IMES DISCUSSION PAPER SERIES

Market Concentration and Sectoral Inflation under Imperfect Common Knowledge

Ryo Kato and Tatsushi Okuda

Discussion Paper No. 2017-E-11

IMES

INSTITUTE FOR MONETARY AND ECONOMIC STUDIES

BANK OF JAPAN

2-1-1 NIHONBASHI-HONGOKUCHO
CHUO-KU, TOKYO 103-8660
JAPAN

IMES Discussion Paper Series 2017-E-11
December 2017

Market Concentration and Sectoral Inflation under Imperfect Common Knowledge

Ryo Kato* and Tatsushi Okuda**

Abstract

We show empirical evidence that sectoral inflation persistence, measured by autocorrelation of monthly changes in US producer prices, is starkly dispersed and negatively correlated with market concentration across sectors. To account for such empirical observation, we develop a dynamic stochastic model of firms' pricing strategy in which monopolistically competitive firms set their prices while receiving private signals on cost shocks. In the model, an increase in the number of competing firms raises strategic complementarity among the firms in the same sector. Using the model, we analytically show that, under imperfect common knowledge, sectoral inflation persistence is monotonically decreasing in market concentration.

Keywords: Imperfect common knowledge; Inflation persistence; Market concentration

JEL classification: E31, D40, D82, L16

* Head of Economic Studies Group, Institute for Monetary and Economic Studies (currently Head of Global Economic Research Division, International Department), Bank of Japan (E-mail: ryou.katou@boj.or.jp)
** Economist, Institute for Monetary and Economic Studies, Bank of Japan (E-mail: tatsushi.okuda@boj.or.jp)

The authors thank Ryan Chahrour, Jeffrey Fuhrer, Mark Gertler, Yukinobu Kitamura, John McDermott, Benjamin Moll, Satoshi Nakada, Taisuke Nakata, Kiyohiko G. Nishimura, Mototsugu Shintani, Takeki Sunakawa, Yuki Teranishi, Takayuki Tsuruga, Takashi Ui, Rosen Valchev, Toshiaki Watanabe, and the participants at the 2017 BOJ-IMES Conference, CEF 2017, JEA 2017 Autumn Meeting, JSME 2017 Autumn Annual Meeting, and 2017 BOJ/IMES-BOK/ERI Workshop for their invaluable comments. The views expressed in this paper are those of the authors and do not necessarily reflect the official views of the Bank of Japan.

1 Introduction

Inflation persistence has received considerable attention in the macroeconomics literature. A number of empirical studies indicate that inflation dynamics are highly persistent and argue that standard sticky price models tend to fail in replicating such highly persistent inflation dynamics.[1] Prompted by the arguable shortfalls of the existing models, Mankiw and Reis (2002) and Woodford (2002) ignited a stream of literature highlighting the role of information rigidities to account for inflation persistence. More recently, empirical studies on price rigidities and inflation persistence look into disaggregate data and report several important findings. A seminal paper by Bils and Klenow (2004) articulates size and frequency of price changes using micro-data provided by Bureau of Labor Statistics (BLS) and, based on their findings, they conclude that popular time-dependent sticky price models cannot replicate industry-level inflation persistence observed in the data.[2]

Against this backdrop, we focus on industry-level inflation persistence, measured by the (first-order) autocorrelation of monthly changes in the US producer price index (PPI)—hereafter, we denote this measure as sectoral inflation persistence—and (i) reconfirm that there is stark dispersion in sectoral inflation persistence across sectors and (ii) examine what factor(s) can account for such cross-sectoral dispersion.[3] Relying on the NAICS (North American Industry Classification System) six-digit level disaggregate dataset provided by Boivin, Giannoni, and Mihov (2009) and our own extended dataset, we report empirical evidence that sectoral inflation persistence of the US PPI is negatively correlated with market concentration in the sector. Namely, if a few of the largest firms in terms of sales share donimate the market, sectoral inflation persistence in the sector tends to be low. In this regard, Figure 1 shows scatterplots of sectoral inflation persistence and market concentration. Although the data are somewhat noisy, a cursory look reveals that sectoral inflation persistence tends to decline as market concentration increases. Motivated by this empirical fact, we build a model with information rigidity that can explain why highly concentrated

[1] A number of early empirical studies report highly persistent dynamics of US aggregate inflation dynamics. See Fuhrer and Moore (1995), Gali and Gertler (1999) and Mankiw (2001), among others. More recently, Cogley and Sargent (2002, 2005) and Levin and Piger (2003) focus on the role of time-varying mean inflation in creating highly persistent dynamics. In contrast, Pivetta and Reis (2007) argue that aggregate inflation persistence has remained high and broadly stable over time in the US.

[2] See Nakamura and Steinsson (2008) and Midrigan (2011) for similar arguments.

[3] The stark dispersion in sectoral inflation persistence has been reported by many empirical studies, such as Clark (2006), Boivin, Giannoni, and Mihov (2009) and Altissimo, Mojon, and Zaffaroni (2007).

markets reduce inflation persistence.

[Figure 1 here]

We develop a dynamic stochastic model in which monopolistically competitive firms set their prices after receiving private signals on cost shocks. In our model, the strategic complementarity among price-setting firms increases as the market concentration decreases in the sector (i.e., the number of firms in the sector increases). As a result of endogenizing strategic complementarity under heterogeneous beliefs, firms in a less concentrated sector respond to a cost shock more slowly, compared with those in a more concentrated sector, because each firm relies more on higher-order expectations in a Bayesian Nash equilibrium. Sectoral inflation in the model exhibits more persistence, typically with hump-shaped dynamic responses, if the sector is less concentrated under imprecise private signals. This cross-industry relationship between market concentration and sectoral inflation persistence is consistent with the empirical facts illustrated by Figure 1 and more formal regression results presented in this paper.

As a key ingredient in our model, we employ consumer preferences that explicitly include a number of differentiated goods (and firms as their producers) in a sector, as proposed by Melitz and Ottaviano (2008). In standard models of monopolistic competition, if other firms were to raise their prices incrementally more than one firm, the firm's best response is to raise its own price, but to a lesser extent, which means that strategic complementarity is playing out in the market. A notable feature of Melitz and Ottaviano's (2008) setup is that the degree of strategic complementarity, which is reflected in the cross-price elasticity of demand, is not a constant parameter, but endogenously varies depending on market concentration (proxied by the number of firms in the sector). As the number of firms increases in a sector/industry, the firms respond more to changes in prices set by others. The interpretation of the variable strategic complementarity is straightforward. As argued by standard industrial organization theories, coordination is easier among a small number of firms. By contrast, each firm responds to the prices set by others more sensitively if there are many competitors because coordination is more difficult in a less concentrated market.

This property of variable strategic complementarity depending on market concentration remains the same, even if firms cannot observe cost shocks, and only receive private signals. That is, each firm's price-setting strategy is affected more strongly by the prices set by others in less concentrated markets where cross-price elasticities are higher. A crucial difference arising from private signals is that the information set held by each firm is heterogeneous—in other words, cost shocks are no longer common knowledge. Given private signals, each firm sets its own price, taking into consideration its *expectations* of the average price. Importantly, these expectations vary across firms as a result of their heterogeneous beliefs. Hence, each firm's expectations of the average price are explicitly affected by the expectations formed by other firms. Because this process of guessing other firms' expectations of others' expectations continues, which are also based on guessing others' expectations, each firm's pricing strategy is affected by higher-order expectations. Through the entire process, the interdependency among higher-order expectations is strengthened if the cross-price elasticity is high. Hence, higher-order expectations matter more in a less concentrated sector, where many firms are engaged in monopolistic competition with high cross-price elasticity.

As emphasized by Woodford (2002) and Angeletos and La'O (2009), higher-order expectations respond to shocks more slowly because, intuitively, higher-order expectations reflect only a small fraction of newly available private signals via Bayesian updating. Recall that, in less concentrated markets, monopolistically competitive firms rely more on higher-order expectations that are updated relatively slowly. Accordingly, the actual average prices set by firms in a less concentrated sector tend to react more slowly to cost shocks, typically with some lags, in a symmetric Bayesian Nash equilibrium.

Similar attempts to build models that generate empirically plausible persistent inflation dynamics are made by a number of early studies in the literature on information rigidity.[4] Mankiw and Reis (2002), Dupor, Kitamuta, and Tsuruga (2010), and Reis (2006), develop sticky-information models, in which only a fraction of firms update their information sets in every period, because of fixed costs incurred in the acquisition of information. In the meantime, noisy-information models have been developed by Woodford (2002), Sims (2003), Adam (2007), Fukunaga (2007), Nimark (2008), Mackowiak and Wiederholt (2009), and Angeletos and La'O (2009) among others. The common feature of these noisy-information models is that, because firms cannot observe the true state, they continuously update their

[4] Apart from the literature on information rigidity, models incorporating mechanical inflation indexation can create inflation inertia, as demonstrated by Gali and Gertler (1999), Gali, Gertler, and Lopez-Salido (2001, 2005), and Christiano, Eichenbaum, and Evans (2005). Some of these models present hybrid New Keynesian Phillips Curves (NKPCs), which include a few lagged, "backward-looking" inflation terms. In a related vein, Levine and Piger (2003) argue that observed inflation persistence is "inherited" rather than "intrinsic" in the sense that persistent/inertial monetary policy generates the seemingly persistent dynamics of inflation. Blanchard and Gali (2007), among others, demonstrate that models with real wage rigidity can generate inflation inertia, in a similar manner to a hybrid NKPC.

beliefs via signal extraction. The signal extraction process results in slow reactions by firms compared with the changes in the true state.[5] Our study is classified in the camp of noisy-information models, and the most closely related works are Woodford (2002) and Angeletos and La'O (2009).[6]

Both Woodford (2002), Angeletos and La'O (2009) and our model highlight the role of imperfect common knowledge in generating slow adjustments in firms' price-setting responses. The important difference is that, because we employ Melitz and Ottaviano's (2008) consumer preferences, instead of the Dixit–Stiglitz preferences used in other similar models, strategic complementarity changes explicitly depending on market concentration in our model. We acknowledge that Angeletos and La'O (2009) already argued that, using a generalized Woodford (2002) model, greater strategic complementarity among firms makes price dynamics more sticky. We reaffirm their argument by comparing data and the prediction of our model which translates the degree of strategic complementarity into an observable indicator; i.e., market concentration in the sector. In addition, because of the linear-quadratic nature of Melitz and Ottaviano's (2008) setup, our model is fully tractable and does not require linear approximation to obtain the explicit solution form of the equilibrium. The linear demand system in our model assures the uniqueness of the Bayesian Nash equilibrium, as demonstrated by Morris and Shin (2002) in a broader context.

The paper is organized as follows. Section 2 provides empirical evidence on sectoral inflation persistence and market concentration. Section 3 introduces the setup of the model under perfect information. Section 4 allows heterogeneous beliefs because of private signals in the model and shows some key results including comparative statics. Section 5 assesses the dynamics of sectoral inflation using numerical illustrations with various parameter assumptions, including mark-up shocks. Section 6 discusses related issues, such as a robustness check and relation to preceding studies. Section 7 concludes the paper.

[5]Empirical analyses by Coibion (2010), Coibion and Gorodnichenko (2012, 2015), Lahiri and Sheng (2008), and Mankiw, Reis, and Wolfers (2004) broadly point to the importance and substantial impacts of information rigidities.

[6]A number of extensions have been presented following Woodford (2002). Fukunaga (2007), Nimark (2008) and Angeletos and La'O (2009) develop dynamic general equilibrium models in which firms set their prices subject to nominal rigities. Combined with sticky-pricing, the imperfect common knowledge in their models successfully generates some empirical observations, including aggregate inflation dynamics and average price durations. Adam (2007) analyzes the optimal monetary policy in an economy where some fundamentals are imperfect common knowledge. Sims (2003) and Mackowiak and Wiederholt (2009) emphasize the role of the constrained and limited information processing capacity of firms, rather than the limited information availability focused on by Woodford (2002) and others.

2 Inflation Persistence and Market Concentration

2.1 Data

We use two datasets to explore the empirical characteristics of sectoral inflation persistence and market concentration. The first dataset was created originally by Boivin, Giannoni, and Mihov (2009) and was downloaded from the AEA website.[7] We use US producer prices at the six-digit level of the NAICS code. The dataset includes monthly prices of over 150 items, and the sample period runs from February 1976 to June 2005. For clarification, we calculate the first-order autocorrelation of the monthly changes of seasonally adjusted prices and define this measure as "sectoral inflation persistence." This measure of inflation persistence is exactly the same as that used in Bils and Klenow (2004).[8] We focus on PPI rather than CPI because our primary aim is to highlight the linkage between inflation persistence of a particular good/service and the degree of market concentration of that sector. For this aim, matching the PPI item classification and industry/sector classification of market concentration data is pivotal. In terms of item/industry, the PPI classification and that of the 2007 Economic Census can be fully matched, while if we use CPI, item/industry matching with the census requires much more complicated and arbitrary data processing.

While Boivin, Giannoni, and Mihov's (2009) dataset is quite accessible and ready for use, we build a similar dataset that is more recent. Our second dataset contains the same PPI but its sample period runs from January 2004 to February 2017. The first dataset has 272 manufacturing sectors while the second dataset includes 383 sectors, including nonmanufacturing sectors. We present descriptive statistics of the sectoral inflation persistence of the two datasets in Table 1.

With respect to the measurement of market concentration, two indicators are available. The most popular indicator of market concentration is the share of the top four largest firms in the sector, which is often noted as the C4 ratio.[9] As a robustness check, we employ another indicator of market concentration, the classic Herfindahl–Hirschman Index (HHI). Both are

[7] https://www.aeaweb.org/articles?id=10.1257/aer.99.1.350

[8] Fuhrer (2011) discusses a battery of measures of inflation persistence, including first-order autocorrelation. Among alternative measures, for instance, Boivin, Giannoni, and Mihov (2009) use the sum of all AR coefficients including 13 lags. As a robustness check, we also use this alternative measure and confirm that our main finding, i.e., negative correlation with market concentration, remains unchanged.

[9] Carlton (1986), Bils and Klenow (2004), and Boivin, Giannoni, and Mihov (2009) use this C4 ratio as a measure of market concentration.

taken from the 2007 Economic Census. While the C4 ratio is available for all sectors, the HHI is available only for the manufacturing sectors.

[Table 1 here]

2.2 Regression results

Before proceeding to regression analysis, we present scatterplots in Figures 2 using the four-digit NAICS code data. Together with Figure 1 introduced in Section 1, both figures indicate a negative correlation between sectoral inflation persistence and market concentration at a moderately disaggregate level.[10] With this casual observation in place, we move on to formal regression analysis based on more disaggregated NAICS six-digit level data.[11]

[Figures 2 here]

We run weighted least squares (WLS) regressions using sectoral inflation persistence as the dependent variable. Our explanatory variables include indicators of market concentration (i.e., C4 ratio and HHI) but also several control variables in various specifications for robustness checks. In Tables 2, 3 and 4, all the regression results, using both of the two datasets, indicate that our measures of market concentration have statistically significant power with negative coefficients in accounting for sectoral inflation persistence. One proviso may be that the correlation is even clearer if we focus on the manufacturing sector. In fact, Boivin, Giannoni, and Mihov's (2009) dataset does not cover nonmanufacturing sectors. While both price measurement and industry classification are done more simply and accurately for manufacturing sectors, data collection, measurement, and aggregation for nonmanufacturing sectors are likely to involve various more complex issues.

Now we turn to the model to account for the empirical findings in the next section. After comparing the model's predictions with the facts, we will address some remaining issues, in relation to some early empirical studies in Section 6.

[10] Bils and Klenow (2004) examine sectoral inflation persistence, defined in the same manner as in our study, and report that sectoral inflation persistence does not have statistically significant correlation with frequency of price changes, which implies inconsistent prediction with Calvo pricing. We will discuss these issues related to the findings of previous studies later in Section 6.

[11] If idiosyncratic shocks are large, then inflation persistence in highly concentrated sector can be lowered by the effects of the shocks on individual firms' prices. However, we suppose that the effects of the idiosyncratic shocks on our empirical finding can be disregarded because the regression result for sub-samples (using NAICS four-digit level data) as C4 ratio $\leq 50\%$ exhibits the same level of negative correlation.

[Tables 2, 3 and 4 here]

3 The Model Set-up

3.1 Shock Process, Preference and Firms as Price-Setters

There are two types of players, one representative consumer and a continuum of firms, indexed by $i \in [0, N]$. Every firm operates under monopolistic competition, and the population of firms implies a mass of product varieties. The game has infinite periods, indexed by $t \in \{0, 1, 2, ...\}$. The consumer maximizes his/her utility u by choosing the quantity of consumption of the goods, $q_t(i) \in \mathbb{R}$ for $i \in [0, N]$ and $q_A \in \mathbb{R}$, given his/her endowment $\overline{q_A} \in \mathbb{R}_{++}$ in each period. The firm $i \in [0, N]$ maximizes its profit $\pi(p_t(i), c_t) : \mathbb{R}_+^2 \to \mathbb{R}$ by producing its good with variable cost $c_t \in \mathbb{R}_+$, for any $t \in \{0, 1, 2, ...\}$,[12] which follows the stochastic process given by:

$$
\begin{aligned}
c_t &= c_{t-1} + \epsilon_t, &(1)\\
\epsilon_t &= \rho\epsilon_{t-1} + \eta_{t,} &(2)
\end{aligned}
$$

where $\eta_t \sim \mathcal{N}(0, \sigma^2)$ and $0 < \rho < 1$. For the sake of simplicity, there is no time discounting. This simplification makes little difference to our results because both firms and the consumer solve period-by-period optimization problems and, as a result, the equilibrium is characterized as a unique Nash Equilibrium of a one-shot game, as shown later in this section.

Following Melitz and Ottaviano (2008), the consumer enjoys consumption of many varieties of products, $q_t(i)$ for $i \in [0, N]$ and q_A. Specifically, the consumer's utility function u, is given by:

$$
u\left(q_t\left(i\right),\ q_A\right) \equiv \alpha_t \int_{i\in[0,N]} q_t(i)di - \frac{\beta}{2}\int_{i\in[0,N]} (q_t(i))^2 di - \frac{\gamma}{2}\left[\int_{i\in[0,N]} q_t(i)di\right]^2 + q_A, \quad (3)
$$

for any $t \in \{0, 1, 2, ...\}$. The parameter $\alpha_t \in \mathbb{R}_{++}$ and $\gamma \in \mathbb{R}_{++}$ govern the substitutability of

[12] In our model, we implicitly assume that c_0 is sufficiently large. This assumption effectively allows us to ignore the possibility that $c_t < 0$.

product varieties with q_A.[13] A larger α_t shifts out the demand for differentiated varieties i.e., as α_t increases, the consumer consumes $q_t(i)$ more for any $i \in [0, N]$.[14] As γ increases, the total demand for the differentiated goods compared to the numeraire decreases, amplifying the degree of competition across firms. The parameter $\beta \in \mathbb{R}_+$ indicates the degree of the love of variety. If $\beta = 0$, the differentiated varieties are perfect substitutes for each other and, hence, the consumer is interested only in the total consumption of the differentiated goods ($\int_{i\in[0,N]} q_t(i)di$). As β increases, the consumer cares more about the consumption distribution across varieties. The budget constraint of the consumer for $t \in \{0, 1, 2, ...\}$ is given by:

$$\int_{i\in[0,N]} q_t(i)p_t(i)di + q_A \leq \overline{q_A},$$

where $p_t(i)$ is the price of each good and $\overline{q_A}$ is the endowment of the numeraire, which is exogenously provided with the same amount in each period. Note that this inequality always binds because of the monotonicity of the utility function u. The consumer has no option to save the endowment in this economy. Given the setting for the consumer, the consumer makes his/her decisions on $q_t(i)$ for $t \in \{0, 1, 2, ...\}$ and $i \in [0, N]$. Then, the following linear demand function is obtained:[15]

$$q_t(i) = \left(\frac{1}{\beta + \gamma N}\right)\alpha - \frac{1}{\beta}p_t(i) + \left(\frac{\gamma N}{\beta\left(\beta + \gamma N\right)}\right)\overline{p}_t, \tag{4}$$

where:

$$\overline{p}_t \equiv \frac{1}{N}\int_{i\in[0,N]} p_t(i)di.$$

As $\gamma N/\left[\beta(\beta + \gamma N)\right]$ in the third term on the right-hand side of (4) is positive, there are strategic complementarities between a firm's own price $p_t(i)$ and the prices of others $\overline{p}_t$. The degree of the complementarities increases as γ and N increase. As only the combined term γN matters in our model, we define $\tilde{N} \equiv \gamma N \in \mathbb{R}_{++}$ for brevity. Note that $\tilde{N}$ is the parameter indicating the degree of concentration in the market. A smaller $\tilde{N}$ means that the market is highly concentrated. Also, note that α is a shift parameter affecting the demand

[13] The consumer problem has an interior solution only if γ is strictly positive. If γ is zero, then the consumer consumes only q_A or $q_t(i)$ (for $i \in [0, N]$). In this case, we cannot define the price.

[14] For the moment, we assume the variable is constant (i.e., $\alpha_t = \alpha$) for the sake of analytical simplicity. However, in Section 5.3, the case in which α_t follows a random walk process ($\alpha_t = \alpha_{t-1} + \xi_t$ where $\xi_t \sim \mathcal{N}(0, \zeta^2)$) is to be numerically examined.

[15] See Appendix 1 for the derivation of the demand function given by (4).

independently from the prices $p_t(i)$ and $\overline{p}_t$. Hence, hereafter, we refer to α as the demand parameter.

Now, the profit function of firm i can be written as $\pi(p_t(i), c_t) = (p_t(i) - c_t)q_t(i)$. In the following analysis, we examine the price-setting strategies of the firms, given the linear demand function shown in (4).

3.2 Perfect Information Equilibrium as a Benchmark Case

In this subsection, we analyze the benchmark case where the history of marginal costs $\{c_s\}_{s=0}^t$ is perfectly observable in period $t \in \{0, 1, 2, ...\}$. In this case, every firm solves for a unique Nash equilibrium of the game, which is defined as follows.

Definition 1 *A Nash equilibrium in period $t \in \{0, 1, 2, ...\}$ is given by a set of the prices of each firm, $p_t^{**}(i)$ for $i \in [0, N]$, where each firm maximizes $\pi(p_t(i), c_t)$, by choosing $p_t(i)$, given the exogenously determined marginal cost c_t and the demand system $q_t(i)$ for $i \in [0, N]$.*

We denote $\kappa(\tilde{N}) \equiv \beta/(2\beta + \tilde{N})$. Then, given the maximization problem of firm i:

$$\max_{\{p_t(i)\}} (p_t(i) - c_t)q_t(i), \tag{5}$$

subject to (4), the individual price $p_t^{**}(i)$ and the average price $\overline{p}_t^{**}$ in a Nash equilibrium are written as:

$$p_t^{**}(i) = \overline{p}_t^{**} = \kappa(\tilde{N})\alpha + [1 - \kappa(\tilde{N})]c_t. \tag{6}$$

Note that $\kappa(\tilde{N}) \in [0, 1/2]$ and $\kappa'(\tilde{N}) < 0$. Therefore, the weight on the marginal cost of unitary production c_t is at least $1/2$ and increases to one as $\tilde{N}$ increases.

4 Imperfect Information Equilibrium

4.1 Information Structure and Equilibrium Characterization

We define the information set of firm i as $\mathcal{H}_t(i)$, which includes all of the parameters and variables other than $\{c_s\}_{s=1}^t$. This section examines the case where firm i observes private information $x_t(i)$, instead of c_t, given by:

$$x_t(i) = c_t + \delta_t(i), \tag{7}$$

where $\delta_t(i) \sim \mathcal{N}(0, \tau^2)$. Therefore, $c_0 \in \mathcal{H}_t(i)$ and $\{x_s(i)\}_{s=1}^t \in \mathcal{H}_t(i)$, but $\{c_s\}_{s=1}^t \notin \mathcal{H}_t(i)$.[16] Note that, because $x_t(i)$ varies across firms, firms are receiving private signals in this economy. It could be interpreted that every firm observes accounting information $x_s(i)$ on its own cost c_s for $s \in \{1, ..., t-1, t\}$, yet the firm needs to guess the true cost based on the accounting information. In this case, every firm solves for a unique Bayesian Nash equilibrium of the game, defined as follows.

Definition 2 *A Bayesian Nash equilibrium in period $t \in \{0, 1, 2, ...\}$ is given by a set of the prices of each firm, $p_t^*(i)$ for $i \in [0, N]$, where each firm maximizes $E_i[\pi(p_t(i), c_t)|\mathcal{H}_t(i)]$ by choosing $p_t(i)$ given the beliefs regarding marginal cost c_t and the demand system, $q_t(i)$ for $i \in [0, N]$.*

In period t, the firm's problem with the information set $\mathcal{H}_t(i)$ for each $i \in [0, N]$ is now written as:

$$\max_{\{p_t(i)\}} \; E_i[(p_t(i) - c_t)q_{i,t}|\mathcal{H}_t(i)],$$

subject to (4). We denote $r(\tilde{N}) \equiv \tilde{N}/(2\beta + 2\tilde{N}) \in (0, 1/2)$ and let λ_t express the ratio of variance of the prior belief held by individual firms divided by the total variance, i.e., the sum of variances of prior and private signals. That is:

$$\lambda_t \equiv \frac{Var[c_t|\mathcal{H}_{t-1}(i)]}{Var[c_t|\mathcal{H}_{t-1}(i)] + \tau^2}, \tag{8}$$

where $Var[c_t|\mathcal{H}_{t-1}(i)] = Var[c_{t-1} + \rho\epsilon_{t-1}|\mathcal{H}_{t-1}(i)] + \sigma^2$ from (1).

Definition 3 *Given the (first-order) expectations E_i, let the average of the expectations across all firms be:*

$$\overline{E} \equiv \overline{E}^1 \equiv \frac{1}{N}\int_{i\in[0,N]} E_i di.$$

[16] The marginal costs c_t may effectively be observable in period $t+s$. For instance, firms may try to infer the true costs $\{c_s\}_{s=1}^{t-1}$ based on the information about observable prices $\{\overline{p}_s\}_{s=1}^{t-1}$. In Section 6, we discuss an alternative case with private and noisy public signals.

We use $E_i\overline{E}^{j-1}$ to denote the j-th order expectations. Then, let the averages of the j-th order expectations for $j \in \{2, 3, 4, ...\}$ be:

$$\overline{E}^j \equiv \frac{1}{N} \int_{i\in[0,N]} E_i \overline{E}^{j-1} di.$$

Now, we are ready to state the following results.

Proposition 1 *The average price in a symmetric Bayesian Nash Equilibrium in period t, $\overline{p}_t^*$, is given by:*

$$\overline{p}_t^* = \kappa(\tilde{N})\alpha + \left(1 - \kappa(\tilde{N})\right)\left(1 - r(\tilde{N})\right)\sum_{j=0}^{\infty} r(\tilde{N})^j \overline{E}^{j+1}[c_t|\mathcal{H}_t(i)], \tag{9}$$

where:

$$\overline{E}^{j+1}[c_t|\mathcal{H}_t(i)] = \lambda_t^{j+1} c_t + (1 - \lambda_t)\sum_{k=1}^{j+1} {\lambda_t}^{j+1-k}\overline{E}^k[c_t|\mathcal{H}_{t-1}(i)], \tag{10}$$

and λ_t is given by (8).

Sketch of the Proof: For firm i, the best-response function to choose $p_t(i)$ is given by:

$$p_t(i) = \frac{\alpha\beta + \tilde{N}E_i[\overline{p_t}|\mathcal{H}_t(i)]}{2(\beta + \tilde{N})} + \frac{E_i[c_t|\mathcal{H}_t(i)]}{2}. \tag{11}$$

By taking the average of (11) over $i \in [0, N]$, $\overline{p_t}$ is expressed as:

$$\overline{p}_t = \frac{\alpha\beta + \tilde{N}\overline{E}[\overline{p_t}|\mathcal{H}_t(i)]}{2(\beta + \tilde{N})} + \frac{\overline{E}[c_t|\mathcal{H}_t(i)]}{2}. \tag{12}$$

By plugging (12) into (11) repeatedly, we obtain (9).

Next, we show the process by which the expectations are updated. The prior belief regarding c_t is denoted as $\mu_{i,t-1}(c_t|\mathcal{H}_{t-1}(i))$, which follows $\mu_{i,t-1}(c_t|\mathcal{H}_{t-1}(i)) \sim \mathcal{N}\left(E_i[c_t|\mathcal{H}_{t-1}(i)], Var[c_t|\mathcal{H}_{t-1}(i)]\right)$. Accordingly, the posterior belief $\mu_{i,t}(c_t|\mathcal{H}_t(i))$ after observing the private signal $x_t(i)$, is written as $\mu_{i,t}(c_t|\mathcal{H}_t(i)) \sim \mathcal{N}\left(E_i[c_t|\mathcal{H}_t(i)], Var[c_t|\mathcal{H}_t(i)]\right)$. By Bayes' rule, the updating processes of $E_i[c_t|\mathcal{H}_t(i)]$ and $Var[c_t|\mathcal{H}_t(i)]$ are given by:

$$E_i[c_t|\mathcal{H}_t(i)] = \lambda_t x_t(i) + (1 - \lambda_t)\, E_i[c_t|\mathcal{H}_{t-1}(i)], \tag{13}$$

and $Var[c_t|\mathcal{H}_t(i)] = (1-\lambda_t)Var[c_t|\mathcal{H}_{t-1}(i)]$, respectively.

Thus far, we have derived the first-order expectations on c_t for $t \in \{1, 2, 3, ...\}$. Next, we focus on the higher-order expectations in period $t \in \{1, 2, 3, ...\}$. Bearing in mind that $\int_{i\in[0,N]} x_t(i)/N = c_t$, from (13), the average expectations of c_t with posterior beliefs, $\overline{E}_t[c_t|\mathcal{H}_t(i)]$, can be expressed as:

$$\overline{E}[c_t|\mathcal{H}_t(i)] = \lambda_t c_t + (1-\lambda_t)\overline{E}_t[c_t|\mathcal{H}_{t-1}(i)].$$

For expository purposes, we denote the information set of firm $\hat{\imath} \in [0, N]$ as $\mathcal{H}_t(\hat{\imath})$. Then, firm i's expectations of the average expectations (i.e., the second order expectations), $E_i[\overline{E}[c_t|\mathcal{H}_t(\hat{\imath})]|\mathcal{H}_t(i)]$, is given by:

$$E_i[\overline{E}[c_t|\mathcal{H}_t(\hat{\imath})]|\mathcal{H}_t(i)] = \lambda_t^2 x_t(i) + (1-\lambda_t)\left\{E_i[\overline{E}_t[c_t|\mathcal{H}_{t-1}(\hat{\imath})]|\mathcal{H}_t(i)] + \lambda_t E_i[c_t|\mathcal{H}_t(i)]\right\}.$$

Therefore, by the same calculation, the j-th order expectations can be calculated as:

$$E_i[\overline{E}^j[c_t|\mathcal{H}_t(\hat{\imath})]|\mathcal{H}_t(i)] = \lambda_t^{j+1} x_t(i) + (1-\lambda_t)\sum_{k=1}^{j+1} \lambda_t^{j+1-k} E_i[\overline{E}_t^{k-1}[c_t|\mathcal{H}_{t-1}(\hat{\imath})]|\mathcal{H}_t(i)]. \qquad (14)$$

Taking the average of (14) over $i \in [0, N]$, we obtain (10).□

A formal proof is available upon request. Here, two remarks are in order. One notable feature is the proximity of (6) and (9). In the imperfect information equilibrium, (9) indicates that the price is set as the weighted sum of expectations of the parameter on demand (α) and the marginal cost (c_t), whereas, under perfect information, (6) is a weighted sum of actual α and c_t. In both cases, the weights on each factor, $\kappa(\tilde{N})$ and $1-\kappa(\tilde{N})$, sum to one. The other remark is that, in the second term on the right-hand side of (9), the weights on higher-order expectations can be added up to one. These features confirm the insight that (6) is a special case of (9); namely, (6) arises when τ^2 is zero, or equivalently, when λ_t is unity.

4.2 Comparative Statics

4.2.1 Equilibrium Prices at the Steady State

In the previous subsection, we derive the explicit solution form (9) by applying the "brute force" solution method demonstrated by Morris and Shin (2002). While (9) is an intuitively clear expression, working with the infinite sum in (9) is intractable. In the meantime, because the firms' optimization problem is linear-quadratic and the noises follow Gaussian, the problem can be transformed into a Bayesian potential game and the objective function can be expressed by a Bayesian potential function. In this section, we apply formula to solve our model which is redefined as a Bayesian potential game and derive an alternative, more tractable solution form of the equilibrium prices.[17] Then, using the alternative expression at the steady state, where t takes a sufficiently large number and λ_t converges to a constant value λ, we analytically show how sectoral inflation persistence depends on market concentration and information structure.

In parallel with (9), the alternative expression of the average equilibrium price in the steady state $(t \to \infty)$ is summarized in the following proposition.

Proposition 2 *The average price in a symmetric Bayesian Nash equilibrium in the steady state is given as follows:*

$$\overline{p}_t^* = \kappa(\tilde{N})\alpha + \left(1 - \kappa(\tilde{N})\right)\left(c_0 + \sum_{s=0}^{\infty}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)\left[1 - (1-\omega(\tilde{N}))^{s+1}\right]\eta_{t-s}\right), \tag{15}$$

where

$$\omega(\tilde{N}) = \frac{(1-r(\tilde{N}))\lambda}{1-r(\tilde{N})\lambda}, \tag{16}$$

and λ satisfies,

$$\lambda = \frac{(1+\rho(1-\lambda))\sigma^2}{(1+\rho(1-\lambda))\sigma^2 + \lambda(1-\rho(1-\lambda))(1-\rho^2(1-\lambda))\tau^2}. \tag{17}$$

Proof: See Appendix B.1.□

To acquire intuition behind (15), suppose $\rho = 0$ for simplicity. Then, (15) is simplified as $\overline{p}_t^* = \kappa(\tilde{N})\alpha + \left(1 - \kappa(\tilde{N})\right)\left(c_0 + \sum_{s=0}^{\infty}\left[1 - (1-\omega(\tilde{N}))^{s+1}\right]\eta_{t-s}\right)$. In period t, $\overline{p}_t^*$ reflects

[17] To apply the formula, "interim prior" needs to be calculated. For details see Appendix B.1.

information of the entire sequence of $\left\{\eta_{t-s}\right\}_{s=0}^{\infty}$ and $\omega(\tilde{N})$ plays a critical role in determining the weights on each η_{t-s}. If $\omega(\tilde{N})$ is close to zero, price adjusts more slowly because a very old shock (i.e., η_{t-s} with a large s) is not fully reflected in $\overline{p}_t^*$ even after long period of time passed since period $t-s$. As can be confirmed by (16), $\omega(\tilde{N})$ can take the maximum value at λ. That is, $\omega(\tilde{N}) \leq \lambda$. Recall that λ governs the speed of signal extraction where a single agent is engaged in Bayesian updating. By contrast, because many firms are collectively updating their beliefs in this model, it is not optimal for each firm to simply make the best guess of the true state (at the speed of λ), but all firms share additional incentive in this game to coordinate not to deviate from the average behavior. Because of this additional incentive held by each firm, the optimal speed of collective updating $\omega(\tilde{N})$ is always smaller than λ.

As discussed so far, the coordination motives make ω smaller than λ. Angeletos and La'O (2009) and other early studies point out that $\omega'(r) < 0$, which implies that higher strategic complementarity, represented by r, reduces the speed of price adjustment. Our value-added here is that because ω (and r) explicitly depends on $\tilde{N}$, our model's prediction can be taken to observable data, namely, market concentration.

4.2.2 Inflation Persistence, Information Structure and Market Concentration

Using (15), now we analytically show how sectoral inflation persistence is related to market concentration. Define $\widetilde{\pi}_t \equiv \overline{p}_t^* - \overline{p}_{t-1}^*$ and $\widetilde{\pi}_{t-1} \equiv \overline{p}_{t-1}^* - \overline{p}_{t-2}^*$. Accordingly, define $\widetilde{\rho}(\widetilde{\pi}_t, \widetilde{\pi}_{t-1}) \equiv Cov(\widetilde{\pi}_t, \widetilde{\pi}_{t-1})/Var(\widetilde{\pi}_{t-1})$ as the first-order quasi-autocorrelation function. We are ready to state the set of main results as follows.

Lemma 1 *$\widetilde{\pi}_t$ and $\widetilde{\pi}_{t-1}$ are respectively given by,*

$$
\begin{aligned}
\widetilde{\pi}_t &= \left(1-\kappa(\tilde{N})\right)\left\{\omega(\tilde{N})\eta_t + \sum_{s=1}^{\infty}\Theta_{t-s}(\tilde{N})\eta_{t-s}\right\}, \\
\widetilde{\pi}_{t-1} &= \left(1-\kappa(\tilde{N})\right)\sum_{s=1}^{\infty}\Theta_{t-s+1}(\tilde{N})\eta_{t-s},
\end{aligned}
$$

where $\Theta_{t-s}(\tilde{N})$ *is given by,*

$$\Theta_{t-s}(\tilde{N}) \equiv \left[\left(\frac{1-\rho^s}{1-\rho}\right)(1-\omega(\tilde{N}))^s + \rho^s\left(\frac{1-(1-\omega(\tilde{N}))^{s+1}}{1-(1-\omega(\tilde{N}))}\right)\right]\omega(\tilde{N}).$$

Proof: See Appendix B.2.□

Lemma 2 *The following inequalities hold:*

$$\begin{array}{lcl} (i) \quad \dfrac{\partial \omega(\tilde{N})}{\partial \widetilde{N}} & < & 0, \\ (ii) \quad \dfrac{\partial \omega(\tilde{N})}{\partial \tau^2} & < & 0. \end{array}$$

Proof: (i) From (16) and $r(\tilde{N}) \equiv \tilde{N}/(2\beta + 2\tilde{N})$,

$$\frac{\partial \omega(\tilde{N})}{\partial \widetilde{N}} = -\frac{2\beta\lambda(1-\lambda)}{\left(2\beta + \widetilde{N}(2-\lambda)\right)^2} < 0.$$

(ii) Similarly, $\partial\omega(\tilde{N})/\partial\tau^2$ can be written as,

$$\frac{\partial \omega(\tilde{N})}{\partial \tau^2} = \frac{\partial \omega(\tilde{N})}{\partial \lambda}\frac{\partial \lambda}{\partial \tau^2} = \frac{1-r(\tilde{N})}{\left(1-r(\tilde{N})\lambda\right)^2}\frac{\partial \lambda}{\partial \tau^2},$$

where $\partial\omega(\tilde{N})/\partial\lambda > 0$. It suffices to show $\partial\lambda/\partial\tau^2 < 0$. From (17), we obtain,

$$\tau^2 = \frac{(1-\lambda)(1+\rho-\rho\lambda)\sigma^2}{(1-\rho+\rho\lambda)(1-\rho^2+\rho^2\lambda)\lambda^2},$$

which clearly assures $\partial\lambda/\partial\tau^2 < 0$.□

Proposition 3 *The following inequalities hold:*

$$\begin{array}{lcl} \dfrac{\partial \widetilde{\rho}(\widetilde{\pi}_t, \widetilde{\pi}_{t-1})}{\partial \widetilde{N}} & > & 0, \\ \dfrac{\partial \widetilde{\rho}(\widetilde{\pi}_t, \widetilde{\pi}_{t-1})}{\partial \tau^2} & > & 0. \end{array}$$

Proof: See Appendix B.3.□

Proposition 3 formally confirms the economic intuition discussed so far in this section. To help understand the analytical structure, two limiting cases worth noting. If $\omega(\tilde{N}) = 1$, then, $\widetilde{\rho}(\widetilde{\pi}_t, \widetilde{\pi}_{t-1}) = \rho$. Further, if $\rho = 0$, $\widetilde{\rho}(\widetilde{\pi}_t, \widetilde{\pi}_{t-1}) = 1 - \omega(\tilde{N})$, which is obviously decreasing in $\omega(\tilde{N})$, and thus increasing in $\tilde{N}$ and τ^2.

5 Inflation Dynamics: Numerical Assessment

5.1 Computation and Parameterization

In this section, we numerically illustrate the inflation dynamics generated by our model. Because the equilibrium average price $\overline{p}_t^*$ has an explicit solution form in our model as shown in (15), we do not need to rely on any approximation to compute the dynamics.

We summarize the benchmark parametrization used for numerical illustrations: $\alpha = 200$, $c_0 = 100$, and $\beta = 200$. We show results under different values for ρ and τ/σ. The extrinsic persistence is determined by ρ, which takes values of 0, 0.25, and 0.5 in our simulations. Even in the case where ρ is zero, our model generates fairly persistent inflation dynamics, as discussed in the next subsection. The remaining key parameter is τ/σ, which indicates the noisiness of private information (normalized by σ). We employ a fairly wide range of τ/σ in numerical illustrations and confirm that, quite naturally, imprecise private signals (i.e., a large τ/σ) give rise to noticeably persistent inflation dynamics. With the parameter set, we focus on how the results quantitatively change along with the degree of market concentration represented by $\tilde{N}$ in this section.

5.2 Impulse Response Functions

First, we examine the impulse response of *changes* in average prices in one sector—i.e., sectoral inflation—to a temporary shock to the *level* of marginal cost. Specifically, we give a one-period negative impulse for η_t in (1) and inspect the inflation dynamics. Note that this impulse in η_t gives rise to a permanent (downward) shift in the level of marginal cost c_t whereas it has no effect on the rate of changes of the marginal cost as depicted in panel (a) of Figure 3.

Figure 3(b) shows the resulting dynamic responses in sectoral inflation. Not surprisingly, inflation rates respond to the cost shock immediately on impact and die out very quickly under the perfect information case, as shown in the center panel in Figure 4. By contrast, the right-hand panel clearly indicates that negative inflation rates continue for substantially multiple periods under the imperfect information case. In addition, the bold line on the right-hand panel points to a several-periods delay in the maximum impact on inflation, or hump-shaped dynamics similar to those generated by Mankiw and Reis (2002). The underlying mechanism of our model that generates the difference between the center panel and the right-hand panel is broadly the same as Woodford (2002) and its variants despite different consumer preferences and a different market structure.

Another notable observation in our simulation results, shown in Figure 3, is the discrepancy between the bold lines and the dotted lines in each panel. The bold and dotted lines indicate, respectively, the inflation response in markets with low concentration ($\log(\tilde{N}) = 4$) and high concentration ($\log(\tilde{N}) = 0$). Clearly, the magnitude of the inflation responses are amplified under the case with a large $\tilde{N}$ compared to that with a small $\tilde{N}$.

[Figure 3 here]

This numerical result can be summarized in the following proposition.

Proposition 4 *Define the impulse response function of inflation* $\Psi_{\pi,k} \equiv \partial\widetilde{\pi}_{t+k}/\partial\eta_t$. *Then, the following inequality holds for any* $k \in \{0, 1, 2, ...\}$:

$$\frac{\partial\Psi_{\pi,k}}{\partial\tilde{N}} > 0.$$

Proof: See Appendix B.4.□

We will discuss related issues to this proposition later in Section 6.

5.3 Inflation Persistence, Market Concentration and Mark-up Shocks

To quantitatively assess the causal relationship between market concentration and inflation persistence, we perform Monte Carlo simulations with the parameter sets listed in subsection 5.1. Specifically, the cost shocks $\{\eta_t\}_{t=1}^{2,000}$ in (1) are drawn from a standard normal distribution (i.e., $\sigma = 1$) and are given to the model. The model generates the sequence of the average prices and the first-order autocorrelations of the inflation rates (the log differences of the prices) are calculated. The relationship between the autocorrelation of the simulated path of inflation and the parameter capturing the market concentration ($\tilde{N}$) is plotted in Figure 4.

In Figure 4, the vertical axis shows the first-order autocorrelation of the inflation rates. The horizontal axis indicates $\tilde{N}$ in natural logs. Recall that a smaller $\tilde{N}$ means higher concentration. An important takeaway from Figure 4 is that downward-sloping curves are observed unless the private signal is very precise (i.e., τ/σ is 0.1 in Figure 4). The simulation results broadly replicate the observed relationship between inflation persistence and market concentration, as observed in Section 2 and analytically shown in Section 4.[18]

[Figure 4 here]

Finally, we incorporate mark-up shocks into our model to check the robustness of the model's prediction in comparison with the data. Specifically, we consider the case in which α_t in (3) follows a random walk process $\alpha_t = \alpha_{t-1} + \xi_t$ where $\xi_t \sim \mathcal{N}(0, \zeta^2)$ represents a mark-up shock in our model.[19] Figure 5 maps the prediction of the model with the mark-up shocks onto the data. The shaded region in Figure 5 shows the model prediction including mark-up shocks. The results tend to show noisier relationship between market concentration and inflation persistence, particularly if a sector is less concentrated (i.e., N is large in the sector). The model with the mark-up shocks replicates the data reasonably well.

[Figure 5 here]

6 Discussion

6.1 Price as Endogenous Public Information

In the model presented in Section 4, firms cannot observe the true marginal cost even after an infinitely long period of time.[20] Regarding this assumption, it may be argued that firms can infer the true marginal cost after observing the average prices set by others. In other

[18] As the relationship in Section 2 is unconditional (i.e., other sector-specific shocks and parameters, including information precision τ/σ, are not controlled), it is not surprising that the negative correlations appear to be weaker than predicted by Figure 4.

[19] As noted in Section 3.1, α_t represents a demand shift parameter in Melitz and Ottaviano's (2008) preference while it can effectively be interpreted as a mark-up shock as shown in (6).

[20] We follow Woodford (2002) on this assumption.

words, while the true cost may remain unobservable for a long period of time, actual average prices are likely to be publicly available data. This subsection demonstrates that our main results remain intact even if firms try to infer c_t in period $t+1$ based on publicly available data of average price $\widehat{p}_t^*$ as long as the publicly available data contain a certain amount of noise and/or measurement error.[21].

We denote noisy public information on the average price ($\overline{p}_t^*$) as $\widehat{p}_t^* \sim \mathcal{N}(\overline{p}_t^*, e^2)$ such as price and deflator statistics and assume that $\widehat{p}_t^*$ is made public/observable in period $t+1$. Here, $\widehat{p}_t^*$ contains measurement error of which the variance is e^2. In period $t+1$, the unbiased public signal on marginal cost denoted by $y_t \sim \mathcal{N}\left(c_t, \mathcal{S}e^2\right)$ can be formed based on $\widehat{p}_t^*$ where,

$$\mathcal{S} \equiv \left(\frac{\left(1 - \kappa(\tilde{N})\right)(1 - r(\tilde{N}))\lambda_t}{1 - r(\tilde{N})\lambda_t} \right)^{-2}.$$

Note that $\mathcal{S}$ is decreasing in $\lambda_t \in (0,1)$, and $\mathcal{S} \to \infty$ as $\lambda_t \to 0$ holds. This property means that imprecision of y_t stemming from e^2 is amplified by a smaller λ_t (i.e., the private signal in period t is very noisy). If the private signals arriving in every period are noisy enough, firms' beliefs in period $t+1$ are marginally affected by the public signal y_t and because y_t has very little information value, the updating process of the beliefs with public signal y_t remains broadly the same as that without y_t articulated in our main results.

Figures 6 and 7 confirm that, as long as the price data contain certain measurement error, our main results remain almost the same quantitatively.[22] In Figure 6, panel (a) is the same as that in Figure 3. Panel (b) indicates that the dynamic responses of sectoral inflation with public information are similar to those without such information as depicted in Figure 3. In terms of the correlation between sectoral inflation persistence and market concentration, Figure 7 confirms that the model's prediction remains almost unchanged compared with those in Figure 4.

[Figures 6 and 7 here]

[21] The following argument follows Amador and Weill (2010).

[22] As for the derivation of the average price for this numerical illustration, see Appendix C.

6.2 Relations to Early Studies

6.2.1 Empirical Evidence of Sticky Prices

Our model does not include nominal rigidities in firms' price-setting to crystallize our main results focusing on inflation persistence. In a related vein, an empirical study by Bils and Klenow (2004) provides an important result. They show scatterplots of sectoral inflation persistence and frequency of price changes.[23] Based on the scatterplots, they point out that, there is almost no correlation found between sectoral inflation persistence and frequency of price changes across sectors although these two should be negatively correlated if firms are following Calvo pricing. In other words, if prices are more sticky in a Calvo model (i.e., the Calvo parameter measured by the monthly frequency of price changes is lower), higher inflation persistence would be observed. They report, however, that sectoral inflation persistence and frequency of price changes have a *positive* correlation of 0.26 using a sample from January 1995 to June 2000. Their findings imply that there exists evidence of sticky prices, however, sticky prices may not be a major determinant/source of inflation persistence. In line with their view, we do not include price stickiness in our model, not because prices are flexible, but because, bearing in mind that sticky prices are not confirmed as a major source of inflation persistence, we aim to show our results without relying on sticky prices.[24]

6.2.2 Market Concentration and Price Dynamics

A prima facie idea is that large firms in a highly concentrated market—conceivably with strong monopolistic power—may change their prices less frequently. Namely, prices will be stickier in more concentrated sectors. This hypothesis has been repeatedly tested in the literature and mixed results have been reported. Bils and Klenow (2004) examine how the frequency of price changes is related to market concentration, proxied by the same indicator as ours, the C4 ratio, across sectors. They conclude that there is no robust relationship between frequency of price changes and market concentration. As argued by Bils and Klenow (2004) themselves, we reemphasize that there is no clear empirical relationship between price

[23] See Figures 2 and 3 in Bils and Klenow (2004). Table 4 shows the correlation between (sectoral) inflation persistence and frequency of price changes.

[24] Angeletos and La'O (2009) discuss the interaction of sticky prices, imperfect common knowledge and strategic complementarity in full detail. Our model can be interpreted as a special case of the Calvo parameter in their framework, while it also has an additional dimension of strategic complementarity proxied by market concentration. An early attempt in a similar context was made by Nishimura (1986).

change frequency and inflation persistence. Hence, the fact that price change frequency has no correlation with the C4 ratio provides no direct implication for our findings reported in Section 2.

In a closely related vein, Boivin, Giannoni, and Mihov (2009) estimate the impulse responses of the *price levels* of goods to an aggregate shock and argue that *more competitive* sectors have higher price flexibility.[25] At a glance, their finding contradicts ours as illustrated in Section 2. Conversely, the fact is that our model replicates their empirical findings with respect to the impulse response functions of price levels. As illustrated by the center and right panels in Figures 3, the solid lines always show larger swings in terms of size (not persistence in inflation) than those of the dotted lines. The solid lines indicate the responses of sectoral inflation in, let us say, *more competitive*, sectors. If we focus on the size of the changes in price levels or the deviation of price levels from the initial level, our model's prediction is consistent with the fact that more competitive sectors have higher price (level) flexibility as noted by Boivin, Giannoni, and Mihov (2009).

This argument is formally summarized in the following proposition.

Proposition 5 *Define the impulse response function of price levels* $\Psi_{p,k} \equiv \partial \overline{p}^{*}_{t+k}/\partial \eta_t$*. Then, the following inequality holds for any* $k \in \{0, 1, 2, ...\}$*:*

$$\frac{\partial \Psi_{p,k}}{\partial \tilde{N}} > 0.$$

Proof: see Appendix B.5.□

7 Concluding Remarks

This paper presents empirical facts that point to negative correlations between inflation persistence (measured autocorrelation of monthly price changes) and market concentration using US PPI data. Then, to provide a possible explanation for this observation, we build a dynamic stochastic model in which monopolistically competitive firms set their prices while receiving private signals on cost shocks. In the model, firms in a less concentrated sector respond to a cost shock more slowly as they rely more on higher-order expectations which

[25] Boivin, Giannoni, and Mihov (2009) separately estimate the impulse responses of price levels to "aggregate shocks" and "sector-specific shocks" which they identified using their factor vector autoregressions.

tend to be updated only slowly. As a result, the model generates highly persistent sectoral inflation, typically with hump-shaped dynamic responses, if the sector is less concentrated under imprecise private signals. Our model is broadly successful in replicating the cross-industry relation between market concentration and inflation persistence observed in the US PPI data.

Our model can be extended in multiple directions. One extension is to develop a general equilibrium model following Melitz and Ottaviano's (2008) approach to explore the implications for aggregate inflation dynamics. Another extension could be to endogenize firms' information acquisition choice, following models of rational inattention. Such extensions may provide more helpful insights into the inflation dynamics observed in the data.

References

[1] Adam, Klaus, 2007, "Optimal Monetary Policy with Imperfect Common Knowledge," *Journal of Monetary Economics* 54, pp. 267-301.

[2] Altissimo, Filippo, Benoit Mojon, and Paolo Zaffaroni, "Can Aggregation Explain the Persistence of Inflation?," *Journal of Monetary Economics* 56(2), pp. 231-241.

[3] Amador, Manuel, and Pierre-Olivier Weill, 2010, "Learning from Prices: Public Communication and Welfare," *Journal of Political Economy* 118(5), pp. 866-907.

[4] Angeletos, George-Marios, and Jennifer La'O, 2009, "Incomplete Information, Higher-Order Beliefs and Price Inertia," *Journal of Monetary Economics* 56, pp. S19-S37.

[5] Angeletos, George-Marios, and Alessandro Pavan, 2007, "Efficient Use of Information and Social Value of Information," *Econometrica* 75, pp. 1103-1142.

[6] Bils, Mark, and Pete Klenow, 2004, "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy* 112(5), pp. 947-985.

[7] Blanchard, Olivier, and Jordi Gali, 2007, "Real Wage Rigidities and the New Keynesian Model," *Journal of Money, Credit, and Banking* 39(s1), pp. 35-65.

[8] Boivin, Jean, Marc Giannoni, and Ilian Mihov, 2009, "Sticky Prices and Monetary Policy: Evidence from Disaggregated US Data," *American Economic Review* 99(1), pp. 350-384.

[9] Carlton, Dennis, 1986, "The Rigidity of Prices," *American Economic Review* 76 (September), pp. 637-658.

[10] Christiano, Lawrence , Martin Eichenbaum, and Charles Evans, 2005, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy* 113(1), pp. 1-45.

[11] Clark, Todd, 2006, "Disaggregate Evidence on the Persistence of Consumer Price Inflation," *Journal of Applied Econometrics* 21(5), pp. 563-587.

[12] Cogley, Timothy, and Thomas J. Sargent, 2002. "Evolving Post-World War II U.S. Inflation Dynamics," *NBER Chapters in NBER Macroeconomic Annual* 2001, vol. 16, pp. 331-388.

[13] Cogley, Timothy, and Thomas J. Sargent, 2005, "Drifts and Volatilities: Monetary Policies and Outcomes in the Post WWII US," *Review of Economic Dynamics* 8(2) pp. 262-302.

[14] Coibion, Olivier, 2010, "Testing the Sticky Information Phillips Curve," *Review of Economics and Statistics* 92(1), pp. 87-101.

[15] Coibion, Olivier, and Yuriy Gorodnichenko, 2012, "What Can Survey Forecasts Tell Us about Information Rigidities," *Journal of Political Economy 120*(1), pp. 116-159.

[16] Coibion, Olivier, and Yuriy Gorodnichenko, 2015, "Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts," *American Economic Review* 105(8), pp. 2644-2678.

[17] Dupor, Bill, Tomiyuki Kitamura, and Takayuki Tsuruga, 2010, "Integrating Sticky Prices and Sticky Information," *Review of Economics and Statistics* 92, pp. 657-669.

[18] Fuhrer, Jeffrey, 2011, "Inflation Persistence," in *Handbook of Monetary Economics 3A*, Friedman, B. and Woodford, M. eds., North-Holland, San Diego CA.

[19] Fuhrer, Jeffrey, and George Moore, 1995, "Inflation Persistence," *Quarterly Journal of Economics* 110(1), pp. 127-159.

[20] Fukunaga, Ichiro, 2007, "Imperfect Common Knowledge, Staggered Price Setting, and the Effects of Monetary Policy," *Journal of Money, Credit, and Banking* 39(s1), pp. 35-65.

[21] Gali, Jordi, and Mart Gertler, 1999, "Inflation Dynamics: A Structural Econometric Analysis," *Journal of Monetary Economics* 44(2), pp. 195-222.

[22] Gali, Jordi, Mark Gertler, and David Lopez-Salido, 2001, "European Inflation Dynamics," *European Economic Review* 45, pp. 1237-1270.

[23] Gali, Jordi, Mark Gertler, and J. David Lopez-Salido, 2005, "Robustness of the Estimates of the Hybrid New Keynesian Phillips curve," *Journal of Monetary Economics* 52(6), pp. 1107-1118.

[24] Lahiri, Kajal, and Xuguang Sheng, 2008, "Evolution of Forecast Disagreement in a Bayesian Learning Model," *Journal of Econometrics* 144, pp. 325-340.

[25] Levin, Andrew, and Jeremy Piger, 2003, "Time-Variation in U.S. Inflation Persistence," Manuscript, Federal Reserve Bank of St. Louis.

[26] Mackowiak, Bartosz, and Mirko Wiederholt, 2009, "Optimal Sticky Prices under Rational Inattention," *American Economic Review* 99, pp. 769-803.

[27] Mankiw, Gregory, 2001, "The Inexorable and Mysterious Tradeoff between Inflation and Unemployment," *Economic Journal* 111(471), pp. 45-61.

[28] Mankiw, Gregory, and Ricardo Reis, 2002, "Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve," *Quarterly Journal of Economics* 117(4), pp. 1295-1328.

[29] Mankiw, Gregory, Ricardo Reis, and Justin Wolfers, 2004, "Disagreement about Inflation Expectations," *NBER Macroeconomics Annual 2003*, pp. 209-248.

[30] Melitz, Marc, and Gianmarco Ottaviano, 2008, "Market Size, Trade, and Productivity," *Review of Economic Studies* 75, pp. 295-316.

[31] Midrigan, Virgiliu, 2011, "Menu Costs, Multiproduct Firms, and Aggregate Fluctuations," *Econometrica* 79(4), pp. 1139-1180.

[32] Morris, Stephen, and Hyun Song Shin, 2002, "Social Value of Public Information," *American Economic Review* 92(5), pp. 1521-1534.

[33] Nakamura, Emi, and Jon Steinsson, 2008, "Five Facts about Prices: A Reevaluation of Menu Cost Models," *Quarterly Journal of Economics* 123(4), pp. 1415-1464.

[34] Nimark, Kristoffer, 2008, "Dynamic Pricing and Imperfect Common Knowledge," *Journal of Monetary Economics* 55, pp. 365-382.

[35] Nishimura, Kiyohiko, 1986, "Rational Expectations and Price Rigidity in a Monopolistically Competitive Market," *Review of Economic Studies* 53(2), pp. 283-292.

[36] Radner, Roy, 1962, "Team Decision, Problems," *Annuals of Mathematical Statistics* 33, pp. 857-881.

[37] Reis, Ricardo, 2006, "Inattentive Producers," *Review of Economic Studies* 73, pp. 793-821.

[38] Pivetta, Frederic, and Ricardo Reis, 2007, "The persistence of inflation in the United States," *Journal of Economic Dynamics and Control* 31(4), pp. 1326-1358.

[39] Sims, Christopher, 2003, "Implications of Rational Inattention," *Journal of Monetary Economics* 50(3), pp. 665-690.

[40] Ui, Takashi, 2009, "Bayesian Potentials and Information Structures: Team Decision Problems Revisited," *International Journal of Economic Theory* 5, pp. 271-291.

[41] Ui, Takashi, 2016, "Bayesian Nash Equilibrium and Variational Inequalities," *Journal of Mathematical Economics* 63, pp. 139-146.

[42] Ui, Takashi, and Yasunori Yoshizawa, 2013, "Radner's Theorem on Teams and Games with a Continuum of Players," *Economics Bulletin* 33(1), pp. 73-77.

[43] Woodford, Michael, 2002, "Imperfect Common Knowledge and the Effects of Monetary Policy," in *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honour of Edmund S. Phelps*, Phelps, P. Aghion, R. Frydman, J. Stiglitz, and M. Woodford, eds., Princeton: Princeton University Press.

A Consumer and Demand

The maximization problem of the consumer in period $t \in \{0, 1, 2, ...\}$ is given by

$$\max_{\{\{q_t(i)\}_{i\in[0,N]},\, q_A\}} \alpha \int_{i\in[0,N]} q_t(i)di - \frac{\beta}{2}\int_{i\in[0,N]} (q_t(i))^2 di - \frac{\gamma}{2}\left[\int_{i\in[0,N]} q_t(i)di\right]^2 + q_A,$$

$$\textit{subject to} \int_{i\in[0,N]} q_t(i)p_t(i)di + q_A = \overline{q_A}.$$

By substituting the budget constraint into the objective function, the problem in period $t \in \{0, 1, 2, ...\}$ is simplified as,

$$\max_{\{\{q_t(i)\}_{i\in[0,N]},\, q_A\}} \alpha \int_{i\in[0,N]} q_t(i)di - \frac{\beta}{2}\int_{i\in[0,N]} (q_t(i))^2 di - \frac{\gamma}{2}\left[\int_{i\in[0,N]} q_t(i)di\right]^2 + \overline{q_A} - \int_{i\in[0,N]} q_t(i)p_t(i)di.$$

By taking the first derivatives of the function with respect to $q_t(i)$, we derive the first-order condition for $q_t(i)$ as,

$$\alpha - \beta q_t(i) - \gamma \int_{i\in[0,N]} q_t(i)di - p_t(i) = 0 \Leftrightarrow p_t(i) = \alpha - \beta q_t(i) - \gamma \int_{i\in[0,N]} q_t(i)di.$$

By integrating $p_t(i)$ over $i \in [0, N]$ and dividing with N, we obtain

$$\begin{aligned} \frac{1}{N}\int_{i\in[0,N]} p_t(i)di &= \overline{p}_t = \alpha - \frac{\beta}{N}\int_{i\in[0,N]} q_t(i)di - \gamma \int_{i\in[0,N]} q_t(i)di \\ &\Leftrightarrow \int_{i\in[0,N]} q_t(i)di = \frac{N}{\beta + \gamma N}\left(\alpha - \overline{p}_t\right). \end{aligned}$$

By substituting $\int_{i\in[0,N]} q_t(i)di$ into the condition, we obtain the (linear) inverse demand function and the (linear) demand function:

$$\begin{aligned} p_t(i) &= \frac{\beta}{\beta + \gamma N}\alpha - \beta q_t(i) + \frac{\gamma N}{\beta + \gamma N}\overline{p}_t, \\ q_t(i) &= \frac{1}{\beta + \gamma N}\alpha - \frac{1}{\beta}p_t(i) + \frac{\gamma N}{\beta(\beta + \gamma N)}\overline{p}_t. \end{aligned}$$

B Proofs

B.1 Proof of Proposition 2

Because c_t is composed of the initial value c_0 and the accumulation of stochastic shocks η_{t-s} for $s = \{0, 1, .., t-1\}$ ($c_t = c_{t-1} + \epsilon_t$, $\epsilon_t = \rho\epsilon_{t-1} + \eta_t$), c_t has a representation only with the variables c_0 and η_{t-s} as $c_t = c_0 + \sum_{s=0}^{t-1} \left(\frac{1-\rho^{s+1}}{1-\rho}\right) \eta_{t-s}$. Denote $\Gamma_{t|t}$ by the value which is incorporated into the average price in period t (i.e., posterior in period t) as $\overline{p}_t^* = \kappa(\tilde{N})\alpha + \left(1 - \kappa(\tilde{N})\right) \Gamma_{t|t}$. We reasonably conjecture that $\Gamma_{t|t}$ is the linear composition of c_0 and η_{t-s} for $s = \{0, 1, .., t-1\}$ as,

$$\Gamma_{t|t} \equiv c_0 + \sum_{s=0}^{t-1} \left(\frac{1-\rho^{s+1}}{1-\rho}\right) \Phi_{t,t-s}\eta_{t-s}.$$

Similarly, denote $\Gamma_{t|t-1}$ by the firms' average expectations, in period $t-1$ (i.e., prior in period t), about the value which will be incorporated into the average price in period t as,

$$\Gamma_{t|t-1} \equiv c_0 + \sum_{s=1}^{t-1} \left(\frac{1-\rho^{s+1}}{1-\rho}\right) \Phi_{t-1,t-s}\eta_{t-s}.$$

The term $\Phi_{t,t-s}$ is the weight on η_{t-s} given the information set in period t and $\Phi_{t-1,t-s}$ is the weight on η_{t-s} given the information set in period $t-1$.

To begin with, we build a lemma about the updating process from $\Gamma_{t|t-1}$ to $\Gamma_{t|t}$.

Lemma 3 *The following relationship holds:*

$$\Gamma_{t|t} = \omega_t(\tilde{N})c_t + (1 - \omega_t(\tilde{N}))\Gamma_{t|t-1},$$

where,

$$\omega_t(\tilde{N}) = \frac{(1 - r(\tilde{N}))\lambda_t}{1 - r(\tilde{N})\lambda_t}.$$

Proof: Because the objective function of firms has linear quadratic nature and the shocks follow Gaussian, the problem can be restated as Bayesian potential games with the payoff function:

$$
\begin{aligned}
& -\frac{1}{\beta}p_t(i)^2 + \frac{\tilde{N}}{\beta(\beta+\tilde{N})}\overline{p}_t p_t(i) + \left(\frac{1}{\beta+\tilde{N}}\alpha + \frac{c_t}{\beta}\right)p_t(i) - \frac{1}{\beta+\tilde{N}}\alpha c_t - \frac{\tilde{N}}{\beta(\beta+\tilde{N})}c_t\overline{p}_t \\
= \;& -Ap_t(i)^2 - 2B\int_{j\in[0,N],\; j\neq i} p_t(j)p_t(i)dj + 2\phi(c_t)p_t(i)
\end{aligned}
$$

where $A \equiv 1/\beta$, $B \equiv -\tilde{N}/(2\beta(\beta+\tilde{N}))$, and $\phi(c_t) \equiv \alpha/(2(\beta+\tilde{N})) + c_t/(2\beta)$. Define $C \equiv A+B = (2\beta+\tilde{N})/(2\beta(\beta+\tilde{N}))$, $\Lambda(\tilde{N}) \equiv A/C(\tilde{N}) = (2\beta+2\tilde{N})/(2\beta+\tilde{N})$ and $\omega_t(\tilde{N}) = Var[c_t|\mathcal{H}_{t-1}(i)]/(Var[c_t|\mathcal{H}_{t-1}(i)] + \Lambda(\tilde{N})\tau^2)$. Denote $\Gamma_{t|t-1}(i)$ by each firm's expectations, in period $t-1$, about the value which will be incorporated into its own price in period t.

Then, following Ui and Yoshizawa (2013),[26] we obtain each firm's price and the average price in a unique Bayesian Nash equilibrium as,

$$
\begin{aligned}
p_t(i)^* &= \left(\frac{\lambda_t}{C(\tilde{N})\lambda_t + A\tau^2}\right)\left(\frac{x_t(i)-\Gamma_{t|t-1}(i)}{2\beta}\right) + \frac{\Gamma_{t|t-1}(i)}{C(\tilde{N})} \\
&= \kappa(\tilde{N})\alpha + \left(1-\kappa(\tilde{N})\right)(\omega_t(\tilde{N})x_t(i) + (1-\omega_t(\tilde{N}))\Gamma_{t|t-1}(i)), \\
\overline{p}_t^* &= \kappa(\tilde{N})\alpha + \left(1-\kappa(\tilde{N})\right)(\omega_t(\tilde{N})c_t + (1-\omega_t(\tilde{N}))\Gamma_{t|t-1}),
\end{aligned}
$$

which leads to,

$$
\Gamma_{t|t} = \omega_t(\tilde{N})c_t + (1-\omega_t(\tilde{N}))\Gamma_{t|t-1}.
$$

From $1/\Lambda(\tilde{N}) = 1 - r(\tilde{N})$ and $\lambda_t = \tau^2/(Var[c_t|\mathcal{H}_{t-1}(i)] + \tau^2)$, we obtain

$$
\omega_t(\tilde{N}) = \frac{\frac{1}{\Lambda(\tilde{N})}\frac{Var[c_t|\mathcal{H}_{t-1}(i)]}{Var[c_t|\mathcal{H}_{t-1}(i)]+\tau^2}}{\frac{1}{\Lambda(\tilde{N})}\frac{Var[c_t|\mathcal{H}_{t-1}(i)]}{Var[c_t|\mathcal{H}_{t-1}(i)]+\tau^2} + \frac{\tau^2}{Var[c_t|\mathcal{H}_{t-1}(i)]+\tau^2}} = \frac{(1-r(\tilde{N}))\lambda_t}{1-r(\tilde{N})\lambda_t}.\square
$$

Given lemma 3, we have the following lemma.

Lemma 4 *The average price in a symmetric Bayesian Nash equilibrium is given as follows:*

$$
\overline{p}_t^* = \kappa(\tilde{N})\alpha + \left(1-\kappa(\tilde{N})\right)\left\{c_0 + \sum_{s=0}^{t-1}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)\sum_{u=1}^{s+1}\left[\frac{\omega_{t-u+1}(\tilde{N})}{1-\omega_{t-u+1}(\tilde{N})}\prod_{v=0}^{u-1}(1-\omega_{t-v}(\tilde{N}))\right]\eta_{t-s}\right\},
$$

[26] Radner (1962), Angeletos and Pavan (2007) and Ui (2009, 2016) show the same results in the games with finite number of players.

where,

$$\omega_t(\tilde{N}) = \frac{(1-r(\tilde{N}))\lambda_t}{1-r(\tilde{N})\lambda_t}, \;\; \lambda_t = \frac{1}{\tau^2}\sum_{s=0}^{t-1}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)^2 Var[\eta_{t-s}|\mathcal{H}_t(i)],$$

$$\begin{aligned} & \sum_{s=0}^{t-1}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)^2 Var[\eta_{t-s}|\mathcal{H}_t(i)] \\ = \;\; & \sigma^2 \sum_{s=0}^{t-1}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)^2 \prod_{u=0}^{s}\left\{\frac{\tau^2}{\sigma^2 + \sum_{u=0}^{s-1}\left(\frac{1-\rho^{u+2}}{1-\rho}\right)^2 Var[\eta_{t-1-u}|\mathcal{H}_{t-1}(i)] + \tau^2}\right\}. \end{aligned}$$

Proof: From lemma 3, in a symmetric Baysian Nash equilibrium, $\Gamma_{t|t}$ and $\Gamma_{t|t-1}$ satisfy the following equation: $\Gamma_{t|t} = \omega_t(\tilde{N})c_t + (1-\omega_t(\tilde{N}))\Gamma_{t|t-1}$. By substituting $\Gamma_{t|t}$, c_t, and $\Gamma_{t|t-1}$ into the equation above, we have the equation:

$$c_0 + \sum_{s=0}^{t-1}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)\Phi_{t,t-s}\eta_{t-s} = c_0 + \sum_{s=0}^{t-1}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)\left(\omega_t(\tilde{N}) + (1-\omega_t(\tilde{N}))\Phi_{t-1,t-s}\right)\eta_{t-s}.$$

By repeating the same calculation, $\Phi_{t,t-s}$ for any $s \in \{0,1,2,...\}$ is uniquely determined as,

$$\begin{aligned} \Phi_{t,t-s} &= \omega_t(\tilde{N}) + (1-\omega_t(\tilde{N}))\Phi_{t-1,t-s} = \omega_t(\tilde{N}) + (1-\omega_t(\tilde{N}))(\omega_{t-1}(\tilde{N}) + (1-\omega_{t-1}(\tilde{N}))\Phi_{t-2,t-s}) \\ &= \omega_t(\tilde{N}) + (1-\omega_t(\tilde{N}))\omega_{t-1}(\tilde{N}) + (1-\omega_t(\tilde{N}))(1-\omega_{t-1}(\tilde{N}))\omega_{t-2}(\tilde{N}) + ... \\ &= \sum_{u=1}^{s+1}\left[\frac{\omega_{t-u+1}(\tilde{N})}{1-\omega_{t-u+1}(\tilde{N})}\prod_{v=0}^{u-1}(1-\omega_{t-v}(\tilde{N}))\right], \end{aligned}$$

where $\Phi_{t-s-v,t-s} = 0$ holds for any $v \in \{1,2,3,...\}$.

By substituting $\Phi_{t,t-s}$ into $\Gamma_{t|t}$, we obtain

$$\begin{aligned} \Gamma_{t|t} &\equiv c_0 + \sum_{s=0}^{t-1}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)\Phi_{t,t-s}\eta_{t-s} \\ &= c_0 + \sum_{s=0}^{t-1}\left\{\left(\frac{1-\rho^{s+1}}{1-\rho}\right)\sum_{u=1}^{s+1}\left[\frac{\omega_{t-u+1}(\tilde{N})}{1-\omega_{t-u+1}(\tilde{N})}\prod_{v=0}^{u-1}(1-\omega_{t-v}(\tilde{N}))\right]\right\}\eta_{t-s}, \end{aligned}$$

$$\overline{p}_t^* = \kappa(\tilde{N})\alpha + \left(1-\kappa(\tilde{N})\right)\left\{c_0 + \sum_{s=0}^{t-1}\left\{\left(\frac{1-\rho^{s+1}}{1-\rho}\right)\sum_{u=1}^{s+1}\left[\frac{\omega_{t-u+1}(\tilde{N})}{1-\omega_{t-u+1}(\tilde{N})}\prod_{v=0}^{u-1}(1-\omega_{t-v}(\tilde{N}))\right]\right\}\eta_{t-s}\right\}.$$

Next, we characterize $\omega_t(\tilde{N})$ for $t \in \{1, 2, 3, ...\}$. Because $\omega_t(\tilde{N}) = (1-r(\tilde{N}))\lambda_t / \left(1 - r(\tilde{N})\lambda_t\right)$ holds, and $r(\tilde{N})$ is a fixed parameter, it is enough to write out λ_t for $t \in \{1, 2, 3, ...\}$. Based on the equation $c_t = c_0 + \sum_{s=0}^{t-1} \left(\frac{1-\rho^{s+1}}{1-\rho}\right) \eta_{t-s}$, $Var[c_t|\mathcal{H}_{t-1}(i)]$ can be expressed as,

$$Var[c_t|\mathcal{H}_{t-1}(i)] = \sum_{s=0}^{t-1} \left(\frac{1-\rho^{s+1}}{1-\rho}\right)^2 Var[\eta_{t-s}|\mathcal{H}_{t-1}(i)].$$

Following Bayes' theorem for the variance of normal distribution, we obtain,

$$\begin{aligned}
Var[c_t|\mathcal{H}_t(i)] &= \left(\frac{\tau^2}{Var[c_t|\mathcal{H}_{t-1}(i)] + \tau^2}\right) Var[c_t|\mathcal{H}_{t-1}(i)] = (1-\lambda_t) Var[c_t|\mathcal{H}_{t-1}(i)] \\
\Longleftrightarrow \quad \sum_{s=0}^{t-1} \left(\frac{1-\rho^{s+1}}{1-\rho}\right)^2 Var[\eta_{t-s}|\mathcal{H}_t(i)] &= (1-\lambda_t) \sum_{s=0}^{t-1} \left(\frac{1-\rho^{s+1}}{1-\rho}\right)^2 Var[\eta_{t-s}|\mathcal{H}_{t-1}(i)] \\
&= \sum_{s=0}^{t-1} \left(\frac{1-\rho^{s+1}}{1-\rho}\right)^2 \prod_{u=0}^{s} (1-\lambda_{t-u}) Var[\eta_{t-s}|\mathcal{H}_0(i)] \\
&= \sigma^2 \sum_{s=0}^{t-1} \left(\frac{1-\rho^{s+1}}{1-\rho}\right)^2 \prod_{u=0}^{s} \left\{ \frac{\tau^2}{\sigma^2 + \sum_{u=0}^{s-1} \left(\frac{1-\rho^{u+2}}{1-\rho}\right)^2 Var[\eta_{t-1-u}|\mathcal{H}_{t-1}(i)] + \tau^2} \right\}.
\end{aligned}$$

By substituting the following equality

$$\sum_{s=0}^{t-1} \left(\frac{1-\rho^{s+1}}{1-\rho}\right)^2 Var[\eta_{t-s}|\mathcal{H}_{t-1}(i)] = \frac{\tau^2 \sum_{s=0}^{t-1} \left(\frac{1-\rho^{s+1}}{1-\rho}\right)^2 Var[\eta_{t-s}|\mathcal{H}_t(i)]}{\tau^2 - \sum_{s=0}^{t-1} \left(\frac{1-\rho^{s+1}}{1-\rho}\right)^2 Var[\eta_{t-s}|\mathcal{H}_t(i)]},$$

λ_t is determined as,

$$\lambda_t = \frac{\sum_{s=0}^{t-1} \left(\frac{1-\rho^{s+1}}{1-\rho}\right)^2 Var[\eta_{t-s}|\mathcal{H}_{t-1}(i)]}{\sum_{s=0}^{t-1} \left(\frac{1-\rho^{s+1}}{1-\rho}\right)^2 Var[\eta_{t-s}|\mathcal{H}_{t-1}(i)] + \tau^2} = \frac{1}{\tau^2} \sum_{s=0}^{t-1} \left(\frac{1-\rho^{s+1}}{1-\rho}\right)^2 Var[\eta_{t-s}|\mathcal{H}_t(i)]. \square$$

Finally, given lemma 4, we derive the average price in the steady state ($t \to \infty$). Because

$1-\lambda_t \in (0,1)$ and $\left(1-\rho^{s+1}\right)/\left(1-\rho\right)$ is bounded above by $1/\left(1-\rho\right)$,

$$\begin{aligned}
&\sum_{s=0}^{t-1}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)^2 Var[\eta_{t-s}|\mathcal{H}_t(i)] \\
= \; & \sigma^2\sum_{s=0}^{t-1}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)^2\prod_{u=0}^{s}\left\{\frac{\tau^2}{\sigma^2+\sum_{u=0}^{s-1}\left(\frac{1-\rho^{u+2}}{1-\rho}\right)^2 Var[\eta_{t-1-u}|\mathcal{H}_{t-1}(i)]+\tau^2}\right\} \\
= \; & \sigma^2\sum_{s=0}^{t-1}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)^2\prod_{u=0}^{s}(1-\lambda_t),
\end{aligned}$$

converges to a certain value, and consequently λ_t and $\omega_t(\tilde{N})$ also converge. Define the values as λ and $\omega(\tilde{N})$. Then, for $t \to \infty$,

$$\begin{aligned}
\overline{p}_t^* &= \kappa(\tilde{N})\alpha+\left(1-\kappa(\tilde{N})\right)\left\{c_0+\sum_{s=0}^{t-1}\left\{\left(\frac{1-\rho^{s+1}}{1-\rho}\right)\sum_{u=1}^{s+1}\left[\frac{\omega_{t-u+1}(\tilde{N})}{1-\omega_{t-u+1}(\tilde{N})}\prod_{v=0}^{u-1}(1-\omega_{t-v}(\tilde{N}))\right]\right\}\eta_{t-s}\right\} \\
&\to \kappa(\tilde{N})\alpha+\left(1-\kappa(\tilde{N})\right)\left\{c_0+\sum_{s=0}^{\infty}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)\left[1-(1-\omega(\tilde{N}))^{s+1}\right]\eta_{t-s}\right\}.
\end{aligned}$$

Finally, we characterize $\omega(\tilde{N})$. We have the equation to identify λ as,

$$\begin{aligned}
1-\lambda &= \frac{\tau^2}{\sum_{s=0}^{t-1}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)^2 Var[\eta_{t-s}|\mathcal{H}_{t-1}(i)]+\tau^2} = \frac{\tau^2}{\sigma^2\sum_{s=1}^{\infty}(1-\lambda)^{s-1}\left(\frac{1-\rho^s}{1-\rho}\right)^2+\tau^2} \\
&\iff \lambda = \frac{(1+\rho-\rho\lambda)\sigma^2}{(1+\rho-\rho\lambda)\sigma^2+\lambda(1-\rho+\rho\lambda)(1-\rho^2+\rho^2\lambda)\tau^2}.
\end{aligned}$$

Because $\lambda \in (0,1)$ holds, the left hand side of the equation is monotonically increasing in λ from 0 to 1, and the right hand side is monotonically decreasing in λ from 1 to $\tau^2/(\sigma^2+\tau^2)$. Therefore there exists a unique λ which satisfies the equation above. $\square$

B.2 Proof of lemma 1

From proposition 2, $\widetilde{\pi}_t$ and $\widetilde{\pi}_{t-1}$ are given by,

$$\begin{aligned}
\widetilde{\pi}_t &= \left(1-\kappa(\tilde{N})\right)\sum_{s=0}^{\infty}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)\left(\left[1-(1-\omega(\tilde{N}))^{s+1}\right]\eta_{t-s}-\left[1-(1-\omega(\tilde{N}))^{s+1}\right]\eta_{t-1-s}\right) \\
&= \left(1-\kappa(\tilde{N})\right)\left\{\omega(\tilde{N})\eta_t+\sum_{s=0}^{\infty}\omega(\tilde{N})\left[\left(\frac{1-\rho^s}{1-\rho}\right)(1-\omega(\tilde{N}))^s+\rho^s\left(\frac{1-(1-\omega(\tilde{N}))^{s+1}}{1-(1-\omega(\tilde{N}))}\right)\right]\eta_{t-s}\right\}, \\
\widetilde{\pi}_{t-1} &= \left(1-\kappa(\tilde{N})\right)\sum_{s=0}^{\infty}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)\left(\left[1-(1-\omega(\tilde{N}))^{s+1}\right]\eta_{t-s-1}-\left[1-(1-\omega(\tilde{N}))^{s+1}\right]\eta_{t-2-s}\right) \\
&= \left(1-\kappa(\tilde{N})\right)\sum_{s=1}^{\infty}\omega(\tilde{N})\left[\left(\frac{1-\rho^{s-1}}{1-\rho}\right)(1-\omega(\tilde{N}))^{s-1}+\rho^{s-1}\left(\frac{1-(1-\omega(\tilde{N}))^{s}}{1-(1-\omega(\tilde{N}))}\right)\right]\eta_{t-s}.\square
\end{aligned}$$

B.3 Proof of Proposition 3

Lemma 5 *The first-order quasi-autocorrelation function $\widetilde{\rho}(\widetilde{\pi}_t, \widetilde{\pi}_{t-1})$ is given by*

$$\widetilde{\rho}(\widetilde{\pi}_t, \widetilde{\pi}_{t-1}) = \frac{\Xi_N}{\Xi_D},$$

where,

$$\begin{aligned}
\Xi_N &\equiv (1-\omega(\tilde{N}))\zeta_1+\rho\zeta_2+\rho(1-\omega(\tilde{N}))\zeta_3+\left(\rho+(1-\omega(\tilde{N})\right)\zeta_4 \\
&\quad -(1-\omega(\tilde{N}))\,(1+\rho)\,\zeta_5-\rho(1+(1-\omega(\tilde{N})))\zeta_6, \\
\Xi_D &\equiv \zeta_1+\zeta_2+\zeta_3+2\zeta_4-2\zeta_5-2\zeta_6, \\
\zeta_1 &\equiv \frac{\left(1-(1-\omega(\tilde{N}))\right)^2}{1-(1-\omega(\tilde{N}))^2},\ \zeta_2\equiv\frac{(1-\rho)^2}{1-\rho^2}, \\
\zeta_3 &\equiv \frac{\left(1-\rho(1-\omega(\tilde{N}))\right)^2}{1-\rho^2(1-\omega(\tilde{N}))^2},\ \zeta_4\equiv\frac{(1-\rho)\left(1-(1-\omega(\tilde{N}))\right)}{1-\rho(1-\omega(\tilde{N}))}, \\
\zeta_5 &\equiv \frac{\left(1-(1-\omega(\tilde{N}))\right)\left(1-\rho(1-\omega(\tilde{N}))\right)}{1-\rho(1-\omega(\tilde{N}))^2},\ \zeta_6\equiv\frac{(1-\rho)\left(1-\rho(1-\omega(\tilde{N}))\right)}{1-\rho^2(1-\omega(\tilde{N}))}.
\end{aligned}$$

Proof: First, from lemma 1, the following equalities hold:

$$
\begin{aligned}
Cov(\widetilde{\pi}_t, \widetilde{\pi}_{t-1}) &= \left(1-\kappa(\tilde{N})\right)^2 \left[\sum_{s=1}^{\infty} \Theta_{t-s}(\tilde{N})\Theta_{t-s+1}(\tilde{N})\right] \sigma^2, \\
Var(\widetilde{\pi}_{t-1}) &= \left(1-\kappa(\tilde{N})\right)^2 \left[\sum_{s=1}^{\infty} \Theta_{t-s+1}^2(\tilde{N})\right] \sigma^2.
\end{aligned}
$$

Thus, the first-order quasi-autocorrelation function $\widetilde{\rho}(\widetilde{\pi}_t, \widetilde{\pi}_{t-1}) \equiv Cov(\widetilde{\pi}_t, \widetilde{\pi}_{t-1})/Var(\widetilde{\pi}_{t-1})$ is given by,

$$
\widetilde{\rho}(\widetilde{\pi}_t, \widetilde{\pi}_{t-1}) = \left[\sum_{s=1}^{\infty} \Theta_{t-s}(\tilde{N})\Theta_{t-s+1}(\tilde{N})\right] \left[\sum_{s=1}^{\infty} \Theta_{t-s+1}^2(\tilde{N})\right]^{-1}
$$

$$
= \left[\begin{array}{c}
(1-\omega(\tilde{N}))\frac{\left(1-(1-\omega(\tilde{N}))\right)^2}{1-(1-\omega(\tilde{N}))^2} + \rho\frac{(1-\rho)^2}{1-\rho^2} \\
+\rho(1-\omega(\tilde{N}))\frac{\left(1-\rho(1-\omega(\tilde{N}))\right)^2}{1-\rho^2(1-\omega(\tilde{N}))^2} + \left(\rho + (1-\omega(\tilde{N})\right)\frac{(1-\rho)\left(1-(1-\omega(\tilde{N}))\right)}{1-\rho(1-\omega(\tilde{N}))} \\
-(1-\omega(\tilde{N}))\,(1+\rho)\,\frac{\left(1-(1-\omega(\tilde{N}))\right)\left(1-\rho(1-\omega(\tilde{N}))\right)}{1-\rho(1-\omega(\tilde{N}))^2} - \rho(1+(1-\omega(\tilde{N})))\frac{(1-\rho)\left(1-\rho(1-\omega(\tilde{N}))\right)}{1-\rho^2(1-\omega(\tilde{N}))}
\end{array}\right]
$$

$$
\left[\begin{array}{c}
\frac{\left(1-(1-\omega(\tilde{N}))\right)^2}{1-(1-\omega(\tilde{N}))^2} + \frac{(1-\rho)^2}{1-\rho^2} + \frac{\left(1-\rho(1-\omega(\tilde{N}))\right)^2}{1-\rho^2(1-\omega(\tilde{N}))^2} + 2\frac{(1-\rho)\left(1-(1-\omega(\tilde{N}))\right)}{1-\rho(1-\omega(\tilde{N}))} \\
-2\frac{\left(1-(1-\omega(\tilde{N}))\right)\left(1-\rho(1-\omega(\tilde{N}))\right)}{1-\rho(1-\omega(\tilde{N}))^2} - 2\frac{(1-\rho)\left(1-\rho(1-\omega(\tilde{N}))\right)}{1-\rho^2(1-\omega(\tilde{N}))}
\end{array}\right]^{-1}
$$

which is the composition of the symmetric functions of ρ and $(1-\omega(\tilde{N}))$. $\square$

According to lemma 5, $\widetilde{\rho}(\widetilde{\pi}_t, \widetilde{\pi}_{t-1})$ is the composition of the symmetic functions of ρ and $(1-\omega(\tilde{N}))$. Because a symmetric polynomial function can be transformed into the function of elementary symmetric polynomials, the combinations of polynominal functions $(\widetilde{\rho}(\widetilde{\pi}_t, \widetilde{\pi}_{t-1}))$ can be also transformed into the function of elementary symmetric polynomials. Suppose the function $f(\rho, (1-\omega(\tilde{N})), \overline{\rho}) \equiv \widetilde{\rho}(\widetilde{\pi}_t, \widetilde{\pi}_{t-1}) - \overline{\rho}$ where $\overline{\rho}$ is an arbitraty number in $[0, 1]$. Then, there exists a unique function with elementary symmetric polynominals $\overline{\alpha} \equiv \rho + (1-\omega(\tilde{N}))$ and $\overline{\beta} \equiv \rho(1-\omega(\tilde{N}))$, given by $g(\overline{\alpha}, \overline{\beta}, \overline{\rho}) \equiv \widetilde{\rho}(\widetilde{\pi}_t, \widetilde{\pi}_{t-1}) - \overline{\rho}$. Therefore, there exists the correspondence satisfying $g(\overline{\alpha}, \overline{\beta}, \overline{\rho}) = f(\rho, (1-\omega(\tilde{N})), \overline{\rho})$.

For $\rho = (1-\omega(\tilde{N})) \equiv \theta$, $f(\theta, \theta, \overline{\rho})$ has a unique solution because

$$
f(\theta, \theta, \overline{\rho}) = \frac{4\theta\frac{(1-\theta)^2}{1-\theta^2} - 2\theta\,(1+\theta)\,\frac{(1-\theta)\left(1-\theta^2\right)}{1-\theta^3} + \theta^2\frac{\left(1-\theta^2\right)^2}{1-\theta^4}}{4\frac{(1-\theta)^2}{1-\theta^2} - 4\frac{(1-\theta)\left(1-\theta^2\right)}{1-\theta^3} + \frac{\left(1-\theta^2\right)^2}{1-\theta^4}} - \overline{\rho},
$$

leads to $f(0,0,\overline{\rho}) = -\overline{\rho} < 0$, $f(1,1,\overline{\rho}) = 1-\overline{\rho} > 0$ and,

$$\frac{\partial}{\partial\theta}f(\theta,\theta,\overline{\rho}) = 2\frac{(1-\theta)(1+\theta^2+5\theta^4+\theta^5+8\theta^6-\theta^7+\theta^8)}{(-1+\theta-4\theta^2+\theta^3-\theta^4)^2} > 0.$$

Because $f(\theta,\theta,\overline{\rho})$ is monotonic, by Intermediate Value Theorem, there exists a unique solution for $f(\theta,\theta,\overline{\rho}) = 0$ for any $\overline{\rho}$. Because $\overline{\alpha} \equiv 2\theta$, $\overline{\beta} \equiv \theta^2$, and θ is unique, the combination of $\overline{\alpha}$ and $\overline{\beta}$ for θ is unique too. Therefore, $g(\overline{\alpha},\overline{\beta},\overline{\rho}) = 0$ has a unique solution for any $\overline{\rho}$ satisfying

$$\overline{\alpha} \equiv \rho + (1-\omega(\tilde{N})),\ \overline{\beta} \equiv \rho(1-\omega(\tilde{N})).$$

There may exist multiple combinations of ρ and $(1-\omega(\tilde{N}))$ for a unique combination of $\overline{\alpha}$ and $\overline{\beta}$.

Our argument in the following is, for a fixed $(1-\omega(\tilde{N}))$, there exists a unique solution ρ for the unique combination of $\overline{\alpha}$, $\overline{\beta}$ for any fixed $(1-\omega(\tilde{N}))$. To satisfy the equations above,

$$\begin{aligned}\rho^2 - \overline{\alpha}\rho + \overline{\beta} &= 0 \Leftrightarrow \left(\rho - \frac{1}{2}\overline{\alpha}\right)^2 - \frac{1}{4}\overline{\alpha}^2 + \overline{\beta} = 0\\ &\Leftrightarrow \rho = \frac{\overline{\alpha} \pm \sqrt{\overline{\alpha}^2 - 4\overline{\beta}}}{2},\end{aligned}$$

must hold. Therefore, there possibly exist two solutions, and one of the solution is greater than $\overline{\alpha}/2$ and the other is less than $\overline{\alpha}/2$. Fix, without loss of generality, $(1-\omega(\tilde{N}))$ and consider two cases, (i) $1 > \rho > (1-\omega(\tilde{N})) > 0$ and (ii) $0 < \rho < (1-\omega(\tilde{N})) < 1$ (the uniqueness of the solution in the case with $\rho = (1-\omega(\tilde{N}))$ has already been shown). In case (i) $1 > \rho > (1-\omega(\tilde{N})) > 0$, $\overline{\alpha} > \rho > \overline{\alpha}/2$ and $\rho > \overline{\beta}$ hold ($\rho \in (\overline{\alpha}/2, \overline{\alpha})$). Therefore, the solution is unqiue $\rho = (\overline{\alpha} + \sqrt{\overline{\alpha}^2 - 4\overline{\beta}})/2$. On the other hand, in case (ii), $0 < \rho < (1-\omega(\tilde{N})) < 1$, by the same logic, $\overline{\alpha}/2 > \rho >$ holds ($\rho \in (\overline{\beta}, \overline{\alpha}/2)$). Therefore, the solution is unqiue $\rho = (\overline{\alpha} - \sqrt{\overline{\alpha}^2 - 4\overline{\beta}})/2$. Thus, for any fixed $(1-\omega(\tilde{N}))$, there exists a unique ρ satisfying $f(\rho, (1-\omega(\tilde{N})), \overline{\rho}) = 0$. By the symmetricity of ρ and $(1-\omega(\tilde{N}))$, for any fixed ρ, there exists a unique $(1-\omega(\tilde{N}))$ satisfying $f(\rho,(1-\omega(\tilde{N})),\overline{\rho}) = 0$. Because $\widetilde{\rho}(\widetilde{\pi}_t, \widetilde{\pi}_{t-1})$ is a continuous function over $[0,1]^2 \to [0,1]$ and has the one-to-one correspondence with $(1-\omega(\tilde{N}))$ for a fixed ρ, this function is monotonic to $(1-\omega(\tilde{N}))$. Finally, because $\widetilde{\rho}(\widetilde{\pi}_t, \widetilde{\pi}_{t-1}) = \rho$ for $(1-\omega(\tilde{N})) = 0$, and $\widetilde{\rho}(\widetilde{\pi}_t, \widetilde{\pi}_{t-1}) = 1$ for $(1-\omega(\tilde{N})) = 1$, it is increasing in $(1-\omega(\tilde{N}))$. Thus, together with lemma 2, we finish the proof. □

B.4 Proof of Proposition 4

First, from lemma 1 and the assumption that $\eta_t \neq 0$ and $\eta_{t-s} = 0$ for $s \in \{..., -2, -1, 1, 2, ...\}$ hold, $\widetilde{\pi}_t$ and $\widetilde{\pi}_{t+k}$ for $k \in \{1, 2, 3, ...\}$ are given as follows:

$$\begin{aligned}
\widetilde{\pi}_t &= \left(1 - \kappa(\tilde{N})\right) \omega(\tilde{N})\eta_t = \left(\frac{\lambda(\beta + \widetilde{N})}{2\beta + \widetilde{N}(2-\lambda)} \right) \eta_t, \\
\widetilde{\pi}_{t+k} &= \left(1 - \kappa(\tilde{N})\right) \left[\left(\frac{1-\rho^{k+1}}{1-\rho} \right) \left[1 - (1-\omega(\tilde{N}))^{k+1} \right] - \left(\frac{1-\rho^{k}}{1-\rho} \right) \left[1 - (1-\omega(\tilde{N}))^{k} \right] \right] \eta_t \\
&= \left(\frac{\lambda(\beta + \widetilde{N})}{2\beta + \widetilde{N}(2-\lambda)} \right) \left\{ \begin{array}{c} \rho^k \left[1 + ... + \left[\frac{2\beta(1-\lambda)+\widetilde{N}(2-2\lambda)}{2\beta+\widetilde{N}(2-\lambda)} \right]^{k-1} \right] \\ + \left[\frac{2\beta(1-\lambda)+\widetilde{N}(2-2\lambda)}{2\beta+\widetilde{N}(2-\lambda)} \right]^{k} \left(1 + ... + \rho^{k-1}\right) + \rho^k \left[\frac{2\beta(1-\lambda)+\widetilde{N}(2-2\lambda)}{2\beta+\widetilde{N}(2-\lambda)} \right]^{k} \end{array} \right\} \eta_t.
\end{aligned}$$

Therefore, $\Psi_{\pi,0}$ and $\Psi_{\pi,k}$ for $k \in \{1, 2, 3, ...\}$ are given by,

$$\begin{aligned}
\partial\Psi_{\pi,0} &\equiv \frac{\partial \widetilde{\pi}_t}{\partial \eta_t} = \frac{\lambda(\beta + \widetilde{N})}{2\beta + \widetilde{N}(2-\lambda)} \\
\partial\Psi_{\pi,k} &\equiv \frac{\partial \widetilde{\pi}_{t+k}}{\partial \eta_t} = \left(\frac{\lambda(\beta + \widetilde{N})}{2\beta + \widetilde{N}(2-\lambda)} \right) \left\{ \begin{array}{c} \rho^k \left[1 + ... + \left[\frac{2\beta(1-\lambda)+\widetilde{N}(2-2\lambda)}{2\beta+\widetilde{N}(2-\lambda)} \right]^{k-1} \right] \\ + \left[\frac{2\beta(1-\lambda)+\widetilde{N}(2-2\lambda)}{2\beta+\widetilde{N}(2-\lambda)} \right]^{k} \left(1 + ... + \rho^{k-1}\right) \\ + \rho^k \left[\frac{2\beta(1-\lambda)+\widetilde{N}(2-2\lambda)}{2\beta+\widetilde{N}(2-\lambda)} \right]^{k} \end{array} \right\}.
\end{aligned}$$

Here, the following inequalities hold:

$$\begin{aligned}
\frac{\partial}{\partial \widetilde{N}} \left(\frac{\lambda(\beta + \widetilde{N})}{2\beta + \widetilde{N}(2-\lambda)} \right) &= \frac{\beta\lambda}{\left(2\beta + \widetilde{N}(2-\lambda)\right)^2} > 0, \\
\frac{\partial}{\partial \widetilde{N}} \left(\frac{2\beta(1-\lambda) + \widetilde{N}(2-2\lambda)}{2\beta + \widetilde{N}(2-\lambda)} \right) &= \frac{2\lambda(1-\lambda)\beta}{\left(2\beta + \widetilde{N}(2-\lambda)\right)^2} > 0.
\end{aligned}$$

Therefore, $\Psi_{\pi,0}$ and $\Psi_{\pi,k}$ for $k \in \{1, 2, 3, ...\}$ are monotonically increasing in $\tilde{N}$. $\square$

B.5 Proof of Proposition 5

First, using proposition 2 and the assumption that $\eta_t \neq 0$ and $\eta_{t-s} = 0$ for $s \in \{..., -2, -1, 1, 2, ...\}$ hold, $\overline{p}^*_{t+k}$ for $k \in \{0, 1, 2, ...\}$ are given as follows:

$$\overline{p}^*_{t+k} = \kappa(\tilde{N})\alpha + \left(1 - \kappa(\tilde{N})\right) \left\{ c_{t-1} + \left(\frac{1-\rho^{k+1}}{1-\rho}\right) \left[1 - (1 - \omega(\tilde{N}))^{k+1}\right] \eta_t \right\}.$$

Therefore, by substituting $1-\kappa(\tilde{N}) = (\beta+\widetilde{N})/(2\beta+\widetilde{N})$ and $\omega(\tilde{N}) = \left(\left(2\beta + \widetilde{N}\right)\lambda\right) / \left(2\beta + \widetilde{N}(2-\lambda)\right)$ into $\overline{p}^*_{t+k}$, $\Psi_{p,k}$ is expressed as,

$$\begin{aligned}
\Psi_{p,k} &\equiv \frac{\partial \overline{p}^*_{t+k}}{\partial \eta_t} = \left(1 - \kappa(\tilde{N})\right) \left(\frac{1-\rho^{k+1}}{1-\rho}\right) \left[1 - (1 - \omega(\tilde{N}))^{k+1}\right] \\
&= \left(1 - \kappa(\tilde{N})\right) \left(\frac{1-\rho^{k+1}}{1-\rho}\right) \omega(\tilde{N}) \sum_{s=0}^{k} (1 - \omega(\tilde{N}))^s \\
&= \left(\frac{\beta + \widetilde{N}}{2\beta + \widetilde{N}}\right) \left(\frac{\left(2\beta + \widetilde{N}\right)\lambda_t}{2\beta + \widetilde{N}(2-\lambda_t)}\right) \left(\frac{1-\rho^{k+1}}{1-\rho}\right) \sum_{s=0}^{k} (1 - \omega(\tilde{N}))^s.
\end{aligned}$$

Next, we examine the sign of $\partial\Psi_{p,k}/\partial\tilde{N}$. Here, the term $\sum_{s=0}^{k}(1-\omega(\tilde{N}))^s$ must be monotonically increasing in $(1-\omega(\tilde{N}))$ and $(1-\omega(\tilde{N}))$ is monotonically decreasing in $\omega(\tilde{N})$. Thus, from lemma 2, the term $\sum_{s=0}^{k}(1-\omega(\tilde{N}))^s$ is monotonically increasing in $\widetilde{N}$. Moreover, because

$$\frac{\partial}{\partial \widetilde{N}} \left\{ \left(\frac{\beta + \widetilde{N}}{2\beta + \widetilde{N}}\right) \left(\frac{\left(2\beta + \widetilde{N}\right)\lambda}{2\beta + \widetilde{N}(2-\lambda)}\right) \right\} = \frac{\partial}{\partial \widetilde{N}} \left(\frac{\lambda(\beta + \widetilde{N})}{2\beta + \widetilde{N}(2-\lambda)}\right) = \frac{\beta\lambda_t}{\left(2\beta + \widetilde{N}(2-\lambda)\right)^2} > 0,$$

holds, $\Psi_{p,k}$ is increasing in $\widetilde{N}$ for any $k \in \{0, 1, 2, ...\}$. $\square$

C Average Price with Endogenous Public Information

For Figure 6 and 7 in subsection 6.1, this appendix derives the average price when firms obtain endogenous public information from the average price in the previous period.

From lemma 3, in a symmetric Baysian Nash equilibrium, $\Gamma_{t|t}$ and $\Gamma_{t|t-1}$ satisfy the following equation:

$$\Gamma_{t|t} = \omega_{A,t}c_t + (1-\omega_{A,t})\Gamma_{t|t-1},$$

where $\omega_{A,t} \equiv (1-r(\tilde{N}))\lambda_t/(1-r(\tilde{N})\lambda_t)$. Define the fraction of updating from the ex post observable signal y_t as $\omega_{B,t}$. Then, we have,

$$\sum_{s=0}^{t-1}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)\Phi_{t,t-s}\eta_{t-s} = \sum_{s=0}^{t-1}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)\left(\omega_{A,t} + (1-\omega_{A,t})\Phi_{t-1,t-s}\right)\eta_{t-s},$$

and we can indentify $\Phi_{t,t-s}$ for any $s \in \{0,1,2,...\}$ as,

$$\begin{aligned}
\Phi_{t,t-s} &= \omega_{A,t} + (1-\omega_{A,t})\Phi_{t-1,t-s} \\
&= \omega_{A,t} + (1-\omega_{A,t})\left(\omega_{B,t-1} + (1-\omega_{B,t-1})\Phi_{t-2,t-s}\right) \\
&= \begin{cases} \omega_{A,t} \text{ for } s=0 \\ \omega_{A,t} + (1-\omega_{A,t})\sum_{u=1}^{s}\left[\frac{\omega_{B,t-u}}{1-\omega_{B,t-u}}\prod_{v=0}^{u-1}(1-\omega_{B,t-v-1})\right] \text{ for } s>0. \end{cases}
\end{aligned}$$

Note that $\Phi_{t-s-v,t-s} = 0$ obviously holds for any $v \in \{1,2,3,...\}$ and that $\omega_{B,t} = \omega_{A,t}$ always holds in period t because the endogenous public signal has not been generated yet. Therefore, we have

$$\begin{aligned}
\overline{p}_t^* =& \kappa(\tilde{N})\alpha + \left(1-\kappa(\tilde{N})\right)c_0 \\
&+ \left(1-\kappa(\tilde{N})\right)\sum_{s=0}^{t-1}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)\left\{\omega_{A,t} + (1-\omega_{A,t})\sum_{u=1}^{s}\left[\frac{\omega_{B,t-u}}{1-\omega_{B,t-u}}\prod_{v=0}^{u-1}(1-\omega_{B,t-v-1})\right]\right\}\eta_{t-s}.
\end{aligned}$$

Denote λ_t' by the value satisfying $\omega_{B,t} \equiv (1-r(\tilde{N}))(\lambda_t+\lambda_t')/(1-r(\tilde{N})(\lambda_t+\lambda_t'))$. Then, the following equality holds: $\lambda_t \equiv Var[c_t|\mathcal{H}_{t-1}(i)]/\left(Var[c_t|\mathcal{H}_{t-1}(i)] + \tau^2\right)$, and

$$\begin{aligned}\lambda_t' &= (1-\lambda_t)\frac{\tau^2(1-\lambda_t)}{\tau^2(1-\lambda_t) + \frac{e^2}{\left(1-\kappa(\tilde{N})\right)^2\left(\frac{(1-r(\tilde{N}))\lambda_t}{1-r(\tilde{N})\lambda_t}\right)^2}} \\ &= \frac{\tau^2(1-\lambda_t)^2(1-r(\tilde{N}))^2\lambda_t^2\left(1-\kappa(\tilde{N})\right)^2}{\tau^2(1-\lambda_t)(1-r(\tilde{N}))^2\lambda_t^2\left(1-\kappa(\tilde{N})\right)^2 + e^2\left(1-r(\tilde{N})\lambda_t\right)^2}.\end{aligned}$$

Because $r(\tilde{N})$ is a fixed parameter, it is enough to write out λ_t for $t \in \{1, 2, 3, ...\}$. Note that $\lambda_t' = 0$. Further, by using the same step as before, we have,

$$\begin{aligned}&\sum_{s=0}^{t-1}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)^2 Var[\eta_{t-s}|\mathcal{H}_t(i)] \\ =\ & (1-\lambda_t)\sum_{s=0}^{t-1}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)^2 Var[\eta_{t-s}|\mathcal{H}_{t-1}(i)] \\ =\ & \sum_{s=0}^{t-1}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)^2 \prod_{u=0}^{s}(1-\lambda_{t-u}-\lambda_{t-u}')Var[\eta_{t-s}|\mathcal{H}_0(i)] \\ =\ & \sigma^2\sum_{s=0}^{t-1}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)^2 \prod_{u=0}^{s}(1-\lambda_{t-u})\left(\frac{e^2\left(1-r(\tilde{N})\lambda_t\right)^2}{\tau^2(1-\lambda_t)(1-r(\tilde{N}))^2\lambda_t^2\left(1-\kappa(\tilde{N})\right)^2 + e^2\left(1-r(\tilde{N})\lambda_t\right)^2}\right),\end{aligned}$$

where $\lambda_t = \left[\sigma^2 + \sum_{u=0}^{s-1}\left(\frac{1-\rho^{u+2}}{1-\rho}\right)^2 Var[\eta_{t-1-u}|\mathcal{H}_{t-1}(i)]\right]\left[\sigma^2 + \sum_{u=1}^{s-1}\left(\frac{1-\rho^{u+2}}{1-\rho}\right)^2 Var[\eta_{t-1-u}|\mathcal{H}_{t-1}(i)] + \tau^2\right]^{-1}$.

Therefore, there is the unique value for $Var[c_t|\mathcal{H}_t(i)]$, λ_t, λ_t', $\omega_{A,t}$ and $\omega_{B,t}$, respectively. Next, we derive the price in the steady state when $\lambda_t \to \lambda$ and $\lambda_t' \to \lambda'$ (and thus $\omega_{A,t} \to \omega_A$ and $\omega_{B,t} \to \omega_B$). The price is given by,

$$\begin{aligned}\overline{p}_t^* =& \kappa(\tilde{N})\alpha + \left(1-\kappa(\tilde{N})\right) \\ &+ \left(1-\kappa(\tilde{N})\right)\sum_{s=0}^{\infty}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)\{\omega_A + (1-\omega_A)\left[1-(1-\omega_B)^s\right]\}\,\eta_{t-s}.\end{aligned}$$

Because

$$
\begin{aligned}
&\sum_{s=0}^{\infty}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)^2 Var[\eta_{t-s}|\mathcal{H}_t(i)] \\
= \;& \sigma^2\sum_{s=0}^{t-1}(1-\lambda)^s\left(\frac{e^2\left(1-r(\tilde{N})\lambda\right)^2}{\tau^2(1-\lambda)(1-r(\tilde{N}))^2\lambda^2\left(1-\kappa(\tilde{N})\right)^2+e^2\left(1-r(\tilde{N})\lambda\right)^2}\right)^{s-1}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)^2,
\end{aligned}
$$

and $\lambda=\left[\sigma^2+\sum_{u=0}^{s-1}\left(\frac{1-\rho^{u+2}}{1-\rho}\right)^2 Var[\eta_{t-1-u}|\mathcal{H}_{t-1}(i)]\right]\left[\sigma^2+\sum_{u=1}^{s-1}\left(\frac{1-\rho^{u+2}}{1-\rho}\right)^2 Var[\eta_{t-1-u}|\mathcal{H}_{t-1}(i)]+\tau^2\right]^{-1}$, we have the equation:

$$
\begin{aligned}
1-\lambda &= \frac{\tau^2}{\sum_{u=0}^{\infty}\left(\frac{1-\rho^{u+1}}{1-\rho}\right)^2 Var[\eta_{t-u}|\mathcal{H}_{t-1}(i)]+\tau^2} \\
&= \frac{\tau^2}{\sigma^2\sum_{s=1}^{\infty}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)^2(1-\lambda)^{s-1}\left(\frac{e^2\left(1-r(\tilde{N})\lambda\right)^2}{\tau^2(1-\lambda)(1-r(\tilde{N}))^2\lambda^2\left(1-\kappa(\tilde{N})\right)^2+e^2\left(1-r(\tilde{N})\lambda\right)^2}\right)^{s-1}+\tau^2} \\
\iff\; & \lambda=\frac{\sigma^2\left(\frac{1}{1-\rho}\right)^2\left[\frac{1}{1-\Omega}-\frac{2\rho^2}{1-\rho\Omega}+\frac{\rho^4}{1-\rho^2\Omega}\right]}{\sigma^2\left(\frac{1}{1-\rho}\right)^2\left[\frac{1}{1-\Omega}-\frac{2\rho^2}{1-\rho\Omega}+\frac{\rho^4}{1-\rho^2\Omega}\right]+\tau^2},
\end{aligned}
$$

for $\Omega\equiv\left[(1-\lambda)\,e^2\left(1-r(\tilde{N})\lambda\right)^2\right]\left[\tau^2(1-\lambda)(1-r(\tilde{N}))^2\lambda^2\left(1-\kappa(\tilde{N})\right)^2+e^2\left(1-r(\tilde{N})\lambda\right)^2\right]^{-1}$. Here, because

$$
\frac{1}{\Omega}\left(=\frac{\tau^2(1-r(\tilde{N}))^2\lambda^2\left(1-\kappa(\tilde{N})\right)^2}{e^2\left(1-r(\tilde{N})\lambda\right)^2}+\frac{1}{1-\lambda}\right)
$$

is monotonically increasing in λ (and the lower bound is 1), Ω is monotonically decreasing in λ.

$$
\frac{1}{1-\Omega}-\frac{2\rho^2}{1-\rho\Omega}+\frac{\rho^4}{1-\rho^2\Omega}=(1-\rho)^2\left[\rho+\rho^3\Omega^2\right]>0,
$$

leads to $\sigma^2\left(\frac{1}{1-\rho}\right)^2\left[\frac{1}{1-\Omega}-\frac{2\rho^2}{1-\rho\Omega}+\frac{\rho^4}{1-\rho^2\Omega}\right]\left\{\sigma^2\left(\frac{1}{1-\rho}\right)^2\left[\frac{1}{1-\Omega}-\frac{2\rho^2}{1-\rho\Omega}+\frac{\rho^4}{1-\rho^2\Omega}\right]+\tau^2\right\}^{-1}\in(0,1)$.

Thus, there exists a unique solution of λ. Similarly, the sum of λ and λ' is given by,

$$\lambda + \lambda' = \frac{\tau^2(1-\lambda)(1-r(\tilde{N}))^2\lambda^2\left(1-\kappa(\tilde{N})\right)^2 + e^2\lambda\left(1-r(\tilde{N})\lambda\right)^2}{\tau^2(1-\lambda)(1-r(\tilde{N}))^2\lambda^2\left(1-\kappa(\tilde{N})\right)^2 + e^2\left(1-r(\tilde{N})\lambda\right)^2}.$$

Finally, ω_A and ω_B are respectively given by,

$$\omega_A = \frac{(1-r(\tilde{N}))\lambda}{1-r(\tilde{N})\lambda},\ \omega_B = \frac{(1-r(\tilde{N}))\left[\frac{\tau^2(1-\lambda)(1-r(\tilde{N}))^2\lambda^2\left(1-\kappa(\tilde{N})\right)^2+e^2\lambda\left(1-r(\tilde{N})\lambda\right)^2}{\tau^2(1-\lambda)(1-r(\tilde{N}))^2\lambda^2\left(1-\kappa(\tilde{N})\right)^2+e^2\left(1-r(\tilde{N})\lambda\right)^2}\right]}{1-r(\tilde{N})\left[\frac{\tau^2(1-\lambda)(1-r(\tilde{N}))^2\lambda^2\left(1-\kappa(\tilde{N})\right)^2+e^2\lambda\left(1-r(\tilde{N})\lambda\right)^2}{\tau^2(1-\lambda)(1-r(\tilde{N}))^2\lambda^2\left(1-\kappa(\tilde{N})\right)^2+e^2\left(1-r(\tilde{N})\lambda\right)^2}\right]}.$$

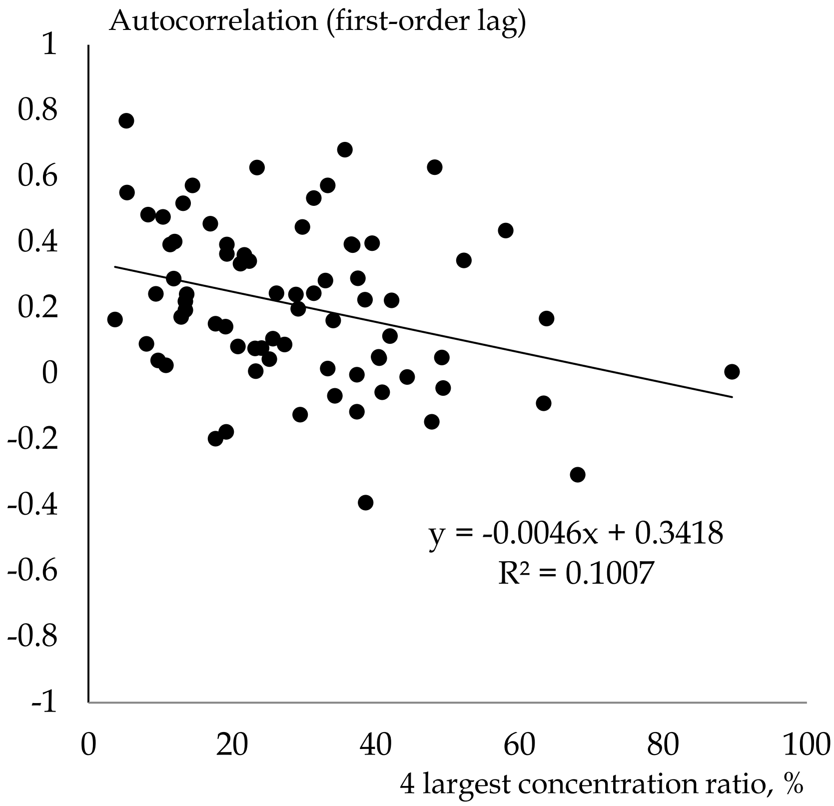
Therefore, the average price with endogenous public information is given by,

$$\overline{p}_t^* = \kappa(\tilde{N})\alpha + \left(1-\kappa(\tilde{N})\right)\left\{c_0 + \sum_{s=0}^{\infty}\left(\frac{1-\rho^{s+1}}{1-\rho}\right)\left\{\omega_A + (1-\omega_A)\left[1-(1-\omega_B)^s\right]\right\}\eta_{t-s}\right\},$$

where

$$\begin{aligned}
\omega_A &= \frac{(1-r(\tilde{N}))\lambda}{1-r(\tilde{N})\lambda}, \\
\omega_B &= \frac{(1-r(\tilde{N}))\left[\frac{\tau^2(1-\lambda)(1-r(\tilde{N}))^2\lambda^2\left(1-\kappa(\tilde{N})\right)^2+e^2\lambda\left(1-r(\tilde{N})\lambda\right)^2}{\tau^2(1-\lambda)(1-r(\tilde{N}))^2\lambda^2\left(1-\kappa(\tilde{N})\right)^2+e^2\left(1-r(\tilde{N})\lambda\right)^2}\right]}{1-r(\tilde{N})\left[\frac{\tau^2(1-\lambda)(1-r(\tilde{N}))^2\lambda^2\left(1-\kappa(\tilde{N})\right)^2+e^2\lambda\left(1-r(\tilde{N})\lambda\right)^2}{\tau^2(1-\lambda)(1-r(\tilde{N}))^2\lambda^2\left(1-\kappa(\tilde{N})\right)^2+e^2\left(1-r(\tilde{N})\lambda\right)^2}\right]}, \\
\lambda &= \frac{\sigma^2\left(\frac{1}{1-\rho}\right)^2\left[\frac{1}{1-\Omega} - \frac{2\rho^2}{1-\rho\Omega} + \frac{\rho^4}{1-\rho^2\Omega}\right]}{\sigma^2\left(\frac{1}{1-\rho}\right)^2\left[\frac{1}{1-\Omega} - \frac{2\rho^2}{1-\rho\Omega} + \frac{\rho^4}{1-\rho^2\Omega}\right] + \tau^2}, \\
\Omega &\equiv \frac{(1-\lambda)\,e^2\left(1-r(\tilde{N})\lambda\right)^2}{\tau^2(1-\lambda)(1-r(\tilde{N}))^2\lambda^2\left(1-\kappa(\tilde{N})\right)^2 + e^2\left(1-r(\tilde{N})\lambda\right)^2}.
\end{aligned}$$

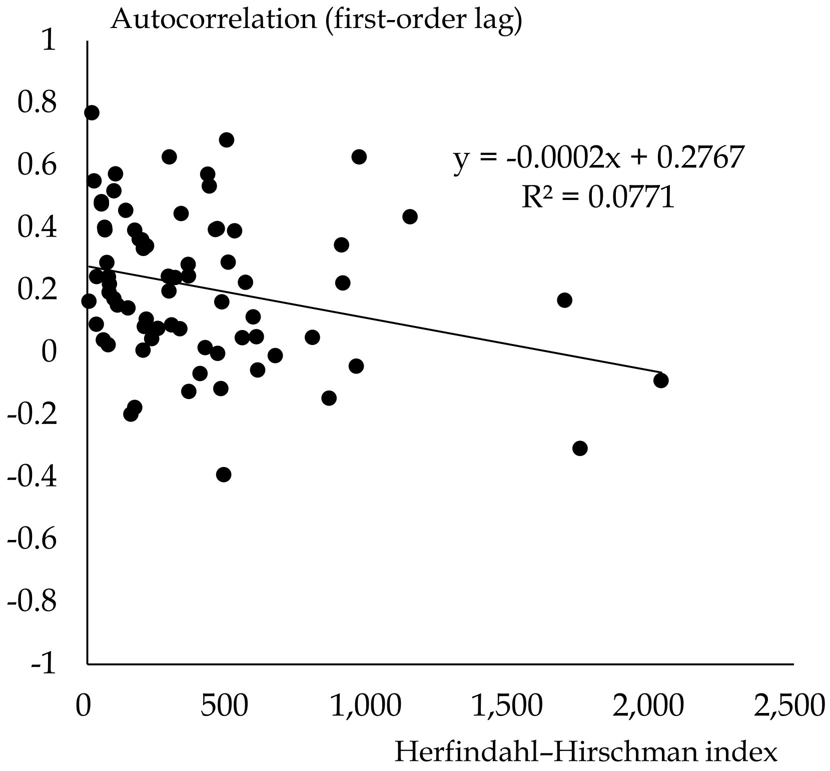
Figure 1: Inflation Persistence and Market Concentration of Manufacturers, C4 ratio



Note: Four-digit level NAICS disaggregation. "Petroleum and coal products manufacturing," "Nonmetallic mineral product manufacturing," and "Primary metal manufacturing" are excluded. Sample period is from January 2004 to November 2016.

Sources: Census Bureau "Economic Census," Bureau of Labor Statistics "Producer Price Index"

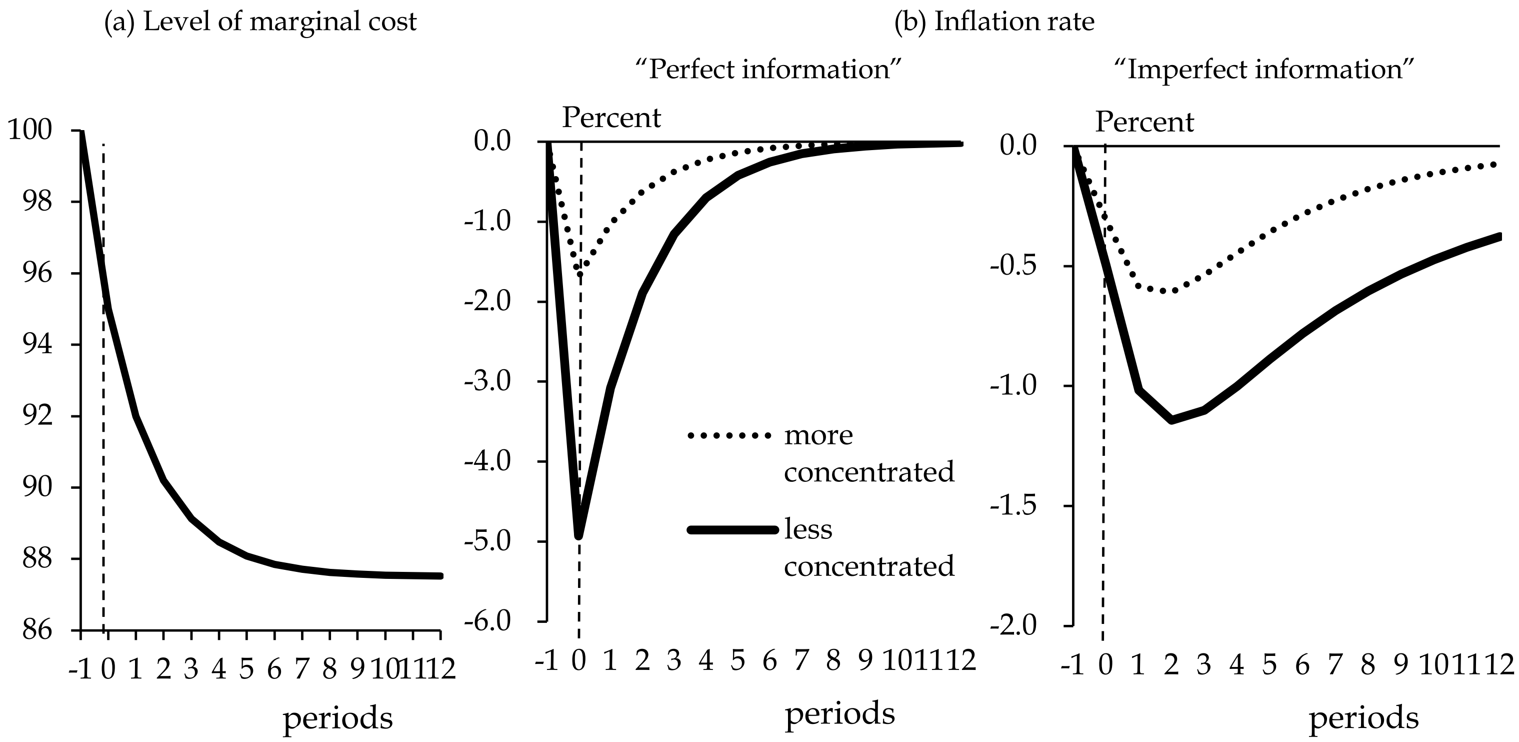
Figure 2: Inflation Persistence and Market Concentration of Manufacturers, HHI



Note: Four-digit level NAICS disaggregation. “Petroleum and coal products manufacturing,” “Nonmetallic mineral product manufacturing,” and “Primary metal manufacturing” are excluded. Sample period is from January 2004 to November 2016.

Sources: Census Bureau “Economic Census,” Bureau of Labor Statistics “Producer Price Index”

Figure 3: Impulse Response Functions

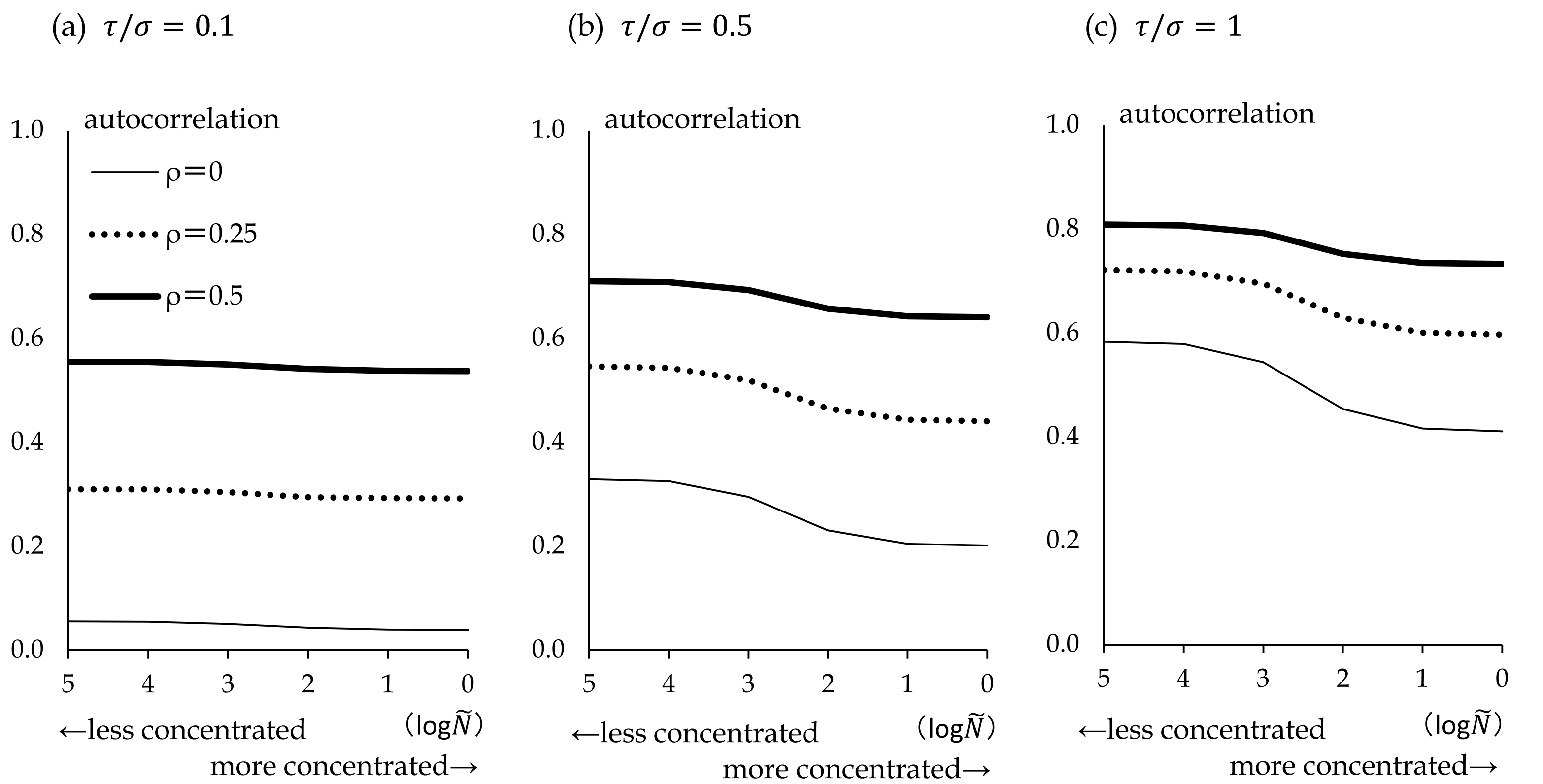


Note: Parameters are $\rho = 0.6, \beta = 200, \alpha = 200, c_0 = 100$.

τ/σ is 0 under perfect information and 10 under imperfect information.

$\log(\tilde{N}) = 0$ for "more concentrated" market, and $\log(\tilde{N}) = 4$ for "less concentrated" market.

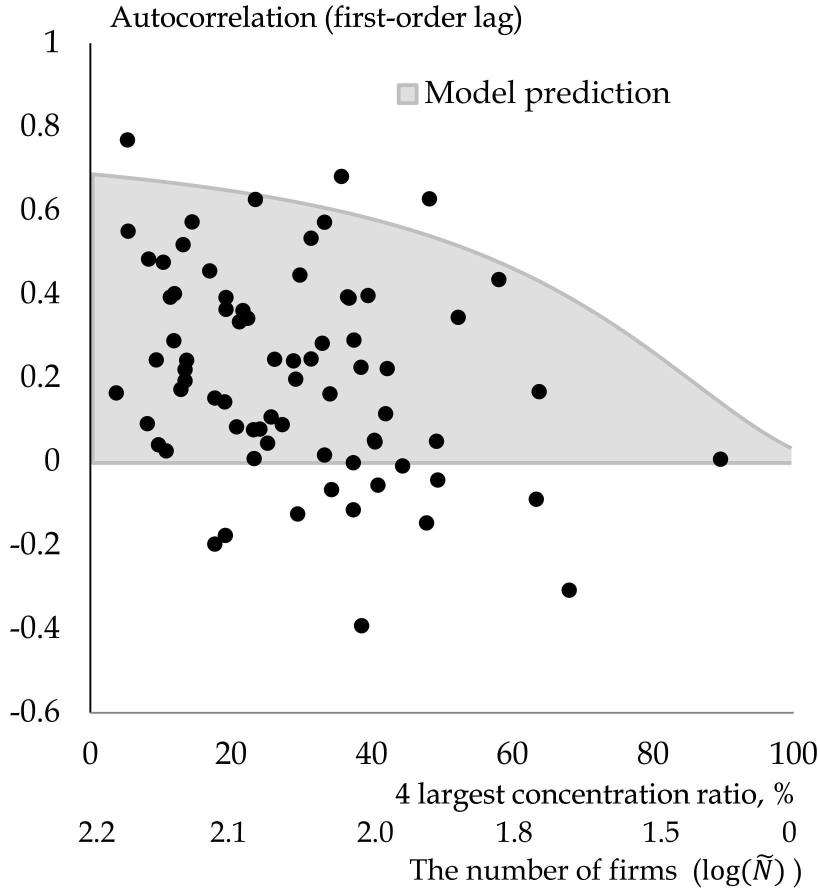
Figure 4: Inflation Persistence and Market Concentration (Simulations)



Note: ρ is the parameter of persistence of cost shocks. The number of simulated periods is 2,000.

Parameters are $\beta = 200, \alpha = 200,\ c_0 = 100$.

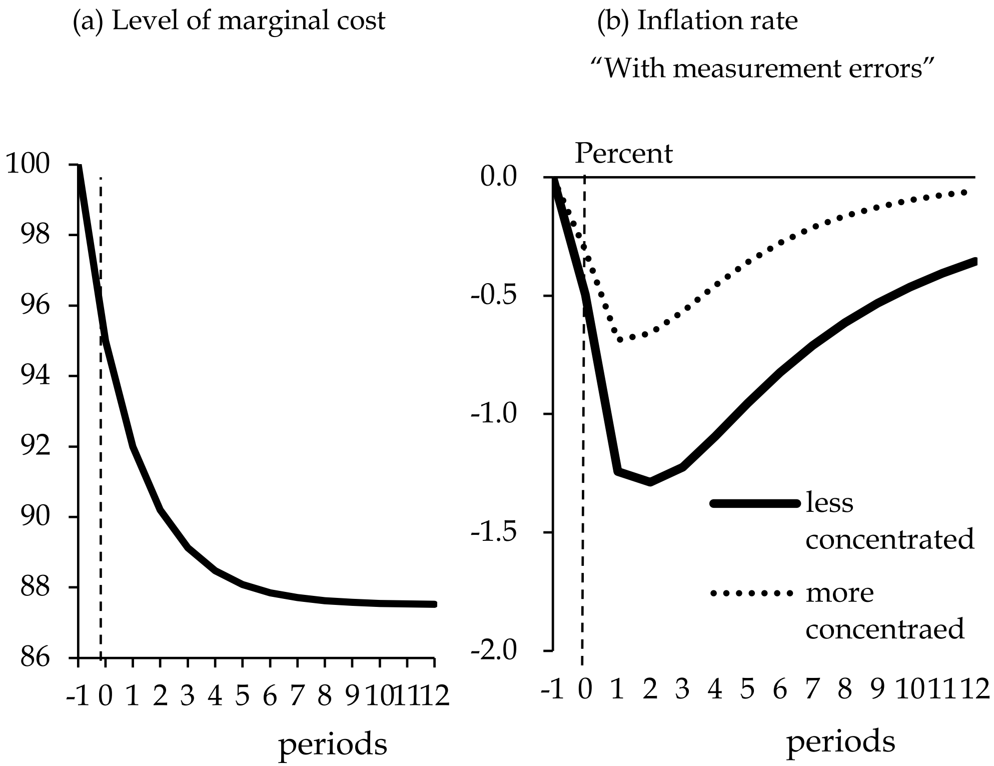
Figure 5 : Actual Observation and Theoretical Prediction with Mark-up Shocks; Inflation Persistence and Market Concentration of Manufacturers, C4 ratio



Note: The plotted data are replication of Figure 1. The shaded region indicates the "model prediction" based on simulation result with the parameterization: $\rho = 0, \beta = 10, \sigma = 1, \xi = \sqrt{5}$, and $\tau \in [0, 2]$.

Sources: Census Bureau "Economic Census," Bureau of Labor Statistics "Producer Price Index"

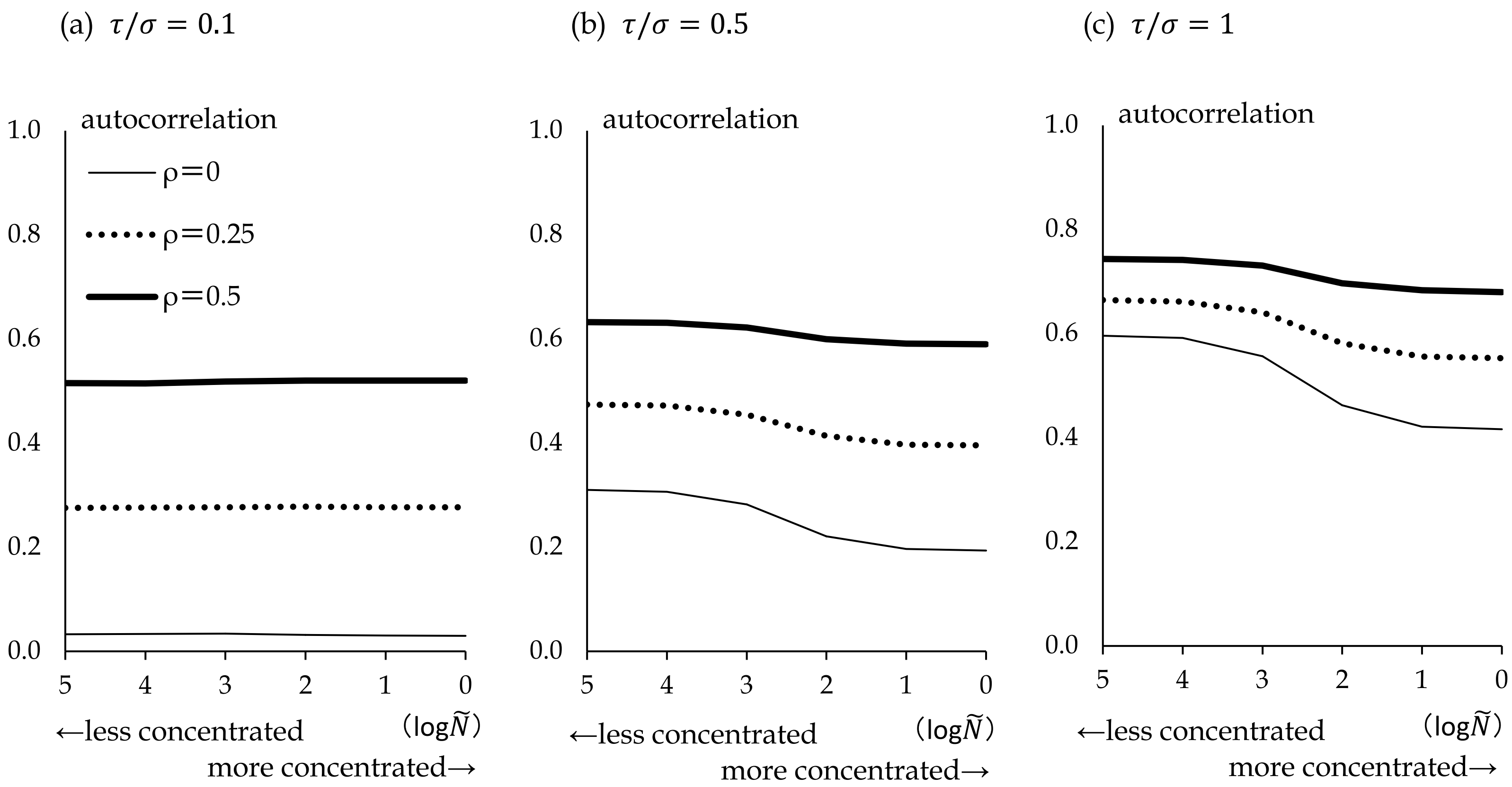
Figure 6: Impulse Response Functions with Public Information



Note: Parameters are $\rho = 0.6$, $\beta = 200, \alpha = 200$, $c_0 = 100$, $\tau/\sigma = 10$ and $e = 5$.

$\log(\widetilde{N}) = 0$ for "more concentrated" market, and $\log(\widetilde{N}) = 4$ for "less concentrated" market.

Figure 7: Inflation Persistence and Market Concentration with Public Information



Note: ρ is the parameter of persistence of cost shocks. The number of simulated periods is 2,000.

Parameters are $\beta = 200,\ \alpha = 200,\ c_0 = 100,$ and $e = 1$.

Table 1: Descriptive Statistics of Sectoral Inflation Persistence

	Boivin, Giannoni, and Mihov (2009)	*Extended dataset*	
Industries	*Manufacturers*	*Manufacturers*	*Nonmanufacturers*
Sample Period	*1976/2M-2005/6M*	*2004/1M-2017/2M*	
Average	*0.14*	*0.11*	*-0.08*
Median	*0.12*	*0.06*	*-0.08*
Minimum	*-0.44*	*-0.59*	*-0.60*
Maximum	*0.61*	*0.75*	*0.50*
Standard Deviation	*0.19*	*0.23*	*0.18*
Observations	*152*	*272*	*111*

Note: The inflation persistence is estimated by an AR(1) model using seasonally-adjusted monthly log-difference of sectoral prices (NAICS six-digit Classification).

Table 2: Regression Results (1976−2005, Manufacturers)

Dataset: Boivin, Giannoni, and Mihov (2009)

Manufacturers, 1976/2M−2005/6M, NAICS six-digit Classification

Dependent Variable: First-order autocorrelation of monthly inflation (ρ_i)

	(1)	(2)	(3)	(4)	(5)	(6)
Constant	0.225***	0.226***	0.231***	0.190***	0.192***	0.199***
	(0.042)	(0.041)	(0.041)	(0.029)	(0.029)	(0.030)
$C4_i$	−0.194**	−0.192**	−0.175**			
	(0.078)	(0.081)	(0.082)			
$HHI_i/1000$				−0.062***	−0.062***	−0.063***
				(0.023)	(0.023)	(0.024)
$\hat{\sigma}_i^2$		−1758.36			−3437.28	
		(6340.93)			(6283.96)	
d1			−0.073			−0.072
			(0.045)			(0.042)
d2			0.010			0.033
			(0.058)			(0.056)
Observations	152	152	152	145	145	145
Adjusted-R^2	0.031	0.024	0.047	0.034	0.028	0.059
SE	0.188	0.189	0.187	0.189	0.189	0.186

Note: Estimated by weighted-least-squares. The standard errors are HAC estimators. The dependent variable ρ_i is estimated by an AR(1) model using seasonally adjusted monthly log-difference of sectoral prices. C4 in the 2002 census is used in the regression. $\hat{\sigma}_i^2$ is the sample variance of the residuals of the AR(1) model. d1 is a dummy variable for the broad category of food and textiles (NAICS codes starting with 31) and d2 is a dummy variable for the broad category of paper, wood, and chemicals (NAICS codes starting with 32).

**** Significant at the 1 percent level.*

*** Significant at the 5 percent level.*

Table 3: Regression Results (2004−2017, Manufacturers)

Dataset: Manufacturers, 2004/1M−2017/2M, NAICS six-digit Classification						
Dependent Variable: First-order autocorrelation of monthly inflation (ρ_i)						
	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*
Constant	*0.177****	*0.179****	*0.151****	*0.146****	*0.141****	*0.116****
	(0.032)	*(0.031)*	*(0.039)*	*(0.024)*	*(0.024)*	*(0.032)*
$C4_i$	*−0.160***	*−0.199****	*−0.185****			
	(0.066)	*(0.061)*	*(0.064)*			
$HHI_i/1000$				*−0.048****	*−0.056****	*−0.055****
				(0.018)	*(0.018)*	*(0.018)*
$\hat{\sigma}_i{}^2$		*11713.56****			*9798.93****	
		(4210.32)			*(3537.75)*	
d1			*0.080*			*0.085*
			(0.051)			*(0.052)*
d2			*0.074*			*0.071*
			(0.044)			*(0.042)*
Observations	272	272	272	264	264	264
Adjusted-R^2	0.015	0.043	0.035	0.014	0.031	0.035
SE	0.227	0.224	0.225	0.225	0.223	0.223

Note: Estimated by weighted-least-squares. The standard errors are HAC estimators. The dependent variable ρ_i is estimated by an AR(1) model using seasonally adjusted monthly log-difference of sectoral prices. C4 and HHI in the 2007 census are used in the regression. $\hat{\sigma}_i{}^2$ is the sample variance of the residuals of the AR(1) model. d1 is a dummy variable for the broad category of food and textiles (NAICS codes starting with 31) and d2 is a dummy variable for the broad category of paper, wood and chemicals (NAICS codes starting with 32).

**** Significant at the 1 percent level.*

*** Significant at the 5 percent level.*

Table 4: Regression Results (2004−2017, All industries)

Dataset: All industries, 2004/1M−2017/2M, NAICS six-digit Classification			
Dependent Variable: First-order autocorrelation of monthly inflation (ρ_i)			
	(1)	(2)	(3)
Constant	0.087***	0.089***	−0.028
	(0.031)	(0.031)	(0.030)
$C4_i$	−0.080	−0.079	−0.156***
	(0.064)	(0.063)	(0.049)
$\hat{\sigma}_i^2$		−1003.69***	
		(357.8964)	
d			0.203***
			(0.031)
Observations	383	383	383
Adjusted-R^2	0.003	0.004	0.154
SE	0.232	0.232	0.214

Note: Estimated by weighted-least-squares. The standard errors are HAC estimators. The dependent variable ρ_i is estimated by an AR(1) model using seasonally adjusted monthly log-difference of sectoral prices. C4 in the 2007 census is used in the regression. $\hat{\sigma}_i^2$ is the sample variance of the residuals of the AR(1) model. d is a dummy variable for manufacturers.

**** Significant at the 1 percent level.*

*** Significant at the 5 percent level.*