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Choice of Collateral Asset and the Cross-Border Effect of Automatic Stays

Hiroshi Fujiki* and Charles M. Kahn**

Abstract

We analyze the choice of collateral for use in repurchase agreement (repo) contracts based on an optimal contract model, and examine when regulatory restrictions are appropriate on the use of types of collateral in repo contracts, especially on the exemption to the automatic stay requirements. Our analyses imply that the crucial characteristics of the assets more desirable as collateral in an optimal contract are lower opportunity cost to create the collateral, lower hazard rate in the distribution of valuations of the collateral assets, and higher negative correlation of valuations between borrower and lender. We consider an externality based on divergence between market price and buyer's surplus in a resale asset market, and analyze its effect on the welfare consequences of the exemption to the automatic stay requirements. We also consider the welfare consequences when national regulators evaluate the welfare benefits only from their national perspectives.

Keywords: repo market; the exemption from automatic stays; collateral; cross-border financial transaction

JEL classification: G15, G17, G18

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1 Introduction

The purpose of this paper is to analyze the choice of collateral for use in repurchase agreement (repo) contracts, and to understand what regulatory restrictions are appropriate on the use of types of collateral in repo contracts. Repo contracts are favored by current bankruptcy law in many countries through an exemption to the automatic stay requirements. This makes them an attractive vehicle for short term liquid lending. However, regulators have been concerned with the possible damage that can arise in markets for the asset used as underlying collateral should systemic defaults lead to "fire sales" of these assets by collateral holders. Indeed there has been a movement in recent years toward extending the automatic stay more generally to limit the possibility of damage. But there is a liquidity benefit provided by being able to bypass the automatic stay. Thus there are important costs and benefits to extending the automatic stay. We will examine the differences in characteristics between the assets that are most preferred as collateral by participants in repo contracts and the assets whose use would cause least disruption to their markets in the event of fire sales. Thus the analysis will provide guidance to regulators about appropriate restrictions on exemptions from automatic stays.

Why do we care about the exemption from automatic stays in the U.S. repo contract? This is because the U.S. repo market played a central role in the global financial crisis. For example, the Lehman Brothers experienced problems borrowing in this market before its collapse, and the losses associated with the failure of the Lehman Brothers triggered a fire sale of assets seized from the Lehman Brothers. According to [4], the composition of securities posted as collateral in tri-party repo (a major segment of the repo market) as a percent of the total from 2007 to 2010 was as follows; about 80 percent was U.S. government collateral, such as U.S. treasuries and strips and government agency MBS, and the remaining 20 percent was non government collateral, such as non government corporate bonds or asset-backed securities. Various kinds of securities posted as collateral could be subject to a fire sale, and the falling asset prices could transmit to the other markets in the world. Since the U.S. repo contracts are not subject to the automatic stays in the U.S. bankruptcy code, both U.S. regulators and international institutions including G-20, worried about that this special legal treatment might trigger an asset fire sale as we will explain in details later. See the details on the U.S. repo market in [3].

The exemption from automatic stays benefits one class of market participant at the expense of another class. Decisions about the correct limits on such exemptions will therefore involve trade-offs between the importance of two different markets to an economy. In fact because of difference in the size of these markets in various economies, it is likely that regulators in different countries will come to different conclusions about the correct limitations to place on them—and indeed regulations do vary among jurisdictions. Thus a second goal of this paper is to understand the cross-border differences: can we link differences in rules to the economic importance (or at least the political weight) of the two markets in different countries? What are the consequences of having different restrictions in different countries on the cross-border choices of international agents? What are the relative merits of attempting restrictions on the use of repos, restrictions on the choices of collateral protected by repos, and restrictions on the activities of collateral holders in the resale market for the collateral—particularly when some participants in these markets have flexibility in their choice of jurisdiction for some financial transactions?

To answer these questions, we provide models for lending markets and collateral markets and obtain the following results:

First, in an optimal borrowing contract with collateral, the borrower will default even though the asset is worth more to the borrower than to the lender in some states of the economy.

Second, in such an optimal contract with collateral, the crucial characteristics of the assets more desirable as collateral, in a sense that a large amount of the assets will be used as collateral in equilibrium, are likely to be the following: (1) lower opportunity cost to create the collateral, (2) lower hazard rate in the distribution of valuations of the collateral assets, and (3) higher negative correlation of valuations between borrower and lender.

Third, we define the externality in the resale market as the departure from the market price and the buyer's surplus when there is no entry of the lenders to the resale market. We then ask if the externality alone justifies the intervention to stop the fire sales of lenders experiencing the default of their borrowers in the resale market, and find that it is beneficial to introduce the automatic stays in repo transactions if the effects of the automatic stays on the social value through the externality is positive and larger than the regular seller's cost of creating the asset in the resale market.

Finally, when some of the market participants are foreigners, the policy implication discussed above may change substantially, because national regulators evaluate the welfare benefits from the lending market and the resale market from the national perspectives, even though the whole welfare benefit of those markets should better be evaluated from the global perspective. For example, suppose that the regular sellers and buyers in the resale market and the market participants in the lending market live in different countries. If the regulators in the resale market country favor the automatic stays, our model suggests that the regulators in the borrower/lender country will favor the exemption from the automatic stays. In such a case, we need international coordination on the implementation of the automatic stay requirements to resolve the conflicts of interest between two regulators.

Our analyses go a few steps further than the conventional argument that if the benefit of the liquidity in the normal times outweigh the cost of market disruption in the event of default in the resale market, that would be the case for the exemption to the automatic stay requirements. First, we make explicit the nature of the assets that could be suitable as collateral. Second, we weigh the externality of demand for the asset in the resale market against the cost of creating that asset in the resale market to determine the desirability of the automatic stay requirements. Finally, we discuss the case for international regulatory coordination when some of the market participants in the lending and the resale markets live in different economies. To the best of our knowledge, we are the first to discuss such a cross-border issues in a tractable model.

After discussing the historical background on the discussion on the exemptions from the automatic stays and summarizing the literature in the next two subsections, the rest of the paper is organized as follows. Section 2 explains our model for lending market and collateral asset market in turn. Section 3 analyzes the cross-border effect of automatic stays by reinterpreting our models in section 2. Section 4 concludes.

1.1 Background

To understand the importance of analyzing the costs and benefits to extending the automatic stay, we provide the background on this issue.

A goal of U.S. bankruptcy law is to avoid the chaotic effects of a breakup of a potentially viable and valuable institution. One important measure within the code is the *automatic stay*, which prohibits holders of collateral from exercising their rights until the bankruptcy process has provided the opportunity to develop a rescue plan. The automatic stay means that the secured creditor will be unable to take his security to the detriment of the other stakeholders. In principle at least, the automatic stay could be of benefit to the secured creditor as well, since it prevents potentially mutually-destructive runs to grab secured portions and thereby reduce the value of claims on the bankrupt firm.

However, while the automatic stay can arguably enhance the value of the firm's stakeholders as a whole, it is in fact likely to reduce the value of the secured claim–particularly if the claim was designed to be liquid. If we focus on maintaining the bankrupt entity this is stabilizing, but if we focus on the potential damage to the creditors this can in fact be destabilizing. Bankruptcy proceedings are lengthy and complicated, and the creditor's claim will be tied up and largely unusable during the time. Short term, liquid claims, intended to be reversed over the course of a day, become long term, illiquid claims, tied up in court. Disruption in the liquidity of these claims can have damaging effects beyond the creditor himself; if the creditor possesses a variety of interlocking and mutually offsetting financial claims, the problem can spillover to his counterparties, and to the markets in which these claims trade. Such considerations lay behind the changes in the U.S. bankruptcy code in 1978, which allowed the exemption from the automatic stay for Treasury repo contracts and a few listed future contracts. Due to this exemption, if a counter-party of the Treasury repo contracts went bankrupt, the creditor to this counter-party could terminate the repo contract, could offset credit and debit positions, and could liquidate collateral without waiting for bankruptcy proceedings. Thanks to this exemption, the repo market became more liquid, and the contagion risks in the repo market was contained in those days.

As might be expected, the advantage of this bypass to the basic rules for certain favored instruments has led to a pressure over time to increase the scope of the covered instruments. After a series of amendments, the U.S. bankruptcy law now exempts several qualified financial contracts from the automatic stay. Those exempted contracts include: repos, securities contracts, forward contracts, commodity contracts, and swaps. (See details in [11], [5]).

Other nations have also exempted several qualified financial contracts from the automatic stay as in the U.S. For example, in the EU, a series of EU Directives ensures uniformity across bankruptcy codes of all member countries. In the EU Financial Collateral Directive of 6, June 2002 says "Member States shall ensure that a financial collateral arrangement can take effect in accordance with its terms notwithstanding the commencement or continuation of winding-up proceedings or reorganization measures in respect of collateral provider or collateral taker (Article 4, 5)" and "Member States shall ensure that a financial collateral arrangement, as well as the provision of financial collateral under such arrangement, may not be declared invalid or void or be reversed on the sole basis that the financial collateral arrangement has come into existence, or financial collateral has been provided on the day of the commencement of winding-up proceedings or reorganization measures, but prior to the order or decree making that commencement (Article 8, 1, (a))." The financial collateral arrangements in the EU Directive include a title transfer financial collateral arrangement such as repurchase agreements (Article 2, 1, (b)), and the revisions of Article 4 in 2009 adds safe harbour provisions by the exemption from the automatic stay to credit claims.[1]

In effect, it would seem that there would be no limitation to the avoidance of the automatic stay if all that is required is to recast the form of the contract: to declare the arrangement a repo rather than a collateralized loan. In fact the crucial distinction probably boils down to the nature of the collateral: if the collateral is sufficiently illiquid and "real" the automatic stay will apply and if the collateral is sufficiently liquid and "financial" the automatic stay will not apply. From the point of view of bankruptcy law this is natural; financial assets are less likely to be part of a joint production technology leading to fundamental loss of value in separating them. In the case where the cost of the avoidance of the automatic stay falls on non-secured creditors, it might be argued that the avoidance is not a major consideration; after all, if it is known that non favored credits will fall lower in the priority list, the main remedy is simply a rejiggering of the interest rates paid on secured and non-secured lending.

But there is one important class of individuals for whom it cannot be assumed that such contract modification will undo the damage: namely, other participants in the market for the assets in question. By encouraging or discouraging the avoidance of automatic stays we make the repo less or more attractive, encouraging or discouraging its use, and thereby even further increasing or decreasing its use in the market. This consideration was probably primary in the development of the law for exemption from the automatic stay.

Before the global financial crisis, partly because of the exemption from the automatic stay, the repo market provided major short-term funding for

[1]See Keijser [9] for recent discussion about the EU Finanical Collateral Directive.

systemically important financial institutions, especially those in the "shadow banking" sector. However, after the financial turmoil around the collapse of Lehman Brothers, regulators cast doubt on the conventional benign view on the exemption of repo contracts from the automatic stay. The financial turmoil gave them an important lesson: when the bankrupt firm itself is a systemically important financial institution which plays a large role in the repo market, there are potential disadvantages to the exemption which regulators must weigh against the previously recognized benefits. With the benefit of hindsight, it is easy to list some potential disadvantages: First, the priority of repo market lenders is obtained at the cost of less privileged lenders, in particular, government-guranteed lending. Second, the ease of exit from the repo contracts means that the lenders do not monitor the debtor in comparison with the other financial contract. Third, the short term nature of the trade means that distressed borrowers can simultaneously be forced to sell other assets, as we have seen before the collapse of Lehman Brothers, and large amounts of repossessed collateral could be dumped into the market simultaneously. Should such fire sales occur, one may end up in preserving the liquidity of repo market by destroying the liquidity of the markets of collateral assets and the liquidity of repo market itself in the end.

Among those disadvantages, the effects on the market for the underlying asset due to possible disruptions in the event of massive fire sales of collateral become important for policy purposes recently. If we take the disadvantages seriously, should we go back to a financial system without exemption from the automatic stay? The answer is no, because the need for exemption from bankruptcy rules arise largely because of the slowness and inappropriateness of bankruptcy regime to troubled financial institutions. We need a new rule for allowing the exemption from the automatic stay that is consistent with the new regimes for resolving financial institutions.

For example, the principles for resolution regimes proposed by the FSB strives to achieve the right balance between the benefit of liquidity in the repo contract due to the exemption from the automatic stay and the benefit of smooth resolutions ([6]). On the one hand we want the takeover of a financial institution by a resolution regime not to trigger any contractual arrangement for accelerating the contract. Of course, this is precisely what any creditor would want to have incorporated into such a contract. Thus the rule would seem to work like an automatic stay. On the other hand, upon entry of a firm into resolution, the standard financial contracts allow the creditor will trigger early termination rights. In the case of SIFI, the termination of large volumes of financial contracts upon entry into resolution could result in a disorderly rush for the exits that creates further market instability. Thus, the principles also specify that the resolution authority should have the power to stay temporarily (See detail in [6], 4.3, and Annex 4). The stay should be of short duration, say a period not exceeding 2 business days, and it should meet several conditions, such as a "no cherry picking" rule.

Observing the proposal by the FSB ([6]), a Working Group under the Financial System Council of Japanese Financial Service Agency suggested the need for the temporary stay of the early termination clause in its report on January 2013. The report said, for a troubled financial institution with large numbers of derivative contracts or repo contracts which are subject to an orderly resolution regime, simultaneous termination of contracts could destabilize financial markets and damage value of troubled institutions. Therefore, it was appropriate to allow restrictions to the early termination clause to the extent necessary for preventing sever market disruption. The proposal of the report led to the revision of Deposit Insurance Act enacted in June 2013.[2]

1.2 Literature

Before moving on to the details of our model, we review the related literature. As summarized in FSB ([7]), three potential disadvantages for allowing the exemption from the automatic stay in repo contracts are recognized. First, the exemption may encourage the overuse of repos. Second, it may encourage fire sales of collateral on default, and third, it may reduce creditors' incentive to monitor credit quality of repo counterparties.

It is the second of these potential disadvantages upon which we will focus, and there are several related works discussing the appropriateness of exemption of automatic stays for repo contract which are not backed by liquid asset given the fact those assets are most likely to be subject to fire sales in the upcoming financial crisis. Note the difference in focus–the first disadvantage of whether the exemption from the automatic stay encourage the use of repos in general is a general issue. But once it is determined that repos in general are worth encouraging, it becomes of interest to see which sorts of collateral might be suitable or unsuitable as the second leg of the repo. It is also the case that some repos are not intended as collateralized lending,

[2]We thank Keiko Yamamoto for her useful suggestions about this paragraph.

but as “rental” of particular financial assets to be used temporarily for other purposes. In this case the choice of collateral is much more central to the existence of the contract.

Duffie and Skeel [5] have recommended that repos that are backed by liquid securities should be exempted from automatic stays. Repos backed by illiquid assets should not be given this safe harbor. Acharya and Öncü [1] have recommended requiring the collateral to be sold to a “Repo resolution Authority” which could insulate them from the market problem at pre-spcified haircuts. FSB ([7], recommendation 11) acknowledges the theoretical importance of these considerations but argues that it is too difficult to change bankruptcy law. Perotti [11] also thinks that it is impossible to repeal the bankruptcy exemptions, and recommends a tax to be able to discourage the buildup in liquidity risks through the overuse of repo transactions.

Our analysis is most closely related to Antinolfi et al., [2]. They examine the trade-off associated with exemption from the automatic stay; the benefit to improve the value and effectiveness of collateralized lending and the cost of fire sales in the market through a search externality for collateral in the event of default. More specifically, in their model, giving liquid assets exemption from automatic stays increases their value by reducing the cost of dealing in them. Thus agents are more willing to use them. On the other hand, the fact that they will be sold precipitately into an illiquid market in the event of a financial disruption imposes an externality on participants in that market. That externality is modeled by assuming trades occur in a matching market in which benefits are affected in non-competitive fashion by introduction of additional agents on the same side of the market. Based on this model, they suggest that exemption from the automatic stay causes no harm if the market is liquid in the following sense: There are large numbers of potential purchasers of assets, so that the competition from collateral sellers does not affect the position of other sellers in the asset market. Our analysis derives from their framework, but simplifies it in some dimensions in order to focus on and extend its implications for variation in the characteristics of candidate collateral assets.

Infante [8] considers the effects of automatic stay on the collateral asset price. Fire sales of collateral asset will occur because of limited liquidity available to solvent firms to purchase assets after a default event. Thus, the repo’s exemption from the automatic stay alters firms’ investment opportunity sets, giving them the opportunity to purchase assets at discounted prices. This creates an incentive for firms to hold their initial endowment to take advantage of the potential fire sales, and create a premium for having liquidity and reducing the initial price of the risky asset. In our model, lenders with collateral of defaulting borrowers compete with ordinary sellers of collateral asset. Ordinary sellers choose supply of collateral before the default outcomes are known. Therefore, the competition from lenders may also disrupt supplies in non-default periods.

2 Model

There are two periods and two markets—a period-one market for lending and a period-two asset market. The asset is produced in period 1. The asset can also be used in period 1 as collateral for loans. Borrowers and lenders participate in the lending market. A separate set of traders are the primary participants in the market for the collateral asset, but agents from the lending market who have excess holdings of collateral may also desire to participate in the period-two asset market. Whether they are permitted to do so will depend on the rules established by the regulators.

Figure 1 presents a summary timeline of the economy.

2.1 Lending Market

For simplicity there is no time discounting. Lenders and borrowers enter a relationship in period 1. The terms of the relationship specify the amount borrowed, c_1, the amount promised to be repaid c_2, and the amount of the collateral good a to be provided by the borrower, and retained by the lender in the event that the borrower does not repay the loan.

The borrower has a project available, which provides $v(c_1)$, where v is an increasing concave function satisfying the Inada conditions. The borrower can create collateral at an opportunity cost k. (We can also think of k as the payment that the borrower could receive, net of transactions costs, were he to attempt to dispose of collateral he already possessed in period 1.)

Thus the remaining information needed is the stochastic value of the collateral to borrower and lender in period 2. Let ε be a non-negative random variable representing the value of a unit of collateral to the borrower if returned in period 2. In general the value to the lender will be imperfectly correlated with the value to the borrower, and will also depend on the legal structure which determines how quickly the collateral can be liquidated when the borrower is in default. Denote the expected value to the lender conditional on ε by the continuous function $u(\varepsilon) \geq 0$.

Ex post, the borrower will decide to pay back if the value of the collateral to him exceeds the value of the repayment. We will assume the lender is completely reliable, so that there is no issue of repayment if the value of the collateral to the lender exceeds the repayment.[3]

Thus the realized utility of the borrower is

$$v(c_1) - ka$$

if he does not repay the loan and

$$v(c_1) + (\varepsilon a - c_2) - ka$$

if he does. He repays whenever

$$\varepsilon a \geq c_2$$

and so his utility is

$$v(c_1) + I(\varepsilon a \geq c_2)(\varepsilon a - c_2) - ka$$

where I is the indicator function.

The utility of the lender is

$$au(\varepsilon) - c_1 + I(\varepsilon a \geq c_2)(c_2 - au(\varepsilon)).$$

The contract is chosen to solve the following problem:

$$\max_{c_1, c_2, a, \varepsilon_0} -c_1 + \int_{\varepsilon < \varepsilon_0} au(\varepsilon)\, dF(\varepsilon) + \int_{\varepsilon \geq \varepsilon_0} c_2\, dF(\varepsilon) \tag{1}$$

subject to

$$\begin{aligned} \varepsilon_0 a &= c_2 && (2) \\ v(c_1) + \int_{\varepsilon \geq \varepsilon_0} (\varepsilon a - c_2)\, dF(\varepsilon) - ka &\geq 0, && (3) \end{aligned}$$

[3] For an examination of the problem when both borrower and lender are imperfectly reliable, see Mills and Reed [10].

where $F(.)$ is the distribution of ε, for convenience assumed continuous with density function $f(.)$. For this problem to have bounded solutions, Appendix A demonstrates that the following condition is necessary and sufficient:

$$\int_{\varepsilon<\varepsilon_0} u(\varepsilon)\, dF(\varepsilon) + \int_{\varepsilon\geq\varepsilon_0} \varepsilon\, dF(\varepsilon) < k \text{ for almost all } \varepsilon_0 \tag{4}$$

This condition has a natural interpretation: A contract ultimately allocates the asset to the borrower in some states and to the lender in other states. Roughly speaking this condition says that no matter how the asset is allocated production is costly. That is, the sum of expected asset values to borrower and lender is not enough to directly justify the cost of creating the asset. We will maintain this assumption throughout.

The variable ε_0 is the cutoff level below which the borrower defaults. It can also be interpreted as the contracted "price" for returning the collateral.

Note that ε is not the fire-sale price itself; instead it is a productivity shock on the value of the collateral to some of the participants in the market, and so it feeds into the pricing. When ε is too low it can induce fire-sale pricing as we explain in section 2.2.

Theorem 1 *The cutoff level ε_0 is chosen to solve*

$$\max_{\varepsilon_0} \frac{\int_{\varepsilon<\varepsilon_0} u(\varepsilon)\, dF(\varepsilon) + \int_{\varepsilon\geq\varepsilon_0} \varepsilon_0\, dF(\varepsilon)}{k - \int_{\varepsilon<\varepsilon_0} u(\varepsilon)\, dF(\varepsilon) - \int_{\varepsilon\geq\varepsilon_0} \varepsilon\, dF(\varepsilon)} \tag{5}$$

Proof. See Appendix A for this and the remaining proofs in this section. ■

The maximand in the theorem has a natural interpretation: Per unit of collateral, the numerator is the payoff the lender receives (ε_0 if the loan is repaid, $u(\varepsilon)$ if it is not) and the denominator is the net cost of creating the unit of collateral. Thus the objective can be described as pricing the collateral to maximize the payoff to the lender per unit expenditure in creating the collateral.

As a bonus, this same quantity can be used to compare the suitability of various assets for use as collateral. Suppose we have a variety of assets, each with its own associated distribution F, cost k, and seller utility $u(.)$. The optimized expression (5) can be calculated for each different asset type, with the following result:

Corollary 2 *Assume all assets satisfy condition (4). Then the asset which will be chosen for use as collateral is the one which maximizes (5).*

Thus condition (5) describes the desirability of an asset as collateral. If the condition (4) is violated (so that the denominator of (5) is negative), the asset is costless to use as collateral. The closer the asset comes to violating the condition, then in general, the lower the cost of using it. Clearly lower opportunity cost k and higher distributions of valuations $u(.)$ and ε make for more desirable assets. In addition if we hold the distributions of u and ε constant, but make them more negatively correlated, we are able to increase the ex post value of it to the individual who ultimately holds it: the usefulness of the asset to the lender only is relevant in states where the asset is not redeemed by the borrower and the usefulness of the asset to the borrower is only relevant in states where the lender does not retain it.

On the one hand, this corollary implies that assets which are likely to be of particular value to lenders in states where borrowers are troubled (for example T-bills or other flight-to-quality-refuges) are likely to be favored as collateral. As we explain at the beginning of this paper, consistent with this corollary, U.S. Treasuries are used as collateral. On the other hand, it implies that assets that are likely to be of high value to the borrower relative to the market as a whole (real assets with borrower-specific productivity) are also likely to be favored. In addition, assets with low costs of transfer will also be desirable.

In order to analyze the comparative statics of the problem, we consider the case of an exponential distribution for ε_0, namely $F(\varepsilon_0) = 1 - e^{-\tau\varepsilon_0}$. Let $V(\varepsilon_0)$ be the mean of $u(\varepsilon)$ conditinal on $\varepsilon < \varepsilon_0$,and assume further that the expected value of the collateral to the lender is uncorrelated with ε_0, so that $V(\varepsilon_0) = u$, a constant. Then (4) reduces to:

$$u + e^{-\tau u}\tau^{-1} \leq k, \tag{6}$$

and the optimum $(\varepsilon_0, c_1, a, c_2)$ is characterized by

$$
\begin{aligned}
\mu^{-1} &= v'(c_1) && (7)\\
(\varepsilon_0 - u)\tau &= 1-\mu && (8)\\
\mu k &= (1-e^{-\tau\varepsilon_0})u + \lambda\varepsilon_0 + \mu e^{-\tau\varepsilon_0}(\varepsilon_0 + 1/\tau) && (9)\\
\lambda &= (1-\mu)e^{-\tau\varepsilon_0} && (10)\\
c_2 &= \varepsilon_0 a && (11)\\
v(c_1)/a &= k - e^{-\tau\varepsilon_0}(1/\tau). && (12)
\end{aligned}
$$

where λ and μ are, respectively, the Lagrange multipliers of the two constraints (2)-(3).

After some algebraic manipulations, we get the following four equations which can be solved for ε_0, c_1, a,and c_2 as functions of τ, k and u.

$$
\begin{aligned}
\frac{u + e^{-\tau\varepsilon_0}(1/\tau)}{(1-(\varepsilon_0 - u)\tau)} &= k && (13)\\
v'(c_1) &= \frac{1}{1-(\varepsilon_0 - u)\tau} && (14)\\
a &= \frac{v(c_1)}{k - e^{-\tau\varepsilon_0}(1/\tau)} && (15)\\
c_2 &= \varepsilon_0 a. && (16)
\end{aligned}
$$

Specifically, we first obtain the solution of ε_0 from equation (13) given τ, k and u. Then, the remaining equations (14) through (16) give us the solution for c_1, a, and c_2 in turn.

Equation (13) is illustrated in Figure 2: the solid upward line is the expression on the left side of equation (13) as a function of ε_0 given parameters $\tau = 1.1$, and $u = 0.8$. The intersection with the horizontal solid line yields the solution $\varepsilon_0 = 0.8252$ when $k = 1.2$.

Since equation (13) is the first order condition for the ε_0, differentiating this first order condition with respect to ε_0 yields the second order condition $k\tau - e^{-\tau\varepsilon_0} \geq 0$. Combining this condition with (13) and (6) we find that

$$\varepsilon_0 > u$$

More generally, Appendix A shows the following result for the general solution to problem (1):

Theorem 3 *In the optimal borrowing contract, with positive probability the borrower will default even though the asset is worth more to the borrower than to the lender, $\varepsilon > u(\varepsilon)$.*

Because the collateral is valuable for its incentive properties, it is produced even though its direct usefulness does not justify the cost of production. In the absence of its use as collateral, the asset once produced would simply be left in the hands of whichever party valued it more. But its use as an incentive for repayment means that it will sometimes be left with the lender when the borrower fails to repay, even though the borrower values it more highly.

In the absence of the problems of enforcing repayment, the first best level of investment is determined by $v'(c_1) = 1$. The incentive problem reduces the level of investment:

Theorem 4 *In the optimal borrowing contract, c_1 is lower than the first-best level.*

For the exponential case, Appendix A establishes the following table, which summarizes the results of comparative statics (for example, the positive entry in the cell (ε_0, τ) means $\frac{\partial \varepsilon_0}{\partial \tau} > 0$.):[4]

	τ	k	u
ε_0	$+$	$+$	$+$
c_1	$-$	$-$	$+$
a	$-$	$-$	?
c_2	?	?	$+$

For ε_0 we have:

$$0 < \frac{d\varepsilon_0}{du} < 1.$$

This and the other results for ε_0 can be verified based on Figures 2 through 4.

We do not get an unambiguous sign for $\frac{da}{du}$. Appendix A shows that a sufficient condition for $\frac{da}{du}$ to be positive is that

$$v'(c_1)c_1[\frac{dc_1}{du}\frac{u}{c_1}] > (c_2 e^{-\tau\varepsilon_0})[\frac{d\varepsilon_0}{du}\frac{u}{\varepsilon_0}]. \tag{17}$$

[4] The comparative statics for u and τ are necessarily specific to the case examined here, but the comparative statics for k generalize to the full model.

In other words, if the impact of u on the additional lending evaluated at the borrower's marginal value to the project (the left hand side) is larger than the impact of u on the cutoff value, evaluated at the expected value of the repayment (the right hand side), then, it makes sense to use more collateral a as u increases. This is because if that condition holds, the lender is more happy to seize the collateral in the event of default.

Finally, the effects of τ and k on c_2 are ambiguous. Because $c_2 = \varepsilon_0 a$, the effects of τ and k on ε_0 and a offset each other, as the comparative statics thus far show. However, we know the effects of u on c_2 will be positive. To see this point, Figure 5 is helpful; the appendix provides details.

Given the limitation of comparative statics, it is useful to have some numerical examples to examine the effects of u on a and the effects of τ and k on c_2. Supposing a constant relative risk aversion function,

$$v(c_1) = \frac{c_1^{1-\sigma}}{1-\sigma}, \tag{18}$$

equations (13) through (16) simplify as follows:

$$\begin{aligned}
\frac{u + e^{-\tau\varepsilon_0}(1/\tau)}{(1-(\varepsilon_0 - u)\tau)} &= k && (19)\\
c_1 &= (1-(\varepsilon_0 - u)\tau)^{\frac{1}{\sigma}} && (20)\\
a &= (\frac{c_1^{1-\sigma}}{1-\sigma})/(k - e^{-\tau\varepsilon_0}(1/\tau)) && (21)\\
c_2 &= \varepsilon_0 a && (22)
\end{aligned}$$

The results are summarized in Figure 8 of Appendix A for specific parameter values. In Figure 8 we also report the value of $(c_2/c_1) - 1$,which corresponds to an interest rate in this model. While the effects of τ and k on c_1 are negative, the effects of τ and k on c_2 are ambiguous. The effects of u on c_1 and c_2 are both positive, but we cannot compare the impact of u on c_1 and c_2. Figure 8 shows that the impacts of τ, k and u on $(c_2/c_1) - 1$ are positive, positive and negative.

In general the costliness of collateral will inhibit the amount of lending that takes place, as shown by the comparative statics results regarding the increase in the value of k. Because holding collateral is expensive, less is held than would be necessary to guarantee the promised payment in all states where return of collateral would be efficient. In other words an inevitable feature of the collateralized debt contract is that it sometimes leaves the collateral with individuals who do not value it as much and prefer to resell it.

It is also immediate that to the extent that allowing the lender the option of selling the asset increases its value to the lender in period 2, welfare for the lender is increased by allowing immediate resale in case of default. We want to examine the effects of the resale in case of default in the collateral asset market in the next section.

2.2 Collateral Asset Market

In Antinolfi et al. [2], the effect of the automatic stay on the resale market in collateral asset arises through an externality affecting the search frictions in the market. In this paper we will treat the externality more generally, but the reader can, if s/he prefers, continue to think of the externality as arising through a search friction. The underlying crucial features are that

1. Lenders, when they show up in the resale market, are in competition with the regular sellers in that market.

2. Regular sellers make decisions about participating in the market before the realization of the extent of default.

3. Regular sellers' social contribution to the economy in the periods in which lenders do not show up exceeds the profits they obtain during those periods.

4. Thus loss of profits during periods in which lenders show up discourages participation in periods where lenders do not show up, and thereby potentially damages the market.

Appendix B shows how these considerations apply in the original Antinolfi et al. [2] model.

There are three types of agent in the collateral asset resale market in period 2 as we have shown in the third row of Figure 1: Regular sellers create S units of the asset in period 1 at a cost of K per unit. Regulation permitting, and depending on the rate of failure of borrowing in period 1, lenders may bring a supply A of collateral asset to the resale market. Assume θ, $0 < \theta < 1$, is the fraction of lenders that are allowed to use the collateral of their defaulting. An exemption from an automatic stay on collateral for all lenders implies that $\theta = 1$, and automatic stays means $\theta = 0$. Under this assumption, the supply of collateral assets to the resale market is θA.

The price at which assets can be sold in period 2, $p(S+\theta A,\varepsilon)$ depends on the total supply provided and can in addition vary with the same shocks that affect the borrowers. Regular asset buyers obtain consumer surplus from purchasing the assets equal to $P(S+\theta A,\varepsilon)$.

Assume that the use value of the asset to the lender is zero (it was held solely as an incentive). We also assume that when the borrower defaults, he would not be able to purchase replacement collateral on that asset market—a natural assumption if there is a transaction cost for the borrower to enter the asset market, as there would most certainly be in the presence of default.

Under these assumptions, S is chosen such that

$$E_\varepsilon p(\theta A(\varepsilon)+S,\varepsilon)=K$$

while efficiency in the resale market would set

$$E_\varepsilon P_S(\theta A(\varepsilon)+S,\varepsilon)=K.$$

where P_S is the partial derivative of consumer surplus with respect to an additional unit of asset provided. The existence of an externality depends on the mismatch between these two values.

Theorem 5 *If $P_S(.,.)=p(.,.)$ always, then there is no externality.*

The crucial effect to make the externality arise is the reduction in participation by the suppliers; if the number of suppliers was not affected there would be no externality. Below we consider, as an example, the case where there are only two states–either $\varepsilon=1$ and no one defaults, or $\varepsilon=0$ and everyone defaults. For specificity we will assume that when $\varepsilon=0$, $P_S\equiv p$, but when $\varepsilon=1$, they diverge.

To see this point, consider linear demand and surplus functions. If $\varepsilon=1$, no one defaults, $p=\alpha-\beta s$ and $P=\hat{\alpha}-\hat{\beta}s$ where $s=A+S$. If $\varepsilon=0$ then $p=\alpha-\beta s-\gamma-\delta s$ and $P=\hat{\alpha}-\hat{\beta}s-\hat{\gamma}-\hat{\delta}s$.

Let ϕ be the probability that no default occurs, hence $\varepsilon=1$. Then $S(\theta A)$ is chosen so that

$$\phi p(S(\theta A),1)+(1-\phi)p(\theta A+S(\theta A),0)=K$$

Since we are interested in the effects of the imposition of automatic stays in the U.S. repo market, we need to compare two polar cases where $\theta=1$

(exemption from automatic stays) and $\theta = 0$ (imposition of automatic stays). Hereafter, we set $\theta = 1$ to approximate the status quo, and examine what will happen if we set $\theta = 0$.

Letting $S(A)$ be the amount of sales by regular sellers with the fire sales by the lenders in the amount A under the assumption that $\theta = 1$, the quantity is defined by

$$\phi(\alpha - \beta S(A)) + (1-\phi)[(\alpha - \beta(A + S(A)) - \gamma - \delta(A + S(A))] = K$$

or

$$S(A) = \frac{\alpha - (1-\phi)(\beta A \ + \gamma + \delta A) - K}{\beta + (1-\phi)\delta}$$

and so the changes in the supply ΔS, from no fire sales by the lenders ($\theta = 0$) to the case where there fire sales by the lenders are permitted ($\theta = 1$) is,

$$\Delta S \equiv S(A) - S(0) = -A\left(\frac{(1-\phi)(\beta + \delta)}{\beta + (1-\phi)\delta}\right)$$

Social value of the resale market, given A and S is

$$V(A, S) \equiv \phi(\hat{\alpha} - \hat{\beta}(S)) + (1-\phi)(\hat{\alpha} - \hat{\beta}(A + S) - \hat{\gamma} - \hat{\delta}(A + S)) - KS$$

This is the social value if we include the value to both buyers and lenders. When we want to consider issues of political economy, we will also be interested in the surplus of the buyers alone, ignoring the costs of production, KS. We will denote this by

$$B(A, S) \equiv \phi(\hat{\alpha} - \hat{\beta}(S)) + (1-\phi)(\hat{\alpha} - \hat{\beta}(A + S) - \hat{\gamma} - \hat{\delta}(A + S))$$

Suppose regulators want to stop the fire sales by the lenders after the default of borrowers through the imposition of automatic stay. The effect of the automatic stay is the change in social value arising by excluding the lenders from participating in the resale market:

$$\begin{aligned}\Delta V &\equiv V(0, S(0)) - V(A, S(A)) \\ &= A(1-\phi)\frac{\phi(\beta\hat{\delta} - \hat{\beta}\delta) - K(\beta + \delta)}{(\beta + (1-\phi)\delta)}\end{aligned}$$

(the calculation is verified in the appendix).

Under our assumptions in this linear example, $\beta+\delta=\hat{\beta}+\hat{\delta}$. Suppose in addition that $\beta > \hat{\beta}$. Then, compared with the situation when lenders enter, the price of the asset falls faster than the buyer's surplus when lenders do not show up. This condition is consistent with the assumption that regular sellers' social contribution to the economy in the periods in which lenders do not show up (namely,P_S) exceeds the profits they obtain during those period (namely, the market price, p). This condition is also consistent with the existence of externality in the other model in Appendix B, which captures the mechanism proposed by Antinolfi et al. [2]. Appendix B shows that for the externality to exist it must be the case that the suppliers extract less than full rent in the state when they are the only ones around.

With $\beta > \hat{\beta}$, it is convenient to rewrite the expression ΔV as the sum of two terms, the first one positive and the second one negative:

$$\frac{\phi(\beta\hat{\delta}-\hat{\beta}\delta)}{(\beta+(1-\phi)\delta)} - \frac{K(\beta+\delta)}{(\beta+(1-\phi)\delta)}$$

The first term is associated with the externality. If this term is zero, then the exclusion of lenders is a bad idea, because ΔV is always negative. If this is an important consideration in a sense that $\phi(\beta\hat{\delta}-\hat{\beta}\delta) - K(\beta+\delta) > 0$, then the exclusion can be desirable, but this depends on the cost of the production—if the cost is high and $\phi(\beta\hat{\delta}-\hat{\beta}\delta) - K(\beta+\delta) < 0$, then again exclusion is a bad idea, since it means that the resale market must always rely on costly suppliers spending cost of K per unit of supply of the asset while the lenders provide the assets with zero cost.

Note that in this set-up, if default always occurs (namely, $\phi = 0$), the externality is irrelevant (since always $P_S(.,.) = p(.,.)$), and the exclusion of the lenders from the resale market is a bad idea. If the borrower never defaults (namely, $\phi = 1$), exclusion is irrelevant, since $S = 0$.

From the perspective of buyers (therefore ignoring K) the benefit of the automatic stay is

$$\Delta B = A(1-\phi)\frac{\phi(\beta\hat{\delta}-\hat{\beta}\delta)}{(\beta+(1-\phi)\delta)}$$

Hence, with the restriction that $\beta+\delta=\hat{\beta}+\hat{\delta}$ and $\beta > \hat{\beta}$, benefit to the buyers goes up with increasing β, down with increasing δ, up with decreasing $\hat{\beta}$, and up with increasing $\hat{\delta}$. The surplus increases with A and biggest for mid level ϕ.

One should not forget that the borrowers in the lending market pay a cost k in the earlier period for the asset. By "zero cost of supply in the resale market," we mean that, given the default, the lenders will supply the collateral asset without incurring any further cost. The increase in the value of k decreases the use of collateral asset a in the lending contract as Appendix A shows. Since the potential supply to the resale market is determined by the size of a, the value of k affects the value of ΔV through the changes in A.

To examine the effects of changes in the parameters on ΔB and ΔV, let us move on to some numerical examples.

Figure 6 reports the value of ΔB (the solid line) and $A(1-\phi)\frac{K(\beta+\delta)}{(\beta+(1-\phi)\delta)}$ (the dashed line) assuming that $\beta = 60$, $\delta = 40$, $\hat{\beta} = 45$, $\hat{\delta} = 50$, $A = 10$, and $K = 1$ by changing the value of ϕ from 0 to 1 in solid line as our benchmark. When the solid line is above the dashed line, the social value of excluding the lenders are positive, and the value is the highest around $\phi = 0.5$. It is obvious when K increases, the social value will decrease given ϕ.

Figure 7 reports the value of ΔB with higher values of β, δ, and A, which confirms the effects of changing those parameter values discussed above. The social value arising from the automatic stay goes up with increasing β, because the price will fall faster than the social benefit when none default and thus the lenders do not participate in the resale market.

We summarize the above discussions as the following theorem:

Theorem 6 *It is beneficial to introduce the automatic stay in the repo contract if the nature of the externality, as captured by the discrepancy between P and p when no one defaults (and thus the lenders never participate in the collateral asset market) is positive and large relative to the regular seller's cost of creating S supply of asset K.*

Our discussion above also highlights that the argument that the collateral should better be traded in a deep and liquid market requires clarity regarding the meaning of the term "liquidity," especially whether liquidity is originated from the demand side of asset (say, a lot of buyers and price does not fall substantially even if a large amount of assets are supplied) or originated from the supply side of asset (say, lower cost of supply).

Theorem 7 *The effects of policies making the collateral asset market more liquid on the desirability of automatic stays differ depending on the regulator's* *tools to enhance market liquidity. If a regulator takes steps to make the externality term towards zero to make the resale market liquid, the regulator would be against the automatic stays. If a regulator takes steps to make both k and K smaller to make both lending market and resale market liquid, ΔB would be larger through the increase in A due to the smaller k, and falling K would make ΔV more likely to be positive. In such a case, the regulator would favor automatic stays even though he/she does effort to make both the lending market and the resale market liquid.*

This calculation still ignores the cost the automatic stay imposes on the borrowing/lending market. Adding in these costs further decreases the value of the automatic stay.

3 Cross-border Effect of Automatic Stays

So far we have assumed that the regulator who is determining whether or not to honor automatic stays takes into account the welfare of all four participants in the market. But in important examples, it may be the case that not all of the agents in the economy receive equal welfare weights in the calculations of the regulator. In particular, when the regulator is in one country and some of the agents are foreigners, the calculations for regulatory purposes may put lower (in the extreme, zero) weights on the surpluses that those agents receive.

Suppose that repo contracts are a mechanism for liquidity used by international financial agents. For example the legal structure of the U.S. might make it an attractive place in which to engage in financial transactions, even though the individuals using the U.S. financial assets as security would have no interest in holding them for long term. The U.S. regulators may wish to reduce the possibility of fire sales caused by a failure of a foreign bank in the U.S. repo market by regulation of cross-border bank structure. For example, the U.S. regulators may force foreign banks to set up subsidiaries in the U.S. rather than branches, or by allowing only well regulated foreign banks in their home country to open branches. However, as long as the subsidiaries or branches are following the U.S. law, once the U.S. regulators allow them to participate in the U.S. market, there is no way for U.S. regulators to eliminate the possibility that the failures of some foreign banks (say, Greek banks) subject to foreign shocks (say, some shocks in Greece) become triggers for the fire sales in the collateral market. If the failing foreign banks are borrowing money using relatively illiquid U.S. bonds for the collateral for repo contract, the impact of fire sales to that illiquid U.S. bonds market could be huge.[5]

To consider the implications of such cross-border misalignments, we can return to the analysis of section 2.2 and make some minor modifications in the welfare calculations. As a simple example, assume that the buyers of the asset are in one country, and the other agents in the model are in another country. From the point of view of a regulator solely concerned with the buyers, the social value of the resale market is ΔB, rather than ΔV, because we now omit the value KS representing costs to sellers. In such a economy, the regulator is always in favor of the automatic stays when $\beta > \hat{\beta}$, because $\Delta B > 0$.

More subtly consider the case where both buyers and sellers in the resale market are in one country and the participants in the borrowing/lending market are in the other country. Now, the welfare calculation by the country with the resale market will use the expression ΔV because now the regulator cares about both buyers and sellers.

Regarding the welfare calculation by the country with the lending market, note that the additional expected utility for the lender due to the use of the resale market is an increasing function of a, which coincide with A in this case. This is because the lender does not have any additional values from the holdings of that collateral asset, and thus the best he can get is the amount he sells in the resale market multiplied by the fire sales price of that collateral asset. Note also that we can ignore the benefit to the borrower, because the borrower's expected utility is always zero due to the binding constraints in the optimal lending contract as we see in Figure 8 in the appendix.

Now, the regulator in charge of the resale market would like to have automatic stays for repo contracts if they observe high value of A because his benefit from the automatic stay, ΔV, is an increasing function of A,when $\Delta V > 0$.

However, the regulator of the borrower/lender country hopes that the fire sales of the lender will help the financial situation of the lender, and thus he does not like to have automatic stays especially if he observes high value of A. The two regulators have completely opposite preferences regarding the

[5]See the composition of collaterals used in the U.S. tri-party market before and after financial turmoil in Copeland et al [4], Page 11, Table 1, http://www.newyorkfed.org/research/staff_reports/sr506.pdf.

choice of automatic stay exemption to repo contracts. We summarize the results as below.

Theorem 8 *If the regulators in the collateral asset resale market country favor the automatic stays for repo contracts ($\Delta V > 0$), the regulators in the borrower/lender country would favor the exemption from the automatic stays for repo contracts. In such a situation, the borrower/lender would like to make their initial lending contract in economies where regulators favor the exemptions to the automatic stays.*

There is a conflict between the interests of the agents in the two markets. Clearly coordination between the two countries is desirable, but the real question is, in the absence of coordination, what are likely to be the outcomes–in effect, which country really has control over the decisions to permit or bypass automatic stays? The answer to that depends on the location of the bankrupt firm.

For example, in the Lehman dispute, bankruptcy proceedings occurred in both the U.S. and the U.K. Each jurisdiction used its own standards for determining degree to which creditors could obtain access to collateral from the defunct institution. Indeed, some U.S. legal experts have been wondering if the special treatment of repo contract in the U.S. bankruptcy law might induce financial institutions to use repo contract even though the creditworthiness of the counterparty is deteriorated because they have the safe harbor treatment, even before the Lehman's failure.[6] Thus we would expect that jurisdictions which provide for generous exemptions from automatic stays would be the jurisdictions which both borrowers and lenders would prefer to use for their financial contracts. Being able to exploit these bankruptcy provisions provides an incentive for incorporation under those laws. In the case of large financial institutions domiciled in multiple jurisdictions, the temptation is to use the jurisdiction with the most liberal exemptions as the location for the trade in repos.[7]

[6] See Yamamoto (2013) on the survey of this point.

[7] For example, three overseas affiliated subsidiaries of Japanese large security firms: Nomura Securities International, Inc., Daiwa Capital Markets America Inc., Mizuho Securities USA Inc., are active players in the U.S. repo market. While there are many sound economic reasons for such activity, the differing bankruptcy rules can also be a consideration in these firms' choice of jurisdictions.

If the assets being used in these transactions are primarily of use to individuals in a different country, then that country has a difficult dilemma. It is not easy to protect itself from the consequences of a fire sales induced by failures of foreign firms. Restricting sales of assets on the domestic market is not likely to be effective, particularly if it is possible to observe shadow prices for equivalent assets bought and sold abroad. Putting in place a financial insurer to guarantee a floor price for the domestic assets in the presence of foreign panics will be expensive, and only can be effective if the guarantor institutions pockets are extremely deep.

4 Conclusion

To analyze the effects of the automatic stay, we provided models for lending market and collateral market respectively, and obtained the following results.

First, in an optimal borrowing contract with collateral, the borrower will default even though the asset is worth more to the borrower than to the lender in some states of the economy.

Second, in such an optimal contract with collateral, the crucial characteristics of the assets more desirable as collateral, in a sense that a large amount of the assets will be used as collateral in equilibrium, will tend to have the following characteristics: (1) lower opportunity cost to create the collateral, (2) lower hazard rate in the distribution of valuations of the collateral assets, and (3) higher negative correlation of valuations between borrower and lender.

Third, we define the externality in the resale market as the departure from the market price and the buyer's surplus when there is no entry of the lenders to the resale market. We then ask if the externality alone justifies the intervention to stop the fire sales of lenders experiencing the default of their borrowers in the resale market, and find that it is beneficial to introduce the automatic stays in repo transactions if the effects of the automatic stays on the social value through the externality is positive and larger than the regular seller's cost of creating the asset in the resale market.

Finally, we have investigated the effect on optimal policy of changing the weights of different participants in the objective function. For example, national regulators are likely to evaluate the welfare benefits from the lending market and the resale market from the national perspectives. For example, suppose that the regular sellers and buyers in the resale market and the market participants in the lending market live in different countries. If the regulators in the resale market country favor the automatic stays, our model suggests that the regulators in the borrower/lender country will favor the exemption from the automatic stays. In such a case, we need international coordination on the implementation of the automatic stay requirements to resolve the conflicts of interest between two regulators. To the best of our knowledge, we are the first to discuss such a cross-border issues in a tractable model.

Our analysis could provide guidance to regulators about appropriate restrictions on exemptions from automatic stays. If they want to expand or reduce the list of financial assets under safe harbour provision, they should weigh the possibly opposing requirements from the participants in repo contracts and participants in the asset market. Relevant considerations would include characteristics of the collateral asset that help to define the liquidity of its market–according to our model, such characteristics include the opportunity cost of holding the asset, the difference between the average values of the asset from the point of view of borrower and lender, and the correlation of these values. We speculate that centrally and publicly cleared contracts, with small cost of creation, would be relatively easy to satisfy the necessary characteristics of collateral asset.

5 Appendix A. Calculations for the collateralized loan contract

Appendix A will explain the details of the derivation of the first order conditions, comparative statics and numerical examples in Section 2.1. We will present the problem in a slightly expanded version by considering the possibility that the lender (who is assumed to be reliable) also contracts to pay a separate fixed amount c_3 to the borrower.

Let H be the hazard function for F, let $G(\varepsilon_0)$ be the mean of ε conditional on $\varepsilon \geq \varepsilon_0$, and let $V(\varepsilon_0)$ be the mean of $u(\varepsilon)$ conditional on $\varepsilon < \varepsilon_0$. Then we can write the problem (1) as

$$\max_{c_1, c_2, c_3, a, \varepsilon_0} -c_1 + aF(\varepsilon_0)V(\varepsilon_0) + (1 - F(\varepsilon_0))c_2 - c_3$$

subject to

$$\begin{aligned}
\varepsilon_0 a &= c_2 && (23)\\
a &\geq 0 && (24)\\
c_3 &\geq 0 && (25)\\
v(c_1) + ((1 - F(\varepsilon_0))G(\varepsilon_0) - k)a + c_3 &\geq (1 - F(\varepsilon_0))c_2. && (26)
\end{aligned}$$

Holding the value of ε_0 fixed, the problem is convex (and linear in a, c_2, c_3).

The problem is unbounded unless

$$F(\varepsilon_0)V(\varepsilon_0) + (1 - F(\varepsilon_0))G(\varepsilon_0) < k \tag{27}$$

(To see this consider Figure 9, which shows the objective function and the feasible combinations of a and c_3, given that $\varepsilon_0 a = c$. The light lines are isoprofit lines for the lender; highest profit lies to the south and east. The heavy line represents the combinations for which (26) is binding, and feasible combinations (positive profit for the borrower) lie above the line. The solution is bounded if and only if the slope of the feasible boundary is at least as steep as the slope of the isoprofit lines.)

Condition (4) of the main text is equivalent to (27). If this condition is satisfied, then a solution to the problem for each ε_0 occurs with conditions (25) and (26) binding (point A in Figure 9). For this reason that we have omitted the possibility of $c_3 > 0$ in our description in the text.

Eliminating c_2 and c_3, condition (26) reduces to

$$v(c_1) = a[(1 - F(\varepsilon_0))\varepsilon_0 - (1 - F(\varepsilon_0))G(\varepsilon_0) + k]$$

and the problem therefore reduces to finding c_1, ε_0 to maximize

$$\max_{c_1,\varepsilon_0} -c_1 + \frac{v(c_1)}{(1 - F(\varepsilon_0))\varepsilon_0 - (1 - F(\varepsilon_0))G(\varepsilon_0) + k}(F(\varepsilon_0)V(\varepsilon_0) + (1 - F(\varepsilon_0))\varepsilon_0)$$

Thus the correct ε_0 maximizes

$$Z(\varepsilon_0) \equiv \frac{F(\varepsilon_0)V(\varepsilon_0) + (1 - F(\varepsilon_0))\varepsilon_0}{(1 - F(\varepsilon_0))\varepsilon_0 - (1 - F(\varepsilon_0))G(\varepsilon_0) + k} \tag{28}$$

thereby proving Theorem 1 of the text. (Expression (5) in the text is in fact equal to $Z(\varepsilon_0)/(1 - Z(\varepsilon_0))$ which is an increasing monotonic transformation of $Z(\varepsilon_0)$ by the following lemma. Corollary 2 in the text is an immediate consequence of the theorem.)

Lemma 9 *The expression (28) is positive and less than* 1 *for all* ε_0.

Proof. (27) implies

$$\begin{aligned} 0 &\leq F(\varepsilon_0)V(\varepsilon_0) \leq k - (1 - F(\varepsilon_0))G(\varepsilon_0) \\ F(\varepsilon_0)V(\varepsilon_0) + (1 - F(\varepsilon_0))\varepsilon_0 &\leq k - (1 - F(\varepsilon_0))G(\varepsilon_0) + (1 - F(\varepsilon_0))\varepsilon_0. \end{aligned}$$

■

And so an optimal ε_0 exists in the set $[0, \infty]$ (where $\varepsilon_0 = \infty$ in effect means that the "borrower" simply sells collateral to the "lender").

This lemma implies that

$$v'(c_1) > 1$$

demonstrating Theorem 4.

The first order condition for ε_0 is (using $N(\varepsilon_0)$ and $D(\varepsilon_0)$ as temporary shorthand for the numerator and denominator of the expression (28)):

$$\begin{aligned} & D(\varepsilon_0)\frac{d}{d\varepsilon_0}[\int^{\varepsilon_0} u(\varepsilon)\, dF(\varepsilon) + (1 - F(\varepsilon_0))\varepsilon_0] \\ & -N(\varepsilon_0)\frac{d}{d\varepsilon_0}[(1 - F(\varepsilon_0))\varepsilon_0 - \int_{\varepsilon_0} \varepsilon\, dF(\varepsilon) + k] \\ = \; & D(\varepsilon_0)[(u(\varepsilon_0) - \varepsilon_0)f(\varepsilon_0) + 1 - F(\varepsilon_0)] - N(\varepsilon_0)[1 - F(\varepsilon_0) - \varepsilon_0 f(\varepsilon_0) + \varepsilon_0 f(\varepsilon_0)] \\ = \; & D(\varepsilon_0)(u(\varepsilon_0) - \varepsilon_0)f(\varepsilon_0) + (D(\varepsilon_0) - N(\varepsilon_0))(1 - F(\varepsilon_0)) = 0 \end{aligned}$$

and since $D(\varepsilon_0) > N(\varepsilon_0) > 0$ by the above lemma, we conclude that $u(\varepsilon_0) < \varepsilon_0$ at the optimum. This proves Theorem 3 of the text.

Letting λ and μ represent the multipliers, we can write the Lagrangian for the problem as

$$c_1 + \int_{\varepsilon<\varepsilon_0} au(\varepsilon)dF(\varepsilon) + \int_{\varepsilon\geq\varepsilon_0} c_2 dF(\varepsilon) + \lambda(\varepsilon_0 a - c_2) + \mu(v(c_1) + \int_{\varepsilon\geq\varepsilon_0} (\varepsilon a - c_2)dF(\varepsilon) - ka)$$

and the first order conditions are

$$\begin{aligned} -1 + \mu v'(c_1) &= 0 \\ 1 - F(\varepsilon_0) - \lambda - \mu(1 - F(\varepsilon_0)) &= 0 \\ \int_{\varepsilon<\varepsilon_0} u(\varepsilon)\, dF(\varepsilon) + \lambda\varepsilon_0 + \mu \int_{\varepsilon\geq\varepsilon_0} \varepsilon\, dF(\varepsilon) &= k \\ (au(\varepsilon_0) - c_2)f(\varepsilon_0) + \lambda a - \mu(\varepsilon_0 a - c_2)f(\varepsilon_0) &= 0 \end{aligned}$$

Using the definitions of $V(\varepsilon_0)$ and $G(\varepsilon_0)$ we can write those first order conditions and constraints as follows:

$$\begin{aligned}
-1+\mu v'(c_1) &= 0\\
1-F(\varepsilon_0)-\lambda-\mu(1-F(\varepsilon_0)) &= 0\\
V(\varepsilon_0)F(\varepsilon_0)+\lambda\varepsilon_0+\mu G(\varepsilon_0)(1-F(\varepsilon_0)) &= \mu k\\
(au(\varepsilon_0)-c_2)f(\varepsilon_0)+\lambda a-\mu(\varepsilon_0 a-c_2)f(\varepsilon_0) &= 0\\
\varepsilon_0 a &\geq c_2\\
v(c_1)+(1-F(\varepsilon_0)(aG(\varepsilon_0)-c_2)-ka &\geq 0
\end{aligned}$$

and assuming the inequality constraints are binding, we get

$$\begin{aligned}
\mu &= \frac{1}{v'(c_1)}\\
\lambda &= (1-\mu)(1-F(\varepsilon_0))\\
\mu k &= \int_{\varepsilon<\varepsilon_0} u(\varepsilon)\, dF(\varepsilon)+\lambda\varepsilon_0+\mu\int_{\varepsilon\geq\varepsilon_0}\varepsilon\, dF(\varepsilon)\\
(1-\mu) &= \frac{(\varepsilon_0-u(\varepsilon_0))f(\varepsilon_0)}{1-F(\varepsilon_0)}\\
c_2 &= \varepsilon_0 a\\
ka &= v(c_1)+\int_{\varepsilon\geq\varepsilon_0}(\varepsilon a-c_2)dF(\varepsilon)
\end{aligned}$$

Consider the case of an exponential distribution for ε_0. That is,

$$\begin{aligned}
F(\varepsilon_0) &= 1-e^{-\tau\varepsilon_0}\\
G(\varepsilon_0) &= \varepsilon_0+1/\tau\\
H(\varepsilon_0) &= \tau
\end{aligned}$$

and assume that $V(\varepsilon_0)=u$, a constant. Then $Z(\varepsilon_0)$ becomes

$$\frac{(1-e^{-\tau\varepsilon_0})u+e^{-\tau\varepsilon_0}\varepsilon_0}{k-e^{-\tau\varepsilon_0}(1/\tau)} \tag{29}$$

and the condition for the bounded solution (27) becomes

$$u + e^{-\tau\varepsilon_0}(\varepsilon_0 - u + 1/\tau) \leq k \text{ for all } \varepsilon_0 \tag{30}$$

The left side of the condition is maximized at $\varepsilon_0 = u$, so the the condition for the bounded solution is equivalent to

$$u + \tau^{-1}e^{-\tau u} \leq k$$

which implies also

$$1/\tau \leq k \tag{31}$$

The first order condition for a maximum to (29) is

$$(k - e^{-\tau\varepsilon_0}\tau^{-1})e^{-\tau\varepsilon_0}(1 + \tau u - \tau\varepsilon_0) - [(1 - e^{-\tau\varepsilon_0})u + e^{-\tau\varepsilon_0}\varepsilon_0]e^{-\tau\varepsilon_0} = 0$$

$$\begin{aligned}
[(1 - e^{-\tau\varepsilon_0})u + e^{-\tau\varepsilon_0}\varepsilon_0] + e^{-\tau\varepsilon_0}\tau^{-1}(1 + \tau u - \tau\varepsilon_0) &= k(1 + \tau u - \tau\varepsilon_0) \\
[(1 - e^{-\tau\varepsilon_0})u] + e^{-\tau\varepsilon_0}\tau^{-1}(1 + \tau u) &= k(1 + \tau u - \tau\varepsilon_0) \\
u + e^{-\tau\varepsilon_0}\tau^{-1} &= k(1 + \tau u - \tau\varepsilon_0)
\end{aligned}$$

Note that the last equation is the same as the first order condition for ε_0 obtained in the main text (equation (13)). Moreover, differentiating this first order condition with respect to ε_0 yields the second order condition

$$-k\tau + e^{-\tau\varepsilon_0} \leq 0, \tag{32}$$

which is satisfied by condition (31). Hence the solution yields the maximum.

Then, the above first order conditions will be

$$\begin{aligned}
\mu^{-1} &= v'(c_1) \\
(\varepsilon_0 - u)\tau &= 1 - \mu \\
\mu k &= (1 - e^{-\tau\varepsilon_0})u + \lambda\varepsilon_0 + \mu e^{-\tau\varepsilon_0}(\varepsilon_0 + 1/\tau) \\
\lambda &= (1 - \mu)e^{-\tau\varepsilon_0} \\
c_2 &= \varepsilon_0 a \\
v(c_1)/a &= k - e^{-\tau\varepsilon_0}(1/\tau).
\end{aligned}$$

Inserting the second equation into the first equation, and inserting the second and fourth equations into the third equation, we get equations (13) through (16) of the text.

Combining the bounded solution condition (30) with the first order conditions for ε_0, we get the following inequality;

$$u + e^{-\tau\varepsilon_0}\tau^{-1} - k = k\tau(u - \varepsilon_0) \leq e^{-\tau\varepsilon_0}(u - \varepsilon_0)$$

which simplifies to

$$(k\tau - e^{-\tau\varepsilon_0})(u - \varepsilon_0) \leq 0,$$

and since the second order condition (32) implies the first term is positive, we conclude that $\varepsilon_0 > u$.

From condition (13) we have

$$u + e^{-\tau\varepsilon_0}\tau^{-1} = k(1 + \tau u - \tau\varepsilon_0)$$

and totally differentiating, we have that

$$\begin{aligned}
0 &< \frac{d\varepsilon_0}{du} = \frac{k\tau - 1}{k\tau - e^{-\tau\varepsilon_0}} < 1 \\
\frac{d\varepsilon_0}{dk} &= \frac{1 - \tau(\varepsilon_0 - u)}{k\tau - e^{-\tau\varepsilon_0}} = \frac{(u + e^{-\tau\varepsilon_0}\tau^{-1})(1/k)}{k\tau - e^{-\tau\varepsilon_0}} > 0 \\
\frac{d\varepsilon_0}{d\tau} &= \frac{e^{-\tau\varepsilon_0}(\varepsilon_0 + \tau^{-1}) - k\tau(\varepsilon_0 - u)}{\tau(k\tau - e^{-\tau\varepsilon_0})} = \frac{e^{-\tau\varepsilon_0}\varepsilon_0 + k - u}{\tau(k\tau - e^{-\tau\varepsilon_0})} > 0
\end{aligned}$$

What can be said about the changes in the exogenous variables τ, k and u on the other endogenous variables c_1, a and c_2? We summarize the results of comparative statistics in turn.

First, regarding c_1, we can get the following clear results. By totally differentiating condition (14) we get

$$v''(c_1)dc_1 = \frac{(\varepsilon_0 - u)d\tau + \tau(d\varepsilon_0 - du)}{(1 - (\varepsilon_0 - u)\tau)^2}$$

and using the previous results for the derivatives of ε_0, we obtain the following expressions:

$$\begin{aligned}
\frac{dc_1}{d\tau} &= \frac{1}{v''(c_1)} \frac{(\varepsilon_0 - u) + \tau\frac{d\varepsilon_0}{d\tau}}{(1 - (\varepsilon_0 - u)\tau)^2} < 0 \\
\frac{dc_1}{dk} &= \frac{1}{v''(c_1)} \frac{\tau\frac{d\varepsilon_0}{dk}}{(1 - (\varepsilon_0 - u)\tau)^2} < 0 \\
\frac{dc_1}{du} &= \frac{1}{v''(c_1)} \frac{\tau(\frac{d\varepsilon_0}{du} - 1)}{(1 - (\varepsilon_0 - u)\tau)^2} > 0
\end{aligned}$$

In obtaining the sign, we use the conditions that $v''(c_1) < 0, \varepsilon_0 - u > 0, \frac{d\varepsilon_0}{d\tau} > 0, \frac{d\varepsilon_0}{dk} > 0, \frac{d\varepsilon_0}{du} - 1 < 0$.

Second, regarding a, the changes in τ and k have clear effects, while the effects of u are ambiguous as follows. By totally differentiating the first order conditions, $v(c_1)/a = k - e^{-\tau\varepsilon_0}(1/\tau)$, we get

$$\frac{av'(c_1)dc_1 - v(c_1)da}{a^2} = dk + e^{-\tau\varepsilon_0}d\varepsilon_0 + \frac{e^{-\tau\varepsilon_0}}{\tau}(\varepsilon_0 + \frac{1}{\tau})d\tau$$

or

$$\begin{aligned}
\frac{da}{d\tau} &= \frac{a^2}{v(c_1)}\left[\frac{v'(c_1)}{a}\frac{dc_1}{d\tau} - \frac{e^{-\tau\varepsilon_0}}{\tau}(\varepsilon_0 + \frac{1}{\tau})\right] < 0 \\
\frac{da}{dk} &= \frac{a^2}{v(c_1)}\left[\frac{v'(c_1)}{a}\frac{dc_1}{dk} - 1\right] < 0 \\
\frac{da}{du} &= \frac{a^2}{v(c_1)}\left[\frac{v'(c_1)}{a}\frac{dc_1}{du} - e^{-\tau\varepsilon_0}\frac{d\varepsilon_0}{du}\right]
\end{aligned}$$

Regarding the sign of $\frac{da}{du}$, the first term in the right hand side is negative, and the second term in the right hand side is positive. We do not have particular restrictions regarding the size of those two terms, and thus the sign of $\frac{da}{du}$ is ambiguous. However, if $\frac{v'(c_1)}{a}\frac{dc_1}{du} > e^{-\tau\varepsilon_0}\frac{d\varepsilon_0}{du}$, then $\frac{da}{du} > 0$. The condition (17) of the text follows. This conditions says that as u (the expected value of collateral to the lender) increase, the initial loan (c_1) increase, if the impact of u on the additional lending, evaluated at the borrower's marginal value to the project is larger than the impact of u on the cut off value, evaluated at the expected value of the repayment. In such a situation, it makes sense to use more precious collateral for the lender (since u increases) for the sake of risky investment.

Finally, regarding c_2, the effects of changes in τ, k and u are ambiguous. This is because $c_2 = \varepsilon_0 a$, and we prove that the changes in τ and k have opposite effects on ε_0 and a. We do not know the sign of the effect of u on both ε_0 and a, however, we do know about the sign of the effect of u on c_2. To see this point, Figure 8 is useful. Figure 8 plots two equations, which corresponds to the incentive constraints and the participation constraints.

$$\begin{aligned}
\varepsilon_0 a &= c_2 \\
\frac{v(c_1)}{e^{-\tau\varepsilon_0}} + \left[\varepsilon_0 + (1/\tau) - \frac{k}{e^{-\tau\varepsilon_0}}\right]a &= c_2.
\end{aligned}$$

The panel A of Figure 8 shows that the intersections of the thin solid upward sloping curve, $\varepsilon_0 a = c_2$, and the solid downward sloping curve, $\frac{v(c_1)}{e^{-\tau\varepsilon_0}} + \left[\varepsilon_0 + (1/\tau) - \frac{k}{e^{-\tau\varepsilon_0}}\right] a = c_2$, becomes the solution of a and c_2, the point A. $\left[\varepsilon_0 + (1/\tau) - \frac{k}{e^{-\tau\varepsilon_0}}\right] < 0$ follows from the boudedness conditions.

If an increase in the value of parameters τ, k and u occurs, the thin solid line $c_2 = \varepsilon_0 a$ will shift upward, and moves towards the thick solid upward line. This is because we know that the all increase in these parameters lead to the increase in $c_2$given ε_0 and a.

Regarding the relationship $\frac{v(c_1)}{e^{-\tau\varepsilon_0}} + \left[\varepsilon_0 + (1/\tau) - \frac{k}{e^{-\tau\varepsilon_0}}\right] a = c_2$, we have the following results for the intercept term.

$$\begin{aligned}
\frac{d[\frac{v(c_1)}{e^{-\tau\varepsilon_0}}]}{d\tau} &= \frac{1}{e^{-\tau\varepsilon_0}}\left[v'(c_1)\frac{dc_1}{d\tau} + v(c_1)(\tau\frac{d\varepsilon_0}{dk} + \varepsilon_0)\right] \\
\frac{d[\frac{v(c_1)}{e^{-\tau\varepsilon_0}}]}{dk} &= \frac{1}{e^{-\tau\varepsilon_0}}\left[v'(c_1)\frac{dc_1}{dk} + v(c_1)\tau\frac{d\varepsilon_0}{dk}\right] \\
\frac{d[\frac{v(c_1)}{e^{-\tau\varepsilon_0}}]}{du} &= \frac{1}{e^{-\tau\varepsilon_0}}\left[v'(c_1)\frac{dc_1}{du} + v(c_1)\tau\frac{d\varepsilon_0}{du}\right] > 0
\end{aligned}$$

Regarding the slope term, we get the following results.

$$\begin{aligned}
\frac{d[\varepsilon_0 + (1/\tau) - \frac{k}{e^{-\tau\varepsilon_0}}]}{d\tau} &= \frac{d\varepsilon_0}{d\tau}(1 - \frac{k\tau}{e^{-\tau\varepsilon_0}}) - (\frac{1}{\tau^2} + \frac{\varepsilon_0 k}{e^{-\tau\varepsilon_0}}) < 0 \\
\frac{d[\varepsilon_0 + (1/\tau) - \frac{k}{e^{-\tau\varepsilon_0}}]}{dk} &= \frac{d\varepsilon_0}{d\tau}(1 - \frac{k\tau}{e^{-\tau\varepsilon_0}}) - \frac{1}{e^{-\tau\varepsilon_0}} < 0 \\
\frac{d[\varepsilon_0 + (1/\tau) - \frac{k}{e^{-\tau\varepsilon_0}}]}{du} &= \frac{d\varepsilon_0}{du}(1 - \frac{k\tau}{e^{-\tau\varepsilon_0}}) < 0
\end{aligned}$$

Therefore, if we increase the value of parameters τ and k, the slope becomes flatter, but the effects on the intercept term is ambiguous. Hence, as the panel A shows, depending on the changes in the intercept term, c_2 may increase or decrease. For example, if the intercept term goes up, the equilibrium would be point C, and c_2 increases. If the intercept term goes down, the equilibrium would be point B, and c_2 decreases.

Regarding the increase in u, the slope becomes flatter, and the intercept term increases (as changes from C to D in panel B of Figure 8), and thus we are sure that c_2 increases (as changes from A to B of Figure 8).

The table in the text summarizes the results of comparative statics.

5.0.1 Numerical examples

Figure 8 reports numerical examples for the case of an exponential distribution and a constant relative risk averse utility function (18), based on the first order conditions (19-22)

The second row of Figure 8 reports the benchmark numerical solutions for the equilibrium values of ε_0, c_1, a,and c_2, probability of default (the column labeled as P. of default) and the expected utility of a borrower (the column labelled as EU (Borrower)) and that of a lender (the column labelled as EU (Lender))given $u = 0.8$, $\tau = 1.1$, $k = 1.2$, and $\sigma = 0.5$. The probability of default is $(1 - e^{-\tau\varepsilon_0})$ and the expected utility of a lender is defined as $EU(Lender) = -c_1 + a(1 - e^{-\tau\varepsilon_0})u + e^{-\tau\varepsilon_0}c_2$. All numerical solutions ε_0 in the table satisfy the condition for bounded solution, equation (3). Moreover, due to the binding incentive constraint, equation (3), the expected utility of a borrower is always zero.

The fifth column of the second row of Figure 8 (the column labeled as P. of default) shows that in the optimal borrowing contract with collateral, the probabilities of default are positive. Note that the value of the unit of collateral to the borrower if returned, $\varepsilon_0 = 0.8252$, is positive and greater than the value of the collateral for lenders when the borrower defaults, $u = 0.8$. Namely, borrowers will default in some states even though the asset is worth more to the borrower than to the lender, which means that the optimal level of ε_0 is higher than the level which efficiently allocates the collateral ex post. Given $u = 0.8$, $\tau = 1.1$, $k = 1.2$, and $\sigma = 0.5$, the seventh to the eleventh column of the first row of Figure 8 report the numerical solutions for c_1, a,and c_2, and the expected utility of a borrower (the column labelled as EU (Borrower)) and that of a lender (the column labelled as EU (Lender)) as follows: $c_1 = 0.9454, a = 2.3339, c_2 = 1.9258$, the expected utility of a borrower is zero and that of a lender is equal to 0.9454.

The remaining rows of Figure 8 examine the effects of changes in the parameter values of τ, k, u and σ on the endogenous variables. We will explain those effects in turn.

The third and fourth rows of Figure 8 examine the changes in the value of hazard rate τ from 1.1 to 1.2 and 1.3 given $k = 1.2$ and $u = 0.8$. Comparisons of these rows with the second row show that higher value of hazard rate increases the probability of default and the value of unit collateral if returned ε_0. The size of initial loan c_1 and the amount of collateral a decrease, as theoretically expected. In these numerical examples, the amount promised to repay and the expected utility of the lender decrease.

The fifth and seventh rows of Figure 8 examine the effects of increase in the costliness of collateral for the borrower represented by higher values of k, from 1.2 to 1.3 and 1.4 given $\tau = 1.1$ and $u = 0.8$. Comparing with the second rows, if we increase the value of k, ε_0 increases, c_1 decreases, and a, decreases as theoretically expected. The amount promised to repay, c_2, which equals to $\varepsilon_0 a$, and the utility of the lender decreases in those examples.

The seventh and eighth rows of Figure 8 examine the effects of the increase in the value of u, from 0.8 to 0.81 and 0.82 given $\tau = 1.1$ and $k = 1.2$. We find that the increase in the value of u yields higher values of the unit of collateral to the borrower if returned, ε_0, and the probability of default. The amount of initial loan, c_1, and the amount promised, c_2, increase as theoretically expected. In these numerical examples, the amount of collateral, a, and the expected value of the utility of the lender increases.

5.0.2 Welfare comparative statics

Welfare increases with decreases in k, with increases in the value of the collateral to the lender ($V(\varepsilon_0)$) or to the borrower ($G(\varepsilon_0)$), holding constant the probability in the region of ε_0. It also increases with movement of collateral value from below ε_0 to above.

$$\max_{c_1,\varepsilon_0} -c_1 + \frac{v(c_1)}{(1-F(\varepsilon_0))\varepsilon_0 - (1-F(\varepsilon_0))G(\varepsilon_0) + k}(F(\varepsilon_0)V(\varepsilon_0) + (1-F(\varepsilon_0))\varepsilon_0)$$

Any of these improvements reduce the shadow cost of collateral and increase the amount borrowed.

If we consider the case of an exponential distribution exponential distribution for ε_0, in equilibrium, the welfare statistics will take the following functional form:

$$EU(Lender) = -c_1 + a(1 - e^{-\tau\varepsilon_0})u + e^{-\tau\varepsilon_0}c_2$$

Note that due to the incentive constraint, the expected utility of a borrower is always zero in equilibrium. Hence the welfare of the lending market will be summarized by the expected utility of a lender alone.

6 Appendix B. Comparison to Antinolfi et al. resale market

(Note that in this Appendix, B stands for number of buyers and does not correspond to the buyers' surplus in the rest of this paper.)

The purpose of this appendix is to verify that the features listed in section 2.2 are characteristics of the model of Antinolfi et al. (ACetal). In that model, the key assumption is of a bilateral search and bargaining structure for the final market. This assumption generates the features described.

Specifically, they assume a bilateral matching technology in which individuals on the "tight" side of the market are matched with certainty, and individuals on the other side are all matched with equal probability. In a successful match the payout from the trade is split in fixed proportions, s to the seller, $1 - s$ to the buyer. All agents want to buy or sell one unit of the good.

Sellers come in two versions: "suppliers" and "lenders." Suppliers make a decision before the period begins as to whether to join the market. If they join, they pay a cost k to supply the good. Lenders have a stochastic probability of joining the market (in the full ACetal model, depending on whether their borrower has defaulted, leaving them with collateral to sell). The number joining depends on, among other things, a state realization ε. The payout the seller obtains from sale if successful is sP, and the reservation value if unsuccessful is p. If there are B buyers, S suppliers and L lenders, then the odds that a seller is matched is $\min\{1, B/(S + L)\}$.

Thus a supplier's realized profit from producing a unit is

$$p + \frac{B}{S+L}(sP - p) - k$$

which is decreasing in L. Suppliers will choose to join the market if the expectation of this quantity is positive.

If there is a shortage of suppliers, then the social value of the appearance of a supplier is $P - k$. The private value is $sP - k$. If there is a surplus of lenders, then the social value of joining the market is $p - k$. The private value is $p + \frac{B}{S}(sP - p) - k$ (provided, as we should assume, that $sP > p$), less than the profits in the lenders' absence.

7 Appendix C: Linear functional form for the collateral market

Appendix C verifies the derivation of section 2.2.

The change in social value arising by excluding the L lenders from participating in the resale market (or, social value arising from the imposition of automatic stay), ΔV, is

$$\begin{aligned} & \Delta V \equiv V(0, S(0)) - V(A, S(A)) \\ = \; & [\phi(\hat{\alpha} - \hat{\beta} S_0) + (1-\phi)(\hat{\alpha} - \hat{\beta} S_0 - \hat{\gamma} - \hat{\delta} S_0) - K S_0] \\ & -[\phi(\hat{\alpha} - \hat{\beta}(S_L)) + (1-\phi)(\hat{\alpha} - \hat{\beta}(A + S_L) - \hat{\gamma} - \hat{\delta}(A + S_L)) - K S_L] \end{aligned}$$

or

$$\Delta V = A(1-\phi)(\hat{\beta} + \hat{\delta}) - \Delta S[\hat{\beta} + (1-\phi)\hat{\delta} + K]$$

Substituting for ΔS,

$$\begin{aligned} \Delta V = \; & A(1-\phi)(\hat{\beta} + \hat{\delta})[1 - \left(\frac{(\beta/\delta + 1)}{\beta/\delta + (1-\phi)} \right) \frac{\hat{\beta}/\hat{\delta} + (1-\phi) + K/\hat{\delta}}{\hat{\beta}/\hat{\delta} + 1}] \\ = \; & A(1-\phi)(\hat{\beta} + \hat{\delta}) \frac{(\beta/\delta + (1-\phi))(\hat{\beta}/\hat{\delta} + 1) - (\beta/\delta + 1)(\hat{\beta}/\hat{\delta} + (1-\phi) + K/\hat{\delta})}{(\beta/\delta + (1-\phi))(\hat{\beta}/\hat{\delta} + 1)} \\ = \; & A(1-\phi) \frac{\phi(\beta\hat{\delta} - \hat{\beta}\delta) - K(\beta + \delta)}{(\beta + (1-\phi)\delta)} \end{aligned}$$

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Figure 1 Market Participants and Time Line

	Borrower	Lender		Asset Seller	Asset Buyer
Period 1	Enter Lending Contract				
	Creates Asset	Receives Asset as Collateral		Creates Asset	
			Project Payoffs and Asset Values Realized		
Period 2	Default Decision				
		Collateral Sold (if default occurs and if no automatic stay)		Asset Sold	Asset Bought

Figure 2 Benchmark solution and comparative statistics with respect to τ

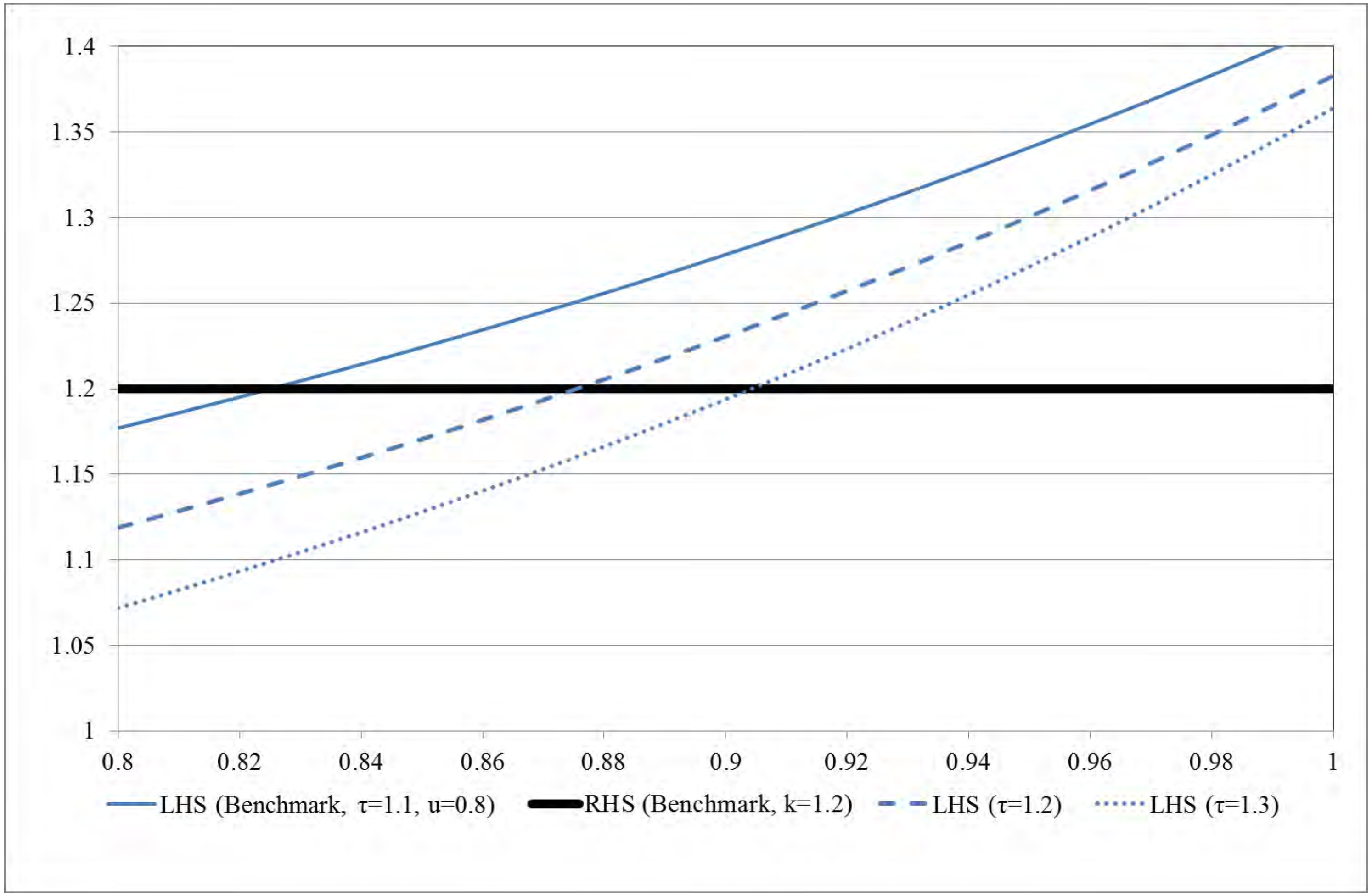

Figure 3 Benchmark solution and comparative statistics with respect to *k*

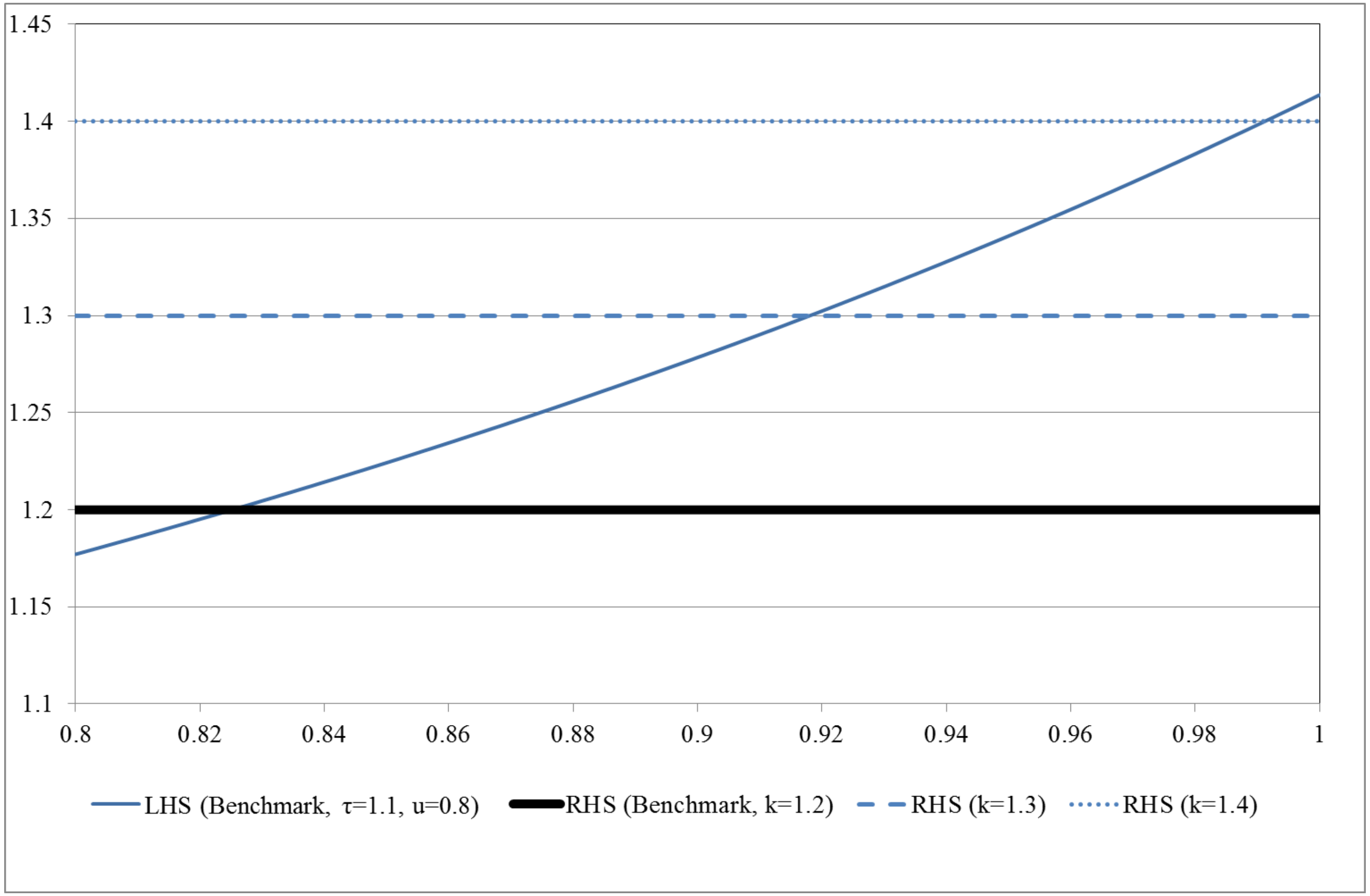

Figure 4 Benchmark solution and comparative statistics with respect to *u*

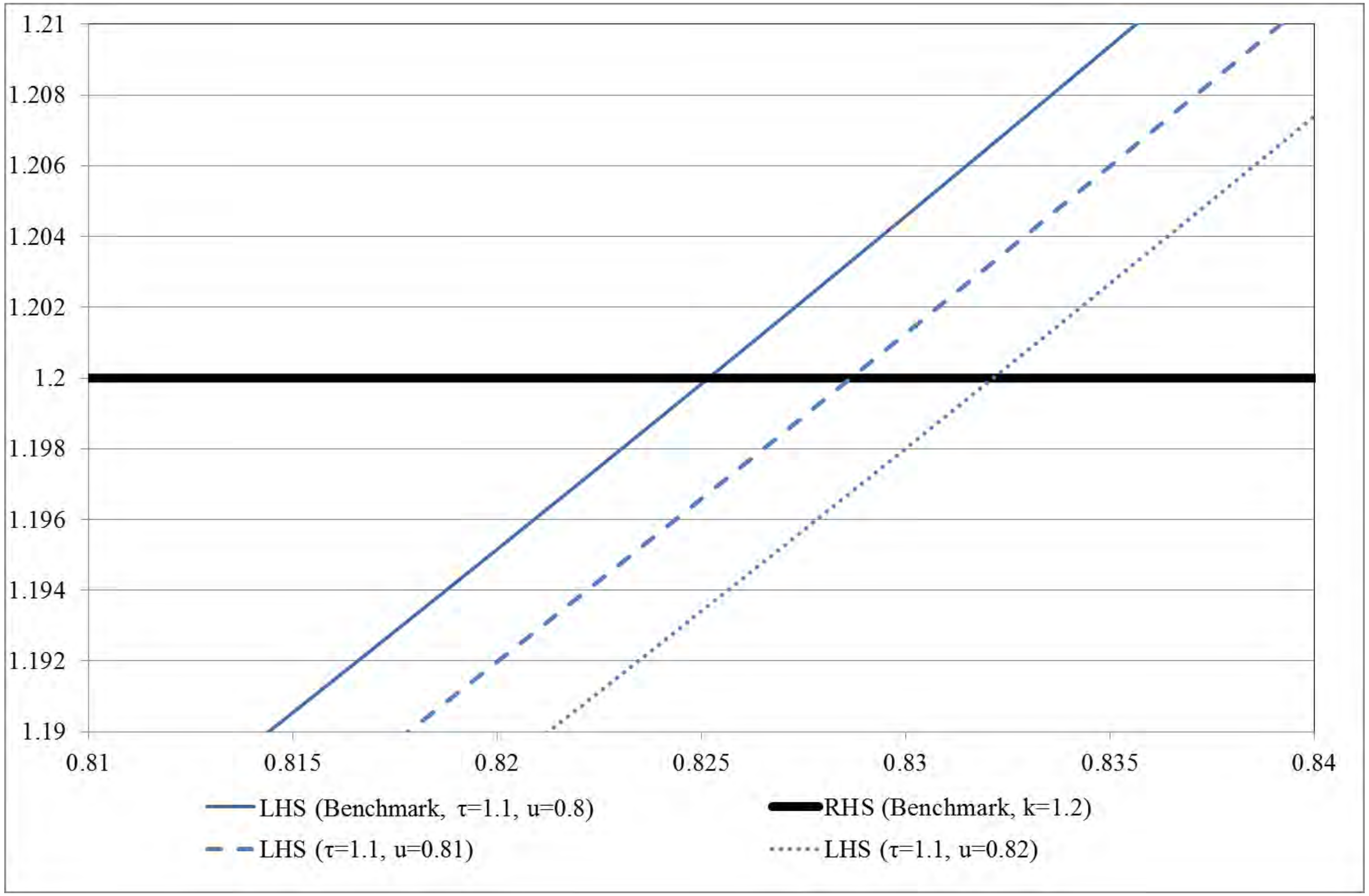

Figure 5 Solutions for c_2 and a

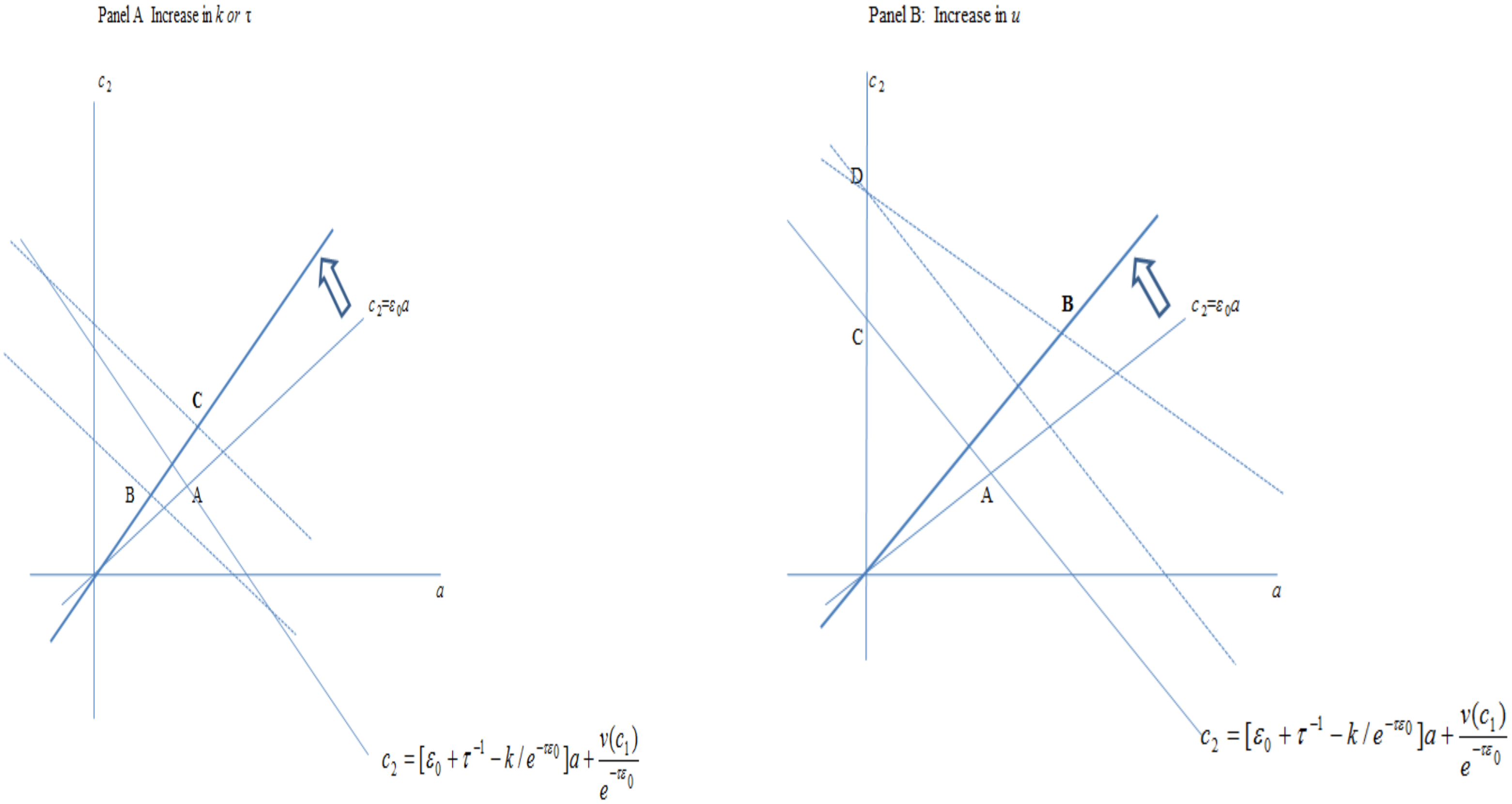

Figure 6 Social value of excluding the lenders

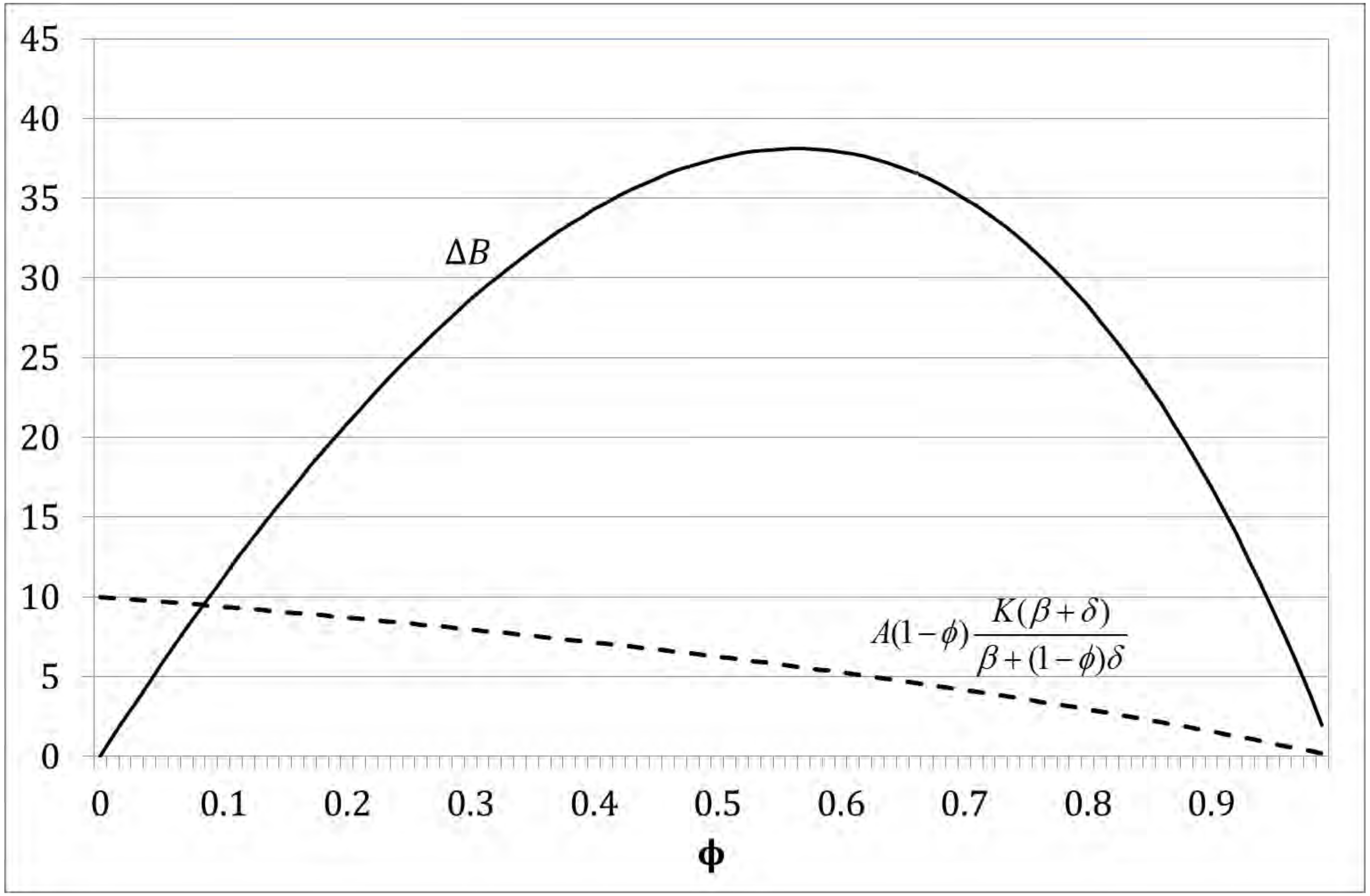

Figure 7 Benefits of Automatic Stays

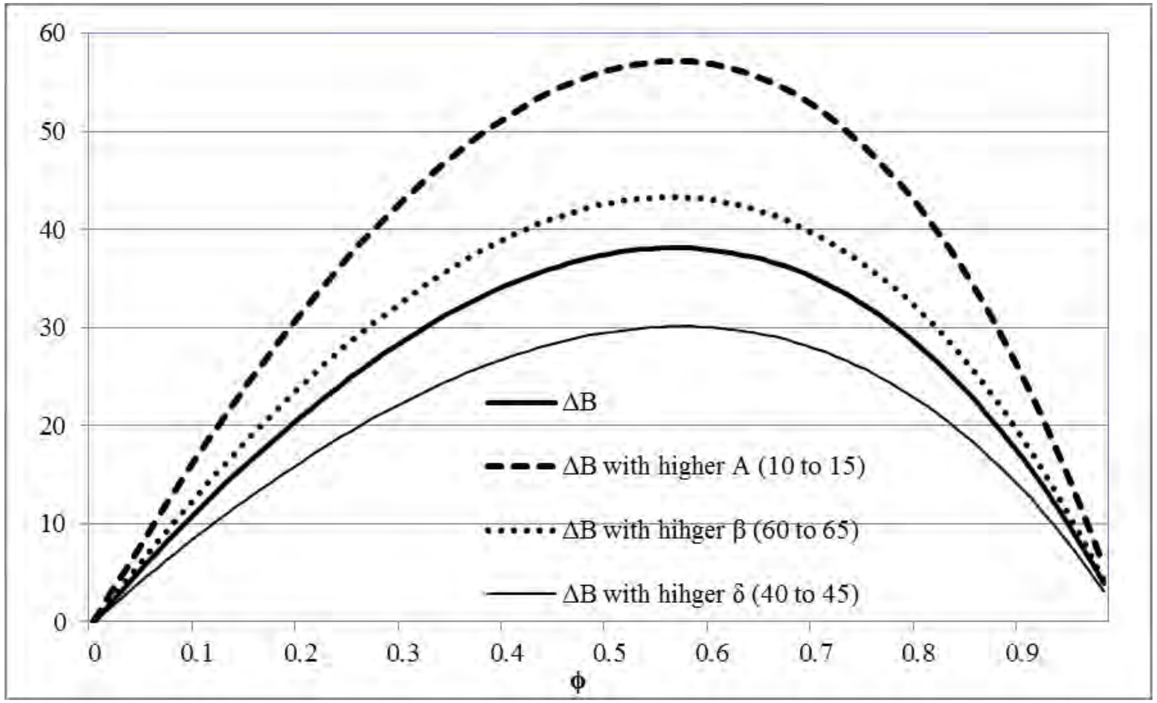

Figure 8 Numerical Examples

u	τ	k	e_0	P. of defalut	σ	c_1	a	c_2	c_2 / c_1 -1	EU(Borrower)	EU(Lender)
0.8	1.1	1.2	0.8252	0.5965	0.5	0.9454	2.3339	1.9258	1.0371	0	0.9454
0.8	1.2	1.2	0.8754	0.6502	0.5	0.8272	2.0022	1.7527	1.1188	0	0.8273
0.8	1.3	1.2	0.9042	0.6913	0.5	0.7474	1.7963	1.6243	1.1731	0	0.7474
0.8	1.1	1.3	0.9181	0.6357	0.5	0.7571	1.7961	1.6490	1.1782	0	0.7571
0.8	1.1	1.4	0.9912	0.6639	0.5	0.6236	1.4431	1.4304	1.2937	0	0.6236
0.81	1.1	1.2	0.8287	0.5981	0.5	0.9594	2.3471	1.9449	1.0273	0	0.9594
0.82	1.1	1.2	0.8321	0.5996	0.5	0.9735	2.3603	1.9641	1.0177	0	0.9735
0.8	1.1	1.2	0.8252	0.5965	0.6	0.9543	2.9448	2.4299	1.5463	0	1.4314
0.8	1.1	1.2	0.8252	0.5965	0.7	0.9607	3.9527	3.2616	2.3951	0	2.2416

Figure 9 Feasible combinations of c_2 and a