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Abstract

To what extent does population ageing limit fiscal capacity and affect fiscal sustainability? We answer this question through the lens of a fiscal space defined by the budgetary room between the current tax revenue and the peak of a Laffer curve. We use a dynamic general equilibrium, overlapping generations model calibrated to data from Japan and the US. Our findings show that the evolution of underlying demographic structures plays an important role in shaping a country's fiscal capacity. There will be significant contractions in the fiscal space of Japan and the US when the two countries enter the late stage of demographic transition in 2040. In particular, the results from the model calibrated to Japan indicate that an increase in the old-age dependency ratio to over 70 percent can reduce Japan's fiscal space by 36 percent. The existing design of Japan's tax-transfer system is not fiscally sustainable by 2040 when factoring in the growing fiscal cost of the social security program.

Keywords: Population Ageing; Laffer Curve; Fiscal Limit; Sustainability; Heterogeneity; Dynamic General Equilibrium
JEL classification: E62, H20, H60, J11

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1 Introduction

The issue of fiscal sustainability has special importance in the aftermath of the global financial crisis in advanced economies. There have been sharp increases in debt to GDP ratios as the prolonged and deep recession generated automatic budget deficits and induced many countries to implement sizeable fiscal stimulus packages. Unfortunately, the fiscal situation in advanced economies is heading toward more trouble. The debt-to-GDP projections indicate increasing public debt is a prominent worry in all advanced economies (e.g., see Figure 1 and Cecchetti, Mohanty and Zampolli (2010)). Yet, providing funding to meet the need of age-related government spending will be even more of a pressing challenge in years to come. It has become clear that many governments will have a hard time fulfilling all existing fiscal commitments (e.g., see IMF (2010)). Advanced economies that have committed to generous ageing-related public programs are heading into a phase of fiscal unsustainability.

There is no doubt that every government's capacity for spending is grounded by its own revenue. However, we are far from fully understanding how such fiscal capacity evolves as the population ages. In this paper, we address this question through the lens of a fiscal space defined by the budgetary room between the current tax revenue and the maximum tax revenue. Intuitively, fiscal space is a relative measure of how capable a government is at generating more tax revenue. Holding government spending constant, fiscal space describes the government's fiscal capacity to meet spending commitments without compromising fiscal sustainability.

We use a dynamic general equilibrium model to quantify the effects of an ageing demographic structure on the fiscal space in two steps. First, we measure government capacity of raising revenue in terms of the budgetary room between the current tax revenue and the peak of an estimated Laffer curve. Basically, the peak of a Laffer curve defines the maximum revenue that a government can raise in order to cover its spending programs and repay government debt, i.e. fiscal limit. Next, we quantitatively assess how the size and shape of the fiscal space is affected by the ageing demographic structure.

More specifically, we formulate an overlapping generations model with uninsurable idiosyncratic risk and incomplete markets. The model consists of heterogeneous households, a perfectly competitive representative firm and a government. The Laffer curve fiscal limits are country-specific and determined by model fundamentals including preferences, endowments, technologies and market structure in our setting. Demographic factors, as part of the underlying model fundamentals, affect household behaviors and equilibrium conditions, and therefore play an important role in shaping a country's fiscal limit.

Among advanced economies Japan has a fast and large demographic transition to an older society and the highest debt to GDP ratio. For that reason, we choose Japan as a benchmark economy for our quantitative analysis. We calibrate our benchmark model to the data from Japan in 2010. Our model is capable of matching key patterns of life-cycle behavior and essential features of the Japanese macroeconomy. Next, we use the benchmark model to conduct a quantitative analysis. Our main results are summarized as follows.

First, we construct Laffer curves for labor and capital income taxes, and consumption taxes. We find that Laffer curves for labor and capital taxes have a single peak, while the Laffer curve for consumption tax has no peak and monotonically increases as the tax rate increases. For both labor and capital taxes, the benchmark economy lies to the left of the peaks of the Laffer curves. These findings indicate that Japanese government can increase one of the two taxes to raise revenue. In order to quantify exactly how much tax revenue is left we construct the fiscal space for Japan. We use a gap between the 2010 tax revenue and the maximum revenues defined by the three Laffer curves. We find that the government can, by altering either labor income tax or capital income tax while keeping the other constant, increase the fiscal spaces for labor and capital income taxes by 43 percent and 17 percent, respectively. Moreover, when the labor and capital income taxes are allowed to vary at the same time we find that the two dimensional fiscal space for Japan, measured in terms of Laffer hill, can be expanded further up to 45 percent.

Second, we quantitatively characterize how changing demographic structures affect the size and shape of the fiscal space. We employ two alternative demographic structures: the younger one of 1980 and the older one of 2040. We find that aging demographic structure shifts the Laffer curves downward and causes a significant contraction in Japan's fiscal space by 2040. Specifically, the demographic shift leads to a contraction in the fiscal space for the capital income tax by 65 percent, compared to the 2010 level. Meanwhile, the fiscal space for the labor income tax is contracted by 38 percent and the fiscal space for the consumption tax is decreased by 11 percent. Even though the fiscal space is diminished significantly, the government still has some room to raise tax revenues. The government can increase tax revenues by an additional 51 percent, when the labor income tax rate is set to 0.68. By raising labor and capital tax rates simultaneously, the government can raise revenue by an additional 54 percent, compared to the benchmark level of 2010 tax revenue.

In addition, we construct the net fiscal space where we account for the increased government outlay due to commitments to the age pension program. Expectedly, the net fiscal space will shrink even more. When the government can only alter one tax instrument at a time, the net fiscal spaces for labor and capital income taxes will be contracted by 59.0 percent and 13 percent, respectively. Most notably, the net fiscal space for the capital income tax disappears completely, and only 96 percent of the 2010 fiscal space can be maintained at the peak of 2040 Laffer curve for the capital income tax.

Moreover, we identify the relative importance of the fertility and mortality rates in contributing to the decrease in the fiscal space. We find that over the 1980 to 2010 period, the increase in survival probabilities and decrease in fertility rate have contributed equally to ageing the population and decreasing fiscal space. However, the contribution will be asymmetrical over the 2010 to 2040 period, with fertility as the main driver for further decreases in the fiscal space. We also note that a purely fertility rate driven change in the dependency ratio will shift the fiscal space to a greater extent than a purely mortality rate driven change in the old age dependency ratio of the same size.

Finally, we explore how demographic shifts affect the US fiscal space. We re-calibrate our benchmark model to match the US data in 2010. We conduct similar experiments and find that the effects of the US demographic shift on the US fiscal space are quite different. Unlike Japan, there was a large expansion in the US fiscal space from 1980 to 2010. This expansion is driven mainly by increase in the population share of workers in their 40s. However, as the population ageing is accelerated, the fiscal space for the US will be contracted by 40 percent in 2040. These results suggest that fiscal limits and fiscal spaces vary greatly across countries and over time, depending on the evolution of underlying demographic factors.

Related literature. By its focus, our paper is connected to the recent literature attempting to quantify fiscal space. Ostry, Ghosh, Kim and Qureshi (2010) define fiscal limit by the notion of the debt limit above which the debt becomes unsustainable. They estimate the debt limits using a reduced-form model for a country's policy reaction function. They construct fiscal space in terms of a distance between the current debt levels and the debt limits. Similarly, Ghosh, Kim, Mendoza, Ostry and Qureshi (2013) estimate a country's debt limit using a stochastic ability-to-pay model of sovereign default. They use data from 23 advanced economies to estimate the response of primary surpluses to debt levels and compute a debt limit for each country that is fully determined by the risk-free interest rate, the recovery rate, and the range of the shock to primary balances. They define fiscal space in terms of the difference between the long-run average debt ratio and the debt limit. Notice that the empirically-based approach to calculating fiscal limit and space is grounded in historical data. Their calculations rely on the underlying assumption that the government always follows its historically estimated rule and that there is no structural change in the economic environment. Any change in policy rules as well as economic fundamentals would alter the country's fiscal limit, destabilizing the backward measure of fiscal space.

Our paper is also related to the growing literature on fiscal limits in general equilibrium models. Bi and Leeper (2013) show that a country's fiscal limit varies systematically with the economic environment, including the specification of policy behavior. They formulate a real business cycle model and map the economic environment into a distribution for the maximum sustainable debt-GDP ratio. Bi and Traum (2014) use Bayesian methods to estimate the fiscal limit distribution for Greece, using a real business cycle model that allows for interactions among fiscal policy instruments, the stochastic fiscal limit and sovereign default. Richter (2015) uses a perpetual youth model to examine how intergenerational redistributions of wealth, the average duration of government debt, and entitlement reform impact the consequences of explosive government transfers. These studies focus on interactions between monetary and fiscal policies. They emphasize that future fiscal deficits are driven by the growth in age-related government spending programs. However, these studies abstract from mapping out an explicit link between demographic structures and fiscal limit, which is the main focus of our paper.

Our paper contributes to a growing body of literature using dynamic general equilibrium models to quantify Laffer curves. Trabandt and Uhlig (2011) construct Laffer curves for the US and the EU 14 in a infinitely lived representative agent model. They find that the classic Laffer curve shape exists for labor and capital income taxes. Park (2012) applies the Trabandt and Uhlig representative framework to examine the effects of ageing on the G-7 nations' capacities for raising tax revenue. However, Park abstracts from foundations of demographic structure and models population ageing as an exogenous shift in the dis-utility from supplying labor over time. Mendoza, Tesar and Zhang (2014) formulate a two-country representative agent model with cross-country tax externality to quantify Laffer curves for eurozone countries. They focus on the positive and normative effects of alternative tax strategies that countries could follow to restore fiscal solvency in response to debt shocks. Instead, we construct a life-cycle model that explicitly accounts for underlying demographic factors. This modelling approach allows us to quantify the effects of demographic changes on fiscal solvency through the lens of the Laffer curve and fiscal space.

Our work is related with recent work on the Laffer curve in heterogenous agents model. Feve, Matheron and Sahuc (2013) extend Trabandt and Uhlig (2011) to incorporate heterogeneous agents, market incompleteness and government debt. They show that the Laffer curve depends on the level of outstanding government debt. Holter, Krueger and Stepanchuk (2014) consider a life-cycle model with realistically calibrated wage heterogeneity and risk, extensive margin labor supply choice as well as endogenous human capital accumulation. They find that household heterogeneity and the degree of tax progressivity matter for the level and location of the peak of the Laffer curve. Guner, Lopez-Daneri and Ventura (2016) quantify the extent to which a more progressive income tax system can raise tax revenues in a life-cycle economy. Differently, we demonstrate that population ageing will significantly affect the limit to which a government can raise tax revenues.

Since Auerbach and Kotlikoff (1987) there is a vast literature that uses overlapping generations (OLG) models with inter- and intra-generational heterogeneity and population dynamics to study the dynamic effects of fiscal policy. Recently, that literature has been extended to quantifying the consequences of population ageing and fiscal adjustments in advanced economies (e.g., see Kitao (2015), Braun and Joines (2015), Nishiyama (2015) and Kudrna, Tran and Woodland (Forthcoming)). Notice that most of these studies focus on quantifying the expenditure-side effects of population ageing as well as the effects of counterfactual fiscal adjustments. We connect that literature on the fiscal implications of ageing to the literature on quantifying Laffer curves.

The remaining paper is organized as follows. Section 2 provides an overview of the model. Section 3 outlines the calibration exercise. Section 4 presents the quantitative analysis and results. Section 5 offers a sensitivity analysis and extension. Section 6 offers a conclusion. The additional information, Tables and Graphs are included in the Appendix.

2 Model

The model is a stochastic dynamic general equilibrium model, consisting of overlapping households, a perfect competitive representative firm, and a government with full commitment technology. The economy is assumed to be on a balanced-growth path with a constant labor-augmenting productivity growth rate g and a constant population growth rate g^n.

2.1 Demographics

In each discrete time period t, the economy is populated by J overlapping generations of households of generations $j = 1, ..., J$. Each period, a new cohort of households of the generation is born. Households of a particular generation share a common chance of dying before reaching the next period. The probability of surviving to generation $j+1$ conditional on belonging to generation j is denoted by sp_j. We denote the size of the generation j cohort at the beginning of time t as P_j. The cohort share of the generation

j households at time t is given by $\mu_j = \frac{P_j}{\sum_{j=1}^{J} P_j}$. Demographic structure is driven by two factors: (i) the age-dependent survival probability (sp_j), and (ii) the population growth rate (g^n). When the demographic pattern is stationary, the population share of the cohort age j is constant at any point in time and can be recursively defined as $\mu_j = \mu_{j-1} sp_j / (1 + g^n)$. The share of agents who do not survive to age j is $\widetilde{\mu}_j = \mu_{j-1} (1 - sp_j) / (1 + g^n)$.

2.2 Preferences

All households have identical lifetime preferences over consumption $c_j > 0$ and leisure l_j, where household leisure time per period for household j is constrained by $0 \leq l_j \leq 1$. Preferences are time-separable with a constant subjective discount factor β and are given by the expected utility function

$$E\left[\sum_{j=1}^{J} \beta^j u(c_j, l_j)\right]. \tag{1}$$

2.3 Endowments

In each period of life households are endowed with one unit of labor time that has labor efficiency (or working ability) denoted by e_j. The efficiency unit e_j is skill and age dependent and follows a Markov switching process with $\pi_j(e_{j+1}|e_j)$ denoting the conditional probability that an individual of working ability e_j at age j will have working ability e_{j+1} when at age $j+1$. According to this specification, agents have working abilities that vary by age and change stochastically over the life cycle; they therefore face idiosyncratic earnings risk, which is assumed to be non-insurable.

Households devote l_j units of time to leisure and supply $n_j = (1 - l_j)$ units of time to the labor market. The quantity of effective labor supplied is given by $h_j = (1 - l_j)e_j = n_j e_j$, and labor earnings are $w_t h_j$. For the retired periods $j = J^w + 1, ..., J$, households are out of the labor force and consume $l_j = 1$ units of leisure time, and do not have labor earnings.

We let a_j denote asset holdings of a typical agent at age j. We assume households enter the economy with no assets, so $a_1 = 0$. We let $x_j = \{a_j, e_j\}$ denote the state variable of a typical household at age j and $\mu(x_j)$ denote the measure of households in state x_j.

2.4 Technology

The production sector consists of a large number of competitive firms, and can be proxied by a single producer that maximizes profits. It produces a single output (Y_t) each period from two inputs, capital (K_t) and effective labor (H_t), based on a constant returns to scale production function $Y_t = A_t F(K_t, H_t)$ where A_t is the total factor productivity. The firm is a price taker in the input markets for capital and labor, and aims to maximize its profit given the rental rate (q_t) and market wage rate (w_t) by choosing K_t and H_t such that

$$\max_{K_t,\, H_t} \{A_t F(K_t, H_t) - q_t K_t - w_t H_t\} \tag{2}$$

2.5 Fiscal policy

The government runs a social security system and two other spending programs.

Social security system. The government provides the retirement benefits p_j to all retiring households at age $j = J^w + 1, ..., J$. The social security benefits are given by a replacement rate Ψ and an average life-time labor earning wH_{J^w} at time t, so that $p_j = \Psi w H_{J^w}$. The total social security payment for all retirees at time t is $SS_t = \sum\limits_{j=J^w+1}^{J} \int_{x_j} p_j \mu_j (x_j)$. The social security system is partially funded by social security tax revenue and partially funded by the general government budget. Let θ^{ss} denote a fraction of the total social security payment funded by the general government budget. The social security tax τ^{ss} adjusts to clear the rest of the social security payment

$$\tau^{ss} \sum_{j=1}^{J^w} \int_{x_j} w_t h_j \mu_j (x_j) = (1 - \theta^{ss}) SS_t, \tag{3}$$

with $\theta^{ss} \in [0, 1]$. There are two special cases. When $\theta^{ss} = 0$, the social security system is fully self-financed. When $\theta^{ss} = 1$, the social security system is part of overall government operations.

Government budget. The government has two spending programs: general government purchases, G_t, and transfers to households, $Tr_t = \sum\limits_{j=1}^{J} \int_{x_j} tr_j \mu_j (x_j)$, where tr_j is individual lump-sum transfers received by households. In order to finance its expenditures the government collects revenues through taxes on consumption (τ^c), labor income (τ^l) and capital income (τ^k). The total tax revenue is given by

$$Tax_t = \tau^c \sum_{j=1}^{J} \int_{x_j} c_j \mu_j (x_j) \; + \tau^l \sum_{j=1}^{J^w} \int_{x_j} w_t h_j \mu_j (x_j) + \tau^k \sum_{j=1}^{J} \int_{x_j} r_t a_j \mu_j (x_j),$$

where w_t is the market wage rate and r_t is the market interest rate.

The government maintains budget balance each period, using a combination of the taxation revenue and issuance of new debt D_{t+1} to fund interest and principle payments on existing debt $(1+r_t^d)D_t$ with r_t^d the interest rate for government debt. The government inter-temporal budget is given by

$$(1+g^n)(1+g)D_{t+1}+Tax_t=(1+r_t^d)D_t+G_t+\theta^{ss}SS_t+Tr_t. \tag{4}$$

2.6 Market structure

Markets are incomplete and households cannot insure against the idiosyncratic labor income and mortality risks by trading state contingent assets. They can, however, hold one-period riskless assets to imperfectly self-insure against idiosyncratic risks. We assume that households are not allowed to borrow against future income, implying asset holdings are non-negative, i.e., $a_j \geq 0$ for all j.

The economy is closed, and the domestic interest rate is determined endogenously. It is related to the rental price of capital by $r_t = q_t - \delta$, where q_t is determined by the demand and supply for capital in the economy, and δ is the depreciation rate of capital.

2.7 Household problem

In this model, households are heterogeneous with respect to their age, working ability and asset holdings. Over their lifetime, households have different sources of income. First, during working period between 1 and J^w, households supply labor to the firm in return for wage income, $w_t h_j$. Labor income is subjected to a social security tax (τ^{ss}) and labor income tax (τ^l). Next, households exit the labor market and retire at age $j > J^w$. Households' savings are rented to the firm in the form of capital the follow period. It earns interest at the rate r_t and is taxed at the rate τ^k. Henceforth, the post-tax return is $R_t^k = 1+(1-\tau^k)r_t$. During retirement, households receive a public pension benefit (p_j) from the government. Households also receive an equal share of lump-sum transfers from the government (tr_j). Finally, as there are no annuity markets, the savings (including interest return) of households who die each period are shared equally amongst the remaining households as accidental bequests (b_j).

At the beginning of age j the household realizes its individual state x_j and chooses its optimal consumption, c_j, leisure time, l_j, or working hours, $(1-l_j)$, and the end-of-period asset holdings, a_{j+1}, taking the transition law for working ability, $\pi_j(e_{j+1}|e_j)$, conditional survival probabilities, sp_j, the wage and interest rates, and government tax and pension policies as given. Formally, the Bellman equation for a household of age j

is given by

$$V_j(x_j) = \max_{c_j,\, l_j,\, a_{j+1}} \{u(c_j, l_j) + sp_j \beta E\left[V_{j+1}(x_{j+1})\right]\} \tag{5}$$

subject to

$$(1+g)\, a_{j+1} + (1+\tau^c)c_j = \begin{cases} R_t^k a_j + (1-\tau^l-\tau^{ss})w_t h_j + tr_j + b_j & \text{if } j = 1,..,J^w \\ \\ R_t^k a_j + p_j + tr_j + b_j & \text{if } j > J^w, \end{cases}$$

where $V_j(x_j)$ is the value function of a household at age j conditional on the given state variable x_j and $E\left[V_{j+1}(x_{j+1})\right]$ is the expected value function. Additional constraints are $a_1 = 0$, $a_{J+1} = 0$, $a_j \geq 0$ and $0 < l_j \leq 1$.

2.8 Equilibrium

Given a set of exogenous demographic parameters $\{sp_j\}_{j=1}^{J}$ and $\{g^n\}$, exogenous growth rate $\{g\}$ and fiscal policy variables $\{\Psi, \theta^{ss}, \tau^{ss}, \tau^k, \tau^l, \tau^c, G, \frac{D}{Y}, tr, p, r^d\}$, a competitive equilibrium consists of a collection of household decisions $\{c_j\left(x_j\right), l_j\left(x_j\right), a_{j+1}\left(x_j\right)\}_{j=1}^{J}$ for each state vector x_j, factor prices $\{w, r\}$, consumption tax $\left\{\tau^l\right\}$, the measure of individual state $\{\mu\left(x_j\right)\}$ such that

(a) the households solve the household problem (5);

(b) the firm chooses labor and capital inputs to solve the profit maximization problem (2);

(c) factor prices are determined competitively, i.e., $w = F_L\left(K, L\right)$, $q = F_K\left(K, L\right)$ and $r = q - \delta$; and the domestic markets for capital and labor clear

$$\begin{aligned} K &= \sum_{j=1}^{J} \int_{x_j} a_j\left(x_j\right) \mu_j\left(x_j\right) + B - D, \\ H &= \sum_{j=1}^{J} \int_{x_j} \left(1 - l_j\right) e_j\left(x_j\right) \mu_j\left(x_j\right), \end{aligned}$$

where $B = \sum_{j=1}^{J} \int_{x_j} a_j\left(x_j\right) \widetilde{\mu}_j\left(x_j\right)$ is the total amount of assets left by all the deceased agents;

(d) the labor tax $\left\{\tau^l\right\}$ adjusts, so that the government budget constraint defined in Eq. (4) is satisfied;

(f) the aggregate resource constraint is given by $C + I + G = Y$, where

$$C = \sum_{j=1}^{J} \int_{x_j} c_j(x_j)\,\mu_j(x_j) \text{ and } I = \sum_{j=1}^{J} \int_{x_j} a_{j+1}(x_j)\,\mu_j(x_j).$$

3 Calibration

In this section we describe the parameterization and calibration of the benchmark model. Among advanced economies Japan has a fast and large demographic transition to an older society with the highest debt to GDP ratio. We choose Japan as a benchmark economy for our quantitative analysis. We calibrate our model to match the Japanese economy in an artificial steady state in 2010. We source the values of model parameters from the previous literature and the macroeconomic data on government tax and fiscal policy, and population dynamics. We calibrate some structural parameters and fiscal policy variables to replicate life-cycle profiles of labor supply and asset holdings and targeted macroeconomic aggregates in the base year. The values of key parameters of the benchmark model is in Table 1.

3.1 Demographics

The model economy is populated by 16 overlapping generations of households in each discrete time period t, with each period lasting for 5 years. Households become economically active at age 20, and the $j = 1$ generation correspond to ages 20 to 24, $j = 2$ to 25 to 29 and so forth, with $J = 16$ – the oldest generation – corresponding to ages 95 to 99. We use data from the Japanese National Institute of Population and Social Security Research (IPSS) to construct the demographic structures for Japan. We use actual 5×5 (age interval $\times$ year interval) life tables (for both sexes) to construct the conditional survival probabilities. The demographic pattern is not stationary in 2010, we are not able to replicate the shape of population distribution based on the population transition equation $\mu_j = \mu_{j-1} sp_j / (1 + g^n)$ in the model. We choose to adjust population growth rates to replicate the actual age-distribution in 2010 in our benchmark calibration.

3.2 Preferences

The instantaneous utility from consumption and leisure is given by

$$u(c_j, l_j) = \frac{1}{1-\sigma} \left\{ [c_j]^{1-\sigma} \left[1 - \kappa(1-\sigma)(1-l_j)^{1+\frac{1}{\phi}} \right]^{\sigma} - 1 \right\}, \tag{6}$$

where κ represents the dis-utility from work and $1/\sigma$ is the inter-temporal elasticity of substitution. This functional form of Constant Frisch Elasticity (CFE) preferences is specified in Trabandt and Uhlig (2011).

Iiboshi, Nishiyama and Watanabe (2006) estimated a mean of σ at 2.041 in their dynamic stochastic general equilibrium model of the Japanese economy. Kuroda and Yamamoto (2007) estimated the Frisch elasticity (ϕ) for Japan on the extensive and intensive margins combined at between 0.7 and 1.0 for both sexes over the 1990 period. These estimates are in the value range used in the literature (e.g., see Hall (2009) and Trabandt and Uhlig (2011)).

We calibrate the value of parameter κ so that the average hours worked per working age person ($\bar{n}$) as a fraction of total time is 0.30. The value of parameter β is set to match a capital output ratio $\left(\frac{K}{Y}\right)$ of 3.

3.3 Endowments

The labor productivity, e_j, of an age j household in the model economy evolves over the life cycle according to $\ln e_j = \ln \overline{e}_j + \ln \widetilde{z}_j$ for $j = 21, ..., 65$. The evolution of labor productivity has two components: deterministic, $\overline{e}_j$, and stochastic, $\widetilde{z}_j$.

The deterministic component $\{\overline{e}_j\}$ is based on the estimates of the age-profile for Japan in Braun, Ikeda and Joines (2009). That age-profile of time-invariant labor productivity is constructed from Japanese data on employment, wages, and weekly hours from 1990 to 2000. The age-specific labor productivities are values over five-year age groups, starting from age $20 - 24$, to ages 65 and over. We set $\overline{e}_j = 0$ for $j = J^w + 1, ..., J$. The results are plotted in Figure 2. The life-cycle profile is hump-shaped, reflecting the productivity gains as households gain experience, before declining at the end of the working life.

The idiosyncratic component $\widetilde{z}_j$ of labor productivity is specified as a first-order autoregressive process in log as

$$\ln \widetilde{z}_j = \rho \ln \widetilde{z}_{j-1} + \epsilon_j,$$

where the temporary shock, ϵ_j, is normally distributed. We set the persistence parameter $\rho = 0.97$ and the variance of the white noise $\sigma_\epsilon^2 = 0.03$, which lie in the range of estimates in Lise, Sudo, Suzuki, Yamada and Yamada (2014). We approximate this continuous process with a three-state, first-order discrete Markov process.

3.4 Technology

We assume the production function has the Cobb-Douglas functional form

$$Y = AK^{\alpha}H^{1-\alpha}. \tag{7}$$

We set the capital share α at 0.4 and the depreciation δ at 0.082, using estimates from Muto, Oda and Sudo (2016). These are also close to the values from Hayashi and Prescott (2002). We set A to grow at a constant rate.

3.5 Fiscal policy

Social security. The replacement rate Ψ is set at 0.33 to match the size of the social security system as a proportion of output in 2010. The fraction of the social security payment contributed by the general government budget is determined by

$$\theta^{ss} = 1 - \frac{\text{total pension contribution}}{\text{total pension transfers}} \tag{8}$$

As in Muto, Oda and Sudo (2016) we choose $\theta^{ss} = 0.413$. The social security tax rate is adjusted to keep the social security fund in balance. In our benchmark model, the equilibrium social security tax rate is around 10%.

Other government expenditures and debt. Government expenditures including the spending for health care and long-term care are 20% of aggregate output according to the National Accounts of Japan (SNA) in 2010. We set $\frac{G}{Y}$ at 20% to match that size.

The net government debt to GDP ratio $\frac{D}{Y}$ is calculated using the net debt to GDP ratio from the IMF. The government debt to GDP is set at 110%. The average number of years to maturity of outstanding government bonds is about 7 years and the average real interest rate on 7 year government bonds is 1.0% in $2000 - 2010$. The interest rates on government bonds (r^d) is set at 0.01, matching the implied yield on 10-year Japanese government bonds. This is set exogenously as they are significantly lower than the endogenous interest rate on private capital (r).

Taxes. The consumption tax is set at 5% in the initial steady state. Capital income tax is set at 30%, which is in the range of estimates of effective tax rates on capital income, for example, in Braun and Joines (2015). The labor income tax rate that clears the government budget constraint is 26% in the benchmark calibration. The combining rate of the labor income tax rate and the social security tax rate is around 36%, which is very close to the labor tax rate in Kitao (2015).

3.6 Benchmark model performance

In this section, we present the calibration results of the benchmark model based on 2010 demographics, and discuss how well the model matches the data in describing the Japanese economy.

The calibration results are reported in Figure 3. The first panel depicts the life-cycle asset holdings of households, relative to asset holdings at age 50. The hump-shape is consistent the life-cycle hypothesis, where households acquire assets over their working life and run down their savings over retirement. We discipline the household sector to data from the 2009 National Survey of Family Income and Expenditure (NSFIE). Our model is able to replicate the accumulation of asset holdings over the households' working life, with households maximising their asset holdings at age 60.

The second panel presents the average number of labor hours supplied by households of different ages per week. We compare this to data from the 2010 Labor Force Survey, using the average number of hours worked by employed persons of different age groups across all industries. As we do not distinguish between employed and unemployed households in our model, the hours worked are adjusted by the employment rate within each particular age group.[1] Our model is able to generate the lifecycle shape of labor supply, with households supplying less hours at younger ages, supplying more during the middle ages, and supplying less again as they approach retirement age.

The last panel presents the labor earning profile of different age groups, relative to the age 50 group. Our model can replicate the life cycle pattern of average labor income for workers in the data from the 2009 NSFIE. Specifically, the model can generate the hump-shape of the labor incomes. However, labor income is maximized for households at age 50 in the data, whereas it is maximized at age 45 in the model. Furthermore, the drop off in labor income is both earlier and steeper in the model. This is especially apparent for ages 65 and over where we assumed exogenous retirement and the labor income is zero. This is in contrast to the data, where the surveyed households by definition continue to work in those periods and earn wages income.

4 Quantitative analysis

In this section, we first discuss how we quantify the fiscal limit and measure the fiscal space. We next analyze how the evolution of demographic structures over time affects

[1] Average hours worked per week per household (per generation) = $\frac{\text{Total number employed}}{\text{Total population}} \times$ Average hours worked per week for each employed person. Further, for the model labor supply, average hours per week is derived by multiplying the result by 14×7, where 14 is the number of total free hours we assumed households have per day from the calibration section.

the size and shape of fiscal space.

4.1 Fiscal space

Laffer curve. In a fundamentals-based model, higher distorting taxes diminish household incentives to work and save, and subsequently shrinks tax bases. In this setting, government faces a trade-off when increasing the tax rate to raise revenues. There are tax rates that balance out the trade-off and maximize tax revenues. This Laffer curve reasoning implies a natural limit on the government capacity to raise tax revenues. When the economy is placed at the peak of Laffer curve, the maximum tax revenue is defined.

We base on the Laffer curve approach to define fiscal limit and fiscal space. In our setting, there are three taxes in the benchmark model. Accordingly, we construct three Laffer curves. To do so, we vary only one tax rate of interest at a time, while holding the other two tax rates constant at the benchmark level. In our baseline analysis, we assume that the government adjusts general government purchases (G) to keep its budget in balance. We call it the g-Laffer curve.[2]

More specifically, we construct the Laffer curves for the labor and capital income taxes, and the Laffer curve for the consumption tax. Figure 4 presents the three g-Laffer curves. To ease our comparison we normalize the tax revenue in the benchmark model to 100.

The Laffer curves for the labor and capital income taxes have the classic single-peaked concave shape, while the Laffer curve for consumption tax does not have a peak in the range of tax rates between 1 and 100 percent. The benchmark economy lies to the left of the peaks of the Laffer curves for labor and capital income taxes. The maximum tax revenues are reached when setting the labor income tax rate (τ^l) at 63 percent and the capital income tax rate (τ^k) at 82 percent. On other hand, the Laffer curve for the consumption tax does not have a peak, we note that by setting $\tau^c = 0.95$, the government can raise up to 206 percent of the current tax revenue.

The differences in the shape of three Laffer curves are mainly driven by interaction between the tax rate and the tax base.[3] The graph for the decomposition can be found in Figure 6. As the labor income tax rate increases, the labor income tax base decreases almost linearly as the hours worked decrease. The multiplication of two linear factors with opposing signs gives rise to the inverse-U shape. For the capital income tax, the

[2]In our general equilibrium model, there are several options to keep the government budget in balance when raising taxes: decreasing general government purchases (G) and government debt (D), or increasing transfers (Tr) and public pension (p). We will consider the other budget balancing rules in section 5.

[3]The labor tax, capital and consumption tax bases are given by $(1-\tau^{ss}(\tau))\times w(\tau)\times H(\tau)$, $r(\tau)\times K(\tau)$, and $C(\tau)$, respectively, where $\tau = \{\tau^l, \tau^k, \tau^c\}$.

tax base is mostly flat at lower tax rates, before decreasing at higher rates. This gives rise to the asymmetrical shape. Interestingly, both labor and capital tax bases are barely affected by the increase in the consumption tax rate.

The overall shape of the Laffer curves in our heterogeneous agents model are quite similar to the ones in representative agent models (e.g., see Trabandt and Uhlig (2011)). However, there are some noticeable differences. The Laffer curve for the capital income tax is not flat to the left of the peak. Whereas the labor income tax revenue decreases in a representative agent model, it is flatter in our overlapping generations model. The main reason is that households supply is more elastic to decreases in wage rates caused by an increase in the capital income tax rate.

The results from Park (2012) based on a infinite horizon, representative agent framework indicate that tax revenues can be raised by increasing labor income tax rates while lowering capital income tax rates in the US and other G7 countries. Differently, we find that both labor and capital income taxes can be used to raise revenues. This discrepancy likely arises from the different shape of the capital income tax Laffer curve in our overlapping generations framework. Notice that, the capital income tax Laffer curve is mostly flat to the left of the peak in a representative agent model. Lowering capital income tax would have minimal impact on tax revenue; meanwhile, it significantly reduces tax distortion and expands the tax bases for labor supply and consumption in the economy. This in return enables the government to raise more revenue at a lower capital income tax rate. Differently, lowering the capital income tax rate would decrease revenue more in our model, as the slope is steeper to the left of the peak in our overlapping generations model.

Fiscal space. We use the Laffer curves to formulate a fiscal space in terms of the budgetary room between the current (benchmark) tax revenue level and the maximum tax revenue level.[4] Our fiscal space consists of the current fiscal situation (current tax revenue) and fiscal limit (peak of Laffer curve). Intuitively, it is a relative measure of how capable a government is of generating more tax revenues. Holding government spending constant, fiscal space describes the government's fiscal capacity to raise revenue to meet its spending commitments without compromising fiscal sustainability. Figure 5 depicts our definition of fiscal space.

In our benchmark model, the tax revenue ($\overline{Tax}$) is computed directly from this equation: $\overline{Tax} = \bar{\tau}^l \overline{w}\overline{H} + \bar{\tau}^k \bar{r}\overline{K} + \bar{\tau}^c \overline{C}$, where $\bar{\tau}^l, \bar{\tau}^k$, and $\bar{\tau}^c$ are the labor, capital and consumption tax rates, respectively, $\overline{w}$ is the wage rate, $\bar{r}$ is the interest rate, and $\overline{H}$, $\overline{K}$

[4] Alternatively, fiscal space can be defined in terms of a distance between the current debt levels and the debt limits above which the debt becomes unsustainable (e.g. see Ostry et al. (2010)). We abstract from political economy arguments that are more likely to determine fiscal limits in democratic societies.

and $\overline{C}$ are aggregate human capital, physical capital and consumption, respectively. Let $Tax_{\tau}^{\max}$ denote the maximum tax revenue for the labor and capital income taxes and consumption tax with $\tau = \{\tau^l, \tau^k, \tau^c\}$, respectively. The fiscal space ($FS^{\{\tau\}}$) is defined by the gap between the maximum tax revenue ($Tax_{\tau}^{\max}$) and the benchmark tax revenue ($\overline{Tax}$) according to $FS^{\{\tau\}} = \left[Tax_{\tau}^{\max} - \overline{Tax}\right]$.

Figure 4 depicts the fiscal spaces for labor and capital income taxes and consumption tax. We find that the fiscal space for the labor income tax is 43 percent of the current level of tax revenue. This implies that the government can raise more revenue by increasing the labor income tax rate up to 63 percent and collect a maximum revenue of 143 percent of the benchmark revenue. The government can also generate more tax revenue through raising the capital income tax rate. However, the fiscal space for the capital income tax is much smaller. The additional revenue is only 11.8 percent of the current revenue, achieved when increasing the capital income tax rate to $\tau^k = 0.82$.

Laffer hill and fiscal space. We now consider another measure of fiscal space using the Laffer hill for the joint labor-capital income taxes. We construct the Laffer hill by jointly varying the labor and capital income tax rates, while keeping the consumption tax rate unchanged at the benchmark level. The peak of the Laffer hill is the maximum tax revenue when adjusting the labor income tax $\left(\tau^l\right)$ and the capital income tax $\left(\tau^k\right)$. We use our model to map out the Laffer hill for Japan in 2010. Figure 7 presents the Laffer hill and fiscal space in 2010.

The contour lines present the different tax rate combinations that raise the same level of revenue, relative to the benchmark revenue level that is set at 100. The dotted lines mark the benchmark tax rates. As the peak of the hill lies in the north-east quadrant, the government can maximise the level of tax revenue by raising both the capital and labor income tax rates. Specifically, the peak of the Laffer hill is achieved when setting $\tau^l = 0.63$ and $\tau^k = 0.55$.

We compute a two-dimensional fiscal space based on the Laffer hill. Technically, the fiscal space for joint capital and labor income taxes is measured as a gap between the peak of the Laffer hill and the benchmark tax revenue in 2010. Let $FS^{\left\{\tau^l,\tau^k\right\}}$ be a Laffer hill fiscal space. We can write $FS^{\left\{\tau^l,\tau^k\right\}} = Tax_{\left\{\tau^l,\tau^k\right\}}^{\max} - \overline{Tax}$.

As shown in Figure 7, the Laffer hill fiscal space is about 45 percent of the revenue level in the 2010. The Japanese government can raise at most 45 percent of the current revenue when it is allowed to increase both labor and capital income taxes. Compared to the Laffer curve fiscal space, the Laffer hill fiscal space is slightly larger by about 2 percent.

Consumption tax and fiscal space. So far we have kept the consumption tax rate unchanged at 5 pecent as in the benchmark economy. We investigate how the Laffer hill fiscal space can be expanded further when the consumption tax rate is increased. Figures 8 and 9 demonstrate the effects of increasing the consumption tax rate to 10 percent and 20 percent. We find that an increase in the consumption tax shifts the peak of the Laffer hill upward and enlarges the fiscal space based on capital and labor income taxation. However, the additional revenue diminishes quickly as the consumption tax increases further.

4.2 Demographic shift and fiscal space

In this section, we isolate the effects of a demographic shift on Japan's fiscal space. We start from the benchmark calibrated model and vary the demographic structure, while keeping all other non-demographic variables unchanged at the benchmark level.

Demographic shift. We use the survival and fertility rates from IPSS demographic data to construct two alternative stationary demographic structures for Japan: the younger one in 1980 and the older one in 2040. Note that, because of non-stationary demographic pattern we are not able to replicate the shape of population distributions in 1980 and 2040 based on the population transition equation in the model. Instead, we adjust population growth rates to match the actual age-distributions in 1980 and 2040. Panels a) and b) of Figure 10 present the age distributions and conditional survival probabilities used in our experiments. Panel c) of Figure 10 demonstrates how the old-age dependency ratio in the model fits the data.

Behavioral responses and aggregate effects. We first examine the impacts of the demographic shift on the household and macroeconomic variables. The aggregate results are summarised in Table 2. The 2010 results from the model are given, while most of the 1980 and 2040 figures are reported as percentage deviations from their 2010 (per capita) levels, with the exception of the tax rate changes, which are expressed in percentage point changes instead.

Households adjust their behavior to respond to the improvement in longevity over the 1980 to 2040 period. Despite the decrease at the aggregate level, individuals tend to hold on to their asset for longer time. Asset decumulation in retirement occurs at a slower rate than in 1980. In 2010, household savings surpass the 1980 levels at age 80, while in 2040, household savings surpass the 1980 levels at age 85. As the survival probabilities continue to improve at the higher ages between 2010 and 2040, asset decumulation for households aged 85 and above occurs even slower, and catches up to 2010 levels by age 95.

Labor supply is affected disproportionally by the decrease in disposable wages. From 1980 to 2010, the labor supply falls during the middle ages between 30 to 39, before rising for the remaining working ages between 40 to 64. This can be explained by the interaction of the income and substitution effects triggered by the fall in disposable wages as a result of ageing. In the younger periods when the decrease in effective wages is relatively larger once we account for the lower age-specific labor productivity, the substitution effect dominates the income effect, so households consequently work less. In the older periods when the decrease in effective wages is relatively smaller due to the higher age-specific productivity, the income effect dominates and households are willing to work more. From 2010 to 2040, as disposable wages fall further, the substitution effect dominates, and the labor supply falls for all ages. Overall, we find that households aged of 40 to 64 still supply more labor than in 1980.

The labor tax base was 17.67 percent higher in 1980, and will be 18.48 percent lower in 2040, compared to the benchmark level in 2010. This arises from the decreases in the aggregate labor supply (N_t) and effective labor supply (H_t). As labor supply is mostly unchanged at the individual level, the decrease is mostly driven by the smaller share of working age cohorts as the age-dependency ratio rises. The per capita labor supply was 19.02 percent higher in 1980, and will be 15.24 percent lower in 2040, compared to 2010 levels. The effective labor supply changes to a greater extent, arising from demographic shift and behavioral changes. The age-specific labor productivity is asymmetrical, peaking at age 50 with the decrease over ages 50 to 64 sharper than the rise from ages 20 to 50. While ageing leads to more labor supply at age 50, it also increases labor supply at the beginning and end of working ages (20 to 25, and 50 to 65) where households are relatively less productive, compared with ages 30 to 40. As the cohort share is concentrated between ages 20 to 50 in 1980, it captures the additional effective labor supply of households aged between 30 to 40. In 2040, as cohort shifts to the 50 to 80 age range, it captures the lower effective supply of the 50 to 65 age range.

The capital tax base is affected significantly by the demographic shift, with an expansion of 20.3 percent in 1980, and a reduction of 15.7 percent in 2040, compared to the 2010 level. This change is driven by the change in composition of savers as well as behavioral response. As a result of ageing, the proportion of working age households (savers) decreases, while that of retired age households (dissavers) increases. This is compounded by the decrease in disposable wages and the fall of household savings, as the population ages. The asymmetrical decrease from 1980 to 2010 and 2010 to 2040 arises from the different asset levels over the life cycle. From 1980 to 2010, the effects of ageing was masked as in 2010, since the most populous cohorts were concentrated in the high-assets age range between ages 50 and 70. However, with the shift to the 2040 demographic structure, the 60 to 80 age range will be more populous, and as they lie to the right of the asset holdings peak, total capital stock within the economy will fall to a greater extent. While the labor supply decreases proportionally less than the labor tax base, the capital tax base decreases more than the capital stock. This arises from the simultaneous fall in interest rates (and rise in wage rates) driven by capital deepening.

The consumption tax base is negatively affected by population ageing. Specifically, it is 20.27 percent higher in 1980 when the old age-dependency ratio is low, and 15.66 percent lower in 2040 when the old age-dependency ratio is higher, compared to 2010. The asymmetrical decrease results from the life-cycle behavior of households, with the 1980 to 2010 effects softened by the increase in households with high disposable incomes. As the proportion of low income retired households continue to increase, the consumption tax base will decrease at a faster rate.

Despite a decrease in the pension benefits received by each retired household in 2040, the effects of the demographic shift will dominate, and the social security system will expand by 29.48 percent as a result of population ageing. To support the growing social security system, both τ^{ss} and τ^{l} have to be increased to keep the level of government debt and spending to remain constant. At the aggregate level, the ageing population has a negative effect on output, and GDP per capita will be 13.80 percent lower in 2040.

Laffer curve fiscal space. We now examine how the demographic shift affects Japan's fiscal capacity.

We first construct the g-Laffer curves for the labor and capital income taxes, and consumption tax under two alternative demographic structures in 1980 and 2040. Panels a) to c) of Figures 11 present the corresponding Laffer curves. Japan's fiscal limit measured by the peak of the Laffer curves decreases significantly since 1980. In absolute terms, the Laffer curve for the labor income tax is affected most by the population ageing in Japan. The maximum tax revenue raised by the labor income tax decreases from 156 percent of the baseline tax revenue in 1980 to 143 percent and 128 percent in 2010 and 2040, respectively. There is a smaller decrease in the maximum revenue raised by the capital income tax. The government could raise at most 127 percent of the baseline tax revenue at $\tau^{k} = 0.83$ in 1980. However, that maximum revenue decreases to 106 percent at $\tau^{k} = 0.86$ in 2040. The tax revenue raised by the consumption tax at the rate of 95 percent is also decreased from 210 percent in 1980 to 192 percent in 2040. Thus, the peak of all three Laffer curves shifts down significantly due to the decrease in the tax bases caused by ageing population.

Next, we calculate the percentage changes in the fiscal spaces due to the changes in the underlying demographic structure. To ease comparison, we normalize the fiscal space in 2010 to 100. Panel d) of Figure 11 presents the contraction in Japan's fiscal spaces in 1980, 2010 and 2040. It appears that the fiscal space for the capital income tax is the most sensitive to increases in old-age dependency ratio, with the fiscal space contracted by 64 percent by 2040. This is followed closely by the labor income tax, with the fiscal space lower by 34 percent. The fiscal space for the consumption tax is the least sensitive since it only contracts by 13 percent by 2040. The sensitivity of the fiscal space for the capital income tax is again driven by the compounded fall of both the capital stock and interest rate.

To understand how elastic fiscal space is when the population ages, we compute the relative movements of the fiscal space between 1980 and 2040. We find that the fiscal spaces for the labor and capital income taxes are almost linear, while the fiscal space for the consumption tax is kinked and will decrease at a faster rate. Lastly, while the fiscal space will shrink as a result of population ageing, it will remain positive for all three taxes. This implies that the Japanese government could raise tax revenue by increasing either one of the labor, capital, and consumption tax rates.

Laffer hill fiscal space. We construct g-Laffer hills as a two-dimensional metric to gauge the impact of the demographic shift on fiscal space.

Panels $a) - c)$ of Figure 12 depict the g-Laffer hills for 1980, 2010 and 2040, respectively. The contour lines present all combinations of capital and labor income tax rates that result in a similar level of tax revenue. We find that the contour lines shift north-east as a result of population ageing. This indicates that higher labor and capital tax rates are needed to generate the same level of income as before. For instance, at the benchmark labor and capital tax rates, 116 percent of the current revenue could be raised in 1980, and only 98 percent will be raised in 2040. The maximum tax revenue decreases from 160 percent in 1980 to 145 percent in 2010 and to 129 percent in 2040.

We compute the fiscal space based on the g-Laffer hill approach and present it in the last panel of Figure 12. We find that there is a significant contraction in government revenue generating capacity since 1980. Compared to the 2010 level, the Laffer hill fiscal space was 34 percent larger in 1980, but will be 36 percent smaller in 2040. The slope of the dependency ratio-fiscal space curve implies how elastic the fiscal space is as the population ages. It looks like there is an almost linear relationship between the dependency ratio and the change in fiscal space. Approximately, a 10 percentage point increase in the old age dependency ratio is associated with an 12 percentage point decrease in the g-Laffer hill fiscal space.

4.3 Role of each demographic factor

In our analysis so far, we have assumed both the age-specific survival probabilities and fertility rate change according to either their actual or projected paths. This allows us to quantify the total effect of the two demographic factors on the fiscal space. In order to isolate the quantitative importance of each demographic factor, we conduct the following decomposition exercise.

We keep the 2010 economy based on the actual time series of survival probabilities and fertility rates up until 2010 as the baseline scenario. We consider four counter-scenarios: (i) a "high fertility" scenario, where the survival probabilities are kept at the benchmark values, and the fertility rates revert to their time sequence up until 1980; (ii) a "low fertility" scenario with 2010 survival probabilities and fertility rates at their 2040 levels; (iii) a "high mortality" scenario with benchmark fertility rates and 1980 survival probabilities; and (iv) a"low mortality" scenario with benchmark fertility rates and 2040 survival probabilities.

The effects of the alternative demographic assumptions on the population composition are plotted in Figure 13. For ease of comparison, we have included the benchmark 2010 as the baseline.

Both high fertility and high mortality rates increase the proportion of young households relative to the baseline, while low fertility and mortality rates decrease the proportion. Over the 1980 to 2040 period, the decrease in mortality rate is the single biggest driver of the demographic composition, with the dependency ratio at 0.33 in the high mortality scenario, compared to 0.51 in the baseline. This is followed closely by a decrease in fertility rate. The dependency ratio is 0.37 in the high fertility scenario. As the demographic changes slow down over the 2010 to 2040 period, so does the relative change in cohort sizes. This is especially true for the mortality rate (see Figure 13), and the dependency ratio only rises to 0.60 in the low mortality scenario. This compares to 0.64 for the low fertility scenario.

To isolate the effects of changes in the fertility rate only on the gross fiscal space, we compare the g-Laffer curve under the baseline economy with the high fertility and low fertility cases. Similarly, we compare the g-Laffer curve under the baseline economy with the high mortality and low mortality cases to isolate the effects of the changes in survival probabilities. The results are summarized in Figure 14 (a) to (c), and (d) to (f) respectively.

All three Laffer curves shift down as either the fertility rate or mortality rate decreases. This is consistent with our intuition. Using the size of the effect on the fiscal space as a measure of the relative sizes of the demographic trends, we can compare the relative sizes of the fertility trends with the mortality trends. Over the 1980 to 2010 period, the size effect of the isolated fertility and mortality changes on the gross fiscal spaces are similar. In the high fertility counter-scenario where the fertility rate decrease does not occur, the gross labor fiscal space would be 95.33 percent of current revenue levels, compared to 94.58 percent for the high mortality counter-scenario where the survival probability improvements do not occur. Interestingly, we find that the change in fertility has a bigger effect on the fiscal space despite affecting the population composition (through the dependency ratio) to a lesser degree. This is reversed for the capital Laffer curve, with the fiscal space under the high mortality scenario slightly higher than the high fertility scenario, at 64.38 and 63.73 percent respectively.

Over the 2010 to 2040 period, the changes in both the fertility and mortality rates will slow down, as measured by a smaller shift in the Laffer curves between the high fertility and the baseline scenarios, compared to the baseline and low fertility scenario. The fiscal space for the labor income tax under the high fertility scenario would have been 23 percent larger, compared to 20 percent lower for the low fertility scenario when comparing to the baseline fiscal space. Similarly, the fiscal spaces for the capital income tax and the consumption tax under the low fertility scenario will decrease less, by 21 percent and 5.5 percent, respectively. This compares to an increase of 23 percent and 6.5 percent under the high fertility scenario.

The relative scale of the fertility and mortality changes were similar over the 1980 to 2010 period. However, this is not the case for the 2010 to 2040 period. Improvements in mortality rate contribute less to the fiscal space contraction experienced by the economy in 2040. Under the low mortality case, the fiscal spaces for the labor, capital and consumption taxes are 9.7 percent, 11.9 percent, and 2.8 percent lower than the baseline levels, respectively. As fertility rates are projected to remain low, the contraction in fiscal space will be increasingly driven by the fertility rate.

It is expected that a larger change in the scale of the demographic factors would lead to a corresponding larger movement in the Laffer curves. To control for the size effect, we quantify the relative importance of the two demographic factors on a per unit basis, with the dependency ratio used as a general measure of the level of ageing. We plot the fiscal space of the various scenarios against the dependency ratio that they produce, with the results summarised in Figure 15. The "fertility only" case refers to the low and high fertility scenarios, the "mortality only" case refers to the low and high fertility scenarios, and the "total" case refers to the actual population dynamics we originally used.

There are some minor qualitative differences on how the mortality and fertility rates affect the Laffer curves on a per unit basis. The Laffer curve for the labor income tax is relatively more sensitive to changes in the fertility rate. This is depicted in the first panel by the position of the "fertility only" points relative to the benchmark 2010 point and the trend line. For the high fertility point to the left of the benchmark, the point lies above the trend line, indicating that for a given decrease in the dependency ratio driven purely by a fertility rate increase, the fiscal space would increase more than the trend suggests. Similarly for the low fertility point to the right of the benchmark, the point lies below the trend line, indicating that for a given increase in the dependency ratio driven by purely by a fertility rate decrease, the fiscal space decreases more than the trend suggests. In comparison, the low mortality point lies above the trend line, while the high mortality point lies below it, indicating a lesser impact on fiscal space than suggested by the movements in the dependency ratio. Looking at the fiscal spaces for the capital income tax and consumption tax, the relative importance of fertility on a per unit basis also holds true, abiet to a lesser extent.

5 Sensitivity analysis and extension

In this section, we conduct sensitivity analysis.

5.1 Alternative Laffer curves

We consider two additional options that the government can use to balance its budget: lump-sum transfers to households and public debt reduction.

tr-Laffer curve. We assume that the government redistributes additional tax revenue back to all households in the form of a lump-sum transfer (tr). For a given combination of tax rates $\bar{\tau}^c, \bar{\tau}^k, \bar{\tau}^l$, the transfer is given by

$$tr = \frac{\bar{\tau}^c C + \bar{\tau}^k rK + \bar{\tau}^l(1-\tau^{ss})wH - \bar{G} - r^d\bar{D} - \bar{\theta}^{ss}SS}{\sum_j^J \int_{x_j} \mu_j\left(x_j\right)} \tag{9}$$

d-Laffer curve. In this case, we relax the assumption that debt is exogenously fixed at $\bar{D}$ and allow the government to use the additional tax revenue to repay outstanding government debt. The level of public debt is decreased according to

$$D = \frac{\bar{\tau}^c C + \bar{\tau}^k rK + \bar{\tau}^l(1-\tau^{ss})wH - \bar{G} - \bar{\theta}^{ss}SS}{r^d} \tag{10}$$

Notice that, in our setting the d-Laffer and g-Laffer curves are equivalent as the interest rate on government bonds (r^d) is exogenous. Adjusting either the debt level or government spending is just an accounting exercise to balance the government budget. There is no real effect on households and tax revenue.

We present tr- Laffer curves for the labor, capital and consumption taxes in Figure 16. We also present the d-Laffer/g-Laffer for comparison. Compared to the g-Laffer curves, the tr-Laffer curves have lower peaks in both the labor and capital income tax curves, while a higher peak for the consumption tax Laffer curve. The difference arises from the additional revenue that is fed back into the economy through transfers. The transfers increase household incomes and result in positive income effects, which subsequently induces households to consume more leisure and consumption, while saving less. The labor and capital tax bases are contracted, and the entire curve shifts down as a result. Conversely, the increase in the consumption base expands consumption tax revenue under the tr-Laffer specification. The effects become more pronounced at the higher tax rates as the size of the transfers increases.

5.2 Alternative preferences

We examine whether our results are robust to different functional forms of preferences. Following Trabandt and Uhlig (2011), we consider two alternative preference specifications. We first set the inverse of the IES (σ) to 1, so that the CFE preferences has an alternative form of

$$u(c_j, l_j) = \log(c_j) - \kappa(1 - l_j)^{1+\frac{1}{\phi}} \tag{11}$$

In the second case, we use Cobb-Douglas (CD) preferences as one of the most commonly used functional forms. In particular, we use the following form

$$u_{c_j,l_j} = \rho\log(c_j) + (1 - \rho)\log(l_j) \tag{12}$$

where ρ is the weight on consumption relative to leisure.

We re-calibrate the model under the two alternative preference specifications. The budget constraints and equilibrium conditions of the model hold as before. The values of deep parameters under both the original and the alternative specifications are given in Table 3. All other parameters for demographics, endowments, technology, and fiscal policy remained unchanged as in Table 1. Sugo and Ueda (2008) estimates the mean of the IES $1/\sigma = 1.25$ for Japan, in contrast to the higher value found by Iiboshi, Nishiyama and Watanabe (2006). We set the Frisch labor supply elasticity $\phi = 3$ and labor supply is more elastic to changes in wage rate. Under Cobb-Douglas preferences, the Frisch elasticity is no longer constant. Instead, the elasticity decreases as the labor supply increases (leisure decreases). The calibrated value of the discount factor is $\beta = 0.962$. This similarity stems from the separability of the consumption and leisure in the utility function, and the common $\log(c_j)$ term. The discount factor is lower, while the disutility from work is higher in the alternative CFE case.

We compute the g-Laffer curves in under the two alternative preference specifications. The results are reported in Figure 17. The benchmark revenue level under the CFE preferences is normalized to 100. As seen in Figure 17, the Laffer curves for all three taxes are quite similar. However, there are differences at the high tax rates for labor and capital incomes. The slope of the Laffer curve under Cobb-Douglas preferences is greater than in both CFE preferences at the higher tax rates. This results from the changing Frisch elasticity. Households decrease their labor supply less when increasing tax rates at the higher end. Consequently, the total tax revenue falls more slowly in the Cobb-Douglas preference case.

The Laffer curve peaks for the labor income tax are higher in the case of modified CFE and Cobb-Douglas preferences, compared to the benchmark CFE preference case. This is qualitatively different to the result in Trabandt and Uhlig (2011) in a representative agent framework, where the Laffer curve for the labor income tax has a higher peak in the case of the CFE preferences. This results mainly from the difference in the calibrated value of time discount β in an overlapping generations framework. The capital income tax raises relatively less revenues at lower tax rates in the benchmark CFE case, compared to the Cobb-Douglas case. However, the Laffer curve peaks at a higher tax rate. Notice that, the Laffer curves for the capital income tax under the benchmark and alternative preferences are almost identical in the representative agent model. The difference arises again from the calibrated β value, with a higher β implying households are patient and willing save more even as the return on capital decreases. This leads to a slower decrease in the capital tax base and the total tax revenue level is higher. The consumption Laffer curves for all three specifications are similar, especially the two CFE specifications. The Cobb-Douglas preferences produces the highest maximum tax revenue at a higher tax rate.

5.3 Net fiscal space

We have demonstrated how population ageing undermines fiscal capacity. So far, we have abstracted from the effects of ageing on the spending side, which might overestimate the true degree to which fiscal manoeuvrability can be compromised in the context of ageing. We now consider the increased fiscal cost of the social security program due to population ageing.[5]

We construct a net fiscal space by deducting the “committed” contribution to the social security system from the (gross) fiscal space measured in the previous part. Specifically, the net fiscal space (NFS) in terms of the pb-Laffer hill for the joint capital and

[5]We abstract from the fiscal costs of other age-related governemnt spending programs such health care and long-term care.

capital income taxes is given by this formula

$$NFS^{\{\tau^l,\tau^k\}} = \left[Tax^{\max}_{\{\tau^l,\tau^k\}} - \overline{Tax} - \theta^{ss} \sum_{j=J^w+1}^{J} \int_{x_j} p_j^{2010} \mu_j (x_j)\right],$$

where $Tax^{\max}_{\{\tau^l,\tau^k\}}$ is the maximum tax revenue when letting both labor and capital income taxes vary, $\overline{Tax}$ is the current tax revenue in 2010, θ^{ss} is the policy variable that determines the government contribution to the social security program in 2010, and p_j^{2010} is the pension benefit committed in 2010. The net fiscal space is interpreted as the "uncommitted" tax revenue.[6]

Figure 18 depicts pb-Laffer hills for 1980, 2010 and 2040, respectively. Notice that, the net fiscal space in 2010 is the benchmark tax revenue in this experiment. As seen in the first three panels of Figure 18, the Laffer hills shift downward as Japan moves toward an ageing society. Our results indicate that the increased cost of the social security program decreases the peaks of the pb-Laffer hills. This is due to two reasons. In the 1980 pre-ageing economy, there is a smaller proportion of retired households. The government contribution to the social security system is relatively smaller because the contribution rate is fixed at the 2010 level. As a direct consequence, the net fiscal space is relatively larger than the gross fiscal space in 1980. The net fiscal space will be smaller in 2040 as the government has to contribute more to the social security system to meet the pension payment commitments to a relatively larger retiree population. Notice that, if the government commits to the same level of pension benefits in 2010, the net fiscal space for Japan will be negative in 2040. This implies that the Japanese government will not be able to generate sufficient tax revenue to finance its existing commitments in 2040.

This finding has important implications for Japan. It points out that the government budget currently follows a unsustainable path. Structural fiscal reforms will be required to move Japan away from fiscal insolvency in 2040.

5.4 Cross-country comparison

In this section we analyze how differences in demographic structure lead to differences in fiscal space across countries, while taking into account cross-country differences in endowments, preferences, technologies and tax-transfer systems.

We choose the U.S. as another case for cross country comparison. We re-calibrate our model to match the US economy in an artificial steady state in 2010. We source the

[6] We also compute the pb-Laffer curves for the labor and capital income taxes and consumption tax separately. These results are availale upon request.

values of model parameters from (i) the previous literature for specifying preferences; (ii) the macroeconomic data on government tax and fiscal policy, and population dynamics. We calibrate some structural parameters and fiscal policy variables to replicate life-cycle profiles of labor supply and asset holdings and targeted macroeconomic aggregates in the base year. The values of key parameters of the benchmark model are in Table 4. We consider two alternative demographic structures: one in 1980 and one in 2040. Figure 19 presents the US age distributions and conditional survival probabilities in our experiments. We conduct a similar experiment in which we vary the demographic structure and quantify the effects of demographic shift on the US fiscal limit and fiscal space.

Figure 20 presents the labor, capital and consumption g-Laffer curves in the benchmark economy in 2010 and two alternative cases in 1980 and 2040. Note that the values of non-demographic parameters for the US economy are kept constant at their levels in 2010. To easy our comparison we normalize the benchmark tax revenue level to 100. The percentage changes in the fiscal spaces for labor, capital and consumption taxes are computed accordingly.

The US benchmark economy lies to the left of the Laffer curve peaks for the labor and capital income taxes. This indicates that the US government still has room to raise tax revenue by increasing labor and capital tax rates. The peak of the g-Laffer curve for labor income taxation is achieved at the tax rate of 63 percent, $\tau^l = 0.63$. The peak of the g-Laffer curve for the capital income tax is achieved when the capital income tax rate is set at 86 percent, $\tau^k = 0.86$. The g-Laffer curve for the consumption tax does not have a peak.

Interestingly, the evolution of the fiscal spaces are quite different in the US. The shift in the US demographic structure from 1980 to 2010 drives a large expansion in the fiscal space, while causing a sharp contraction in 2040. This result is mainly driven by the change in the share of households in their 40s and 50s. As seen in Panel a) of Figure 19, there is a relatively larger fraction of Americans aged 40s and 50s in 2010. The households in these age groups are at the peak of their labor productivities. The increase in relative share of these age groups causes a significant expansion of the labor and capital income tax bases in 2010. Indeed, the US government is in a better position to collect tax revenue in 2010 relative to 1980. However, as the US moves to the late phase of population aging in 2040 its fiscal capacity will be reduced. Such change in the US demographic structure shifts the Laffer curve peaks down significantly, and subsequently results in a sharp contraction in the fiscal space. Precisely, the fiscal spaces for the labor, capital and consumption taxes are decreased by 36%, 41% and 13% in 2040, respectively.

Thus, our findings show that the change in the underlying age structure of the US significantly affects the size of the labor force, capital accumulation and consumption bases over time, and subsequently leads to an expansion/contraction in the fiscal space.

6 Conclusion

In this paper we assess government capacity to raise tax revenue under demographic shift through the lens of fiscal space. We use a stochastic dynamic general equilibrium, overlapping generations model calibrated to Japan and US data. Our findings indicate that the size of the fiscal space varies greatly over time and across countries, depending on country-specific fundamentals and the timing of the demographic transition. The fiscal spaces for Japan and USA will contract sharply by 2040, as the two countries enter their late stage of demographic transition. Our results also indicate that the contraction in the fiscal space will be predominantly driven by the fertility rate, as mortality improvements slow while fertility rates remain below replacement rate.

Our findings show that there is an almost linear negative relationship between the old age dependency ratio and the fiscal space. In particular, our model calibrated to Japan indicates that the increase in dependency ratio from around 40 percent in 2010 to over 70 percent in 2040 leads to a contraction in the capital-labor fiscal space by around 36 percent. The net fiscal limit will be gone when factoring in the increased fiscal cost of age-related spending commitments. The fiscal space for capital income taxation is the most sensitive to ageing, with a 10 percentage point increase in the dependency ratio associated with a 14 percentage point decrease in the gross fiscal space relative to the benchmark fiscal space.

There are several matters that we abstract from in this analysis. First, we have kept policies constant in the simulations and altered the population dynamics only. This does not account for policy reaction to the ageing trend – including increasing the retirement age, labor force participation and pension reform. Second, the peak of the Laffer curves does not necessarily maximize the welfare of the population, and the results do not imply that raising labor and capital income tax rates would be optimal. Third, while the peaks of the Laffer curves offer the theoretical maximum revenue that can be raised, the "effective" maximum revenue that is politically feasible is likely much lower. Other factors that matter for government capacity to raise taxes include the size of informal sector, international tax competition and tax havens. These are assumed away in this analysis and we leave them for future research.

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7 Appendix

7.1 An algorithm to solve the model

We use dynamic programming to solve the model. The general procedure can be summarized as follows:

1. Discretize the state space of assets as $[a_0, ..., a_{max}]$ and the space for tax rates as $[0, ..., 1]$.

2. Choose a combination of τ^l, τ^k, and τ^c by either setting two constant and looping 1 over the taxation grid, or setting one constant and looping two over the taxation grid.

3. Guess for the initial wage rate w, post-tax interest rate R^k, pension benefit pb, bequest b and capital stock K.

4. Work backwards from J to period 1 to obtain optimal decision rules for consumption, savings, labor supply, and the value and marginal value functions of the household.

5. Iterate forwards to obtain the vector of optimal consumption, savings, and labor supply choices for the households across different generations, using $a_1 = 0$ and following the optimal decision rules.

6. Compute the stationary distribution of households μ_j; Compute the new aggregate variables and new wage rate and interest rates from the market clearing conditions, new bequest, and pension values; Balance government and social security budgets to determine endogenous tax financing variables.

7. Check the relative changes in the aggregate variables after each iteration and stop the algorithm when the change is sufficiently small. Otherwise, repeat from Steps 3 to 6.

7.2 Tables and Figures

Variable	Description	Comments/Source
Demographics		
J =16	Maximum lifetime periods	Equivalent to 80 years
J^w= 9	Maximum working periods	Equivalent to 45 years
Preferences		
$\sigma = 2$	Inverse of IES	Literature
$\phi = 1$	Frisch labour supply elasticity	Literature
$\kappa = 3.3$	Weight of labor	Calibrated to match labor supply
$\beta = 0.998$	Time discount factor	Calibrated to match K/Y
Endowments		
$\overline{e}$	Age-specific labor productivity	Braun (2009)
$\widetilde{z}$	Stochastic component of labor productivity	Lise et al (2014)
Technology		
$A = 1$	Total productivity factor	Normalization
$\alpha = 0.406$	Capital share in production	Data (40.6%)
$\delta = 0.082$	Annual depreciation rate	Data (8.2%)
Fiscal policy		
θ^{ss}= 0.413	Govt coverage of pension benefits	Data
$\psi = 0.33$	Pension replacement rate	Calibrated
$G/Y = 0.20$	Government purchase to GDP ratio	Data (20%)
$D/Y = 1.10$	Government debt to GNP ratio	Data (110%)
$\tau^c = 0.05$	Consumption tax	Data (5%)
$\tau^i = 0.25$	Labor income tax	Data (14%)
$\tau^k = 0.40$	Capital income tax	Data (27%)
$tr_t = 0$	Transfers	Balances Laffer curve
r^d= 0.01	Interest rate on government bonds	Data (1%)

Table 1: The parameter values for demographics, preferences, technology and fiscal policy for the benchmark economy calibrated to Japan

Variable		1980	2010	2040
Labor tax base	%	17.67	0.00	-18.48
Labor supply	%	19.02	0.00	-15.24
Effective labor supply	%	20.10	0.00	-15.66
Wage rate	%	-6.42	0.00	1.7
Capital tax base	%	20.27	0.00	-15.66
Capital stock	%	1.06	0.00	-11.64
Interest rate	%	17.10	0.00	-4.17
Consumption (tax base)	%	6.92	0.00	-9.91
Social security system	%	-46.70	0.00	29.48
Pension benefit	%	-8.41	0.00	0.89
Pension contribution rate	percentage point	-5.10	0.00	4.91
Labor income tax rate	percentage point	-5.46	0.00	5.18
Output (GDP)	%	11.37	0.00	-13.80

Table 2: The effects of demographic shift on macroeconomic variables. Note that, all variables other than the wage and interest rates, and pension contribution and labour income tax rates, are expressed in *per capita* terms. The changes are caculated in terms of percentage deviations from the 2010 benchmark levels.

Variable	**Description**	**Comments**
CFE preferences (Benchmark)		
σ =2	Inverse of IES	–
ϕ= 1	Frisch labor supply elasticity	–
$\kappa = 3.3$	Weight of labor	$\bar{n} = 0.3$
$\beta = 0.975$	Time discount factor	$K/Y = 3.05$
$\psi = 0.33$	Pension replacement rate	$SS/Y = 0.10$
CFE preferences (Alternative)		
σ =1	Inverse of IES	–
ϕ= 3	Frisch labor supply elasticity	–
$\kappa = 3.53$	Weight of labor	$\bar{n} = 0.3$
$\beta = 0.962$	Time discount factor	$K/Y = 3.05$
$\psi = 0.34$	Pension replacement rate	$SS/Y = 0.10$
Cobb-Douglas preferences		
$\rho = 0.325$	Weight of consumption	$\bar{n} = 0.3$
$\beta = 0.962$	Time discount factor	$K/Y = 3.05$
$\psi = 0.33$	Pension replacement rate	$SS/Y = 0.10$

Table 3: Model parameter values for alternative preferences

Variable	Description	Comments/Source
Demographics		
$J = 16$	Maximum lifetime periods	Equivalent to 80 years
$J^w = 9$	Maximum working periods	Equivalent to 45 years
Preferences		
$\sigma = 2$	Inverse of IES	Literature
$\phi = 1$	Frisch labor supply elasticity	Literature
$\kappa = 3.3$	Weight of labor	Calibrated to match labour supply
$\beta = 0.985$	Time discount factor	Calibrated to match K/Y
Endowments		
e_j	Age-specific labor productivity	Kitao (2014)
Technology		
$A = 1$	Total productivity factor	Normalization
$\alpha = 0.36$	Capital share in production	Literature
$\delta = 0.071$	Annual depreciation rate	Literature
Fiscal policy		
$\theta^{ss} = 0$	Govt coverage of pension benefits	Data
$G/Y = 0.20$	Government purchase to output ratio	Data (20%)
$D/Y = 0.75$	Government debt to output ratio	Data (110%)
$\tau^{SS} = 0.10$	Social security rate	Data
$\tau^c = 0.05$	Consumption tax	Data (5%)
$\tau^k = 0.30$	Capital income tax	Data (40%)
$\tau^l = 0.28$	Labor income tax	Endogenous

Table 4: Model parameter values for the US

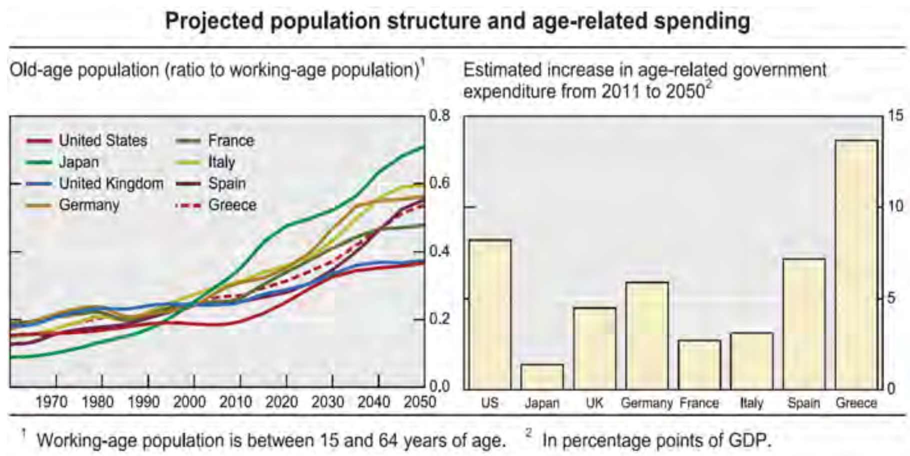

Figure 1: Ageing and Fiscal Costs from Cecchetti et. al. (2010)

Figure 2: Age-specific Labour Productivities in the Benchmark Model

Figure 3: Lifecycle Profiles: Benchmark Model vs Data

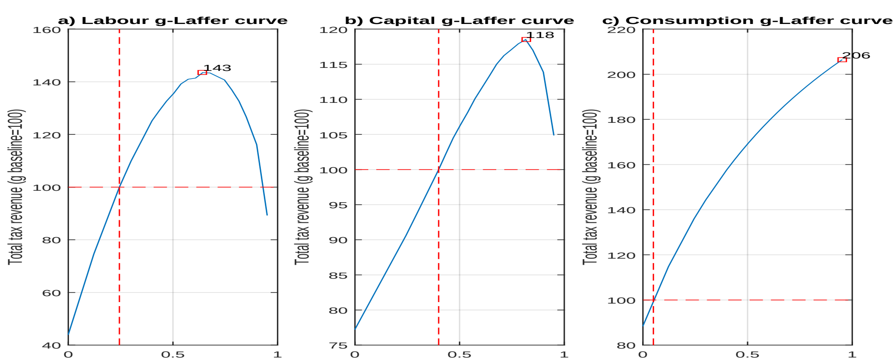

Figure 4: Benchmark g-Laffer Curves, Fiscal Limit and Fiscal Space. Note that, the total tax revenue in the benchmark model is normalized to 100.

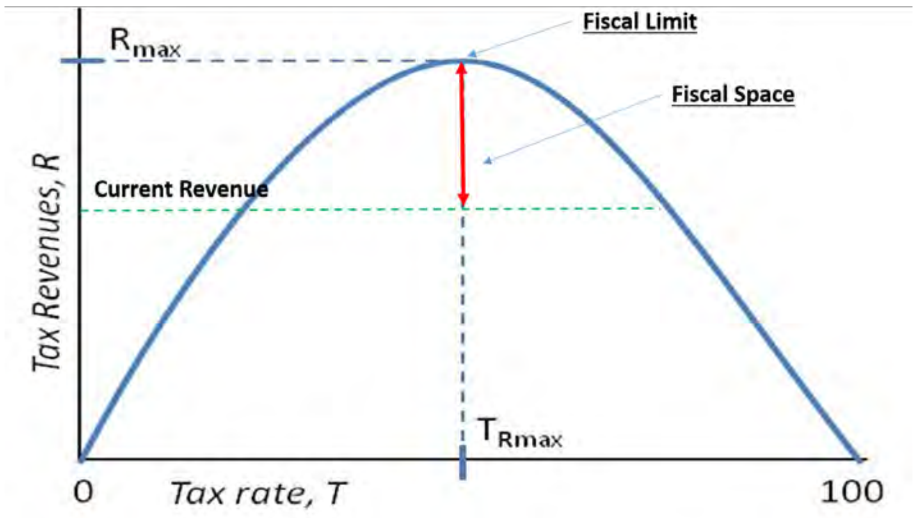

Figure 5: Laffer Curve Approach to Measuring Fiscal Limit and Space

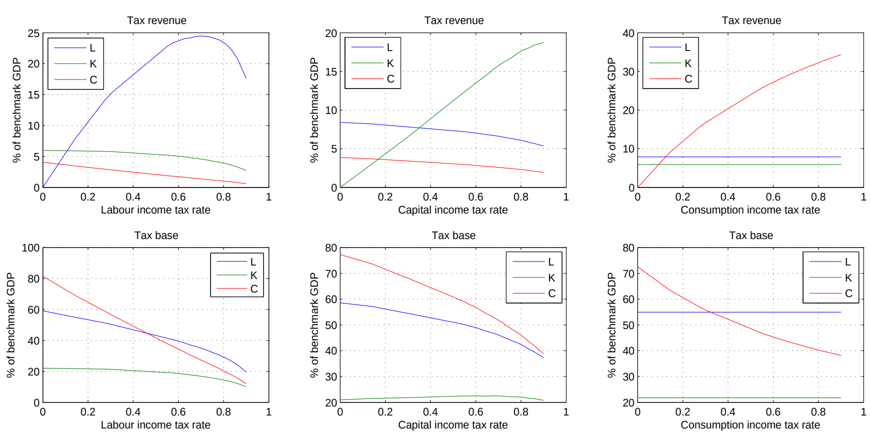

Figure 6: Tax Revenue Decomposition and Tax Bases for Benchmark g-Laffer Curve

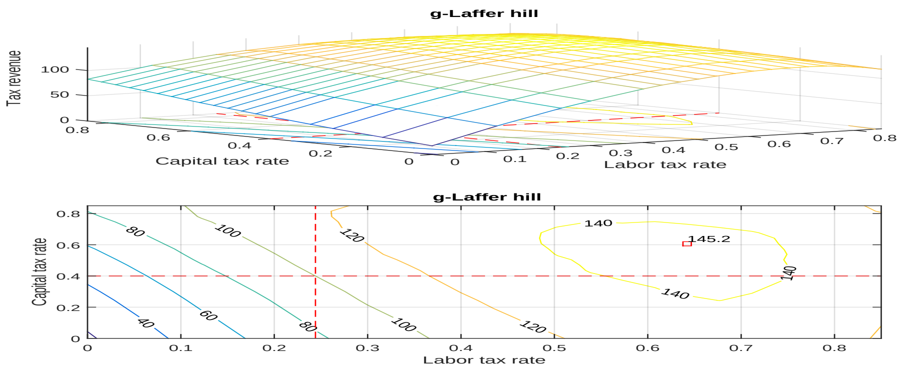

Figure 7: g-Laffer Hill, Fiscal Limit and Fiscal Space in 2010

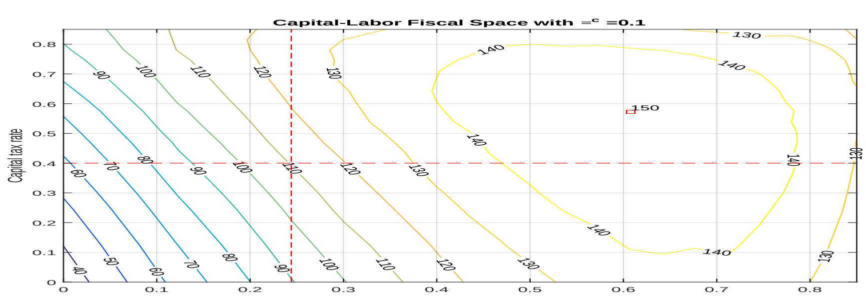

Figure 8: The g-Laffer Hill for Labor and Capital Income Taxation in Japan with the consumption tax set at $\tau^c = 0.1$.

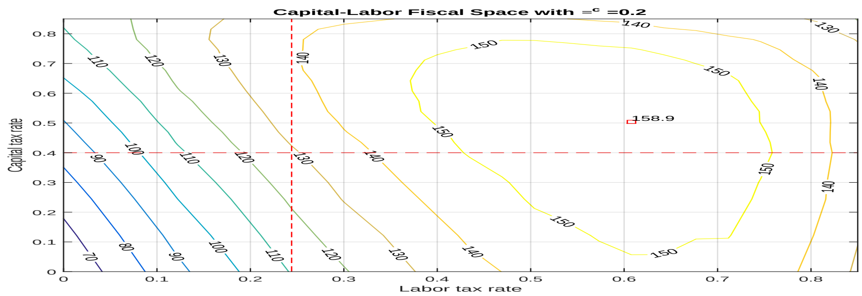

Figure 9: The g-Laffer Hill for Labor and Capital Income Taxation in Japan with the consumption tax set at $\tau^c = 0.2$.

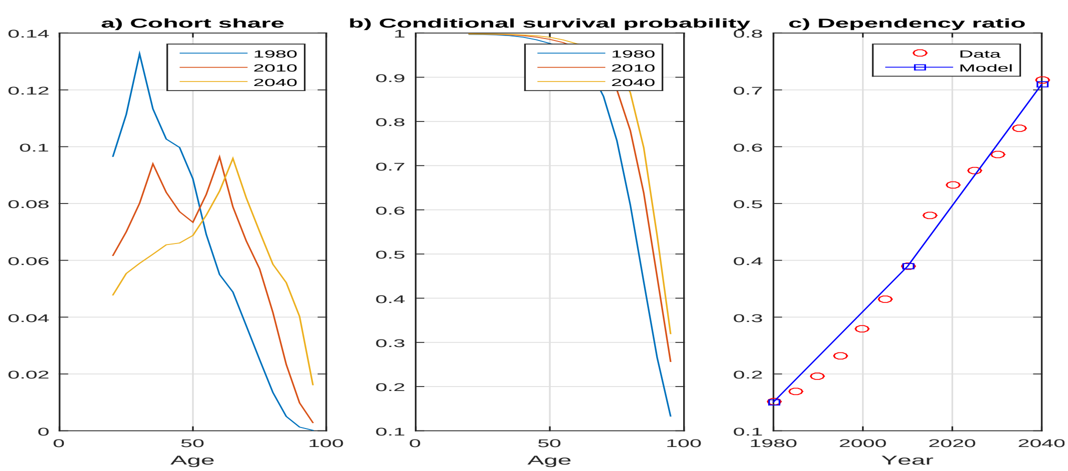

Figure 10: Demographic Shift in Japan: Model and Data

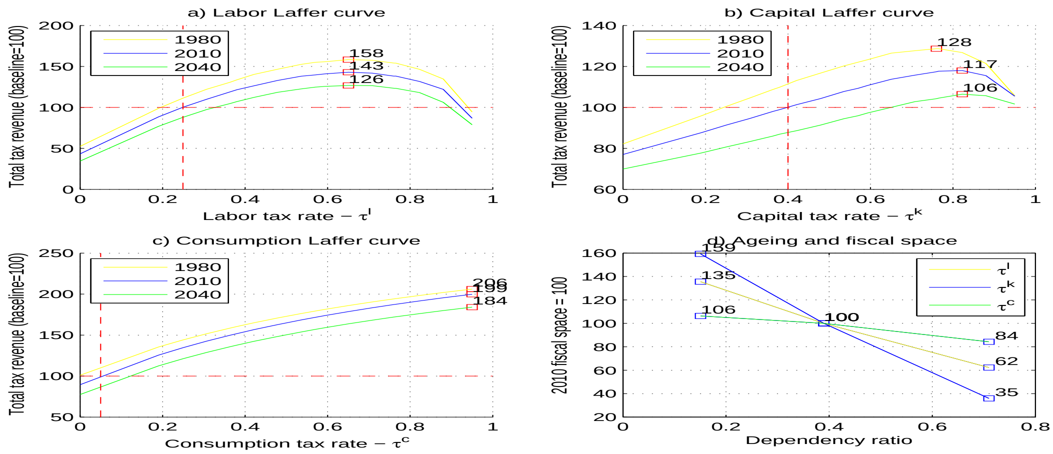

Figure 11: Demographic Shift and Fiscal Spaces

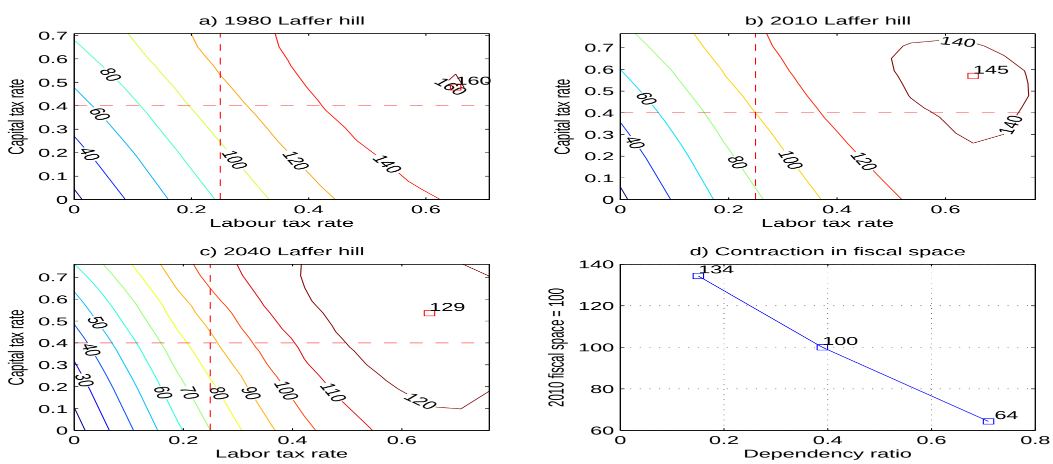

Figure 12: Demographic Shift and Fiscal Space: Labor-Capital Laffer Hill

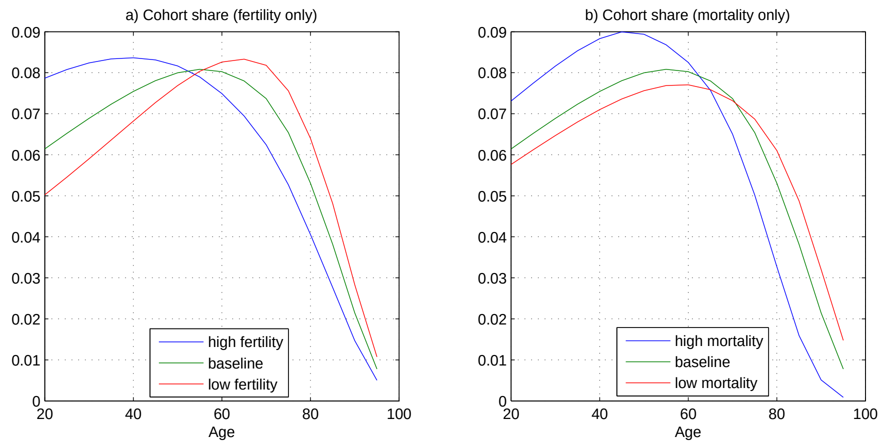

Figure 13: Demographic Composition under Different Fertility and Mortality Assumptions. Note that, the low mortality case and low fertility case uses 2040 surivval probabilities and fertility rates from the IPSS's medium-fertility medium-mortality population projections.

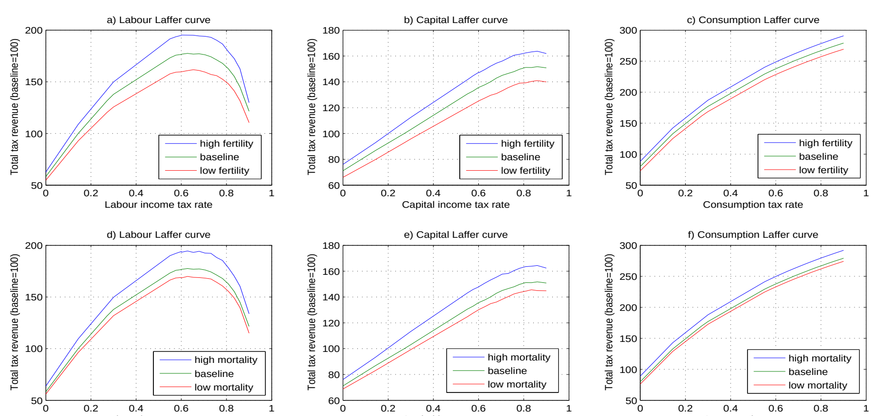

Figure 14: g-Laffer Curves under Different Fertility and Mortality Assumption.

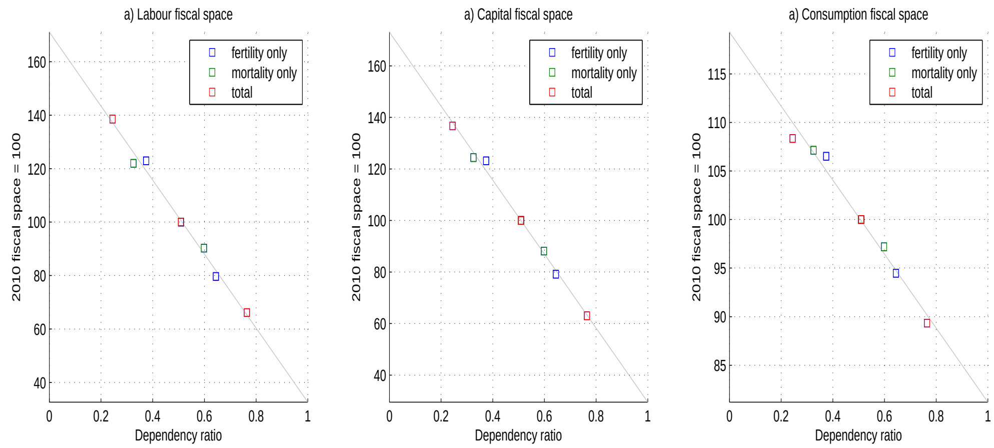

Figure 15: Demographic Factors and Fiscal Space

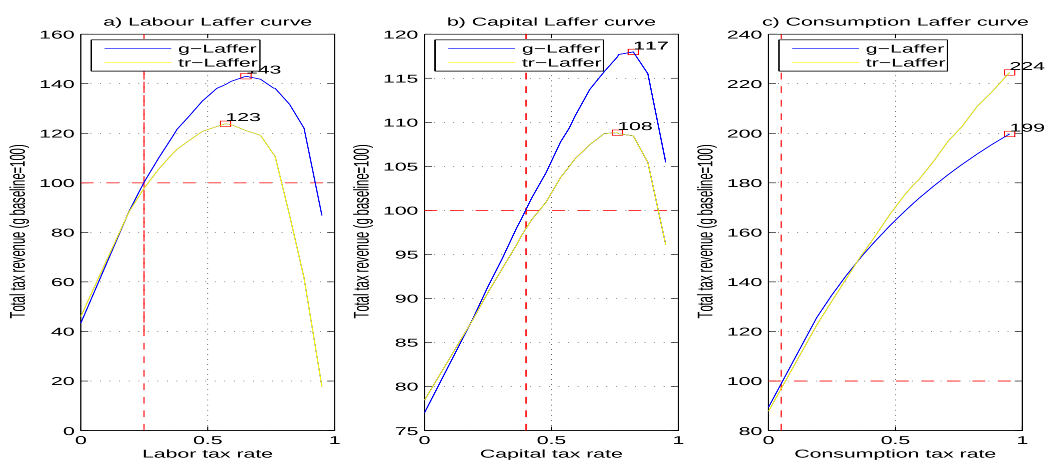

Figure 16: g-Laffer curves vs. tr-Laffer curves

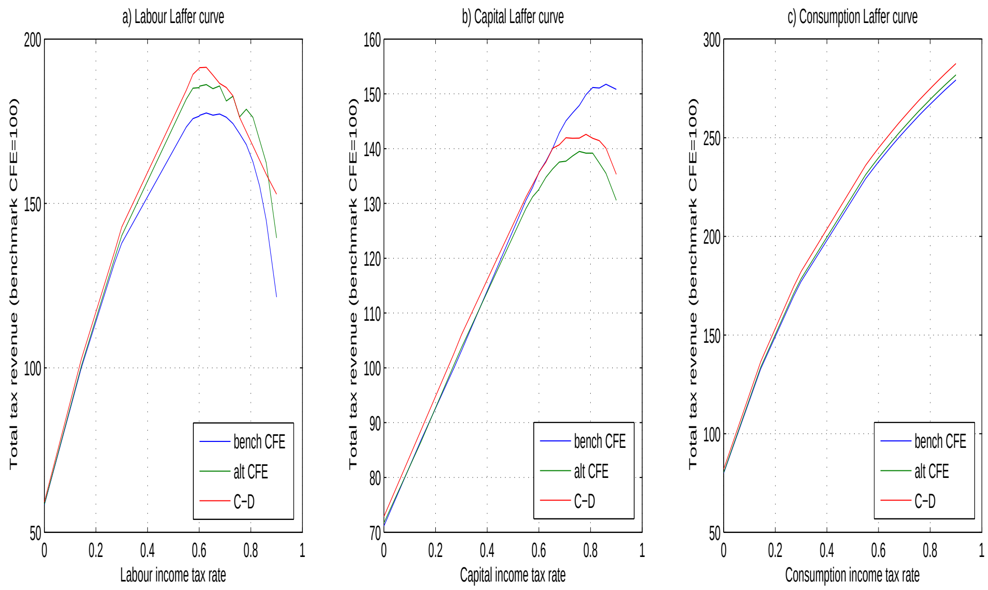

Figure 17: Comparing the 2010 g-Laffer curves under alternative preferences

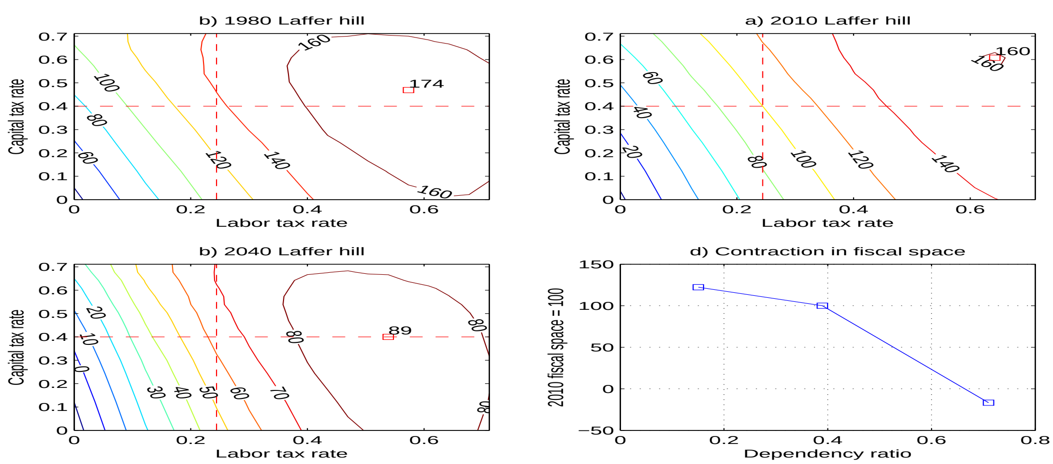

Figure 18: Demographic Shift and Net Fiscal Space

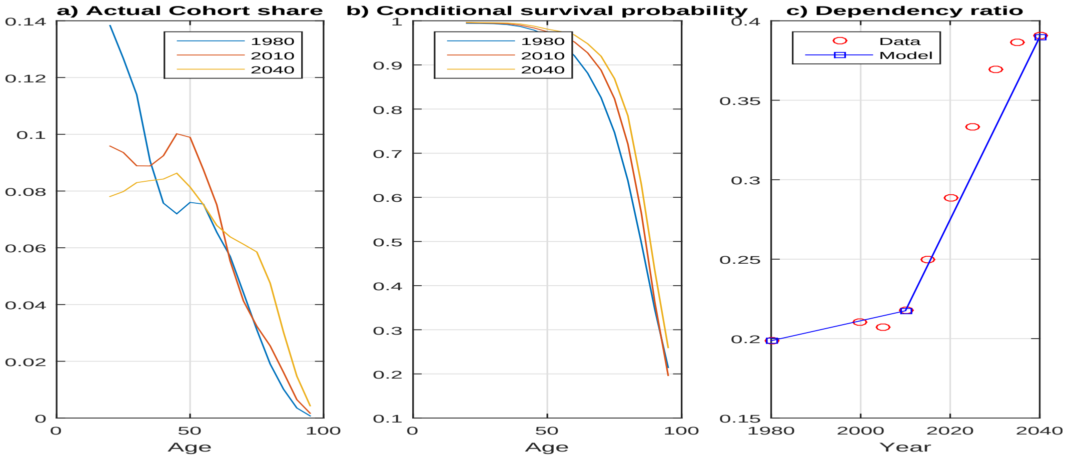

Figure 19: Demographic Shift in the US: Model and Data

Figure 20: Laffer Curves and Fiscal Spaces in the US