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Discussion Paper No. 2018-E-6

IMES

INSTITUTE FOR MONETARY AND ECONOMIC STUDIES

BANK OF JAPAN

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JAPAN

IMES Discussion Paper Series 2018-E-6
June 2018

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Abstract

The zero lower bound (ZLB) constraint on interest rates makes speed limit policies (SLPs)—policies aimed at stabilizing the output growth—less effective. Away from the ZLB, the history dependence induced by a concern for output growth stabilization improves the inflation-output tradeoff for a discretionary central bank. However, in the aftermath of a deep recession with a binding ZLB, a central bank with an objective for output growth stabilization aims to engineer a more gradual increase in output than under the standard discretionary policy. The anticipation of a more restrained recovery exacerbates the declines in inflation and output when the lower bound is binding.

Keywords: Liquidity Traps; Markov-Perfect Equilibrium; Speed Limit Policy; Zero Lower Bound
JEL classification: E52, E61

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This paper was prepared in part while Taisuke Nakata was a visiting scholar at the Institute for Monetary and Economic Studies, Bank of Japan. The authors thank seminar participants at the Bank of Japan and the University of Tokyo for comments and suggestions. The authors also thank Shigenori Shiratsuka, Yoichi Ueno, and an anonymous referee for the IMES Discussion Paper Series for useful suggestions and Katherine Arnold for editorial assistance. Philip Coyle, Philipp Lieberknecht, and Jonathan Yu provided excellent research assistance. The views expressed in this paper are those of the authors and do not necessarily reflect the official views of the Bank of Japan, the Federal Reserve Board of Governors, the Federal Reserve System, or the European Central Bank.

1 Introduction

Economists have long studied the efficacies of various monetary policy strategies. Several studies on the design of monetary policy have emphasized the desirability of central bank objectives that assign a role to the stabilization of output growth. First and foremost, a seminal paper by Walsh (2003) demonstrated that, in standard sticky-price models, a discretionary central bank that is concerned with stabilization of inflation and output growth, rather than output, improves welfare by creating endogenous inertia in the interest rate policy that is akin to the inertia observed under the optimal commitment policy. Others made a similar point by showing that interest rate feedback rules involving an output growth stabilization term can generate economic outcomes similar to those achieved under optimal commitment policy in various sticky-price models (Blake (2012); Giannoni and Woodford (2003); Stracca (2007)). These policies that aim to reduce output growth volatility are known as speed limit policies (SLPs).

In this paper, we revisit the desirability of SLPs in an economy with an occasionally binding zero lower bound (ZLB) constraint on nominal interest rates. Following Walsh (2003), we conduct our analyses in the context of policy delegation where the discretionary central bank's objective function is modified to include an output growth stabilization motive.[1] Our model features both demand and cost-push shocks. In the model without the ZLB constraint, the presence of the cost-push shock makes it desirable to assign a strictly positive weight to the output growth stabilization motive, as shown by Walsh (2003). The presence of the demand shock would have no implication for the desirability of SLPs when we abstract from the ZLB constraint because the demand shock can be fully neutralized by the adjustment in the policy rate regardless of whether or not the central bank pursues a SLP. However, in the model with the ZLB, a sufficiently large negative demand shock forces the central bank to reduce the policy rate to the ZLB, causing inflation and the output gap to decline. In this case, demand disturbances are no longer inconsequential for the assessment of SLPs.

Our main exercise is to compare the desirability of SLPs in the models with and without the ZLB constraint. We begin our analysis with a stylized model in which the cost and benefit of the SLP can be transparently described. We then move on to a richer model calibrated to match key features of the U.S. economy to examine the quantitative relevance of SLP.

Our main finding is that the optimal weight on the output growth stabilization term is smaller in the model with the ZLB constraint than in the model that abstracts from the constraint. The ZLB constraint makes SLPs less desirable because the central bank under a SLP acts in a way that is almost a polar opposite of what the central bank would do under the optimal commitment policy. Under the optimal commitment policy, the central bank

[1] A key insight of the literature on policy delegation is that delegating policy to an agent with a different objective function than society can help to solve time inconsistency problems and improve welfare. Prominent examples in the context of monetary policy include Rogoff (1985), Persson and Tabellini (1993), Walsh (1995), Walsh (2003), and Svensson (1997).

keeps the policy rate low for long with the explicit goal of overshooting inflation and the output gap above their longer-run targets. The overshooting of inflation and the output gap in the future mitigates the declines in inflation and the output gap while the ZLB is a binding constraint through improved expectations.

Under the SLP, the central bank has an incentive to raise the policy rate more rapidly in the aftermath of a crisis than it does under the standard discretionary policy in order to keep the output gap close to the low level that prevailed during the crisis. Since households and firms are forward-looking, the expectations of low output gap and low inflation associated with it amplify the declines in inflation and the output gap at the ZLB constraint.

The extent to which the ZLB constraint reduces the optimal weight on the output growth stabilization term depends on the specifics of the model under investigation. In the stylized model, the optimal weight on the output growth term becomes smaller when the ZLB constraint is accounted for, but remains strictly positive under all parameter values we consider in this paper. In the quantitative model calibrated to match key features of the U.S. economy, we find that the optimal weight is zero in the model with the ZLB, while it is positive in the model without the ZLB.

This paper builds on previous studies that have examined the desirability of SLPs in sticky-price models. As mentioned in the introductory paragraph, the benefits of SLPs arising from its ability to generate desirable history dependence in the policy rate have been documented by several studies (Blake (2012); Giannoni and Woodford (2003); Stracca (2007); Walsh (2003)).[2] Others have emphasized the desirability of SLP in models where the target variable—be it the output gap or unemployment gap—is measured with errors. Examples are Orphanides, Porter, Reifschneider, Tetlow, and Finan (2000), Orphanides and Williams (2002), and Orphanides and Williams (2007). In such models with measurement errors, policies aimed at reducing the volatility of the mismeasured target variable can cause a higher-than-intended volatility in the policy rate, which in turn leads to higher volatilities of inflation and the output gap. All these papers, however, abstract from the ZLB constraint: our contribution is to examine the desirability of speed-limit policy in models with the ZLB constraint.

The message of our paper echoes those of Blake, Kirsanova, and Yates (2013) and Brendon, Paustian, and Yates (2013) which provide caveats to speed-limit policies in different setups. Blake, Kirsanova, and Yates (2013) show that, when a persistent endogenous variable—such as capital—is introduced into an otherwise standard New Keynesian model without the ZLB, there are multiple equilibria under the standard discretionary policy. They find that SLPs often improve welfare in one equilibrium, but worsen welfare in another. Brendon, Paustian, and Yates (2013) examine the desirability of adding an output growth stabilization term in an interest rate feedback rule in the New Keynesian model with the ZLB constraint. They find that the combination of SLP and the ZLB constraint can generate belief-driven recessions.

[2]See Bilbiie (2014) and Bodenstein and Zhao (2017) for more recent contributions.

Our analysis is different from Blake, Kirsanova, and Yates (2013) because our result does not require the presence of any natural endogenous state variables and our analysis features the ZLB constraint. Our analysis differs from Brendon, Paustian, and Yates (2013) because we assume the central bank is optimizing, as opposed to following an interest-rate feedback rule, and we focus on recessions driven by fundamental shocks.

Finally, this paper is related to a set of papers that consider policy delegation approaches to mitigate the adverse consequences of the ZLB under the discretionary central bank. Nakata and Schmidt (2014) demonstrate that the appointment of an inflation-conservative central banker is welfare-improving, as it mitigates the deflationary bias problem facing the discretionary central bank in the economy with occasionally binding ZLB constraint. Billi (2013) shows that modifying the central bank's objective function to include a nominal-income stabilization objective or a price-level stabilization objective can improve welfare, as both frameworks generate a form of history dependence that is similar to the history dependence attained under the optimal commitment policy. Finally, Nakata and Schmidt (2016) show that it is welfare-improving to assign an interest rate smoothing objective to the central bank, as the desire to smooth the path of interest rates creates a desirable history dependence, leading the central bank to keep the policy rate low for long in the aftermath of a recession. Our work is a useful reminder that introducing history dependence *per se* does not improve welfare; one needs the right kind of history dependence.

As the U.S. economy and monetary policy normalize, there is a growing interest in re-examining monetary policy strategies when the policy rate is subject to the ZLB (see, for example, Williams (2017)). Accordingly, a careful re-examination of how policy strategies known to perform well in the economy without ZLB would perform once the ZLB is taken into account is as relevant as ever. Although there are a growing number of papers on this question of how to best conduct monetary policy in the presence of the ZLB constraint, so far there is no paper examining how SLP performs in the model with ZLB. Thus, our paper fills in an important gap in the literature.

The rest of the paper is organized as follows. Section 2 describes the stylized model and formulates the problem or the central bank. Section 3 presents the results on how the ZLB constraint alters the desirability of SLPs. Section 4 extends the analysis to a calibrated quantitative model. Section 5 examines the the desirability of SLPs in the context of modified interest rate feedback rules. Section 6 concludes.

2 The model

This section presents the model, lays down the policy problem of the central bank and defines the equilibrium.

2.1 Private sector

The private sector of the economy is given by the standard New Keynesian structure formulated in discrete time with infinite horizon as described in detail in Woodford (2003) and Gali (2015). A continuum of identical, infinitely-living households consumes a basket of differentiated goods and supplies labor in a perfectly competitive labor market. The consumption goods are produced by firms using (industry-specific) labor. Firms maximize profits subject to staggered price-setting as in Calvo (1983). Following the majority of the literature on the ZLB, we put all model equations except for the ZLB constraint in semi-loglinear form.

The equilibrium conditions of the private sector are given by the following two equations:

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t \pi_{t+1} + e_t \tag{1}$$

$$y_t = \mathbb{E}_t y_{t+1} - \sigma \left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n \right), \tag{2}$$

where π_t is the inflation rate between periods $t-1$ and t, y_t denotes the output gap, i_t is the level of the nominal interest rate between periods t and $t+1$, r_t^n is an exogenous natural real rate of interest, and e_t is an exogenous cost-push shock.. Equation (1) is a standard New Keynesian Phillips curve and equation (2) is the Euler equation. Parameters satisfy $\beta \in (0,1)$ and $\sigma, \kappa > 0$.[3]

We assume that the natural real rate follows a two-state Markov process. In particular, r_t^n takes the value of either r_H^n or r_L^n where we refer to $r_H^n > 0$ as the high (non-crisis) state and $r_L^n < 0$ as the low (crisis) state. The transition probabilities are given by

$$\text{Prob}(r_{t+1}^n = r_L^n | r_t^n = r_H^n) = p_H \tag{3}$$

$$\text{Prob}(r_{t+1}^n = r_L^n | r_t^n = r_L^n) = p_L. \tag{4}$$

p_H is the probability of moving to the low state in the next period when the economy is in the high state today and will be referred to as the *frequency* of the contractionary shocks. p_L is the probability of staying in the low state when the economy is in the low state today and will be referred to as the *persistence* of the contractionary shocks. We will also refer to high and low states as non-crisis and crisis states, respectively.

We assume that the cost-push shock e_t follows a three-state Markov process, taking e_H, e_M, and e_L. For simplicity, we now assume that the process is independent and the probability of each state is identical. In other words, $\text{Prob}(e_{t+1} = e_i | e_t = e_j) = 1/3$ for any i and j in $\{H, M, L\}$. For simplicity, we also assume the shock is symmetric in size. That is, $e_H = c$, $e_M = 0$, and $e_L = -c$ where c is the magnitude of the cost-push shock. In Appendix A, we show the robustness of our result in a version of the stylized model in which the demand and cost-push shocks follow AR(1) processes.

[3] κ is related to the structural parameters of the economy as follows: $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\eta\theta)} \left(\sigma^{-1} + \eta \right)$, where $\alpha \in (0,1)$ denotes the share of firms that cannot reoptimize their price in a given period, $\eta > 0$ is the inverse of the elasticity of labor supply, and $\theta > 1$ denotes the price elasticity of demand for differentiated goods.

2.2 Society's objective and the central bank's problem

We assume that society's value, or welfare, at time t is given by the expected discounted sum of future utility flows,

$$V_t = u(\pi_t, y_t) + \beta \mathbb{E}_t V_{t+1}, \tag{5}$$

where society's contemporaneous utility function, $u(\cdot, \cdot)$, is given by the standard quadratic function of inflation and the output gap,

$$u(\pi, y) = -\frac{1}{2}\left(\pi^2 + \lambda y^2\right). \tag{6}$$

This objective function can be motivated by a second-order approximation to the household's preferences. In such a case, λ is a function of the structural parameters and is given by $\lambda = \kappa/\theta$.

The value for the central bank is given by

$$V_t^{CB} = u^{CB}(\pi_t, y_t, y_{t-1}) + \beta \mathbb{E}_t V_{t+1}^{CB}. \tag{7}$$

where the central bank's contemporaneous utility function, $u^{CB}(\cdot, \cdot)$, is given by

$$u^{CB}(\pi_t, y_t, y_{t-1}) = -\frac{1}{2}\left(\pi_t^2 + \lambda y_t^2 + \alpha (y_t - y_{t-1})^2\right). \tag{8}$$

Note that the central bank's objective function has the output growth stabilization term $\alpha(y_t - y_{t-1})^2$ in addition to the inflation and output stabilization terms.[4] We assume that the central bank does not have a commitment technology. When $\alpha = 0$, the central bank's contemporaneous utility function is identical to that of the society. We will refer to the optimal discretionary policy associated with $\alpha = 0$ as the standard discretionary policy. Each period t, the central bank chooses the inflation rate, the output gap, and the nominal interest rate in order to maximize its objective function subject to the behavioral constraints of the private sector, with the policy functions at time $t+1$ taken as given. The problem of the central bank is thus given by

$$V_t^{CB}(r_t^n, y_{t-1}) = \max_{\pi_t, y_t, i_t} \quad u^{CB}(\pi_t, y_t, y_{t-1}) + \beta \mathbb{E}_t V_{t+1}^{CB}(r_{t+1}^n, y_t). \tag{9}$$

subject to the ZLB constraint,

$$i_t \geq 0, \tag{10}$$

and the private-sector equilibrium conditions (1) and (2) described above.

A *Markov-Perfect equilibrium* is defined as a set of time-invariant value and policy functions $\{V^{CB}(\cdot, \cdot), y(\cdot, \cdot), \pi(\cdot, \cdot), i(\cdot, \cdot)\}$ that solves the central bank's problem above, together

[4] In Walsh (2003), the central bank's objective function is modified so that the output growth stabilization term *replaces* the output stabilization term. We choose to keep the output stabilization term because, by doing so, the central bank's objective function nests the society's objective function and thus the mechanism is clear.

with society's value function $V(\cdot,\cdot)$, which is consistent with $y(\cdot,\cdot)$ and $\pi(\cdot,\cdot)$.

The main exercise of the paper will be to examine the effects of α on welfare. We quantify the welfare of an economy by the perpetual consumption transfer (as a share of its steady state) that would make a household in the economy indifferent to living in the economy without any fluctuations. This is given by

$$W := (1-\beta)\frac{\theta}{\kappa}\left(\sigma^{-1}+\eta\right)\mathbb{E}[V]. \tag{11}$$

where the mathematical expectation is taken with respect to the unconditional distribution of d_t and y_{t-1}.

2.3 Parameter values and solution methods

Parameter values for the stylized model are shown in Table 1. The values for the model's behavioral parameters are from Woodford (2003). The parameters governing the shock processes are chosen to illustrate the key forces of the model in a transparent way. We will conduct sensitivity analyses with respect to key parameters at the end of Section 3.

Table 1: Parameter Values for the Stylized Model

Parameter	Value	Economic interpretation
β	0.99	Subjective discount factor
σ	1	Interest-rate elasticity of output
κ	0.024	Slope of the Phillips curve
λ	0.003	Weight on the output stabilization term
r^n_H	0.01	Natural-rate shock in the high state
r^n_L	-0.015	Natural-rate shock in the low state
p_H	0.005	Frequency of contractionary demand shock
p_L	0.5	Persistence of contractionary demand shock
$100c$	0.15	Magnitude of cost-push shocks

We solve the model by a variant of the standard time-iteration method that is modified to take it into consideration that the partial derivatives of future policy show up in the system of nonlinear equations characterizing the model's equilibrium conditions. The details of the solution method and the solution accuracy are described in Appendix B and C, respectively.

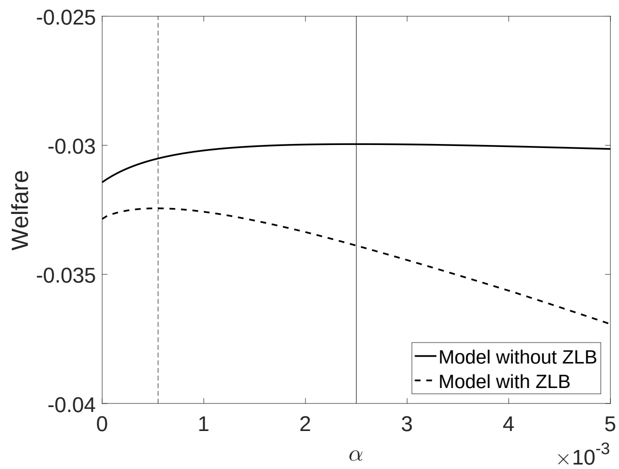
3 Results

This section describes how the introduction of an output growth objective affects welfare and the propagation of economic shocks in the models with and without the ZLB.

3.1 Welfare effects of speed-limit policies

Dashed and solid black lines in Figure 1 show how the welfare of the economies with and without the ZLB depends on the weight on the output growth stabilization term, respectively.

Figure 1: Output Growth Stabilization and Welfare in the Stylized Model with the ZLB



Note: The dashed and solid black lines show how welfare as defined in equation 11 varies with the relative weight on the output growth objective in the model with and without the ZLB, respectively. The dashed and solid vertical black lines show the optimal weights from the model with and without the ZLB constraint, respectively.

According to the figure, while putting some weight on the output growth stabilization term increases welfare in both economies with and without the ZLB, the optimal weight on the output growth stabilization term is smaller in the model with the ZLB than in the model without it. Also, the resulting welfare gain is smaller in the model with the ZLB than in the model without it.

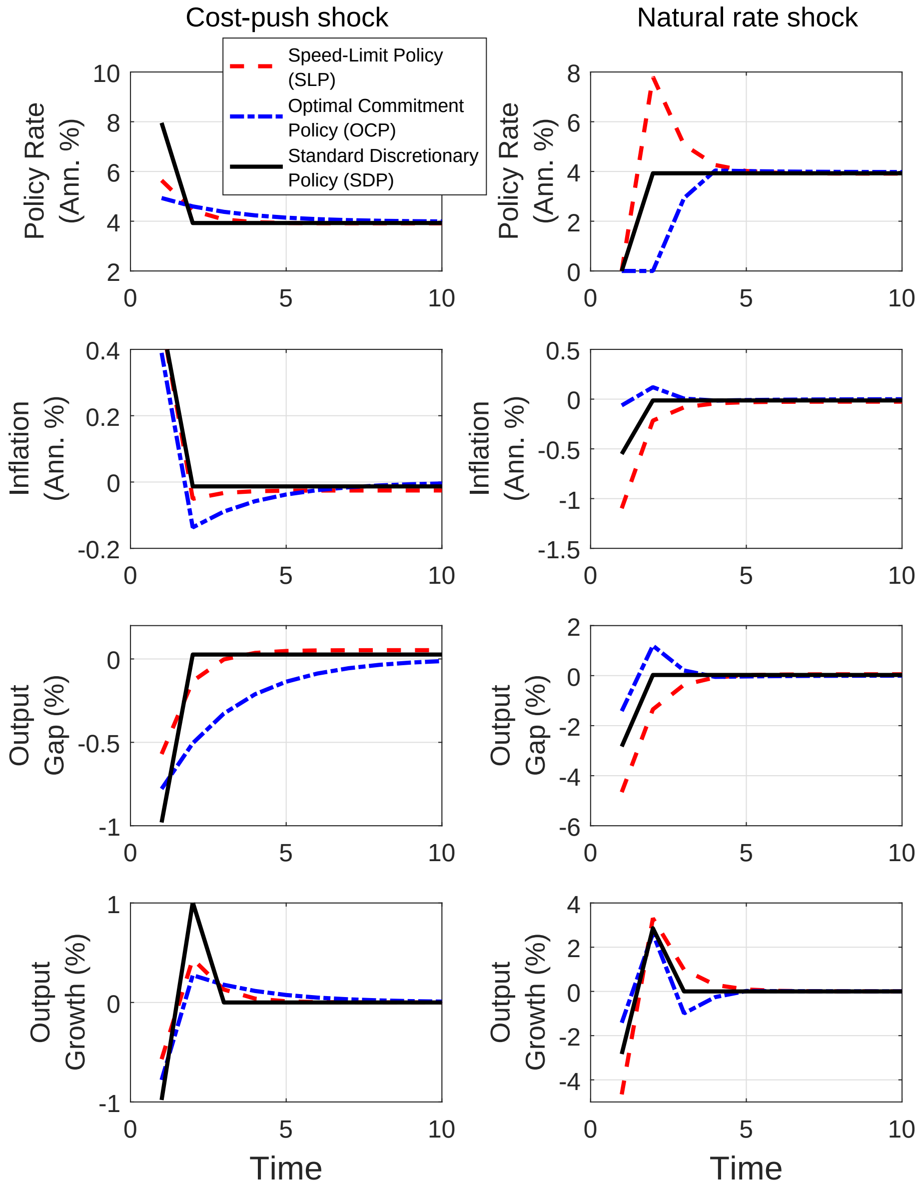
To understand why the ZLB makes the SLP less effective, left and right columns of Figure 2 show how the economy with the ZLB constraint responds to a positive cost-push and a negative natural rate shock—lasting for one period—under alternative assumptions about monetary policy, respectively. In each figure, solid black, dashed red, and dash-dotted blue lines show the impulse response functions under the standard discretionary policy (SDP), the speed limit policy (SLP) with $\alpha = 0.003$, and the optimal commitment policy (OCP).[5]

3.2 Benefits of Speed Limit Policy

According to the left column of Figure 2, under the SDP—shown by the black lines—the central bank raises the policy rate at the time of the positive cost-push shock. After the shock is gone, the policy rate, inflation and the output gap immediately return to their

[5]The formulation of the central bank's problem under commitment is standard and is described in Appendix E.

Figure 2: Impulse Responses in the Stylized Model
(One-Period Cost-Push and Natural Rate Shocks)



Note: "Standard Discretionary Policy (DSP)" shows the dynamics of the economy with the discretionary central bank with $\alpha = 0$ and "Speed Limit Policy (SLP)" shows the dynamics of the economy with the discretionary central bank with $\alpha = 0.003$. "Optimal Commitment Policy (OCP)" shows the dynamics of the economy under the optimal commitment policy.

steady-state values. Under the OCP—shown by the blue lines—the central bank raises the policy rate by less at the time of the shock and lowers the policy rate gradually to its steady state thereafter. Consistent with this policy path, the output gap is negative and inflation undershoots its long-run target level after the positive cost-push shock disappears. The future inflation undershooting improves the inflation-output tradeoff for the central bank in the first period through expectations. As a result, both the rise in inflation and the fall in the output gap in the initial period are smaller under the OCP than under the SDP. The benefit of more stable inflation and output in the first period dominates the less-stabilized inflation and output afterwards in terms of welfare. This welfare gain from policy commitment in sticky-price models with a cost-push shock is well known in the literature (Woodford (2003) and Gali (2015)).

Under the SLP—shown by the red lines—the policy rate path is qualitatively similar to that under the OCP; the central bank raises the policy rate initially by less than it would under the standard discretionary policy and lowers it gradually to its steady-state value afterwards. Output declines less than under the SDP in the first period and gradually returns to its steady state. This path of the output gap leads to a small temporary undershooting of inflation from the second period on, with the initial increase in inflation being about the same as that under the standard discretionary policy. With the inflation path roughly unchanged from the SDP, but the output gap path substantially more stabilized, the welfare costs associated with cost-push shocks are smaller under the SLP than under the SDP. This beneficial effect of the SLP in response to a cost-push shock was first pointed out by Walsh (2003) and is present in both the model with the ZLB and the model without the ZLB.

3.3 Costs of Speed Limit Policy

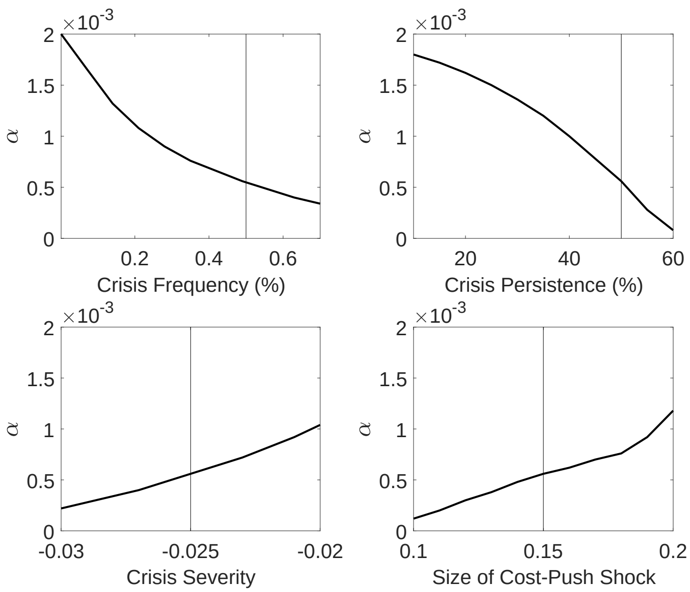
We now turn to the right column of Figure 2 which shows how the economy with the ZLB responds to a negative demand shock. Under the SDP—shown by the black lines—the central bank keeps the policy rate at the ZLB as long as the crisis shock lasts, and inflation and output return to their steady-state values immediately after the crisis shock disappears. Under the OCP—shown by the blue lines—the central bank keeps the policy rate at the ZLB for two extra periods after the crisis shock is gone, which generates a temporary overshooting of inflation and output above their long-run targets. Since households and firms are forward-looking, the anticipation of the overshooting mitigates the declines in inflation and output while the policy rate is constrained at the ZLB. These dynamics of the economy under the SDP and the OCP just described have been studied by many, most notably by Eggertsson and Woodford (2003).

Under the SLP—shown by the blue lines—the policy rate path in the aftermath of a crisis shock is almost a mirror image of what prevails under the OCP: The central bank under the SLP raises the policy rate above its steady-state level after the crisis shock disappears in order to let output return gradually to its steady-state level. As a result, inflation also returns to its steady state gradually. Through expectations, more gradual returns of inflation and output exacerbate the initial declines in inflation and output when the ZLB is a binding constraint. Ironically, in equilibrium, the output growth term is less stabilized under the SLP than under the SDP, as shown by the bottom-right panel.

Note that, in the absence of the ZLB constraint, output and inflation are fully stabilized in the presence of a demand shock under the SDP, the OCP, and the SLP. Thus, it is the interaction of the ZLB constraint and a large negative shock that makes SLP undesirable.

3.4 Sensitivity Analysis

Figure 3: Sensitivity Analysis



Note: In the bottom left panel, crisis severity is given by the difference between the natural rate shock in the high state and that in the low state (that is, $r_L^n - r_H^n$).

The extent to which the ZLB constraint makes the SLP less desirable depends on the relative importance of the natural rate shock and the cost-push shock. On the one hand, as shown in the top two panels and the bottom-left panel of Figure 3, when the crisis state of the natural rate shock is more frequent, more persistent, or more severe, the inefficiency arising from the ZLB constraint is larger, and thus the optimal weight on the output growth stabilization term, as well as the welfare gain from the SLP are smaller. On the other hand, as shown in the bottom-right panel of Figure 3, when the magnitude of the cost-push shock is larger, the inefficiency arising from the cost-push shock is larger and the optimal weight as well as the welfare gain from the SLP is larger.

4 Quantitative Model

This section extends the analysis of the previous section to a more elaborate model. The model provides an empirically more plausible framework to quantify the implications of SLP for economic dynamics and welfare.

4.1 Model and Calibration

The quantitative model features nominal price and wage rigidities as in Erceg, Henderson, and Levin (2000), and non-reoptimized prices and wages that are partially indexed to past price inflation. As in the stylized model of Sectoin 2 and 3, two exogenous shocks—an aggregate demand shock and a cost-push shock—buffet the economy.

The aggregate private sector behavior of the quantitative model is summarized by the following system of equations:

$$\pi_t^p - \iota_p \pi_{t-1}^p = \kappa_p w_t + \beta\left(\mathbb{E}_t \pi_{t+1}^p - \iota_p \pi_t^p\right) + u_t, \tag{12}$$

$$\pi_t^w - \iota_w \pi_{t-1}^p = \kappa_w\left(\left(\frac{1}{\sigma}+\eta\right) y_t - w_t\right) + \beta\left(\mathbb{E}_t \pi_{t+1}^w - \iota_w \pi_t^p\right), \tag{13}$$

$$\pi_t^w = w_t - w_{t-1} + \pi_t^p, \tag{14}$$

$$y_t = \mathbb{E}_t y_{t+1} - \sigma\left(i_t - \mathbb{E}_t \pi_{t+1}^p - r_t^n\right), \tag{15}$$

$$i_t \geq i_{ELB}. \tag{16}$$

Equation (12) captures the price-setting behavior of firms, where w_t is the real wage and u_t is a price mark-up shock. Equation (13) summarizes the nominal wage setting behavior of households, where π_t^w denotes wage inflation between periods $t-1$ and t. Parameters ι_p and ι_w represent the degree of indexation of prices and wages to past price inflation. Equation (14) relates nominal wage inflation to the change in the real wage rate and the price inflation rate, and equation (15) is the familiar consumption Euler equation. Finally, equation (16) represents the effective lower bound (ELB) constraint on the policy rate.[6] Parameters satisfy $\kappa_p = \frac{(1-\alpha_p)(1-\alpha_p\beta)}{\alpha_p}$ and $\kappa_w = \frac{(1-\alpha_w)(1-\alpha_w\beta)}{\alpha_w(1+\eta\theta_w)}$, where $\alpha_p \in (0,1)$ and $\alpha_w \in (0,1)$ denote the share of firms and households that cannot reoptimize their price and wage in a given period, respectively. $\theta_p > 1$ is the price elasticity of demand for differentiated goods, whereas $\theta_w > 1$ is the wage elasticity of demand for differentiated labor services. The notations for η, σ, and β are the same as in the stylized model.

Society's welfare at time t is given by the expected discounted sum of future utility flows. That is,

$$V_t = u(\pi_t^p, y_t, \pi_t^w, \pi_{t-1}^p) + \beta E_t V_{t+1}, \tag{17}$$

where society's contemporaneous utility function $u(\cdot)$ is given by the following second-order approximation to the household's utility:[7]

$$u(\pi_t^p, y_t, \pi_t^w, \pi_{t-1}^p) = -\frac{1}{2}\left[\left(\pi_t^p - \iota_p \pi_{t-1}^p\right)^2 + \lambda y_t^2 + \lambda_w\left(\pi_t^w - \iota_w \pi_{t-1}^p\right)^2\right], \tag{18}$$

[6] In this section, we use the term "the effective lower bound," instead of "the zero lower bound," because the lower bound on the federal funds rate was slightly positive in the U.S., a country to which we calibrate our model.

[7] We assume that the deterministic steady-state distortions associated with imperfect competitions in goods and labor markets are eliminated by appropriate subsidies.

The relative weights are functions of the structural parameters.[8]

The central bank acts under discretion. The central bank's contemporaneous utility function $u^{CB}(\cdot)$ is given by,

$$u^{CB}(\pi_t^p, y_t, y_{t-1}, \pi_t^w, \pi_{t-1}^p) = -\frac{1}{2}\Big\{(1-\alpha)\left[\left(\pi_t^p - \iota_p \pi_{t-1}^p\right)^2 + \lambda y_t^2 + \lambda_w \left(\pi_t^w - \iota_w \pi_{t-1}^p\right)^2\right] + \alpha(y_t - y_{t-1})^2\Big\}, \quad (19)$$

where $\alpha \geq 0$ is the weight on the output growth stabilization term. When $\alpha = 0$, the central bank's objective function collapses to society's objective function.

Each period t, the central bank chooses the price and wage inflation rate, the output gap, the real wage, and the nominal interest rate to maximize its objective function subject to the private-sector equilibrium conditions (equations (12) - (16)) and the ELB constraint, with the value and policy functions at time $t+1$ taken as given,

$$V_t^{CB}(u_t, r_t^n, y_{t-1}, \pi_{t-1}^p, w_{t-1}) = \max_{\pi_t^p, \pi_t^w, y_t, w_t, i_t} u^{CB}(\pi_t^p, y_t, y_{t-1}, \pi_t^w, \pi_{t-1}^p) + \beta \mathbb{E}_t V_{t+1}^{CB}(u_{t+1}, r_{t+1}^n, y_t, \pi_t^p, w_t). \quad (20)$$

The first-order necessary conditions are presented in Appendix D.

We quantify the effects of gradualism on society's welfare by the perpetual consumption transfer (as a share of its steady state) that would make a household in the artificial economy without any fluctuations indifferent to living in the economy just described. This welfare-equivalent consumption transfer is given by

$$W := (1-\beta)\frac{\theta_p}{\kappa_p} E[V]. \quad (21)$$

Parameter values, shown in Table 2, are chosen so that the key moments implied by the model under $\alpha = 0$ are in line with those in the U.S. economy over the last two decades. The model-implied standard deviations of inflation, output, and the policy rate are 0.63 percent (annualized), 2.9 percent, and 2.3 percent. The same moments from the U.S. data are 0.52 percent (annualized), 2.8 percent, and 2.2 percent.[9] The model-implied probability of being at the ELB is about 28 percent, while the federal funds rate was at the ELB constraint 35 percent of the time over the past two decades.

[8]Specifically, $\lambda = \kappa_p \left(\frac{1}{\sigma} + \eta\right) \frac{1}{\theta_p}$ and $\lambda_w = \lambda \frac{\theta_w}{\kappa_w \left(\frac{1}{\sigma} + \eta\right)}$.

[9]Our sample is from 1997:Q3 to 2017:Q2. Inflation rate is computed as the annualized quarterly percentage change (log difference) in the personal consumption expenditure core price index. The measure of the output gap is based on the FRB/US model. The quarterly average of the (annualized) federal funds rate is used as the measure for the policy rate.

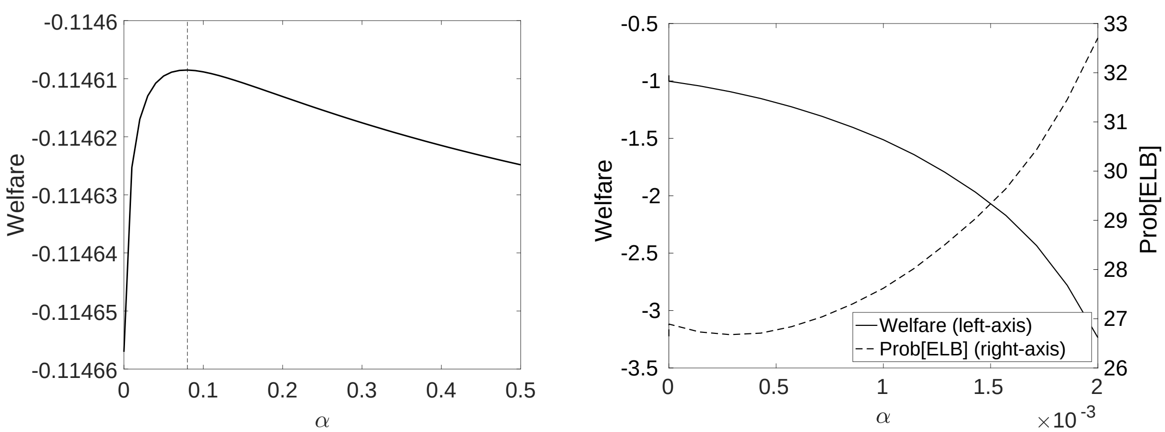
Table 2: Calibration for the Quantitative Model

Parameter	Description	Parameter Value
β	Discount rate	0.9925
σ	Intertemporal elasticity of substitution in consumption	4
η	Inverse labor supply elasticity	2
θ_p	Price elasticity of substitution among intermediate goods	11
θ_w	Wage elasticity of substitution among labor services	11
α_p	Share of firms per period keeping prices unchanged	0.9
α_w	Share of households per period keeping wages unchanged	0.9
ι_p	Degree of indexation of prices to past price inflation	0.1
ι_w	Degree of indexation of wages to past price inflation	0.1
i_{ELB}	Effective lower bound on interest rates	0.125/400
ρ_r	AR(1) coefficient for natural real rate shock	0.85
σ_r	Standard deviation of natural real rate shock	$\frac{0.31}{100}$
ρ_u	AR(1) coefficient for cost-push shock	0
σ_u	Standard deviation of cost-push shock	$\frac{0.17}{100}$

4.2 Results

Solid black lines in Figures 4(a) and 4(b) show how the welfares of the economies without and with the ELB constraint depend on the weight on the output growth stabilization term, respectively.

Figure 4: Output Growth Stabilization and Welfare in the Quantitative Model



(a) Model without the ELB

(b) Model with the ELB

Consistent with our earlier analysis based on the stylized model, the optimal weight in the model without the ELB constraint is positive. In the model with the ELB constraint, the optimal α is zero and the economy's welfare decreases monotonically as α increases from zero; as in the stylized model, the optimal α is lower in the model with the ELB constraint than without it.

Figure 5: A Recession Scenario in the Quantitative Model with ELB

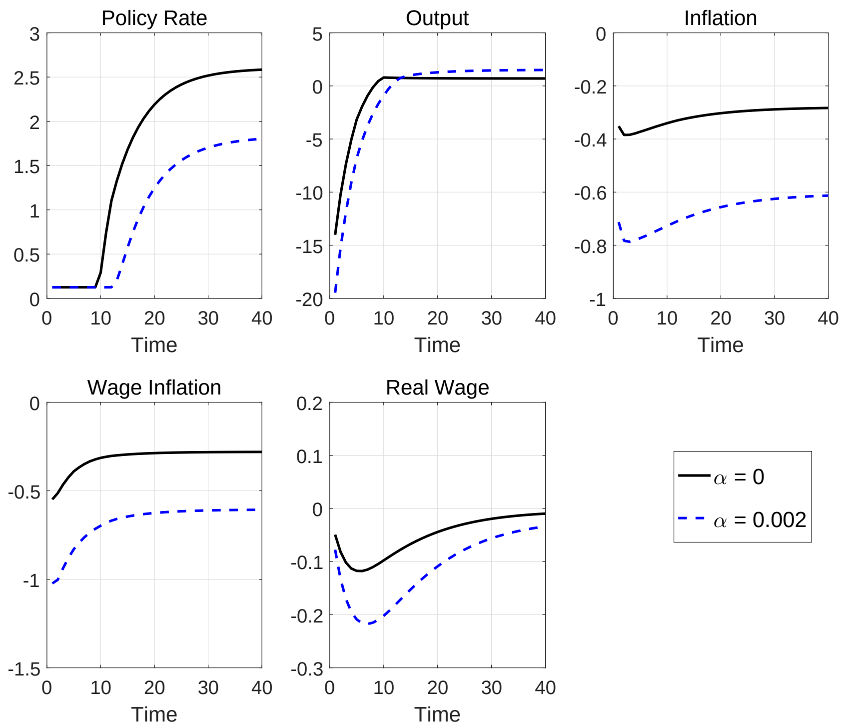


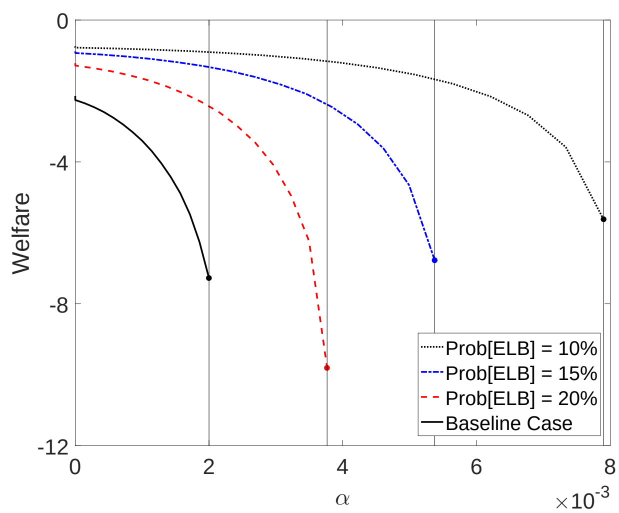
Figure 5 illustrates the effect of SLP on the dynamics of the economy with the ELB. Solid black and dashed blue lines show the impulse response functions from the economy with the ELB constraint under $\alpha = 0$—the standard discretionary case—and $\alpha = 0.002$—a SLP case—respectively, when the initial demand shock is 2.5 unconditional standard deviations below the steady state. The declines in inflation and output are larger under the SLP than under the SDP. While the concern for the output growth stabilization gives the central bank an incentive to speed up the increase in the policy rate in the aftermath of a recession, a more severe recession caused by the SLP exerts a force to push down the policy rate path. In equilibrium, the second force dominates the first; the policy rate lifts off the ELB one quarter later and rises more gradually after the liftoff under $\alpha = 0.002$ than under $\alpha = 0$. The prolonged ELB duration translates into a higher probability of being at the ELB. As shown by the dashed blue line in Figure 4(b), the probability of being at the ELB is higher under $\alpha = 0.002$ than under $\alpha = 0$.

Figure 5 also demonstrates that the weight on the output growth stabilization term affects the risky steady state of the economy—the point where the economy converges once the exogenous shocks disappear. In particular, due to the higher possibility of being at the ELB, price and wage inflation are nontrivially lower, and output and real wages are slightly higher, at the risky steady state with $\alpha = 0.002$ than with $\alpha = 0$. These effects of the increased ELB risk on the economy's risky steady state are consistent with the closed-form analysis of

Nakata and Schmidt (2014) and the numerical analysis of Hills, Nakata, and Schmidt (2016).

Note that the range of α we show in Figure 4(b) is very narrow, ranging from $\alpha = 0$ to $\alpha = 0.002$. As α increases, the probability of being at the ELB constraint increases, as discussed earlier and as shown by the dashed black line in Figure 4(b). In sticky-price models, no equilibrium exists when the probability of being at the ELB is sufficiently high (Richter and Throckmorton (2015) and Nakata and Schmidt (2014)). In our quantitative model with the ELB constraint, the equilibrium ceases to exist for $\alpha > 0.002$.

Figure 6: Output Growth Stabilization and Welfare with Different Shock Variances



Note: The figure shows how welfare varies with the relative weight on the output growth objective in the model with the ELB for alternative variances for the natural rate shock.

To examine the robustness of our result in environments in which the range of α consistent with the equilibrium existence is wider, in Figure 6, we reduce the standard deviation of the demand shock so that, when $\alpha = 0$, the probability of being at the ELB is 10, 15, and 20 percent, respectively—all lower than the baseline case of 28 percent. When the natural rate shock is less volatile and thus the probability of being at the ELB is lower, the equilibrium existence is achieved under a wider range of α; the maximum value of α consistent with the equilibrium existence—indicated by the vertical lines—is higher when the volatility is lower. Not surprisingly, for any given α, welfare is higher when the shock is less volatile. In all three low-volatility scenarios, the optimal α remains zero.[10]

5 Analysis based on an interest rate feedback rule

Throughout the paper, we have followed Walsh (2003) and have taken the policy delegation approach to study the interaction of the ZLB constraint and SLP. However, it is also possible

[10]In the model without the ELB constraint, the optimal α are unaffected by the standard deviation of the natural rate shock, as the effect of the natural rate shock is fully neutralized in the absence of the ELB constraint.

to study the interaction of the ZLB and SLP using an interest rate feedback rule with an output growth term, as some authors have done in the past (Blake (2012); Giannoni and Woodford (2003); Stracca (2007)).

In this section, we assume that the central bank sets the interest rate according to the following rule and examines the desirability of SLP by varying the weight on the output growth term. For both stylized and quantitative model, we assume that the interest rate feedback rule is given by

$$i_t = r_t^n + \phi_\pi \pi_t + \alpha(y_t - y_{t-1}) \tag{22}$$

in the economy without the ZLB constraint, and

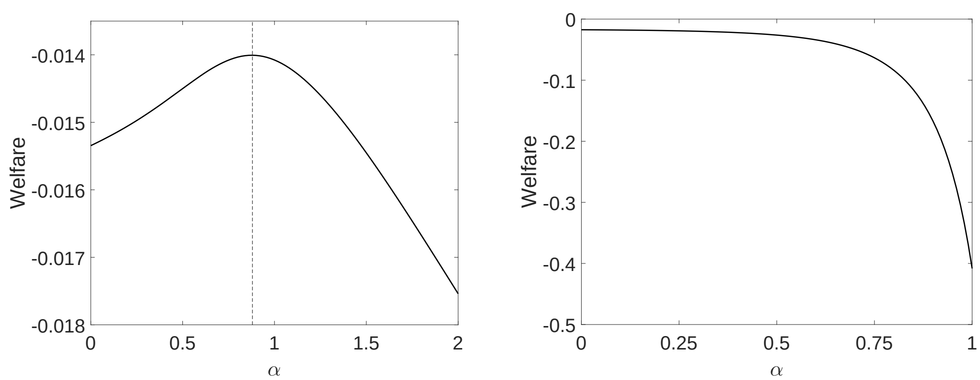
$$i_t = \max[i_{ELB}, i_t^*] \tag{23}$$

$$i_t^* = r_t^n + \phi_\pi \pi_t + \alpha(y_t - y_{t-1}) \tag{24}$$

in the economy with the ELB constraint. In the following experiment, the coefficient on inflation, ϕ_Π, is set to 3, but all the results that ensue are robust to alternative values.

Left and right panels of Figure 7 show how welfare varies with the weight on the output growth term in the stylized model of Section 3 without and with the ZLB constraint, respectively. According to the figure, consistent with our baseline analysis based on policy delegation—shown in Figure 1—the optimal weight is lower in the economy with the ZLB than in the economy without the ZLB (0.9 versus 0).

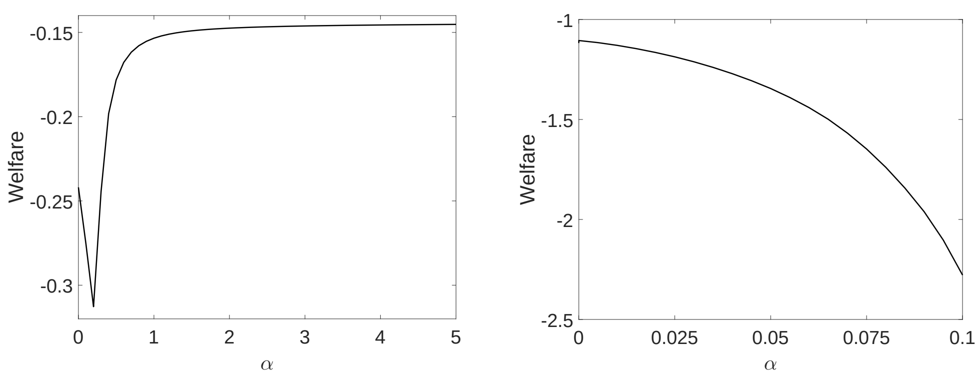
Figure 7: Output Growth Stabilization and Welfare in the Stylized Model (based on a simple interest rate feedback rule)



(a) Model without the ZLB (b) Model with the ZLB

Figure 8 shows how welfare varies with the weight on the output growth term in the quantitative model considered in the previous section. Left and right panels are for the economy without and with the ELB constraint, respectively. According to the left panel, in the model without the ELB constraint, welfare initially declines slightly as α increases from zero, then starts increasing thereafter and flattens out eventually at a level of welfare higher than the level with $\alpha = 0$. While this welfare pattern is qualitatively different from that based on the policy delegation approach considered in the previous section—shown in Figure 4(a)—the result is consistent with the idea that SLP is desirable in economies without the ELB. Finally, in the economy with ELB constraint, shown in the right panel, the optimal weight is zero, consistent with our policy delegation analysis.

Figure 8: Output Growth Stabilization and Welfare in the Quantitative Model (based on a simple interest rate feedback rule)



(a) Model without the ELB (b) Model with the ELB

All in all, the key insight of our paper—the lower bound constraint on nominal interest rates makes SLP less effective—is robust to the alternative approach based on an interest-rate feedback rule.

6 Conclusion

We have examined the desirability of SLPs in sticky-price models with a lower bound constraint on nominal interest rates. Our main finding is that the lower bound constraint reduces the desirability of SLPs. In the aftermath of a deep recession, the central bank under a SLP aims to engineer more gradual increases in output and inflation than under the standard discretionary policy. Through expectations, such gradual recoveries in inflation and output exacerbate the declines in inflation and output when the ZLB constraint is binding. In our quantitative model calibrated to match key features of the U.S. economy, the optimal weight on the output growth stabilization term becomes zero when we account for the lower bound constraint. Our analysis underscores the importance of accounting for the lower bound constraint on nominal interest rates in the assessment of any monetary policy strategy.

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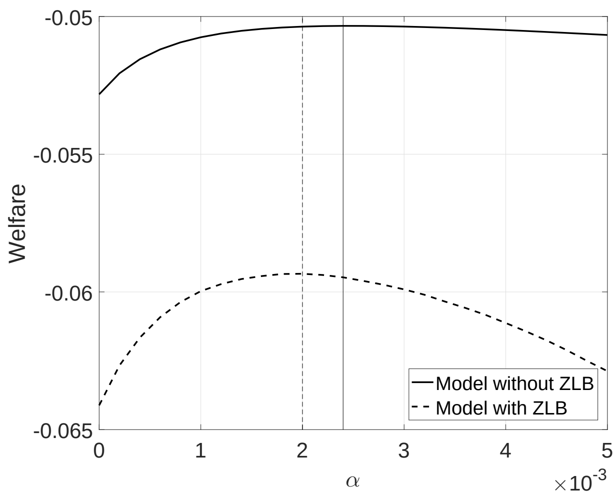
Technical Appendix

A Stylized model with AR(1) cost-push and natural rate shocks

In the main body of the paper, we have used a stylized model with a two-state Markov natural-rate shock and a three-state Markov cost-push shock to examine how the ZLB constraint affects the desirability of speed-limit policy. In Figure 9, we conduct the same analysis in the stylized model in which the natural-rate and cost-push shocks both follow AR(1) processes. According to the figure, the optimal weight on the output growth stabilization term is lower in the economy with the ZLB than in the economy without the ZLB, consistent with the analysis with two-state and three-state Markov shocks.

Figure 9: Output Growth Stabilization and Welfare in the Stylize Model with AR(1) Shocks



B The Solution Method

We use a time-iteration method to solve stylized and quantitative models. In this section, we illustrate our time-iteration method using the quantitative model. It is straightforward to apply the method to the stylized model.[11]

There are total of five state variables, which we denote by $\mathbb{S}_t \ni [u_t, r_t^n, \pi_{t-1}^p, w_{t-1}, y_{t-1}]$. The problem is to find a set of policy functions, $\{\pi^p(\mathbb{S}_t)$, $\pi^w(\mathbb{S}_t)$, $y(\mathbb{S}_t)$, $w(\mathbb{S}_t)$, $i(\mathbb{S}_t)$, $\phi_1(\mathbb{S}_t)$,

[11]One simply needs to apply the same method to different equilibrium conditions, recognizing that there are only three state variables (u_t, r_t^n, and y_{t-1}).

$\phi_2(\mathbb{S}_t)$, $\phi_3(\mathbb{S}_t)$, $\phi_4(\mathbb{S}_t)\}$, and $\phi_5(\mathbb{S}_t)\}$ that solves the following system of *functional* equations.

$$\pi^p(\mathbb{S}_t) - \iota_p \pi^p_{t-1} = \kappa_p \left(\frac{\gamma}{1-\gamma} y(\mathbb{S}_t) + w(\mathbb{S}_t) \right) + \beta \left(\mathbb{E}_t \pi^p(\mathbb{S}_{t+1}) - \iota_p \pi^p(\mathbb{S}_t) \right) + u_t, \tag{B.1}$$

$$\pi^w(\mathbb{S}_t) - \iota_w \pi^p_{t-1} = \kappa_w \left(\left(\frac{1}{\sigma} + \frac{\eta}{1-\gamma} \right) y(\mathbb{S}_t) - w(\mathbb{S}_t) \right) + \beta \left(\mathbb{E}_t \pi^w(\mathbb{S}_{t+1}) - \iota_w \pi^p(\mathbb{S}_t) \right), \tag{B.2}$$

$$\pi^w(\mathbb{S}_t) = w(\mathbb{S}_t) - w_{t-1} + \pi^p(\mathbb{S}_t), \tag{B.3}$$

$$y(\mathbb{S}_t) = \mathbb{E}_t y(\mathbb{S}_{t+1}) - \sigma \left(i(\mathbb{S}_t) - \mathbb{E}_t \pi^p(\mathbb{S}_{t+1}) - r^n_t \right), \tag{B.4}$$

$$i(\mathbb{S}_t) \geq i_{ELB}. \tag{B.5}$$

$$\begin{aligned} 0 = {} & -\lambda y(\mathbb{S}_t) - \alpha(y(\mathbb{S}_t) - y_{t-1}) + \beta\alpha(\mathbb{E}_t y(\mathbb{S}_{t+1}) - y(\mathbb{S}_t)) \\ & + \phi_1(\mathbb{S}_t) \left(1 - \frac{\partial \mathbb{E}_t y(\mathbb{S}_{t+1})}{\partial y(\mathbb{S}_t)} - \sigma \frac{\partial \mathbb{E}_t \pi^p(\mathbb{S}_{t+1})}{\partial y(\mathbb{S}_t)} \right) \\ & - \phi_2(\mathbb{S}_t) \left(\kappa_w \left(\frac{1}{\sigma} + \frac{\eta}{1-\gamma} \right) + \beta \frac{\partial \mathbb{E}_t \pi^w(\mathbb{S}_{t+1})}{\partial y(\mathbb{S}_t)} \right) - \phi_3(\mathbb{S}_t) \left(\kappa_p \frac{\gamma}{1-\gamma} + \beta \frac{\partial \mathbb{E}_t \pi^p(\mathbb{S}_{t+1})}{\partial y(\mathbb{S}_t)} \right), \end{aligned} \tag{B.6}$$

$$0 = -\lambda_w(\pi^w(\mathbb{S}_t) - \iota_w \pi^p_{t-1}) + \phi_2(\mathbb{S}_t) + \phi_4(\mathbb{S}_t), \tag{B.7}$$

$$\begin{aligned} 0 = {} & -(\pi^p(\mathbb{S}_t) - \iota_p \pi^p_{t-1}) + \beta \iota_p (\mathbb{E}_t \pi^p(\mathbb{S}_{t+1}) - \iota_p \pi^p(\mathbb{S}_t)) + \beta \lambda_w \iota_w (\mathbb{E}_t \pi^w(\mathbb{S}_{t+1}) - \iota_w \pi^p(\mathbb{S}_t)) \\ & - \phi_1(\mathbb{S}_t) \left(\frac{\partial \mathbb{E}_t y(\mathbb{S}_{t+1})}{\partial \pi^p(\mathbb{S}_t)} + \sigma \frac{\partial \mathbb{E}_t \pi^p(\mathbb{S}_{t+1})}{\partial \pi^p(\mathbb{S}_t)} \right) - \phi_2(\mathbb{S}_t) \beta \left(\frac{\partial \mathbb{E}_t \pi^w(\mathbb{S}_{t+1})}{\partial \pi^p(\mathbb{S}_t)} - \iota_w \right) - \beta \iota_w \mathbb{E}_t \phi_2(\mathbb{S}_{t+1}) \\ & + \phi_3(\mathbb{S}_t) \left(1 - \beta \left(\frac{\partial \mathbb{E}_t \pi^p(\mathbb{S}_{t+1})}{\partial \pi^p(\mathbb{S}_t)} - \iota_p \right) \right) - \beta \iota_p \mathbb{E}_t \phi_3(\mathbb{S}_{t+1}) - \phi_4(\mathbb{S}_t), \end{aligned} \tag{B.8}$$

$$\begin{aligned} 0 = {} & -\phi_1(\mathbb{S}_t) \left(\frac{\partial \mathbb{E}_t y(\mathbb{S}_{t+1})}{\partial w(\mathbb{S}_t)} + \sigma \frac{\partial \mathbb{E}_t \pi^p(\mathbb{S}_{t+1})}{\partial w(\mathbb{S}_t)} \right) + \phi_2(\mathbb{S}_t) \left(\kappa_w - \beta \frac{\partial \mathbb{E}_t \pi^w(\mathbb{S}_{t+1})}{\partial w(\mathbb{S}_t)} \right) \\ & - \phi_3(\mathbb{S}_t) \left(\kappa_p + \beta \frac{\partial \mathbb{E}_t \pi^p(\mathbb{S}_{t+1})}{\partial w(\mathbb{S}_t)} \right) - \phi_4(\mathbb{S}_t) + \beta \mathbb{E}_t \phi_4(\mathbb{S}_{t+1}), \end{aligned} \tag{B.9}$$

$$0 = \phi_1(\mathbb{S}_t)\sigma + \phi_5(\mathbb{S}_t). \tag{B.10}$$

Following the idea of Christiano and Fisher (2000), we decompose these policy functions into two parts using an indicator function: one in which the policy rate is allowed to be less than 0, and the other in which the policy rate is assumed to be 0. That is, for any variable Z,

$$Z(\cdot) = I_{\{R(\cdot) \geq 0\}} Z_{NZLB}(\cdot) + (1 - I_{\{R(\cdot) \geq 0\}}) Z_{ZLB}(\cdot), \tag{B.11}$$

The problem then becomes finding a set of a *pair* of policy functions, $\{[\pi^p_{NZLB}(\cdot), \pi^p_{ZLB}(\cdot)]$, $[\pi^w_{NZLB}(\cdot), \pi^w_{ZLB}(\cdot)]$, $[y_{NZLB}(\cdot), y_{ZLB}(\cdot)]$, $[w_{NZLB}(\cdot), w_{ZLB}(\cdot)]$, $[i_{NZLB}(\cdot), i_{ZLB}(\cdot)]$, $[\phi_{1,NZLB}(\cdot)$, $\phi_{1,ZLB}(\cdot)]$, $[\phi_{2,NZLB}(\cdot), \phi_{2,ZLB}(\cdot)]$, $[\phi_{3,NZLB}(\cdot), \phi_{3,ZLB}(\cdot)]$, $[\phi_{4,NZLB}(\cdot), \phi_{4,ZLB}(\cdot)]$, $[\phi_{5,NZLB}(\cdot)$ $\phi_{5,ZLB}(\cdot)]\}$ that solves the system of functional equations above. The approach of Christiano and Fisher (2000) can achieve a given level of accuracy with a considerable less number of grid points relative to the standard approach.

The time-iteration method aims to find the values for the policy and value functions consistent with the equilibrium conditions on a finite number of grid points within the predetermined grid intervals for the model's state variables. Let $X(\cdot)$ be a vector of policy functions that solves the functional equations above and let $X^{(0)}$ be the initial guess of such policy functions.[12] At the s-th iteration, given the approximated policy function $X^{(s-1)}(\cdot)$, we solve the system of nonlinear equations given by equations (B.1)-(B.10) to find today's $\pi_t^p, \pi_t^w, y_t, w_t, i_t, \phi_{1,t}, \phi_{2,t}, \phi_{3,t}, \ \phi_{4,t}$, and $\phi_{5,t}$ at each grid point. In solving the system of nonlinear equations, we use Gaussian quadrature (with 10 Gauss-Hermite nodes) to discretize and evaluate the expectation terms in the Euler equation, the price and wage Phillips curves, and expectational partial derivative terms.

The values of the policy function that are not on any of the grid points are interpolated or extrapolated linearly. The values of the partial derivatives of the policy functions not on any of the grid points are approximated by the slope of the policy functions evaluated from the adjacent two grid points. That is, for any variable X and Z,

$$\frac{\partial X(\delta_{t+1},_t)}{\partial Z_t} = \frac{X(\delta_{t+1}, Z^{''}) - X(\delta_{t+1}, Z^{'})}{Z'' - Z'}. \tag{B.12}$$

where $Z^{'}$ and $Z^{''}$ are two adjacent grid points to Z_t such that $Z^{'} < Z_t < Z^{''}$. When Z_t is outside the grid interval, the partial derivative is approximated by the slope evaluated at the edge of the grid interval.

The system is solved numerically by using a nonlinear equation solver, dneqnf, provided by the IMSL Fortran Numerical Library. If the updated policy functions are sufficiently close to the previously approximated policy functions, then the iteration ends. Otherwise, using the former as the guess for the next period's policy functions, we iterate on this process until the difference between the guessed and updated policy functions is sufficiently small ($\left\|vec(X^s(\delta) - X^{s-1}(\delta))\right\|_\infty <$ 1E-12 is used as the convergence criteria). The solution method can be extended to models with multiple (non-perfectly correlated) exogenous shocks and with multiple endogenous state variables in a straightforward way.

C Solution Accuracy

In this section, we report the accuracy of our numerical solutions for the stylized and empirical models. Following Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2015) and Maliar and Maliar (2015), we evaluate these residuals functions along a simulated equilibrium path. The length of the simulation is 100,000.

C.1 Stylized model

For the stylized model, there are three key residual functions of interest. The first two residual functions, denoted by $R_{1,t}$ and $R_{2,t}$, are associated with the sticky-price equation and the Euler equation, respectively (equations (1) and (2)). The last residual function, denoted by $R_{3,t}$, is associated with the first-order conditions of the central bank's optimization problem with respect to price inflation. For each equation, the residual function is defined as the absolute value of the difference between the left-hand side and the right-hand side of the equation. Table 3 shows the average and the 95th percentile of the six residual functions over the 100,000 simulations. The size of the residuals are comparable to those reported in other numerical works on the New Keynesian model with the ELB constraint, such as Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramírez (2015), Hills, Nakata, and Schmidt (2016), Hirose and Sunakawa (2015), and Maliar and Maliar (2015).

[12]For all models and all variables, we use flat functions at the deterministic steady-state values as the initial guess.

Table 3: Solution Accuracy: Simple Model with $\alpha = \alpha^{opt}$

	Mean$\left[\log_{10}(\mathrm{R}_{k,t})\right]$	95th-percentile of $\left[\log_{10}(\mathrm{R}_{k,t})\right]$
$k = 1$: Phillips curve error	−8.28	−2.52
$k = 2$: Euler equation error	−18.53	−17.76
$k = 3$: Error in the FONC w.r.t output	−20.16	−19.87

C.2 Quantitative Model

For the quantitative model, there are six key residual functions of interest. The first three residual functions, denoted by $R_{1,t}$, $R_{2,t}$, and $R_{3,t}$, are associated with the sticky-price equation, the sticky-wage equation, and the Euler equation, respectively (equations (12), (13), and (15)). The last three residual functions, denoted by $R_{4,t}$, $R_{5,t}$, and $R_{6,t}$, are associated with the first-order conditions of the central bank's optimization problem with respect to output, price inflation, and real wage, respectively (equations (D.1), (D.3), and (D.4)). For each equation, the residual function is defined as the absolute value of the difference between the left-hand side and the right-hand side of the equation. Table 4 shows the average and the 95th percentile of the six residual functions over the 100,000 simulations. The size of the residuals are comparable to those reported in the aforementioned numerical works on the New Keynesian model with the ELB constraint.

Table 4: Solution Accuracy: Quantitative Model with $\alpha = \alpha^{opt}$

	Mean$\left[\log_{10}(\mathrm{R}_{k,t})\right]$	95th-percentile of $\left[\log_{10}(\mathrm{R}_{k,t})\right]$
$k = 1$: Sticky-price error	−6.46	−5.84
$k = 2$: Sticky-wage error	−6.05	−5.29
$k = 3$: Euler equation error	−4.10	−2.93
$k = 4$: Error in the FONC w.r.t price inflation	−5.07	−4.50
$k = 5$: Error in the FONC w.r.t real wage	−3.90	−3.83
$k = 6$: Error in the FONC w.r.t. policy rate	−2.94	−2.90

D First-Order Necessary Conditions for Central Bank's Problem in the Quantitative Model

Including private-sector equilibrium conditions (equation (12) - (16)), first-order necessary conditions for the central bank's maximization problem are enumerated as follows:

$$
\begin{aligned}
0 &= -\lambda y_t - \alpha(y_t - y_{t-1}) + \beta\alpha(\mathbb{E}_t y_{t+1} - y_t) + \phi_{1,t}\left(1 - \frac{\partial \mathbb{E}_t y_{t+1}}{\partial y_t} - \sigma \frac{\partial \mathbb{E}_t \pi^p_{t+1}}{\partial y_t}\right) \\
&\quad -\phi_{2,t}\left(\kappa_w\left(\frac{1}{\sigma} + \frac{\eta}{1-\gamma}\right) + \beta \frac{\partial \mathbb{E}_t \pi^w_{t+1}}{\partial y_t}\right) - \phi_{3,t}\left(\kappa_p \frac{\gamma}{1-\gamma} + \beta \frac{\partial \mathbb{E}_t \pi^p_{t+1}}{\partial y_t}\right), \qquad \text{(D.1)} \\
0 &= -\lambda_w(\pi^w_t - \iota_w \pi^p_{t-1}) + \phi_{2,t} + \phi_{4,t}, \qquad \text{(D.2)} \\
0 &= -(\pi^p_t - \iota_p \pi^p_{t-1}) + \beta\iota_p(\mathbb{E}_t \pi^p_{t+1} - \iota_p \pi^p_t) + \beta\lambda_w \iota_w(\mathbb{E}_t \pi^w_{t+1} - \iota_w \pi^p_t) \\
&\quad -\phi_{1,t}\left(\frac{\partial \mathbb{E}_t y_{t+1}}{\partial \pi^p_t} + \sigma \frac{\partial \mathbb{E}_t \pi^p_{t+1}}{\partial \pi^p_t}\right) - \phi_{2,t}\beta\left(\frac{\partial \mathbb{E}_t \pi^w_{t+1}}{\partial \pi^p_t} - \iota_w\right) - \beta\iota_w \mathbb{E}_t \phi_{2,t+1} \\
&\quad +\phi_{3,t}\left(1 - \beta\left(\frac{\partial \mathbb{E}_t \pi^p_{t+1}}{\partial \pi^p_t} - \iota_p\right)\right) - \beta\iota_p \mathbb{E}_t \phi_{3,t+1} - \phi_{4,t}, \qquad \text{(D.3)} \\
0 &= -\phi_{1,t}\left(\frac{\partial \mathbb{E}_t y_{t+1}}{\partial w_t} + \sigma \frac{\partial \mathbb{E}_t \pi^p_{t+1}}{\partial w_t}\right) + \phi_{2,t}\left(\kappa_w - \beta \frac{\partial \mathbb{E}_t \pi^w_{t+1}}{\partial w_t}\right) \\
&\quad -\phi_{3,t}\left(\kappa_p + \beta \frac{\partial \mathbb{E}_t \pi^p_{t+1}}{\partial w_t}\right) - \phi_{4,t} + \beta \mathbb{E}_t \phi_{4,t+1}, \qquad \text{(D.4)} \\
0 &= \phi_{1,t}\sigma + \phi_{5,t}, \qquad \text{(D.5)}
\end{aligned}
$$

where $\phi_{1,t}$ - $\phi_{5,t}$ are Lagrangian multipliers for equation (12) - (16), respectively. First-order necessary conditions for simpler models than the above quantitative model with partial indexation can be easily derived by dropping extra variables and private-sector equilibrium conditions from above equations.

E The Ramsey Problem in the Stylized Model

In period 0, the Ramsey policymaker chooses the state-contingent plan for output, inflation, and the policy rate that maximizes society's welfare subject to the private sector behavioral constraints (1)-(2) and the ZLB constraint (10). Formally:

$$
\begin{aligned}
\max_{\{\pi_t, y_t, i_t\}_{t=0}^{\infty}} \mathbb{E}_0 \beta^t \Big\{ & -\frac{1}{2}\left(\pi_t^2 + \lambda y_t^2\right) \\
& + \mu_{1,t}\left[\pi_t - \kappa y_t - \beta \mathbb{E}_t \pi_{t+1} - e_t\right] \\
& + \mu_{2,t}\left[y_t - \mathbb{E}_t y_{t+1} + \sigma\left(i_t - \mathbb{E}_t \pi_{t+1} - r^n_t\right)\right] \\
& + \mu_{3,t} i_t \Big\}
\end{aligned}
$$

The first-order necessary conditions are:

$$0 = \pi_t - \mu_{1,t} + \mu_{1,t-1} + \frac{\sigma}{\beta}\mu_{2,t-1} \tag{E.1}$$

$$0 = \lambda y_t + \kappa \mu_{1,t} - \mu_{2,t} + \frac{1}{\beta}\mu_{2,t-1} \tag{E.2}$$

$$0 = \sigma \mu_{2,t} + \mu_{3,t} \tag{E.3}$$

$$\mu_{3,t} \geq 0, \quad i_t \geq 0 \tag{E.4}$$

where $\mu_{1,-1}, \mu_{2,-1} = 0$. The Ramsey equilibrium can then be characterized by a set of time-invariant policy functions $[\pi(\mathbb{S}_t^R), y(\mathbb{S}_t^R), i(\mathbb{S}_t^R), \mu_1(\mathbb{S}_t^R), \mu_2(\mathbb{S}_t^R), \mu_3(\mathbb{S}_t^R)]$, where $\mathbb{S}_t^R = [r_t^n,\ e_t,\ \phi_{1,t-1},\ \phi_{2,t-1}]$, that solve the system of equilibrium conditions consisting of the private sector behavioral constraints (1)-(2) and (E.1)-(E.4).