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Toshiyuki Sakiyama and Tetsuya Yamada

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Market Liquidity and Systemic Risk in Government Bond Markets: A Network Analysis and Agent-Based Model Approach

Toshiyuki Sakiyama* and Tetsuya Yamada**

Abstract

Recently, market liquidity in government bond markets has been attracting attention by market participants and central bankers since interest rate spikes have become frequent under unconventional monetary easing. We analyze network structures in the JGB (Japanese government bond) market using daily data from the BOJ-NET (the Bank of Japan Financial Network System). To our knowledge, this is the first network analysis on the government bond market. We studies how QQE (quantitative and qualitative monetary easing) has affected JGB market structure. We also conduct event studies for the spikes in interest rates (the shock after the introduction of QQE and the so-called VaR [Value at Risk] shock in 2003). In addition, we propose an agent-based model that accounts for the findings of the above event studies, and show that not only the capital adequacy of market participants but also the network structure are important for financial market stability.

Keywords: Market Liquidity; Government bond markets; Quantitative and Qualitative Easing; Network analysis; Systemic risk; Agent-based model
JEL classification: C58, G12, G18

* Deputy Director, Economic and Financial Studies Division, Institute for Monetary and Economic Studies (currently Financial Markets Department), Bank of Japan (E-mail: toshiyuki.sakiyama@boj.or.jp)
** Director, Economic and Financial Studies Division, Institute for Monetary and Economic Studies, Bank of Japan (E-mail: tetsuya.yamada@boj.or.jp)

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1. Introduction

Recently, market liquidity of government bond markets has been declining because central bank purchases huge amounts of government bonds in many advanced economies. The market liquidity, therefore, has been attracting more attention from market participants, central bankers and policymakers.

Under the circumstances, we have seen frequent spikes in interest rates in advanced economies. For example, U.S. Treasury securities experienced an unusually high level of volatility, with prices pulling off a high-speed day return, on October 15, 2014. A similar spike was observed in Germany in mid-April 2015. A U.S. Treasury report provides details about how and why the spike occurred on October 15, 2014, concluding that it was caused by high frequency trading, as well as strict financial regulation (U.S. Department of the Treasury *et al.* [2015]). *The IMF Global Stability Report* also points out that market participants have become worried that both the level of market liquidity and its resilience may be declining, especially for bonds, and that as a result the risks associated with a liquidity shock may be rising (International Monetary Fund [2015]). A BIS publication reaches similar conclusions (Bank for International Settlements [2016]). In Japan, a similar spike was observed in the Japanese government bond (JGB) market after the introduction of QQE on April 4, 2013, for the first time since the famous fire sale in 2003 known as the "VaR (Value at Risk) shock." As mentioned above, the market liquidity of government bond markets has been getting a lot of attention, especially in advanced economies. Market liquidity and systemic risk in these markets has become correspondingly hot issue.

In this paper, we study the network structure of the JGB market using daily settlement data from the BOJ-NET (the Bank of Japan financial network system) and study how QQE has affected JGB market structure. In addition, we investigate systemic risk and financial market stability from a network perspective, using event studies of fire sales and agent-based model simulations.

Network analysis has become popular among central bankers, policy makers and other researchers. However, most research to date has focused on interbank money markets (Inaoka *et al.* [2004], Boss *et al.* [2004] , Upper and Worms [2004], Müller [2006], Lelyveld and Liedorp [2006], Soramäki *et al.* [2006], Hans and Grégory [2007], Iori *et al.* [2008], Imakubo and Soejima [2010], Cont, Moussa, and Santos [2013], Minoiu and Reyes [2013], Squartini, Lelyveld and Garlaschelli [2013], Veld and Lelyveld [2014], Beltran, Bolotnyy, and Klee [2015]). The pioneering papers on interbank money markets studied the network structure only at a few points in time, because of the absence of time series data or the burden of computation (Inaoka *et al.* [2004], Müller [2006], Soramäki *et al.* [2006], Imakubo and Soejima [2010]). Recently, however, several papers have introduced time series analysis of network indicators (Squartini, Lelyveld and Garlaschelli [2013], Beltran, Bolotnyy, and Klee [2015]). For example, Beltran, Bolotnyy, and Klee (2015) analyzed link numbers in the federal funds market from daily data. They showed that link numbers experienced a remarkable decline after Lehman Brothers' bankruptcy because market participants suspended transactions with those with lower credit ratings. In this paper, we also analyze the daily dynamics of network indicators to investigate how QQE has affected the JGB market. To our knowledge, our paper is the first network analysis on government bond markets.

In this paper, we offer original contributions in the following ways. First, we revealed the network structure of the JGB market. Second, we directly analyze the daily dynamics of government bond markets using JGB "spot" data, although many papers adopt a more indirect approach using JGB "future" data (Kurosaki *et al*. [2015]). Third, we investigate market liquidity and systemic risk in terms of network indicators as well as standard liquidity indicators such as trade volume and price range over volume. Finally we construct a systemic risk simulation model that takes account of the market network structure.

The rest of the paper is organized as follows. Section 2 explains the fundamental network structure of the JGB market, and studies the structural changes in the markets after the introduction of QQE. Section 3 presents event studies of two particular fire sales. Section 4 introduces an agent-based model and simulates some illustrative cases that provide insight for macroprudential policy. Section 5 concludes.

2. Analysis of the basic network structure in the JGB market

In this section, we provide an overview of network structure in the JGB market using daily settlement data from the BOJ-NET.[1,2] Applying network analysis to government bond market, we should note the differences from the interbank money market. Financial institutions use the money market to smooth their daily cash positions. As a result, each of them uses the money market almost every day with a steady set of institutions (its steady customers). In government bond markets, financial institutions also deal with a steady set of institutions, but they do not transact every day. Transactions take place on an *ad hoc* basis, their timing sensitive to an institution's trading needs and counterparty availability. The network structure, therefore, tends to be changeable. If we analyze the normal situations, we are not interested in such potentially large but short-term variation in the network structure. Thus, we overview the monthly network structures not daily structures. On the other hand, during situations of stress events the sudden shifts in network structure become more important. In section 3, therefore, we study the dynamics of the daily network structure in order to investigate the effects of stress events.

A. Network structure in the JGB market

First, we give an overview of the network structure in the JGB market under normal circumstances. Figure 1 plots the network structure of the JGB market in June 2015. We

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[1] The Bank of Japan provides a system not only to settle financial transactions but also to transfer Japanese government bonds. In order to eliminate settlement risk, a DVP (delivery-versus-payment) mechanism has been introduced for JGB transactions, where the delivery of a JGB occurs simultaneously with the corresponding transfer of funds. Each JGB transaction can therefore be observed from the data on DVP settlements.

[2] This is the settlement data for JGB transactions, which is different from the transaction data. It includes not only outright transactions (buying and selling bonds) but also repos and collateralized call trades. Transactions between the Bank of Japan and the Ministry of Finance are not included.

aggregate all the transactions in June 2015 to reveal the basic network structure. The network can be seen to have a core-periphery structure, where some highly connected institutions play the role of hubs (situated in the network "core") while others are located around the hubs (in the network "periphery"). The core-periphery structure is also widely observed in interbank money markets.[3]

Now we introduce two network measures widely used in network analyses: degree (number of links) and local clustering coefficient.

- **Degree/Number of Links:** The degree of a node is defined by the number of links attached to it. Figure 2 shows that many financial institutions have a small number of links, while a few have a large number. Only 14 institutions had more than 50 links in June 2015. In addition, Figure 3 reveals a strong correlation between degree and transaction value; in other words, financial institutions with high degree tend to transact large amounts of securities.

- **Local Clustering Coefficient:** The local clustering coefficient for a given node "X" is defined as a ratio: the proportion of all the nodes attaching to X that are also attached to each other. This takes a maximal value of one when all the nodes attached to node X also share links with every other node in the cluster; and a minimal value zero when there are no other links between any of them. Local clustering coefficients for the majority of financial institutions are between 0.1 and 0.3 (Figure 4).

Financial institutions can be broadly divided into the following three types (Figure 4). The first type is financial institutions located in area A. These have an extremely high degree and a low local clustering coefficient, and are regarded as the institutions that comprise the core.[4] Major banks and major security companies are typically situated in the core. The second type is financial institutions in area B. These have a low degree but a high local clustering coefficient, which means that they tend to transact only with a

[3] See Imakubo and Soejima (2010).
[4] In this paper, we define financial institutions which have more than 40 links as core institutions.

small number of habitual counterparties. The third type is financial institutions in area C. They have a low degree and a low local clustering coefficient, which means that they tend to transact only with some particular core institutions. In this paper, we define financial institutions in areas B and C as periphery institutions; regional banks and small security companies are included among these.

B. Structural changes in the JGB market after the introduction of QQE

In this section, we study how QQE has affected JGB market structure. After the introduction of QQE, the outstanding amounts of both long-term and short-term bonds declined due to large-scale purchases by the Bank of Japan (Figure 5). Figure 6 shows the interest rates for repos, 3-month, and 10-year bonds from 2013 to 2016. We can see that the 10-year rate spiked just after the introduction of QQE on April 4, 2013; thereafter, all three interest rates basically trend downwards, with the short-term repos and 3-month rates sometimes falling into negative territory after the expansion of QQE on October 31, 2014. Under the background, we examine the daily dynamics of selected network indicators using JGB "spot" data.[5]

- **Number of players** (Figure 7): The number of players is one simple network indicator. The number of players in the T-Bill market has witnessed a drastic decline since the introduction of QQE. In the long-term JGB market, there has been no structural change, but the number of buyers tends to increase rapidly when the interest rate spikes. We will address this tendency in more detail in Section 3.

- **Network links** (Figures 8–12): The number of links is a popular network indicator as mentioned in Section 2-A. The number of links in the long-term JGB market tends to increase when the interest rate spikes, just as the number of buyers does

[5] A 10-day backward moving average is applied to the daily dynamics of network indicators, because we wish to analyze the trend for transactions in the JGB market. We also obtained similar results when we applied a 5-day backward moving average. There are other methods of estimating the trend; for example, Inaoka *et al*. (2004) calculated the number of links using only transactions of over 10 million yen.

(Figure 8). In contrast, the number of links has decreased in the T-Bill market since the introduction of QQE, reflecting the fact that the T-bill rate has reached zero (Figures 8-9).

In more detail, the number of links decreased mainly in core-periphery pairs (Figure 10). The core-core pairs, curiously, recovered for a while after the expansion of QQE, but this temporary recovery was mainly due to foreign investors (Figure 11). After the expansion of QQE, foreign investors could borrow Japanese yen at a negative rate of around -0.3% via currency swaps between U.S. dollars and yen, and because of the spread they earned (about 0.3%), they could purchase T-bills in spite of their very low rate. This caused the T-bill rate to fall into negative territory—a result consistent with BOJ (2015).

Finally, Figure 12 shows that, since the expansion of QQE, even the number of links in medium-term markets (1-4year, 4-7year) have begun to decrease. It is thought that some market participants have cut down on transactions because medium-term rates have reached zero. On the other hand, long-term markets (7-10year, over 10-year) are still unchanged structurally.

From the view point of market liquidity, how should we interpret this decrease in the link number? There are 4 categories of market liquidity indicator: tightness, resiliency, depth, and volume (Engle and Lange [1997], Kurosaki *et al*. [2015]). Volume indicators include trade volume which shows how easily large transactions can be made. We show the daily dynamics of trade volume for our BOJ-NET data in Appendix 1 (Figure 27). Indicators of tightness include bid-ask spread, which indicates the difference between the price at which traders can execute a transaction when they wish to sell and the price at which they can execute that transaction when they wish to buy. Indicators of resiliency include price range over volume, which is a measure of how rapidly traders can execute transactions even if there are shocks to prices. We show the daily dynamics of price range over volume for our BOJ-NET data in Appendix 1 (Figure 28). Depth indicators typically include the volume of limit orders at the best-ask (-bid); this indicates the size of the limit order book at the current price level. From this categorization, we could interpret the link number (and the number of participants) as a kind of depth indicator: "depth of the market". More precisely, the link number is not just the volume of orders but of potential ask (bid) orders. We should therefore call it the "potential depth" of the market. Our findings above can thus be reinterpreted as showing that after the introduction of QQE the potential depth of the market has decreased in T-bill and medium-term markets.

3. Network analysis and systemic risk: event studies for the "VaR shock" and the shock after the introduction of QQE

In Section 2-B, we confirm that the long-term JGB market experienced no basic structural change, although it was noted that the number of links tended to increase when the interest rate spiked. In this section, we present some event studies of fire sales in the long-term JGB market to show what happened during these interest rate spikes. Figure 13 shows the 10-year interest rate from 1995 to 2015. It can be observed that the interest rate sometimes spikes up rapidly just after reaching an extremely low level. Two well-known fire sale events are the so-called "government investment policy shock" in 1997 and the "VaR shock" in 2003. A similar event was experienced immediately after the introduction of QQE in 2013. In this section, we investigate the dynamics of the market structure for the VaR shock and the shock after the introduction of QQE, by applying network analysis to BOJ-NET data.

Our analysis reveals that:

- **Number of transactions (settlements)** (Figure 14): Before interest rate spikes, the number of transactions tended to decrease significantly, implying a corresponding drop in market liquidity.

- **Number of players** (Figure 15): During interest rate spikes, the number of players participating in the market increased; increases in the number of buyers were especially notable.
- **Number of links** (Figures 16–17): During interest rate spikes, numbers of network links increased rapidly, especially in core-periphery pairs. The implication is that core institutions are selling and periphery institutions buying bonds during interest rate spikes. It also suggests that core institutions needed to find new periphery counterparties in order to complete their desired sales, while new periphery institutions entered the market to buy them. This is why the number of buyers increased drastically.
- **Number of Transactions per player** (Figure 18): During interest rate spikes, the number and volume of purchases per periphery buyer decreased rapidly. It may suggest that the new periphery participants in the market are small institutions such as small banks and security companies. It is also possible that periphery institutions buy bonds in smaller transaction sizes because they are more risk sensitive than usual.

From the event studies presented above, we can observe that core institutions needed to find new periphery counterparties in order to complete their desired bond sales, while periphery purchasers obtained their bonds at a discounted price. This suggests that the capacity to absorb losses not only at core but also at periphery institutions is important for understanding the vulnerability of financial markets.

4. Systemic risk simulation using an agent-based model

In this section, we introduce an agent-based model which replicates the fire sales of securities in financial markets. The model can explain the relationship between market vulnerability and systemic risk. The mechanism driving fire sales in our model is simply the market participants' VaR-constraint, as used in some previous papers (Adrian and

Shin [2012], Adrian and Shin [2015], Bookstaber, Paddrik and Tivnan [2014]). Our model takes account of the findings of the event studies, such as the need for core institutions to find new periphery counterparties to complete their desired bond sales during interest rate spikes. It therefore replicates the potential for the network structure to change in situations of stress. With this model, we seek to show that the impact of fire sales depends not only on participants' capacity to absorb loss but also on the vulnerability of the market network structure.

We consider a bond market where players are linked based on their transaction network structure. The main ideas of our model are as follows (details can be found in Appendix 2).

- First, the market price declines due to an external shock and market players suffer capital losses.
- Some players have to sell their bonds due to the binding of their VaR-constraint, namely that VaR must be less than equity capital. Others with sufficient capacity continue to buy bonds until their VaR reaches an optimal level which depends on the bond price.
- If selling orders are less than buying orders, the price P_t does not change. If selling orders exceed buying orders, market equilibrium requires the price to decline to the point where selling orders are matched by buying orders, following the linear demand curve.[6] If other players hit their VaR-constraints, then the above process is repeated.

The model described above does not yet account for the network structure of the market, which is often seen in previous works (Adrian and Shin [2012], Adrian and Shin [2015], Bookstaber, Paddrik and Tivnan [2014]). In order to take the network

[6] For example, if players faced with a bond price of 100 can buy bonds until their VaR hits 40 percent of equity capital, a price fall to 90 will allow them to continue buying bonds up to 50 percent of equity capital. In other words, the VaR level, as a percentage of equity capital, up to which players can continue to buy bonds is related to the bond price. We assume this relationship to be linear demand function.

structure into consideration, we add the following processes to the model. This is our original contribution.

- First, VaR-constrained market players sell their bonds to the players already connected with them in the transaction network. These already connected players are called “Tier 1 players” (Figure 19).

- Tier 1 players can purchase bonds until their VaR reaches a certain level (e.g. 20 percent of equity capital). Note that there is no change in the price P_t at this stage. If these Tier 1 players have already exhausted their capacity, we go to the next stage.

- Second, market players sell their remaining bonds to players with whom they are not already connected. These previously unconnected players are called “Tier 2 players” (Figure 20). We assume that Tier2 players are more risk sensitive and, therefore, have a smaller capacity to purchase than Tier1 players. Tier 2 players can purchase bonds until their VaR reaches a certain level (e.g. 10 percent of equity capital).[7] If buying orders exceed the remaining selling orders, the price does not change. Otherwise, we go to the next stage.

- If selling orders still exceed buying orders, market equilibrium requires the bond price to fall along the linear demand curve.[8] The price decline means that both Tier 2 and Tier 1 players are able to increase their purchases. Equilibrium is achieved when the price has fallen far enough for the remaining selling orders to be matched by buying orders so the market clears. If other players hit their VaR-constraints, then the above process is repeated.

Now, we present our simulation results.

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[7] For instance, when the bond price is 100, Tier 1 players will purchase bonds until their VaR rises to 20 percent whereas Tier 2 players will stop when their VaR hits 10 percent.

[8] See footnote 6.

Example 1

For simplicity, our first example has only six players: two core and four periphery players. In line with intuition, we call a player who has minimal capacity to absorb losses due to its VaR-constraint a "red player", one with some limited remaining capacity a "yellow player", and one with sufficient capacity "blue player." Two networks are presented in this example (Figure 21). The first network has four blue periphery players and the second network has four red periphery players. The two core players are the same in both networks.

Figure 22 shows the simulation results. For each external shock to the security price, shown on the horizontal axis, the vertical axis shows the corresponding final price of the security. In the first network, the decline in the security price is directly proportional to the size of the external shock. In the second network, however, the decline in the security price magnifies the size of the external shock once the external shock exceeds a certain level. In this paper, we call the additional decline in the price "systemic risk" and we find that systemic risk is likely to materialize when the network has insufficient capacity.

Example 2

In example 2, two networks are presented, each with two core and four periphery players (Figure 23). Both networks comprise the same nodes: one red core, one yellow core, two red periphery, and two blue periphery nodes. This means that total capacity is the same in both networks. The disposition of capacity, however, is different between them. That is to say, where in the first network the red core player enjoys links with two blue periphery counterparties, in the second network the red core is linked with two red periphery counterparties. Thus the disposition of capacity in the second network may be said to be less "balanced": capacity is arranged within the network in such a way that the network is less "stable".

Figure 24 gives the results of simulation. In comparison with the first network, a drastic decline in the final price is observed in the second network, even in response to a small external shock. This means that the second network is more vulnerable than the first. It suggests that the disposition of capacity within the network as well as the total capacity of the network can affect market stability.

Example 3

Finally, we provide a more realistic example where there are 100 players in the market (Figure 25). We consider three types of network structure, in all of which players' profiles such as the amounts of their security holdings and equity capital are the same. Only the transaction network structures are different. All three networks have a core-periphery structure (Figure 25). However, red core players are given red periphery counterparties in the second network, while they have blue periphery counterparties in the third network.

Figure 26 gives the results of simulation. We can draw the same inferences as from example 2 above. That is to say, the third network is more robust than the second network in the face of external shocks. The implication is that the network structure is an important factor to consider when monitoring systemic risk and market stability.

Implications for Macroprudential Policy

It is important to discuss the role of the central bank in reducing systemic risk from a network perspective. Should the central bank take steps to reduce systemic risk or is this better left to market mechanisms? While we cannot yet obtain a definitive answer, the event studies provide us with some useful insights.

After the global financial crisis, central banks gained the idea of the central bank playing a role as "MMLR"(market maker of last resort) compatible to the role as lender of last resort.[9] During the global financial crisis, central banks provided liquidity

[9] See Nakaso (2013).

directly to market participants; for example, the Federal Reserve introduced measures to provide funds to issuers of CP (commercial paper) and holders of asset-backed securities. In the euro area, when the sovereign bond market of the so-called peripheral countries crashed, the European Central Bank purchased bonds issued by the peripheral countries due not to deteriorating solvency but to the impairment of market liquidity. The Bank of Japan has similarly purchased CPs, asset-backed commercial paper, and corporate bonds during periods of rapid deterioration of market liquidity. In addition, during the VaR-shock in 2003, the Bank of Japan provided longer term cash (via operations) in the interbank market in order to reduce interest rate risk for VaR-constrained banks.[10] Such provision of liquidity, which in effect turned central banks into market makers, aided the recovery of market functioning. In this regard, central banks played the role of MMLR.

Examples 2 and 3 show that the network structure, not merely the distribution of capacity among players but also the disposition of capacity within the network, affects the impact of shocks and the extent of systemic risk. It suggests that if there are no concerns about the "balance" of network capacity, the central bank should leave matters to the market mechanism. But if capacity-constrained players have made the network "unbalanced", the central bank should exercise MMLR functions such as purchasing bonds or providing longer term cash in the interbank market.

5. Concluding Remarks

In this paper, we study network structures in the JGB market using settlement data from the BOJ-NET and study how the QQE has affected JGB market structure. In addition, we investigate systemic risk and financial market stability from a network perspective, specifically through event studies of fire sales and agent-based model simulations.

[10] The minutes of the monetary policy meeting in 2003 have been published by the Bank of Japan. They state that the Bank of Japan provided longer term cash to the interbank market in order to reduce interest rate risk for VaR-constrained banks.

The main results are as follows. First, from a network analysis perspective, the JGB market is seen to possess a core-periphery structure. We find that, since the introduction of QQE, the number of participants and links have been decreasing in the T-Bill market. In addition, since the expansion of QQE, the number of links in even the medium-term market has begun to decrease as the medium-term rate has reached zero. These results can be interpreted that the potential depth of the market has decreased since the introduction of QQE.

Second, there is a notable increase in the number of links in the long-term JGB market when the interest rate spikes. Event studies of fire sales show that core institutions sell bonds which are bought by periphery counterparties. The number of links experienced a rapid increase among core-periphery pairs in particular, although the number and volume of purchases per periphery buyer declined sharply. This means core institutions need to find new periphery counterparties in order to complete their desired bond sales, while periphery purchasers obtain their bonds at a discounted price. These purchases by periphery institutions prevent markets from evaporating. It suggests that the capacity to absorb losses not only at core but also at periphery institutions is important for understanding financial market stability.

Third, we propose an agent based-model which can replicate the fire sales of securities and account for the results of the event studies. The simulation results show that the impact of systemic risk depends not only on the capacity constraints of participants but also on the market network structure. Thus, we should pay attention to the market network structure when we monitor financial market stability. Our findings could help the central bank in deciding when to exercise their function as MMLR.

We leave some issues for future research. First, a 10-day backward moving average is applied to the daily dynamics of network indicators in order to analyze transaction trends in the JGB market. However, there are other methods of estimating such trends. Moreover, a greater number of scenarios should be employed in the stress simulations, and their implications considered for macroprudential policy. Finally, the development of a new technique for network analysis called a "temporal network" (Takaguchi et al. [2012], Holme and Saramäki [2013]) enables the dynamics of a network structure to be examined in more detail. Applying this technique to the JGB market, we will undoubtedly generate interesting findings.

Appendix 1: Indicators of Market Liquidity from BOJ-NET Data

Trade volume (Figure 27)

In the long-term JGB market, transaction volume fell sharply immediately after the introduction of QQE reflecting the decline in market liquidity, but started to recover gradually a few months later, once the BOJ improved its purchasing methods.[11] Volumes seem once again to be decreasing, however, since the expansion of QQE. This is consistent with what was happening in the JGB-futures (10-year) market (Kurosaki *et al*. [2015]). In the T-Bill market, transaction volume was initially unresponsive to the introduction of QQE, but it started to decrease about 1 year later. It recovered after the expansion of QQE due to the activities of foreign investors, for the same reasons as the link number, as explained in Section 2-B.

Price range over volume (Figure 28)

Price range over volume is defined as the ratio of the 1-day absolute change in the price to the 1-day trade volume. In the long-term JGB market, price range over volume increased immediately following both the introduction of QQE and the expansion of QQE, indicating a decline in market liquidity. But it recovered about half a year later. This is consistent with what was happening in the JGB-futures (10-year) market (Kurosaki *et al*. [2015]). In the T-bill market, price range over volume did not change after the introduction of QQE, but it started to increase after the expansion of QQE.

[11] The BOJ improved its purchasing policy by: 1) increasing purchasing frequency; 2) decreasing the size of each purchase; 3) reducing variability in the size of purchases. Iwatsubo and Taishi (2016) analyze how these policies improved JGB market liquidity.

Appendix2: Systemic Risk Model in a Core-Periphery Network under the VaR Constraint

In this appendix, we present an agent-based systemic risk model which replicates fire sales of securities in financial markets. We assume that there are n players (i = 1, 2, … , n) in the bond market and they are linked with each other in a transaction network. We define some notation as follows:

- $P(t)$: the price of the bond at time t.
- $y_i(t)$: the number of securities held by player "i" at time t.
- $y_i(t)\,P(t)$: the value of securities held by player "i" at time t.
- $\mathrm{VaR}_i(t)$: the value at risk of player "i" at time t.
- C_i : the initial equity capital of player "i".

In this paper, we define the value at risk as the sum of two terms. One is a proportion of the value of security holdings, namely a fixed percentile of the loss distribution; the other is the capital losses (profits) on securities.

The procedures for simulation in our model are as follows.

VaR-constraint and fire sales:

- The market price declines due to an external shock from $P(t–1)$ to $P(t)$. Market players face a capital loss $–(P(t)–P(t–1))y(t–1)$.
- The value at risk of each player thus changes in accordance with the following:

$$VaR_i(t) = VaR_i(t-1) + \alpha(P(t)y_i(t) - P(t-1)y_i(t-1)) - (P(t) - P(t-1))y_i(t-1) \tag{1}$$

and

$$VaR_i(0) = \alpha P(0)y_i(0), \tag{2}$$

where α is a quantile coefficient such as the 99 percentile of the normal distribution. This is set at 0.7 in our paper.

- When value-at-risk exceeds equity capital, a player must sell bonds until its value-at-risk falls to 90 percent of its equity capital. The player's selling order is determined by the following equation (3):

$$90\%\mathrm{C}_i = VaR_i(t-1) + \alpha(P(t)y_i(t) - P(t-1)y_i(t-1)) - (P(t) - P(t-1))y_i(t-1). \tag{3}$$

  From (3) we can calculate total selling orders in the market.

- If value-at risk is less than equity capital, a player purchases bonds until its value-at-risk reaches a certain level defined by equation (4). This level depends on the bond price, and the relationship is assumed to be linear (linear demand curve).

$$VaR_{f,i} \equiv [10\% - 80\% \times (P(t)/P(0) - 1)]\ \mathrm{C}_i. \tag{4}$$

  For example, at the initial bond price, players will buy bonds only until value-at-risk hits 10 percent of equity capital; however, with the bond price at zero, they will be far more eager to hold bonds, and will willingly continue to purchase until their value-at-risk is 90 percent of their equity capital.

- If total selling orders exceed total buying orders, so that desired sales cannot be fulfilled at the current price, the bond price declines to the price at which the market clears. If other players hit their value-at-risk constraints due to the price decline, the above process is repeated. If total selling orders are less than total buying orders, so that all selling orders are fulfilled at the current price, the price does not change and the simulation is complete.

Accounting for network structure:

The model described above does not account for the network structure of the market. To do so, we extend the procedures of the model as follows.

- First, a VaR-constrained market player sells bonds to players with whom it is already connected in the transaction network, called “Tier 1 players” (Figure 18).

  Tier 1 players buy bonds until their value-at-risk reaches 20 percent of their equity capital. If the total buying orders of Tier1 players are less than total selling orders, we go to the second stage as follows.

- The VaR-constrained player now sells its remaining bonds to players with whom it was not previously connected, called “Tier 2 players”. (Figure 19). We assume that Tier2 players are more risk sensitive and, therefore, have a smaller capacity to purchase than Tier1 players. Tier 2 players buy bonds until their value-at-risk hits 10 percent of their equity capital. If selling orders exceed the total buying orders of Tier 2 players, the price declines in accordance with the demand curve (4).

  The price decline means that both Tier 2 players and Tier 1 players are able to increase bond purchases so that the market clears. Here, we define the demand curve for Tier 1 players as equation (5), which is similar to equation (4). The level of equity capital up to which they will continue purchasing bonds is 20 percent for these Tier 1 players.

  $$VaR_{g,i} \equiv \max\{[10\% - 80\% \times (P(\mathrm{t})/P(0) - 1)]\mathrm{C}_i, 20\% \times \mathrm{C}_i\}, \qquad (5)$$

- If other players hit their value-at-risk constraints due to the decline of the price, then the above process is repeated. If the remaining selling orders do not exceed the total buying orders of Tier 1 players and Tier 2 players together, so that all desired bond sales can be made, the price does not change and the simulation is complete.

References

Adrian, Tobias and Hyun Song Shin, “Liquidity and Leverage,” *Journal of Financial Intermediation*, 19(3) 2012, pp. 418–437.

Adrian, Tobias and Hyun Song Shin, “Procyclical Leverage and Value at Risk,” *Review of Financial Studies*, 27(2) 2015, pp. 373–403.

Bank for International Settlements, “Fixed income market liquidity,” CGFS Papers No 55, January 2016.

Bank of Japan, “Money Market Operations in fiscal 2014,” BOJ Reports & Research Papers, July 2015.

Beltran, Daniel O., Valentin Bolotnyy, and Elizabeth C. Klee, “Un-Networking: The Evolution of Networks in the Federal Funds Market,” Finance and Economics Discussion Series 055, Federal Reserve Board, July, 2015.

Bookstaber, Rick, Mark Paddrik, and Brian Tivnan, “An Agent-based Model for Financial Vulnerability,” Office of Financial Research Working paper, July, 2014.

Boss, Michael, Helmut Elsinger, Martin Summer, and Stefan Thurner, “Network topology of the interbank market”, *Quantitative Finance*, 4(6), 2004, pp.677–684.

Cont, Rama, Amal Moussa, and Edson B. Santos, “Network Structure and Systemic Risk in Banking Systems,” in Jean-Pierre Fouque and Joseph A. Langsam (eds.), *Handbook of Systemic Risk*, Cambridge University Press, 2013, pp. 327–368.

Craig, Ben, and Goetz von Peter, “Interbank tiering and money center banks,” *Journal of Financial Intermediation*, 23(3), 2014, pp.322–347.

Degrysea, Hans, and Grégory Nguyen, “Interbank Exposures: An Empirical Examination of Contagion Risk in the Belgian Banking System,” *International Journal of Central Banking*, 3(2), 2007, pp.123–171.

Engle, R. F., and J. Lange, “Measuring, Forecasting and Explaining Time Varying Liquidity in the Stock Market, ” NBER Working Paper No. 6129,1997.

Nakaso, Hiroshi, “Financial Crises and Central Banks' "Lender of Last Resort" Function,” 2013.

Holme, Petter, and Jari Saramäki (Eds.), *Temporal Network*, Springer 2013.

Imakubo, Kei, and Yutaka Soejima, “The Transaction Network in Japan’s Interbank Money Markets,” *Monetary and Economic Studies*, Institute for Monetary and Economic Studies, Bank of Japan, 28 (S-1), 2010, pp. 107–150.

Iori, G., G. de Masi, O. Precup, G. Gabbi, and G. Caldarelli, “A network analysis of the Italian overnight money market,” *Journal of Economic Dynamics and Control*, 32(1), 2008, pp.259–278.

Inaoka, H., T. Ninomiya, K. Taniguchi, T. Shimizu, and H. Takayasu, “Self-similarity of banking network,” *Physica A*, 339, 2004, pp.621–634.

International Monetary Fund, “Global Financial Stability Report,” October 2015.

Iwatsubo, Kentaro, and Tomoki Taishi, “Quantitative Easing and Liquidity in the Japanese Government Bond Market,” IMES Discussion paper series No.2016-E-12, 2016.

Kurosaki, Tetsuo, Yusuke Kumano, Kota Okano, and Teppei Nagano, “Liquidity in JGB Markets: An Evaluation from Transaction Data,” Bank of Japan Working Paper Series No.15-E-2, 2015.

Lelyveld, Iman van, and Franka Liedorp, “Interbank Contagion in the Dutch Banking Sector,” *International Journal of Central Banking*, 2(2), 2006, pp.99–133.

Müller, Jeannette, “Interbank Credit Lines as a Channel of Contagion,” *Journal of Financial Services Research*, 29 (1), 2006, pp. 37–60.

Minoiu, Camelia, and Javier A. Reyes, “A network analysis of global banking: 1978–2010,” *Journal of Financial Stability*, 9(2), 2013, pp.168–184.

Soramäki, Kimmo, Morten L. Bech, Jeffrey Arnold, Robert J. Glass, and Walter E. Beyeler, “The topology of Interbank Payment Flows,” *Physica A*, 379, 2007, pp. 317–337.

Squartini, Tiziano, Iman van Lelyveld and Diego Garlaschelli, “Early-warning signals of topological collapse in interbank networks,” Scientific Reports 3-3357, 2013, pp.1–9

Takaguchi, Taro, Nobuo Sato, Kazuo Yano and Naoki Masuda, “Importance of individual events in temporal networks,” *New Journal of Physics*, 14, 2012, pp.1–15.

Tellez, Eduardo, “Mapping the Australian Banking System Network,” Bulletin June Quarter 2013, Reserve Bank of Australia, 2013.

Upper, C., and A. Worms, “Estimating Bilateral Exposures in the German Interbank Market: Is There a Danger of Contagion?,” *European Economic Review*, 48(4), 2004, pp.827–849.

U.S. Department of the Treasury, Federal Reserve System, Federal Reserve Bank of

New York, U.S. Securities and Exchange Commission, and U.S Commodity Futures Trading Commission, “The U.S. Treasury market on October 15, 2014,” Joint Staff Report, 2015.

Veld, Dann in't, and Iman van Lelyveld, “Finding the core: Network structure in interbank markets,” *Journal of Banking and Finance*, 49, 2014, pp.27–40.

Figure 1 Network Structure of JGB markets in June 2015

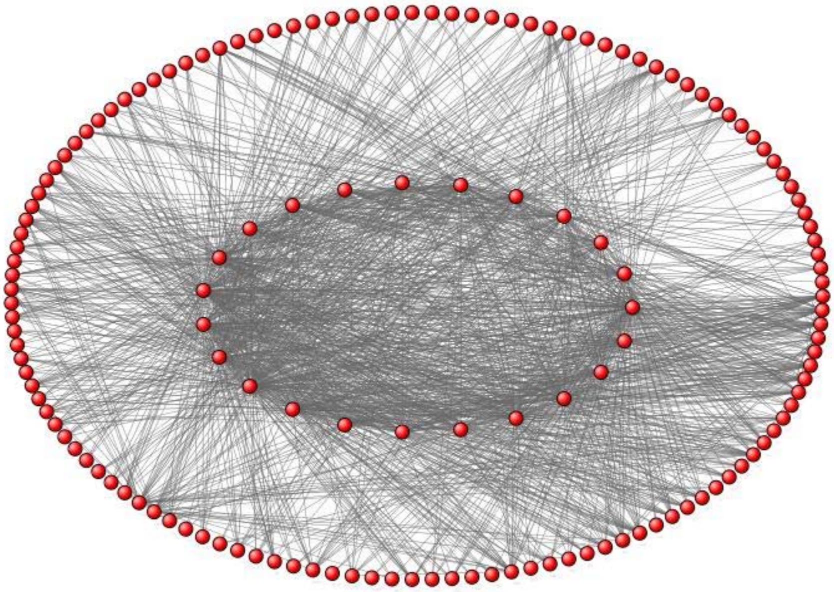


Figure 2 Distribution of Number of Links

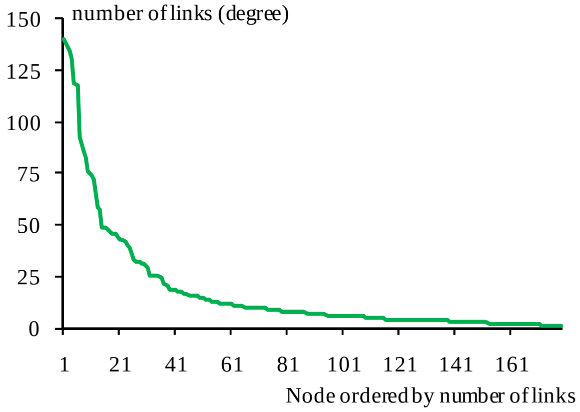


Figure 3 Transaction Value and Degree

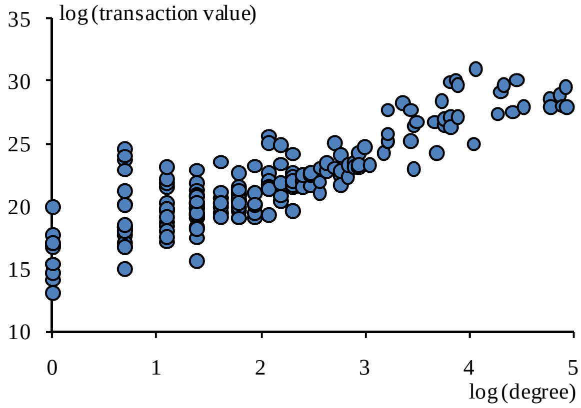


Figure 4 Clustering Coefficient and Degree

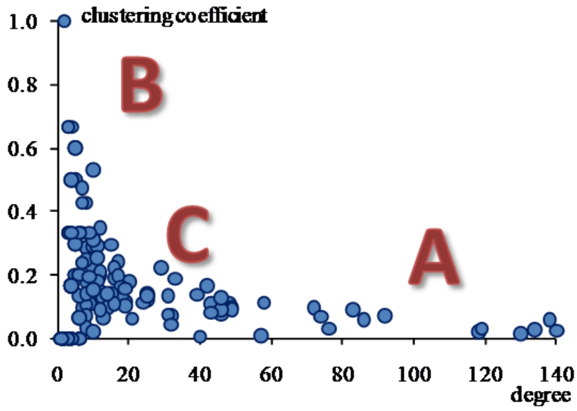


Figure 5 Outstanding Amount of JGBs

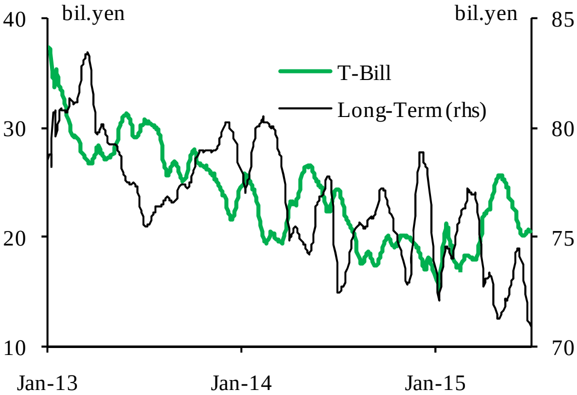


Figure 6 Interest rates after QQE

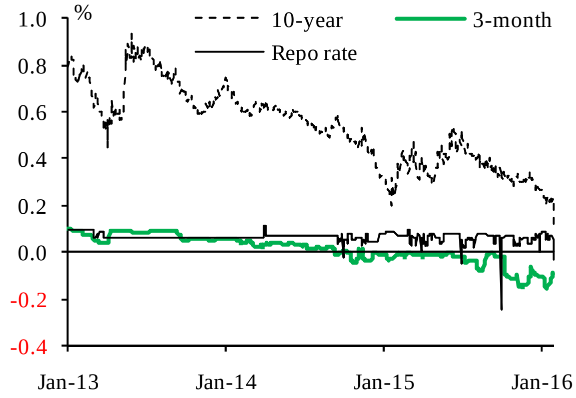
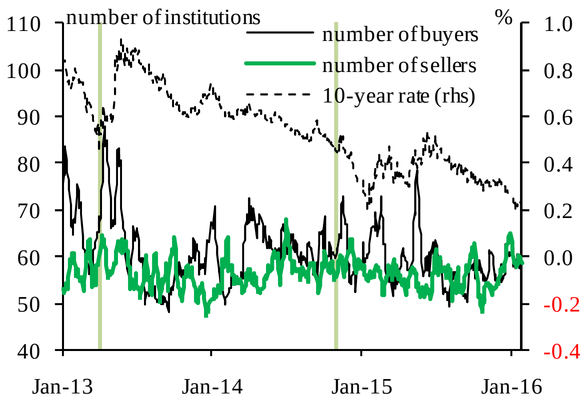


Figure 7 Number of Players

Long-Term JGB Market



Short-Term JGB Market

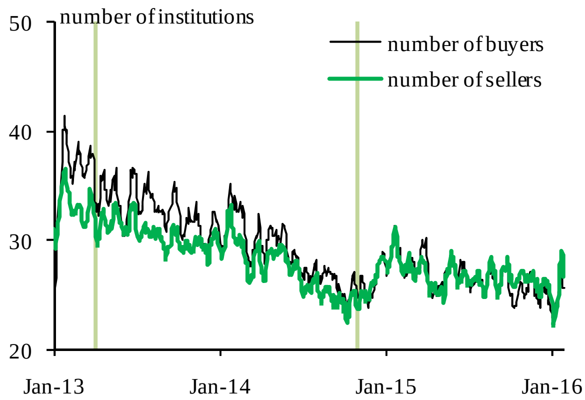
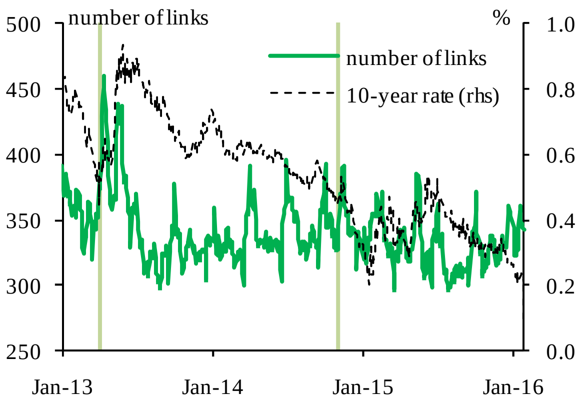


Figure 8 Number of Links

Long-Term JGB Market



Short-Term JGB Market

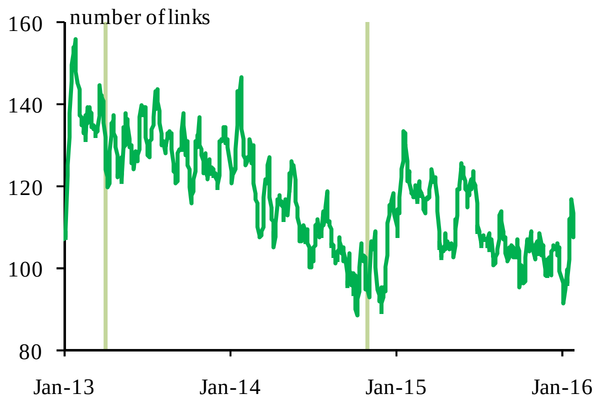


Figure 9 Network Structure of Short-Term JGB Market

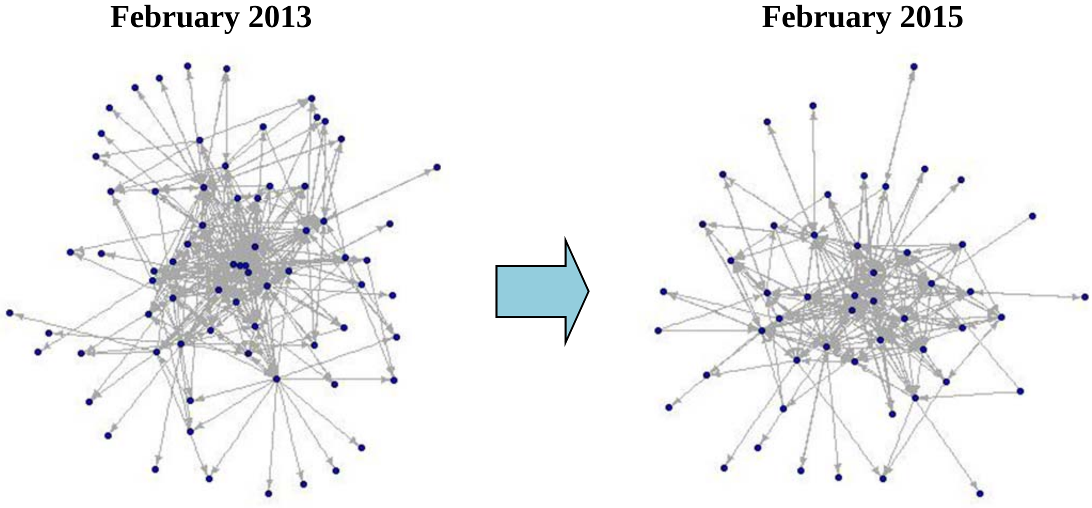
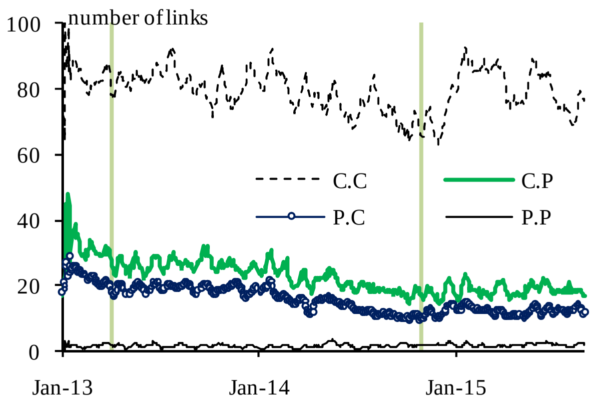


Figure 10 Short-Term JGB Market



Note: C.C. denotes links from core players to other core players, C.P. from core to periphery players, P.C. from periphery to core players, and P.P. from periphery to other periphery players.

Figure 11 Foreign Investor Links

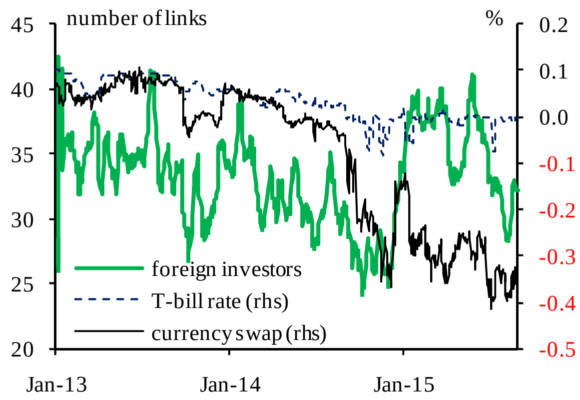


Figure 12 Number of Links by Maturity

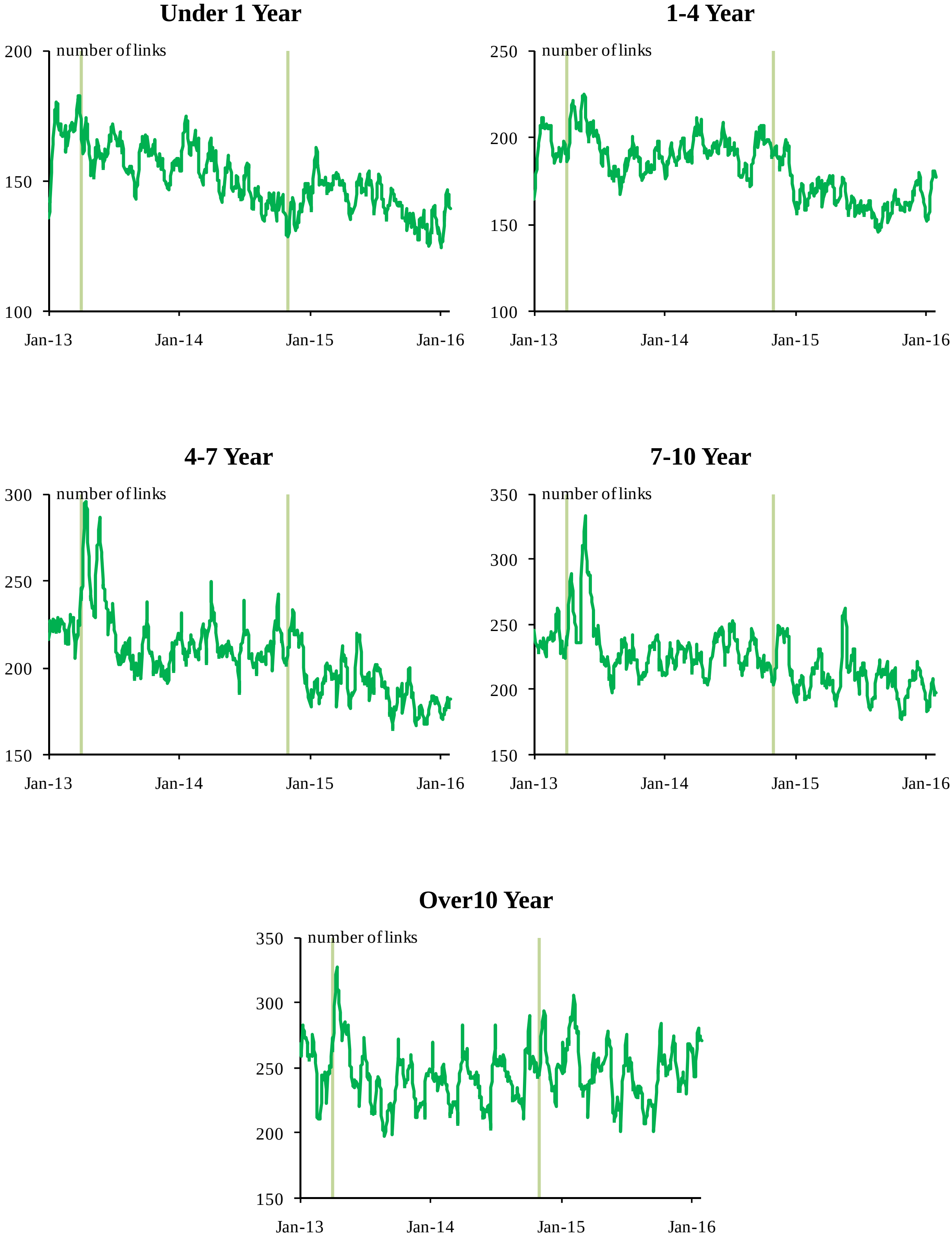


Figure 13 Interest Rate on 10-year JGB

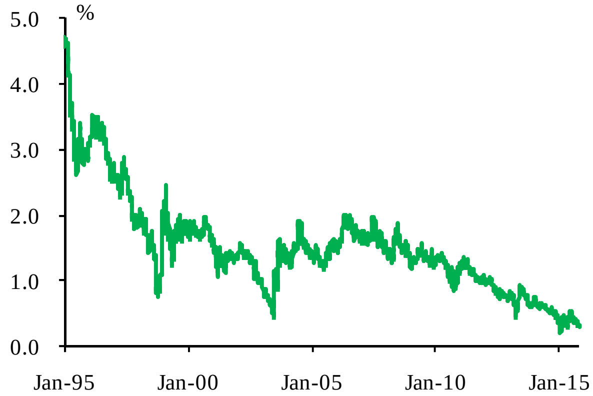
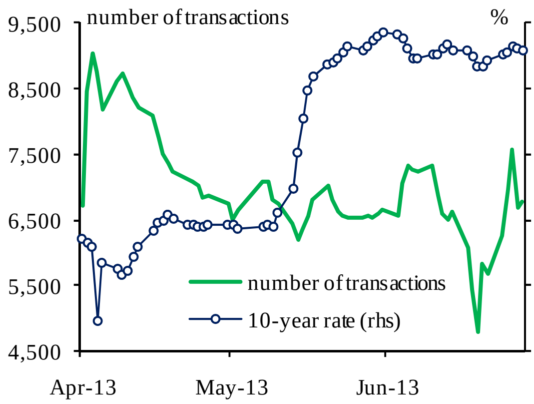


Figure 14 Number of Transactions (Settlements)

After the introduction of QQE



VaR Shock

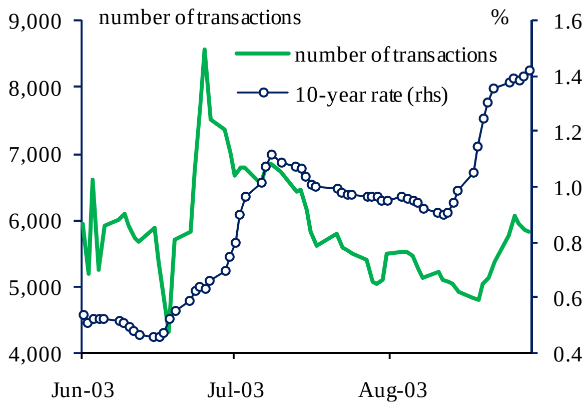
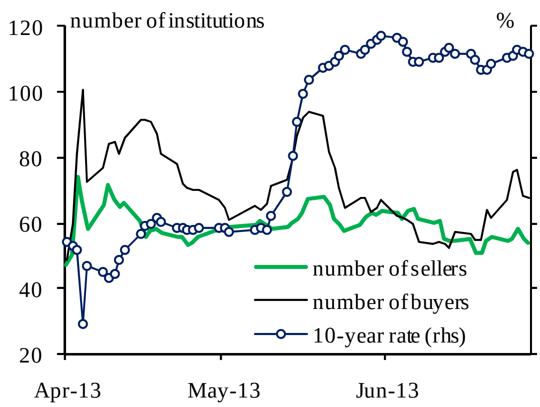


Figure 15 Number of Players

After the introduction of QQE



VaR Shock

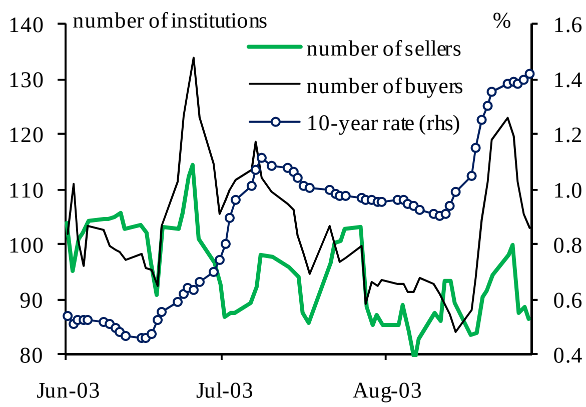


Figure 16 Number of Links

After the introduction of QQE | VaR Shock

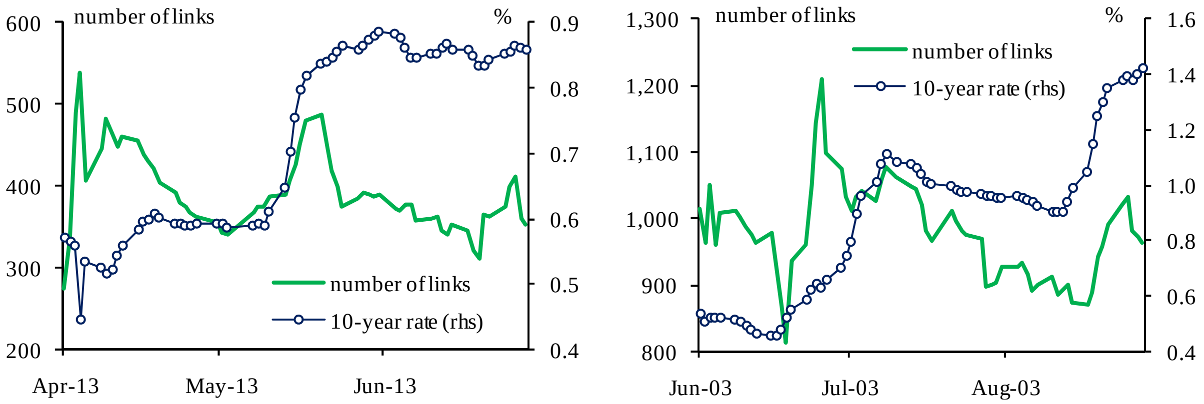
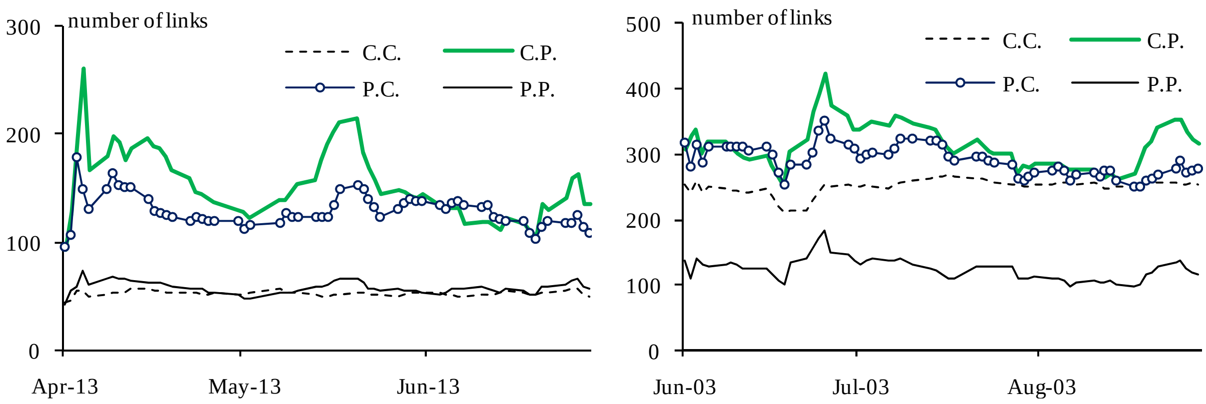


Figure 17 Number of Links in Core-Periphery Pairs

After the introduction of QQE | VaR Shock



Note: C.C. denotes links from core players to other core players, C.P. from core to periphery players, P.C. from periphery to core players, and P.P. from periphery to other periphery players.

Figure 18 Number of Transaction per Player

After the introduction of QQE | VaR Shock

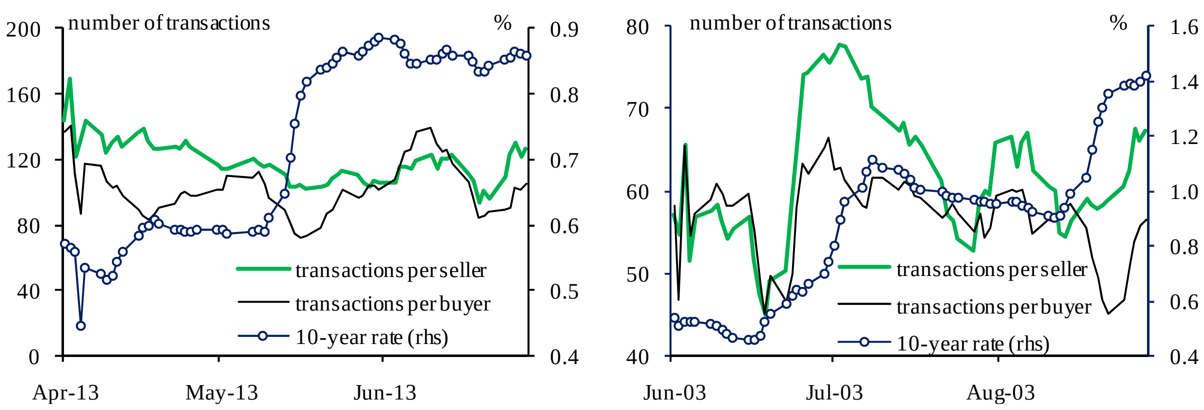


Figure 19 Selling to Tier 1 Players

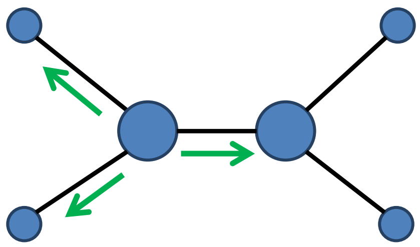


Figure 20 Selling to Tier 2 Players

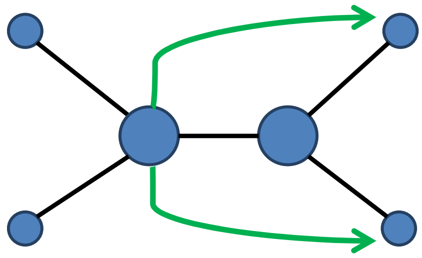


Figure 21 Networks in Example 1

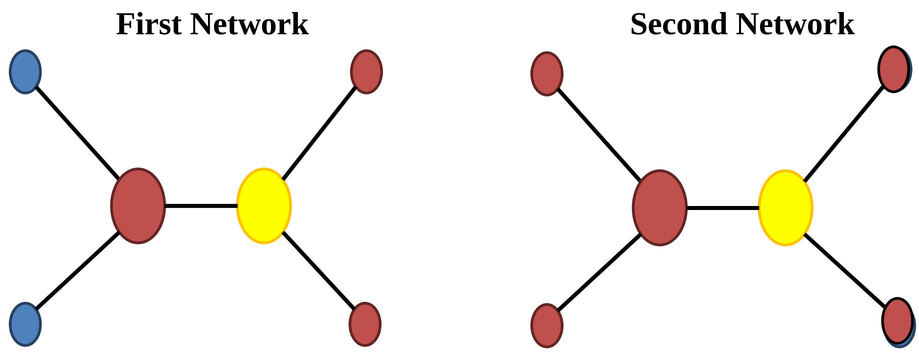


Figure 22 Final Price and External Shock in Example 1

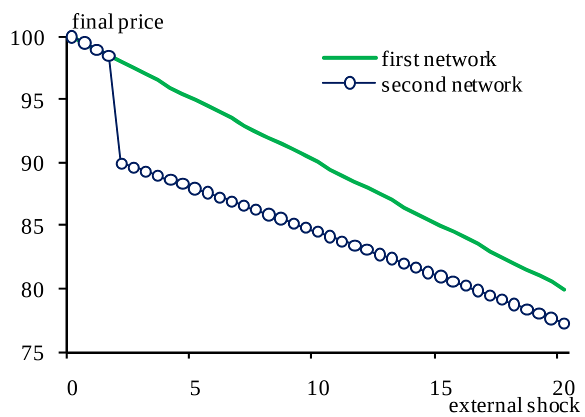


Figure 23 Networks in Example 2

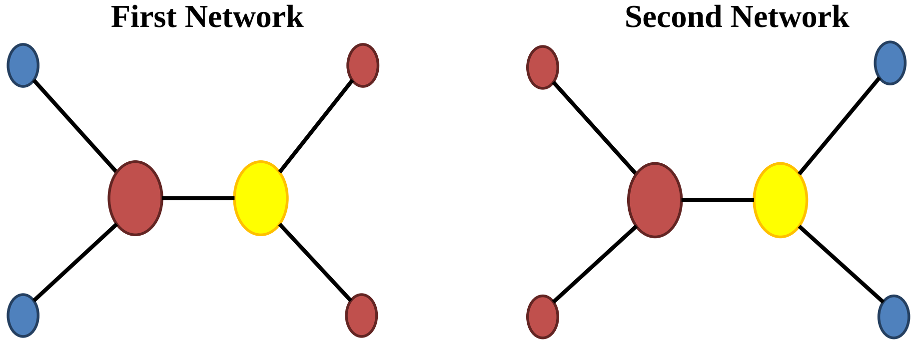


Figure 24 Final Price and External Shock in Example 2

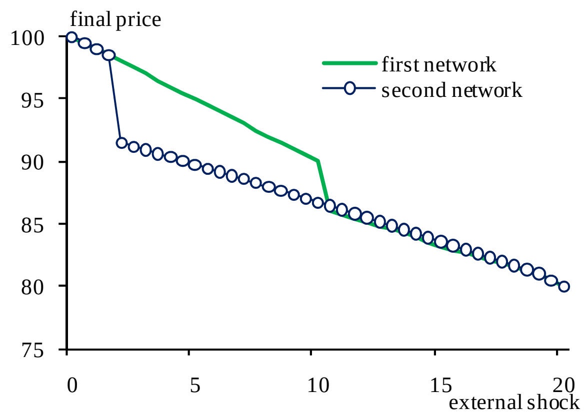


Figure 25 Networks in Example 3

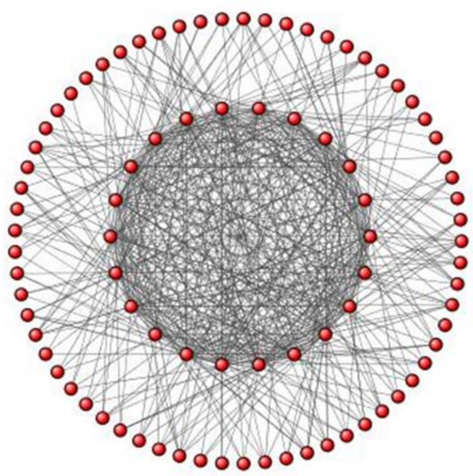


Figure 26 Final Price in Example 3

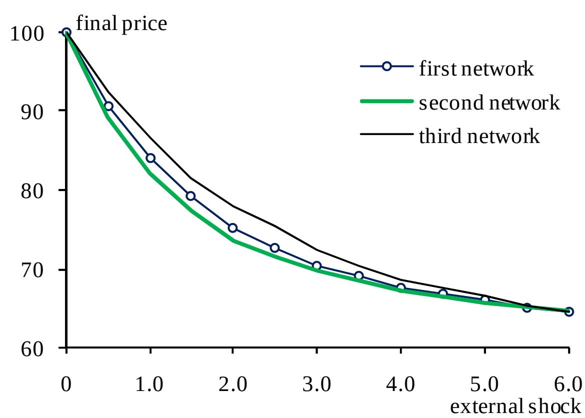
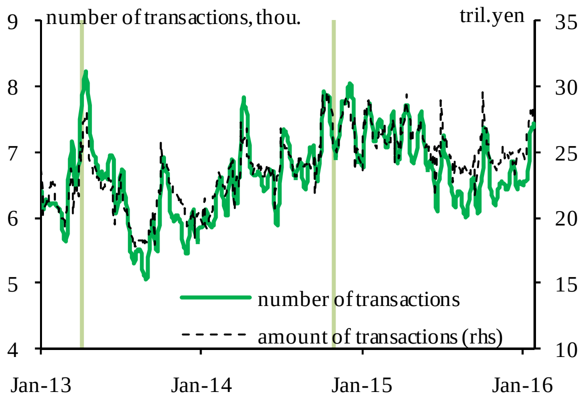
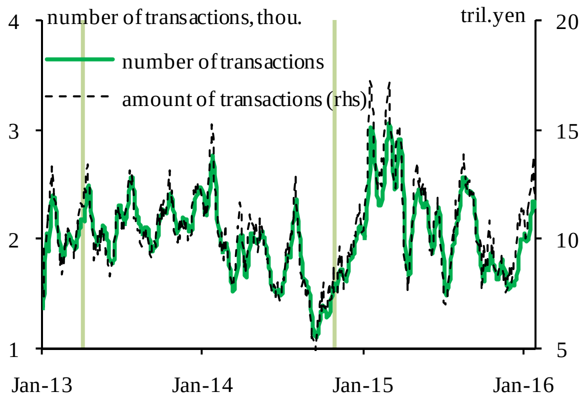


Figure 27 JGB Transaction Volumes

Long-Term JGB Market



Short-Term JGB Market



JGB Futures Market (Kurosaki *et al*. [2015])

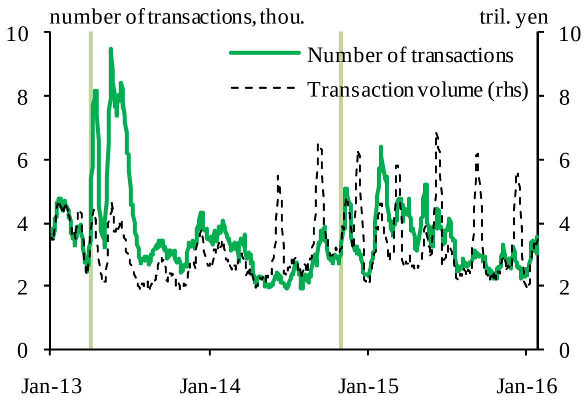


Figure 28 Price range over trade volume for JGBs