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Guiding the Economy Toward the Target Inflation Rate: An Evolutionary Game Theory Approach

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Abstract

Under what condition is the target inflation rate attainable even after the monetary policy rate hits its lower bound? This study examines the question using a dynamic model based on evolutionary game theory. In the model, entrepreneurs and workers iteratively play a stage game to make investment decisions. In the presence of complementarity between entrepreneurs' and workers' investments, two long-run equilibria exist: all players invest or no player invests. The study shows two conditions for successfully guiding the economy toward the long-run equilibrium that all players invest at the target inflation rate. First, the type of entrepreneurs' investments needs to be demand-creating innovation rather than cost-reducing innovation. Second, the proportions of entrepreneurs and workers currently investing must be sufficiently large.

Keywords: Target inflation rate; Evolutionary game; Best-response dynamics; Perfect-foresight dynamics; Multiple long-run equilibria; Capital–skill complementarity; Demand-creating innovation

JEL classification: C72, C73, E31, E52

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1 Introduction

In the wake of the Global Financial Crisis, central banks in many advanced economies moved aggressively in response to deflationary pressures by introducing various unconventional monetary policy measures. In that process, central banks also lowered their policy rates swiftly, but were soon constrained by the effective lower bound of nominal interest rates. Central banks became concerned over the risk of falling into and being trapped in deflation, first raised by Benhabib, Schmitt-Grohé, and Uribe (2001).[1] The authors argue that once the policy rate is stuck at its lower bound, the economy may fall into a long-run equilibrium (or steady state) at a lower inflation rate than its target. They employ a simple model consisting of an interest-rate feedback rule of the sort proposed by Taylor (1993) and the Fisher relation among inflation and the nominal and real rates of interest. They then show that the active rule leads to two long-run equilibria in the presence of a lower bound on the policy rate.[2] One equilibrium is consistent with the inflation target, but the other is not. In the second equilibrium, the policy rate stays at its lower bound and the inflation rate is below its target, possibly negative. Moreover, they demonstrate that an infinite number of equilibrium paths exist starting from regions close to the first equilibrium, then converging to the second equilibrium.

This study revisits the issue initiated by Benhabib, Schmitt-Grohé, and Uribe (2001). In particular, this study examines conditions for achieving the target inflation rate even after hitting the lower bound of policy rates. More precisely, the study investigates what economic circumstances are required to establish credibility of the inflation target and how the economy evolves from its current state to a long-run equilibrium at the target inflation rate. Our study examines the questions by using a dynamic model based on evolutionary game theory starting from Maynard-Smith and Price (1973).[3] The study analyzes transition

[1] St. Louis Fed President Bullard (2013) applies the analytical framework of Benhabib, Schmitt-Grohé, and Uribe (2001) to the U.S. and Japanese data, and discusses the possibility that the U.S. economy may become enmeshed in a Japanese-style deflationary outcome within the next several years.

[2] A Taylor (1993)-type monetary policy rule is called active if it raises the policy rate by more than the increase in inflation. Otherwise, it is called passive.

[3] Another modeling approach is to use a global game as in, for example, Carlsson and van Damme (1993), Morris and Shin (1998), and Katagiri (2016). In that game, an equilibrium is determined solely on the basis of signals about future fundamentals regardless of the current state of the economy. Moreover, the selection from multiple equilibria is done without considering transition dynamics of the economy. Our study chooses to employ an evolutionary game because it allows us to take into account the current state and transition dynamics in the analysis of multiple long-run equilibria.

mechanism of the economy on the basis of both the best-response dynamics (Gilboa and Matsui [1991]) and the perfect-foresight dynamics (Matsui and Matsuyama [1995]).[4] To the best of our knowledge, this study is the first to apply evolutionary game theory to the analysis of the macroeconomic issue on multiple long-run equilibria.

The model includes two types of players: entrepreneurs and workers. Each entrepreneur owns a firm and hires a worker to produce a good under monopolistic competition. Entrepreneurs and workers iteratively play a stage game to make investment decisions.[5] As in Levin and Reiss (1988), these investments are a combination of demand-creating and cost-reducing innovations.[6] Moreover, complementarity exists between entrepreneurs' and workers' investments, just like complementarity between physical and human capital investments discussed in Bartel and Lichtenberg (1987) and Blundell *et al.* (1999). This complementarity is in line with the capital–skill complementarity analyzed by Griliches (1969) and Krusell *et al.* (2000). Note, however, that our study ignores the accumulation of physical and human capitals for the sake of analytical simplicity.

As is similar to Acemoglu (1997) and Redding (1996), the presence of the complementarity results in producing two (pure-strategy) long-run equilibria: all players invest or no player invests. In these equilibria the inflation rate is not necessarily consistent with the target. Under certain conditions, however, the inflation rate converges to its target in the equilibrium where all players invest.

This study shows two necessary conditions for achieving a long-run equilibrium at the target inflation rate. The first condition is that entrepreneurs choose investments in demand-creating innovation rather than cost-reducing innovation. If entrepreneurs engage in cost-reducing innovation, the economy is expected to suffer from deflation.[7] Thus, even in the long-run equilibrium where all players invest, no player views the inflation target as credible.

[4]Roughly speaking, the perfect-foresight dynamics corresponds to equilibrium paths under rational expectations in macroeconomics, while the best-response dynamics corresponds to those under adaptive expectations.

[5]Workers' investments can be interpreted in various ways. For example, workers incur costs to acquire general or firm-specific skills or make more effort to increase demand for goods or reduce production costs.

[6]For demand-creating innovation, see, e.g., Jovanovic and Rob (1987), Romer (1986, 1990), Grossman and Helpman (1991), Aghion and Howitt (1992), and Kesteloot and De Bondt (1993). For cost-reducing innovation, see, e.g., Dasgupta and Stiglitz (1980), Reinganum (1983), and Spence (1984). Following the arguments of Capon *et al.* (1992), Athey and Schmutzler (1995), and Gilbert (2006), our model considers both types of innovation.

[7]Cost-reducing innovation leads to fierce competition among firms by decreasing prices of their products to increase their market shares.

The second condition is that, according to the analysis with the best-response dynamics, the proportions of entrepreneurs and workers currently investing are sufficiently large. If only a small part of players invest in the current state of the economy, each player expects its investment counterpart to be less likely to invest and as a consequence, the player's profit increase arising from its investment is presumed to be lower. Therefore, the economy continues to evolve toward the long-run equilibrium in which no player invests, and inflation never occurs. Thus, even if the first condition is met, no player views the inflation target as credible and the target inflation rate is not achievable.

These two conditions are needed to guide the economy successfully toward the long-run equilibrium where all players invest and the inflation rate is at its target. Unless both conditions are satisfied, policies to stimulate investments in demand-creating innovation are called for.

The remainder of this paper is organized as follows. Section 2 presents the model setting. Section 3 analyzes equilibrium strategies of the stage game, while Section 4 investigates the dynamics of the strategies using the evolutionary game theory approach. Section 5 concludes.

2 Model Setting

There are two types of players: entrepreneurs and workers. The population of each type of player is a continuum of mass one. One entrepreneur and one worker are randomly chosen from the population. The entrepreneur owns a firm and hires the worker to produce a good under monopolistic competition. They play the two-period stage game, which is iteratively conducted for an infinite number of continuous rounds indexed by $\tau \in [0, \infty)$.

[Figure 1 here]

The timing of the stage game is summarized in Figure 1. In period 0, the worker chooses its amount of (human capital) investment $h \in \mathbb{R}_+$, while the entrepreneur chooses its amount of (physical capital) investment $k \in \mathbb{R}_+$ and (its firm chooses) output in period 0 and 1, $(x_0, x_1) \in \mathbb{R}^2_+$.[8] Both players make their decisions simultaneously.

[8]It is implicitly assumed that in period 1, the firm can change its current-period output $x_1 \in \mathbb{R}_+$ on the basis of currently available information, although this is not explicitly analyzed in the model.

In period 1, the entrepreneur pays a fixed fraction $\beta \in (0, 1)$ of its firm's total profit to the worker after production. The firm's total real profit $\pi \in \mathbb{R}$ is given by

$$\pi = \frac{p_0 - v_0}{\gamma_0} x_0 + \frac{1}{1+r} \frac{p_1 - v_1}{\gamma_1} x_1,$$

where $(p_0, p_1) \in \mathbb{R}^2_{++}$ are prices of the firm's product in period $t = 0, 1$, $(v_0, v_1) \in \mathbb{R}^2_{+}$ are its variable production costs other than its payments to the worker, $(\gamma_0, \gamma_1) \in \mathbb{R}^2_{++}$ are the price levels in the economy, and $r \in \mathbb{R}$ is the real interest rate. The value of γ_1 is not observable until the stage game ends, and thus each player is assumed to set $\gamma_1 = \tilde{\gamma}_1$, where $\tilde{\gamma}_1$ is calculated from the period-0 price level γ_0 and the target inflation rate announced by the central bank.[9] All players are assumed to be risk neutral.[10] Then, the entrepreneur's payoff is $(1 - \beta)\pi - k$, while the worker's payoff is $\beta\pi - h$.

The demand functions for the good in period $t = 0, 1$ are assumed to take the form

$$x_t = d_t - \epsilon(p_t - \gamma_t),$$

where $d_t \in \mathbb{R}_{++}$ denotes the "fundamental" amount of demand, which is demand for the good when $p_t = \gamma_t$, and $\epsilon \in \mathbb{R}_{++}$ is the elasticity of demand with respect to the relative price $(p_t - \gamma_t)$.

Based on the above setting, the inverse demand functions are given by

$$p_0 = \gamma_0 + \frac{1}{\epsilon}(d_0 - x_0), \tag{1}$$
$$p_1 = \gamma_1 + \frac{1}{\epsilon}(d_1 - x_1). \tag{2}$$

These functions are linearly decomposed into the *price-competition effect* (γ_0, γ_1) and the *goods-property effect* $(d_0/\epsilon, d_1/\epsilon)$. From (2), the expected period-1 price in period 0 is given by

$$E[p_1] = \tilde{\gamma}_1 + \frac{1}{\epsilon}(E[d_1] - x_1). \tag{3}$$

[9] This assumption is related to rational inattention analyzed by, for example, Sims (2003). That is, if information acquisition on future prices is sufficiently costly, players rationally gather no information on their own and rely heavily on the central bank's target inflation rate.

[10] We confirm that qualitatively the same result is obtained even in the case of risk-averse players.

We assume that each player's investment is comprised of demand-creating innovation and cost-reducing innovation.[11] Specifically, the fundamental demand and variable cost in period 0, d_0 and v_0, are constants, whereas those in period 1, d_1 and v_1, are random variables whose distribution depends on the amount of investments k and h, that is, d_1 and v_1 follow the probability density functions $f(d_1|k,h)$ and $g(v_1|k,h)$, respectively.[12]

Further, we assume that when the amount of each type of investment is greater than the respective threshold value $\underline{k} \in \mathbb{R}_+$ or $\underline{h} \in \mathbb{R}_+$, the investment stochastically leads to an increase in the fundamental demand and a reduction in the variable cost. That is, the cumulative density functions $F(d_1|k,h)$ and $G(v_1|k,h)$ are assumed to satisfy

$$\begin{aligned} &F(d_1|k \in [0,\underline{k}), h \in [0,\underline{h})) > F(d_1|k \in [\underline{k},\infty), h \in [0,\underline{h})) \\ &= F(d_1|k \in [0,\underline{k}), h \in [\underline{h},\infty)) > F(d_1|k \in [\underline{k},\infty), h \in [\underline{h},\infty)) \end{aligned}$$

for all d_1, and

$$\begin{aligned} &G(v_1|k \in [0,\underline{k}), h \in [0,\underline{h})) < G(v_1|k \in [\underline{k},\infty), h \in [0,\underline{h})) \\ &= G(v_1|k \in [0,\underline{k}), h \in [\underline{h},\infty)) < G(v_1|k \in [\underline{k},\infty), h \in [\underline{h},\infty)) \end{aligned}$$

[11] Our model implicitly assumes that innovation can be both demand-creating and cost-reducing in different combinations and that entrepreneurs face a continuous interval of innovation types $w \in [0,1]$ from which to choose.

[12] The probability density functions $f(d_1|k,h)$ and $g(v_1|k,h)$ may also depend on the expected inflation rate $\tilde{\gamma}_1/\gamma_0$, because there may exist income effects and intertemporal substitution effects on d_1 and because v_1 may depend on $\tilde{\gamma}_1$. We do not explicitly incorporate these effects into our model for the sake of analytical simplicity. As for $g(v_1|k,h)$, if it depends fully on $\tilde{\gamma}_1/\gamma_0$, non-linearity arises in the calculations presented in the following sections. However, we confirm that this does not detract from our arguments. Regarding $f(d_1|k,h)$, future work should aim to incorporate the effects listed above. Yet note that even when we endogenize the consumption choices of entrepreneurs and workers, the income effect on d_1 is only half of the increase in real income and d_0 also increases as $1/(2+2r)$ (see Appendix A.1). Therefore, the ratio of the increment of d_1 to that of d_0 is always $(1+r) \approx 1$.

for all v_1. In addition, the mean of each distribution is denoted by

$$d_I \equiv E[d_1|k \in [\underline{k}, \infty), h \in [\underline{h}, \infty)],$$
$$d_N \equiv E[d_1|k \in [0, \underline{k}), h \in [0, \underline{h})],$$
$$v_I \equiv E[v_1|k \in [\underline{k}, \infty), h \in [\underline{h}, \infty)],$$
$$v_N \equiv E[v_1|k \in [0, \underline{k}), h \in [0, \underline{h})].$$

The values of d_I and v_I represent, respectively, the expected fundamental demand and variable cost when the player and its counterpart both invest, while d_N and v_N represent, respectively, those when no player invests. Consequently, when only one player invests more than the threshold, the expected fundamental demand and variable cost are denoted respectively by

$$\alpha_d d_I + (1-\alpha_d)d_N = E[d_1|k \in [\underline{k}, \infty), h \in [0, \underline{h})] = E[d_1|k \in [0, \underline{k}), h \in [\underline{h}, \infty)],$$
$$\alpha_v v_I + (1-\alpha_v)v_N = E[v_1|k \in [\underline{k}, \infty), h \in [0, \underline{h})] = E[v_1|k \in [0, \underline{k}), h \in [\underline{h}, \infty)],$$

where α_d and α_v are assumed to satisfy $d_I > \alpha_d d_I + (1-\alpha_d)d_N > d_N$ for $d_I > d_0 \geq d_N$ and $v_N > \alpha_v v_I + (1-\alpha_v)v_N > v_I$ for $v_0 \geq v_N > v_I$. For the sake of analytical simplicity, it is assumed that $\alpha_d = \alpha_v = \alpha$.

A smaller value of α implies a greater degree of complementarity between both players' investments. We assume that $\alpha \in (0, 1/2)$. If only one player invests, the fundamental demand increases little and the variable cost decreases only slightly. For the fundamental demand and the variable cost to change substantially, both players need to invest. In the analysis, d_I and v_I are exogenously given; however, we will discuss each entrepreneur's choice of the type of innovation in Section 3.3.2.

3 Stage Game

This section analyzes equilibrium strategies of the stage game presented in the preceding section.

3.1 Best Responses

3.1.1 Entrepreneur

Each entrepreneur's problem is given by[13]

$$\max_{x_0,x_1,k} \quad (1-\beta)\left(\frac{p_0-v_0}{\gamma_0}x_0+\frac{1}{1+r}\,E\left[\frac{p_1-v_1}{\tilde{\gamma}_1}\right]x_1\right)-k.$$

This problem can be separated into three problems, because the optimal choices of x_0, x_1, and k can be independently determined. The value of x_0 is determined by

$$x_0^* \equiv \arg\max_{x_0} \frac{p_0-v_0}{\gamma_0}x_0 = \arg\max_{x_0}\left(1+\frac{1}{\epsilon\gamma_0}(d_0-x_0)-\frac{v_0}{\gamma_0}\right)x_0 = \frac{1}{2}\left[d_0+\epsilon\left(\gamma_0-v_0\right)\right]. \quad (4)$$

Substituting (4) into the inverse demand function (1) yields

$$p_0^* = \frac{1}{2}\left(\frac{d_0}{\epsilon}+\gamma_0+v_0\right).$$

We assume only symmetric equilibria in which all members of each population (i.e., entrepreneurs and workers) choose the same strategy. Then, $p_0^*=\gamma_0$ must hold, and thus $p_0^*=\gamma_0=(d_0+\epsilon v_0)/\epsilon$. Because $\tilde{\gamma}_1$ is exogenously determined and (d_1,$\tilde{\gamma}_1$) and (v_1,$\tilde{\gamma}_1$) are independent, the optimal amount of production in period 1 is determined by

$$\begin{aligned} x_1^* &\equiv \arg\max_{x_1} E\left[\frac{p_1-v_1}{\tilde{\gamma}_1}\right]x_1 = \arg\max_{x_1}\left(1+\frac{1}{\epsilon\tilde{\gamma}_1}(E[d_1]-x_1)-\frac{E[v_1]}{\tilde{\gamma}_1}\right)x_1 \\ &= \frac{1}{2}\left[E[d_1]+\epsilon\left(\tilde{\gamma}_1-E[v_1]\right)\right]. \end{aligned} \quad (5)$$

Substituting (5) into (2) yields

$$E[p_1^*] = \frac{1}{2}\left(\frac{E[d_1]}{\epsilon}+\tilde{\gamma}_1+E[v_1]\right).$$

Turning to the maximization problem for k, although k can take a continuous value, the optimal choice of k is actually binary, that is, $k\in\{0,\underline{k}\}$. A strategy $k\in(0,\underline{k})$ is always strictly dominated by $k=0$, because this investment does not affect d_1 but the entrepreneur

[13] The problems of each entrepreneur and each worker are consistent with maximization problems of indirect utility functions derived from log utility, as shown in Appendix A.1.

needs to pay k. Similarly, $k \in (\underline{k}, \infty)$ is strictly dominated by $k = \underline{k}$. Thus it suffices to consider the binary choice $k \in \{0, \underline{k}\}$. For the same reason, each worker faces the binary choice $h \in \{0, \underline{h}\}$. Letting π_1 denote the firm's real profit in period 1, we have

$$E[\pi_1|k,h] \equiv \frac{1}{1+r} E\left[\frac{p_1 - v_1}{\tilde{\gamma}_1}\right] x_1 = \frac{1}{4\tilde{\gamma}_1(1+r)\epsilon}(\epsilon\tilde{\gamma}_1 + E[d_1|k,h] - \epsilon E[v_1|k,h])^2.$$

Moreover, let A and B be defined as follows.

$$\begin{aligned}
A &\equiv E[\pi_1|k = \underline{k}, h \in [0, \underline{h})] - E[\pi_1|k = 0, h \in [0, \underline{h})] \\
&= \frac{\alpha\left[(d_I - \epsilon v_I) - (d_N - \epsilon v_N)\right]}{4\tilde{\gamma}_1(1+r)\epsilon}\left[2\epsilon\tilde{\gamma}_1 + \alpha\left(d_I - \epsilon v_I\right) + (2-\alpha)\left(d_N - \epsilon v_N\right)\right], \\
B &\equiv E[\pi_1|k = \underline{k}, h \in [\underline{h}, \infty)] - E[\pi_1|k = 0, h \in [\underline{h}, \infty)] \\
&= \frac{(1-\alpha)\left[(d_I - \epsilon v_I) - (d_N - \epsilon v_N)\right]}{4\tilde{\gamma}_1(1+r)\epsilon}\left[2\epsilon\tilde{\gamma}_1 + (1+\alpha)\left(d_I - \epsilon v_I\right) + (1-\alpha)\left(d_N - \epsilon v_N\right)\right].
\end{aligned}$$

The value of A represents the increase in the firm's expected profit as a result of the entrepreneur's investment when $h \in [0, \underline{h})$, while B represents that increase when $h \in [\underline{h}, \infty)$. Therefore, given $h \in [0, \underline{h})$, the increase in the entrepreneur's expected payoff from investment is positive if $(1-\beta)A > \underline{k}$, while it is negative if $(1-\beta)A < \underline{k}$. Similarly, given $h \in [\underline{h}, \infty)$, the increase is positive if $(1-\beta)B > \underline{k}$, while it is negative if $(1-\beta)B < \underline{k}$.[14] From the assumption that $\alpha \in (0, 1/2)$, it follows that $B > A$. This is because

$$\begin{aligned}
& B > A \\
\Leftrightarrow\; & \frac{1-\alpha}{\alpha} > \frac{2\epsilon\tilde{\gamma}_1 + \alpha\left[(d_I - \epsilon v_I) - (d_N - \epsilon v_N)\right] + 2\left(d_N - \epsilon v_N\right)}{2\epsilon\tilde{\gamma}_1 + \alpha\left[(d_I - \epsilon v_I) - (d_N - \epsilon v_N)\right] + \left[(d_I - \epsilon v_I) + (d_N - \epsilon v_N)\right]},
\end{aligned}$$

and $\alpha \in (0, 1/2)$ implies that the left-hand side of the last inequality, $((1-\alpha)/\alpha)$, is greater than 1, while the right-hand side is smaller than 1.

Suppose that $(1-\beta)A < \underline{k} < (1-\beta)B$. The entrepreneur chooses $k = \underline{k}$ if it expects the worker to choose $h \in [\underline{h}, \infty)$; otherwise (i.e., it expects the worker to choose $h \in [0, \underline{h})$), the entrepreneur chooses $k = 0$. Moreover, the entrepreneur may choose a mixed strategy in which $k = \underline{k}$ with probability $\sigma_e \in [0, 1]$ and $k = 0$ otherwise. Similarly, the worker may

[14] If $(1-\beta)A = \underline{k}$ holds for $h \in [0, \underline{h})$ or $(1-\beta)B = \underline{k}$ holds for $h \in [\underline{h}, \infty)$, then the entrepreneur's expected payoff from $k = \underline{k}$ and the one from $k = 0$ are the same.

choose a mixed strategy in which $h = \underline{h}$ with probability $\sigma_w \in [0,1]$ and $h = 0$ otherwise. Based on these mixed strategies, the entrepreneur's expected payoff from investment is given by

$$(1-\beta)\left(\sigma_w E[\pi_1|k=\underline{k}, h=\underline{h}] + (1-\sigma_w)E[\pi_1|k=\underline{k}, h=0]\right) - \underline{k}, \tag{6}$$

while the expected payoff from no investment is given by

$$(1-\beta)\left(\sigma_w E[\pi_1|k=0, h=\underline{h}] + (1-\sigma_w)E[\pi_1|k=0, h=0]\right). \tag{7}$$

The entrepreneur strictly prefers to invest when (6) is greater than (7), i.e., $(1-\beta)[(1-\sigma_w)A + \sigma_w B] > \underline{k}$. Thus, the entrepreneur invests when $\sigma_w > \sigma_w^*$, where

$$\sigma_w^* \equiv \frac{\underline{k}/(1-\beta) - A}{B - A}.$$

Note that $\sigma_w^* \in (0,1)$, because $(1-\beta)A < \underline{k} < (1-\beta)B$. Then, the best-response correspondence $\sigma_e(\sigma_w)$ is given by

$$\sigma_e(\sigma_w) \begin{cases} = 1 \quad if \quad \sigma_w > \sigma_w^* \\ \in [0,1] \quad if \quad \sigma_w = \sigma_w^* \\ = 0 \quad if \quad \sigma_w < \sigma_w^*. \end{cases}$$

Consequently, the best-response correspondence of the entrepreneur is given by

$$k^* = \begin{cases} \underline{k} \quad if \quad \underline{k} < (1-\beta)A \\ \sigma_e(\sigma_w) \quad if \quad (1-\beta)B > \underline{k} > (1-\beta)A \\ 0 \quad if \quad \underline{k} > (1-\beta)B. \end{cases}$$

3.1.2 Worker

Each worker's problem is given by

$$\max_h \quad \beta\left(\frac{p_0 - v_0}{\gamma_0} x_0 + \frac{1}{1+r} E\left[\frac{p_1 - v_1}{\tilde{\gamma}_1}\right] x_1\right) - h.$$

In what follows, we ignore the period-0 payoff $\beta([p_0 - v_0]/\gamma_0)x_0$, because it is not affected by h. Thus, when $k \in [0, \underline{k})$, the expected increase in the firm's profit as a result of the worker's investment is given by

$$E[\pi_1 | k \in [0, \underline{k}), h = \underline{h}] - E[\pi_1 | k \in [0, \underline{k}), h = 0] = A;$$

otherwise (i.e., $k \in [\underline{k}, \infty)$), it is given by

$$E[\pi_1 | k \in [\underline{k}, \infty), h = \underline{h}] - E[\pi_1 | k \in [\underline{k}, \infty), h = 0] = B.$$

Thus, given $k \in [0, \underline{k})$, the increase in the worker's expected payoff from investment is positive if $\beta A > \underline{h}$, while it is negative if $\beta A < \underline{h}$. Similarly, given $k \in [\underline{k}, \infty)$, the increase is positive if $\beta B > \underline{h}$, while it is negative if $\beta B < \underline{h}$.[15]

When $\beta A < \underline{h} < \beta B$, the worker's best response depends on the entrepreneur's mixed strategy. Define σ_e^* as

$$\sigma_e^* \equiv \frac{\underline{h}/\beta - A}{B - A}.$$

Note that $\sigma_e^* \in (0, 1)$, because $\beta A < \underline{h} < \beta B$. Then, from a similar calculation to that in the entrepreneur's problem, it follows that the best-response correspondence $\sigma_w(\sigma_e)$ is given by

$$\sigma_w(\sigma_e) \begin{cases} = 1 \;\; if \;\; \sigma_e > \sigma_e^* \\ \in [0, 1] \;\; if \;\; \sigma_e = \sigma_e^* \\ = 0 \;\; if \;\; \sigma_e < \sigma_e^*. \end{cases}$$

Therefore, the best-response correspondence of the worker is given by

$$h^* = \begin{cases} \underline{h} \;\; if \;\; \underline{h} < \beta A \\ \sigma_w(\sigma_e) \;\; if \;\; \beta B > \underline{h} > \beta A \\ 0 \;\; if \;\; \underline{h} > \beta B. \end{cases}$$

3.2 Equilibrium

Based on the best responses of the entrepreneur and the worker, the Nash equilibria are determined as follows.[16]

[15] When $\beta A = \underline{h}$ holds for $k \in [0, \underline{k})$ or $\beta B = \underline{h}$ holds for $k \in [\underline{k}, \infty)$, the worker's expected payoff from $h = \underline{h}$ and the one from $h = 0$ are the same.

[16] When $\underline{k} = (1 - \beta)A$ ($\underline{h} = \beta A$) holds, players choose $k^* = \underline{k}$ ($h^* = \underline{h}$) if $\underline{h} \leq \beta B$ ($\underline{k} \leq (1 - \beta)B$); otherwise, they are indifferent between $k^* = \underline{k}$ ($h^* = \underline{h}$) and $k^* = 0$ ($h^* = 0$). When $\underline{k} = (1 - \beta)B$ ($\underline{h} = \beta B$)

Proposition 1 *Suppose* $\underline{k} \notin \{(1-\beta)A, (1-\beta)B\}$ *and* $\underline{h} \notin \{\beta A, \beta B\}$. *Then, the stage game has the following equilibria.*

1. *Both the entrepreneur and the worker invest* ($k^* = \underline{k}$ *and* $h^* = \underline{h}$) *if (i)* $\underline{k} < (1-\beta)A$ *and* $\underline{h} < \beta B$ *or (ii)* $\underline{k} < (1-\beta)B$ *and* $\underline{h} < \beta A$.

2. *Only the entrepreneur invests* ($k^* = \underline{k}$ *and* $h^* = 0$) *if* $\underline{k} < (1-\beta)A$ *and* $\underline{h} > \beta B$.

3. *Only the worker invests* ($k^* = 0$ *and* $h^* = \underline{h}$) *if* $\underline{k} > (1-\beta)B$ *and* $\underline{h} < \beta A$.

4. *No one invests* ($k^* = 0$ *and* $h^* = 0$) *if (i)* $\underline{k} > (1-\beta)B$ *and* $\underline{h} > \beta A$ *or (ii)* $\underline{k} > (1-\beta)A$ *and* $\underline{h} > \beta B$.

5. *The following three equilibria exist if* $A < \underline{k}/(1-\beta) < B$ *and* $A < \underline{h}/\beta < B$:

   (a) *Both the entrepreneur and the worker invest* ($k^* = \underline{k}$ *and* $h^* = \underline{h}$).

   (b) *No one invests* ($k^* = 0$ *and* $h^* = 0$).

   (c) *Both the entrepreneur and the worker choose the mixed strategies* σ_e^* *and* σ_w^*, *respectively.*

Proof: When $(1-\beta)B < \underline{k}$ ($\beta B < \underline{h}$), the entrepreneur (worker) does not invest regardless of its counterpart's strategy. When $(1-\beta)A > \underline{k}$ ($\beta A > \underline{h}$), the entrepreneur (worker) invests regardless of its counterpart's strategy. When $A < \underline{k}/(1-\beta) < B$ ($A < \underline{h}/\beta < B$), the entrepreneur (worker) invests if the probability that the worker (entrepreneur) will invest is sufficiently high and vice versa. There also exists a mixed-strategy equilibrium, because both players' strategies are the best responses to each other when $\sigma_e = \sigma_e^*$ and $\sigma_w = \sigma_w^*$. □

These results are summarized in Table 1 and Figure 2. If the marginal increases in the expected payoffs from investing (A and B) are sufficiently high, both players invest. However, if both values are low, they do not invest. When $A < \underline{k}/(1-\beta) < B$ and $A < \underline{h}/\beta < B$, there exist multiple equilibria, meaning that the complementarity between both players' investments plays a crucial role.

[Figure 2 here]

holds, players choose $k^* = 0$ ($h^* = 0$) if $\underline{h} \geq \beta A$ ($\underline{k} \geq (1-\beta)A$); otherwise, they are indifferent between $k^* = \underline{k}$ ($h^* = \underline{h}$) and $k^* = 0$ ($h^* = 0$). Note that a mixed-strategy equilibrium (σ_e^*, σ_w^*) exists only when $A < \underline{k}/(1-\beta) < B$ and $A < \underline{h}/\beta < B$ even when the cases in which players are indifferent between $k^* = \underline{k}$ ($h^* = \underline{h}$) and $k^* = 0$ ($h^* = 0$) are included.

Table 1: Ranges of $\underline{k}$ and $\underline{h}$ and equilibrium strategy (k^*, h^*)

	$\underline{h} < \beta A$	$A < \underline{h}/\beta < B$	$\underline{h} > \beta B$
$\underline{k} < (1-\beta)A$	$(\underline{k}, \underline{h})$	$(\underline{k}, \underline{h})$	$(\underline{k}, 0)$
$A < \underline{k}/(1-\beta) < B$	$(\underline{k}, \underline{h})$	$(\underline{k}, \underline{h})$, $(0,0)$, or (σ_e^*, σ_w^*)	$(0,0)$
$\underline{k} > (1-\beta)B$	$(0, \underline{h})$	$(0,0)$	$(0,0)$

3.3 Discussion

3.3.1 Types of Innovation

This subsection analyzes the effects of each type of innovation on the inflation rate. As explained above, $d_1 = d_I$ and $v_1 = v_I$ hold in the equilibrium with $k^* = \underline{k}$ and $h^* = \underline{h}$, whereas $d_1 = d_N$ and $v_1 = v_N$ hold in the equilibrium with $k^* = 0$ and $h^* = 0$. Therefore, the expected price level is given by $\left(\tilde{\gamma}_1 + d_I/\epsilon + v_I\right)/2$ in the former equilibrium, while it is given by $\left(\tilde{\gamma}_1 + d_N/\epsilon + v_N\right)/2$ in the latter equilibrium.

Suppose that all entrepreneurs and workers invest in the economy. Then, each entrepreneur-worker pair's expected price level should be the same as the expected price level in the economy when all players have rational expectations, i.e.,

$$\tilde{\gamma}_1 = \frac{d_I + \epsilon v_I}{\epsilon}.$$

When none of the entrepreneurs or workers invest, the expected price level in the economy is given by

$$\tilde{\gamma}_1 = \frac{d_N + \epsilon v_N}{\epsilon}.$$

While all pairs do not necessarily choose the same strategies, the range of $\tilde{\gamma}_1$ should be

$$\tilde{\gamma}_1 \in \left[\min\left\{\frac{d_N + \epsilon v_N}{\epsilon}, \frac{d_I + \epsilon v_I}{\epsilon}\right\},\ \max\left\{\frac{d_N + \epsilon v_N}{\epsilon}, \frac{d_I + \epsilon v_I}{\epsilon}\right\}\right].$$

Thus, the expected inflation rate between period 0 and period 1, $\tilde{\gamma}_1/\gamma_0 - 1$, meets

$$\frac{\tilde{\gamma}_1}{\gamma_0} - 1 \in \left[\min\left\{\frac{d_N + \epsilon v_N}{d_0 + \epsilon v_0}, \frac{d_I + \epsilon v_I}{d_0 + \epsilon v_0}\right\} - 1, \max\left\{\frac{d_N + \epsilon v_N}{d_0 + \epsilon v_0}, \frac{d_I + \epsilon v_I}{d_0 + \epsilon v_0}\right\} - 1\right].$$

Note that $(d_N + \epsilon v_N)/(d_0 + \epsilon v_0) - 1 \leq 0$ always holds. If $d_I + \epsilon v_I > d_0 + \epsilon v_0$ or, equivalently,

$d_I - d_0 > \epsilon(v_0 - v_I)$, the inflation rate is positive in the equilibrium with $k^* = \underline{k}$ and $h^* = \underline{h}$, while in the one with $k^* = 0$ and $h^* = 0$ the inflation rate is at most zero and likely to be negative. Otherwise (i.e., $d_I - d_0 \leq \epsilon(v_0 - v_I)$), the inflation rate is at most zero even in the equilibrium with $k^* = \underline{k}$ and $h^* = \underline{h}$ and therefore the inflation rate is at most zero in both of the equilibria with $k^* = \underline{k}$ and $h^* = \underline{h}$ and with $k^* = 0$ and $h^* = 0$. Thus, the following observation is obtained.

Observation 1 *Both equilibria, that with a positive inflation rate and that with a zero or negative inflation rate, exist if and only if $d_I - d_0 > \epsilon(v_0 - v_I)$. Inflation occurs in the equilibrium with $k^* = \underline{k}$ and $h^* = \underline{h}$ (where both types of players invest), while no inflation occurs in the equilibrium with $k^* = 0$ and $h^* = 0$ (where no player invests). When $d_I - d_0 \leq \epsilon(v_0 - v_I)$, there is no inflation in either of the equilibria. Therefore, a positive inflation rate is attainable in an equilibrium only if $d_I - d_0 > \epsilon(v_0 - v_I)$.*

This observation highlights one of the necessary conditions for achieving the (positive) target inflation rate in an equilibrium. When entrepreneurs invest in demand-creating innovation rather than cost-reducing innovation (a high value of d_I such that $d_I - d_0 > \epsilon(v_0 - v_I)$ holds), players can view the target inflation rate as credible. On the contrary, when entrepreneurs stick to cost-reducing innovation (a low value of v_I such that $d_I - d_0 < \epsilon(v_0 - v_I)$ holds), no player views the target rate as credible. This is because in this case the upper bound of $\tilde{\gamma}_1$ is zero. Note that $d_N + \epsilon v_N \leq d_0 + \epsilon v_0$ always holds from the assumptions $d_N \leq d_0$ and $v_N \leq v_0$. In such a case, policies to stimulate investments in demand-creating innovation (e.g., structural reforms, tax incentives, subsidies) are called for so that players could view the target inflation rate as credible (see Figure 3). In what follows, it is assumed that $d_I - d_0 > \epsilon(v_0 - v_I)$.

[Figure 3 here]

3.3.2 Choice of Innovation Types

This subsection simply supposes that entrepreneurs can decide the type of innovation they engage in, that is, $w \in \{0, 1\}$, where $w = 1$ denotes demand-creating innovation and $w = 0$ denotes cost-reducing innovation.[17] When demand-creating innovation is chosen, expected

[17] For the sake of analytical simplicity, this subsection assumes the binary choice, i.e., $w \in \{0, 1\}$.

demand creation is $d^*(> d_N)$ and the expected production cost is v_N in the case of $k \in [\underline{k}, \infty)$ and $h \in [\underline{h}, \infty)$. On the contrary, if cost-reducing innovation is selected, expected demand creation is d_N and the expected production cost is $v^*(< v_N)$. Therefore, d^* and v^* represent the expected fundamental demand and variable cost achieved through each investment, respectively. Note that d^* and v^* may correspond to d_I and v_I in the baseline model presented above when $d_I = d^* > d_N$ and $v_I = v_N$ for $w^* = 1$ as well as $d_I = d_N$ and $v_I = v^* < v_N$ for $w^* = 0$. From the firm's expected period-1 profit $E[\pi_1|k,h] = \{1/[4\tilde{\gamma}_1(1+r)\epsilon]\}(\epsilon\tilde{\gamma}_1 + E[d_1|k,h] - \epsilon E[v_1|k,h])^2$, the optimal choice of innovation types w is given as follows.

Corollary 1 *Given the expected demand-creation effect ($d^* - d_N$) and the expected cost-reduction effect ($v_N - v^*$), the entrepreneurs' decision with regard to $w^* \in \{0,1\}$ is given by*

$$w^* = \begin{cases} 1 \;\; if \;\; d^* - d_N > \epsilon(v_N - v^*) \\ 0 \;\; if \;\; d^* - d_N < \epsilon(v_N - v^*). \end{cases}$$

Proof: See Appendix A.2. □

The elasticity of demand (ϵ) plays a key role here. As the elasticity increases, firms are more likely to choose cost-reducing innovation and vice versa. If demand for a good is elastic to its relative price, a decrease in the price leads to an increase in revenue, meaning that it is more attractive for entrepreneurs to reduce costs. On the contrary, if demand is inelastic, entrepreneurs can raise prices and at the same time, increase demand for their product through demand-creating innovation, as pointed out by Kamien and Schwartz (1970) and Spence (1975).

Therefore, to satisfy the condition in Observation 1, entrepreneurs should have an incentive to choose demand-creating innovation, which means that ϵ should be low enough. A sufficiently low value of ϵ is necessary for inflation to occur in an equilibrium. If this is not the case, policies to raise d^* are called for so that the condition $d^* - d_N > \epsilon(v_N - v^*)$ could hold.

3.3.3 Economic Performance

The performance of the economy can be measured by output growth

$$g \equiv \frac{x_1}{x_0} - 1 = \frac{\epsilon\gamma_1 + d_1 - \epsilon v_1}{\epsilon\gamma_0 + d_0 - \epsilon v_0} - 1. \tag{8}$$

Because there are an infinite number of entrepreneurs and workers, the range of γ_1 is identical to that of $\tilde{\gamma}_1$ given by $[(d_N + \epsilon v_N)/\epsilon, (d_I + \epsilon v_I)/\epsilon]$. Thus, by substituting the values of $\gamma_1 \in [(d_N + \epsilon v_N)/\epsilon, (d_I + \epsilon v_I)/\epsilon]$, $\gamma_0 = (d_0 + \epsilon v_0)/\epsilon$, and $d_1 - \epsilon v_1 \in [d_N - \epsilon v_N, d_I - \epsilon v_I]$ into (8), the range of g is given by

$$g \in \left[\frac{d_N}{d_0} - 1, \frac{d_I}{d_0} - 1\right].$$

Therefore, the economic performance in the equilibrium with $k^* = \underline{k}$ and $h^* = \underline{h}$, which is given by $d_I/d_0 - 1$, is greater than that in the equilibrium with $k^* = 0$ and $h^* = 0$, given by $d_N/d_0 - 1$. Consequently, the following observation holds.

Observation 2 *The output growth rate in the equilibrium with $k^* = \underline{k}$ and $h^* = \underline{h}$, where both players invest, is higher than that in the equilibrium with $k^* = 0$ and $h^* = 0$, where neither player invests.*

Therefore, as long as investments contain demand-creating innovation, the equilibrium with $k^* = \underline{k}$ and $h^* = \underline{h}$ has better economic performance than that with $k^* = 0$ and $h^* = 0$. In this sense, the former equilibrium is the one policymakers should target and aim to guide the economy toward.[18]

4 Evolutionary Game

This section investigates the dynamic transition of strategies of the stage game played iteratively for continuous round $\tau \in [0, \infty)$ by applying the method of evolutionary games.[19] We concentrate on the case in which $(1-\beta)A < \underline{k} < (1-\beta)B$ and $\beta A < \underline{h} < \beta B$ hold, because

[18] Letting the social surplus $\Pi \equiv \pi_1(k, h) - k - h$ be defined as the sum of entrepreneurs' and workers' surplus, the social surplus in the equilibrium with $k^* = \underline{k}$ and $h^* = \underline{h}$ is higher than that in the equilibrium $k^* = 0$ and $h^* = 0$ unless the potential increase in demand $d_I - d_N$ or the potential decrease in variable cost $v_N - v_I$ is too small to cover the total investment cost $\underline{k} + \underline{h}$.

[19] In our evolutionary game, an entrepreneur and a worker are randomly matched from each population and they play the same stage game iteratively for all the rounds.

in the other cases the Nash equilibrium is unique and players' strategies simply converge to the equilibrium in the long run.

4.1 Forward-Looking versus Backward-Looking

This section analyzes the evolutionary dynamics of players' strategies between investment and non-investment, using the perfect-foresight dynamics introduced by Matsui and Matsuyama (1995) and the best-response dynamics initiated by Gilboa and Matsui (1991). In the perfect-foresight dynamics, players are forward-looking about other players' strategies and take into account the whole future path of the economy. By contrast, in the best-response dynamics, players are backward-looking about others' strategies and presuppose that the same situation continues in the future.

In existing studies using evolutionary game theory approach (e.g., Vega-Redondo [1997], Kaneda [2003], Oyama [2009]), the perfect-foresight dynamics might be more popular, because the uniquely absorbing and globally accessible equilibrium is determined solely on the base of the payoff matrix and is independent of the initial state of the economy.[20] However, our study focuses on the best-response dynamics because in the macroeconomics literature there has a limited understanding of how the economy evolves from its initial state toward an equilibrium in the long run, given that the initial state is interpreted as the round of the stage game right after the implementation of a policy. The comparison of the dynamics illustrates the effects of players' expectation formation processes not only on the transition dynamics but also on the conditions for achieving a long-run equilibrium at the target inflation rate.

Our approach is closely related to the macroeconomics literature that compares results under rational expectations (forward-looking) and those under adaptive expectations (backward-looking). While rational expectations have been widely used, they impose the strong assumption that economic agents exploit all currently available information in forming their expectations. Thus, some pioneering studies (e.g., Mankiew and Reis [2002], Sims [2003], Woodford [2002]) investigate modeling that postulates deviations from rational expectations, particularly the implications of information frictions for the expectation formation

[20] An equilibrium is uniquely absorbing if it is the only absorbing state, that is, the only state from which it is impossible to leave once the state is entered. An equilibrium is globally accessible if, for any initial state, there exists a path toward the equilibrium. Therefore, if an equilibrium is uniquely absorbing and globally accessible, then the economy will converge to the equilibrium in the long run regardless of its initial state. Because of this property, the perfect-foresight dynamics is widely used for equilibrium selection.

and their macroeconomic consequences. Our research is in the same vein as these studies.

In both the cases of the perfect-foresight dynamics and the best-response dynamics, there are certain conditions for guiding players to a long-run equilibrium at the target inflation rate. This implies that, given the exogenously determined upper bound of $\tilde{\gamma}_1$, both of the dynamics confirm that the target inflation rate is not always attainable in the long run. However, the effects of the current state of the economy (i.e., history) on the conditions are totally different between the two dynamics: with the perfect-foresight dynamics the current state of the economy never affects for the conditions, whereas with the best-response dynamics the conditions become more severe the closer the current state of the economy is to the equilibrium with a zero or negative inflation rate.[21]

In preparing for the derivation of the dynamics, we define π_H, π_M, and π_L as

$$\begin{aligned}
\pi_H &\equiv \frac{1}{4\tilde{\gamma}_1(1+r)\epsilon}\left(\epsilon\tilde{\gamma}_1 + d_I - \epsilon v_I\right)^2, \\
\pi_M &\equiv \frac{1}{4\tilde{\gamma}_1(1+r)\epsilon}\left[\epsilon\tilde{\gamma}_1 + \alpha(d_I - \epsilon v_I) + (1-\alpha)(d_N - \epsilon v_N)\right]^2, \\
\pi_L &\equiv \frac{1}{4\tilde{\gamma}_1(1+r)\epsilon}\left(\epsilon\tilde{\gamma}_1 + d_N - \epsilon v_N\right)^2,
\end{aligned}$$

where π_H is the expected total profit when both players invest, π_M is that when only one player invests, and π_L is that when no player invests. Table 2 presents the expected payoff matrix for entrepreneurs and workers.

Table 2: Expected payoff matrix

	$h = \underline{h}$	$h = 0$
$k = \underline{k}$	$((1-\beta)\pi_H - \underline{k}, \beta\pi_H - \underline{h})$	$((1-\beta)\pi_M - \underline{k}, \beta\pi_M)$
$k = 0$	$((1-\beta)\pi_M, \beta\pi_M - \underline{h})$	$((1-\beta)\pi_L, \beta\pi_L)$

From Proposition 1, it follows that when $(1-\beta)A < \underline{k} < (1-\beta)B$ and $\beta A < \underline{h} < \beta B$ hold, there exist two pure-strategy Nash equilibria $(k^*, h^*) \in \{(\underline{k}, \underline{h}), (0,0)\}$ and one mixed-strategy Nash equilibrium (σ_e^*, σ_w^*). Using π_H, π_M, and π_L, the mixed-strategy equilibrium can be rewritten as

[21] If the size of each player's sample is very small, the equilibrium to which the economy converges from its initial state depends weakly on the initial state but strongly on the payoff matrix (Oyama, Sandholm, and Tercieux [2015]). However, generally speaking, the sample size of entrepreneurs and workers is less likely to be small when they decide their investment plans.

$$(\sigma_e^*, \sigma_w^*) = \left(\frac{\underline{h}/\beta - (\pi_M - \pi_L)}{\pi_H + \pi_L - 2\pi_M}, \frac{\underline{k}/(1-\beta) - (\pi_M - \pi_L)}{\pi_H + \pi_L - 2\pi_M} \right).$$

In what follows, we continue to assume that the population of each type of player is a continuum of mass one. Therefore, we denote the share of entrepreneurs investing by $s_e \in [0, 1]$ and that of workers investing by $s_w \in [0, 1]$.

4.2 Forward-Looking Players: Perfect-Foresight Dynamics

Under the perfect-foresight dynamics, players are highly rational and choose their strategies to maximize their expected discounted payoffs. The key assumption is that players cannot switch actions at each instantaneous time. Every player must commit to a particular strategy in the short run. We assume that the opportunity to switch strategies arrives randomly and that the arrival time follows a Poisson process with a mean arrival rate of one. Moreover, we assume that players' discount rate in each round is $\delta > 0$. Then, from Matsui and Matsuyama (1995), we have the following result.

Proposition 2 *Denote $s_e^* \equiv \sigma_e^*$ and $s_w^* \equiv \sigma_w^*$. For $\delta \to 0$, the equilibrium with $k = \underline{k}$ and $h = \underline{h}$, where the inflation rate is positive, is a unique absorbing and globally accessible equilibrium if and only if $A + B - \underline{h}/\beta - \underline{k}/(1-\beta) > 0$ holds. By contrast, the equilibrium with $k = 0$ and $h = 0$, where the inflation rate is zero or negative, is a unique absorbing and globally accessible equilibrium if and only if $A + B - \underline{h}/\beta - \underline{k}/(1-\beta) < 0$ holds.*

Proof: See Appendix A.3. □

Therefore, when the expected increment in firms' profits from investment is sufficiently large, the economy converges to the equilibrium with $k = \underline{k}$ and $h = \underline{h}$ in the long run, regardless of the initial state of the economy. Note that Proposition 2 can be restated: the economy converges eventually to the risk-dominant Nash equilibrium (Harsanyi and Selten [1988]). Intuitively, the risk-dominant Nash equilibrium is the Nash equilibrium that is less risky for players.

As shown in Proposition 2, if players are provident ($\delta \to 0$), then the equilibrium to which the economy eventually converges is uniquely determined by the condition above. In contrast, if players are myopic ($\delta \to \infty$), the initial state of the economy matters for the equilibrium selection as follows.

Corollary 2 *(Proposition 2 of Matsui and Matsuyama [1995]) For $\delta \to \infty$, both the equilibrium with a positive inflation rate and that with a zero or negative inflation rate are absorbing for any (s_e^*, σ_e^*).*

4.3 Backward-Looking Players: Best-Response Dynamics

This section derives the dynamics of backward-looking players' strategies and explicitly provides the initial conditions used to select the equilibrium to which the economy eventually converges. To analyze the dynamics of the game with backward-looking players, the best-response dynamics introduced by Gilboa and Matsui (1991) is used, because it is natural to assume that the players best know the structure of the game. This assumption is equivalent to the situation in which players are strongly interested in the payoffs and therefore try to procure all the necessary information.[22]

In the best-response dynamics, each player observes the *ex post* share of each action and revises its belief about the payoffs from its own action in the next round. The key difference from the perfect-foresight dynamics is that players in the best-response dynamics only consider expected payoffs in the round (i.e., this is the case if $\delta \to \infty$ as in the previous subsection), which can be interpreted that players consider future rounds but presuppose that the same situation continues in the future. At each instantaneous time, $d\tau$ players revise their strategies, and each player chooses its best response given s_e and s_w in each game. The arrival time of the opportunity to switch strategies follows a Poisson process with a mean arrival rate of one. Thus, the following proposition holds.

Proposition 3 *Letting $s_e^* \equiv \sigma_e^*$ and $s_w^* \equiv \sigma_w^*$, the best-response dynamics of the game is given by*

$$\frac{ds_e}{d\tau} \begin{cases} = 1 - s_e \quad if \quad s_w > s_w^* \\ \in [-s_e, 1 - s_e] \quad if \quad s_w = s_w^* \\ = -s_e \quad if \quad s_w < s_w^*, \end{cases}$$

[22] In other dynamics, such as trial and error dynamics (e.g., Roth and Erev [1995, 1999]) and imitation dynamics (e.g., Bjornerstedt and Weibull [1996]), it does not seem easy to derive the transition dynamics from a model with uncertainty.

$$\frac{ds_w}{d\tau}\begin{cases} = 1 - s_w \quad if \quad s_e > s_e^* \\ \in [-s_w, 1 - s_w] \quad if \quad s_e = s_e^* \\ = -s_w \quad if \quad s_e < s_e^*. \end{cases}$$

The stable stationary equilibria are $(s_e, s_w) \in \{(1,1), (0,0)\}$.

Proof: See Appendix A.4. □

Therefore, the stable stationary equilibrium to which the economy converges depends on initial shares of the strategies. By solving the differential equation with the initial shares s_{e_0} and s_{w_0}, we have the trajectories

$$s_{e_\tau} = \begin{cases} 1 - e^{-\tau}(1 - s_{e_0}) \quad if \quad s_{w_0} > s_w^* \\ e^{-\tau} s_{e_0} \quad if \quad s_{w_0} < s_w^*, \end{cases}$$
$$s_{w_\tau} = \begin{cases} 1 - e^{-\tau}(1 - s_{w_0}) \quad if \quad s_{e_0} > s_e^* \\ e^{-\tau} s_{w_0} \quad if \quad s_{e_0} < s_e^*. \end{cases}$$

Therefore, the speed of the convergence to each equilibrium is positively related to the distance between the current (initial) shares and the equilibrium shares. When the current shares are far from the equilibrium shares, the economy evolves rapidly toward the equilibrium. By contrast, when they are close to the equilibrium shares, the economy converges slowly to the equilibrium.

An exception is the case in which $s_{e_0} = s_e^*$ and $s_{w_0} = s_w^*$. In this case, the economy can converge to both $(s_e, s_w) = (1,1)$ and $(s_e, s_w) = (0,0)$ with a possible delay. Thus, the trajectories are not unique if the economy reaches (s_e^*, s_w^*), whereas other trajectories converge to one of the pure-strategy Nash equilibria of the stage game.

Corollary 3 *The equilibrium to which the economy eventually converges is uniquely determined by initial shares* (s_{e_0}, s_{w_0}) *except in the case of initial shares whose trajectories reach* (s_e^*, s_w^*).

Proof: See Appendix A.5. □

This corollary highlights another necessary condition for achieving the (positive) target inflation rate in an equilibrium. The economy converges to the equilibrium with the target inflation rate when the initial shares of entrepreneurs and workers who choose to invest are sufficiently large. Otherwise, it converges to the equilibrium with a zero or negative inflation rate.

To be precise, two main sets of initial shares exist: S^D for converging to the equilibrium with a zero or negative inflation rate and S^I for converging to the equilibrium with a positive inflation rate. In addition, another set of initial shares exists which the economy passes through (s_e^*, s_w^*) and eventually converges to both equilibria. Such a trajectory that passes (s_e^*, s_w^*) is a line, and this line divides S^I and S^D, as shown in Figure 4. In this figure, the horizontal axis represents the share of entrepreneurs currently investing s_e, which increases from right to left. The vertical axis represents the share of workers currently investing s_w, which increases along the axis. Thus, the upper left point is the equilibrium with a positive inflation rate, while the lower right point is the equilibrium with a zero or negative inflation rate.

[Figure 4 here]

The following proposition can be obtained.

Proposition 4 *Suppose that s_w^* (s_e^*) is fixed. Then, as s_e^* (s_w^*) decreases, S^I becomes larger, whereas S^D becomes smaller.*

Proof: See Appendix A.6. □

Figure 5 shows the simulation results of the best-response dynamics.[23]

[Figure 5 here]

This figure presents two possible cases based on the range of $\tilde{\gamma}_1$ that was discussed in Section 3.3. Panel (a) shows the dynamics with the lower bound of this range (i.e., $\tilde{\gamma}_1 = (d_N + \epsilon v_N)/\epsilon$), where no player invests. In this case, the economy converges from most of the initial shares to the shares in the equilibrium with a zero or negative inflation rate. On the other hand, panel (b) illustrates the dynamics with the upper bound of the range of $\tilde{\gamma}_1$ (i.e., $\tilde{\gamma}_1 = (d_I + \epsilon v_I)/\epsilon$), where all players invest. The economy converges from

[23] Figure 5 was created using Dynamo, developed by Sandholm, Dokumaci, and Franchetti (2012).

most of the initial shares to the shares in the equilibrium with a positive inflation rate. The colors indicate the speed of the evolution along convergence paths, with red representing the highest convergence speed, followed by yellow, green, and blue. As both panels show, the speed declines as the current shares approach the equilibrium shares.

Note that we measure the inflation rate between period 0 and 1, not over τ, because the stage game is the same in all rounds. As a consequence, the price level always declines from period 1 of round $\tau - 1$ to period 0 of round τ, which might be unrealistic. In what follows, it is assumed without loss of generality that the price level is the same between period 1 of round $\tau - 1$ and period 0 of round τ.[24] The price level thus increases over τ.

4.4 Effects of the Target Inflation Rate

To analyze the effects of the target inflation rate, we decompose the real interest rate into the nominal interest rate and the expected inflation rate (i.e., $\tilde{\gamma}_1/\gamma_0$) as follows.

$$1 + r = \frac{1+i}{\tilde{\gamma}_1/\gamma_0}.$$

Then we can examine the policy channels (i.e., intervention to lower the nominal interest rate i and an announcement of the target inflation rate $\tilde{\gamma}_1/\gamma_0$). We assume that at least in the short to medium run, i is fixed at a low rate (possibly zero or slightly negative).[25] When an entrepreneur expects other entrepreneurs to raise prices of their products ($\tilde{\gamma}_1/\gamma_0$ increases), the optimal price for the entrepreneur rises and therefore the equilibrium price level increases. This leads to an increase in expected profit. Then, the comparative statics of $\tilde{\gamma}_1$ are as follows.

[24] Specifically, we assume the following situation. First, at the start of period 1 of round $\tau - 1$, the innovation is realized, firms earn their profits, which are distributed to entrepreneurs and workers. Then, at the end of the period, technological spillovers occur and the effect of the technology of all firms on the price level becomes equal to the average of the effect of the technology of each firm. Therefore, all prices in period 0 of round τ are equal to the price level in period 1 of round $\tau - 1$.

[25] In the long run, a rise in expected inflation may lead to an increase in the nominal interest rate. However, our results remain unchanged as long as it is assumed that the nominal interest rate is fixed for a while. Once the economy embarks on the path toward the equilibrium with a positive inflation rate, it moves closer to the equilibrium over time. Therefore, even if the nominal interest rate rises and entrepreneurs' and workers' expected payoffs decline in the long run, the economy should still evolve to the equilibrium with a positive inflation rate, because players expect the probability of successful coordination between entrepreneurs and workers to be high. For consistency, when the economy is located in the equilibrium with a positive inflation rate, the nominal interest rate is assumed to be endogenously determined to satisfy $(1 + i) = (d_I + \epsilon v_I)/(d_0 + \epsilon v_0)$, given that the real interest rate is assumed to be zero in the long run.

Corollary 4 *(1) A and B increase with $\tilde{\gamma}_1$. (2) S^I becomes larger with $\tilde{\gamma}_1$.*

Proof: See Appendix A.7. □

This corollary shows that the equilibrium with a positive inflation rate is more likely to occur in the long run with the higher $\tilde{\gamma}_1$. First, as the expected period-1 price level $\tilde{\gamma}_1$ increases, both A and B rise, meaning that in Figure 2 the area where only the equilibrium with a positive inflation rate exists becomes larger. Because the increase in B is greater than that in A, the distance $(B-A)$ increases, which means that the area with multiple equilibria ($A < \underline{k}/(1-\beta) < B$ and $A < \underline{h}/\beta < B$) also becomes larger. Moreover, even in the case that there are multiple equilibria in the economy ($A < \underline{k}/(1-\beta) < B$ and $A < \underline{h}/\beta < B$), the increase in $\tilde{\gamma}_1$ transforms the evolutionary dynamics such that the economy is more likely to converge to the equilibrium with a positive inflation rate through the increase in $(A+B)$ under the perfect-foresight dynamics and through the decrease in s_e^* and s_w^* under the best-response dynamics. Therefore, if $\tilde{\gamma}_1$ increases, investment is expected to be more likely.

4.5 Do Players Have an Incentive to Invest More?

This subsection examines whether players necessarily have an incentive to invest more. That is, will the economy always converge to the equilibrium with a positive inflation rate? Our answer is that such a convergence does not necessarily occur for the following reasons.

Players may not expect the inflation target to be realized. Our study assumes that all players fully expect the target period-1 price level $\tilde{\gamma}_1$ to be definitely realized as long as the value is inside the range of $\tilde{\gamma}_1$ rational players view as credible. However, if each player does not view $\tilde{\gamma}_1$ as credible or if it does not consider that other players view $\tilde{\gamma}_1$ as credible, there is little effect of announcing $\tilde{\gamma}_1$.[26] Moreover, if the shares of entrepreneurs and workers currently investing in demand-creating innovation are small, then the upper bound of the range of $\tilde{\gamma}_1$ for rational players may be low, because it may be conditional on the shares of entrepreneurs and workers who invested in the last round. Because only $d\tau$ players revise their strategies at each instantaneous time, the upper bound of the range of $\tilde{\gamma}_1$ may depend

[26] If players perceive uncertainty about γ_1, they may also take into account the covariance terms between the variables γ_1, d_1, and v_1.

on $s_{e_\tau} + d\tau$ and $s_{w_\tau} + d\tau$ even if this leads all players with the opportunity to revise their strategies to invest for certain.

Moreover, the range of $\tilde{\gamma}_1$ is exogenously determined by entrepreneurs' choices of the type of innovations. As long as $d_I - \epsilon v_I$ is sufficiently larger than $d_N - \epsilon v_N$, that is, $d_I - d_N$ or $\epsilon(v_I - v_N)$ is large, investment should increase the expected payoff in period 1. However, if entrepreneurs choose cost-reducing innovation, then the upper bound of $\tilde{\gamma}_1$ for rational players takes a low or possibly negative value. Thus, the real interest rate (the discount rate) becomes high and the present value of the expected payoff will not be high enough to give an incentive for entrepreneurs to invest. As a consequence, investment is not boosted if entrepreneurs select cost-reducing innovation.[27]

Furthermore, even if the upper bound of $\tilde{\gamma}_1$ is positive, the increase in expected inflation may be insufficient to encourage all players to invest. Suppose that the current shares (s_e, s_w) are at or near the equilibrium with a zero or negative inflation rate. If the increase in the expected inflation rate is high and both A and B increase sufficiently, which means that only the equilibrium with a positive inflation rate exists ($\underline{k} < (1-\beta)A$ and $\underline{h} < \beta B$, or $\underline{k} < (1-\beta)B$ and $\underline{h} < \beta A$), all players will then have an incentive to invest and the shares converge to that equilibrium. However, an increase in the expected inflation rate does not ensure that only that equilibrium exists.

Our results show that the main role of raising $\tilde{\gamma}_1$ is to make the payoff structures more likely to stimulate investments. Under the best-response dynamics, changes in $\tilde{\gamma}_1$ shift the threshold between S^I and S^D. However, there is an upper bound of $\tilde{\gamma}_1$ for rational players. Therefore, the policy is not always sufficient to encourage players to invest.

In addition, if players are backward-looking, the effectiveness of the policy depends heavily on the current state of the economy (i.e., the share of players who invested in the most recent round) and is thus history-dependent, whereas the current state does not matter if players are forward-looking. Suppose that the current shares (s_e, s_w) are already near the equilibrium with a zero or negative inflation rate. To give an incentive for players to invest, $\tilde{\gamma}_1$ should be raised so that only the equilibrium with a positive inflation rate would exist. However, this may not be possible, as shown in panel (a) of Figure 6.

[27] Even if the highest $\tilde{\gamma}_1$ within its range for rational players is non-positive, there is still room for stimulating the economy by preventing inflation expectations from falling deeper into negative territory. However, in this case, the more entrepreneurs invest (in cost-reducing innovation), the more deflationary pressure arises, meaning it becomes increasingly difficult to escape from the deflationary environment.

Observation 3 *If the current shares* (s_e, s_w) *in the economy are included in* S^D *even under the upper bound of the expected price* ($\tilde{\gamma}_1 = d_I + \epsilon v_I$)*, then the equilibrium with a positive inflation rate is not achievable.*

[Figure 6 here]

In this case, other policy measures are essential to further lower the threshold, as shown in panel (b) of Figure 6.[28] More specifically, we suggest the following policies. First, the government can expand S^I by lowering the investment cost $\underline{k}$ through investment subsidies or tax incentives for investment.[29] Second, initiatives to resolve any potential mismatch between the needs of entrepreneurs and workers, such as efforts to achieve higher female labor participation, can help expand S^I by lowering $\underline{h}$. The reason is that workers have an incentive to invest in their human capital if they expect to work for longer periods. Third, aggressive public expenditure (investment) and the expansion of social benefits can help expand S^I by directly increasing both the expected fundamental demand d_I and d_N. Yet another potential measure is the so-called "technology push channel" (Sherer [1967], Rosenberg [1974]), in which, for example, the promotion of collaboration among industry, academia, and the government leads to a reduction in investment costs and an increase in expected returns.[30]

Further, when players are backward-looking, the speed of evolution along convergence paths in Figure 5 has a key implication. Suppose that the economy is at S^D but is also close to S^I. Then, the aforementioned policies can easily guide the economy toward the equilibrium with a positive inflation rate. However, because the speed of evolution in that state is higher than that in states near the equilibrium, the economy evolves rapidly from the current state to the equilibrium with a zero or negative inflation rate if the economy does not satisfy the conditions mentioned above. As a consequence, it becomes much harder to achieve the equilibrium with a positive inflation rate, confirming the importance of the prompt

[28] If the condition $A + B - \underline{h}/\beta - \underline{k}/(1-\beta) > 0$ is not met, then other policy measures are required to guide the economy toward the equilibrium with a positive inflation rate even when players are forward-looking.

[29] Many studies provide empirical evidence on the effect of R&D tax credits and subsidies (Hall and Van Reenen [2000], David, Hall, and Toole [2000], Bloom, Griffith, and Van Reenen [2002], Jaumotte and Pain [2005], Harris, Li, and Trainor [2009], Czarnitzki, Hanel, and Rosa [2011], Lokshin and Mohnen [2012], Yang, Huang, and Hou [2012], Czarnitzki and Lopes Bento [2012], Mulkay and Mairesse [2013]).

[30] Several studies show that promoting networked innovation activities can play an important role in boosting innovation (Lundvall [1992], Kamien, Mueller, and Zang [1992], Nelson [1993], Zucker, Darby, and Brewer [1998], Feldman and Audretsch [1999], Etzkowitz and Leydersdorff [2000], Autant-Bernard [2001], Hagedoorn [2002], Cassiman and Veugelers [2002], Adams, Chiang, and Jensen [2003]).

implementation of policies once the economy has started to evolve toward the equilibrium with a zero or negative inflation rate.

5 Conclusion

This study has examined conditions for achieving an equilibrium at the target inflation rate in the long run even after the policy rate hits its lower bound, by employing a dynamic model based on evolutionary game theory. The model consists of an iteratively played stage game where entrepreneurs and workers make investment decisions. In the presence of complementarity between entrepreneurs' and workers' investments, two equilibria exist. In one equilibrium all players invest, whereas no player invests in the other equilibrium. The study has shown two necessary conditions for achieving the target inflation rate in the equilibrium with all players investing in the long run. The first condition is that entrepreneurs choose investments in demand-creating innovation rather than cost-reducing innovation. The second condition is that the proportions of entrepreneurs and workers currently investing are large enough. If these conditions are not in place, policies to meet them are called for.

The model employed in this study is special in several respects. Some important extensions to the model are listed in closing. First, players are to be modeled to utilize all available information in forming expectations of the next-period price level, which should be the average of next-period prices across firms. In the model, the expectations are always consistent with the target inflation rate announced by the central bank as long as the current state of the economy implies that the target rate is attainable. Second, the accumulation of physical and human capital in the economy is to be taken into consideration. Addressing these issues, although they are technically challenging in terms of evolutionary game theory, will lead to a deeper understanding of the conditions for successfully achieving the target inflation rate.

A Appendix

A.1 Indirect Utility Functions

Suppose that an entrepreneur maximizes its utility function as follows.

$$\max_{c_0^e, c_1^e, x_0, x_1, k} \quad \ln(c_0^e) + \ln(c_1^e) \quad s.t. \quad c_0^e + k + a_0^e = (1-\beta)\pi_0, \quad c_1^e = (1-\beta)\pi_1 + (1+r)a_0^e,$$

where a_0^e is the real bond held by the entrepreneur. The budget constraints can be rewritten as

$$c_0^e + k + \frac{1}{1+r}\, c_1^e = (1-\beta)\left(\pi_0 + \frac{1}{1+r}\,\pi_1\right).$$

From the first-order conditions for c_0^e and c_1^{e*}, we have the Euler equation and the optimal c_0^{e*} and c_1^{e*} as follows.

$$\begin{aligned} c_0^e &= \frac{1}{1+r}\, c_1^e, \\ c_0^{e*} &= \frac{1}{2}\left[(1-\beta)\left(\pi_0 + \frac{1}{1+r}\,\pi_1\right) - k\right], \\ c_1^{e*} &= \frac{1}{2}(1+r)\left[(1-\beta)\left(\pi_0 + \frac{1}{1+r}\,\pi_1\right) - k\right]. \end{aligned}$$

By substituting c_0^{e*} and c_1^{e*} into the maximization problem, we obtain the indirect utility functions given by

$$\max_{x_0, x_1, k} \quad 2\ln\left[(1-\beta)\left(\pi_0 + \frac{1}{1+r}\,\pi_1\right) - k\right] + 2\ln\left(\frac{1}{2}\right) + \ln(1+r).$$

Here, the terms $2\ln(1/2)$ and $\ln[1/(1+r)]$ are purely exogenous variables for the entrepreneur. Moreover, $((1-\beta)\{\pi_0 + [1/(1+r)]\pi_1\} - k)$ is monotonic to $2\ln((1-\beta)\{\pi_0 + [1/(1+r)]\pi_1\} - k)$ because of the property of the log function. Therefore, the entrepreneur's maximization problem can be restated as

$$\max_{x_0, x_1, k} \quad (1-\beta)\left(\pi_0 + \frac{1}{1+r}\,\pi_1\right) - k.$$

Similarly, a worker maximizes its utility function as follows.

$$\max_{c_0^w, c_1^w, h} \quad \ln(c_0^w) + \ln(c_1^w) \quad s.t. \quad c_0^w + h + a_0^w = \beta\pi_0, \quad c_1^w = \beta\pi_1 + (1+r)a_0^w,$$

where a_0^w is the real bond held by the worker. Therefore, from a similar argument to the above, we can obtain

$$\max_h \quad \beta\left(\pi_0 + \frac{1}{1+r}\pi_1\right) - h.$$

A.2 Proof of Corollary 1

When $w^* = 1$, we have

$$E[\pi_1|w^* = 1] = \frac{1}{4\tilde{\gamma}_1(1+r)\epsilon}\left[\epsilon\tilde{\gamma}_1 + \sigma_w(d^* - \epsilon v_N) + (1-\sigma_w)(\alpha d^* + (1-\alpha)d_N - \epsilon v_N)\right]^2.$$

On the other hand, when $w^* = 0$, we have

$$E[\pi_1|w^* = 0] = \frac{1}{4\tilde{\gamma}_1(1+r)\epsilon}\left[\epsilon\tilde{\gamma}_1 + \sigma_w(d_N - \epsilon v^*) + (1-\sigma_w)(d_N - \alpha\epsilon v^* - (1-\alpha)\epsilon v_N\right]^2.$$

Thus, $E[\pi_1|w^* = 1] > E[\pi_1|w^* = 0]$ holds if and only if

$$[(d^* - d_N) - \epsilon(v_N - v^*)](\sigma_w + (1-\sigma_w)\alpha) > 0.$$

Under the assumption that $\alpha > 0$, if $d^* - d_N > \epsilon(v_N - v^*)$ holds, we have $E[\pi_1|w^* = 1] > E[\pi_1|w^* = 0]$.□

A.3 Proof of Proposition 2

Define the discounted expected payoffs of entrepreneurs and workers as

$$V_t^e \equiv (1+\delta)\int_0^\infty (s_{w,t+s} - s_w^*)e^{-(1+\delta)s}ds, \quad V_t^w \equiv (1+\delta)\int_0^\infty (s_{e,t+s} - s_e^*)e^{-(1+\delta)s}ds.$$

Then, from Proposition 2 of Matsui and Matsuyama (1995), it follows that an equilibrium is an unique absorbing and globally accessible equilibrium if and only if it is risk-dominant equilibrium. Thus, the equilibrium with $k = \underline{k}$ and $h = \underline{h}$ is a unique absorbing and globally accessible equilibrium if and only if

$$s_e^* + s_w^* < 1 \Leftrightarrow \frac{\underline{h}/\beta - (\pi_M - \pi_L)}{\pi_H + \pi_L - 2\pi_M} + \frac{\underline{k}/(1-\beta) - (\pi_M - \pi_L)}{\pi_H + \pi_L - 2\pi_M} < 1$$
$$\Leftrightarrow A + B - \frac{\underline{h}}{\beta} - \frac{\underline{k}}{1-\beta} > 0.$$

Similarly, the equilibrium with $k = 0$ and $h = 0$ is an unique absorbing and globally accessible equilibrium if and only if $s_e^* + s_w^* > 1 \Leftrightarrow A + B - \underline{h}/\beta - \underline{k}/(1-\beta) < 0$. □

A.4 Proof of Proposition 3

To derive the dynamics of s_e, define entrepreneurs' expected payoffs from choosing $k = \underline{k}$ and $k = 0$ as follows.

$$\pi_{\underline{k}}^e = s_w \left[\pi_H(1-\beta) - \underline{k}\right] + (1 - s_w)\left[\pi_M(1-\beta) - \underline{k}\right],$$
$$\pi_0^e = s_w \pi_M (1-\beta) + (1 - s_w)\pi_L(1-\beta).$$

At each instantaneous time, $d\tau$ players revise their strategies, and each player chooses $k^* = \underline{k}$ ($h^* = \underline{h}$) with probability $\psi \in [0,1]$, which depends on the difference between the expected payoffs from the two actions. Entrepreneurs observe the share of workers currently investing and choose $k^* = \underline{k}$ with probability $\psi(\pi_{\underline{k}}^e - \pi_0^e)$ at τ and $k^* = 0$ with probability $1 - \psi(\pi_{\underline{k}}^e - \pi_0^e)$. Because $\pi_H - \pi_M > \pi_M - \pi_L$, ψ is an increasing function of s_w.

The share of entrepreneurs who change their strategies from $k^* = 0$ to $k^* = \underline{k}$ is given by

$$d\tau(1 - s_e)\psi(\pi_{\underline{k}}^e - \pi_0^e),$$

while the share of entrepreneurs who change their strategies from $k^* = \underline{k}$ to $k^* = 0$ is given by

$$d\tau s_e(1 - \psi(\pi_{\underline{k}}^e - \pi_0^e)).$$

Thus, the net share of entrepreneurs who change their strategies from $k^* = \underline{k}$ to $k^* = 0$ is given by

$$ds_e = d\tau(1 - s_e)\psi(\pi_{\underline{k}}^e - \pi_0^e) - d\tau s_e\left[1 - \psi(\pi_{\underline{k}}^e - \pi_0^e)\right].$$

Dividing by $d\tau$ yields

$$\frac{ds_e}{d\tau} = (1-s_e)\psi(\pi^e_{\underline{k}} - \pi^e_0) - s_e\left[1-\psi(\pi^e_{\underline{k}} - \pi^e_0)\right] = \psi(\pi^e_{\underline{k}} - \pi^e_0) - s_e.$$

Assume that each player who revises its strategy takes the better choice with probability one. That is, $\psi = 1$ if $\pi^e_{\underline{k}} - \pi^e_0 > 0$, $\psi \in [0,1]$ if $\pi^e_{\underline{k}} - \pi^e_0 = 0$, and $\psi = 0$ if $\pi^e_{\underline{k}} - \pi^e_0 < 0$. Then, because

$$\pi^e_{\underline{k}} - \pi^e_0 \begin{cases} > 0 \ \ if \ \ s_w > s^*_w \\ = 0 \ \ if \ \ s_w = s^*_w \\ < 0 \ \ if \ \ s_w < s^*_w, \end{cases}$$

we have

$$\psi(\pi^e_{\underline{k}} - \pi^e_0) \begin{cases} = 1 \ \ if \ \ s_w > s^*_w \\ \in [0,1] \ \ if \ \ s_w = s^*_w \\ = 0 \ \ if \ \ s_w < s^*_w. \end{cases}$$

Finally, the dynamics $ds_{e_t}/d\tau$ are given by

$$\frac{ds_e}{d\tau} \begin{cases} = 1 - s_e \ \ if \ \ s_w > s^*_w \\ \in [-s_e, 1-s_e] \ \ if \ \ s_w = s^*_w \\ = -s_e \ \ if \ \ s_w < s^*_w. \end{cases}$$

Next, to derive the dynamics of s_w, define workers' expected payoffs from choosing $h^* = \underline{h}$ and $h^* = 0$ as follows.

$$\pi^w_{\underline{h}} = s_e\left(\pi_H\beta - \underline{h}\right) + (1-s_e)\left(\pi_M\beta - \underline{h}\right), \quad \pi^w_0 = s_e\pi_M\beta + (1-s_e)\pi_L\beta.$$

Workers observe s_e, the share of entrepreneurs currently investing, and choose $h^* = \underline{h}$ with probability $\psi(\pi^w_{\underline{h}} - \pi^w_0)$ and $h^* = 0$ with probability $1-\psi(\pi^w_{\underline{h}} - \pi^w_0)$. Because $\pi_H - \pi_M > \pi_M - \pi_L$, ψ is an increasing function of s_e.

The share of workers who change their strategy from $h^* = 0$ to $h^* = \underline{h}$ is given by

$$d\tau(1-s_w)\psi(\pi^w_{\underline{h}} - \pi^w_0).$$

On the other hand, the share of workers who change their strategies from $h^* = \underline{h}$ to $h^* = 0$

is given by

$$d\tau s_w(1-\psi(\pi^w_{\underline{h}}-\pi^w_0)).$$

Thus, the net share of workers who change their strategies from $h^*=\underline{h}$ to $h^*=0$ is given by

$$ds_w = d\tau(1-s_w)\psi(\pi^w_{\underline{h}}-\pi^w_0) - d\tau s_w\left[1-\psi(\pi^w_{\underline{h}}-\pi^w_0)\right].$$

Dividing by $d\tau$ yields

$$\frac{ds_w}{d\tau} = (1-s_w)\psi(\pi^w_{\underline{h}}-\pi^w_0) - s_w\left[1-\psi(\pi^w_{\underline{h}}-\pi^w_0)\right] = \psi(\pi^w_{\underline{k}}-\pi^w_0) - s_w.$$

Assume $\psi=1$ if $\pi^w_{\underline{h}}-\pi^w_0>0$, $\psi\in[0,1]$ if $\pi^w_{\underline{h}}-\pi^w_0=0$, and $\psi=0$ if $\pi^w_{\underline{h}}-\pi^w_0<0$. Because

$$\pi^w_{\underline{h}}-\pi^w_0\begin{cases} >0 \;\; if \;\; s_e>s_e^* \\ =0 \;\; if \;\; s_e=s_e^* \\ <0 \;\; if \;\; s_e<s_e^*, \end{cases}$$

we have

$$\psi(\pi^w_{\underline{h}}-\pi^w_0)\begin{cases} =1 \;\; if \;\; s_e>s_e^* \\ \in[0,1] \;\; if \;\; s_e=s_e^* \\ =0 \;\; if \;\; s_e<s_e^*. \end{cases}$$

Finally, the dynamics $ds_w/d\tau$ are given by

$$\frac{ds_w}{d\tau}\begin{cases} =1-s_w \;\; if \;\; s_e>s_e^* \\ \in[-s_w,1-s_w] \;\; if \;\; s_e=s_e^* \\ =-s_w \;\; if \;\; s_e<s_e^*. \end{cases}$$

Based on the dynamics above, $(s_e,s_w)\in\{(1,1),(0,0)\}$ are stable stationary equilibria.

The solution trajectory starting from $(s_e,s_w)=(s_e^*,s_w^*)$ is ambiguous. If we assume that the share never changes from (s_e^*,s_w^*), it might be stationary. However, without such an assumption, it follows a trajectory to $(s_e,s_w)=(1,1)$ or $(0,0)$ possibly with initial delay. □

A.5 Proof of Corollary 3

- Case 1: $s_{e_0} > s_e^*$ and $s_{w_0} > s_w^*$

  Because both s_{e_τ} and s_{w_τ} are increasing and the initial condition satisfies $s_{e_0} > s_e^*$ and $s_{w_0} > s_w^*$, the trajectory is given by

$$s_{e_\tau} = 1 - e^{-\tau}(1 - s_{e_0}), \quad s_{w_\tau} = 1 - e^{-\tau}(1 - s_{w_0}).$$

- Case 2: $s_{e_0} < s_e^*$ and $s_{w_0} < s_w^*$

  Because both s_{e_τ} and s_{w_τ} are decreasing and the initial condition satisfies $s_{e_0} < s_e^*$ and $s_{w_0} < s_w^*$, the trajectory is given by

$$s_{e_\tau} = e^{-\tau} s_{e_0}, \quad s_{w_\tau} = e^{-\tau} s_{w_0}.$$

- Case 3: $s_{e_0} > s_e^*$ and $s_{w_0} < s_w^*$

  First, the trajectory in the region $s_{e_0} > s_e^*$ and $s_{w_0} < s_w^*$ is given by

$$s_{e_\tau} = e^{-\tau} s_{e_0}, \quad s_{w_\tau} = 1 - e^{-\tau}(1 - s_{w_0}).$$

  However, because s_{e_τ} is decreasing and s_{w_τ} is increasing and the initial condition is given by $s_{e_0} > s_e^*$ and $s_{w_0} < s_w^*$, the economy intersects $ds_e/d\tau = 0$ or $ds_w/d\tau = 0$ at some time τ^*. Thus, the economy would be on the trajectory of Case 1 or Case 2 with new initial conditions. Moreover, the economy would be at $s_{e_\tau^*} = s_e^*$ or $s_{w_\tau^*} = s_w^*$ when it passes through $ds_e/d\tau = 0$ or $ds_w/d\tau = 0$. This means that there will be multiple trajectories as shown in Proposition 3. However, it should be noted that the economy eventually converges to $(1, 1)$ if it reaches $s_{w_\tau^*} = s_w^*$, while it converges to $(0, 0)$ if it reaches $s_{e_\tau^*} = s_e^*$. Only if it reaches $(s_{e_\tau^*}, s_{w_\tau^*}) = (s_e^*, s_w^*)$ does it randomly converge to $(s_e, s_w) = (1, 1)$ or $(0, 0)$. In that sense, the equilibrium to which the economy eventually converges is uniquely determined by the initial shares, except for the initial shares which pass through $(s_{e_\tau^*}, s_{w_\tau^*}) = (s_e^*, s_w^*)$.

- Case 4: $s_{e_0} < s_e^*$ and $s_{w_0} > s_w^*$

The trajectory in the region $s_{e_0} < s_e^*$ and $s_{w_0} > s_w^*$ is given by

$$s_{e_\tau} = 1 - e^{-\tau}(1 - s_{e_0}), \quad s_{w_\tau} = e^{-\tau} s_{w_0}.$$

However, because s_{e_τ} is increasing, s_{w_τ} is decreasing, and the initial condition is given by $s_{e_0} < s_e^*$ and $s_{w_0} > s_w^*$, the economy intersects $ds_e/d\tau = 0$ or $ds_w/d\tau = 0$ at some time τ^*. Thus, the economy would be on the trajectory of Case 1 or Case 2 with new initial conditions. Further, the economy would be on $s_{e_{\tau}^*} = s_e^*$ or $s_{w_{\tau}^*} = s_w^*$ when it crosses $ds_e/d\tau = 0$ or $ds_w/d\tau = 0$. Based on the same logic as in Case 3, multiple trajectories exist, but the equilibrium to which the economy eventually converges is still uniquely determined by the initial conditions except for the initial shares which pass through $(s_{e_{\tau}^*}, s_{w_{\tau}^*}) = (s_e^*, s_w^*)$. $\square$

A.6 Proof of Proposition 4

Denote $S \equiv [0,1]^2$. The first lemma proves the existence of the unique line S^M composed of the initial shares which pass through (s_e^*, s_w^*).

Lemma 1 *Denote the set of initial shares which pass through (s_e^*, s_w^*) on their trajectories by S^M. Thus, S^M is a continuous line.*

Proof: From Corollary 3, it follows that the initial shares in the region of $(s_e > s_e^*, s_w > s_w^*)$ and $(s_e < s_e^*, s_w < s_w^*)$ do not move toward (s_e^*, s_w^*). On the other hand, there exist initial shares that reach (s_e^*, s_w^*) in $(s_e > s_e^*, s_w < s_w^*)$ or $(s_e < s_e^*, s_w > s_w^*)$.

Regarding the case of $(s_e > s_e^*, s_w < s_w^*)$, the trajectories are given as $s_{e_\tau} = e^{-\tau} s_{e_0}$ and $s_{w_\tau} = 1 - e^{-\tau}(1 - s_{w_0})$. Therefore, by substituting s_e^* and s_w^* into s_{e_τ} and s_{w_τ} respectively, the initial shares for each fixed τ are uniquely determined as $(e^\tau s_e^*, e^\tau s_w^* + (1 - e^\tau))$. Here, $e^\tau s_e^*$ is increasing in τ, while $e^\tau s_w^* + (1 - e^\tau)$ is decreasing in τ, since $s_w^* < 1$ holds. Thus, based on the constraint $s_{e_0} \in [0,1]$ and $s_{w_0} \in [0,1]$, initial shares that belong to S^M in this region are determined as

$$(e^\tau s_e^*, e^\tau s_w^* + (1 - e^\tau)) \quad for \quad 0 < \tau \le \min\left[\max\left(\ln\left(\frac{s_{e_0}}{s_e^*}\right)\right), \max\left(\ln\left(\frac{1 - s_{w_0}}{1 - s_w^*}\right)\right)\right].$$

Next, as for the case of $(s_e < s_e^*, s_w > s_w^*)$, the trajectories are given by $s_{e_\tau} = 1 - e^{-\tau}(1 - s_{e_0})$ and $s_{w_\tau} = e^{-\tau} s_{w_0}$. Hence, by substituting s_e^* and s_w^* into s_{e_τ} and s_{w_τ}, for each fixed

τ, the unique initial share is given by $(e^{\tau}s_e^* + (1-e^{\tau}), e^{\tau}s_w^*)$. Moreover, $e^{\tau}s_e^* + (1-e^{\tau})$ is decreasing in τ and $e^{\tau}s_w^*$ is increasing in τ. Thus, based on the constraint $(s_{e_0}, s_{w_0}) \in S$, initial shares which are in S^M in this region are given by

$$(e^{\tau}s_e^* + (1-e^{\tau}), e^{\tau}s_w^*) \;\; for \;\; 0 < \tau \leq \min\left[\max\left(\ln\left(\frac{1-s_{e_0}}{1-s_e^*}\right)\right), \max\left(\ln\left(\frac{s_{w_0}}{s_w^*}\right)\right)\right].$$

Needless to say, $(s_e^*, s_w^*) \in S^M$ holds. Thus S^M is given as follows.

$$(s_e^*, s_w^*) \cup \left\{(e^{\tau}s_e^*, e^{\tau}s_w^* + (1-e^{\tau})) \;\; for \;\; 0 < \tau \leq \min\left[\ln\left(\frac{1}{s_e^*}\right), \ln\left(\frac{1}{1-s_w^*}\right)\right]\right\}$$
$$\cup \left\{(e^{\tau}s_e^* + (1-e^{\tau}), e^{\tau}s_w^*) \;\; for \;\; 0 < \tau \leq \min\left[\ln\left(\frac{1}{1-s_e^*}\right), \ln\left(\frac{1}{s_w^*}\right)\right]\right\}.$$

Obviously, the second and third sets are compositions of continuous functions of τ and therefore they are continuous lines as well. Further, because $(e^0 s_e^*, e^0 s_w^* + (1-e^0)) = (e^0 s_e^* + (1-e^0), e^0 s_w^*) = (s_e^*, s_w^*)$ holds, S^M is continuous. $\Box$

The next lemma proves that this line S^M separates S/S^M into S^I and S^D.

Lemma 2 *S^M separates S/S^M into S^I and S^D.*

Proof: Combinations of initial shares can be classified into six cases.

1. $s_{e_0} > s_e^*, s_{w_0} > s_w^*$: The initial shares belong to S^I from Corollary 3.

2. $s_{e_0} < s_e^*, s_{w_0} < s_w^*$: The initial shares belong to S^D from Corollary 3.

3. $s_{e_0} > s_e^*, s_{w_0} = s_w^*$, and $s_{e_0} = s_e^*, s_{w_0} > s_w^*$: The initial shares belong to S^I, since all of them enter the region $(s_e > s_e^*, s_w > s_w^*)$ following the dynamics.

4. $s_{e_0} < s_e^*, s_{w_0} = s_w^*$, and $s_{e_0} = s_e^*, s_{w_0} < s_w^*$: The initial shares belong to S^D, since they enter the region $(s_e < s_e^*, s_w < s_w^*)$ following the dynamics.

5. $s_{e_0} > s_e^*, s_{w_0} < s_w^*$: There are two sub-cases.

   (a) $s_{e_0} \geq e^{\tau}s_e^*$ and $s_{w_0} \geq e^{\tau}s_w^* + (1-e^{\tau})$ with at least one strict inequality for any $t \in (0, \min\{\ln(1/s_e^*), \ln(1/(1-s_w^*))\})$: The initial shares belong to S^I because Lemma 1 implies that such shares intersect one of the points $(s_e > s_e^*, s_w = s_w^*)$ following their trajectories.

(b) $s_{e_0} \leq e^\tau s_e^*$ and $s_{w_0} \leq e^\tau s_w^* + (1 - e^\tau)$ with at least one strict inequality for any $\tau \in (0, \min\{\ln(1/s_e^*), \ln(1/(1 - s_w^*))\})$: The initial shares belong to S^D, since they intersect one of the points $(s_e = s_e^*, s_w < s_w^*)$ following their trajectories by Lemma 1.

6. $s_{e_0} < s_e^*, s_{w_0} > s_w^*$: There are two sub-cases.

(a) $s_{e_0} \geq e^\tau s_e^* + (1 - e^t)$ and $s_{w_0} \geq e^\tau s_w^*$ with at least one strict inequality for any $t \in (0, \min\{\ln(1/(1 - s_e^*)), \ln(1/s_w^*)\})$: The initial shares belong to S^I, since they intersect one of the points $(s_e = s_e^*, s_w > s_w^*)$ following their trajectories by Lemma 1.

(b) $s_{e_0} \leq e^\tau s_e^* + (1 - e^\tau)$ and $s_{w_0} \leq e^\tau s_w^*$ with at least one strict inequality for any $t \in (0, \min\{\ln(1/(1 - s_e^*)), \ln(1/s_w^*)\})$: The initial shares belong to S^D, since they intersect one of the points $(s_e < s_e^*, s_w = s_w^*)$ following their trajectories by Lemma 1.

Therefore, S/S^M is divided into S^I and S^D by S^M. $\square$

Fix s_w^* and pick the new s_e^*, which is denoted by $s_e^{**} < s_e^*$. Then, by Lemma 2, the new S^M with (s_e^{**}, s_w^*) is given by

$$(s_e^{**}, s_w^*) \cup \left\{(e^\tau s_e^{**}, e^\tau s_w^* + (1 - e^\tau)) \;\; for \;\; 0 < \tau \leq \min\left\{\ln\left(\frac{1}{s_e^{**}}\right), \ln\left(\frac{1}{1 - s_w^*}\right)\right\}\right\}$$
$$\cup \left\{(e^\tau s_e^{**} + (1 - e^\tau), e^\tau s_w^*) \;\; for \;\; 0 < \tau \leq \min\left\{\ln\left(\frac{1}{1 - s_e^{**}}\right), \ln\left(\frac{1}{s_w^*}\right)\right\}\right\}.$$

One of the main differences from S^M with (s_e^*, s_w^*) is that for each fixed s_{w_0}, the corresponding s_{e_0} belonging to the new S^M is smaller than s_{e_0} in S^M with (s_e^*, s_w^*) because $e^t(s_e^* - s_e^{**}) > 0$. Further, from the relationship $s_e^{**} < s_e^*$, we have

$$\min\left\{\ln\left(\frac{1}{s_e^{**}}\right), \ln\left(\frac{1}{1 - s_w^*}\right)\right\} \geq \min\left\{\ln\left(\frac{1}{s_e^*}\right), \ln\left(\frac{1}{1 - s_w^*}\right)\right\}$$
$$\min\left\{\ln\left(\frac{1}{1 - s_e^{**}}\right), \ln\left(\frac{1}{s_w^*}\right)\right\} \leq \min\left\{\ln\left(\frac{1}{1 - s_e^*}\right), \ln\left(\frac{1}{s_w^*}\right)\right\}.$$

Then, by Lemma 2, the following set can be transferred from S^D to S^I.

$$\left\{((e^{\tau}s_e^{**},\min\{e^{\tau}s_e^*,1\}],e^{\tau}s_w^*+(1-e^{\tau}))for\ \ 0<\tau\leq\min\left\{\ln\left(\frac{1}{s_e^{**}}\right),\ln\left(\frac{1}{1-s_w^*}\right)\right\}\right\}$$
$$\cup\left\{((\max\{e^{\tau}s_e^{**}+(1-e^{\tau}),0\},e^{\tau}s_e^*+(1-e^{\tau})],e^{\tau}s_w^*)for\ \ 0<\tau\leq\min\left\{\ln\left(\frac{1}{1-s_e^*}\right),\ln\left(\frac{1}{s_w^*}\right)\right\}\right\},$$

which is non-empty for any $s_e^{**}<s_e^*$. This is because (s_e^*,s_w^*) always belongs to this set. Regarding the case that s_w^* decreases for a fixed s_e^*, the same proof applies. □

A.7 Proof of Corollary 4

(1) It is obvious because A and B are given by

$$A\equiv\frac{\alpha\left[(d_I-\epsilon v_I)-(d_N-\epsilon v_N)\right]}{4(1+i)\gamma_0\epsilon}\left[2\epsilon\tilde{\gamma}_1+\alpha\left(d_I-\epsilon v_I\right)+(2-\alpha)\left(d_N-\epsilon v_N\right)\right],$$
$$B\equiv\frac{(1-\alpha)\left[(d_I-\epsilon v_I)-(d_N-\epsilon v_N)\right]}{4(1+i)\gamma_0\epsilon}\left[2\epsilon\tilde{\gamma}_1+(1+\alpha)\left(d_I-\epsilon v_I\right)+(1-\alpha)\left(d_N-\epsilon v_N\right)\right].$$

(2) First, π_H, π_M, and π_L are given by

$$\pi_H\equiv\frac{1}{4(1+i)\gamma_0\epsilon}\left(\epsilon\tilde{\gamma}_1+d_I-\epsilon v_I\right)^2,$$
$$\pi_M\equiv\frac{1}{4(1+i)\gamma_0\epsilon}\left[\epsilon\tilde{\gamma}_1+\alpha(d_I-\epsilon v_I)+(1-\alpha)(d_N-\epsilon v_N)\right]^2,$$
$$\pi_L\equiv\frac{1}{4(1+i)\gamma_0\epsilon}\left(\epsilon\tilde{\gamma}_1+d_N-\epsilon v_N\right)^2.$$

Then, $\pi_H>\pi_M>\pi_L$ and $\partial\pi_H/\partial\tilde{\gamma}_1>\partial\pi_M/\partial\tilde{\gamma}_1>\partial\pi_L/\partial\tilde{\gamma}_1>0$ hold. Moreover, because $\alpha\in(0,1/2)$ and π_H, π_M, and π_L are convex, the following inequalities hold.

$$\frac{\partial\pi_H}{\partial\tilde{\gamma}_1}-\frac{\partial\pi_M}{\partial\tilde{\gamma}_1}>\frac{\partial\pi_M}{\partial\tilde{\gamma}_1}-\frac{\partial\pi_L}{\partial\tilde{\gamma}_1}>0$$

Thus, $\partial s_e^*/\partial\tilde{\gamma}_1<0$ and $\partial s_w^*/\partial\tilde{\gamma}_1<0$ are obtained. From Proposition 4, S^I becomes larger with $\tilde{\gamma}_1$. □

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Figure 1: Timing of the Stage Game

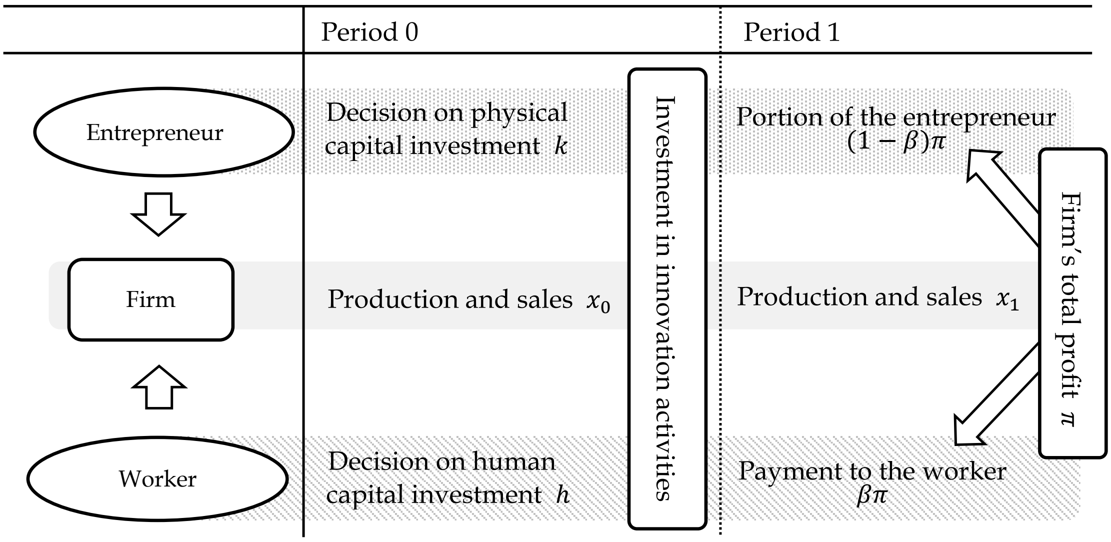


Figure 2: Region of $(\underline{k}, \underline{h})$ and Equilibrium

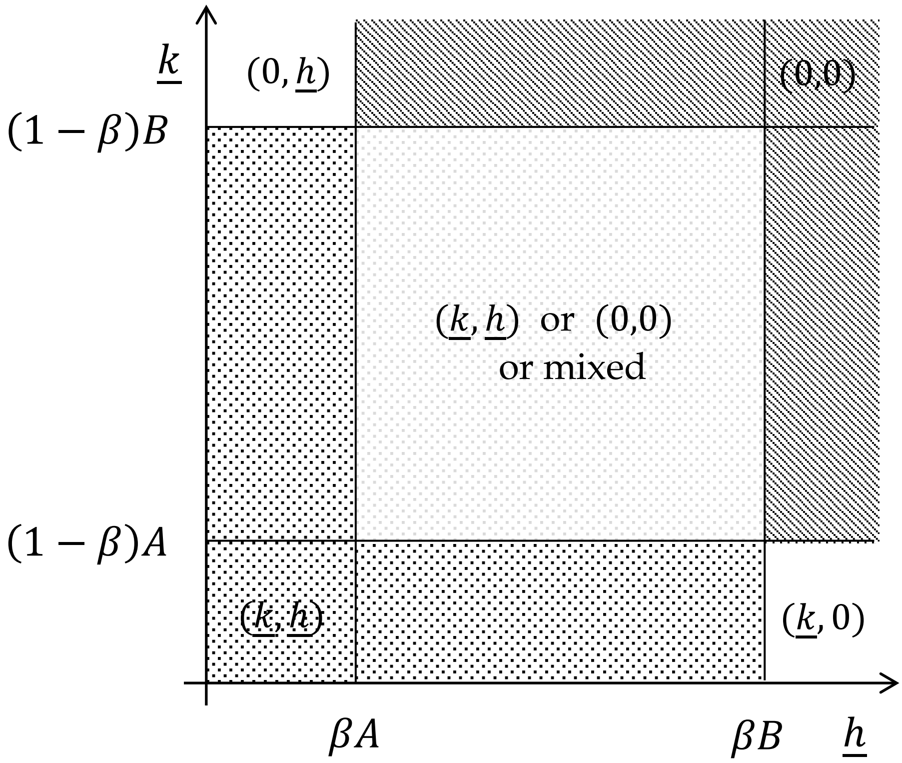


Figure 3: Types of Innovation and Players' Views on the Target Inflation Rate

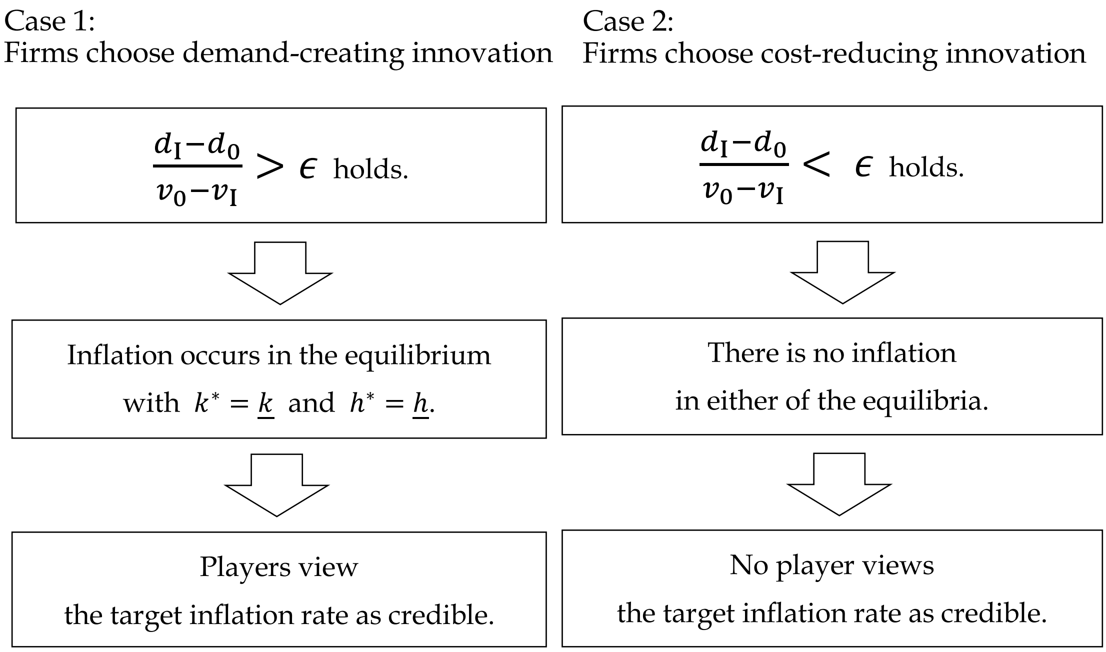


Figure 4: Phase Diagram of The Game

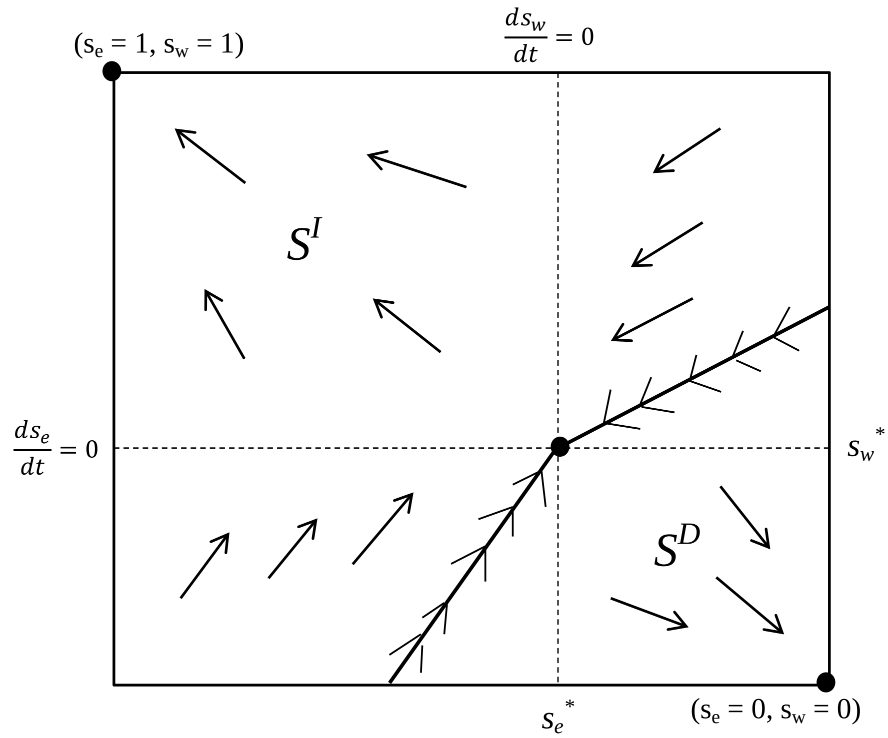
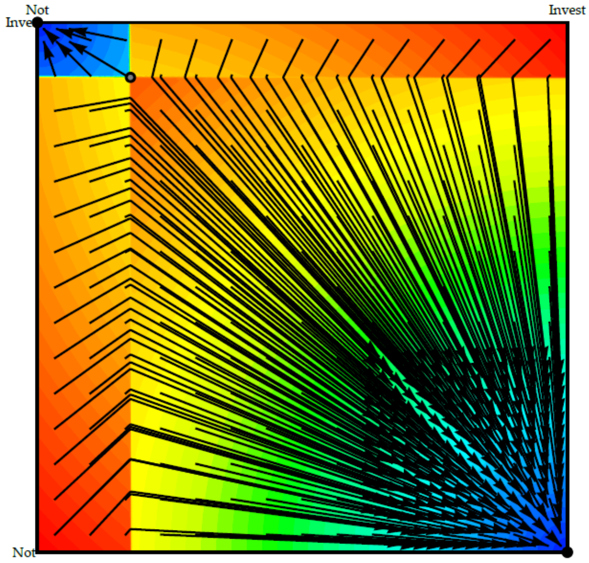
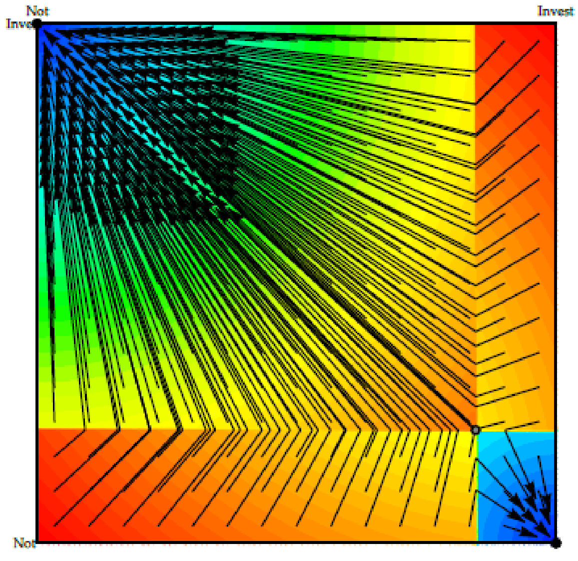


Figure 5: Simulation Results of the Best Response Dynamics

(a) The Expected Inflation Rate is 0%



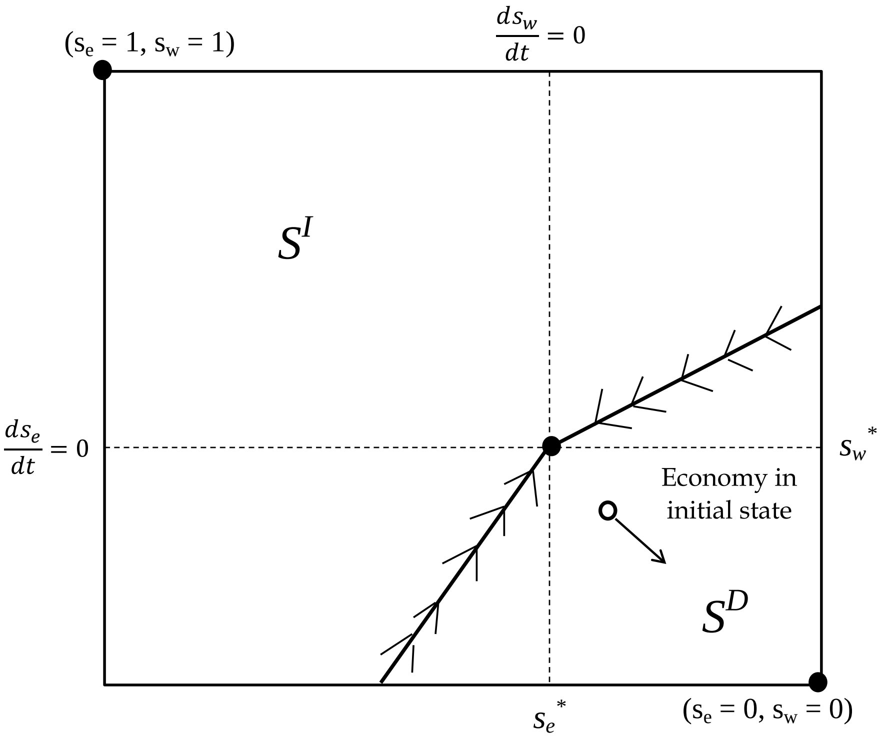
(b) The Expected Inflation Rate is 2%



Note: The parameter values are as follows: $\epsilon = 1, d_0 = 100, d_I = 400, d_N = 100, v_0 = 9900, v_I = 9800, v_N = 9900, \alpha = 0.45, \beta = 0.49, i = 0, \underline{k} = 2.5, \underline{h} = 2.5$. The range of the expected inflation rate is $\widetilde{\gamma_1}/\gamma_0 - 1 \in [0, 0.02]$, which means that the inflation rate is 0% in the equilibrium with $k^* = 0$ and $h^* = 0$, while it is 2% in the equilibrium with $k^* = \underline{k}$ and $h^* = \underline{h}$. Panel (a) shows the best response dynamics for the lower bound of this range (0%), while panel (b) shows those for the upper bound (2%).

Figure 6: Combination of the Target Inflation Rate and Other Measures

(a) Without Other Policy Measures



(b) With Other Policy Measures