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Japanese and U.S. Inflation Dynamics in the 21st Century

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Abstract

This paper examines the behavior of inflation in the U.S. and Japan over the past twenty-five years. The paper estimates structural models of inflation dynamics for both countries, relying on survey measures as proxies for expectations. The results suggest some promising directions for inflation modeling in both countries. First, the use of survey expectations as proxies for the expectations in conventional models appears to be helpful, aiding in identification of the inflation process in both countries. Second, methods for endogenizing such expectations are tractable and replicable. They require the measurement of longer-term expectations, but such data are available for many key variables in many developed economies.[1] Third, models that incorporate such expectations identify a rationale for the behavior of US and Japanese inflation:

a. Long-run expectations anchor the process, although long-run expectations can be influenced over time by persistent deviations of inflation (or output) from their long-run equilibria;

b. Short-run expectations are tied to their long-run counterparts, but they can deviate quite persistently from long-run expectations, due to persistent deviations of output from potential, and due to intrinsic persistence in the expectations;

c. Inflation appears well-explained by short-run expectations and a traditional output gap;

Fourth, the balance sheet actions in Japan appear to have boosted short-run inflation expectations, compared to where they would have been without such actions. The estimates in this paper suggest that this in turn has helped to raise realized inflation by about one-half percentage point.

Keywords: Inflation dynamics; Intrinsic Persistence; Survey expectations
JEL classification: E31, E32, E52, D84

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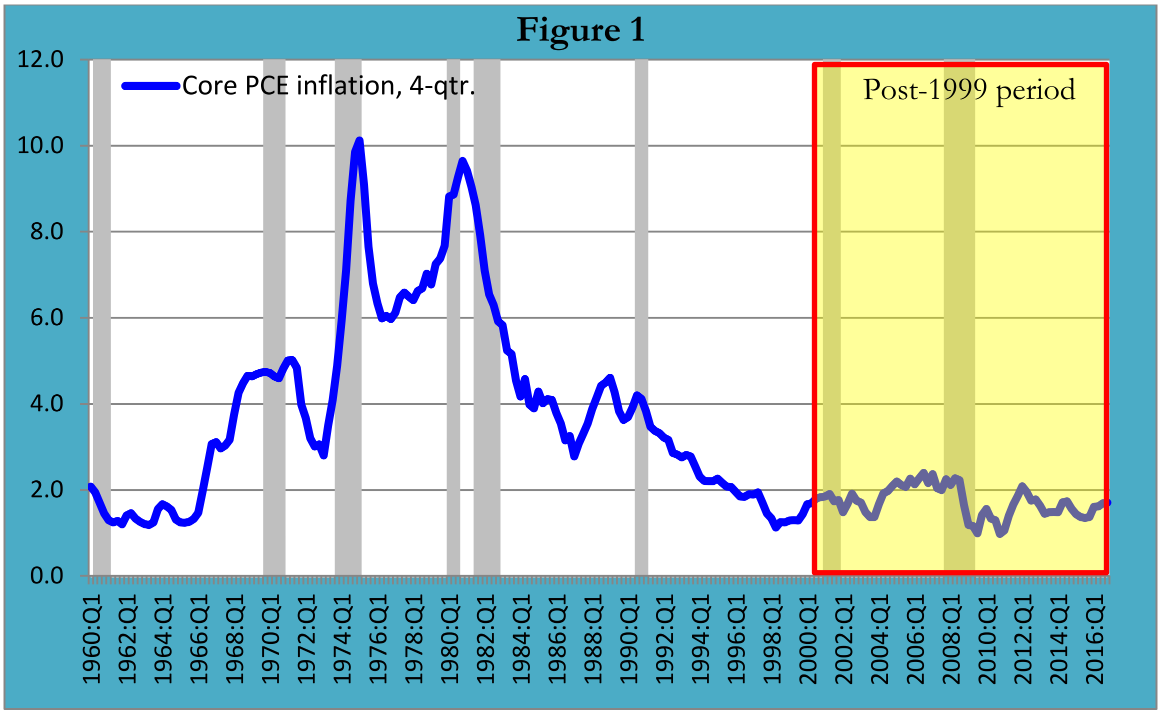
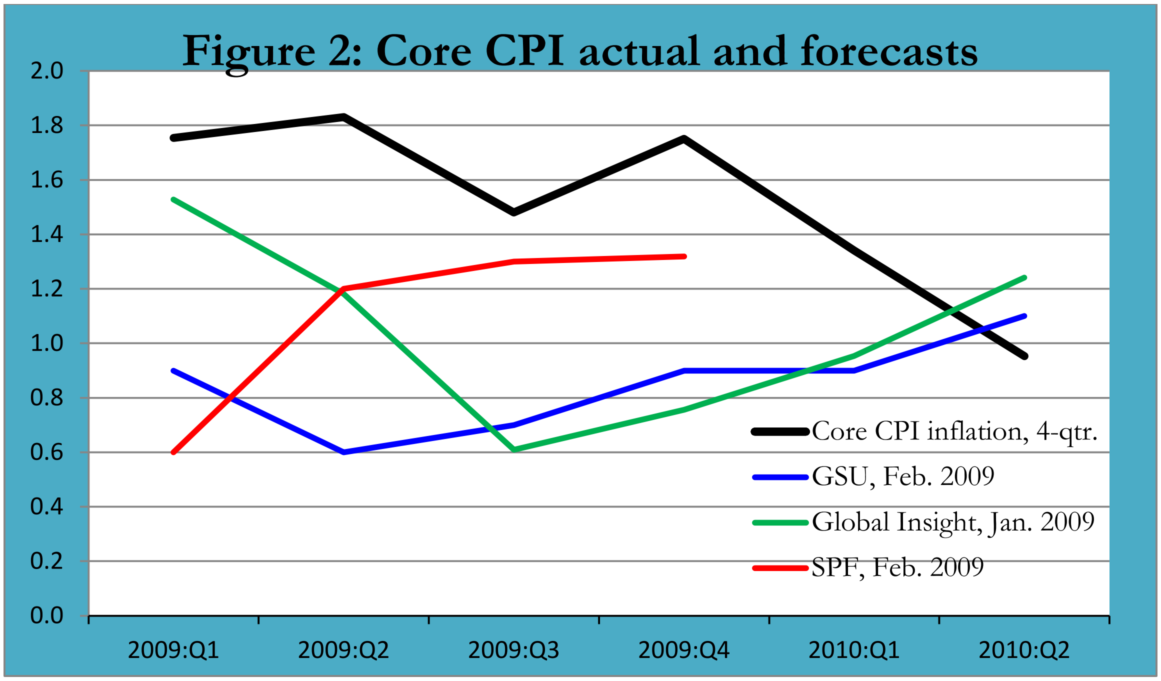
[1] A separate but important issue is whose expectations are measured in the available surveys. To be sure, a survey of important price-setters, rather than professional forecasters, would likely be preferred. Such a dataset is not available for the U.S. or Japan at present, to my knowledge.

1. Introduction

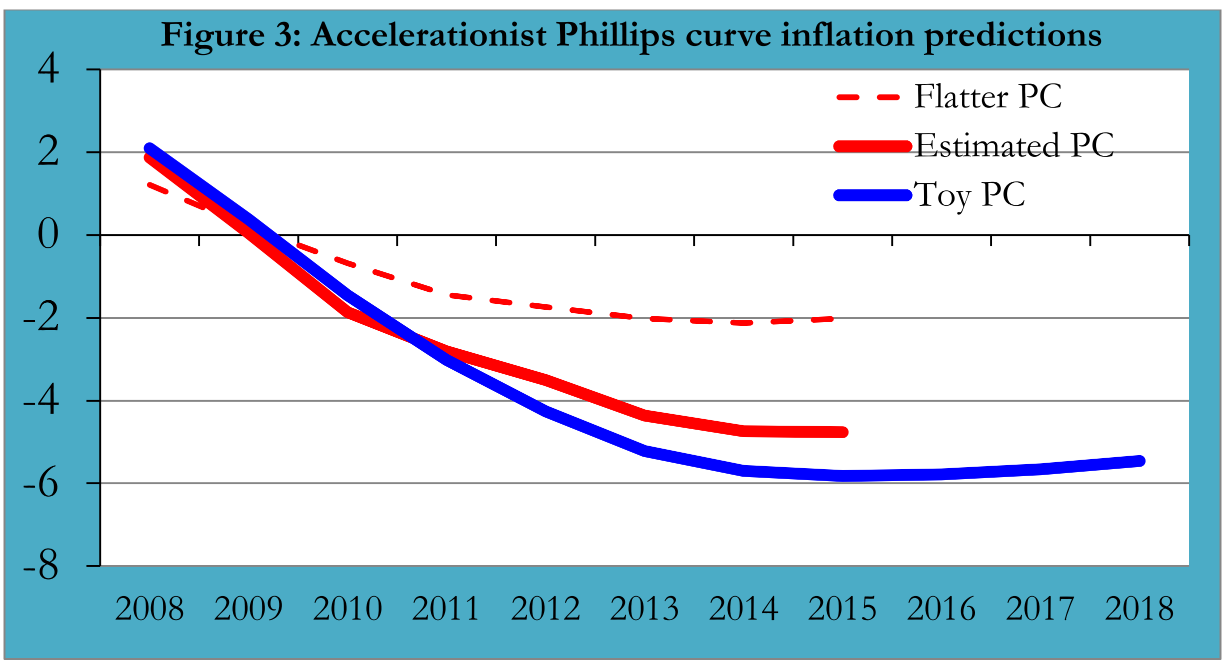
After decades of inflation-fighting around the world, many central banks recently have enjoyed relatively placid inflation behavior. To be sure, the world has faced its share of economic upheaval. But the imprint of these upheavals on inflation has been remarkably small. Figure 1 shows the US experience, highlighting pre-2000 and post-1999 experience, with recession shading indicated by the gray bars.



Remarkably for the U.S., the decline in inflation during the Great Recession is nearly a non-event—consider the trajectory of inflation from 2009 to 2016 in figure 1. Inflation dropped about one percentage point in the wake of the recession, but very quickly rebounded toward two percent, and has fluctuated between 1.4 and 2% since 2012. Many forecasters expected a more pronounced decline in inflation, abstracting from the temporary effects of the large decline in oil prices, as figure 2 shows.

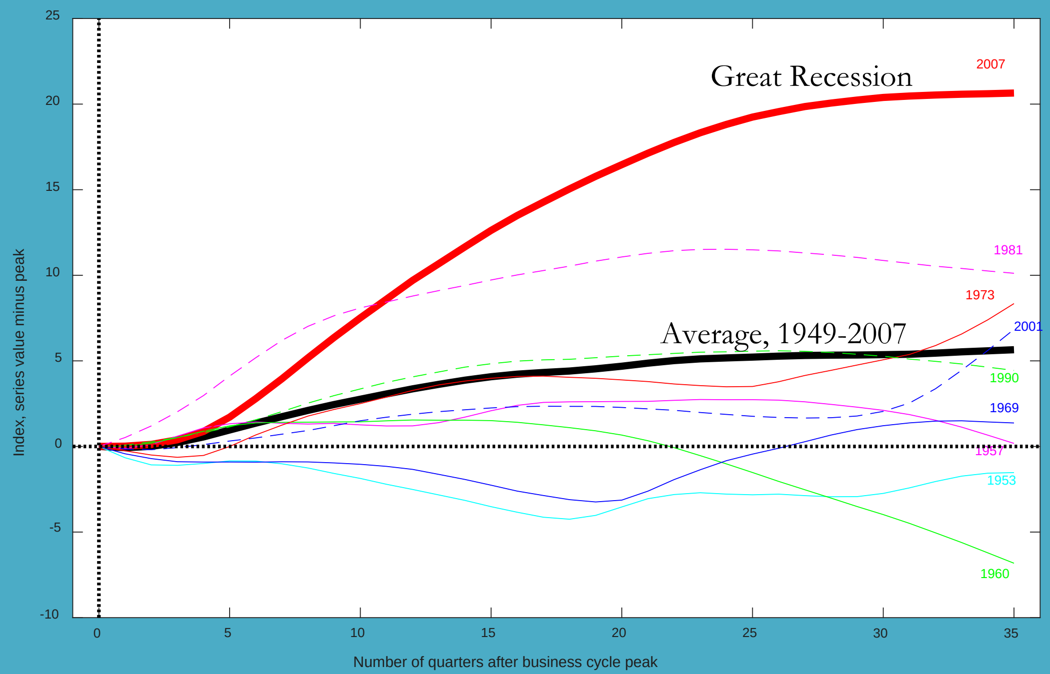


For those who still cling to the old-style accelerationist Phillips curve, the prediction for inflation would have been even more pessimistic, as figure 3 shows.



Such a forecast may seem extreme, but for a recession that featured approximately twenty point-years of cumulative unemployment gaps (see figure 4), the relative stability of inflation is quite remarkable. Even a modest Phillips curve slope of -0.1 would imply a two percentage point drop in inflation over this period (the red dashed line in the figure above). Instead, the decline was barely a percentage point, and quite short-lived at that.[2]

Figure 4: **Cumulative point-years of unemployment gaps, from beginning of recession**



Heavy black line represents average of recessions
Heavy red line represents current experience

In Japan, the story has been somewhat different but in many ways equally puzzling. As figure 5 indicates, despite significant and persistent negative output gaps, inflation fluctuated in a relatively narrow range, averaging about -1% through much of the past twenty years, then rising to positive territory in 2013-4, presumably related to the Bank of Japan's announcements about an even stronger commitment to bringing about positive inflation. The surge to two percent appears to have been temporary, and more recently inflation has been hovering around zero.

Note that this pattern of inflation and output gaps does not conform well to either an accelerationist or a New-Keynesian model of inflation. In the former case, the long sequence of negative output gaps should have resulted in a series of <u>declines</u> in inflation over this period, which evidently was not the case. In the latter case, absent other forces, inflation should have mirrored the output gap, moving toward its trend as the output gap closed. This dynamic is not obvious in the figure either, although there is some tendency for inflation to turn positive in some cases with a lag after the output gap nears or exceeds zero.

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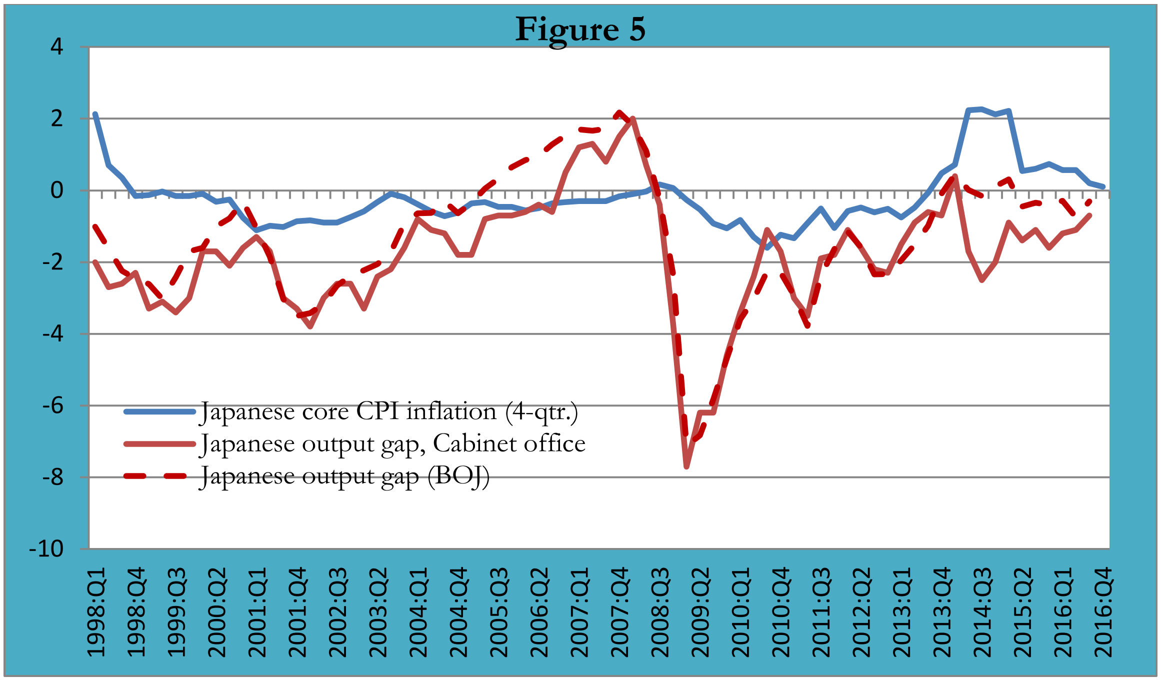
2 In comments delivered at the Federal Reserve Bank of Boston's annual economic conference, I discussed the predictions of NKPC-style models, which rely on the expected future path of real marginal costs. These models fare almost as poorly.

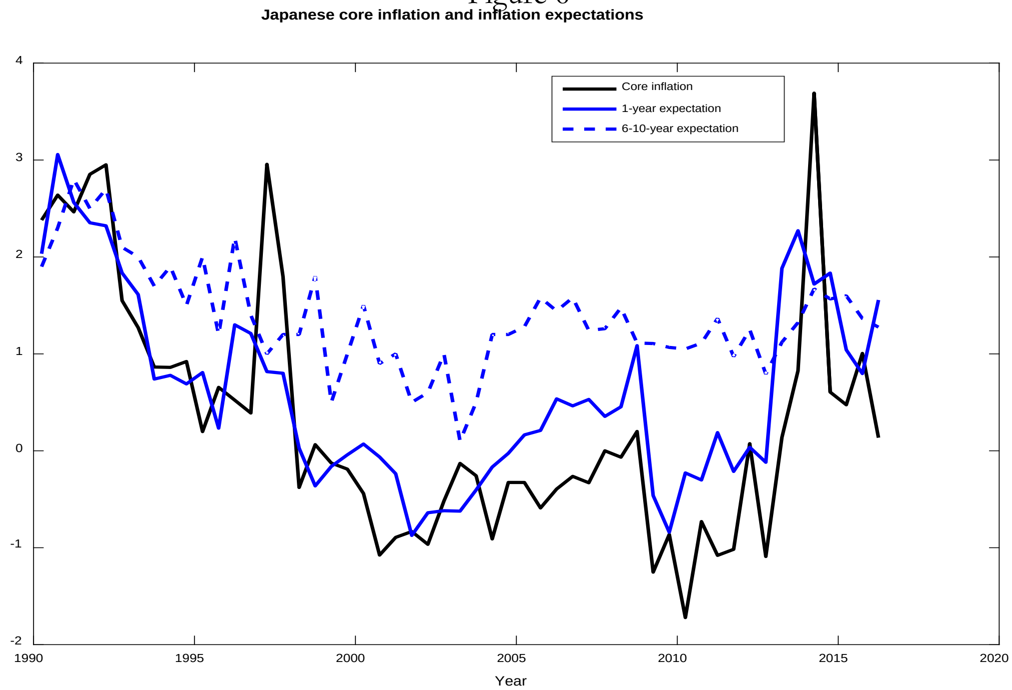


In the wake of this now extended period of stable inflation, despite historic fluctuations in real activity for both Japan and the US, many have turned to a number of explanations. One possibility is that the slope of the Phillips curve has diminished greatly in many countries, which at a high level comports well with the relative inactivity of inflation in the face of large output and employment fluctuations. However, even small Phillips curve slopes would have produced larger responses to the Great Recession in the US, as noted above. Others have proposed nonlinearities in the Phillips curve—both regions of "inactivity" around a zero output gap, and asymmetric gap effects as inflation nears zero.

Most have settled on an "anchored expectations" model of inflation, which will be discussed in more detail below.[3] The next figure presents data for Japanese core inflation along with Consensus Forecasts's one- and 6-to-10-year forecasts for CPI inflation, in the solid blue and dashed blue lines respectively. The sample is restricted by the availability of the Consensus forecasts, which begin semi-annually in 1990.

[3] One exception to this "explanation" for inflation dynamics is the work of Gilchrist *et al* (2017), which emphasizes the role that financial constraints play in determining the price response of firms to recessions.

Figure 6



This figure reveals several important facts about inflation and inflation expectations in Japan: (1) long-run expectations have remained quite stable in the face of prolonged economic weakness; (2) short-run expectations have often deviated quite noticeably, mostly below long-run expectations; (3) actual inflation tends to track the short-run expectations reasonably well; and (4) one might infer a tendency for short-run expectations to revert towards long-run expectations, although the rate of any such reversion is not rapid. A simple error-correction model linking short- and long-run inflation expectations (π_t^{eS}, π_t^{eL}) for this data develops the following error-correction coefficients:

Table 1
Inflation expectations error-correction coefficients
1991-2016 (semiannual data)

$$\pi_t^{ek} - \pi_{t-1}^{ek} = \alpha(\pi_{t-1}^{eS} - \pi_{t-1}^{eL}) + b\Delta\pi_{t-1}^{eS} + c\Delta\pi_{t-1}^{eL} + d \text{ , k = [s,L]}$$

	Error-correction coefficient (α)	*p*-value
Short-run expectations	-0.46	0.005
Long-run expectations	0.036	0.772

Qualitatively similar observations hold for the US, and while the specifics of Japanese inflation history clearly deviate from those of the US, some of the features noted above are common to both the US and Japan. The sections that follow will draw on some of these empirical observations to form a model of inflation dynamics that links actual inflation to short- and long-run expectations, as well as the output gap.[4]

[4] The results for the US data over the same sample period, using SPF inflation expectations on quarterly data, are as follows. Note that because the Japanese data are observed semi-annually, these error-correction coefficients imply an error-correction rate that is more than twice as fast for the U.S.

	Error-correction coefficient (α)	*p*-value
Short-run expectations	-0.53	0.000
Long-run expectations	-0.021	0.49

2. **Anchored expectations models of inflation: Theoretical considerations.**

If one assumes long-run expectations anchoring, then some care must be taken in how one specifies long-run expectations. In the specification for long-run expectations displayed below, inflation will be anchored regardless of what actions monetary policy takes. This seems unlikely to be true. The model comprises an anchored-expectations Phillips curve commonly used in the Federal Reserve System, a trivial IS curve, and a policy rule that targets only the output gap.

$$\begin{aligned} \pi_t &= l_1\pi_{t-1} + (1-l_1)\pi_t^{LR} + \beta(y_t - y_t^*) \\ \pi_t^{LR} &= \rho\pi_{t-1}^{LR} + (1-\rho)\pi^* \\ y_t &= Ey_{t+1} - \sigma(f_t - E\pi_{t+1} - 2) \\ f_t &= 2 + \pi^* + a_y(y_t - y_t^*) \end{aligned} \tag{2.1}$$

Long-run expectations are exogenous in the model—subject to shocks, but always returning to the inflation goal, apart from any action by the central bank. In most standard models with rational expectations, this is not going to work. Unless the policy rule obeys the Taylor principle (or something like it, depending on variations in the specification), the model will be unstable—inflation will not settle to any particular rate from arbitrary initial conditions.[5]

But this model works just fine with no central bank response to inflation.[6] It will always converge to $\pi_t = \pi^*$, as long as the central bank closes the output gap.

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[5] In fact, the stability properties of the model are more complicated than this. For a given value of l_1, the range of value of a_y that stabilizes the model ranges from 0.1 to infinity, given no emphasis on inflation. The greater the weight on the long-run expectation, the lower value of a_y that is required to stabilize the model. Inflation target can also stabilize the model, but that is not the novel feature of the specification.

[6] It behaves fine in the sense that standard solution techniques will produce the right "root count," implying a model that is unique and stable, converging to the inflation goal from arbitrary initial conditions.

Figure 7
Stability region for l_1 and a_y in the reference model
(No response to inflation gap)

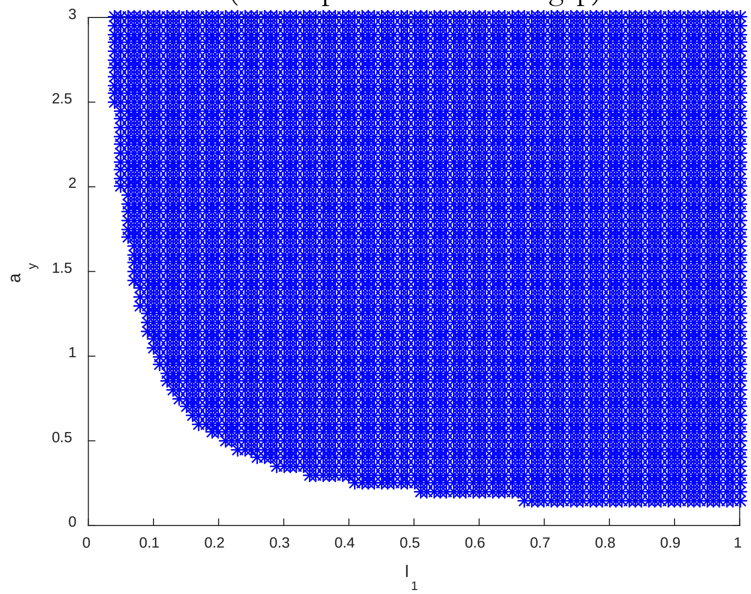
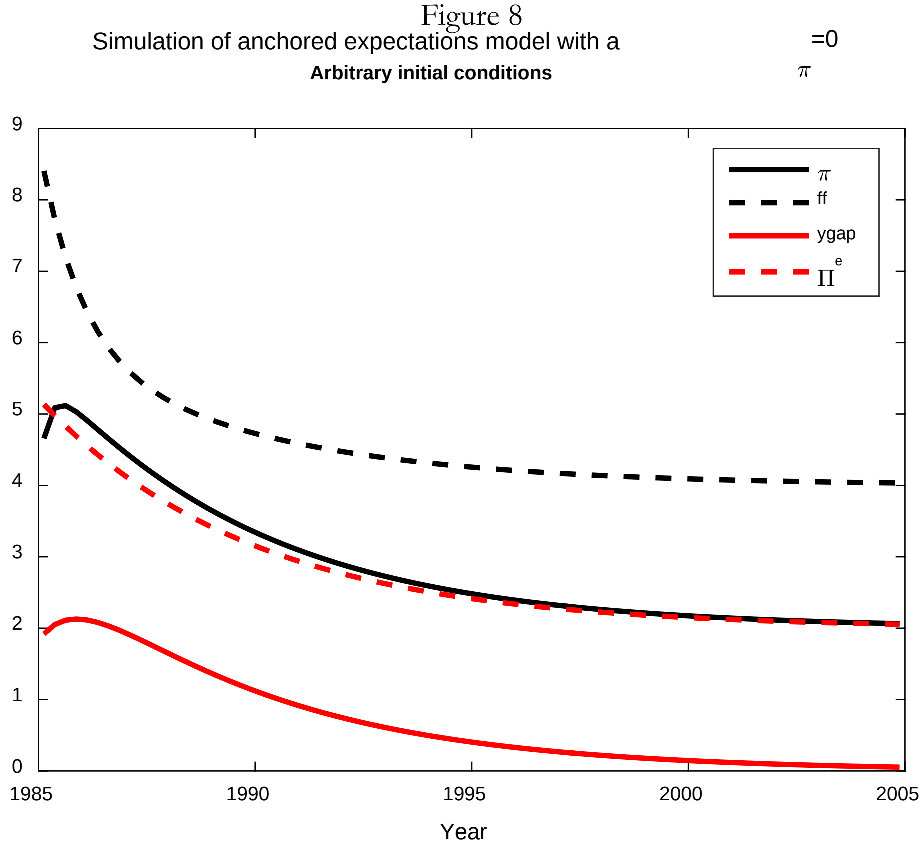


Figure 8 shows the dynamic response of the model to a set of random shocks that push it away from the steady state when the inflation response is 0, l_1=0.7, and a_y=0.5.

Figure 8



This stands standard monetary theory on its head: The central bank provides the real anchor, and that automatically provides a nominal anchor.

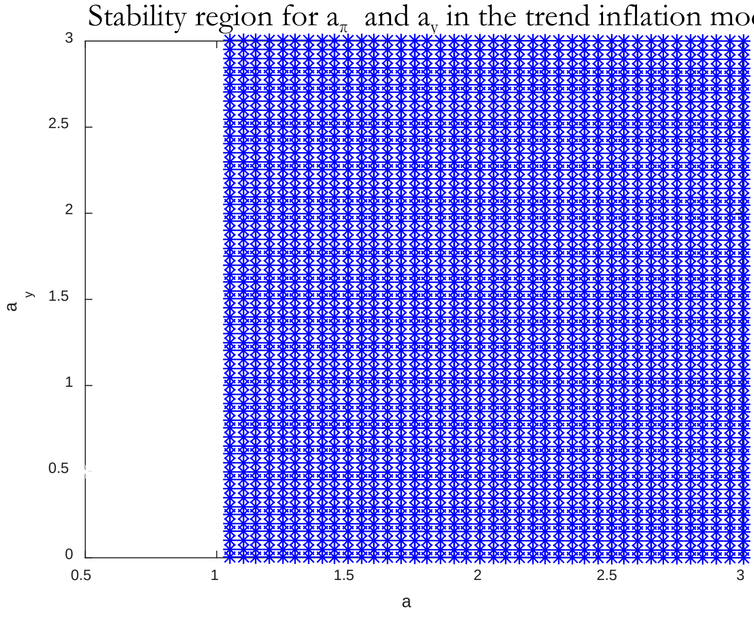
This outcome contrasts sharply with the more conventional incorporation of long-run expectations, in which they represent the trend for inflation, often represented in the following way:

$$\pi_t - \pi_t^{LR} = l_1(E\pi_{t+1} - \pi_{t+1}^{LR}) + (1 - l_1)(\pi_{t-1} - \pi_{t-1}^{LR}) + \beta(y_t - y_t^*) \qquad (2.2)$$

Thus inflation is modeled relative to its long-run trend, here represented by the long-run expectation as is conventional. The stability properties in this case are quite different but well-known: The model converges only if the central bank responds sufficiently to the inflation gap, as illustrated figure 9 below.

Figure 9

Stability region for a_π and a_y in the trend inflation model



This exercise points to what's missing in the anchored-expectations model: Short-term expectations that explicitly link the evolution of inflation to expected monetary policy actions. Alternatively, one can posit a mechanism whereby central bank actions (and expectations about future actions) feed directly into π_t^{LR}. Depending on how this is done, that mechanism can also solve the problem of upside-down monetary theory.

One simple remedy is to make the long-run expectations explicitly dependent on the sequence of short-run expectations that they (presumably) embody. A version of this model that alters equation (2.1) in this way is as follows:

$$\begin{aligned} \pi_t &= l_1 \pi_{t-1} + (1-l_1)\pi_t^{LR} + \beta(y_t - y_t^*) \\ \pi_t^{LR} &= \omega E \pi_{t+1}^{LR} + (1-\omega)\pi_t \end{aligned} \tag{2.3}$$

This model displays exactly the same stability properties as the canonical trend-inflation model above—insufficient responses to the inflation gap do not pin down the inflation rate, but following the Taylor Principle works as expected. The model differs from standard theory in placing primary emphasis on long-run expectations for short-run inflation dynamics. But because long-run expectations will only converge to the target if the central bank is expected to move inflation toward its target, it returns to the domain of sensible monetary policy predictions.

Another version of this model, in which the long-run expectations are modeled as a long moving average of lagged (realized) inflation, also achieves conventional stability properties.

$$\Pi_t^{LR} = a\Pi_{t-1}^{LR} + (1-a)\pi_{t-1} + c \tag{2.4}$$

The reason is that the dependence of inflation on long-run expectations implies dependence of inflation on lagged inflation. Unless the central bank over time guides inflation back towards a specific goal, long-run inflation expectations (and thus realized inflation) will not be anchored, and inflation will not be determinate.

It's unlikely that central bank staff anywhere literally believe the simple anchored expectation model of equation (2.1), and I know that Board staff have in place some mechanisms by which the long-run expectation can become temporarily unanchored from the target, but (presumably, with appropriate policy action) will ultimately return to the target. What we know about such mechanisms as an empirical matter is precious little. That makes reliance on the anchoring power risky, to my eye.

While this section proposed some "fixes" to ensure that the anchored-expectation model behaves sensibly, that model is still divorced from theory. There is no standard model that suggests that inflation should depend <u>only</u> on long-run expectations. Most all incarnations embody a primary role for short-run expectations. Equation (2.2) is a simple version of such a model.

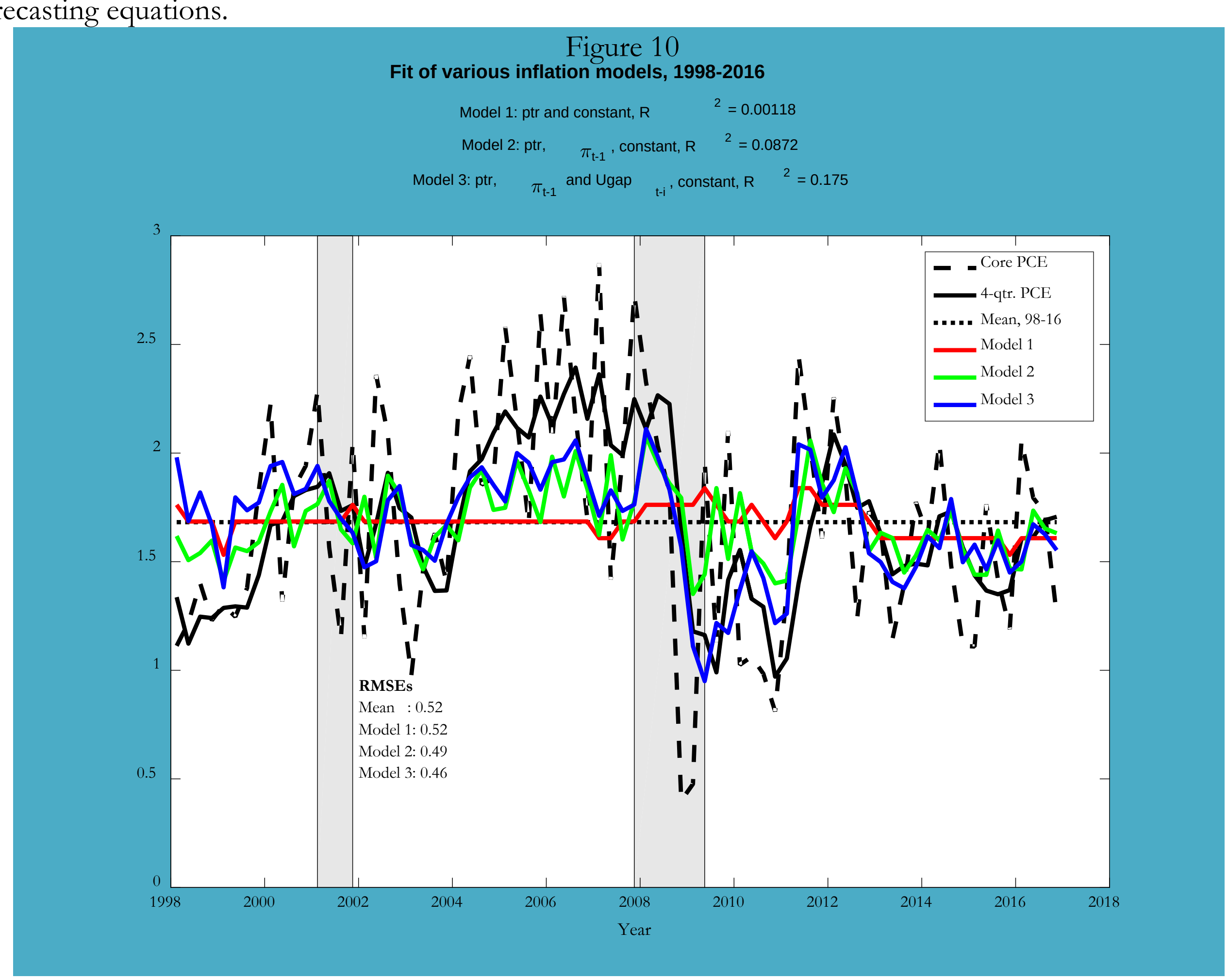
3. <u>Some empirical estimates of the influence of short- and long-run inflation expectations in the U.S.</u>

Much of the rationale for a focus on anchored expectations models lies in their empirical appeal, coupled with a loose connection to central banks' desire that expectations not become un-anchored, as they likely were in the 1970s. Starting with Williams' empirical note (2006), authors documented that inflation since the late-1990s might be well-modeled by an empirical model in which inflation was stationary around a (barely) time-varying long-run trend or (if not varying) constant, perhaps with adjustments for the influence of the output gap and a few transitory relative prices, usually energy and import prices. An extremely simple version of this model, employed by Williams is

$$\pi_t = c + \varepsilon_t \ . \tag{3.1}$$

Williams shows that this model beats both a random walk model of inflation and a simple Phillips curve specification. Perhaps that's because expectations are anchored, and to a first approximation, inflation fluctuates randomly around the central bank's goal?

Figure 10 shows the in-sample fit and RMSE for a few simple models. The first is just the sample mean of core PCE from 1998-2016. The second adds the long-run expectation (PTR), along with an intercept. The third and fourth add a lag of inflation and the change in the unemployment rate. Once one knows the *ex post* average value of inflation, the long-run expectation adds virtually nothing to explaining inflation. Put differently, PTR is essentially an intercept in these simple forecasting equations.



The addition of lags and a gap measure improve the fit and RMSE a bit, but not dramatically. A key point here is that apart from knowing the intercept term, it has been difficult to explain fluctuations in inflation over the past twenty years. That is somewhat less true if one smoothes the data (as in the four-quarter % change, the solid black line above), but it is still true. The change in the unemployment rate picks up some of the fluctuations associated with the recession and its immediate aftermath. Lagged inflation picks up some of the very modest autocorrelation in inflation over the period. But these are small helps.

What about short-run expectations, for which there may be stronger theoretical rationale? There have been many periods during which the inclusion of short-run survey expectations has been essential to inflation forecasting equations (and Phillips curves). It is not the case that such short-run expectations measures enter significantly in all periods, nor that the coefficients on short-run expectations are stable across all sub-periods. But the same is true of long-run expectations. The sign, magnitude and significance of long-run expectations all vary over the past 35 years in a variety of empirical inflation models. Representative empirical results bearing on these points are discussed in the next subsection.

Estimates of Phillips curves with short- and long-run expectations over recent samples:

The model employed in this section allows for the influence of

- Lagged inflation;
- Short-run (survey-based) expectations of inflation;
- Long-run (survey-based) expectations of inflation. Here we use the variable "PTR" from the FRB/US model;
- An unemployment gap;
- Inflation terms that sum to one or not;
- The inclusion of an intercept.

$$\pi_t = a_1\pi_{t-1} + a_2\pi^S_{t+1} + a_3\pi^S_{LR} + b(U_t - U^*_t) + c + \varepsilon_t \qquad (3.2)$$

The intercept is not theoretically required, but it serves as a robustness check on the role that long-run expectations π^S_{LR} play in the inflation model. That is, does inflation track fluctuations in π^S_{LR}, or do either π^S_{LR} or an intercept simply serve as the average inflation rate around which inflation fluctuates in an atheoretic statistical representation of recent inflation, such as

$$\pi_t = c + b(y_t - y^*_t) + \varepsilon_t \text{ or just } \pi_t = c + \varepsilon_t \text{ ?}$$

Short-run expectations π^S_{t+1} are also taken from the SPF; this memo uses the four-quarter expectation in the core CPI, although the results will be similar for the one-quarter or other short-run expectations measures. The unemployment gap is computed using the civilian unemployment rate and the CBO's estimate of the natural rate of unemployment.

The unit sum restriction brings the specification closer to a "structural" relationship; without imposing this restriction, the estimate is more of a practical forecasting equation. For most models, an intercept would be an unnecessary addition, once one has a long-run settling point built into the equation. But as suggested above, in recent years it's a open question whether inflation is best modeled as fluctuating around the long-run expectation, or fluctuating around a constant. Of course, it's a close call because the long-run expectation has been relatively constant in recent years. But one would hope that the long-run expectation would dominate the intercept, if variation in the long-run expectation means anything.

The tables below summarize a few different samples and specifications for simple Phillips curves that are versions of equation (3.2). The key insight from these estimates is that one can obtain quite an array of results, depending on the sample one looks at, and also on the specification one employs. Over the full sample (table 2), short-run survey-based expectations play a quantitatively important role in explaining inflation. The long-run expectations are considerably less reliable. Identification of the unemployment gap is poor.

In samples that focus on the data since the turn of the century (table 3), identification is much weaker. In models that include both short- and long-run expectations, neither consistently enters significantly, and the unemployment gap terms are rarely significant. In many cases, the inclusion of an intercept drives out the long-run expectation, which suggests that it is not variation in that variable, but its relative constancy, that serves as an intercept-anchor in this simple inflation specification.

Table 2
Estimated "Phillips curves", 1982:Q1-2016:Q3
Various specifications, with restrictions (unit sum) or not, intercept or not, short- and long-run expectations
Inflation variable = core PCE

Variable	(1)	(2)	(3)	(4)	(5)	(6)
Lagged π	0.0406	0.437***	0.0438	0.172*	-	-
PTR	0.182	0.331*	0.189	0.306*	0.189	0.194
Short-run π^e	0.777***	0.232	0.757***	0.390**	0.811***	0.799***
Ugap	0.0446	-0.123*	0.0426	-0.0191	0.0473	0.0461
Intercept	**Yes**	No	**Yes**	No	**Yes**	**Yes**
Unit sum restriction?	**Yes**	**Yes**	No	No	**Yes**	No

Newey-West corrected standard errors
* = .05 significance or better
** = .01 significance or better
*** = .001 significance or better

Table 3
Estimated "Phillips curves", 2000-2016:Q3
Various specifications, with restrictions (unit sum) or not, intercept or not, short- and long-run expectations
Inflation variable = core PCE

Variable	1	2	3	4	5	6	7	8
Lagged π	0.139	0.237	0.141	0.139	0.0133	0.247	0.0203	0.00951
PTR	0.861***	0.762***	0.768***	1.33	0.307	0.929***	0.152	0.853
Short-run π^e	-	-	-	-	0.68	-0.176	0.653	0.698
Ugap	-0.1*	-.136***	-0.0997*	-0.107*	-0.0217	-.147***	-0.0231	-0.0272
Intercept	-0.193*	None	None	-1.16	-0.375**	None	None	-1.55
Unit sum restriction?	Yes	Yes	No	No	Yes	Yes	No	No

Newey-West corrected standard errors

* = .05 significance or better ** = .01 significance or better *** = .001 significance or better

While the PCE is the official inflation variable of the Fed, it is still of some interest to see how robust the results above are to the moderately different core CPI measure. The table below shows the full-sample estimates of Phillips curves using the core CPI measure. The key differences are that (a) PTR is never significant, with an estimated coefficient quite close to zero; (b) the short-run expectation is even more uniformly large and significant. The recent sample results (not show) differ relatively little from those for the core PCE.

Table 4
Estimated "Phillips curves", 1982:Q1-2016:Q3
Various specifications, with restrictions (unit sum) or not, intercept or not, short- and long-run expectations
Inflation variable = core CPI

Variable	(1)	(2)	(3)	(4)	(5)	(6)
Lagged π	0.208*	0.208*	0.131	0.191*	-	-
PTR	0.0757	0.0693	-0.0256	0.097	0.0683	-0.0572
Short-run π^e	0.717***	0.722***	1.03***	0.737***	0.932***	1.23***
Ugap	-0.0856	-0.0812	-0.0678	-0.109	-0.101	-0.0709
Intercept	**Yes**	No	**Yes**	No	**Yes**	**Yes**
Unit sum restriction?	**Yes**	**Yes**	No	No	**Yes**	No

Newey-West corrected standard errors
* = .05 significance or better
** = .01 significance or better
*** = .001 significance or better

Figures 11-13 provide rolling-sample estimates of variants of equation (3.2), with only long-run expectations, only short-run expectations, and a horse race (figure 13) that pits short-run against long-run expectations.[7] The size and significance of all of these variables' coefficients have varied over the sample, for all of these specifications. In figure 13, one can see that short- and long-run expectations have both hovered around 0.5, remaining reasonably significant. The significance of the unemployment gap (the green line) is much more consistent in this specification.

[7] Note that the lagged inflation term is excluded here, as it rarely played a prominent role.

Figure 11

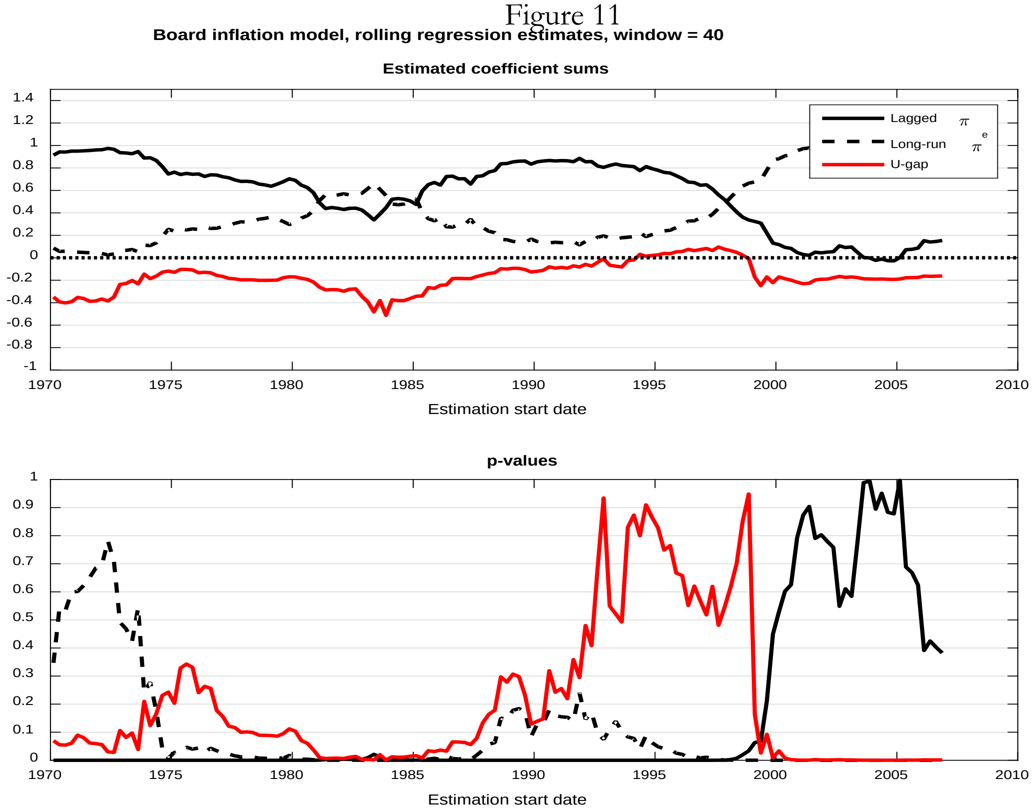


Figure 12

Substitute short-run for long-run expectations

Rolling regression estimates, window = 40 quarters

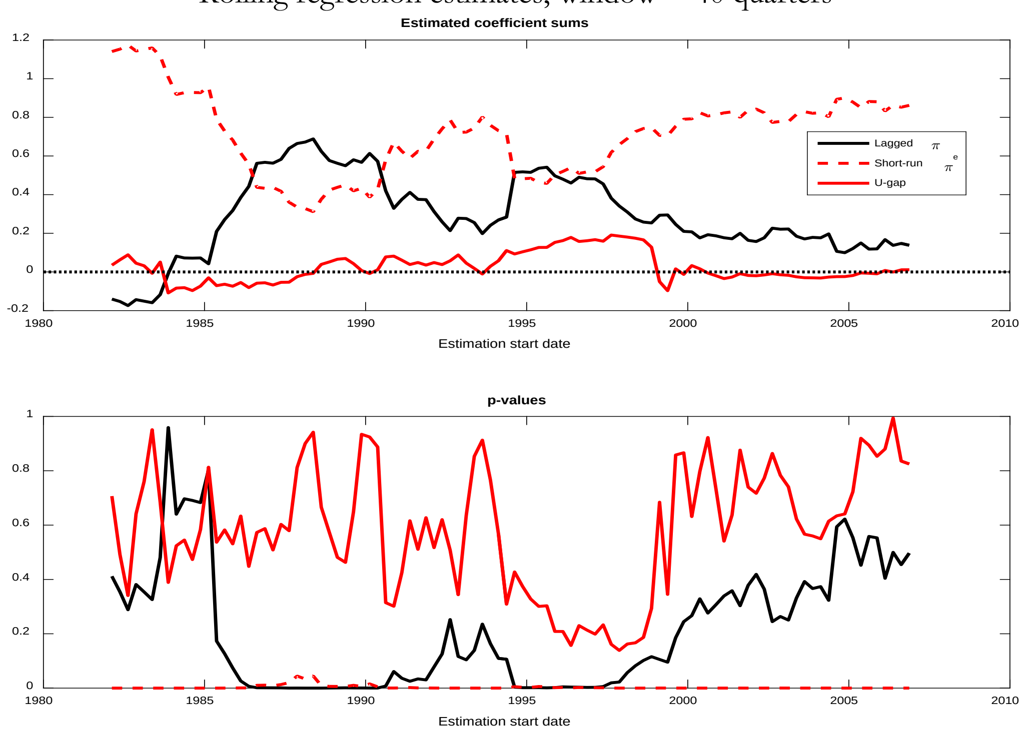
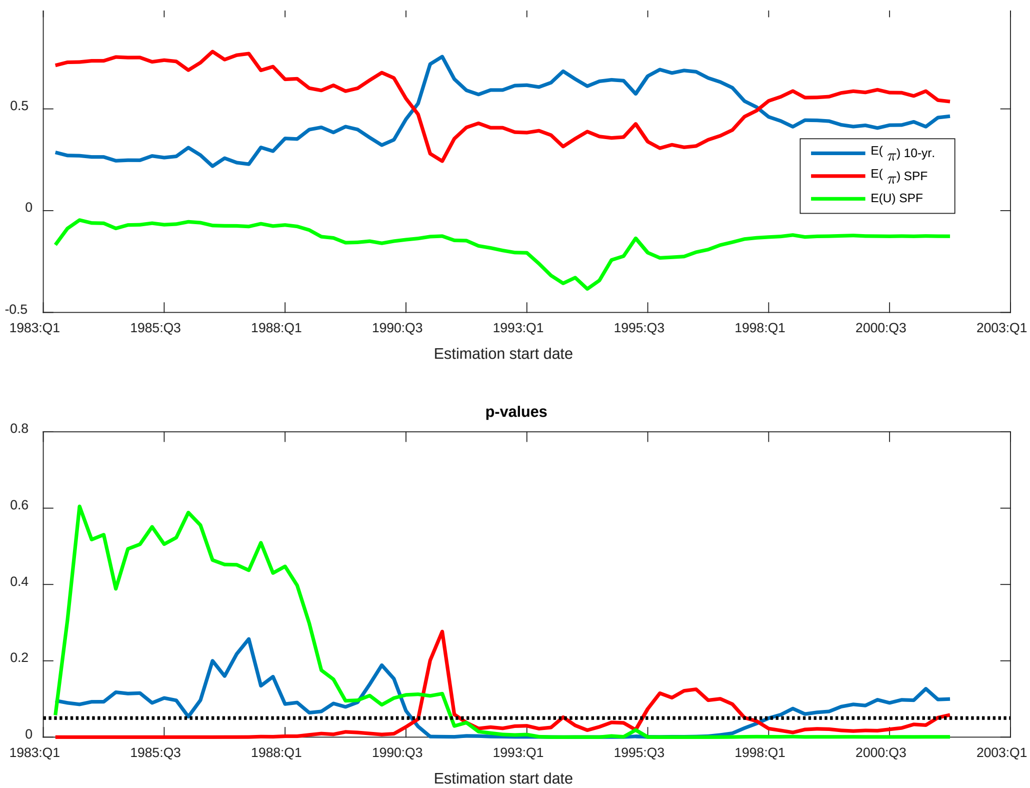


Figure 13

Horse race between short- and long-run expectations, Rolling regression estimates, window = 60 quarters



<u>What to conclude from the empirical evidence?</u>

I would be hard-pressed to argue that only short- or long-run expectations belong in an empirical Phillips curve for U.S. data. My reading is that, especially in recent data (the past twenty years or so), there is not enough variation to distinguish between the influence of short- versus long-run expectations. The fact that our favorite measure of long-run expectations has barely budged since 1998 makes it difficult to know (a) how important that stability is, or (b) how a shift in that measure (or properly-measured long-run expectations) might affect inflation.[8]

In addition, the lack of a theoretical founding for a simple model that includes <u>only</u> long-run expectations should factor in to our search for a preferred specification, at least to some degree. More prosaically, Truman Bewley has surveyed hundreds of producers over the past 10 years, and the number of times that the long-run inflation goal is mentioned (unprompted) as a factor in pricing decisions is precisely zero[9]. If price-setters are not aware that the central bank's inflation goal

[8] As defined, the expected inflation rate on average over the next 10 years really should have moved more than it has. It suggests that the SPF survey respondents think of this not as a ten-year average, but as a proxy for their estimate of the central bank's inflation goal, or at least as a proxy for where inflation will be 10 years hence.

[9] See especially point (5) in the Bewley presentation here: https://www.bostonfed.org/great-recovery2016/agenda/

(or long-run expectations, should they become detached from the inflation target) matters at all in their pricing decisions, then the mechanism by which the goal ends up as a key determinant of aggregate price dynamics is mysterious indeed.

4. A DSGE model with survey expectations

In a recently published paper (Fuhrer 2017),[10] I explore a framework in which both short- and long-run expectations matter, in which long-run expectations are well-anchored, but in which the convergence of inflation to the central bank target is not mechanically guaranteed. The novelty of the paper is that it employs survey measures for all expectations (both inflation and real side), and attempts to endogenize the expectations in a way that is generally consistent with the logic of DSGE models, but importantly, it is not assumed that expectations are rational. It finds a role for both short-run and long-run expectations, in a way that is described below.

The essence of the paper is as follows:

1. Inflation depends on short-run expectations and on a gap measure;
2. The short-run expectations can be iterated forward, and thus depend on longer-run expectations and on short-run expectations of the gap measure;
3. This iteration can of course go on forever, so I proxy for is by using long-run expectations of inflation, which should contain a sequence of shorter-run expectations of the gap measure, if the model is approximately correct. It is not necessary that expectations are rational in order to take this step.
4. This anchors short-run expectations to longer-run expectations, which can be assumed to ultimately return to the central bank's inflation target (or not).
5. A parallel mechanism is employed for output. Output depends on short-run output expectations and a real interest rate. The same iteration arguments imply that short-run output expectations depend on long-run output expectations (which embody a sequence of short-run real interest rate expectations).
6. This model is estimated on US data, and I find that it helps a lot with a number of identification issues.
7. I also find that the inertia that is intrinsic to expectations better explains macro inertia than inflation indexation, or habits, or serially correlated shocks in DSGE models.
8. These results imply that short-run expectations for inflation and output—properly measured, via surveys—are <u>critical</u> inputs to empirical macro models.

An example of the modeling strategy embodied in the paper may be found in the Phillips curve/inflation expectations system below. The first equation posits an expectations-augmented Phillips curve with survey expectations for the one-period-ahead inflation rate:

$$\pi_t = \beta\pi^S_{t+1,t} - \pi^u(U_t - U^*_t).$$

The survey expectations in turn are assumed to follow the law of iterated expectations, so that they may be iterated forward in time. However, lacking an infinite forward sequence of survey

[10] http://www.sciencedirect.com/science/article/pii/S0304393216301362/pdfft?md5=6cf7d546a2ea8ec9a185ec14e28f3143&pid=1-s2.0-S0304393216301362-main.pdf

expectations, we approximate this process by allowing the long-run (10-year) expectation for inflation to proxy for the iterated-forward sequence of expected unemployment rates:

$$\pi^{S*}_{t+1,t} = A\Pi^{S}_{LR,t} - \pi^{u}(U^{S}_{t+1,t} - U^{*}_{t+1}).$$

Finally, one-period survey expectations are assumed to adjust gradually towards this sequence of long-run expectations via an error-correction mechanism, indexed by parameter μ^{π}:

$$\pi^{S}_{t+1,t} = \mu^{\pi}\pi^{S*}_{t+1,t} + (1-\mu^{\pi})(\pi^{S}_{t,t-1}).$$

The "intrinsic" inertia in expectations is captured by the size of the parameter μ^{π}, and its significance is estimated on time series data, allowing for other sources of persistence that are now standard in the DSGE literature (indexation of prices, habit formation, autocorrelated shocks).

Of course, a similar system must be employed to solve for the survey expectations of unemployment $U^{S}_{t+i,t}$. The paper provides more detail.

The paper develops some interesting results, namely:

- Survey expectations, both short- and long-run, enter significantly in the model, with the restrictions indicated above;
- The intrinsic inertia in expectations is strongly preferred by the data as a means of explaining persistence in aggregate time series for inflation and unemployment, as opposed to other methods that the DSGE literature has embraced in recent years;
- To be specific, terms reflecting inflation indexation, habit persistence, and correlated structural errors all but vanish from the model in terms of quantitative and statistical significance;
- In a nested test, the DSGE model with survey expectations strongly dominates the model with rational expectations. The test shows that the data place a small and statistically insignificant weight on rational expectations.

This does not mean that long-run expectations do not matter. They do, inasmuch as they proxy for a sequence of short-run expectations (which appear not to be the rational expectations implied by the model) that are linked to inflation. The model described above allows long-run expectations to deviate from the central bank's goal, but does not provide any deep rationale for such deviations.[11]

Evidence in micro data bearing on the behavior of aggregate survey expectations

In a separate unpublished paper, I show that the micro data for the Michigan, SPF and Euro-SPF surveys exhibit behavior that is fully consistent with intrinsic inertia in expectations formation. Remarkably consistent empirical results show that individual survey participants consistently move their own forecasts toward the lagged central tendency of forecasts, which can impart inertia to the aggregate expectations measure beyond that in the driving processes for inflation, output growth, interest rates or unemployment.

[11] The model allows for shocks that have persistence effects on the long-run inflation expectation, but ultimately, the long-run expectations will converge to the inflation goal. The model does not identify any deeper structural explanation for such shocks.

The figure below provides a scatter plot of the individual observations on forecast revisions (the vertical axis) agains the discrepancy between an individual forecast and the last observed median forecast. A significant negative correlation is evident in the figure; the table following documents the robustness of this negative correlation to the inclusion of many different controls.

Figure 14

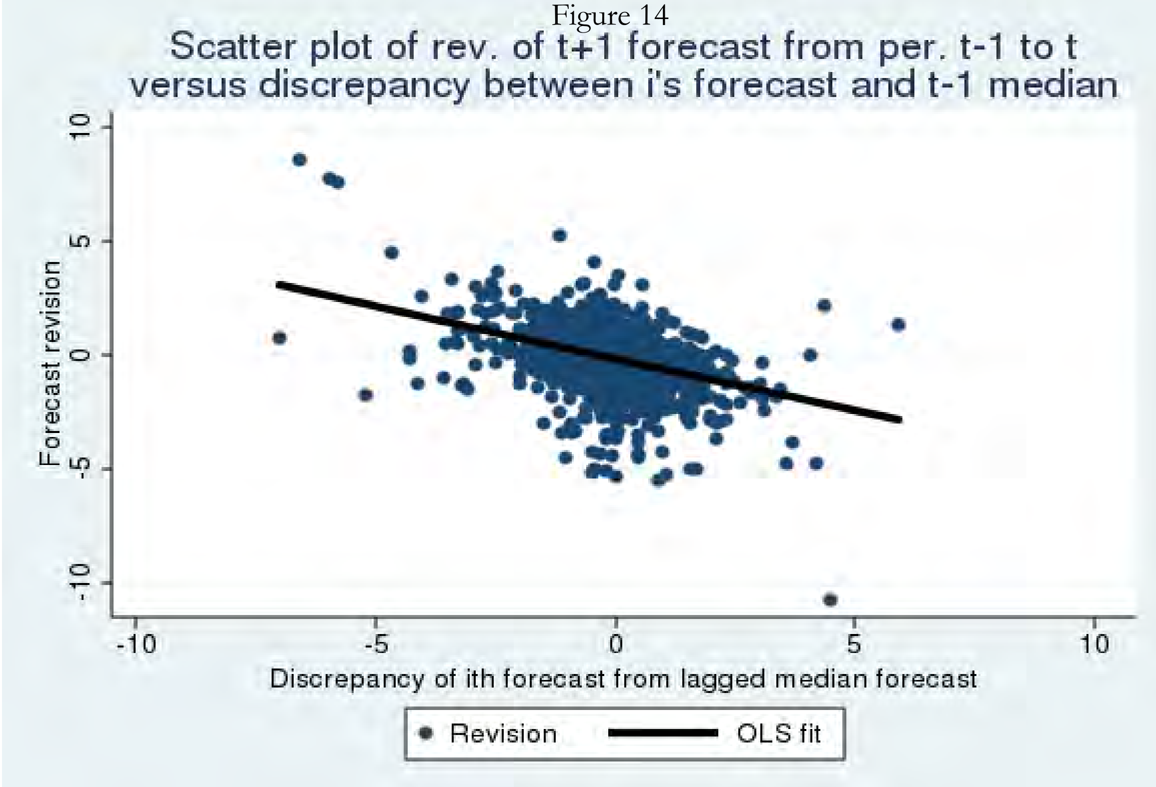


Table 4

Response of forecast revisions to lagged discrepancies between individual forecasts and central tendency measures

$$\pi^{i,SPF}_{t+1,t} - \pi^{i,SPF}_{t+1,t-1} = \delta[\pi^{i,SPF}_{t+1,t-1} - \pi^{Median}_{t+1,t-1}] + a\pi^{i}_{t-1} + cZ^{i}_{t} + \delta_{i} + \varepsilon^{i}_{t}$$

Inflation results

Variable						
$\pi^{i}_{t,t-1} - \pi^{Median}_{t\|t-1}$	-0.30 (0.002)		0.08 (0.193)			
$\pi^{i}_{t+1,t-1} - \pi^{Median}_{t+1\|t-1}$		**-0.58 (0.000)**	**-0.68 (0.000)**	**-0.47 (0.000)**	**-0.56 (0.000)**	**-0.56 (0.000)**
π^{i}_{t-1}					0.01 (0.648)	-0.03 (0.011)
$U^{i,SPF}_{t+1,t}$					0.04 (0.883)	0.33 (0.178)
$U^{i,SPF}_{t+2,t}$					-0.10 (0.690)	-0.02 (0.904)
$\Delta Y^{i}_{t,t}$					0.06 (0.000)	0.03 (0.030)

$\Delta Y^{i}_{t+1,t}$					0.01 (0.640)	0.01 (0.465)
$R^{i}_{t+1,t}$					-0.05 (0.019)	0.04 (0.703)
All controls*						Yes
Adjusted R-squared	0.069	0.206	0.251	0.172	0.224	0.473
Observations	3272	3274	1591	1926	3029	2945
* "All controls" includes real-time estimates of lagged inflation, unemployment, Treasury bill rate, current period, *t*+1, *t*+2, *t*+3 forecasts of inflation, unemployment, Treasury bill rate, output growth.						
Inflation revision on discrepancy from lagged central tendency:other forecast horizons						
	Revision from *t-1* to *t* for forecast period					
	t+1	t+2	t+3	t+1	t+2	t+3
$\pi^{i}_{t+1,t-1} - \pi^{Median}_{t+1\|t-1}$	-0.58 (0.000)			-0.56 (0.000)		
$\pi^{i}_{t+2,t-1} - \pi^{Median}_{t+2\|t-1}$		-0.54 (0.000)			-0.53 (0.000)	
$\pi^{i}_{t+3,t-1} - \pi^{Median}_{t+3\|t-1}$			-0.61 (0.000)			-0.61 (0.000)
π^{i}_{t-1}				0.01 (0.648)	0.03 (0.019)	0.05 (0.000)
Other forecast controls	N	N	N	Y	Y	Y
Observations	3274	3257	3180	3029	3017	2960

Thus expectations inertia is a feature in the micro data for survey expectations. The result for the SPF holds up quite strongly for the Michigan and the Euro-SPF surveys. It cannot be explained as a statistical artifact of forecasters who make rational forecasts about a persistent variable (this is discussed in detail in the paper).

While this work raises questions about how expectations are formed, and what role they play in inflation and output dynamics, the results in these papers appear to be strong and promising.

5. A survey-based DSGE model of inflation dynamics for Japan

As suggested above, although the specifics of Japan's inflation experience differ from those in the U.S., it may be that like the U.S., inflation is well-modeled as depending on both short- and long-run survey expectations of inflation, along with a measure of a resource gap. This section follows the methodology of the preceding sections, using data on Japanese inflation, the output gap, the expected output gap, and short- and long-run inflation expectations. All of the expectations data are taken from the Consensus Forecasts database, which began collecting such data semi-annually in 1990. [A look at anchoring and other key dynamics issues in Japanese data]

Simple Phillips curves with short- and long-run expectations

We begin by examining single-equation relationships among the key data.

Table 5 Dependent variable: core inflation		
	Coefficient	*p*-value
Lagged inflation	0.288*	0.020
Short-run expectations	0.567**	0.007
Long-run expectations	-0.0358	0.904
Output gap	0.102	0.124
Dependent variable: core inflation		
Lagged inflation	0.285*	0.018
Short-run expectations	0.558**	0.004
Output gap	0.1	0.117
Dependent variable: one-year expectation		
Lagged inflation	0.253**	0.002
6-10 year expectations	0.557**	0.006
Output gap	0.14**	0.002

Inflation depends significantly on short-run expectations, lagged inflation and (for the expectations themselves) the output gap. Long-run expectations appear not to be directly important for core inflation, but quite important for anchoring short-run expectations. A system test of the exclusion of long-run expectations from the inflation equation (but not from the short-run expectations equation) cannot reject this restriction, developing a *p*-value of 0.99.

Table 6 presents simple regressions of short-run inflation expectations on lagged inflation, long-run expectations, and an output gap measures, for zero- to five-year-ahead horizons, expanding on the bottom panel of Table 5. The results suggest a very consistent dependence on long-run expectations (perhaps as an "anchor" for short-run expectations), a modest dependence on lagged inflation (depending on the horizon), and modest dependence on the output gap for the first three horizons in the table. Taken together, tables 5 and 6 provide initial evidence that suggests that a survey-based model for inflation may be fruitful for Japanese data. Of course, a more complete assessment of such a model is required to be more confident of this assessment. The next section turns to this task.

Table 6 Dependence of short-run inflation expectations on lagged inflation, long-run expectations and the output gap						
	Horizon (year)					
Coefficients	0	1	2	3	4	5
Lagged inflation	0.48***	0.25**	0.17*	0.091	0.15***	0.076*
6-10 year expectation	0.3	0.55**	0.66***	0.86***	0.74***	0.78***
Output gap	0.17***	0.14**	0.097*	0.027	-0.0045	0.026
Asterisks indicate significance at the 5, 1, and .1% level (*, ** and *** respectively)						

A survey-based model of Japanese inflation and output

In this section, I construct a DSGE-like model for Japanese data, drawing on the approach used for US data in Fuhrer (2017). That paper employs survey expectations in a DSGE model, using methods described therein to endogenize short-run survey expectations of key variables. The model below is a simplified version of the model explored in that paper, and focuses primarily on inflation and inflation expectation dynamics. The description of the IS block of the model is more reduced-form in nature, although it follows the outlines of the approach to expectations taken in the inflation block.

The model comprises

- A New Keynesian Phillips curve in which inflation depends on short-run expected inflation as measured in the Consensus Forecasts survey of forecasters, as well as the current value of the Cabinet Office's estimate of the output gap;
- A semi-structural forward-looking "IS" curve in which an estimate of the output gap depends on a lag of the output gap, the short-run expectation of the output gap as measured by the Consensus survey, and a short-term real interest rate defined as the difference between the policy rate and the one-year inflation expectation from the same survey;
- Equations that link the short-run expectations for inflation and the output gap to long-run survey expectations of inflation and output, as well as survey-based expectations of output and real rates, respectively;
- In addition, we allow for intrinsic inertia in short-run inflation expectations, indexed by the parameter μ_π as indicated in equation (5.1) below;
- Long-run expectations are assumed to follow simple AR(1) processes that revert to sample-estimated long-run values.
- The policy rate follows a simple, inertial policy rule that responds to deviations of inflation from its (for the moment) assumed goal of one percent. The rule is estimated via OLS on a sample prior to the ZLB period (1980-1999), and its parameters are also held fixed for the system estimation.

Thus the model may be represented by the following system of equations:

$$
\begin{aligned}
&\pi_t = \beta\pi^S_{t+1} + \pi_y \tilde{y}^{Cab}_t \\
&\pi^S_{t+1,t} = (1-\mu_\pi)(\beta\Pi^S_{LR,t} + \pi_y \tilde{y}^S_{t+1,t}) + \mu_\pi \pi^S_{t,t-1} \\
&\tilde{y}^{Cab}_t = y_l \tilde{y}^{Cab}_{t-1} + y_E \tilde{y}^S_{t+1,t} - y_\rho(\rho_t - \bar{\rho}) \\
&\rho_t \equiv r_t - \pi^S_{t+1,t} \\
&y^S_{t+1,t} = (1-\mu_y)(y_{LR}\tilde{y}^S_{LR,t} - y_\rho(E_t\rho_{t+1} - \bar{\rho})) + \mu_y \tilde{y}^S_{t,t-1} \\
&\tilde{y}^S_{LR,t} = 0.68\tilde{y}^S_{LR,t-1} \\
&\Pi^S_{LR,t} = 0.95\Pi^S_{LR,t-1} + 0.05c_\Pi \\
&r_t = 0.92r_{t-1} + 0.08[2.5(\pi^S_{t+1,t} - 1) + \bar{r}] \\
&\bar{r} \equiv \bar{\rho} + \bar{\pi}
\end{aligned}
\qquad , (5.1)
$$

in which π denotes inflation, $\tilde{y}$ is the output gap, r is the central bank policy rate (from the IFS database), Π^S_{LR} is the Consensus survey's 6-10 year inflation forecast, r^S_{t+1} is the Consensus survey observation on the one-year forecast of the three-month Treasury yield, the superscript "S" denotes a variable from the Consensus survey, and coefficients that are held fixed at their single-equation OLS estimates are indicated in the model. Critically, we estimate the policy rule over the sample prior to the extended period during which the policy rate was constrained at its effective lower bound. The discount rate β is held constant at 0.98 in the estimation.[12]

This leaves eight key parameters to estimate: π_y, the slope of the Phillips curve, which is common to the Phillips curve and to the one-period-ahead inflation expectations equation; μ_π, the parameter that indexes the intrinsic persistence in inflation expectations; y_l, the coefficient on the lagged output gap in the IS curve; μ_y, the parameter indexing the intrinsic persistence in the output expectation equation; y_E, the coefficient on the one-year-ahead expectation of the output gap in the IS curve; y_{LR}, the coefficient on long-run output expectations in the output expectations equation; y_ρ, the slope of the IS curve; and $\bar{\rho}$, the equilibrium real interest rate, which is common to both the IS curve and to the reduced-form representation of output expectations.[13]

The Consensus Survey does not provide forecasts for the output gap. Instead, I use the Cabinet Office's estimate of the current output gap, combined with the Consensus Survey's forecasts for real GDP growth in ensuing years, along with an estimated trend growth rate, to construct *t+1* and long-run output gap forecast estimates.[14]

Several features are worthy of note from these estimates. First, the degree of "intrinsic persistence" in inflation expectations is significant, with the coefficient on lagged inflation expectations (μ_π) estimated at 0.79 (equivalently, the error-correction coefficient suggests about 21 percent of the gap is closed per half-year). Second, the "slope" parameters in the Phillips and IS curves (π_y, y_ρ) are sizable and reasonably precisely estimated. Third, the estimated value of the equilibrium real interest rate, $\bar{\rho}$ is negative, and the 90% confidence interval lies entirely in the negative domain. Fourth, as suggested by the results in the table, Figure xx shows that for all of the parameters, the data move the estimated values noticeably away from the priors and/or sharpen the precision of the estimates relative to the priors, suggesting that the data contain useful identifying information for this model. For example, the posterior distributions of the inflation error-correction parameter μ_π implies larger and more precisely estimated error-correction than its prior distribution, and the posterior distribution for the equilibrium real rate similarly is more negative than its prior, evincing little overlap with its prior distribution.

[12] Estimating β rather than imposing yields a value above 0.95, using priors that put significant weight on a value above 0.8. Freeing up the priors to allow for lower values, while running counter to the notion of the NKPC, yields lower values of β of 0.6-0.7. We choose to retain the structural interpretation of these equations, and thus keep the higher values that conform better to the notion of a discount rate.

[13] The prior distributions for all of the parameters in this and subsequent estimations are displayed in the Appendix table.

[14] The output gap equals the current output gap plus the cumulative sum of real growth forecasts less the estimate of the trend growth rate. Thus output gaps are constructed using the identity that links the output gap from period to period and the discrepancy between real growth and potential or trend growth over the forecast horizon. The trend growth rate is simply an HP-filtered version of the 6-10 year real GDP growth forecast from the Consensus survey.

Table 7

Bayesian estimates

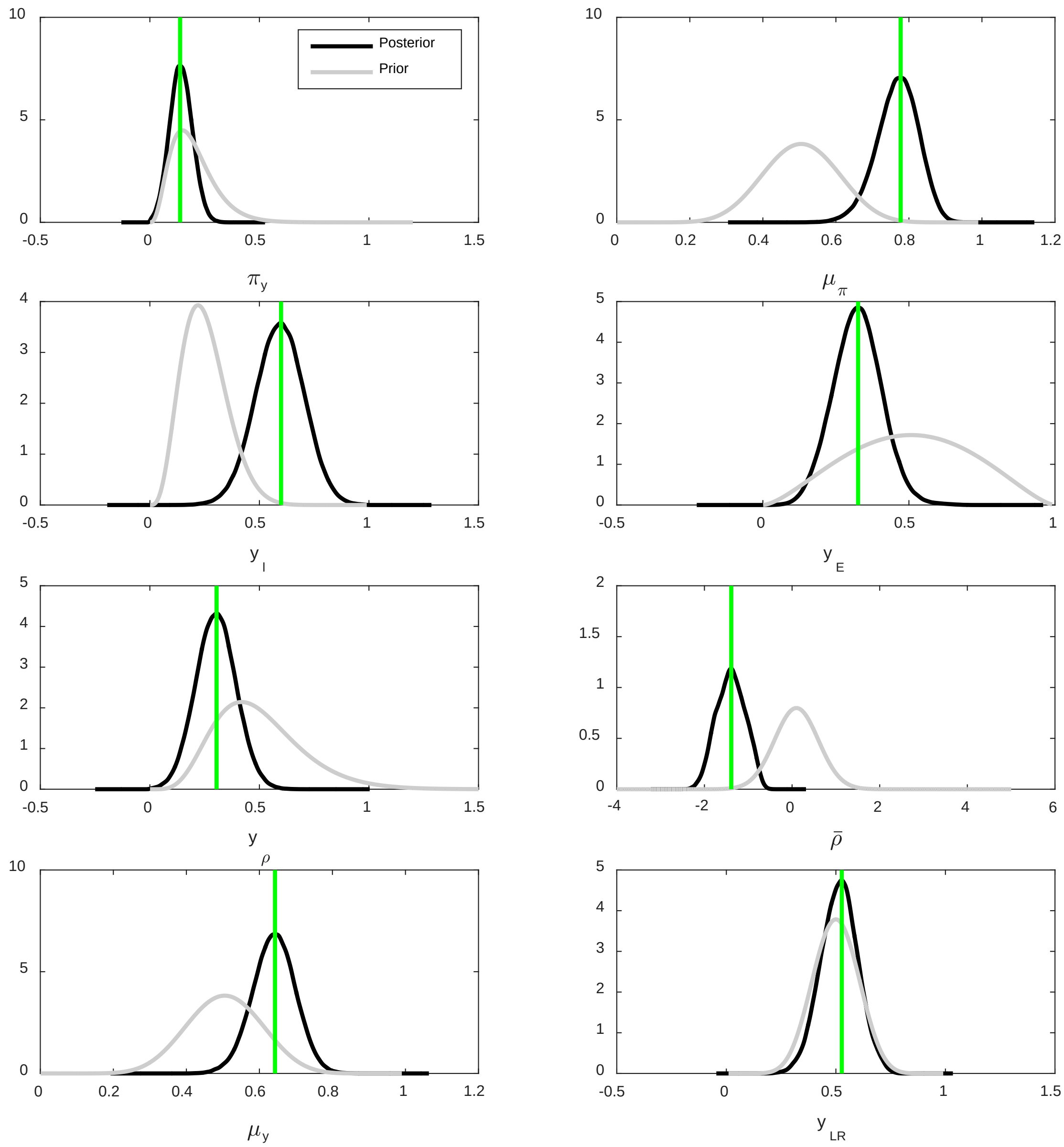
Parameter	Posterior mode	Mean	Median	Standard Deviation	*t*-statistic	5th percentile	95th percentile
π_y	0.14	0.14	0.14	0.051	2.8	0.059	0.23
μ_π	0.78	0.77	0.77	0.057	14	0.67	0.86
y_l	0.59	0.59	0.59	0.11	5.3	0.41	0.77
y_E	0.33	0.32	0.32	0.083	3.9	0.19	0.46
y_ρ	0.3	0.3	0.3	0.092	3.3	0.15	0.45
$\bar{\rho}$	-1.5	-1.4	-1.4	0.32	-4.5	-1.9	-0.87
μ_y	0.64	0.64	0.64	0.058	11	0.54	0.73
y_{LR}	0.52	0.51	0.52	0.084	6.1	0.37	0.65

4 blocks, 65,000 replications per block

Figure 15

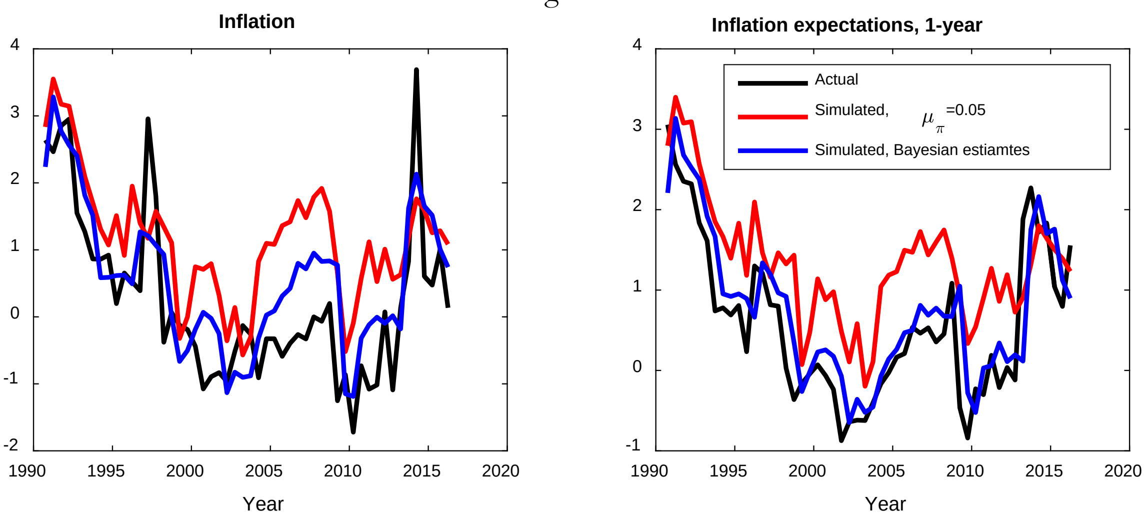
Posterior parameter distributions

260000 replications



By these diagnostics, the model performs quite well. An in-sample simulation of the model verifies that the equations are capturing most of the significant fluctuations in these series over the past thirty-five years. The simulation imposes the zero lower bound constraint for the nominal policy rate. The figure shows the results of these in-sample simulations for inflation and inflation expectations, along with the simulation when the error-correction parameter for inflation expectations is set close to zero. That figure emphasizes the point made in Fuhrer (2017): The intrinsic inertia in inflation expectations is important for explaining the dynamics of inflation and inflation expectations. In Japan, the lowest readings for inflation are best explained as the result of intrinsic persistence in short-run inflation expectations in response to chronically negative output gaps.

Figure 16



Note that at these estimates, the specification for inflation does not require either indexing or autocorrelated structural shocks to match the data reasonably well. The first autocorrelation of the estimated shock processes is 0.089, suggesting little in the way of omitted and highly-correlated structural shocks.

This could suggest that the need to include lagged inflation (via an indexation argument) is obviated, although a more formal test is required to confirm this. Re-estimating the model above, but adding lagged inflation to the Phillips curve equation

$$\pi_t = \omega\pi_{t-1} + (\beta - \omega)\pi_{t+1}^S + \pi_y \tilde{y}_t^{Cab}$$

allows us to test the importance of lagged inflation, once the inertial survey expectations are included. We set the prior for ω to a gamma distribution with mean of 0.1 and standard deviation of 0.05, consistent with the lack of autocorrelation in the estimated structural shocks in the baseline model. The posterior is maximized at a value of ω of 0.10 with an estimated standard deviation of 0.063. The 90% confidence interval for ω extends from 0.032 to 0.24. Other parameter estimates are qualitatively unchanged from those presented above. The data do not suggest moving the estimate from its prior. Put differently, there is no clear role for lagged inflation in this model once survey expectations are incorporated.

<u>Modeling long-run inflation expectations</u>

The model described above includes long-run expectations, which are important as a long-run attractor for short-run expectations (and thus ultimately for realized inflation). But the model simply posits an exogenous AR(1) process for long-run expectations, assuming that they revert ultimately to the central bank's inflation goal. This is not a fully satisfactory description of long-run expectations, particularly not if they are central to determining the steady-state value for inflation. In fact, the model has similar stability properties to the model discussed in section 2 above—because long-run expectations always return to the central bank's goal, the central bank need only return the output gap to zero to stabilize inflation.

One way to model long-run expectations (see, e.g. Clark 2010) is to assume that they ultimately revert to the central bank inflation goal, but in the short-run are influenced by persistent fluctuations in short-run inflation that differ from the central bank target. A simple way to represent this is

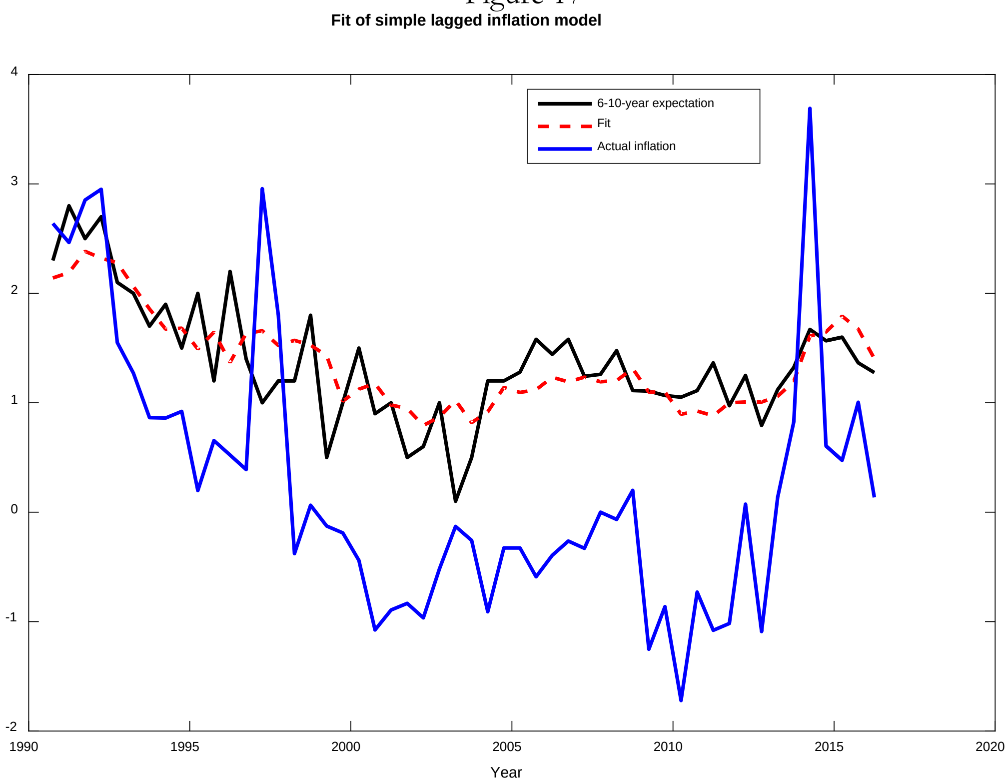
$$\Pi_t^{LR} = a\Pi_{t-1}^{LR} + \sum_{k=0}^{K} b_k \pi_{t-k} + c \tag{5.2}$$

An estimated version of this equation on Japanese data, using Consensus 6-10 year forecasts as proxies for the long-run expectations, yields (p-values in parentheses)

$$\Pi_t^{LR} = \underset{(0.059)}{0.27}\ \Pi_{t-1}^{LR} + \underset{(0.007)}{0.27}\ \Sigma\pi_{t-k} + \underset{(0.000)}{0.91}$$

And the in-sample fit of this equation, displayed below, is quite good. The implied long-run value for the long-run expectation is 1.25, with an asymptotic standard error of 0.072.

Figure 17



This model captures in a simple way the possibility that long-run expectations can be dragged away from their anchor, if actual inflation deviates from target for long enough. If we write the model more generally as a well-defined autoregression

$$\Pi_t^{LR} = a\Pi_{t-1}^{LR} + (1-a)\pi_{t-1} + c\,,$$

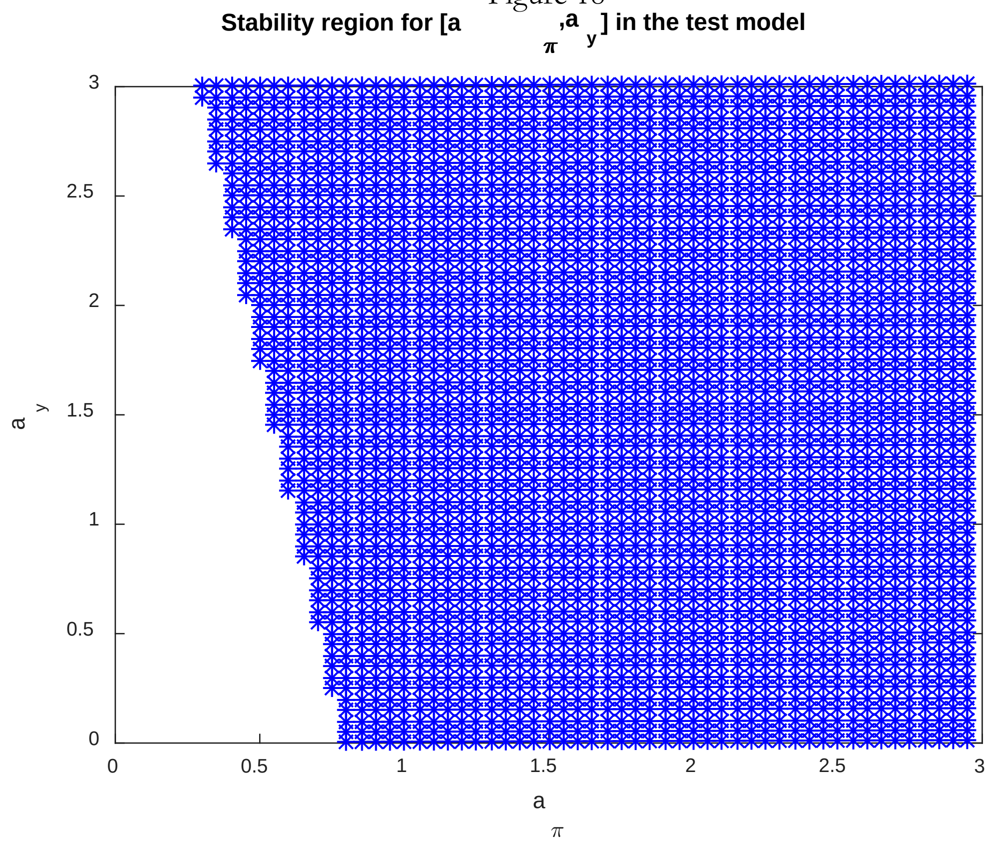
then we can show generically that <u>any</u> coefficient 0<a<1 will yield more sensible stability results for the model. To focus on stability properties associated with long-run inflation expectations, we simplify the model above so that it comprises the following equations

$$\pi_t = \beta\pi^S_{t+1} + \pi_y \tilde{y}^{Cab}_t$$
$$\pi^S_{t+1,t} = \mu_\pi(\beta\Pi^S_{LR,t} + \pi_y \tilde{y}^S_{t+1,t}) + (1-\mu_\pi)\pi^S_{t,t-1}$$
$$\Pi^S_{LR,t} = a\Pi^S_{LR,t-1} + (1-a)\pi_{t-1}$$
$$\tilde{y}^{Cab}_t = b\tilde{y}^{Cab}_{t-1} - y_\rho(\rho_t - \overline{\rho}) \qquad ,$$
$$\rho_t \equiv r_t - \pi^S_{t+1,t}$$
$$r_t = \rho r_{t-1} + (1-\rho)[\overline{r} + a_\pi(\pi^S_{t+1,t} - 1) + a_y \tilde{y}^{Cab}_t]$$

where we simplify the IS curve, and where long-run inflation expectations are now modeled as above, that is as a geometrically weighted average of lagged actual inflation.

The next figure displays the region of stability for this model as we vary the policy response to inflation and output from 0 to 3, holding the lag coefficient *a* from at 0.25, conditioned on a value of $a_y = 0.5, \mu_\pi = 0.25, \pi_y = 0.12, y_\rho = 0.4$, as estimated above.[15] Note that the simple version of the "Taylor principle" does not hold in this model, although clearly for modest values of a_y, the central bank must respond to inflation, unlike the simplest anchored-expectations model.

Figure 18



But this is still a fairly mechanical representation of long-run expectations. An alternative method that captures the spirit of standard models makes long-run expectations a function of the

[15] The value of *a* has no effect on stability, as long as it is strictly greater than zero.

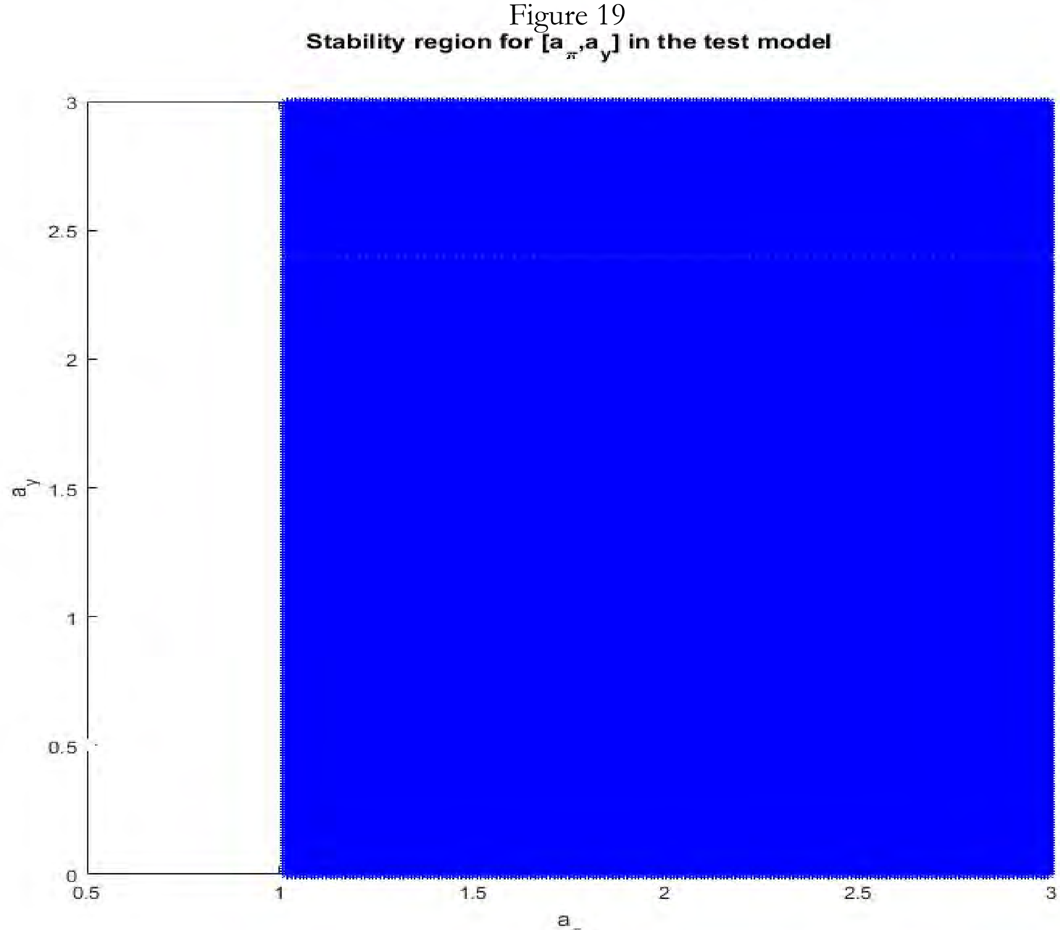
unemployment gap: One can think of this as the implication of a short-run expectations relationship between inflation and expected unemployment. If inflation is high, the central bank raises the policy rate, which raises unemployment, which lowers current and expected (short- and long-run) inflation. This stabilizes the model in exactly the conventional way.

We augment the theory-model above consistent with this notion:

$$\Pi^{S}_{LR,t} = a\Pi^{S}_{LR,t-1} + (1-a)\bar{\pi} + b\tilde{y}^{Cab}_{t}$$

where $\bar{\pi}$ is the inflation goal, which is assumed equal to one. This equation for long-run expectations makes expectations a long moving average of the output gap. Below, we examine the stability properties for values of a_{π}, a_{y}. For this model, we obtain the Taylor Principle exactly:

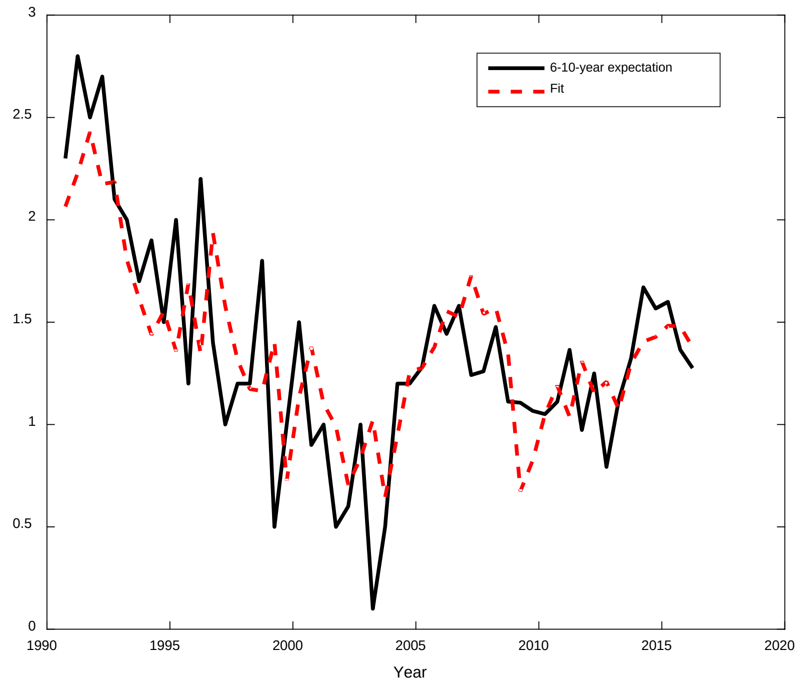
Figure 19



The next figure displays the least-squares fit of this equation for long-run inflation expectations over the past 25 years on Japanese data:

Figure 20

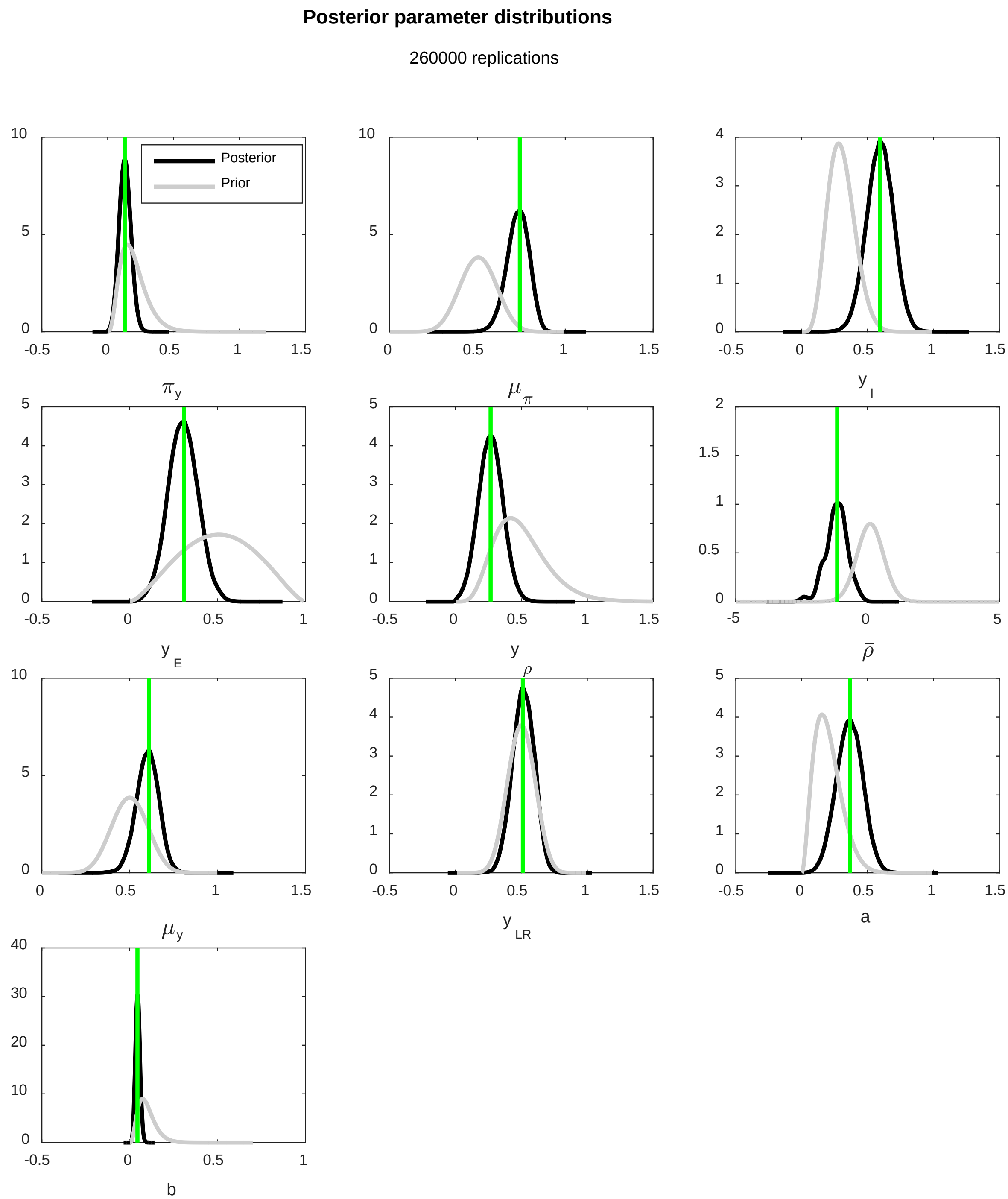
Fit of simple output gap model for long-run expectations



We now estimate the full model with feedback from the output gap to long-run expectations. The results are presented in table 8 and figure 21.

Table 8 **Bayesian estimates**							
Parameter	Posterior mode	Mean	Median	Standard Deviation	*t*-statistic	5^{th} percentile	95^{th} percentile
π_y	0.13	0.13	0.13	0.045	2.9	0.06	0.21
μ_π	0.74	0.73	0.73	0.063	12	0.62	0.83
y_l	0.59	0.59	0.59	0.1	5.9	0.42	0.76
y_E	0.31	0.3	0.3	0.085	3.6	0.16	0.44
y_ρ	0.27	0.27	0.27	0.092	3	0.11	0.42
$\overline{\rho}$	-1.3	-1.2	-1.2	0.42	-3.1	-1.9	-0.52
μ_y	0.6	0.6	0.6	0.064	9.5	0.49	0.7
y_{LR}	0.53	0.51	0.51	0.082	6.4	0.37	0.65
a	0.38	0.36	0.36	0.099	3.8	0.2	0.52
b	0.045	0.046	0.045	0.013	3.5	0.025	0.068
4 blocks, 65,000 replications per block							

Figure 21



As one can see from the estimated parameters and their distributions, the addition of this feedback leaves the other estimated parameter values essentially as they were in the baseline. But now the model has somewhat more desirable stability properties—more specifically, the model incorporates an explicit link from monetary policy to long-run expectations, via its effects on the output gap, so that only if policy devotes sufficient attention to inflation will the economy converge to the central bank's long-run inflation goal.

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The zero lower bound and survey expectations

In rational expectations models, failure to incorporate the zero lower bound in estimation can seriously bias parameter estimates. The logic is straightforward: If the policy rule is estimated over a sample during which the policy rule is binding for part of the period, then the estimated coefficient can imply that the policy rule will take on negative values. While policy rates in some other countries have been set to negative values, rates in Japan over this period were held at or above zero.

This section examines the extent to which the zero lower bound may confound estimates of the model presented thus far. Recall that the model was estimated imposing a policy rule that was estimated on the sample prior to the effective lower bound period that started in 1999. But given low inflation, that rule might imply a policy rate well below zero for much of the sample.

$$r_t = \rho r_{t-1} + (1-\rho)[\overline{r} + a_\pi(\pi^S_{t+1,t} - \overline{\pi})]$$

We look at the model's implication for the policy rate under two policy rules. The first is the one imposed in the sections above. The second is a rule that is estimated over the full sample, including the effective lower bound period. The results of that estimation are presented below. The parameters of the model do not change much, with the exception of the equilibrium real interest rate. While the 90 percent confidence interval for the real rate is still strictly negative, the modal estimate is -0.88, versus -1.4 or lower in the previous estimates. Note that this estimate employs information from both the policy rule and the real side of the model economy ($\overline{\rho}$ enters in the output and output expectations equations as well as the equilibrium level of the policy rate in the policy rule).

The policy rule parameters imply a moderate response to inflation ($a_\pi = 1.2$), a significant degree of interest rate smoothing ($\rho = 0.82$), and an inflation target of about one-half percent ($\overline{\pi} = 0.58$). All are precisely estimated.

Table 9

Bayesian estimates

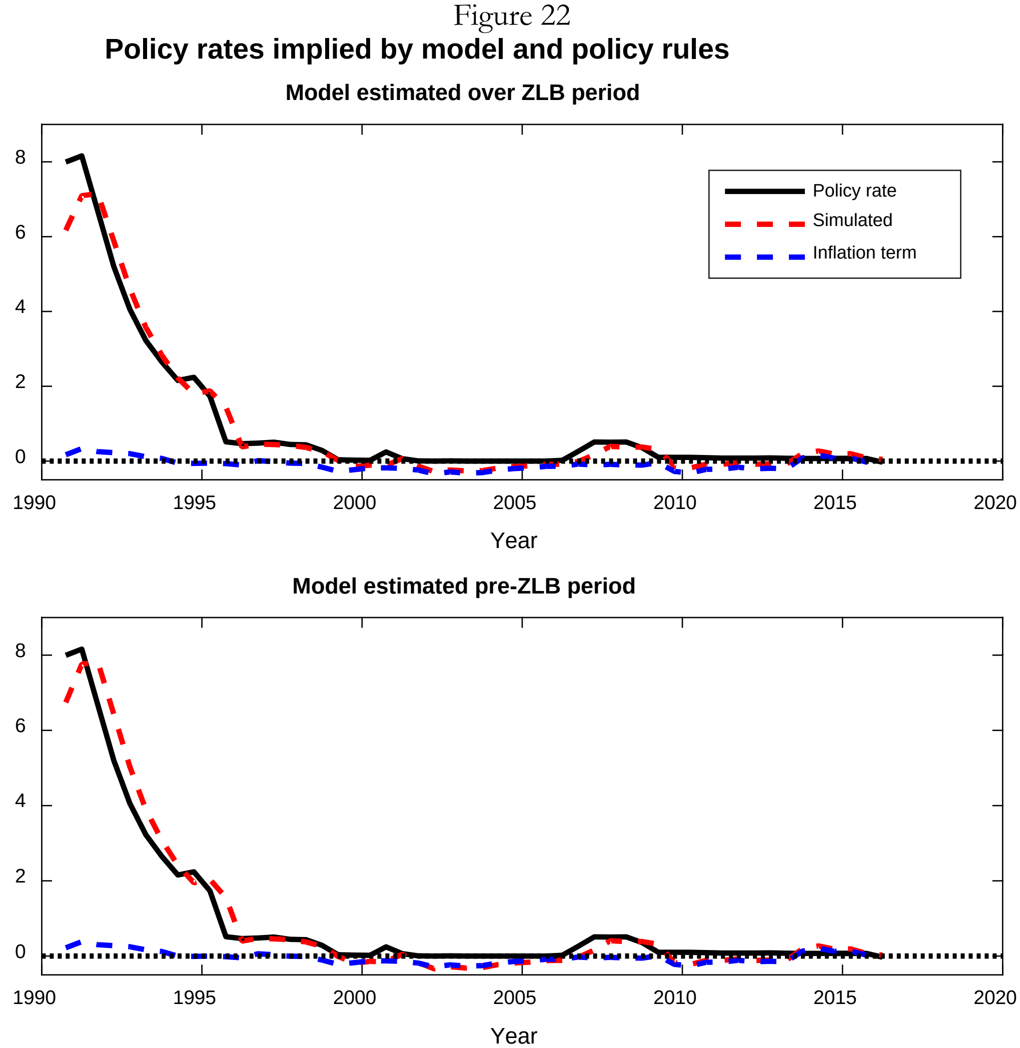
Parameter	Posterior mode	Mean	Median	Standard Deviation	*t*-statistic	5th percentile	95th percentile
π_y	0.14	0.15	0.15	0.053	2.7	0.059	0.23
μ_π	0.77	0.75	0.75	0.062	12	0.64	0.84
y_l	0.64	0.63	0.63	0.099	6.5	0.46	0.79
y_E	0.26	0.27	0.26	0.079	3.3	0.14	0.4
y_ρ	0.32	0.32	0.32	0.089	3.6	0.17	0.47
$\overline{\rho}$	-0.95	-0.95	-0.97	0.32	-2.9	-1.5	-0.41
μ_y	0.6	0.59	0.59	0.062	9.7	0.49	0.69
y_{LR}	0.5	0.49	0.49	0.077	6.4	0.36	0.62
ρ	0.84	0.83	0.83	0.044	19	0.76	0.9
a_π	1.1	1.1	1	0.14	8	0.85	1.3
$\overline{\pi}$	1.3	1.3	1.3	0.12	11	1.1	1.4

a	0.34	0.34	0.34	0.098	3.4	0.18	0.5
b	0.05	0.051	0.051	0.013	3.8	0.03	0.073
4 blocks, 65,000 replications per block							

We then simulate the model over the estimation sample period, taking the long-run expectation as exogenous, performing a period-by-period (static) simulation. The question is whether given the model parameters and data, the model implies significantly negative policy rates during the sample. If so, we will need to re-estimate the model to account for not taking the zero lower bound into account.

As the first figure shows, both policy rules fit the actual policy rate quite well. And the periods of implied sub-zero policy rates are relatively modest in magnitude. The dashed blue line shows the effect of the inflation gap on the policy rate according to the rule, that is, the simulated value of $(1\text{-}\rho)\mathrm{a}_{\pi}(\pi^{S}_{t+1,t}\text{-}\overline{\pi})$. When the policy rate is approximately zero, this is the only term that affects the policy rate, as the interest rate smoothing term ρr_{t-1} is approximately zero.

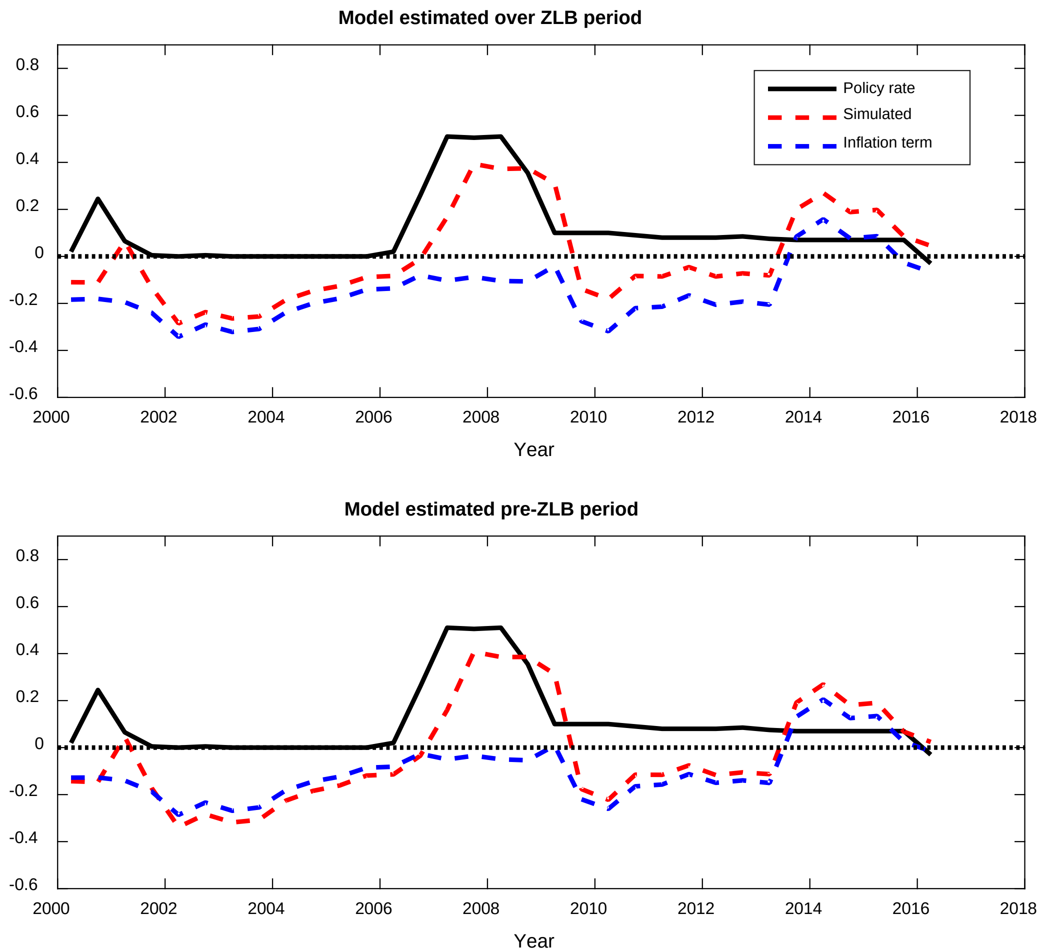
Figure 22



In order to examine more closely these periods, the next figure displays the same simulations, focusing on the period from 2000 forward.

Figure 23

Policy rates implied by model and policy rules



As the figure indicates, the implied policy rates dip slightly negative—from -0.1 to -0.3 percentage points for either of the policy rules. These are within the range of negative rates that the Bank of Japan subsequently and other central banks earlier put into place. In addition, these small deviations below zero are unlikely to have significant effects on the estimates of other model parameters presented above. Of course, interest rate smoothing helps in this regard, as the period-by-period coefficient on the inflation gap is premultiplied by one minus the smoothing coefficient, which is about 0.7 or 0.9 for the two policy rules. Based on these results, one might not expect the imposition of the zero lower bound to have much impact on the estimated parameters. Still, because the constraint implied by the zero lower bound (or near-zero lower bound) played such a prominent role in many developed economies in the post-Great Recession period, we will next consider a method that imposes the zero lower bound in estimation.

<u>Estimating the model with a zero lower bound constraint</u>

Here we estimate the full model that includes feedback from the output gap into long-run inflation expectations, and simultaneously estimated policy parameters, while imposing the zero lower bound. We do so in a way that is similar to the methods of Erceg and Lindé (2014) and Hirose and Inoue (2014). Specifically, we impose the zero lower bound in the filter that maps parameters and data into estimated shocks for the model above.[16] In this way we impose the constraint on the policy rule

$$\begin{aligned} r_t^* &= \rho r_{t-1} + (1-\rho)[\bar{r} + a_\pi(\pi_{t+1,t}^S - \bar{\pi})] \\ r_t &= \max(r_t^*, 0) \end{aligned} \tag{5.3}$$

The results for estimation of the "full" model are presented below. As the first table shows, the parameters for the model are little changed, apart from the policy rule parameters. As the second table shows, imposing the zero lower bound and assuming a constant policy rule from 1990-2016 implies a larger inflation response a_π (1.4 versus 1.1 in the unconstrained model), a lower equilibrium real rate $\bar{\rho}$ (-1.1 versus -0.95), but about the same degree of interest rate smoothing at 0.83 versus 0.84. All other parameters lie within a fraction of a standard deviation of the unconstrained parameter estimates.

Table 10
Bayesian estimates, 1990:H1-2016:H1

Parameter	Posterior mode	Mean	Median	Standard Deviation	*t*-statistic	5th percentile	95th percentile
π_y	0.16	0.16	0.16	0.056	2.8	0.07	0.25
μ_π	0.77	0.76	0.76	0.059	13	0.66	0.85
y_l	0.63	0.58	0.58	0.11	5.6	0.4	0.76
y_E	0.27	0.31	0.31	0.088	3	0.17	0.46
y_ρ	0.32	0.33	0.33	0.088	3.6	0.18	0.47
$\bar{\rho}$	-1.1	-1.3	-1.3	0.34	-3.3	-1.9	-0.82
μ_y	0.59	0.59	0.59	0.062	9.7	0.49	0.69
y_{LR}	0.48	0.48	0.48	0.083	5.8	0.35	0.62
ρ	0.83	0.83	0.83	0.034	25	0.77	0.88
a_π	1.4	1.4	1.4	0.28	5	0.98	1.9
$\bar{\pi}$	1.3	1.2	1.2	0.11	11	1	1.4
a	0.35	0.35	0.35	0.099	3.6	0.18	0.5
b	0.049	0.049	0.049	0.014	3.6	0.027	0.072
4 blocks, 65,000 replications per block							

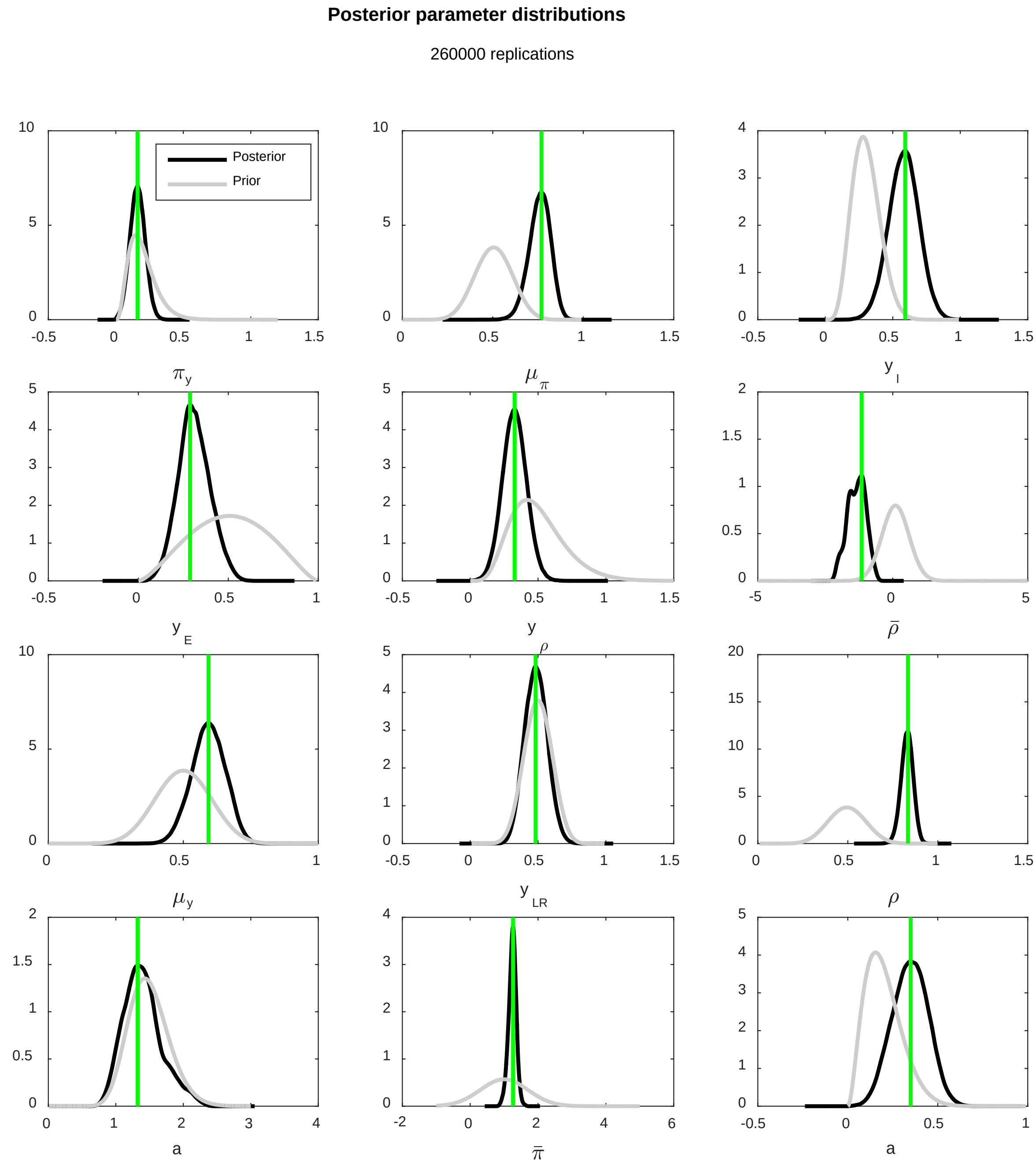
[16] Appendix B provides the details for computing the model solution and residuals under the zero lower bound constraint. The filter employed in my software is a specialized version of the Kalman filter that is optimized for linear rational expectations models. In this implementation, I check for each set of parameter values and each observation whether the implied solution for the policy rate is less than zero. If it is, I impose that the policy rate be zero by resetting the appropriate filter parameters, and then resolve for the set of residuals that are consistent with that constraint. This method allows a simple alteration of the linear Bayesian software that is used to estimate and simulate the models in this paper.

Table 11

Compare estimates with and without imposition of ZLB

Parameter	Posterior mode		Standard Deviation		*t*-statistic	
	with ZLB	without	with ZLB	without	with ZLB	without
π_y	0.16	0.14	0.056	0.053	2.8	2.7
μ_π	0.77	0.77	0.059	0.062	13	12
y_l	0.63	0.64	0.11	0.099	5.6	6.5
y_E	0.27	0.26	0.088	0.079	3	3.3
y_ρ	0.32	0.32	0.088	0.089	3.6	3.6
$\overline{\rho}$	-1.1	-0.95	0.34	0.32	-3.3	-2.9
μ_y	0.59	0.6	0.062	0.062	9.7	9.7
y_{LR}	0.48	0.5	0.083	0.077	5.8	6.4
ρ	0.83	0.84	0.034	0.044	25	19
a_π	1.4	1.1	0.28	0.14	5	8
$\overline{\pi}$	1.3	1.3	0.11	0.12	11	11
a	0.35	0.34	0.099	0.098	3.6	3.4
b	0.049	0.05	0.014	0.013	3.6	3.8

Figure 24



Adding an output response to the policy rule

While price stability is the primary goal of the Bank of Japan, there have no doubt been times when the BOJ has devoted some attention to output. While much of the period has featured both output and inflation below their desired levels, there have been times when they have moved (at least temporarily) in opposite directions. This section estimates a policy rule, imposing the zero lower bound, that allows a response to the one-year-ahead forecasts of both inflation and output, or:

$$r_t = \rho r_{t-1} + (1-\rho)[\bar{r} + a_\pi(\pi^S_{t+1,t} - \bar{\pi}) + a_y \tilde{y}^S_{t+1,t}]$$

The prior distribution for the additional parameter a_y is shown in the table in Appendix A. The results of the estimation are summarized in the following table.

Table 12 **Bayesian estimates, 1990:H1-2016:H1** **Zero lower bound imposed**							
Parameter	Posterior mode	Mean	Median	Standard Deviation	*t*-statistic	5th percentile	95th percentile
π_y	0.14	0.15	0.15	0.054	2.6	0.066	0.24
μ_π	0.78	0.76	0.77	0.056	14	0.67	0.85
y_l	0.59	0.57	0.57	0.086	6.9	0.42	0.71
y_E	0.3	0.34	0.34	0.09	3.4	0.19	0.49
y_ρ	0.31	0.33	0.33	0.069	4.5	0.22	0.45
$\bar{\rho}$	-1.4	-1.5	-1.4	0.27	-5.3	-1.9	-1
μ_y	0.61	0.61	0.61	0.058	11	0.51	0.7
y_{LR}	0.51	0.5	0.5	0.082	6.1	0.37	0.65
ρ	0.75	0.74	0.74	0.059	13	0.64	0.83
a_π	1.2	1.2	1.2	0.22	5.3	0.81	1.5
a_y	1.3	1.2	1.2	0.11	12	1	1.4
$\bar{\pi}$	0.65	0.66	0.68	0.16	4.1	0.33	0.88
a	0.34	0.38	0.38	0.095	3.6	0.23	0.54
b	0.05	0.046	0.046	0.012	4.2	0.027	0.066
4 blocks, 65,000 replications per block							

The estimates suggest a significant role for both inflation and output gap forecasts in the determination of the policy rate. However, the inclusion of the output response does not alter the conclusions about inflation dynamics in Japan during this period. There is still a prominent role for expectations inertia, and a strong link from short-run to long-run expectations.

Balance Sheet effects

Missing from the analysis so far is the effect of the Bank of Japan's significant actions to expand its balance sheet. Using the ratio of total BOJ assets to nominal GNP as a proxy for balance sheet actions, I have estimated their impact on inflation and inflation expectations, augmenting the model above with an additional linear term in the change in the share of BOJ assets to GNP. Using this measure, the BOJ expanded its balance sheet beginning in the late 1990s, rising as a share of GDP from about 10% in 1997 to just over 80% by 2016.

The direct effect on realized inflation appears to be tiny and statistically insignificant.[17] The effect on inflation expectations, which in the model affects realized inflation one-for-one, appears to

[17] This conclusion is the same for both single-equation OLS regressions, and for system estimates in which this effect is included into the model described up to this point.

be more significant. A least-squares regression of one-year inflation expectations on the key variables in the model above, augmented to include the change in the GNP share of BOJ assets yields:

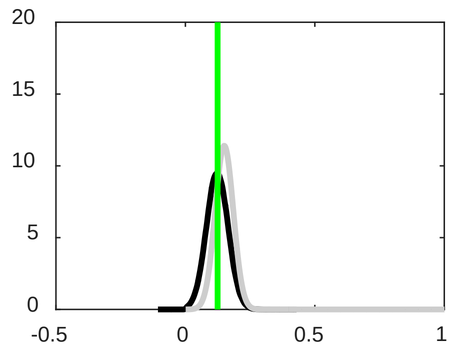
$$\pi^S_{t+1,t} = 0.77\Pi^S_{LR,t} + 0.18\tilde{y}^S_{t+1,t} + 0.11\Delta[\frac{A_{t-1}}{Y_{t-1}}]$$

All of the estimated coefficients have *p*-values of 0.001 or smaller. Thus the estimated effect of balance sheet expansion on inflation expectations is economically and statistically significant. We include this effect into the full model by augmenting the inflation expectations equation as follows:

$$\pi^S_{t+1,t} = (1-\mu_\pi)(\beta\Pi^S_{LR,t} + \pi_y\tilde{y}^S_{t+1,t} + \pi_{BS}\Delta[\frac{A_{t-1}}{Y_{t-1}}]) + \mu_\pi\pi^S_{t,t-1} \ .$$

This yields a Bayesian posterior estimate for the coefficient on the BOJ assets variable of 0.091, with an estimated standard deviation for the posterior distribution of 0.041; the 90% confidence interval ranges from 0.055 to 0.19. Other estimated parameters are not much affected by the inclusion of this variable. The prior and estimated posterior distributions for π_{BS} are displayed below.[18]

Figure 25



π_{BS}

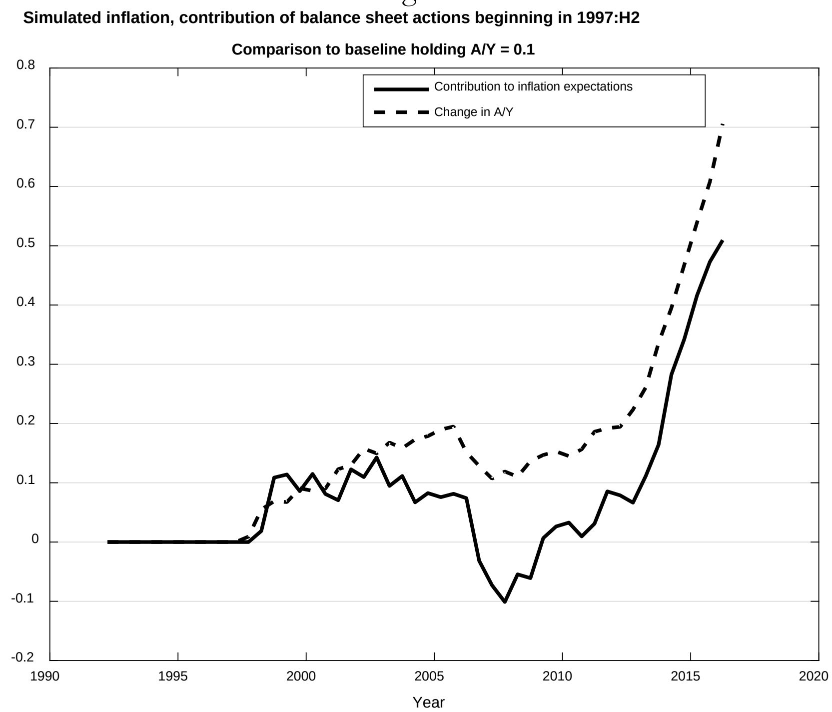
Quantiative importance of balance sheet actions

With the estimated model, we can examine the quantitative impact of the BOJ's balance sheet actions on inflation over time. We compare a baseline simulation of the model that assumes the actual balance sheet trajectory (the realized $\frac{A_t}{Y_t}$ ratio) with an alternative in which the ratio is held fixed at its level in the first half of 1997. Given the coefficient estimated above, the increase in the balance sheet is estimated to have raised short-term inflation expectations by a bit more than one-half percentage point over the period since 1997. This passes directly into higher inflation, according to the estimated model.[19]

[18] The priors for this parameter are somewhat informed by the point estimate and standard error from the OLS estimates above.

[19] This estimated effect is smaller than the coefficient π_{BS} times the increase in the balance sheet-to-GDP ratio, as it is mediated by the error-correction parameter $(1-\mu_\pi)$, and the effect on inflation expectations is also smoothed by the effect of lagged inflation expectations on current inflation expectations.

Figure 26



6. Conclusions

It would be a bit of an over-reach to claim that the preliminary exercises in this paper have explained the inflation puzzles in the U.S. and Japan. However, the results presented herein suggest some promising directions for inflation modeling in both countries:

1. The use of survey expectations as proxies for the expectations in conventional models appears to be of empirical value. That is, surveys aid in identification of the inflation process in both countries;
2. Methods for endogenizing such expectations, which are necessary once one moves away from the rational expectations assumption, are tractable and replicable. They require the measurement of longer-term expectations, but such data are available for many key variables in many developed economies;[20]
3. Models that incorporate such expectations identify the following rationale for the behavior of US and Japanese inflation:
    a. Long-run expectations anchor the process, although long-run expectations can be influenced over time by persistent deviations of inflation (or output) from their long-run equilibria;
    b. Short-run expectations are tied to their long-run counterparts, but they can deviate quite persistently from long-run expectations, due to persistent deviations of output from potential, and due to intrinsic persistence in the expectations;

[20] A separate but important issue is whose expectations are measured in the available surveys. To be sure, a survey of important price-setters, rather than professional forecasters, would likely be preferred. Such a dataset is not available for the U.S. or Japan at present, to my knowledge.

   c. In this regard, while both the U.S. and Japan evince a linkage ("anchoring") from short-run to long-run expectations, the size and duration of deviations between short- and long-run expectations is larger in Japan than in the U.S.
   d. Inflation appears well-explained by short-run expectations and a traditional output gap;
4. The balance sheet actions in Japan appear to have boosted short-run inflation expectations, compared to where they would have been without such actions. Higher expectations in turn have helped to raise realized inflation.

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Appendix A
Prior Distributions

Parameter	Distribution	Prior Mean	Prior Standard Deviation	Support	
				Lower	Upper
π_y	Gamma	0.2	0.1	0.001	1.2
μ_π	Beta	0.5	0.1	0.001	0.99
y_l	Beta	0.3	0.1	0.001	0.99

y_E	Beta	0.5	0.2	0.001	0.99
y_ρ	Gamma	0.5	0.2	0.001	1.5
$\bar{\rho}$	Normal	0.1	0.5	-5	5
μ_y	Beta	0.5	0.1	0.001	1
y_{LR}	Beta	0.5	0.1	0.01	0.99
ρ	Beta	0.5	0.1	0.01	1
a_π	Gamma	1.5	0.3	0.01	3
a_y	Gamma	0.5	0.3	0.01	1.5
$\bar{\pi}$	Normal	1	0.7	-1	5
a	Beta	0.2	0.1	0.001	0.99
b	Gamma	0.1	0.05	0.001	0.7
Shock variances	Diffuse	-	-	-	-

Appendix B
Imposing the zero lower bound in estimation

Each of the models in this paper may be cast in the form

$$\sum_{i=-\tau}^{0} H_i x_{t+i} + \sum_{i=1}^{\theta} H_i E_t(x_{t+i}) = \epsilon_t, \tag{6.1}$$

where τ and θ are positive integers, x_t is a vector of variables, and the H_i are conformable *n*-square coefficient matrices, where n is the number of endogenous variables in the model. The coefficient matrices H_i are completely determined by a set of underlying structural parameters Θ. The expectation operator $E_t(\cdot)$ denotes mathematical expectation conditioned on the process history through period *t* (or *t-1)*,

$$E_t(x_{t+i}) = E(x_{t+i} \mid x_t, x_{t-1}, \ldots). \tag{6.2}$$

The model may include expectational and accounting identities; the former are important because they define variables that can only be observed within the context of the model. Because ϵ_t is white noise, $E_t(\epsilon_{t+k}) = 0, \forall k > 0.$ Leading equation (6.1) by one or more periods and taking expectations conditioned on period-t information yields a deterministic forward-looking equation in expectations,

$$\sum_{i=-\tau}^{\theta} H_i E_t(x_{t+k+i}) = 0, \qquad k > 0. \tag{6.3}$$

We use the AIM procedure detailed above to solve equation (6.3) for expectations of the future in terms of expectations of the present and the past. For a given set of initial conditions, $\{E_t(x_{t+k+i}) : k > 0, i = -\tau, \ldots, -1\}$, if equation (6.3) has a unique solution that grows no faster than a given upper bound, that procedure computes the vector autoregressive representation of the solution path,

$$E_t(x_{t+k}) = \sum_{i=-\tau}^{-1} B_i E_t(x_{t+k+i}), \qquad k > 0. \tag{6.4}$$

In the models we consider here, the roots of equation (6.4) lie on or inside the unit circle.

Using the fact that $E_t(x_{t-k}) = x_{t-k}$ for $k \geq 0$, equation (6.4) is used to derive expectations of the future in terms of the realization of the present and the past. These expectations are then substituted into equation (6.1)to derive a representation of the model that we call the observable structure,

$$\sum_{i=-\tau}^{0} S_i x_{t+i} = \epsilon_t . \tag{6.5}$$

Equation (6.5) is a structural representation of the model because it is driven by the structural disturbance, ϵ_t; the coefficient matrix S_0 contains the contemporaneous relationships among the elements of x_t. It is an observable representation of the model because it does not contain unobservable expectations.

Computing the Model Shocks

Consider concatenating the *n* by *n* coefficient matrices S_i, ordered left to right from $i = -\tau$ to 0. We denote this *n* by $n \times (\tau + 1)$ matrix SCOF. Define the vector stack of the endogenous variables at time *t* as $X_t = [x_{t-\tau}, \dots, x_t]'$. Thus equation (6.5) may be rewritten

$$\text{SCOF } X_t = \epsilon_t \tag{6.6}$$

In computing the residuals, it will be useful to partition SCOF as follows. Denote stochastic equations by the subscript *s*, identity equations by the subscript *i*, and denote data variables with the subscript *d* and "not-data"' variables (such as the unobserved *ex ante* long real rate) with the subscript *n*. Arbitrarily ordering the observable structure so that stochastic equations appear in the top rows and data variables in the left columns of each block, we can write equation (6.6) as

$$\begin{bmatrix} S_{s,t-1} & S_{s,d} \; S_{s,n} \\ S_{i,t-1} & S_{i,d} \; S_{i,n} \end{bmatrix} \begin{bmatrix} X_{t-1} \\ \\ X_{d,t} \\ X_{n,t} \end{bmatrix} = \begin{bmatrix} \epsilon_t \\ 0 \end{bmatrix} \tag{6.7}$$

$S_{s,t-1}$ denotes the coefficient block of SCOF for the lagged variables that enter the stochastic equations; $S_{i,t-1}$ is the corresponding block for identity equations. The right-hand-most *n* by *n* block of equations, representing the coefficients on contemporaneous variables, is further partitioned vertically into its data and not-data components. For each observation *t,* we use this concatenated, partitioned version of the observable structure to solve for the residuals ϵ_t. First, solve for the period-*t* not-data variables as

$$X_{n,t} = -S_{i,n}^{-1}[S_{i,t-1}X_{t-1} + S_{i,d}X_{d,t}] \tag{6.8}$$

Now substitute the solution for $X_{n,t}$ into the top rows of equation (6.7) to solve for ϵ_t:

$$\epsilon_t = S_{s,t-1}X_{t-1} + S_{s,d}X_{d,t} - S_{s,n}S_{i,n}^{-1}[S_{i,t-1}X_{t-1} + S_{i,d}X_{d,t}] \tag{6.9}$$

Equation (6.9) corresponds to the filter equation (it is a specific filter equation for the problem outlined in this appendix) used in standard implementations of likelihood-based and Bayesian estimation. The residuals for each time period $t = 1, \dots, T$ are computed.

One may also define the non-stochastic version of the system in (6.7) as

$$\begin{bmatrix} S^{-} & S_x^0 & S_n^0 \end{bmatrix} \begin{bmatrix} X_{-1} \\ \\ X_{n,t} \\ X_{x,t} \end{bmatrix} = 0 \quad , \tag{6.10}$$

where the partition distinguishes between exogenous and endogenous variables, and the superscripts "-" and "0" denote all lagged observations and the contemporaneous observation, respectively. This partitioning is useful for solving for any of the endogenous variables in terms of lagged data and exogenous variables.[21] If the model (at current parameter settings and lagged data) implies that the constrained variable should be less than zero, then the zero lower bound constraint would be violated.

When one of the variables in the vector *X* is constrained to be strictly non-negative, as in the case of a policy rate that faces a zero lower bound (or an effective lower bound), we modify the computation of the residuals so as to impose the zero lower bound as follows. First, solve for the residuals in period *t* as above. Now compute the solution values for the data variables in *X* using equation (6.10) as

$$\begin{aligned} \hat{X}_t &= \text{-P\_}(\mathrm{S}_x^0 \mathrm{x}_x + \mathrm{S}^{-}\mathrm{x}_{-}) \\ \text{P\_} &\equiv (S_n^0 S_n^0{}')^{-1} S_n^0{}' \end{aligned} ,$$

which follows directly from the partition in equation (6.10). When the zero-bound-constrained element of $\hat{X}_t$ is less than zero, we impose the zero lower bound on this variable by inserting the equation $X_{t,zlb}^k = 0$ into the *H* matrix of equation (6.1) in place of the original equation for that variable in equation (6.7).[22] We then re-solve for SCOF in (6.6); call this new matrix SCOF_{ZLB}, with partitions exactly as in (6.7). We then re-solve for the residuals conditional on this constraint, using equation (6.9) with the appropriate partitions of the SCOF_{ZLB} matrix. This generates a new set of residuals that are consistent with imposing the zero lower bound on the variable of interest.

Using this method, we compute the log-likelihood and the posterior for the model to estimate parameters that maximize the log posterior. The simulated posterior distributions impose the zero lower bound in precisely the same way.

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[21] Here, we take only the constant variable to be exogenous.

[22] That is, we set the *k*, $\tau n + k^{th}$ element of $H_{s,d} = 1$, and the other elements of the *k*th row of $H_{s,d} = 0$.