\lambda_{w,t}$ as a "wage markup shock" below.

Given input prices $W_t\left(j\right)$ and output prices W_t, the optimal demand for each type of labor for the production that maximizes intermediate goods firms' profits is

$$N_t\left(j\right) = \left(\frac{W_t\left(j\right)}{W_t}\right)^{-\frac{1}{\lambda_{w,t}}}N_t. \tag{10}$$

where the aggregate wage index is given by

$$W_t = \left(\int_0^1 W_t(j)^{\frac{\lambda_{w,t}-1}{\lambda_{w,t}}} dj \right)^{\frac{\lambda_{w,t}}{\lambda_{w,t}-1}}. \tag{11}$$

Household j maximizes lifetime utility (3) by taking into account the labor demand schedule (10). The first order condition with respect to nominal wage $W_t(j)$ is obtained as follows:

$$\begin{aligned} 0 &= \frac{-1}{\lambda_{w,t}} U_{N,t}(j) \left(\frac{W_t(j)}{W_t} \right)^{-\frac{1}{\lambda_{w,t}}-1} \frac{N_t}{W_t} \\ &\quad + \lambda_t(j) \left(\frac{W_t(j)}{W_t} \right)^{-\frac{1}{\lambda_{w,t}}} N_t \left((1 - \frac{1}{\lambda_{w,t}}) (1 - \Phi_w(\pi_{w,t}(j))) - \Phi_w'(\pi_{w,t}(j)) \pi_{w,t}(j) \right) \\ &\quad + \beta E_t \frac{d_{t+1}}{d_t} \lambda_{t+1}(j) \left(\frac{W_{t+1}(j)}{W_{t+1}} \right)^{-\frac{1}{\lambda_{w,t+1}}} N_{t+1} \Phi_w'(\pi_{w,t+1}(j)) \pi_{w,t+1}^2(j), \end{aligned} \tag{12}$$

where $U_{N,t}(j)$ denotes the partial derivative of period utility with respect to labor services ($U_{N,t}(j) = \frac{\partial U(C_t, N_t(j))}{\partial N_t(j)} = \chi_t N_t^{\frac{1}{\nu}}(j)$).

2.1.2 Wage adjustment costs

The specification of wage adjustment costs is a key element of our analysis. Following Fahr and Smets (2010), we introduce the following asymmetric wage adjustment cost function:

$$\Phi_w(\pi_{w,t}(j)) = \frac{\phi_w - 1}{2} \left(\frac{\pi_{w,t}(j)}{\overline{\pi}_w} - 1 \right)^2 + \frac{1}{\psi_w^2} \left(\exp\left(-\psi_w \left(\frac{\pi_{w,t}(j)}{\overline{\pi}_w} - 1 \right) \right) + \psi_w \left(\frac{\pi_{w,t}(j)}{\overline{\pi}_w} - 1 \right) - 1 \right), \tag{13}$$

where $\overline{\pi}_w$ is the steady state value of $\pi_{w,t}(j)$. The parameter ϕ_w determines the degree of convexity and ψ_w the degree of asymmetry in wage adjustment costs.

Figure 3 illustrates the properties of wage adjustment cost function (13). If the asymmetry parameter is close to zero ($\psi_w \to 0$), (13) becomes a Rotemberg-style quadratic function (Rotemberg (1982)) where wage rigidity is solely determined by the value of ϕ_w:

$$\lim_{\psi_w \to 0} \Phi_w(\pi_{w,t}(j)) = \frac{\phi_w}{2} \left(\frac{\pi_{w,t}(j)}{\overline{\pi}_w} - 1 \right)^2.$$

However, if ψ_w takes a strictly positive value, the wage adjustment cost is asymmetric and wage cuts are more costly than wage increases. As shown in Figure 3, the increase in the asymmetry parameter ψ_w raises the cost of wage cuts, leaving the cost of wage increases almost unchanged. Therefore, we can interpret ϕ_w as representing the degree of wage rigidity generally seen in both an upward and a downward direction, and ψ_w determines the asymmetry of wage rigidity.

This specification is slightly different from the linex function introduced by Kim and Ruge-Murcia (2009, 2011) and Aruoba, Bocola, and Schorfheide (2017) which is given by:

$$\Phi_w(\pi_{w,t}(j)) = \frac{\phi_w}{\psi_w^2} \left(\exp\left(-\psi_w \left(\pi_{w,t}(j) - \overline{\pi}_w\right)\right) + \psi_w \left(\pi_{w,t}(j) - \overline{\pi}_w\right) - 1 \right). \tag{14}$$

Specification (13) differs from the linex form (14) in that the third derivative is given by the single parameter ψ_w, although it is given by $\phi_w \psi_w$ in the case of (14). Because we carry out Bayesian estimation with second-order approximation, this property of specification (13) is useful since it facilitates the identification of parameters ϕ_w and ψ_w.

2.1.3 Final goods production

We assume perfectly competitive final-goods-producing firms that combine a continuum of intermediate goods indexed by $i \in [0, 1]$ using the technology

$$Y_t = \left(\int_0^1 Y_t\left(i\right)^{1-\lambda_{p,t}} di \right)^{\frac{1}{1-\lambda_{p,t}}}, \tag{15}$$

where $1/\lambda_{p,t}$ denotes the elasticity of demand for each intermediate good. Under the assumption of a perfectly competitive market, profit maximization and free entry imply that the demand for intermediate goods is

$$Y_t\left(i\right) = \left(\frac{P_t\left(i\right)}{P_t} \right)^{-\frac{1}{\lambda_{p,t}}} Y_t, \tag{16}$$

where $P_t(i)$ is the price of intermediate goods. The relationship between $P_t(i)$ and P_t is given by

$$P_t = \left(\int_0^1 P_t(i)^{\frac{\lambda_{p,t-1}}{\lambda_{p,t}}} di \right)^{\frac{\lambda_{p,t}}{\lambda_{p,t-1}}}. \tag{17}$$

The inverse demand elasticity, referred to as the "price markup shock" below, evolves according to a first order autoregressive process in log:

$$\ln \lambda_{p,t} = (1 - \rho_p) \ln \overline{\lambda}_p + \rho_p \ln \lambda_{p,t-1} + \varepsilon_{p,t}, \tag{18}$$

where $\varepsilon_{p,t}$ is an iid normal random variable with mean zero and variance σ_p^2.

2.1.4 Intermediate goods production

Intermediate good i is produced by a monopolist who has access to the following production technology:

$$Y_t(i) = A_t N_t(i). \tag{19}$$

where A_t represents the productivity level, which is exogenous and common to all firms and follows

$$\ln A_t = \ln \gamma + \ln A_{t-1} + \ln z_t, \tag{20}$$

where $\ln z_t = \rho_z \ln z_{t-1} + \varepsilon_{z,t}$ and $\varepsilon_{z,t}$ is an iid normal random variable with mean zero and variance σ_z^2. Intermediate goods producers buy labor services at a nominal wage of W_t. Moreover, they face a Rotemberg-style quadratic adjustment cost with respect to price changes:

$$\Phi_p(\pi_{p,t}(i)) = \frac{\phi_p}{2} \left(\frac{\pi_{p,t}(i)}{\overline{\pi}_p} - 1 \right)^2, \tag{21}$$

where $\pi_{p,t}(i) \equiv P_t(i) / P_{t-1}(i)$ and $\overline{\pi}_p$ is the steady state value of $\pi_{p,t}(i)$.[13]

[13] The Rotemberg-style adjustment cost is a specific form (the case of $\psi_p \to 0$) of the following more general function, which allows upward or downward price rigidity:

$$\Phi_p(\pi_{p,t(i)}) = \frac{\phi_p - 1}{2} \left(\frac{\pi_{p,t}(j)}{\overline{\pi}_p} - 1 \right)^2 + \frac{1}{\psi_p^2} \left(\exp\left(-\psi_p \left(\frac{\pi_{p,t}(j)}{\overline{\pi}_p} - 1 \right) \right) + \psi_p \left(\frac{\pi_{p,t}(j)}{\overline{\pi}_p} - 1 \right) - 1 \right).$$

Given the demand for intermediate goods (16), each firm maximizes profits by choosing its output price and labor input based on the following equation:

$$E_t \left[\sum_{s=0}^{\infty} \beta^s Q_{t+s|t} \left(\frac{P_{t+s}(i)}{P_{t+s}} \left(1 - \Phi_{p,t+s}(\pi_{p,t}(i))\right) Y_{t+s}(i) - \frac{W_{t+s}}{P_{t+s}} N_{t+s}(i) \right) \right], \tag{22}$$

where $Q_{t+s|t}$ is the time t value of a unit of the consumption good in period $t+s$, which is exogenous to intermediate goods producers.

The first order condition with respect to intermediate goods price $P_t(i)$ is derived as follows:

$$\begin{aligned} 0 &= \frac{-\eta_t}{\lambda_{p,t}} \left(\frac{P_t(i)}{P_t} \right)^{-\frac{1}{\lambda_{p,t}}-1} Y_t + A_t N_t \left(1 - \Phi_p(\pi_t(i)) - \Phi_p'(\pi_t(i))\pi_t(i)\right) \\ &\quad + \beta E_t \frac{d_{t+1}}{d_t} Q_{t+1|t} \frac{1}{\pi_{t+1}} A_{t+1} N_{t+1} \Phi_w'(\pi_{t+1}(i)) \pi_{t+1}^2(i), \end{aligned} \tag{23}$$

where η_t is the Lagrange multiplier for the demand for intermediate goods (16).

The first order condition with respect to labor input $N_t(i)$ is given by

$$\eta_t = \frac{P_t(i)}{P_t} \left(1 - \Phi_p(\pi_t(i))\right) - \frac{W_t}{P_t} \frac{1}{A_t}. \tag{24}$$

2.2 Aggregate relationships

2.2.1 Unemployment rate

Next, we derive the relationship between the wage markup and the unemployment rate, following Galí (2011a, b). Consider household member (j,k). Taking nominal wage $W_t(j)$ and price level P_t as given, an individual finds it optimal to provide labor services if and only if

$$\frac{W_t(j)}{P_t} \geq \chi_t A_t \frac{k^{\frac{1}{\nu}}}{(C_t/A_t)^{-\tau}}, \tag{25}$$

Many preceding studies, such as Kim and Ruge-Murcia (2009, 2011), Fahr and Smets (2010), and Abbritti and Fahn (2013), focus on the case of symmetric price adjustment costs ($\psi_p \to 0$), based on empirical analyses using micro data (such as Altissimo, Ehrmann and Smets (2006) and Chen et al. (2008)). Following these studies, we also use a Rotemberg-style quadratic price adjustment cost.

that is, the real wage is above his or her disutility from working, expressed in terms of the marginal utility of consumption. Thus, the marginal supplier of type j labor, which is denoted by $L_t(j)$, is given by

$$\frac{W_t(j)}{P_t} = \chi_t A_t \frac{L_t(j)^{\frac{1}{\nu}}}{(C_t/A_t)^{-\tau}}. \tag{26}$$

Define the aggregate labor force L_t as follows:

$$L_t = \int_0^1 L_t(j)\, dj. \tag{27}$$

In a symmetric equilibrium, the aggregate labor supply condition is given by the following equation:

$$\frac{W_t}{P_t} = \chi_t A_t \frac{L_t^{\frac{1}{\nu}}}{(C_t/A_t)^{-\tau}}. \tag{28}$$

Wage markup μ_t^W, which is defined as the ratio of the real wage to the average marginal rate of substitution, is given by the following equation:

$$\mu_t^W = \frac{W_t}{P_t}\left(\chi_t A_t \frac{N_t^{\frac{1}{\nu}}}{(C_t/A_t)^{-\tau}}\right)^{-1}. \tag{29}$$

Combining these equations,

$$\mu_t^W = \left(\frac{L_t}{N_t}\right)^{\frac{1}{\nu}}. \tag{30}$$

Define the unemployment rate u_t as the deviation of labor input from labor supply ($u_t \equiv \frac{L_t - N_t}{N_t}$). Then the relationship between the wage markup and the unemployment rate is as follows:

$$\mu_t^W = (1 + u_t)^{\frac{1}{\nu}}. \tag{31}$$

2.2.2 Wage inflation dynamics

In a symmetric equilibrium, wages and utilities are the same for all households ($W_t(j) = W_t$, $\pi_{w,t}(j) = \pi_{w,t}$, $\lambda_t(j) = \lambda_t$, $U_{N,t}(j) = U_{N,t}$, and $U_{C,t}(j) = U_{C,t}$). Then, from (6), (12), and (29), aggregate wage inflation dynamics can be obtained as follows:

$$\begin{aligned} 0 &= \frac{1}{\lambda_{w,t}}\frac{1}{\mu_t^W} + \left(\left(1-\frac{1}{\lambda_{w,t}}\right)(1-\Phi_w(\pi_{w,t})) - \Phi'_w(\pi_{w,t})\pi_{w,t}\right) \\ &\quad +\beta E_t\left(\frac{d_{t+1}}{d_t}Q_{t+1|t}\frac{N_{t+1}}{N_t}\frac{1}{\pi_{t+1}}\Phi'_w(\pi_{w,t+1})\pi_{w,t+1}^2\right). \end{aligned} \tag{32}$$

This represents a nonlinear and forward-looking structural relationship between aggregate wage inflation $\pi_{w,t}$ and wage markup μ_t^W.

In the special case where wages are perfectly flexible ($\Phi_w = \Phi'_w = 0$), the above equation becomes

$$0 = \frac{1}{\lambda_{w,t}}\frac{1}{\overline{\mu}_t^W} + \left(1-\frac{1}{\lambda_{w,t}}\right), \tag{33}$$

where $\overline{\mu}_t^W$ denotes the natural level of the wage markup ("natural wage markup" hereafter) that should prevail in a flexible-wage economy. Therefore, the natural wage markup is solely determined by $\lambda_{w,t}$ (i.e. $\overline{\mu}_t^W = \frac{1}{1-\lambda_{w,t}}$). Because (31) holds even in a flexible wage economy, the relationship between the natural wage markup $\overline{\mu}_t^W$ and the natural rate of unemployment u_t^n, which is the unemployment rate that should prevail when wages are rigid, is given by the following equation:

$$\overline{\mu}_t^W = (1+u_t^n)^{\frac{1}{\nu}}. \tag{34}$$

From (33) and (34), the natural rate of unemployment can be expressed in terms of $\lambda_{w,t}$:

$$u_t^n = \left(\frac{1}{1-\lambda_{w,t}}\right)^{\nu} - 1. \tag{35}$$

Therefore, the natural rate of unemployment rate is solely determined by wage markup shock $\lambda_{w,t}$.

By subtracting (33) from (32) and substituting μ_t^W and $\overline{\mu}_t^W$ using (31) and (34), we obtain the following structural relationship between aggregate wage inflation and the unemployment rate:

$$\begin{aligned} 0 &= \frac{1}{\lambda_{w,t}}\left((1+u_t)^{-\frac{1}{\nu}} - (1+u_t^n)^{-\frac{1}{\nu}}\right) - \left(\left(1-\frac{1}{\lambda_{w,t}}\right)\Phi_w(\pi_{w,t}) + \Phi_w'(\pi_{w,t})\pi_{w,t}\right) \\ &\quad +\beta E_t\left(\frac{d_{t+1}}{d_t}Q_{t+1|t}\frac{N_{t+1}}{N_t}\frac{1}{\pi_{t+1}}\Phi_w'(\pi_{w,t+1})\pi_{w,t+1}^2\right). \end{aligned} \tag{36}$$

This can be viewed as a nonlinear version of the New Keynesian wage Phillips curve (NKWPC) in which the current wage inflation rate $\pi_{w,t}$ is related to the unemployment rate (actual rate u_t and natural rate u_t^n) and the expectation for one-period ahead wage inflation $E_t\pi_{w,t+1}$. Similar to the linearized version of the NKWPC derived by Galí (2011a, b), the unemployment gap, namely, the deviation of the actual unemployment rate u_t from the natural rate u_t^n, is the fundamental determinant of wage inflation.[14]

2.2.3 Price inflation dynamics

Aggregate price inflation dynamics are obtained by combining intermediate goods firms' first order conditions (23) and (24), and by assuming a symmetric equilibrium where all intermediate goods producers choose the same prices and labor inputs ($P_t(i) = P_t$ and $N_t(i) = N_t$):

$$0 = \frac{1}{\lambda_{p,t}}\frac{W_t}{P_tA_t} + \left(1-\frac{1}{\lambda_{p,t}}\right)(1-\Phi_p(\pi_t)) - \Phi_p'(\pi_t)\pi_t + \beta E_t\left(\frac{d_{t+1}}{d_t}Q_{t+1|t}\frac{Y_{t+1}}{Y_t}\Phi_p'(\pi_{t+1})\pi_{t+1}\right). \tag{37}$$

This can be viewed as a nonlinear version of the New Keynesian Phillips curve (NKPC) in which current price inflation π_t is related to markup ratio $\frac{W_t}{P_tA_t}$ and the forward-looking expectation for one-period ahead price inflation $E_t\pi_{t+1}$.

[14] When the wage adjustment cost function is quadratic, the linearized version of the NKWPC can be derived as the same functional form as those derived by of Galí (2011a, b), which assumes Calvo-style symmetric wage rigidity (for the derivation of the linearized NKWPC under quadratic wage adjustment costs, see Born and Pfeifer (2016)).

2.2.4 Monetary and fiscal policy

Following Aruoba, Bocola, and Schorfheide (2017), monetary policy is governed by an interest rate feedback rule:

$$R_t = R_t^{*1-\rho_R} R_{t-1}^{\rho_R} e^{\varepsilon_{R,t}}, \tag{38}$$

where $\varepsilon_{R,t}$ is a monetary policy shock given by an iid normal random variable with mean zero and variance σ_R^2 and R_t^* is the target rate of the nominal interest rate. R_t^* is given by

$$R_t^* = r\bar{\pi}_p \left(\frac{\pi_t}{\bar{\pi}_p}\right)^{\psi_1} \left(\frac{Y_t}{\gamma Y_{t-1}}\right)^{\psi_2} \tag{39}$$

where r is the steady state real interest rate ($r = \frac{\gamma}{\beta}$) and $\bar{\pi}$ is the target inflation rate.

The fiscal authority consumes a fraction ζ_t of aggregate output Y_t, where $\zeta_t \in [0, 1]$ follows an exogenous process. Define $g_t = 1/(1 - \zeta_t)$. We assume that

$$\ln g_t = \left(1 - \rho_g\right) \ln g + \rho_g \ln g_{t-1} + \varepsilon_{g,t}, \tag{40}$$

where $\varepsilon_{g,t}$ is an iid normal random variable with mean zero and variance σ_g^2.

2.2.5 Aggregate resource constraint

Intermediate goods producers' total dividend payments to households are given by

$$D_t = (1 - \Phi_p(\pi_{p,t}))\, Y_t - \frac{W_t}{P_t} N_t, \tag{41}$$

Combining the household budget constraint and the government budget constraint, we obtain the following aggregate resource constraint:

$$P_t C_t + \zeta_t P_t Y_t = (1 - \Phi_p(\pi_{p,t}))\, P_t Y_t - \Phi_w(\pi_{w,t}) W_t N_t. \tag{42}$$

2.3 Model Solution

The dynamics of aggregate variables in this model are governed by eleven equations determining the endogenous variables ((5), (7), (13), (19), (21), (35), (36), (37), (38), (39) and (42)) and six equations describing the shock processes ((2), (4), (9), (18), (20) and (40)). Since a closed-form solution is not available, we solve the rational expectation equilibrium using the perturbation method. It should be noted that our model involves a strong nonlinearity generated by the asymmetric wage adjustment costs. For this reason, we employ a second-order approximation of the equilibrium equation and the monetary policy function, instead of a first-order approximation, and solve the local dynamic equations around the steady state. The second-order perturbation method has also been used to solve a similar model with asymmetric price and wage adjustment costs by Kim and Ruge-Murcia (2009) and Aruoba, Bocola, and Schorfheide (2017).[15]

2.4 Wage adjustment costs and wage inflation dynamics

Before conducting our empirical analysis, we examine the relationship between the wage adjustment cost and the wage Phillips curve. Specifically, we show how the asymmetry of the wage adjustment cost function, captured by the parameter ψ_w, affects wage inflation dynamics and eventually the shape of the wage Phillips curve. Because the model is forward-looking and nonlinear, we conduct numerical simulations based on a calibrated model, rather than deriving the relationship analytically.[16]

Figure 4 presents the impulse responses of the unemployment rate and wage inflation to exogenous shocks (productivity and monetary policy shocks) under alternative values of ψ_w. In the case of $\psi_w = 0$, which corresponds to the case of Rotemberg-style quadratic wage adjustment costs, the responses to positive and negative shocks are symmetric. However, if ψ_w has a large positive value (we set $\psi = 5000$ in the figure), meaning that wages are downwardly rigid, the responses to positive and negative

[15] See Schmitt-Grohe and Uribe (2004), and Kim et al. (2008) for the implementation and advantages of the second-order perturbation method.

[16] The parameters of the model are based on the prior distribution presented later in Section 3.3.

shocks are asymmetric. The decline of wage inflation in response to a negative shock is limited and, as a result, the rise of the unemployment rate is much larger, than in the case of symmetric adjustment costs.

Figure 5 shows the relationship between the asymmetry parameter ψ_w and the shape of the wage Phillips curve based on stochastic simulations using the calibrated model.[17] When wage adjustment costs are quadratic ($\psi_w = 0$), the wage Phillips curve is almost linear. However, as the asymmetry parameter ψ_w increases, the wage Phillips curve becomes highly nonlinear. It is almost linear and steep at high levels of wage inflation but is nearly flat at low levels of wage inflation. This shows clearly that in our model DWR bends the wage Phillips curve.

3 Empirical approach

3.1 Data

The data we use for our empirical analysis are presented in Table 1, while Figure 6 shows developments in the key variables. We use (i) GDP per capita, (ii) consumption per capita, (iii) price inflation, (iv) wage inflation, (v) employment per capita, (vi) the unemployment rate, and (vii) the policy interest rate as observable variables. As for wage inflation, we use the nominal wage per worker (rather than per hour) for consistency with the model. For the policy interest rates we use the shadow interest rates estimated by Ueno (2017) for Japan and by Krippner (2015) for the other three economies, in order to capture the effects of unconventional monetary policies. The observation period is from 1970Q2 to 2017Q3.

3.2 Estimation method

We estimate our nonlinear DSGE model with asymmetric wage adjustment costs using a Bayesian approach. Since our model is nonlinear, we cannot use a Kalman

[17] In the stochastic simulation, in order to check the shape of the wage Phillips curve, we add exogenous shocks except for labor supply shocks and wage markup shocks, since these two types of shocks are the factors shifting the wage Phillips curve.

filter to obtain exact likelihood functions. Instead, we use a bootstrap particle filter to evaluate the likelihood function. This strategy is the same as that employed by Fernández-Villaverde and Rubio-Ramírez (2007) to estimate a DSGE model with stochastic volatility. To use the bootstrap particle filter, we need to assume the presence of the measurement error in the measurement equation of the nonlinear state-space representation. We follow Aruoba, Bocola, and Schorfheide (2017) and assume the measurement error follows an iid normal distribution with mean zero and a variance of 10% of the sample variances of GDP growth, inflation, interest rates, and nominal wage growth. The posterior distribution of parameters is evaluated using a single-block MCMC/Metropolis-Hastings algorithm.[18] We calculate the marginal likelihood using the modified harmonic mean estimator proposed by Geweke (1999).

3.3 Bayesian priors

The prior distributions for the Bayesian estimations are presented in Table 2. The priors are mostly based on previous studies. We set the same values for the four economies with a few exceptions.[19] With respect to the risk-aversion parameter τ and the Frisch labor supply elasticity ν, we use gamma distributions with the same means and standard deviations as Aruoba, Bocola, and Schorfheide (2017). Following many previous studies, the constant component of discount factor β is set to 0.99 and $1/g$ is fixed to 0.85.

The prior for wage rigidity ϕ_w is set as a gamma distribution with a range from 0 to 40, which covers the calibrated values set by Fahr and Smets (2010) and Abbritti and Fahr (2013), which are 32 and 37.6, respectively. The degree of asymmetry of wage rigidity ψ_w, which is the central focus of our analysis, potentially takes both positive and negative values, reflecting downward and upward wage rigidity. To take

[18] See Herbst and Schorfheide (2015) and Fernández-Villaverde, Rubio-Ramírez, and Schorfheide (2016) for details of this nonlinear DSGE estimation approach.

[19] For the UK, under the same prior distributions as for the other economies, the natural rate of unemployment coincides with the actual unemployment rate for almost all periods, which seems quite unreasonable. Therefore, we use slightly different prior values for some parameters ($\psi_2 = 0.5$, $\lambda_p = 0.25$, $\lambda_w = 0.15$), while for the remaining parameters we use the same values as for the other economies.

into account that the value and distribution of ψ_w are highly uncertain, we employ a uniform distribution as the prior with respect to ψ_w.

As for price rigidity ϕ_p, we use a gamma distribution with a range from 0 to 60, which covers the calibrated values set by Fahr and Smets (2010) and Abbritti and Fahr (2013), which are 45 and 60, respectively. Regarding the responsiveness of the monetary policy rule, we use prior means of $\psi_1 = 1.5$ and $\psi_2 = 0.2$, which are the same values as those used by Aruoba, Bocola, and Schorfheide (2017). Taking the observed high persistence of the nominal interest rate into account, we set the prior mean for the parameter of interest rate smoothing to $\rho_R = 0.80$.[20]

The priors for the steady state values of the inverse of the demand elasticity of labor and the inverse of the demand elasticity of intermediate goods (λ_w and λ_p, respectively) are set as gamma distributions with 0.10 as the mean 0.05 as the standard deviation.[21] As for the prior distribution of long-run output growth (γ), we use a gamma distribution with 2.0 as the mean and 1.0 as the standard deviation. We calculate sample averages for the steady state price inflation ($\bar{\pi}_p$).[22] The steady state value of wage inflation ($\bar{\pi}_w$) is equal to the sum of $\bar{\pi}_p$ and γ. Finally, the same priors (persistence and variance) are set for the exogenous shock processes of most of the shocks.[23]

[20]This value is higher than the prior mean (0.50) used by Aruoba, Bocola, and Schorfheide (2017), but is close to their posterior mean (0.81).

[21]Aruoba, Bocola, and Schorfheide (2017) use a fixed value of 0.10 for both λ_w and λ_p rather than estimating them.

[22]The sample mean of $\bar{\pi}_p$ is.2.3% for Japan, 3.8% for the euro area, 5.2% for the UK, and 3.9% for the US.

[23]Based on the empirical results of Aruoba, Bocola, and Schorfheide (2017), we set the prior mean for the persistence of productivity shocks to $\rho_z = 0.2$, which is smaller than the persistence of other exogenous shocks.

4 Estimation results for each economy

4.1 Japan

For each economy, we carry out Bayesian estimations based on two models reflecting alternative assumptions regarding the degree of asymmetry of wage adjustment costs ψ_w. In the first model (the "asymmetric model"), we assume that wage adjustment costs are potentially asymmetric and estimate the value of ψ_w based on a diffused prior. In the second model (the "symmetric model"), we assume that wage adjustment costs are symmetric and impose a value of zero for ψ_w *a priori*. We then compare the estimation results of the two models to examine the importance of wage adjustment cost asymmetry.

The posterior distributions for the case of Japan are presented in Table 3. The log marginal likelihood for the asymmetric model is $-1,778.0$, while it is $-1,821.6$ for the symmetric model. Thus, the fit of the asymmetric model is significantly better than that of the symmetric model. In the asymmetric model, the posterior mean of the parameter for the asymmetry of wage rigidity ψ_w, which is the most important parameter considered here, is $3,560$. The 90% confidence interval ranges from $2,367$ to $4,609$, which means that ψ_w is significantly positive. The posterior mean of the degree of convexity in wage adjustment costs (ϕ_w) is 29.4, which is larger than that in the symmetric model (18.1). Figure 7 depicts the estimated wage adjustment cost functions based on the two models. The figure clearly shows that in the asymmetric model wage cuts are more costly than wage increases.

Turning to other parameters, based on the asymmetric model, our estimates of the risk-aversion parameter τ and Frisch labor supply elasticity ν are 5.87 and 0.20, respectively. These values are not far from the estimates obtained by Aruoba, Bocola, and Schorfheide (2017) for the US economy from the mid-1980s onward, which are 4.10 and 0.10, respectively. The posterior mean of price rigidity ϕ_p is 12.4, which is smaller than the estimated wage rigidity ϕ_w.[24] The parameters of the monetary

[24] Let θ_i be the so-called Calvo parameter, namely, the probability that prices or wages ($i = p$ or w) are unchanged for a given period. Then, as shown in Table 1 in Kahn (2005), the following

policy rule ($\rho_R = 0.89$, $\psi_1 = 2.05$ and $\psi_2 = 0.82$) are in line with the values reported in the literature on DSGE models. These structural parameters (except for ψ_w) and the exogenous shock processes (persistence and standard deviations of exogenous shocks) are also estimated in the symmetric model, and they are somewhat - but not substantially - different from those in the asymmetric model.

Next, we consider the natural rate of unemployment and the unemployment gap in Japan, which are presented in Figure 8. The left panel of the figure shows the natural rate of unemployment estimated based on the two alternative models. The bold (red) line depicts the estimated natural rate of unemployment in the asymmetric model, while the thin solid (blue) line depicts that in the symmetric model. The two lines trace a similar path until the late-1990s but then diverge substantially. Furthermore, the natural rate of unemployment in the asymmetric model was substantially lower than the actual unemployment rate (broken black line) from the late 1990s until around 2014, which more or less coincides with the period of deflation in Japan, while the natural rate in the symmetric model has closely followed the actual rate from the early 1990s onward. As a result, as shown in the right panel of Figure 8, the unemployment gap (i.e., the deviation of the actual unemployment rate from the natural rate) obtained based on the asymmetric model took persistently large positive values for a period of almost 15 years from the end of 1990s until the mid-2010s. This is a stark contrast from the symmetric model, in which the unemployment gap was only marginally positive during this period. Since the marginal likelihood strongly prefers the asymmetric model, these results imply that the asymmetry of wage adjustment costs has played quite an important role in determining the tightness of labor market conditions during Japan's so-called *lost decades.* That being said, since the mid-2010s, the unemployment gap has shrunk and turned

relationship holds between the degree of convexity and the Calvo parameter:

$$\phi_i = \frac{\theta_i(1/\lambda_i - 1)}{(1-\theta_i)(1-\theta_i\beta)}.$$

Based on the estimation results of the asymmetric model, we obtain $\theta_p = 0.489$ and $\theta_w = 0.805$ for the Calvo parameters. Therefore, the estimation results imply that in Japan prices are more flexible than wages.

negative, implying that tightness in Japan's labor market has increased in recent years.

Next, Figure 9 shows the wage Phillips curve by plotting wage inflation and the unemployment gap estimated based on the asymmetric model and the symmetric model. We find that the shape of the wage Phillips curve for Japan differs depending on the model. In the symmetric model, the wage Phillips curve is almost linear. In contrast, in the asymmetric model, the wage Phillips curve is L-shaped: the slope is quite steep where the unemployment gap is negative, but almost flat where the unemployment gap is positive. Because the asymmetric model is statistically superior to the symmetric model, we judge that the L-shaped wage Phillips curve provides a better description of Japan's wage inflation dynamics.

4.2 The euro area

The posterior distributions for the case of the euro area are presented in Table 4. The log marginal likelihood is $-1,493.4$ for the asymmetric model and $-1,524.9$ for the symmetric model, which means that, as for Japan, the asymmetric model provides a much better fit than the symmetric model. In the asymmetric model, the posterior mean of the asymmetry parameter ψ_w, is substantially positive $(4,508)$. The posterior mean of the convexity parameter ϕ_w is 51.7. Figure 10 shows the wage adjustment cost function for the euro area. As in Japan, wage adjustment costs are highly asymmetric, with the cost of wage cuts being considerably larger than the cost of wage increases. The other parameters do not differ substantially from those for Japan except for the convexity parameter of price adjustment costs ϕ_p, which is 56.3 and hence slightly larger than the value of ϕ_w. This implies that prices are more flexible than wages.[25]

Figure 11 depicts the natural rate of unemployment in the euro area. The two estimated series deviate substantially especially in the period before the early 1980s, the period from the mid- to late-1990s, and after the global financial crisis. This indicates that DWR had a sizable impact on the actual unemployment rate in these

[25] The Calvo parameters are $\theta_p = 0.741$ and $\theta_w = 0.899$.

three periods. This impact is particularly pronounced from around the mid-1990s onward, with the natural rate of unemployment in the asymmetric model being much lower than the actual rate. As a result, the unemployment gap has been persistently positive since the first half of the 1990s, reaching a peak of more than 5% after the onset of the European debt crisis. We also find that the unemployment gap in the symmetric model turned negative at the end of the observation period (2017Q3), but the gap in the asymmetric model is still large and positive.

Finally, Figure 12 shows the wage Phillips curve for the euro area. While the wage Phillips curve in the symmetric model is quite linear, that in the asymmetric model is fairly nonlinear, since its slope is quite flat for very low levels of wage inflation. Because the log marginal likelihood is higher in the asymmetric model, we judge that, as in the case of Japan, the nonlinear wage Phillips curve in the asymmetric model provides better description of wage inflation dynamics in the euro area.

4.3 The UK

The posterior distributions for UK are presented in Table 5. The log marginal likelihood is $-2,029.4$ for the asymmetric model and $-2,047.5$ for the symmetric model. Therefore, as for Japan and the euro area, the asymmetric model is statistically superior to the symmetric model. In the asymmetric model, the posterior mean of the asymmetry parameter ψ_w is $1,101$, while that of the convexity parameter ϕ_w is 20.1. These values are smaller than those of Japan and the euro area. Figure 13 indicates that the wage adjustment cost function for the UK is fairly asymmetric, meaning that DWR is important. The convexity parameter of price adjustment cost ϕ_p is 35.7, which is larger than the value of ϕ_w.[26]

Figure 14 displays the natural rate of unemployment in the UK. The two estimated series deviate persistently, indicating that DWR had a sizable impact on developments in the unemployment rate in the UK. Specifically, the asymmetric

[26]Nevertheless, in terms of the Calvo parameters, prices are more flexible than wages ($\theta_p = 0.593$ and $\theta_w = 0.874$), because $\overline{\lambda}_w$ is far greater than $\overline{\lambda}_p$. This result is consistent with the findings obtained by Faccini, Millard and Zanetti (2011) who also report that for the UK the Calvo parameter for wages is larger than that for prices.

model shows a large and positive unemployment gap in the early-1980s and again in the early-1990s, when actual unemployment rose sharply. In the wake of the global financial crisis, the actual unemployment rose, but the natural rate also rose sharply. As a result, the unemployment gap was around 0% even during the crisis periods. Meanwhile, in the recovery since the crisis, the actual unemployment rate has declined substantially and fallen below the natural rate. Finally, Figure 15 shows the wage Phillips curve for the UK. The wage Phillips curve is not as clear as for Japan and the euro area. However, it seems more nonlinear in the asymmetric model than in the symmetric model, since the flat part at low wage inflation is much longer in the asymmetric model.

4.4 The US

The posterior distributions for the US are presented in Table 6. In the asymmetric model, the posterior mean of the asymmetric parameter ψ_w is positive and statistically significant. However, the mean value ($\psi_w = 579$) is smaller than those obtained for the other economies and the standard deviation is quite large.[27] We also find that the convexity parameter in the wage adjustment cost function is much larger ($\phi_w = 242$ in the symmetric model) than those for the other economies. Figure 16 displays the shape of the wage adjustment cost function for the US. Since the value of ϕ_w plays a dominant role, the wage adjustment cost function seems to be almost quadratic even in the asymmetric model, which indicates that wage adjustment costs are large but symmetric. This means that, in the case of the US, wages are highly sticky both in an upward and downward direction. In other words, the asymmetry parameter ψ_w does not play a substantial role in wage dynamics, even though it is

[27] These results are in line with those reported by Aruoba, Bocola, and Schorfheide (2017), who similarly find that the asymmetry parameter of the wage adjustment cost function is statistically significant but small in size. Some studies using micro-data argue that the degree of DWR in the US has increased since the global financial crisis (e.g., Fallick, Lettau, and Wascher (2016)). To examine how the degree of DWR has changed in the US, we re-estimate the model using data for the period the global financial crisis only (1970Q2-2007Q4). As shown in Appendix Table 1 and Appendix Figure 2, we find that the asymmetry parameter ψ_w is not statistically significant. This suggests that the degree of DWR has increased somewhat in the post-crisis period, although it is still quite small compared to the other three economies.

statistically significant. Consequently, the log marginal likelihood is $-1,634.9$ for the asymmetric model and $-1,627.2$ for the symmetric model. Therefore, in contrast to the other three economies, for the US the fit is better in the case of the symmetric than the asymmetric model.

Turning to other parameters, in the symmetric model, the risk-aversion parameter τ is 3.37 and the Frisch labor supply elasticity ν is 0.19 - values that are in line with previous studies (such as Aruoba, Bocola, and Schorfheide (2017)). The convexity parameter of price adjustment cost ϕ_p is 69.8, which is lower than the value of ϕ_w. This implies that prices are more flexible than wages.[28] The parameters of the monetary policy rule are $\rho_R = 0.75$, $\psi_1 = 1.06$ and $\psi_2 = 1.06$. As for the properties of exogenous shocks, we find that the standard deviation of labor supply shock is quite large, compared to other economies.

Figure 17 shows the natural rate of unemployment in the US. Both of the estimated series - based on the symmetric and the asymmetric model - move very close together. As a result, developments in the unemployment gap are almost the same in the two models, meaning that DWR has not had a substantial impact on the tightness of labor market conditions in the US.[29] Variations in the natural rate of unemployment are generally smaller than those in the actual unemployment rate. Following the global financial crisis, the natural rate of unemployment jumped, but this jump was less than 2 percentage points and thus only partially accounts for the large increase in the actual unemployment rate of more than 5 percentage points. Finally, Figure 18 plots wage inflation/ unemployment gap observations based on the two alternative models. Because the estimated series of the natural rate of unemployment are very close in these two models, no substantial difference between the two panels can be observed. Furthermore, both panels indicate that, in the case of the US, there is no clear relationship between wage inflation and the unemployment gap.

[28] The Calvo parameters are calculated as $\theta_p = 0.772$ and $\theta_w = 0.913$.

[29] The natural rate of unemployment in the asymmetric model is highly correlated with that estimated by the Congressional Budget Office (the correlation coefficient for the period from 1970Q2 to 2017Q3 is 0.96).

5 Downward wage rigidity and missing wage inflation

5.1 Comparisons of wage adjustment costs

The previous section provided estimation results of the model and the natural rate of unemployment for individual economies. As mentioned, the wage adjustment cost function, which is the key focus of our study, is characterized by two parameters: the degree of convexity (ϕ_w) and the degree of asymmetry (ψ_w). In order to compare the nature of wage rigidity in the four economies, Figure 19 present the shapes of the respective wage adjustment cost functions.

In the case of Japan, wage adjustment costs are highly asymmetric. This means that DWR is very important, since the cost of wage cuts is very high compared to the cost of wage increase. Next, the shape of the adjustment cost function for the euro area is also highly asymmetric, which indicates the importance of DWR in the euro area. We find that wages are generally more rigid in the euro area than in Japan, since adjustment costs are larger for the euro area both in an upward and downward directions. In the case of the UK, wage adjustment costs are relatively small, meaning that wages are relatively flexible. At the same time, however, the adjustment cost function is also asymmetric, although the degree of asymmetry is somewhat smaller than in Japan and the euro area. In contrast, the wage adjustment cost function for the US is symmetric and adjustment costs are quite large both in an upward and downward direction. The results thus indicate that wages are generally very sticky in the case of the US, reflecting the very large value of convexity parameter ϕ_w.

Let us examine whether these results are consistent with the findings of previous studies. Unfortunately, there are relatively few studies that conduct a comprehensive cross-country analysis of wage rigidity or downward rigidity. This is mainly due to the fact that it is not easy to assemble comparable data sets across countries. In particular, analyses using micro-data have to contend with differences in the type of workers included and the definition of wages, and the methods used for detecting downward rigidity often differ across studies. There is also the problem that DWR cannot be detected if the observation period is short. Furthermore, many existing studies judge DWR based solely on the skewness of the distribution of microdata. Because such analysis does not involve identification of shocks hitting to the economy, it is unable to quantify the influence of DWR on aggregate wage inflation dynamics. The studies mentioned below are subject to these issues.

Holden and Wulfsberg (2007), using industry-level wage data for 19 OECD countries for the period 1973-1999, find that DWR is weak in the US and the UK, while it is strong in some euro area countries (such as Italy, Greece, Portugal, and Spain).[30] Another cross-country study is that by Dickens et al. (2007), who report that DWR is relatively weak in the UK but is strong in some euro area countries (such as Germany, Greece, Italy, and Portugal).[31] Meanwhile, Knoppik and Beissinger (2005) compare DWR using micro-data for European households covering the period 1994-2001. They find that the degree of DWR for the UK is smaller than that for the euro area. As for Japan, Kimura and Ueda (2001) and Kuroda and Yamamoto (2003a, 2003b) present empirical results for the emergence of DWR in the 1990s, although they also find that DWR is not observed between the late 1990s and the early 2000s. Yamamoto (2007) also shows that, even in the 2000s, DWR is found in regular wages for full-time workers.[32] With respect to the degree of wage rigidity (not DWR), which is captured by the parameter ϕ_w in this analysis, Muto and Shintani (2018) use aggregate data for Japan and the US to estimate the NKWPC,

[30] They report that strong DWR is typically observed in countries where the unemployment rate is low, employment protection legislation is strong and labor union density is high. However, Dickens et al. (2007) report that these measures do not necessarily explain the degree of DWR. These results indicate that evidence on the relationship between DWR and the characteristics of labor market institutions is still inconclusive.

[31] This study reports the results of the International Wage Flexibility Project (IWFP). The observation period varies across countries. For example, it is 1970-1997 for the US, but 1976-2000 for the UK.

[32] Yamamoto (2008) explains that "wage stickiness should have increased after collapse of the bubble economy in Japan, due to the existence of downward nominal wage rigidity under the low inflation environment. Before collapse of the bubble economy when inflation rate was relatively higher, downward rigidity of nominal wage was not a binding constraint in wage setting. When the inflation rate got lower and even negative after collapse of the bubble economy, however, nominal wage cut became necessary to reduce real wages. Then, downward nominal wage rigidity started to serve as a binding constraint." This interpretation is consistent with our empirical findings.

which assumes Calvo-style symmetric wage rigidity. They report that the slope of the NKWPC is much steeper in Japan than in the US, which suggests that wages are less sticky in Japan than in the US.[33] López-Villavicencio and Saglio (2017) also estimate the NKWPC using data for 15 OECD countries covering the period from 1985Q1 to 2014Q3. Their regression results suggest that wage rigidity is quite low in Japan and very high in the US, while wage rigidity in European countries lies between that Japan and the US. While these empirical studies do not necessarily provide comprehensive analyses of wage rigidity or downward rigidity, our results can be regarded as roughly consistent with previous studies.

5.2 Wage Phillips curve and missing wage inflation

Our remaining task is to investigate whether missing wage inflation observed in the four advanced economies can be explained by DWR. To do so, we present recent developments in wage inflation and the unemployment gap in Figure 20. Specifically, we show how these two variables have evolved since unemployment rates peaked in the wake of the crisis. In the case of Japan, the unemployment rate peaked in 2009Q3. Subsequently, the unemployment gap declined substantially, but wage inflation has remained almost unchanged and has stayed at around 0%, because the wage Phillips curve has been quite flat due to the asymmetric wage adjustment costs. This means that the missing wage inflation observed in Japan's economy very likely is caused by DWR.

In the euro area, the situation is almost the same as in Japan. The unemployment rate reached its peak in 2013Q2. The unemployment gap subsequently declined substantially, reflecting the recovery from the European debt crisis. However, as in Japan, wage inflation has been unresponsive to the decline in unemployment, since the wage Phillips curve has been almost flat due to DWR. In the case of the UK, the unemployment rate peaked in 2011Q3 and the unemployment gap has declined markedly since then. During this period, wage inflation rate has been somewhat

[33] They estimate the NKWPC using data for Japan and the US for the period from 1970Q1 to 2013Q2. They also provide micro-level evidence (Barattieri, Basu, and Gottschalk (2014) for the US and the "*Survey on Wage Increases*" for Japan) supporting this interpretation.

volatile, compared to Japan and the euro area. However, wage inflation and the unemployment gap can be regarded as evolving roughly in line with the flat part of the wage Phillips curve.

Taken together, the empirical results indicate that DWR has been an important source of missing wage inflation in Japan, the euro area and the UK. We find that in these economies wage inflation is not responsive to the degree of labor market slack as long as the unemployment rate is above the natural rate but accelerates when the unemployment rate falls well below the natural rate. The most recent data for 2017Q3 shows that the unemployment gap is approaching the kink of the wage Phillips curve. This suggests that wage inflation is likely to reappear with further improvement in the labor market.

However, the situation seems to be quite different in the US. Since the crisis, the unemployment gap has declined almost monotonically but wage inflation has fluctuated considerably. This raises the fundamental question whether wage inflation dynamics in the US are actually governed by the wage Phillips curve. Our estimation results indicate that the estimated variance of labor supply shocks is particularly large in the US. This raises the possibility that labor supply shocks play an important role in shifting the location of the wage Phillips curve in the case of the US economy. To examine this hypothesis, we investigate the shape of the wage Phillips curve through stochastic simulations using the model estimated with the US data. In Figure 21, we present the results of two alternative simulations. In the first simulation we add all exogenous shocks. In the second simulation we add exogenous shocks except for the labor supply shock. In the first simulation, the relationship between wage inflation and the unemployment gap is quite unclear. However, in the second simulation, the relationship is almost linear. These experiments suggest that there actually exists a linear wage Phillips curve in the US once the distortionary impact of labor supply shocks is eliminated. In other words, in the case of the US, wage inflation has responded only sluggishly to the decline in the unemployment rate because the wage Phillips curve is linear but inherently quite flat.[34] This means that the mechanism

[34] We have also checked the shape of wage Phillips curve by using the employment cost index

behind the missing wage inflation observed in the US is not explained by DWR.[35]

6 Caveats and possible extensions

Our analysis contributes to the literature by providing new empirical findings on the shape of the wage Phillips curve and the natural rate of unemployment in the four advanced economies considered here based on a recently developed particle filter technique for estimating nonlinear DSGE models. However, there are some caveats and possible extensions.

First, our analysis is based on the sticky wage model developed by Galí (2011a, b) and Galí, Smets and Wouters (2012) to incorporate unemployment. A great advantage of the model for our empirical analysis is its simplicity, since the estimation of a nonlinear DSGE model using Bayesian MCMC with a particle filter is quite a computationally demanding task. That being said, a natural extension would be to incorporate search and matching frictions into our model, since the current version of the model is too simple to examine variations in the natural rate of unemployment, in that the natural rate of unemployment is solely determined by exogenous wage markup shocks. In this respect, Abbritti and Fahr (2013) and Abo-Zaid (2013) have already presented sticky wage models with search and matching frictions as well as asymmetric wage adjustment costs. A potentially fruitful avenue would be to estimate these nonlinear models with search and matching frictions and thereby identify the path of the natural rate of unemployment in the four economies.

Second, labor market slack in our model is defined in terms of the unemployment gap. However, an interesting extension would be to incorporate broader measures of labor market slack into the model. One possible extension in this direction would be

published by the Bureau of Labor Statistics, which is free from the influence of employment shifts among occupations and industries. Consequently, we have found that there exists a stable and very flat wage Phillips curve in the US since the beginning of the 2000s. This also suggests that employment shifts are quite important shifting factor of the wage Phillips curve in the US.

[35] While we have highlighted the potential role of labor supply shocks in shifting the wage Phillips curve, other factors affecting the shape of the wage Phillips curve, such as a shift in inflation expectations may also play a role especially in the earlier periods, such as the 1970s and the early 1980s.

to include the intensive margin (endogenous hours) in the model. Another issue is raised by Erceg and Levin (2014), who incorporate households' decision with regard to labor market participation into their model, assuming that labor market exit and reentry decisions are associated with significant adjustment costs. Under this setup, the labor force participation gap, which is defined as the deviation of the labor force participation rate from its potential path implied by demographic and structural factors, enters the Phillips curve in addition to the unemployment gap. Their study suggests that the reason why inflation in the US has been subdued in recent years despite the notable decline in the unemployment rate, is that the labor force participation gap widened markedly after the financial crisis. Our model could be extended by incorporating the labor force participation gap to examine the robustness of our findings.

Third, although our sticky wage model assumes different types of labor services, it does not fully take into account the full variety of workers found in the labor market, such as full-time and part-time workers, incumbent workers and newly hired workers, skilled workers and unskilled workers, and any combination of these characteristics. As highlighted in a number of studies (e.g., IMF (2017)), changes in the composition of the workforce in terms of these characteristics are an important reason for the recent subdued wage inflation in advanced economies. An important task therefore is to develop a model which takes such changes in the workforce composition into account and reassess the importance of DWR as a reason for the recent subdued wage inflation.[36]

Finally, to capture the stance of monetary policy, we used estimates of shadow interest rates obtained in empirical studies on yield curves in our model. Because central banks in the four economies have employed an array of unconventional policies, we think that employing shadow interest rates provides a useful way to measure

[36] As shown in Appendix Figure 3, changes in the composition of the workforce in terms of full-time and part-time workers have persistently restrained aggregate wage inflation in Japan. Similarly, in other economies, too, increases in the share of part-time workers have made a negative contribution to aggregate wage inflation. This means that, if the depressing effect of changes in the workforce composition were excluded, aggregate wage inflation would be higher than it actually was. This implies in turn that DWR might be even more important than our results suggest.

the stance of monetary policy. However, it would be possible to explicitly introduce the zero lower bound (ZLB) or the effective lower bound (ELB) into the model and estimate the model using a Bayesian method with a particle filter, as some recent studies have tried to do (e.g., Hirose and Sunakawa (2016), Iiboshi and Shintani (2017), Iiboshi, Shintani and Ueda (2017), and Aruoba, Bocola, and Schorfheide (2018)). Examining how explicitly introducing the ZLB or ELB affects (or does not affect) the structural parameters, particularly the degree of (downward) wage rigidity, is an issue we hope to tackle in the future.[37,38]

7 Conclusion

Wage inflation in advanced economies has been generally subdued in the aftermath of the global financial crisis. Against this background, the present study examined whether this phenomenon can be explained by downward wage rigidity (DWR). The results indicate that DWR likely is an important source of the flattening wage Phillips curve and the lack of wage inflation in Japan, the euro area, and the UK.

Especially in the case of Japan, an L-shaped wage Phillips curve between wage inflation and the unemployment gap emerges once the presence of DWR is properly taken into account. This indicates that wage inflation is not responsive to the degree of labor market slack as long as the unemployment rate is above the natural rate but accelerates when the unemployment rate falls well below the natural rate. Therefore, our results imply that the missing wage inflation observed in Japan is not a permanent phenomenon and wage inflation is likely to reappear with further improvement in the labor market.

However, our results also indicate that DWR is not necessarily the reason for

[37] To examine how sensitive our results are to the use of alternative data for policy interest rates, we re-estimate the model and the natural rate of unemployment for the case of Japan using the actually observed call rate rather than the shadow interest rate. Using the call rate does not substantially alter the estimation results on the wage adjustment cost function and the natural rate of unemployment (see Appendix Table 2 and Appendix Figures 4 to 6).

[38] Using a time-varying structural VAR model to identify shocks, Debortoli, Galí and Gambetti (2018) provide empirical evidence suggesting that the ZLB constraint is irrelevant for the dynamic response of US macroeconomic variables to such shocks.

the recent missing wage inflation in all advanced economies, since it has no notable impact on the wage inflation dynamics in the US. In the case of the US, wage adjustment costs are large but symmetric, meaning that wages are highly sticky both in an upward and downward direction. This suggests that wage inflation has responded only sluggishly to the decline in the unemployment rate because the wage Phillips curve is inherently quite flat in the case of the US.

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Table 1: Data Sources

	Series	Japan	Euro area	UK	US
1	Output	Cabinet Office, SNA: Real GDP	ECB, Area-wide Model database: GDP; Eurostat, Real Gross Domestic Product for Euro area (19 countries, retrieved from FRED)	Bank of England, Three Centuries of Macroeconomic Data: Real GDP (retrieved from FRED)	U.S. BEA, Gross Domestic Product: Real GDP (retrieved from FRED)
2	Consumption	Cabinet Office, SNA: Real Private consumption	ECB, Area-wide Model database, Individual Consumption Expenditure; OECD, Private Final Consumption Expenditure for the Euro Area(retrieved from FRED)	Bank of England, Three Centuries of Macroeconomic Data: Real Consumption Expenditure (retrieved from FRED)	U.S. BEA, Personal income and outlays: Real Personal Consumption Expenditure (retrived from FRED)
3	CPI inflation	Ministry of Internal Affairs and Communication, Consumer Price Index: CPI less food and energy	ECB, Area-wide Model database: HICP- All items Excluding Energy; Eurostat, Harmonized Index of Consumer Prices: Overall Index Excluding Energy for Euro area (19 countries, retrieved from FRED)	OECD, CPI: All items excluding food and energy for UK	U.S. Bureau of Labor Statistics, Consumer Price Index for All Urban Consumers: All items less food and energy (retrieved from FRED)
4	Wage per head	Monthly Labor Survey; Total cash earning	Calculated by deviding Compensation of employees by employment; ECB, Area-wide Model database: Compensation of employees; OECD, Quarterly National Account	Calculated by deviding Compensation of employees by employment; OECD, Compensation of Employees	Calculated by deviding Compensation of employees by employment; OECD, Compensation of Employees
5	Employment	Labor Force Survey: Employed person	ECB, Area-wide Model database; OECD, Quarterly National Account	Office for National Statistics, Labor Market Statistics: Number of People in Employment	OECD, Employed population
6	Unemployment rate	Labor Force Survey: Unemployment rate	ECB, Area-wide Model database; OECD, Harmonized Unemployment Rate	Office for National Statistics, Labor Force Survey: ILO Unemployment rate	U.S. Bureau of Labor Statistics, Civilian Unemployment Rate
7	Short-term interest rate	Bank of Japan, Overnight call rate	ECB, Area-wide Model database: Euribor 3-month	Bank of England, Bank of England Policy Rate (retrieved from FRED)	Board of Governors of the Federal Reserve System: Effective Federal Funds Rate (retrieved from FRED)
8	Shadow rate	Ueno (2017)	Krippner (2015)	Krippner (2015)	Krippner (2015)
9	Population	Ministry of Internal Affairs and Communication, Populaton estimates	Eurostat	Office for National Statistics, Population estimates (Annual data is converted to quarterly data)	U.S. Bureau of Economic Analysis, Personal income and outlays: Population(retrived from FRED)

Table 2: Prior distributions for Bayesian estimations

Parameters	Description	Distribution	Para1	Para2
τ	Inverse of the intertemporal elasticity of substitution	Gamma	2.00	1.00
ν	Frisch labor supply elasticity	Gamma	0.50	0.25
ϕ_p	Convexity in price adj. cost function	Gamma	30	15
ϕ_w	Convexity in wage adj. cost function	Gamma	20	10
ψ_w	Asymmetricity in wage adj. cost function	Uniform	-10000	10000
ρ_r	Interest rate smoothing	Beta	0.80	0.20
ψ_1	Inflation coefficient in policy rule	Gamma	1.50	0.20
ψ_2	Output coefficient in policy rule	Gamma	0.20	0.10
$\bar{\lambda}_p$	Inverse of demand elasticity of final goods	Gamma	0.10	0.05
$\bar{\lambda}_w$	Inverse of demand elasticity of labor	Gamma	0.10	0.05
γ	Average technology growth	Gamma	2.00	1.00
ρ_g	Autocorrelation of government spending shocks	Beta	0.50	0.20
ρ_z	Autocorrelation of productivity shocks	Beta	0.20	0.10
ρ_p	Autocorrelation of price markup shocks	Beta	0.50	0.20
ρ_w	Autocorrelation of wage markup shocks	Beta	0.50	0.20
ρ_d	Autocorrelation of discount factor shocks	Beta	0.50	0.20
σ_r	Std. dev. of monetary policy shocks	Inv.gamma	0.50	2.00
σ_g	Std. dev. of government spending shocks	Inv.gamma	0.50	2.00
σ_z	Std. dev. of productivity shocks	Inv.gamma	0.50	2.00
σ_p	Std. dev. of price markup shocks	Inv.gamma	0.50	2.00
σ_w	Std. dev. of wage markup shocks	Inv.gamma	0.50	2.00
σ_χ	Std. dev. of labor supply shocks	Inv.gamma	0.50	2.00
σ_d	Std. dev. of discount factor shocks	Inv.gamma	0.50	2.00

Notes: Para(1) and Para(2) denote the means and the standard deviations for Beta, Gamma, and Normal distributions; s and ν for the Inverse Gamma distribution, where $p_{IG}(\sigma|\nu, s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma}$; For Uniform distribution, each denotes the upper and lower bound respectively.

Table 3: Posterior distributions of the parameters for Japan

	Posteriors			
	Asymmetric model		Symmetric model	
	Mean	90% interval	Mean	90% interval
τ	5.87	[5.02, 6.81]	5.05	[4.75, 5.54]
ν	0.20	[0.18, 0.22]	0.16	[0.15, 0.16]
ϕ_p	12.4	[6.89, 18.6]	31.8	[29.7, 33.8]
ϕ_w	29.4	[22.5, 34.2]	18.1	[14.8, 21.9]
ψ_w	3560	[2367, 4609]	-	-
ρ_r	0.89	[0.87, 0.91]	0.86	[0.85, 0.88]
ψ_1	2.05	[1.94, 2.18]	1.85	[1.80, 1.91]
ψ_2	0.82	[0.29, 1.40]	1.36	[1.23, 1.50]
$\bar{\lambda}_p$	0.13	[0.06, 0.20]	0.13	[0.10, 0.16]
$\bar{\lambda}_w$	0.41	[0.36, 0.47]	0.30	[0.25, 0.33]
γ	2.05	[1.94, 2.18]	2.06	[1.91, 2.18]
ρ_g	0.98	[0.97, 0.99]	0.99	[0.99, 0.99]
ρ_z	0.75	[0.66, 0.83]	0.80	[0.78, 0.83]
ρ_p	0.41	[0.13, 0.81]	0.67	[0.44, 0.95]
ρ_w	0.71	[0.70, 0.72]	0.74	[0.73, 0.76]
ρ_d	0.96	[0.96, 0.97]	0.97	[0.97, 0.98]
$100\sigma_r$	0.27	[0.24, 0.30]	0.30	[0.27, 0.33]
$100\sigma_g$	0.21	[0.15, 0.26]	0.31	[0.30, 0.33]
$100\sigma_z$	0.26	[0.19, 0.32]	0.26	[0.23, 0.29]
$100\sigma_p$	0.52	[0.27, 0.83]	0.34	[0.28, 0.42]
$100\sigma_w$	0.54	[0.27, 0.96]	4.00	[3.16, 4.84]
$100\sigma_\chi$	0.50	[0.26, 0.98]	0.51	[0.25, 1.03]
$100\sigma_d$	3.57	[3.10, 4.10]	3.01	[2.54, 3.33]
Log marginal likelihood	-1778.0		-1821.6	

Table 4: Posterior distributions of the parameters for the euro area

	Posteriors			
	Asymmetric model		Symmetric model	
	Mean	90% interval	Mean	90% interval
τ	4.81	[4.17, 5.24]	4.16	[3.84, 4.78]
ν	0.23	[0.21, 0.26]	0.23	[0.13, 0.34]
ϕ_p	56.3	[52.4, 60.0]	49.7	[40.1, 53.7]
ϕ_w	51.7	[46.2, 54.2]	53.9	[50.7, 59.2]
ψ_w	4508	[3802, 6306]	-	-
ρ_r	0.86	[0.84, 0.88]	0.87	[0.85, 0.90]
ψ_1	2.05	[1.77, 2.23]	1.88	[1.68, 2.19]
ψ_2	1.37	[0.79, 1.99]	1.91	[1.58, 2.21]
$\bar{\lambda}_p$	0.16	[0.09, 0.29]	0.17	[0.08, 0.26]
$\bar{\lambda}_w$	0.61	[0.50, 0.74]	0.69	[0.54, 0.77]
γ	2.12	[1.60, 2.81]	2.29	[2,05, 2.48]
ρ_g	0.89	[0.85, 0.93]	0.94	[0.87, 0.97]
ρ_z	0.77	[0.71, 0.86]	0.80	[0.72, 0.84]
ρ_p	0.43	[0.21, 0.71]	0.52	[0.23, 0.85]
ρ_w	0.91	[0.55, 0.99]	0.98	[0.94, 0.99]
ρ_d	0.98	[0.97, 0.98]	0.99	[0.97, 0.99]
$100\sigma_r$	0.24	[0.22, 0.26]	0.22	[0.19, 0.26]
$100\sigma_g$	0.22	[0.19, 0.24]	0.19	[0.15, 0.20]
$100\sigma_z$	0.21	[0.19, 0.23]	0.18	[0.17, 0.20]
$100\sigma_p$	0.57	[0.29, 0.81]	0.91	[0.59, 1.21]
$100\sigma_w$	0.67	[0.29, 0.96]	0.62	[0.41, 0.94]
$100\sigma_\chi$	1.17	[0.96, 1.55]	1.57	[1.07, 2.05]
$100\sigma_d$	4.56	[4.02, 4.92]	6.93	[5.48, 7.69]
Log marginal likelihood	-1493.4		-1524.9	

Table 5: Posterior distributions of the parameters for the UK

	Posteriors			
	Asymmetric model		Symmetric model	
	Mean	90% interval	Mean	90% interval
τ	2.62	[2.32, 2.86]	2.61	[2.36, 2.83]
ν	0.34	[0.24, 0.49]	0.32	[0.12, 0.45]
ϕ_p	35.7	[24.7, 45.5]	48.1	[36.1, 68.2]
ϕ_w	20.1	[17.9, 22.4]	21.2	[20.4, 22.2]
ψ_w	1101	[849, 1289]	-	-
ρ_r	0.58	[0.44, 0.70]	0.51	[0.40, 0.60]
ψ_1	1.20	[1.18, 1.23]	1.12	[1.10, 1.14]
ψ_2	1.38	[1.26, 1.50]	1.22	[1.02, 1.36]
$\bar{\lambda}_p$	0.09	[0.06, 0.12]	0.05	[0.04, 0.08]
$\bar{\lambda}_w$	0.72	[0.56, 0.84]	0.77	[0.64, 0.93]
γ	2.36	[2.19, 2.48]	2.43	[2.28, 2.58]
ρ_g	0.79	[0.75, 0.84]	0.86	[0.82, 0.89]
ρ_z	0.76	[0.67, 0.82]	0.78	[0.72, 0.84]
ρ_p	0.51	[0.19, 0.72]	0.63	[0.32, 0.86]
ρ_w	0.22	[0.14, 0.31]	0.28	[0.15, 0.32]
ρ_d	0.99	[0.98, 0.99]	0.99	[0.99, 0.99]
$100\sigma_r$	0.55	[0.39, 0.78]	0.60	[0.49, 0.72]
$100\sigma_g$	0.49	[0.41, 0.55]	0.43	[0.38, 0.45]
$100\sigma_z$	0.27	[0.25, 0.30]	0.27	[0.26, 0.28]
$100\sigma_p$	0.98	[0.28, 2.21]	0.17	[0.11, 1.40]
$100\sigma_w$	0.46	[0.24, 0.74]	0.54	[0.27, 0.98]
$100\sigma_\chi$	1.19	[1.03, 1.39]	1.36	[1.22, 1.45]
$100\sigma_d$	1.71	[1.52, 1.90]	2.04	[1.79, 2.25]
Log marginal likelihood	-2029.4		-2047.5	

Table 6: Posterior distributions of the parameters for the US

	Posteriors			
	Asymmetric model		Symmetric model	
	Mean	90% interval	Mean	90% interval
τ	3.64	[3.07, 4.01]	3.37	[3.05, 3.75]
ν	0.10	[0.09, 0.12]	0.19	[0.17, 0.27]
ϕ_p	73.8	[67.4, 79.9]	69.8	[64.4, 77.5]
ϕ_w	225	[219, 229]	242	[222, 257]
ψ_w	579	[99, 845]	-	-
ρ_r	0.77	[0.74, 0.79]	0.75	[0.72, 0.77]
ψ_1	1.08	[1.05, 1.11]	1.06	[1.04, 1.07]
ψ_2	0.98	[0.74, 0.79]	1.06	[0.65, 1.27]
$\bar{\lambda}_p$	0.12	[0.07, 0.17]	0.17	[0.11, 0.24]
$\bar{\lambda}_w$	0.50	[0.45, 0.55]	0.31	[0.26, 0.37]
γ	0.71	[0.53, 1.04]	1.21	[0.97, 1.45]
ρ_g	0.98	[0.97, 0.98]	0.97	[0.96, 0.98]
ρ_z	0.53	[0.44, 0.63]	0.57	[0.50, 0.66]
ρ_p	0.41	[0.19, 0.71]	0.51	[0.20, 0.82]
ρ_w	0.44	[0.17, 0.98]	0.39	[0.12, 0.85]
ρ_d	0.81	[0.78, 0.84]	0.80	[0.76, 0.85]
$100\sigma_r$	0.34	[0.28, 0.38]	0.37	[0.32, 0.43]
$100\sigma_g$	0.22	[0.20, 0.25]	0.19	[0.22, 0.26]
$100\sigma_z$	0.48	[0.31, 0.69]	0.24	[0.21, 0.27]
$100\sigma_p$	0.22	[0.20, 0.25]	0.53	[0.29, 0.78]
$100\sigma_w$	0.57	[0.29, 1.17]	0.42	[0.25, 0.77]
$100\sigma_\chi$	4.93	[4.55, 5.33]	2.80	[2.57, 4.03]
$100\sigma_d$	1.41	[1.20, 1.67]	1.12	[1.00, 1.43]
Log marginal likelihood	-1634.9		-1627.2	

Figure 1: Wage inflation and unemployment rate (time-series)

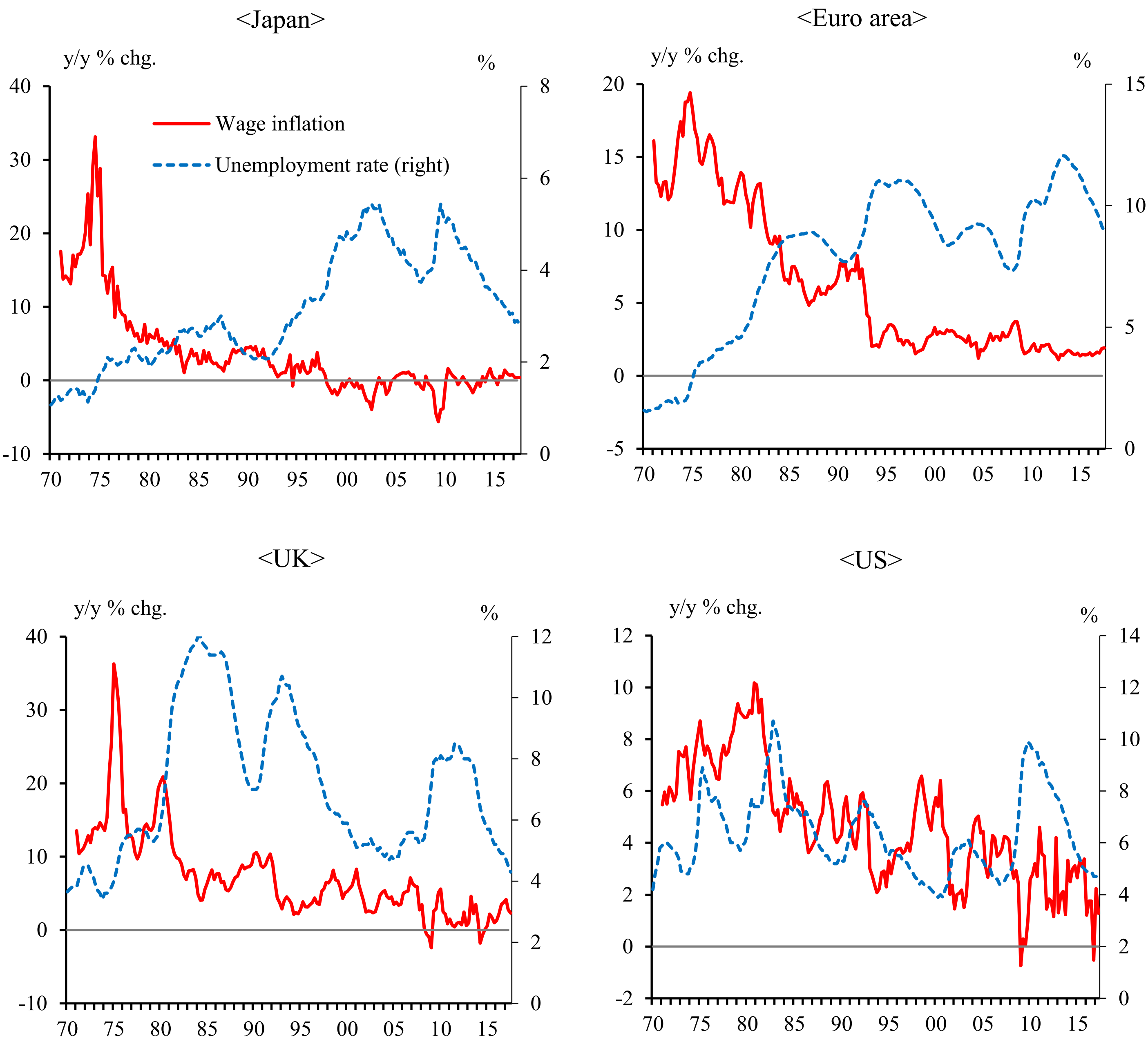


Figure 2: Wage inflation and unemployment rate (scatter plot)

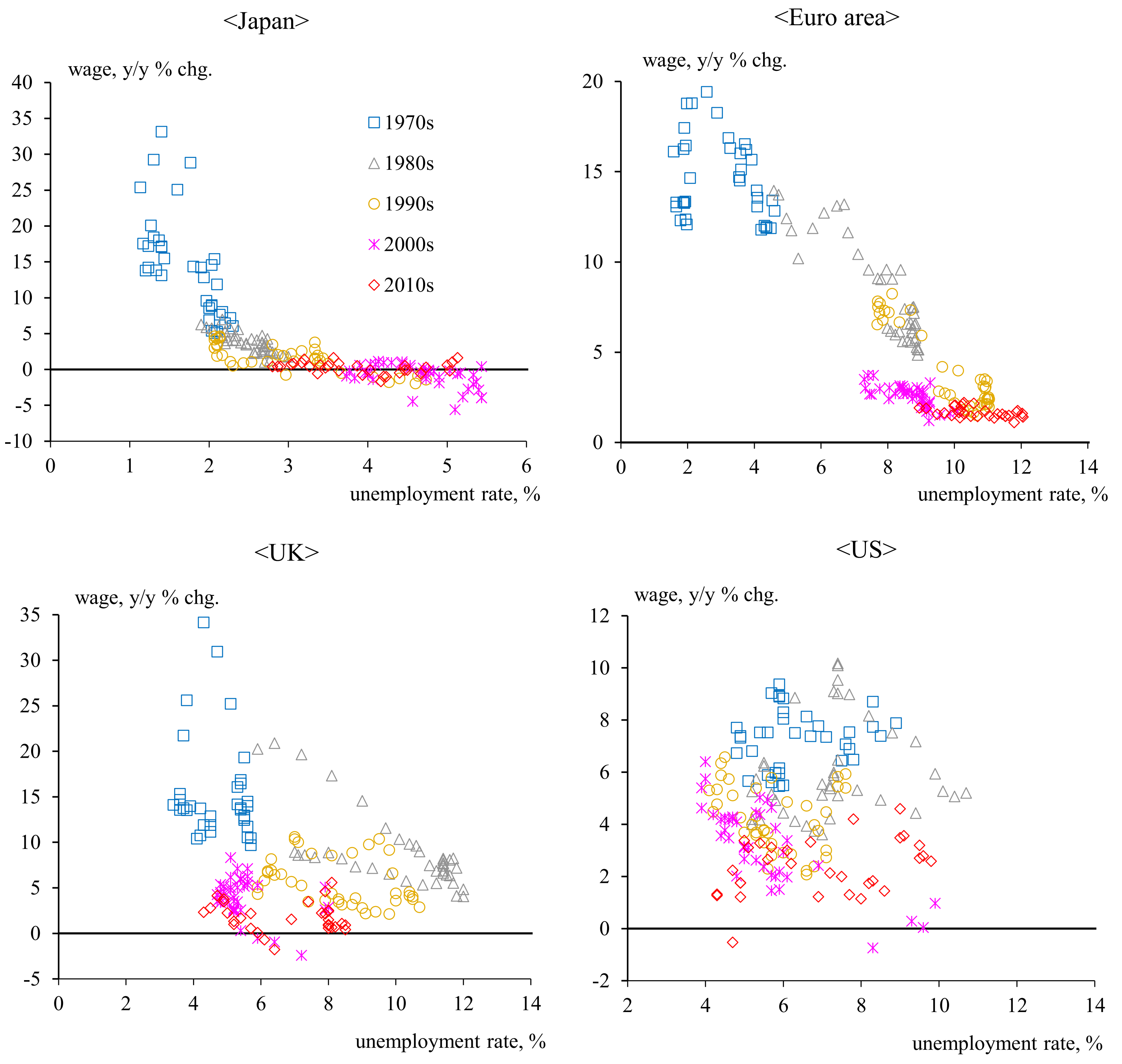
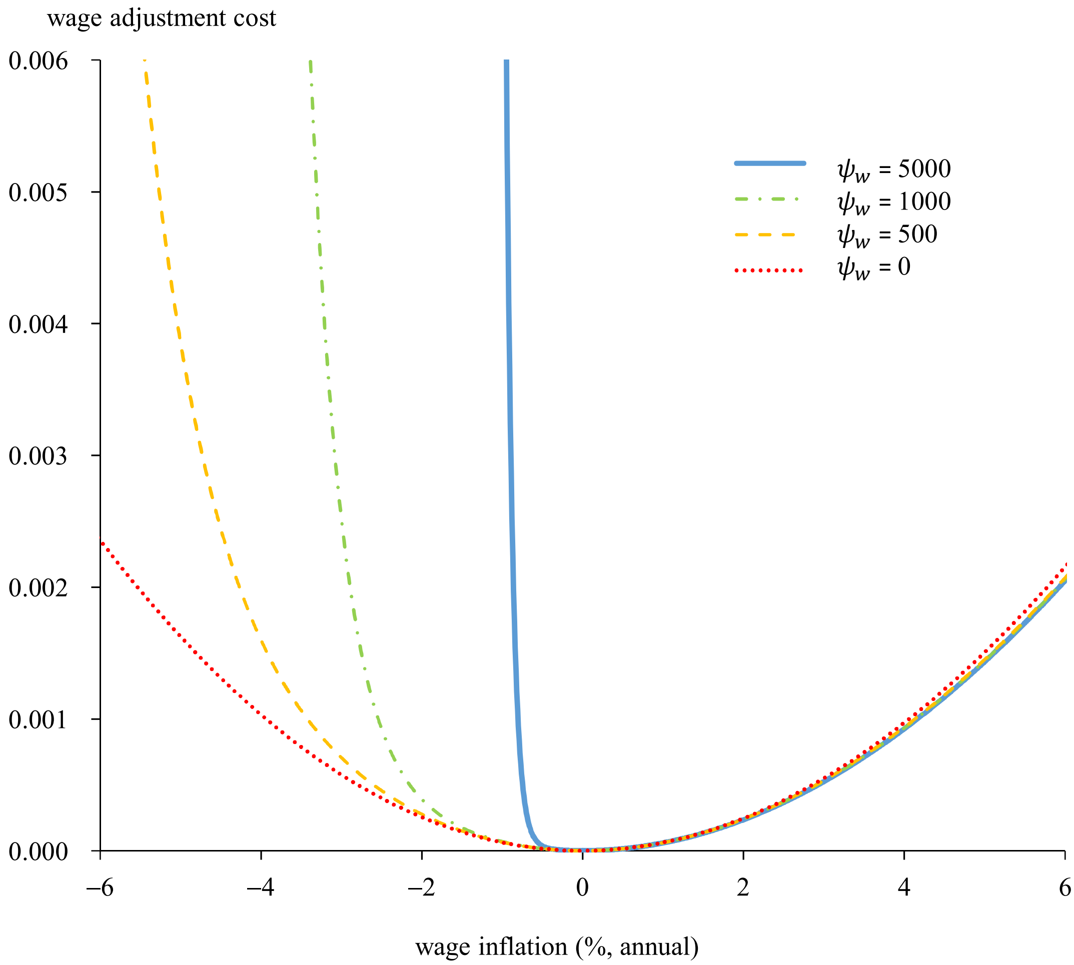


Figure 3: Wage adjustment cost function



Notes: ϕ_w is 20. The horizontal axis depicts deviations from steady-state wage inflation.

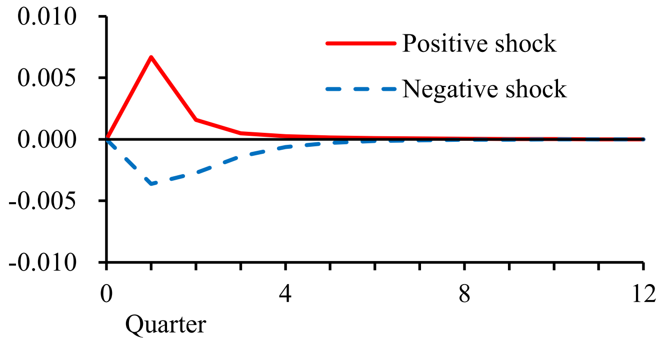
Figure 4: Impulse responses of wage inflation and unemployment

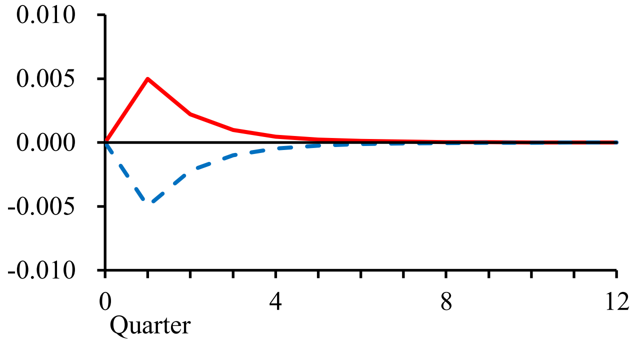
Productivity shock

<Asymmetric model>

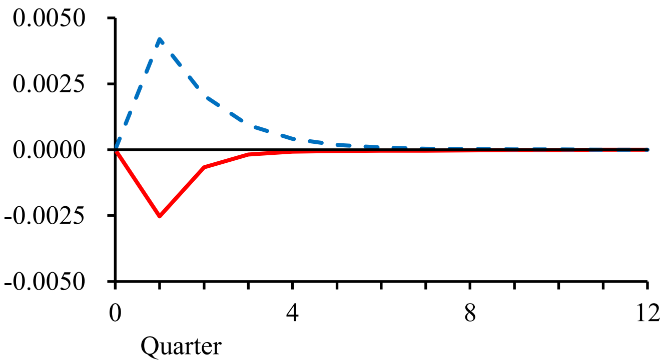
<Symmetric model>

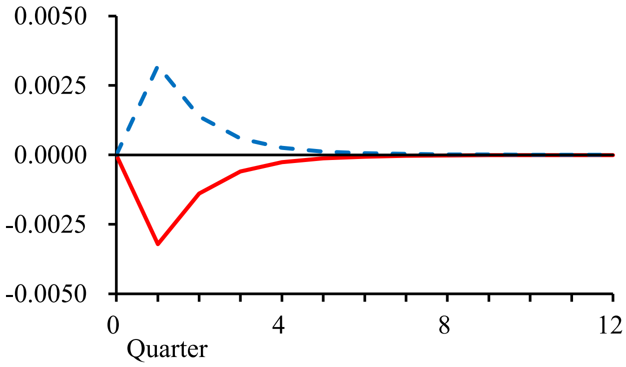
Wage inflation





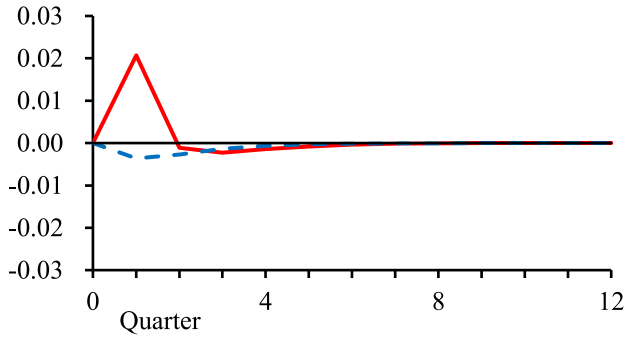
Unemployment rate

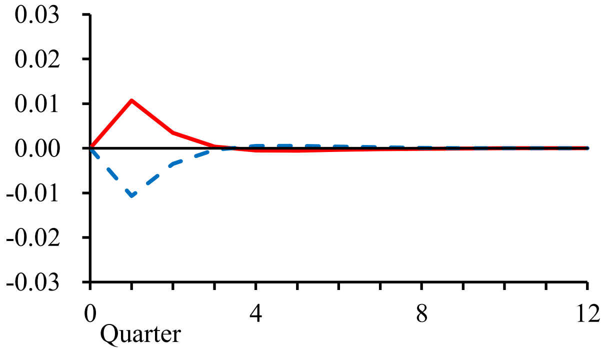




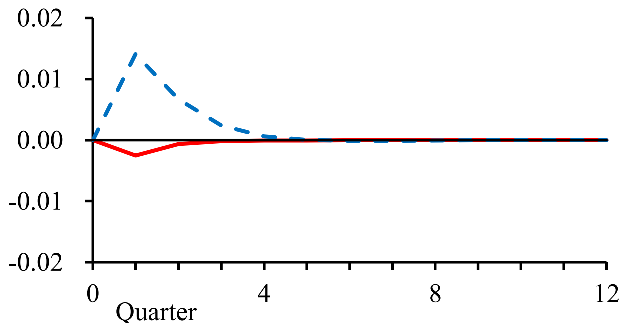
Monetary policy shock

Wage inflation





Unemployment rate



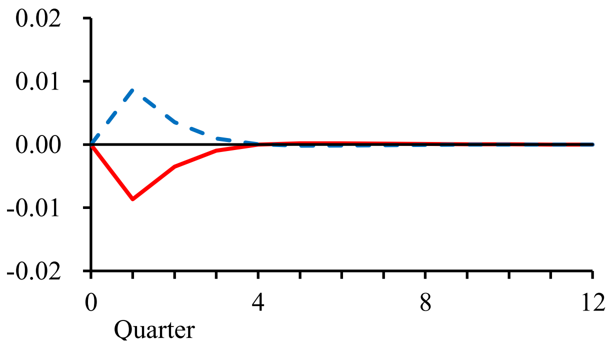


Figure 5: Wage adjustment cost and the wage Phillips curve

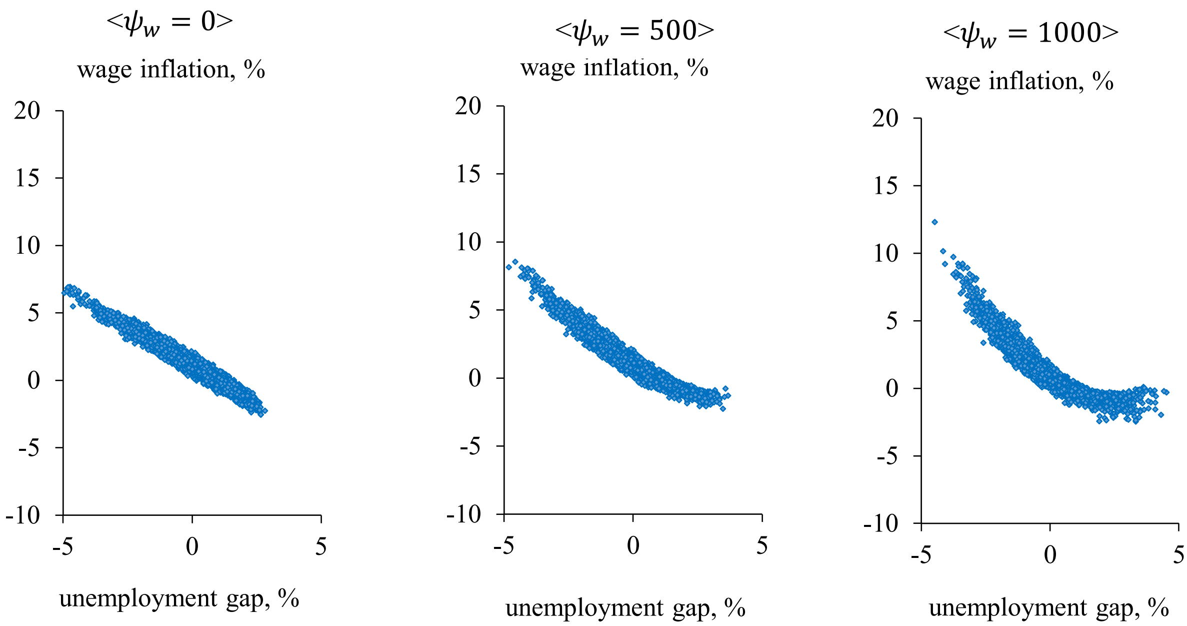
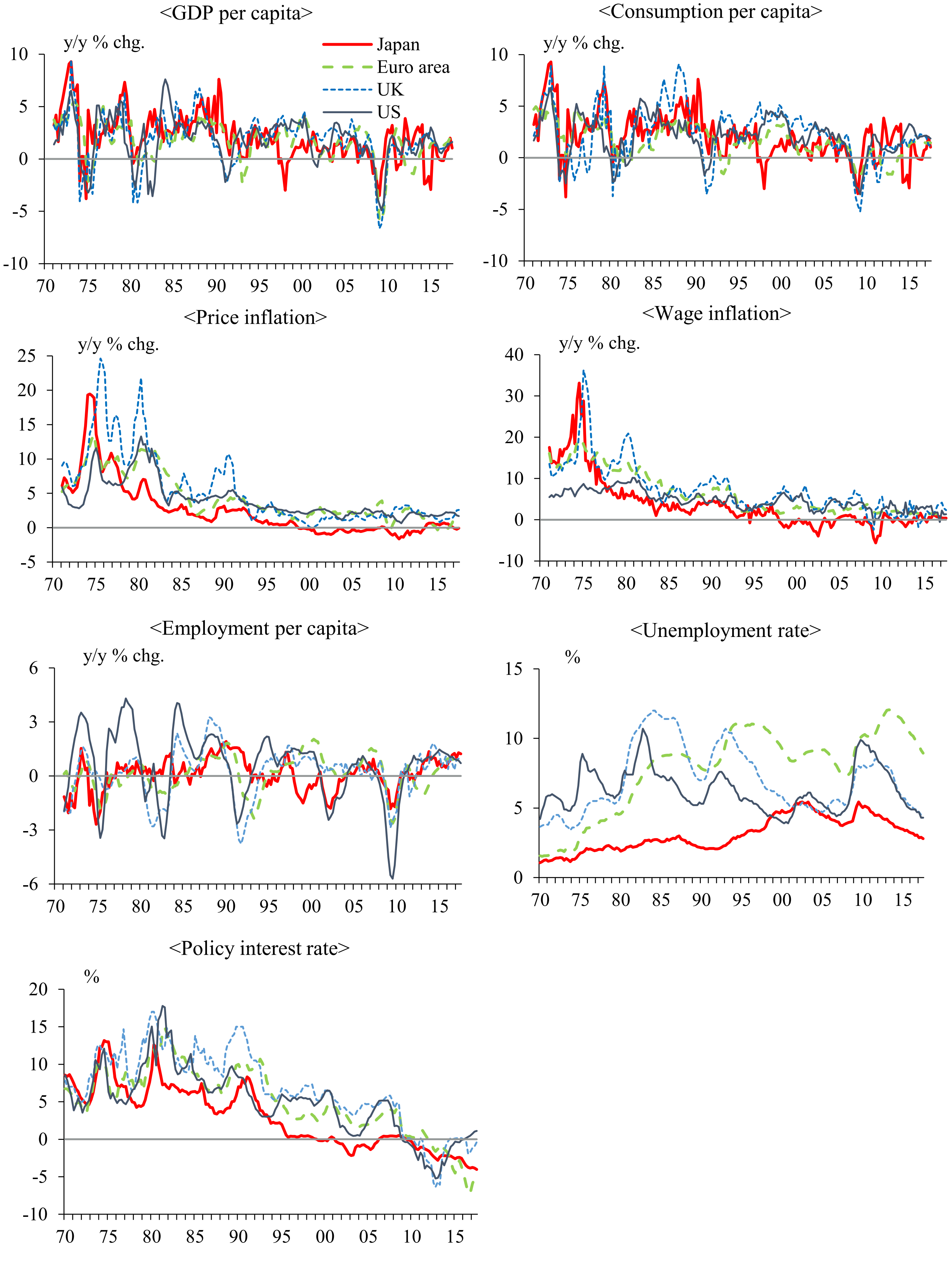
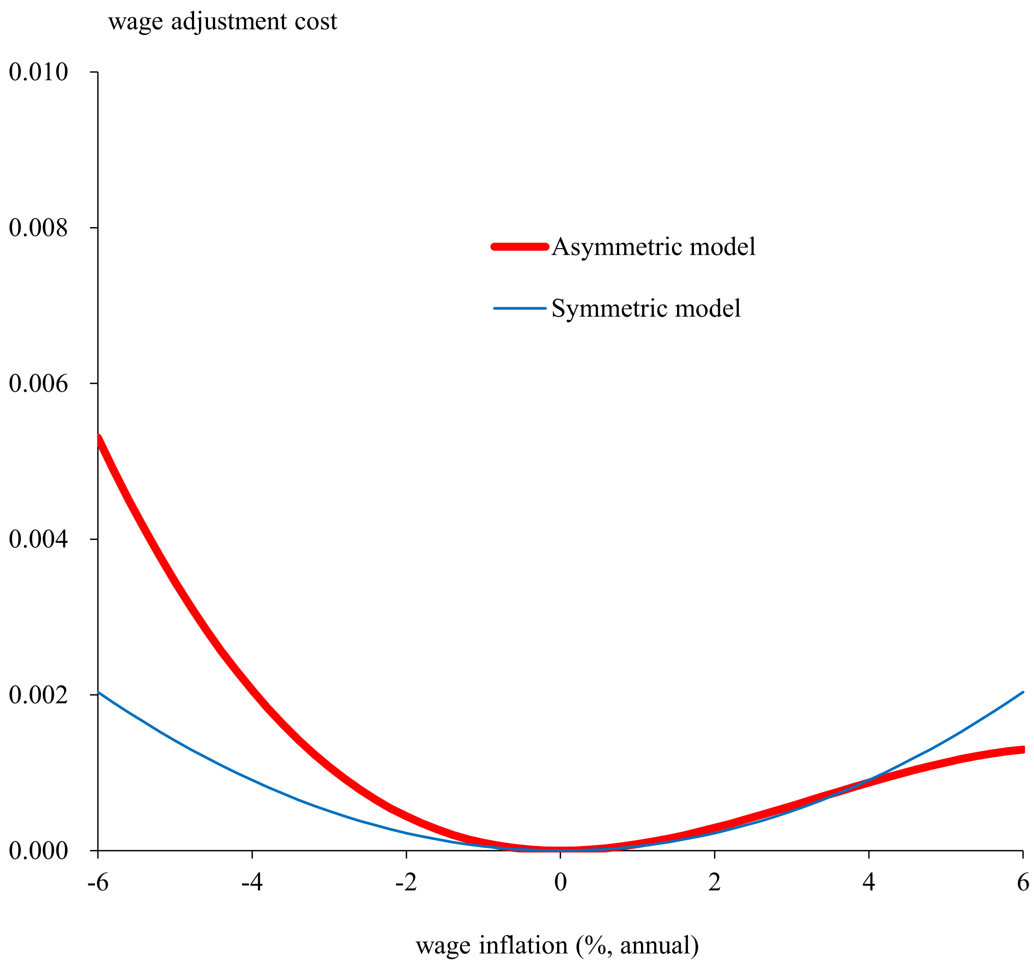


Figure 6: Data used for estimation



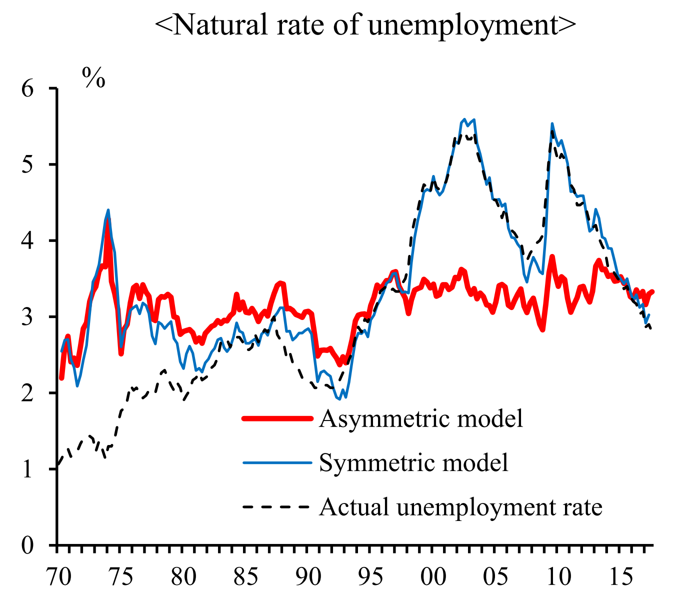
Note: All of the series other than the unemployment rate and the policy interest rate are displayed on a year-on-year basis. However, we use a quarter-on-quarter change rather than a year-on-year change of a variable in our estimation.

Figure 7: Wage adjustment cost function for Japan



Notes: The horizontal axis depicts deviations from steady-state wage inflation.

Figure 8: Natural rate of unemployment in Japan



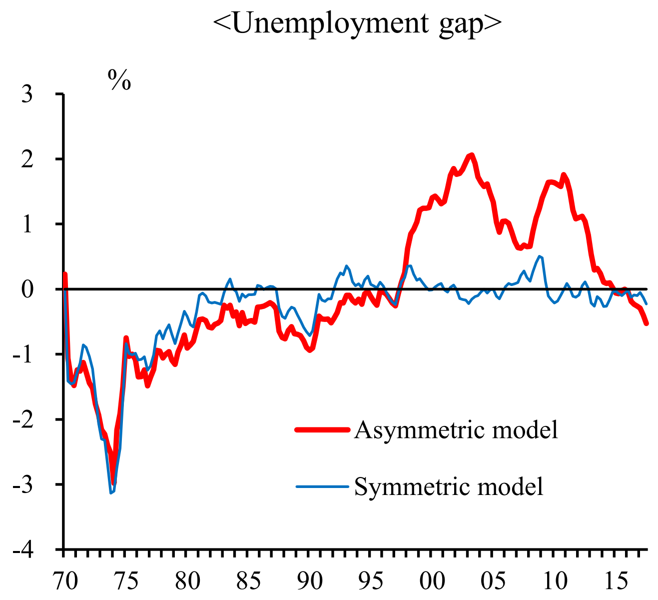
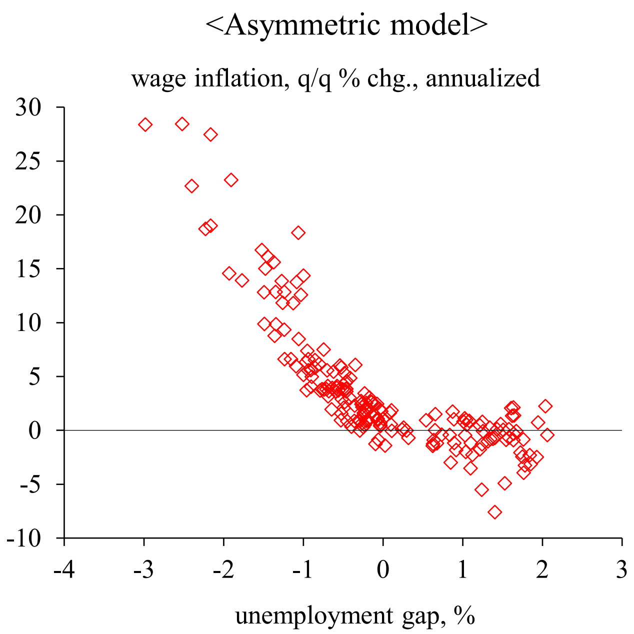


Figure 9: Wage Phillips curve for Japan



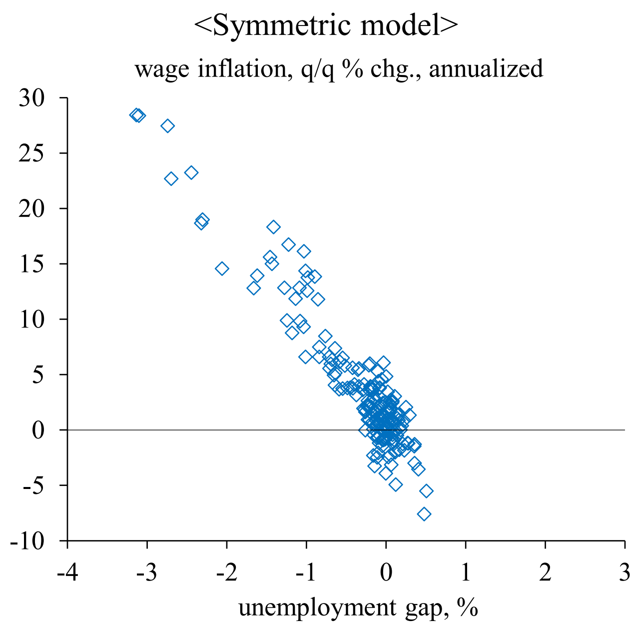
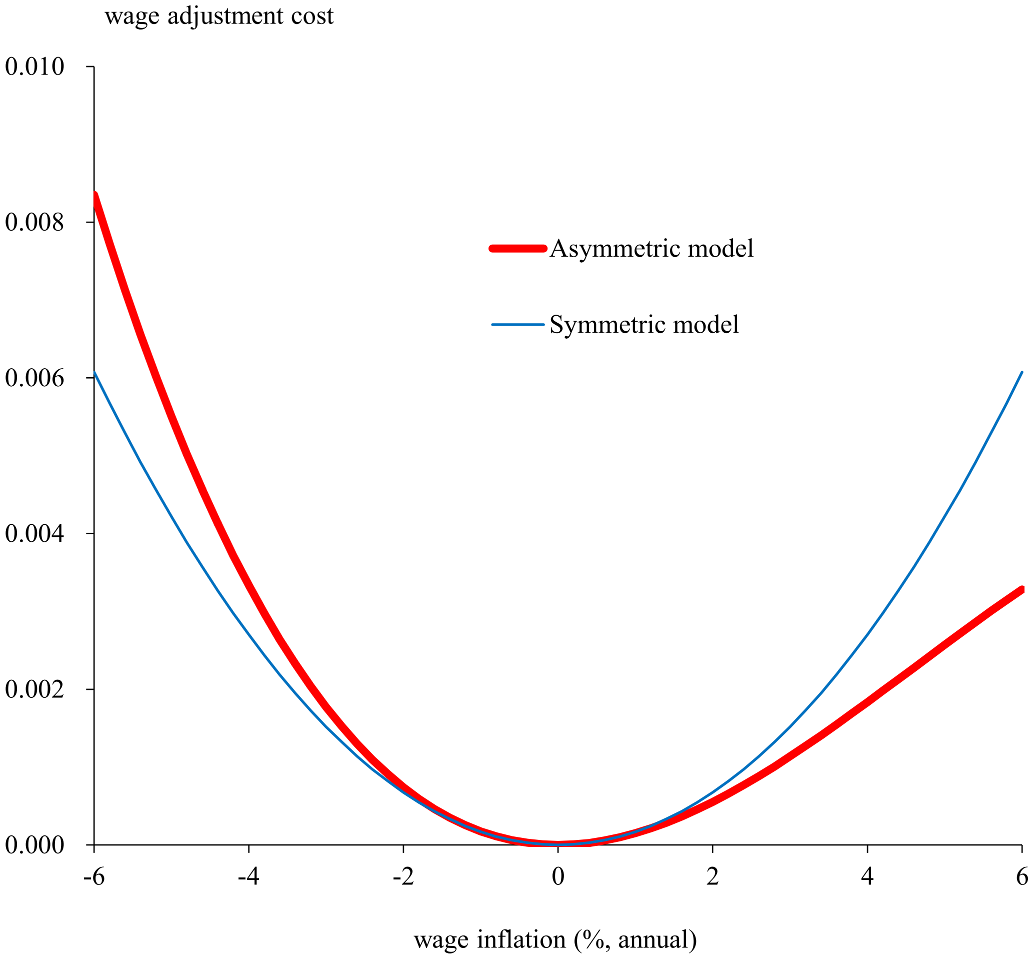


Figure 10: Wage adjustment cost function for the euro area



Notes: The horizontal axis depicts deviations from steady-state wage inflation.

Figure 11: Natural rate of unemployment in the euro area

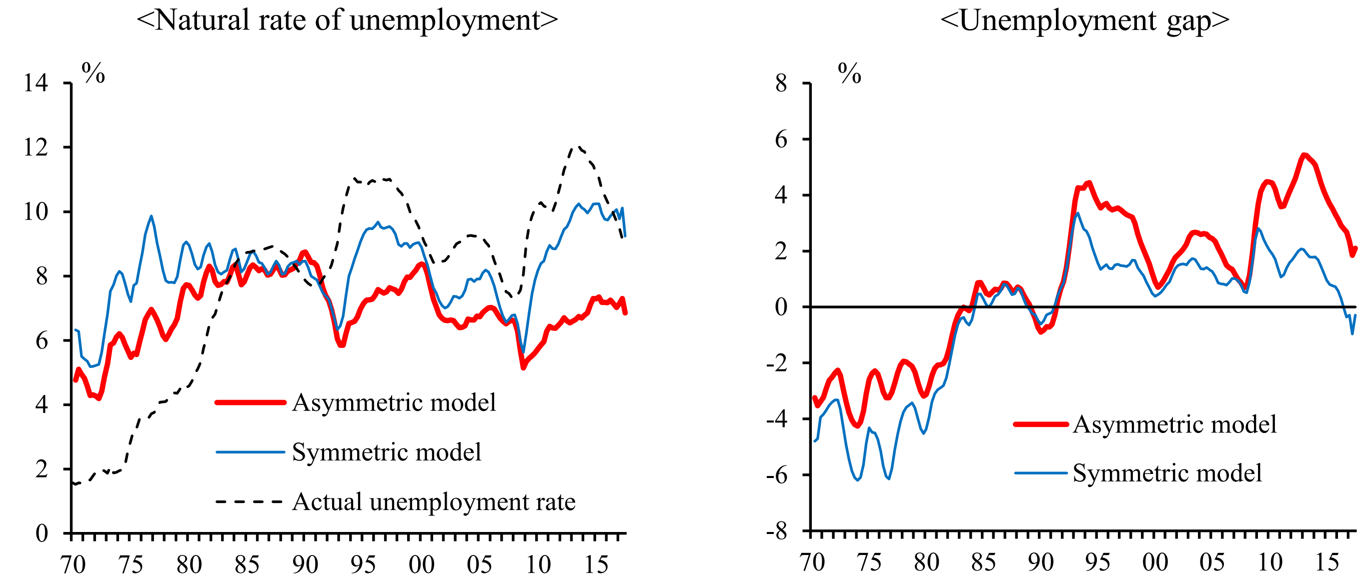


Figure 12: Wage Phillips curve for the euro area

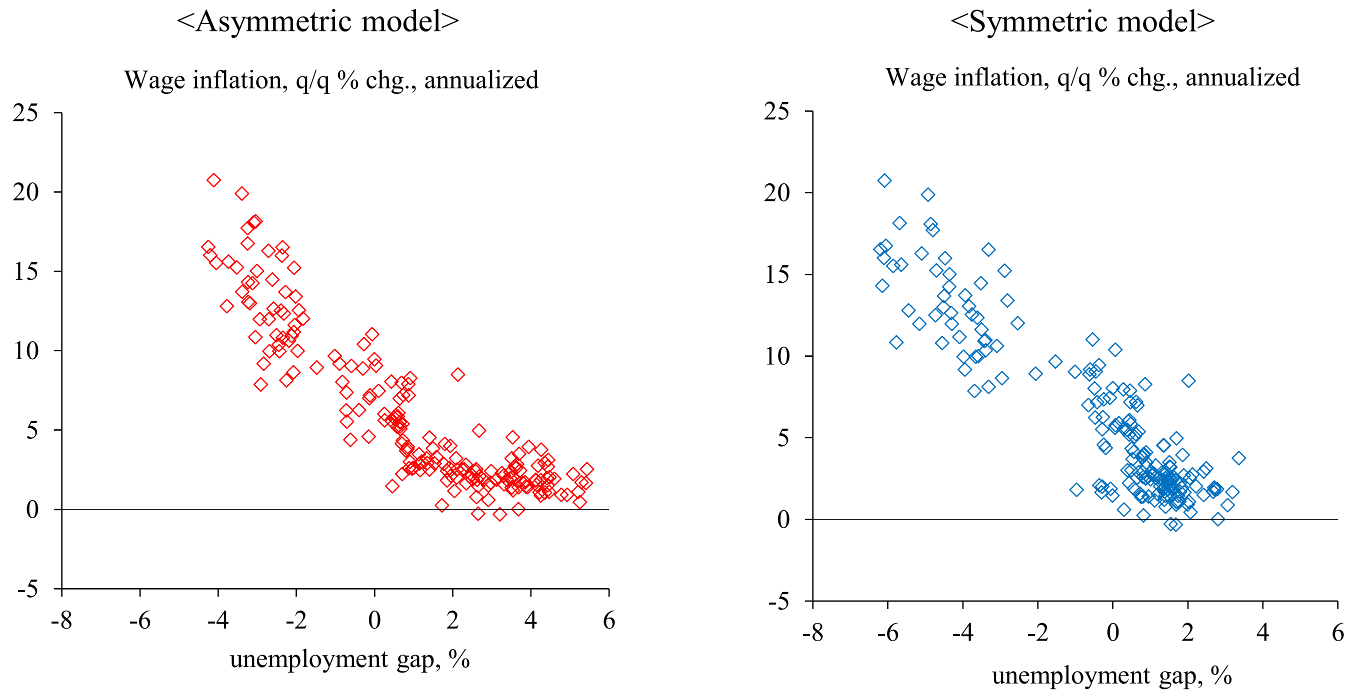
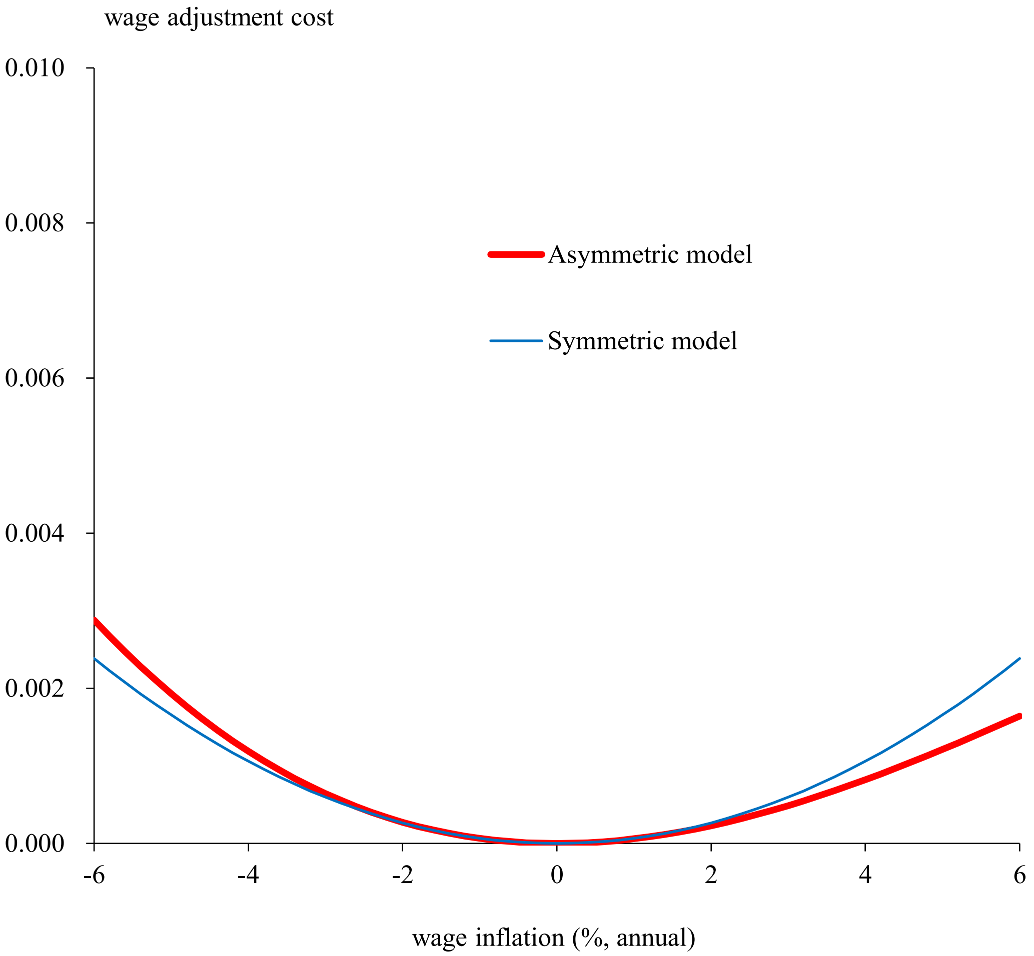


Figure 13: Wage adjustment cost function for the UK



Notes: The horizontal axis depicts deviations from steady-state wage inflation.

Figure 14: Natural rate of unemployment in the UK

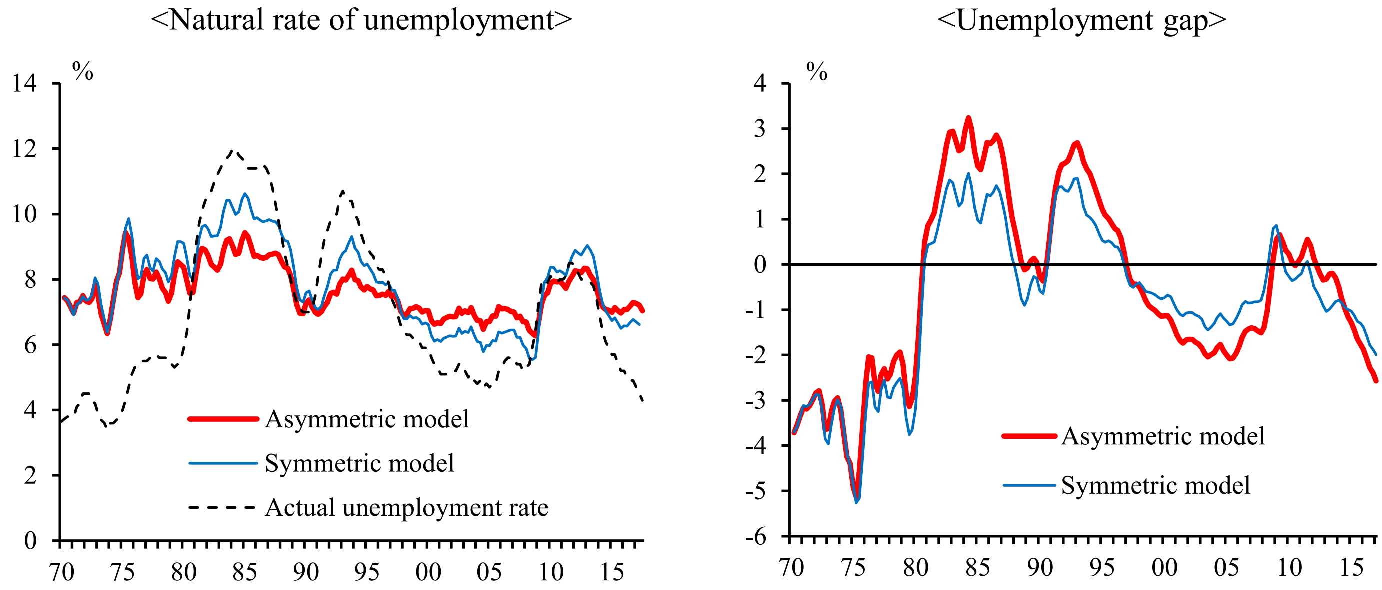


Figure 15: Wage Phillips curve for the UK

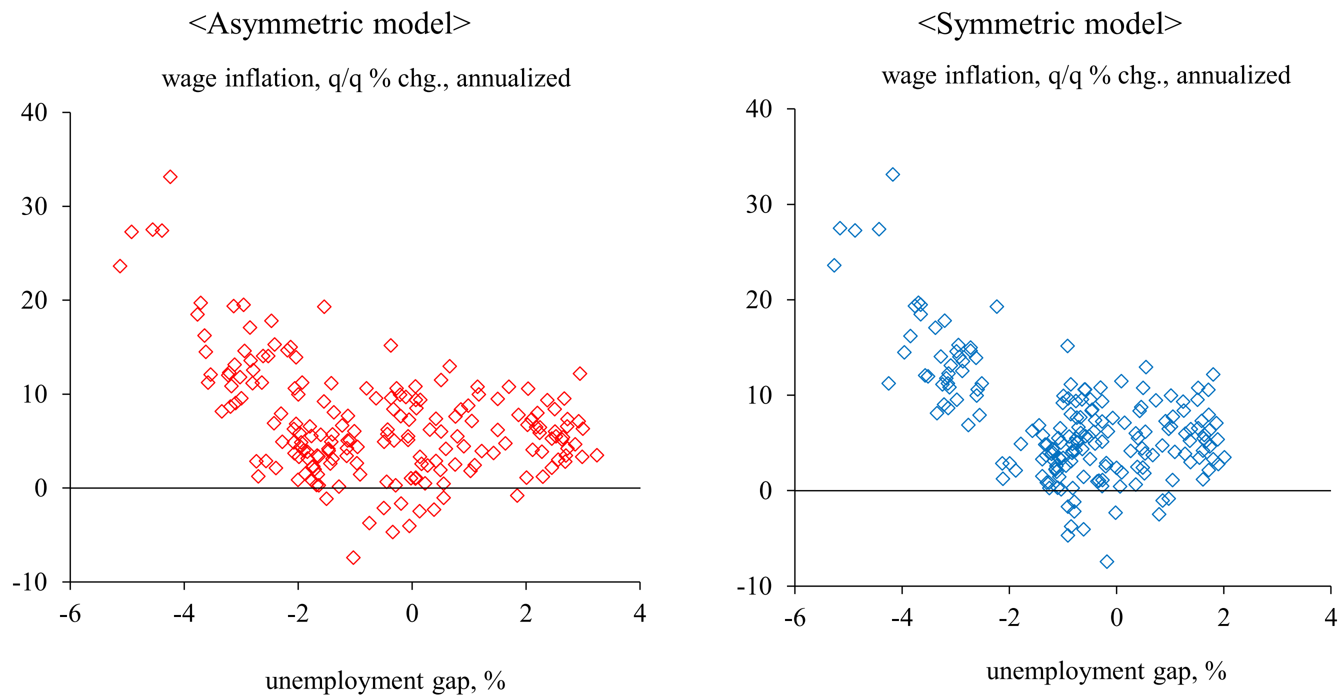
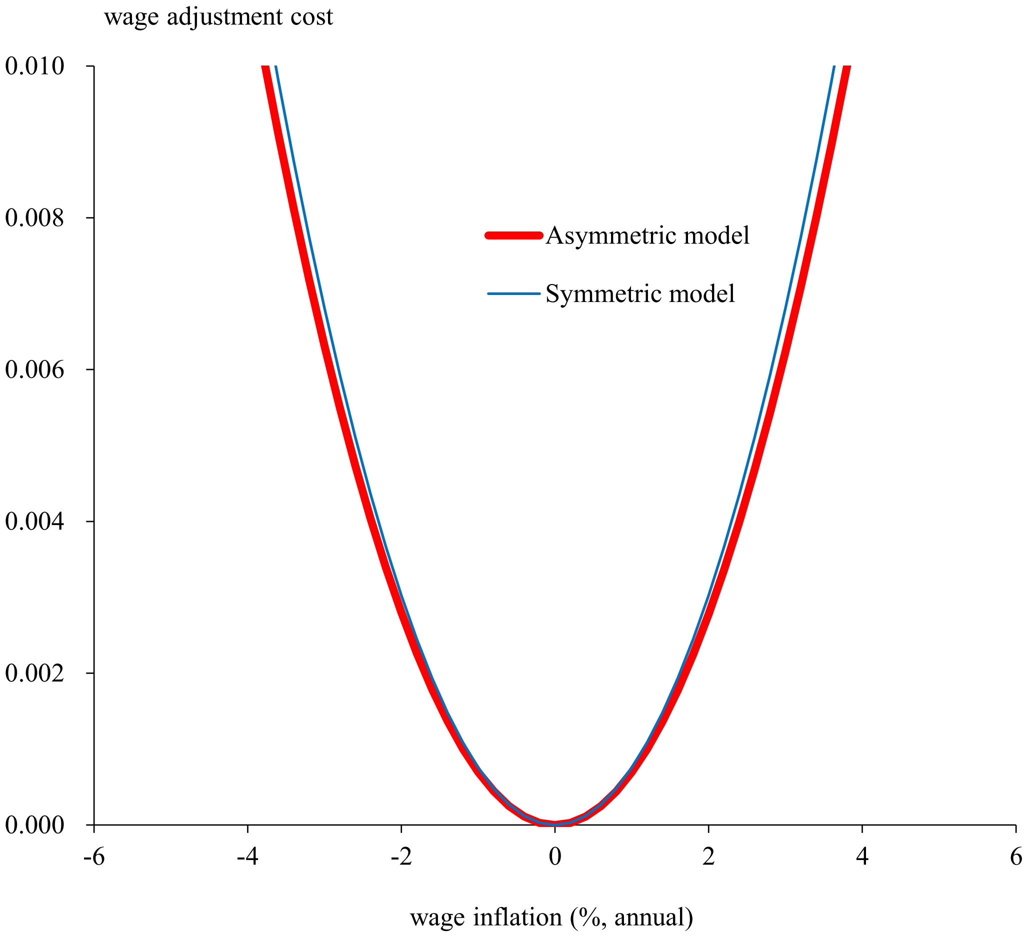


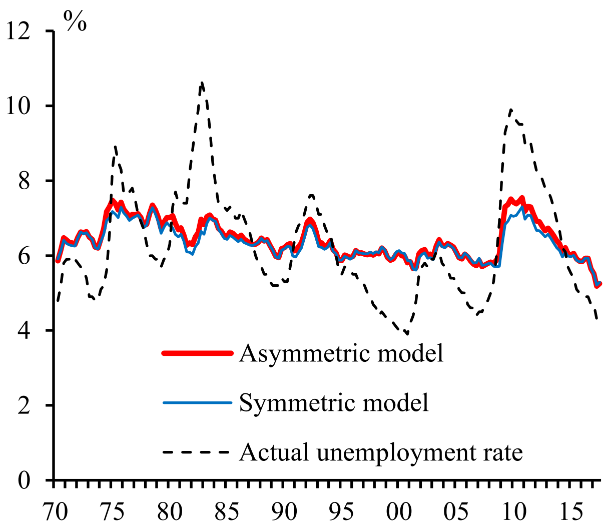
Figure 16: Wage adjustment cost function for the US



Notes: The horizontal axis depicts deviations from steady-state wage inflation.

Figure 17: Natural rate of unemployment in the US

<Natural rate of unemployment>



<Unemployment gap>

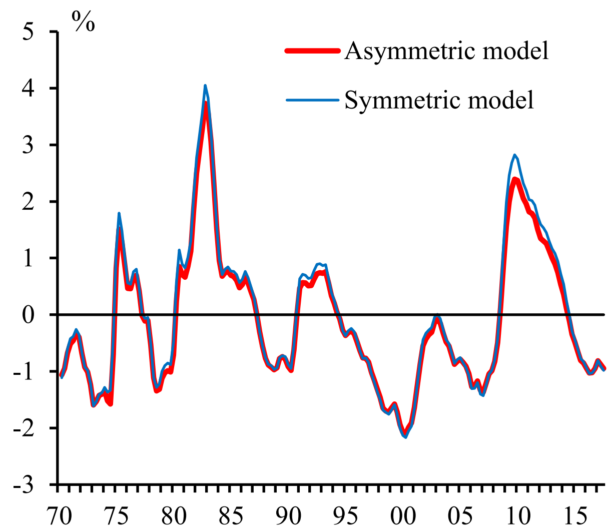
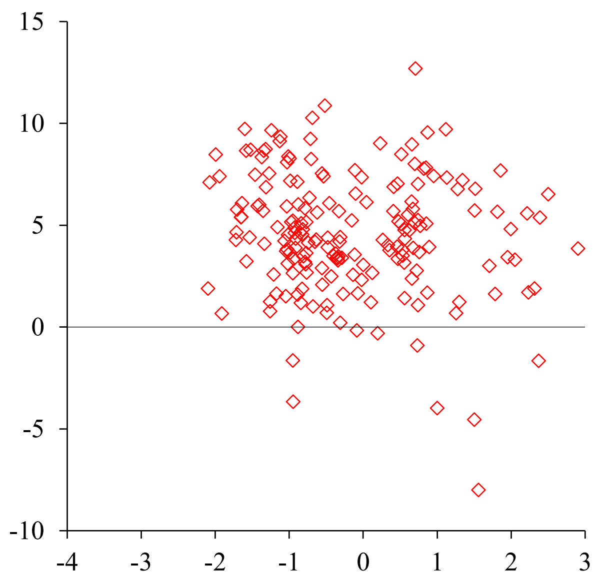


Figure 18: Wage Phillips curve for the US

<Asymmetric model>



<Symmetric model>

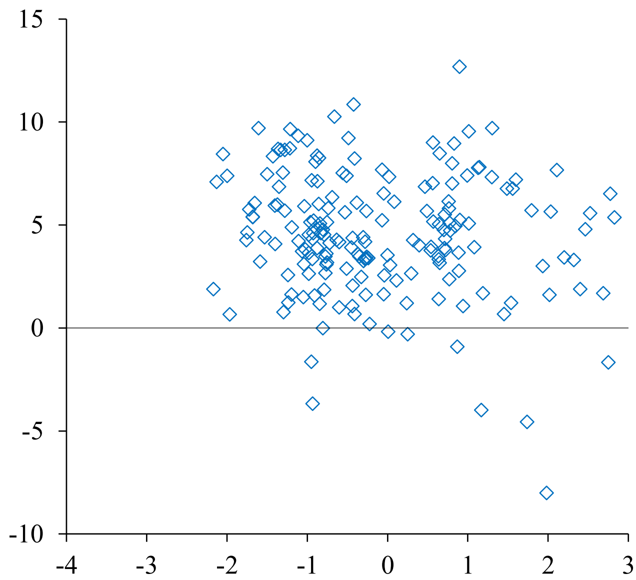
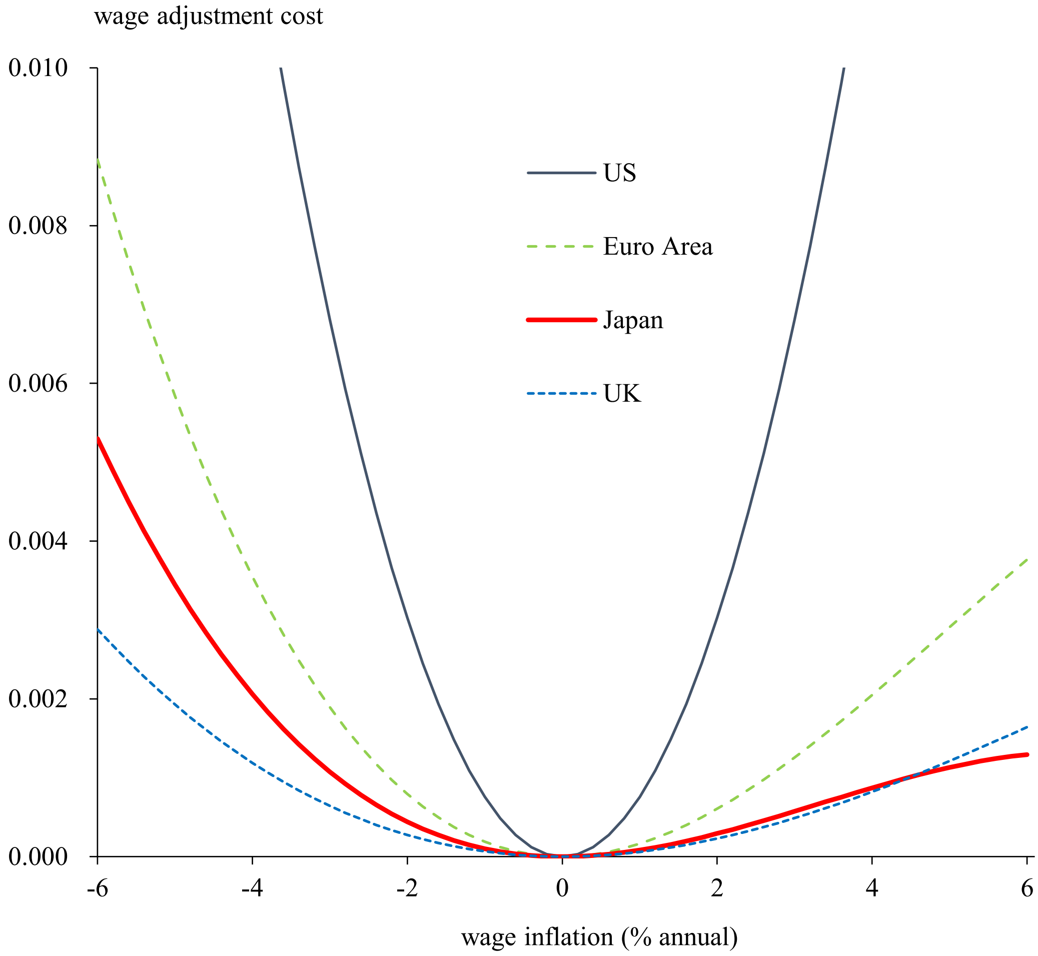


Figure 19: Comparison of wage adjustment cost functions



Notes: The horizontal axis depicts deviations from steady-state wage inflation.

Figure 20: Recent development of wage inflation and unemployment gap

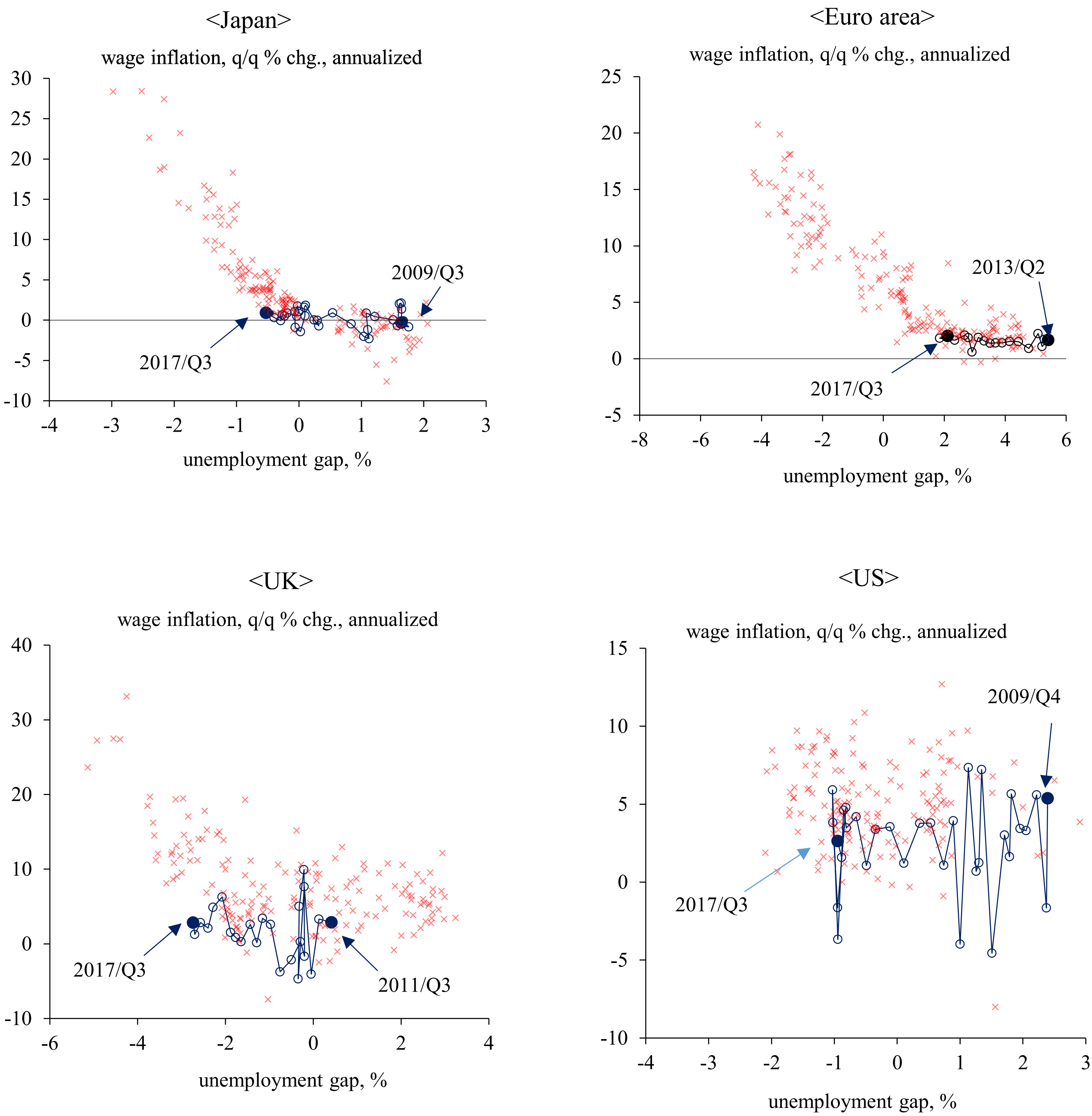
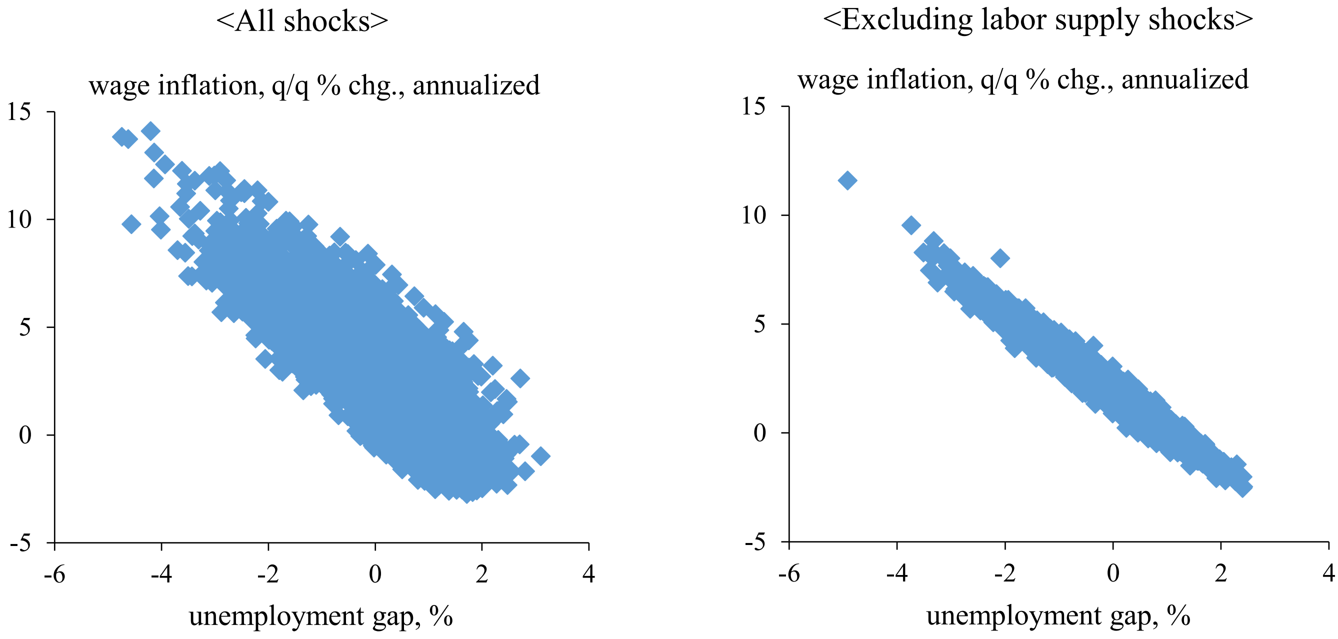


Figure 21: Stochastic simulation of wage Phillips curve for the US



Notes: To obtain simulated series, we use the posterior mean for the symmetric model in Table 6 and randomly generated shocks.

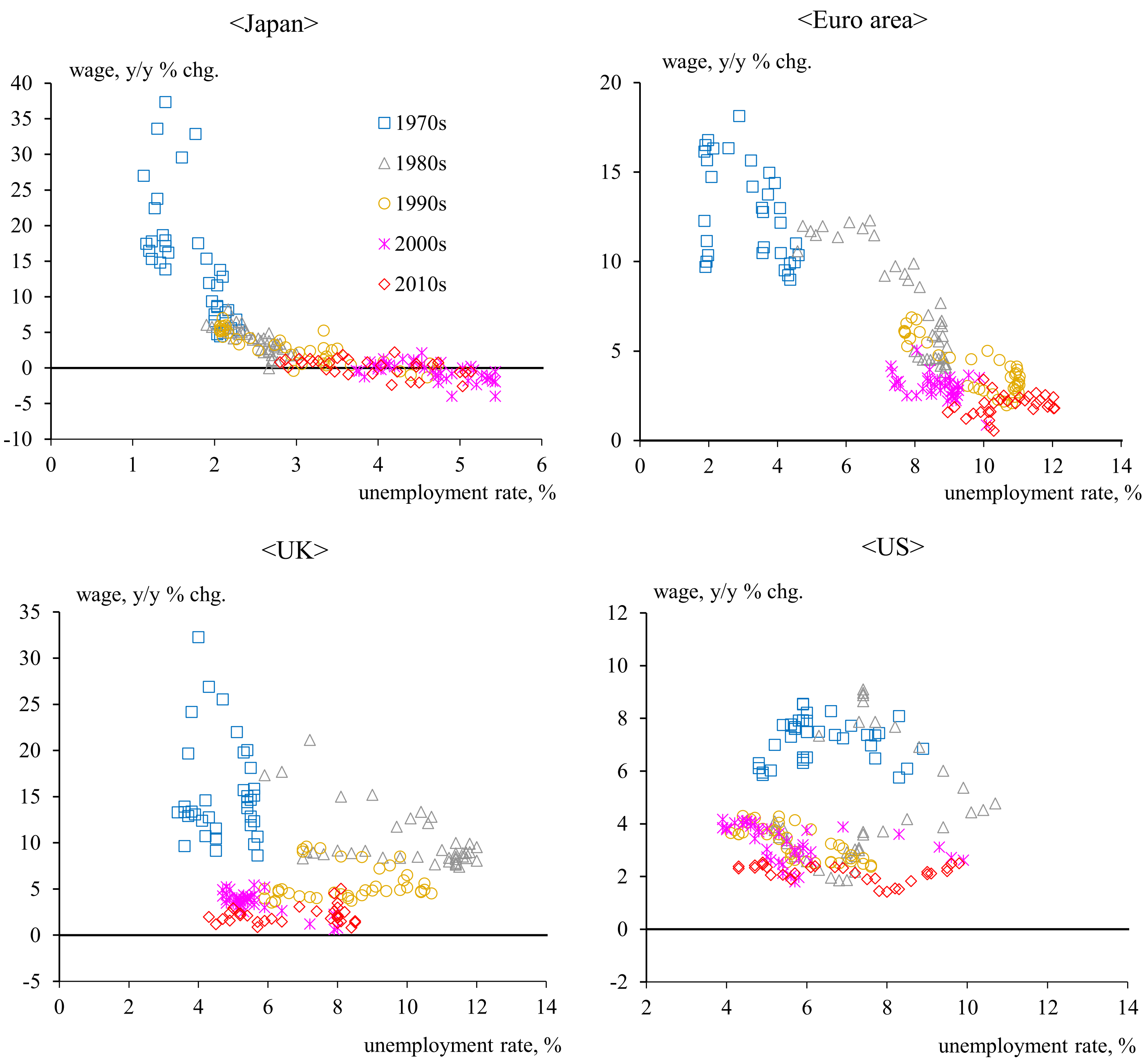
Appendix Table 1: Posterior distributions of the parameters for the US

	Posteriors			
	Full-sample (1970Q2-2017Q3)		Sub-sample (1970Q-2007Q4)	
	Mean	90% interval	Mean	90% interval
τ	3.64	[3.07, 4.01]	3.08	[2.70, 3.72]
ν	0.10	[0.09, 0.12]	0.24	[0.20, 0.28]
ϕ_p	73.8	[67.4, 79.9]	82.9	[79.2, 87.7]
ϕ_w	225	[219, 229]	212	[206, 222]
ψ_w	579	[99, 845]	-60.3	[-117, 19.7]
ρ_r	0.77	[0.74, 0.79]	0.71	[0.67, 0.76]
ψ_1	1.08	[1.05, 1.11]	1.06	[1.04, 1.08]
ψ_2	0.98	[0.74, 0.79]	0.40	[0.20, 0.60]
$\bar{\lambda}_p$	0.12	[0.07, 0.17]	0.08	[0.06, 0.12]
$\bar{\lambda}_w$	0.50	[0.45, 0.55]	0.25	[0.22, 0.30]
γ	0.71	[0.53, 1.04]	0.96	[0.97, 1.23]
ρ_g	0.98	[0.97, 0.98]	0.97	[0.97, 0.98]
ρ_z	0.53	[0.44, 0.63]	0.41	[0.30, 0.53]
ρ_p	0.41	[0.19, 0.71]	0.50	[0.12, 0.84]
ρ_w	0.44	[0.17, 0.98]	0.41	[0.14, 0.61]
ρ_d	0.81	[0.78, 0.84]	0.80	[0.79, 0.83]
$100\sigma_r$	0.34	[0.28, 0.38]	0.37	[0.32, 0.43]
$100\sigma_g$	0.22	[0.20, 0.25]	0.24	[0.22, 0.27]
$100\sigma_z$	0.48	[0.31, 0.69]	0.28	[0.25, 0.32]
$100\sigma_p$	0.22	[0.20, 0.25]	0.67	[0.36, 1.23]
$100\sigma_w$	0.57	[0.29, 1.17]	0.33	[0.28, 0.37]
$100\sigma_\chi$	4.93	[4.55, 5.33]	2.41	[1.90, 3.01]
$100\sigma_d$	1.41	[1.20, 1.67]	0.96	[0.79, 1.23]

Appendix Table 2: Posterior distributions of the parameters for Japan

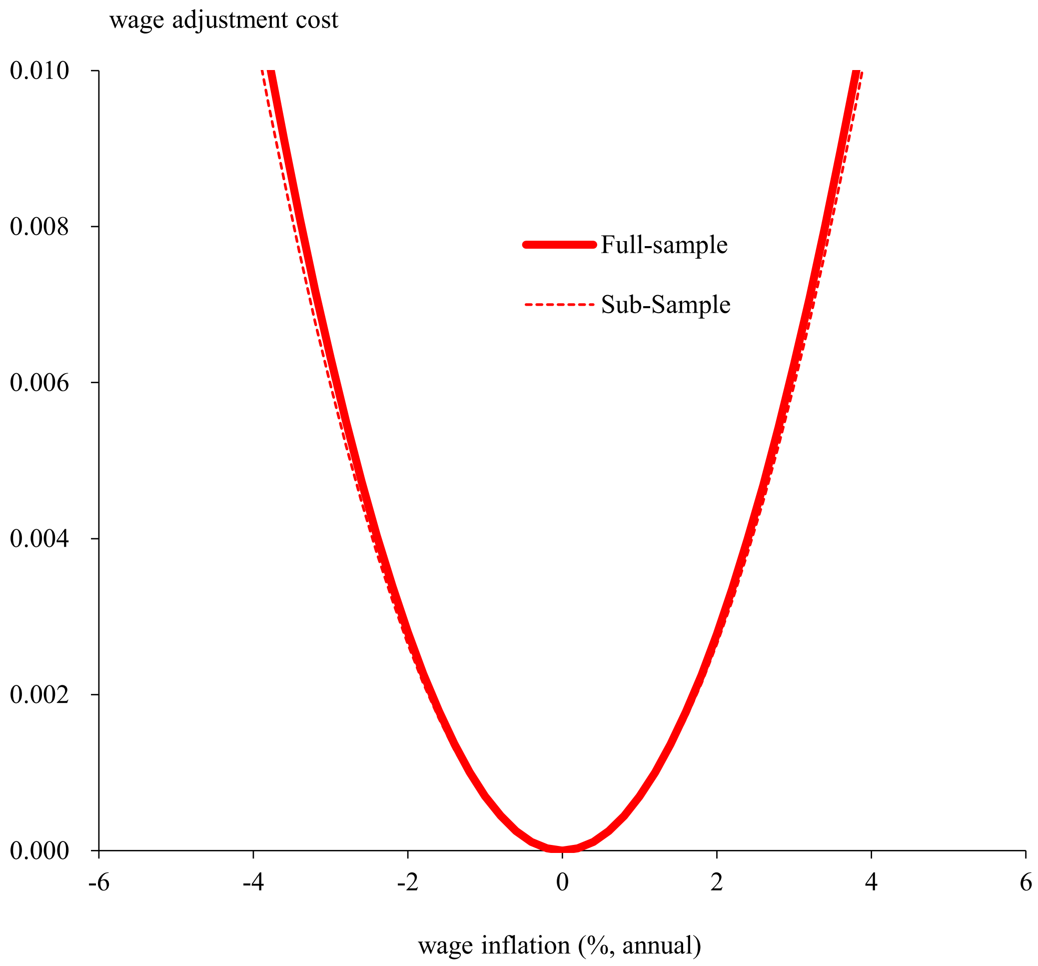
	Posteriors			
	Using shadow rate		Using call rate	
	Mean	90% interval	Mean	90% interval
τ	5.87	[5.02, 6.81]	5.57	[4.71, 6.00]
ν	0.20	[0.18, 0.22]	0.18	[0.16, 0.20]
ϕ_p	12.4	[6.89, 18.6]	29.5	[22.5, 35.3]
ϕ_w	29.4	[22.5, 34.2]	23.7	[16.5, 30.3]
ψ_w	3560	[2367, 4609]	2554	[1240,3567]
ρ_r	0.89	[0.87, 0.91]	0.86	[0.84, 0.89]
ψ_1	2.05	[1.94, 2.18]	1.86	[1.76, 1.97]
ψ_2	0.82	[0.29, 1.40]	0.79	[0.58, 0.97]
$\bar{\lambda}_p$	0.13	[0.06, 0.20]	0.13	[0.08, 0.17]
$\bar{\lambda}_w$	0.41	[0.36, 0.47]	0.38	[0.31, 0.46]
γ	2.05	[1.94, 2.18]	2.06	[1.91, 2.18]
ρ_g	0.98	[0.97, 0.99]	0.99	[0.99, 0.99]
ρ_z	0.75	[0.66, 0.83]	0.72	[0.67, 0.76]
ρ_p	0.41	[0.13, 0.81]	0.47	[0.13, 0.74]
ρ_w	0.71	[0.70, 0.72]	0.71	[0.69, 0.73]
ρ_d	0.96	[0.96, 0.97]	0.97	[0.96, 0.98]
$100\sigma_r$	0.27	[0.24, 0.30]	0.28	[0.24, 0.31]
$100\sigma_g$	0.21	[0.15, 0.26]	0.27	[0.19, 0.30]
$100\sigma_z$	0.26	[0.19, 0.32]	0.33	[0.30, 0.36]
$100\sigma_p$	0.52	[0.27, 0.83]	0.52	[0.28, 0.84]
$100\sigma_w$	0.54	[0.27, 0.96]	0.95	[0.27, 3.51]
$100\sigma_\chi$	0.50	[0.26, 0.98]	0.48	[0.26, 0.96]
$100\sigma_d$	3.57	[3.10, 4.10]	3.23	[2.77, 3.68]

Appendix Figure 1: Hourly wage growth and unemployment rate (scatter plot)



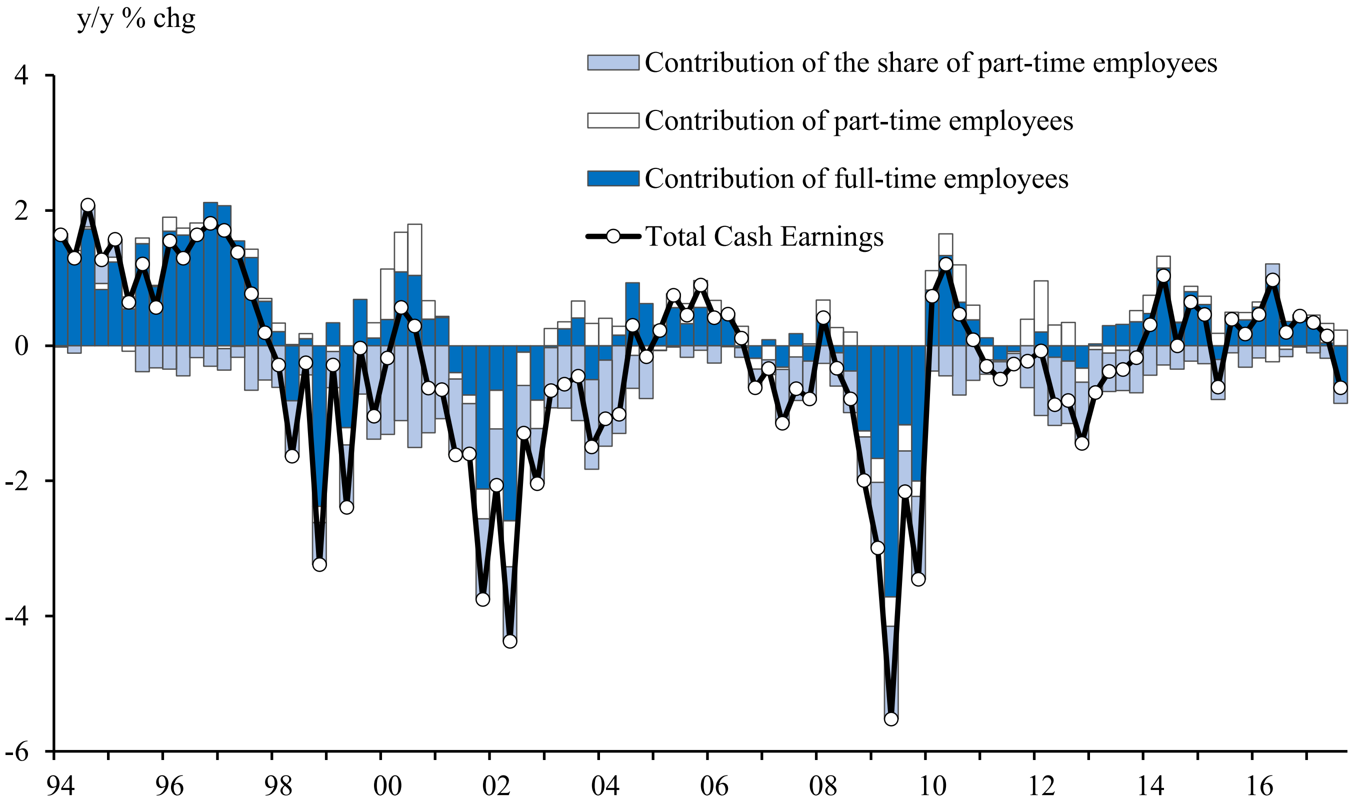
Notes:Figures for Japan and US are those for the overall sectors. Figures for Euro area and UK are those for the maufacturing sector.
Sources: Ministry of Health, Labour and Welfare; OECD

Appendix Figure 2: Wage adjustment cost function in the US (Full-sample vs Sub-sample)



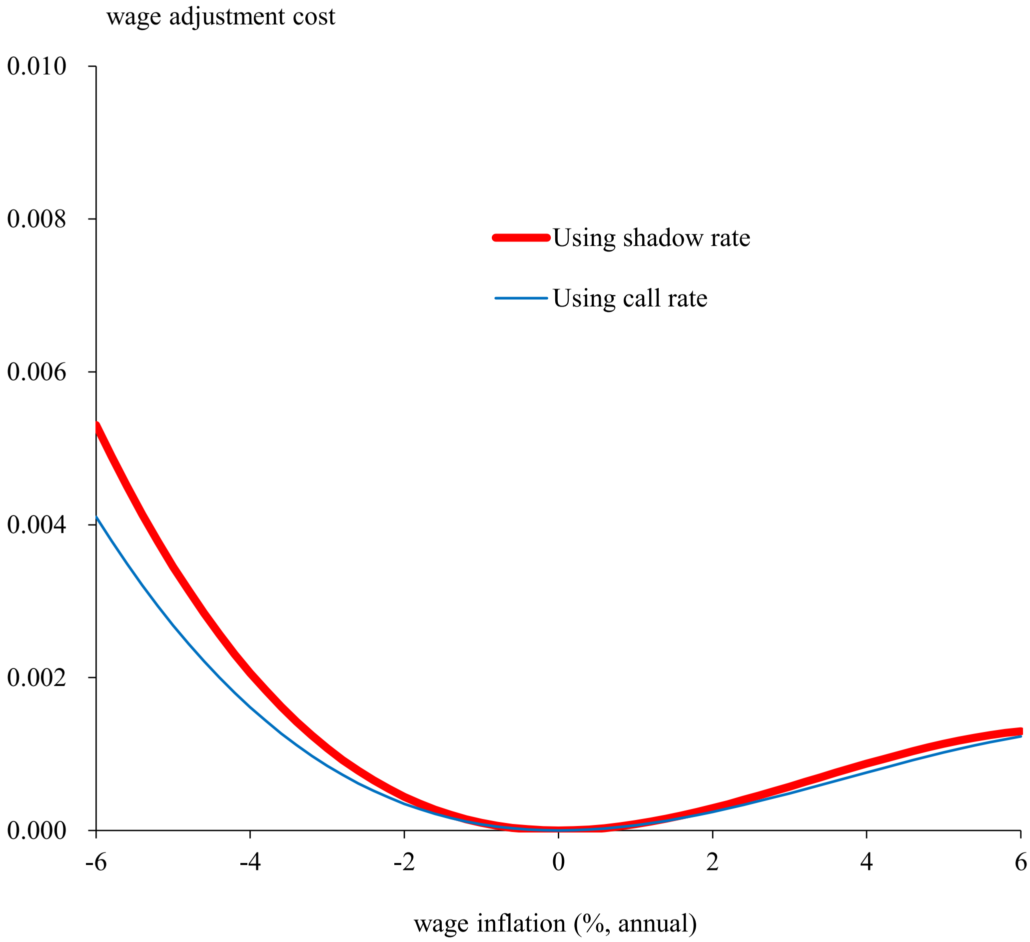
Notes: The horizontal axis depicts deviations from steady-state wage inflation.

Appendix Figure 3: Decomposition of Japan's wage inflation (full-time vs part-time workers)



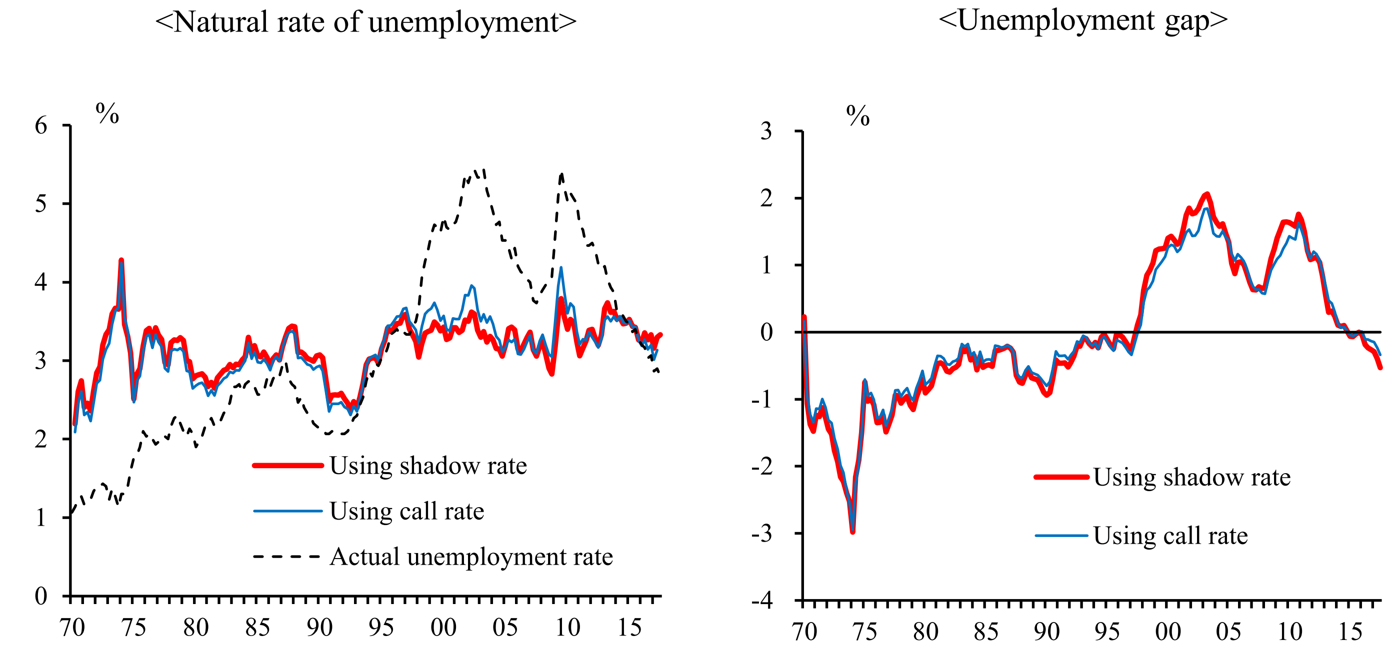
Source: Ministry of Health, Labour and Welfare.
Note: Q1 = March-May, Q2 = June-August, Q3 = September-November, Q4 = December-February.

Appedix Figure 4: Wage adjustment cost function for the Japan (call rate)



Notes: The horizontal axis depicts deviations from steady-state wage inflation.

Appendix Figure 5: Natural rate of unemployment in Japan (call rate)



Appendix Figure 6: Wage Phillips curve for Japan (call rate)